IMAGE PROCESSING HW-2

Rıdvan Demirci 141044070

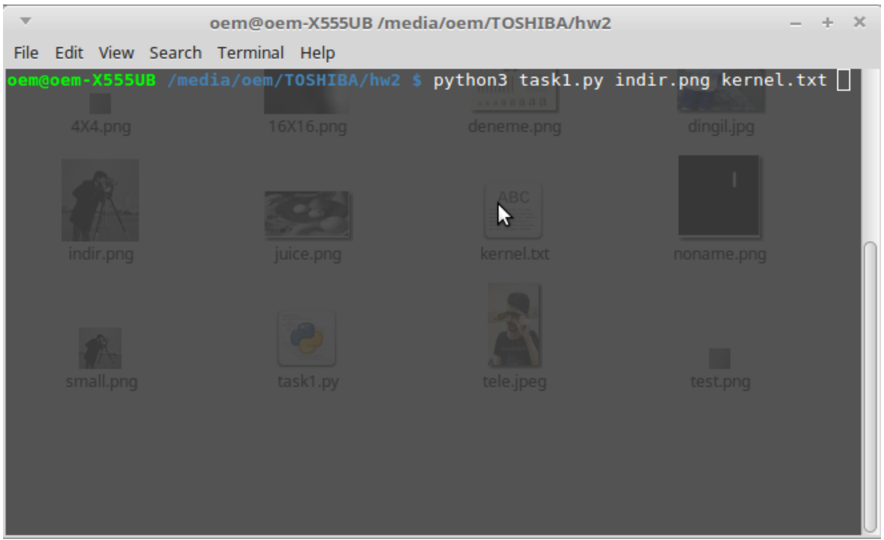
Gebze Teknik Üniversitesi

Task 1

- **Python 3.6.1 ile implement edildi array işlemleri için numpy kullanıldı. (kernel flip)**
- **Programda convolution teorem için 32 pixelden küçük ise kendimin yazdığı fourier dönüşümü kullanıldı.**
- **32 pixelden büyükler için ise numpy'ın fft methodu kullanıldı.**
- **Örnek kernel ve görüntü mevcut.**
- **Programın çalıştırlıması için konsoler arayüz mevcut**

Python3 task1.py image.png kernel.txt

Program çalıştırıldıktan sonra hem convolution theorem hem normal convolution sonuçları ekranda olacak.



Task 2

Rotation matrix $x = x'\cos(\theta) - y'\sin(\theta)$
$y = x'\sin(\theta) + y'\cos(\theta)$

Prove that: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$

Hint: $\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}$ **, Gösterimi kolaylaştırmak için: a =** x' *ve* **b=** y' **diyelim.**

Denklemlerin gösterimi şekildeki gibi değişir:

Rotation matrix $x = a\cos(\theta) - b\sin(\theta)$

$y = a\sin(\theta) + b\cos(\theta)$

Prove that: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial a^2} + \frac{\partial^2 f}{\partial b^2}$

Hint: $\frac{\partial f}{\partial a} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial a} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial a}$

Denklemlerin türevleri alınırsa.

$$\frac{\partial x}{\partial a} = \cos\theta \quad , \frac{\partial y}{\partial a} = \sin\theta$$

$\frac{\partial f}{\partial a} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$ **Denklemin ikinci dereceden türevi alınırsa;**

$$\frac{\partial^2 f}{\partial a^2} = \frac{\partial}{\partial a}\left(\frac{\partial f}{\partial x}\cos\theta\right) + \frac{\partial}{\partial a}\left(\frac{\partial f}{\partial y}\sin\theta\right) = \frac{\partial}{\partial a}\frac{\partial f}{\partial x}\cos\theta + \frac{\partial}{\partial a}\frac{\partial f}{\partial y}\sin\theta$$

Renkli olan bölümler düzenlenir ise (paydalar yer değiştirir);

