COMP90051

Workshop Week 12

About the Workshops

- 7 sessions in total
  - Tue 12:00-13:00 AH211
  - Tue 12:00-13:00 AH108 *
  - Tue 13:00-14:00 AH210
  - Tue 16:15-17:15 AH109
  - Tue 17:15-18:15 AH236 *
  - Tue 18:15-19:15 AH236 *
  - Fri 14:15-15:15 AH211

About the Workshops

- Homepage
  - https://trevorcohn.github.io/comp90051-2017/workshops
- Solutions will be released on next Friday (a week later).

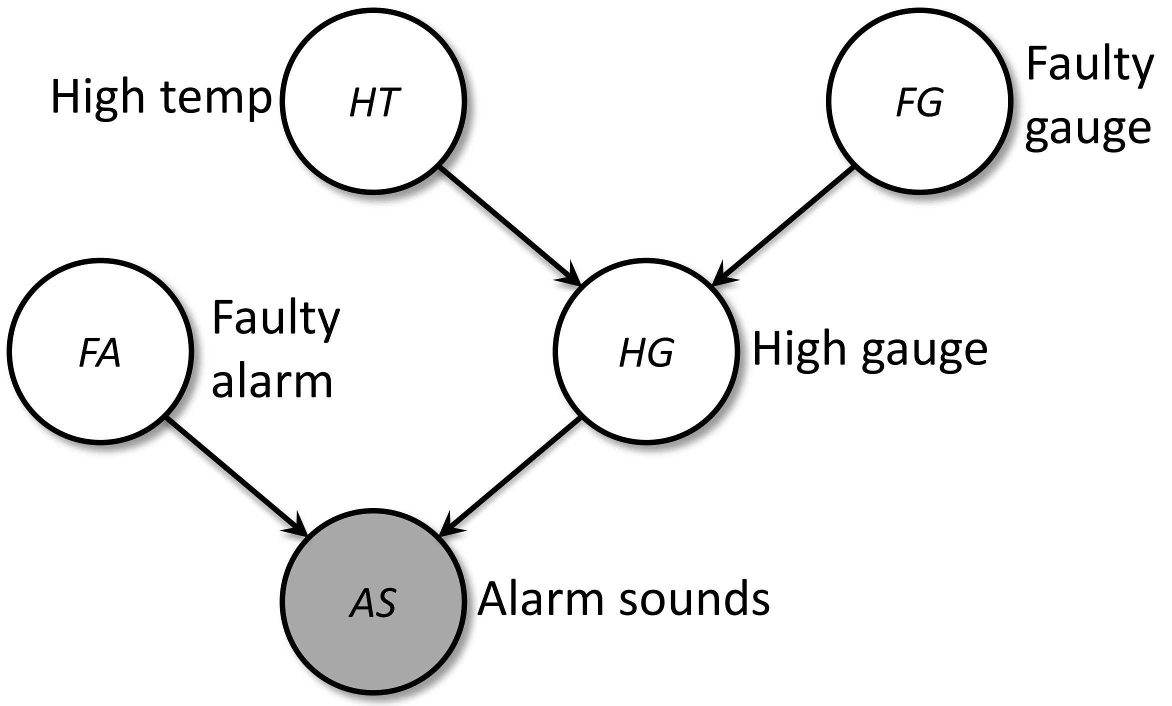
Syllabus

1	Introduction; Probability theory	Probabilistic models; Parameter fitting	
2	Linear regression; Introduction to regularization	Logistic regression; Basis expansion	
3	Optimization; Regularization	Perceptron	
4	Backpropagation	CNNs; Auto-encoders	
5	Hard-margin SVMs	Soft-margin SVMs	
6	Kernel methods	Ensemble Learning	
7	Clustering	EM algorithm	
8	Principal component analysis; Multidimensional Scaling	Manifold Learning; Spectral clustering	
9	Bayesian inference (uncertainty, updating)	Bayesian inference (conjugate priors)	
10	PGMs, fundamentals	PGMs, independence	
11	PGMs, inference	PGMs, EM algorithm	←
12	PGMs, HMMs & message passing	Subject review	

Outline

- Review the lecture, background knowledge, etc.
  - Elimination algorithm
  - Sampling method
  - EM algorithm

Nuclear power plant



- **Alarm sounds**; meltdown?!
- $\Pr(HT|AS=t) = \frac{\Pr(HT, AS=t)}{\Pr(AS=t)}$

$$= \frac{\sum_{FG,HG,FA} \Pr(AS=t, FA, HG, FG, HT)}{\sum_{FG,HG,FA,HT'} \Pr(AS=t, FA, HR, FG, HT')}$$

