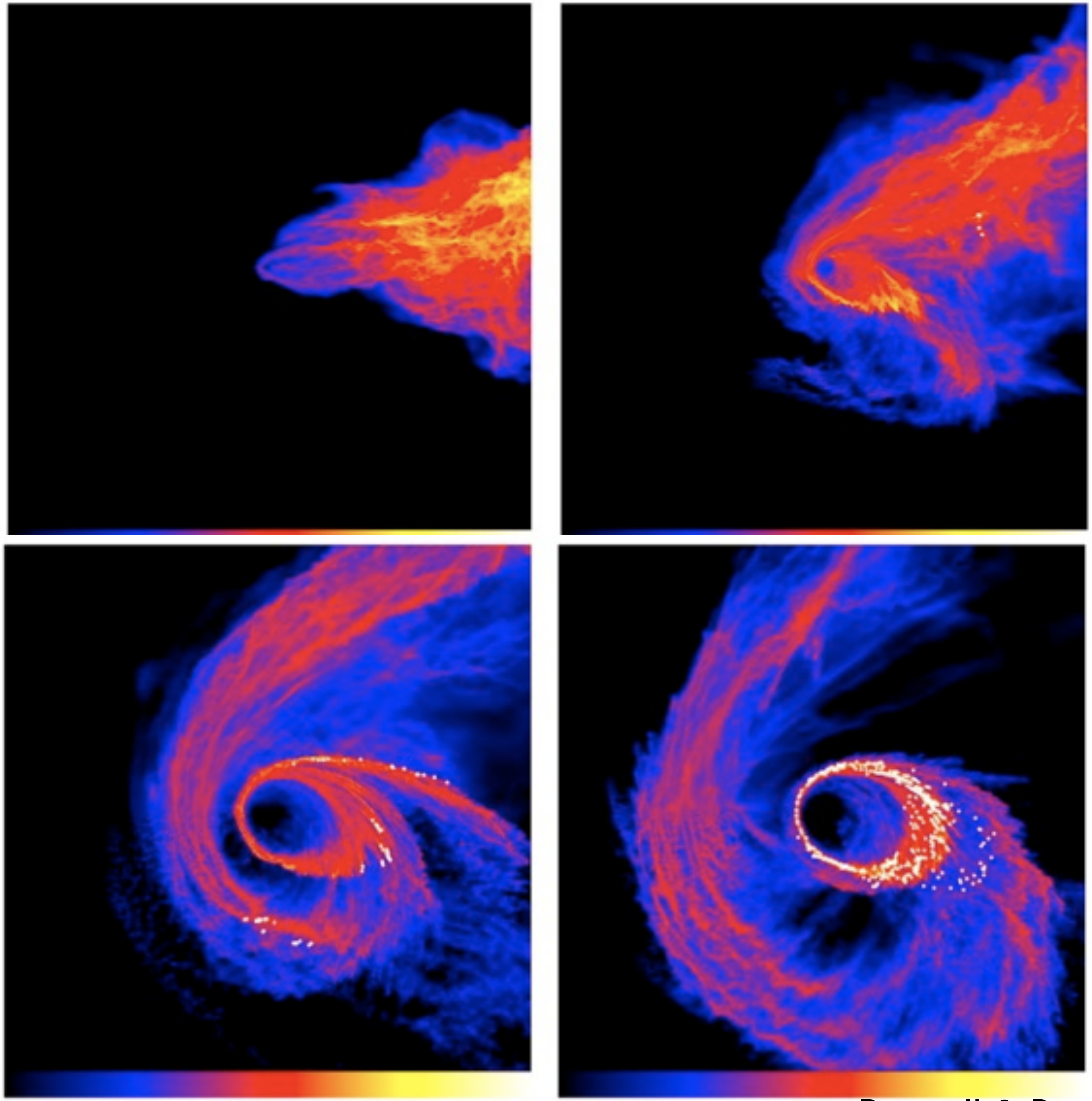
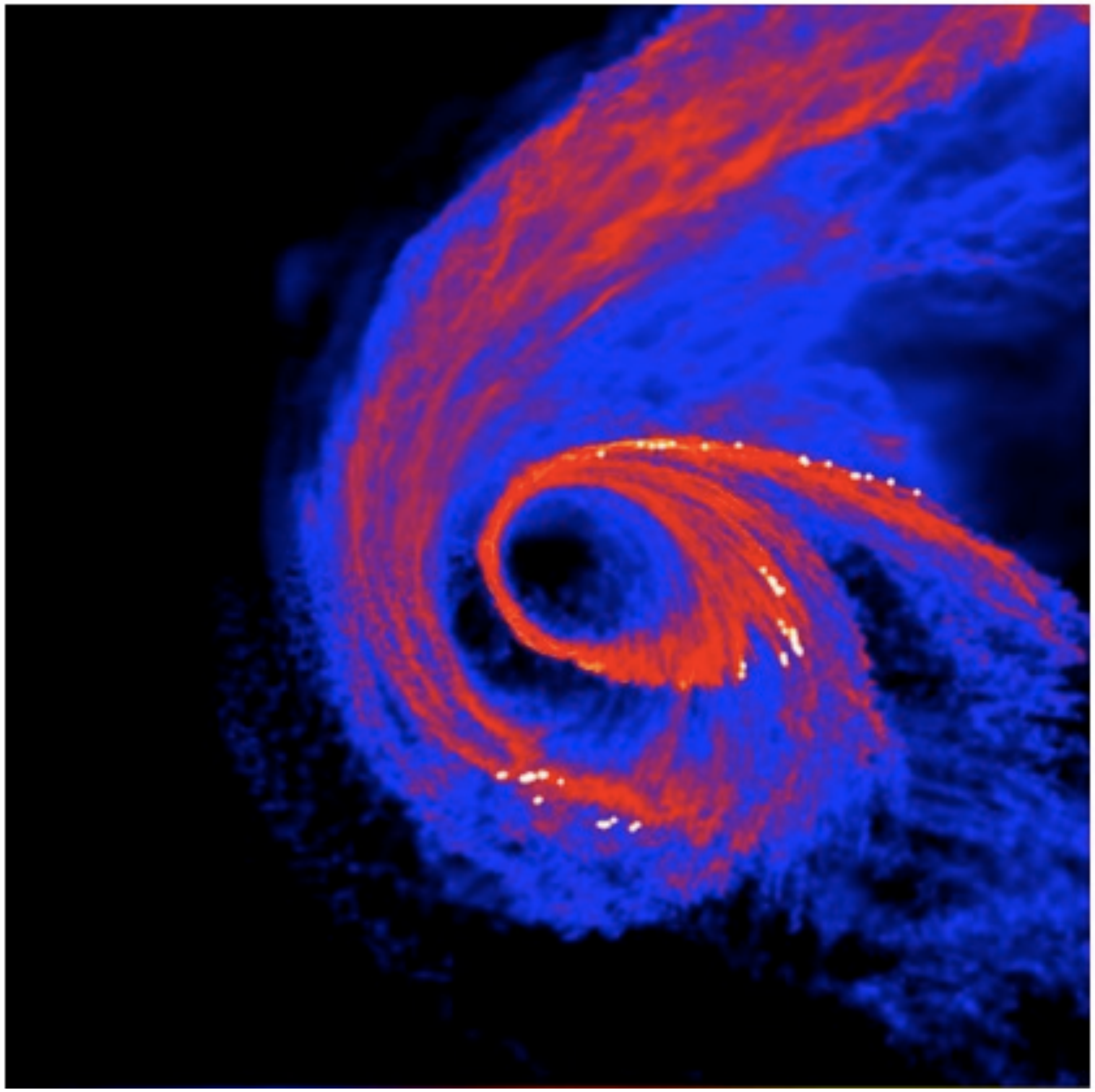
Developments in Particle Weighted Methods

Evghenii Gaburov
Sterrewacht Leiden

Motivation



Bonnell & Rice, Science 2008



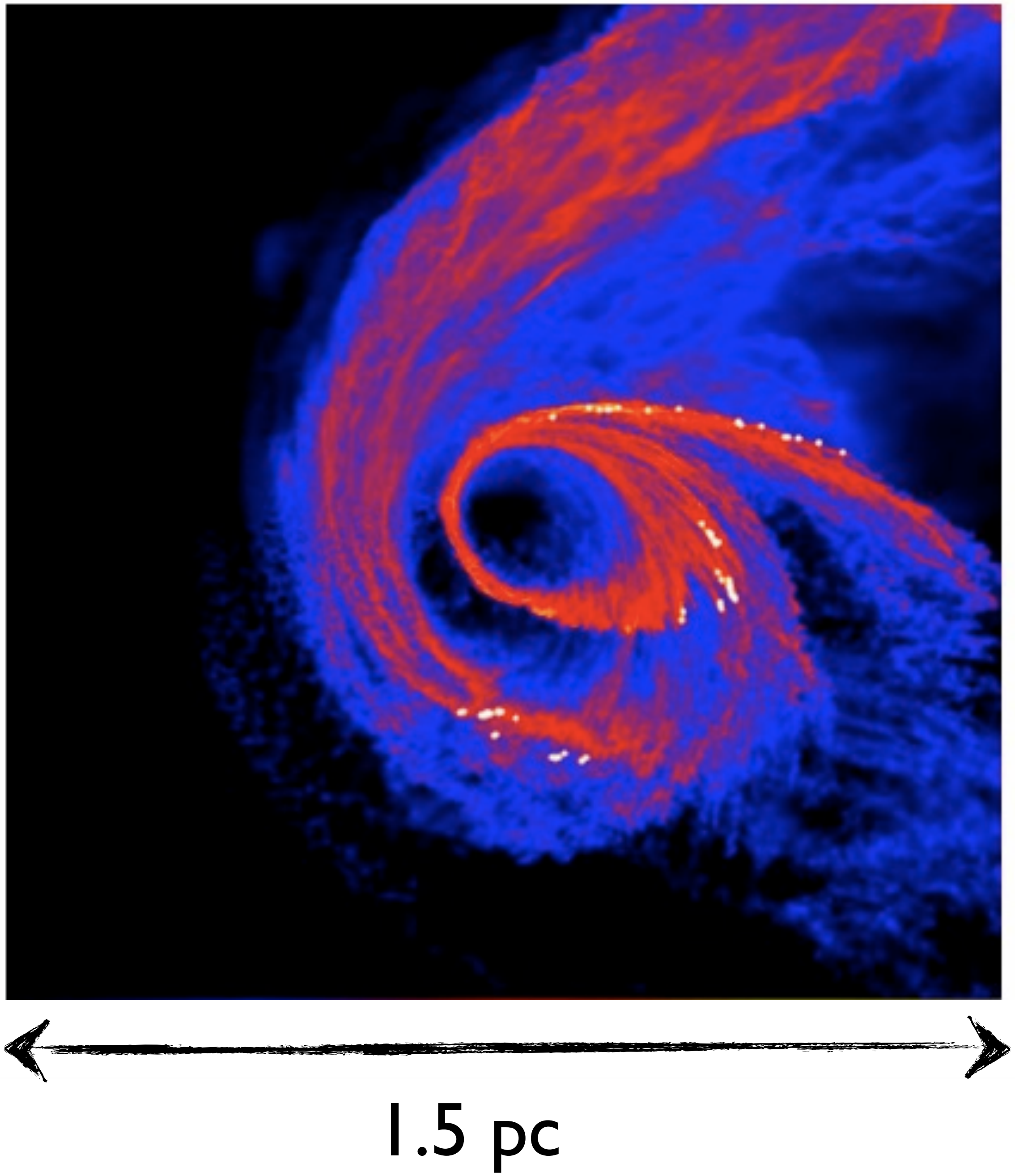
1.5 pc

$T < 100K$: $c_{snd} < 1$ km/s

$v_{orb} > 100$ km/s

$M_{adv} = v_{orb}/c_{snd} > 10^2$

Bonnell & Rice, Science 2008



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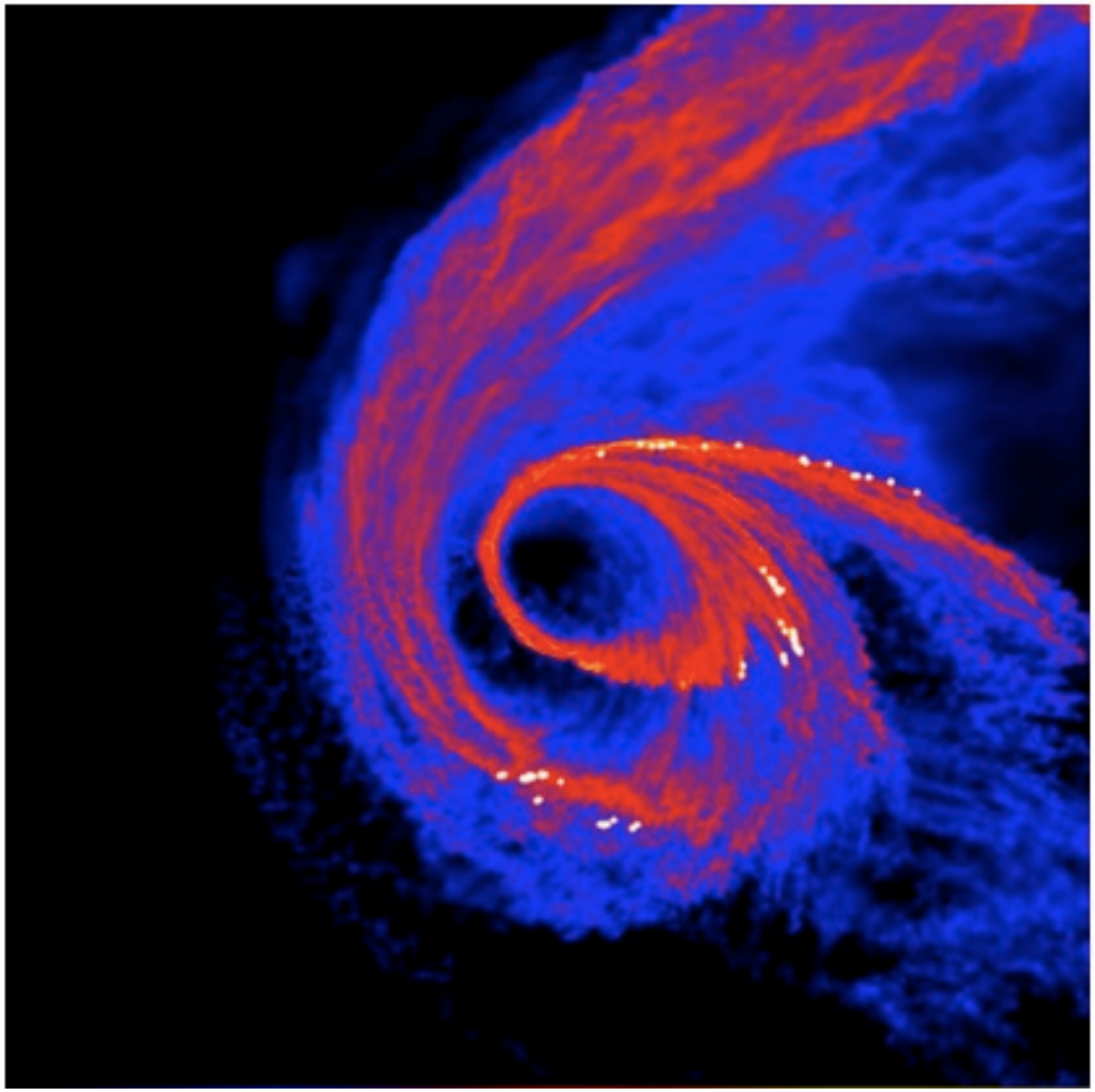
Problem for grid-codes!

Bonnell & Rice, Science 2008



1.5 pc

$T < 100K$: $c_{snd} < 1$ km/s

$v_{orb} > 100$ km/s

$M_{adv} = v_{orb}/c_{snd} > 10^2$

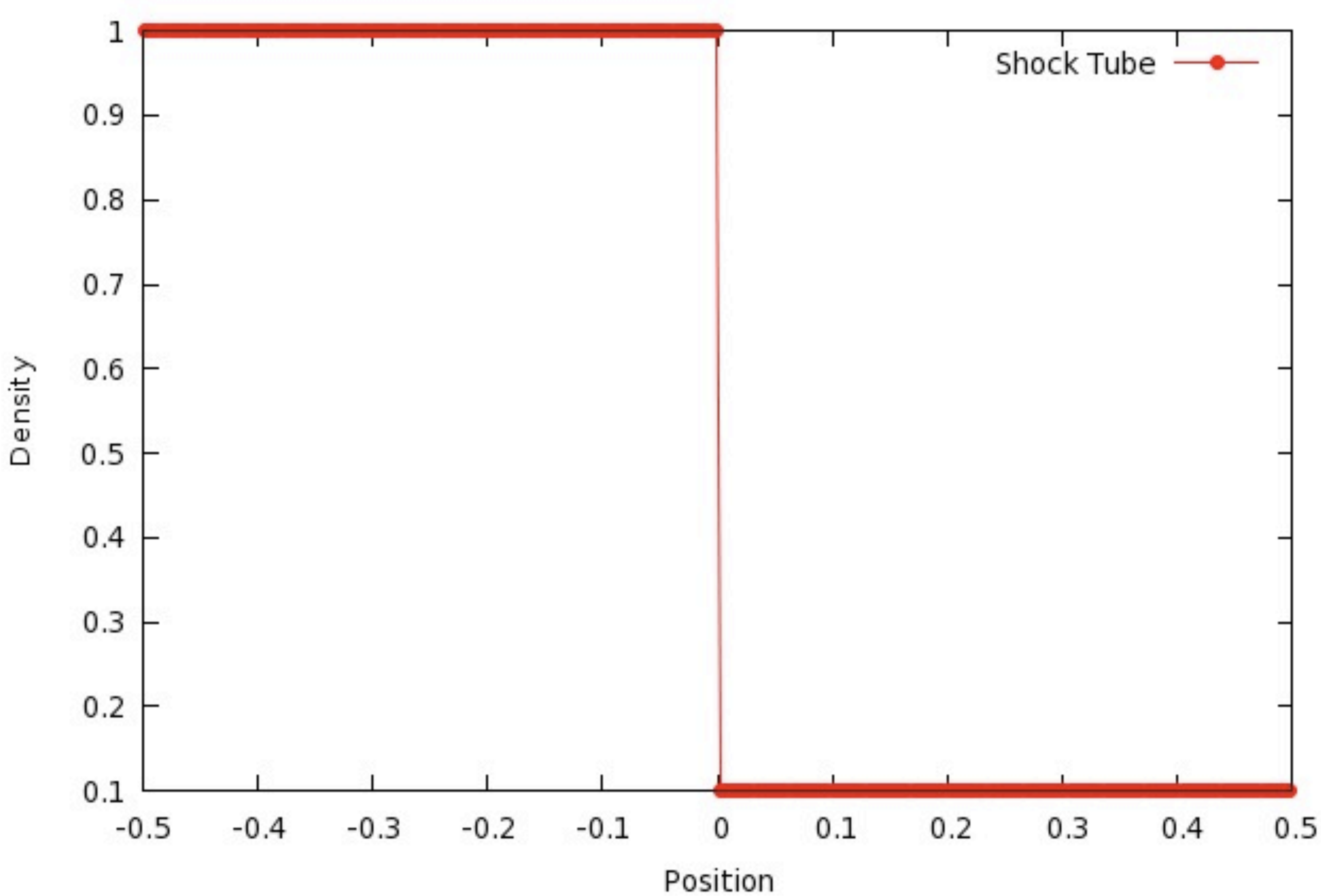
Problem for grid-codes!

