2nd Edition

TRICKS FOR MATHEMATICS

A very helpful Guide for all kind of Engg Entry Tests

This document contains different tricks and formulas to solve the questions of Mathematics portion in Entry Test in short time. Read, Practice and become a MASTER

In Association with

UET TRIBUNE

INTRODUCTION TO THIS DOCUMENT

Assalam O Alaikum..!

Entry tests are very important for the admission in Engineering University. Almost every good and well known university like UET, NUST, PIEAS, GIKI etc. conduct entry tests. So you should have the quality which differs between you and rest of the candidates. You should be hard working and your self confidence should be very high and you should have a firm faith on ALLAH. Many students think that getting admission in UET is first and last thing. No doubt, UET has a name in engineering but there are many other options for you. Apply in every university, appear in all the entry tests, keep your hard work continue and In Sha ALLAH you will get what you deserve.

This document contains different tricks by which you can solve the questions of Mathematics very easily. In Entry test, you don't have to solve the question and show the solution as in board exams. Entry test is different. You have to guess the right option and you will get full marks. If you have good concepts then there will be no difficulties for you.

Before reading and practicing these tricks, we will suggest you to first study the text book very carefully. Take a highlighter and study the theory of the chapter (Many students just solve Exercise). Highlight important points and if you have any *confusion then make it clear. The tricks in this document will only help you if you have studied the text book very carefully.*

The tricks are classified in chapter form. So, you can easily access the document. At the end, we have included some data on logarithms & some bonus tricks because in previous years, 1-2 questions are coming from the logarithms so you should prepare logarithms equally well.

Always remember "PRACTICE MAKES A MAN PERFECT". So, do maximum practice of MCQ. There are many MCQ books available in the market. ILMI is very good and there are many others also. Buy the one you like.

If you find any error in this document or if you know any trick then send us. We'll update this document and add your tricks. Have a good time and many best wishes. Thank You

Regards,

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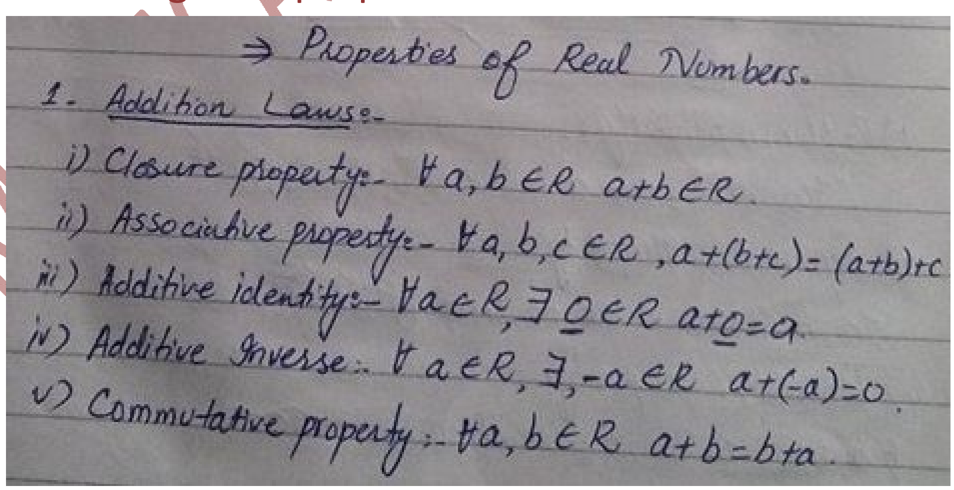
CHAPTER 01
(Number System)

- Number systems are classified into Real numbers and Complex Numbers.
- Real Numbers are further classified into rational and irrational numbers.
- Learn How to Differentiate between a rational and an irrational number in both ways:
  i) By their definitions and **ii)** in the form of decimals.
- $\sqrt{n}$ is rational if n is a perfect square. e.g. $\sqrt{9} = 3$. If n is not a perfect square then it is irrational.
- π (pi) is irrational but it's approximate values like 22/7 etc. are rational.
- Go through the properties of Real numbers.



→ Properties of Real Numbers.

1. Addition Laws:-

i) Closure property:- $\forall a,b \in R \quad a+b \in R$.

ii) Associative property:- $\forall a,b,c \in R, \; a+(b+c) = (a+b)+c$

iii) Additive identity:- $\forall a \in R, \exists \; 0 \in R \quad a+0=a$.

iv) Additive Inverse:- $\forall a \in R, \exists, -a \in R \quad a+(-a)=0$.

v) Commutative property:- $\forall a,b \in R \quad a+b=b+a$.

2- Multiplicative Laws:

i) closure property:- $\forall a,b \in R$, $a.b \in R$

ii) Associative property $\forall a,b \in R$ $a(bc)=(ab)c$

iii) Multiplicative identity:- $\forall a \in R, \exists\ 1 \in R$ $a.1=1.a=a$

iv) Multiplicative inverse:- $\forall a \in R\ \exists a^{-1} \in R$ $a.\frac{1}{a}=\frac{1}{a}.a=1$

v) Commutative property:- $\forall a,b \in R$ $ab=b.a$

3- Distributive Laws:

$\forall a,b,c \in R$

i) $a(b+c)=ab+ac$ (Left dis. law)

ii) $(a+b)c=ac+bc$ (Right dis. law)

Note:- A set of Real number which satisfies all eleven propertyies is called "field".

★ Properties of Equality

i) Reflexive property:-

$\forall a \in R, a=a$. عکس ہمیشہ اپنا ہی ہوتا ہے۔

ii) Symmetric property:-

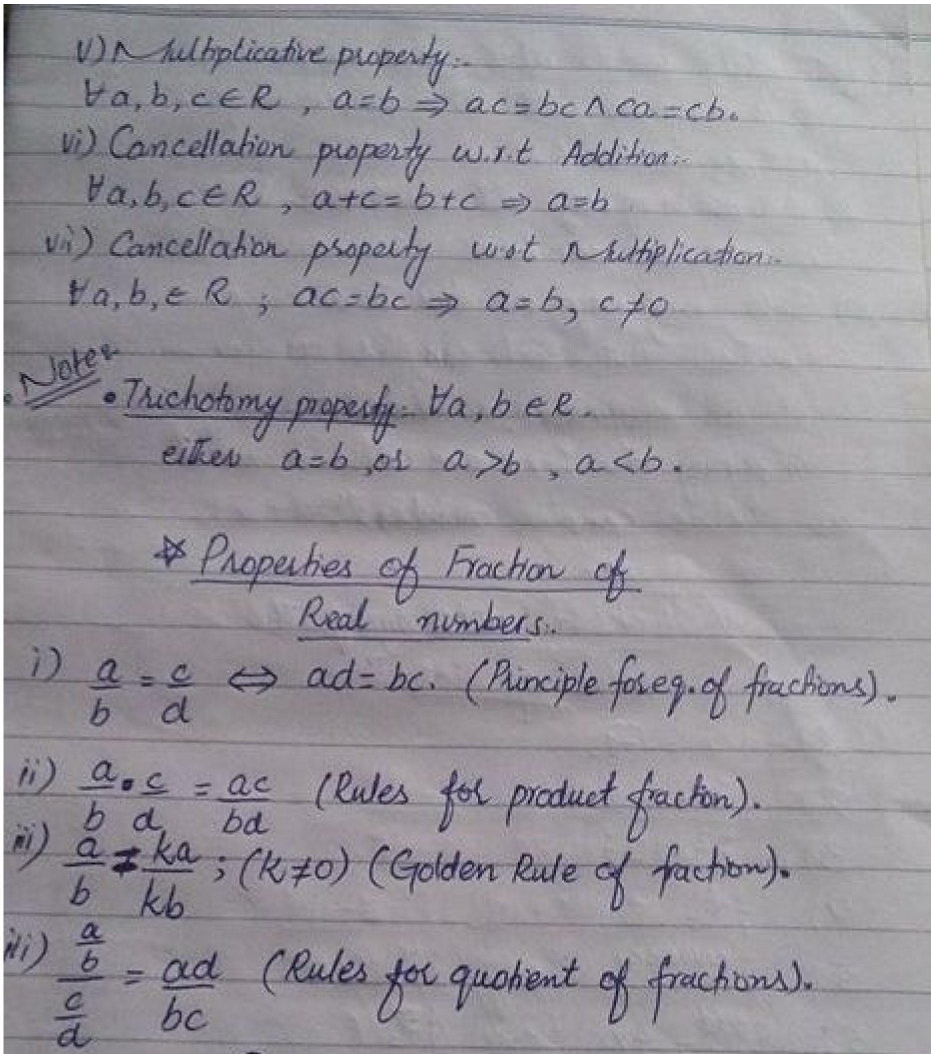
$\forall a,b \in R$, $a=b \Rightarrow b=a$ دوسہیلیوں کی طرح سچی خواہش

iii) Transitive property:-

$\forall a,b,c \in R$ $a=b \wedge b=c \Rightarrow a=c$.

iv) Additive property:-

$\forall a,b,c \in R$, $a=b \Rightarrow a+c=b+c$



v) Multiplicative property:
$\forall a,b,c \in R$, $a=b \Rightarrow ac=bc \wedge ca=cb$.
vi) Cancellation property w.r.t Addition:
$\forall a,b,c \in R$, $a+c=b+c \Rightarrow a=b$
vii) Cancellation property w.r.t Multiplication:
$\forall a,b,c \in R$; $ac=bc \Rightarrow a=b,\ c\neq 0$
Note:
• Trichotomy property: $\forall a,b \in R$.
either $a=b$, or $a>b$, $a<b$.

☆ Properties of Fraction of Real numbers:
i) $\frac{a}{b}=\frac{c}{d} \Leftrightarrow ad=bc$. (Principle for eq. of fractions).
ii) $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$ (Rules for product fraction).
iii) $\frac{a}{b}\neq\frac{ka}{kb}$; $(k\neq 0)$ (Golden Rule of fraction).
iii) $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{ad}{bc}$ (Rules for quotient of fractions).

Properties of Inequalities:

1) Trichotomy Property: For all a,b belongs to R

either a>b or a<b or a=b

2) Transitive Property: For all a,b belongs to R

1) a>b ^ b>c→ a>c

2) a<b ^ b<c → a<c

3) Multiplicative Property:

1) For all a,b,c belongs to R , c>0

$a>b \rightarrow ac>bc$ and $a<b \rightarrow ac<bc$

2) For all a,b,c belongs to R , $c<0$

$a>b \rightarrow ac<bc$ and $a<b \rightarrow ac>bc$

Differentiate b/w Complex numbers and Imaginary Numbers.

Numbers with non-zero real part and zero or non-zero imaginary part are called complex Numbers and Each Real number is a complex number with its imaginary part zero. Means that for a number to be complex, its real part should be non-zero while imaginary part may or may not be zero. On the other hand , Numbers whose imaginary parts are always non-zero and Real parts are always zero are called Imaginary Numbers. For example: " 2+3i " is a complex number but not an Imaginary number but " 3i " is an Imaginary number and also a complex number. So , Each Imaginary number is a complex number but each complex number is not an imaginary number. Similarly , Each Real number is a complex number but no Real number can be Imaginary number. Sometimes Imaginary numbers are also named as pure complex numbers.

Quick Calculation for Mathematical operations on Complex Number

Let two complex number be:

$z_1= a+bi$

$z_2= c+di$

1) Addition:-

$z_1+z_2= (a+c)+i(b+d)$

2) Subtraction:-

$z_1-z_2= (a-c)+i(b-d)$

3) Multiplication:-

$z_1.z_2= (ac-bd)+i(ad+bc)$

4) Division:-

$z_1/z_2= (ac+bd/c^2+d^2)+i(bc-ad/c^2+d^2)$

5)Reciprocal/Multiplicative Inverse:
Let z=a+bi
$1/z= a-bi/a^2+b^2$
6) Square root:-
Let $z= a+bi$

$$\pm\left[\sqrt{\frac{|z|+a}{2}}+i\sqrt{\frac{|z|-a}{2}}\right] for\ b>0$$

$$\pm\left[\sqrt{\frac{|z|+a}{2}}-i\sqrt{\frac{|z|-a}{2}}\right] for\ b<0$$

7)Logarithm:
let z= a+bi
logz=ln|z|+iarg (z)

Now we look at some tricks regarding powers of iota.

You should always keep in mind: $i^{4n}=1$
So, if u have any power of iota, u can simplify as follows:

- Divide power of iota by 4 and check remainder. It will become power of iota. Since u are dividing by 4, the maximum remainder can be 3 and up to i^3, you can simplify easily.
- Solve i^{4245}. If u divide 4245 by 4, remainder is 1 so $i^{4245}= i^1 = i$
- Always remember that:

$$(-1)^{n/2}= i^n \quad \text{(Ex 1.2, Q4, Part iv)}$$

- In simplification, i should not be present in the denominator.

Note:-
1) Each imaginary number is a complex number but each complex number is not an imaginary number.

2) Imaginary numbers are termed as "Pure Complex Number".

More convenient SHORTCUT to find the value of iota when it raises some bigger power:

Let we take an example to learn this shortcut.

Example: $i^{1000000000000000004}$=?

So to guess answer of the Question, just add the power of iota

i.e 1+0+0+0+0+0+0+0+0+0+0+0+0+0+4=5

now; it becomes as i^5

as we know that i^4=1

so that (1). i= i

Take some more example to practice this method.

For Quick guess either your answer is correct or not.
Consider some following points:
Note:
1) If the power of iota is even then it must be equal to 1 or –1. e.g i^4=1 or i^6=-1
2) If the power of iota is odd then it must be equal to i or –i. e.g i^5=i or i^7=-i.
3) If the power of iota is the multiple of 4 then it must equal to 1. e.g i^8=1

Multiplicative and Additive Inverse of Complex Number

Multiplicative Inverse of Complex Number:

$$z^{-1} = (Re(z)/|z|^2, - Im(z)/|z|^2)$$

Additive Inverse of Complex Number:

z =-z

Let z=a+bi

So that additive inverse is -(a+bi)

Condition for Pure and impure Imaginary Number:

1) Impure Im(z) or Pure Re(z):

Let z=a+bi so a≠0, b=0

2) Pure Im(z) or Impure Re(z):

Let z=a+bi so a=0, b≠0

3) z=conj (z) iff z is purely real.

4) z=-conj (z) iff z is purely Imaginary.

Some Important rules of Conversion:

-To convert from Polar Coordinates (r,θ) to Cartesian Coordinates (x,y) :

$x = r \times cos(\theta)$

$y = r \times sin(\theta)$

-To convert from Cartesian Coordinates (x,y) to Polar Coordinates (r,θ):

$r = \sqrt{(x^2 + y^2)}$

$\theta = tan^{-1}(y/x)$

Argument/Amplitude in Different Quadrants:

1) When z is in 1^{st} Quadrant i.e z= x+iy

Θ= arg(z)= arc tan (y/x)

2) When z is in 2^{nd} Quadrant i.e z= -x+iy

Θ= arg(z)= π- arc tan (y/x)

3) When z is in 3^{rd} Quadrant i.e z= -x-iy

Θ= arg(z)= -π+ arc tan (y/x)

4) When z is in 4^{th} Quadrant i.e z=x-iy

Θ= arg(z)= - arc tan (y/x)

When Iota raises to power Iota:

$i^i = e^{-\pi/2}$

Some important Points:

Polar Presentation of Complex Numbers

$a + bi = r(\cos\varphi + i\sin\varphi)$

Modulus and Argument of a Complex Number

If $a + bi$ is a complex number, then

$r = \sqrt{a^2 + b^2}$ (modulus),

$\varphi = \arctan\frac{b}{a}$ (argument).

Product in Polar Representation

$$z_1 \cdot z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1) \cdot r_2(\cos\varphi_2 + i\sin\varphi_2)$$
$$= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

Conjugate Numbers in Polar Representation

$$\overline{r(\cos\varphi + i\sin\varphi)} = r[\cos(-\varphi) + i\sin(-\varphi)]$$

Inverse of a Complex Number in Polar Representation

$$\frac{1}{r(\cos\varphi + i\sin\varphi)} = \frac{1}{r}[\cos(-\varphi) + i\sin(-\varphi)]$$

- Each real number is Self-conjugate. Means Conjugates of 3 and -3 are respectively 3 and -3.
- Learn Formula for the Multiplicative Inverse of a complex number.
- Learn to separate out Real (Re(z)) and Imaginary parts (Im(z)) of a complex number.

De' Moivre's Theorem to solve powers of Complex numbers. In this perspective following is a very important Example. Suppose 'z' is a complex number with 'r' modulus and 'x' argument(angle). Now if z^3 equals iota and *r* is equal to one find 'x'. Now , By De' Moivre's theorem : $z^3 = r(\cos 3x + i \sin 3x)$, put $z^3=i$ and r=1 , we get : i = cos3x + isin3x . Now Obviously we have to find such 3x at which cos3x is zero and sin3x is 1. So , finally we get that 3x = π/2 which implies that *x = π/6*.

Note: **Argument is also called "Amplitude"**

Some Important Points:

Quotient in Polar Representation

$$\frac{z_1}{z_2}=\frac{r_1(\cos\varphi_1+i\sin\varphi_1)}{r_2(\cos\varphi_2+i\sin\varphi_2)}=\frac{r_1}{r_2}[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)]$$

Power of a Complex Number

$$z^n=[r(\cos\varphi+i\sin\varphi)]^n=r^n[\cos(n\varphi)+i\sin(n\varphi)]$$

Formula "De Moivre"

$$(\cos\varphi+i\sin\varphi)^n=\cos(n\varphi)+i\sin(n\varphi)$$

Nth Root of a Complex Number

$$\sqrt[n]{z}=\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n}\right),$$

where

$k=0,1,2,\ldots,n-1$.

Euler's Formula

$$e^{ix}=\cos x+i\sin x$$

Properties of Argument/Amplitude:

1) $\arg(z_1.z_2)=\arg(z_1)+\arg(z_2)$
2) $\arg(z_1/z_2)=\arg(z_1)-\arg(z_2)$
3) $\arg(\bar{z})=-\arg(z)$
4) $\arg(z/\bar{z})=2\arg(z)$
5) $\arg(z^n)=n\arg(z)$

Note: The Nature of Argument is just like Logarithmic Function

Some Productive Points:

1) Every real number is a complex number with zero as its imaginary part.

2) Set of Real Number us a special subset of the complex number.

3) $\sqrt{-a} \times \sqrt{-b} = (i\sqrt{a}) \times (i\sqrt{b}) = i^2(\sqrt{ab}) = -\sqrt{ab}$.

4) Each imaginary Number is a complex number but each complex number not an imaginary number.

5) Imaginary numbers are also named as pure complex number.

6) Sum of four consecutive power of iota is always zero.

7) Product of four consecutive power of iota is always-1.

8) Sum and product of two conjugate numbers are always real number.

9) (0,0) is called additive identity in C.

10) (1,0) is called multiplicative identity in C.

-Locus of a Complex Number:

There are 5 conic which can be formed from complex number in Cartesian plane.

1) Ellipse 2) Circle 3) Hyperbola 4) Parabola 5) Line

1) Circle:

-If $|z|=1$, it forms a unit circle

2-If $|z \pm z_1|=k$, it forms a parabola.

3) Circle or Line:

-If $\left|\frac{z-z_1}{z-z_2}\right| = k$; If $k \neq 1$, represents a circle and $k=1$ represents a line.

4) $|z-z_1|+|z-z_2|=k$

i) If $k>|z_1-z_2|$; it forms an ellipse.
ii) If $k<|z_1-z_2|$; it forms no locus
iii) If $k=|z_1-z_2|$; it forms a line.
5) $|z-z_1|-|z-z_2|=k$
i) If $k>|z_1-z_2|$; it forms no locus
ii) If $k<|z_1-z_2|$; it forms Hyperbola.
iii) If $k=|z_1-z_2|$; it forms a line joining.
(Whereas k=any constant)

Some Important Terms:

Terminating Decimals: A decimal number has only finite number of digits units decimal part, is called a terminating . For Example: 202.04, 0.973 are examples of terminating decimal number.

Recurring Decimals: A recurring or periodic decimal is a decimal in which one or more digits repeat indefinitely these are rational numbers e.g ; 0.3333… etc

Note: Every terminating decimal or recurring decimal represents a rational number.

Non Terminating/Non Recurring Decimal: A non terminating, non-recurring decimal is a decimal which neither terminates nor it is recurring. e.g; 1.124232…, 45.2425…. etc

Note: Every non-terminating, non-recurring decimal represents an irrational number.

Remember that:

e , π and square root which are not perfect squares are Irrational numbers.

Properties of Modulus

1) $|z|=|\bar{z}|=|-z|=|-\bar{z}|$
2) $z.\bar{z}=|z|^2$
3) $|z^n|=|z|^n$
4) $|z_1.z_2|=|z_1||z_2|$
5) $|z_1/z_2|=|z_1|/|z_2|$
6) $|z_1+z_2|\leq|z_1|+|z_2|$
7) $|z_1+z_2|\geq|z_1|-|z_2|$

Triangular inequality

Properties of Conjugate:

Let z_1 and z_2 be two complex numbers.

1) $\overline{z_1+z_2}=\bar{z}_1+\bar{z}_2$
2) $\overline{z_1-z_2}=\bar{z}_1-\bar{z}_2$
3) $\overline{z_1.z_2}=\bar{z}_1.\bar{z}_2$
4) $\overline{z_1/z_2}=\bar{z}_1/\bar{z}_2$;$z_2\neq0$
5) $\overline{(\bar{z})}=z$
6) $(\overline{z^n})=(\bar{z})^n$
7) $z+\bar{z}=2Re(z)$
8) $z-\bar{z}=2Im(z)$
9) $z^2+(\bar{z})^2$ is Real Number
10) $(z-\bar{z})^2$ is Real Number.
11) $\boxed{\frac{1}{z}=\frac{\bar{z}}{|z|^2}}$

Properties of nth Roots of Unity:

1) $1+\alpha+\alpha^2+\ldots\ldots\ldots+\alpha^{n-1}=0$

2) $1.\alpha.\alpha^2+\ldots\ldots\ .\alpha^{n-1}=(-1)^{n-1}$

3) The nth roots of unity lie on the unit circle $|z|=1$

and form the vertices of a regular polygon of n sides

4) nth roots of unity form a G.P with common ratio

$e^{2\pi i/n}$.

Note:

cis =cosx+ isinx

$cis\alpha.cis\beta=cis(\alpha+\beta)$

If $x=\cos\theta+i\sin\theta$ and $1/x=\cos\theta-i\sin\theta$

Then; $x^n-1/x^n=2\sin n\theta$, $x^n+1/x^n=2\cos n\theta$

Properties of Cube Root:

1) $1+w+w^2=0$

2) $w^3=1$

3) $w^{3n}=1$, $w^{3n+1}=w$, $w^{3n+2}=w^2$

4) $\overline{w}=w^2$, $\overline{(w^2)}=w$

5) $w=e^{2\pi i/3}$, $w^2=e^{-2\pi i/3}$

6) The cube root of unity lie on the circle and divide the circumference into three equal parts.

7) If $a+bw+cw^2=0$,then a=b=c provide a, b and c are real

Some Important Concepts...

Equation of Circle:

The equation of circle with center z_0 and radius r is $|z-z_0|=r$

Note:
1) $|z-z_0|< r$ represents interior of circle.
2) $|z-z_0|=r$ represents on the circle.
3) $|z-z_0|>r$ represents exterior of circle.

Important Result:

$|z-z_1/z-z_2|= k$ is a circle if $k\neq 1$ and is a line if $k=1$

Important Points:

1) $i=\sqrt{-1}$
2) $i^2=-1$
3) $1/i= -i$
4) Sum of nth root of unity is zero
5) Product of nth root of unity is $(-1)^{n-1}$
6) Distance between two vertices $z_1, z_2=|z_1-z_2|$

CHAPTER 02
(Sets,Functions,Groups and logics)

Following are the key points that should be kept in mind while preparing this chapter for Entry Test.

- Understand the real meanings of a 'SET'. For example : {1,2,3} is a set but {1,1,2,3,3,1} is not a set by definition.
- Types and Operations on sets.
- Interpretation of Venn diagram especially Results mentioned at page#39 of Text book.
- Explanation and usage of logic Symbols and related terms used frequently in Aristotelian logic.
- Understand the relation between Logic and Set Theory.
- Function, Types of Functions and Inverse of a function.
- Operations on Residue Classes Modulo Sets.
- Understand Complete Group Theory with all the variations .For example : If a semi-group with respect to some binary operation also consists Identity of that operation , It is named as Monoid .
- Real Numbers do not represent a group under Multiplication because Multiplicative Inverse of Zero Doesn't Exist in Real numbers. Moreover if zero is excluded from 'R' it becomes an Abelian Group.

Some Important Points:

If n(A)=n then:

1) Total Subsets= 2^n

2) Total Proper Subsets=2^n-1

3) Total Non-empty Proper Subset=2^n-2

4) Every set is a subset and superset of itself.

5) The empty set is subset of every set

6) "Cardinality" means number of elements present in a set.

7) "Tabular form" for representation of a set is also known as "Roster Form".

8) "Set Builder form" for representation of a set is also known as "Rule Method".

Let n(A)=m and n(B)=n

9) No. of Functions= n^m

10) No. of Relations/Correspondence= $2^{m\times n}$

11) No. of Bijective Function=^nP_n =n! whereas n=m

12) No. of One-to-One Function= ^nP_m ; n≥m

13) If A has n elements and B has m elements then A×B has m×n elements.

14) No. of relation in the distinct set is given by 2^{n^2}

15) Any Conditional and its contrapositive are equivalent.

16) The converse and inverse are equivalent.

17) All sets of residue classes under addition are groups.

18) All sets of residue classes excluding 0 under multiplication are groups.

19) The converse of p→q is q→p.

20) The inverse of p→q is ~p→~q.
21) The contra positive of p→q is ~q→~p.
22) The power set of any empty set is not empty.
23) If a set A has m elements, then its power set P(A) contains exactly 2^m.

Important Results:

Relation b/w A and B	Result Suggested
$A \cap B = \phi$ (A and B are Disjoint)	$n(A \cap B)=0$ $n(A \cup B)=n(A)+n(B)$ $n(A-B)=n(A)$ $n(B-A)=n(B)$
$A \cap B \neq \phi$ (A and B are overlapping)	$n(A \cap B) \neq 0$ $n(A \cup B)=n(A)+n(B)-n(A \cap B)$ $n(A-B)=n(A)-n(A \cap B)$ $n(B-A)=n(B)-n(A \cap B)$
$A \subseteq B$	$n(A \cap B)=n(A)$ $n(A \cup B)=n(B)$ $n(A-B)=0$ $n(B-A)=n(B)-n(A \cap B)$
$B \subseteq A$	$n(A \cap B)=n(B)$ $n(A \cup B)=n(A)$ $n(A-B)=n(A)-n(A \cap B)$ $n(B-A)=0$

Note:

i) $Dom(R^{-1})$= Range

ii) $Ran(R^{-1})$=Domain

iii) $(R^{-1})^{-1}=R$

Function:

-Conditions:

Let X and Y be two non-empty sets such that

1) f is relation from A to B (i.e f $\subseteq A\times B$)

2)Dom f=A

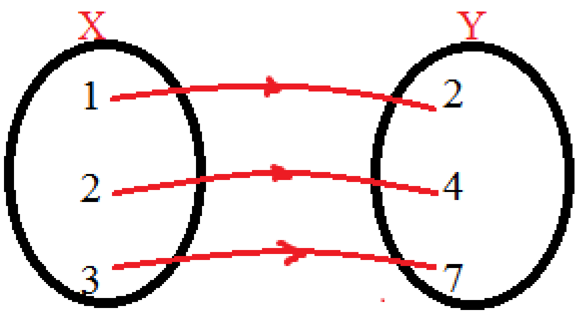
3) First Element of order pairs don't repeat.

Then f is called a function from A to B.

The function f is also written as f:A→B.

Example: X={1,2,3} , Y={2,4,7}

By arrow diagram as shown below:

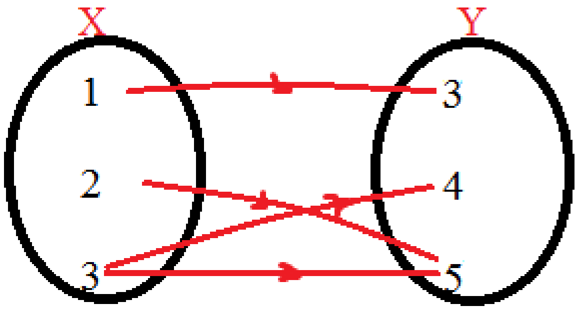


This relation is a function as each element of X is related to exactly element of y.

Example#2:

r={(1,3,(2,5), (3,4),{3,5}

This relation is not a function as "3"element of set is related to two elements of 2^{nd} set.



Types of Function:

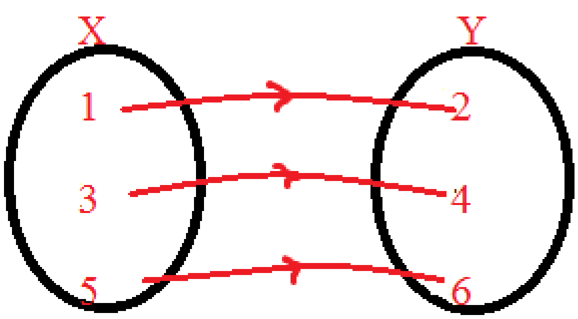
Into Function: A function is called into function if each element of the 1^{st} set is mapped to just one element of the 2^{nd} set.

OR If "f" is a function from set A to set B i.e

$f:A \rightarrow B$ such that Range f is subset of B.

i.e Range $f \neq B$, then the function is called an into function from A to B.

Example: f= {(1,2),(3,4) ,(5,6)



f is a function But Range $f \neq B$

Therefore f is an into function from A to B.

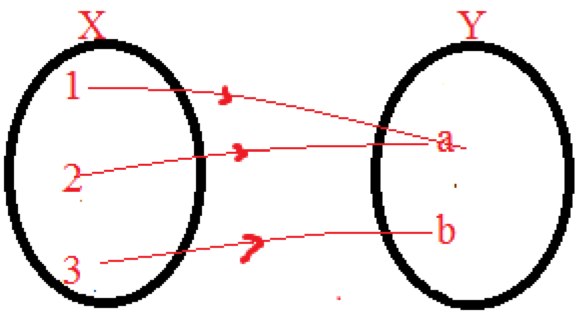
Surjective (Onto) Function:

A function is called onto function if each element of 2nd set is the image of some element of 1st set.

Or If "f" is a function from set Ato B i.e f: A→B

Such that Range f=B , then f is called an onto function from set A to B.

Example: f= {(1,a),(2,a),(3,b)}



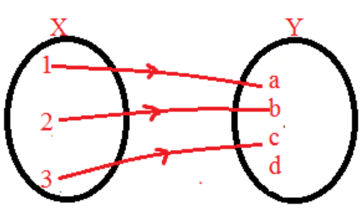
As Range f=B

So that f is called as Onto or surjective function.

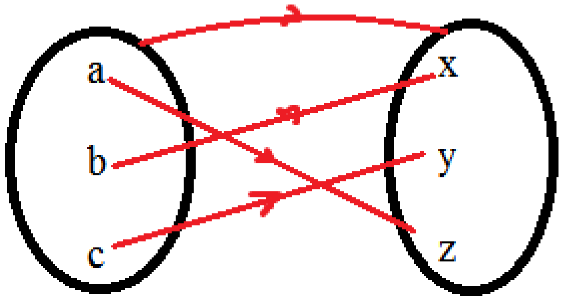
1-1 and into (Injective) Function:

A function is called (1-1) and into function if the different elements have different images.

Example:



Onto ,one-to-one (Bijective) Function:



e.g

F(x)= {(a,z),(b,x),(c,y)}

-Linear and Quadratic Functions:

The function f={|x,y| , y=mx+c is called a linear function and its graph is a straight line.

The function f={|x,y| , $y=ax^2+bx+c$ } is called a quadratic function and its graph is a parabola.

-Inverse of a function:

Inverse of r= f={|x,y| , y=mx+c} is f={|x,y| , x=my+c}

Note: The inverse of a function may or may not be a function but inverse of a bijective function is always a function.

Unary Operation: Any operation which performs on a single number yields another number of the same or different system is called Unary Operation.

For Example:

1) Negation of a given number.

2) Squaring a number etc.

Binary Operation: A binary operation denoted by * (read as star) on a non-empty set G is function which associated with each ordered pair (a,b) of element of G, a unique element denoted as a*b of G.

For Example: Ordinary addition and multiplication are binary operations on N.

Groupoid: A non-empty set on which a binary operatiobn * can be defined is called a groupoid.

Semi-Group:

A non-empty set is semi group if:

1) It is closed with respect to an operation *.

2) The Operation * is assosiative.

Monoid: A semi group having an identity is called a monoid.

Definition of group: A monoid having inverse of each of its element under * is called a group under * i.e.

A non-empty set G is group under * if

1) G is closed under *.

2) The operation * is associative.

3) G has an identity element w.r.t *.

4) Every element of G has an inverse in G w.r.t * If G satisfy the additional condition.

5) For all a,b belongs to G , a*b=b*a
Then G is called an Abelian or commutative group under *.

-Logic Important Points:

1) Conjunction of p and q is represented by p^q if p and q are true , then p^q is true otherwise p^q is false.
2) Disjunction of p and q is represented by pvq if p and q are both false then pvq is false otherwise pvq is true.
3) Implication or Conditional of p and q is represented by p→q iff q is false then p→q is false otherwise p→q is true.
4) Biconditional or equivalence of p and q is represented by p←→ q if both p and q are either true or false then equivalence is true otherwise it is false.
5) A statement which is true for all the possible values of the possible values of the variable involved in it is called a tautology.
6) A statement which is always false is called absurdity or contradiction.
7) Any statement which can be true or false depending upon the truth values is called contingency.

Note: (i) $n(\emptyset) = 0$ (ii) n(A')= n(U)- n(A)
(iii)n(A'UB')=n(A∩B)' (iv) n(A'∩B')= n(AUB)'

CHAPTER 3
(Matrices and Determinants)

Matrix: A rectangular array of numbers enclosed in pair of bracket is called matrix.

Example: $A=\begin{bmatrix} 2 & 5 & 3 \\ 5 & -2 & 0 \end{bmatrix}$

Real Matrix: A matrix is called real matrix if all the entries are real number.

Notes:

If for a matrix $A=[a_{ij}]$, number of rows=number of columns then;

1) Elements a_{ij} ; for all i=j are called main diagonals.
2) Elements a_{ij}; for all $i\neq j$ are called off diagonals
3) Elements a_{ij}; for all $i>j$ are called elements below main diagonals.
4) Elements a_{ij}; for all $i<j$ are called elements above main diagonals.

Dimension/order of a matrix

Order of matrix $=m\times n$

whereas; m= no. of rows and n=no. of Columns

Number of elements/entries in the matrix$=m\times n$

TYPES OF MATRICES

1) Row Matrix:

A matrix having only one row is called row matrix.

For Example: $A=\begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$

In general a matrix of order $1\times n$ is a row matrix.

2) Column Matrix:

A matrix having only one column is called column matrix.

For Example: $A=\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

In general a matrix of order $m\times 1$ is a column matrix.

3) Zero/Null/Void/Empty Matrix:

A matrix each of whose entries is zero is called a zero or null matrix.

For Example: A= $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

In general, it is a matrix of arbitrary order.

4) Square Matrix:

A matrix in which number of rows is equal to number of columns say n is called a square matrix of order n.

For Example: A= $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$

5) Rectangular Matrix:

A matrix in which number of rows is not equal to number of columns is called Rectangular Matrix.

For Example: A= $\begin{bmatrix} 3 & 0 & 5 \\ 5 & 6 & 0 \end{bmatrix}$

-Diagonals of Square Matrix:

Let A be a square matrix of order 3.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 2 & 1 \\ 9 & 6 & 3 \end{bmatrix}$$

In this matrix, 1, 2,3 are main/leading/primary/principal Diagonals and 5,2,9 are secondary diagonals.

6) Scalar Matrix:

A Square matrix in which every non-diagonal element is zero and all main /principal diagonals are equal.

For Example: A= $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Note: Diagonal matrix and scalar are termed to be square matrix if it is not square matrix then it is neither a scalar matrix nor diagonal matrix.

7) Unit/Identity Matrix:

A Square matrix in which every non-diagonal element is zero and every main diagonal element is 1 is called unit or identity matrix.

For Example: A=$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

8) Upper Triangular Matrix:

A square matrix A=$[a_{ij}]_{m\times n}$ is called upper triangular matrix if $a_{ij}=0$, for all $i>j$

For Example: A=$\begin{bmatrix}1 & 4 & 3\\0 & 2 & 3\\0 & 0 & 3\end{bmatrix}$

9) Lower Triangular Matrix:

A square matrix A=$[a_{ij}]_{m\times n}$ is called upper triangular matrix if $a_{ij}=0$, for all $i<j$

For Example: A=$\begin{bmatrix}1 & 0 & 0\\7 & 2 & 0\\8 & 3 & 3\end{bmatrix}$

10) Sub Matrix:

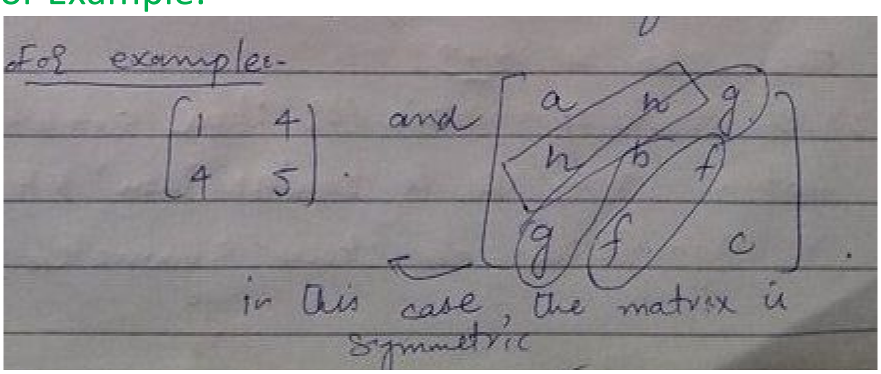
A matrix which is obtained from a given matrix deleting any number of rows or columns or both is called a sub-matrix of the given matrix.

For Example: A=$\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$ is a sub matrix of B=$\begin{bmatrix}5 & 2 & 3\\4 & 1 & 2\\6 & 3 & 1\end{bmatrix}$

11) Symmetric Matrix:

A square matrix A is said to be symmetric if $A^t=A$ that is matrix A=$[a_{ij}]_{m\times n}$ is said to symmetric matrix provided $a_{ij}=a_{ji}$, for all i,j

For Example:

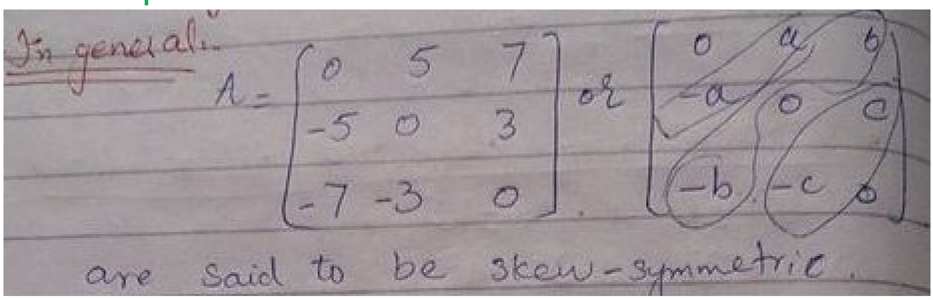


Remarks: For a symmetric matrix , all the leading diagonals are non-zero real number and the diagonal cuts are same.

12) Skew/Anti Symmetric:

A square matrix A is said to be skew symmetric if $A^t=-A$ that is $A=[a_{ij}]_{m\times n}$ is said to be skew symmetric provided $a_{ij}=-a_{ji}$ for all, i,j

For Example:

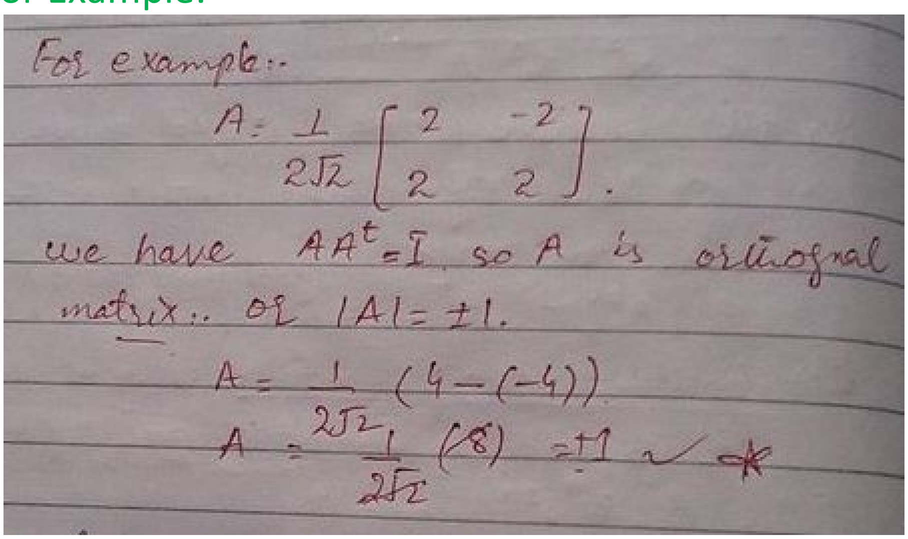


Remarks: Leading diagonals are zero and diagonals cuts are conjugate to each other.

13) Orthogonal Matrix:

A square matrix of n order is said to be orthogonal iff $AA^t=I_n=A^tA$ or $|A|=\pm1$.

For Example:



14) Idempotent Matrix:

A square matrix A is said to be Idempotent if $A^2=A$.

For Example: $A=\begin{bmatrix} 2 & -2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & -3 \end{bmatrix}$ then $A^2=A$

Note: Idempotent matrix is formed from the identity matrix of order greater than 2.

15) Involutory Matrix:

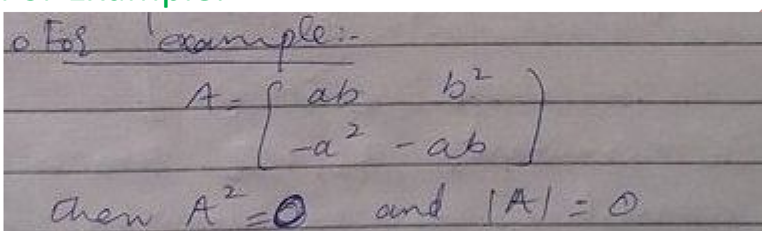
A square matrix A is said to be involutory if $A^2=I$.

For Example: $A=\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ then $A^2=I$

16) Nilpotent Matrix:

A square matrix A is said to be nilpotent matrix if there exists a positive integer m such that $A^m=O$ or $|A|=0$ m is called the nilpotency index of the nilpotent matrix A.

For Example:



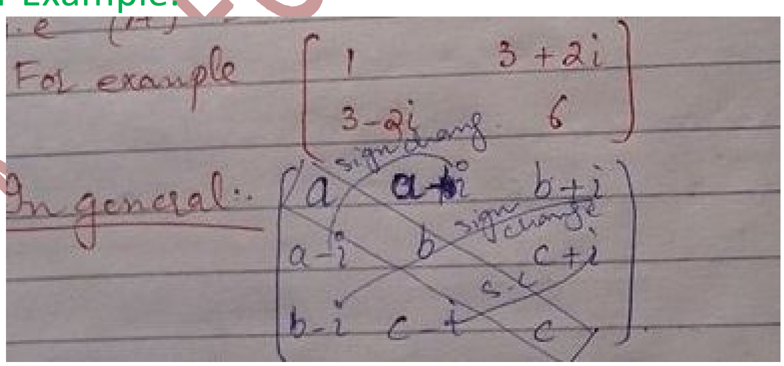
17) Periodic Matrix:

A square matrix A is said to be periodic matrix if $A^{k+1}=A$ for some smallest positive integer "k" then k is called period of A.

18) Hermitian Matrix:

A square matrix A is said to be hermitian matrix if $(\bar{A})^t = A$

For Example:

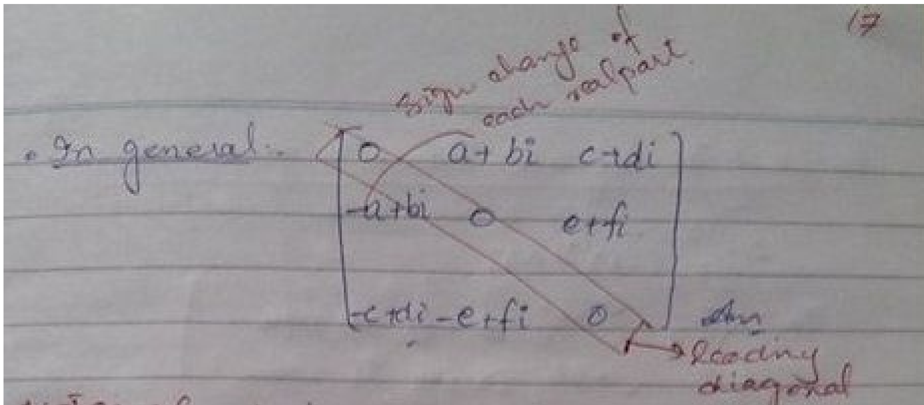


Remarks: For hermitian matrix, all the leading diagonals are non-zero real number and sign of each imaginary part of the complex number change in the diagonal cuts.

19) Skew/anti-Hermitian Matrix:

A square matrix A is said to be skew-hermitian matrix if $(\bar{A})^t = -A$

For Example:



Remarks: For skew-hermitian matrix, all the leading diagonals are zero and sign of each real part of the complex number change in the diagonal cuts.

-Transpose of a matrix:

Let A has a dimension m×n when we transpose it as n×m.

For Example: A=$\begin{bmatrix} 2 & 5 \\ 2 & -5 \end{bmatrix}$, A^t=$\begin{bmatrix} 2 & 2 \\ 5 & -5 \end{bmatrix}$

-Properties of transpose of a matrix:

1) $(A\pm B)^t = A^t \pm B^t$

2) $(AB)^t = B^tA^t$

3) $(kA)^t = k(A)^t$

4) $(A^t)^t = A$

-Properties of Symmetric and skew/anti Symmetric Matrices:

1) If A be a square matrix then

i) $A+A^t$ is symmetric matrix.

ii) $A-A^t$ is skew symmetric matrix.

2) If A and B be two symmetric (or skew symmetric) matrices of the same order then A±B is also a symmetric or skew symmetric matrix.

3) If A is symmetric (or skew symmetric) matrix and k is scalar then kA is symmetric or skew/anti Skew symmetric.

4) If A and B be symmetric matrices of same order then the product is

equal to symmetric iff AB=BA

5)All positive integer power of a symmetric matrix are symmetric.

e.g $A, A^2, A^3, \ldots\ldots A^n$ where n is a natural number.

6) All positive odd integral power of a skew symmetric matrix are skew symmetric.

e.g $A, A^3, A^5, \ldots\ldots A^{\text{odd positive integer}}$

7) All positive even integral power of a skew symmetric are symmetric matrices.

e.g $A^2, A^4, \ldots\ldots A^{\text{even positive integer}}$

8) If AA^t and A^tA are symmetric matrices..

How to multiply two matrices with quick method.

Condition: no. of columns (n) of first matrix= no. of rows (m) of second matrix.

Let have an illustration on behalf of it to learn the shortcut method.

Example:

Let A= $\begin{bmatrix} 2 & 5 & 2 \\ 5 & 3 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 5 & 5 \end{bmatrix}$

Then; AB=?

Rows of 1st matrix	*Columns of 2nd matrix*	
	(2,1,5)	(1,2,5)
(2,5,2)	(4+5+10=19)	(2+10+10=22)
(5,3,1)	(10+3+5=18)	(5+6+5=16)

AB= $\begin{bmatrix} 19 & 22 \\ 18 & 16 \end{bmatrix}$

Notes: For matrices A and B, AB=BA

if: 1) A and B are inverse of each other.
2) A and B are identical
3) Either A or B are identity matrix.

Note:

1) AB does not compulsory imply that A=O or B=O and both are zero. But AB=O
2) If A is square matrix of n order then A^2 is defined as A.A .
In general; A^m=A.A………A(m-times). Where m is a positive integer.
3) If I be an identity matrix then $I=I^2=I^3=\ldots=I$

Determinant of matrix

1) Determinant of matrix 1×1:
Let $A=[a]_{1\times1}$ then $|A|=a$
2) Determinant of matrix 2×2:
Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ so $|A|=ad-bc$

-Minor of an element of matrix;
If A is square matrix then the minor of element aij is determinant of a matrix obtained by deleting ith row and jth column in given matrix. It is denoted by Mij.
For Example: Let |A| be a of 3×3 order determinant.

$|A|=\begin{vmatrix} 1 & 2 & 4 \\ 2 & 4 & 1 \\ 4 & 3 & 4 \end{vmatrix}$ then the minor a_{11} is $M_{11}=\begin{vmatrix} 4 & 1 \\ 3 & 4 \end{vmatrix}$ and so on.

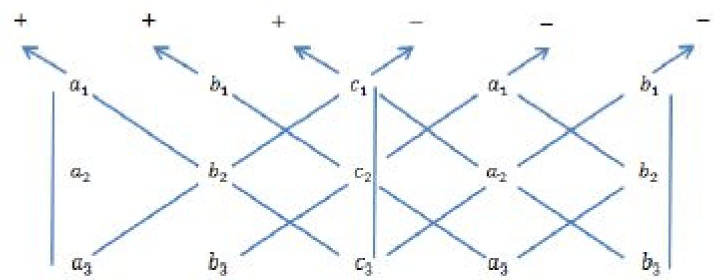
-Cofactor of an element of a matrix:
If A be square matrix then the cofactor of and element aij then the element in the ith row and jth column) is denoted by Aij and is defined by;
$A_{ij}=(-1)^{i+j}M_{ij}$ .Thus;
Aij=(Mij when i+j is even and –Mij when i+j=odd)

-Singular matrix: If |A|=0, then A is said to be singular.
-Non-Singular matrix: If |A|≠0, then A is said to be non-singular.

SHORTCUT TO FIND THE DETERMINAT OF 3×3 ORDER MATRIX
(Sarrus Rule)

After writing the determinant, repeats the first two columns. Mark + and - expands as shown in below $A = a_1$

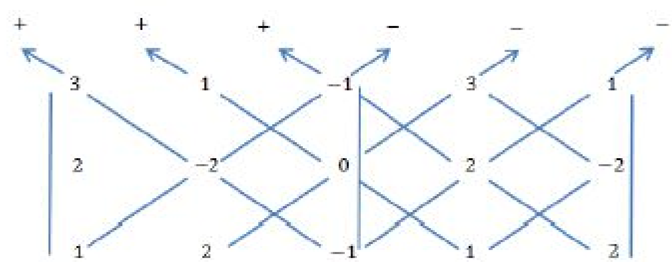
$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$



$$= a_1b_2c_3 + b_1c_2a_3 + a_2b_3c_1 - a_2b_3c_1 - a_1b_3c_2 - a_2b_1c_3$$

Illustration:

Evaluate: $\Delta = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & 1 \end{vmatrix}$



$\Delta = (6 + 0 - 4 - 2 + 0 + 2) = 2$

Remarks: Sarrus Rule does not work for determinants of order greater than 3.

-Properties of Determinants:

Properties of determinants of order three only are stated below. However these properties hold for determinants of any order. These properties help a good deal in evaluation of determinants.

1) The value of determinant reamins unchanged if rows are changed into columns and columns are changed into rows;

e.g. $\begin{vmatrix} 2 & 3 & 4 \\ 2 & 5 & 2 \\ 3 & 5 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 3 \\ 3 & 5 & 5 \\ 4 & 2 & 2 \end{vmatrix}$

i.e $|A| = |A^t|$.

2. If two adjacent rows (or columns) are ~~same~~ interchangeable then the value of determinant is the negative of the value of original determinant

i.e. $\begin{vmatrix} 2 & 5 & 2 \\ 5 & 2 & 5 \\ 2 & 5 & 2 \end{vmatrix} = -\begin{vmatrix} 5 & 2 & 5 \\ 2 & 5 & 2 \\ 2 & 5 & 2 \end{vmatrix}$

but $A = 0$

3. If two rows or columns are same so the value of $|A| = 0$;

$\begin{vmatrix} 2 & 5 & 2 \\ 2 & 5 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0$.

If each element of a row (or column) of a determinant is multiplied by a constant so then the value of a new determinant will k times the value of original determinant

i.e. $\begin{vmatrix} 2 & 5 & 2 \\ 10 & 15 & 20 \\ 2 & 5 & 1 \end{vmatrix} = 5\begin{vmatrix} 2 & 5 & 2 \\ 2 & 3 & 4 \\ 2 & 5 & 1 \end{vmatrix}$.

91

So if any two rows (or columns) of a determinant are proportional then its value is zero.

$$\begin{vmatrix}2&3&4\\10&15&20\\2&2&1\end{vmatrix} = 5\begin{vmatrix}2&3&4\\2&3&4\\2&2&1\end{vmatrix} = 0.$$

6- If each element of a row or column of a determinant is sum of two or more terms then.

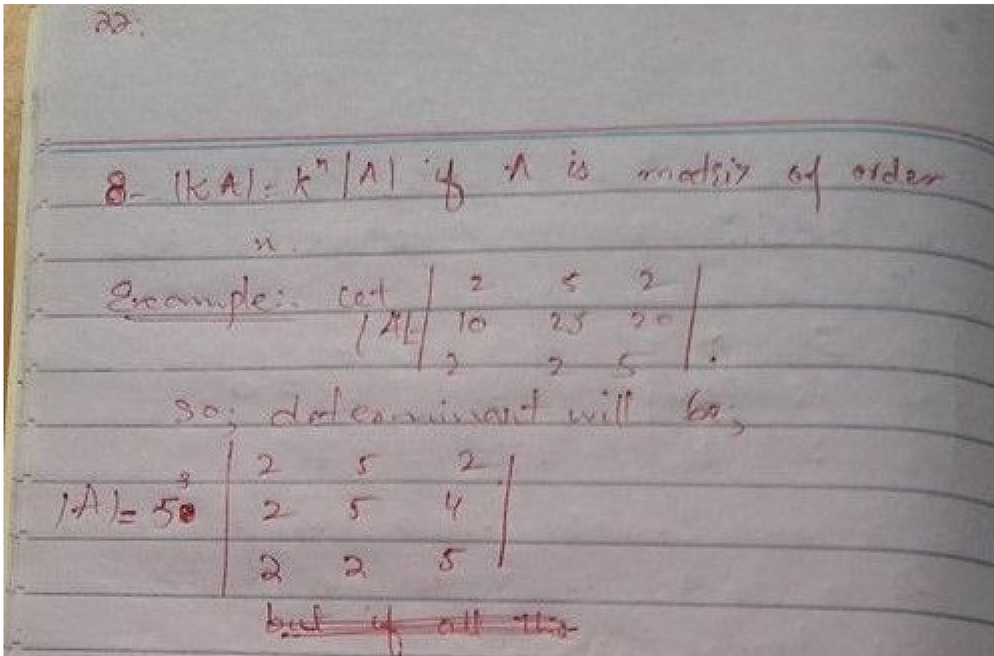
$$\begin{vmatrix}2&3&2\\2x+1&4x+2&4x+3\\2&1&2\end{vmatrix} =$$

$$\begin{vmatrix}2&3&2\\2x&4x&4x\\2&1&2\end{vmatrix} + \begin{vmatrix}2&3&2\\1&2&3\\2&1&2\end{vmatrix}.$$

7. If each element of a row (or column) of a determinant is zero then its value is zero.

i.e $\begin{vmatrix}2&0\\1&2\end{vmatrix} = (2\times2)-(0\times1) = 4.$

but $\begin{vmatrix}2&5&0\\2&5&2\\2&5&1\end{vmatrix}.$ In this case all the determinant will be zero.


22:

8- $|KA| = k^n|A|$ if A is matrix of order n.

Example: let $|A| = \begin{vmatrix} 2 & 5 & 2 \\ 10 & 25 & 20 \\ 2 & 2 & 5 \end{vmatrix}$.

So; determinant will be;

$|A| = 5^3 \begin{vmatrix} 2 & 5 & 2 \\ 2 & 5 & 4 \\ 2 & 2 & 5 \end{vmatrix}$

~~but if all the~~

9) $|AB|=|A||B|$

10) If A is in triangular form then determinant of is equal to the product of its diagonal elements.

For Example: $|A|=\begin{vmatrix} 6 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{vmatrix}$ then $|A|= 6\times1\times1=6$

11) The sum of the product of elements of any row or (column) of a determinant with the co-factor of the corresponding elements of any other row or (Column) is zero.

12) $|A^{-1}|=1/|A|$

13) If A is in diagonal form then product of its diagonal elements is the value of the determinant.

14) $|AB|=|BA|$ although $AB\neq BA$.

ADJOINT OF A MATRIX

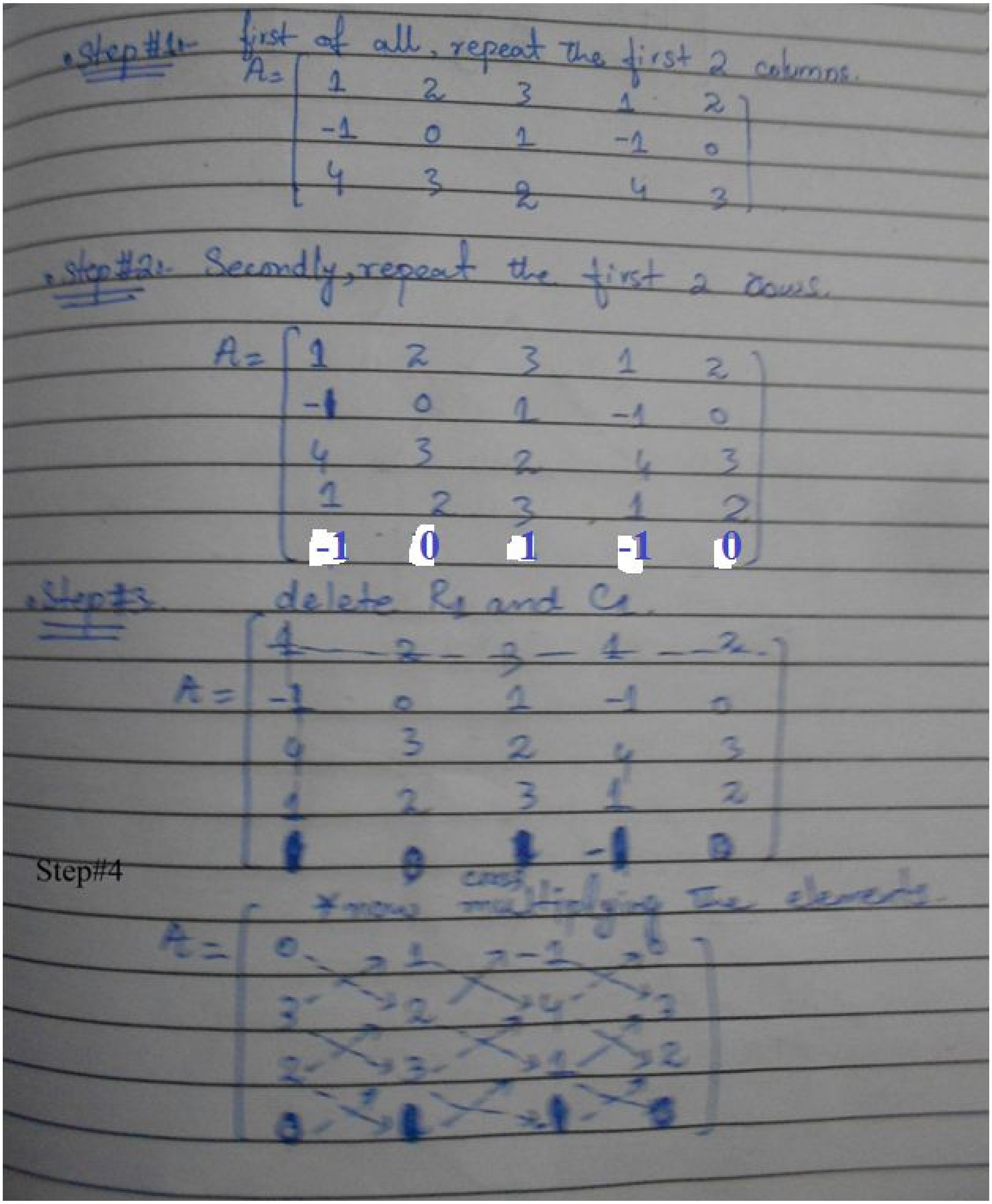
1) Adjoint of 2×2 Matrix:

If A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,then Adj(A)=$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

2) Adjoint of 3×3 Matrix:

There is a shortcut method to find the adjoint of 3×3 Matrix.

