Introduction to Formal Language, Fall 2016	Due: 22-Mar-2016

Homework 2 Solutions

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1. Find all strings in $L((a+b)^*b(a+ab)^*)$ of length less than four.
Answer.

The strings with length 1: $\{\lambda b\lambda\} = \{b\}$;
The strings with length 2: $\{ab\lambda, bb\lambda, \lambda ba\} = \{ab, bb, ba\}$;
The strings with length 3: $\{(aa)b\lambda, (ab)b\lambda, (ba)b\lambda, (bb)b\lambda, (a)b(a), (b)b(a), \lambda b(ab)\} = \{aab, abb, bab, bbb, aba, bba, bab\}$. □

2. Find a regular expression for the set $\{a^n b^m : (n+m) \textit{ is even}\}$.
Answer.

There are two cases:

- n and m are even: $(aa)^*(bb^*)$;
- n and m are odd: $a(aa)^*b(bb^*)$;

Thus, a regular expression for the set $\{a^n b^m : (n+m) \textit{ is even}\}$ is $(aa)^*(bb^*) + a(aa)^*b(bb^*)$. □

3. Give a regular expression for $L = \{a^n b^m : n < 4, m \leq 3\}$.
Answer.

A regular expression for $L_1 = \{a^n : n < 4\}$ is $\lambda + a + aa + aaa$;
A regular expression for $L_2 = \{b^m : m \leq 3\}$ is $\lambda + b + bb + bbb$;
Thus, a regular expression for $L = L_1L_2$ is $(\lambda + a + aa + aaa)(\lambda + b + bb + bbb)$. □

4. What languages do the expressions $(\phi^*)^*$ and $a\phi$ denote?
Answer.

$L((\emptyset^*)^*) = (L(\emptyset^*))^* = (\emptyset^*)^* = \{\lambda\}$;
$L(a\emptyset) = L(a)L(\emptyset) = \{a\}\{\} = \emptyset$. □

5. Find a regular expression for $L = \{vwv : v, w \in \{a,b\}^*, |v| \leq 3\}$.
Answer.

For any string $x \in \{a,b\}^*$, we can treat it as $x = \lambda x \lambda$, where $v = \lambda$, $x = w$. Therefore, any string $x \in \{a,b\}^*$ is in L. Thus, $L = (a+b)^*$. □

6. Give a regular expression for all strings that contain no run of a's of length greater than two. $(\Sigma = \{a, b, c\})$.
Answer.

A regular expression for $\{a^n : n \leq 2\}$ is $\lambda + a + aa$.
Thus, for $\Sigma = \{a, b, c\}$, a regular expression for all strings that contain no run of a's of length greater than two is $((\lambda + a + aa)(b + c))^*(\lambda + a + aa)$. □

7. Give a regular expression for all strings with at most two occurrences of the substring 00. ($\Sigma = \{0, 1\}$).
Answer.

There are three cases for a string with at most two occurrences of 00:

- 0 occurrence of 00: λ;
- 1 occurrence 00: 00;
- 2 occurrences 00: 000, 0011^*00;

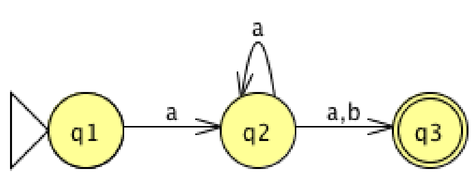
Thus, a regular expression for all strings with at most two occurrences of the substring 00 is $(1 + 01)^*(\lambda + 00 + 000 + 0011^*00)(1 + 10)^*$ □

8. Determine whether or not the following claim is true for all regular expressions r_1 and r_2. The symbol $\equiv$ stands for equivalence regular expressions in the sense that both expressions denote the same language. $r_1^*(r_1 + r_2)^* \equiv (r_1 + r_2)^*$.
Answer.

Since $L(r_1^*(r_1 + r_2)^*) \subseteq L((r_1 + r_2)^*(r_1 + r_2)^*) = L((r_1 + r_2)^*)$ and $L((r_1 + r_2)^*) = L(\lambda(r_1 + r_2)^*) \subseteq L(r_1^*(r_1 + r_2)^*)$, they are equivalent. □

9. Find an nfa that accepts the language $L(aa^*(a + b))$.
Answer.

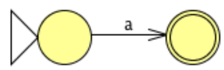
The following graph represents the NFA $M = (\{q1, q2, q3\}, \{a, b\}, \delta, q1, \{q3\})$ that accepts $L(aa^*(a + b))$, where δ is described as in the graph. □



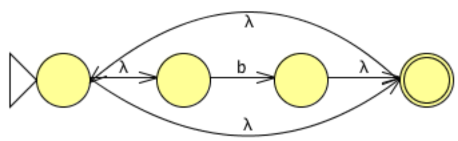
10. Use the construction in Theorem 3.1 to find an nfa that accepts the language $L(ab^*aa+ bba^*ab)$.

