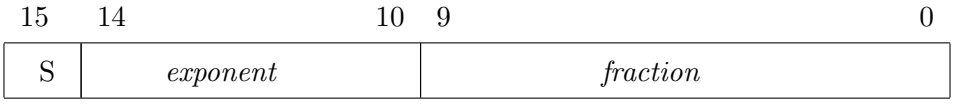
Question 3 (25 points):

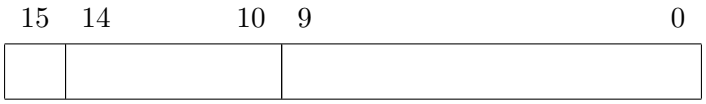
NVIDIA has defined a "half-precision" floating pointing format for use in its Graphics Processing Units (GPUs). A floating-point number is represented in this format in 16 bits as follows: the most significant bit is the sign bit, next there are 5 bits used for the exponent, and 10 bits for the fraction. This format is illustrated below:



The exponent is expressed in excess-16 format (also known as a bias representation). Given the binary representation above, the decimal value of the number represented can be computed by the following expression:

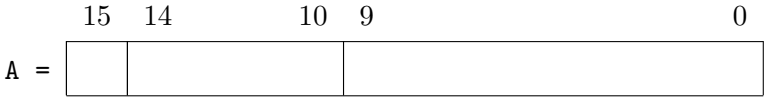
$$N = \begin{cases} (-1)^S \times 0.0 & \text{if } exponent = 0 \text{ and } fraction = 0 \\ (-1)^S \times 0.fraction \times 2^{-14} & \text{if } exponent = 0 \text{ and } fraction \neq 0 \\ (-1)^S \times 1.fraction \times 2^{exponent-15} & \text{if } 0 < exponent < 31 \\ (-1)^S \times \infty & \text{if } exponent = 31 \text{ and } fraction = 0 \\ NaN & \text{if } exponent = 31 \text{ and } fraction \neq 0 \end{cases}$$

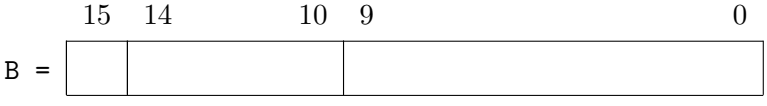
a. (**5 points**) Give the bit pattern for the representation of the number -4.125_{10} in this notation.



Let `A = 0x6C00` and B = 0x4940 be two floating pointing numbers in this format.

b. (**5 points**) What is the value of `A` and what is the value of `B`? Express each of these values both in normalized base-two notation and in decimal notation.





c. **5 points**) What is the true value of $A + B$ expressed in decimal notation? In other words, what is the value of $A + B$ if an infinite precision could be used to compute the addition and to store the result?

d. (**5 points**) Assume a floating-point unit that uses the NVIDIA format presented above. This unit has no guard, no round, and no sticky bits. What is the value of $A + B$, expressed both in normalized base-two notation and in decimal notation, computed by this machine?

e. (**5 points**) Now assume a floating-point unit that uses the NVIDIA format presented above. This unit has one guard, one round, and one sticky bit. What is the value of $A + B$, expressed in normalized base-two notation, computed by this machine?