Florian Rösel, Frauke Liers Winter semester 2021/2022

Mathematics of Learning – Worksheet 3

Basics. [Singular Value Decomposition].
Calculate the SVD of the matrix

$$A = \begin{pmatrix} 2 & -2 & -2 & 0 \\ -1 & -1 & 3 & 4 \\ 2 & -2 & 2 & -2 \end{pmatrix}.$$

The SVD of a matrix $A \in \mathbb{R}^{m \times n}$ consists of matrices $U \in \mathbb{R}^{m \times m}, V \in \mathbb{R}^{n \times n}$ which are orthogonal, i.e., UU^T, VV^T is equal to the corresponding unit matrix, and a matrix $\Sigma \in \mathbb{R}^{m \times n}$ which has only positive, descending entries on the diagonal (it looks like

$$\begin{pmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \end{pmatrix}$$

in our case). $U \cdot \Sigma \cdot V^T = A$ should hold, in case you calculated correctly. Hint: No nice numbers this time.

Exercise 1 [Reading assignment: Association rules].
Read chapter 14.2 of the *Hastie* book, regarding association rules. Discuss the contents with one (or more) fellow student for at least half an hour.

Exercise 2 [Reading assignment: Self organizing maps (SOM)].
Read chapter 14.5 of the *Hastie* book, regarding self organizing maps. Discuss the contents with one (or more) fellow student for at least half an hour.

Exercise 3 [Prerequisites for PCA].
Given a set of data vectors $x_1, ..., x_N \in \mathbb{R}^p$ and a matrix $V_q \in \mathbb{R}^{p \times q}$, $q < p$, with q orthogonal unit vectors as columns. Prove, that

$$\tilde{\mu} = \bar{x}, \quad \tilde{\lambda}_i = V_q^T (x_i - \bar{x})$$

is a minimizer (over μ and λ_i)

$$\sum_{i=1}^{N} ||x_i - \mu - V_q \lambda_i||^2,$$

where $|| \cdot ||$ denotes the euclidean norm. Furthermore, show that the minimizer $\bar{x}$ for μ is not unique and find the set of minimizers for μ.