Answer.

By Theorem 3.1, the automata for $L(a)$ is

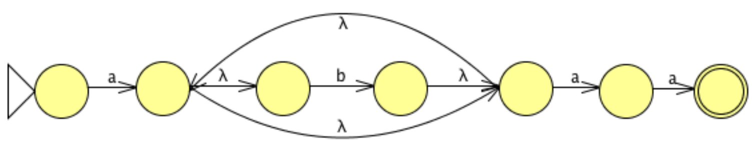


By Theorem 3.1, the automata for $L(a^*)$ is

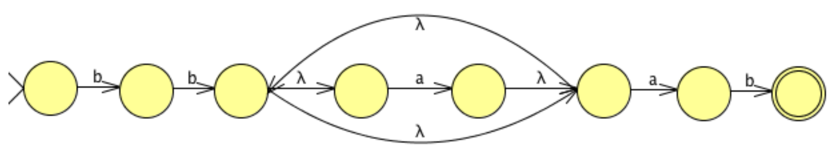


The automata for $L(b)$ and $L(b^*)$ can be constructed in a similar way.

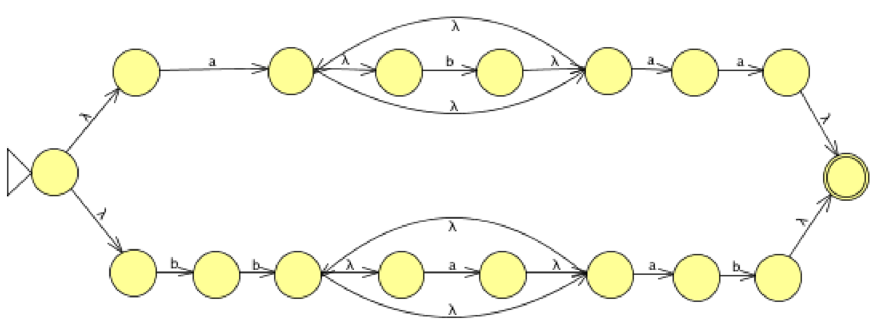
Then by Theorem 3.1, the automata for $L(ab^*aa)$ is



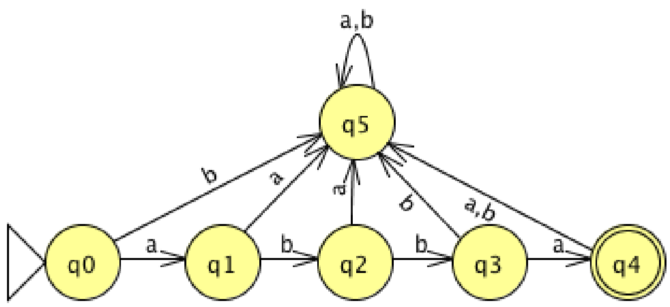
Then by Theorem 3.1, the automata for $L(bba^*ab)$ is



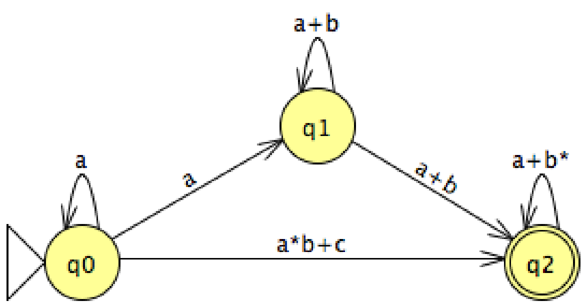
Thus, by Theorem 3.1, the automata for $L(ab^*aa + bba^*ab)$ is $\square$



11. Find dfa that accept $L = L(ab^*a^*) \cap L((ab)^*ba)$.
Answer. $L = L(ab^*a^*) \cap L((ab)^*ba) = \{abba\}$. The following graph represents the DFA $M = (\{q_0, q_1, \ldots, q_5\}, \{a, b\}, \delta, q_0, \{q_4\})$ that accepts L, where δ is described as in the graph. □