Let we have an example to consider this short method.


Step#1: first of all, repeat the first 2 columns.

$$A=\begin{bmatrix}1&2&3&1&2\\-1&0&1&-1&0\\4&3&2&4&3\end{bmatrix}$$

Step#2: Secondly, repeat the first 2 rows.

$$A=\begin{bmatrix}1&2&3&1&2\\-1&0&1&-1&0\\4&3&2&4&3\\1&2&3&1&2\\-1&0&1&-1&0\end{bmatrix}$$

Step#3: delete R_1 and C_1.

$$A=\begin{bmatrix}\cancel{1}&\cancel{2}&\cancel{3}&\cancel{1}&\cancel{2}\\-1&0&1&-1&0\\4&3&2&4&3\\1&2&3&1&2\\-1&0&1&-1&0\end{bmatrix}$$

Step#4 cross multiplying the elements.

$$A=\begin{bmatrix}0&1&-1&0\\3&2&4&3\\2&3&1&2\\0&1&-1&0\end{bmatrix}$$

In the Step#4, we take the determinant of each row and column entries in arrow headed way and then place it as each entry of the matrix in the Step#5.

Step#5:

$$adj(A)=\begin{bmatrix} \begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} \end{bmatrix}$$

$$adj\ (A)=\begin{bmatrix} -3 & 5 & -3 \\ 5 & -10 & 5 \\ 2 & -4 & 2 \end{bmatrix}$$

Step#6: Now take transpose of the Matrix.

$$adj(A)=\begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & -4 \\ -3 & 5 & 2 \end{bmatrix} \text{(Answer)}$$

Note: (This method can be verified by common method.)

Properties of Adjoint:

1) If A is a square matrix of order n then $A(adjA)=|A|I_n=(adjA)A$
2) If A is square matrix of order n then $adj(A^t)=(adjA)^t$
3) If A and B are two square matrices of same order then $adj(AB)=(adjB)(adjA)$
4) $adj\ (adjA)=|A|^{n-2}A$ where A is a non-singular matrix.
5) $|adjA|=|A|^{n-1}$ Whereas A is a matrix of order n.
6) $|Aadj(adjA)|=|A|^{(n-1)^2}$ Where A is non-singular matrix.
7) Adjoint of a diagonal matrix is a diagonal matrix.
8) $|adjAB|=|adjA||adjB|=|adjB||adjA|$
9) $adjA= A^{-1}|A|$
10) For any scalar 'k'
   $adj\ (kA) = k^{n-1}$ Adj A whereas A is a matrix of n order.

-Inverse of a square Matrix:

$A^{-1}=adj(A)/|A|$

It may be noted that $AA^{-1}=A^{-1}A=I$

-Properties of the Inverse:

1) A square matrix is invertible iff it is a non singular.

2) $(A^{-1})^{-1}=A$
3) $(A^{t})^{-1}=(A^{-1})^{t}$
4) $(AB)^{-1}=B^{-1}A^{-1}$
5) If A is non-singular matrix such that A is symmetric then A^{-1} is also symmetric.
6) If A is a non-singular matrix then $|A^{-1}|=|A|^{-1}$

-Rank of a Matrix:

Let A be a non-zero matrix, if r is the number of non-zero rows when it is reduced to the echelon or reduced echelon form then r is called rank (row) of the matrix A.

There are some rules/shortcuts by which finding the rank become very easy.

1) Here is a shortcut for finding rank of a matrix but this method works mostly for rectangular matrices. Keep in mind that rank of a matrix cannot be negative and also cannot be greater than row number of a matrix. The method is that first add the elements of first row and write down the sum then the second row and write down the sum. Proceed until the rows are finished. Now add first column and write down the sum and do the same until columns end. Now you will have these sums (If order of matrix is 3x4 then sums will be 7). Cut negative numbers and the numbers greater than row number of under consideration matrix. All the remaining numbers will be possible of being ranks but the greatest of these will be the rank.

   For Example:

   $A=\begin{bmatrix} 3 & -1 & 3 & 0 & -1 \\ 1 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & 2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{bmatrix}$, find the Rank=?

   Solution:

   Using the shortcut:

   First of we add the rows one by one.

   Row#1: 3-1+3+0-1=4 (accepted)

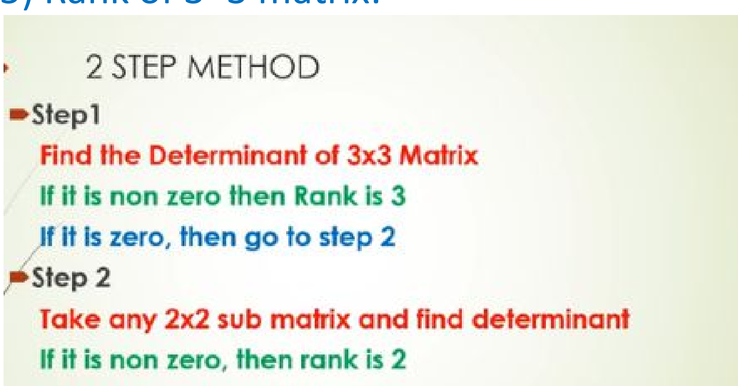
Row#2: 1+2-1-3-2=-3 (Not accepted)
Row#3: 2+3+4+2+5=16 (Not accepted)
Row#4: 2+5-2-3+3=5 (Not accepted)
Column#1: 3+1+2+2=8 (Not accepted)
Column#2: -1+2+3+5=9 (Not accepted)
Column#3: 2+3+4+2+5=16 (Not accepted)
Column#4: 2+5-2-3+3=2=5 (Not accepted)
So that possible sum that can be rank is 4 so that Rank of this matrix=4. (Answer)

2) A matrix have order 2*2 ,
If |A|=0 then its rank is 1
For Example: A=$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, then |A|= 1-1=0 so that Rank=1
If |A|≠0 then its rank is 2
For Example: A=$\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$, then |A|=4 so that rank=2

3) The rank of null matrix is 0.
For Example: A=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then rank=0

4) The rank of identity matrix is equal to order of matrix.
For Emple: A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, Rank=3 as order is equal 3×3.

5) Rank of 3×3 matrix:



Example 1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

$$\begin{aligned} |A| &= 1x(2x2-1x1) - 2x(1x2-1x2) + 3x(1x1-2x2) \\ &= 1x(4-1) - 2x(2-2) + 3(1-4) \\ &= 3-0-9 \\ &= -6 \end{aligned}$$

Non zero Determinant means rank is 3

Example 2

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{aligned} |B| &= 1x(2x6-1x4) - 2x(1x6-1x2) + 3x(1x4-2x2) \\ &= 1x(12-4) - 2x(6-2) + 3(4-4) \\ &= 8-8+0 \\ &= 0 \end{aligned}$$

Take any 2x2 submatrix and find their Determinant

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = -4$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 1 \\ 4 & 6 \end{vmatrix} = 8$$

Det. of any 2x2 submatrix not zero, then rank is 2

Example 3

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0 \qquad \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = 0$$

$|A| = 0$

Determinant of any 2x2 sub matrix of A is zero

Then Rank is 1

6) Note:

1) Rank of a matrix remains unaltered by elementary transformations.
2) No Skew-symmetric matrix can be rank 1.
3) AA^t has the same rank as A.
4) Rank can't be a negative number.
5) Rank can't be greater than number of rows of matrix under consideration.
6) Rank can be less than equal to the number of rows of matrix.
7) Another useful method for rectangular matrix:

For rectangular matrix, if number of rows is less than number of columns then the rank of matrix will be equal to number of linearly independent rows.

Similarly, if number of columns is less than number of rows then rank of matrix will be equal to number of linearly independent columns.

➔Examples

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{bmatrix}$$

Now, you can see, column 1 and 2 are independent because they are not derived form others, but column 3 (C1 + C2) is sum of column 1 and column 2. So there are two linearly independent columns hence its rank is 2.

➔**Linearly Independent Rows/Columns:** The rows/columns which is not derived from other rows/columns (scalar multiple of other rows/columns or sum of two rows/columns) i.e. which don't depend on other rows/columns.

(**Note:** you may also use the long method which is actually smooth to understand)

SYSTEM OF EQUATIONS

HOMOGENEOUS SYSTEM:-

$a_1x+b_1y+c_1z = 0$
$a_2x+b_2y+c_2z = 0$
$a_3x+b_3y+c_3z = 0$

- If, $|A| = 0$, system will have infinite solutions(Non-Trivial), system will be consistent (having solution).
- If, $|A| \neq 0$, system will have only one solution(Trivial Solution).

NON-HOMOGENEOUS SYSTEM:-

$a_1x+b_1y+c_1z = k_1$
$a_2x+b_2y+c_2z = k_2$
$a_3x+b_3y+c_3z = k_3$

- If $|A| = 0$, system will have infinite solutions or no solution.
- If, $|A| \neq 0$, system will have only one solution(Unique Solution) and system will be consistent.

Remarks:

Let AX=O be a homogeneous system of linear equations

1) If rank (A) = number of variables then AX=O have a trivial solution X=O

2) If rank (A) < number of variables then AX=O have a non-trivial

solution. It will be infinitely many solutions.

Note: AA^{-1}=Identity matrix (use this trick where the matrix is given and inverse is to find , just take A^{-1} from all the option and multiply with real to make identity matrix.

CHAPTER 04
(Quadratic Equation)

Given, $ax^2+bx+c = 0$

1) If a+b+c =0, one root will be equal to 1.
2) If a and c are of opposite signs, roots will be of opposite signs. e.g. $-x^2-x+2=0$
3) If b=0, roots will be additive inverse of each other.e.g.$2x^2\pm4=0$
4) If a=c , roots will be reciprocal of each other. e.g. $x^2+2x+1=0$
5) If a+b*i* is one root then a-b*i* will be other root.
6) If $a+\sqrt{b}$ is one root then $a-\sqrt{b}$ will be other root.
7) If $ax^2+bx+c = 0$ has roots α and β, then $cx^2+bx+a = 0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Example

$x^2+7x+12=0$ has roots -3 and -4 & $12x^2+7x+1=0$ has roots $\frac{-1}{3}$ and $\frac{-1}{4}$

8) If a=b=c≠0 then roots are w, w^2.
9) If a=b=c=0 then it becomes an identity.
10) If one root is double of the other then it becomes as $2b^2 =9ac$
11) If one root is square of the other then it becomes as $a^2c+ac^2+b^3=3abc$

12) If a=1, b, c ε Z and the roots are rational numbers then these roots must be integer.
13) The co-efficient of the terms equidistant from beginning and end are equal in Reciprocal Equations that are reducible to quadratic equations.
14) While solving a radical equation we first get a radical-free equation and then solve this radical-free equation. Every solution of Radical-free equation is not necessarily solution of original radical equation but each solution of original radical equation is necessarily a solution of radical-free equation.
15) In standard form of a quadratic equation, if 'b=0' and 'a is not zero' the Quadratic Equation is called PURE quadratic equation.
CUBE ROOTS:-

Number	Cube Roots
1	1, ω, ω^2
-1	-1, - ω, - ω^2
8	2, 2ω, $2\omega^2$
-8	-2, -2ω, $-2\omega^2$
27	3, 3ω, $3\omega^2$

So Cube Roots of any number say n will be: $(n)^{1/3}$= m, mω, $m\omega^2$

ω= $\frac{-1+\sqrt{3}\,i}{2}$ ω^2= $\frac{-1-\sqrt{3}\,i}{2}$
(These values are interchangeable)

- Sum of any cube Roots is 0.

- Product of Cube Roots of n is n.

TRICK:
If you have to solve any power of ω then simply divide the power by 3 and check remainder. It will become the power of ω. $\omega^3 = 1$

If you see such question: $\omega^{56} = ?$

just divide the power of ω by 3 and solve the ω to the remainder power.

In our case.
56/3
Remainder: 2
ω^2 is answer.

FOURTH ROOTS:-

Number	Fourth Roots
1	1, -1, *i*, -*i*
16	2, -2, 2*i*, -2*i*
81	3, -3, 3*i*, -3*i*
625	5, -5, 5*i*, -5*i*
So Fourth Roots of any number say n will be: $(n)^{1/4} = m, -m, mi, -mi$	

- Sum of any fourth roots is 0.

- Product of fourth roots of n is "-n".

Some Important Tricks:

Let us have a notation $ax^2+bx+c=0$ and we have roots of the anonymous equation, by using them, we have to find an equation of those roots.

Type#1: 2α, 2β

Method: Multiply "b" term with 2 and "c" term with square of 2.

Example: Let $ax^2+bx+c=0$

Using the trick:

$ax^2+2(b\,x)+c(2^2)=0$

$ax^2+2(b\,x)+4c=0$

Type#2: 1/α , 1/β

Method: Interchange the co-efficient of "a" and "c"

Example: Let $ax^2+bx+c=0$

Using the trick:

$cx^2+bx+a=0$

Type#3: -1/α , -1/β

Method: Interchange the co-efficient of "a" and "c" and multiply the term "b" with minus " -"

Example: Let $ax^2+bx+c=0$

Using the trick:

$cx^2-b\,x+a=0$

Type#4: 2/α, 2/β

Method: Multiply the "b" term with 2 and "a" term with square of 2 and then interchange the coefficient of "a" an "c".

Example: Let $ax^2+bx+c=0$

Using the trick:

$(2^2)ax^2+(2)b\,x+c=0$

$4ax^2+2bx+c=0$

now interchange the coefficient of a and c.

$cx^2+2bx+4a=0$

Type#5: $1/2\alpha$, $1/2\beta$

Method: Multiply “b” term with 2 and “c” term with its square and then interchange the coefficient of “a” and “c”.

Example: Let $ax^2+bx+c=0$

Using the trick:

$ax^2+(2)b\,x+c(c^2)=0$

$c^3x^2+2bx+a=0$

(Note: These tricks can be verified by usual method).

~Tricks for Solving Simultaneous equation:

Trick#1:

Put the points as given in all the options and satisfied the given equations.

Trick#2:

Let we have two equations:

ax+by=c----- (i)

dx+ey=f------(ii)

For value of x:

$$x=\frac{b(f)-e(c)}{b(d)-a(e)}$$

For value of y:

Put the value of x in eq (i)....

Example:

Find the values of x and y by solving equation:

x+2y=10---- (i)

2x+3y=18----- (ii)

Using the Trick:

For x value:

$$x=\frac{2(18)-3(10)}{2(2)-3(1)}=\frac{6}{1}=6$$

For y value:
Put x in (i):
6+2y=10
y=2
so x=6 and y=2. (Ans)

Judgment of Roots of Any Equation:-
If an equation and its roots are given in MCQ, then you can simply check your answer by putting the roots in the equation instead of solving the equation. The roots of the equation satisfy the equation. e.g.

$$2x^2 \quad 8x \quad 24 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-24)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 \quad (\quad 192)}}{4}$$

$$x = \frac{8 \pm \sqrt{256}}{4}$$

$$x = \frac{8 \pm 16}{4} = 2 \pm 4$$

$$x = 6, \quad 2$$

6 and -2 are the roots of given equation. If we put these roots in the equation, it will satisfy the equation.

x=6: $2(6)^2-8(6)-24 = 0$
x=-2: $2(-2)^2-8(-2)-24= 0$ Both 6 and -2 satisfy the equation.

Relation between Roots and Co-efficient:-

Relation b/w roots and coefficients

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + ax + a_0$

Sum of roots $= -\dfrac{a_{n-1}}{a_n}$

Product $= (-1)^k \dfrac{a_0}{a_n}$

where k = degree

ax^2+bx+c

Sum $= -\dfrac{b}{a}$

Product $= \dfrac{c}{a}$

Difference of roots $= \sqrt{S^2 - 4P}$

S = Sum of roots , P = product of roots

if polynomial $3x^5 - 4x^2 + 1 = 0$ Then Sum and product are

$S = -\dfrac{0}{3} = 0$

product $= (-1)^k \dfrac{c}{a} = (-1)^5 \dfrac{1}{3} = -\dfrac{1}{3}$

if polynomial $6x^6 - 3x^5 + x^4 + 8 = 0$

Sum $= -\dfrac{-3}{6} = \dfrac{1}{2}$

Product $= (-1)^6 \dfrac{8}{6} = \dfrac{4}{3}$

Formation of the Equation by roots:

We can form equation by,

$$x^2 - Sx + P = 0$$

whereas, S = Sum of Roots
P= Product of Roots

- If an equation $ax^2+bx+c=0$ is given and we have to find an equation whose roots are n times the roots of given equation, then equation can simply be found as;
  $ax^2+n(b\,x)+n^2c=0$
- If the sum of the roots is given and we have to find the sum of the roots raises to some power n.
  Sum of roots of nth power = n (S)
  Example: If the sum of the roots is 6 then find the sum of the roots raises to power 3?
  Using the trick:
  Sum of roots of 3rd Power= 3(6)=18 (Answer)
- If the product of the roots is given and we have to find the product of the root raises to some power n.
  Product of roots of nth power= n^2 (P)

Example: If the sum of the roots is 8 then find the product of the roots raises to power 3.

Using the trick:

Sum of roots of 3rd power= $3^2(8)=9(8)=72$ (Answer)

The Quadratic equation when sum and product of roots raises to some power n.

$x^2 - n(S)x + n^2P = 0$

Remainder Theorem:

If a polynomial f(x) of degree $n \geq 1$,where n is non-negative integer, is divided by x-a till no x-term exists in the remainder then f(a) is the remainder.

e.g. If we divide $f(x)= x^2+3x+7$ by $x+1$ then remainder is

$f(-1)= (-1)^2+3(-1)+7=5$

Factor Theorem:

The polynomial x-a is the factor of the polynomial f(x) if and if f(a)=0

Nature Of Roots of the Quadratic Equation:

Nature of roots depends on the expression b^2-4ac which is called discriminant and denoted by D.

a) If D<0, then roots are imaginary

b) If D>0, then roots are real and distinct

c) If D=0, then roots are real and equal

d) Roots are rational iff D is a perfect square.

e) Roots are irrational iff D is positive but not a perfect square.

-Common Roots:

1) One Common Root:

If X is a common root of the equations:

$a_1 x^2+b_1x+c_1=0$ (1)

$a_2 x^2+b_2x+c_2=0$ (2)

then we have common root:

$X=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$

2) Both Common Roots:

If the equations (1) and (2) have both roots common then these equations will be identical. Thus the required condition for both root common is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Note:

1) To find the common root of two equations make the coefficient of second degree terms in two equations equal and subtract . The value of x so obtained is the required root.

2) If two quadratic equations with real coefficients have an imaginary root common then both roots will be common and the equations will be identical. The required condition is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

3) If two quadratic equations have an irrational root common then both roots will be common and the two equations will be identical. The required condition is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Greatest and Least Value of a Quadratic Expression:

1) If a>0, then the quadratic expression $y=ax^2+bx+c$ has no greatest value but it has least value. $\frac{4ac-b^2}{4a}$ at $x=\frac{-b}{2a}$.

2) If a<0, then the quadratic expression $y=ax^2+bx+c$ has no least value but it has greatest value. . $\frac{4ac-b^2}{4a}$ at $x=\frac{-b}{2a}$.

Properties of Cube Root of Unity:

1) Each Complex cube root of unity is square of the other.

If $\frac{-1+\sqrt{3}i}{2}=\omega$ then If $\frac{-1-\sqrt{3}i}{2}=\omega^2$ and If $\frac{-1-\sqrt{3}i}{2}=\omega$ then $\frac{-1+\sqrt{3}i}{2}=\omega$

2) The sum of all the three cube root of unity is zero i.e
$1+w+w^2=0$
3) The product if all the three cube root of unity is 1. i.e $w^3=1$
4) For any $n\epsilon Z$, w^n is equivalent to one of the cube root of unity.
5) $1,w,w^2$ is a geometric sequence with common ratio "w"
6) $\{1,w,w^2\}$ is an Abelian Group under multiplication.
7) Each complex cube root of unity is square, square root, reciprocal and conjugate of other.
8) $w^n+w^{n+1}+w^{n+2}=0$,n ϵZ
9) $w^n.w^{n+1}.w^{n+2}=1$,n ϵZ
-Properties of Forth root of Unity:
1) Sum of all the four roots of unity is zero.
:. 1+(-1)+i+(-1)=0
2) Product of all the four roots of unity is -1
:. 1*(-1)*i*-i=1
3) The real fourth root of unity are additive inverses of each other +1 and -1 are the real fourth root of unity and +1+(-1)=0=(-1)+1
4) Both Complex imaginary fourth roots of unity are conjugate ,additive inverse and multiplicative inverse of each other.
Note: Number of roots of a polynomial is equal to degree of polynomial.
-Quadratic inequalities Tricks:
1) (x-a)(x-b)>0 → x<a or x>b for a<b
2) (x-a)(x-b)<0 → a<x<b, for a<b
3) |x|<a → -a<x<a
4) |x|>a → x<-a or x>a
For Example: The solution set of |x-5|<9=?
So using the trick: -9<x-5<9 adding the 5 on thrice sides so that -4<x<14 i.e (-4,14) (Answer)
For Example: The solution set of $x^2-5x+6<0$=?

First of all factorize it; $(x-2)(x-3)<0$
So by using the trick: $x<2$ or $x<3$ so $]2,3[$ (Answer)
For Example: The Solution set of $x^2-1>0$
As it is $(x+1)(x-1)>0$
So that $-1<x<1$ i.e $(-1,1)$ (Answer)
For Example: The solution set of $|x-1|>2$
So using the Trick: $x-1<-2$ or $x-1>2$ i.e $x<-1$,$x<3$ →$(-1,3)$ ans
Note:
There are many types of question in which the quadratic equation is not given but the method to solve is only by making it a quadratic equation and then to solve them further. For these questions, there are some tricks.

Tricks:

1) $\sqrt{n+\sqrt{n+\sqrt{n+\cdots\ldots\ldots\ldots+\infty}}}=k+1$

2) $\sqrt{n-\sqrt{n-\sqrt{n-\cdots\ldots\ldots\ldots-\infty}}}=k$ **then** $n=k(k+1)$

3) $\sqrt{n+\sqrt{n-\sqrt{n+\cdots\ldots\ldots\ldots\infty}}}=\frac{\sqrt{4n-3}+1}{2}$

4) $\sqrt{n-\sqrt{n+\sqrt{n-\cdots\ldots\ldots\ldots\infty}}}=\frac{\sqrt{4n-3}-1}{2}$

5) If $a^x+b^x=a^k+b^k$; $a,b,x,k\ \epsilon R$ **then** $x=\pm k$

6) If $\sqrt{Q_1}-\sqrt{Q_2}=a-b$,$a-b\epsilon R$ **such that** $Q_1-Q_2=a^2-b^2$ **then** $Q_1=a^2$

For example: $x=\sqrt{2+\sqrt{2+\sqrt{2+\cdots\ldots\ldots\ldots+\infty}}}$ **then** $x=?$

Usual Method: squaring both sides.

$x^2=2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots\ldots\ldots\ldots+\infty}}}$

$x^2=2+x \Rightarrow x^2-x-2=0 \Rightarrow x=-1$ **and** $x=2$

By trick: $n=k(k+1)$ ➔ $2\times1=k(k+1)$ **so that** $x=2$

For Example: $x=\sqrt{2+\sqrt{2-\sqrt{2+\cdots\ldots\ldots\ldots\infty}}}$ **Then Find the value of x?**

By trick: $x=\frac{\sqrt{4P-3}+1}{2}$ **so that** $x=\frac{\sqrt{4(2)-3}+1}{2}=\frac{\sqrt{5}+1}{2}$

For Example: $(4+\sqrt{15})^x+(4-\sqrt{15})^x=62$, **find the value of x?**

Solution: As $(4+\sqrt{15})^x+(4-\sqrt{15})^x=8^2+(-(\sqrt{2})^2)$,

Here k=2 so that Using the trick; x=±k ➔x=±2 (Answer)
For Example: $\sqrt{x^2-5x+10}-\sqrt{x^2-5x+7}=1$ **, x=?**
As $x^2-5x+10-x^2+5x-7$**=3 such that** $3=2^2-1^2$
So Using the trick $x^2-5x+10=2^2$ **→** $x^2-5x+6=0$ **,so x=2,3 (Answer)**

Entry Test Typed MCQs:
1.The quadratic equation whose roots are 4 and -4 is given by :

A. $x^2+3x-28=0$
B. $x^2-3x+28=0$

C. $x^2-3x-28=0$
D. $x^2+3x+28=0$

Answer: Option C.

Explanation:

Let α =7 and β =-4.

Then, α+ β =3, α β = -28.

$x^2-(\alpha+\beta)x+\alpha\beta=0$.

or $x^2-3x-28=0$.

2.If some of the roots of a quadratic equation is 6 and the product of its roots is also 6, then the equation is
A. $x^2+6x-6=0$
B. $x^2-6x+6=0$
C. $x^2-6x-6=0$
D. $x^2+6x+6=0$

Answer: Option B.

Explanation:

α+ β =6 and α β = 6

The equation is :

$x^2-(\alpha+\beta)x+\alpha\beta=0$.

or $x^2-6x+6 = 0$.

3. The quadratic equation with rational coefficients and having $(2-\sqrt{3})$, as one of its roots is :

A. $x^2+4x+1=0$
B. $x^2+4x-1=0$

C. $x^2-4x-1=0$
D. $x^2-4x+1=0$

Answer: Option D.

Explanation:

Let $\alpha=(2-\sqrt{3})$

Then $\beta=(2+\sqrt{3})$

$\alpha+\beta=4$ and $\alpha\beta=(4-3) = 1$.

The equation is :

$x^2-(\alpha+\beta)x+\alpha\beta=0$.

or $x^2-4x+1 = 0$.

4.The quadratic equation with rational coefficients, one of whose roots is $(3+2\sqrt{3})$, is:

A. $x^2+6x-3=0$
B. $x^2-6x-3=0$

C. $x^2+6x+3=0$
D. $x^2-6x+3=0$

Answer: Option B.

Explanation:

Let α =(3+ 2√ 3)

Then β =(3- 2√ 3)

α+ β =6 and α β =(9-12) = -3.

The equation is :

x^2-(α+ β)x+ α β =0.

or x^2-6x-3 = 0.

5. The quadratic equation with rational coefficients and having (√ 2 -1) as one of its roots is :

A. $x^2-2\sqrt{2}x+1=0$
B. $x^2-2\sqrt{2}x-1=0$

C. $x^2+2x-1=0$
D. $x^2-2x+1=0$

Answer: Option C.

Explanation:

Let α =(-1+ √ 2)

Then β =(-1- √ 2)

α+ β =-2 and α β = (1-2) =-1.

The equation is :

x^2-(α+ β)x+ α β =0.

or x^2+2x-1 = 0.

6.The quadratic equation with real coefficients and having (2+3i)as one of its roots, is

A. $x^2+4x+13=0$
B. $x^2+4x-13=0$
C. $x^2-4x-13=0$
D. $x^2-4x+13=0$

Answer: Option C.

Explanation:

Let $\alpha=(2+3i)$ and $\beta=(2-3i)$.

Then, $\alpha+\beta=4$, $\alpha\beta=(4+9)=13$.

$x^2-(\alpha+\beta)x+\alpha\beta=0$.

or $x^2-3x-28=0$.

7. The quadratic equation with real coefficients and having $(4+\sqrt{-3})$ as one of its roots, is :

A. $x^2-8x+19=0$
B. $x^2-8x-19=0$

C. $x^2+8x+19=0$
D. None of these

Answer: Option A.

Explanation:

Let $\alpha=(4+\sqrt{3}i)$

Then $\beta=(4-\sqrt{3}i)$

$\alpha+\beta=8$ and $\alpha\beta=(16+3)=19$.

The equation is :

$x^2-(\alpha+\beta)x+\alpha\beta=0$.

or $x^2-8x+19=0$.

8. If α, β are the roots of the equation $x^2-q(1+x)-r=0$, then the value of $(1+\alpha)(1+\beta)$ is

A. (1-r)
B. (1+r)

C. (q-r)
D. (q+r)

Answer: Option A.

Explanation:

$x^2-qx-(q+r) = 0.$

$\alpha+\beta=q, \alpha\beta=-(q+r).$

$(1+\alpha)(1+\beta)$

$= 1+(\alpha+\beta)+\alpha\beta$

$=1+q-(q+r)=(1-r).$

9.If α, β are the roots of the equation $x^2-px+q = 0$, then the value of $(\alpha^2+\beta^2)$ is

A. p^2+2q
B. p^2-2q

C. $p(p^2-3q)$
D. p^2-4q

Answer: Option B.

Explanation:

$\alpha+\beta=p, \alpha\beta=q$

$(\alpha^2+\beta^2)=(\alpha+\beta)^2-2\alpha\beta$

$=(p^2-2q)$

10.If the product of the roots of x^2-3x+k is -2, then find the value of k?

A. -2
B. 8

C. -8
D. 12

Answer: Option B.

Explanation:

Given equation is $x^2-3x+(k-10)=0$.

∴ $k-10=-2$

⇒ $k=8$.

11.If one root of $3x^2-6kx+8k = 0$ is 4, the other root is:

A. 2
B. -2
C. -4
D. 3

Answer: Option A.

Explanation:

$x=4$ satisfies $3x^2-6kx+8k=0$.

∴ $3*16-6*k*4+8k = 0$

⇒ $16k=48$ i.e $k=3$.

∴ The equation is $3x^2-18x+24=0$.

⇒ $x^2-6x+8=0$.

12.The quadratic equation whose roots are the reciprocals of the roots of the equation $ax^2+bx+c = 0$ is

A. $cx^2 +bx+c = 0$
B. $bx^2 +cx+a = 0$

C. $ax^2+bx^2+c = 0$
D. None of these

Answer: Option A.

Self Explanatory.

18.If α, β be the irrational roots of $ax^2+bx+c = 0$, where a, b, c are rational and $a \neq 0$, then

A. $\alpha = \beta$
B. $\alpha,\beta = 1$
C. $\alpha^2+ \beta^2$
D. 1

Answer: Option D.

Explanation:

Clearly Option D is true.

13. The coefficient of x in the equation $x^2+px+q = 0$ was taken as 17 in place of 13 and its roots were found to be -2 and -15. The roots of the original equation are:

A. 2, 15
B. 10, 3

C. -2, 15
D. -10, -3

Answer: Option D.

Explanation:

Let α,β be the roots of the original equation. then,

$\alpha+\beta = -13$ and $\alpha\beta =(-2)(-15) =30$.

∴ original equation is

$x^2+13x+30 = 0$.

$\Rightarrow x^2+10x+3x+30 = 0$.

$\Rightarrow x(x+10)+3(x+10) = 0$.

⇒ (x+10) (x+10) = 0.

⇒ x =-10 or x =-3.

14.The number of real solutions of $x^2-3|x|+2 = 0$ is

A. 1
B. 2

C. 3
D. 4

Answer: Option D.

Explanation:

Case I: When $x \geq 0$

In this case, $|x| = x$.

So the equation is

$x^2-3x+2 = 0$.

∴ $(x-2)(x-1) = 0$.

∴ $x = 2$ or $x = 1$.

Case II: When $x<0$

In this case, $|x| = -x$.

So the equation is

$x^2+3x+2 = 0$.

∴ $(x+2)(x+1) = 0$.

∴ $x = -2$ or $x = -1$.

Hence, the given equation has 4 solutions.

15.For the equation $|x^2|+|x|-6 = 0$, the roots are

A. coincident
B. real with sum zero

C. real with sum 1
D. none of these

Answer: Option B.

Explanation:

$|x^2| = x^2$

∴ Given equation is

$x^2+|x|-6 = 0$.

Case I: When $x \geq 0$

In this case, $|x| = x$.

So the equation is

$x^2+x-6 = 0$.

∴ $(x+3)(x-2) = 0$.

∴ $x = -3$ or $x = 2$.

∴ $x = 2$.

Case II: When $x<0$

In this case, $|x| = -x$.

So the equation is

$x^2-x-6 = 0$.

∴ $(x-3)(x+2) = 0$.

∴ $x = 3$ or $x = -2$.

∴ $x = -2$.

16 .If α, β are the roots of the equation x^2-3x+k = 0, then the value of k for which α = 2β is

A. 2
B. -3

C. 3
D. 1

Answer: Option A.

Explanation:

α+ β =3, α = 2β

⇒ 2β =3,

⇒ beta; =1.

∴ x =1 is a root of x^2-3x+k =0.

∴ 1-3+k =0

⇒ k =2

17. If the sum of the squares of the roots of the equation x^2+2x-p = 0 is 10, then the value of p is :

A. -3
B. 3

C. 6
D. -6

Answer: Option B.

Explanation:

α+ β =-2, α β = -p.

Also, α^2+ β^2 =10.

$\Rightarrow (\alpha+\beta)^2-2\alpha\beta = 10.$

$\Rightarrow 4+2p = 10.$

$\Rightarrow p = 3.$

18.The sum of 2 numbers is 9 and the sum of their squares is 41. The numbers are:

A. 4, 5
B. 1, 8

C. 3, 6
D. 2, 7

Answer: Option A.

Explanation:

$\alpha+\beta = 9$, $\alpha^2+\beta^2 = 41$.

$\therefore \alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow 41 = 81-2\alpha\beta$

$\Rightarrow \alpha\beta = 20.$

$(\alpha-\beta)^2 = (\alpha+\beta)^2-4\alpha\beta$

$\Rightarrow (81-80) = 1.$

$\Rightarrow \alpha-\beta = 1.$

Solving $\alpha+\beta = 9$,$\alpha-\beta = 1$, we get $\alpha = 5$ and $\beta = 4$.

19. The roots of $ax^2+bx+c = 0$ will be reciprocal to each other , if

A. $a = 1/c$
B. $a = c$
C. $b = ac$
D. $a = b$

Answer: Option B.

Explanation:

Let the roots of $ax^2+bx+c=0$ be α and β.

Then, $\alpha \beta = c/a$.

When $\beta = 1/\alpha$, we have $\alpha \beta = 1$.

$\therefore c/a = 1$.

$\Rightarrow c = a$.

20.The value of k for the which roots α, β of the equation $x^2-6x+k = 0$ satisfy the relation $3\alpha+2\beta = 20$, is

A. -8
B. 8

C. 16
D. -16

Answer: Option D.

Explanation:

$\alpha + \beta = 6$.

$3\alpha + 2\beta = 20$

$\Rightarrow \alpha + 2*6 = 20$.

$\Rightarrow \alpha = 8$.

Now, $\alpha = 8$ satisfies $x^2-6x+k=0$.

$\therefore 64-6*8+k=0 \therefore k=-16$.

21.If the roots of the equation $x^2-px+q = 0$ differ by unity, then

A. $p^2 = 4q+1$
B. $p^2 = 4q-1$

C. $q^2 = 4p+1$
D. $q^2 = 4p-1$

Answer: Option A.

Explanation:

$\alpha + \beta = p$, $\alpha \beta = q$ and $\alpha - \beta = 1$.

$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4 \alpha \beta$

$\Rightarrow p^2 - 1 = 4q$.

$\Rightarrow p^2 = 4q + 1$.

22. For what value of p, the difference between the roots of the equation $x^2 - px + 8 = 0$ is 2?

A. ±2
B. ±4
C. ±6
D. ±8 $= p$, $\alpha \beta = 8$ and $\alpha - \beta = 2$.

Answer: C

Explanation:

$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4 \alpha \beta$

$\Rightarrow p^2 - 4 = 32$.

$\Rightarrow p^2 = 36$. i.e $p = \pm 6$

CHAPTER 05

- Difference between a Conditional Equation and an Identity Equation. Conditional is true for some specific values while Identity is a universal Equation.
- In a universal Equation the sign of Equality used is " ≡ ".
- Difference between a Proper and Improper Rational Function.
- Theorem of Equality of Polynomials.(Page:180).

Now to solve MCQ's of Partial Fractions Quickly, you have two options.

1) Just keep one thing in mind that as we can obtain partial fractions of a rational Function conversely we can get original Function by adding Partial fractions. For example if Partial fractions of a function are asked with four options, add partial fractions in each option to get original fraction.
2) you can solve the MCQ of Partial Fraction by following Method:

- Put x=0 in the question and check what value is coming (Note this value). If 0/0 form is becoming by putting x=0 then put x=1,2 or 3 etc. and note the value.
- Now put that value of x in the options of MCQ for which you have noted the value in 1st step and evaluate. For one option, the value will match with the value of the question you calculated. It will be the answer.

  -Proper Rational Fraction:

  Let P(x)/Q(x) be a rational fraction then P(x)<Q(x) is a proper Rational Fraction.

-Improper Rational Fraction:

Let P(x)/Q(x) be a rational fraction then P(x)≥ Q(x) is an improper Rational Fraction.

Note: For making improper Rational Fraction, a proper Fraction, we divide it.

For Example, please see Textbook page# 179.(Def. of Improper Fraction)

Case#1: Resolution of P(x)/Q(x) into partial Fractions when Q(x) has only non-repeated factor:

The Polynomial Q(x) may be written as:

$Q(x)=(x-a_1)(x-a_2)......(x-a_n)$ whereas $a_1 \neq a_2 \neq ..... \neq a_n$

Then $P(x)/Q(x)=\frac{A}{x-a_1}+\frac{B}{x-a_2}+\cdots+\frac{Real\ Number}{x-a_n}$

For this case, there is a shortcut (Cover Up Method) to find the Partial Fraction.

Example 1. Decompose $\frac{x-7}{(x-1)(x+2)}$ into partial fractions.

Solution. We know the answer will have the form

$$\frac{x-7}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}\,. \tag{1}$$

To determine A by the cover-up method, on the left-hand side we mentally remove (or cover up with a finger) the factor $x-1$ associated with A, and substitute $x=1$ into what's left; this gives A:

$$\left.\frac{x-7}{(x+2)}\right|_{x=1} = \frac{1-7}{1+2} = -2 = A\,. \tag{2}$$

Similarly, B is found by covering up the factor $x+2$ on the left, and substituting $x=-2$ into what's left. This gives

$$\left.\frac{x-7}{(x-1)}\right|_{x=-2} = \frac{-2-7}{-2-1} = 3 = B\,.$$

Thus, our answer is

$$\frac{x-7}{(x-1)(x+2)} = \frac{-2}{x-1} + \frac{3}{x+2}\,. \tag{3}$$

See the Example#1, page#181 (Alternative Method)

Case#2: When Q(x) has repeated linear factors:

If the polynomial has a factor $(x\text{-}a)^2$,x≥2 and n is a +ive integer then P(x)/Q(x) may be written as the following identity,

:. P(x)/Q(x)= $\frac{A}{x-\mathrm{a}_1}+\frac{B}{x-a_2}+\cdots+\frac{Real\ Number}{x-a_n}$

For Example, please check the Textbook page#184,Example#1

Case#3: When Q(x) contains non-repeated irreducible Quadratic Factor:

If the polynomial Q(x) contains non-repeated irreducible quadratic factor then P(x)/Q(x) may be written as identity having partial fractions of the form $\frac{Ax+B}{ax^2+bx+c}$, where A and B are the numbers to be found.

See the example on page#186 (Text Book), Example#1.

Case#4: When Q(x)= has repeated irreducible Quadratic Factors:

If The polynomial Q(x) contains repeated irreducible quadratic factors $(ax^2+bx+c)^n$, n≥2 and n is +ive integer then P(x)/Q(x) may be written as the identity.

(See the example on Page#188, Example#1)

CHAPTER 6
(Series and Sequence)

Sequence of a Function: A sequence of a function whose domain is the subset of set of natural numbers and range is subset of real or complex numbers.

Real Sequence: A sequence whose range (terms) is a subset of real numbers is called real sequence.

Series: A series is obtained by adding or subtracting the terms of a sequence.

Progression: If the terms of a sequence follow certain pattern then the sequence is called a progression.

Types of Progression:

1) Arithmetic progression (A.P)
2) Geometric progression (G.P)
3) Harmonic progression (H.P)

1) Arithmetic Progression:

A sequence whose terms increase or decrease by a fixed number is called Arithmetic Progression. The fixed number is called common difference of the A.P.

Note:

1) Common difference (d) can be a +ive or –ive number.
2) Total terms (n) can never be a negative number.

-Important Results:

1) If an A.P has n terms then nth term is called last term or limiting value of A.P and it is denoted by l or a_n. It is given by $a_n=a_1+(n-1)d$

For Example: 1,3,5,7,9,.... Then find a_{11}

Solution:

Using the formula:-

$a_n=a_1+(n-1)d$, as d=3-1=2

$a_{11}=1+(11-1)(2)$

$a_{11}=1+20$

$a_{11}=21$ (Answer)

2) Three numbers a,b,c are in A.P if and only if b-a=c-b i.e a+c=2b

3) If a,b,c are in A.P then Common Difference (d) is given by:

d=c-a/2

For Example: If 2,4,6,8 then find the d?

Solution:

Using the formula:-

d= 6-2/2 ➔ d=2 (Answer)

4) If a_m and a_n are two terms of an A.P whereas a_m is bigger term and a_n is smaller term. Also m is no. of bigger term and n is no. of smaller term. Then d=$\frac{a_m-a_n}{m-n}$

For Example: If $a_{23}=76$ and $a_{19}=20$ then find the d?

Solution:

Using the formula:

d= 76-20/23-19

d=56/4 ➔ d=14 (Answer)

5) If 1/a,1/b, 1/c are in A.P then b=2ac/a+c

6) If 1/a, 1/b, 1/c are in A.P then d=a-c/2ac

7) If a is the first term and d be the common difference of an A.P having m terms then nth term from the end is (m-n+1)th term from the beginning. Thus nth term from the end is given by $a_n=a_1+(m-n)d$ whereas m is total terms and n is specific term from the end.

For Example: If 2,4,……,10 then find the 4th term from the end?

Solution:

Using the formula:

$a_4=2+(5-4)(2)$

$a_4=2+2 \rightarrow a_4=4$ (Answer)

8) For total no. of terms of A.P, we use the formula $n=\frac{a_n-a_1}{d}+1$

For Example: If 2,4,………,100 then find the total no. of terms?

Solution:

Using the formula:

n= 100-2/2 +1

n= 49+1 ➔ n=50 (Answers)

9) Any three numbers in an A.P can be taken as a-d,a,a+d and any four numbers in A.P can be a-3d,a-d,a+d, a+3d, any five numbers in A.P can be taken as a-2d,a-d,a,a+d,a+2d.

For Example: The sum of three numbers in A.P is 15 and their product is 80, the largest number is? A) 2 B) 5 C) 8 (Correct) D) 11

Solution: As we know that three numbers are a-d,a,a+d

So that Sum of three number will be a-d+a+a+d=15 ➔ a=5

and the product will be $(a-d)(a)(a+d)=80 \rightarrow a(a^2-d^2)=80 \rightarrow (5)(25-d^2)=80$

$\rightarrow 125-5d^2=80 \rightarrow -5d^2=80-125 \rightarrow -5d^2=-45 \rightarrow d=3$

Now; a-d,a,a+d=15 ➔ 5-3,5,5+3 so that Largest number is 8 (Answer)

10) Middle term of three consecutive terms of A.P. is A.M. between the extreme terms.

-Sum of First n terms of an A.P:

The sum of first n terms of an A.P with first term and d is given by:

$S_n=n/2\ [2a+(n-1)d]$

For Example: If 2,4,6,8,10 are in A.P then find the Sum of these terms?

Solution:

Using the formula:

As n=5 , d=2 ,$a_1=2$

$S_5=5/2\ [2(2)+(5-1)(2)]$

$S_5=5/2\ [4+8]$

$S_5=30$ (Answer)

Note:

1) If Sn is the sum of first n terms of an A.P whose first term is a and last term is l or an then;

Sn= n/2 [a+l] Or Sn=n/2 [a+an]

2) If Common difference is d, number of terms are n and the last terms is l are given then; Sn=n/2 [2l- (n-1)d]

3) If $S_n=a_n+S_{n-1}$

-Important Note:

1) If nth term of a sequence is a linear expression in n then sequence is an A.P

For Example: If 1+3+5+.......+(2n-1) then find nature of progression?

Solution: As nth term is linear expression so that it is an A.P

2) If the sum of the first n term of a sequence is a quadratic expression in n then the sequence is an A.P.

For Example: The sum of n terms of series is n(5n-1), find the nature of the progression?

Solution:

As the sum of n term is quadratic expression so that it is an A.P

3) When the middle term of A.P is given and no. of total terms is an odd number then sum is given by

Sum =Middle term× Total terms

For Example:

If seventh term of A.P is 10 find the sum of first thirteen terms.

Solution:

Using the Formula:

Sum=10*13

Sum=130 (Answer)

4) When the no. of total terms is an even number and the middle terms are two then Sum is given by:

Sum = Total Terms * (middle terms/2)

For Example: If 23+24+25+26+27+28+29+30

Solution:

Here in this case Total terms are 8 and the middle terms are 26 and 27.

Sum= 8*(26+27/2)

Sum=8*(26.5)

Sum=212 (Answer)

Note: These both tricks (3,4) are only for sum of consecutive terms.

5) Shortcut process for finding the sum of an A.P:

Once we get the corresponding terms for any A.P, we cane easily find the sum of an A.P by using the property of averages.

Sum= Number of terms × A.M of that A.P

For Example:

For an A.P, 2,6,10,14,18,22. Sum=?

Solution:

As A.M = 22+2/2 ➔ A.M= 12

So that Sum = 6×12=72 (Answer)

6) When d=0 then S_n=a+a+a+……… n terms = na

For Example:- If 2+2+2+………..78 terms is an A.P then find the sum and common difference?

Solution: S_{78}=2+2+2+…….= 78(2)= 156 (Answer)

-Trick For Guessing the answer of the sum of A.P when it is given in n-form in the options:

For Quick Guessing, use the trick:

Firstly, Put n=1 in the given options and see the sum of 1st term.

Then, Put n=2 in the given options and see the sum of first 2 terms.

Then, Put n=3 in the given options and see the sum of first 3 terms.

For Example:

If 1,3,5,7,……..+(2n-1) is in A.P then sum of this A.P is:

A) n^2 (Correct) B) n(n+1) C) 2n+1 D) None of these

Solution:

As Correct Answer is Option So we use this trick on it. But when we are in Exams we check it for each option.

Using the trick:

Put n=1 in given options i.e

Option. A: $(1)^2$=1 and check the sum of 1st term that is 1

Put n=2 in given Option i.e

Option A: $(2)^2$=4 and check the sum of First 2 terms that is 4

and so on.

Note: We don't need to check it for more than first 2 values of n because if it's satisfied with the given option then that's our Correct Answer. (Checking for more than first 2 values (i.e n=1,2) is just waste of time).

-Important Note:

(This trick is also valid for sum of n-term of G.P)

- Note:

Sum of infinite series of an A.P does not exist.

-Properties of A.P:

1) If $a_1,a_2,a_3,\ldots\ldots a_n$ are in A.P then;

a) $a_1+k,a_2+k,a_3+k,\ldots\ldots\ldots a_n+k$ are in A.P

b) $a_1-k,a_2-k,a_3-k,\ldots\ldots\ldots\ldots a_n-k$ are in A.P

c) $a_1.k, a_2.k, a_3.k\ldots\ldots\ldots\ldots a_n.k$ are in A.P

d) $a_1/k, a_2/k,a_3/k,\ldots\ldots\ldots a_n/k$ are in A.P , $k\neq 0$

2) If $a_1,a_2,a_3,\ldots\ldots a_n$ and $b_1,b_2,b_3,\ldots\ldots b_n$ are in A.P then the basic mathematical Operations can be operated on it.

3) If $a_1,a_2,a_3,\ldots\ldots a_n$ are in A.P then;

a) $a_1+a_n=a_2+a_{n-1}=a_3+a_{n-2}$

Note: In an A.P. of finitely many terms, sum of terms equidistant from the beginning and end is constant equal to the sum of the first and last terms and so on.

For Example: If $a_1+a_5+a_{11}+a_{15}+a_{20}+a_{24}=225$ are in A.P then find the Sum of first 24 terms?

Solution: Using the property:

As, If $a_1+a_5+a_{11}+a_{15}+a_{20}+a_{24}=225$

So; $a_1+a_{24}=a_5+a_{20}=a_{11}+a_{15}$

Hence $a_1+a_{24}+a_1+a_{24}+a_1+a_{24}=225$

$3(a_1+a_{24})=225$

$a_1+a_{24}=75$

Then; we know that Sn=n/2 [a1+an]

So that S24=24/2 [75]

$S_{24}=12[75]$

$S_{24}= 900$ (Answer)

b) $a_n=(a_{r-k} + a_{r+k})/2$, $0\leq k\leq n-r$

For Example: 2,4,6,8,10,12 then find the third term?
$a_3=(a_{3-2}+a_{3+2})/2$
$a_3= a_1+a_5/2$
$a_3=2+10/2$
$a_3=6$ (Answer)
-Important Result:
1) If a^2,b^2,c^2 are in A.P then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.
2) If l,m,n be the pth,qth and rth terms of an A.P then
l(q-r)+m(r-p)+n(p-q)=0
and p(m-n)+q(n-l)+r(l-m)=0
-Inserting Single Arithmetic Mean (A.M):
A.M= $\frac{a+b}{2}$
For Example: If 2,10,b then find the value of b:
Solution:-
As we know that:
A.M=$\frac{a+b}{2}$
10=$\frac{2+b}{2}$
20=2+b
20-2=b
So b=18 (Answer)
-Inserting n-Arithmetic Means b/w two given numbers.
$A_m=a+m\left(\frac{b-a}{n+1}\right)$
Whereas;
m=no. of specific A.M
n=no. of A.M b/w a and b
➔ Find the problem related to it yourself ☺
-Note:
General Formula of nth A.M b/w a & b is also given by; $A_n=\left(\frac{a+nb}{n+1}\right)$
-Important Notes:
1) The sum of n arithmetic means b/w two given numbers is n times

the single A.M b/w a and b. i.e.

$A_1+A_2+......+A_n=n(A.M)$ i.e. Sum of n-A.M= n(a+b/2)

For example: If 2,A1,A2,10 then find the sum of Arithmetic mean b/w 2 and 10?

Solution:

As we know that:

A1+A2+.....An= n(a+b/2)

So that; A1+A2= 2(2+10/2)

A1+A2=2(6)

A1+A2=12 (Answer)

2) Sum of Alternating Series:

a) Sum of Series a-a+a-a+a... having even number of terms is equal to 0.

b) Sum of series a-a+a-a+a… having odd number of term is equal to a.

-Word Problems on A.P:-

-Entry Test MCQ:

A clock strikes twice when its hour hand is at one, four time when it is at 2 and on . How many times does the clock strike in 12 hours?

Solution:

Using the condition:-

When clock strikes 1 it is twice i.e 1*2=2

When clock strikes 2 it is four times to prior i.e 2*2=4

When clock strikes 3 it is six times to prior i.e 3*2=6

and so on…

From this condition, we conclude that it's an A.P with common difference=2

2+4+6+…..+24

Using the Sum of first n terms:

Sn=n/2 [a1+an]➔ S12= 12/2[2+24]➔ S12=6[26]➔ S12=156 (Answer)

Note: (Every Clock Related Problems are in A.P so we use the sum of n-terms formula in A.P.

2- Geometric Progression:

A sequence of non-zero numbers in which every term except the first one bears a constant ratio with its preceding term is called a geometric

progression.
The constant ratio is called the common ratio and it is denoted by r.
Note:
1) $r=\frac{a_n}{a_{n-1}}, \forall n \epsilon N, n > 1$
2) 0 cannot be common ratio of G.P
3) No term of G.P can be 0.
Important Results:
1) If a G.P has n-terms then nth term is called last term or limiting term and it is denoted by l or a_n that is given by $l=ar^{n-1}$
For Example: If 2,4,8,16,...a8 then find the a8?
Solution:
As we know that:
$a_n=ar^{n-1}$
As $a_8=ar^{8-1}$
$a8=ar^7$
Here r= 4/2➔ r=2
So that; $a_8=2(2)^7$
$a_8=2(128)$➔ a8=256 (Answer)
2) Three numbers a,b,c are in G.P if and only if b/a=c/b i.e $b^2=ac$
3) If a_m and a_n are two terms of an G.P whereas a_m is bigger term and a_n is smaller term. Also m is no. of bigger term and n is no. of smaller term. Then; $r=\left(\frac{a_m}{a_n}\right)^{\frac{1}{m-n}}$
For Example: If $a_2=9$ and $a_5=243$ then r=?
Solution:
Using the formula:
$r=\left(\frac{243}{9}\right)^{\frac{1}{5-2}}$
$r=(27)^{\frac{1}{3}}$
r=3 (Answer)

4) If 1/a,1/b,1/c are in G.P then common ratio is $r=\pm\sqrt{\frac{a}{c}}$

5) The nth term from the end of a G.P with last term l and common ratio is $\frac{l}{r^{n-1}}$.
6) If a is the first term and r is the common ratio of a finite G.P consisting of m terms then the nth term from the end is given by ar^{m-n}
For Example: If 2,4,8,16,32 then find the 4^{th} term from the last?
Solution:
Using the formula:
a4= (2).$(2)^{5-4}$
a4=2(2)➔a4=4 (Answer)
7) For total terms of G.P , the formula is given by $r^n=\left(\frac{a_n}{a_1}\right).r$
For Example: If 2,4,8,16,..,1024 is in G.P then find the total number of terms?
Solution:
As we know that;
$r^n=\left(\frac{a_n}{a_1}\right).r$
$2^n=\left(\frac{1024}{2}\right).2$
$2^n=\left(\frac{1024}{2}\right).2$
$2^n=(1024)$
$2^n=2^{10}$ (As the bases are same so that n=10)
n=10 (Answer)
8) Three numbers in G.P can be taken as a/r , a, ar, Four numbers in G.P can be taken as a/r^3, a/r,ar,ar^3. Five numbers in G.P can be taken as a/r^3,a/r,a,ar,ar^3.
For Example:
The sum of three numbers in G.P is 19 and their product is 216 then r=?
A) 1.5 (Correct) B) 0.5 C) 1.25 D) 2.25
Solution:
As we know that:
Three numbers in G.P can be taken as a/r , a, ar
So that; Sum➔ a/r+a+ar=19➔ $(a+ar+ar^2)/r=19$

and product ➔ a/r.a. ar=216➔ a^3=216➔a=6

so that (a+ar+ar^2)/r=19

Taking common; a(1+r+r^2)/r= 19

so 6(1+r+ r^2)=19r

6+6r+6r^2=19r

6r^2+6r-19r+6=0

6r^2-13r+6=0

6r^2-9r-4r+6=0

3r(2r-3)-2(2r-3)=0

3r-2=0 , 2r-3=0

r=2/3 , r=3/2

So the r=3/2 is the answer in this case.

9) Sum of Infinite Geometric Series when common ratio is 1/2 then Sum= 2×(First Term)

For Example: 1,½ ,1/4, 1/8,1/32……is in G.P, then find S_∞?

Solution:-

Using the formula:

Sum =2×1➔ Sum=2 (Answer)

For Example: $\sum_{n=1}^{\infty}(\frac{1}{2})^n$ =?

Solution:

Using the Formula:

Sum=2×1/2 =1 (Answer)

-Sum of first Terms of a G.P

The Sum of first n terms of a G.P with first term and common ratio r is given by:

$S_n = \frac{a(r^n-1)}{r-1}, |r| > 1$

Or $S_n = a + ar + ar^2 + \cdots \ldots\ldots + ar^{n-1}$

For Example: 2,4,……….a10 , Find the sum of first 10 terms of G.P?

Solution:-

a=2 , r=2 , n=10

$S_{10} = \frac{2(2^{10}-1)}{2-1}, |r| > 1$

$S_{10} = \frac{2(1023)}{1}$

$S_{10} = 2046$ (Answer)

Note:

1) $S_n = \frac{a(1-r^n)}{1-r}$, $|r| < 1$

2) $S_n = \frac{a(1-lr)}{1-r}, |r| < 1$

3) $S_n = \frac{lr-a}{r-1}, |r| > 1$

4) $S_n = \frac{a(r^n-1)}{r-1}$, $|r| > 1$

-How to solve the Sum of n-terms when Summation sign comes in the Question:

If $\sum_{n=1}^{n}(a^n)$

then Using the easy method, we can solve it within seconds.

Method:

Put first value as given below in the series n=1

then put second value n=2

and put third value n=3

now take the common ratio and then apply the Sum of n-term formula.

Note: (We start putting the value of n from where it is given, in this case we just suppose to begin with n=1)

For Example:

If $\sum_{n\to1}^{5}(\frac{1}{i})^n = ?$

Solution:

Using the easy method:

Put n=1; $(1/i)^1=1/i=-i$

Put n=2; $(1/i)^2=1/i^2=-1$

Put n=3; $(1/i)^3=1/i^3=i$

Put n=4; $(1/i)^4=1/i^4=1$

Put n=5; $(1/i)^5=1/i^5=i$

so that r=1/i

As it can be simplified by simple method here we don't need to use the formula:

S5= -i-1+i+1-i

S5= -I (Answer)

-Sum of Infinite Series of G.P:

The sum of an infinite G.P with first term a and common ratio r is:

$$S_\infty = \frac{a}{1-r}, |r| < 1 \ \ or -1 < r < 1$$

For Example: If 27,9,3,1,………∞ then find the sum?

Solution:-

As we know that;

$$S_\infty = \frac{a}{1-r}, |r| < 1$$

So that $S_\infty = \frac{1}{1-\frac{1}{3}}, |r| < 1,$

Now; $S_\infty = \frac{1}{\frac{2}{3}}, |r| < 1$

Hence; $S_\infty = \frac{3}{2} \ \ (Answer)$

And this series is convergent as |r|<1

-Note:

1) An Infinite geometric Series is convergent if;

$\lim_{x\to\infty} S_n$ exists i.e. |r|<1

(Example is solved as above)

2) An Infinite geometric Series is Divergent if |r|≥1

For Example: 1,3,9,27,………… find the nature of series?

Solution:-

As; r=3/1 ➔ r=3 and it is greater than 1 so that it's a divergent Series.

3) An Infinite geometric Series is Oscillatory if r=-1

For Example: If 2-2+2-2+……, find the nature of Series?

A) Oscillatory (correct) B) Divergent C) Fluctuating D) Convergent

Solution:-

As we know that r=-2/2 =-1 so it's an oscillatory series.

4) An Infinite geometric Series is fluctuating if r does not remain constant.

For Example:-

The series 1/3.4 + 1/4.5 +.... 1/(n+2)(n+3)+.....is

(a) Divergent (b) Convergent (c) Fluctuating (Correct) (d) N.O.T

Solution:-

As it is; 1/12+1/20+1/30 +1/42+……… +1/(n+2)(n+3) +…..

r= (1/20)/(1/12)=3/5

r= 1/30/1/20= 2/3

…. And so on.

From this, we conclude that r doesn't remains same, therefore it's a fluctuating series..

Note: S∞ does not exist if |r|≥1

-Ratio Test For Determining the nature of the Series:

1) If $Sn = u_1 + u_2 + u_3 + \ldots\ldots + u_n$ *converges as* $n \to \infty$ $\lim_{x\to\infty}\left|\frac{u_n+1}{u_n}\right|<1$

2) If $Sn = u_1 + u_2 + u_3 + \ldots\ldots + u_n$ *diverges as* $n \to \infty$ $\lim_{x\to\infty}\left|\frac{u_n+1}{u_n}\right|>1$

For Example: $\sum_{n=1}^{\infty}(\frac{-1}{5})^n$ Then find the nature of Series?

Solution:

Using the Exponent Rule:

$$:.\lim_{x\to\infty}\left|\frac{(\frac{-1}{5})^n+1}{(\frac{-1}{5})^n}\right|$$

$$:.\lim_{x\to\infty}\left|\frac{(\frac{-1}{5})^n+1}{(\frac{-1}{5})^n}\right|$$

$$:.\lim_{x\to\infty}\left|(\frac{-1}{5})^{n+1-1}\right|$$

$\therefore \lim_{x\to\infty} \left|\left(\frac{-1}{5}\right)^n\right|$

:.limx→∞|(-1/5)^n| as |-a|=a

So applying the limit it is (1/5) which is less than 1 hence it is convergent series.

Note: (This ratio test is valid when nth term is also given in the infinite geometric series)

-How to solve the Question when Summation sign comes in the Question sum of Infinite geometric Series.

If $\sum_{n=1}^{\infty}(a^n)$

Method:-

Firstly, Put n=1

secondly, put n=2

and then put n=3

and later on take the common ratio and use the Sum of infinite Geometric Series formula.

For Example: $\sum_{n=1}^{\infty}(-\frac{3}{7})^{2n}$ =?