- Numerator (denominator similar)

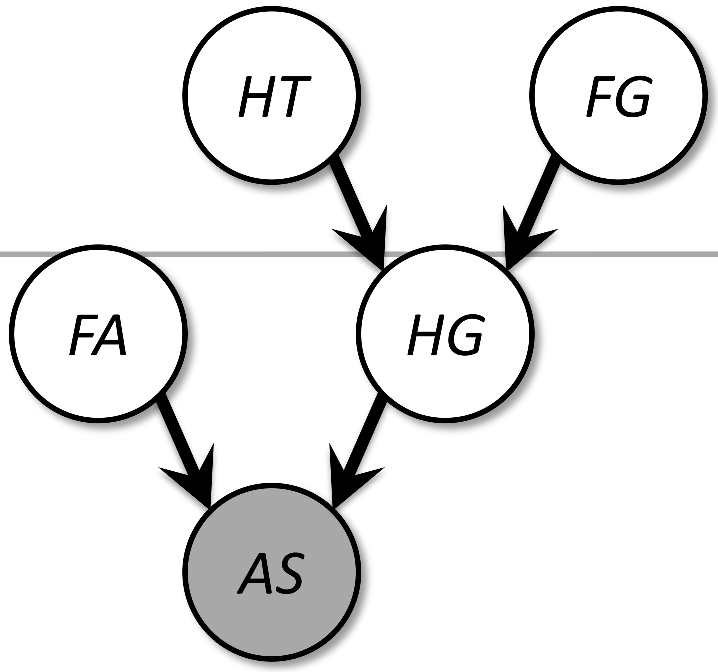
expanding out sums, joint *summing once over 2^5 table*

$$= \sum_{FG} \sum_{HG} \sum_{FA} \Pr(HT) \Pr(HG|HT, FG) \Pr(FG) \Pr(AS=t|FA, HG) \Pr(FA)$$

distributing the sums as far down as possible *summing over several smaller tables*

$$= \Pr(HT) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG) \sum_{FA} \Pr(FA) \Pr(AS=t|FA, HG)$$

To calculate $P(HT|AS = 1)$



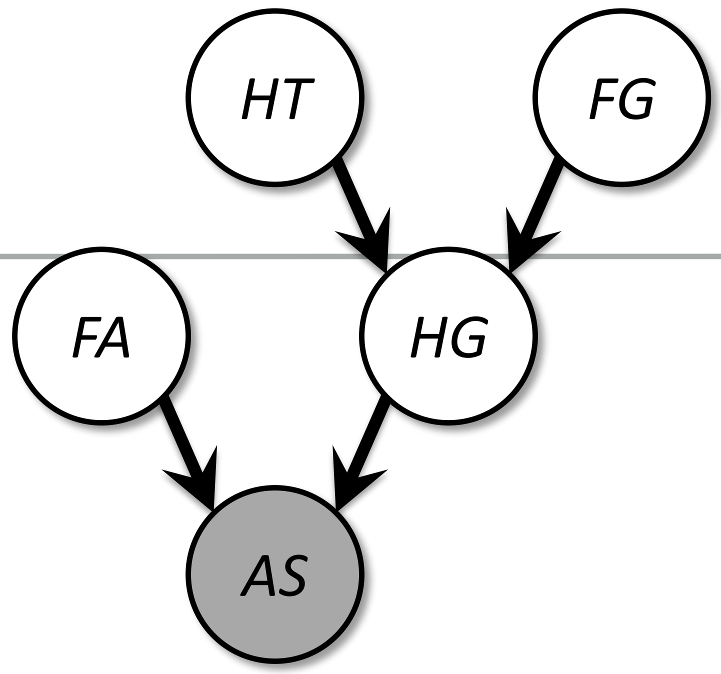
- Joint
$$P(AS, FA, HG, HT, FG) = P(AS|FA, HG)P(FA)P(HG|HT, FG)P(HT)P(FG)$$

- Step 1. $P(HT|AS = 1) \propto P(AS = 1, HT)$
- Step 2. $P(AS = 1, HT) = \sum_{FG,HG,FA} P(AS = 1, FA, HG, HT, FG)$
- Step 3. Normalize $P(AS = 1, HT) \rightarrow P(HT|AS = 1)$

- $P(AS = 1, HT)$ has two numbers
  - $P(AS = 1, HT = 0)$ and $P(AS = 1, HT = 1)$
  - can be calculated together