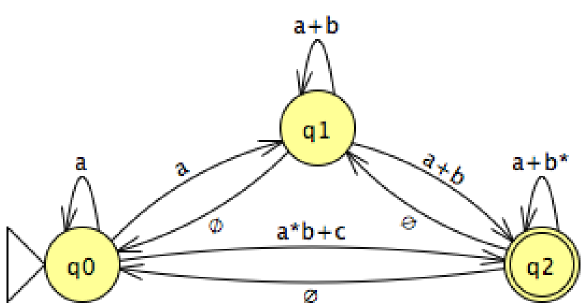


12. Find regular expression for the language accepted by the following automata.

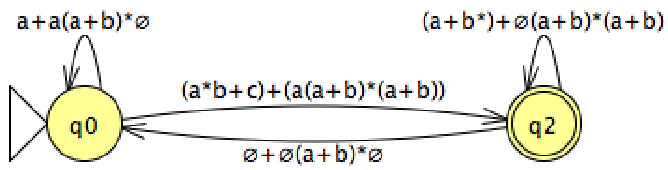


Answer.

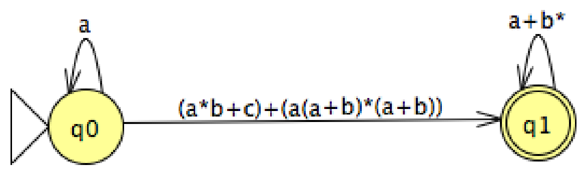
We first convert the given nfa to a complete GTG as follows.



Then we reduce the state q_1 to obtain a two states one as follows.



Then we have:

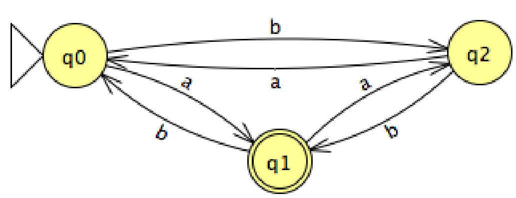


Finally, we obtain a regular expression: $a^*(ab^* + c + a(a+b)^*(a+b))(a+b^*)^*$ $\square$

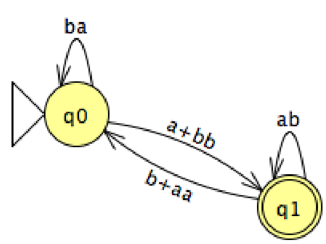
13. Find a regular expression for $L = \{w : (n_a(w) - n_b(w)) \mod 3 = 1\}$ on $\{a, b\}$.
Answer.

The following figure shows the dfa of the language L, where

- q_0: the state for $n_a(w) - n_b(w) \bmod 3 = 0$;
- q_1: the state for $n_a(w) - n_b(w) \bmod 3 = 1$;
- q_2: the state for $n_a(w) - n_b(w) \bmod 3 = 2$;



Then, we use reduce the steps to obtain the following nfa:



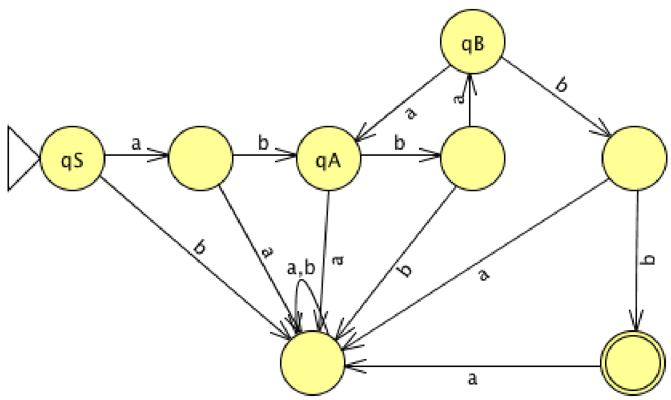
Finally the regular expression is $((ba)^*(a+bb))((ab)^* + (b+aa)(ba)^*(a+bb))^*$. $\square$

14. Construct a dfa that accepts the language generated by the grammar

$$S \to abA,$$
$$A \to baB,$$
$$B \to aA|bb.$$

Answer.

The dfa is constructed as follows, where q_x corresponds to variable x, $x \in \{S, A, B\}$. □



15. Find a regular grammar for $L = \{w : |(n_a(w) - n_b(w))|\ is\ odd\}$ on $\{a, b\}$.
Answer.

$$S \to aA|bA$$
$$A \to aS|bS|\lambda$$

□