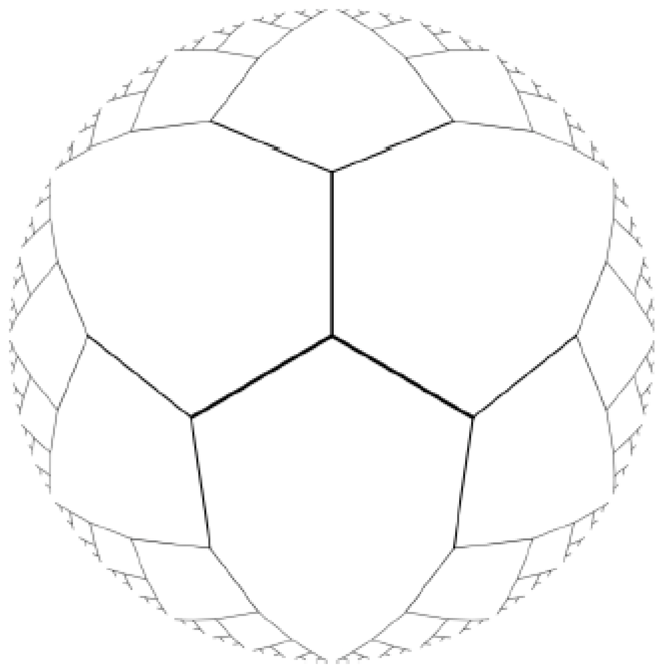
PSG COLLEGE OF TECHNOLOGY, COIMBATORE – 641 004
Department of Applied Mathematics and Computational Sciences
15XD98 – Network Science Lab
ProblemSheet-3

1. A Cayley tree is a symmetric regular tree in which each vertex is connected to the same number k of others, until we get out to the leaves, like the figure below, with k = 3. Show that the number of vertices reachable in d steps from the central vertex is $k(k-1)^{d-1}$ for d ≥ 1. Then give an expression for the diameter of the network in terms of k and the number of vertices n. State whether this network displays the "small-world effect," defined as having a diameter that increases as O(log n) or slower.



2. Let A be the adjacency matrix of a simple graph (un-weighted, undirected edges with no self-loops) and 1 be the column vector whose elements are all 1. In terms of these quantities, multiplicative constants and simple matrix operations like transpose and trace, write expressions for
   i) the vector k whose elements are the degrees ki of the vertices
   ii) the number m of edges in the network
   iii) the matrix N whose elements Nij is equal to the number of common neighbors of vertices i and j
   iv) the total number of triangles in the network, where a triangle means three vertices, each connected by edges to both of the others.

3. Consider the random graph G(n, p) with average degree c. Show that in the limit of large n the expected number of triangles in the network is $\frac{1}{6}c^3$. In other words, show that the number of triangles is constant, neither growing nor vanishing in the limit of large n