Solution:-

Using the easy method as described earlier:-

$\therefore \sum_{n=1}^{\infty}(-\frac{3}{7})^{2n} = (-\frac{3}{7})^2 + (-\frac{3}{7})^4 + (-\frac{3}{7})^6 + \cdots$

As r=9/7

So that; S∞=$\frac{9/49}{1-9/49}$=9/40 (Answer)

Note: (We start putting the value of n from where it is given, in this case we just suppose to begin with n=1)

MCQ:-

Here is most frequently asked MCQ in the entry tests but this method sometimes be not in consideration.

If y=1+2x+4x^2+8x^3+……. Then find x=?

Solution:

Use the Sum of infinite series formula on Right Hand side:-

As r=2x

So; y= 1/1-2x

Now; y(1-2x)= 1

y-2xy=1

-2xy=1-y

Taking common Both sides.

-2xy= -1(-1+y)

2xy=y-1

x=y-1/2y (Answer)

-WORDS PROBLEMS ON G.P:

-Tricky MCQs:

An Object is dropped from a building having "h" vertically downward , after one drop it jumps to height $(\frac{a}{b})^{th}$ of height (initial height),find the distance before it comes to the rest?

Trick: Distance= $\frac{h(1+(\frac{a}{b})}{1-(\frac{a}{b})}$

For Example: What distance will a ball travel before coming to rest if it dropped from a height of 75m and after each fall it rebounds 2/5 th of the distance it fell

Solution:

Using the trick:

As a=2 , b=5 so that;

Distance= $\frac{75(1+(\frac{2}{5})}{1-(\frac{2}{5})}$

Distance=175m (Answer)

-Properties Of G.P:

1) If $a_1,a_2,$......are in G.P then;

a) a_1 k,a_2 k,a_3k ,...... are also in G.P

b) a_1/k,a_2/k,a_3/k,....... are also in G.P , k≠0

c) 1/a_1 ,1/a_2 ,1/a_3 ,..... are also in G.P

d) a_1^k, a_2^k,...... are also in G.P

2) If a_1,a_2,a_3,..... and b_1,b_2,b_3,....are in G.P then;

a) a_1b_1,a_2b2,...... are also in G.P.

b) a_1/b_1,a_2/b_2 ,.......also in G.P.

3) If $a_1, a_2, a_3, \ldots\ldots a_n$ then ;

a) $a_1a_n = a_2a_{n-1} = \ldots\ldots$

Note: In a G.P. of finitely many terms, the product of terms equidistant from the beginning and end
is constant equal to the sum of the first and last terms.

-Inserting Single Geometric Mean:

G.M= $\pm\sqrt{ab}$

For Example: If 3,G,27, then find G?

Solution:-

As we know that;

$G = \pm\sqrt{3.27}$

$G = \pm\sqrt{81}$

$G = \pm 9$ (Answer)

-Note: Here we take the +9 as the G.M as all the terms are positive

-Inserting n-Geometric Means b/w two given Numbers:

$\therefore G_m = a\left(\frac{b}{a}\right)^{\frac{m}{n+1}}$

Whereas n=no. of G.M b/w a and b and m=specific G.M which is to find.

➔ Find the problem related to it yourself. ☺

-Note:-

The general formula of nth G.M b/w a & b is also given by $G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

-Important Results:

1) The Product of n geometric means b/w two given numbers is nth power of the single G.M b/w them i.e. If a and b are two given numbers and $G_1, G_2, \ldots\ldots G_n$ are n geometric b/w them then

$G_1.G_2.\ldots\ldots G_n = (\sqrt{ab})^n$ or $a^n (b/a)^{n/2}$

For Example: If $2, G_1, G_2, G_3, 32$ then find the Product of G.M?

Solution:

As we know that:-

$G_1.G_2.G_3 = (\sqrt{2.32})^3$

$G_1.G_2.G_3 = (8)^3$

$G_1.G_2.G_3$ =512 (Answer)

2) If A and G are respectively arithmetic and geometric means b/w two positive numbers a and b then:

a) A>G

b) The quadratic equation having a and b as its roots is x^2-2Ax+G^2=0.

For Example: Find the quadratic equation in such the arithmetic mean of its roots is 4 and its geometric mean is 9?

Solution:-

Using the formula:

As x^2-2Ax+G^2=0

So x^2-2(4)x+$(9)^2$=0

x^2-8x+81=0 (Answer)

c) The two positive numbers are $A \pm \sqrt{A^2 - G^2}$

For Example: If arithmetic mean and geometric mean are 5 and 4 respectively then find two numbers on extreme of these means?

Solution:-

As we know that:

The two positive numbers are $5 \pm \sqrt{5^2 - 4^2}$

then; two positive numbers are 8 and 2 (Answer).

Entry Test MCQ

If 0.2+0.02+0.002+0.0002+........is in G.P Find the sum of Infinite Series?

Solution: First of all, simply it.

Take common 2 for all the terms:

2(0.1+0.01+0.001+0.0001+.......)

Now in the brackets find the common ratio i.e 0.01/0.1= 0.1

Now Again multiply 2 in the brackets and solve it for Sum of series.

As $S_\infty = \frac{a_1}{1-r}$

So; $S_\infty = \frac{0.2}{1-0.1} = \frac{0.2}{0.9} = \frac{2}{9}$ (Answer)

-Arithmetic-o-Geometric Series (A.G.P.)

This is combination of Arithmetic and Geometric Series.

If P_1, P_2, P_3 ... be an A.P. and a_1, a_2, a_3 ... be a G.P. then p_1q_1, p_2q_2, p_3q_3, ... is said to be an arithmetic-o-geometric progression. A general A.G.P. is $a,(a+d)r,(a+2d)r^2,(a+3d)r^3,...$

-Nth term of an A.G.P:

The nth term of an A.G.P is given by:-

$T_n=\{a+(n-1)d\}.r^{n-1}$

Note:- A sequence is both an A.P. and a G.P. iff it is a constant sequence.

3) Harmonic Progression (H.P):

A sequence of numbers reciprocal of whose terms form an A.P. is called harmonic Progression.

For Example: ¼,1/7,1/10 .... Is a H.P since 4,7,10,....is an A.P of nth term= 1/nth term of the corresponding A.P = $1/a_1+(n-1)d$

-Note:

1) No term of H.P can be zero.

2) Three number a, b, c are in H.P if and only if b=2ac/a+c .

3) There is no general formula of finding the sum of n terms of H.P.

-Inserting Single Harmonic Mean:-

H.M= $\frac{2ab}{a+b}$

For Example: If 2, H, 10 then find H.M?

Solution:

Using the formula:

H.M= $\frac{2(2)(10)}{2+10}$

H.M= $\frac{40}{12}$

H.M= $\frac{10}{3}$ (Answer)

-Inserting n-Harmonic Mean b/w a and b:

$H_n=\frac{ab(n+1)}{na+b}$

For Example: 2, H_1,H_2, 10 then find H.M?

Solution:-

Using the formula;

$H_n = \frac{20(2+1)}{2(2)+10}$

$H_n = \frac{60}{14}$

$H_n = \frac{30}{7}$ (Answer)

-Relation b/w A.M, G.M and H.M:

If A,G and H be the harmonic ,geometric and harmonic means b/w a and b , then;

1) G^2=A.H

For Example: If G and A are 6 and 4 respectively then find the value of H?

Solution:

Using the formula:

G^2=A.H

$(6)^2$=4.H

36/4=H➔ H=9 (Answer)

2) For a and b distinct positive real numbers.

A≥G≥H, where G>0

3) For a and b distinct negative real numbers.

A≤G≤H , where G<0

4) A,G,H are in G.P

-Important Result:

1) Reciprocal of the term of G.P forms a G.P

2) Reciprocal of the term of A.P may or may not form a H.P

3) Reciprocal of the term of H.P forms a A.P

4) If a,b,c are in G.P then x,y,z are in H.P

5) If a,b,c are in G.P then 1/x,1/y,1/z are in A.P

6) If b=c then a^2,b^2,c^2 are in G.P.

7) The number $\frac{1}{a}, \frac{a+b}{2ab}, \frac{1}{b}$ are in G.P.

8) The reciprocal of whose terms form again same type of sequence is H.P.

9) If a,b,c are in A.P then 1/a,1/b,1/c are in H.P.

10) If the first term of an infinite geometric series is equal to twice of

the sum of all the terms of that follows it, then the value of r is 1/3

11) If a, b, c form a G.P with common ratio r (0<r<1).If a,2b,3c form an A.P then r= 1/3

12) Three numbers of G.P. If we double the middle number we get an A.P the common ratio of G.P is $2\pm\sqrt{3}$.

13) Non zero terms are in A.P, G.P and H.P.

14) If a,b,c is in A.P then b is A.M.

15) If a,b,c is in G.P then b is G.M.

16) If a,b,c is in H.P then b is H.M.

17) Every term of G.P is the logarithm of each term of A.P.

18) Every term of A.P is the anti logarithm of term of G.P.

19) If a,b,c are in H.P then bc, ca, ab are in A.P.

20)If a,b,c are in A.P then 1/1-a , 1/1-b , 1/1-c are in H.P.

-Some Important Points:

If the expression $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is:

1) A.M when n=1

2) G.M when n=1/2

3) H.M when n=0

If the expression $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is:

1) H.M when n=-1

2) A.M when n=0

3) G.M when n=-1/2

-Important Results:

If α and β be roots of the quadratic equation $x^2-2Ax+G^2=0$ then;

1) A.M= $\frac{\alpha+\beta}{2}$

2) G.M= $\sqrt{\alpha\beta}$

3) H.M= $\frac{2\alpha\beta}{\alpha+\beta}$

For Example: Form an quadratic equation if A.M is p and G.M is q and also find the H.M?

Solution:

As we know that:-

A.M= $\frac{\alpha+\beta}{2}$

$\alpha+\beta=2p$----- (1)

As we also know that:-

G.M=$\sqrt{\alpha\beta}$

$\alpha\beta=q^2$------(2)

As x^2-Sx+P=0

So that $x^2-2px+q^2=0$

For H.M, we know that:

H.M=$\frac{2\alpha\beta}{\alpha+\beta}$

From (1) and (2)

H.M=$\frac{2(q^2)}{2p}$

H.M=$\frac{q^2}{p}$ (Answer)

Note:

Let a and b are two numbers and if a=b then;

G.M=H.M=A.M

-Some Derived Results:

When Quadratic equation is $ax^2+bx+c=0$ then;

1) A.M=$\frac{-b}{2a}$

2) H.M=$\frac{-2c}{b}$

3) G.M=$\sqrt{\frac{c}{a}}$

For Example: If $x^2-10x+5=0$ then find H.M ,A.M and G.M?

Solution:-

As we know that:

A.M=$\frac{-b}{2a}$

A.M=$\frac{-(-10)}{2(1)}$ ➔ A.M=5 (Answer)

As we also know that:-

H.M=$\frac{-2(5)}{(-10)}$ ➔ H.M=1 (Answer)

As we also know that:-

G.M=$\sqrt{\frac{c}{a}}$

G.M=$\sqrt{\frac{5}{1}}$

G.M=$\sqrt{5}$ (Answer)

-Sum of Some Special Sequences:-

1) $\sum_{k=1}^{n}[k^m-(k-1)^m]=n^m$

2) $\sum_{k=1}^{n}1=n$

3) The Sum of first n natural number is given by:-

∴$\sum_{k=1}^{n}k=1+2+3+\cdots..+n=\frac{n(n+1)}{2}$

4) The sum of first n even natural number is given by n(n+1)

5) The sum of first n odd natural number is given by n^2.

6) The sum of squares of first n natural numbers is given by:-

∴$\sum_{k=1}^{n}k^2=1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$

7) The sum of squares of first n odd natural numbers is given by $\frac{n(4n^2-1)}{3}$

8) The sum of squares of first n even natural number is given by $\frac{2n(n+1)(2n+1)}{3}$

9) The sum of cube of first n natural number is given by

∴$\sum_{k=1}^{n}k^3=1^3+2^3+3^3+\cdots+n^3=\left[\frac{n(n+1)}{2}\right]^2$

10) The sum of cube of first n even natural number is given by $2n^2(n+1)^2$

11) The sum of cube of first n odd natural number is given by $n^2(2n^2-1)$

12) 1+5+9+…….+(4n-3)=n(2n-1)

13) 1+4+7+…..+(3n-2)=$\frac{n(3n-1)}{2}$

14) 2+6+18+…..+2×3^{n-1}=3^n-1

15) 1+2+4+…….+$2^{n-1}$$2^n$-1

16) 1×3+2×5+3×7+…….+n×(2n+1)=$\frac{n(n+1\_(4n+5)}{6}$

17) $1\times2+2\times3+\ldots\ldots\ldots\ldots+n\times(n+1)=\frac{n(n+1)(n+2)}{3}$

18) $1\times2+3\times4+\ldots\ldots\ldots\ldots+(2n-1)(2n)=\frac{n(n+1)(4n+1)}{3}$

19) $1^2-2^2+3^2-4^2+\ldots\ldots\ldots\ldots+(-1)^{n-1}(n)^2=\frac{(-1)^{n-1}.n(n+1)}{2}$

20) $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots+\frac{1}{n(n+1)}=1-\frac{1}{n+1}$

(Find yourself the examples of these points and apply on it)

Note: These formulas are only valid for A.P.

Note:

1) nth term of b+bb+bbb+…(form) is

$a_n=\frac{b}{9}(10^n-1)$,whereas b∈{1,2,…..9}

For Example:

Find third 5th term of 3+33 ?

Solution:

Using the trick:

$a_3=\frac{3}{9}(10^3-1)$

$a_3=\frac{1}{3}(1000-1)$,

$a_3=\frac{1}{3}(999)$=333 (Answer)

2) nth term of 0.b+0.bb+0.bbb+… (form) is

$a_n=\frac{b}{9}(1-10^n)$,whereas b∈{1,2,…..9}

For Example: Find the 4th term of 0.2+0.22+..?

Solution:

Using the trick:

$a_4=\frac{2}{9}(1-10^{-4})$

$a_4=\frac{2}{9}\left(1-\frac{1}{10000}\right)$

$a_4=\frac{2}{9}\left(\frac{9999}{10000}\right)$

a_4=0.2222 (Answer)

-Rules to solve decimal fraction (forming a common/vulgar Fraction)

Here are the key points of working out Vulgar fraction within seconds.

The sign of repeating is dot over the digit.

Case-I (When all Digits in Decimal part are repeating)

In this case , in the denominator of vulgar Fraction , the number of nines is equal to number of repeating digits and numerator is actually the complete given number without decimal minus the number before decimal.

Examples:-

1) **Vulgar Fraction** of $2.\dot{3}\dot{4}\dot{2}$ will be **: 2342-2/999 = 2340/999.**
2) $13.\dot{4}\dot{2}\dot{3}\dot{5}$ will be : **134235-13/9999 = 134222/9999**.
3) $0.\dot{2}\dot{7}\dot{1}$ will **be 0271-0/9999** = **271/999.**

Case-II (When all digits are not repeating)

In this case, in the denominator of vulgar fraction, the number of nines is equal to the number of repeating digits and after nines we put zeros and the number of zeros is equal to the number of non-repeating digits in decimal part. The numerator is the whole given number without decimal minus the number before repeating digits.

Examples:

1) **Vulgar fraction of** $2.1\dot{3}\dot{4}\dot{1}$ **will be 21341-21/9990 ➔21320/9990 (Answer)**
2) **Vulgar fraction of 0.02**$\dot{1}$ **will be 0021-002/900➔ 19/900 (Answer)**

CHAPTER#7 (COMBINATION, PERMUTATION AND PROBABILITY

SUMMARY OF PERMUTATIONS & COMBINATIONS

Formulas:
No. of ways of selecting r objects out of n objects $= {}^nC_r$
No. of ways of arranging n objects $= n!$
No. of ways of arranging n objects with a identical & b identical $= \dfrac{n!}{a!\,b!}$
No. of ways of arranging r objects out of n objects $= n(n-1)(n-2)\ldots(n-(r-1)) = {}^nP_r = {}^nC_r\,r!$
No. of ways of arranging n objects in a circle $= (n-1)!$

Choosing People
A team of 4 is to be chosen from a group consisting of Anne and Bob and 4 other people. In how many ways can this be done if
(i) there are no restrictions?
(ii) Anne must be in the team?
(iii) Anne and Bob must both be in the team?
(iv) at most one of Anne and Bob in the team?
(v) Anne or Bob or both are in the team?

(i) No. of ways of choosing 4 people $= {}^6C_4 = 15$
(ii) No. of ways of choosing the other 3 people $= {}^5C_3 = 10$
(iii) No. of ways of choosing the other 2 people $= {}^4C_2 = 6$
(iv) Total no. of ways – no. of ways with Anne & Bob both in the team $= 15 - 6 = 9$
(v) No. of ways with Anne in the team + no. of ways with Bob in the team – no. of ways with both in the team $= 10 + 10 - 6 = 14$

Choosing from Different Types of People (e.g. Boys & Girls)
A team of 3 is to be chosen from a group of 3 boys and 4 girls. How many ways can this be done if
(i) there are no restrictions?
(ii) there must be exactly 1 boy?
(iii) there must be at least 1 boy?
(iv) there must be at least 1 boy and at least 1 girl?

(i) No. of ways of choosing 4 people $= {}^7C_3 = 35$
(ii) No. of ways of choosing 1 boy and 2 girls $= {}^3C_1\,{}^4C_2 = 18$
(iii) No. of ways = Total no. of ways – no. of ways with no boys $= 35 - {}^4C_3 = 35 - 4 = 31$
Note: It is wrong to say no. of ways $= {}^3C_1\,{}^6C_2 = 45$
(iv) No. of ways = Total no. of ways – no. of ways with no boys – no. of ways with no girls
$= 35 - {}^4C_3 - {}^3C_3 = 35 - 4 - 1 = 30$
Note: It is wrong to say no. of ways $= {}^3C_1\,{}^4C_1\,{}^5C_1 = 60$

Choosing People to form Groups
Find the no. of ways in which 6 people can be divided into
(i) two groups consisting of 4 & 2 people.
(ii) two groups consisting of 3 people each.
(iii) group 1 and group 2, with 3 people in each group.

(i) No. of ways $= {}^6C_4\,{}^2C_2 = 15$
(ii) No. of ways $= \dfrac{{}^6C_3\,{}^3C_3}{2!} = 10$ **Note:** Divide by 2! since the 2 groups are of equal size.
(iii) No. of ways $= {}^6C_3\,{}^3C_3 = 20$ **Note:** Don't divide by 2! since the 2 groups are labeled.

Choosing Letters, Balls or other Identical Objects
Seven cards each bear a single letter, which together spells the word "MINIMUM". Three cards are to be selected. The order of selection is disregarded. Find the number of different selections.

Case 1: No. of ways of choosing 3 identical letters = no. of ways of choosing "MMM" = 1
Case 2: No. of ways of choosing "MM" + 1 other letter = no. of ways of choosing I, N, U = ${}^3C_1 = 3$
Case 3: No. of ways of choosing "I I" + 1 other letter = no. of ways of choosing M, N, U = ${}^3C_1 = 3$
Case 4: No. of ways of choosing 3 different letters = no. of ways of choosing M, I, N, U = ${}^4C_3 = 4$
$\therefore$ total no. of selections = 1 + 3 + 3 + 4 = 11

Arranging People
Anne and Bob and 2 other people are to sit in a row. How many ways can this be done if
(i) there are no restrictions?
(ii) Anne must sit on the left and Bob on the right?
(iii) Anne and Bob must sit together?
(iv) Anne and Bob must be separate?

(i) No. of ways of arranging 4 people = 4! = 24
(ii) No. of ways of arranging the other 2 people = 2! = 2
(iii) Treat Anne and Bob as 1 item.
No. of ways of arranging 3 items × no. of ways of arranging Anne & Bob = 3! 2! = 12
(iv) No. of ways = 24 – 12 = 12

Arranging Different Types of People (e.g. Boys & Girls)
In how many ways can 3 boys & 3 girls be arranged in a row if
(i) there are no restrictions?
(ii) the 1st person on the left is a boy?
(iii) the person on each end is a boy?
(iv) the boys are together?
(v) the boys are separate?

(i) No. of ways of arranging 6 people = 6! = 720
(ii) No. of arrangements = ${}^3C_1 \times$ no. of ways of arranging the other 5 people = ${}^3C_1\ 5! = 360$
(iii) No. of arrangements = ${}^3C_2 \times 2! \times$ no. of ways of arranging the other 4 people
$= {}^3C_2\ 2!\ 4! = 144$
(iv) Treat the 3 boys as 1 item.
No. of ways of arranging 4 items = 4!
The boys can be arranged among themselves in 3! ways
$\therefore$ no. of arrangements = 4! 3! = 144
(v) $\uparrow \quad G_1 \quad \uparrow \quad G_2 \quad \uparrow \quad G_3 \quad \uparrow$
The 3 girls can be arranged in 3! ways.
From the 4 spaces, choose 3 places for the boys in ${}^4C_3$ ways, and arrange them in 3! ways.
$\therefore$ no. of arrangements = $3!\ {}^4C_3\ 3! = 144$
Note: It is wrong to subtract no. of ways where boys are together from the total no. of ways.

Arranging Letters, Balls or other Identical Objects
(a) Find the number of arrangements of all 7 letters of the word "MINIMUM" in which
(i) there are no restrictions.
(ii) the 3 letters M are next to each other
(iii) the 3 letters M are separate

(iv) the first letter is M
(v) the first & last letters are M
(vi) the first letter is M or the last letter is M or both

(b) Find the number of 4–letter code–words that can be made from the letters of the word "MINIMUM".

(ai) No. of arrangements $= \dfrac{7!}{3!\,2!} = 420$

(ii) Treat "MMM" as 1 item.

No. of arrangements of "MMM", I, N, I, U $= \dfrac{5!}{2!} = 60$

(iii) $\quad \uparrow \; \text{I} \; \uparrow \; \text{N} \; \uparrow \; \text{I} \; \uparrow \; \text{U} \; \uparrow$

The letters I, N, I, U can be arranged in $\dfrac{4!}{2!}$ ways

From the 5 spaces, choose 3 places for the 3 M's in ${}^5C_3$ ways.

$\therefore$ no. of arrangements $= \dfrac{4!}{2!}\,{}^5C_3 = 120$

(iv) M _ _ _ _ _ _

No. of ways of arranging I, N, I, M, U, M $= \dfrac{6!}{2!\,2!} = 180$

(v) M _ _ _ _ _ M

No. of ways of arranging I, N, I, M, U $= \dfrac{5!}{2!} = 60$

(vi) No. of arrangements = 180 + 180 – 60 = 300

(b) Case 1: M, M, M & 1 other letter: No. of words $= {}^3C_1\,\dfrac{4!}{3!} = 12$

Case 2: M, M, I, I: No. of words $= \dfrac{4!}{2!\,2!} = 6$

Case 3: M, M & 2 different letters: No. of words $= {}^3C_2\,\dfrac{4!}{2!} = 36$

Case 4: I, I & 2 different letters: No. of words $= {}^3C_2\,\dfrac{4!}{2!} = 36$

Case 5: 4 different letters: No. of words = 4! = 24

Total no. of code–words = 12 + 6 + 36 + 36 + 24 = 114

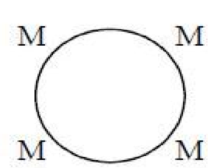
Arranging People Around a Table

4 men and 3 women are to sit at a round table. Find the number of ways of arranging them if
(i) there are no restrictions.
(ii) the 3 women must sit together.
(iii) the 3 women must be separate.
(iv) the 3 women must be separate and the seats are numbered 1 to 7.

(i) No. of ways of arranging 7 people around a table = (7 – 1)! = 720

(ii) Treat the 3 women as 1 item.
No. of ways of arranging 5 items around a table = (5 – 1)!
The 3 women can be arranged among themselves in 3! ways
$\therefore$ no. of arrangements = (5 – 1)! 3! = 144

(iii) Let the 4 men sit down first in (4 – 1)! ways.
From the 4 spaces, choose 3 places for the women in ${}^4C_3$ ways.
Arrange the 3 women in 3! ways.
$\therefore$ total no. of arrangements $= (4 – 1)!\,{}^4C_3\,3! = 144$



(iv) No. of arrangements = 144 × 7 = 1008

Let have a start up with Permutation and some basics

Factorial Notation:

Let n be a positive integer.

Factorial is represented by n! or Ln...

Important Points..:

n is only positive integer.

Factorial of fraction and negative integer can't be defined

n!=n(n-1)!

Note:

(a+b)!≠a!+b!

(a-b)! ≠a!-b!

(ab)!≠a!. b!

(a/b)!≠a!/b!

n!=n(n-1).(n-2).(n-3)!.

Some important factorial notations:

1) n!=n(n-1)!
2) 0!=1
3) 1!=1
4) 2!=2
5) 3!=6
6) 4!=24
7) 5!=120
8) 6!=720
9) 7!=5040
10) 8!=40320

-Fundamental Principal of Counting:

Multiplication Principle:

If an operation can be performed in m different ways following which a

second operation can be performed in n different ways then the two operations in succession can be performed in mn different ways,
The word "And" in statement represents that we have to multiply in that case.

For example:

In how many ways can select 2 persons of different gender of 6 man and 5 woman.

Solution:

=6C1*5C1

=6*5=30

Addition Principle:

If an operation can be performed in m ways and another operation which is independent of the first operation can be performed in n different ways . Then either of the two operations can be performed in (m+n) ways.
The word "either/or" represents that there is an addition rule which is to be operated.

For Example:

In how many ways can I select 3 persons of same gender out of 6 men and 5 women.

Note:

Here the question is raised that why I didn't use the word "either". The reason is quite simple there are two genders either Woman or man. The meanings of the same gender will emphasize me to operate here the "Addition Rule" While in the case of different genders we have to operate the multiplication rule.
So let us explain this example with solution.

Solution:

$^{6}C_3 + {}^{5}C_3 = 30$

Note:

The above two mentioned principles can be extended for any finite number of operations.

-Permutation:

Each of the different arrangements which can be made by taking the some or all the given numbers of things or objects at a time is called a Permutation:

Note#1:

Permutation of objects actually means arrangements of objects. The word "Arrangements" is used if order of objects is taken into account. Thus if the order of different Objects changes then their arrangements also changes.

Note#2:

Permutation means arrangements , standing or sitting in a row or in a circle Problems regarding digits , letters (A,B,C.....etc) Formation of words, numbers etc.

The General formula for the permutation is given by nPr =n!/(n-r)!

Whereas:

n=total number of objects

r=Specific selected Objects

Note#:

$^{n}P_n = n!$

$^{n}P_0 = n!$

$^{n}P_r = {}^{n}P_{r-1}$

$^{n}P_r / {}^{n}P_{r-}1 = n-r+1$

-Most Important Formulas to be known:

1) Numbers of arrangements of n different objects takes r at a time is

given by:
$^nP_r=n!/(n-r)!$
For example:
How many 4 digits numbers can be made with 0,1,2,3,4,5?
Solution:-
Using suitable formula according to the Condition:
$^nP_r=n!/(n-r)!$
As n=6, r=4
so 6P4= 6!/(6-4)!= 6.5.4.3.2!/2! =6.5.4.3=360 Numbers can be formed..
2) Numbers of Arrangements of n different objects taken all at time is given by $^nP_n=n!$ as r=n
For Example:
How many 5 digits numbers can be formed with 1,2,3,4,5?
Solution:
Using the formula:
$^nP_n=n!$
As n=5
and r=5
So n!=5!=120 (5 digits numbers can be formed).
3) The number of permutations of n objects taken all at a time out of which p are alike and are of one type , q are are alike and are of second type and rest are all different is given by n!/p!.q!
For example:
How many words can be made with letters of word "PAKISTAN"
Solution:-
Using the formula:
n=7
p=2 (As (A)s are alike in the Pakistan)

so 7!/2!= 720 words can be formed with different arrangements.

4) The numbers of permutation of n different objects taken r at a time when each may be repeated any number of times (i.e Repetition is allowed) given by n^r.

For Example:

How many 4 digits numbers can be formed with 0,1,2,3,4,5 where as repetition of digit is allowed.

Solution:-

Using the formula:

n=6

r=4

so that $(6)^2$=1296 (Answer).

5) Number of permutation of n different objects taken r at a time when a particular thing is to be always included in each arrangement is given by r.($^{n-1}P_{r-1}$)

-For Example:

How many numbers can be formed when 0,1,3 is always included in the particular arrangement when the digits are 0,1,2,3,4,5?

Solution:

Using the formula:

As n=6

r=3 (i.e 0,1,3)

so 3. $^{6-1}P_{3-1}$= 3* $^5P_2$=3* 5!/3! =3*5!/6=60 numbers can be formed in which specific digits are always included.

6) Number of the permutations of n different objects taken r at a time when s particular things are to be always included in each arrangement is given by s!(r-(s-1). $^{n-s}P_{r-s}$

For Example:

How many 3 digits numbers can be formed 0,1,2,3,4 when 2 and 3 is always included in each arrangement.

Solution:

Using the formula:

n=5

r=3

s=2

so that:

$=2!(3-(2-1)).\,^{5-2}P_{3-2}$

$=2.(3-1).\,^{3}P_{1}$

=2(2). 3!/2!

=4.3

=12 (Answer)

7) Number of permutations of n different objects taken r at a time when a particular objects can never be taken in each arrangements is given by $^{n-1}P_r$

For Example:

How many numbers can formed when 0,2 can never be included in each number 0,1,2,3?

Solution:

Using the formula:

n=4

r=2

so that:

$= \,^{4-1}P_2$= 3P2=3!/1!= 6/1=6 (Answer)

8) Numbers of permutations of n different objects taken all at a time when m specified objects always put together is m!(n-m+1)!

For Example:

In a party, management invites 7 sophisticated persons for business meeting. In how many ways can be seated if 3 of them are in favor to sit together?

Solution:

Using the formula:

n=7

m=3

So that

=3!(7-3+1)!

=3!(5)!

=6(120)

=720 (Answer)

9) Numbers of permutations of n different objects taken all at a time when m specified objects can never put together is n!-m!(n-m+1)!

For Example:

In a party, management invites 5 sophisticated persons for business meeting. In how many ways can be seated if 2 of them are not in favor to sit together?

Solution:

Using the formula:

n=5

m=2

So that

=5!-2!(5-3+1)!

=120-2(6)

=120-12

=108 (Answer)

-Circular Permutation:

10) Number of circular permutations when of n different objects is (n-1)! . e.g arrangements of living things around a circular table.

Note:- (This formula is restricted for living things only who are arranged in round way)

For Example:

In a round table, there are six members are seated so that how many circular arrangements can be made in this condition?

Solution:

As we know that:

n=6

So that:

=(6-1)!

=5!=120 (Answer)

11) Number of circular permutations of n different objects when clockwise and counter/anti- clockwise arrangements are not different i.e When observations can be made from both sides is given by:

1/2 (n-1)! (e.g the arrangements of beads in the necklace)

Note: (This formula is restricted for arrangements of non-living things into the circular way)

-For Example:

In how many arrangements, 7 beads can be arranged in the circular way?

Solution:

As we know that:

n=7

So that:

=1/2 (7-1)!

=1/2 (6)!

=1/2(720)

=360 (Answer)

12) Number of circular permutations of n different objects taken r at a time when clockwise and anti-clockwise orders are taken different is given by $^nP_r/r$.

For Example:

In how many ways can we arrangements 5 persons on the round table when 2 particular persons are taken at a time (circular order might be different)?

Solution:

As we know that:

n=5

r=2

So that:

$={}^5P_2/2$

=(5!/3!)/2

=5.4/2

=20/2

=10 (Answer)

13) Number of circular permutations of n different objects taken r at a time when clockwise and anti-clockwise orders are not taken different is given by $^nP_r/2r$.

For Example:

In how many ways can we arrangements 4 persons on the round table when 2 particular persons are taken at a time (circular order might not be different)?

Solution:

As we know that:

$n=4$

$r=2$

So that:

$={}^{4}P_2/2(2)$

$=(4!/2!)/4$

$=(4.3.2!/2!)/4$

$=12/4$

$=3$ (Answer)

-Combination:

Each of different groups or selection which can be made by taking all or a number of objects (irrespective of order) is called a combination.

Note#1:

Combination of objects means selection of the objects, Obviously in selection of objects order of the objects has no importance. Thus with the change of order of objects selection of objects does not change.

Note#2:

Combination means selections, choices, draws etc. Distribution, formation of groups committee, team etc. Problems regarding the geometry.

The general formula of the combination is given by:

${}^{n}C_r=n!/r!(n-r)!$

Some Important Results:

1) ${}^{n}C_r={}^{n}C_{n-r}$ (Complimentary Combination)

2) ${}^{n}C_0={}^{n}C_n=1$

3 ${}^{n}C_1={}^{n}C_{n-1}=n$

4) If ${}^{n}C_x={}^{n}C_y$ then either $x=y$ or $x+y=n$

5) ${}^{n}C_r+{}^{n}C_{r+1}={}^{n+1}C_{r+1}$

6) ${}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$

7) If 0<r<n, n, rϵN, then r. ^nC_r=n.$^{n-1}C_{r-1}$
8) ^nP_r=r!.^nC_r
9)$^nC_r/^nC_{r-1} = \frac{n-r+1}{r}$
10) $^{n-1}C_r+^{n-1}C_{r-1}=^nC_r$
-Some Important Formula to remember:
1) Number of combination of n different objects taken r at a time is given by $^nC_r=\frac{n!}{r!(n-r)!}$
For Example:
How many groups can be made if there 5 persons in hall and 2 are taken at time in the group in the group?
Solution:-
As we know that:-
n=5 , r=2
so that; Number of groups formed= $^nC_r=^5C_2$=10 (Answer)
2) Number of combinations of n different objects taken r at a time when p particular objects are always included is given by $^{n-p}C_{r-p}$.
Whereas
n=no. of total objects
r=no. of selected objects
p=no. of particular objects
For Example:
How many groups are formed if there are 2 boys and 3 girls in the school hall , each group contain 2 students in it and 1 boy is always present in each group?
Solution:
As we know that:
n=2+3=5
r=2

p=1

So that:

No. of groups formed = ${}^{5-1}C_{2-1}= {}^{4}C_{1}.4$ (Answer)

3) Number of combinations of n different objects taken r at a time when p particular objects are never included in the selection is given by ${}^{n-p}C_{r}$

For Example:

How many groups can be made if there are 2 boys and 2 girls in a quiz contest if each group contains 2 members in it and 1 girl can never present in each group?

Solution:-

As we know that:

n=2+2=4

r=2 , p=1

Numbers of groups formed= ${}^{4-1}C_{2}= {}^{3}C_{2}=3$ (Answer)

4) Number of combinations of n different objects taken r at a time when p particular are not together in any selection is given by ${}^{n}C_{r}-{}^{n-p}C_{r-p}$.

For Example:

How many groups can be formed if there are 2 boys and 3 girls in a hall and each group must contain 2 members in it and 1 girl and 1 boy can never be together in each group?

Solution:-

As we know that:-

n=2+3=5

r=2 , p=2

Number of group formed= ${}^{5}C_{2}-{}^{5-2}C_{2-2}= {}^{5}C_{2}-{}^{3}C_{0}=10-1=9$ (Answer)

5) Number of selections of r consecutive objects out of n objects in a

row is given by n-r+1

For Example:

How many selections can be made if there are 7 boys are standing in a row whereas 3 boys are stood together.

Solution:-

No. of selection= 7-3+1=5 (Answer)

6) Number of selections of r consecutive objects out of n objects along a circle is given:-

1) n when r<n

2) 1 when r=n

Example#1:

How many selections can be made if there are 3 consecutive boys out of 5 boys standing in circular way.

Solution:-

As we know that:

r=3 , n=5 (r<n)

So we use here the condition:

No. of selections =n➔ 5 (Answer)

Example#2:

How many selections can be made if there 5 consecutive boys out of 5 boys standing in a circular way.

Solution:-

As we know that r=n

So No. of selections= 1 (Answer)

7) Number of selections of n different objects taken r at a time and repetition of the objects is allowed is given by $^{n+r-1}C_r$.

For Example:

How many selections of 3 digits numbers can be made when 0,1,2,3,4, and repetition is allowed?

Solution:

As we know that:

n=5 , r=3

So that:

No. of Selections= $^{n+r-1}C_r \rightarrow {}^{5+3-1}C_3 \rightarrow {}^{7}C_3 \rightarrow 35$ (Answer)

8) Number of selections of zero or more objects out of n different objects is given by 2^n

For Example:

How many groups of both genders can be made up of 5 member?

Solution:-

No. of selection= $2^n=2^5=32$ (Answer)

9) Number of selections of n different objects , selecting at least one of then is given by 2^n-1.

For Example:

How many selections of coin sides can be made if the coin is tossed 6 times and selecting head at least of them?

Solution:-

As n=6

So, Number of selections made= $2^6-1=63$ (Answer)

10) Sum of all even and odd binomial coefficients is given by 2^{n-1}

For Example:

Find the sum of even binomial coefficient in the expansion of $(ax+by)^8$?

Solution:-

Sum of even binomial Coefficient=$2^{8-1}=128$(Answer)

11) Sum of all binomial coefficient is given by 2^n.

For Example: Find the sum of all binomial coefficient in $(3x+2y)^3$?

Solution:-

Sum of all binomial Coefficient=2^3=8 (Answer)

12) Number of selections of r objects (r≤n) out of n identical objects is 1 (Find the example of this point yourself)

13) Number of selections of one of more objects out of n identical objects is n.

For Example:

How many selections of 6 balls in 10 identical color balls?

Solution:-

As n=10, So No. of Selections=10 (Answer)

14) Number of selections of zero or more objects out of n identical objects is n+1.

15) Number of ways of dividing m+n different objects in two groups containing m and n groups respectively is given by $\frac{(m+n)!}{m!.n!}$

For Example:

In how many ways can divide 1 boy and 2 girls into 2 group?

Solution:-

Using the formula:

Number of groups=$\frac{(1+2)!}{1!.2!}$=3 (Answer)

16) Number of ways of dividing m+n+p different objects in three groups containing m,n and p objects respectively is given by $\frac{(m+n+p)!}{m!.n!.p!}$,m≠n≠p

For Example:

In how many ways can we divide 3 women,1 child and 2 men in three group?

Solution:-

Using the formula:

Number of groups=$\frac{(3+1+2)!}{3!.1!.2!}$=$\frac{(6)!}{3!.1!.2!}$=60 (Answer)

Some most Useful Results:

1) If n distinct points are given in the plane such that no three of which are collinear are then number of straight line segments formed=$^nC_2$

For Example:

How many lines can be drawn with 7 points?

Solution:- As n=7

So Number of line segments= $^7C_2$=21 (Answer)

2) If m of these points are collinear in n different points (m≥3)then the number of line segments is given by $^nC_2$-$^mC_2$+1

For Example:

How many line segments are drawn with 5 points when 3 out of them are collinear?

Solution:-

As n=5, m=3, So that:

Number of line segments= $^5C_2$-$^3C_2$+1=8 (Answer)

3) No. of triangles formed in n sided Polygon =$^nC_3$

For Example:-

How many triangles can be formed in 4 sided polygon?

Solution:-

Using the formula:

No. of triangles= $^4C_3$=4 (Answer)

4) If m of these points are collinear (m≥3) then the number of number of triangles formed =$^nC_3$-$^mC_3$

5) Number of diagonals in an n sided closed polygon=$^nC_2$-n

6) Number of hand shake of n people in a party=$^nC_2$

If n distinct points are given on the circumference of a circle ,then:

7) Number of straight line = $^nC_2$

8) Number of triangles =$^nC_3$

9) Number of Quadrilaterals=$^nC_4$

10) If there are n teams and each team will face every other only once total number of matches=$^{n}C_2$

11) Product of n consecutive integer =n!

In such type of question.

How many 3-digit numbers can be formed from digits 1,2,3,4 {repetition allowed}

BOX-TRICK:-

Then, just make 3 boxes and fill first box by number that you can put in it and so on...

In our case

We can fill first box by 4 , as we have 4 digits to fill in,

Now i can fill second box by 4 , as i have also 4 digits for it, and same for third box..(Repetition is allowed).

If repetition is not allowed then,

I should fill first box by 4, second by 3, third by 2.

Then multiply the digits in the boxes, you will get your answer.

-Understand the Deck of Cards.

1) Normal Deck Contains 52 total Cards.
2) There are 26 Red and 26 Black Cards.
3) There are four types of cards i.e. Spade, Diamond, Heart and Club.
4) Red Cards contain 13 heart and 13 Diamond Cards.
5) Black Cards contain 13 Club and 13 Spade Cards.
6) Each type of the cards contains 3 cards and 1 Ace card.
7) There are 12 Face cards in a deck of cards.
8) Face Cards are;

(i) One King (ii) One Queen (iii) One Jack

Probability and Types of operations on Probability

Simply, Probability is just the way you can express numerically that how much is the chance of a specific event to occur or not under some specific circumstances.

Basic Terminology:-

1) Experiment: An Operation which results in some well defined outcome is called an experiment.

2) Random Experiment: An experiment whose outcome can't be predicted with certainty is called random experiment.

3) Trial: A single performance of an experiment is called a trial.

4) Sample Space: The set of all possible outcomes of a random experiment is called the sample Space.

a. No. of Sample Space when a coin is tossed n times= 2^n

b. No. of Sample Space when a dice is rolled n times=6^n

5) Event: It is a subset of the sample space.

6) Impossible Event: ϕ is also subset of Sample Space.

7) Sure/Certain Event: It is also subset of Sample Space.

8) Simple Event: An Event having single sample point.

9) Mixed Event: A subset of sample space which contains more then one element.

10) Equally Likely Events: A set of events is said to be equally likely if none of them are expected to occur in preference to other.

11) Exhaustive Events: A set of events is said to be exhaustive when a random experiment always results in the occurrence of at least one of them.

12) Mutually Exclusive Events: A set of events is said to be mutually exclusive if occurrence of one of them precludes the occurrence of any of the remaining events. (In other words, Events, E_1, E_2…E_n are mutually

exclusive if and only if $E_i \cap E_j = \phi$

13) Independent Events: Two events are said to be independent if the occurrence of the one does not depend on the occurrence of the others..

14) Compliment of Event: It is denoted by E' and it is given by: $E \cup \overline{E} = S$

15) Fair Coin: A coin whose one side is head and other is tail.

16) Biased Coin: A coin whose both sides are either heads or tails.

Some Important Results on Probability:

- Numerically, Probability (P) of an event (E) say: 'P (E)' lies in interval $0 \le P(E) \le 1$.
- Probability of an event can't be negative.
- Probability of occurrence of an impossible event is 0.
- Probability of occurrence of a sure event is 1.
- $P(E)=\frac{n(E)}{n(S)} = \frac{Number\ of\ favourable\ ways\ of\ event\ E}{Total\ Number\ of\ ways}$
- $P(A' \cap B') = 1 - P(A \cup B)$
- $P(A' \cup B') = 1 - P(A \cap B)$
- If $A_1, A_2, \ldots, A_n$ are independent events, then probability that all of the events occur is given by: $P(A_1 \cap A_2 \cap \ldots A_n) = P(A_1).P(A_2)\ldots.P(A_n)$
- $P(A' \cap B') = P(A').P(B')$
- There are two types of operations on Probability:

Addition of Probability has a general formula for two events A and B as:
1) P (AUB) = P (A) + P (B). (When A and B are totally Independent or Disjoint)
For Example:-
A card is drawn from a pack of 52 cards. Find the probability of getting a king or a red 9?
Solution:-

Using the formula:
P(AUB)=P(King)+P(Red 9)➔ P(AUB)=4/52 + 2/52= 3/26 (Answer)
2) P(AUB) = P(A) + P(B) - P(A∩B) (When A and B are Dependent or Over-Lapping)
For Example:
In a class, 30% of the students offered English, 20% offered Urdu and 10% offered both. If a student is selected at random. What is probability that he has offered English or Urdu?
Solution:-
P(Urdu)= 30%
P(English)=20%
P(Urdu ∩ English)=10%
Now using the formula:
P(AUB)=P(A)+P(B)-P(A∩B)
P(Urdu U English)= $\frac{3}{10}+\frac{2}{10}-\frac{1}{10}$➔Probability=$\frac{2}{5}$ (Answer)
3) Multiplication of Probability has a general formula for two independents events A and B as:
P(A∩B) = P(A) x P(B)
For Example:-
Two cards are drawn with replacement from a deck of 52 cards. Find the probability of first card being a red card and the second card being a club card?
Solution:-
P (Red face)=6/52
P (Club)=13/52
Using the formula:
P(Red Card ∩ Club) = 6/52 x 13/52= 3/104 (Answer)
4) The probability of getting k successes and n-k failures within n tails is given by:
Pr(X=k) =$^{n}C_{k}\, P^{k}(1-P)^{n-k}$
For Example:-
Suppose a biased coin comes up heads with probability 0.3 when tossed. What is probability of achieving 1 heads after six tosses?
Solution:-

Here n=6 and k=1

Using the Formula:-

$Pr(X=k) = {}^6C_1\,(0.3)^1(1-0.3)^{6-1} = 0.3025$ (Answer)

5) Probability of at least one head/tail is given by:

Probability$=\frac{2^n-1}{2^n}$

For Example:-

If a fair coin is tossed 6 times. Find the probability of getting at least one head?

Solution:-

Here; n=6

Probability$=\frac{2^n-1}{2^n}=\frac{2^6-1}{2^6}=\frac{63}{64}$ (Answer)

6) If a coin is tossed n times. Then the probability of getting r head and n-r tails is given by:

Probability$=\frac{nCr}{2^n}$

For Example: If a fair coin is tossed 3 times then find the probability of getting 2 head is given?

Solution:-

Here; n=3 , r=2

Probability$=\frac{nCr}{2^n}=\frac{3C2}{2^3}=\frac{3}{8}$ (Answer)

7) Probability when an event does not occur is given by:

$P(\overline{A}) = 1 - P(A)$

For Example: If P(A)=1/8 ,then find the probability of event when it does not occur?

Solution:-

$P(\overline{A}) = 1 - P(A)$

$P(\overline{A}) = 1 - \frac{1}{8}$

$P(\overline{A}) = \frac{7}{8}$ (Answer)

8) When one thing is to be selected out of two, the Probability is given by:

Probability$= P(A)P(\overline{B}) + P(B)P(\overline{A})$

For Example:-

A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and the probability of wife's selection is 1/5. What is the probability that only one of them is selected?

Solution:- P(A)=1/7 and P(B)=1/5

Now; $P(\overline{A}) = 1 - P(A) \Rightarrow P(\overline{A}) = 1 - \frac{1}{7} = \frac{6}{7}$

Also, $P(\overline{B}) = 1 - P(B) \Rightarrow P(\overline{B}) = 1 - \frac{1}{5} = \frac{4}{5}$

Now using the formula:

Probability that only one of them is selected=$P(A)P(\overline{B}) + P(B)P(\overline{A})$

Probability=$\frac{1}{7}\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{6}{7}\right)=\frac{2}{7}$(Answer)

-Important Points:

1) No. of rectangles that can be made on the chess board= 1296

:. It is given by $\left(\frac{n(n+1)}{2}\right)^2$

$\Rightarrow$In case of 8*8 chess board it is given by$\left(\frac{8(8+1)}{2}\right)^2 = 1296$

2) No. of squares that can be made on chess board =204

-Shortcut Tricks on Sum/Product of (2 or 3) dices when it is rolled

@Optimist

SUM OF TWO
DICES when these are rolled (Shortcut Trick)

Sum of 2 dices	No.of times come
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

Increase the no.of times (1-6) on the sum of 2 dices from (2-7)

decrease the no.of times (5-1) on the sum of 2 dices from (8-12)

@OptimiSt

Sum of Three dices when these are rolled

(Shortcut Trick)

Sum of 3 dices	Place magic digits from (4-10) and then add the previous number's sum		No.of times come
3	1	As (1+1+1)=3 only 1 time	1
4	+2 M A	1+2=3	3
5	+3 G	3+3=6	6
6	+4 I C	6+4=10	10
7	+5	10+5=15	15
8	+6 D I	15+6=21	21
9	+4 G	21+4=25	25
10	+2 I T	25+2=27	27
11	Now, inverse the sequence of result of no.of time from (27-3) on the sum from (11-18)	27	27
12		25	25
13		21	21
14		15	15
15		10	10
16		6	6
17		3	3
18		1	1

➔No. of same dots when 2 dices are rolled=6

PRODUCT OF TWO DICES
when these are rolled

Product of dices	No.of times come
1	1
2	2
3	2
4	3
5	2
6	4
8	2
9	1
10	2
12	4
15	2
16	1
18	2
20	2
24	2
25	1
30	2
36	1

Remember these no.of occurance in the certain form i.e 12-23-24-21-24-21-22-21-21 according to product of two dices.

-Entry Test Typed MCQ:

1.If ${}^{n}C_{10} = {}^{n}C_{14}$, then n=?

A. 14
B. 10

C. 6
D. 24

Answer: Option D

Explanation:

${}^{n}C_{p} = {}^{n}C_{q}$

$\Rightarrow p+q=n.$

$\therefore {}^{n}C_{10} = {}^{n}C_{14}$

$\Rightarrow n = (10+14) = 24$

2. If ${}^{n}C_{r}+{}^{n}C_{r+1} = {}^{n+1}C_{x}$, then x=?

A. r-1
B. r
C. r+1
D. n

Answer: Option C.

Explanation:

${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$.

So x =r+1

3. If ${}^{12}P_{r}=1320$, then r=?
A. 3
B. 4
C. 5
D. 6

Answer: Option A.

Explanation:

$^{12}P_r = 1320$

$\Rightarrow 1320 = 12*11*10 \Leftrightarrow = 3.$

4. If $^nP_5=20.^nP_3$, then n=?

A. 8
B. 9
C. 10
D. 11

Answer: Option A.

Explanation:

$^nP_5 = 20.^nP_3$

$\Leftrightarrow \ ^nP_5/^nP_3=20$

$\Leftrightarrow (n-4)(n-3) = 20.$

$\Leftrightarrow n^2 -7n-8 = 0.$

$\Leftrightarrow (n-8)(n+1) = 0.$

$\Leftrightarrow n = 8.$

5. If $^nC_3=220$, then n=?
A. 10
B. 11
C. 12
D. 9

Answer: Option C.

Explanation:

$^nC_3 = 220.$

$\Rightarrow \frac{n(n-1)(n-2)}{6}=220$

$\Rightarrow$ n(n-1) (n-1) = 1320.

$\Rightarrow$ n = 12.

6. If $^{15}P_{r-1}$:$^{16}P_{r-2}$=3:4, then r=?
A. 8
B. 14
C. 12
D.10

Answer: Option B.

Explanation:

$^{15}P_{r-1}$: $^{16}P_{r-2}$ = 3:4.

$\Rightarrow$ (r^2-35r+294) = 0

$\Rightarrow$ (r-21)(r-14) = 0.

$\Rightarrow$ r =14. ($\because$ r ≤ 16.)

7. How many 3 digit numbers are there with no digit repeated?
A. 729
B. 648
C. 720
D. None

Answer: Option B.

Explanation:

Hundred's place can be filled by any of non-zero digits. So, there are 9 ways of filling this place. The ten's digit can be filled by any of the remaining 9 digits.

So, there are 9 ways of filling of the ten's place.

The unit place can now be filled by any of the remaining 8 digits.

So, there are 8 ways of filling the unit digit.

∴ Required number of numbers = (9X9X8)=648

8. How many 10 digit numbers can be formed by using digits 1 and 2?

A. 10
B. ${}^{10}C_2$
C. ${}^{10}P_2$
D. 2^{10}

Answer: Option D.

Explanation:

Each place of the number can be filled in 2 ways.

∴ Required number of numbers = 2^{10}

9. How many 4 digit numbers can be formed with no digit repeated by using the digits 3, 4, 5, 6, 7, 8 and 0?

A. 280
B. 720

C. 840
D. 660

Answer: Option B.

Explanation:

Thousand's place can be filled by any of the 6 non-zero digits.So, there are 6 ways

of filling

this place.The ten's digit can be filled by any of the remaining 9 digits.

Hundred’s place can be filled by any of the remaining 6 digits.

So, there are 6 ways of filling of the ten’s place.

Ten’s place can be filled by any of the remaining 5 digits.

So, there are 5 ways of filling of the ten’s place.

The unit place can now be filled by any of the remaining 4 digits.

So, there are 4 ways of filling the unit digit.

∴ Required number of numbers = (6X6X5X4) =720.

10. How many 3 digit even numbers can be formed with no digit repeated, by using the digits, 0, 1, 2, 3, 4 and 5?

A. 48
B. 50
C. 52
D. 56

Answer: Option C.

Explanation:

Numbers with 0 at unit place = 5*4*1 = 20.

Numbers with 2 at unit place = 4*4*1 = 16.

Numbers with 4 at unit place = 5*4*1 = 16.

Total numbers = (20+16+16) = 52.

11. The number of positive integers greater than 6000 and less than 7000 which are divisible by 5, with no digit repeated, is:

A. 28
B. 56

C. 112
D. 84

Answer: Option C.

Explanation:

Clearly, thousand's digit is 6.

Number of numbers with unit digit 0 = 1*8*7*1 = 56.

Number of numbers with unit digit 5 = 1*8*7*1 = 56.

∴ Required number of numbers = (56+56) =112.

12.How many words beginning with T and ending with E can be made (with no letter repeated) out of the letters of the word 'TRIANGLE'?

A. ${}^{8}P_{6}$
B. 720

C. 1440
D. 772

Answer: Option B.

Explanation:

Fixing T at the beginning and E at the end, the remaining 6 letters

can be arranged at 6 places in 6! = 720 ways.

∴ Required number of words = 720.

13. How many words can be formed from the letters of word 'DAUGHTER' so that the vowels always come together?

A. 720
B. 726

C. 4320
D. None

Answer: Option C.

Explanation:

Take all the vowels A, U, E together and take them as one letter.

Then, the letters to be arranged are D, G, H, T, R, (AUE).

These 6 letters can be arranged at 6 places in 6! ways.

Now 3 letters A, U, E among themselves can be arranging in 3! ways.

∴ Required number of words = (6!)*(3!) = 4320.

14. How many words can be formed from the letters of word 'LAUGHTER' so that the vowels are never together?

A. 4320
B. 3600

C. 40320
D. 36000

Answer: Option D.

Explanation:

Total number of words formed by using all the 8 letters at at a time = ${}^{8}P_{8}$ =8! =40320.

Number of words in which vowels are never together = (40320 – 4320) = 36000

15. In how many ways can 10 books be arranged on a shelf so that a particular pair of books shall be always together?

A. 9!
B. 2*9!

C. 8!
D. None

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Answer: Option B.

Explanation:

Let us tie the 2 particular books together and treat it as one.

Then, there are 9 books in all which can be arranged among themselves in 9! ways.

Also, 2 books can be arranged among themselves in 2! ways.

∴ Required number of ways = (2*9!)

16. In how many ways can 10 books be arranged on a shelf so that a particular pair of books shall be never together?

A. 8!
B. 9!
C. 2*9!
D. 8*9!

Answer: Option C.

Explanation:

Number of ways in which 10 books may be arranged = 10!

Number of ways in which 10 books may be arranged with 2 particular books

together = (2*9!)

17. There are 6 English, 4 Arabic and 5 Chinese books.In how many ways can hey be arranged on a shelf so as to keep all the books of the same language together?

A. 720
B. 120
C. 870
D. (6*720*24*120)

Answer : Option D

3 packets can be arranged in 3!=6 ways.

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6 English books can be arranged in 6!=720 ways.

4 Arabic books can be arranged in 4!=24 ways.

5 Chinese books can be arranged in 5!=120 ways.

∴ Required number of ways=(6*720*24*120).

18. In how many ways can the word 'PENCIL' be arranged so that N is always next to E?

A. 1440
B. 720
C. 240
D. 120.

Answer : Option D

Keeping EN together and considering it as 1 letter,

we have to arrange 5 letters at 5 places.

This can be done in $^5P_5$ =5! = 120 ways.

19.In how many ways can the letter of the word 'MACHINE' be arranged so that the vowels may occupy only odd positions?

A. (4*7!)
B. 576

C. 288
D. None

Answer : Option B

There are 7 letters in the given word, out of which there are

3 vowels and 4 consonants.Let us mark the positions of the

letters as :

(1) (2) (3) (4) (5) (6) (7)

Now, the 3 vowels can be placed at any of the three places out of four, marked 1, 3, 5, 7.

So, the number of ways of arranging the vowels = $^4P_3$ = 4*3*2 = 24.

Also, the 4 consonants at the remaining 4 position can be arranged in $^4P_4$ = 24 ways.

∴ Required number of ways = (24*24) = 576.

20.In how many ways can the letter of the word ‘APPLE’ be arranged?

A. 720
B. 120

C. 60
D. 180

Answer : Option C.

There are in 5 letters in the given word, out of which there are

2 Ps , 1 A, 1 L and 1 E

∴ Required number of ways = 5!/(2!)(1!)(1!)(1!) = 60.

21.How many words can be formed using the letters A thrice, the letter B twice and the letter C once?

A. 60
B. 120

C. 90
D. 6

Answer : Option A.

There are in 6 letters in all, out of which A is repeated thrice,

B is repeated twice and C is taken only once.

∴ Required number of words = 6!/(3!)(2!)(1!) = 60.

22.How many words can be formed by using all the letters of the word 'ALLAHABAD'?

A. 3780
B. 1890

C. 7560
D. 9!

Answer : Option C.

There are 9 letters in all, out of which A is repeated 4 times,

L is repeated twice and rest are different.

∴ Required number of words = 9!/(4!)(2!) = 7560.

23.In how many ways 6 rings of different type can be had in 4 fingers?

A. 6^4
B. 4^6

C. ${}^6P_4$
D. None

Answer : Option B.

The first ring can be worn in any of the 4 fingers.

So, there are 4 ways of wearing this ring.

Similarly, each one of the other rings may be worn in 4 ways.

∴ Required number of ways = 4^6

24.How many 4 digit numbers are there, when a digit may be repeated any number of times in each number?

A. 5040
B. 9000

C. 10000
D. None

Answer : Option B.

Clearly 0 can't be placed at the thousand' place.So, this place can be filled in 9

ways.

Each of the hundred's , ten's and unit digit can be filled in 10 ways.

∴ Required number of numbers = 9*10*10*10 = 9000.

25.How many 4 digit numbers can be formed by using the digits 1,2,3,4,5,6 when a digit may be repeated any number of times in each number?

A. 4^6
B. 6^4

C. 1440
D. None

Answer : Option B.

Each of the thousand's, hundred's , ten's and unit's place can be filled in 6 ways.

∴ Required number of numbers = 6*6*6*6 = 6^4.

26.In how many ways can 8 students be arranged in a row?

A. 8!
B. 7!

C. 8
D. 2*7!

Answer : Option A.

8 students may be arranged in a row in 8! ways.

27.In how many ways can 8 students be seated in a circle?

A. 8!
B. 7!

C. 8
D. 2*7!

Answer : Option B.

8 students may be arranged in a circle in 7! ways.

28.In how many ways can a party of 4 men and 4 women be seated at circular table so that no 2 women are adjacent?

A. (24*24)
B. (2*4)!
C. 144
D. 4^4

Answer : Option C.

4 men can be seated at the circular table such that

there is a vacant seat between every pair of men.

This can be done in 3! = 6 ways.

Now, 4 vacant seats may be occupied by 4 women in ${}^{n}P_4$ = 4! =24 ways.

29.In how many ways a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

A. 25
B. 50

C. 100
D. 200

Answer : Option D.

(3 men out of 6) and (2 ladies out of 5) can be selected in ($^6C_3$ * $^5C_2$) = 200 ways.

30.Out of 5 men and 2 women, a committee of 3 is to be formed.In how many ways can it be formed if at least 1 women is included in each committee?

A. 25
B. 50

C. 21
D. 32

Answer : Option A.

We may have

(i) 1 women and 2 men or (ii) 2 women and 1 man.

Required number of ways = ($^2C_1$ * $^5C_2$) + ($^2C_2$ * $^5C_1$) = 25.

31. A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways can this be done when each committee may have at the most 2 ladies?

A. 120
B. 160
C. 180
D. 186.

Answer : Option C.

We may have

(i) 1 lady out of 4 and 4 gents out of 6 or

(ii) 2 ladies out of 4 and 3 gents out of 6.

Required number of ways = $({}^4C_1 * {}^6C_4) + ({}^4C_2 * {}^6C_3) = 180$.

32.An examination paper containing 12 questions consist of 2 parts, A and B. Part A contains 7 questions and part B contains 5 questions. A candidates is required to attempt 8 questions, selecting at least 3 from each part.In how many ways the questions can be selected?

A. 420
B. 360

C. 720
D. 180.

Answer : Option A.

The selection can be made as under:

(i) (3 out of 7 from A) and (5 out of 5 from B) or

(ii) (4 out of 7 from A) and (4 out of 5 from B) or

(iii) (5 out of 7 from A) and (3 out of 5 from B).

