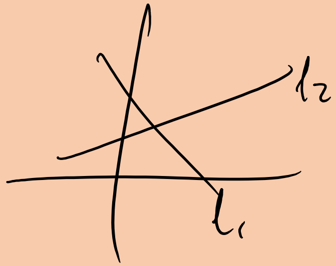
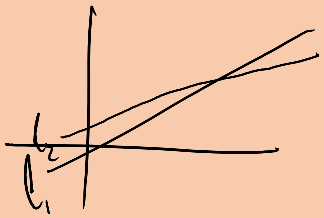
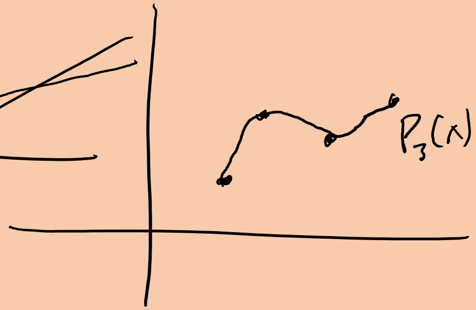
Interpolasi → Polinomial



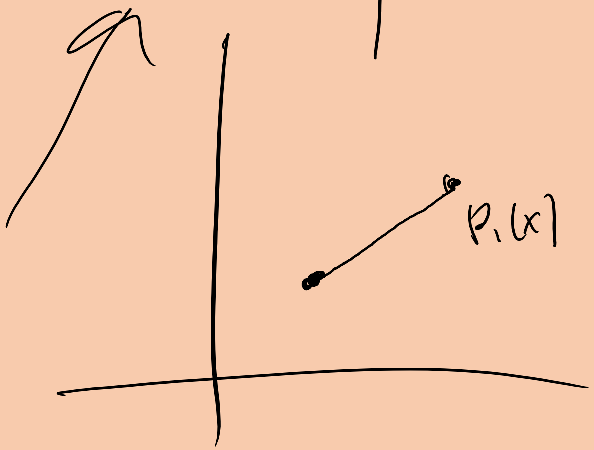




* Polinom sederhana

$$P_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$$

$f(x) \approx P_n(x)$ → dibutuhkan $n+1$ data/titik.



2 data/titik → (x_0, y_0) dan (x_1, y_1) → $P_1(x)$

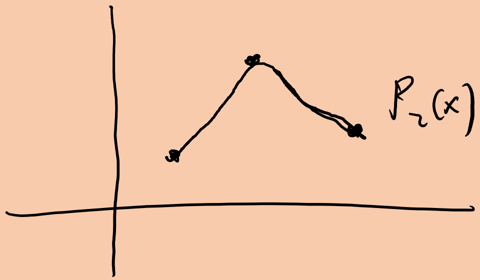
$P_1(x) = a_0 + a_1 x$

a_0, a_1 ?

3 titik → (x_0, y_0), (x_1, y_1) dan (x_2, y_2) → $P_2(x)$

$P_2(x) = a_0 + a_1x + a_2x^2$ → $P_2(x) \approx f(x)$

a_0, a_1 dan a_2 ?



1. Polinom Lagrange
2. Polinom Newton
3. Polinom Newton-Gregory

} $P_n(x)$

$$\boxed{P_1(x) = y_0 L_0 + y_1 L_1}$$

dgn

$$L_0 = \frac{x - x_1}{x_0 - x_1} \quad \text{dan}$$

$$L_1 = \frac{x - x_0}{x_1 - x_0}$$

1. Polinom Lagrange

$$P_1(x) = a_0 + a_1 x \longrightarrow (x_0, y_0) \text{ dan } (x_1, y_1)$$

$$P_1(x) = y_0 + \frac{(y_1 - y_0)}{(x_1 - x_0)}(x - x_0)$$

modifikasi aljabar

$$P_1(x) = y_0 \underbrace{\frac{(x - x_1)}{x_0 - x_1}}_{L_0} + y_1 \underbrace{\frac{(x - x_0)}{x_1 - x_0}}_{L_1}$$

$$P_2(x) = y_0 L_0 + y_1 L_1 + y_2 L_2$$

Lagrange orde 2

Polinom Lagrange orde n

$$P_n(x) = y_0 L_0 + y_1 L_1 + y_2 \cdot L_2 + \cdots + y_n \cdot L_n$$

$$= \sum_{i=0}^{n} y_i L_i$$

$$L_i = \prod_{\substack{j=0 \\ i \neq j}}^{n} \frac{(x - x_j)}{(x_i - x_j)} \rightarrow \text{koef. Lagrange}$$

$$P_1(x) = y_0 L_0 + y_1 L_1 \neq P_1(x)$$

$$P_2(x) = y_0 L_0 + y_1 L_1 + y_2 L_2$$

Data berubah $\longrightarrow$ $P_n(x)$ dibangun dari awal.