$\frac{\partial}{\partial a}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial a}$ **,** $\frac{\partial}{\partial a}\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial a}$ **her iki denklemdeki** $\frac{\partial f}{\partial a}$ **kısım önceden hesaplanmıştı.** $\frac{\partial f}{\partial a} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$ Yerlerine yazılır ise;

$$\frac{\partial}{\partial x}\frac{\partial f}{\partial a} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) = \frac{\partial^2 f}{\partial x^2}\cos\theta + \frac{\partial\,\partial f}{\partial x\partial y}\sin\theta$$

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial a} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) = \frac{\partial\,\partial f}{\partial x\partial y}\cos\theta + \frac{\partial^2 f}{\partial y^2}\sin\theta$$

Bulunan sonuçlar $\frac{\partial^2 f}{\partial a^2}$ denkleminde yerine yazılırsa.

$$\frac{\partial^2 f}{\partial a^2} = \left(\frac{\partial^2 f}{\partial x^2}\cos\theta + \frac{\partial\,\partial f}{\partial x \partial y}\sin\theta\right)\cos\theta + \left(\frac{\partial\,\partial f}{\partial x \partial y}\cos\theta + \frac{\partial^2 f}{\partial y^2}\sin\theta\right)\sin\theta$$

Denklem toparlanırsa;

$$\frac{\partial^2 f}{\partial a^2} = \left(\frac{\partial^2 f}{\partial x^2}(\cos\theta)^2 + \frac{\partial\,\partial f}{\partial x \partial y}\sin\theta\cos\theta\right) + \left(\frac{\partial\,\partial f}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y^2}(\sin\theta)^2\right)$$

Aynı işlemler $\frac{\partial f}{\partial b} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial b} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial b}$ için uygulanırsa; $\frac{\partial x}{\partial b} = (-\sin\theta)$, $\frac{\partial y}{\partial b} = \cos\theta$

$\frac{\partial f}{\partial b} = \frac{\partial f}{\partial x}(-\sin\theta) + \frac{\partial f}{\partial y}\cos\theta$ ikinci dereceden türevi alınırsa.

$$\frac{\partial^2 f}{\partial b^2} = \frac{\partial}{\partial b}\left(\frac{\partial f}{\partial x}(-\sin\theta)\right) + \frac{\partial}{\partial b}\left(\frac{\partial f}{\partial y}\cos\theta\right) = \frac{\partial}{\partial b}\frac{\partial f}{\partial x}(-\sin\theta) + \frac{\partial}{\partial b}\frac{\partial f}{\partial y}\cos\theta$$

$\frac{\partial}{\partial b}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial b}$, $\frac{\partial}{\partial b}\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial b}$ her iki denklemdeki $\frac{\partial f}{\partial b}$ kısım önceden hesaplanmıştı.

$\frac{\partial f}{\partial b} = \frac{\partial f}{\partial x}(-\sin\theta) + \frac{\partial f}{\partial y}\cos\theta$ yerlerine yazılırsa.

$$\frac{\partial}{\partial x}\frac{\partial f}{\partial b} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}(-\sin\theta) + \frac{\partial f}{\partial y}\cos\theta\right) = \frac{\partial^2 f}{\partial x^2}(-\sin\theta) + \frac{\partial\,\partial f}{\partial x \partial y}\cos\theta$$

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial b} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}(-\sin\theta) + \frac{\partial f}{\partial y}\cos\theta\right) = \frac{\partial\,\partial f}{\partial x \partial y}(-\sin\theta) + \frac{\partial^2 f}{\partial y^2}\cos\theta$$

Bulunan sonuçlar $\frac{\partial^2 f}{\partial b^2}$ denkleminde yerine yazılırsa;

$$\frac{\partial^2 f}{\partial b^2} = \left(\frac{\partial^2 f}{\partial x^2}(-\sin\theta) + \frac{\partial\,\partial f}{\partial x \partial y}\cos\theta\right)(-\sin\theta) + \left(\frac{\partial\,\partial f}{\partial x \partial y}(-\sin\theta) + \frac{\partial^2 f}{\partial y^2}\cos\theta\right)\cos\theta$$

