

CMPUT 229 - Quiz #4 — Winter 2014

Name: Solution

The following is a format for the binary representation of a floating-point number:

10	9		5	4		0
S	exponent				fraction	

The exponent is expressed using a bias of 15. Given the binary representation above, the decimal value of the number represented can be computed by the following expression:

$$N = \begin{cases} 0.0 & \text{if } \text{exponent} = 0 \text{ and } \text{fraction} = 0 \\ (-1)^S \times 0.\text{fraction} \times 2^{-15} & \text{if } \text{exponent} = 0 \text{ and } \text{fraction} \neq 0 \\ (-1)^S \times 1.\text{fraction} \times 2^{-15} & \text{if } 0 < \text{exponent} < 31 \\ (-1)^S \times \infty & \text{if } \text{exponent} = 31 \text{ and } \text{fraction} = 0 \\ NaN & \text{if } \text{exponent} = 31 \text{ and } \text{fraction} \neq 0 \end{cases}$$

Question 1 (3 points):

Let $X = 32_{10}$. Give the normalized binary representation for X and the bit pattern representation of X in this notation.

$$32_{10} = 100000_2 \Rightarrow 1.0 \times 2^5$$

$$\begin{aligned} \text{Sign} &= 0 \\ \text{Exponent} &= 5 + \text{bias} = 20 \Rightarrow 10100 \\ \text{Fraction} &= 00000 \end{aligned}$$

$$0 \ 10100 \ 00000$$

Question 2 (3 points):

Let $Y = 6.75_{10}$. Give the normalized binary representation for Y and the bit pattern representation of Y in this notation.

$$6.75_{10} = 110.11_2 \Rightarrow 1.1011 \times 2^2$$

$$\begin{aligned} \text{Sign} &= 0 \\ \text{Exponent} &= 2 + \text{bias} = 17 \Rightarrow 10001 \\ \text{Fraction} &= 10110 \end{aligned}$$

$$0 \ 10001 \ 10110$$

Question 3 (4 points): If this machine has an adder with a round bit, a guard bit and a sticky bit, what is the value of $X + Y$ computed by this machine? Give both the normalized binary notation and the decimal value.

$$\begin{array}{r} 1.00000 \\ + 0.0011011 \\ \hline 1.00110 \end{array} \times 2^5 \quad \left. \begin{array}{l} \text{guard} = 1 \\ \text{round} = 1 \\ \text{sticky} = 0 \end{array} \right\} \text{round}$$

$$1.00111 \times 2^5$$

$$100111_2 = 32_{10} + 4_{10} + 2_{10} + 1_{10} = 39_{10}$$