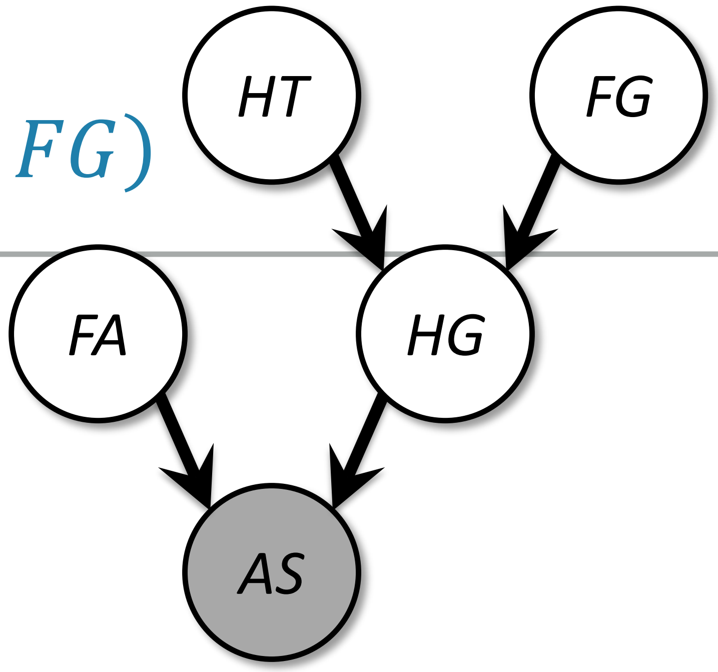
- We will first see a Naive way to calculate them

Define some tables



```
table_FG = np.asarray([0.1, 0.9]) ← P(FG)
table_HT = np.asarray([0.2, 0.8]) ← P(HT)
table_FA = np.asarray([0.3, 0.7]) ← P(FA)

table_HG_HT_FG = np.empty((2, 2, 2))      ← P(HG|HT,FG)
table_HG_HT_FG[:, 0, 0] = [0.35, 0.65]
table_HG_HT_FG[:, 0, 1] = [0.25, 0.75]
table_HG_HT_FG[:, 1, 0] = [0.15, 0.85]
table_HG_HT_FG[:, 1, 1] = [0.05, 0.95]

table_AS_FA_HG = np.empty((2, 2, 2))      ← P(AS|FA,HG)
table_AS_FA_HG[:, 0, 0] = [0.45, 0.55]
table_AS_FA_HG[:, 0, 1] = [0.55, 0.45]
table_AS_FA_HG[:, 1, 0] = [0.65, 0.35]
table_AS_FA_HG[:, 1, 1] = [0.75, 0.25]
```

$P(AS = 1, HT) = \sum_{FG,HG,FA} P(AS = 1, FA, HG, HT, FG)$



```
AS = 1

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            for FA in [0, 1]:
                prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                                table_HG_HT_FG[HG, HT, FG] *
                                table_FA[FA] *
                                table_AS_FA_HG[AS, FA, HG]
                               )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

```
m_AS = table_AS_FA_HG[1, :, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            for FA in [0, 1]:
                prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                                table_HG_HT_FG[HG, HT, FG] *
                                table_FA[FA] *
                                m_AS[FA, HG]
                               )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Any ideas to reduce #multiplications?

```
m_AS = table_AS_FA_HG[1, :, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            for FA in [0, 1]:
                prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                                table_HG_HT_FG[HG, HT, FG] *
                                table_FA[FA] * m_AS[FA, HG]
                               )
print(prob_HT)
```

```
---------------
[ 0.0672  0.2528]
```

Loop unrolling

```
m_AS = table_AS_FA_HG[1, :, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_HG_HT_FG[HG, HT, FG] *
                            table_FA[0] * m_AS[0, HG] ←
                           )
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_HG_HT_FG[HG, HT, FG] *
                            table_FA[1] * m_AS[1, HG] ←
                           )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Rearranging the parentheses

```
m_AS = table_AS_FA_HG[1, :, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_HG_HT_FG[HG, HT, FG] *
                            (table_FA[0] * m_AS[0, HG] + ←
                             table_FA[1] * m_AS[1, HG]   ←
                            )
                           )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Define a message function for *FA*

```
m_AS = table_AS_FA_HG[1, :, :]

def m_FA(HG): return (table_FA[0] * m_AS[0, HG] +
                      table_FA[1] * m_AS[1, HG])

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_HG_HT_FG[HG, HT, FG] *
                            m_FA(HG)
                           )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Better to precompute m_FA