Denklem toparlanırsa;

$$\frac{\partial^2 f}{\partial b^2} = \frac{\partial^2 f}{\partial x^2}(\sin\theta)^2 - \frac{\partial\,\partial f}{\partial x \partial y}(\sin\theta\cos\theta) - \frac{\partial\,\partial f}{\partial x \partial y}(\sin\theta\, cos\theta) + \frac{\partial^2 f}{\partial y^2}(\cos\theta)^2$$

Hesaplanan denklemler alt alta yazılır ve denklemler alt alta toplanırsa bir birinin tersi ifadeler götürür.

$$\frac{\partial^2 f}{\partial a^2}=\frac{\partial^2 f}{\partial x^2}(\cos\theta)^2+\cancel{\frac{\partial\,\partial f}{\partial x\partial y}\sin\theta\cos\theta}+\cancel{\frac{\partial\,\partial f}{\partial x\partial y}\cos\theta\sin\theta}+\frac{\partial^2 f}{\partial y^2}(\sin\theta)^2$$

$$\frac{\partial^2 f}{\partial b^2}=\frac{\partial^2 f}{\partial x^2}(\sin\theta)^2-\cancel{\frac{\partial\,\partial f}{\partial x\partial y}(\sin\theta\cos\theta)}-\cancel{\frac{\partial\,\partial f}{\partial x\partial y}(\sin\theta\cos\theta)}+\frac{\partial^2 f}{\partial y^2}(\cos\theta)^2$$

$+$

$$\frac{\partial^2 f}{\partial a^2}+\frac{\partial^2 f}{\partial b^2}=\frac{\partial^2 f}{\partial x^2}(\cos\theta)^2+\frac{\partial^2 f}{\partial x^2}(\sin\theta)^2+\frac{\partial^2 f}{\partial y^2}(\sin\theta)^2+\frac{\partial^2 f}{\partial y^2}(\cos\theta)^2$$

Ortak paranteze alınırsa;

$$\frac{\partial^2 f}{\partial a^2}+\frac{\partial^2 f}{\partial b^2}=\frac{\partial^2 f}{\partial x^2}\left((\cos\theta)^2+(\sin\theta)^2\right)+\frac{\partial^2 f}{\partial y^2}\left(((\sin\theta)^2+(\cos\theta)^2)\right)$$

$$((\sin\theta)^2+(\cos\theta)^2)=1\quad \boldsymbol{olduğundan};$$

$$\frac{\partial^2 f}{\partial a^2}+\frac{\partial^2 f}{\partial b^2}=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\quad ,$$

a = x' *ve* **b=** y' **tekrar yerlerine yazılırsa**

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=\frac{\partial^2 f}{\partial x'^2}+\frac{\partial^2 f}{\partial y'^2}$$ **, Denklemi sağlanmış olunur.**

TASK 3

$h(x, y)=3f(x, y)+2f(x\text{-}1, y)+2f(x\text{+}1, y)\text{-}17 f(x, y\text{-}1)+99f(x, y\text{+}1)$

A-) ***is h linear? Prove your answers.***

Linearlik formülü : $h(\alpha f_1 + Bf_2) = \alpha h(f_1) + Bh(f_2)$ ***olmalı***

$h(\alpha f_1 + Bf_2)$ ***için;***

$$h(\alpha f_1 + Bf_2) = 3\big(\alpha f_1(x,y) + Bf_2(x,y)\big) + 2\big(\alpha f_1(x-1,y) + Bf_2(x-1,y)\big) + 2\big(\alpha f_1(x+1,y) + Bf_2(x+1,y)\big) - 17\big(\alpha f_1(x,y-1) + Bf_2(x,y-1)\big) + 99(\alpha f_1(x,y+1) + Bf_2(x,y+1))$$

Denklemlem toparlanırsa;

$$h(\alpha f_1 + Bf_2) = \alpha\big(3f_1(x,y) + 2f_1(x-1,y) + 2f_1(x+1,y) - 17f_1(x,y-1) + 99f_1(x,y+1)\big) + B\big(3f_2(x,y) + 2f_2(x-1,y) + 2f_2(x+1,y) - 17f_2(x,y-1) + 99f_2(x,y+1)\big)$$