$E_{th} = E_{tot} - E_{kin}$

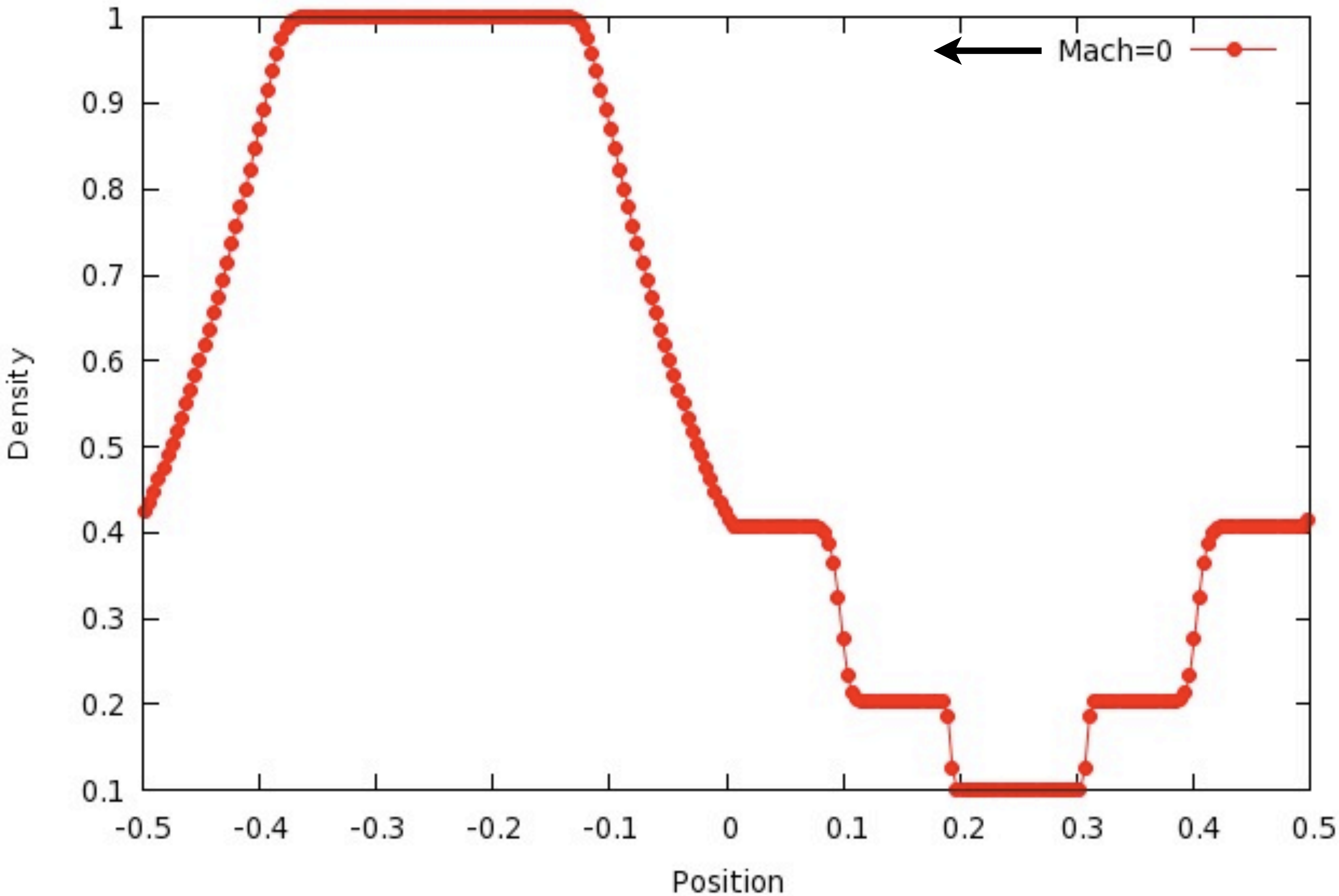
$c_{snd}^2 << E_{tot} - v_{orb}^2$

Bonnell & Rice, Science 2008

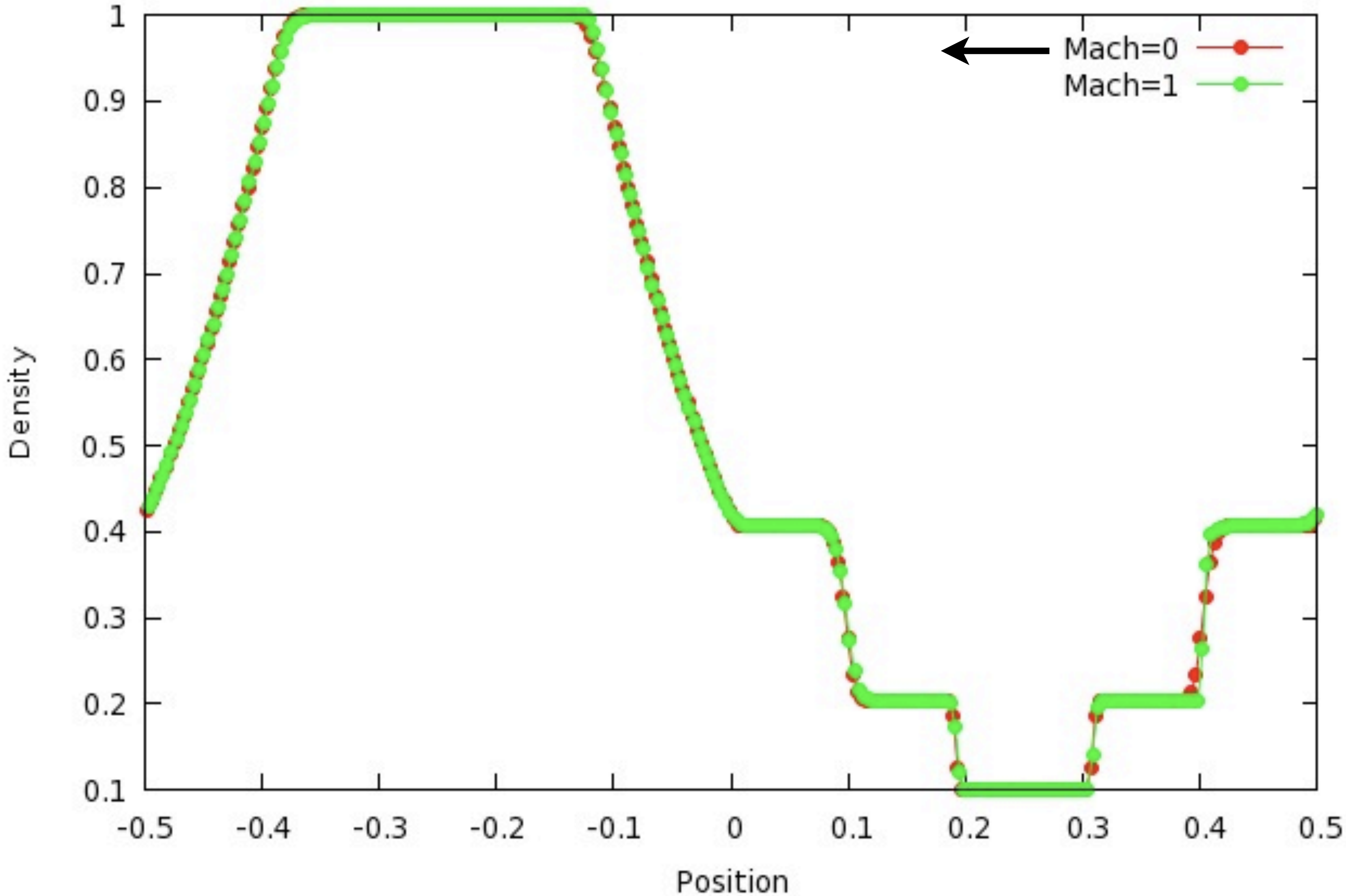
Examples



Computed with ATHENA code



Computed with ATHENA code



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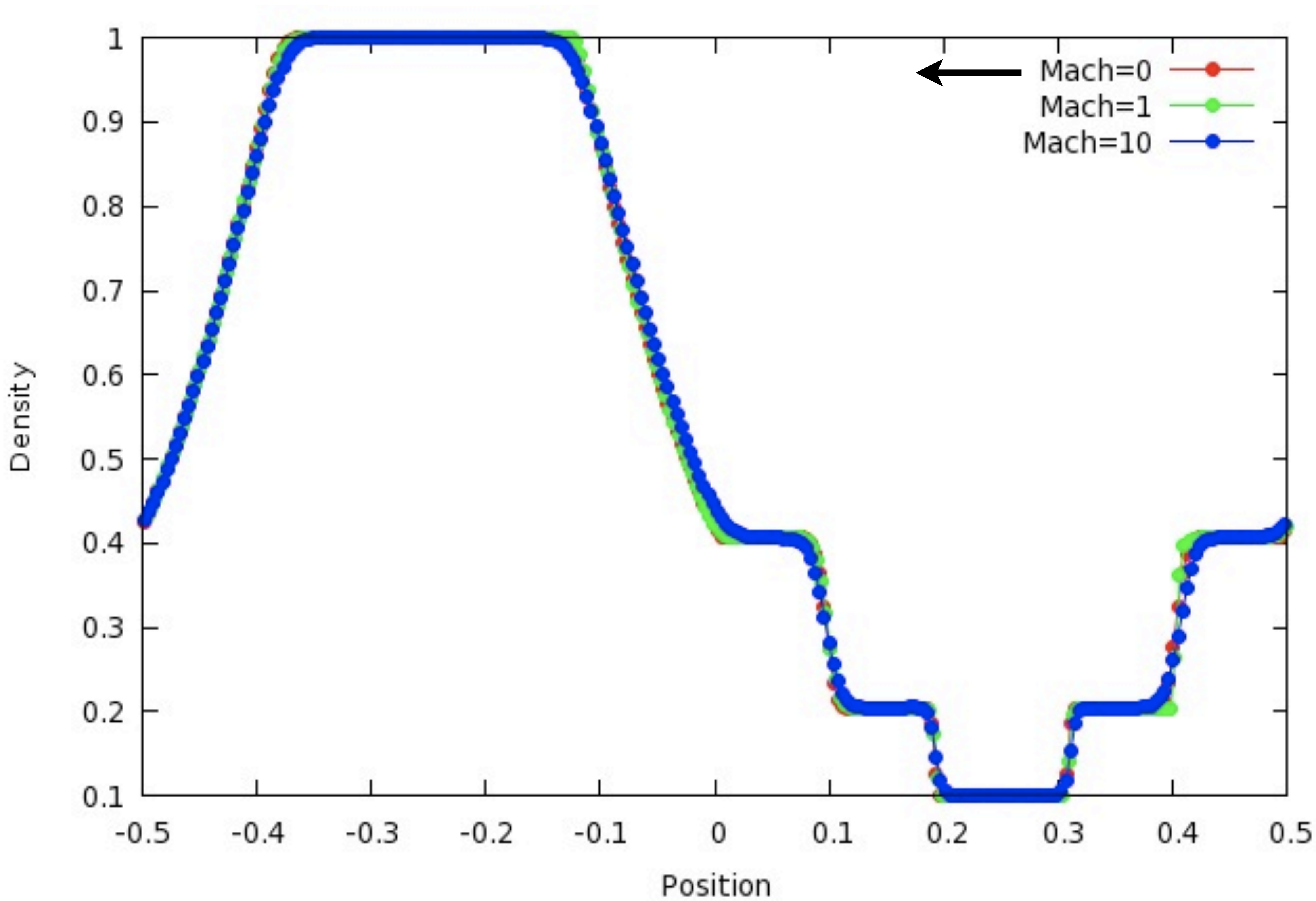
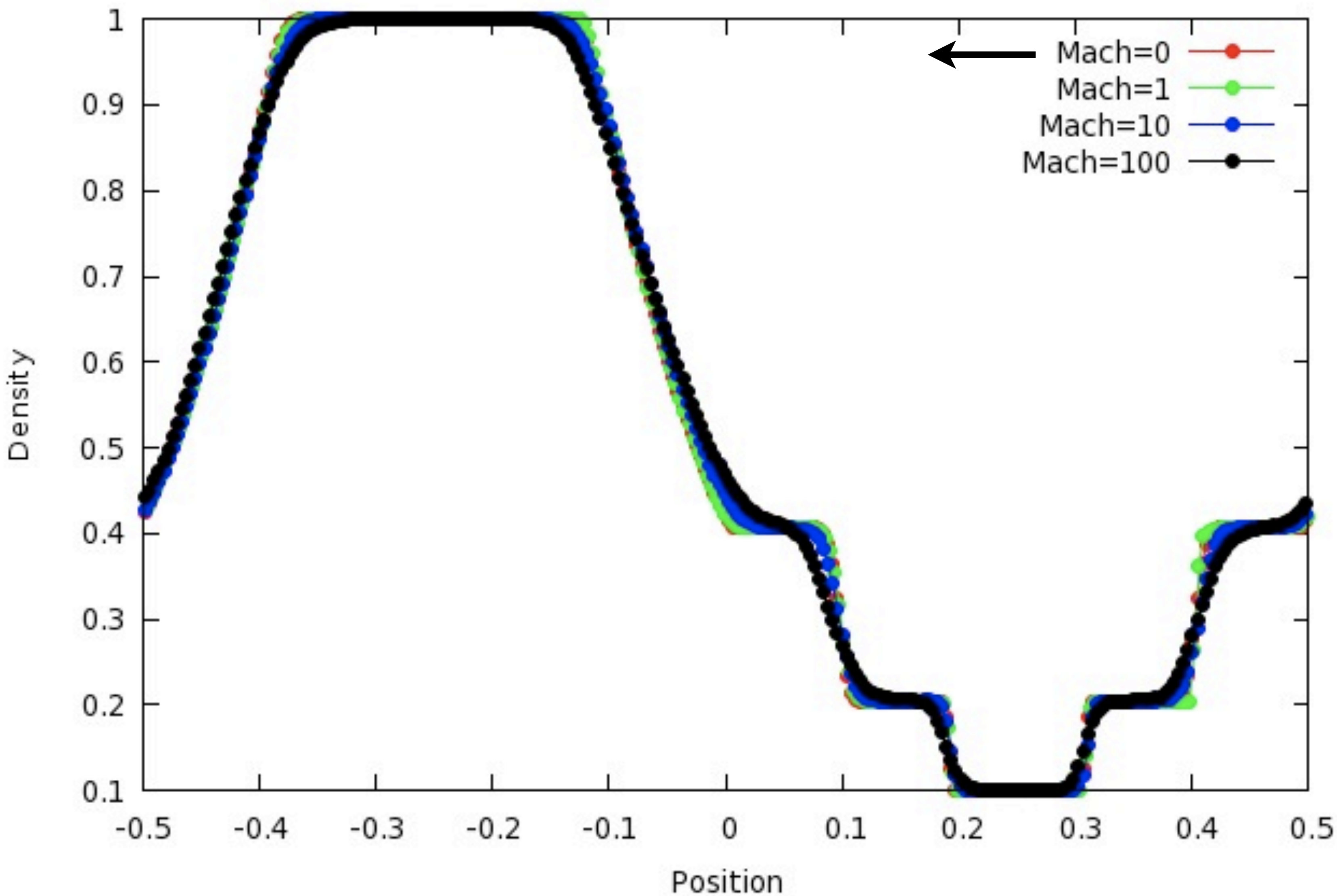
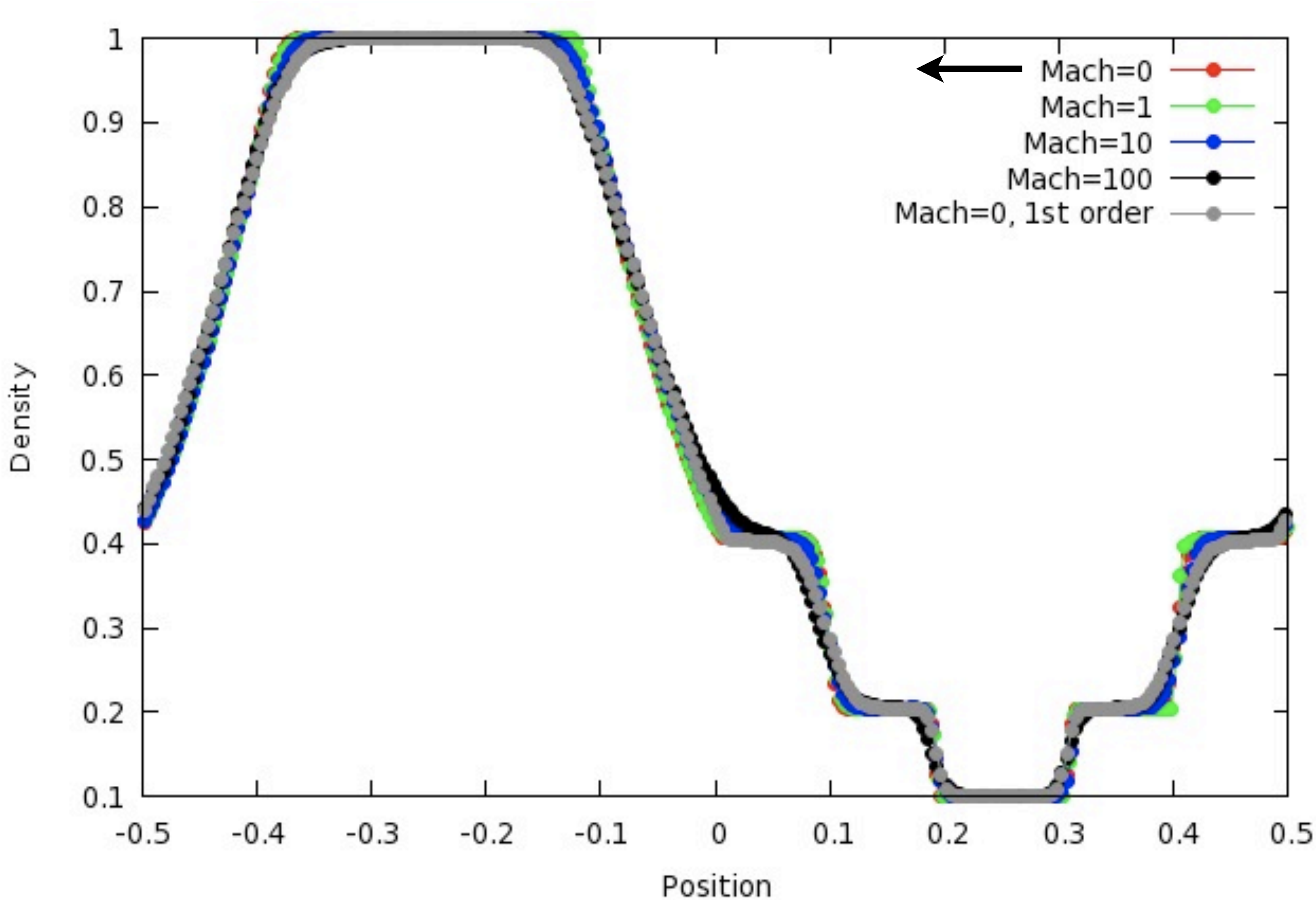