Diketahui : $\ln(8) = 2.0794$
$\ln(9) = 2.1972$
$\ln(9.5) = 2.2513$

$$L_i(x_j) = \prod_{\substack{j=0 \\ i \neq j}} \frac{(x - x_j)}{x_i - x_j}$$

Ditanya : a. $P_2(x) \rightarrow$ lagrange orde 2
b. prediksi $\ln(9.2)$ dgn $P_2(x)$

Jawab : $(x_0, y_0) = (8, 2.0794)$
$(x_1, y_1) = (9, 2.1972)$
$(x_2, y_2) = (9.5, 2.2513)$

$$L_2 = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

$$\cancel{L_0} = \frac{(x-8)(x-9)}{(9.5-8)(9.5-9)} = \frac{(x-8)(x-9)}{0{,}75}$$

$$P_2(x) = y_0 \cdot L_0 + y_1 L_1 + y_2 \cdot L_2$$

$$L_0 = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} \rightarrow L_0 = \frac{(x-9)(x-9.5)}{(8-9)(8-9.5)} = \frac{(x-9)(x-9.5)}{(-1)(-1.5)} = \frac{(x-9)(x-9.5)}{1.5}$$

$$L_1 = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} \rightarrow L_1 = \frac{(x-8)(x-9.5)}{(9-8)(9-9.5)} = \frac{(x-8)(x-9.5)}{-0.5}$$

Jadi Polinom Lagrange orde 2 $(P_2(x))$ adalah

a. $P_2(x) = 2.0794 \dfrac{(x-9)(x-9.5)}{1.5} + 2.1972 \dfrac{(x-8)(x-9.5)}{-0.5} + 2.2513 \dfrac{(x-8)(x-9)}{0.75}$

b. $\ln(9,2) \approx P_2(9,2)$

$= 2.0794 \dfrac{(9,2-9)(9,2-9.5)}{1.5} + 2.1972 \dfrac{(9,2-8)(9,2-9.5)}{-0.5} + 2.2513 \dfrac{(9,2-8)(9,2-9)}{0,75}$

$8 — 9,5 \to 9,6$

$= 1,3863\,(0.2)(-0.3) - 4,3944\,(1,2)(-0,3) + 3,0017\,(1,2)(0,2)$

$\ln(8,5)$?

$= -0,0832 + 1,5820 + 0,7204$

$= 2,2192 \longrightarrow$

nilai $\ln(9,2) \approx 2,2192$ (by Lagrange orde 2)

by Calculator $\ln(9,2) = 2,2192034841 \leftarrow$ algoritma data: $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ dan (x_3, y_3)

$$P_3(x) = y_0 l_0 + y_1 l_1 + y_2 l_2 + y_3 l_3$$

$$l_0 = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$P_2(x) = y_0 l_0 + y_1 l_1 + y_2 l_2$$

$$l_0 = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$x_0 = \cdots \rightarrow$ Taylor

$x_0 = 0 \rightarrow$ Mclaurin

→ Kelemahan Lagrange

u/ mencari $P_3(x)$ tdk bisa digunakan $P_2(x)$

→ Newton $\longrightarrow$ $P_3(x) = P_2(x) +$ sesuatu

$P_4(x) = P_3(x) +$ sesuatu

Interpolasi Newton

$$P_1(x) = a_0 + a_1(x)$$

titik (x_0, y_0) dan (x_1, y_1) mk

$$P_1(x) = y_0 + \frac{(y_1 - y_0)}{x_1 - x_0}(x - x_0)$$

$$= a_0 + a_1(x - x_0)$$

$$a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$ → selisih terbagi (divided difference)

$$= f[x_1, x_0]$$

mk $P_2(x) = \underbrace{a_0 + a_1(x - x_0)} + a_2(x - x_0)(x - x_1)$

$$= P_1(x) + a_2(x - x_0)(x - x_1)$$

$$a_2 = \frac{f(x_2) - a_0 - a_1(x_2 - x_0)}{(x_2 - x_0)(x_2 - x_1)}$$

$$a_2 = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0}$$

$$a_0 = f(x_0)$$

$$a_1 = f[x_1, x_0]$$

$$a_2 = f[x_2, x_1, x_0]$$

$\vdots$

$$a_n = f[x_n, x_{n-1}, \ldots, x_0]$$

Tabel Selisih Terbagi

		$\to a_0$	$\to a_1$	$\to a_2$	$\to a_3$
i	x_i	$y_i = f(x_i)$	ST 1	ST-2	ST-3
0	x_0	$f(x_0)$	$f[x_1, x_0]$	$f[x_2, x_1, x_0]$	$f[x_3, x_2, x_1, x_0]$
1	x_1	$f(x_1)$	$f[x_2, x_1]$	$f[x_3, x_2, x_1]$	
2	x_2	$f(x_2)$	$f[x_3, x_2]$		
3	x_3	$f(x_3)$			

$$P_3(x) = \underbrace{a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)}_{P_2(x)} + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$= P_2(x) + a_3(x-x_0)(x-x_1)(x-x_2)$$

Contoh : diketahui : $\ln(8) = 2,0794$

$\ln(9) = 2,1972$

$\ln(9,5) = 2,2513$

ditanya : $\ln(9,2)$ dgn ~~polinom~~ polinom Newton orde 2. $(P_2(x))$.

Jawab :

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

i	x_i	$f(x_i)$	ST-1	ST-2
0	8	2,0794	0,1178	$-0,0064$
1	9	2,1972	0,1081	
2	9,5	2,2513		

$$f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{2,1972 - 2,0794}{9 - 8} = 0,1178$$

$$f[x_2, x_1] = \frac{2,2513 - 2,1972}{9,5 - 9} = 0,1081$$

$$f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0}$$

$$= \frac{0,1081 - 0,1178}{9,5 - 8}$$

$$= -0,0064$$

Sehingga

$$P_2(x) = 2.0794 + 0{,}1178\,(x-8) - 0{,}0064\,(x-8)(x-9)$$

$$\ln(9{,}2) \approx P_2(x) = 2{,}0794 + 0{,}1178\,(9{,}2-8) - 0{,}0064\,(9{,}2-8)(9{,}2-9)$$

$$= 2{,}0794 + 0{,}14136 - 0{,}0015$$

$$= 2{,}21926 \rightarrow \text{Newton orde } 2$$

Lagrange orde 2

Kuadratik orde 2