$\alpha h(f_1) + Bh(f_2)$ ***için ise;***

$$\alpha h(f_1) + Bh(f_2) = 3\alpha f_1(x,y) + 2\alpha f_1(x-1,y) + 2\alpha f_1(x+1,y) - 17\alpha f_1(x,y-1) + 99\alpha f_1(x,y+1) + 3Bf_2(x,y) + 2Bf_2(x-1,y) + 2Bf_2(x+1,y) - 17Bf_2(x,y-1) + 99Bf_2(x,y+1)$$

Denklemlem toparlanırsa;

$$\alpha h(f_1) + Bh(f_2) = 3\left(\alpha f_1(x,y) + Bf_2(x,y)\right) + 2\left(\alpha f_1(x-1,y) + Bf_2(x-1,y)\right) + 2\left(\alpha f_1(x+1,y) + Bf_2(x+1,y)\right) - 17\left(\alpha f_1(x,y-1) + Bf_2(x,y-1)\right) + 99(\alpha f_1(x,y+1) + Bf_2(x,y+1))$$

çıkan denkem sonuçları $h(\alpha f_1 + Bf_2) = \alpha h(f_1) + Bh(f_2)$ ***eşit olduğu için h linear dir.***

Matrix örneklerinde deneme:

$$h = \begin{bmatrix} 0 & -17 & 0 \\ 2 & 3 & 2 \\ 0 & 99 & 0 \end{bmatrix} \quad f_1 = \begin{bmatrix} 3 & 5 & 8 \\ 12 & 15 & 17 \\ 13 & 1 & 4 \end{bmatrix} \quad f_2 = \begin{bmatrix} 2 & 4 & 86 \\ 0 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$h(\alpha f_1 + Bf_2) = \alpha h(f_1) + Bh(f_2)$ ***eşitlik matrixlerde uygulanırsa***

$$\alpha f_1 + B f_2 = \begin{bmatrix} 3\alpha + 2B & 5\alpha + 4B & 8\alpha + 6B \\ 12\alpha + 0B & 15\alpha + 4B & 17\alpha + 2B \\ 13\alpha + B & \alpha + 2B & 4\alpha + 3B \end{bmatrix}$$

$$h(\alpha f_1 + B f_2) = \begin{bmatrix} 0 & -17(5\alpha + 4B) & 0 \\ 2(12\alpha + 0B) & 3(15\alpha + 4B) & 2(17\alpha + 2B) \\ 0 & 99(\alpha + 2B) & 0 \end{bmatrix}$$

$\alpha h(f_1) + \ Bh(f_2)$ için ise;

$$\alpha h(f_1) = \begin{bmatrix} 0 & -17(5\alpha) & 0 \\ 2(12\alpha) & 3(15\alpha) & 2(17\alpha) \\ 0 & 99(\alpha) & 0 \end{bmatrix}, Bf_2 = \begin{bmatrix} 0 & -17(4B) & 0 \\ 2(0B) & 3(4B) & 2(2B) \\ 0 & 99(B) & 0 \end{bmatrix}$$

$$\alpha h(f_1) + \ Bh(f_2) = \begin{bmatrix} 0 & -17(5\alpha + 4B) & 0 \\ 2(12\alpha + 0B) & 3(15\alpha + 4B) & 2(17\alpha + 2B) \\ 0 & 99(\alpha + 2B) & 0 \end{bmatrix}$$

matrixler eşit çıktığı için h filtresi linear'dir

B-)

$h(x,y)$=3$f(x,y)$+2$f(x$-1$,y)$+2$f(x$+1$,y)$-17 $f(x,y$-1$)$+99$f(x,y$+1$)$ filter denkleminden

H = $\begin{bmatrix} 0 & 99 & 0 \\ 2 & 3 & 2 \\ 0 & -17 & 0 \end{bmatrix}$ **olur denklemi convolution tipinde yazmak için flip attırılır ve maske aşağıdaki gibi olur.**

H = $\begin{bmatrix} 0 & -17 & 0 \\ 2 & 3 & 2 \\ 0 & 99 & 0 \end{bmatrix}$