Computed with ATHENA code

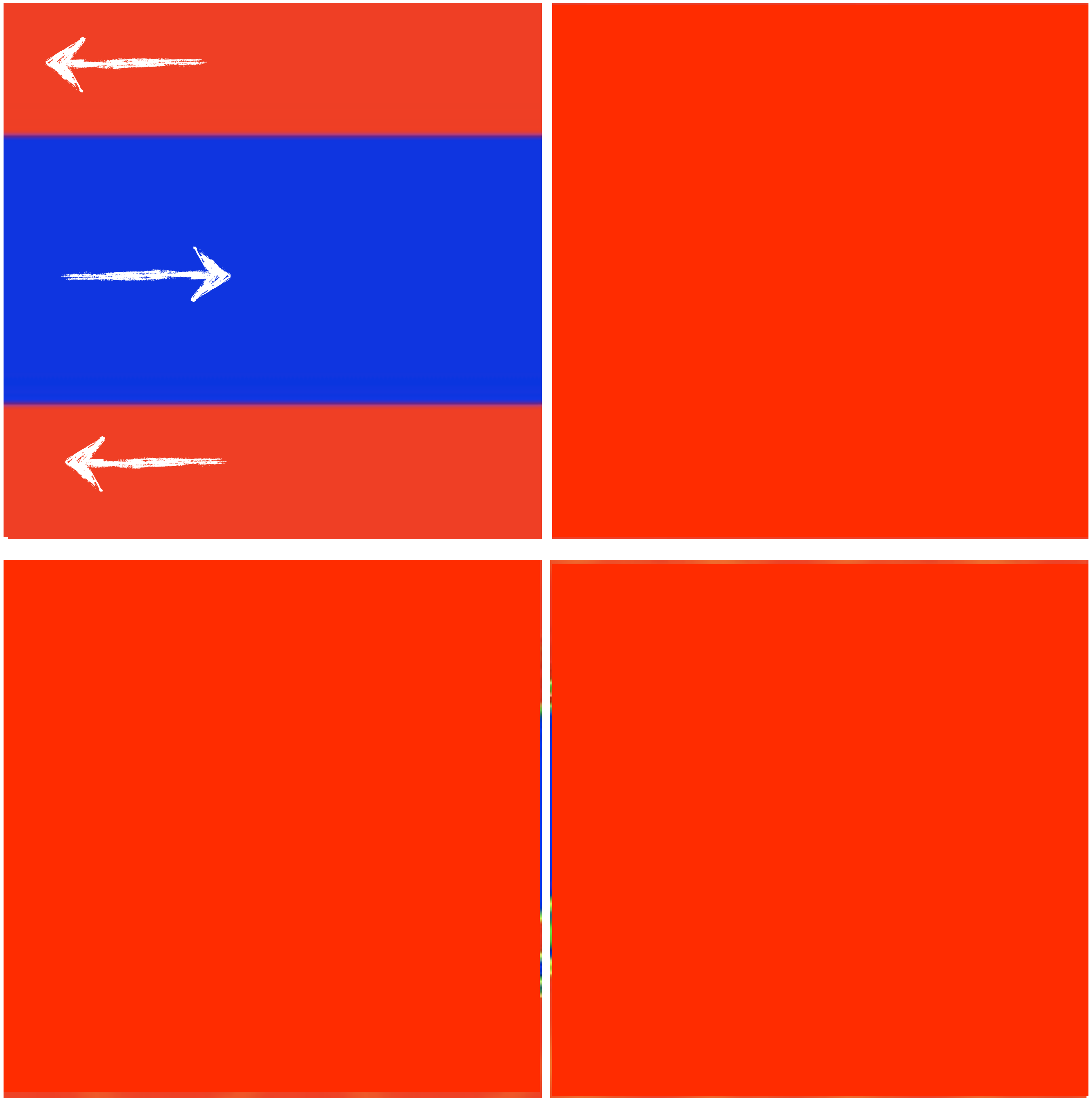


Computed with ATHENA code

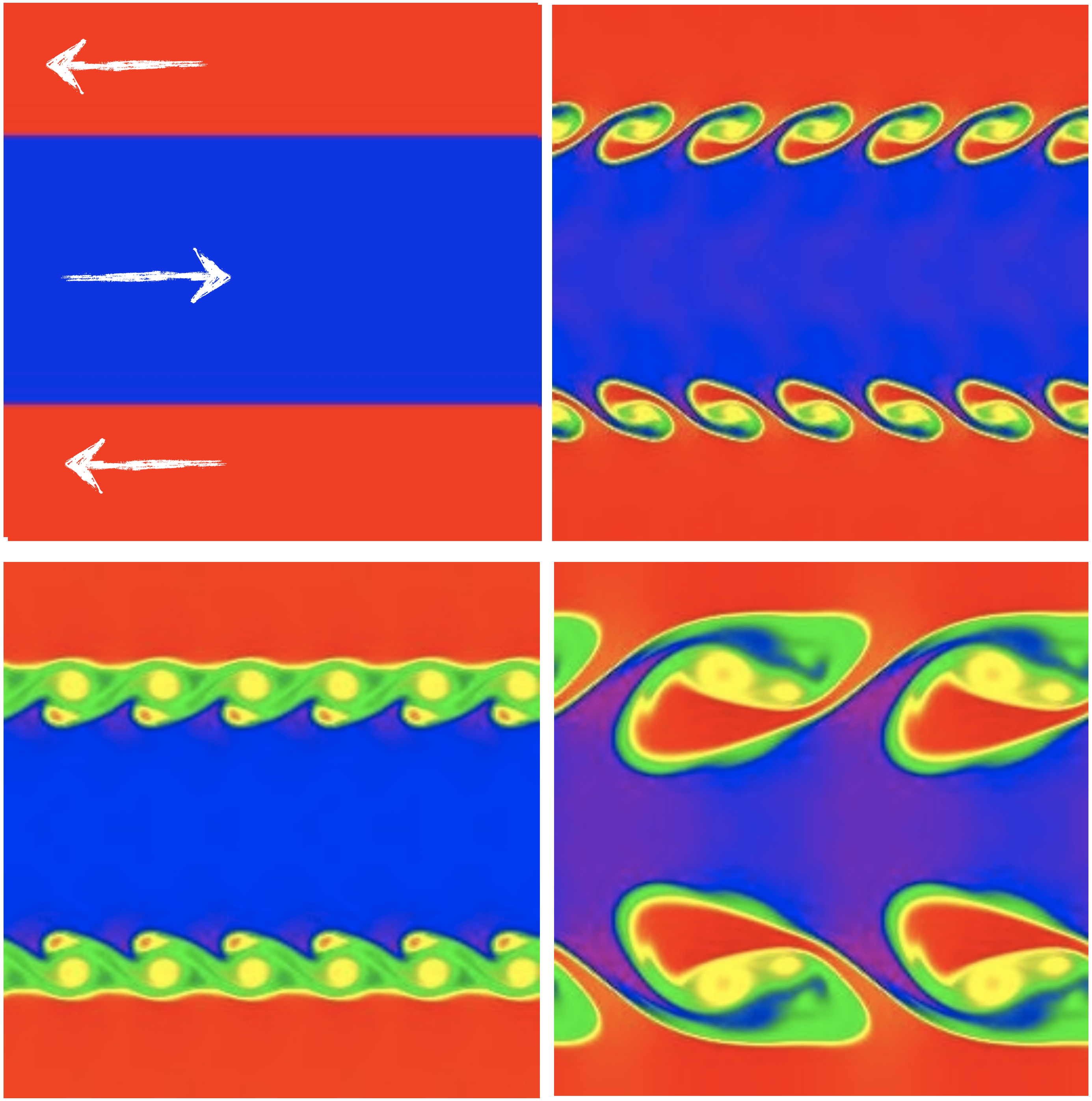


Computed with ATHENA code

KH Instability

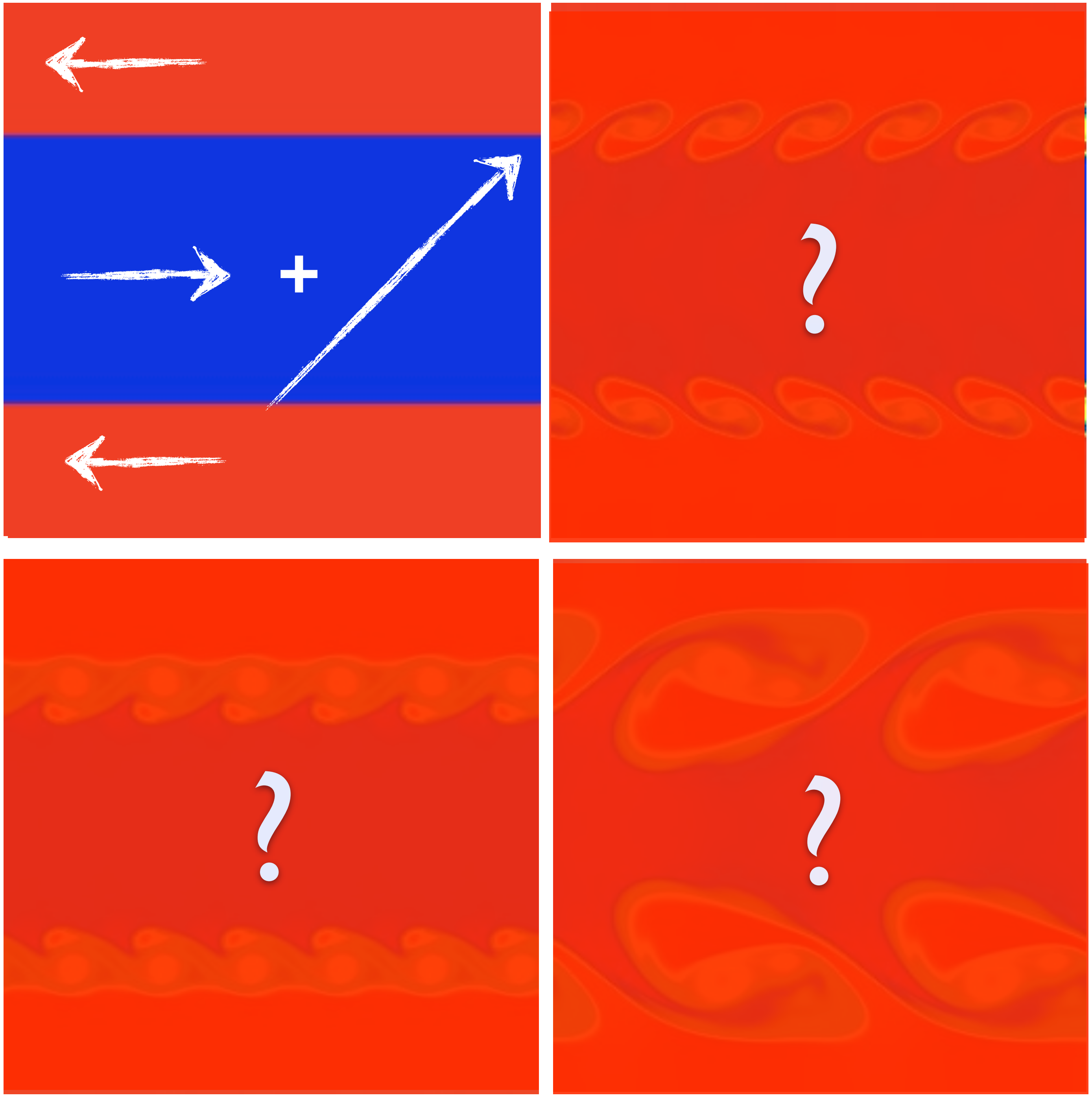


Computed with ATHENA code



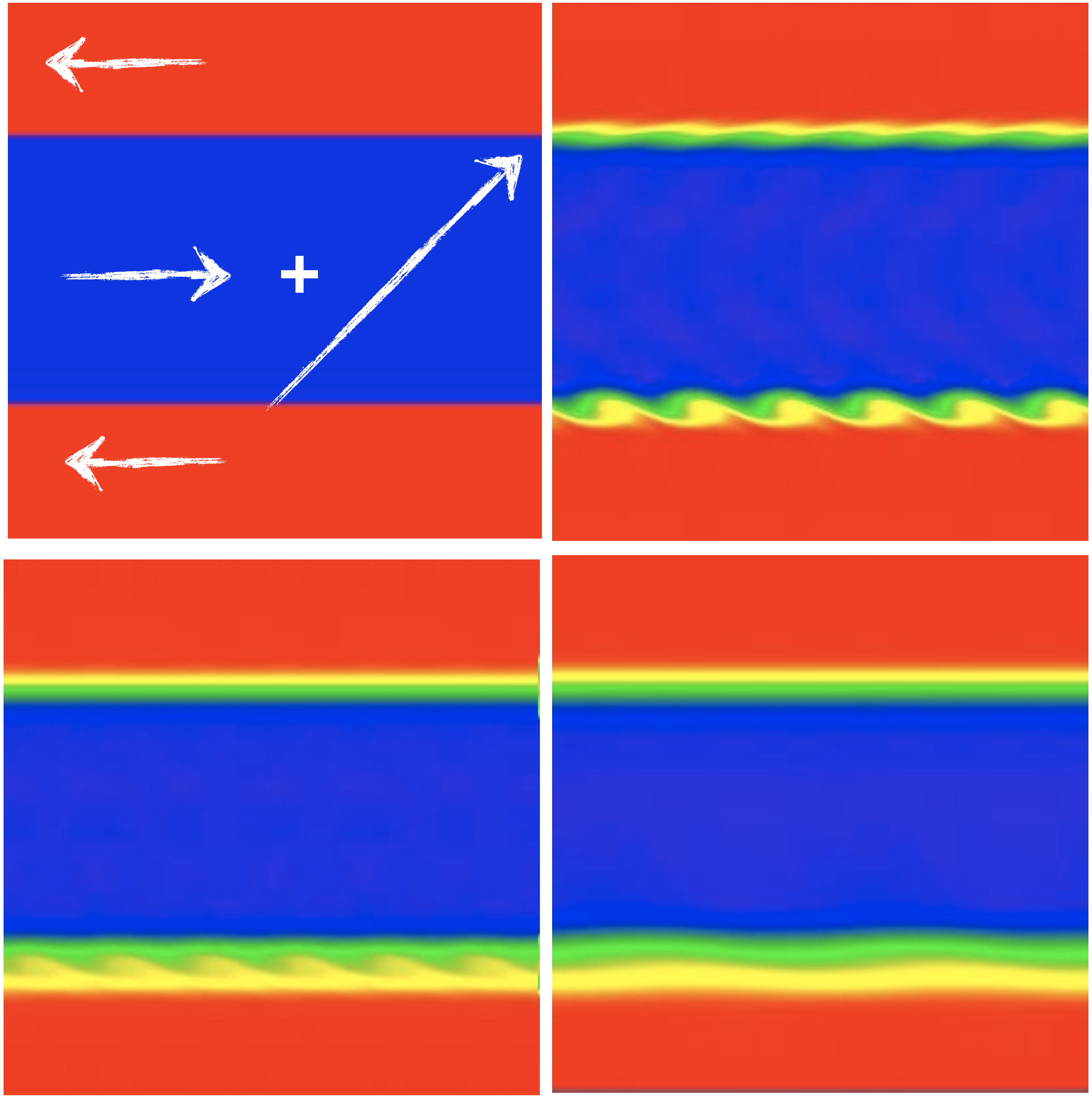
Computed with ATHENA code

Mach 10 advection



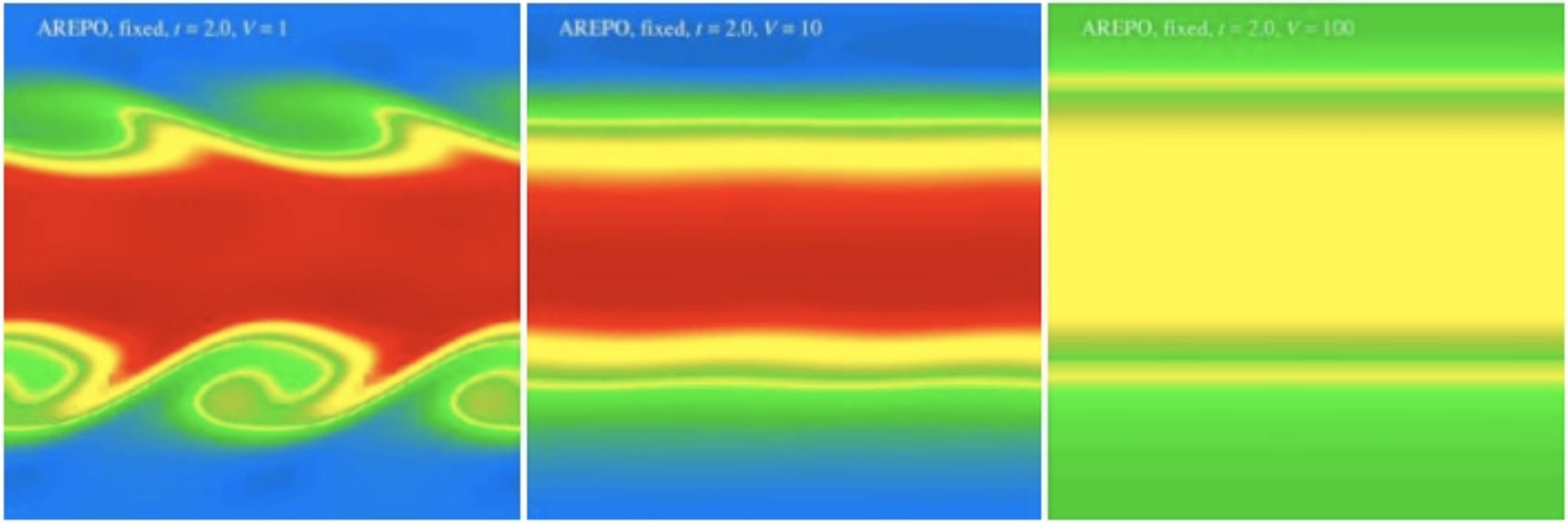
Computed with ATHENA code

Mach 10 advection



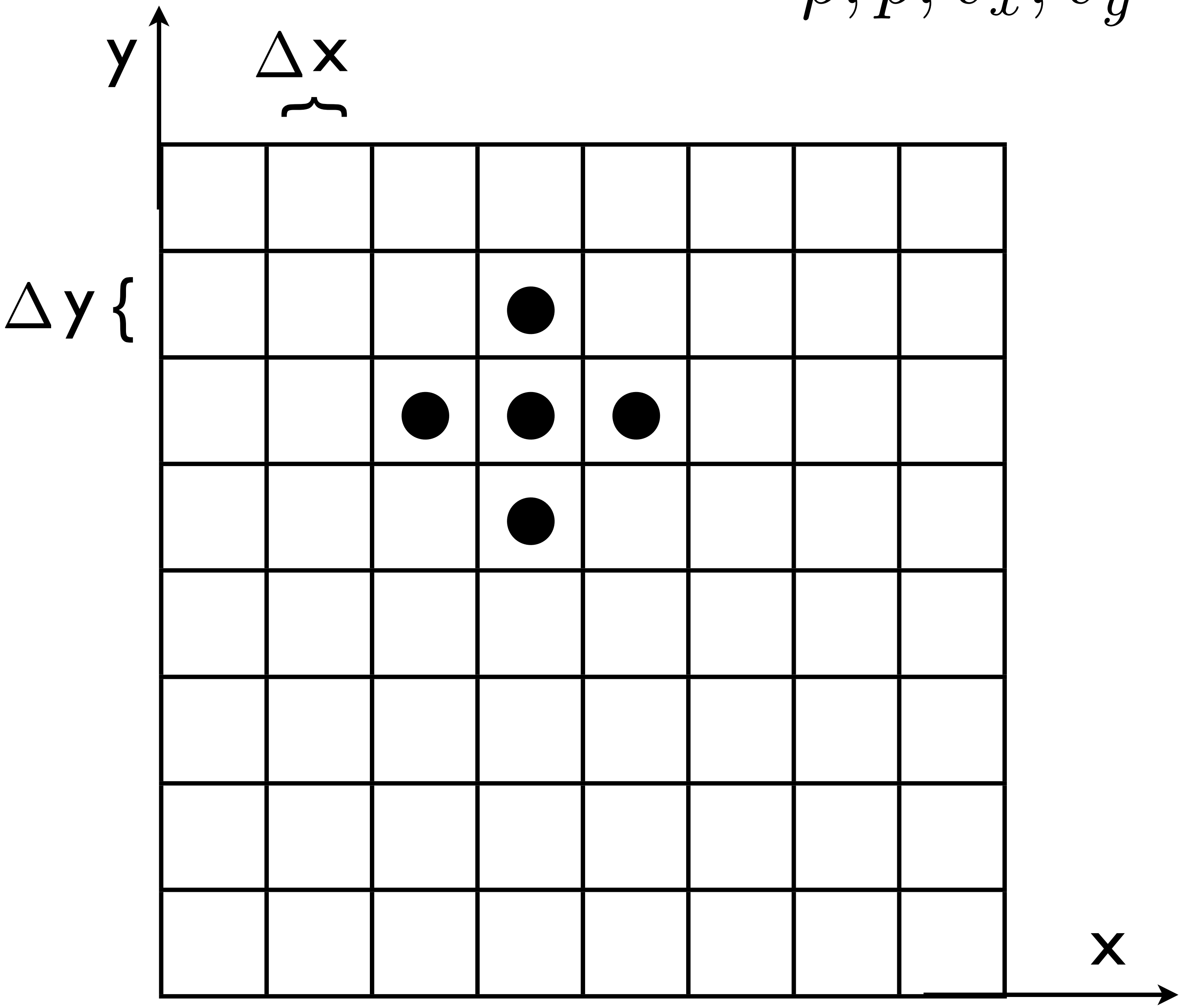