Required number of ways

$= ({}^7C_3 * {}^5C_5) + ({}^7C_4 * {}^5C_4) + ({}^7C_5 * {}^5C_3) = 420$.

33. In how many ways can 21 books on English and 19 books on Hindi be placed in a row on a shelf so that 2 books on Hindi may not be together?

A. 770
B. 385

C. 1540
D. 399

Answer : Option C.

Explanation:

In order that two books on Hindi are never together, we must place all these books as under :

X E X E X E X.......X E X

where E denotes the position of an English book x that of Hindi book.

Since there are 2 books on English, the number of places marked x are therefore, 22.

Now, 19 places out of 22 can be chosen in

${}^{22}C_9 = {}^{22}C_3$ =1540 ways.

34. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

A. 1050
B. 330
C. 25200
D. 6300

Answer : Option C.

Explanation:

Number of ways of selecting 3 consonants out of 7 and 2 vowels out of 4.

${}^{7}C_3 * {}^{4}C_2$ =210 ways.

Number of groups of 3 consonants and 2 vowels = 210.

Now 5 latters can be arranged among themselves in 5! = 120 ways.

35.How many diagonals are there in a decagon?

A. 10
B. 25
C. 35
D. 45

Answer : Option C.

Explanation:

Number of diagonals = 1/2 n(n-3) = 1/2 * 10 * 7 = 35.

36.A polygon has 54 diagonals. Number of sides of this polygon is?

A. 12
B. 15

C. 16
D. 9

Answer : Option A.

Explanation:

Number of sides = 1/2 * n(n-3) = 54.

⇒ n^2-3n-108 = 0.

⇒ (n-12)(n+9) = 0

⇒ n = 12.

37.There are 10 points in a plane, out of which 4 points are collinear. The number of lined obtained from the pairs of these points is?

A. 45
B. 41

C. 40
D. 39

Answer : Option C.

Explanation:

Number of lines formed by joining pairs of 10 points = ${}^{10}C_2$ = 45.

Number of lines formed by joining pairs of 4 points = ${}^4C_2$ = 6.

But, these four points being collinear give only 1 line.

∴ Required number of lines = (45-6+1) =40

38. There are 10 points in a plane, out of which 4 points are collinear. The number of triangles formed with vertices as these points is:

A. 20
B. 120

C. ${}^6C_3$
D. 116

Answer : Option D.

Explanation:

Number of triangles obtained from 10 points = ${}^{10}C_3$ = 120.

Number of triangles obtained from 4 points = ${}^4C_3$ = 4.

But, these four points being collinear give no triangle.

∴ Required number of ▲S = (120-4) =116.

39.In how many ways can a cricket eleven be selected from 17 players, in which 5 players can bowl, each cricket team must include 2 bowlers?
A. 550
B. 1100
C. 1650
D. 2200

Answer : Option D.

Explanation:

The selection can be made as under:

(2 bowlers out of 5) and (others out of 12)

∴ Required number of ways = $^5C_2$ * $^{12}C_9$ = 2200.

40.How many triangles can be drawn through n given points on a circle?
A. $^nC_3$
B. ($^6C_3$-n)
C. n
D. None

Answer : Option A.

Explanation:

Any 3 points on a circle are non – collinear.

∴ Required number of triangles = $^nC_3$.

-Some Productive Points:
1) Maximum point of intersection for line to line .nC2
2) Maximum point of intersection for circle to circle is 2* (mC2)
3) Maximum point of intersection for circle to line is (m*n)*2
Whereas m= no. of circles
n=no. of lines
4) Common point of intersection of line to circle:
2*mC2+nC2 +2mn
Double Factorial
n!!= n(n-2)(n-4)….
For n=even then
n!!= n(n-2)(n-4)…..4.2
Example: 4!! =4(4-2)(4-4)…..4.2=0 (Answer)
For n=odd n!!= n(n-2)(n-4)…… 3.1
Example: 9!!=9(9-2)(9-4)….3.1=945 (Answer)

CHAPTER 8 (Mathematical Induction and Binomial Theorem)

Look Exercise 8.1. The Questions are like this:

1. $1+5+9+....+(4n-3) = n(2n-1)$
2. $1+3+5+....+(2n-1) = n^2$ and so on..

-TRICK:-

On the right side of equality, the formulas of Sum of series are given and to the left side of equality nth term/General Term of series is given. In Question 1, nth term is (4n-3) and formula for sum of series is n(2n-1). $a_n= (4n-3)$

$S_n = n(2n-1)$

Given is $a_1=1$, $a_2=5$ and $a_3=9$.

Now if you put n= 1,2 and 3 in a_n you will get 1,5 and 9 respectively.

$a_1=[4(1)-3]=1$

$a_2=[4(2)-3]=5$

$a_3=[4(3)-3]=9$

If you put n=1 in S_n, you will get the first term as it is. $S_1=a_1$

$S_1= 1[2(1)-1]= 1$

If you put n=2 in S_n, you will get the sum of first two terms of series.

$S_2=a_1+a_2$

$S_2= 2[2(2)-1]= 6$

If you put n=3 in S_n, you will get the sum of first three terms of the series. $S_3= a_1+a_2+a_3$

$S_3= 3\{2(3)-1] = 15$

I hope this is clear to you. Now in MCQ's, they will give you left side of the question and you have to guess the right side from the options.

Example:-

1x3+2x5+3x7+.....+nx(2n+1)= ?

a) $\frac{n(n+1)(n+2)}{3}$ b) $\frac{n(n+1)(4n+5)}{3}$ c)____ d)_____

Now to guess which is the right option, do following steps:

Calculate S_1, S_2 and S_3 by $S_1=a_1$, $S_2=a_1+a_2$, $S_3= a_1+a_2+a_3$

$S_1=3$, $S_2= 13$, $S_3= 34$

Now put n=1, 2 and 3 in all the given options and check for which option your answers matches with S_1, S_2 and S_3 you first calculated. It will be the answer. In above question option b is correct. Check minimum 3 values by putting n. Practice it maximum.

-Binomial Expression:-

An algebraic expression consisting of only two terms is called a binomial expression.

For Example:

$(x + y)^2= x^2+2xy+y^2={}^2C_0x^2+{}^2C_1xy+{}^2C_2y^2$

Binomial Theorem:-

This theorem gives a formula by which any power of a binomial expression can be expanded. It was first given by Sir Isaac Newton.

Important Points:-

1) There must be two terms or made them two.

e.g. a+b or (a+b+c)➔ (a+(b+c))

2) Power must be positive integer. e.g. Z={0,1,2,3,….}

3) Last term of binomial theorem exists.

4) Total number of terms of binomial expansion is always one more than index. i.e. n+1

e.g. ➔ No. of terms in $(x+y)^2$ is 2+1

➔Easy Way to learn this theorem:-

Words:- Power or a wali term ko mila k power pehly, a wali term ki power me se aek km or b wali term ki power me izafa krna hae, Mtlb ye k pehli 2 laqmo ko km krna hae or tisri ko brhana hae. Or Last pe, pehli term se 0! , Second pe 1! And so on. Divide krna hae…

In general:-

$(a+b)^n = \frac{a^n}{0!} + \frac{na^{n-1}b^1}{1!} + \frac{n(n-1)a^{n-2}b^2}{2!} + \cdots + \frac{b^n}{n!}$

Illustration:-

Let us have an example:-

$(x+y)^2 = \frac{x^2}{0!} + \frac{2(x^{2-1})y^1}{1!} + \frac{2(2-1)(x^{2-2})y^2}{2!} = x^2 + 2xy + y^2$(Answer)

➔Even Binomial Coefficient:-

If r=0,2,4,6,8…. then ^nC_r is called even places or even binomial coefficient.

➔Odd Binomial Coefficient:-

If r=1, 3, 5, 7, 9… then ^nC_r is called odd places or odd binomial coefficient.

➔Sum of Binomial Coefficients:-

$^nC_0+^nC_1+^nC_2+\ldots+^nC_n=2^n$

Example:- Find the Sum of binomial Coefficient in the binomial expansion of $(x+y)^5$?

Solution:- Sum of Binomial Coefficient= $2^5=32$ (Answer)

➔Sum of Even Binomial Coefficients is equal to sum of odd binomial Coefficient.

$^nC_0+^nC_2+^nC_4+\ldots+^nC_n= {}^nC_1+^nC_3+^nC_5+\ldots+^nC_{n-1}=2^{n-1}$

Example:- Find the sum of Even Binomial Coefficients of $(2x+3y)^6$?

Solution:- Sum of Even Binomial Coefficients=$2^{6-1}=2^5=32$(Answer)

➔Sum of all exponents in $(x+y)^n = n^2+n$

Examples:- Find the sum of all exponents of $(a+x)^3$?

Solution:- Sum of all exponents=$3^2+3=12$ (Answer)

➔ The binomial coefficient of the term equidistant from the beginning and the end are equal i.e ${}^nC_r={}^nC_{n-r}$

➔ General term/Specific Term of $(a+b)^n$: $T_{r+1}={}^nC_r\, a^{n-r}.b^r$

a) T_{r+1} is the (r+1)th term from beginning and (n-r+1)th term from end.

b) Value of r is one less than number of term

c) ${}^nC_r$ is the binomial coefficient of (r+1)th term.

➔ rth term of $(a+b)^n$: $T_r={}^nC_{r-1}.a^{n-r+1}.b^{r-1}$

➔ nth term of $(a+b)^n$: $T_n={}^nC_{n-1}.ab^{n-1}$

➔ Last term/(n+1)th term of $(a+b)^n$:$T_{n+1}=b^n$

➔Middle term/s

a) The middle term in the binomial expansion of $(a+b)^n$ depends upon the value of n

b) If n is even then there is only one middle term. i.e $\left(\frac{n}{2}+1\right)^{th}$

c) If n is odd then there are two middle terms i.e $\left(\frac{n+1}{2}\right)^{th}$ and $(\frac{n+3}{2})^{th}$

➔ Coefficient of Middle terms:

a) If n is even then binomial Coefficient of middle term is ${}^nC_{n/2}$

b) If n is odd then binomial Coefficient of middle terms are $nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$

➔ pth term from the end in the Binomial Expansion of $(a+b)^n$ is (n-p+1)th term from the beginning.

➔Properties of ${}^nC_r$

1) If n is even then greatest coefficient= ${}^nC_{n/2}$

2) If n is odd then greatest coefficient=${}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$

3) If 0<r<n , n,rεN, then ${}^nC_r=n/r.{}^{n-1}C_{r-1}$

4) ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$

5) ${}^nC_x={}^nC_y$➔ x=y or x+y=n

6) If l,m are integers then sum of coefficients in the expansion of $(lx+my)^n$ is $(l+m)^n$

CONVERGENCE OF BINOMIAL SERIES:-

- Binomial Series is valid if it is convergent.
- In convergent series, next term is smaller i.e we can neglect bigger terms of order x^4, x^5,.......

Condition:-

- Binomial series converges if,
  $|x|<1$ or $-1<x<1$ or $x \in (-1,1)$

Examples:-

1. $(1+3x)^{-7}$, Series is convergent if x=?
   Applying condition,
   $|3x|< 1 \Rightarrow |x|<\frac{1}{3}$
2. $(1-\frac{3}{4}x)^{1/2}$, Series is convergent if x=?
   Neglect –ve sign
   $|\frac{3}{4}x| < 1 \Rightarrow |x|<\frac{4}{3}$
3. $(3+4x)^{-2}$

In this case, first take out 3 common and then check

$3^{-2}(1+\frac{4}{3}x)^{-2}$ Now, $|\frac{4}{3}x|< 1 \rightarrow |x|< \frac{3}{4}$

Some particular cases of expansion $(1+x)^n$ when n<0

Binomial Expansion	(r+1)th term
$(1+x)^{-1}$	$(-1)^r x^r$
$(1+x)^{-2}$	$(-1)^r(r+1)x^r$
$(1+x)^{-3}$	$(-1)^r\frac{(r+1)(r+2)}{2}x^r$
$(1-x)^{-1}$	x^r
$(1-x)^{-2}$	$(r+1)x^r$
$(1-x)^{-3}$	$\frac{(r+1)(r+2)}{2}x^r$

-Entry Test Type MCQ:-

Q#1: The number of terms of $(a-b)^{17}$ is:

A) 2 B) 17 C) 18 (Correct) D) 20

Solution: The total terms in binomial expansion is n+1

In this case: n=17 so Total terms =17+1 $\rightarrow$ =18 (Answer)

Q#2: The middle term in the expansion of $(x+y)^{30}$ is:

A) 13^{th} B) 14^{th} C) 15^{th} D) 16^{th} (Correct)

Solution: As we know that n=30

so middle term will be $(\frac{n}{2}+1)^{th} \rightarrow =(\frac{30}{2}+1)^{th} \rightarrow =16^{th}$ (Answer)

Q#3: The binomial coefficient of 21^{st} term in the expansion of $(a+b)^{23}$ is:

A) 1771 (Correct) B) 2891 C) 3421 D) 1563

Solution: As 21^{st} term is asked so r=20

Using the general term formula:

$T_{r+1}= nCr.\ a^{n-r}.b^r$

T_{20+1}= 23C20 . $a^{23-20}b^{20}$

T_{21}= 23!/20!. 3! a^3b^{20}

T_{21}= 23.22.21/6 a^3b^{20}

T_{21}= 23.11.7 a^3b^{20}

T_{21}= 1771 a^3b^{20} (Answer)

Q#4: The total number of terms in the expansion of $(1-x+x^4)^4$ is:

A) 5 B) 15 (Correct) C)10 D) 8

Solution: As we know that:

Total number of terms in $(x_1+x_2+.....+x_r)^n$ is ${}^{n+r-1}C_{r-1}$

whereas r= no. of terms

And also for trinomial expansion total terms is $\frac{(n+1)(n+2)}{2}$

In this case:

M#1: $(1-x+x^4)^4$

As r=3 so Total number of terms ${}^{4+3-1}C_{3-1}={}^6C_2$=15 (Answer)

M#2: Number of terms in trinomial expansion= $\frac{(4+1)(4+2)}{2}=\frac{5(6)}{2}$=15 (Ans)

Q#5: The 6th term from the end in the expansion $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$ is:

A) 16x B) 77x C) $\frac{16}{77}x$ D) $\frac{77}{16}x$ (Correct)

Solution: The pth term from the end in the expansion is $(n-p+1)^{th}$ term from beginning.

so $(11-5+1)^{th}$ ➔ 7^{th} so r=6

Now. Using the general term formula:

$T_{6+1}={}^{11}C_6\left(\frac{3}{2}x\right)^{11-6}\left(-\frac{1}{3x}\right)^6$

$T_{6+1}={}^{11}C_6\left(\frac{3^5}{2^5}x^5\right)\ \frac{(-1)^6}{3^6x^6}$

T_{6+1} =11!/5!.6! $\left(\frac{1}{32}\right)\left(-\frac{1}{3x}\right)$

T_{6+1} =11.10.9.8.7.6!/5.4.3.2.1.6! $\left(\frac{1}{32}\right)\left(-\frac{1}{3x}\right)$

T_{6+1} =11.3.2.7$\left(\frac{1}{32}\right)\left(-\frac{1}{3x}\right)$

T_{6+1} =11.7$\left(\frac{1}{16}\right)\left(-\frac{1}{x}\right)$

T_{6+1} =$(-\frac{77}{16x})$(Answer)

Q#6: The coefficient of the term involving x^5 in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$? A) 15309 B) -15309 C) -15309/8(Correct) D) 15309/8

Explanation:

Trick:

The coefficient of the term involving x^m in the expansion of $(x^p+(a/x^q))^n$ is $T_{r+1}={}^nC_r\, a^{n-r}.b^r$ whereas r=$\frac{np-m}{p+q}$

Apply the trick: r= $\frac{2(10)-5}{2+1}$=$\frac{15}{3}$ =5

So T5+1= ${}^{10}C_5\,(x^2)^{10-5}.(-3/x^2)^5$= -15309/8 (Answer)

Q#7: The term free from Z in the expansion of $\left(\sqrt{\frac{Z}{3}}+\frac{3}{2z^2}\right)^{10}$ is:

A) 5/4 B) ¼ C) 9/4 D) 4/5

Explanation:

Trick:

The coefficient of the term involving x^m in the expansion of $(x^p+(a/x^q))^n$ is $T_{r+1}={}^nC_r\, a^{n-r}.b^r$ whereas r=$\frac{np-m}{p+q}$

Here in this case m=0 as independent term/term free from variable/constant term contain x^0 always so that:

r=$\frac{\frac{1}{2}(10)-0}{1/2+2}$=$\frac{10}{5}=2$

T2+1= ${}^{10}C_2\,(z/3)^4.(3/2z^2)^2$ =5/4 (Answer)

Q#8: The term free from t in $(4t+5/t)^8$ is?

Explanation:

Using trick: r= np/p+q➔ r= 1(8)/1+1➔ 4

so Tr+1= T5 ➔ 5^{th} term (Answer)

Q#9: Total number of terms in the expansion of $(x+z)^{50}+(x-z)^{50}$ are?

Explanation:

Trick: Total terms of $(x+y)^n+(x-y)^n$ are:

1) If n=odd then total terms are (n+1)/2

2) If n=even then total terms are (n/2)+1

In this case: (50/2)+1= 26 (Answer)

Q#10:The coefficient of x^5 in the expansion of $(1+x^2)(1+x)^4$ is?

Explanation:

Using binomial theorem on $(1+x)^4$

$=(1+x^2)(1+4x+6x^2+4x^3+x^4)$

$=1+4x+7x^2+8x^3+4x^5+x^6$

So coefficient of x^5 is 4 (Answer)

Q#11:

The coefficient of terms x^{100} in the expansion of $(1-x)^{-3}$ is?

Explanation:

Using the formulas:

(r+1)th term of $(1-x)^{-3}$ is $\frac{(r+1)(r+2)}{2}$➔ $\frac{(100+1)(100+2)}{2} = 5151$

Q#12:

If the coefficient of (2r+4)th and (r-2)th term in the expansion of $(1+x)^{18}$ are equal?

Explanation:

Using the property"

nCx=nCy ➔ n=x+y

so: 2r-3+r-3=18➔3r=18➔r=6 (Answer)

Q#13:The sum of coefficient of last 3 last term in the expansion of $(8-3x)^{1/2}$?

Explanation: The last term of the binomial series does not exist so can't be determined.

Q#14: If the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are same then a=?

Explanation:

Using the formula:

$T_{r+1}= nCr.\ a^{n-r}b^r$

$T_{r+1}= {}^9Cr\ (3)^{9-r}(ax)^r$

For $x^2 \rightarrow T_3= 78732(ax)^2$

For $x^3 \rightarrow T_4=61236(ax)^3$

As the coefficient are equal so that:

$78732(ax)^2=61236(ax)^3$

$\rightarrow$ a= 9/7 (Answer)

Q#15:The greatest positive integer divides $11^n-10n-1$ is?

Explanation:

Put n=1 it gives the notation n=0

Put n=2 it gives the $11^2-10(2)-1= 121-20-1=121-21=100$

which means greatest integer is 100.. (Answer)

Q#16:

If $S(n)= n^2-n+41$ is a proposition then which of the following is not prime

A) S(1) B) S(1) and S(10) C) S(10) D) S(41)

Explanation:

On putting the value of n from the given options and satisfies the condition.

so option D is satisfying.

$S(41)= (41)^2-(41)(41)$

$S(41)=41^2=1681$

As we know that prime number is one which has at most 2 divisor i.e itself and 1.

In this case there are three divisor of 1681 i.e 1681 ,41 and 1 (Answer)

Q#17:

If $a^3+b^3+c^3+....+(2n-1)^3=n^2(2n^2-1)$ then value of (a,b,c)=?

Explanation:

Put n=1 in both $(2n-1)^3$ and $n^2(2n^2-1)$ and get the answer.

As in this case $(2(1)-1)^3=1$ so $n^2(2n^2-1)$ is also 1

Put n=2 in both notation so:

$(2(2)-1)^3=27$ so $n^2(2n^2-1)=1+27=28$

Put n=3 in both notation so:

$(2(3)-1)^3=125$ so $n^2(2n^2-1)=1+27+125=153$

so $(1,27,125)=(a^3,b^3,c^3)$

taking cube root on both sides so:

(1,3,5)=(a,b,c) (Answer)

CHAPTER 9-14 TRIGONOMETRY

PUT AND CHECK TRICK:-

Suppose you have to solve an identity or any trigonometric equation but you don't remember the formula then there is a very easy method to guess the correct answer.
Just put any value from the domain of a trigonometric function in the question, you will get some value (Note that). Now put the same value in all the given options and check for which option your answer matches. It will be the answer.

Example:-

$2\sin\theta = ?$

a. $\cos 3\theta$ **b.** $\sin 3\theta$ **c.** $\tan 3\theta$ **d.** $\cot 3\theta$

Now put $\theta=30°$ in $2\sin\theta = 2\sin 30° = 2(1/2) = 1$ Note it…

Now put $\theta=30°$ in all options

$\cos 3\theta = \cos 3(30°) = \cos 90° = 0$
$\sin 3\theta = \sin 3(30°) = \sin 90° = 1$
$\tan 3\theta = \sin 3\theta / \cos 3\theta = 1/0 =$ Undefined
$\cot 3\theta = \cos 3\theta / \sin 3\theta = 0/1 = 0$

So, right option is b. $2\sin\theta = \sin 3\theta$
Note:
This trick is very helpful. Especially in 9.3, 9.4 and 10.3, 10.4 also in chapter 13, 14. Where a long equation come to solve practice it. But remember the domain and try it that a function should not become undefined as tan90 and cot0 etc.

Otherwise if in equation there come only sine and cos functions, solutions with 90,0 is more easy. Practice it maximum.

SOME USEFUL RESULTS TO REMEMBER:-

1. tan x + tan(180-x) = 0
2. cot x + cot(180-x) = 0
3. sin x – sin(180-x) = 0
4. cos x – sin(90-x) = 0
5. sin x – cos (90-x) = 0
6. cos x + cos(180-x) = 0
7. $\sin^2\alpha+\cos^2\beta=1$ if $\alpha+\beta= 90°$
8. $\sin n\pi = 0$
9. $\cos n\pi= (-1)^n$ $n \in Z$
10. $\sin(2n+1)\frac{\pi}{2} = (-1)^n$
11. $\cos(2n+1)\frac{\pi}{2} = 0$
12. $\tan n\pi= 0$
13. $\cot n\pi= \infty$
14. $\tan(2n+1)\frac{\pi}{2}= \infty$

15. $\cot(2n+1)\frac{\pi}{2} = 0$

$\sin(n\pi+\alpha) = (-1)^n \sin\alpha$, $\cos(n\pi+\alpha) = (-1)^n \cos\alpha$

$$\sin\left(\frac{n\pi}{2}+\theta\right) = \begin{cases} (-1)^{n-1/2} \cos\alpha, \text{ if n is odd} \\ (-1)^{n/2} \sin\alpha \text{ if n is even} \end{cases}$$

$$\cos\left(\frac{n\pi}{2}+\theta\right) = \begin{cases} (-1)^{n-1/2} \sin\alpha, \text{ if n is odd} \\ (-1)^{n/2} \cos\alpha \text{ if n is even} \end{cases}$$

SHORTCUT TO FIND DOMAIN, RANGE, PERIOD & Frequency

Let,

Y = A (Trigonometric Function) Bx+C ------------------------------(1)

Then,

- Domain = $\frac{Domain\ of\ Function}{B}$
- Range = A (Range of Function)
- Period = $\frac{Period\ of\ Function}{B}$
- Frequency = $\frac{1}{Period}$
- Phase shift= $\frac{-C}{B}$

e.g.,

y = 3 sin 4x+9

Comparing it with eq. 1,

A=3, B=4, C=9

We know the Domain of sinx is R(All real numbers), Range is [-1,1] and Period is 2π. So, for y=3sin4x

- Domain = $\frac{R}{4}$ = R
- Range = 3 [-1,1] = [-3,3]
- Period = $\frac{2\pi}{4} = \frac{\pi}{2}$
- Frequency = $\frac{1}{Period} = \frac{2}{\pi}$
- Phase Shift= -9/4

Quick Guessing Points for the Period of any function:

- Constant Function is a function with no fundamental period. e.g; 1 has no period.
- If f(x) is periodic with period T, then 1/f(x) and $\sqrt{f(x)}$ are also periodic with the same period. e.g; $\sqrt{2sin2x+3}$ has π as period.
- sinx, cosx, secx, and cosecx are periodic functions with period 2π.
- tanx and cotx are periodic functions with period π.
- |sinx|,|cosx|,|secx|,|cscx|,|cotx| and |tanx| are periodic functions with period π.
- $Sin^n x$, $cos^n x$, $sec^n x$ and $csec^n x$ are periodic function with period 2π when n is odd and π when n is even.
- $tan^n x$ and $cot^n x$ are periodic functions with period π.

Period of $a\,f(x) + b\,g(x) = L.C.M\ of\ periods\ of\ f(x)\ \&\ g(x)$

E.g.

* Period of $2sinx + 3tanx = L.C.M\ of\ periods\ of\ sinx\ \&\ tanx$

$$= L.C.M\ of\ 2\pi\ \&\ \pi = 2\pi$$

* Period of $\cos(2x) + tan\frac{x}{3} = L.C.M\ of\ periods\ of\ \cos(2x)\ \&\ tan\frac{x}{3}$

$$= L.C.M\ of\ \pi\ \&\ 3\pi = 3\pi$$

Range of Function of type af(x)+b if f(x) is a trigonometric Function.

- asinx+b=[b-a,b+a]
  e.g range of 2sinx+5 is [5-2,5+2]=> [3,7]
- acosx+b= [b-a,b+a]
- atanx+b= (-∞,∞)
- acotx+b= (-∞,∞)
- asecx+b= (-∞,b-a]U[a+b,+∞)
  e.g 2secx+9= (-∞,7]U[11,+∞)
- acsecx+b=(-∞,b-a]U[a+b,+∞)
  Range of sin^2x, cos^2x, tan^2x, sec^2x, $csec^2x$ and cot^2x can be got by eliminating negative portion of range of function without square.
  e.g sin^2x has range [-1,1] now eliminate the negative part of range of trigonometric function and get the real range of sin^2x=[1].

https://www.facebook.com/groups/698559670280811/

--Tricks on Clock Related Problem:

1. An hour hand covers half degree (1/2) in one minute.
2. Hour hand and Minute hand overlaps 11 times in 12 hours and 22 times in 24 hours.
3. Hour hand and Minute hand makes straight line 11 times in 12 hours and 22 times in 24 hours.
4. To find the Mirror Image of any given time, substract the given value from **11:60**.
5. A minute hand covers 6 degrees in One Minute
6. Minute hand runs 55 minutes more than Hour hand in 60 Minute. So we can also say that:

   1 Minute gain= 60/55 = 12/11 minutes

Shortcut Trick for finding Degrees, Minutes and Hours

$$\theta = \left| \frac{11}{2} m - 30 \times H \right|$$

Here **m** = Minutes and **H** = Hours.

- When value of θ becomes more than 360, subtract 360 from the value of θ and complete the calculation.

1) An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?

(a) 144 (b)150
(c) 168 **(d)180**

Sol - **8** o'click in morning - **2** o'clock = **6** hours.

using trick,

In 1 minute, an hour hand covers 1/2 degree and 1 hour = 60 minute. So, an hour hand will cover (1/2) x 60 = 30 degrees in 1 hour. In total, we have to find the degrees movement of an Hour clock in 6 hour so, it should be :

30 x 6= 180

(2) How many times are the hands of a clock at right angle in a day?

(a)22 (b) 24
(c)44 (d) 48

Sol - In 12 hours, they are at right angles 22 times so in 24 hours, they will at right angle at 44 times.

(3) How many times do the hands of a clock coincide in a day?

(a) 22 (b)24
(c) 44 (d) 48

Sol- Have a look on above show data, it is clearly written than hands of a clock overlap each other 22 times in a day so the answer will be 22.

(4) **A clock is started at noon. By 10 minutes past 5, the hour hand has turned through:**

(a) 145 degrees (b) 150 degrees
(c)155 degrees (d)160 degrees

Sol- 10 minutes past 5 means :: 5 : 10. It can also be said that it is 5 hours and 10 minutes.

For Minute Hand

(i) 1 min = 1/2 degree so 10 min = 5 degrees
(ii) 1 hour = 60 minute = 30 degree
(iii) 5 hour = 5 x 30 degree = 150 degree

But we have to find degree subtended by 5 Hours and 10 Minutes = 150 + 5 = 155 degrees

(5) The angle between the minute hand and the other hour hand of a clock when the time is 8:30 is

(a) 80 degrees (b) 75 degrees
(c) 60 degrees (d) 105 degrees

Sol - In this kind of question, use the formula which has been written above.

Degree required = (11/2) *30 - 30*8
= 15*11 - 240
=165 -240
= 75 degree

Note - *Here please note that, you can get answer in negative form but never consider negative answer because it applies mod value*

➔*Maximum and Minimum Value of special case:*

acosx+bsinx+c

Maximum value: $c+\sqrt{a^2+b^2}$

Minimum value: $c-\sqrt{a^2+b^2}$

➔ *Maximum value of acosx+bsinx-c* $=\sqrt{a^2+b^2-c}$

-Maximum and Minimum values of trigonometric expressions.

Trigonometric identities	Maximum Value	Minimum Value
(1) n sin θ / ncos θ	+n	-n
(2) n sin θ cos θ	$+\frac{n}{2}$	$-\frac{n}{2}$
(3) $\text{Sin}^n\,\theta$ - $\text{Cos}^n\,\theta$	$(1/2)^n$	$(-1/2)^n$
(4) a Sin θ + b Cos θ a Sin θ - b Cos θ	$+\sqrt{a^2+b^2}$	- $\sqrt{a^2+b^2}$
(5) a $\text{Sin}^2\,\theta$ + b $\text{Cos}^2\,\theta$	**Bigger Value** If a>b , the answer will be a If b>a, the answer will be b	**Smaller Value** If a>b , the answer will be b If b>a, the answer will be a
(6) a $\text{Sin}^2\,\theta$ + b $\text{Cosec}^2\,\theta$ a $\tan^2\theta$ + b $\cot^2\theta$ a $\cos^2\theta$ + b $\sec^2\theta$	Infinity	$2\sqrt{ab}$
(7) a sec2 θ + b cosec2 θ	Infinity	$(\sqrt{a} + \sqrt{b})^2$

Type -1

$$a \sin^2\theta + b\cos^2\theta$$

1. If **a>b**, maximum value = **a** and Minimum Value = **b**
2. If **b>a**, maximum value = **b** and Minimum Value = **a**

Example - **(1) Find the maximum and Minimum Value of 3 $\sin^2$x + 4 $\cos^2$x**

Sol- Here the 4> 3 so

- Maximum Value = 4
- Minimum Value = 3

Example - (2) **Find the maximum and Minimum Value of 5 $\sin^2$x + 3 $\cos^2$x**

Sol - Here 5>3

- Maximum Value = 5
- Minimum Value = 3

Type -2

$$a \sin\theta + b \cos\theta$$

1. Maximum Value = $\sqrt{a^2 + b^2}$
2. Minimum Value = - $\sqrt{a^2 + b^2}$

Example - (1) **Find the Maximum and Minimum Value of 3 sin x + 4 cos x**

Sol- If you find the question of this kind, apply the above formulae.

- Maximum Value = √ 9 + 16 = √ 25 = 5
- Minimum Value = - √ 9 + 16 = - √ 25 = - 5

(2) **Find the Maximum and Minimum Value of 3 sin x + 2 cos x**

Sol- If you find the question of this kind, apply the above formulae.

- Maximum Value = √ 9 + 4 = √ 13
- Minimum Value = - √ 9 + 4 = - √ 13

Type-3

In case of **$\sec^2$x, cosec^2x, $\cot^2$x and $\tan^2$x**, we cannot find the maximum value because they can have infinity as their maximum value. So in question containing these trigonometric identities, you will be asked to find the minimum values only. The typical question forms are listed below:

$$a \sin^2\theta + b\, \text{cosec}^2\theta$$

$$a \cos^2\theta + b \sec^2\theta$$

$$a \tan^2\theta + b \cot^2\theta$$

- Minimum Value = $2\sqrt{ab}$

Example- (1) **Find the Minimum value of 3 sin ^{2}x + 4 cosec ^{2}x**
sol - this equation is a typical example of our type-3 so apply the formula 2√ab so,

- Minimum Value = 2√ 3 x 4= 2√ 12

Example (2) **Find the Minimum value of 9 cos ^{2}x + 2 sec ^{2}x**
sol - this equation is a typical example of our type-3 so apply the formula 2√ab so,

- Minimum Value = $2\sqrt{9 \times 2} = 2\sqrt{18}$

Example (3) -**Find the Minimum value of 8 tan ^2x + 7 cot ^2x**

sol - this equation is a typical example of our type-3 so apply the formula $2\sqrt{ab}$ so,

- Minimum Value = $2\sqrt{8 \times 7} = 2\sqrt{56}$

Type-4

In this type, we will give you the explanation of question which are different from type-1, type-2 and type-3. If you find this kind of questions, you will have to convert these questions into type-1,2 or 3 by using trigonometric formulas

Example - Find the Minimum Value of Sec ^2x + cosec ^2x

Sol - 1 + tan ^2x **+ cosec ^2x** -- (Sec ^2x = 1 + tan ^2x)

= 1+ tan ^2x + 1 + cot ^2x --- (cosec ^2x = 1 + cot ^2x)

=2 + tan ^2x **+ cot ^2x**---apply type-3 formula

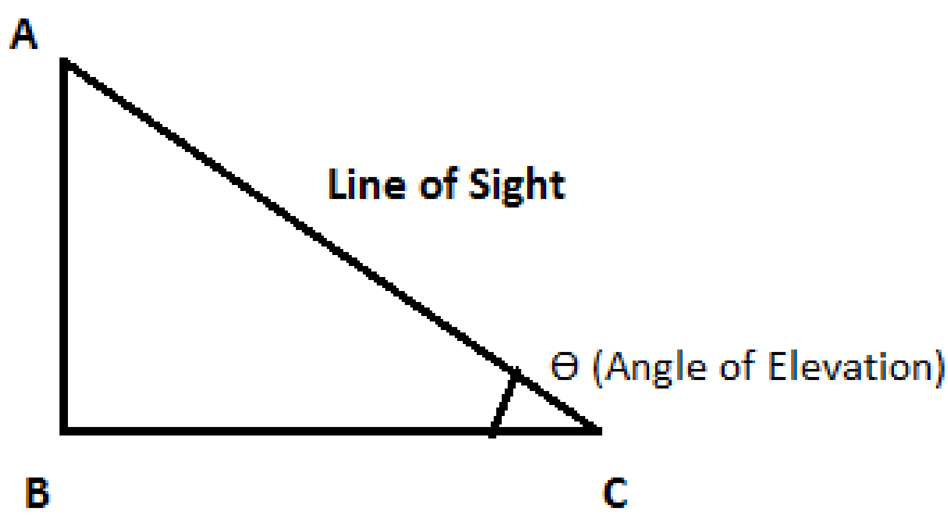
$=2 + 2\sqrt{1 \times 1}$

= 2 + 2

=4 (Answer)

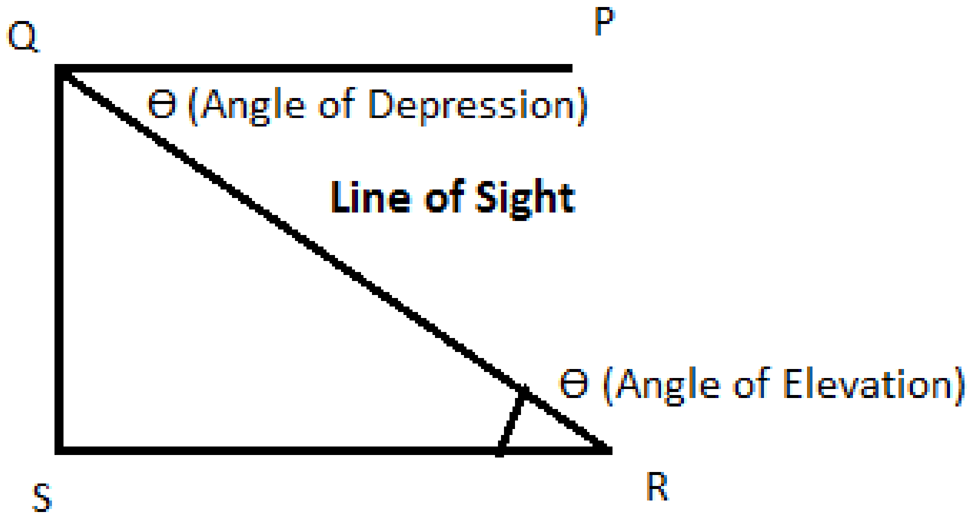
Important Notes & Short Tricks on Height & Distance

Angle of Elevation: Let AB be a tower/pillar/shell/minar/pole etc.) standing at any point C on the level ground is viewing at A.



The angle , which the line AC makes with the horizontal line BC is called angle of elevation .so angle ACB is angle of elevation.

Angle of Depression: If observer is at Q and is viewing an object R on the ground , then angle between PQ and QR is the angle of depression .so angle PQR is angle of depression.

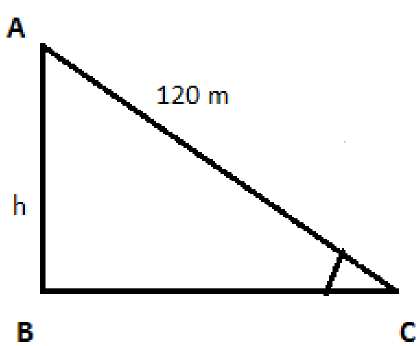


Numerically angle of elevation is equal to the angle of depression.
Both the angles are measured with the horizontal.

Angle	0°	30°	45°	60°	90°
Sin	0	½	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	½	0
Tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

1. **The thread of a kite is 120 m long and it is making 30° angular elevation with the ground .What is the height of the kite?**

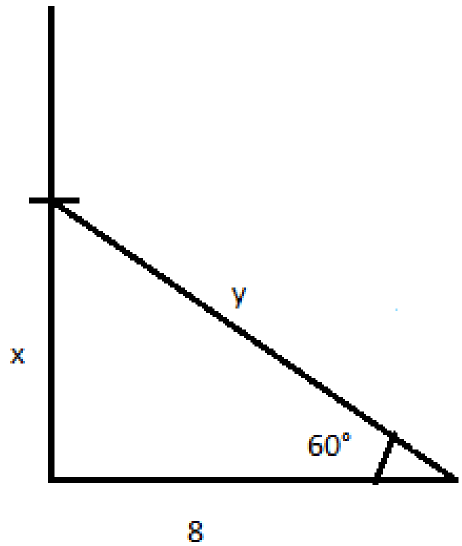
Solution:



Sin 30° = h/120
1/2 = h/120
h = 60m

2. **A tree bent by the wind .The top of the tree meets the ground at an angle of 60°.If the distance between the top of the foot be 8 m then what was the height of the tree?**

Solution:



tan 60° = x/8

√3 = x/8

x = 8 √3

y cos 60° = 8/y

1/2 = 8/y

y = 16

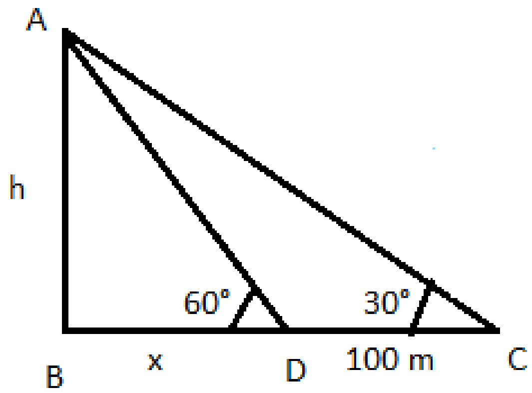
therefore height of the tree = x+y

= 8√3+16

= 8(√3+2)

3. **The angle of elevation of the top of a tower from a point on the ground is 30° . On walking 100m towards the tower the angle of elevation changes to 60° . Find the height of the tower.**

Solution:



In right triangle ABD,

tan 60° = h/x

√3 x = h

x = h/√3

Again , in right triangle ABC ,

tan 30 = h/x+100

1/√3 = h/x+100

√3 h = x+100

√3 h = h/√3 + 100

√3 h – h/√3 =100

3 h - h/√3 =100

2 h = 100√3

h = 50√3

By short trick:

d = h (cot ϴ1 - cot ϴ2)

h = 100/(√3-1/√3) = 100*√3/2 = 50√3

ϴ1 = small angle

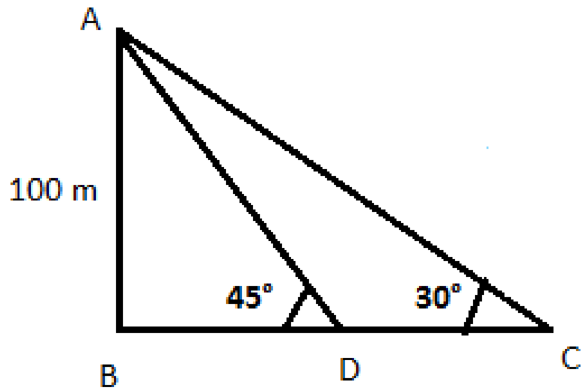
ϴ2 = large angle

d = distance between two places

h = height

4. **From the top of a temple near a river the angles of depression of both the banks of river are 45° & 30°. If the height of the temple is 100 m then find out the width of the river.**

Solution:



tan 45° = AB/BD

1 = 100/BD

BD = 100

tan 30 ° = AB/BC

1/√3 = 100/BC

BC = 100 √3

Width of the river , CD = BC - BD = 100 (√3-1)

When height of tower is 1 m then width of river is √3-1

Since height of tower is 100 m

Therefore ,

Width of river is 100(√3-1)m

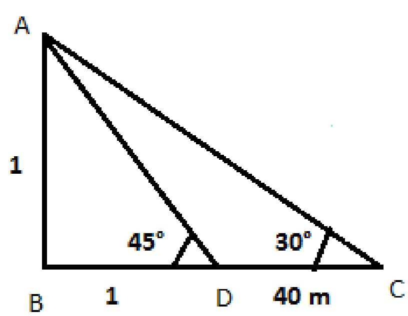
By short trick:

Same formula can be used in this question too i.e.

d= h (cot ϴ1 - cot ϴ2)

5. **The angle of elevation of the top of a tower from a point is 30 °. On walking 40 m towards the tower the angle changes to 45°.Find the height of the tower?**

Solution:



$\tan 45° = AB/BD$

$1 = AB/1$

Therefore $AB = 1$

$\tan 30° = AB/BC \Rightarrow 1/\sqrt{3} = 1/BC$

therefore $BC = \sqrt{3}$

Now $CD = \sqrt{3}-1$ m and height of tower is 1 m

$1 \text{ m} = 1/\sqrt{3}-1$

Therefore $40 \text{ m} = 1/\sqrt{3}-1 . 40 = 40/\sqrt{3}-1$

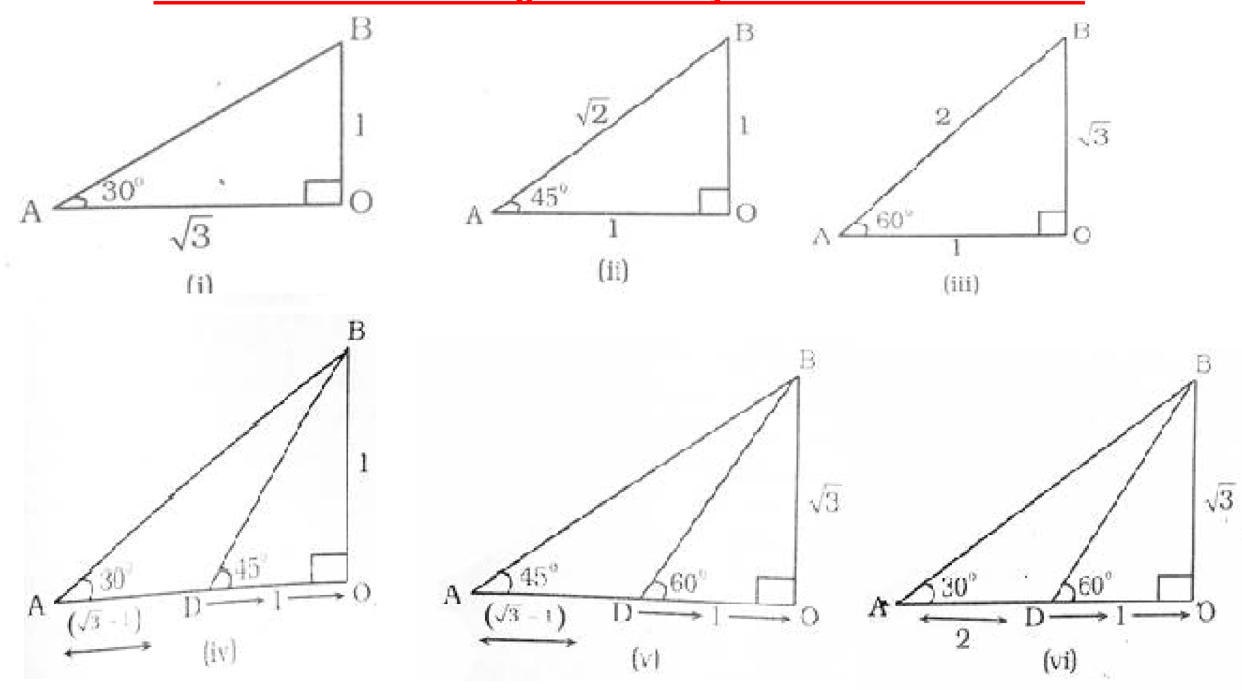
$= 20(\sqrt{3}+1)\text{m}$

By trick:

$40 = h(\sqrt{3}-1)$

$H = 40/(\sqrt{3}-1) = 20(\sqrt{3}+1)\text{m}$

Here are some ratio figure which you have to remember



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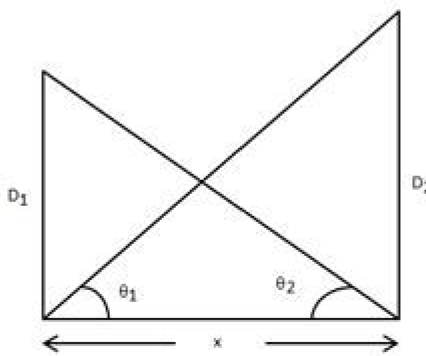
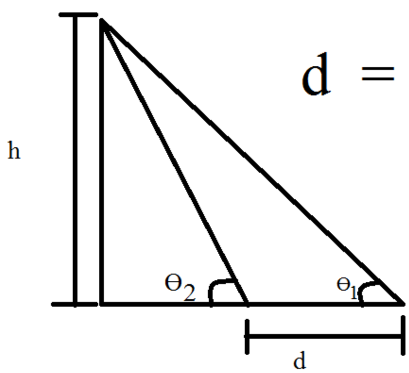
Important short trick are :



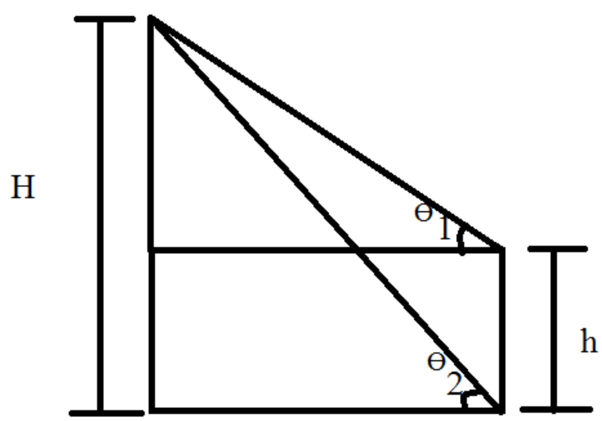
Note: $x = \sqrt{D_1 \cdot D_2}$ only when the sum of angle i.e $q_1 + q_2 = 90^0$



$$d = h(\cot \Theta_1 - \cot \Theta_2)$$

d = distance between two point

h = height of the tower or any object



$$H = \frac{h\cot\Theta 1}{\cot\Theta 1 - \cot\Theta 2}$$

H = height of tall tower or object

h = height of small tower or object

Some Important question are as follows:

Example 1: The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30°. The height of tower is :

(a) $\frac{500(\sqrt{3}-1)}{3}m$

(b) $\frac{500(\sqrt{3}+1)}{3}$

(c) 500

(d) $\frac{500\sqrt{3}}{3}m$

Ans. (d)

$\tan 30^\circ = \frac{BC}{AB} = \frac{h}{500}$

$\Rightarrow h = 500 \times \frac{1}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$

Short trick:

Solve it with ratio , as the angle of elevation is 30° then ratio between P: B: H is 1:√3:2 so √3= 500 then 1= 500/√3 and height is equal to $\frac{500\sqrt{3}}{3}m$

Example 2: The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45^0 and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is :

(a) 20 m

(b) 28.28 m

(c) 14.14 m

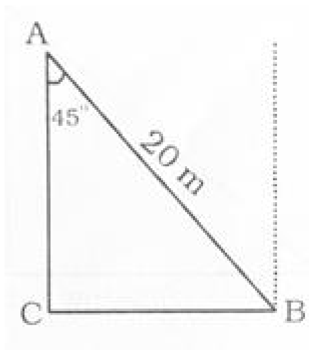
(d) 40 m

Ans. (c) 14.14 m

Solution:

Let A be the starting point and B, the end point of the swimmer. Then AB = 20m & $\angle BAC = 45^0$

$\sin 45^\circ = \frac{BC}{AB}$



$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{20} \Rightarrow BC = 10\sqrt{2} = 14.14m$

Short Method;

AS the angle of elevation is 45° then the ratio of P: B : H i.e. 1:1:√2

here √2 =20 then 1 =20/√2

Question 3: A man from the top a 50m high tower, sees a car moving towards the tower at an angle of depression of 30^0. After some time, the angle of depression becomes 60^0. The distance (in m) travelled by the car during this time is –

(a) $50\sqrt{3}$

(b) $\frac{50\sqrt{3}}{3}$

(c) $\frac{100\sqrt{3}}{3}$

(d) $100\sqrt{3}$

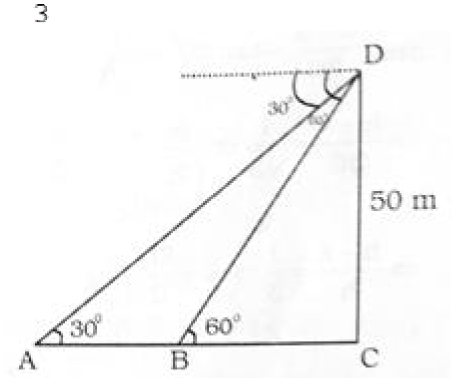
Ans. (c)

Solution:

$BC = \frac{50}{\sqrt{3}}, AC = 50\sqrt{3}$

AB = AC – BC

$= \frac{100\sqrt{3}}{3}$



Example 4: A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite side of the bank is 60^0. When he moves 50m away from the bank, the angle of elevation becomes 30^0. The height of the tree and width of river respectively are :

(a) $25, 25\sqrt{3}\,m$

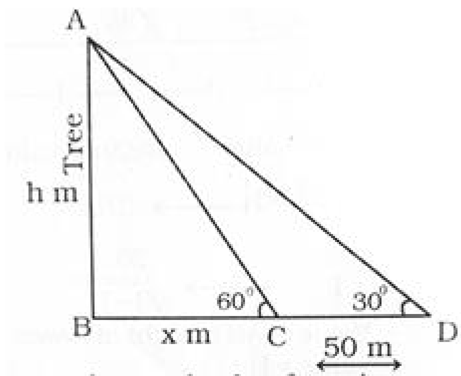
(b) $25\sqrt{3}, 25\sqrt{3}\,m$

(c) $25\sqrt{3}\,m, 25\,m$

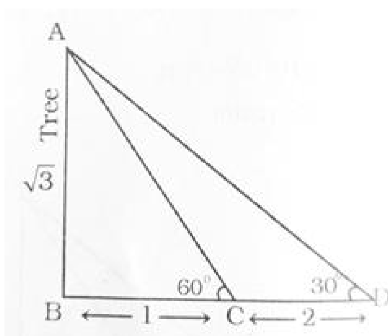
(d) None of these

Answer: c)

Solution:



(original value figure)



(Ratio value figure)

Ratio value original value

$2 \rightarrow 50$

$\therefore\ 1 \rightarrow 25$

and $\sqrt{3} \rightarrow 25\sqrt{3}$

height of the tree= h (ratio value = $\sqrt{3}$)= $25\sqrt{3}\ m$

and width of the river = x (ratio value = 1) = 25 m

Example 5: From the top of a pillar of height 80 m the angle of elevation and depression of the top and bottom of another pillar are 30^0 and 45^0respectively. The height of second pillar (in metre) is:

(a) $80\sqrt{3}$ m

(b) $\frac{80}{\sqrt{3}}(\sqrt{3}-1)m$

(c) $\frac{80}{\sqrt{3}}(\sqrt{3}+1)m$

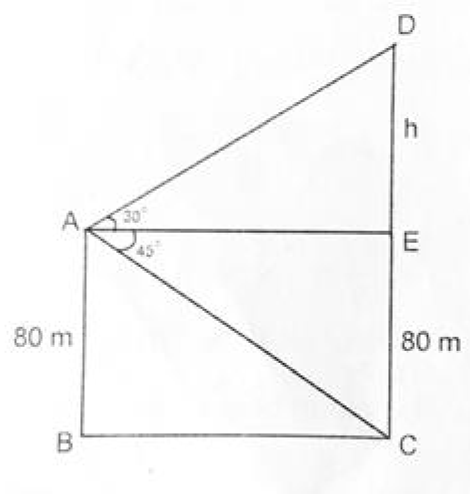
(d) $\frac{80}{\sqrt{3}}m$

Answer: (c)

Solution:

Let AB and CD are pillars.

Let DE = h



In $\Delta ADE, \tan 30^0 = \frac{h}{AE}$

$\Rightarrow AE = h\sqrt{3}$ ....(i)

$In\ \Delta ACE, \tan 45^0 = \frac{80}{AE}$

$\Rightarrow AE = 80 \Rightarrow h\sqrt{3} = 80\ [From(i)]$

$\Rightarrow h = \frac{80}{\sqrt{3}}$

Required height

$= 80 + \frac{80}{\sqrt{3}} = \frac{80}{\sqrt{3}}\left(\sqrt{3}+1\right)m$

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Some Important Results:

If α+β=90*

1) sinα=cos β

2) cosα=sinβ

3) tanα=cotβ

4) cotα=tanβ

5) secα=cosecβ

6) cosecα=secβ

Important Points:

If α+β=90*

1) sinα/cos β= cosα/sinβ=1

2) tanα/cotβ=cotα/tanβ= 1

3) secα/cosecβ= cosecα/secβ=1

-Results:

If A+B+C=π ,then:

1) tanA+tanB+tanC=tanA.tanB.tanC

2) cotA.cotB+cotB.cotC+cotC.cotA=1

3) $tan\frac{A}{2}.tan\frac{B}{2}+tan\frac{B}{2}.tan\frac{C}{2}+tan\frac{C}{2}.tan\frac{A}{2}=1$

4) $cot\frac{A}{2}+cot\frac{B}{2}+cot\frac{C}{2}=cot\frac{A}{2}.cot\frac{B}{2}.cot\frac{C}{2}$

-Domains of trigonometric Function:

Function	Domain
Y=sinx	R
Y=cosx	R
Y=tanx	R-{(2n+1)π/2 , nεZ}
Y=cotx	R-{nπ; nεZ}
Y=secx	R-{(2n+1)π/2 , nεZ}
Y=cosecx	R-{nπ ; nεZ}

-Domain of Identities:

Identity	Domain
$Sin^2x+cos^2x=1$	R
$1+tan^2x=sec^2x$	R-{(2n+1)π/2 , nεZ}
$1+cot^2x=cosec^2x$	R-{nπ; nεZ}

-Domain of Reciprocal:

Function	Domain
$Y=\frac{1}{cosx}$	R-{(2n+1)π/2 , nεZ}
$Y=\frac{1}{sinx}$	R-{nπ ; nεZ}
$Y=\frac{1}{sinx.cosx}$	$x\neq\frac{n\pi}{2}$, nεZ

-Application of Trigonometry:

Types of triangles:

Name of triangle	Sides	Angles
Scalene	3 sides are different	3 angles are different
Isosceles	2 sides are same	2 angles are same
Equilateral	3 sides are same	3 angles are same
Obtuse	May be scalene or isosceles but not equilateral	1 angle obtuse
Right	May be scalene or isosceles but not equilateral	1 angle is 90*
Oblique	May be scalene or isosceles but not equilateral	No angle of 90*

Acute	May be scalene, isosceles or equilateral	3 angles are acute(less than 90*)

-Types of Angles:
1) Acute angle (Less than 90*)
2) Obtuse angle (greater than 90*)
3) Straight angle (180*)
4) Reflex angle (360*)
5) Right angle (90*)
6) Complementary angle (Sum of two angles is 90*)
7) Supplementary angle (sum of two angles is 180*)

Trigonometric Triplets Trick:

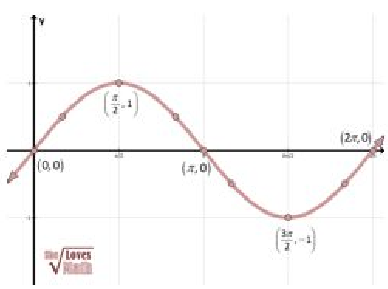
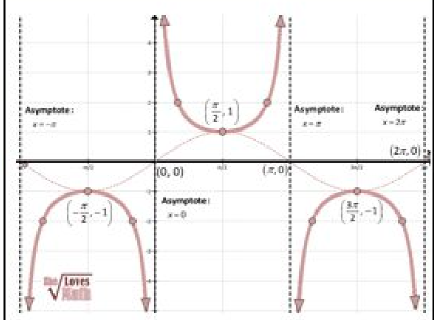
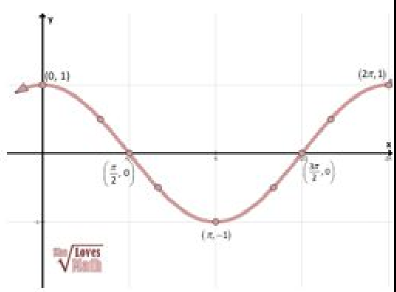
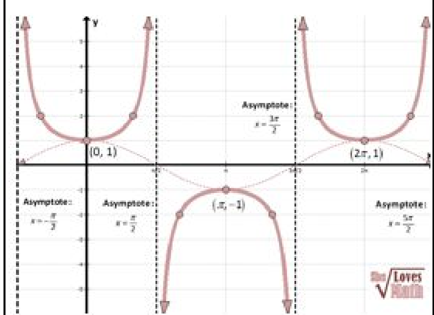
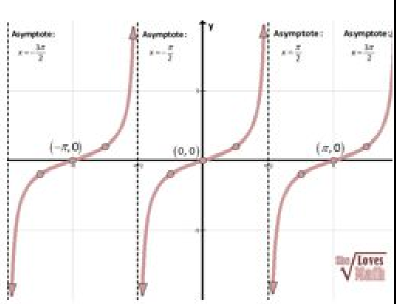
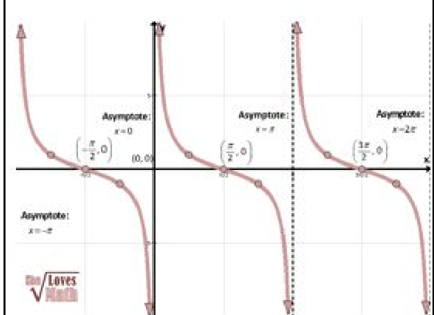
1) If Hypotenuse is an odd number then follow the following method.

For Example: Let us have number "9"
Step#1: Square it first of all i.e $9^2=81$
Step#2: Now divide 81/2 which is 40.5
Step#3: Now add and subtract 0.5 to 40.5 i.e:
➔ 40.5-0.5=40
➔ 40.5+0.5=41
So, the triplets are 9,40, 41

2) If hypotenuse is an even number then follow the following method:

For Example: Let us have number "8"
Step#1: Divide the number by 2 i.e 8/2 =4
Step#2:Now square it. i.e; $4^2=16$
Step#3: Now add and subtract 1 from it. i.e;
➔ 16-1=15
➔ 16+1=17
So, the triplets are 4,15,17.

