Question 1 (20 points): In this question you will demonstrate that you understand how the data stored in caches is organized. Assume that each cache operates in a machine with a 32-bit virtual address.

- (**10 points**) For each of the cache configurations below, indicate how many bits are used for byte offset, for cache index, and for tag.

Cache capacity	Associativity	Size of Cache Block	Tag Size	Index Size	Byte offset size
64 KB	Direct mapped	128 bytes	16	9	7
32 KB	2-way	64 bytes	18	8	6
32 MB	8-way	512 bytes	10	13	9

$$\begin{aligned}
\#\text{ of blocks in cache} &= \frac{\text{cache capacity}}{\text{block size}} \\
\#\text{ of entries in cache} &= \frac{\#\text{ of blocks in cache}}{\text{associativity}} \\
\#\text{ of index bits} &= log_2(\#\text{ of entries in cache}) \\
\#\text{ of offset bits} &= log_2(\text{size of cache block}) \\
\#\text{of tag bits} &= 32 - \#\text{ of index bits} - \#\text{ of offset bits}
\end{aligned}$$

For first cache:

$$\begin{aligned}
\#\text{ of blocks in cache} &= \frac{64KB}{128} = \frac{2^{16}}{2^7} = 2^9 \\
\#\text{ of entries in cache} &= \frac{2^9}{1} = 2^9 \\
\#\text{ of index bits} &= log_2(2^9) = 9 \text{ bits} \\
\#\text{ of offset bits} &= log_2(128) = 7 \text{ bits} \\
\#\text{of tag bits} &= 32 - 9 - 7 = 16 \text{ bits}
\end{aligned}$$

For second cache:

$$\begin{aligned}
\#\text{ of blocks in cache} &= \frac{32KB}{64} = \frac{2^{15}}{2^6} = 2^9 \\
\#\text{ of entries in cache} &= \frac{2^9}{2} = 2^8 \\
\#\text{ of index bits} &= log_2(2^8) = 8 \text{ bits} \\
\#\text{ of offset bits} &= log_2(64) = 6 \text{ bits} \\
\#\text{of tag bits} &= 32 - 8 - 6 = 18 \text{ bits}
\end{aligned}$$

For third cache:

$$\begin{aligned} \#\text{ of blocks in cache} &= \frac{32MB}{512} = \frac{2^{25}}{2^9} = 2^{16} \\ \#\text{ of entries in cache} &= \frac{2^{16}}{8} = 2^{13} \\ \#\text{ of index bits} &= log_2(2^{13}) = 13 \text{ bits} \\ \#\text{ of offset bits} &= log_2(512) = 9 \text{ bits} \\ \#\text{of tag bits} &= 32 - 13 - 9 = 10 \text{ bits} \end{aligned}$$

- **10 points** The table below contains the same cache organizations listed in the table above. Using binary notation, provide the value of the tag and the value of the cache index for a memory reference to the address 0xABCD EF78. Also indicate which word within the block is being accessed. The word at offset 0 is word #0, the word at offset 4 is word #1, etc.

Cache capacity	Associativity	Cache Block	Tag	Index	word accessed
64 KB	Dir. map.	128 bytes	1010 1011 1100 1101	1110 1111 0	30
32 KB	2-way	64 bytes	1010 1011 1100 1101 11	1011 1101	14
32 MB	8-way	512 bytes	1010 1011 11	00 1101 1110 111	94

64KB, Direct, 128 bytes:

A	B	C	D	E	F	7	8
1010	1011	1100	1101	1110	1111	0 111	1000
tag				index		offset	

32KB, 2-way, 64 bytes:

A	B	C	D	E	F	7	8
1010	1011	1100	1101	11 10	1111	01 11	1000
tag				index		offset	

32MB, 8-way, 512 bytes:

A	B	C	D	E	F	7	8
1010	1011	11 00	1101	1110	111 1	0111	1000
tag		index				offset	