Computed with ATHENA code

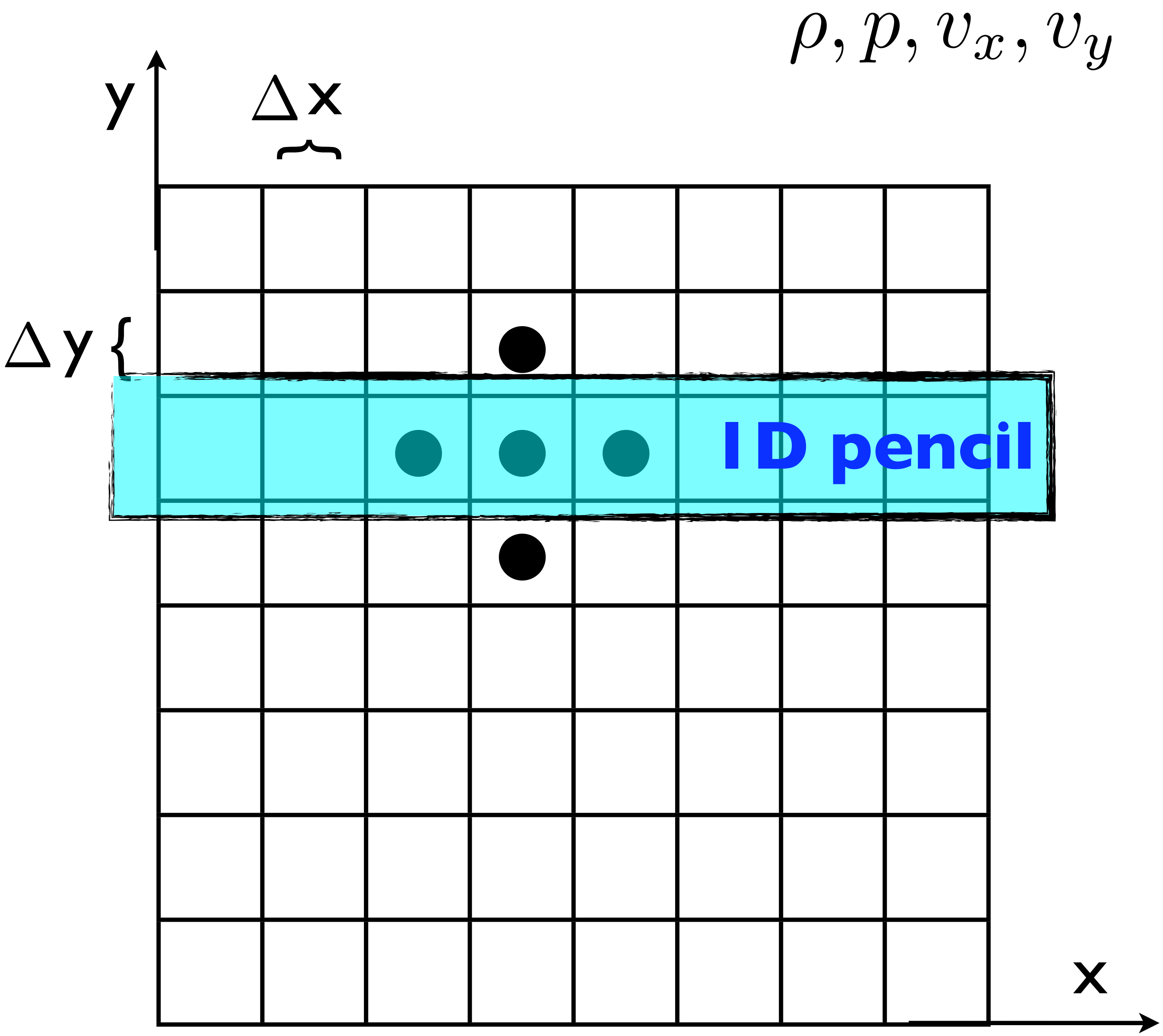
M ~ 1 M ~ 10 M ~ 100



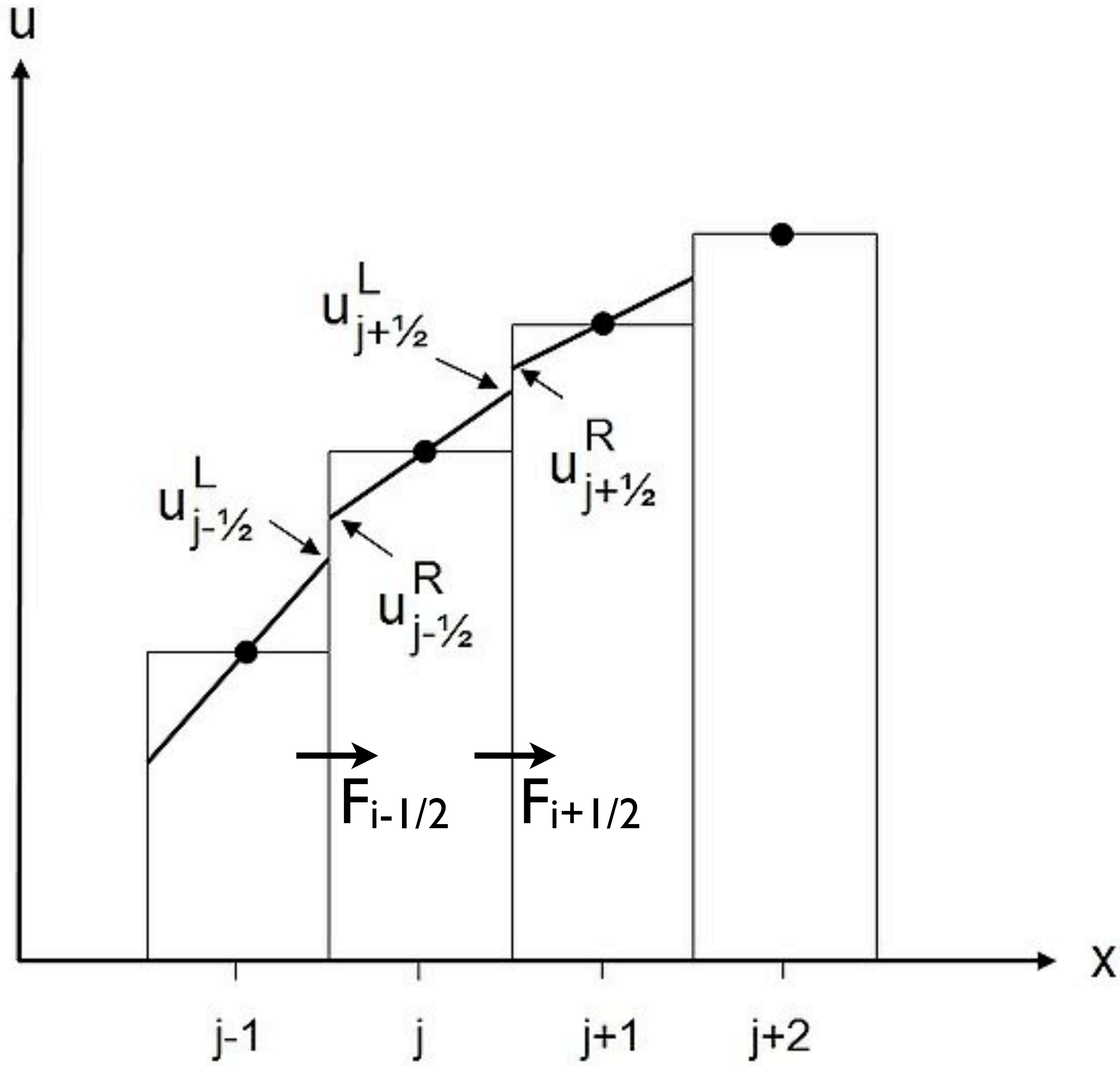
Springel, arXiv:0901.4107

$$\rho, p, v_x, v_y$$



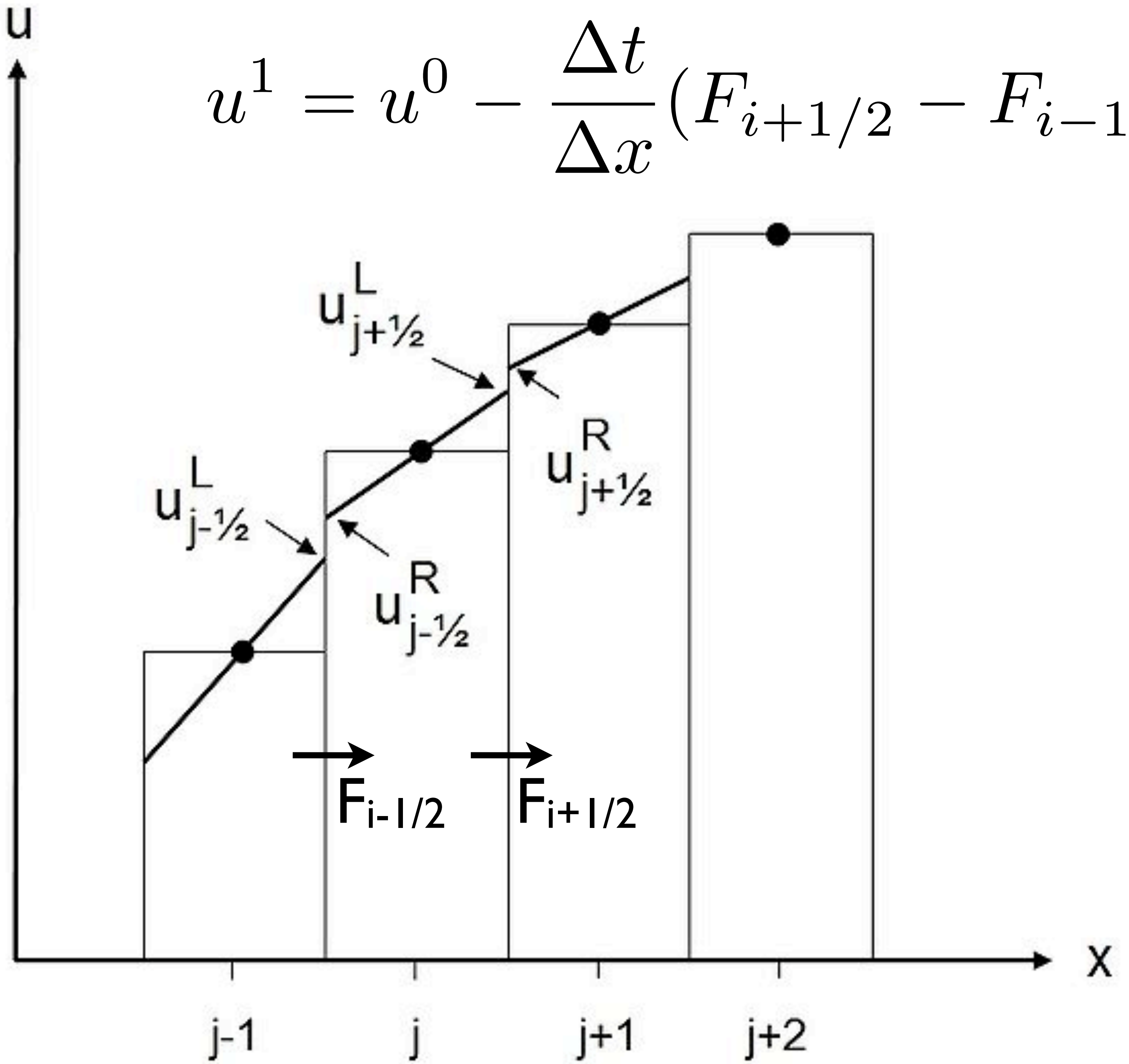

ρ, p, v_x, v_y
y
Δx
Δy
1D pencil
x

Subgrid model: Linear reconstruction

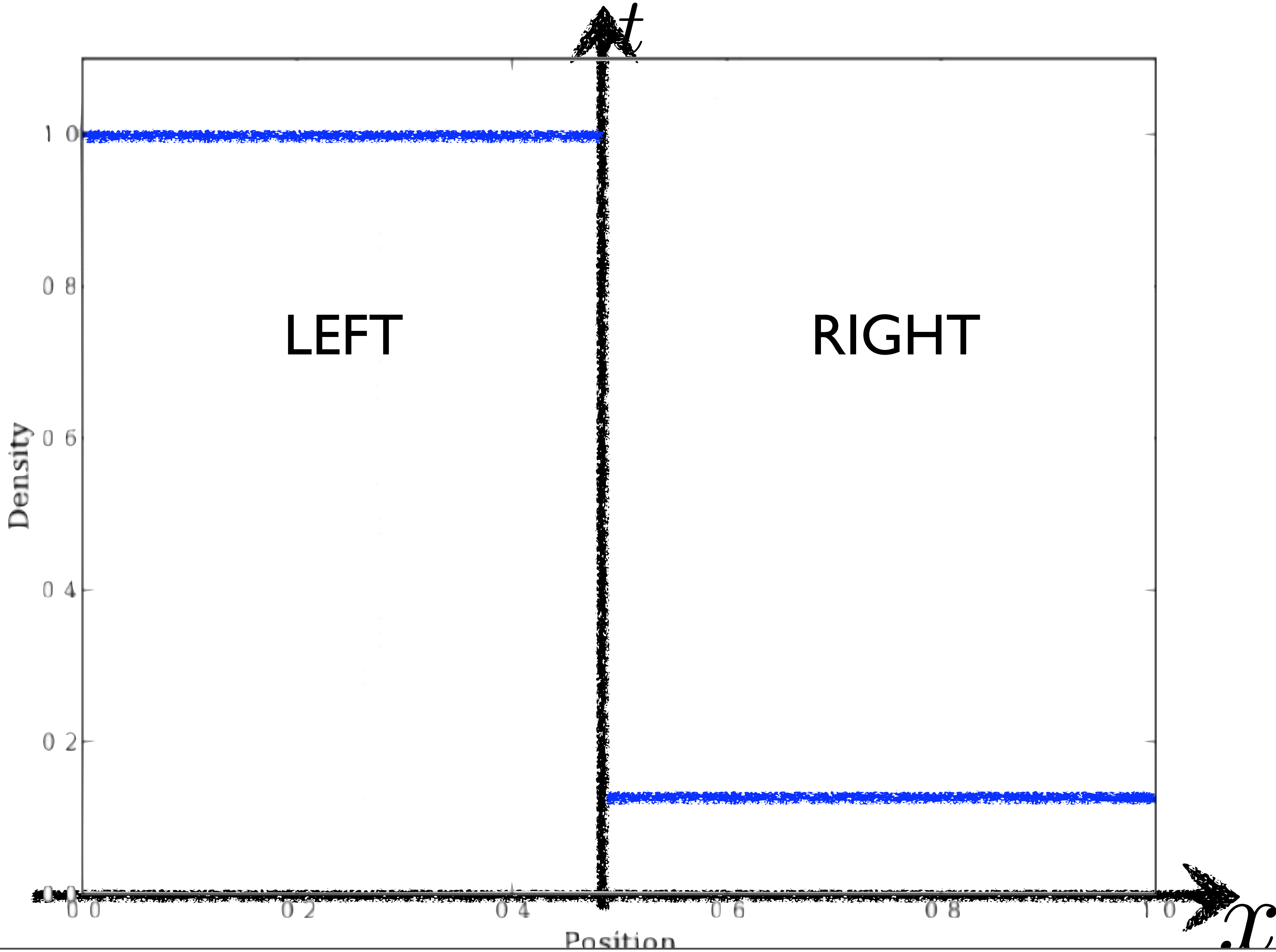


Subgrid model: Linear reconstruction

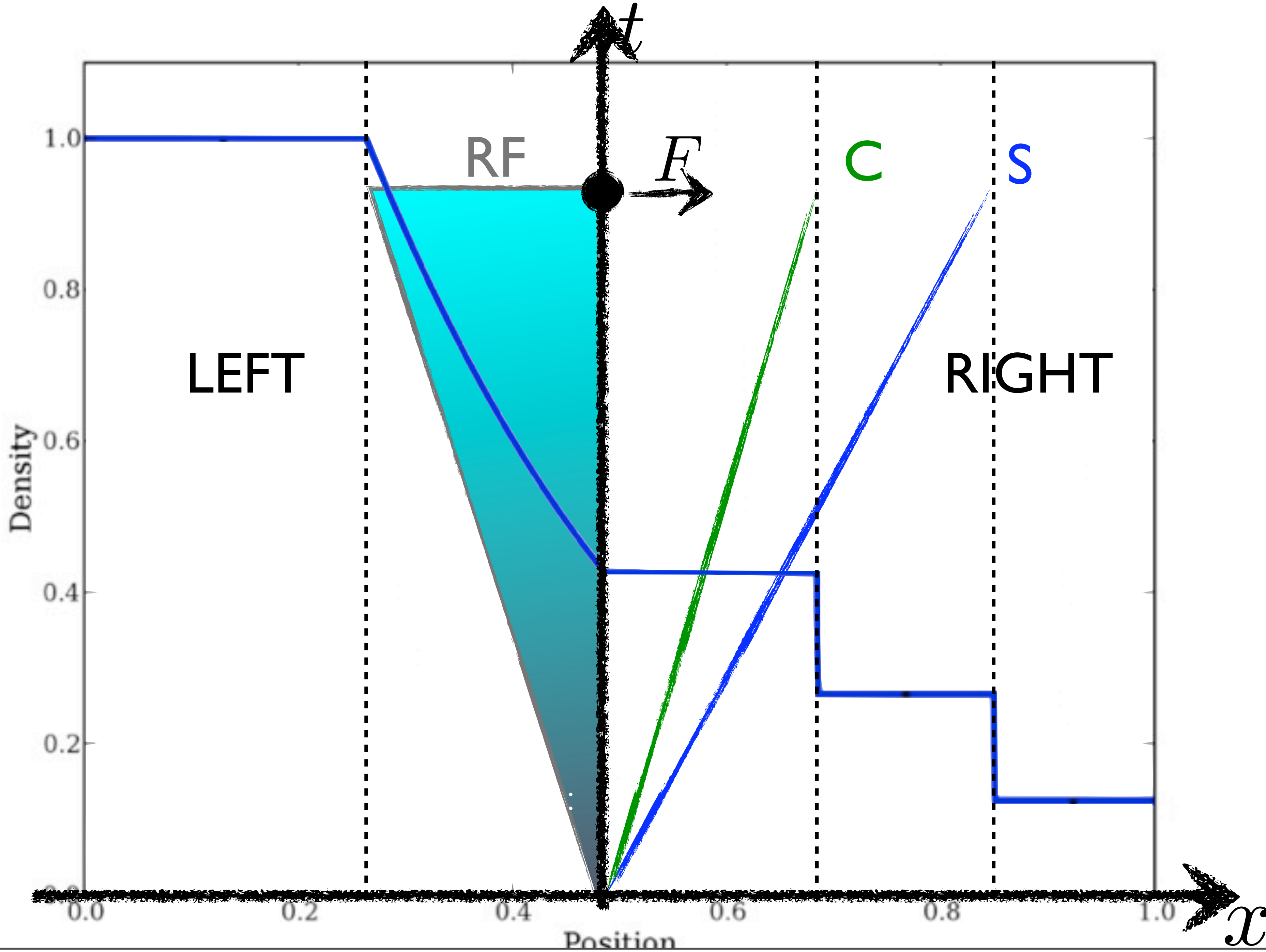
$$u^1 = u^0 - \frac{\Delta t}{\Delta x}(F_{i+1/2} - F_{i-1/2})$$