Graph of Trigonometric Functions:

Parent Function	Graph	Parent Function	Graph
$y = \sin(x)$ Odd **Domain:** $(-\infty, \infty)$ **Range:** $[-1, 1]$ **Period:** 2π **Zeros:** $(\pi k, 0)$, $k \in$ Integers		$y = \csc(x)$ Odd **Domain:** $x \neq \pi k$ **Range:** $(-\infty, -1] \cup [1, \infty)$ **Asymptotes:** $x = \pi k$ **Period:** 2π **Zeros:** None	
$y = \cos(x)$ Even **Domain:** $(-\infty, \infty)$ **Range:** $[-1, 1]$ **Period:** 2π **Zeros:** $\left(\frac{\pi}{2} + \pi k, 0\right)$		$y = \sec(x)$ Even **Domain:** $x \neq \frac{\pi}{2} + \pi k$ **Range:** $(-\infty, -1] \cup [1, \infty)$ **Asymptotes:** $x = \frac{\pi}{2} + \pi k$ **Period:** 2π **Zeros:** None	
$y = \tan(x)$ Odd **Domain:** $x \neq \frac{\pi}{2} + \pi k$ **Range:** $(-\infty, \infty)$ **Asymptotes:** $x = \frac{\pi}{2} + \pi k$ **Period:** π **Zeros:** $(\pi k, 0)$		$y = \cot(x)$ Odd **Domain:** $x \neq \pi k$ **Range:** $(-\infty, \infty)$ **Asymptotes:** $x = \pi k$ **Period:** π **Zeros:** $\left(\frac{\pi}{2} + \pi k, 0\right)$	

-Domain and Range of Trigonometric function:

Function	Domain	Range
y = sin(x)	$-\infty < x < \infty$	$-1 \leq y \leq 1$
y = cos(x)	$-\infty < x < \infty$	$-1 \leq y \leq 1$
y = tan(x)	$-\infty < x < \infty$, $x \neq n\pi + \pi/2$; n is an integer	$-\infty < y < \infty$
y = csc(x)	$-\infty < x < \infty$, $x \neq n\pi$; n is an integer	$y \leq -1$ or $y \geq 1$
y = sec(x)	$-\infty < x < \infty$, $x \neq n\pi + \pi/2$; n is an integer	$y \leq -1$ or $y \geq 1$
y = cot(x)	$-\infty < x < \infty$, $x \neq n\pi$; n is an integer	$-\infty < y < \infty$

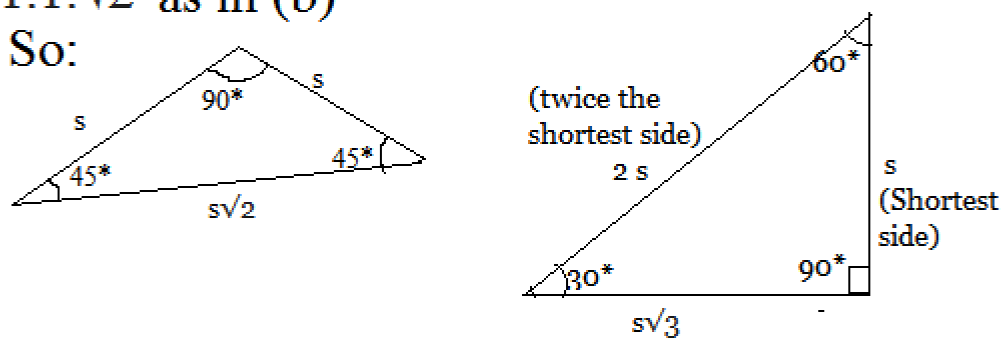
-Domain & Range of Inverse trigonometric Function:-

Function	Domain	Range
y = arcsin(x)	$-1 \le x \le 1$	$-\pi/2 \le y \le \pi/2$
y = arccos(x)	$-1 \le x \le 1$	$0 \le y \le \pi$
y = arctan(x)	$-\infty < x < \infty$	$-\pi/2 < y < \pi/2$
y = arccsc(x)	$x \le -1$ or $x \ge 1$	$-\pi/2 \le y \le \pi/2,\ y \ne 0$
y = arcsec(x)	$x \le -1$ or $x \ge 1$	$0 \le y \le \pi,\ y \ne \pi/2$
y = arccot(x)	$-\infty < x < \infty$	$0 < y < \pi$

1) If Angles are 30*:60*:90* then sides are 1:√3:2 as in (a)

2) If angles are 45*:45*:90* then sides are 1:1:√2 as in (b)

So:



TRICK FOR GUESSING THE RELATIONS OF EXERCISE 12.8

There are so many relations in exercise 12.8 and it is not possible to memorize all of them for the mcq's. But with the following trick it is very easy to guess the relation.

If a relation comes in mcq and you have to guess, then first assume that the relation is for equilateral triangle.

Cram the following values.

1. $\alpha + \beta + r = 60°$ (As it is equilateral triangle)
2. $s = \frac{3a}{2}$
3. $\Delta = \frac{\sqrt{3}}{4} a^2$
4. $r = \frac{a}{2\sqrt{3}}$
5. $R = \frac{a}{\sqrt{3}}$
6. $r_1 = r_2 = r_3 = \frac{\sqrt{3}}{2} a$

Now, whatever the relation is, you can easily answer.

Example:-

$r:R:r_1 = ?$

Put the values from above

$\frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2} a$

Multiply by $2\sqrt{3}$

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a : 2a : 3a

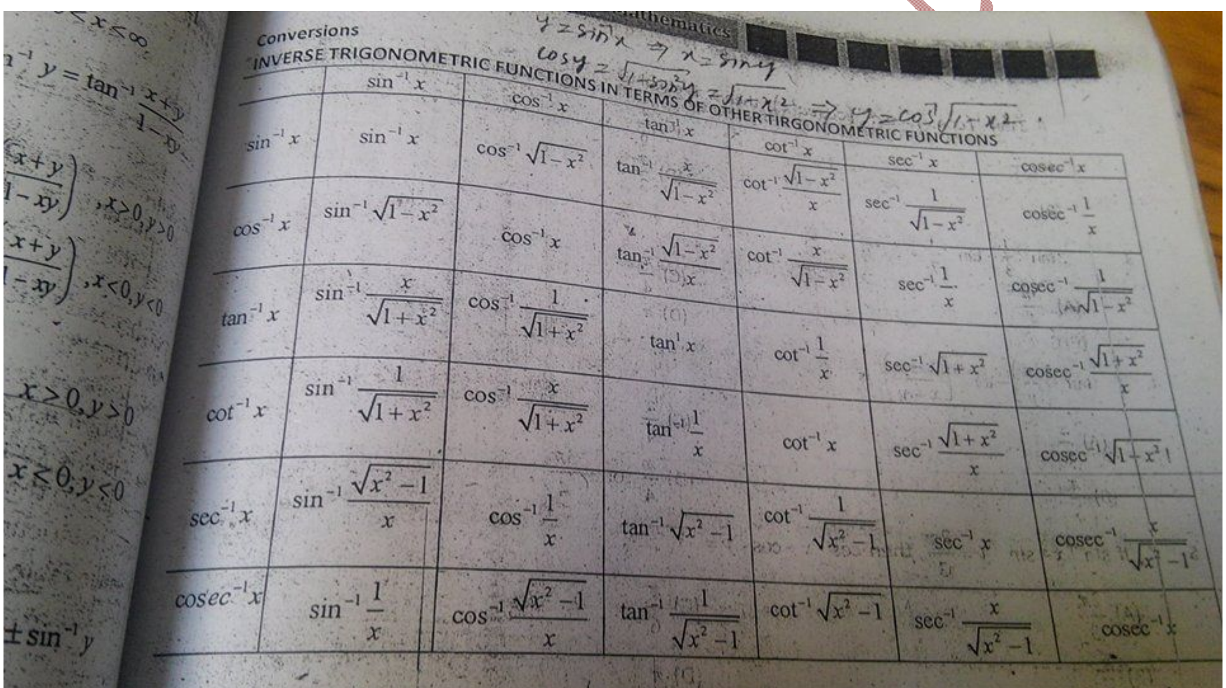
a will be cancelled

1 : 2 : 3

So, $r : R : r_1 = 1 : 2 : 3$

Practice this on all questions of 12.8

Important Points:



Conversions

INVERSE TRIGONOMETRIC FUNCTIONS IN TERMS OF OTHER TIRGONOMETRIC FUNCTIONS

	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$	$\cot^{-1}x$	$\sec^{-1}x$	$\text{cosec}^{-1}x$
$\sin^{-1}x$	$\sin^{-1}x$	$\cos^{-1}\sqrt{1-x^2}$	$\tan^{-1}\frac{x}{\sqrt{1-x^2}}$	$\cot^{-1}\frac{\sqrt{1-x^2}}{x}$	$\sec^{-1}\frac{1}{\sqrt{1-x^2}}$	$\text{cosec}^{-1}\frac{1}{x}$
$\cos^{-1}x$	$\sin^{-1}\sqrt{1-x^2}$	$\cos^{-1}x$	$\tan^{-1}\frac{\sqrt{1-x^2}}{x}$	$\cot^{-1}\frac{x}{\sqrt{1-x^2}}$	$\sec^{-1}\frac{1}{x}$	$\text{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1}x$	$\sin^{-1}\frac{x}{\sqrt{1+x^2}}$	$\cos^{-1}\frac{1}{\sqrt{1+x^2}}$	$\tan^{-1}x$	$\cot^{-1}\frac{1}{x}$	$\sec^{-1}\sqrt{1+x^2}$	$\text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}$
$\cot^{-1}x$	$\sin^{-1}\frac{1}{\sqrt{1+x^2}}$	$\cos^{-1}\frac{x}{\sqrt{1+x^2}}$	$\tan^{-1}\frac{1}{x}$	$\cot^{-1}x$	$\sec^{-1}\frac{\sqrt{1+x^2}}{x}$	$\text{cosec}^{-1}\sqrt{1+x^2}$
$\sec^{-1}x$	$\sin^{-1}\frac{\sqrt{x^2-1}}{x}$	$\cos^{-1}\frac{1}{x}$	$\tan^{-1}\sqrt{x^2-1}$	$\cot^{-1}\frac{1}{\sqrt{x^2-1}}$	$\sec^{-1}x$	$\text{cosec}^{-1}\frac{x}{\sqrt{x^2-1}}$
$\text{cosec}^{-1}x$	$\sin^{-1}\frac{1}{x}$	$\cos^{-1}\frac{\sqrt{x^2-1}}{x}$	$\tan^{-1}\frac{1}{\sqrt{x^2-1}}$	$\cot^{-1}\sqrt{x^2-1}$	$\sec^{-1}\frac{x}{\sqrt{x^2-1}}$	$\text{cosec}^{-1}x$

-Important Results:

1) When two sides and the angle opposite to one of them are given;
In this case, either no triangle or one triangle or two triangles are possible. For this reason, it is called ambiguous case.

Let b, c and B are given:

i) When B is acute and b<c sinB , no triangle is formed.

ii) When B is acute and b=c sinB, then only one triangle is formed which is right triangle.

iii) When B is acute and b>c sin B, then two triangles are formed if b<c and only one triangle is possible iff b≥c.

iv) When B is obtuse, there is no triangle iff b<c and only one triangle iff b>c.

Remarks:

1) The mid point of the hypotenuse of the right angle triangle is equidistant from the three vertices of triangle.

2) In a right angle triangle, the orthocenter coincides with vertex containing the right angle.

3) In a right triangle, the mid point of the hypotenuse is the circum center of the triangle.

4) Point of congruency of three altitude is orthocenter of triangle.

5) Point of congruency of three median is called centroid triangle.

Important points:

1) $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

2) 2S=a+b+c

3) $r:R:r_1:r_2:r_3=1:2:3:3:3$ (For Equilateral Triangle)

4) $r_1r_2+r_2r_3+r_3r_1= s^2$

5) $rr_1r_2r_3=\Delta^2$

6) $r_1+r_2+r_3-r=4R$

7) $r_1r_2r_3=rs^2$

8) r=R/2 (For equilateral triangle)

9) cosA/a= cosB/b =cosC/c (forms an equilateral triangle)

How to convert the radian into degree.

Example: 2π/3 radian into degree?

Solution: As we know that 1 rad= 180/π

so: 2π/3 * 180/π = 120*

How to convert degree into radian:

Example: 54*45’ into radian

Solution:

As we know that 1*=60’

so (54 . 45/60)*= (54.3/4)*= (219/4)*

As we know that 1*=π/180

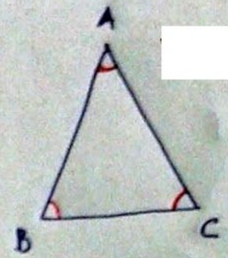
so; (219/4) (π/180)

=219π/720

= 0.955 (Answer)

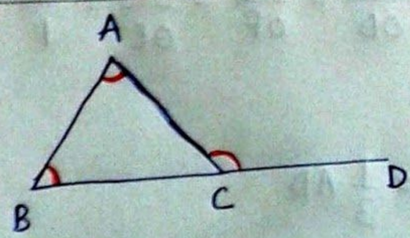
Properties of Triangle:-

TRIANGLE



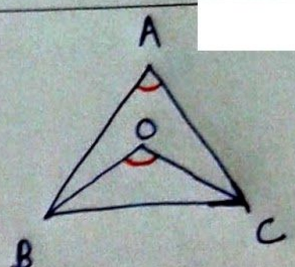
1) Sum of angles of a triangle is always 180°.

$$\angle A + \angle B + \angle C = 180^\circ$$



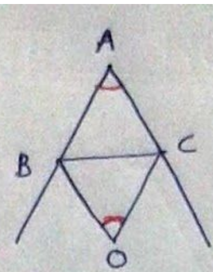
$$\angle A + \angle B = \angle ACD$$

II) Sum of any two interior angles of a triangle equal to exterior angle of third angle

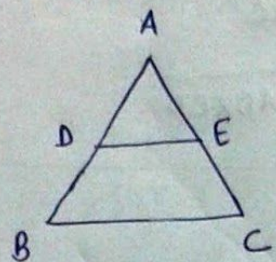


III) if O is a point called incenter of a triangle then,

$$\angle BOC = 90^\circ + \frac{\angle A}{2}$$



iv) $\angle BOC = 90^\circ - \dfrac{\angle A}{2}$

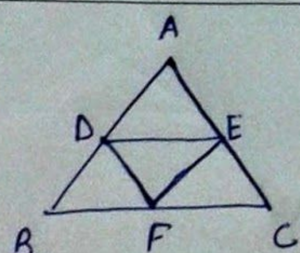


v) D is the mid point of AB

E is the mid point of AC

$DE \parallel BC$

then, $DE = \dfrac{BC}{2}$



vi) D, E and F is the mid point of AB, AC & BC resp.

$\angle DEF$ formed by joining these mid points.

then, Area of $\Delta DEF = \dfrac{1}{4}$ area of ΔABC

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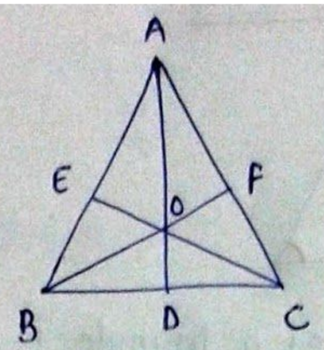


vii) The angle bisector of ΔABC meets at O.

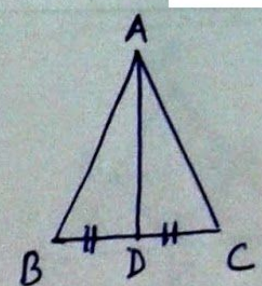
then $\dfrac{OA}{OD} = \dfrac{OB}{OF} = \dfrac{OC}{OE} = \dfrac{2}{1}$

and

$OD = \frac{1}{3} AD$

$OE = \frac{1}{3} CE$

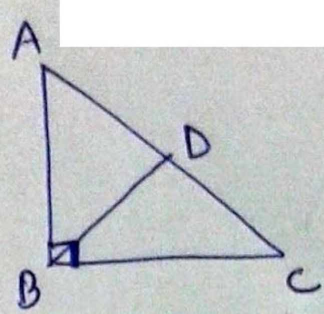
$OF = \frac{1}{3} BF$



viii) In ΔABC

$BD = CD$

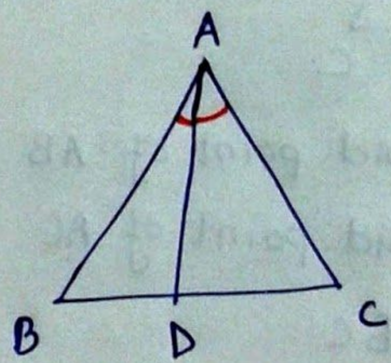
then, area of ΔABD = area of ΔACD



ix) If $\angle ABC = 90^\circ$

$BD \perp AC$

then, $BD = \sqrt{AD \times CD}$



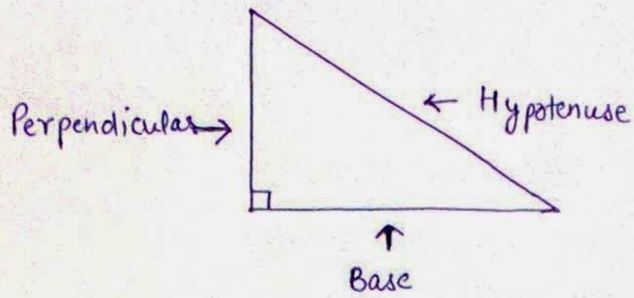
x) If ΔABC is equilateral triangle.

$AD \perp BC$

then, $\dfrac{AB^2}{AD^2} = \dfrac{4}{3}$

$\therefore\ 3AB^2 = 4AD^2$

Some Trigonometric Tricks:



The hypotenuse of a right-angled triangle is the side opposite its right angle.

first remember this formulas

$\sin\theta = \frac{P}{H}$ | $\operatorname{cosec}\theta = \frac{H}{P}$

$\cos\theta = \frac{B}{H}$ | $\sec\theta = \frac{H}{B}$

$\tan\theta = \frac{P}{B}$ | $\cot\theta = \frac{B}{P}$

P = Perpendicular
B = Base
H = Hypotenuse

Basically you have to learn only value of $\sin\theta$, $\cos\theta$ and $\tan\theta$

the value of others are opposite to that values.

Ex i) $\sin 30^\circ = \frac{1}{2}$ than, $\operatorname{cosec} 30^\circ$ value is opposite to that which is $\frac{2}{1} = 2$.

ii) $\tan 60^\circ = \frac{\sqrt{3}}{1}$ than, $\cot 60^\circ = \frac{1}{\sqrt{3}}$

Some basic identities of Trigonometry

$\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$

i) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

ii) $\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

iii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

1) $5\tan\theta = 4$, $\dfrac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta}$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

$\tan\theta = \dfrac{4}{5}$, $\sin\theta = 4$ & $\cos\theta = 5$

Putting these value

$$= \frac{5\times4 - 3\times5}{4 + 2\times5}$$

$$= \frac{20-15}{14} = \frac{5}{14}$$

ii) $\tan\theta = 1$, then $\dfrac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$

$\tan\theta = 1 \cdot \theta = 45^\circ$

$\sin 45 = \dfrac{1}{\sqrt2}$, $\cos 45^\circ = \dfrac{1}{\sqrt2}$

Putting above

$$= \frac{8\times\frac{1}{\sqrt2} + 5\times\frac{1}{\sqrt2}}{\left(\frac{1}{\sqrt2}\right)^3 - 2\left[\frac{1}{\sqrt2}\right]^3 + 7\frac{1}{\sqrt2}}$$

$$= \frac{\frac{8}{\sqrt2} + \frac{5}{\sqrt2}}{\frac{1}{2\sqrt2} - \frac{2}{2\sqrt2} + \frac{7}{\sqrt2}} = \frac{\frac{13}{\sqrt2}}{\frac{2\sqrt2}{13}} = \frac{13}{\sqrt2}\times\frac{2\sqrt2}{13} = 2$$

iii) $\tan\theta + \cot\theta = 2$ then, $\tan^5\theta + \cot^{10}\theta = ?$

$\tan 45^\circ = 1 = \cot 45^\circ$

Putting value

$\tan(1)^5 + \cot(1)^{10} = 2$

iv) $\sin\theta + \operatorname{cosec}\theta = 2$ then, $\sin^{100}\theta + \operatorname{cosec}^{100}\theta = ?$

$\boxed{\sin^n\theta + \operatorname{cosec}^n\theta = 2}$

then,

$\sin^{100}\theta + \operatorname{cosec}^{100}\theta = \underline{\underline{2}}$

v) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then $2\cos^2\theta - 1 = ?$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\boxed{\sin^2\theta = 1 - \cos^2\theta}$

$\cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$2\cos^2\theta - 1 = \underline{\underline{\frac{2}{3}}}$

Trick to find the Numerical value of trigonometric series:

Q#1:

sin1*. Sin2*. Sin3*........ sin270=?

A) -1

B) 0 (Correct)

C) 1

D) None of these

Solution:

As we know that:

sin180*=0

The value of expression is 0. (Answer)

Q#2:

$\sin^2 1^* + \sin^2 2^* + \ldots\ldots + \sin^2 89 = ?$

Solution:

Trick: (Last angle/1st angle)= Product/2 = Answer

In this case;

89/1= 89/2= 44 ½ (Answer)

Q#3:

$\cos^2 5 + \cos^2 10 + \ldots\ldots \cos^2 90 = ?$

Solution:

Using the trick: (Last angle/1st angle)= Product/2= Answer.

In this case:

As $\cos^2 90 = 0$

so: $\cos^2 5 + \ldots\ldots + \cos^2 85 = ?$

Now; 85/5= 17/2= 8 ½ (Answer)

Q#4:

The value of tan5*. Tan10*....tan85*=?

Solution:

As we know that:

$\tan\alpha=\cot\beta$ when $\alpha+\beta=90^*$

so that:

tan5=cot85

tan10= cot80

....

so that tan45 is one which can't be paired with anyone so that

tan45*=1 (Answer)..

Q#5:

$\sin\pi/9 . \sin2\pi/9 .\sin\pi/3 .\sin4\pi/9=?$

A) ¾ B) 3/5 C) 3/17 D) 3/16 (Correct)

Solution:

~Trick:

(Only, if one value is known)

Step#1: (Trigonometric value of angle)= P

Step#2: Take the square of its numerator.

Step#3: Take the power of denominator to the number of angles involve in the notation.

Using the trick:

S#1: $\sin60= \frac{\sqrt{3}}{2}$

S#2: Squaring the numerator i.e $(\sqrt{3})^2$

S#3: Take the denominator power as the number of angles in notation are 4 so: $(2)^4=16$

Final result is 3/16 (Answer)

Q#6:

cos20.cos40. cos60.cos80=?

Solution:

As we know that cos60=1/2
so using the trick:
= $(1)^2/2^4$ = 1/16 (Answer)
Q#7: sin10.sin30.sin50.sin70=?
Solution: As we know that: sin30=1/2
So using the trick:
= $(1)^2/2^4$=1/16 (Answer)
Q#8:
cos20.cos40.cos80=?
Solution:
There is a type where the exact angle is not mentioned in the question so in that case we make it by adding any two angles whose sum is known value. If the sum is from 30, 60, 90 etc then take the $1/2^n$ for final value whereas n=no. of angles involve in notation.
In this case; 20+40=60 so $½^3$=1/8 (Answer)
Q#9:
cosπ/5.cos2π/5.cos3π/5. Cos4π/5=?
Solution:
As 2π/5+ 3π/5= 5π/5= π
As we know that no. of angles in the notation are 4 so $1/2^4$=1/16 (Answer).

Some Basic Trigonometric Formulas:

Relation Between l & θ

$$l = r\theta$$

Conversion of Radian

$$1^\circ = \frac{\pi}{180}\,radian \quad \& \quad 1\,radian = \frac{180^\circ}{\pi}$$

Fundamental Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$

Signs of Trigonometric Function

$\sin$ & $\csc$ $+ve$	$all + ve$
II-Quad.	I-Quad.+
III-Quad.	IV-Quad.
$\tan$ & $\cot + ve$	$\cos$ & $\sec + ve$

Values of Trigonometric Functions

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
$\sin$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Fundamental Laws of Trigonometry

1. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double Angle Identities

1. $\sin 2\alpha = 2\sin\alpha\cos\alpha$
2. $\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$
3. $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

Triple Angle Identities

1. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$
2. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
3. $\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$

Sum, Difference & Product

1. $2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
2. $2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
3. $2\cos\alpha\cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
4. $-2\sin\alpha\sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$
5. $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$
6. $\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$
7. $\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$
8. $\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$

Domain & Range of Trig. Functions

Functions	Domain	Range
$y = \sin x$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \le y \le 1$
$y = \tan x$	$-\infty < x < +\infty$, $x \ne \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty$, $x \ne \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty$, $x \ne n\pi$	$y \ge 1$ or $y \le -1$
$y = \csc x$	$-\infty < x < +\infty$, $x \ne n\pi$	$y \ge 1$ or $y \le -1$

where $n \in z$.

Period of Trigonometric Functions

Function	Period
$\sin a\theta$	$\frac{2\pi}{a}$
$\cos a\theta$	
$\csc a\theta$	
$\sec a\theta$	
$\tan a\theta$	$\frac{\pi}{a}$
$\cot a\theta$	

The Law of Sines

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

The Law of Cosine

1. $a^2=b^2+c^2-2bc\cos\alpha$
2. $b^2=c^2+a^2-2ca\cos\beta$
3. $c^2=a^2+b^2-2ab\cos\gamma$

The Law of Tangents

1. $\frac{a-b}{a+b}=\frac{\tan(\frac{\alpha-\beta}{2})}{\tan(\frac{\alpha+\beta}{2})}$
2. $\frac{b-c}{b+c}=\frac{\tan(\frac{\beta-\gamma}{2})}{\tan(\frac{\beta+\gamma}{2})}$
3. $\frac{c-a}{c+a}=\frac{\tan(\frac{\gamma-\alpha}{2})}{\tan(\frac{\gamma+\alpha}{2})}$

Half Angle Formulas

1. $\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$
2. $\sin\frac{\beta}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}$
3. $\sin\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$
4. $\cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}}$
5. $\cos\frac{\beta}{2}=\sqrt{\frac{s(s-b)}{ca}}$
6. $\cos\frac{\gamma}{2}=\sqrt{\frac{s(s-c)}{ab}}$
7. $\tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
8. $\tan\frac{\beta}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
9. $\tan\frac{\gamma}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Area of Triangles (Δ)

1. $\Delta=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ca\sin\beta=\frac{1}{2}ab\sin\gamma$
2. $\Delta=\frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}=\frac{b^2\sin\gamma\sin\alpha}{2\sin\beta}=\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
3. $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

   where $s=\frac{a+b+c}{2}$

Circum – Circle (R)

1. $R=\frac{a}{2\sin\alpha}=\frac{b}{2\sin\beta}=\frac{c}{2\sin\gamma}$
2. $R=\frac{abc}{4\Delta}$

In – Circle (r)

$$r=\frac{\Delta}{s}$$

Escribed – Circle

$$r_1=\frac{\Delta}{s-a},\ r_2=\frac{\Delta}{s-b},\ r_3=\frac{\Delta}{s-c}$$

Inverse Trigonometr Formulas

$\sin^{-1}A+\sin^{-1}B=\sin^{-1}(A\sqrt{1-B^2}+B\sqrt{1-A^2})$

$\sin^{-1}A-\sin^{-1}B=\sin^{-1}(A\sqrt{1-B^2}+B\sqrt{1-A^2})$

$\cos^{-1}A+\cos^{-1}B=\cos^{-1}(AB-\sqrt{(1-A^2)(1-B^2)})$

$\cos^{-1}A-\cos^{-1}B=\cos^{-1}(AB+\sqrt{(1-A^2)(1-B^2)})$

$\tan^{-1}A+\tan^{-1}B=\tan^{-1}(\frac{A+B}{1-AB})$

$\tan^{-1}A-\tan^{-1}B=\tan^{-1}(\frac{A-B}{1+AB})$

$2\tan^{-1}A=\tan^{-1}(\frac{2A}{1-A^2})$

Principal Trig. Functions

Function	Domain	Range
$y=\sin x$	$-\frac{\pi}{2}\le x\le\frac{\pi}{2}$	$-1\le y\le 1$
$y=\sin^{-1}x$	$-1\le y\le 1$	$-\frac{\pi}{2}\le x\le\frac{\pi}{2}$
$y=\cos x$	$0\le x\le\pi$	$-1\le y\le 1$
$y=\cos^{-1}x$	$-1\le y\le 1$	$0\le x\le\pi$
$y=\tan x$	$-\frac{\pi}{2}<x<\frac{\pi}{2}$	R
$y=\tan^{-1}x$	R	$-\frac{\pi}{2}<x<\frac{\pi}{2}$
$y=\cot x$	$0<x<\pi$	R
$y=\cot^{-1}x$	R	$0<x<\pi$
$y=\sec x$	$[0,\pi], x\ne\frac{\pi}{2}$	$y\le -1$ or $y\ge 1$
$y=\sec^{-1}x$	$x\le -1$ or $x\ge 1$	$[0,\pi], y\ne\frac{\pi}{2}$
$y=\csc x$	$[-\frac{\pi}{2},\frac{\pi}{2}], x\ne 0$	$y\le -1$ or $y\ge 1$
$y=\csc^{-1}x$	$x\le -1$ or $x\ge 1$	$[-\frac{\pi}{2},\frac{\pi}{2}], y\ne 0$

Best of Luck

nano physics point timargara dir lower
0301487450,03458868826

Angles: α, β
Real numbers (coordinates of a point): x, y
Whole number: k

4.1 Radian and Degree Measures of Angles

362. $1 \text{ rad} = \frac{180°}{\pi} \approx 57°17'45''$

363. $1° = \frac{\pi}{180} \text{ rad} \approx 0.017453 \text{ rad}$

364. $1' = \frac{\pi}{180 \cdot 60} \text{ rad} \approx 0.000291 \text{ rad}$

365. $1'' = \frac{\pi}{180 \cdot 3600} \text{ rad} \approx 0.000005 \text{ rad}$

366.

Angle (degrees)	0	30	45	60	90	180	270	360
Angle (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

4.2 Definitions and Graphs of Trigonometric Functions

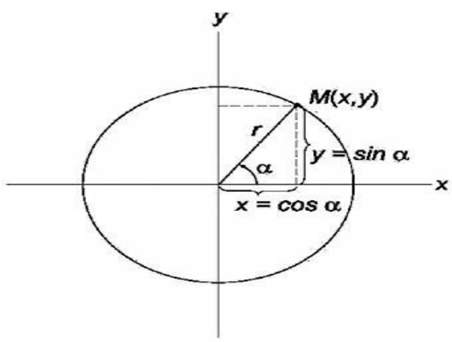


Figure 58.

367. $\sin\alpha = \frac{y}{r}$

368. $\cos\alpha = \frac{x}{r}$

369. $\tan\alpha = \frac{y}{x}$

370. $\cot\alpha = \frac{x}{y}$

371. $\sec\alpha = \frac{r}{x}$

372. $\operatorname{cosec}\alpha = \frac{r}{y}$

373. Sine Function
$y = \sin x,\ -1 \le \sin x \le 1.$

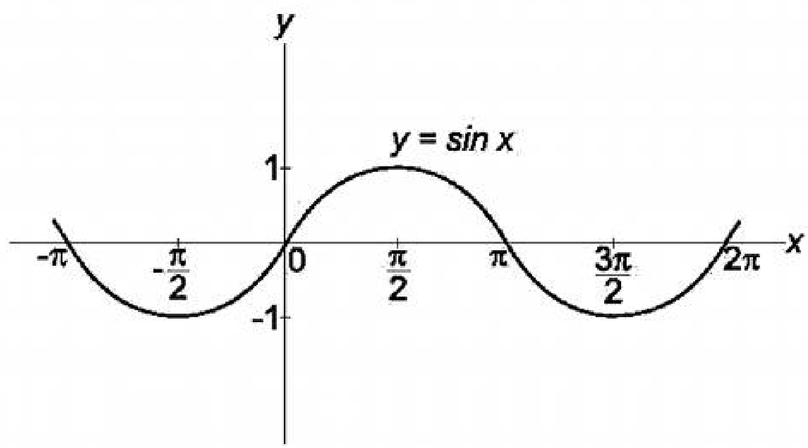


Figure 59.

374. Cosine Function
$y = \cos x,\ -1 \le \cos x \le 1.$

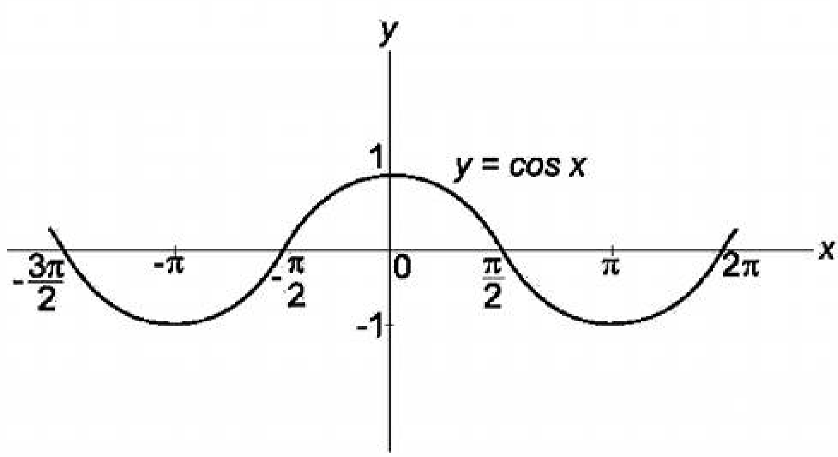


Figure 60.

375. Tangent Function

$y = \tan x,\ x \neq (2k+1)\frac{\pi}{2},\ -\infty \leq \tan x \leq \infty.$

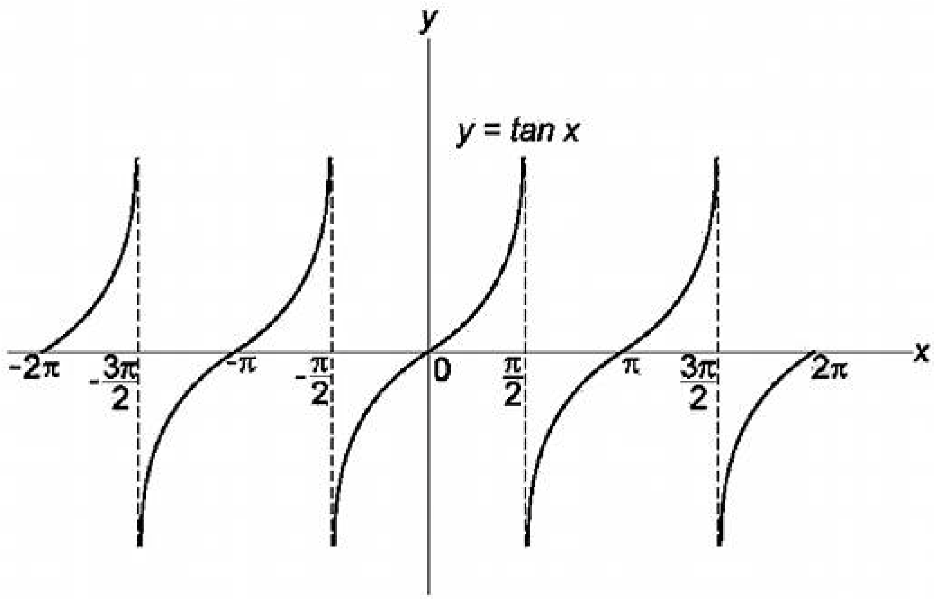
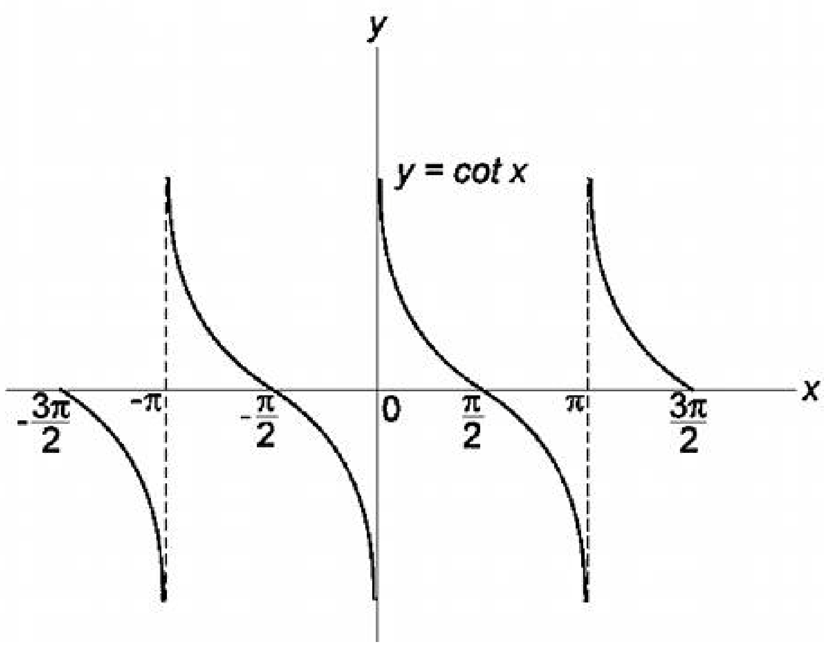


Figure 61.

376. Cotangent Function

$y = \cot x,\ x \neq k\pi,\ -\infty \leq \cot x \leq \infty.$



377. Secant Function

$y = \sec x\,,\ x \neq (2k+1)\frac{\pi}{2}.$

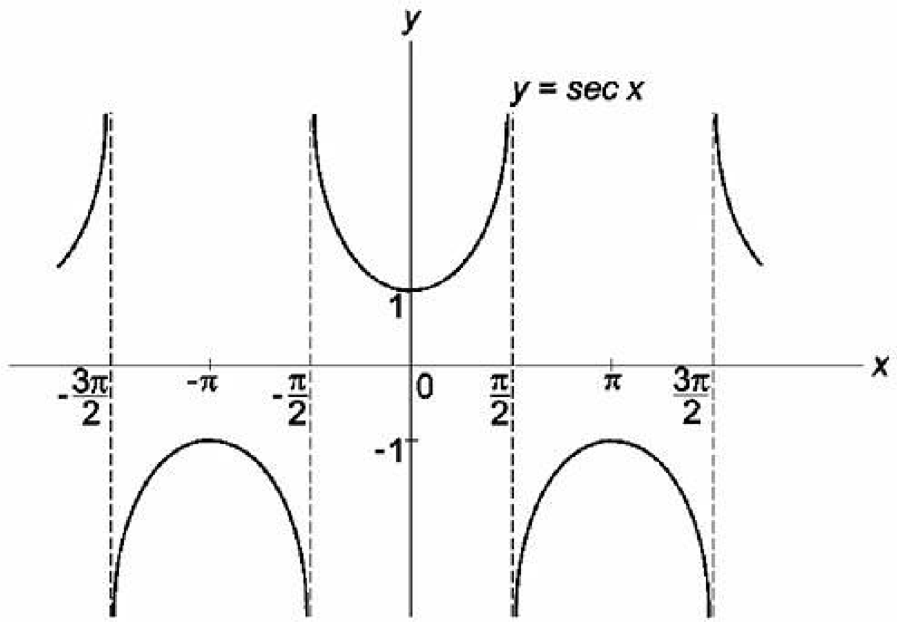


Figure 63.

378. Cosecant Function

$y = \operatorname{cosec} x\,,\ x \neq k\pi.$

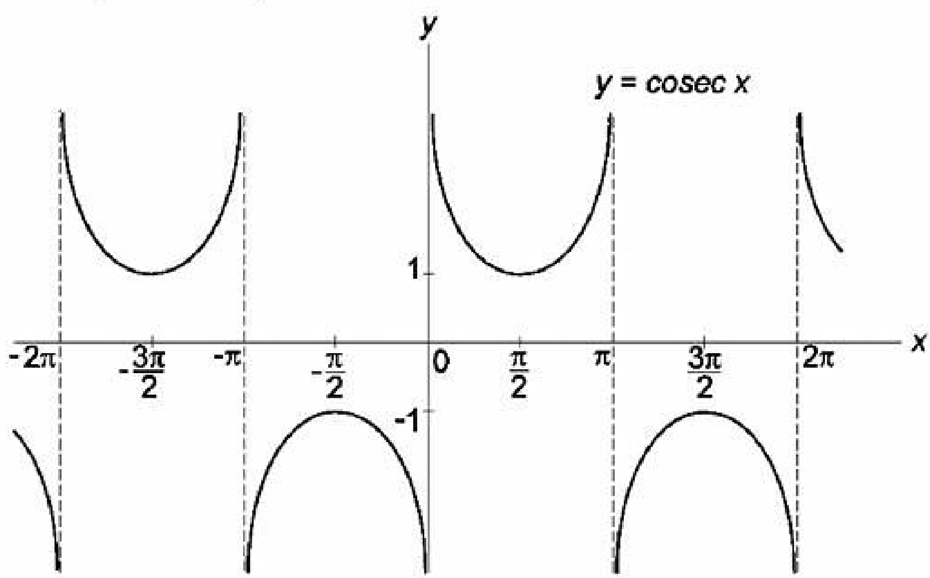


Figure 64.

4.3. Signs of Trigonometric Functions

379.

Quadrant	Sin α	Cos α	Tan α	Cot α	Sec α	Cosec α
I	+	+	+	+	+	+
II	+					+
III			+	+		
IV		+			+	

380.

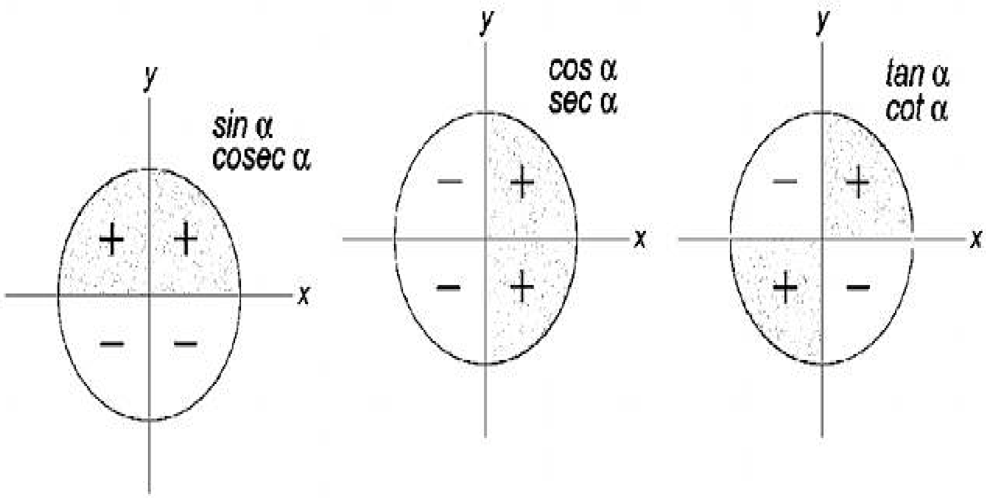


Figure 65.

382.

$\alpha°$	α rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
15	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
36	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
54	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
72	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
75	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

4.5 Most Important Formulas

383. $\sin^2\alpha + \cos^2\alpha = 1$

384. $\sec^2\alpha - \tan^2\alpha = 1$

385. $\csc^2\alpha - \cot^2\alpha = 1$

386. $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

387. $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

4.7 Periodicity of Trigonometric Functions

392. $\sin(\alpha \pm 2\pi n) = \sin\alpha$, period 2π or $360°$.

393. $\cos(\alpha \pm 2\pi n) = \cos\alpha$, period 2π or $360°$.

394. $\tan(\alpha \pm \pi n) = \tan\alpha$, period π or $180°$.

395. $\cot(\alpha \pm \pi n) = \cot\alpha$, period π or $180°$.

4.8 Relations between Trigonometric Functions

396. $\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \pm\sqrt{\frac{1}{2}(1-\cos 2\alpha)} = 2\cos^2\left(\frac{\alpha}{2}-\frac{\pi}{4}\right)-1$

$$= \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

397. $\cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \pm\sqrt{\frac{1}{2}(1+\cos 2\alpha)} = 2\cos^2\frac{\alpha}{2}-1$

$$= \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

398. $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \pm\sqrt{\sec^2\alpha - 1} = \frac{\sin 2\alpha}{1+\cos 2\alpha} = \frac{1-\cos 2\alpha}{\sin 2\alpha}$

$$= \pm\sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

399. $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha} = \pm\sqrt{\csc^2\alpha - 1} = \dfrac{1+\cos 2\alpha}{\sin 2\alpha} = \dfrac{\sin 2\alpha}{1-\cos 2\alpha}$

$$= \pm\sqrt{\frac{1+\cos 2\alpha}{1-\cos 2\alpha}} = \frac{1-\tan^2\dfrac{\alpha}{2}}{2\tan\dfrac{\alpha}{2}}$$

400. $\sec\alpha = \dfrac{1}{\cos\alpha} = \pm\sqrt{1+\tan^2\alpha} = \dfrac{1+\tan^2\dfrac{\alpha}{2}}{1-\tan^2\dfrac{\alpha}{2}}$

401. $\csc\alpha = \dfrac{1}{\sin\alpha} = \pm\sqrt{1+\cot^2\alpha} = \dfrac{1+\tan^2\dfrac{\alpha}{2}}{2\tan\dfrac{\alpha}{2}}$

4.9 Addition and Subtraction Formulas

402. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

403. $\sin(\alpha-y) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

404. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

405. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

406. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

407. $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

408. $\cot(\alpha+\beta)=\dfrac{1-\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

409. $\cot(\alpha-\beta)=\dfrac{1+\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

4.10 Double Angle Formulas

410. $\sin 2\alpha = 2\sin\alpha\cdot\cos\alpha$

411. $\cos 2\alpha = \cos^2\alpha-\sin^2\alpha = 1-2\sin^2\alpha = 2\cos^2\alpha-1$

412. $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2}{\cot\alpha-\tan\alpha}$

413. $\cot 2\alpha = \dfrac{\cot^2\alpha-1}{2\cot\alpha} = \dfrac{\cot\alpha-\tan\alpha}{2}$

414. $\sin 3\alpha = 3\sin\alpha-4\sin^3\alpha = 3\cos^2\alpha\cdot\sin\alpha-\sin^3\alpha$

415. $\sin 4\alpha = 4\sin\alpha\cdot\cos\alpha-8\sin^3\alpha\cdot\cos\alpha$

416. $\sin 5\alpha = 5\sin\alpha-20\sin^3\alpha+16\sin^5\alpha$

417. $\cos 3\alpha = 4\cos^3\alpha-3\cos\alpha = \cos^3\alpha-3\cos\alpha\cdot\sin^2\alpha$

418. $\cos 4\alpha = 8\cos^4\alpha-8\cos^2\alpha+1$

419. $\cos 5\alpha = 16\cos^5\alpha-20\cos^3\alpha+5\cos\alpha$

420. $\tan 3\alpha = \dfrac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$

4.12 Half Angle Formulas

426. $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

427. $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

428. $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \csc\alpha - \cot\alpha$

429. $\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha} = \csc\alpha + \cot\alpha$

4.14 Transforming of Trigonometric Expressions to Product

434. $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

435. $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

436. $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

437. $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

-Range of Modulus of trigonometric function:

Ranges of: |sinx|≤1 , |cosx|≤1 , |secx|≥1 and |cscx|≥1

-Period of special cases:

The period of: 1)|sinx|+|cosx|=π/2

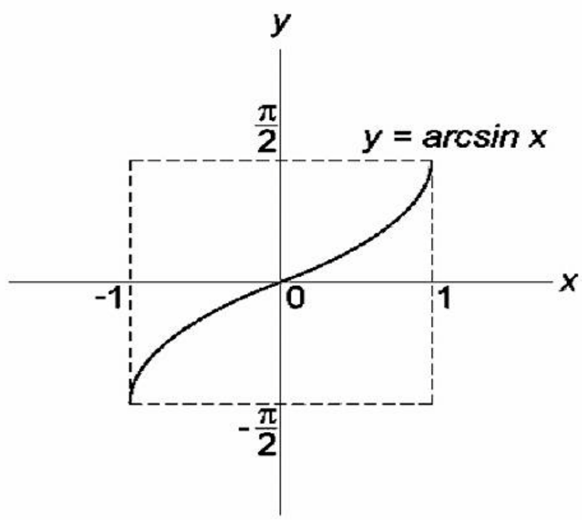
2) |tanx|+|cotx|=π/2

3)|secx|+|cscx|=π/2

4.17 Graphs of Inverse Trigonometric Functions

466. Inverse Sine Function

$$y = \arcsin x,\ -1 \le x \le 1,\ -\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}.$$



467. Inverse Cosine Function

$y = \arccos x$, $-1 \le x \le 1$, $0 \le \arccos x \le \pi$.

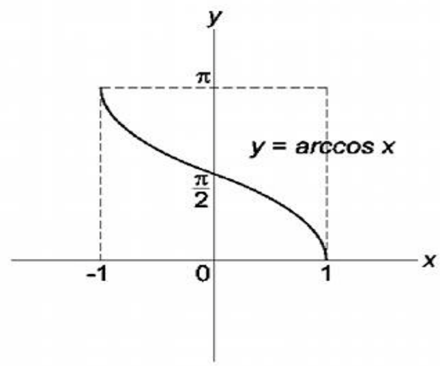
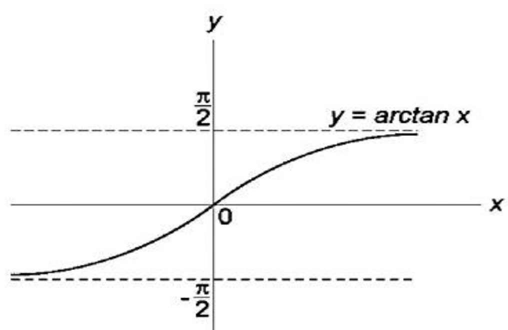


Figure 67.

468. Inverse Tangent Function

$y = \arctan x$, $-\infty \le x \le \infty$, $-\dfrac{\pi}{2} < \arctan x < \dfrac{\pi}{2}$.



469. Inverse Cotangent Function

$y = \operatorname{arccot} x$, $-\infty \le x \le \infty$, $0 < \operatorname{arccot} x < \pi$.

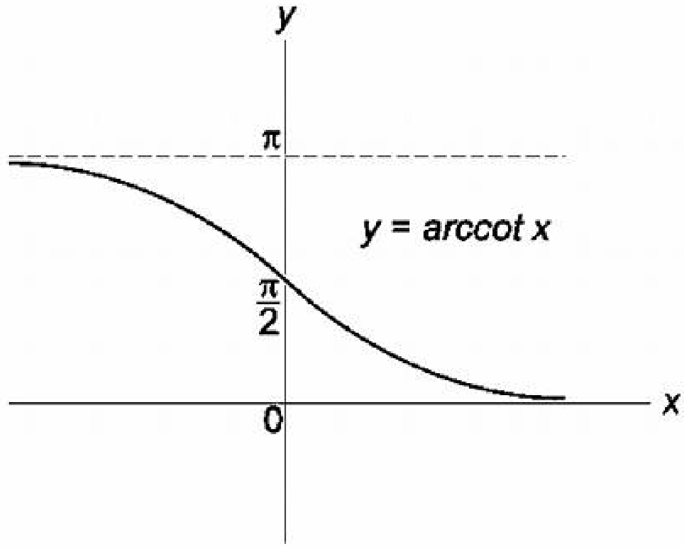


Figure 69.

470. Inverse Secant Function

$y = \operatorname{arcsec} x,\ x \in (-\infty, -1] \cup [1, \infty),\ \operatorname{arcsec} x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right].$

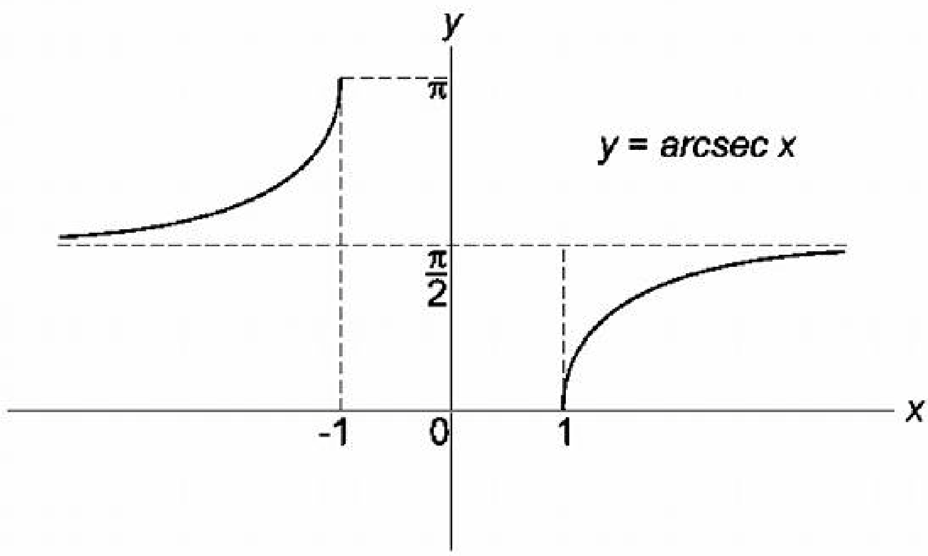


Figure 70.

471. Inverse Cosecant Function

$y = \operatorname{arccsc} x,\ \ x \in (-\infty, -1] \cup [1, \infty),\ \ \operatorname{arccsc} x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right].$

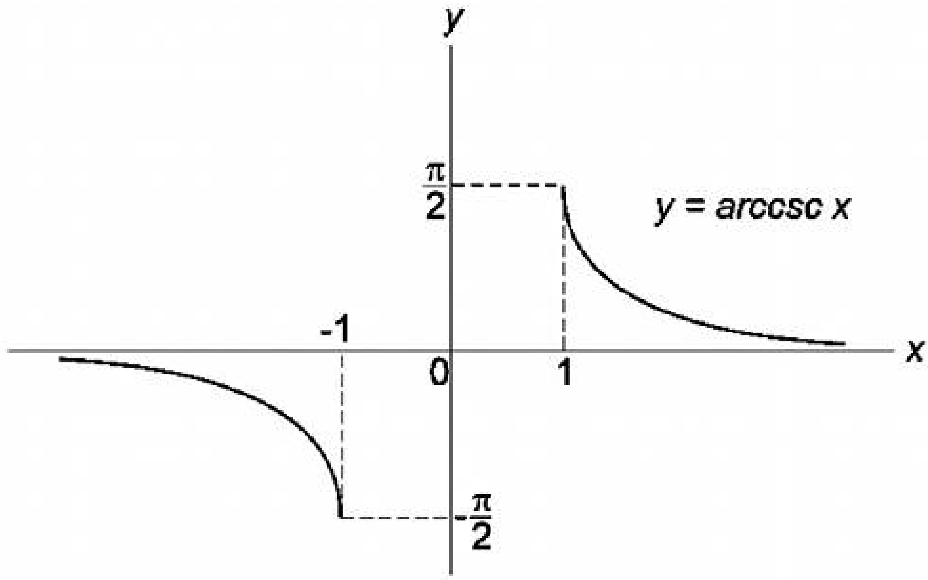


Figure 71.

4.18 Principal Values of Inverse Trigonometric Functions

472.

x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
arcsin x	0°	30°	45°	60°	90°
arccos x	90°	60°	45°	30°	0°
x	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	
arcsin x	−30°	−45°	−60°	−90°	
arccos x	120°	135°	150°	180°	

473.

x	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$
$\arctan x$	$0°$	$30°$	$45°$	$60°$	$-30°$	$-45°$	$-60°$
$\operatorname{arccot} x$	$90°$	$60°$	$45°$	$30°$	$120°$	$135°$	$150°$

4.19 Relations between Inverse Trigonometric Functions

474. $\arcsin(-x) = -\arcsin x$

475. $\arcsin x = \frac{\pi}{2} - \arccos x$

476. $\arcsin x = \arccos\sqrt{1-x^2}$, $0 \le x \le 1$.

477. $\arcsin x = -\arccos\sqrt{1-x^2}$, $-1 \le x \le 0$.

478. $\arcsin x = \arctan\frac{x}{\sqrt{1-x^2}}$, $x^2 < 1$.

479. $\arcsin x = \operatorname{arccot}\frac{\sqrt{1-x^2}}{x}$, $0 < x \le 1$.

480. $\arcsin x = \operatorname{arccot}\frac{\sqrt{1-x^2}}{x} - \pi$, $-1 \le x < 0$.

481. $\arccos(-x) = \pi - \arccos x$

482. $\arccos x = \frac{\pi}{2} - \arcsin x$

483. $\arccos x = \arcsin\sqrt{1-x^2}$, $0 \le x \le 1$.

484. $\arccos x = \pi - \arcsin\sqrt{1-x^2}$, $-1 \le x \le 0$.

485. $\arccos x = \arctan\dfrac{\sqrt{1-x^2}}{x}$, $0 < x \le 1$.

486. $\arccos x = \pi + \arctan\dfrac{\sqrt{1-x^2}}{x}$, $-1 \le x < 0$.

487. $\arccos x = \operatorname{arccot}\dfrac{x}{\sqrt{1-x^2}}$, $-1 \le x \le 1$.

488. $\arctan(-x) = -\arctan x$

489. $\arctan x = \dfrac{\pi}{2} - \operatorname{arccot} x$

490. $\arctan x = \arcsin\dfrac{x}{\sqrt{1+x^2}}$

491. $\arctan x = \arccos\dfrac{1}{\sqrt{1+x^2}}$, $x \ge 0$.

492. $\arctan x = -\arccos\dfrac{1}{\sqrt{1+x^2}}$, $x \le 0$.

493. $\arctan x = \dfrac{\pi}{2} - \arctan\dfrac{1}{x}$, $x > 0$.

494. $\arctan x = -\dfrac{\pi}{2} - \arctan\dfrac{1}{x}$, $x < 0$.

495. $\arctan x = \operatorname{arccot}\frac{1}{x},\ x > 0.$

496. $\arctan x = \operatorname{arccot}\frac{1}{x} - \pi,\ x < 0.$

497. $\operatorname{arccot}(-x) = \pi - \operatorname{arccot} x$

498. $\operatorname{arccot} x = \frac{\pi}{2} - \arctan x$

499. $\operatorname{arccot} x = \arcsin\frac{1}{\sqrt{1+x^2}},\ x > 0.$

500. $\operatorname{arccot} x = \pi - \arcsin\frac{1}{\sqrt{1+x^2}},\ x < 0.$

501. $\operatorname{arccot} x = \arccos\frac{x}{\sqrt{1+x^2}}$

502. $\operatorname{arccot} x = \arctan\frac{1}{x},\ x > 0.$

503. $\operatorname{arccot} x = \pi + \arctan\frac{1}{x},\ x < 0.$

4.21 Relations to Hyperbolic Functions

Imaginary unit: i

508. $\sin(ix) = i\sinh x$

509. $\tan(ix) = i\tanh x$

510. $\cot(ix) = -i\coth x$

511. $\sec(ix) = \operatorname{sech} x$

512. $\csc(ix) = -i\operatorname{csch} x$

4.6 Reduction Formulas

391.

