CMPUT 229 (A2) - Quiz # 5 - Fall 2013

Name: Solution

The following is a format for the binary representation of a floating-point number:

10	9		5	4		0
S		*exponent*			*fraction*	

The exponent is expressed in excess-8 format (also known as a bias representation). Given the binary representation above, the decimal value of the number represented can be computed by the following expression:

$$
N = \begin{cases}
0.0 & \text{if } exponent = 0 \text{ and } fraction = 0 \\
(-1)^S \times 0.fraction \times 2^{-14} & \text{if } exponent = 0 \text{ and } fraction \neq 0 \\
(-1)^S \times 1.fraction \times 2^{exponent-15} & \text{if } 0 < exponent < 30 \\
(-1)^S \times \infty & \text{if } exponent = 31 \text{ and } fraction = 0 \\
NaN & \text{if } exponent = 31 \text{ and } fraction \neq 0
\end{cases}
$$

1. (30 points) Let $X = 128_{10}$. Give the normalized binary representation for X and the bit pattern for the representation of X in this notation.

   $128_{10} = 2^7 = 1.0 \times 2^7 \Rightarrow exponent - 15 = 7 \Rightarrow exponent = 22$

10	9		5	4		0
0		10110			00000	

2. (30 points) Let $Y = 12.5_{10}$. Give the normalized binary representation for Y and the bit pattern for the representation of Y in this notation.

   $12.5_{10} = 2^3 + 2^2 + 2^{-1} = 1100.1 = 1.1001 \times 2^3 \Rightarrow exponent - 15 = 3 \Rightarrow exponent = 18$

10	9		5	4		0
0		10010			10010	

3. (40 points) If this machine has an adder with a round bit, a guard bit, and a *sticky* bit, what is the value of $X + Y$ computed by this machine? Give both the normalized binary notation and the decimal value.

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         Mantissa   Guard Round    Sticky      exponent

X  =     1.00000      0      0        0           7

Y =      0.00011      0      0        1           7

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X+Y =    1.00011      0      0        1           7
```

With teh guard/round bits equal 00 we must round down:

$$
\begin{aligned}
X + Y &= 1.00011 \times 2^7 \\
X + Y &= 10001100 = 2^7 + 2^3 + 2^2 = 128 + 8 + 4 = 140
\end{aligned}
$$