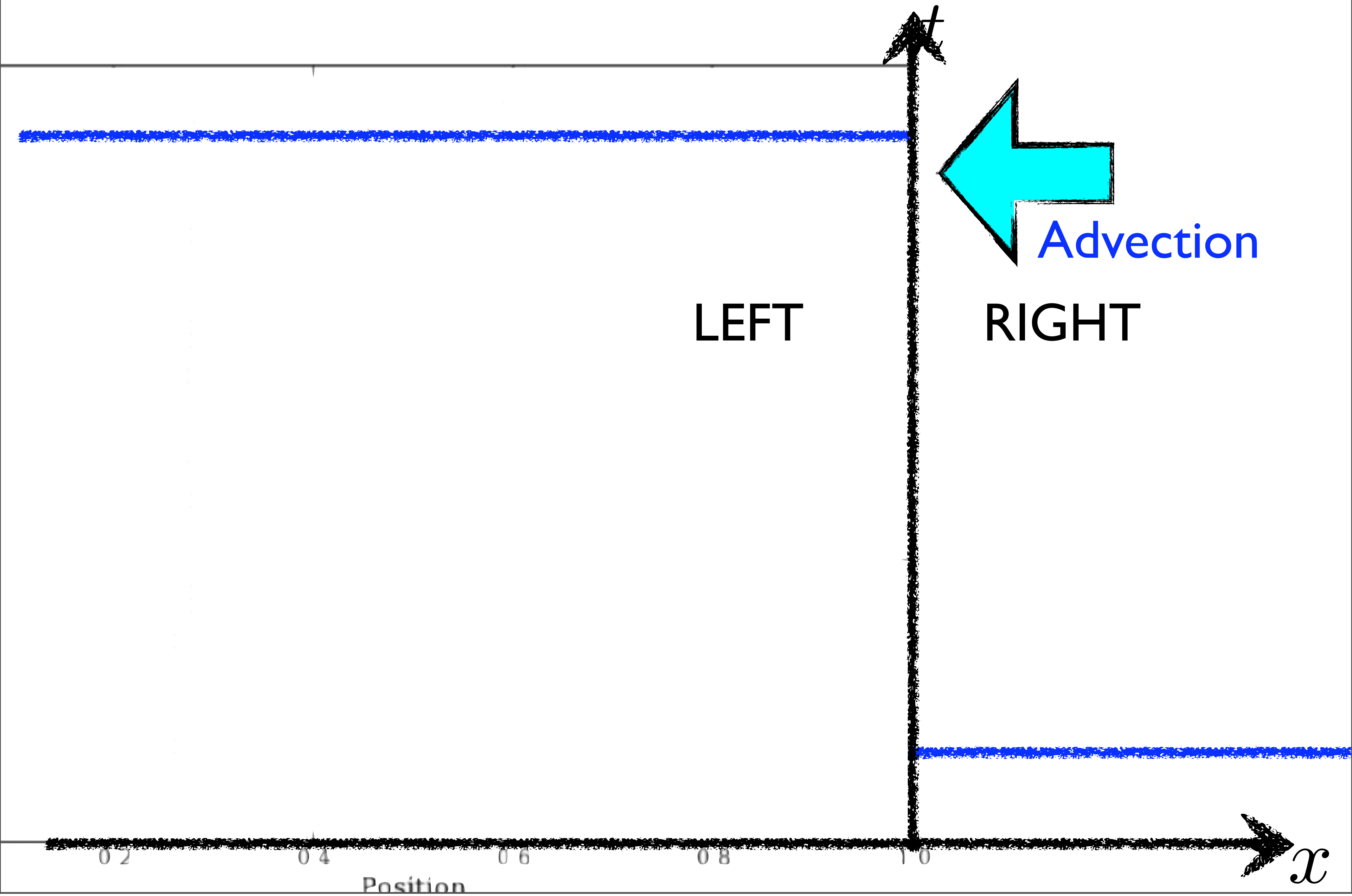
Flux computations: Riemann problem



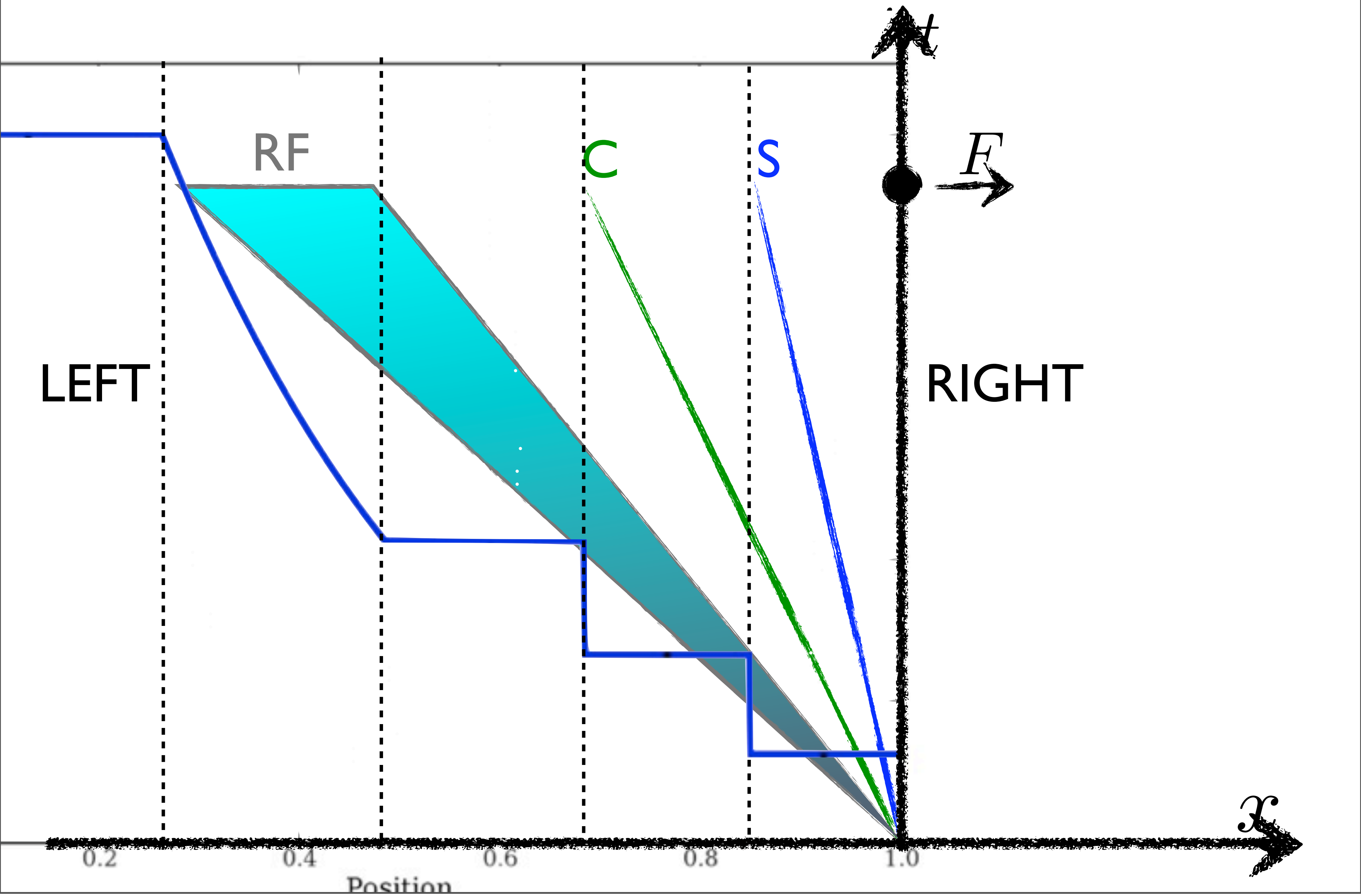
Flux computations: lab frame

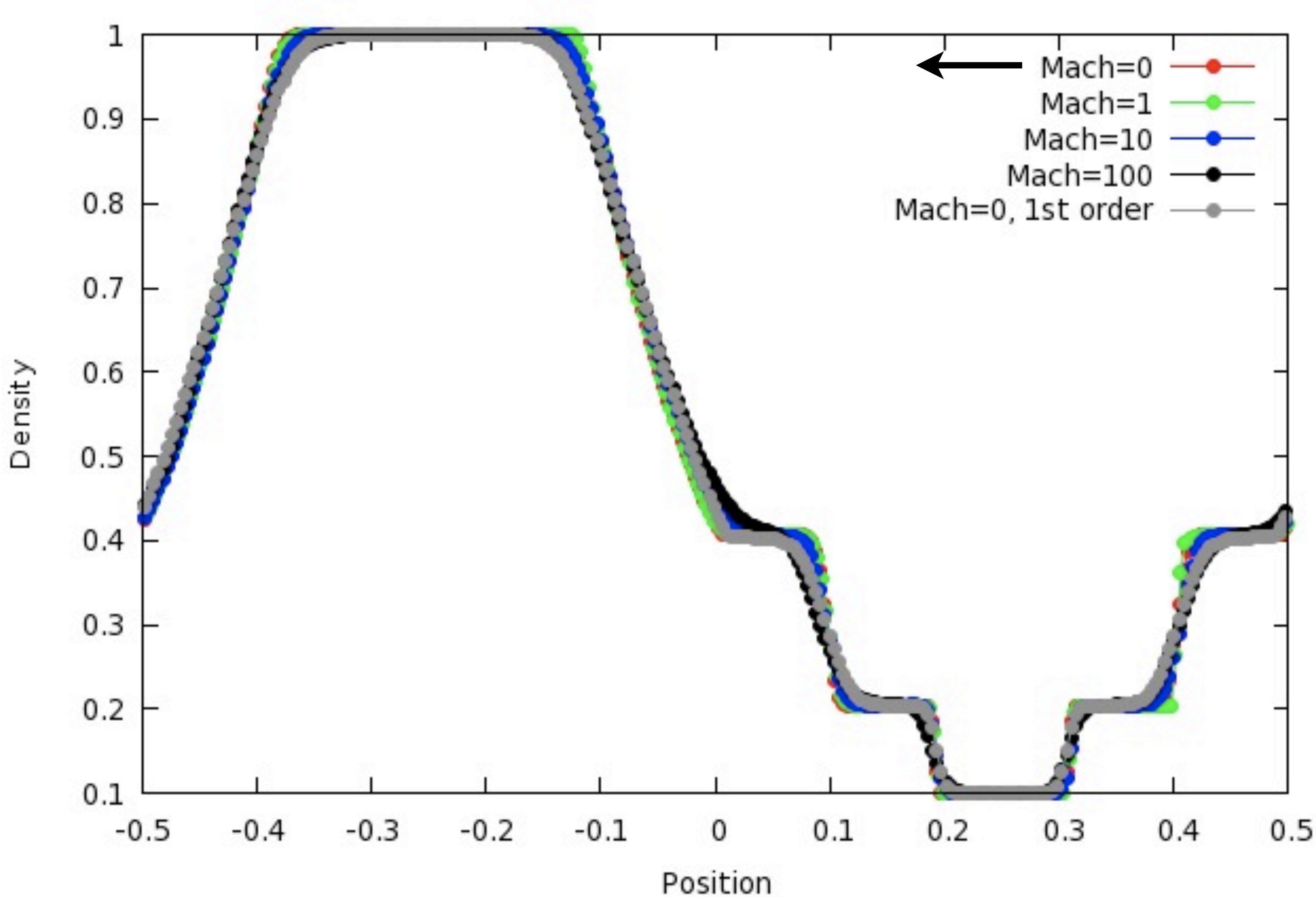


Flux computations: lab frame +advection



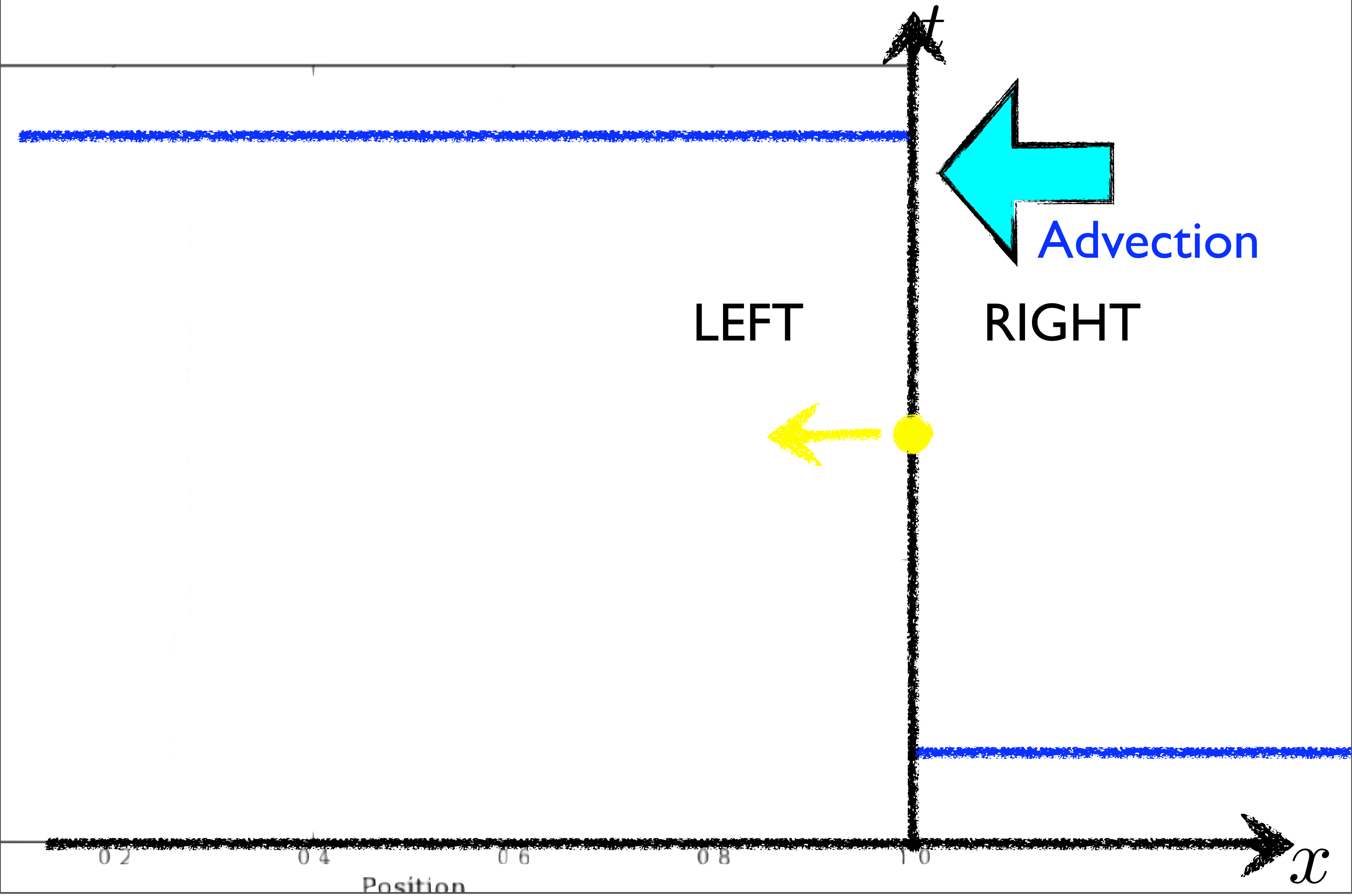
Flux computations: lab frame +advection



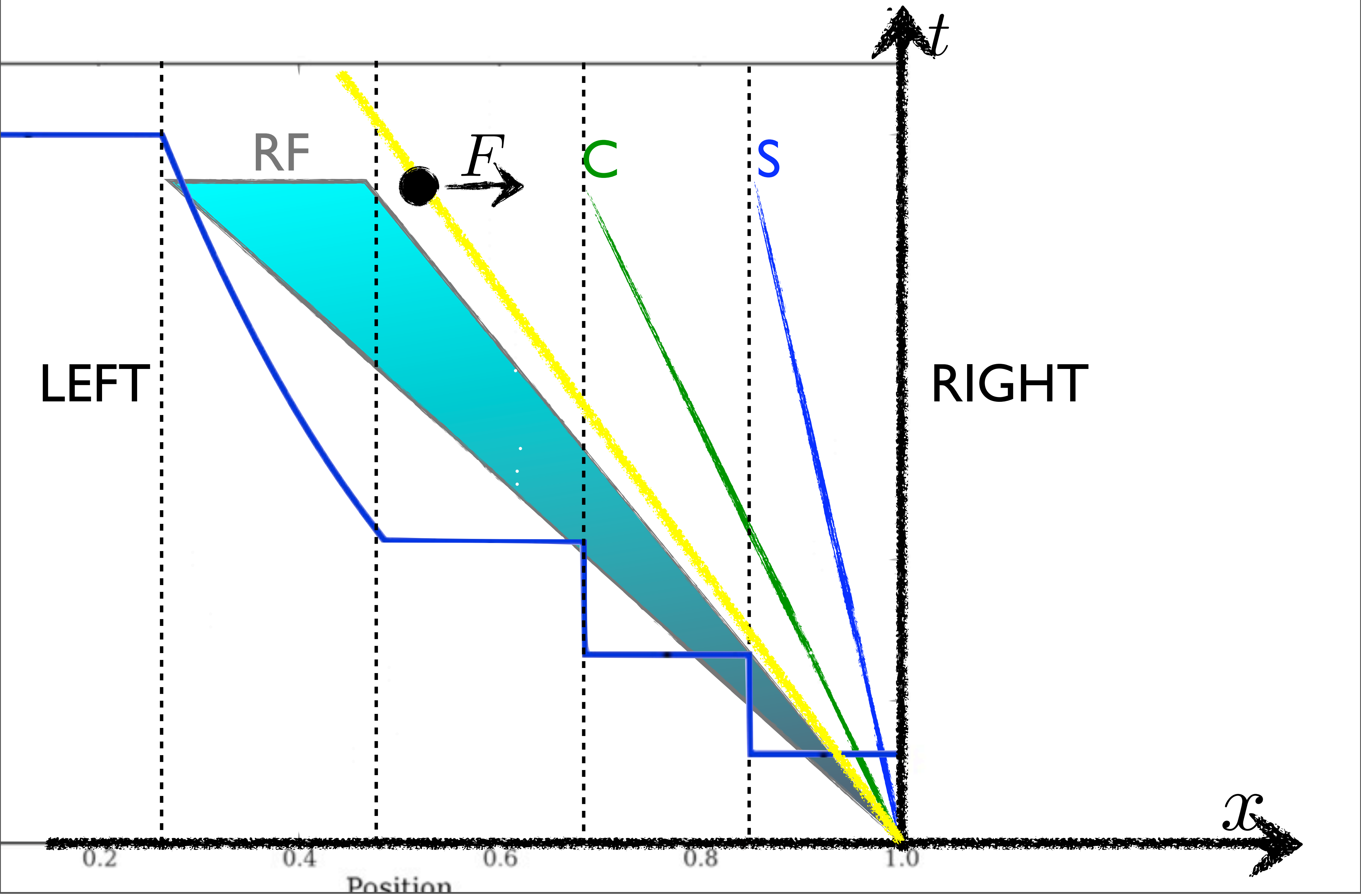


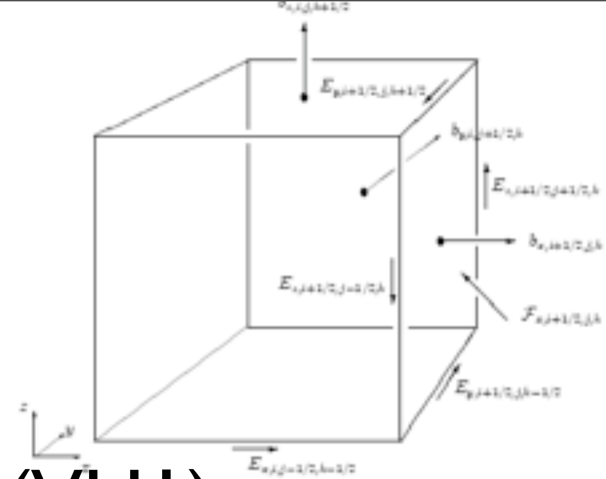
Computed with ATHENA code

Flux computations: moving frame



Flux computations: moving frame





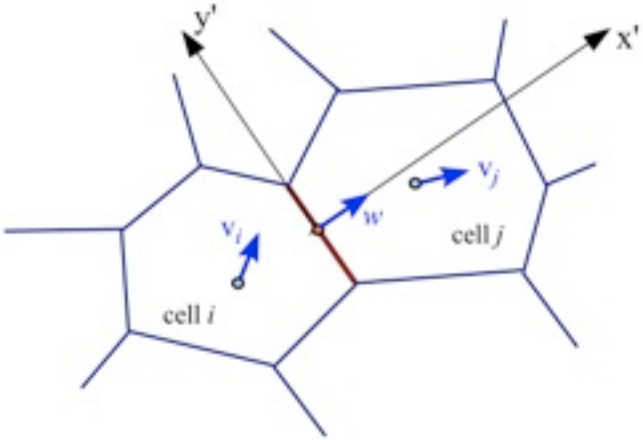
Methods:

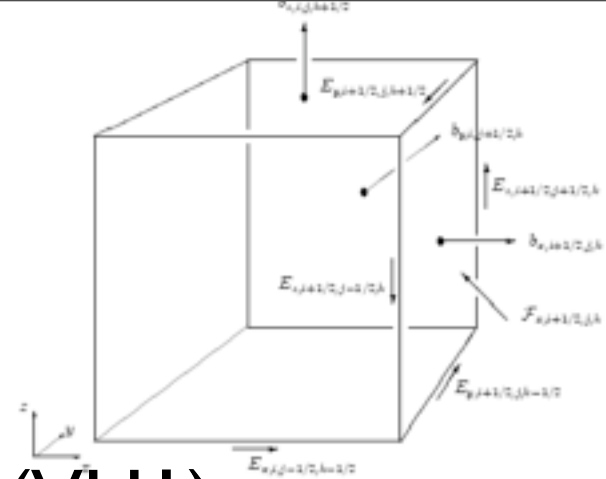
Lagrangian 1D +-> remap for 2/3-D (VH1)

Various ALE-methods (moving mesh method)

AREPO - Volker's new hydro-code on Voronoi

arXiv:0901.4107





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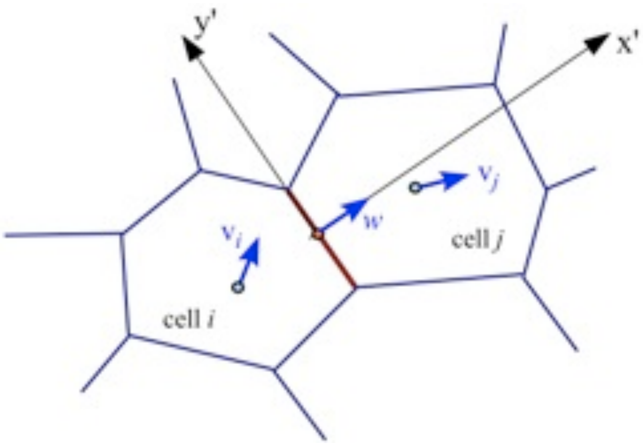
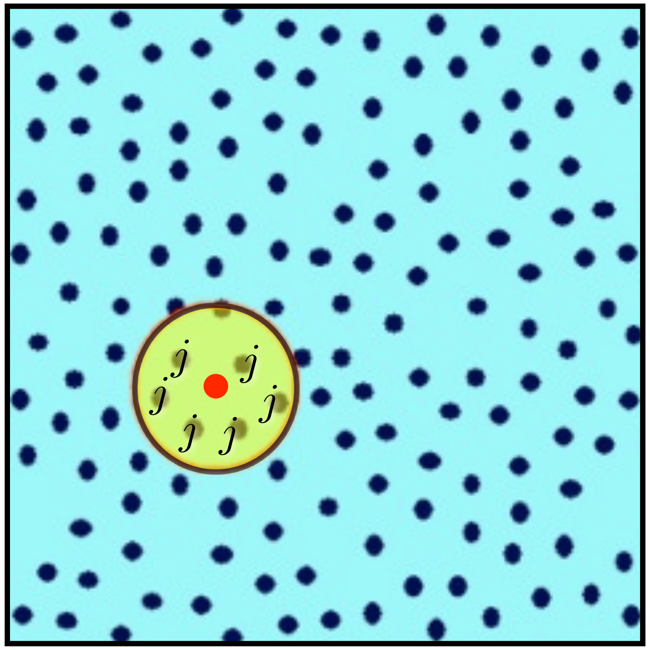


How about meshless schemes?

Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics



Dynamical model:

$$L_{\mathrm{SPH}} = T - (U + \Omega)$$

$$T = \frac{1}{2}\sum_i m_i v_i^2 \quad U = \sum_i m_i u(\rho_i, S_i)$$

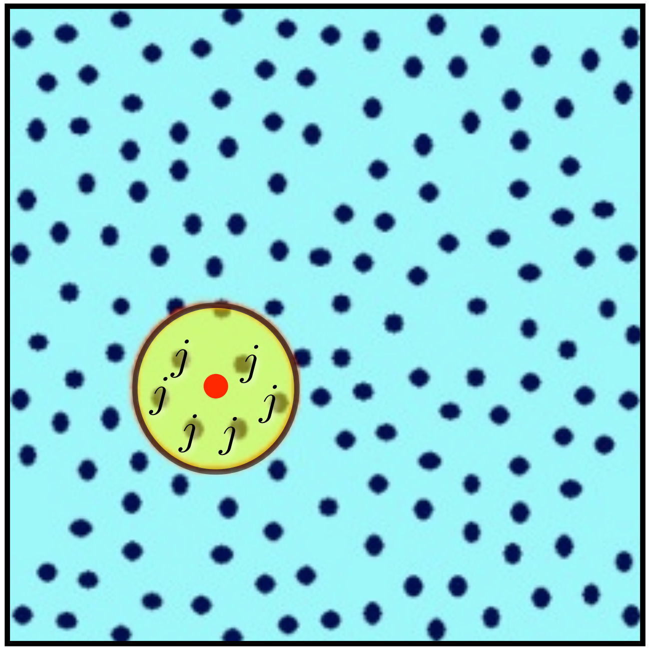
$$\rho_i = \sum_j m_j W(\mathbf{r}_{ij}, h_i)$$

$$(m_i, v_i, S_i)$$

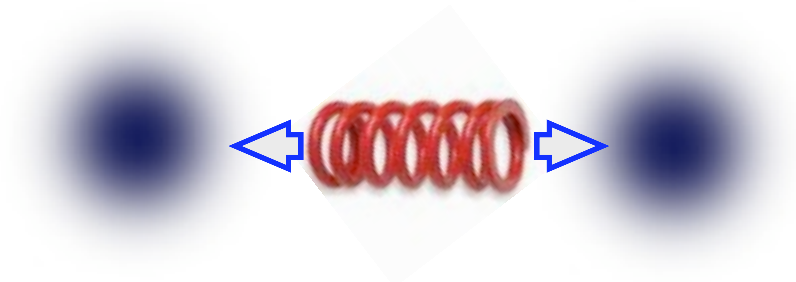
Monaghan, Phys.Rep.,2005

Smoothed Particle Hydrodynamics

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left(\frac{f_i P_i}{\rho_i^2} \nabla W_{ij} + \frac{f_j P_j}{\rho_j^2} \nabla W_{ji} \right)$$



$$\frac{dS_i}{dt} = 0$$



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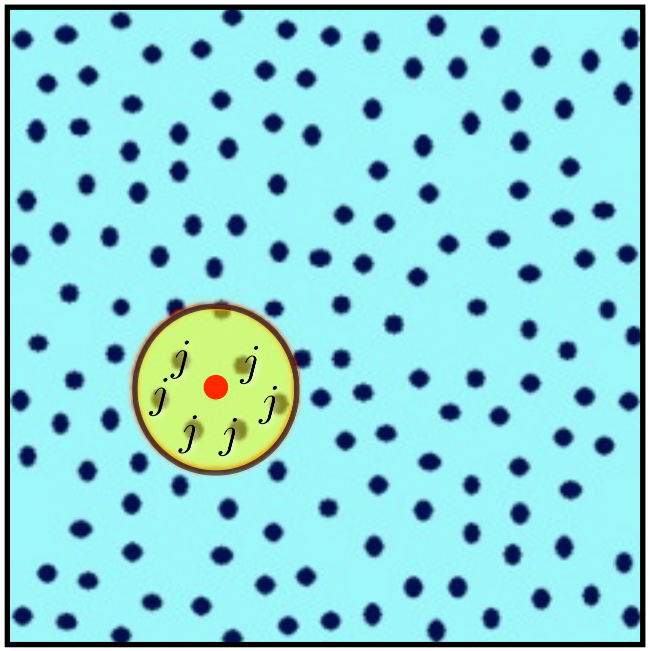

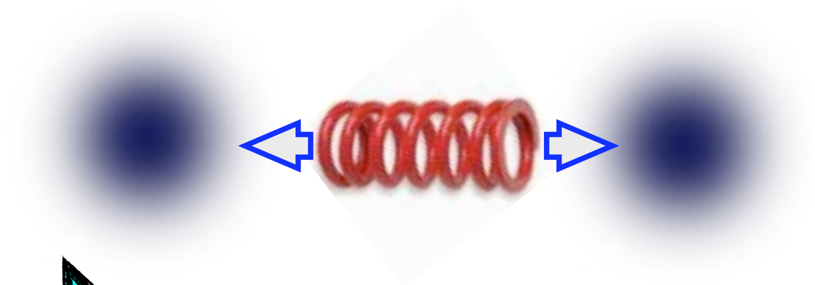

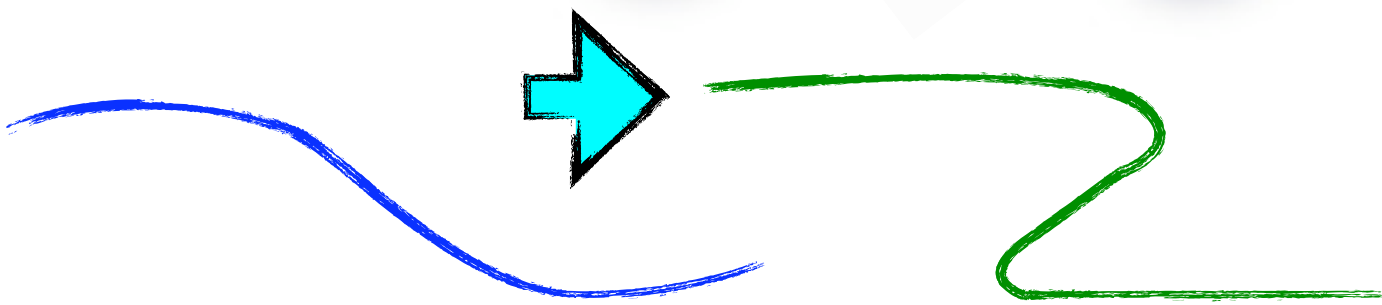

Monaghan, Phys.Rep.,2005

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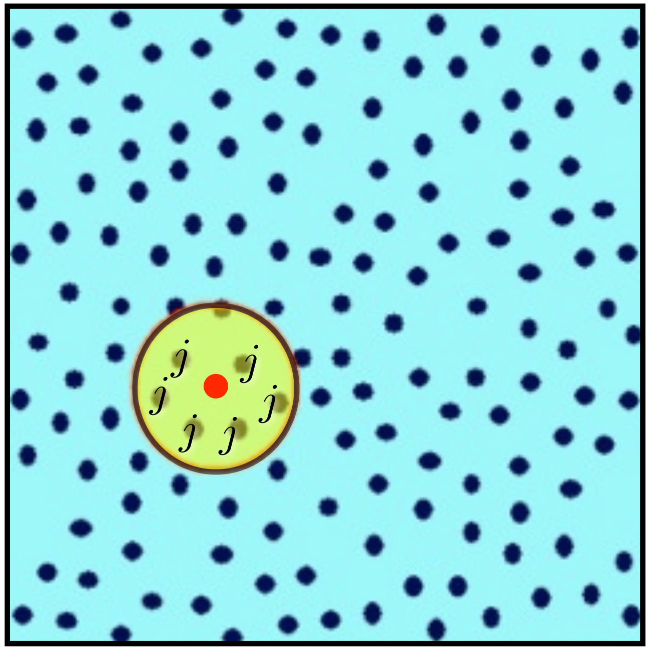
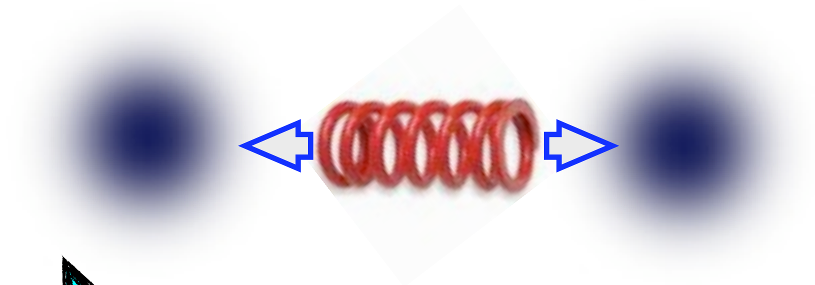
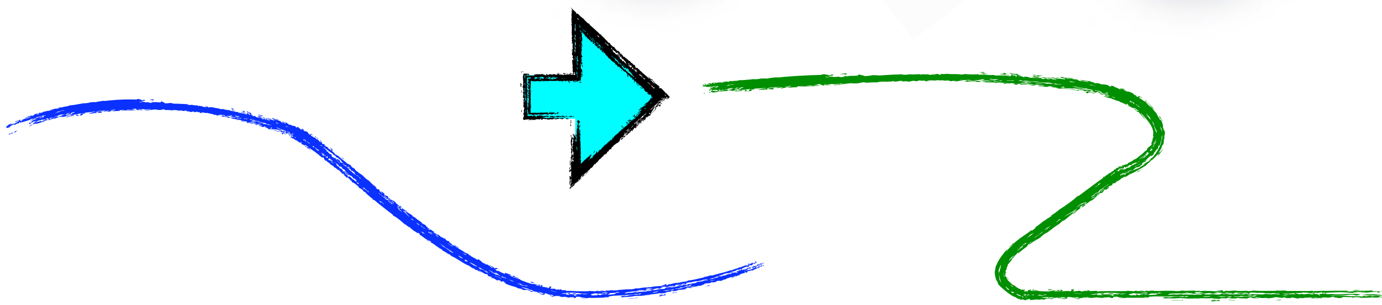
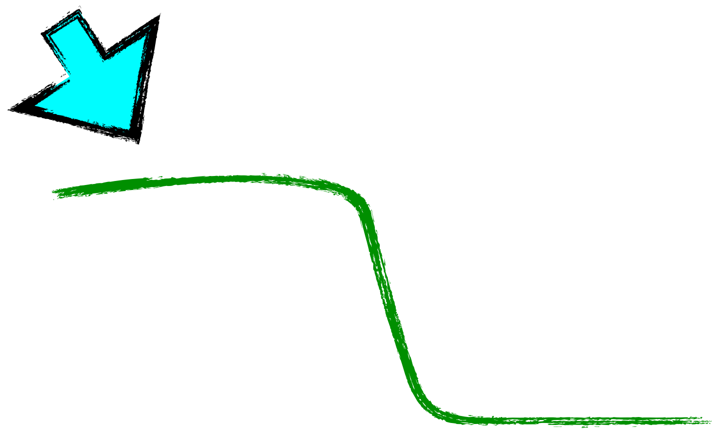


$$\frac{dS_i}{dt} = 0$$





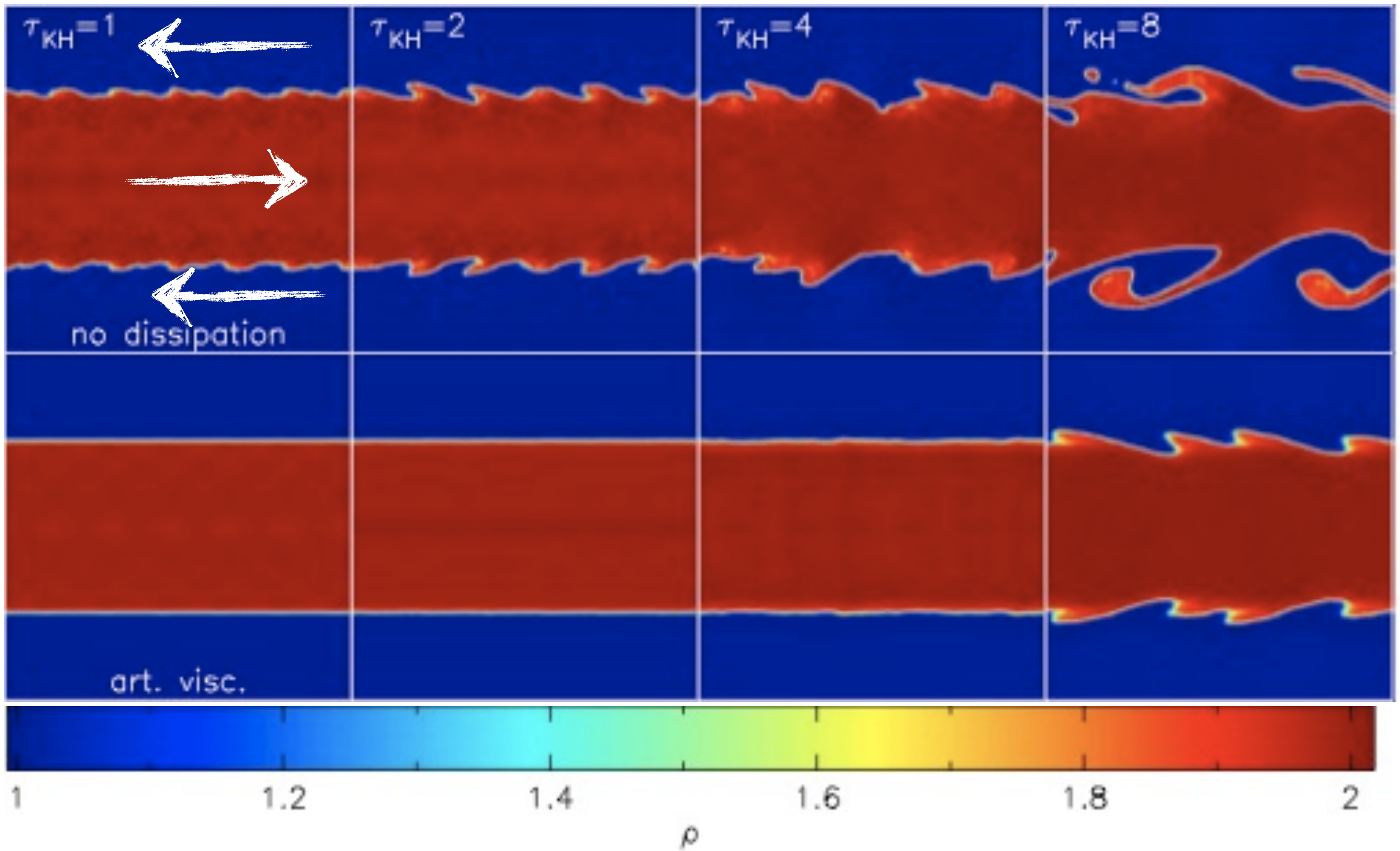
Dissipation (artificial):