β	$\sin\beta$	$\cos\beta$	$\tan\beta$	$\cot\beta$
$-\alpha$	$-\sin\alpha$	$+\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$90°-\alpha$	$+\cos\alpha$	$+\sin\alpha$	$+\cot\alpha$	$+\tan\alpha$
$90°+\alpha$	$+\cos\alpha$	$-\sin\alpha$	$-\cot\alpha$	$-\tan\alpha$
$180°-\alpha$	$+\sin\alpha$	$-\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$180°+\alpha$	$-\sin\alpha$	$-\cos\alpha$	$+\tan\alpha$	$+\cot\alpha$
$270°-\alpha$	$-\cos\alpha$	$-\sin\alpha$	$+\cot\alpha$	$+\tan\alpha$
$270°+\alpha$	$-\cos\alpha$	$+\sin\alpha$	$-\cot\alpha$	$-\tan\alpha$
$360°-\alpha$	$-\sin\alpha$	$+\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$360°+\alpha$	$+\sin\alpha$	$+\cos\alpha$	$+\tan\alpha$	$+\cot\alpha$

Some Important Points

r	r_1	r_2	r_3
$r=\Delta/S$	$r_1=\Delta/S-a$	$r_2=\Delta/S-b$	$r_3=\Delta/S-c$
$r=(s-a)\tan\frac{A}{2}$ $=(s-b)\tan\frac{B}{2}$ $=(s-c)\tan\frac{C}{2}$	$r_1=s\tan\frac{A}{2}$	$r_2=s\tan\frac{B}{2}$	$R_3=s\tan\frac{C}{2}$
$r=\frac{a\sin\frac{B}{2}.\sin\frac{C}{2}}{\cos\frac{A}{2}}$ $r=\frac{b\sin\frac{C}{2}.\sin\frac{A}{2}}{\cos\frac{B}{2}}$ $r=\frac{c\sin\frac{A}{2}.\sin\frac{B}{2}}{\cos\frac{C}{2}}$	$r_1=\frac{a\cos\frac{B}{2}.\cos\frac{C}{2}}{\cos\frac{A}{2}}$	$r_2=\frac{b\cos\frac{C}{2}.\cos\frac{A}{2}}{\cos\frac{B}{2}}$	$r_3=\frac{c\cos\frac{A}{2}.\cos\frac{B}{2}}{\cos\frac{C}{2}}$
R= $4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$	$r_1=$ $4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$	$r_2=$ $4R\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$	$r_3=$ $4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Projection Law:

In any ΔABC,

i) a=bcosB+c cosB

ii) b=acosC+c cosA

iii) c=acosB+bcosA

-Some Important Formulas:

If θ is in restricted domain of corresponding trigonometric function then:

1-

a) arc sin(sinx)=x

b) arc cos(cosx)=x

c) arc tan (tanx)=x

d) arc cot (cotx)=x

e) arc sec (secx)=x

f) arc csec (csecx)=x

If x is in domain of corresponding inverse trigonometric function then:

2-

a) sin(arc sinx)=x

b) cos (arc cosx)=x

c) tan (arc tanx)=x

d) cot (arc cotx)=x

e) sec (arc secx)=x

f) csec (arc csecx)=x

3-

a) arc sin(-x)= -arcsinx

b) arc cos(-x)= π-arc cosx

c) arc tan(-x)= -arc tanx

d) arc cot(-x)= π-arc cotx

e) arc sec(-x)= π- arc secx

f) arc csec(-x)= -csec x

4-

a) arc cosec(x)= arc sin(1/x) , x≥1 or x≤-1

b) arc sec(x)= arc cos (1/x), x≥1 or x≤-1

c) arc cot(x)= $\left\{\begin{matrix} arc\ tan\ \left(\frac{1}{x}\right), x > 0 \\ \pi + arc\tan\left(\frac{1}{x}\right), x < 0 \end{matrix}\right\}$

5-

a) arc sinx+ arc cosx= π/2

b) arc secx+ arc csecx=π/2

c) arc tanx+ arc cotx=π/2

Some useful General Solutions:

Trigonometric Equation	General Solution
sinx=0	x=nπ , nεI
cosx=0	x=(2n+1)π/2 , nεI
tanx=0	X=nπ, n εI
asinx+bcosx=c	
Note : If $\lvert c\rvert \le \sqrt{a^2+b^2}$ is not satisfied then no real solution exists	x=nπ+α±β, nεI Where cosα=$\frac{a}{\sqrt{a^2+b^2}}$ sinα=$\frac{b}{\sqrt{a^2+b^2}}$ cosβ=$\frac{c}{\sqrt{a^2+b^2}}$

➔Some Important Hints for solving Trigonometric equations.

1) Squaring should be avoided as far as possible if square is done then check for extra solutions.

For example, consider equation sinx+cosx=1 on squaring we get $\sin^2x+\cos^2x+2\sin x.\cos x=1$ ➔ sin2x=1 ➔x=nπ/2, n=0,±1,±2,…

The value of angle x are x=π and x=3π/2 don't satisfy the given equation. So we get extra solution. Thus if squaring is done must verify each of the solution.

2) Never cancel a common factor containing "x" from both sides of an equation.

For example, consider an equation tanx=√2sinx which is not equivalent to the given equation as the solutions obtained by sinx=0 are lost. Thus, instead of dividing an equation by a common factor take this factor out as a common factor from all the terms of the equation.

3) Make sure that the answer should not contain any value of unknown "x" which makes any of terms undefined.

4) If tanx and secx is involved in the equation , x should not be a

multiple of π/2.

5) All the solutions should satisfy the given equation and lie the domain of the variable in the given condition.

Some Important Points:

1 right angle= 90*

1*= 60'

1'= 60''

Measurement of angles:

There are three systems for measurement of an angle.

1) Sexagesimal System or English System:

In this system angle is measured in degrees, minutes and seconds

Note: If we divide the circumference of a circle into 360* equal parts the angle subtended at the center by each part of circle is one degree angle.

One complete rotation anticlockwise= 360*

½ rotation anticlockwise= 180*

¼ rotation anticlockwise= 90*

2) Centesimal or French system:

In this system angle is measured in grades , minutes and seconds.

One rotation= 400^{g}

½ rotation= 200^{g}

1 right angle= 100^{g}

1^{g}= 100'

3) Radian or Circular measure:

A radian is angle subtended at the center of circle whose length is equal to the radius of circle.

Note :

Radian is unit to measure angle and it should not be interrupted that π stands for 180* and π is a real number whereas π* stands for 180*

Remember: π=180*=200^g

1’=100’’

Note:

1’ of centesimal system≠1’ of sexagesimal

1’’ of centesimal system≠ 1’’ of sexagesimal

Relation between different system of measurement or measurement of angles

1*=10/9 grades

1^g= 9/10 degree

1*=π/180 radian= 0.01745 radian

1 radian= 180/π=57.295*

Thus if the measure of an angle in degree , grades and radians be F.G and θ respectively.

D/180= G/200=θ/π

Relation between sides and interior angles of a regular polygon.

Sum of interior angles of polygon of n sides:

=(n-2)*180

Each interior angle of regular polygon of n sides:

=(n-2)/n * 180

➔ θ=l/r or l=rθ

➔ Area of sector=A= ½ r^2θ= ½ rl

➔ Perimeter of sector= (2r+l)

Coterminal angles :

The angles whose initial and terminal sides are same are called coterminal angles. i.e

θ,θ+2π,…. Are coterminal.

General Angles:

θ+2kπ, belongs to integers. Is called general angle.

Quadrant Angle:

If the terminal side of an angle fall on x-axis or y-axis is called quadrant angle.

i.e 0, 90,180,270,360 ..

- Trigonometric equation contains at least one trigonometric function
- Trigonometric function are periodic
- A trigonometric equation has infinite solutions
- To solve a trigonometric equation , firstly we find the solution over the interval which is period.

Remember that:

1) $\sin^{-1}x \neq (\sin x)^{-1}$

2) $\cos^{-1}x \neq (\cos x)^{-1}$

3) $\tan^{-1}x \neq (\tan x)^{-1}$

Some important results:

For 1st quadrant: θ=θr

For 2nd quadrant: θ=180-θr

For 3rd quadrant: θ=180+θr

For 4th quadrant: θ=360-θr

- Graphical approach

A function is invertible if and only if no horizontal line intersects its graph more than once.

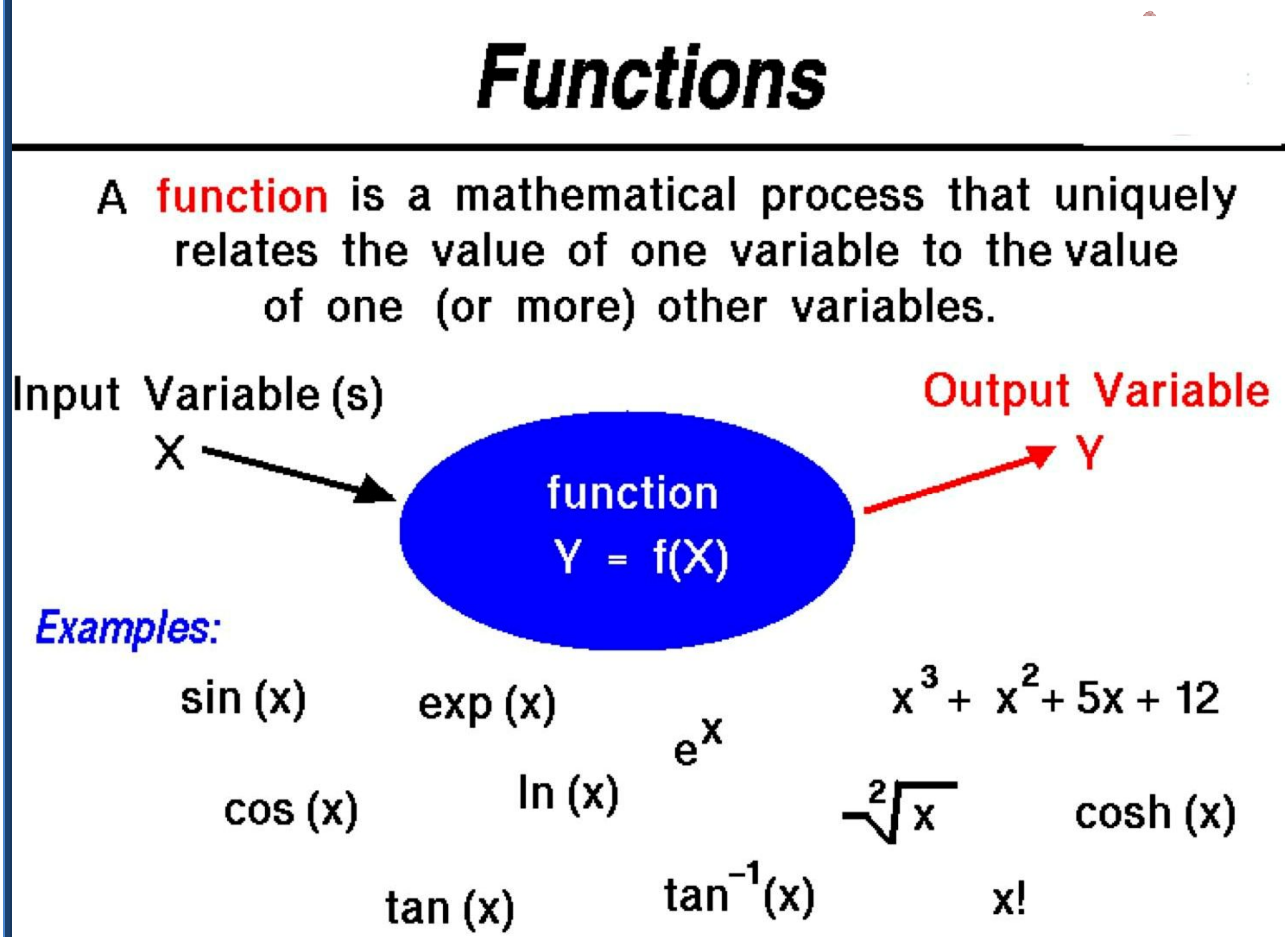
Note#1: The least numerical value among all the values of angle whose sine is x is called principal vale of arc sinx

Note#2: i) $\sin^{-1}x$ =arc

ii) Graph of y=arcsinx is obtained by reflecting the restricted portion of graph of y=sinx about y=x

Book - II

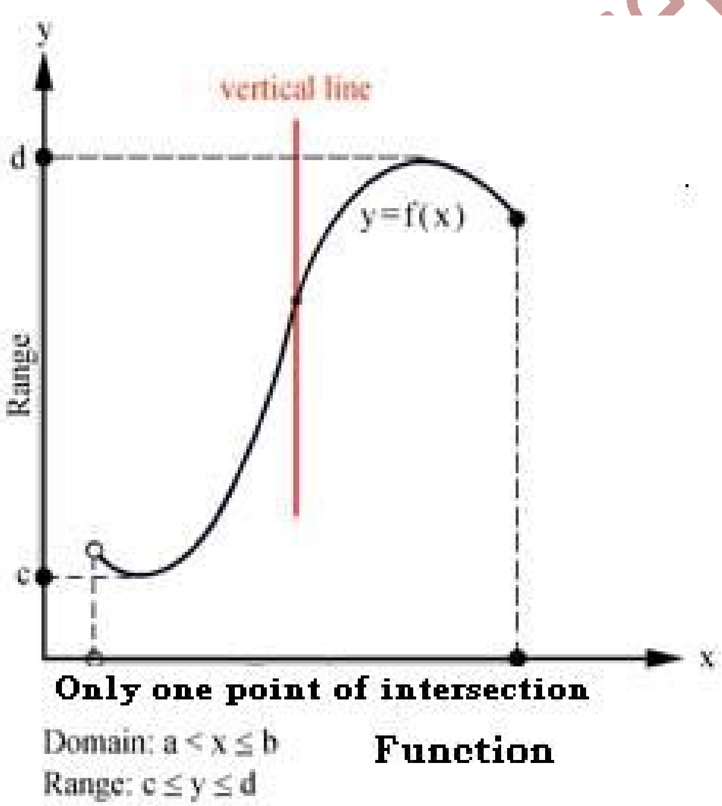
CHAPTER 1
(FUNCTIONS AND LIMITS)

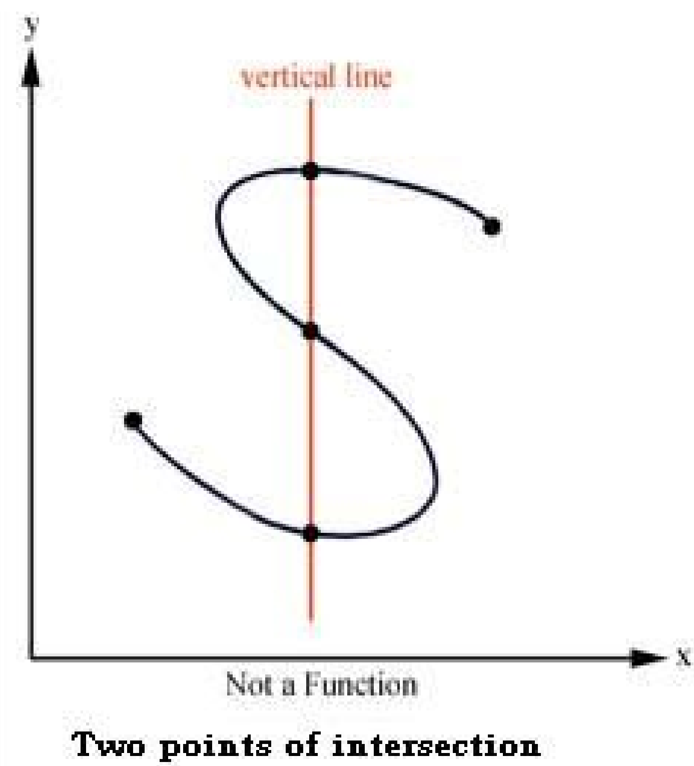


- A function gives an output for a unique input. If there is more than one output for one input then there will be no function. e.g., $y=4x$ is a function but $y^2=4x$ is not a function.

VERTICAL LINE TEST:-

- This test is used to test that whether the given graph is of function or not. Graphically, if we draw a straight vertical line on the graph and it intersects a curve at one point only then that curve will be showing function. If there are more than one intersection points then the curve will not represent the function. You can say it a relation but not a function.





TRICK FOR GUESSING INTO & ONTO FUNCTION:-

- Every polynomial function having degree an odd number is a ONTO function.
- Every polynomial function having degree an even number is a INTO function.

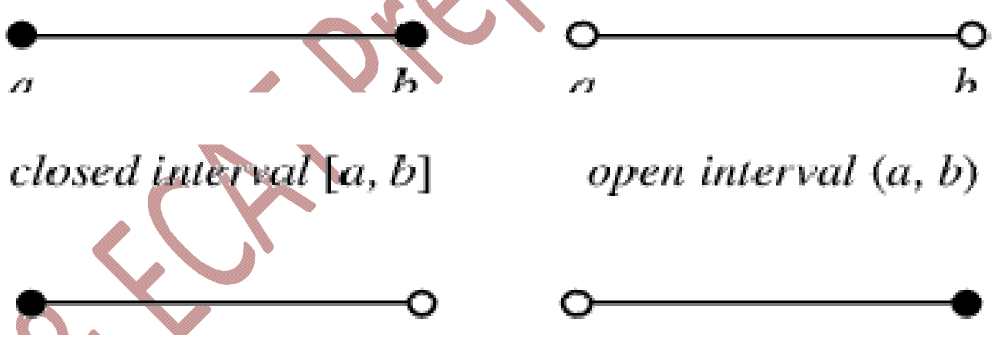
e.g.,

$y=x^3+x+1$ is a ONTO function

$y=x^4+x^3$ is a INTO function

CONCEPT OF INTERVALS:-

An interval is a connected portion of the real line. If the endpoints a and b are finite and are included, the interval is called closed and is denoted [a,b]. If the endpoints are not included, the interval is called open and denoted (a,b). If one endpoint is included but not the other, the interval is denoted [a,b) or (a,b] and is called a half-closed (or half-open interval).



Now we are defining some types of intervals for any two real numbers a and b, where a < b.

(i) Open interval
$(a, b) =]\,a, b\,[= \{x \in R : a < x < b\}$

(ii) Closed interval
$[a, b] = \{x \in R : a \leq x \leq b\}$

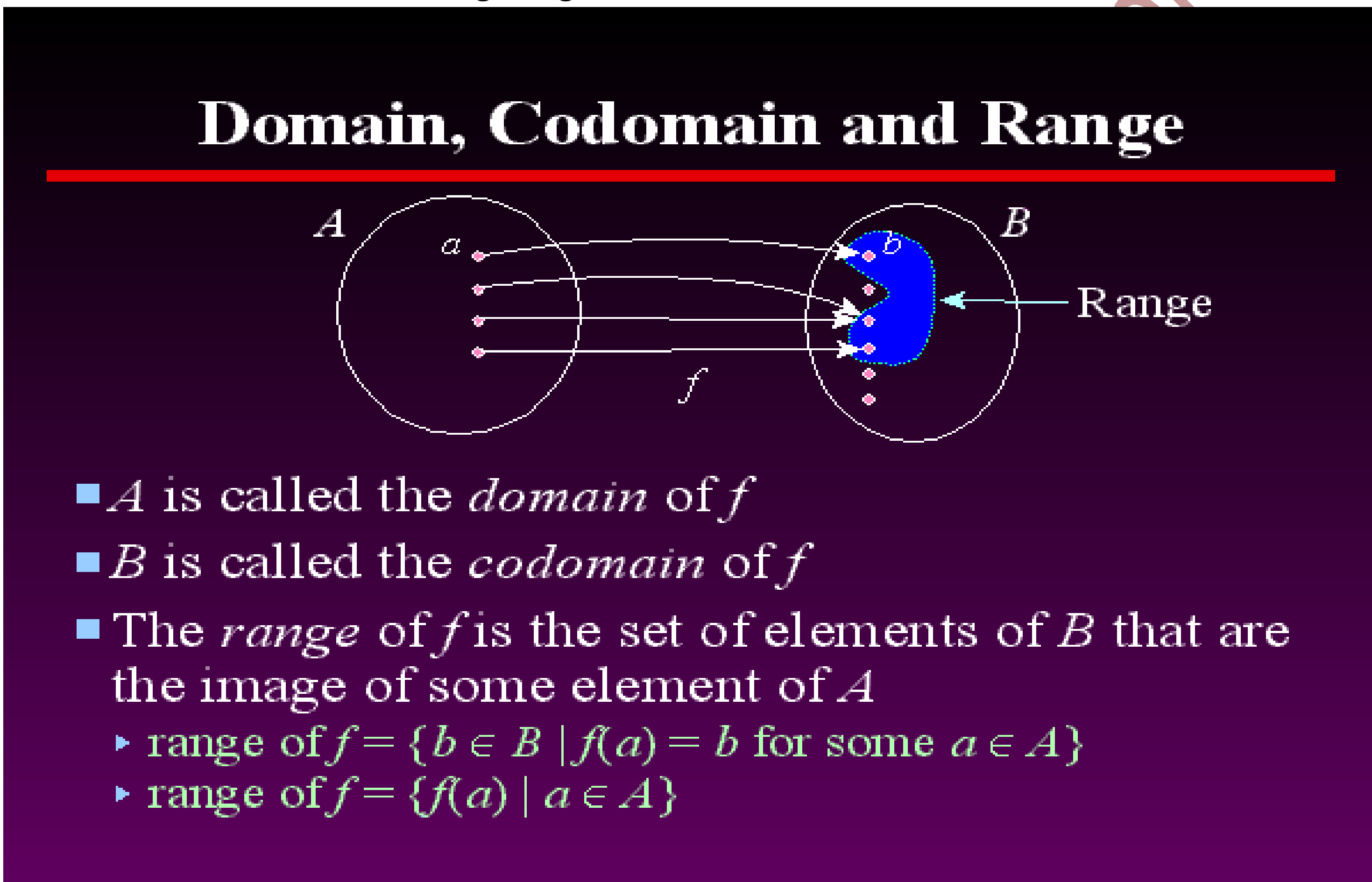
(iii) Right half open interval
$[a, b) = [a, b\,[= \{x \in R : a \leq x < b\}$

(iv) Left half open interval
$(a, b] =]\,a, b] = \{x \in R: a < x \leq b\}$ say.

Now, we look at the concept of Domain, Co-Domain and Range.

DOMAIN, CO-DOMAIN and RANGE

Understand the following image



There are many functions given in chapter 1. You have to determine their domains and ranges. For few functions, it is very easy but for few functions it is very difficult task.

For few functions there are some tricks to find Domain & Range which we have discussed below.

DOMAIN:-

Whatever the function is, check for two things:

1. Fraction
2. Square Root

If fraction or square root is not present, then the domain of the function will be R (All real numbers). But if Fraction or square root is present, then domain will not be R. It will be changed.

Examples:-

1. y= X^2+4
2. y= x^3-x^2+5x

In above two functions, whatever you put in place of x, you will get some definite value. Hence the domain of above two functions is R.

3. y= $\frac{1}{2x-2}$

In case of fractions, you have to see that for which value of x, you are getting 0 in the denominator. Subtract this value from R. It will be your domain. e.g. If I put x=1 in above function then denominator will become 2(1)-2=0 and hence our function will be undefined. So, I cannot put x=0. I can put everything except 0 and function will give some definite value. S, the domain of above function becomes:
Domain: {R-0}

4. y= $\sqrt{Square\ Root\ Function}$

In case of square root function, you have to check for which value of variable, you are getting –ve sign in square root as it yields iota. Those values of variable, for which we get iota, will not be included in the domain.

Some Rules related to Domain of f(x).

- f(x)/g(x) is defined when $g(x)\neq 0$
  e.g To find domain of the function $y= x^2-16/x-4$
  so $x-4\neq 0$ ➔ $x\neq 4$ So Domain is R-{4}.
- $\sqrt{f(x)}$ is defined when $f(x)\geq 0$
  e.g To find the domain of function $y=\sqrt{x+1}$
  $x+1\geq 0$ ➔ $x\geq -1$ so Domain is $[-1,\infty)$
- $\frac{1}{\sqrt{f(x)}}$ is defined when $f(x)>0$
  e.g To find the domain of function $y=\frac{1}{\sqrt{x+1}}$
  $x+1>0$ ➔ $x>-1$ so Domain is $(-1,\infty)$
- $X^2>a^2$ ➔ $x<-a$ and $x>a$,e.g.
  To find the domain of function $y=\sqrt{x^2-4}$
  $x^2-4\geq 0$ ➔ $x^2\geq 4$ ➔ $x\leq -2$ and $x\geq 2$
- $X^2<a^2$ ➔ $-a<x<a$,e.g.
  To find the domain of function $y=\sqrt{4-x^2}$
  $4-x^2\geq 0$➔ $x^2\leq 4$ ➔ $-2\leq x\leq 2$ (Answer)

  Rule for Finding range of a function:-
  1) If (x)=ax+b then range is R .e.g. when f(x)=2x-5, Ran (f)=R
  2) To find the range of expressions under the square root we find the least and greatest values of y for values of x in the domain.
  e.g.
  when $f(x)=\sqrt{x+1}$, Dom(f)= $[1,\infty)$, Range (f)= $[0,\infty)$

  Graph of functions:
  1) The graph of linear function is straight line.
  2) The graph of identity function is a straight line passing through

origin and inclined at angle of 45* with x-axis.

3) The graph of constant function is a straight line parallel to x-axis.

4) The graph of quadratic function is a parabola.

5) The graph of an even function is symmetric about y-axis.

6) The graph of an odd function is symmetric about origin.

- ➔ Modulus function is also type of linear function
- ➔ Constant function is always even function
- ➔ Identity function is also type of linear function.

-Important Results:

As we know that:

f(x)= $\frac{|x|}{x}$ ➔ $\left\{\begin{pmatrix} 1, & x > 0 \\ -1, & x < 0 \end{pmatrix}\right\}$

1) When Limx→0⁺ $\frac{|x|}{x}$=1

2) When Limx→0⁻ $\frac{|x|}{x}$=-1

3) When Lim x→0 $\frac{|x|}{x}$=does not exist

For f(x)= $\frac{x}{|x|}$

1) Limx→0⁻ $\frac{x}{|x|}$=-1

2) Limx→0⁺ $\frac{|x|}{x}$=1

-Vertical Asymptotoes:

1) Limx→a^- f(x)=-∞

2) Limx→a^+ f(x)=+∞

Important Results to remember:

Trigonometric limits:

If x is in radian and 0<|x|< π/2 , then:

1) $\lim_{x\to 0}\frac{sinx}{x}$=1

2) $\lim_{x\to 0}\frac{\sin^{-1}x}{x}$=1

3) $\lim_{x\to 0}\frac{sinax}{sinbx}=\frac{a}{b}$

4) $\lim_{x\to 0}\frac{sinx^{\circ}}{x}=\pi/180^{\circ}$

5) $\lim_{x\to 0}\frac{tanx}{x}$=1

6) $\lim_{x\to 0}\frac{\tan^{-1}x}{x}$=1

7) $\lim_{x\to \infty}\frac{sinx}{x}$=0

8) $\lim_{x\to \infty}\frac{cosx}{x}$=0

9) $\lim_{x\to 0}\frac{sinpx}{qx}=\frac{p}{q}$

Exponential and Logarithmic Limits:

1) $\lim_{x\to 0}\frac{a^x-1}{x}$ =$\log_e$a , a>0

2) $\lim_{x\to 0}\frac{e^x-1}{x}$= 1

3) $\lim_{x\to 0}\left(1+\frac{1}{n}\right)^n=\lim_{x\to\infty}(1+x)^{1/x}=e$

4) $\lim_{x\to 0}\frac{(1+x)^n-1}{x}=n$

5) $\lim_{x\to 0}\frac{x^n-a^n}{x-a}=na^{n-1}$

6) $\lim_{x\to 0}\frac{a^x-b^x}{x}$ =$\log_e$(a/b)

Limit at Infinity:

1) $\lim_{x\to \pm\infty}(\frac{a}{x})$=0

2) $\lim_{x\to \pm\infty}\frac{a}{x^p}=0$

3) $\lim_{x\to +\infty}e^x=\infty$

4) $\lim_{x\to -\infty}e^x=0$

5) $\lim_{x\to \infty}\frac{1}{x}$=0

-Some special type of functions:

1) Modulus or absolute value function:

y= f(x)= |x| As we know that |-3|=3

Here we have to well know about:

$|x| \rightarrow \begin{Bmatrix} x; x \geq 0 \\ -x, x < 0 \end{Bmatrix}$

e.g: $|x-a| \rightarrow \begin{matrix} x-a; x \geq a \\ -(x-a), x < a \end{matrix}$

Range: [0,∞)

Domain = R

2) Greatest value function or floor value function:

y=[x]

Concept: if[5]=5

[5.99]=5

[6.99]=6

Here we draw Number line system and evaluate the left side value of respective decimal as in above case [5.99]→ we select the closest left side value of 5.99 which is 5 and in case of [5] we remain the value unchanged.

More Examples:

[-1.5]=-2

[-0.5]=-1

[2.5]=-3

[0.01]=0 etc..

3) Least value function:

In this case, we select the right side value of number from number line system.

As ⌈5.5⌉=6

⌈−3.5⌉=4 etc

-Asymptotes in case of rational function.

Let f(x)=$\frac{ax^n+\cdots..(nth\ degree\ polynomial)}{bx^m+\ .....(mth\ degree\ polynomial)}$

1) If n<m then the x-axis is horizontal asymptotes.

2) If n=m then the horizontal asymptotes is y=a/b

3) If n>m then there is no horizontal asymptotes (There is a slant diagonal or oblique asymptotes)

Example#1:

f(x)= $\frac{4x^2-3x+1}{2x^2-1}$

Then asymptotes will be as m=n so: 4/2=2

Example#2:

f(x)=$\frac{2x}{x^2-5x-3}$

If n<m then $\frac{2x}{x^2}=\frac{2}{x}$ so y=0 (Answer)

Example#3:

f(x)=$\frac{3x^3+2}{x^2-x-7}$

If n>m so y=3x+3 after division so (Slant asymptotes)

Vertical Asymptotes:

If f(x)/g(x) then g(x)is vertical asymptotes

Example: The vertical Asymptotes of f(x)=$\frac{x}{x-2}$?

As g(x)=0 is vertical asymptotes

x-2=0 ➔ x=2 (Answer)

➔When y= logx ,x=0 so it forms the asymptotes.

-Shortcut to find the inverse of a function:

1) f(x)= ax+b then inverse of f(x) is $\frac{x-b}{a}$

2) f(x)= $\frac{ax+b}{cx+d}$ then inverse of f(x) is $\frac{dx-b}{-cx+a}$

3) f(x)= ax^n+b then inverse of f(x) is $\left(\frac{x-b}{a}\right)^{\frac{1}{n}}$

Examples:

E#1: The inverse of f(x)=4x+5 is?

Using the trick: inverse of f(x) is $\frac{x-5}{4}$

E#2: The inverse of f(x)= $\frac{4x+2}{5x+1}$ is?

Using the trick: inverse of f(x) is $\frac{x-2}{-5x+4}$

E#3: The inverse of f(x)= $3x^3+7$?

Using the trick: inverse of f(x) is $\left(\frac{x-7}{3}\right)^{\frac{1}{3}}$

Important results:

1) Inverse of function y=lnx is e^x

2) Inverse of function $y=10^x$ is $\log_{10}x$

Important result:

$f(f^{-1}(x))=x$ (Identity)

Example: f(x)= -2x+8 then inverse is?

A) x-8/2 B)8-x/2 (Correct) C) 8-x D) x-8

Applying the relation:

Checking the options:

$f(f^{-1}(x))= -2\,(8-x/2)+8 \rightarrow =x$ (identity) Answer)

RANGE OF QUADRATIC FUNCTION:-

$f(x) = ax^2+bx+c \qquad a\neq0$

1) If a>0 then Range: $[\frac{4ac-b^2}{4a},+\infty]$

2) If a<0 then Range: [-∞,$\frac{4ac-b^2}{4a}$]

MCQ:

What is the range of f(x)= x²+2x+2

a) R b) [0,+∞) c) [1, +∞) d) (-∞,1]

If a>0 then Range: [$\frac{4(1)(2)-(2)^2}{4(1)}$,+∞]= [1,+∞](Answer)

RATIONAL FUNCTION

If you have a function like this:

f(x) = $\frac{ax+b}{cx+d}$

then you can simply find the domain and range as:

Domain = R- {$\frac{-d}{c}$}

Range = R- {$\frac{a}{c}$}

SYMMETRY OF FUNCTIONS

1) To check which function is symmetric about x-axis, put (x,-y) in place of (x,y), if no change occur in the original function, then this function is symmetric about x-axis. (not a function)

2) To check which function is symmetric about y-axis, put (-x,y) in place of (x,y), if no change occur in the original function, then this function is symmetric about y-axis. (Even function)

3) To check which function is symmetric about origin , put (-x,-y) in place of (x,y), if no change occur in the original function, then

this function is symmetric about origin. (Odd function)

4) If we replace x by y and y by x in a given equation and there is no change i.e f(x,y)= f(y,x), then the graph is symmetric about a line y=x (Identity Function)

Examples:

1) $y = x^2+3$

Put (-x,y) in place of (x,y)

$(y)=(-x)2+3$

$y=x^2+3$

No change occur, this is symmetric about y-axis

2) $y^2+x^2 = 25$

put (-x,-y) in place of (x,y)

$(-y)2+(-x)2=25$

$y2+x2=25$

no change occur, this is symmetric about origin.

Quick Rule for finding the limits of Rational Function as x goes to +∞ or –∞:-

$$\lim_{x\to+\infty}\frac{c_nx^n+\ \ldots\ldots\ldots\ldots+c_1x+c_0}{d_nx^n+\ \ldots\ldots\ldots\ldots+d_1x+d_0}=\lim_{x\to+\infty}\frac{c_nx^n}{d_nx^n}$$

$$\lim_{x\to-\infty}\frac{c_nx^n+\ \ldots\ldots\ldots\ldots+c_1x+c_0}{d_nx^n+\ \ldots\ldots\ldots\ldots+d_1x+d_0}=\lim_{x\to-\infty}\frac{c_nx^n}{d_nx^n}$$

Example:

$$\lim_{x\to+\infty}\frac{3x^6+2x^4+7x^3+3x+2}{x^7+2x^3+2x+4}=?$$

Solution:

Using the Quick Rule:

$= \lim_{x\to+\infty} \frac{3}{x}$

As we also know that (Any number/∞=0)

=0 (Answer)..

L-HOSPITAL RULE:-

If you find any limit problem, first of all try to put the value and solve it.

If you find $\frac{0}{0}$ form or $\frac{\infty}{\infty}$ form then L-Hospital will use.

The Rule is very simple.

i) Take derivative of numerator and denominator separately.

ii) Now apply the limit and check whether you are getting the answer or not. If you get the answer then it is good but if you are not getting the answer then again take derivative of numerator and denominator and again check by applying the limit. Continue this procedure and finally will get your answer.

Note: Sometimes we have to repeat the process if the form 0/0 and ∞/∞ again.

Examples:-

Evaluate each of the following limits.

(a) $\lim_{x\to 0} \frac{\sin x}{x}$

(b) $\lim\limits_{t\to 1}\dfrac{5t^4-4t^2-1}{10-t-9t^3}$

(c) $\lim\limits_{x\to\infty}\dfrac{e^x}{x^2}$

(d) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$

By applying limit we come to know that this is a 0/0 indeterminate form so let's just apply L'Hospital's Rule. The derivative of sinx is cosx and derivative of x is 1.

$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}=\frac{1}{1}=1$$

(b) $\lim\limits_{t\to 1}\dfrac{5t^4-4t^2-1}{10-t-9t^3}$

In this case we also have a 0/0 indeterminate form so apply L'Hospital's Rule.

$$\lim_{t\to 1}\frac{5t^4-4t^2-1}{10-t-9t^3}=\lim_{t\to 1}\frac{20t^3-8t}{-1-27t^2}=\frac{20-8}{-1-27}=\frac{3}{7}$$

(c) $\lim\limits_{x\to\infty}\dfrac{e^x}{x^2}$

We know that it's the indeterminate form ∞/∞. So let's apply L'Hospital's Rule.

$$\lim_{x\to\infty}\frac{e^x}{x^2}=\lim_{x\to\infty}\frac{e^x}{2x}$$

Now we have a small problem. This new limit is also a ∞/∞ indeterminate form. However, it's not really a problem. We know how to deal with these kinds of limits. Just apply L'Hospital's Rule again.

$$\lim_{x\to\infty}\frac{e^x}{x^2}=\lim_{x\to\infty}\frac{e^x}{2x}=\lim_{x\to\infty}\frac{e^x}{2}=\infty$$

Sometimes we will need to apply L'Hospital's Rule more than once.

TRICK FOR QUESTIONS OF THE TYPE $\mathbf{lim}_{x\to 0}\left(\mathbf{1}+\frac{\mathbf{3}x}{\mathbf{2}}\right)^{\mathbf{5}/\mathbf{3}x}$

Just Multiply the second term and power. It will become the power of e.

$\frac{3x}{2}$ x $\frac{5}{3x}$ = 5/2.

So,

$$\lim_{x\to 0}\left(1+\frac{3x}{2}\right)^{5/3x} = e^{5/2}$$

EVEN & ODD FUNCTIONS:-

- f(-x) = f(x) EVEN e.g. cosx
- f(-x) = -f(x) ODD e.g. sinx
- f(-x) ≠ ±f(x) Neither EVEN nor ODD

POINTS TO REMEMBER:-

- Constant function is always even
- Even function is symmetric about y-axis.
- Odd function is symmetric about origin.
- The graph of odd functions occur in 1st and 3rd Quadrant or 2nd and 4th Quadrant.
- $f(x) = 0$ is both Even & Odd function.

COMPOSITION OF FUNCTION:-

- $fog(x) = f[g(x)]$
- $fog \neq gof$
- $fof^{-1}(x) = f^{-1}of(x) = x$ Identity
- $f^3(x) = fofof(x) = f(f(f(x)))$

POINTS TO REMEMBER:-

- If f is even and g is odd then fog or gof is even.
- If f is odd and g is even then fog or gof is even.
- If f is even and g is even then fog or gof is even.
- If f is odd and g is odd then fog or gof is odd.

Continuity of a function at a point:

A function f(x) is said to be continuous at an interior point x=a of its domain if limx→a f(x)= f(a). In other words a function f(x) is said to be continuous at a point provided that left hand limit right limit and value of function are equal.

A function f(x) is continuous at a point x=a if Limh→a f(a-h)= Limx→a f(a+h)=a

Continuity of a function on an interval

Continuity on an open interval:

A function f(x) is said to be continuous on an open interval (a,b) if it is continuous at each point of (a,b).

Continuity on a closed interval:

A function is said to be continuous on a closed interval [a,b] if

1) f(x) is continuous from right at x=a i.e

$\lim_{h\to 0} f(a+h)=f(a)$

2) f(x) is continuous from left at x=a i.e

$\lim_{h\to 0} f(b-h)=f(b)$

3) f(x) is continuous at each point of the open interval (a,b).

Note: For continuity of f(x) at the end of an interval [a,b] we must have

1) $\lim_{h\to 0} f(a+h)= f(a)$ at x=a

2) $\lim_{h\to 0} f(b-h)= f(b)$ at x=b

Geometrical meaning of continuity:

1) A function f(x) will be continuous at a point x=a , if there is not break or cut or gap in the graph of the function y=f(x) at the point (a,f(a)) . other, it discontinuous at that point.

2) A function f(x) will be continuous on the closed interval [a,b] if the graph of the function y=f(x) is an broken line (curved or straight) from the point (a,f(a)) to point (b,f(b)).

Note: Broken straight line in the graph is an example of discontinuity.

Note: f(x) approaches l means the absolute difference between f(x) and l i.e |f(x)-l| can be made as small as we please.

When the values of f(x) don't approach a single finite value as approaches "a" we say that the limit does not exist.

CHAPTER#2
"Differentiation"

y = f(x) (Function)

$y+\delta y = f(x+\delta x)$

$\delta y=f(x+\delta x) - f(x)$

$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$ (Average Rate of Change or slope of secant)

$\lim_{\delta x\to 0} \frac{\delta y}{\delta x} = \lim_{\delta x\to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$

$\frac{dy}{dx} = f'(x)$ (Instantaneous Rate of change or slope of tangent)

DIFFERENT NAMES OF $\frac{dy}{dx}$:

- Derivative of f(x)
- Slope of Tangent
- Instantaneous Rate of change
- Derived function of f(x)
- Differential Co-efficient of f(x)
- Gradient of f(x)

- Rate of change

PARTIAL DERIVATIVES

A partial derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant (as opposed to the total derivative, in which all variables are allowed to vary). For example, If I have a function like z = f(x,y) and I want to find derivative of the function w.r.t y only, then I will assume that x is constant.

NOTATION:

$\frac{dy}{dx}$ *is used for simple differentiation..*

$\frac{\partial y}{\partial x}$ *is used for partial differentiation.*

Example:-

If $z = f(x,y) = y^2+2x^2y+2$ then find $\frac{\partial z}{\partial y}$.

I will take x as constant.

$\frac{\partial z}{\partial y} = 2y + 2x^2$

That's it.....

Some other symbols are also used for it. You should be familiar with that.

$\frac{\partial z}{\partial x}$ = F_x $\frac{\partial z}{\partial y}$ = F_y

I hope you are now familiar with it.

IMPLICIT DIFFERENTIATION:-

Implicit functions are those in which there is mixing of the variables. Their differentiation is little lengthy but with a simple formula, you can find $\frac{dy}{dx}$ in short time. The formula is:

$$\frac{dy}{dx} = (-1)\ F_x / F_y$$

F_x & F_y are partial derivatives as I discussed above.

Example:-

Find $\frac{dy}{dx}$ if $f(x,y) = x^2y + x^2 + y^2x + y^2 = 0$

So, Find F_x & F_y and put in the formula.

F_x (treating y as constant) = $2xy + 2x + y^2$

F_y (treating x as constant) = $x^2 + 2xy + 2y$

Put in formula,

$\frac{dy}{dx} = (-1)\ F_x / F_y$

$\frac{dy}{dx} = -\frac{2xy+2x+y^2}{x^2+2yx+2y}$ Answer.

-Differentiation of Rational Function:-

$f(x) = \frac{ax+b}{cx+d}$ ➔ $f'(x) = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$

-For Absolute Functions:-

- f(x) = |x|

f'(x) does not exist at x=0

- f(x) = |x+2|

f'(x) does not exist at x=-2

See the result carefully. The derivative does not exist at that value of the variable for which your function gives 0.

Derivative of modulus /absolute value function:

d/dx |u|= $\frac{u}{|u|}\left(\frac{du}{dx}\right)$

As we know that |x|=$\sqrt{x^2}$

Example: y=|2x-3|+1

Putting |2x-3|=$\sqrt{(2x-3)^2}$

y= $((2x-3)^2)^{1/2}$ +1

dy/dx= ½ $((2x-3)^2)^{-1/2}$d/dx$(2x-3)^2$

dy/dx=$\frac{1}{2\sqrt{(2x-3)^2}}$ $.2(2x-3).(2)$

dy/dx= 2(2x-3)/$\sqrt{(2x-3)^2}$ at x=1

dy/dx= 4x-6/$\sqrt{(2x-3)^2}$

Now:

dy/dx= 4x-3/|2x-3| (Answer)

Derivative of inverse of a function:

$[f^{-1}]'$ (x)= $\frac{1}{f'(f^{-1}(x)]}$

Example:

f(x)=3x-2

inverse of f(x)= x+2/3

Now $[f^{-1}]'(x) = \frac{1}{f'(f^{-1}(x))}$

Direct: f'(x)=3

$[f^{-1}](x)$= 1/3

By rule: $[f^{-1}]'(x) = 1/f'(f^{-1}(x))$ ➔ 1/3 (Answer)

Example:

$f(x)=y= x^3+1$

$f^{-1}(x)= (x-1)^{1/3}$

so: $f'(x)=3x^2$

Direct: $[f^{-1}]'(x)= 1/3\ (x-1)^{1/3-1} = \frac{1}{3(x-3)^{\frac{2}{3}}}$

Now by rule: $(f^{-1})(x) = \frac{1}{3\left(\sqrt[3]{x-1}\right)^2}$ (Answer)

-Derivative of composite Function:

d/dx (f(g(x))= f'(g(x)) g'(x)

Derivative of different notations:

1) $\frac{d}{dx}\left(\sqrt{f(x)}\right) = \frac{f'(x)}{2\sqrt{f(x)}}$

2) $\frac{d}{dx}\sqrt{f(x)+\sqrt{f(x)+\sqrt{f(x)+\cdots..+\infty}}} = \frac{f'(x)}{(2y-1)}$

3) If $y=f(x)^{f(x)^{f(x)\ldots\ldots\infty}}$ $then\ , \frac{dy}{dx} = \frac{y^2(f'(x))}{f(x)(1-y\log f(x)}$

4) If $y=\sqrt{\frac{1+g(x)}{1-g(x)}}$ then $\frac{dy}{dx} = \frac{g'(x)}{(1-g(x))^2}.\sqrt{\frac{1-g(x)}{1+g(x)}}$

5) If y= 1/g(x) then dy/dx= $\frac{-g\prime(x)}{[g(x)]^2}$, g≠0

HIGHER ORDER DERIVATIVES

1) If $y = x^n$ then y_{n+1} and all higher derivatives will be zero.

Example:

$y = x^{10}+x^2+2$; Find y_{11}

y_{11} will be zero.

2) If $y = e^x$ then all higher derivatives will be same.

3) If $y = e^{ax}$ then $y_n = a^n e^{ax}$

4) If $y = a^x$ then $y_n = a^x.(\ln a)^n$

5) If $y = a^{bx}$ then $y_n = (b \ln a)^n a^{bx}$

6) If $y = \log_a x$ then;

- If n is odd $y_n= (n-1)! / x^n \ln a$
- If n is even $y_n = -(n-1)! / x^n \ln a$

7) If $y = \ln x$

- If n is odd then $y_n = (n-1)! / x^n$
- If n is even then $y_n = -(n-1)! / x^n$

8) Find out 6th derivative of $y= 3x^6+2x^3-9$.
If you have asked the same order derivative as the degree of polynomial, then no need to take derivative again and again. Just do following:
Co-efficient of Highest Power Term * Order of Polynomial !
In this question, 3*6! { 3 is coefficient of highest power term and 6 is the order of polynomial }
3*(6*5*4*3*2*1) this is answer.

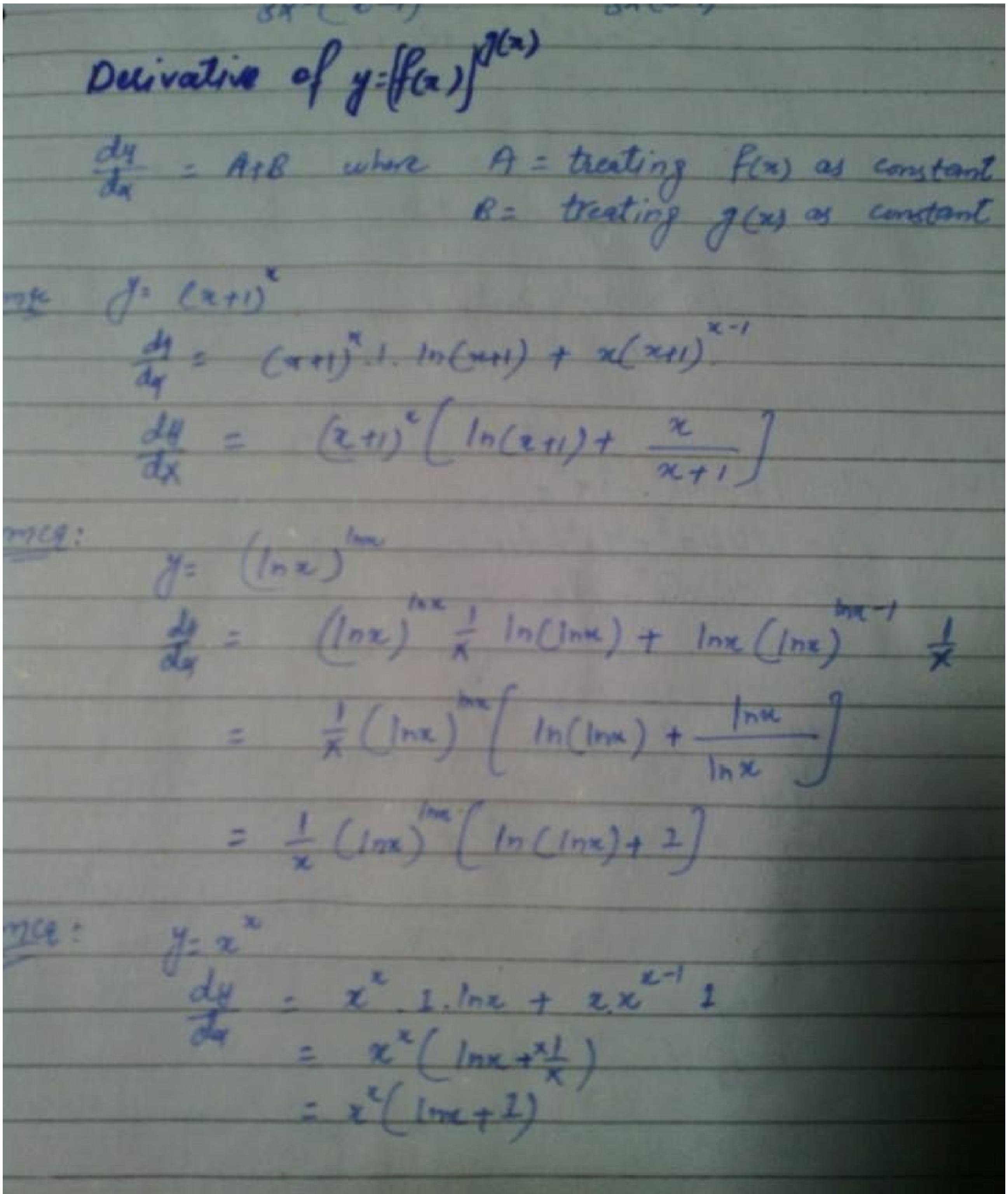
9) sin and cos functions Repeats their values after 4th Derivative

If f(x) = cos x then find $f^{105}(x)$
No need to take derivative again n again just divide the power by 4 ,and take remainder order derivative.

Divide 105 by 4, remainder is 1. So, instead of finding 105th derivative of cos x, we will take the first derivative of cosx.

$f^{105}(\cos x) = f^{1}(\cos x) = -\sin x$

DERIVATIVES OF SPECIAL FUNCTIONS



Derivative of $y = [f(x)]^{g(x)}$

$\frac{dy}{dx} = A + B$ where A = treating $f(x)$ as constant
B = treating $g(x)$ as constant

$y = (x+1)^x$

$\frac{dy}{dx} = (x+1)^x \cdot 1 \cdot \ln(x+1) + x(x+1)^{x-1}$

$\frac{dy}{dx} = (x+1)^x \left[\ln(x+1) + \frac{x}{x+1}\right]$

$y = (\ln x)^{\ln x}$

$\frac{dy}{dx} = (\ln x)^{\ln x} \frac{1}{x} \ln(\ln x) + \ln x (\ln x)^{\ln x - 1} \frac{1}{x}$

$= \frac{1}{x}(\ln x)^{\ln x}\left[\ln(\ln x) + \frac{\ln x}{\ln x}\right]$

$= \frac{1}{x}(\ln x)^{\ln x}\left[\ln(\ln x) + 1\right]$

$y = x^x$

$\frac{dy}{dx} = x^x \cdot 1 \cdot \ln x + x \cdot x^{x-1} \cdot 1$

$= x^x\left(\ln x + x \cdot \frac{1}{x}\right)$

$= x^x(\ln x + 1)$

https://www.facebook.com/groups/698559670280811/

Nth derivative of some more functions:

1) $\frac{d^n}{dx^n}$(sin(ax+b))= a^nsin(nπ/2 +ax+b)

2) $\frac{d^n}{dx^n}$(cos(ax+b))= a^ncos(nπ/2 +ax+b)

3) $\frac{d^n}{dx^n}(ax+b)^m = \frac{m!}{(n-m)!} a^n (ax+b)^{m-n}$

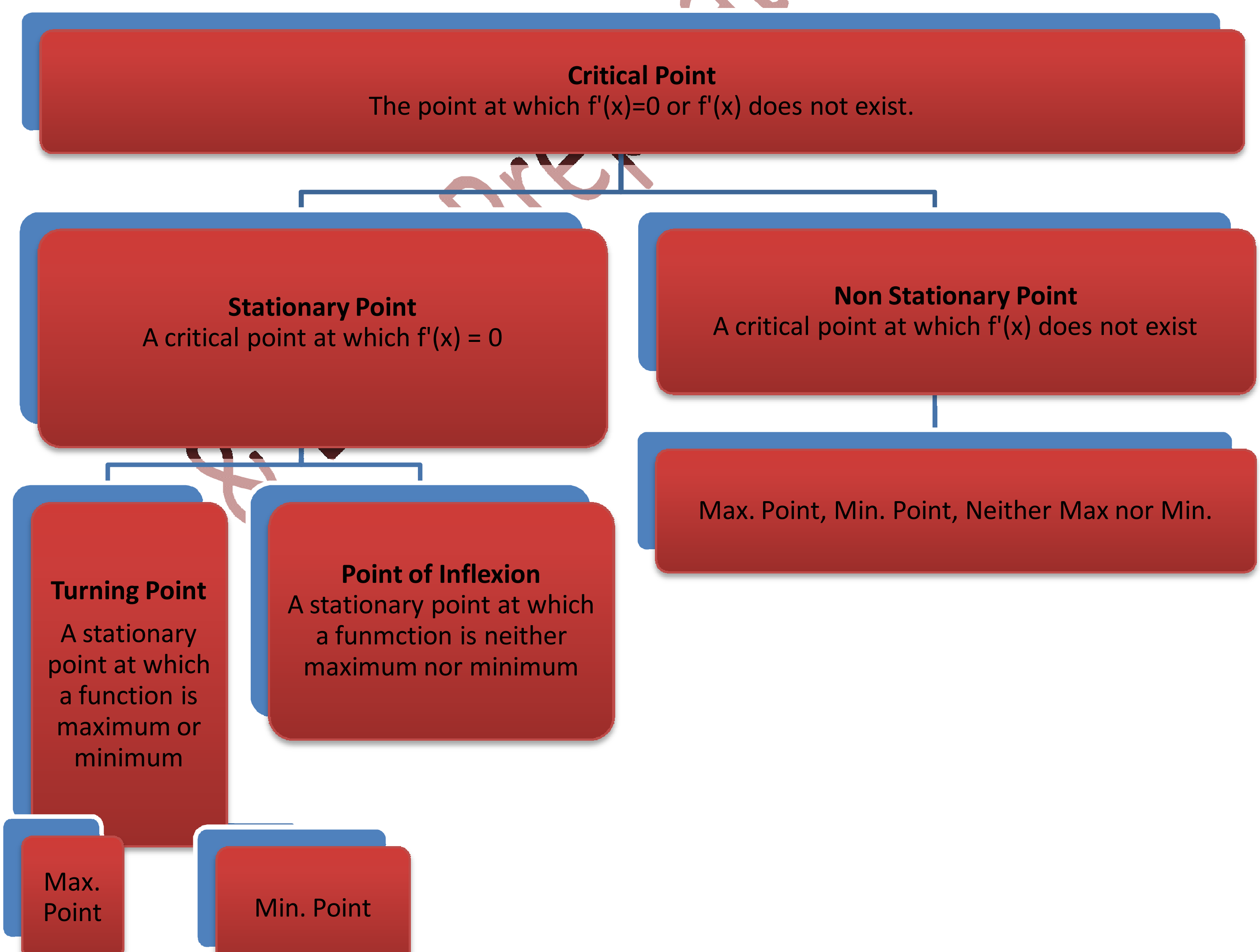
4) If $y=\frac{1}{ax+b}$ then $y_n = \frac{(-1)^n n!}{(ax+b)^n a^n}$

Find derivatives of following using above procedure:

i. x^x

ii. $\ln x^{\ln x}$

CONCEPT MAP OF CRITICAL POINTS



Derivative of Determinant:

y= $\begin{vmatrix} f(x) & g(x) \\ \delta(x) & \phi(x) \end{vmatrix}$

dy/dx= $\begin{vmatrix} f'(x) & g'(x) \\ \delta(x) & \phi(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ \delta'(x) & \phi'(x) \end{vmatrix}$

Derivative of a matrix:

Let A= $\begin{bmatrix} 2x & 2x^2 \\ 2 & 5x^3 \end{bmatrix}$

Then d(A)/dx= $\begin{bmatrix} 2 & 4x \\ 0 & 15x^2 \end{bmatrix}$ (Answer)

Decreasing and Increasing function (Shortcut)

Let us have an example to understand the shortcut:

f(x)= $\frac{4x^2+1}{x}$; x=0

f'(x)=4x+1/x ➔ 4-1/x^2

f'(x)=0

$\frac{4x^2-1}{x^2}$=0

$\frac{(2x+1)(2x-1)}{x^2}$=0

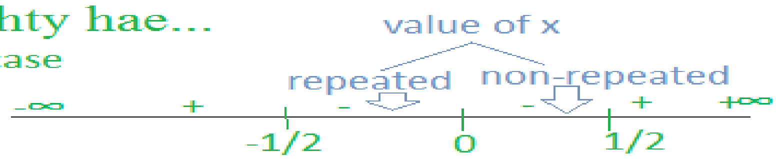
So x=1/2 , x=-1/2 ,x=0

Now using the number line system:

Shortcut:

Agr higher degree k sath + aa jaye tu hum number line system pe right side se values clear krty or jb interval banaty hae tb left hand side se likhty hae...

In this case



So that: Intervals are:

(-∞,-1/2) (Increasing)

(-1/2 , 0) (decreasing)

(0, -1/2) (decreasing)

(1/2,0) (Increasing)

Note: here the +sign shows increasing and - sign shows decreasing

Note: Agr higher degree k coefficient me –ive sign aa jaye tu phr hum value –ive se show krty hae.. left hand side pe..

1) Let f(x)=sinx ; xε(-π,π)

Determine the increasing and decreasing order.

As we know that:

f'(x)=cosx

cosx=0➔x=π/2 ,-π/2

So Interval becomes as: -π,-π/2,π/2,π

For (-π, -π/2) interval f'(x)<0

as cos (-π)<0 ➔-1<0 (Decreasing)

Now out for f'(-π/2)=cos(-π/2)<0 (Decreasing)

For (-π/2, π/2)

f'(x)=cosx>0 whereas xε(-π/2, π/2) (increasing)

f'(x)=cosx<0 decresing x ε(π/2, π)

Maximum and minimum value:

Example: y=lnx/x has maximum value at x=e

Solution: dy/dx= $\frac{x.\frac{1}{x}-lnx.1}{x^2}=\frac{1-lnx}{x^2}$

f'(x)=0➔ 1-lnx=0➔ -lnx=-1 ➔lnx=lne➔x=e

f''(x) =$\frac{x^2.\left(-\frac{1}{x}\right)-(1-lnx)2x}{x^4}$=$\frac{-3x-2xlnx}{x^4}$

f''(e)= $\frac{-(3+2lnx)}{x^3}$

f''(e)=$-\frac{5}{e^3}<0$ (then f(x) is maximum at x=e)

Maxima and Minima (Applied Problems) (Shortcut Trick)

Find the number whose sum is 26 and product is as large as possible

Solution:

Let number be x,y

x+y=36 , xy=?

Trick: Check exponents of x and y and add them too i.e 1+1=2 in this case. So now divide 36 by 2 =18

Now: take the power of x and multiply with 18 and in the same way for y. (1*18), (1*18)➔ (18,18)

so x+y=18+18=36

xy=18*18=324 (Answer)

Important Results:

1) If dy/dx >0 , the tangent line makes an acute angle (less than 90*) with x-axis

2) If dy/dx<0, the tangent line makes an obtuse angle (greater than 90*) with x-axis

3) If dy/dx=0 the tangent is parallel to x-axis

4) If tangent is perpendicular to x-axis then dy/dx=∞ i.e dx/dy=0

5) If tangent is equally inclined to the axes then dy/dx=tan45*=tan135=±1

Test for monotonicity of functions:

1) f(x) is increasing on [a,b] if f'(x)≥0 for all x belongs to [a,b]

2) f(x) is strictly increasing on [a,b] if f'(x)>0 for all x belongs to [a,b]

3) f(x) is decreasing on [a,b]if f'(x)≤0 for all x belongs to [a,b]

4) f(x) is strictly decreasing on [a,b] of f'(x)<0.....

-Important Points:

1) When function y=f(x) is concave up the graph of its derivative y=f'(x) is increasing.

2) When function y=f(x) is concave down the graph of its derivative y=f'(x) is decreasing.

3) When the function y=f(x) has a point of inflection (Changes from concave up to concave down) the graph of its derivative y=f'(x) has maximum or minimum (also changes form increasing to decreasing

respectively)

4) The graph of y=f(x) is concave upward on those interval y=f''(x)>0 .

5) The graph of y=f(x) is concave downward on those interval y=f''(x)<0

6) The graph of y=f(x) has no point of inflection y=f''(x)=0

Important Results:

1) Derivative of a cubic function is quadratic function.

2) Derivative of a quadratic function is a linear function.

3) Derivative of a linear function is a constant function

4) Derivative of a constant function is always zero.

IMPORTANT POINTS TO REMEMBER:-

1) If degree of polynomial function is n then maximum number of bends in graph are (n-1).

2) If degree of polynomial function is n then maximum number of critical points are (n-1), except constant function. Every point of constant function is maximum, minimum, critical & point of inflexion.

3) $f(x) = x^{2n+1} + c$ where n∈N has always one bend and has always one point of inflexion

To find maximum and minimum values of Special Type of Function

f(x) = a sinx + b cosx

- Maximum Value of Function = $\sqrt{a^2 + b^2}$
- Minimum Value of Function = - $\sqrt{a^2 + b^2}$

CHAPTER 3
(Integration)

Guessing Answer from the options:-

You should know that differentiation and Integration are reverse of each other. If you have to evaluate an integral and there are four options in the mcq and you are unable to do integration, then take derivatives of all four options. 1 option will match with your question and that will be your answer.

Other Tricks:-

If the upper and lower limits of integration are additive inverse of each other i.e. their sum is zero then check the integrant function:

- If function is odd then no need to evaluate the integral, its answer will be zero.

  $\int_{-2}^{2} sinx\ dx = 0$

  sinx is an odd function.

- If function is even then change the limits .

$$\int_{-\pi}^{\pi} cosx\ dx$$

Now cosx is even. Now I will change the limits and Multiply the integral by 2 and I will get the answer. The thing you need to focus here is to change the limits. Given limits are [–π to π] and I am changing them with [0 to π] and multiply by 2.