```
m_AS = table_AS_FA_HG[1, :, :]

m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]


prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_HG_HT_FG[HG, HT, FG] *
                            m_FA[HG]
                           )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

FA is removed, then remove HG

```
m_AS = table_AS_FA_HG[1, :, :]
m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_HG_HT_FG[HG, HT, FG] *
                            m_FA[HG]
                           )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Loop unrolling, rearranging the parentheses

```
m_AS = table_AS_FA_HG[1, :, :]
m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                         (table_HG_HT_FG[0, HT, FG] * m_FA[0] +
                          table_HG_HT_FG[1, HT, FG] * m_FA[1]
                         )
                        )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Define a message function for *HG*

```python
m_AS = table_AS_FA_HG[1, :, :]
m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]

def m_HG(HT, FG): return (table_HG_HT_FG[0, HT, FG] * m_FA[0] +
                          table_HG_HT_FG[1, HT, FG] * m_FA[1])

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                        m_HG(HT, FG)
                       )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Again, better to precompute m_HG

```
m_AS = table_AS_FA_HG[1, :, :]
m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]

m_HG = (table_HG_HT_FG[0, :, :] * m_FA[0] +
        table_HG_HT_FG[1, :, :] * m_FA[1])

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                         m_HG[HT, FG]
                        )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Then FG, directly define the message func

```
m_AS = table_AS_FA_HG[1, :, :]
m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]
m_HG = (table_HG_HT_FG[0, :, :] * m_FA[0] +
         table_HG_HT_FG[1, :, :] * m_FA[1])

def m_FG(HT): return (table_FG[0] * m_HG[HT, 0] +
                      table_FG[1] * m_HG[HT, 1])

prob_HT = np.zeros(2)
for HT in [0, 1]:
    prob_HT[HT] += m_FG(HT) * table_HT[HT]

print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Finally

```
m_AS = table_AS_FA_HG[1, :, :]

m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]

m_HG = (table_HG_HT_FG[0, :, :] * m_FA[0] +
        table_HG_HT_FG[1, :, :] * m_FA[1])

m_FG = table_FG[0] * m_HG[:, 0] + table_FG[1] * m_HG[:, 1]

prob_HT = m_FG[HT] * table_HT[HT]

print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Finally (how many multiplications?)

```
m_AS = table_AS_FA_HG[1, :, :] 0

m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]
                   2                           2
m_HG = (table_HG_HT_FG[0, :, :] * m_FA[0] + 4
        table_HG_HT_FG[1, :, :] * m_FA[1])  4

m_FG = table_FG[0] * m_HG[:, 0] + table_FG[1] * m_HG[:, 1]
                   2                           2
prob_HT = m_FG[HT] * table_HT[HT]
                   2
print(prob_HT)            in total 2*2 + 4*2 + 2*2 + 2 = 18
--------------
[ 0.0672  0.2528]
```

Naive way (how many multiplications?)

```
AS = 1

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for HG in [0, 1]:
            for FA in [0, 1]:
                prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                                table_HG_HT_FG[HG, HT, FG] *
                                table_FA[FA] *
                                table_AS_FA_HG[AS, FA, HG]
                               )    4
print(prob_HT)                   in total 4 * 16 = 64
---------------
[ 0.0672  0.2528]
```

What we have done mathematically?

```
m_AS = table_AS_FA_HG[1, :, :]
```

$$m_{AS}(FA, HG) = P(AS = 1|FA, HG)$$

```
m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]
```

$$m_{FA}(HG) = \sum_{FA} P(FA) m_{AS}(FA, HG)$$

```
m_HG = (table_HG_HT_FG[0, :, :] * m_FA[0] +
         table_HG_HT_FG[1, :, :] * m_FA[1])
```

$$m_{HG}(HT, FG) = \sum_{HG} P(HG|HT, FG) m_{FA}(HG)$$

```
m_FG = table_FG[0] * m_HG[:, 0] + table_FG[1] * m_HG[:, 1]
```

$$m_{FG}(HT) = \sum_{FG} P(FG) m_{HG}(HT, FG)$$

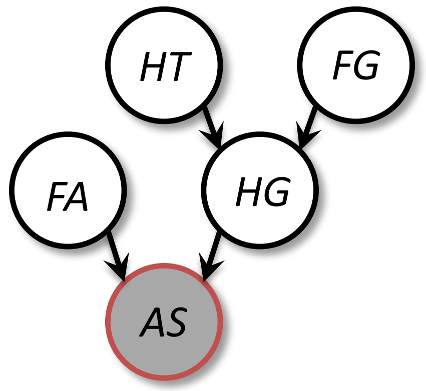
```
prob_HT = m_FG[HT] * table_HT[HT]
```

$$P(AS = 1, HT) = m_{FG}(HT) P(HT)$$

Nuclear power plant (cont.)