$$\frac{dS_i}{dt} = \text{art. viscosity} \propto \nu \nabla^2 v$$

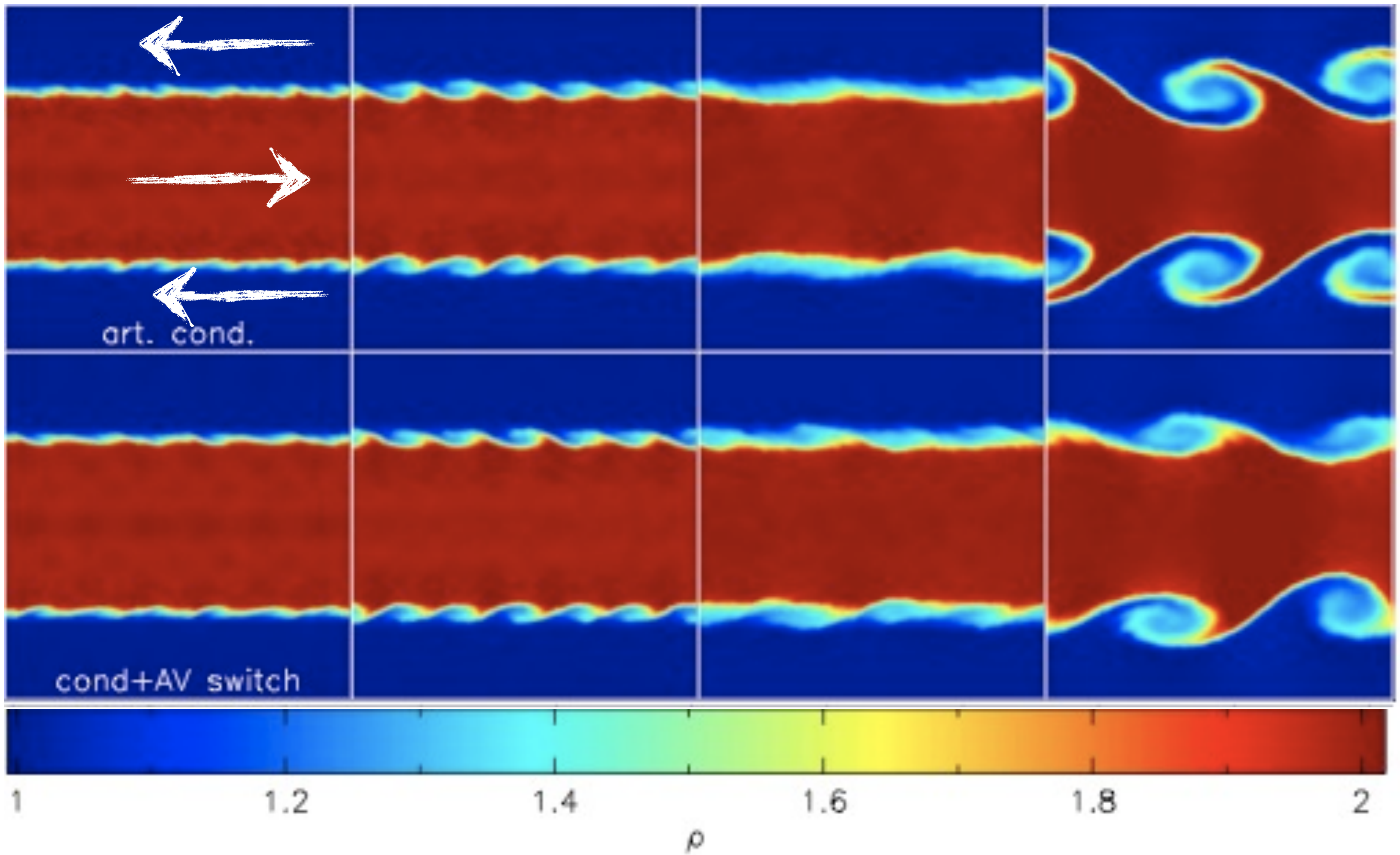
Monaghan, Phys.Rep.,2005

Smoothed Particle Hydrodynamics



Price, JCP 2009

Smoothed Particle Hydrodynamics



Price, JCP 2009

Where else will SPH fail?

Different approach:

Particle Weighted Method

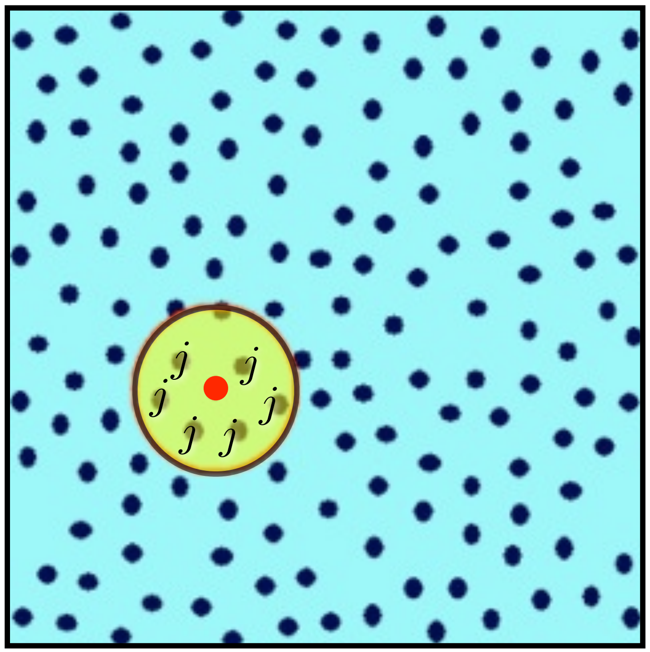
Vila 1999, Math. Model. Meth. App. Sc. 9, 161

Particle Weighted Method

Particles: *are interpolation points*

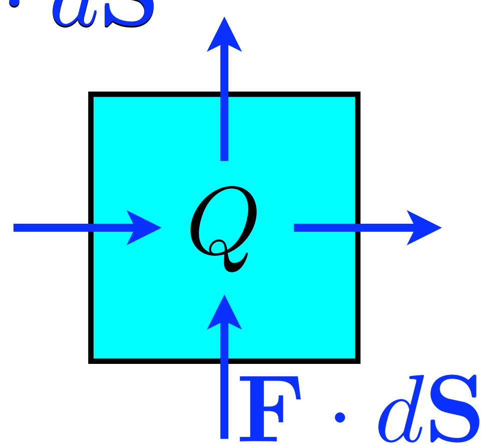
*have **no** physical meaning*

Conservative formulation:



$$Q = (m, \mathbf{p}, E)$$

$$\frac{dQ}{dt} = -\int \mathbf{F} \cdot d\mathbf{S}$$

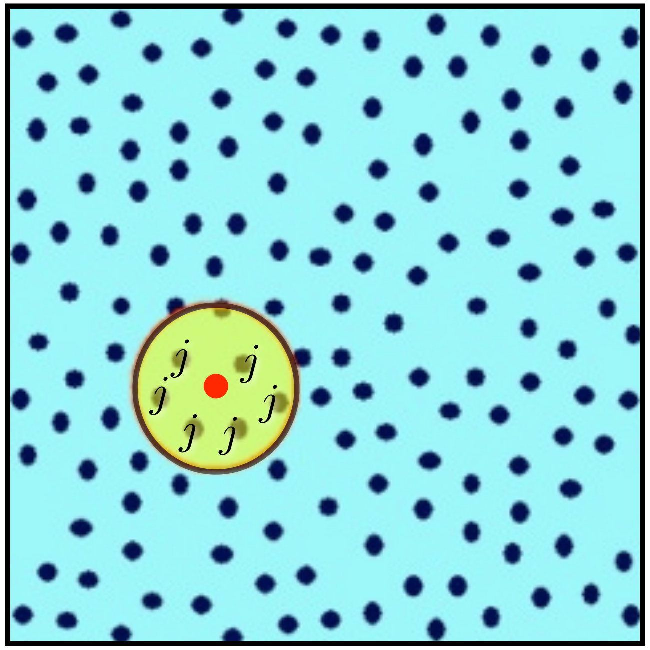


Vila 1999, Math. Model. Meth. App. Sc. 9, 161

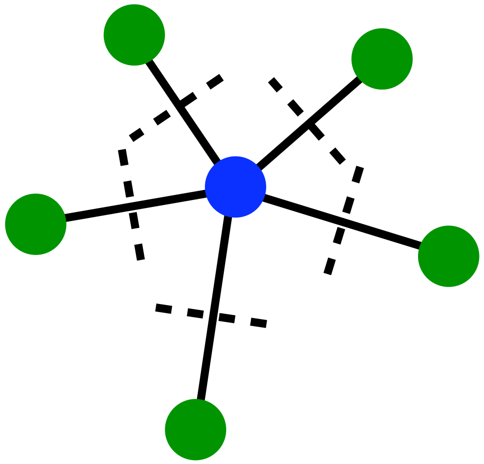
Particle Weighted Method

equations

$$\dot{Q}_i = -V_i \sum_j V_j (\mathbf{F}_{ij} \cdot \mathbf{B}_{ij} + \mathbf{F}_{ij} \cdot \mathbf{B}_{ji})$$



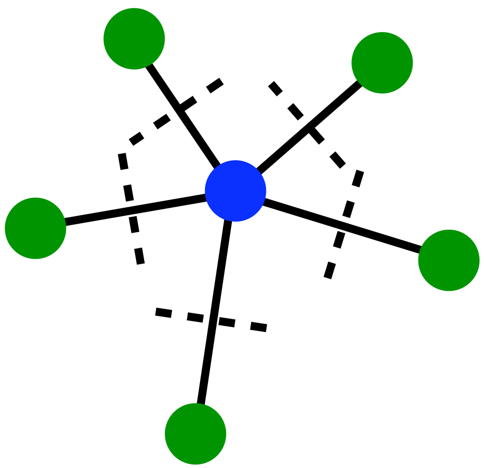
$$V_i = \sum_j W(\mathbf{r}_{ij}, h_i)$$

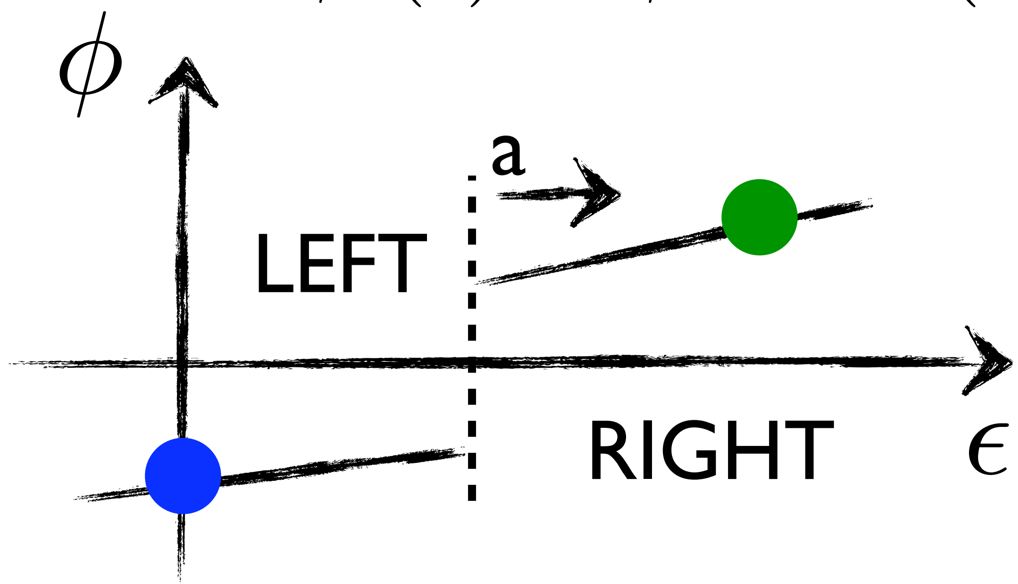


Lanson & Vila, 2008, SIAM J. Num. Anal. 46, 1912

Particle Weighted Method
reconstruction

$$\phi_i(\epsilon) = \phi_i + \epsilon \cdot (\nabla\phi)_i$$

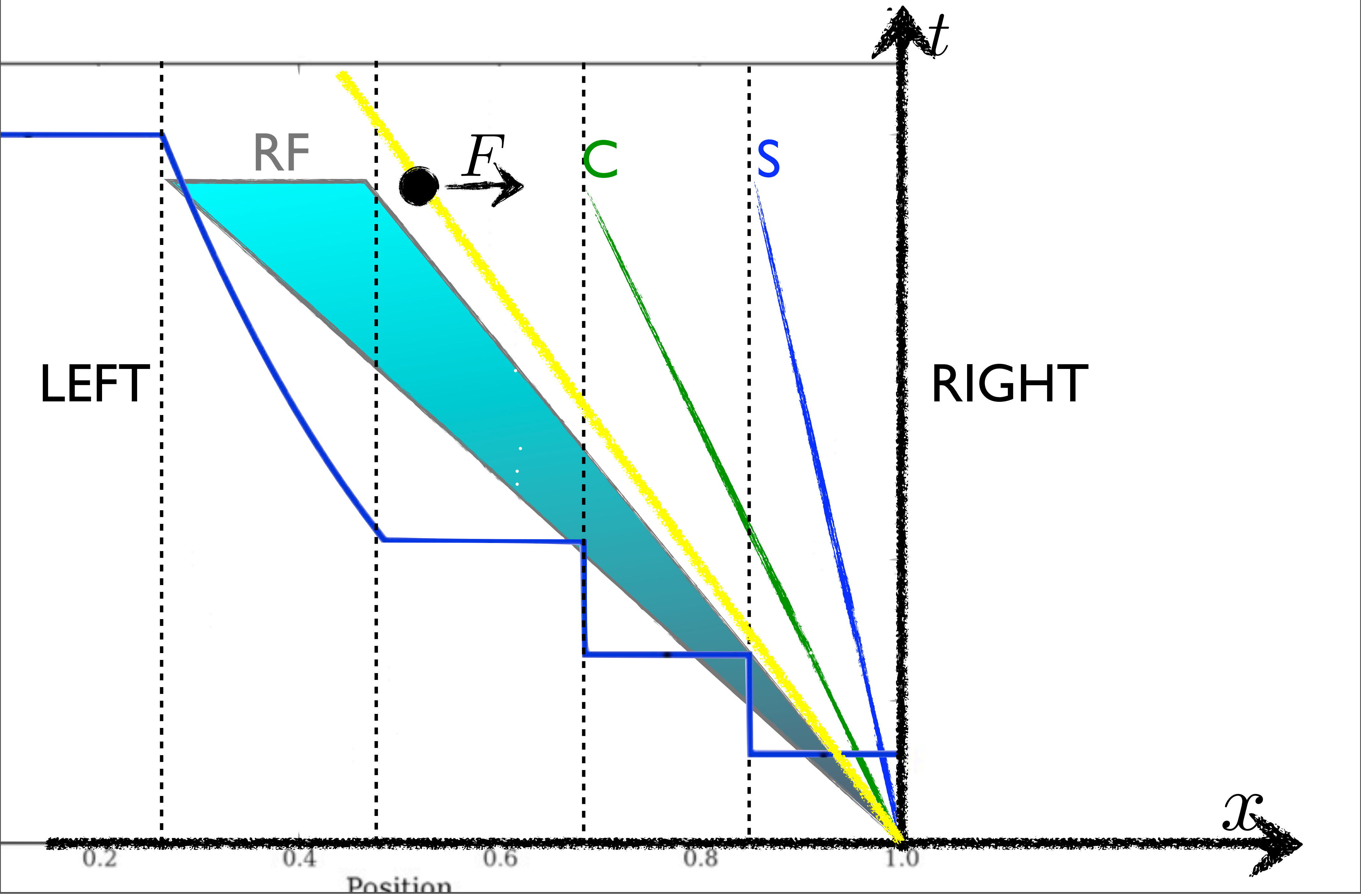




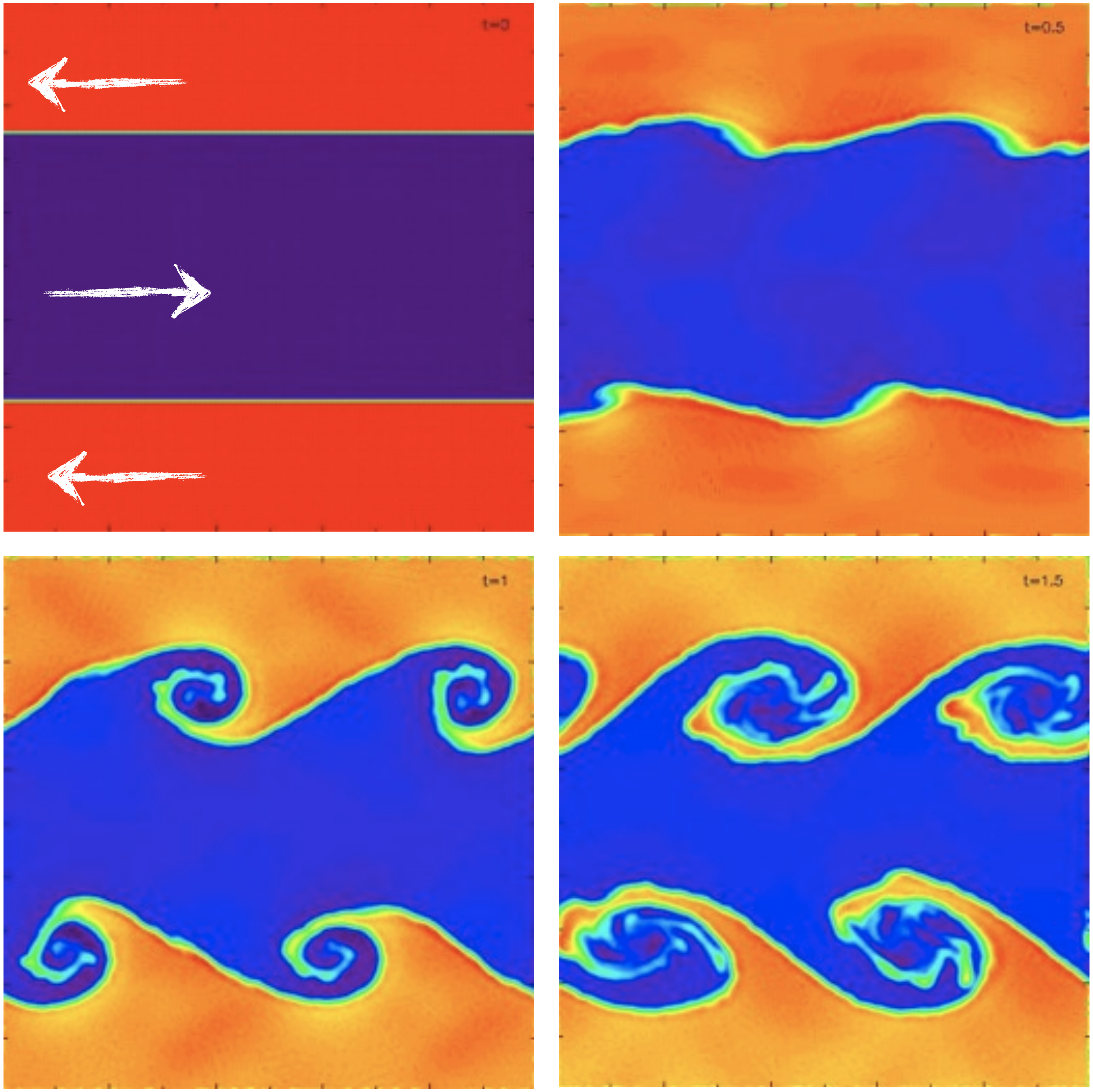
$$(\nabla\phi)_i = \sum_j w_j(\phi_j - \phi_i)\mathbf{B}_{ij} + \mathcal{O}(\Delta r)^2$$