$2x\int_0^{\pi} cosx\, dx$

Solve it and you will get the answer.

Standard formulae of Integration:

1) $\int 0\, dx = radial\ constant$
2) $\int 1\, dx = x + c$
3) $\int e^x\, dx = e^x + c$
4) $\int a^x\, dx = \frac{a^x}{lna} + c$
5) $\int \frac{1}{x}\, dx = lnx + c$
6) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$+c
7) $\int sinx\, dx = -cosx + c$
8) $\int cosx\, dx = sinx + c$
9) $\int tanx.secx\, dx = secx + c$
10) $\int cosecx.cotx\, dx = cosecx + c$
11) $\int cosec^2x\, dx = -cotx + c$
12) $\int \sec^2 x\, dx = tanx + c$
13) $\int secx\, dx = \ln|secx + tanx| + c$
14) $\int cosecx\, dx = \ln|cosecx - cotx| + c$
15) $\int \frac{1}{\sqrt{1-x^2}}\, dx = arc\ sinx + c$
16) $\int \frac{1}{1+x^2}\, dx = arc\ tanx + c$
17) $\int \frac{1}{\sqrt{x^2-1}} dx = arc\ secx + c$
18) $\int tanx\, dx = \ln|secx| + c$
19) $\int cotx\, dx = \ln|sinx| + c$

20) $\sin^2 mx = \frac{1-cos2mx}{2}$

21) $\cos^2 mx = \frac{1+cos2mx}{2}$

22) $tan^2\, mx = \sec^2 mx - 1$

23) $cot^2\, mx = \csc^2 mx - 1$

24) -2sinA.sinB=cos(A+B)-cos(A-B)

25) 2 sinA.cosB= sin(A+B)+sin(A-B)

26) 2 cosA.sinB= sin(A+B) –sin (A-B)

27) 2 cosA.cosB= cos (A+B) + cos (A-B)

Some Useful Subtitution:

1) $\sqrt{a^2 - x^2}$ $then\ x = asin\theta$

2) $\sqrt{x^2 - a^2}$ $then\ x = asec\theta$

3) $\sqrt{a^2 + x^2}$ $then\ x = atan\theta$

Some Integrand to remember:

1) $\int \frac{dx}{\sqrt{a^2-x^2}} = arcsin\frac{x}{a} + c$

2) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\ |x + \sqrt{x^2 - a^2}| + c$

3) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + c$

4) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\,arc\tan\frac{x}{a} + c$

5) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\,\text{arcsec}\frac{x}{a} + c$

6) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\left|\frac{x-a}{x+a}\right| + c$

7) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\left|\frac{a+x}{a-x}\right| + c$

8) $\int e^{ax}[af(x) + f'(x)]dx = e^{ax}f(x) + c$

Some special cases:

1) $\int e^{ax} sinbx\ dx = \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin(bx - arc\tan(\frac{b}{a})) + c$

2) $\int e^{ax} cosbx\ dx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx) + c$

3) $\int lnxdx = xlnx - x + c$

Definite Integral:

If ϕ(x) is any anti-derivative of f(x), then the difference $\phi(b) - \phi(a)$ is called definite Integral of f(x) from a to b is denoted by:

$$\int_a^b f(x)dx = \phi(b) - \phi(a)$$

Whereas a is called upper limit and b is called lower limit.

Geometrically, the definite integral $\int_a^b f(x)dx$ represents the area under the curve y=f(x) from a to b and above x-axis.

Fundamental Theorem of Calculus:

If f is continuous on [a,b] and $^{\phi}$(x) is any anti-derivative of f on [a,b] then

$$\int_a^b f(x)dx = \phi(b) - \phi(a)$$

Properties of Definite Integrals:

1) $\int_a^a f(x)dx = 0$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx\,, a < c < b$

4) If f(x) is an even , then $\int_{-a}^a f(x) = 2\int_0^a f(x)\,dx$

5) If f(x) is odd , then $\int_{-a}^a f(x)dx = 0$

Area Bounded by the curve and x-axis:

Area bounded by the curve y=f(x) and the x-axis from x=a to x=b is given as:

Area= $\int_a^b y\,dx = \int_a^b f(x)dx$

If f(x)≤0 for a≤x≤b , then area is below the x-axis and above the curve y=f(x) is given as:

Area = $-\int_a^b f(x)dx$

Shortcut:

Area between two parabola= 16/3 ab

U-Substitution (To solve the indefinite Integral-Easy Method)

Let us have some examples to understand this rule:

Example: $\int 3x^2(x^3+5)^7dx$----(1)

The question is just raised how to solve it with easy method.

Let we consider: put u= x^3+5

Now the take differential of it:

du= $3x^2$dx

From (1) we got the some notation like it:

=$\int u^7.du$

Now integrate it easily by using the power rule so:

=$\int \frac{u^8}{8}+c$

As we want our expression in x form so put back x^3+5=u

so: =$\int \frac{(x^3+5)^8}{8}+c$ (Answer)

Here the question must be raise out that all the questions of indefinite integral are followed by it. So answer will be yes but there will be arise some variation so here would like to explain those types too.

Example:

$\int \frac{x^3}{\sqrt{1-x^4}}\,dx$=? -------(1)

So Here we not actually got the situation as earlier but we have to make it as:

Put u= 1-x^4

so take the differential of it as:

du= -4x^3dx

As we don't want -4 in our expression upper side of integrand so we just divide but both the sides of differential by “-4”

so we got -du/4= x^3dx

So in putting the values in (1)

$=\int \frac{-\frac{1}{4}du}{\sqrt{u}} = -\frac{1}{4}\int u^{-1/2}\ du$= -1/2 $u^{1/2}$ +c

So now put back value : 1-x^4=u

= -1/2 (1-x^4)$^{1/2}$+c (Answer)

Example: Integrate $x\sqrt{x+2}$

Solution: Now put u=x+2

now take differential of it: du= dx

Now u=x+2➔ u-2=x

So $\int (u-2)\sqrt{u}$ du

= $\int (u-2)u^{1/2}\,du$

$=\int u^{\frac{3}{2}}$du-$\int u^{1/2}\,du$

$=\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}} + c$

Put back x+2=u

so:

$=\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$(Answer)

DIFFERENTIAL EQUATION:-

An equation containing at least one derivative of one or more dependent variable w.r.t an independent variable.

e.g. $x\frac{dy}{dx} + y\frac{d^2y}{dx^2} = 5$

ORDER: It is the highest derivative involved in the differential equation.

DEGREE: Positive integral exponent of highest order derivative.(There should be no radical sign in the Differential Equation while checking the degree).

MCQ

What is the order and degree of following Differential Equation?

$5(\frac{d^2y}{dx^2})^{1/3}$ - x $(\frac{d^3y}{dx^3})^{3/2}$ = 0

First do simplification,

$5(\frac{d^2y}{dx^2})^{1/3}$ = x $(\frac{d^3y}{dx^3})^{3/2}$

Take square on both sides

$5^2(\frac{d^2y}{dx^2})^{2/3} = x^2(\frac{d^3y}{dx^3})^3$

Now take cube on both sides

$(5^2)^3(\frac{d^2y}{dx^2})^2 = x^6 (\frac{d^3y}{dx^3})^9$

Highest derivative is of 3rd order and exponent of that is 9, So

Order=3 , Degree=4

SOLUTION OF DIFFERENTIAL EQUATION:-

An equation involving dependent and independent variables that satisfy the differential equation.

GENERAL SOLUTION OF DIFFERENTIAL EQUATION:-

A solution of differential equation which contains as many arbitrary constants as order of Differential Equation.

PARTICULAR SOLUTION:-

If order of Differential Equation > No. of Arbitrary Constants in solution

INITIAL VALUE CONDITIONS:-

- Conditions required to find arbitrary constants.
- To find n arbitrary constants, we need n initial value conditions.

More Entry Test Type MCQ:

Find the $\int_{-1}^{5}|x-3|dx$

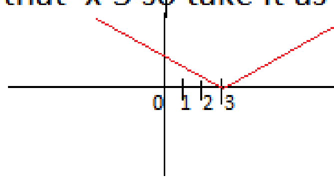
Solution: Shortcut to find the integral of modulus function when limits are given.. This shortcut is based upon the graphing and then to solve through area of triangle formula.

Using the shortcut:

As the graph of modulus function is always V-shaped and V-shaped based on the x-a=0 ==> x=a value.

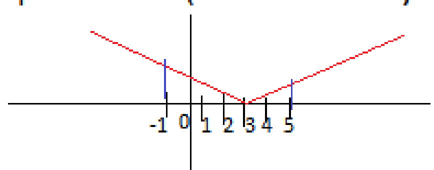
As we know that x-3 so take it as x=3 so

STEP#1:



STEP#2: Now check for the limits as x=5 and x=-1

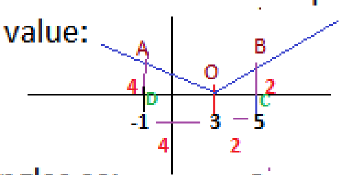
So graph it on the (Number Line System)



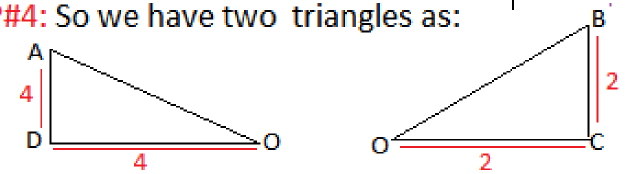
STEP#3: Now check for difference between V-shaped value and upper or low limit value:

As 3-(-1)=3+1=4

5-(3)=2



STEP#4: So we have two triangles as:



STEP#5: Using the formula of area of triangle:

Area of triangle AOD=1/2 (4)(4)= 8 sq.units

Area of triangle BOC= 1/2 (2)(2)= 2 sq.units

STEP#6: So area of both triangles = 8+2=10 sq.units

MCQ: Find the $\int_3^5 [x]dx$ whereas [x] is greatest value function.

Solution: Using the property:

We break notation into two parts:

$$\int_3^5 [x]dx = \int_3^4 [x]dx + \int_4^5 [x]dx$$

$\int_3^4 [x]$ is expression under consideration. Hence greatest value b/w 4 and 3 is left side positive integer on the number line system. So we take the left side value which is 3.

$\int_4^5 [x]$ is expression under consideration now. Hence greatest value b/w 5 and 4 is left side positive integer on the number line system. So we take the left side value which is 4.

So Expression becomes simplified as:

$=\int_3^4 3dx + \int_4^5 4dx$

$= 3\int_3^4 1dx + 4\int_4^5 1dx$

=3 x(3→4) + 4x(4→5)

= 3(4-3)+ 4(5-4)

= 3(1)+4=7 (Answer)

WORDS PROBLEM ON INTEGRALS:

Example: Find the Area b/w x-axis and the curve $y=x^2+1$ from x=1 and x=2

Solution: Using the Formula:

Area under the curve= $\int_a^b f(x)dx$

As $f(x) = x^2+1$ so:

Area=$\int_1^2 x^2 + 1\, dx$

Area =$\int_1^2 x^2 dx + \int_1^2 1\, dx$

Area= $x^3/3$|(1→2) + x|(1→2)

Area= 1/3 (2^3-1^3)+(2-1)

Area= 1/3 (7)+1

Area= 7/3 +1

Area= 10/3 sq. units

Example: Find the area of curve under the y=cosx , x=-π/2 to x=π/2

Solution: Area= $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosx\, dx$

Area=sinx|(-π/2→π/2)

Area= sinπ/2 – sin(-π/2)

Area = 1-(-1)

Area= 2 (Answer)

Example: Find the area between the x-axis and the curve y=4x-x^2?

Solution:

As x-axis , y=0

so put it: 4x-x^2=0→ x(4-x)=0→ x=4 and x=0

Area = $\int_0^4 4x - x^2 dx$

Area=4 $\int_0^4 xdx - \int_0^4 x^2 \quad dx$

Area= 4(x^2/2)(4→0) – (x^3/3)(4→0)

Area= 2(4^2-0)-1/3 (4^3-0)

Area= 2(16)- (1/3)(64)

Area= 32- 64/3

Area= (96-64)/3

Area=32/3 sq. units (Answer)

Differential Equations:

Example:

xdy/dx= 1+y

1/1+y dy= 1/x dx

Taking integral on both sides.

$\int \frac{1}{1+y}\, dy = \int \frac{1}{x} dx.$

ln(1+y)= ln(x)+lnc

ln(1+y)= lncx

1+y=cx

y=cx-1 (Answer)

Find the δy and dy in the following case:

1)y=x^2-1 ---(i) when x changes from 3 to 3.02

Solution:

Add increment on both sides:

y+δy= $(x+\delta x)^2$ -1 (ii)

y+δy= x^2+$(\delta x)^2$+2δx-1

Now (ii) –(i)

y+δy-y= x^2+$(\delta x)^2$+2δx-1- x^2+1

δy= $(x+\delta x)^2 - x^2$ -----(iii)

Now we let x+δx=3.02 and put x=3

Thus: dx= $\delta x = 0.02$

Put the value of x and δx in (ii)

δy =$(3+0.02)^2$-3^2

$\delta y = 9.1204 - 9$

δy= 0.1204

Now Original statement :

y=x^2-1

Taking differential on both sides:

dy= 2x dx

so dy= 2(3) (0.02)

dy= 0.12 (Answer)

2) Find the y= $\sqrt[4]{17}$

Let f(x)=$x^{1/4}$ .... (1)

Take x=16

so x+δx=17

δx= 1

Put x=16 in (i)

f(16)= $(16)^{1/4}$

f(16)=2

Differentiate w.r.t "x" to (1)

f'(x)= ¼ $x^{-3/4}$ dx

Now put x=16

f'(16)= 1/3$(16)^{3/4}$=1/32

f'(16)= 0.01325

Using formula:

f(x+δx)=f(x)+ f'(x)dx

As we know that all:

f(16+1)= f(16)+f'(16) dx

f(17) =2 +(0.03125)(1)

f(17)= 2.03125 (Answer)

NOTE:

Theorem On Integration:

1) d/dx $(\int f(x)dx = f(x)$

2) $\int f'(x)dx = \int \frac{d}{dx}(f(x)dx = f(x)$

Geometrically, an infinite integral is a family of curves that are vertical translation of one another.

Integration is also called **Anti derivation.**

A function Φ(x) is called **primitive** or integral or **anti derivative** of f(x)

INTEGRATION BY PARTS – THE D–I METHOD

This is a short cut to integration by parts and is especially useful when one has to integrate by parts several times. It is a schematic method of the traditional udv method that usually is written as $\int u dv = uv - \int v du$ in calculus books.

We will illustrate and demonstrate by using examples. Remember that integration by parts is usually indicated when you have to integrate a product of two functions. This method works exceedingly well when one of these functions is a polynomial and the other is successively integrate. In this case it is possible to successively differentiate until the last derivative is zero.

Example 1. Find $\int xe^x dx$. The format is as follows:

D	***I***	
x	e^x	+
1	e^x	-
0	e^x	+

D means to differentiate the functions in that column. ***I*** means to integrate the functions in that column. Form the diagonal products as indicated by the arrows in the above table alternating algebraic signs as you move down the table. Finally, you integrate the product of the last entry in the ***D*** column and the last entry in the ***I*** column, with the appropriate sign, of course.

It follows that the solution to the above problem is: $\int xe^x dx = +x \cdot e^x - 1 \cdot e^x + \int 0 \cdot e^x dx$

which is: $\int xe^x dx = xe^x - e^x + C$.

Example 2. Find $\int x^2 e^x dx$. The format is as follows:

D	***I***	
x^2	e^x	+
2x	e^x	-
2	e^x	+
0	e^x	-

and therefore the answer to the problem is: $\int x^2 e^x dx = +x^2 \cdot e^x - 2x \cdot e^x + 2 \cdot e^x - \int 0 \cdot e^x dx$

or $\int x^2 e^x dx = x^2 e^x - 2xe^x + 2xe^x + C$.

Example 3. Find $\int x\sin x dx$.

D	I	
x	sinx	+
1	-cosx	-
0	-sinx	+

and therefore the solution to the problem is: $\int x\sin x dx = -x\cos x + \sin x + C$.

Here we have not written the last term as $\int 0\cdot(-\sin x)dx$ but rather just as an arbitrary constant of integration C

Example 4. Find $\int e^{-2x}\cos(3x)dx$.

Notice that here neither function is a polynomial. Since we have been picking the polynomial, x's to powers, and differentiated until we reached zero we were able to discard the last integral of a product which was zero and merely write *C*. We can use the D–I method to work out the above integral in the following fashion:

D	I	
e^{-2x}	cos3x	+
$-2e^{-2x}$	sin3x/3	-
$4e^{-2x}$	-cos3x/9	+

Notice we stopped the process when the product of the functions in the ***D*** and ***I*** columns in the third row was the same as the product of the functions in the ***D*** and ***I*** columns in the first row, except for constants. That is we have $e^{-2x}\cos 3x$ in the first row and we have $-\frac{4}{9}e^{-2x}\cos 3x$ in the third row. They are the same except for the constant multiple $-\frac{4}{9}$.

We have: $\int e^{-2x}\cos(3x)dx = +\frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x - \frac{4}{9}\int e^{-2x}\cos(3x)dx$.

Solving for the integral we wish to evaluate, and adding an arbitrary constant of integration, we have:

$$\int e^{-2x}\cos(3x)dx = \frac{3}{13}e^{-2x}\sin 3x - \frac{2}{13}e^{-2x}\cos 3x + C.$$

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CHAPTER 4

TRICK FOR GUESSING REQUIRED VERTEX:

The coordinates of 3 vertices of a rectangle are (5,7) (2,-2) and (8,4) the forth vertex ?????

Just draw a simple graph and plot these points and join them by a line. You will easily know about the position of fourth vertex. Similarly, technique can be applied for Triangle.

Formulas and tricks Sheet:

Distance Formula:

The distance between two point $A(x_1,y_1)$ and $B(x_2,y_2)$ is given by

$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Note:

1) Distance of a point (x ,y) from origin = $\sqrt{x^2+y^2}$
2) Distance of a point (x,y) from x-axis = |y|
3) Distance of a point (x,y) from y-axis = |x|
4) Directed distance of (x,y) from x-axis= y
5) Directed distance of (x,y) from y axis= x

Point dividing the join of two points:

If $A(x_1,y_1)$ and $B(x_2,y_2)$ are two given points in the plane. The coordinate of the point dividing segment AB internally in the ratio k_1: k_2 are:

$\left(\frac{k_1x_2+k_2x_1}{k_1+k_2},\frac{k_1y_2+k_2y_1}{k_1+k_2}\right)$

Note:

1) If the directed distances AP and PB have opposite signs i.e P is beyond AB. Then their ratio is negative and P is said to divide AB externally. Coordinate of P are given as:-
$\left(\frac{k_1x_2-k_2x_1}{k_1-k_2}, \frac{k_1y_2-k_2y_1}{k_1-k_2}\right)$.

2) If k_1:k_2=1:1 then P becomes mid point of AB and coordinate of P are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$(Mid Point Formula)

3) The ratio is which y-axis divides the line joining points A (x_1,y_1) and $B(x_2,y_2)$ is $-\frac{x_1}{x_2}$

4) The ratio is which x-axis divides the line joining points A (x_1,y_1) and $B(x_2,y_2)$ is $-\frac{y_1}{y_2}$.

Centroid of a Triangle:

The point of intersection of medians of a triangle is called centroid of triangle. The centroid divides each median in the ratio 2:1

If $A(x_1,y_1)$,$B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of triangle then coordinates of centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

In center of a Triangle:

The point of intersection of angle bisectors of a triangle is called in center of triangle.

If $A(x_1,y_1)$,$B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of triangle then coordinates of in center are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$.

Translation of axes:

X=x-h

Y=y-h

Rotation of axes:

X=xcosθ+ysinθ

Y=ycosθ-xsinθ

Inclination of a line:

The angle α (0<α<180*) measured counterclock wise from positive x-axis to non-horizontal straight line is called inclination of a line.

1) If a line parallel to x-axis then α=0*

2) If a line is parallel to y-axis then α=90*

➔ m= tanα (Angle of inclination)

➔If α be the inclination of a non-vertical straight line P(x_1,y_1) and Q(x_2,y_2) is given by: m=$\frac{y_2-y_1}{x_2-x_1}$ $or\ m = \frac{y_1-y_2}{x_1-x_2}$

Note:

1) If the line is horizontal then its slope is zero.

2) If the line is vertical then its slope is infinity

3) If 0*<α<90* then slope is positive

4) If 90*<α<180* then slope is negative.

5) Three points A, B and C are collinear if slope of AB=Slope of BC=Slope of CA

6) Two lines l_1 and l_2 with respective slope m_1 and m_2 are:

a) parallel iff m_1=m_2

b) Perpendicular iff m_1m_2=-1 or m_1=-1/m_2

c) Non-parallel (Intersected) iff $m_1 \neq m_2$

Equation of straight line parallel to x-axis

The equation of the straight line parallel to x-axis is y=a

Note:

i) If a>0, then the line is above the x-axis

ii) If a<0 , then the line is below the x-axis

iii) If a=0 then the line becomes the x-axis. Thus equation of the x-axis is y=0

Equation of straight line parallel to y-axis

The equation of the straight line parallel to y-axis is x=b

Note:

i) If b>0, then the line is above the y-axis

ii) If b<0 , then the line is below the y-axis

iii) If b=0 then the line becomes the y-axis. Thus equation of the y-axis is x=0

Intercepts of the line on axis:

a) If a line intercepts x-axis at (a,0) , then a is called x-intercepts of the lines

b) If a line intercepts at y-axis at (0,b) then b is called y-intercepts of the lines

1) Slope-intercept Form:

Equation of non-vertical straight line with slope m and y-intercept is given by:

y=mx+c

2) Point-slope form:

The equation of a straight line passing through the point (x_1,y_1) and having the slope m is given by:

$y-y_1=m(x-x_1)$

3) Symmetric Form:

The equation of a straight line passing through the point (x_1,y_1) and making an angle with positive direction of x-axis is given by:

$\frac{x-x_1}{cos\alpha}=\frac{y-y_1}{sin\alpha}=r$

4) Two point form:

The equation of a straight line passing through two points (x_1,y_1) and (x_2,y_2) is given by: $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

5) Intercept form:

The equation of a straight line whose non-zero x and y-intercepts are a and b respectively is given by: $\frac{x}{a}+\frac{y}{b}=1$

6) Normal Form:

The equation of a straight line such that length of the perpendicular from the origin to the line is p and α is the inclination of this perpendicular is given by; $x\cos\alpha + y\sin\alpha = p$

➔Slope of a line:

It is give by $m = -a/b$

➔x-Intercept of line=$-c/a$

➔y-intercept of line= $-c/b$

➔Distance of line from the origin =$\frac{|c|}{\sqrt{a^2+b^2}}$

Position of a point:

Consider a non-vertical line $l=ax+by+c=0$ where $b>0$ in xy plane . A point (x_1,y_1) lies:

a) Above the line l if $ax_1+by_1+c>0$

b) Below the line l if $ax_2+by_2+c<0$

Conditions for two lines to be parallel , perpendicular or coincident:

a) Parallel if $a_1b_2-a_2b_1=0$

b) Perpendicular if $a_1a_2+b_1b_2=0$

c) Coincident if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

d) Intersected if $a_1b_2-a_2b_1\neq 0$

➔Distance of a point from line:

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

➔Distance B/w two non-parallel lines:

If $m_1\neq m$ then $d=0$

➔ Distance between two parallel line: $(m_1=m_2)$

$d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

Note: If the lines are not in actual form that they can be converted into parallel form by suitable multiplication if need.

If $c_1 = c_2$, lines will be co-incident i.e. distance will be zero

➔If points P,Q and R are collinear then Δ=0

➔ Point of Intersection of Two Lines:

PIL=$\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}\right)$

Note: In MCQ, the paper setter give the options so you can solve it back solving technique more conveniently.

➔ Condition of Concurrency of Three lines:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

Note: An infinite number of lines can be passed through the point of intersection of two lines.

☺ Circumcenter of Triangle:

The point of intersection of right bisector of the sides of triangle is called circumcenter.

☺ Orthocenter of Triangle:

The point of intersection of altitudes of the sides of a triangle is called orthocenter.

Note: Centroid, Circumcenter and Orthocenters are collinear

Note: Centroid divides the line joining orthocenter and circumcenter in ratio 2:1.

SHORTCUTS IN RECTANGULAR COORDINATES:

➔How to find the area of triangle within second:

Let $A(x_1,y_1)$,$B(x_2,y_2)$ and $C(x_3,y_3)$ are given then Area of triangle by it.

Area of triangle = $\frac{1}{2}\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_3 & y_2-y_3 \end{vmatrix}$

Example:

Let us have points as A(-1,3) ,B(2,4) and (5,6)

Using trick:

Area of triangle = $\frac{1}{2}\begin{vmatrix} -1-5 & 3-6 \\ 2-5 & 4-6 \end{vmatrix}$

Area of triangle= $\frac{1}{2}\begin{vmatrix} -6 & -3 \\ -3 & -2 \end{vmatrix}$

Area of triangle= ½ (12-9)= 3/2 sq.units

➔How to find the 3rd unknown vertex in case of isosceles right angled triangle within seconds:

Let A (x_1,y_1) , $B(x_2,y_2)$ are given and C is unknown to form a triangle so it is given by:

$x_3=\frac{(x_1+x_2)\pm (y_1-y_2)}{2}$

$y_3= \frac{(y+y_2)\mp(x_1-x_2)}{2}$

Example:

Let we have two points A(2,0) and B(-2,0) and we have one unknown point to form a isosceles right angled triangle

Using the trick:

$x_3=\frac{(2-2)\pm (0-0)}{2}=0$

$y_3= \frac{(0+0)\mp(2+2)}{2}=-2\ and\ 2$

So (0,±2) is unknown C point. (Answer)

➔How to find the 3rd unknown vertex in case of equilateral triangle within seconds:

Let A (x_1,y_1) , $B(x_2,y_2)$ are given and C is unknown to form a triangle so it is given by:

$x_3=\frac{(x_1+x_2)\pm \sqrt{3}(y_1-y_2)}{2}$

$y_3= \frac{(y+y_2)\mp\sqrt{3}(x_1-x_2)}{2}$

Example:

Let we have two points A(2,0) and B(-2,0) and we have one unknown point to form a isosceles right angled triangle

Using the trick:

$x_3=\frac{(2-2)\pm\sqrt{3}(0-0)}{2}=0$

$y_3=\frac{(0+0)\mp\sqrt{3}(2+2)}{2}=-2\sqrt{3}\ and\ 2\sqrt{3}$

So $(0,\pm2\sqrt{3})$ is unknown C point. (Answer)

➔How to find the vertices of a triangle when the mid points of each vertex is given:

A=$(x_1+x_3-x_2, y_1+y_3-y_2)$

B= $(x_1+x_2-x_3, y_1+y_2-y_3)$

C= $(x_2+x_3-x_1, y_2+y_3-y_1)$

Example:

Let us have three mid points of a triangle (3,-1),(3,3) and (1,-1) on each vertex. So find the values of each vertex.

Using the trick:

A=$(3+3-1, -1-1-3)$=(5,-5)

B= $(3+1-3, -1+3+1)$=(1,3)

C= $(3+1-3, 3-1+1)$=(1,3) (Answer)

➔How to find the fourth unknown vertex of a parallelogram within seconds:

Concept: The opposite vertex to unknown vertex should be minus signed. And the two known vertices which are opposite to each other are added up in parallelogram/quadrilateral so that condition becomes as:

If A is unknown then A= B+D-C

If B is unknown then B=A+C-D

If C is unknown then C= B+D-A

If D is unknown then D= A+C-B

Example:

If the A(1,6) , B(2,5), C(5,1) and D(x,y) forms a parallelogram then find the forth unknown vertex?

Using the trick:

As D is unknown so we use the condition as D= A+C-B

So: (x,y)= (1,6)+(5,1)- (2,5)

(x,y)= (1+5-2, 6+1-5)

(x,y)= (4,2) (Answer)

So fourth vertex is (4,2)

➔ How to find the circumcenter of right angled triangle within second:

Concept: The circumcenter is midpoint of hypotenuse of triangle.

Example:

If A(3,2) ,B(3,4) and C(7,2) forms a right angle triangle so find the circumcenter of it?

Using the trick:

On graphing,the hypotenuse is the side BC so take the mid point of it as: Circumcenter= (B+C)/2 = $\left(\frac{7+3}{2},\frac{2-4}{2}\right)$=(5,-1) (Answer)

➔ How to find the Centroid if Orthocenter and Circumcenter are given within seconds :

Concept: As we know that Centroid , orthocenter and circumcenter are collinear .Furthermore the centroid divides the orthocenter and circumcenter in 2:1.

So Here using the concept:

Here orthocenter= (x_1,y_1) and circumcenter= (x_2,y_2)

Centroid➔ $x= (x_1+2x_2)/3$ and $y= (y_1+2y_2)/3$

Example:

Orthocenter (3,-5) ,Circumcenter (6,2) so centroid=?

Using trick:

Centroid➔ x= (3+2(6))/3 and y= (-5+2(2))/3➔(15/3 , -1/3) ➔ (5,-1/3) (Answer)

Note: You can find anyone of it , if two of them are given in this fashion.

➔ Area of triangle on line joining of the circumcenter , orthocenter and centroid is always zero.

➔ How to find the orthocenter of a right angled triangle within seconds:

Concept: Orthocenter is a point where the right angle is formed:

Example:

If A(3,2) , B(3,-4) and C(7,2) are forming a right angled triangle so find the orthocenter of triangle.

Using the trick:

On graphing the A(3,2) is vertex where the right angle is formed so it is orthocenter of triangle.

Note: In equilateral triangle, centroid, circumcenter and orthocenter coincides on the same line.

Note:

Area of Quadrilateral= ½ $\begin{vmatrix} x_1 - x_2 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$

Position of point with respect to line:

The point P is above or below l respectively if ax_1+by_1+c and b have same sign or opposite sign.

1) above if b have same sign

2) below if b have opposite sign

Example: Check whether point (2,-4) lies above or below the line : 4x+5y-3=0

Here b=5 so:

so; 4(-2)+5(4)-3= 9>0 so (b and expression has same sign so point is above the line)

Note:

The point $P(x_1,y_1)$ and the origin are

1) on the same side of line l according as $ax_1+by_1+c=0$ and c has same sign

2) on the opposite side of line l according as $ax_2+by_2+c=0$ and c has opposite sign

Example:

Check whether the origin and the point P(5,-8) lies on the same side or opposite side.

3x+7y+15=0

Here c=15

and P(5,-8)

so 3(5)+7(-8)+15=0

-26<0

So that c and $ax_1+by_1+c=0$ has opposite signs so it lies opposite to origin.

➔ For checking the position we make the coefficient of y positive by multiplying the equation by (-1) if need.

-Some Important Results:

1) Altitudes are concurrent

2) Median are concurrent

3) Right bisector are concurrent

Area of Trapezoidal Regian is given by:

½ (sum of ||sides) (distances b/w ||sides)

➔ Angel between two lines:

1) Acute angle between two lines:

$\tan\theta = \left|\frac{a_1b_2 - b_1a_2}{a_1a_2 + b_1b_2}\right|$

2) Angle between two lines when m_1 and m_2 are given:

$\tan\theta = \frac{m_2 - m_1}{1 + m_2m_1}$

3) Acute angle between l_1 and l_2 when m_1 and m_2 are given:

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2m_1}\right|$

➔ Equations of line:

1) The equation of a line parallel to a given line ax+by+c=0 is ax+by+k=0 whereas k is constant.

2) The equation of a line perpendicular to a given ax+by+c=0 is given by by-ax+k=0 where k is constant.

3) The area of triangle formed by these lines with axis is:

Area= $\frac{c^2}{2\sqrt{ab}}$

Note: Two non-parallel lines are intersected at one and only one point.

General Homogeneous equation of second degree.

The equation $ax^2+2hxy+by^2=0$ is called the general homogeneous equation of second degree. It represents the pair of straight lines both passing through the origin.

➔ The lines are real and distinct if $h^2-ab>0$

➔ The lines are real and coincident if $h^2-ab=0$

➔ The lines are imaginary if $h^2-ab<0$

☺To Find the angle between the lines represented by $ax^2+2hxy+by^2=0$

Let $y=m_1x$ and $y=m_2x$ be the lines represented by $ax^2+2hxy+by^2=0$ then

$m_1+m_2=-2h/b$ and $m_1*m_2=a/b$

whereas: $m_1=\frac{-h+\sqrt{h^2-ab}}{b}$

$m_2=\frac{-h-\sqrt{h^2-ab}}{b}$

If θ is the measure of angle b/w two lines represented by $ax^2+2hxy+by^2=0$ then:

$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$

For Acute angle , it is given by:

$\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

Some Important Results:

1) The two lines are coincident if $h^2-ab=0$

2) The two lines are orthogonal or perpendicular if $a+b=0$

Important Points:

1) The equation of pair of straight lines passing through the origin and perpendicular to the pair of lines is represented by $ax^2+2hxy+by^2=0$ is given by $bx^2-2hxy+ay^2=0$

2) The equation of pair of straight lines passing through the origin and parallel to the pair of lines is represented by $ax^2+2hxy+by^2=0$ is same.

3) The equation of bisector of angles b/w the pair of lines is represented by $ax^2+2hxy+by^2=0$ us $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$

Important Result:

The area of triangle formed by $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is given by $\left|\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|$

Example:

Find the area of the region bounded by:

$10x^2 - xy - 21y^2 = 0$ and $x+y+1=0$

Solution:

Here n=1 , l=1,m=1 a=10 , 2h=-1 ➔h=-1/2 and b=-21

Using the trick:

Area of triangle=$\left|\dfrac{1^2\sqrt{(\frac{-1}{2})^2-(10)(-21)}}{(10)1^2-2(-\frac{1}{2})(1)(1)+(-21)1^2}\right| = \left|\dfrac{\sqrt{\frac{1}{4}+210}}{(10)+1-21)}\right| =$

$\left|\dfrac{\sqrt{\frac{841}{4}}}{-10}\right| = \left|\dfrac{29}{-20}\right|$=29/24sq.units (Answer)

CHAPTER 5

What is an Equation?

An equation in two variables defines a path, line, parabola or something else.

What is Inequality?

An equality in two variables defines a region whose boundary is defined by the associated equation of that inequality.

Inequality: $x-y \geq 1$ Associated Equation: $x-y = 1$

Keep in mind,

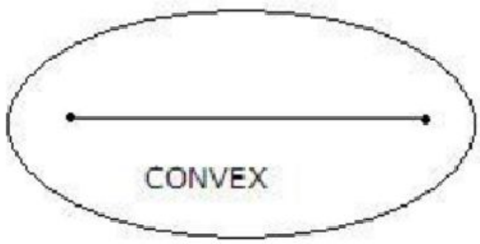
- Region defined by one inequality has never a corner point.
- Two or more inequalities may have or may not have a corner point.

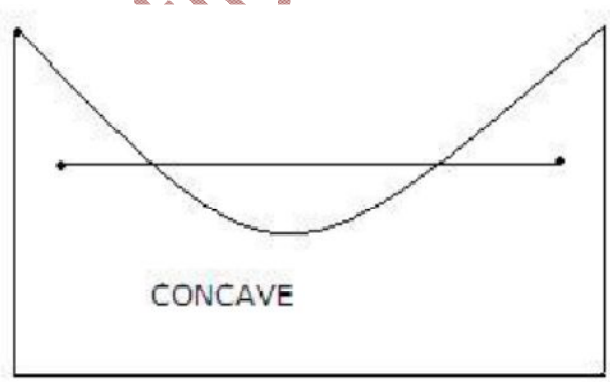
BOUNDED REGION:-

- A region that can be enclosed in a circle of finite radius.
- Region defined by one inequality can never be bounded.
- Region defined by two inequalities can never be bounded.
- Region defined by three or more inequalities may or may not be bounded.

CONCAVE & CONVEX REGION:-

- If the line joining any 2 points of the region lie entirely in the region then it is convex region.
- If the line joining any 2 points of the region does not lie entirely in the region then it is concave region.
- Region defined by one inequality is always convex.
- Region defined by linear inequalities is convex but region defined by Quadratic or high degree inequalities may or may not be convex.





IMPORTANT POINTS:-

1) X=c is a vertical line. x>c : Right half open. x≥c : Right half closed

2) Y=c is a horizontal line. y<c : Lower half open. y≤c : Lower half closed.

3) ax+by=c is a slant straight line. ax+by>c: Upper ax+by<c: Lower

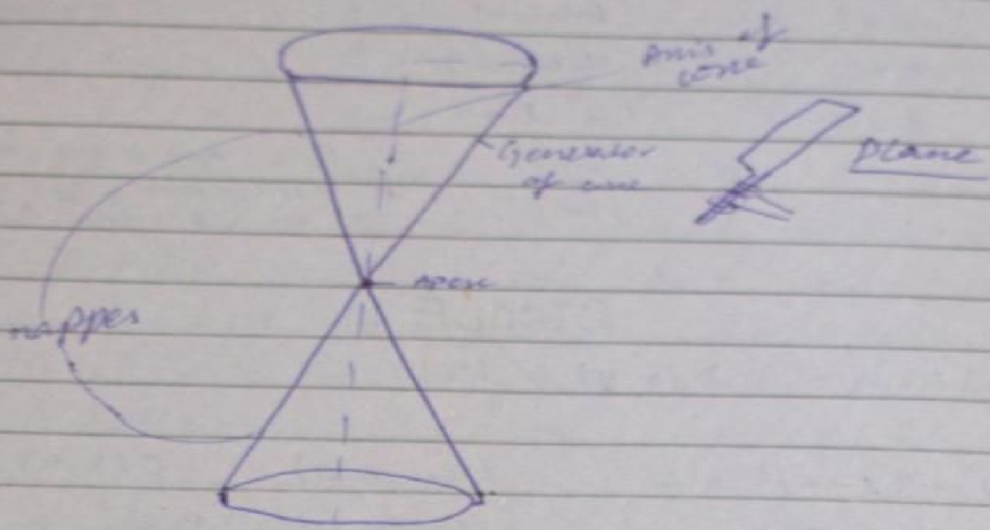
4) x+y ≥ 0 x+y = 0 If, 0≥0, origin test fails. Choose any other point.

CHAPTER 6

CH 6

CONIC SECTIONS

Double Right Circular cone :-



(1) If cutting plane is ⊥ to axis or horizontal, resulting section will be a circle (conic)

b) If cutting plane is ⊥ to axis but passes through apex, resulting section is point circle. (Degenerate conic)

(2) If cutting plane is slightly tilted & does not pass through apex, resulting section will be ellipse.

b) .... pass through apex → point.

(3) If cutting plane is ∥ to generator, cuts one nap & does not pass through apex, resulting section is parabola.

b) through apex → line

4) If cutting plane is ∥ to axis of cone, cuts both naps & does not pass through apex, resulting section is hyperbola

b) through apex → two intersecting lines.

Chapter #6:-

Conic Section:.

• Circle:.

1) The general equation of the circle is given by $x^2+y^2+2gx+2fy+c=0$.
2) The standard equation of the circle is given by $x^2+y^2=a^2$.
(whereas this is called polar form).
3) The equation of circle at center (h,k) is given by $(x-h)^2+(y-k)^2=a^2$.
4). The set notation form of circle is given by $S(h,k)=\{P(x,y) \mid |PC|=r\}$ where (h,k) is center.

• Radius of circle:-

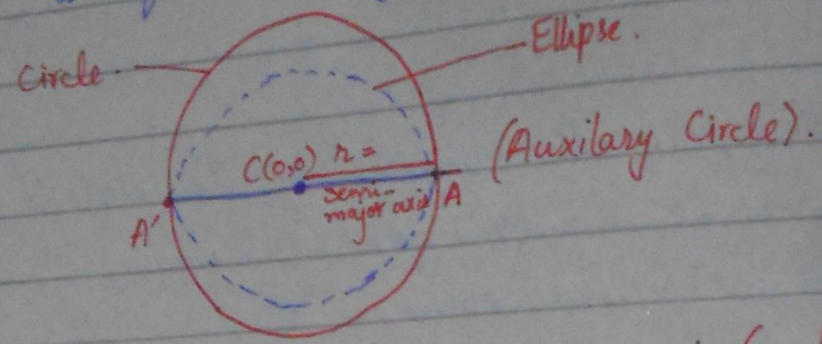
⇒ 1) Radius of equation of general form of circl is $\sqrt{g^2+f^2-c}$.
2) Radius of standard equation of circl is given by $x^2+y^2=r^2$.

* Properties of circle:.

* If $r=1$, circle is called "unit circle".
* If $r=0$, circle is called "point circle".
* If $r>1$, circle is known "Real circle".
* If $r<1$, circle is known "Imaginary circle".

*Auxiliary circle is one whose diameter

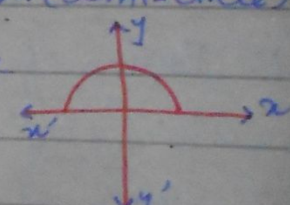
is equal to major axis of the "ellipse".



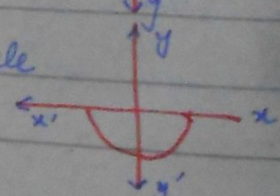
* Real circle is one whose radius ($r=\sqrt{g^2+f^2-c}>0$) with center $C(-g,-f)$.
* Imaginary circle is one whose radius ($r=\sqrt{g^2+f^2-c}<0$) with center $C(-g,-f)$ (virtual circle).
* point circle is one whose radius ($r=\sqrt{g^2+f^2-c}=0$) with center $C(-g,-f)$

* Standard equation of ~~circle~~ : (semi-circle).

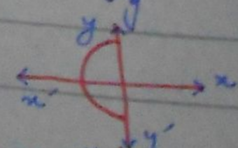
1) The equation of upper semi-circle is given by $y=\sqrt{a^2-x^2}$.



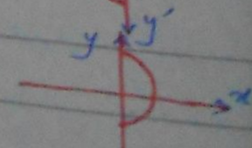
2) The equation of lower semi-circle is given by $y=-\sqrt{a^2-x^2}$.



3) The equation of right-semi circle is given by $x=\sqrt{a^2-y^2}$.



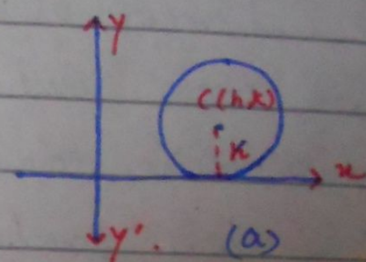
4) The equation of left semi-circle is given by $x=-\sqrt{a^2-y^2}$.



* Different forms of Standard equation of circle:

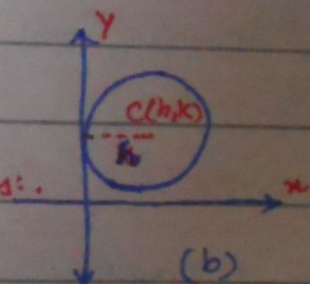
1) Circle withe center C(h,k) which touches the x-axis:

$$(x-h)^2+(y-k)^2=k^2.$$

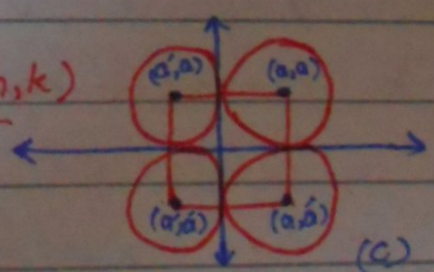


2) Circle with center C(h,k) which touches the y-axis:

$$(x-h)^2+(y-k)^2=h^2.$$



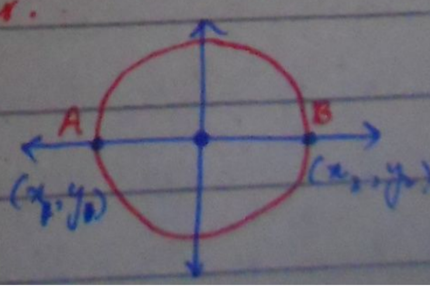
3) circle with center C(h,k) which touches both axes:



$$(x\pm a)^2+(y\pm a)^2=a^2.$$

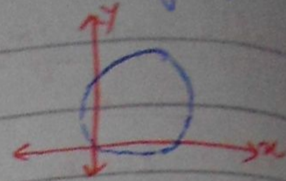
4) Equation of circle on line joining Two points as Diameter.

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$



*** Intercept ... the axes:**

a) The length of intercept made by equation $x^2+y^2+2gx+2fy+c=0$



1) On x-axis: $2\sqrt{g^2-c}$.

2) on y-axis: $2\sqrt{f^2-c}$.

b) Intercept always be positive.

c) If circle touches x-axis then $g^2=c$.

d) If circle touches y-axis then $f^2=c$.

e) If circle touches both axes then $g^2=f^2=c$.

f) If $c=0$, then circle passes through origin.

*** Parametric Equations of a circle:**

1) The parametric equation of a circle $x^2+y^2=a^2$ is $(a\cos\theta, a\sin\theta)$.
$\quad\quad x \quad\quad y$

2) The parametric equation of circle with the center $C(h,k)$ $(x-h)^2+(y-k)^2=a^2$ is given by;

$$x=h+a\cos\theta.$$
$$y=k+a\sin\theta.$$

*** Position of point with respect to a circle:**

- $x^2+y^2+2gx+2fy+c>0$ (outside)
- $x^2+y^2+2gx+2fy+c<0$ (inside)
- $x^2+y^2+2gx+2fy+c=0$ (on).

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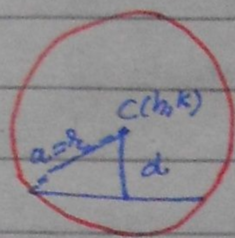
Note:- • Co-efficient of x^2 and y^2 is equal to 1 in equation

• Gen eq of circle contains no term involving xy so $xy=0$

• **Intersection of a line and Circle:**

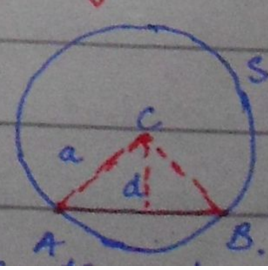


Let l be a line in the plane of a circle and d be a perpendicular distance from the center $C(h,k)$ of a circle.

* l intersects S in two distinct points iff $d<a$
* l intersects S in one and only point iff $d=a$
* l does not exist iff $d>a$.
* The line $y=mx+c$ intersects the circle $x^2+y^2=a^2$ in the most two points, these points are;
  * Real and distinct if $a^2(1+m^2)>c^2$.
  * Real and coincident if $a^2(1+m^2)=c^2$.
  * Imaginary if $a^2(1+m^2)<c^2$.

• **Length of intercept made by a circle on a line:-**

$$|AB| = 2\sqrt{a^2-d^2} \quad \text{—(i)}$$



• Example:- If C(4,3) be

* Contact of two circles:

a) If intersect in two real and distinct points if and only if $|r_1 - r_2| < |AB| < r_1 + r_2$.

b) Touch each other externally if and only if $|AB| = r_1 + r_2$ and the point of contact is given by $C = \left(\dfrac{r_1 x_2 + r_2 x_1}{r_1 + r_2}, \dfrac{r_1 y_2 + r_2 y_1}{r_1 + r_2}\right)$.

c) Touch each other internally if and only if $|AB| = |r_1 - r_2|$ and the point of contact is given by $C = \left(\dfrac{r_1 x_2 - r_2 x_1}{r_1 - r_2}, \dfrac{r_1 y_2 - r_2 y_1}{r_1 - r_2}\right)$

d) One circle lies outside the other if; $|AB| > r_1 + r_2$.

e) One circle is contained in other if; $|AB| < |r_1 - r_2|$.

* Tangent to a circle at a Line point:

1* The tangent to a curve is a line that cuts a curve in two coincident point. Equation tangent at (x_1, y_1) $y - y_1 = m_{(x_1, y_1)}(x - x_1)$.

2* Equation of tangent to the circle $x^2 + y^2 = a^2$ is given by $xx_1 + yy_1 = a^2$.

3* Equation of tangent to the general form of circle is given by:.

4* Equation of tangent of standard form at $(a\cos\theta, a\sin\theta)$ is given by:

$$x\cos\theta + y\sin\theta = a.$$

• <u>Equation of tangent in Slope form:</u>

1) The equation of tangent of slope to circle $x^2+y^2=a^2$ is $y = mx \pm a\sqrt{(1+m^2)}$.

2) The coordinates of points of contact are.

$$\left(\pm \frac{am}{\sqrt{1+m^2}}, \mp \frac{a}{\sqrt{1+m^2}}\right)$$

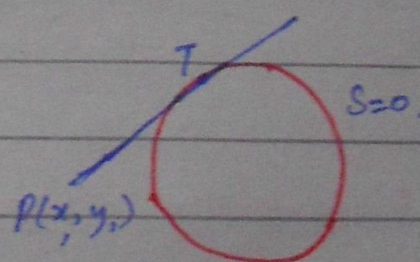
* <u>Condition of Tangency:</u>

$$c^2 = a^2(1+m^2).$$

⇒ <u>NOTE:</u> A line will be tangent to circle if and if the length of perpendicular from the center of circle to line is equal to the radius of the circle.

* <u>Length of tangent:</u>

$$|PT| = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$



* <u>Pair of tangents:</u> Two tangents can be drawn from a point to a circle, the tangent will be a;

1) Real or distinct. (outside)

2) Real and ~~imaginary~~ coincident. (on)

3) Imaginary (inside).

Note: The pair of tangent from (0,0) to the circle $x^2+y^2+2gx+2fx+c=0$ are at right angle if $g^2+f^2=2c$.

* **Equation of Normal to circle:**

- The equation of normal to the equation of circle $x^2+y^2+2gx+2fx+c=0$ is given by:

$$\frac{x-x_1}{x_1+g}=\frac{y-y_1}{y_1+f}.$$

- The equation of normal to the standard equation of circle $x^2+y^2=a^2$ is given by:

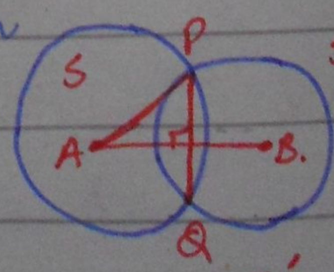
$$\frac{x}{x_1}=\frac{y}{y_1}$$

- Equation of normal to any curve at the point $P(x_1,y_1)$ is given by:

$$y-y_1=\frac{-1}{dy/dx|_{(x_1,y_1)}}(x-x_1).$$

* **Common chord of two circles:**

- The equation of common chord of two circles is given by:



(b)

$$\boxed{S-S'=0}$$

As $S=0$

$S'=0$

NOTE: In order to get equation of common chord, the co-efficient of x^2 and y^2 should be 1.

* **Length of Common Chord:**

⇒ It is given by:

$$PQ = 2\sqrt{r^2 - d^2}.$$

(See figure (b) for explanation).

r = radius of circle.

d = perpendicular distance from center to the common chord.

* **Some important points:** [common CHORD].

1) The length of the common chord becomes maximum when it has a diameter of smaller one between them.

2) If the length of common chord becomes zero; it means the common chord becomes the common tangent to each other as both circle touches each other at the common point of contact.

* **Some properties of Circle:**

1) Perpendicular dropped from the center on the chord bisects the chord.

2) The perpendicular bisector of a chord of any circle passing through the center of circle.

3) The line joining the center of circle to the mid point of a chord is perpendicular to the circle/chord.

* **NOTE:** Mid point of chord is given by:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

4) The chord equidistant from the center are congurent.

5) An angle in semi-circle is a right angle

6) The tangent to any point of the circle is perpendicular to the radial segment at that point.

7) Normal lines of a circle passes through the ~~center of a circle~~ "point of tangency".

8). Perpendicular dropped from a point of a circle on a diameter is mean proportional b/w the segments into which it divides the diameter.

- Tangent on the circle: One real tangent can be drawn
- Tangent in the circle: no real tangent can be drawn
- Tangent out of the circle: Two real tangent can be drawn.

* Area of a circle = πr^2. • Circumference = $2\pi r$

* Area of a Semi-circle = πr^2

*Parabola:

"Some important Points"

- **Axis:** The straight line passing through the focus and perpendicular to the directix of the conic.
- **Vertex:** The point of intersection of a conic with its axis.
- **Center:** The point which bisects every chord of the conic passing through it.
- **Focal Chord:** A chord passing through the focus of the conic.
- **Latusrectum:** The focal chord which is perpendicular to the axis.
- **Double ordinate:** A chord of conic which is perpendicular to the axis.

• Eccentsities of Conic:

1) $e = 0$ (Circle).
2) $e = 1$ (Parabola)
3) $e = \infty$ (Pair of stright line).
4) $e > 1$ (Hyperbola).
5) $0 < e < 1$ (Ellipse).
6) $e = \sqrt{2}$ (Rectangular Hyperbola).
7) $e = 0.0167$ $(e < 1)$ (Eccentricity of Earth). (Earth is elliptical).

* Eccentricity of conic $= \frac{c}{a}$.
$e = \frac{c}{a} \Rightarrow c = ae$.

* Note: Two parabolas are said to be equal if they have equal length of Latus rectum.

* Standard forms of equation of parabola:

• Standard eq.	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = 4ay$
• Distance b/w vertex and DTX.	a	a	a	a
• Distance b/w Focus and DTX.	$2a$	$2a$	$2a$	$2a$.
• Distance b/w vertex and focus.	a	a	a	a.
• Length of Latus rectum.	$4a$	$4a$	$4a$	$4a$.
• Length of semi-Latusrectum.	$2a$	$2a$	$2a$	$2a$.
• Extremeties of Latusrectum.	$(a, \pm 2a)$	$(-a, \mp 2a)$	$(\pm 2a, a)$	$(\pm 2a, -$
• Vertex.	$O(0,0)$	$O(0,0)$	$O(0,0)$	$O(0,0)$
• Focus.	$S(a,0)$	$S(-a,0)$	$S(0, a)$	$S(0,-$

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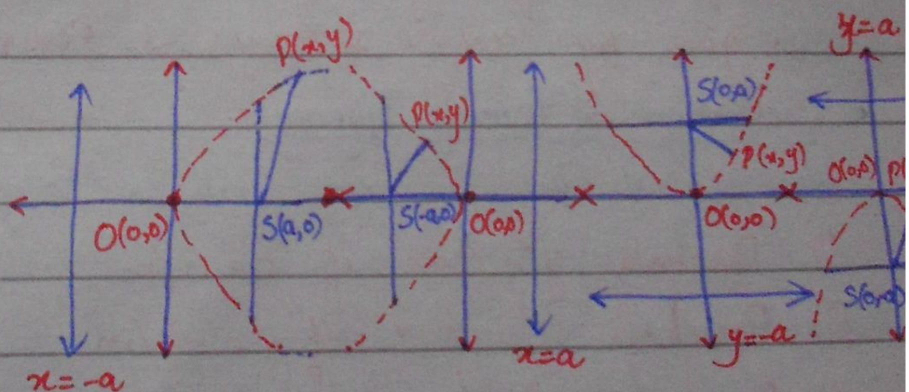
• Equation of Directrix.	$x=-a$	$x=a$	$y=-a$	$y=a$.
• Equation of Axis	$y=0$	$y=0$	$x=0$	$x=0$.
• Equation of Latus rectum.	$x=a$	$x=-a$	$y=a$	$y=-a$.
• Equation of tangent to the vertex.	$x=0$	$x=0$	$y=0$	$y=0$.
• Focal distance of point $P(x,y)$.	$x+a$	$x-a$	$y+a$	$y-a$.
• Parametric coordinates.	$(at^2, 2at)$	$(-at^2, 2at)$	$(2at, at^2)$	$(2at, -at^2)$

• Shape of a Parabola.



* The Equations of Parabola at Point $P(h,k)$ is given by:

$(x-h)^2 = 4a(y-k)$. (Upper Parabola).

$(x-h)^2 = -4a(y-k)$. (Lower Parabola)

$(y-k)^2 = 4a(x-h)$. (Right Parabola)

→ The general equation of Parabola is given by $(x-h)^2+(y-k)^2 = \frac{(lx+my+n)^2}{l^2+m^2}$.

[whereas $F(h,k)$ be the focus and $lx+my+n=0$ be a directrix of parabola].

• Equations of tangent:-

1) Equation of tangent for the parabola $y^2=4ax$ is given by $yy_1 = 2a(x+x_1)$ at $P(x_1,y_1)$.
2) Equation of tangent for the parabola $y^2=-4ax$ is given by $yy_1=-2a(x+x_1)$ at $P(x_1,y_1)$
3) Equation of tangent for the parabola $x^2=4ay$ is given by $xx_1 = 2a(y+y_1)$ at $P(x_1,y_1)$
4) Equation of tangent for the parabola $x^2=-4ay$ is given by $xx_1=-2a(y+y_1)$ at $P(x_1,y_1)$.

• Condition of [contig] Tangency:

A straight line $y=mx+c$ will be tangent to:

1) Parabola $y^2=4ax$ if $c=a/m$.
2) Parabola $y^2=-4ax$ if $c=-a/m$.
3) Parabola $x^2=4ay$ if $c=am^2$.
4) Parabola $x^2=-4ay$ if $c=-am^2$.

• Equations of Normal:

1) Equation of normal for the parabola $y^2=4ax$ is given by $y-y_1=\frac{-y_1}{2a}(x-x_1)$ at $P(x_1,y_1)$

2) Equation of normal for the parabola $y^2 = -4ax$ is given by $y - y_1 = \frac{y_1}{2a}(x - x_1)$ at $P(x_1, y_1)$.

3) Equation of normal for the parabola $x^2 = 4ax$ at $P(x_1, y_1)$ is given by $x - x_1 = \frac{-x_1}{2a}(y - y_1)$.

4) Equation of normal for the parabola $x^2 = -4ax$ at $P(x_1, y_1)$ is given by $x - x_1 = \frac{x_1}{2a}(y - y_1)$.

* Coordinates of contact's point:

The line $y = mx + c$ touches the parabola $y^2 = 4ax$ if $c = \frac{a}{m}$ and coordinates of point of contact are $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

* The equation of tangent to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $ty = x + at^2$.

* The gradient/slope of normal of parabola $y^2 = 4ax$ at $(at_1^2, 2at_1)$ is given by $-t_1$.

* The equation of normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ is given by:

$$y + tx = 2at + at^3.$$

* The equation of normal to the parabola $y^2 = 4ax$ in terms of slope is given by:

$$y = mx - 2am - am^3.$$

- whereas $c = -2am - am^3$.

* The coordinates of Point of contacts are $(am^2, -2am)$
* If three ~~parabo~~ normal drawn to any point of parabola $y^2 = 4ax$ from a given point (h,k) be a real; then $h > 2a$.

* **Equation of Chord:**

1) The equation of chord joining the point $P(x_1, y_1)$ and (x_2, y_2) of the parabola $y^2 = 4ax$ is given by: $y(y_1 + y_2) = 4ax + y_1 y_2$

2) The equation of chord joining the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ of the parabola $y^2 = 4ax$ is given by: $y(t_1 + t_2) = 2(x + at_1 t_2)$.

* **Condition for the chord to be focal chord:**

The chord joining the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ passes through the focus provided $t_1 t_2 = -1$.

* **Length of chord:**

$y = mx + c$ to the parabola $y^2 = 4ax$ is given by $\frac{4}{m^2}\sqrt{(1+m^2)}\sqrt{a(a-mc)}$.

* **Focal chord's Length:**

The length of focal chord at the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is given by: $(t_1 + t_2)^2$.

* And at point "t" is given by $\left(t + \frac{1}{t}\right)^2$.

• Solving a parabola with respect to a standard parabola $y^2 = 4ax$. e.g find the focus of $y^2 = 8x$. So first of all compare it with S.P $4a = 8 \Rightarrow a = 2$ and focus is $(2,0)$.

* Position of Point with Respect to Parabola:

The point lies ($P(x_1, y_1)$)

1) On the Parabola $y_1^2 - 4ax_1 = 0$
2) outside the parabola $y_1^2 - 4ax_1 > 0$.
3) Inside the parabola. $y_1^2 - 4ax < 0$.

* Number of Tangents drawn from a point to a parabola:

1) Two tangents are real and distinct (outside the parabola).
2) Two tangents are coincident. (on the parabola).
3) Two tangents are imaginary. (inside the parabola).

⇒ Area of Parabola:

$$A = \frac{2}{3}ab.$$

a = length of parabola b = width of parabola.

⇒ Theorems on Parabola:

1) The point closest to the focus is vertex.
2) The ordinate at any point P of parabola is a mean proportional to the length of latus rectum and abscissa of P.
3). The tangent at any point P of a parabola makes equal angles with the line PF and the line P through P parallel to the axis of parabola, F being focus.

