
Homework - Introduction to Frontiers of Computational Science

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Problem 1.
Conservation or continuity in Maxwell's equation can be derived from Ampere's law and Gauss' law. Suppose we write out a vector identity that is always true, which states that the divergence of the curl of any vector field is always zero:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0$$

If we apply the divergence to Ampere's Law, we obtain:

$$\begin{aligned} \nabla \cdot \left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \right) &= \nabla \cdot (\nabla \times \mathbf{H}) = 0 \\ \frac{\partial (\nabla \cdot \mathbf{D})}{\partial t} + \nabla \cdot \mathbf{J} &= 0 \end{aligned}$$

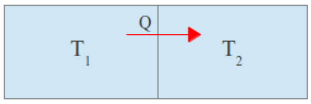
and from the Gauss' law ($\nabla \cdot \mathbf{D} = \rho$):

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} &= 0 \\ \nabla \cdot \mathbf{J} &= -\frac{\partial \rho}{\partial t} \end{aligned}$$

The left side of the equation is the divergence of the Electric Current Density ($\mathbf{J}$). This is a measure of whether current is flowing into a volume (i.e. the divergence of $\mathbf{J}$ is positive if more current leaves the volume than enters). If charge is exiting, then the amount of charge within the volume must be decreasing. This is exactly what the right side is a measure of - how much electric charge is accumulating or leaving in a volume. Hence, the continuity equation is about continuity - if there is a net electric current is flowing out of a region, then the charge in that region must be decreasing. If there is more electric current flowing into a given volume than exiting, than the amount of electric charge must be increasing. (ref: `http://maxwells-equations.com/`).

Problem 2.

a. Solve this problem:



$$\begin{aligned} Q &= k(T_1 - T_2) \\ \frac{d}{dt}(cT_1) &= -Q \\ \frac{d}{dt}(cT_2) &= +Q \end{aligned}$$

if the constants and initial value is given, find $T_1(t)$ and $T_2(t)$!

Answer:

Analitically:

$$\begin{aligned} \frac{d}{dt}(c(T_1 + T_2)) &= 0 \\ \frac{d}{dt}(c(T_1 - T_2)) &= -2Q \end{aligned}$$

then,

$$\begin{aligned} \frac{d}{dt}(T_1 + T_2) &= 0 \\ \frac{d}{dt}(T_1 - T_2) &= -2\mu(T_1 - T_2) \end{aligned}$$

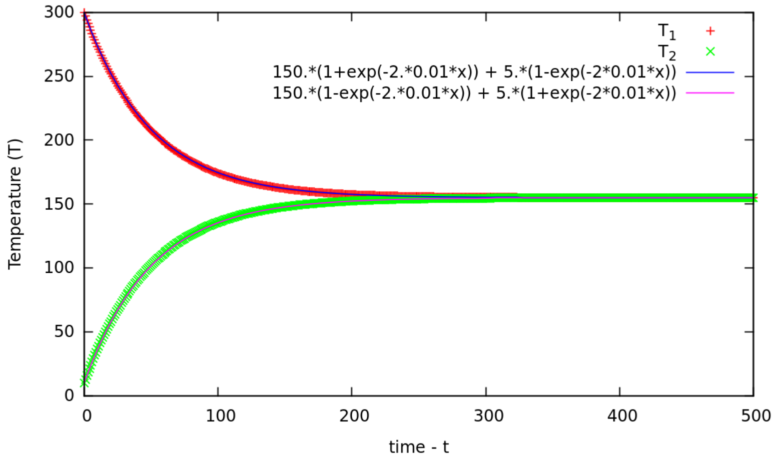
with $\mu = k/c$,

$$\begin{aligned} (T_1 + T_2)_{(t)} &= (T_1 + T_2)_{(0)} \\ (T_1 - T_2)_{(t)} &= (T_1 - T_2)_{(0)} e^{-2\mu t} \end{aligned}$$

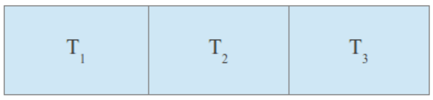
using addition and substraction,

$$\begin{aligned} T_1 &= \frac{T_1^{(0)}}{2}\left(1 + e^{-2\mu t}\right) + \frac{T_2^{(0)}}{2}\left(1 - e^{-2\mu t}\right) \\ T_2 &= \frac{T_1^{(0)}}{2}\left(1 - e^{-2\mu t}\right) + \frac{T_2^{(0)}}{2}\left(1 + e^{-2\mu t}\right) \end{aligned}$$

Numerically:
Using euler method directly from the problem, and using analitically solution above,



b. Write the heat transfer equation for this problem:



Answer:

$$
\begin{aligned}
\frac{d}{dt}(T_1) &= -\mu(T_1 - T_2) \\
\frac{d}{dt}(T_2) &= \mu(T_1 - T_2) - \mu(T_2 - T_3) \\
\frac{d}{dt}(T_3) &= \mu(T_2 - T_3)
\end{aligned}
$$