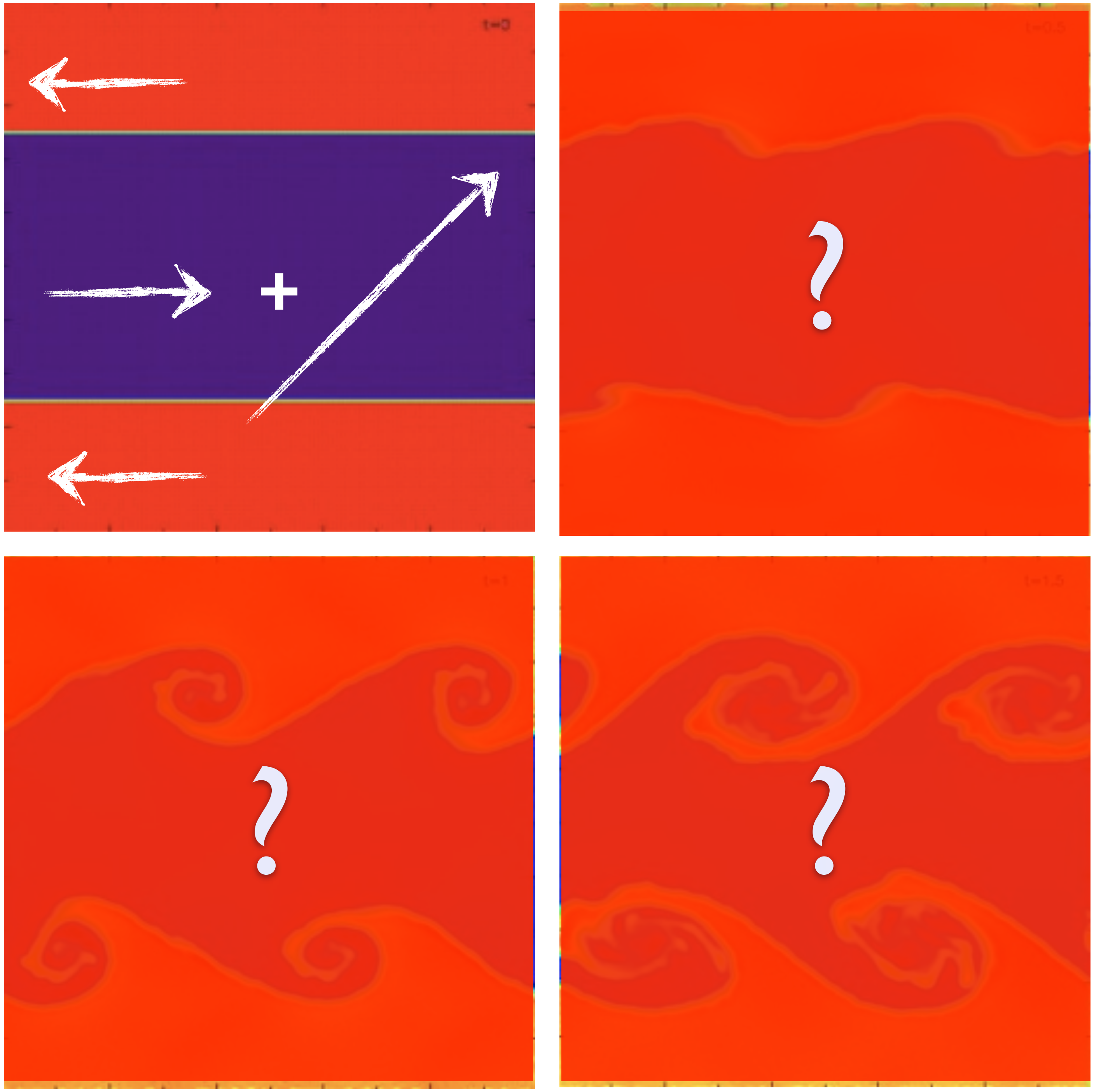
Lanson & Vila, 2008, SIAM J. Num. Anal. 46, 1912

Flux computations: moving frame

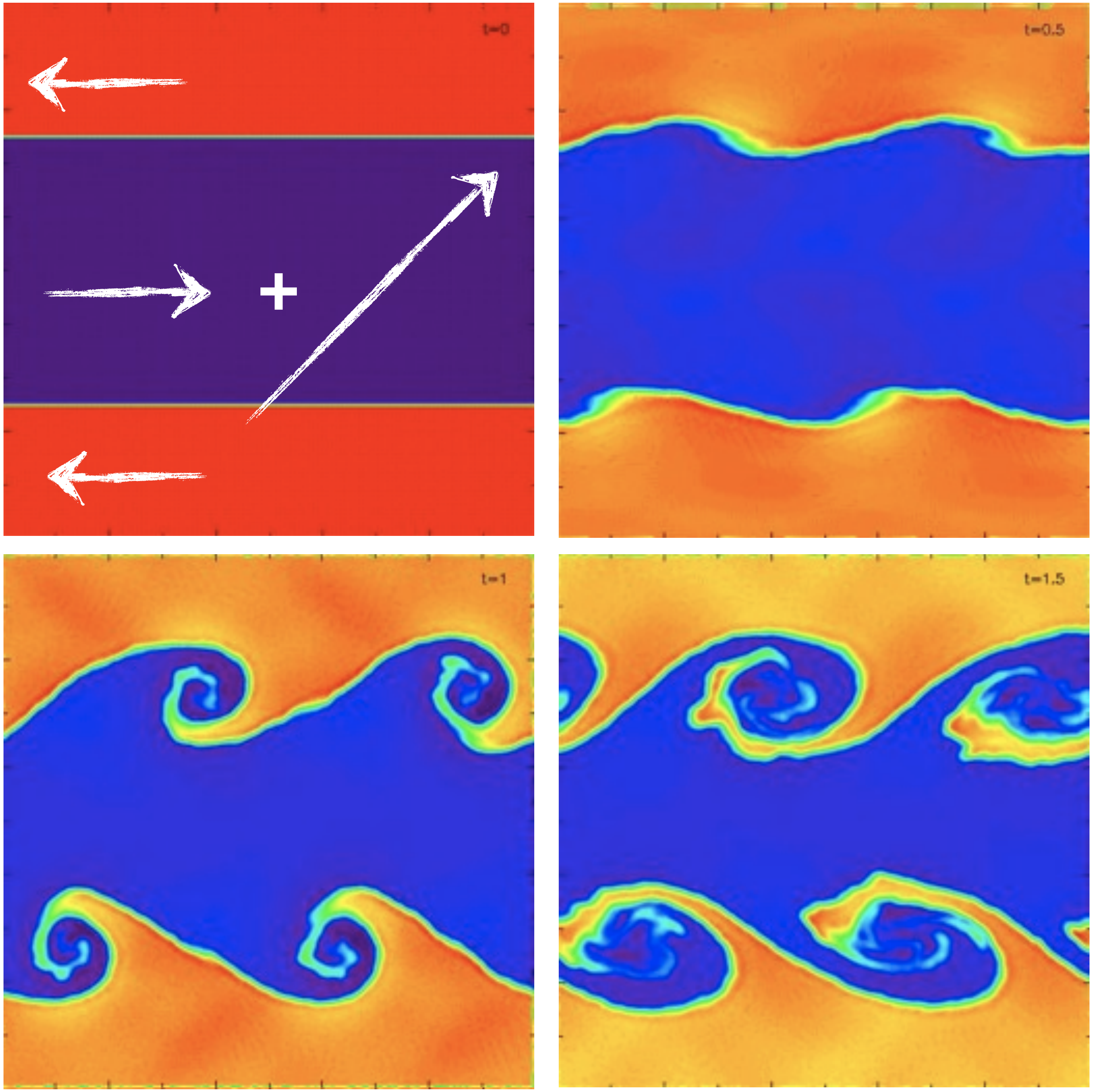


Example


t=0
t=0.5
t=1
t=1.5



Mach 10 advection



Work in progress

GPU implementation

Magnetic fields via **A**

already works in 1D

Possible extension to higher order

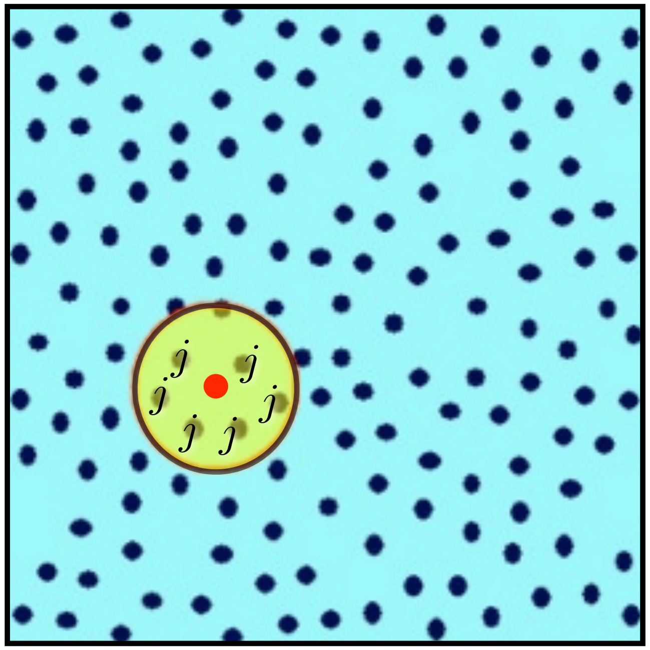
à la Gradient Particle MHD (Maron & Howes, ApJ 2003) + WENO

Extra slides, not shown

Particle Weighted MHD

Particles: *interpolation points*

no *physical meaning*



Conservative formulation (HD):

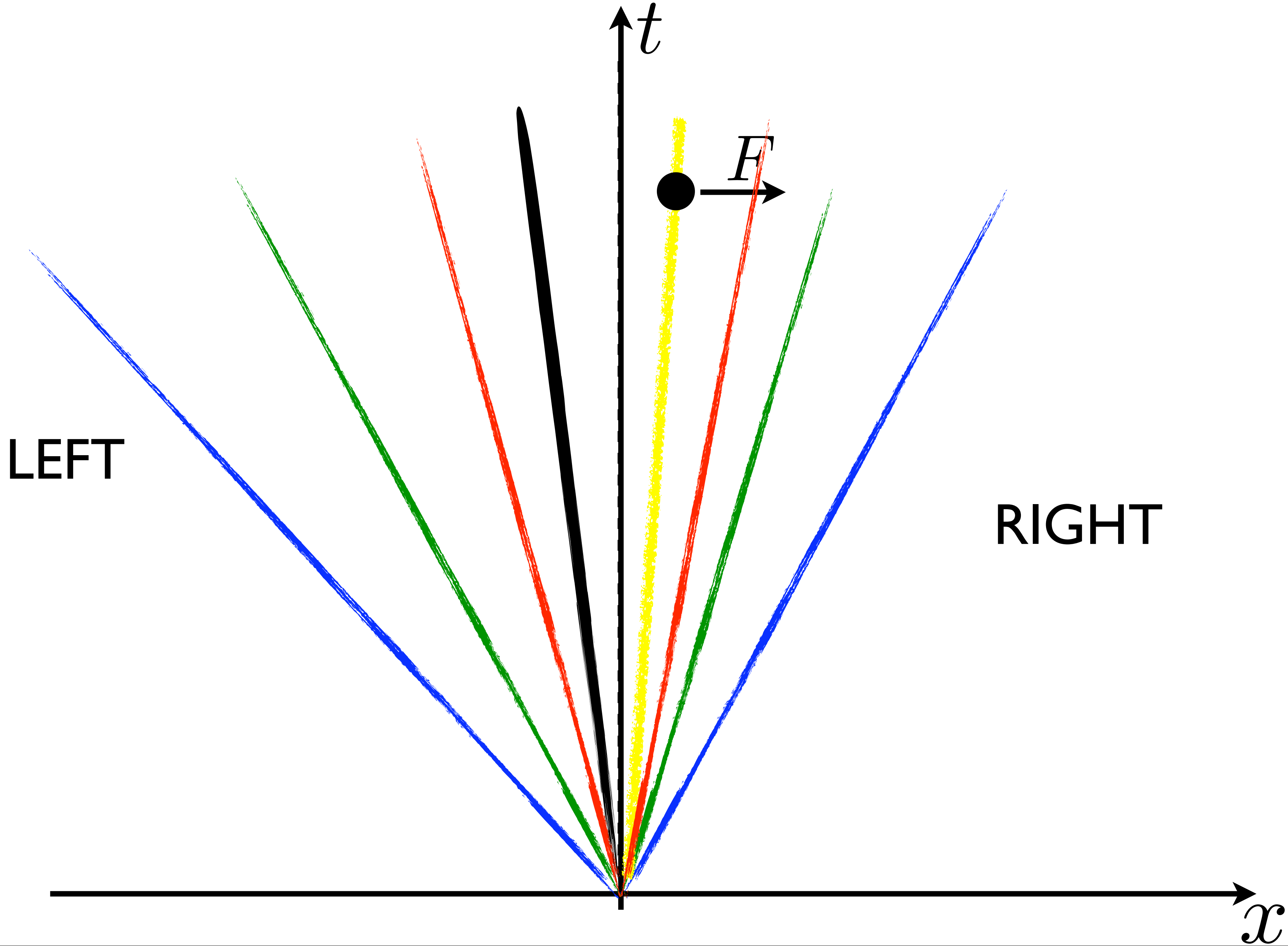
$$Q = (m, \mathbf{p}, E)$$

$$\frac{dQ}{dt} = -\int \mathbf{F} \cdot d\mathbf{S}$$

Vector potential (MHD):

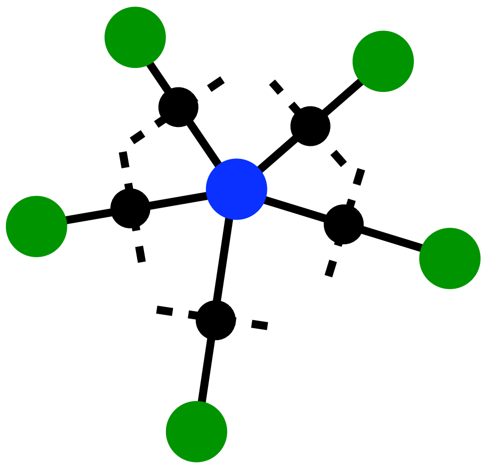
$$\frac{dA^\alpha}{dt} = \mathcal{E}^\alpha + a^\beta \nabla^\alpha A^\beta$$

MHD solution: Riemann solver



Particle Weighted MHD

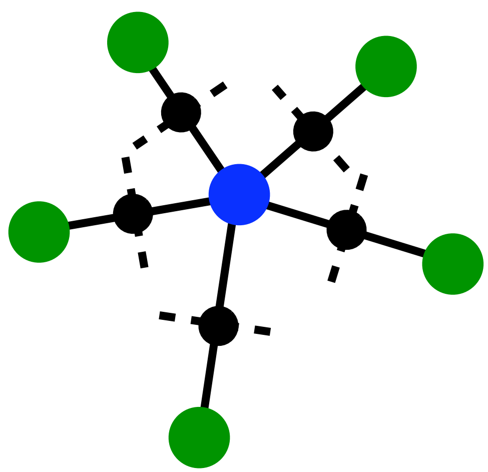
Least-square fit : $\mathcal{E}^{\alpha}(\mathbf{r}) = \mathcal{E}^{\alpha}_c + \mathbf{r} \cdot (\nabla \mathcal{E}^{\alpha})_c$



Minimise: $\mathcal{L}_i = \sum_j V_j \left[\mathcal{E}^{\alpha}_j n^{\alpha}_{ij} - \mathcal{E}^{\alpha}(\mathbf{r}_{ij}) n^{\alpha}_{ij} \right]^2$

Particle Weighted MHD

Least-square fit : $\mathcal{E}^\alpha(\mathbf{r}) = \mathcal{E}_c^\alpha + \mathbf{r}\cdot(\nabla\mathcal{E}^\alpha)_c$

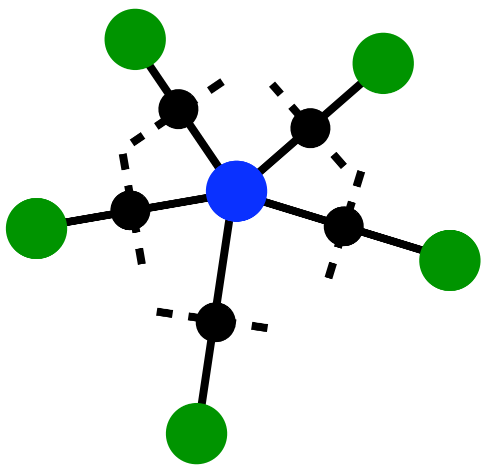


Minimise: $\mathcal{L}_i = \sum_j V_j \left[\boxed{\mathcal{E}_j^\alpha n_{ij}^\alpha} - \mathcal{E}^\alpha(\mathbf{r}_{ij}) n_{ij}^\alpha\right]^2$

From RP

Particle Weighted MHD

Least-square fit : $\mathcal{E}^\alpha(\mathbf{r}) = \mathcal{E}^\alpha_c + \mathbf{r}\cdot(\nabla\mathcal{E}^\alpha)_c$



$$\frac{dA^\alpha}{dt} = \mathcal{E}^\alpha_c + a^\beta \nabla^\alpha A^\beta$$

Minimise: $\mathcal{L}_i = \sum_j V_j \left[\boxed{\mathcal{E}^\alpha_j n^\alpha_{ij}} - \mathcal{E}^\alpha(\mathbf{r}_{ij}) n^\alpha_{ij}\right]^2$

From RP