* Ellipse:-

"Terms related to Ellipse"

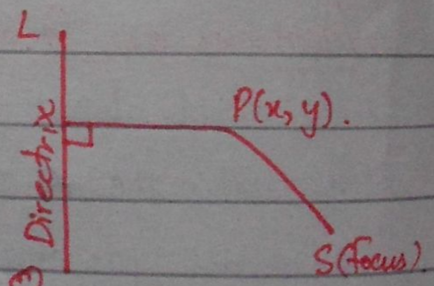
- **Vertices** The points on standard ellipse where it crosses x-axis
- **Covertices** " " " " where it crosses y-axis
- **Centre** The midpoint of line joining vertices (co-vertices or foci)
- **Major Axis** The line joining vertices
- **Minor Axis** The line joining co-vertices.
- **Latusrectum.** The chord perpendicular to the major axis is called latusrectum.

- **Eccentricity of Ellipse:** It is given by;

$(0<e<1)$ $e=\frac{c}{a}$ or $c=ae$.

- **Definition of Ellipse:**

The ellipse is a locus which moves in such a way that the sum of distance between two focii is equal to constant.



$$|PS|+|PS'|=2a.$$

⇒ For standard equation of ellipse;

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a>b)$$

$$c^2=a^2-b^2 \quad \text{and} \quad c=ae.$$

⇒ Standard Forms of Ellipse.

• Standard equation.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ (Horizontal ellipse)	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ $(a>b)$. (Vertical Ellipse)
• Symmetry	Symmetry about (x, y axis)	about x, y axis.
• Center of symmetry.	$(0,0)$ is center of symmetry	$(0,0)$ is " " ".
• Equation of Latusrectum.	$x=\pm c$ (OR) $x=\pm ae$	$y=\pm c$ (OR) $y=\pm ae$.
• Center	$O(0,0)$	$O(0,0)$.
• Equation of Major axis	$y=0$	$x=0$.
• Equation of Minor axis.	$x=0$	$y=0$.
• Length of major axis	$2a$	$2a$.
• Length of minor axis.	$2b$	$2b$.
• Focii	$(\pm c,0)$	$(0,\pm c)$.
• Vertices	$(\pm a,0)$	$(0,\pm a)$.
• Co-vertices.	$(0,\pm b)$	$(\pm b,0)$.
• Length of Latusrectum.	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

• Equation of directrix (DTX)	$x = \pm \frac{a}{e}$	$y = \pm \frac{a}{e}$.
• Eccentricity.	$e = \sqrt{1 - b^2/a^2}$	$e = \sqrt{1 - \frac{b^2}{a^2}}$
• Ends of Latusrectum.	$\left(\pm c, \pm \frac{b^2}{a^2}\right)$	$\left(\pm \frac{b^2}{a^2}, \pm c\right)$.
• Parametric coordinates and eq.	$(a\cos\theta, b\sin\theta)$ $x = a\cos\theta, y = b\sin\theta$	$(a\cos\theta, b\sin\theta)$ $x = a\cos\theta, y = b\sin\theta$.
• Distance b/w focii.	$2c$	$2c$
• Distance b/w (DTX)	$2a/e$	$\frac{2a}{e}$.
• Tangent to the vertices.	$x = \pm a$	$y = \pm a$.
• Apogee (greatest distance)	$a+c$	$a+c$
• Perigee. (least distance)	$a-c$.	$a-c$.

* The equation of Ellipse when center is (h,k) ;

1) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. (Horizontal ellipse).

2) $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$. (Vertical Ellipse).

* Circle as a special case of Ellipse :.

If we take $a = b$ then equation of ellipse will take the form $x^2 + y^2 = a^2$, which is a circle with $e=0$

NOTE: Ellipse has a symmetry (central) so called central symmetric.

* Equations of tangent:
* The equation of tangent of horizontal ellipse is given by $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$.
* The equation of tangent of vertical ellipse is given by $\frac{xx_1}{b^2}+\frac{yy_1}{a^2}=1$.
* The equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta, b\sin\theta)$ is given by: $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$.
* The equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in terms of slope is given by:

$$y=mx\pm\sqrt{a^2m^2+b^2}.$$

whereas $c=\pm\sqrt{a^2m^2+b^2}$.

* The coordinates of point of contact is given by:

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right).$$

⇒ Position of a point with respect to ellipse:

The point $P(x,y)$ lies.

- On the ellipse. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$.
- Inside the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1<0$.
- Outside the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1>0$.

* NUMBER of tangents drawn from a point.

- Two tangent are real, and distinct. (Outside ellipse)
- Two tangents are coincident (On ellipse)
- Two tangents are imaginary (Inside ellipse).

NOTE:- The product of perpendiculars from the foci on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. is equal to b^2 (constant).

- Equations of normal :-

1) The equation of normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(x_1, y_1)$ is given by:

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2-b^2.$$

2) The equation of normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(a\cos\theta, b\sin\theta)$ is given by;

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2.$$

3) The coordinate of point of contact are;

$$\left(\pm\frac{a^2}{\sqrt{a^2+b^2m^2}}, \pm\frac{mb^2}{\sqrt{a^2+b^2m^2}}\right) \text{ (if require).}$$

Some important points:-

On solving the equation of ellipse if this form appears (Then) $25x^2+16y^2=400$ then we make R.H.S $=1$ always.

Solution:- $$\frac{25x^2}{400}+\frac{16y^2}{400}=\frac{400}{400}.$$

$$\boxed{\frac{x^2}{16}+\frac{y^2}{25}=1.}$$

- But sometimes the equation is usually incomplete. for example;

$$x^2+4x+y^2+6y-3=0.$$

- In this case, we first of all, completed square; i-e $(x^2+4x+2^2)+(y^2+6y+3^2)=3+4+9.$

$$(x+2)^2+(y+3)^2=16.$$

then again apply the same prior discussed method;

$$\frac{(x+2)^2}{16}+\frac{(y+3)^2}{16}=1.$$

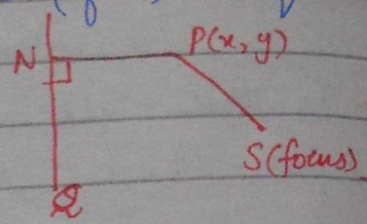
⇒ But this example is totally case of of ellipse in terms of circle. You may consider more examples in the same fashion.

⇒ Area of Ellipse $= \pi ab.$

Hyperbola :-

A hyperbola is locus which moves in the plane in such a way the magnitude of difference of its distance from fixed points (focii) is equal to constant.

$$|SP| - |SP'| = 2a$$



-Note: $\Rightarrow$ $e = c/a$ or $c = ae$.

- Equations of Hyperbola :-

1) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ (x-axis)

2) $+\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$ (y-axis).

3) $c^2 = a^2 + b^2$.

4) $c = ae$.

- Standard Forms of Hyperbola :-

Standard Equation.	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.
Symmetry	Symmetry about x and y axis	" " "
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Co-vertices.	$(0, \pm b)$	$(\pm b, 0)$.

• Center	$O(0,0)$	$O(0,0)$.
• Equation of Transverse Axis	$y=0$	$x=0$.
• Equation of Conjugate Axis	$x=0$	$y=0$.
• Length of conjugate axis	$2a$	$2a$.
• Length of Transverse axis.	$2a$	$2a$.
• Foci	$(\pm c, 0)$	$(0, \pm c)$.
• Equation of directrix.	$x = \pm \frac{a}{e}$	$y = \pm \frac{a}{e}$.
• Distance b/w directrix.	$2a/e$	$2a/e$.
• Eccentricity	$e = \sqrt{1+\frac{b^2}{a^2}}$	$e = \sqrt{1+\frac{b^2}{a^2}}$.
• Length of Latusrectum.	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$.
• eq. Tangents at vertices.	$x = \pm a$	$y = \pm a$.
• Distance b/w foci.	$2c$	$2c$.
• Ends of Latusrectum.	$\left(\pm c, \pm \frac{b^2}{a}\right)$	$\left(\pm \frac{b^2}{a}, \pm c\right)$.

• NOTE: Hyperbola has a central Symmetry so it is called "Central conic".

• **Conjugate Hyperbola:**

The hyperbola whose Transver[se] and conjugate axes are respectively the conjugate and transverse axes of given hype[rbola] is called the conjugate Hyperbola of a given Hyperbola.

It is given by:-

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$

- Position of a point with respect to a Hyperbola:
  - The point $P(x_1, y_1)$ lies inside the Hyperbola if $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 < 0$.
  - On the Hyperbola if $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$.
  - Outside the Hyperbola if $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 > 0$.

- Equations of tangent:-

* The equation of the tangent at $P(x_1, y_1)$.

1) $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

2) $\frac{yy_1}{a^2} - \frac{xx_1}{b^2} = 1$.

* The equation of the Tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(a\sec\theta, b\tan\theta)$ is

$$\frac{x\sec\theta}{a} + \frac{y\tan\theta}{b} = 1.$$

* The equation of tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in terms of slope "m" is given by:

$$y = mx \pm \sqrt{a^2m^2 - b^2}.$$

whereas $c = \pm\sqrt{a^2m^2 - b^2}$.

→ Equations of Normal:-

* The equation of normal to hyperbola at $P(x_1, y_1)$ is given by: $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2+b^2$.

* The equation of normal to hyperbola at $P(a\sec\theta, b\tan\theta)$ is given by: $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2+b^2$.

• Note:- In general, four normals can be drawn to a hyperbola from a point in its plane, i.e. there are four points on the hyperbola the normals at which will pass through a given point.

• Equation of Asymptotes of Hyperbola:-

$$y = \pm \frac{bx}{a}$$

• Note:- The curve comes close to these lines as $x \to \infty$ & $x \to -\infty$ but never meet them.

• Angle b/w the asymptotes of Hyperbola:-

It is given by $\theta = 2\tan^{-1}\left(\frac{b}{a}\right)$ or $\theta = 2\cos^{-1}\left(\frac{1}{e}\right)$.

• Asymptotes are diagonals of the rectangles passing through A, B, A', B' with side parallel to axes.

• A hyperbola and its conjugate hyperbola have same asymptotes.

4) Asymptotes pass through the center of the Hyperbola.

5) The product of the perpendicular from any point on hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is constant i.e. $\frac{a^2 \times b^2}{a^2+b^2}$.

6) Any line drawn parallel to the asymptotes of Hyperbola would meet curve only at one point.

• Rectangular Hyperbola:-

If asymptotes of standard hyperbola are perpendiculars to each other then equation is given by:.

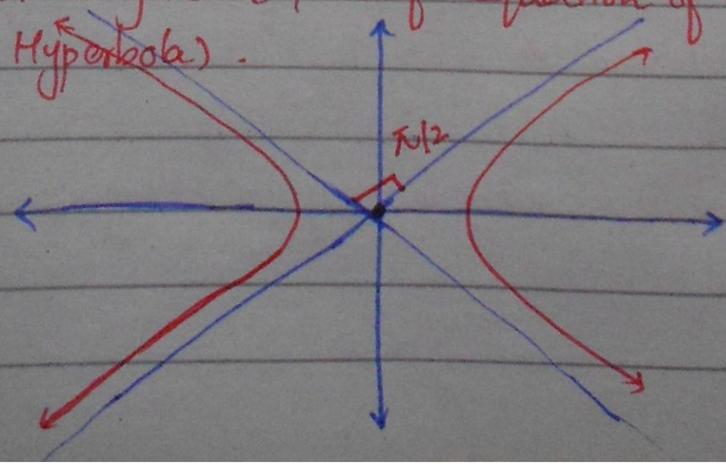
(Standard Hyperbola) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

when $a=b$

$$\frac{x^2}{a^2}-\frac{y^2}{a^2}=1.$$

$$x^2-y^2=a^2.$$

(It is general form of equation of Rectangular Hyperbola).



• Angle between asymptotes of Rectangular Hyperbola

$$\theta = 2\tan^{-1}\left(\frac{b}{a}\right)$$

As $a = b$.

$$\theta = 2\tan^{-1}\left(\frac{\cancel{a}}{\cancel{a}}\right)$$

$$\theta = 2 \times 45$$

$$\theta = 90^\circ \text{ or } \pi/2$$

3) Eccentricity of Rec. Hyperbola:.

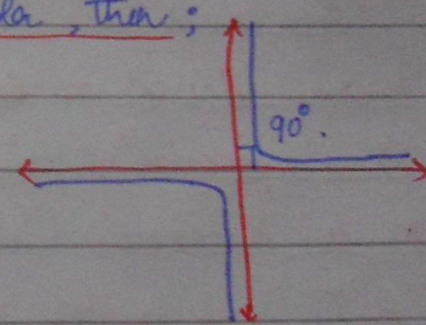
$$e = \sqrt{1 + \frac{b^2}{a^2}}.$$

As $b = a$.

So; $e = \sqrt{1+1} = \sqrt{2}$.

• Equation of Rectangular Hyperbola when we Take coordinates axes along the asymptotoes of a rectangular Hyperbola, then;

$$xy = c^2.$$



• Parametric form:.

$$x = ct$$

$$y = \frac{c}{t}.$$

• Equation of Asymptotes of Rec. Hyperbola:.

As we know that;

$$y = \pm \frac{b}{a} x.$$

So; $b = a$ $\quad y = \pm \frac{\cancel{a}}{\cancel{a}} x \Rightarrow y = \pm x$.

* Equation of hyperbola at $C(h,k)$:

1) $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$.
2) $-\dfrac{(y-k)^2}{a^2}+\dfrac{(x-h)^2}{b^2}=1$.

• General equation of Second Degree:.

The general equation of second degree $Ax^2+By^2+Gx+Fy+C=0$ represents:.

1) a circle if $A=B\neq 0$.
2) an ellipse if $A\neq B$ but both have same signs.
3) a hyperbola if $A\neq B$ but both have opposite signs.
4) a parabola if either $A=0$ or $B=0$.

• Classification of conic by the discriminant.

The most general equation of the second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents

1) an ellipse or circle; $h^2-ab<0$.
2) a parabola; $h^2-ab=0$
3) a Hyperbola; $h^2-ab>0$.

• To eliminate the xy-term from equation $ax^2+2hxy+by^2+2gx+2fy+c=0$, the axis is rotated through angle (θ) then:

$$\tan 2\theta=\frac{2h}{a-b}.$$

- The condition that the equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents pair of straight lines if:

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

Intersection of conic:.

1) Two conics will always intersect in four points. These points may be real, distinct two real and two imaginary, all imaginary two or more points may be coincide.

2) Two conics will be said to touch other if they intersect in two or more coincident points.

⇒ Center of Conics:.

The center of conic of $ax^2+2hxy+by^2+2gx+2fy+c=0$ is given by:

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$$

- **Translation of Axis:.** To transform the given equation referred to the new origin $O'(h,k)$

Put $x = X+h$

$y = Y+k.$

- **Rotation of Axis:.** $x = X\cos\theta - Y\sin\theta.$

$y = X\sin\theta + Y\cos\theta.$

-Equation of Director Circle is $x^2+y^2=2r^2$

Some Useful Information:

Areas:

_ Area of triangle= 1/2 (Width)(Height)
_ Area of Square= $(\text{Side of Square})^2$
_ Area of Rectangle= Length × Width
_ Area of Trapezium=1/2 (length×breadth×height)
_ Area of Circle= πr^2
_ Area of Ellipse= πab
_ Area of Parabola= 2/3 (Width)×(Height)
_ Area of Sector=1/2 r^2 ϴ
_ Area of Rhombus= 1/2 (Product of Diagonals)=a+b/2
_ Area of Parallelogram= Breadth×Height.
_Area of Semi Circle= $\pi r^2/2$

Perimeters:

_ Perimeter of Sector= 2r+l
_ Perimeter of Square= 4l
_ Perimeter of Rectangle=2(l+w)
_ Perimeter of Triangle= a+b+c
_ Perimeter of Semi-Circle= πd/2+d
_ Perimeter of Rhombus= 4a
_ Perimeter of Ellipse= $2\pi\sqrt{(a^2+b^2)/2}$
_ Perimeter of Parallelogram=2(a+b)
_ Perimeter of Circle= πd

Circumferences:

_ Circumference of Circle=2πr
_ Circumference of Semi Circle=πr

LOCUS:

A locus is a set of points satisfying given conditions.

2-Dimensional	3-Dimensional
1) The set of all points in a plane at a constant distance from a fixed point is a Circle.	1) The set of all points in space at a constant from a fixed point is a sphere.
2) The set of all points in a plane at a constant distance from a given straight line is a pair of straight lines parallel to the given line.	2) The set of all points in space at a constant distance from a given straight line is a cylinder.
3) The set of all the points in a plane such that the sum of their distances from two fixed points is a constant is an Ellipse.	3) In space an ellipse is changed into an ellipsoid.
4) The set of all points in a plane such that the differences of their distances from two fixed points are a constant is a hyperbola.	4) In space hyperbola is shifted to a hyperboloid.
5) The set of all points that their distances from a fixed point are equal to their distances from a fixed straight line is a parabola.	5) In space parabola is called paraboloid.
6) The locus of a point in a plane that is equidistant from two fixed points is the perpendicular bisector (mediator) of the straight line joining those two points.	
7) The locus of a point in a plane equidistant from two intersecting straight lines is pair of straight lines bisects the angles b/w two given straight lines.	

CHAPTER 7

The solution of vector product is comparatively tough. Determinant method and other method are very long.

As we know the result of vector product is also a vector and its perpendicular.

So in MCQ we can take dot product of any one given vector to the option. The option in which we get 0 result is the right answer as both are perpendicular i.e. θ =90° and Cos90° = 0

Example:-

Given Force F = 2i + j - 3k acting at a point A(1,-2,1). Find moment of force about point B(2,0,-2)

A) 3i+4j+3k b) 3i+3j+5k c) ans3i+3j+3k d) none

As we know Torque = r x F

Now r is not given in this question. So if you try to solve the MCQ by determinant method, then first you have to find r and then solve. But it is very time consuming. So, do the following to guess the right option.

Take dot products of Force with options A , B and C.

For A) 1
For B) -6
For B) 0

so c is the answer

This technique can be applied on every cross product .

Some Important Results:

-Vector Triple product:

A×(B×C)= (AC)B-(AB)C

➔ i.(j×i)=0

➔ i.(j×k)=1

-Component of a along b and perpendicular to vector a

➔ Component of b along a= $\left(\frac{a.b}{|a|^2}\right).a$

➔Component of b perpendicular to a= b-$\left(\frac{a.b}{|a|^2}\right).a$

➔Any vector r can be expressed as:

r= (r.i)I +(r.j).j + (r.k)k

➔ Some Useful Identities:

$(a+b)^2=a^2+b^2+2ab$

$(a-b)^2=a^2+b^2-2ab$

$(a+b)(a-b)=a^2-b^2$

➔ Work done by a force:

Work done by Force F in displacing a particle from A to B is defined by W=F.(AB)

Note: If no. of forces acting on the particle the sum of work done by the separate forces is equal to the work done by resultant force.

➔ |a×b|= |a||b|sinθ

➔ n^= $\frac{a\times b}{|a\times b|}$

➔ Area of triangle ABC= ½ |AB×AC|

➔ Area of Parallelogram with diagonals a and b is given by |a×b|

➔The area of quadrilateral ABCD is given by ½ |AC×BD| whereas AC amd BD are its diagonal

➔ (a.b)×c is meaningless.

Formulas and concepts:

Unit Vector: (Magnitude is one in a given direction)

v^=**v**/|v|

Note: |v|=$\frac{v}{\vec{v}}$ is false because a vector can't be divided by another vector.

Null vector: (Magnitude is zero with arbitrary direction)

Note Vector is also known as zero, void and empty vector.

Parallel (Collinear vectors)

Two vectors are parallel if and only if they are non-zero multiple of each other i.e

The vectors u and v are parallel if u=cv

1) If c>0 then vectors are in same direction i.e. θ=0*

2) If c<0 then vectors are in opposite direction i.e. θ=180*

Note: For any vectors: AB= OB-OA

➔ Magnitude of a vector is also known as norm and length of vector.

➔Ratio Formula: for p:q ➔r=$\frac{q\ a+pb}{p+q}$

Note: If P is mid point of AB then p:q=1:1 position vector of P=r=a+b/2

➔Distance between two points in the space:

If $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ be the points in the space then

$P_1P_2=OP_2-OP_1=[x_2-x_1 , y_2-y_1 , z_2-z_1]$

Distance between two points:

$|P_1P_2|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Direction angle s and direction cosines of a vector:

Let r=OP =xi+yj+zk be a non-zero vector , let α,β and ω denote the angles formed b/w r and unit coordinate vectors i , j and k respectively such that 0≤α≤π , 0≤β ≤π and 0≤ω≤π Then:

1) The angle α ,ω and β are called as direction angles:

Note:

1) Let v=xi+yj+zk be a vector and |v| be its magnitude

-Angle of vector with x-axis:

cosα=$\frac{coefficient\ of\ x-component}{|v|}$

$\cos\beta = \frac{cofficient\ of\ y-component}{|v|}$

cosω=$\frac{coefficient\ of\ z-component}{|v|}$

These are known as direction cosines

2) $\sin^2\alpha+\sin^2\beta+\sin^2\omega=2$

3) $\cos^2\alpha+\cos^2\beta+\cos^2\omega=1$

➔Scalar product (Dot or inner)

u.v= |u||v| cosθ

➔Cosine of angle b/w u and v is given by:

$cos\theta = \frac{u.v}{|u||v|}$.

If $u=a_1i+b_1j+c_1k$ and $v=a_2i+b_2j+c_2k$ are two non-zero vectors in space then dot product is

$u.v=a_1a_2+b_1b_2+c_1c_2$

Note: i.i=j.j=k.k=1 and i.j=j.k=k.i=0

Perpendicular (Orthogonal) Vector)

condition: a.b=0

Properties of dot product:

Let u and v be the vectors and let c be a real number then:

1) u.v=0 ➔ u=0 or v=0

2) u.v=v.u

3) u.(v+w)= u.v+v.w

4) (cu)v= c(u.v)

5) $(u.u)=|u|^2$

➔ Projection of v along u= |v|cosθ=$\frac{u.v}{|u|}$

➔ Projection of u along v= |u|cosθ=$\frac{u.v}{|v|}$

➔Triangle is formed when a+b+c=0 or c=a+b

➔ Projection laws are:

a=bcosC+c cosB

b=c cos A+ a cosC

c= a cosB+b cos A

➔ Laws of cosines are:

$a^2=b^2+c^2-2bc\cos A$

$b^2=c^2+a^2-2ca\cos B$

$c^2=a^2+b^2-2ab\cos C$

➔Cross (Vector) Product:

u×v=|u||v|sinθ n^

Whereas n^ is a unit vector perpendicular the plane of u and v

Note: u×v is perpendicular to both u and v.

➔Sine of angle between two vector:

sinθ=$\frac{|u\times v|}{|u||v|}$

u×v=$\begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

Note: i×i=j×j=k×k=0 and i×j=k ,j×k=i and k×i=j

whereas j×i=-k ,k×j=-i and i×k=-j

➔If two vectors are parallel then:

Let u=ai+bj+ck and v= xi+yj+zk then two vectors are parallel iff:

a/x= b/y=c/z

➔ Null vector is one which both parallel and perpendicular to every vector.

➔ Parallel vector:

a×b=0

➔Properties of cross product:

1) u×v=0 iff u=0 or v=0

2) u×v=-v×u

3) u×(v+w)= u×v+ u×w

4) u×(kv)= (ku)×v=k(u×v)

5) u×u=0

➔ Area of parallelogram= |u×v|

➔ Area of triangle= ½ |u×v|

➔ u.(v×w)= $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Note: u.(v×w)= v.(w×w)=w.(u×v)

➔ **Volume of parallelepiped= u .(v×w)**

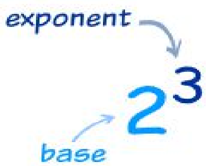
➔**Volume of Tetrahedron= 1/6 (u.(v×w))**

➔ **Coplanar Vectors: u.(v×w)=0**

LOGARITHMS

There are 1-2 questions of log in ECAT. So, you should revise its basic concepts.

What is an Exponent?



The **exponent** of a number says **how many times** to use the number in a multiplication.

In this example: $2^3 = 2 \times 2 \times 2 = 8$

(2 is used 3 times in a multiplication to get 8)

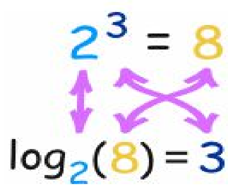
What is a Logarithm?

A Logarithm goes the other way.

It asks the question "what exponent produced this?":

$$2^{?} = 8$$

And answers it like this:



In that example:

- The Exponent takes **2 and 3** and gives **8** *(2, used 3 times, multiplies to 8)*
- The Logarithm takes **2 and 8** and gives **3** *(2 makes 8 when used 3 times in a multiplication)*

> A Logarithm says **how many** of one number to multiply to get another number

So a logarithm actually gives you the exponent as its answer:

The laws of logarithms

Introduction

There are a number of rules known as the **laws of logarithms**. These allow expressions involving logarithms to be rewritten in a variety of different ways. The laws apply to logarithms of any base but the same base must be used throughout a calculation.

The laws of logarithms

The three main laws are stated here:

First Law

$$\log A + \log B = \log AB$$

This law tells us how to add two logarithms together. Adding $\log A$ and $\log B$ results in the logarithm of the product of A and B, that is $\log AB$.

For example, we can write

$$\log_{10} 5 + \log_{10} 4 = \log_{10}(5 \times 4) = \log_{10} 20$$

The same base, in this case 10, is used throughout the calculation. You should verify this by evaluating both sides separately on your calculator.

Second Law

$$\log A - \log B = \log \frac{A}{B}$$

So, subtracting $\log B$ from $\log A$ results in $\log \frac{A}{B}$.

For example, we can write

$$\log_e 12 - \log_e 2 = \log_e \frac{12}{2} = \log_e 6$$

The same base, in this case e, is used throughout the calculation. You should verify this by evaluating both sides separately on your calculator.

Third Law

$$\log A^n = n \log A$$

So, for example

$$\log_{10} 5^3 = 3 \log_{10} 5$$

You should verify this by evaluating both sides separately on your calculator.

Two other important results are

$$\log 1 = 0, \qquad \log_m m = 1$$

The logarithm of 1 to any base is always 0, and the logarithm of a number to the same base always 1. In particular,

$$\log_{10} 10 = 1, \qquad \text{and} \qquad \log_e e = 1$$

Exercises

1. Use the first law to simplify the following.

   a) $\log_{10} 6 + \log_{10} 3$,
   b) $\log x + \log y$,
   c) $\log 4x + \log x$,
   d) $\log a + \log b^2 + \log c^3$.

2. Use the second law to simplify the following.

   a) $\log_{10} 6 - \log_{10} 3$,
   b) $\log x - \log y$,
   c) $\log 4x - \log x$.

3. Use the third law to write each of the following in an alternative form.

   a) $3 \log_{10} 5$,
   b) $2 \log x$,
   c) $\log(4x)^2$,
   d) $5 \ln x^4$,
   e) $\ln 1000$.

4. Simplify $3 \log x - \log x^2$.

Answers

1. a) $\log_{10} 18$, b) $\log xy$, c) $\log 4x^2$, d) $\log ab^2c^3$.

2. a) $\log_{10} 2$, b) $\log \frac{x}{y}$, c) $\log 4$.

3. a) $\log_{10} 5^3$ or $\log_{10} 125$, b) $\log x^2$, c) $2 \log(4x)$, d) $20 \ln x$ or $\ln x^{20}$,
e) $1000 = 10^3$ so $\ln 1000 = 3 \ln 10$.

4. $\log x$.

1. **Logarithm:**

   If a is a positive real number, other than 1 and $a^m = x$, then we write:
   $\boldsymbol{m = \log_a x}$ and we say that the value of log x to the base a is m.

   Examples:

   (i). 10^3 1000 $\Rightarrow$ $\log_{10} 1000 = 3$.

   (ii). $3^4 = 81$ $\Rightarrow$ $\log_3 81 = 4$.

   (iii). $2^{-3} = \frac{1}{8}$ $\Rightarrow$ $\log_2 \frac{1}{8} = -3$.

   (iv). $(.1)^2 = .01$ $\Rightarrow$ $\log_{(.1)} .01 = 2$.

2. **Properties of Logarithms:**

   1. $\log_a (xy) = \log_a x + \log_a y$

   2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

   3. $\log_x x = 1$

   4. $\log_a 1 = 0$

   5. $\log_a (x^n) = n(\log_a x)$

   6. $\log_a x = \dfrac{1}{\log_x a}$

   7. $\log_a x = \dfrac{\log_b x}{\log_b a} = \dfrac{\log x}{\log a}$.

3. **Common Logarithms:**

Logarithms to the base 10 are known as common logarithms.

4. The logarithm of a number contains two parts, namely 'characteristic' and 'mantissa'.

Characteristic: The internal part of the logarithm of a number is called its **characteristic.**

Case I: When the number is greater than 1.

In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II: When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of -1, -2 etc. we write 1 (one bar), 2 (two bar), etc.

Examples:-

Number	Characteristic	Number	Characteristic
654.24	2	0.6453	1
26.649	1	0.06134	2
8.3547	0	0.00123	3

Mantissa:

The decimal part of the logarithm of a number is known is its **mantissa.** For mantissa, we look through log table.

SOME QUESTIONS FOR PRACTICE

1. Which of the following statements is not correct?

A. $\log_{10} 10 = 1$

B. $\log (2 + 3) = \log (2 \times 3)$

C. $\log_{10} 1 = 0$

D. $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

Answer & Explanation

Answer: Option **B**

Explanation:

(a) Since $\log_a a = 1$, so $\log_{10} 10 = 1$.

(b) $\log (2 + 3) = \log 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3$

$\therefore \log (2 + 3) \neq \log (2 \times 3)$

(c) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$.

(d) $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$.

So, (b) is incorrect.

2. If $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$, then:

A. $a + b = 1$ **B.** $a - b = 1$

C. $a = b$ **D.** $a^2 - b^2 = 1$

Answer & Explanation

Answer: Option **A**

Explanation:

$$\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$$

$$\Rightarrow \log (a + b) = \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1.$$

So, $a + b = 1$.

3. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to:

A. $-(1 + a)$ **B.** $(1 + a)^{-1}$

C. $\frac{a}{10}$ **D.** $\frac{1}{10a}$

Answer & Explanation

Answer: Option **A**

Explanation:

$$\log_{10}\left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70$$

$$= -\log_{10}(7 \times 10)$$

$$= -(\log_{10} 7 + \log_{10} 10)$$

$$= -(a + 1).$$

4. If $\log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$, then x is equal to:

A. 1

B. 3

C. 5

D. 10

Answer & Explanation

Answer: Option **B**

Explanation:

$\log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + \log_{10} 10$

$\Rightarrow \log_{10} [5(5x + 1)] = \log_{10} [10(x + 5)]$

$\Rightarrow 5(5x + 1) = 10(x + 5)$

$\Rightarrow 5x + 1 = 2x + 10$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3.$

5. The value of $\left(\dfrac{1}{\log_3 60} + \dfrac{1}{\log_4 60} + \dfrac{1}{\log_5 60}\right)$ is:

A. 0 **B.** 1

C. 5 **D.** 60

Answer & Explanation

Answer: Option **B**

Explanation:

Given expression $= \log_{60} 3 + \log_{60} 4 + \log_{60} 5$

$= \log_{60} (3 \times 4 \times 5)$

$= \log_{60} 60$

$= 1.$

Some Natural Logarithm Properties:

1) $\log_a(e) \times (\log_e(a)) = 1$

2) $\log_e(e) = 1$

Note: These properties also hold in case of common logarithm.

6. If $\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to:

A. $-\frac{3}{4}$

B. $\frac{3}{4}$

C. $\frac{81}{256}$

D. $\frac{256}{81}$

Answer & Explanation

Answer: Option **D**

Explanation:

$$\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$$

$$\Rightarrow x^{-1/2} = \frac{9}{16}$$

$$\Rightarrow \frac{1}{x} = \frac{9}{16}$$

$$\Rightarrow x = \frac{16}{9}$$

$$\Rightarrow x = \left(\frac{16}{9}\right)^2$$

$$\Rightarrow x = \frac{256}{81}$$

7. If $a^x = b^y$, then:

A. $\log \frac{a}{b} = \frac{x}{y}$

B. $\frac{\log a}{\log b} = \frac{x}{y}$

C. $\frac{\log a}{\log b} = \frac{y}{x}$

D. None of these

Answer & Explanation

Answer: Option **C**

Explanation:

$a^x = b^y$

$\Rightarrow \log a^x = \log b^y$

$\Rightarrow x \log a = y \log b$

$\Rightarrow \frac{\log a}{\log b} = \frac{y}{x}.$

8. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is:

A. 2^{10} **B.** 2^{100}

C. 2^{1000} **D.** 2^{10000}

Answer & Explanation

Answer: Option **C**

Explanation:

$\log_2 x = 10 \quad \Rightarrow \quad x = 2^{10}$.

$\therefore \log_x y = 100$

$\Rightarrow y = x^{100}$

$\Rightarrow y = (2^{10})^{100}$ [put value of x]

$\Rightarrow y = 2^{1000}$.

9. The value of $\log_2 16$ is:

A. $\frac{1}{8}$ **B.** 4

C. 8 **D.** 16

Answer & Explanation

Answer: Option **B**

Explanation:

Let $\log_2 16 = n$.

Then, $2^n = 16 = 2^4 \quad \Rightarrow \quad n = 4$.

$\therefore \log_2 16 = 4$.

➔For more practice of logarithmic problem please click on the following link:

http://bit.do/logarithmquiz

Shortcut Trick for Finding the Units Digits of Large Powers

In NET, Every year this question must ask to puzzle the candidate so here is the trick to solve in short time.

(a) Find the Units Place in $(785)^{98} + (342)^{33} + (986)^{67}$

(b) What will come in Units Place in $(983)^{85} - (235)^{37}$

These questions can be time consuming for those students who are unaware of the fact that there is a shortcut method for solving such questions. Don't worry if you don't know the shortcut already because we are providing it today.

Finding the Unit Digit of Powers of 2

1. First of all, divide the Power of 2 by 4.

2. If you get any remainder, put it as the power of 2 and get the result using the below given table.
3. If you don't get any remainder after dividing the power of 2 by 4, your answer will be $(2)^4$ which always give 6 as the remainder

Power	**Unit Digit**
$(2)^1$	2
$(2)^2$	4
$(2)^3$	8
$(2)^4$	6

Let's solve few Examples to make things clear.

(1) Find the Units Digit in $(2)^{33}$
Sol -
Step-1:: Divide the power of 2 by 4. It means, divide 33 by 4.
Step-2: You get remainder 1.
Step-3: Since you have got 1 as a remainder , put it as a power of 2 i.e $(2)^1$.
Step-4: Have a look on table, $(2)^1=2$. So, **Answer will be 2**

(2) Find the Unit Digit in $(2)^{40}$
Sol -
Step-1:: Divide the power of 2 by 4. It means, divide 40 by 4.
Step-2: It's completely divisible by 4. It means, the remainder is 0.
Step-3: Since you have got nothing as a remainder , put 4 as a power of 2 i.e $(2)^4$.
Step-4: Have a look on table, $(2)^4=6$. So, **Answer will be 6**

Finding the Unit Digit of Powers of 3 (same approach)

1. First of all, divide the Power of 3 by 4.
2. If you get any remainder, put it as the power of 3 and get the result using the below given table.
3. If you don't get any remainder after dividing the power of 3 by 4, your answer will be $(3)^4$ which always give 1 as the remainder

Power	**Unit Digit**

$(3)^1$	3
$(3)^2$	9
$(3)^3$	7
$(3)^4$	1

Let's solve few Examples to make things clear.

(1) Find the Units Digit in $(3)^{33}$
Sol -
Step-1:: Divide the power of 3 by 4. It means, divide 33 by 4.
Step-2: You get remainder 1.
Step-3: Since you have got 1 as a remainder , put it as a power of 3 i.e $(3)^1$.
Step-4: Have a look on table, $(3)^1=3$. So, **Answer will be 3**

(2) Find the Unit Digit in $(3)^{32}$
Sol -
Step-1:: Divide the power of 3 by 4. It means, divide 32 by 4.
Step-2: It's completely divisible by 4. It means, the remainder is 0.
Step-3: Since you have got nothing as a remainder , put 4 as a power of 3 i.e $(3)^4$.
Step-4: Have a look on table, $(3)^4=1$. So, **Answer will be 1**

Finding the Unit Digit of Powers of 0,1,5,6

The unit digit of 0,1,5,6 always remains same i.e 0,1,5,6 respectively for every power.

Finding the Unit Digit of Powers of 4 & 9

In case of 4 & 9, if powers are Even, the result will be **6 & 4.** However, when their powers are Odd, the result will be **1 & 9.** The same is depicted below.

- If the Power of 4 is Even, the result will be **6**
- If the Power of 4 is Odd, the result will be 4
- If the Power of 9 is Even, the result will be **1**
- If the Power of 9 is Odd, the result will be **9.**

For Example -

- **$(9)^{84} = 1$**
- **$(9)^{21} = 9$**
- **$(4)^{64} = 6$**
- **$(4)^{63} = 4$**

Finding the Unit Digit of Powers of 7 (same approach)

1. First of all, divide the Power of 7 by 4.
2. If you get any remainder, put it as the power of 7 and get the result using the below given table.
3. If you don't get any remainder after dividing the power of 7 by 4, your answer will be $(7)^4$ which always give 1 as the remainder

Power	Unit Digit
$(7)^1$	7
$(7)^2$	9
$(7)^3$	3
$(7)^4$	1

Let's solve few Examples to make things clear.

(1) Find the Units Digit in $(7)^{34}$
Sol -
Step-1:: Divide the power of 7 by 4. It means, divide 34 by 4.
Step-2: You get remainder 2.
Step-3: Since you have got 2 as a remainder , put it as a power of 7 i.e $(7)^2$.
Step-4: Have a look on table, $(7)^2=9$. So, **Answer will be 9**

(2) Find the Unit Digit in $(7)^{84}$
Sol -
Step-1:: Divide the power of 7 by 4. It means, divide 84 by 4.
Step-2: It's completely divisible by 4. It means, the remainder is 0.
Step-3: Since you have got nothing as a remainder , put 4 as a power of 7 i.e $(7)^4$.
Step-4: Have a look on table, $(7)^4=1$. So, **Answer will be 1**

Finding the Unit Digit of Powers of 8 (same approach)

1. First of all, divide the Power of 8 by 4.
2. If you get any remainder, put it as the power of 8 and get the result using the below given table.
3. If you don't get any remainder after dividing the power of 8 by 4, your answer will be $(8)^4$ which always give 6 as the remainder

Power	Unit Digit
$(8)^1$	8
$(8)^2$	4
$(8)^3$	2
$(8)^4$	6

Let's solve few Examples to make things clear.

(1) Find the Units Digit in $(8)^{34}$
Sol -
Step-1:: Divide the power of 8 by 4. It means, divide 34 by 4.
Step-2: You get remainder 2.
Step-3: Since you have got 2 as a remainder , put it as a power of 8 i.e $(8)^2$.
Step-4: Have a look on table, $(8)^2$=4. So, **Answer will be 4**

(2) Find the Unit Digit in $(8)^{32}$
Sol -
Step-1:: Divide the power of 8 by 4. It means, divide 32 by 4.
Step-2: It's completely divisible by 4. It means, the remainder is 0.
Step-3: Since you have got nothing as a remainder , put 4 as a power of 8 i.e $(8)^4$.
Step-4: Have a look on table, $(8)^4$=1. So, **Answer will be 6**

Now, you can easily solve questions based on finding the Unit's Digit of large powers. Lets try at least a few.

(a) Find the Units Place in $(785)^{98} + (342)^{33} + (986)^{67}$

Sol : 5 + 2 + 6 = 13 . So answer will be 3 .

(a) Find the Units Place in $(983)^{85}$ - $(235)^{37}$

Sol : 3 - 5 = 13 - 5 = 8 . So answer will be 8 . In this question, we have considered 3 as 13 because 3-5= -2 which is negative which is not possible.

Some Useful Information:

Areas:

→ Area of triangle= 1/2 (Width)(Height)
→Area of Square= $(\text{Side of Square})^2$
→Area of Rectangle= Length × Width
→ Area of Trapezium=1/2 (length×breadth×height)
→Area of Circle= πr^2
→Area of Ellipse= πab
→Area of Parabola= 2/3 (Width)×(Height)
→Area of Sector=1/2 r^2 ϴ
→Area of Rhombus= 1/2 (Product of Diagonals)=a+b/2
→Area of Parallelogram= Breadth×Height.
→Area of Semi circle= $\pi r^2/2$
→Lateral Surface Area of cylinder= 2πrh
→Total surface Area of Cylinder=$2\pi rh+2\pi r^2$
→Total Surface area of Cone=$\pi r\sqrt{r^2+h^2}+\frac{1}{2}d$
Where as d denotes diagonal.
→Area of Surface area sphere= $4\pi r^2$

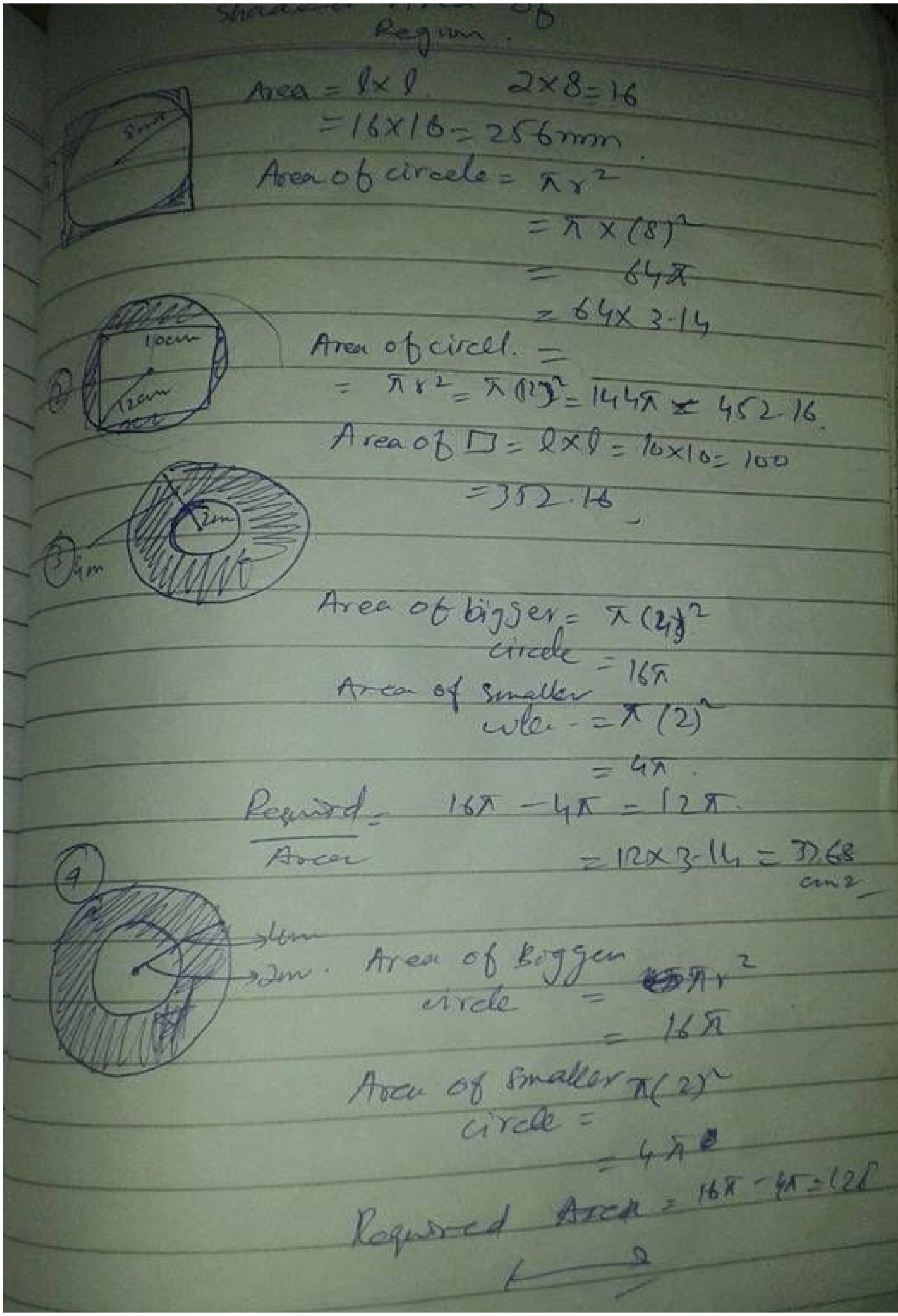
Perimeters:

→Perimeter of Sector= 2r+l
→ Perimeter of Square= 4l
→Perimeter of Rectangle=2(l+w)
→Perimeter of Triangle= a+b+c
→Perimeter of Semi-Circle= πd/2+d
→Perimeter of Rhombus= 4a
→Perimeter of Ellipse= $2\pi\sqrt{(a^2+b^2)/2}$

→Perimeter of Parallelogram=2(a+b)
→Perimeter of Circle= πd
Circumferences:
→Circumference of Circle=2πr
→Circumference of Semi Circle=πr

Volume:
→Volume of Cylinder= $\pi r^2 h$
→Volume of Sphere= 4/3 πr^3
→Volume of Cone=1/3 $\pi r^2 h$
SOME THEOREM ON PROPORTION:
Let us have a:b::c:d
1) Inventendo-Theorem:
a:b=c:d→ a/b= c/d → b/a= d/c
Note: Invert the ratio simply both side.
2) Alternendo- Theorem:
a:b=c:d→a/b=c/d → a/c= b/d
Note: Just interchange the place of "b" with "c" in proportion
3) Dividendo Theorem:
a:b=c:d → a/b= c/d→ a-b/b = c-d/d
Note: Just Subtract the numerator value by denominator value (In numerator)
4) Componendo Theorem:
a:b=c:d→ a/b=c/d → a+b/b= c+d/d
Note: Just add the numerator value to the denominator value (In numerator)
5) Componendo-Dividendo Theorem:
a:b=c:d → $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
Note: follow the same fashion of dividendo and componendo theorem.

HOW TO FIND THE SHADED AREA OF GEOMETRICAL SHAPES:



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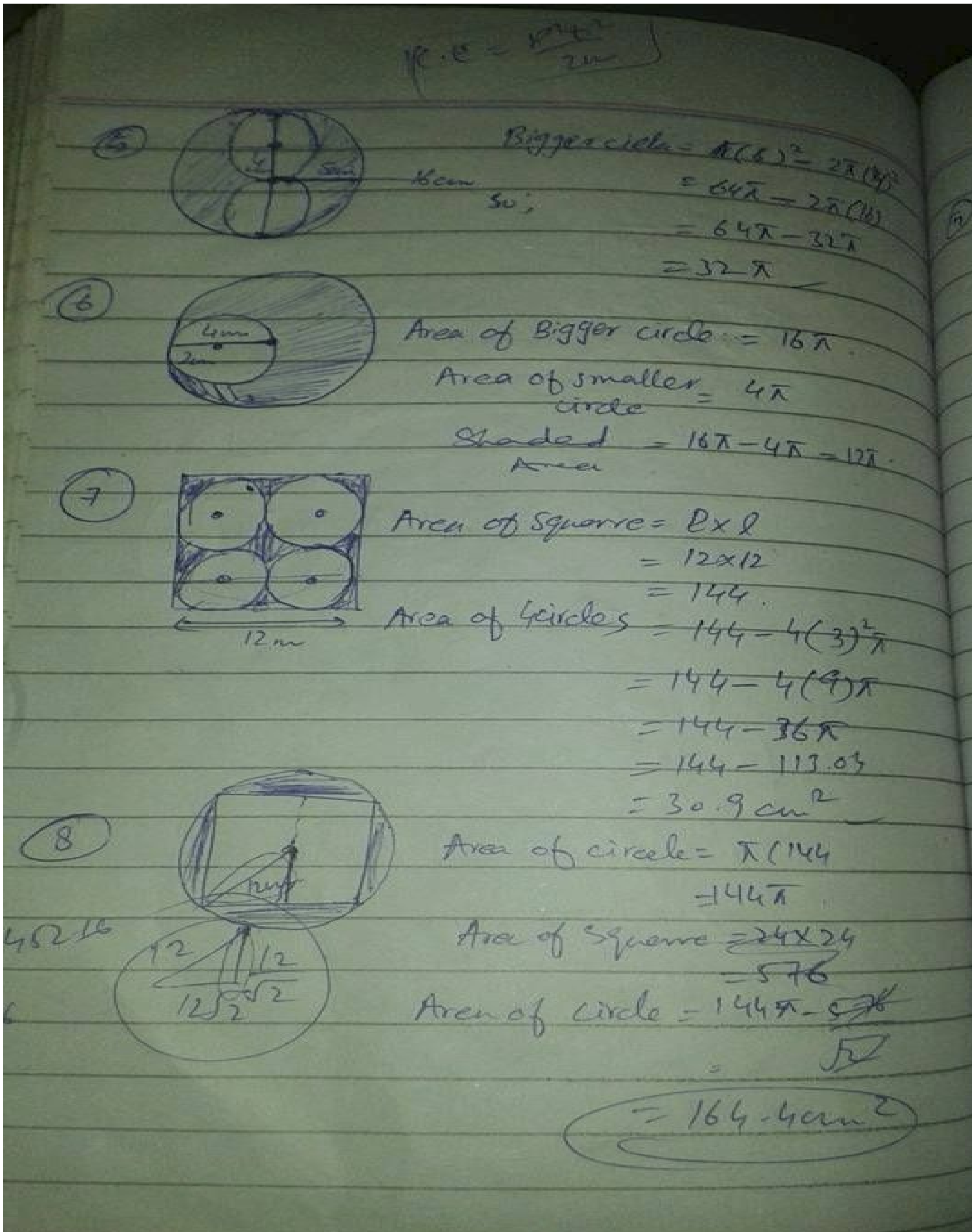
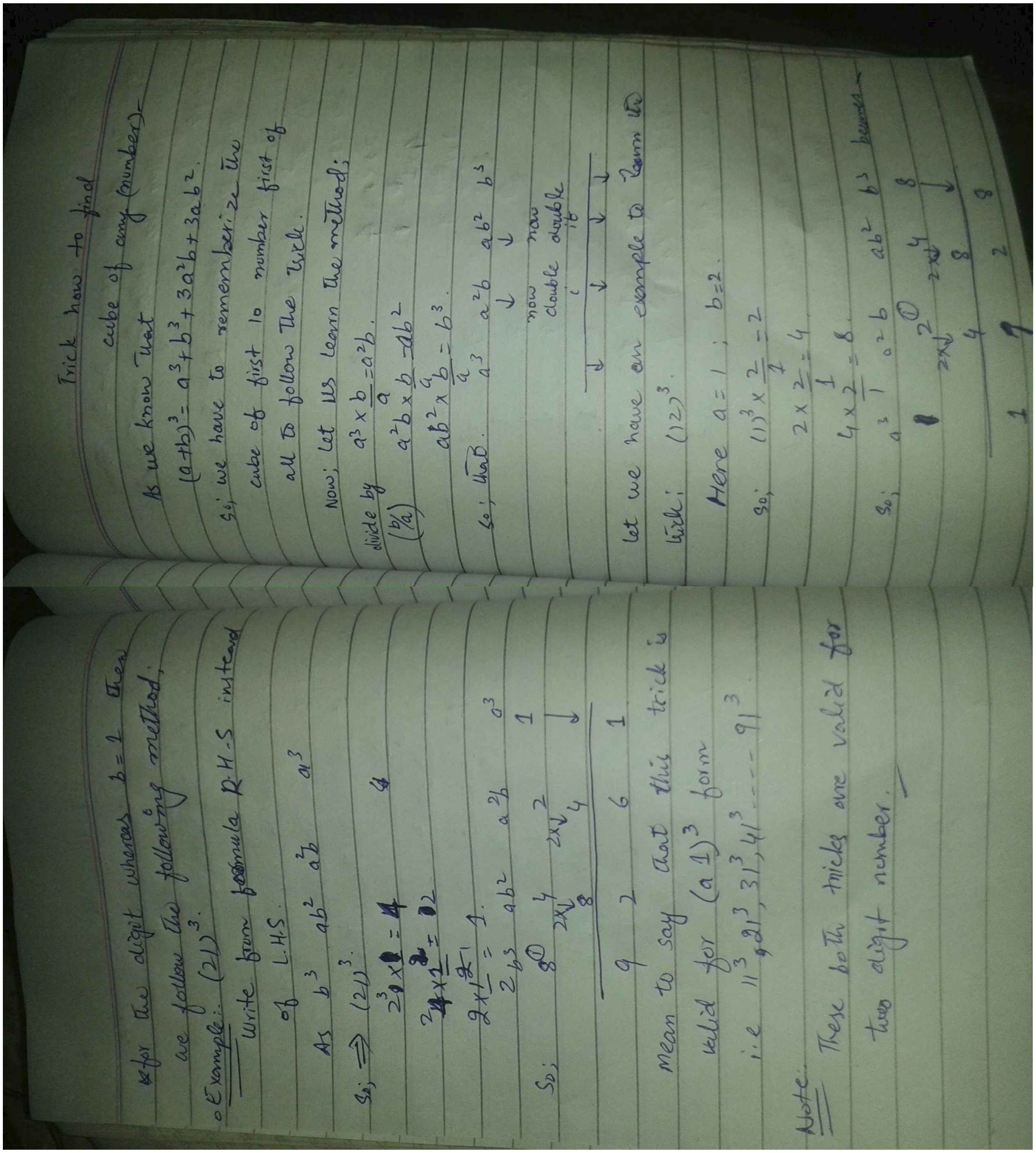


Note: This Question is always asked in the NET series to check the geometrical skills of the candidates..

HOW TO FIND THE CUBE OF A NUMBER.



Square root (Some values)

$\sqrt{1} = 1$ $\tan 30 = 0.577$

$\sqrt{2} = 1.414$ $\tan 60 = 1.732$

$\sqrt{3} = 1.732$ $\sin 45 = 0.707$

$\sqrt{5} = 2.236$ $\cos 45 = 0.707$

$\sqrt{6} = 2.449$ $\sin 60 = 0.866$

$\sqrt{7} = 2.649$ $\cos 30 = 0.866$

$\sqrt{8} = 2.828$

$\sqrt{10} = 3.162$

Squares of 1-30			Cubes of 1-10
$1^2 = 1$	$14^2 = 196$	$28^2 = 784$	$1^3 = 1$
$2^2 = 4$	$15^2 = 225$	$29^2 = 841$	$2^3 = 8$
$3^2 = 9$	$16^2 = 225$	$30^2 = 900$	$3^3 = 27$
$4^2 = 16$	$17^2 = 289$		$4^3 = 64$
$5^2 = 25$	$18^2 = 324$		$5^3 = 125$
$6^2 = 36$	$19^2 = 361$		$6^3 = 216$
$7^2 = 49$	$20^2 = 400$		$7^3 = 343$
$8^2 = 64$	$21^2 = 441$		$8^3 = 512$
$9^2 = 81$	$22^2 = 484$		$9^3 = 729$
$10^2 = 100$	$23^2 = 529$		$10^3 = 1000$
$11^2 = 121$	$24^2 = 576$		
$12^2 = 144$	$25^2 = 625$		
$13^2 = 169$	$26^2 = 676$		

SOME HELPFUL POINT **TO REMEMBER BEFORE ANY ENTRANCE TEST.**

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PRODUCTIVE TRICK FOR FINDING ANY TRIGONOMETRIC WHEN ANGLE (30 >) IS GIVEN:

As we know that: 1*= 0.01745 rad

The Question here comes sin31=?

As we know that sin30=0.5

So: sin 30+1*= 0.5+0.01745=~0.51745

Note This trick gives you approximate value which is enough to guess the Answer.

But here the Question should arise that sin33=?

So Simply 1*=0.01745

So we 3*= 1*3=3*0.01745=0.05235

Hence sin (30) +3*= 0.5+0.05325= ~0.55325 Answer)

NOTE: same the trend is used for tan and cos ☺

HOW TO FIND THE VALUE OF LOGARITHM OF NUMBER OTHER THAN PRIME NUMBER:

Prime number: A number which has at most two divisors i.e 1 and itself

Composite number: A number which has at least more than two division.

So log (4)=?

Here We should some values of logarithm

i.e $\log_{10} 1=0$, $\log_{10}2=0.3010$, $\log_{10}3=0.4771$ and $\log_{10}5=0.6989$

So that:

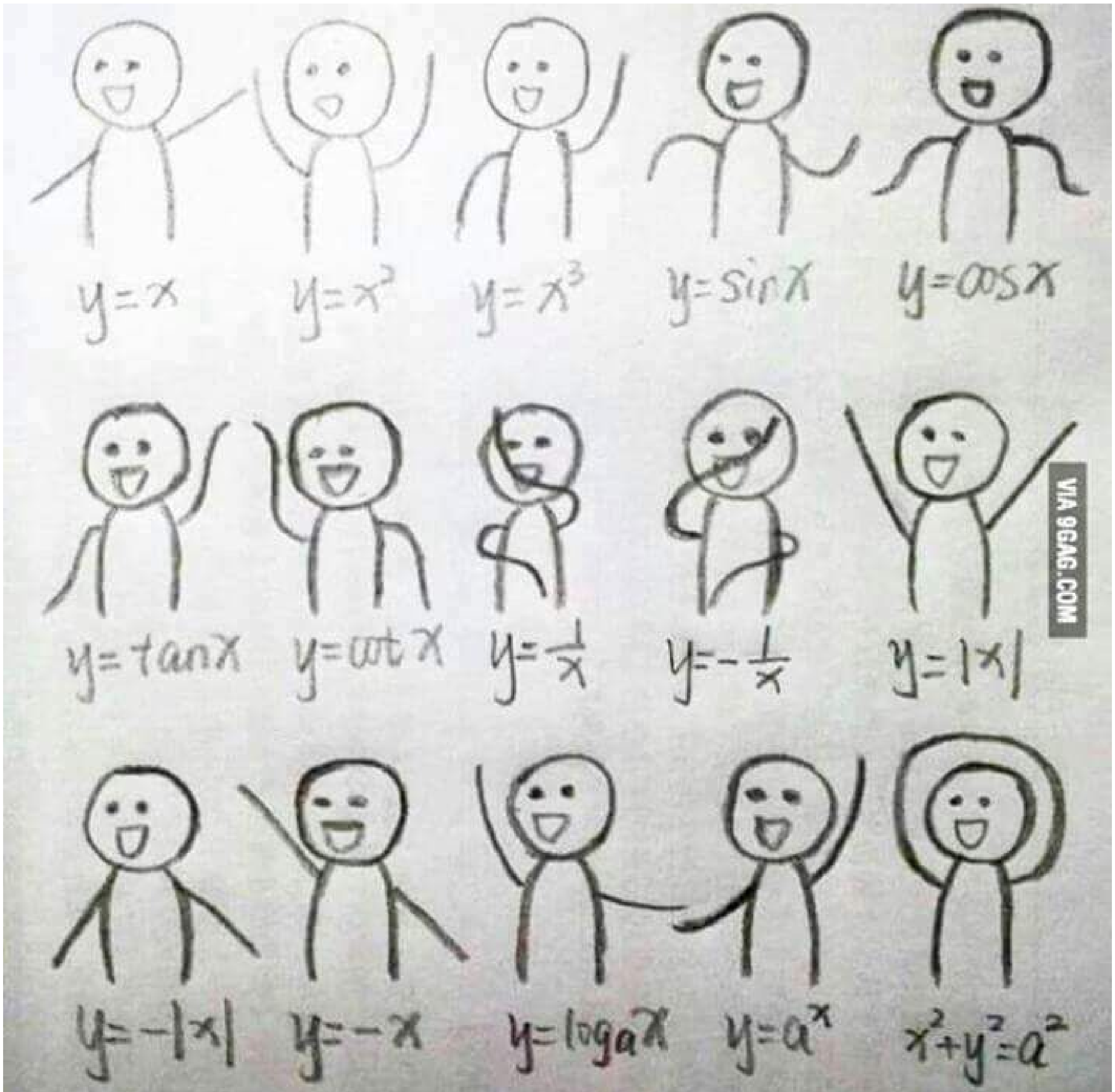
log (4)

Using the property:

log (2*2)= log2+log2= 0.3010+0.3010=0.6020 (Answer)

➔With same fashion we can find the logarithm..

GRAPHICAL REPRESENTATION OF DIFFERENT FUNCTIONS IN COOL STYLE:



HOW TO QUICKLY TAKE SQUARE OF ANY NUMBER OF THE FORM $(A5)^2$

Remember this:

$(A5)^2 = [A(A+1)25]$

Examples:-

$(65)^2 = 6(6+1)25 = 6(7)25 = 4225$

$(95)^2 = 9(9+1)25 = 9(10)25 = 9025$

Best of luck for Luminous Future..