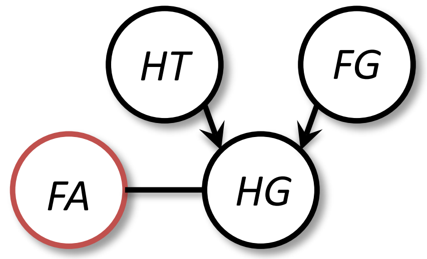
$$= \Pr(HT)\sum_{FG}\Pr(FG)\sum_{HG}\Pr(HG|HT,FG)\sum_{FA}\Pr(FA)\Pr(AS=t|FA,HG)$$

eliminate *AS*: since *AS* observed, really a no-op



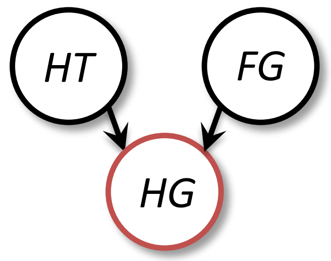
$$= \Pr(HT)\sum_{FG}\Pr(FG)\sum_{HG}\Pr(HG|HT,FG)\sum_{FA}\Pr(FA)\,m_{AS}(FA,HG)$$

eliminate *FA*: multiplying 1x2 by 2x2



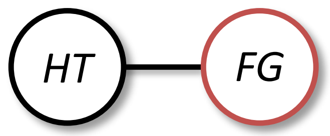
$$= \Pr(HT)\sum_{FG}\Pr(FG)\sum_{HG}\Pr(HG|HT,FG)\,m_{FA}(HG)$$

eliminate *HG*: multiplying 2x2x2 by 2x1



$$= \Pr(HT)\sum_{FG}\Pr(FG)\,m_{HG}(HT,FG)$$

eliminate *FG*: multiplying 1x2 by 2x2



$$= \Pr(HT)\,m_{FG}(HT)$$

Multiplication of tables, followed by summing, is actually matrix multiplication

$m_{FA}(HG)$=

FA f	*FA* t
0.6	0.4

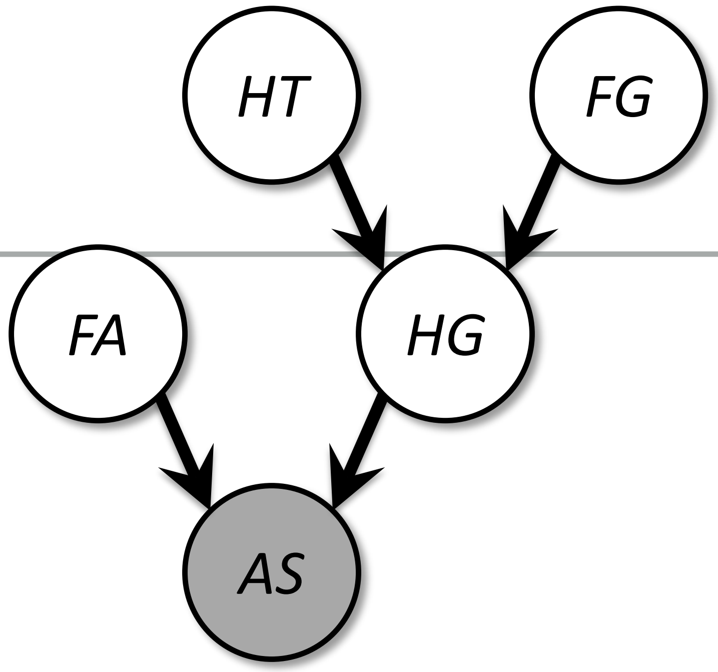
x

FA		*HG* f	*HG* t
F	f	1.0	0
A	t	0.8	0.2

But why the order $FA \rightarrow HG \rightarrow FG \rightarrow HT$?

```
m_AS = table_AS_FA_HG[1, :, :] 0

m_FA = table_FA[0] * m_AS[0, :] + table_FA[1] * m_AS[1, :]
                    2                            2
m_HG = (table_HG_HT_FG[0, :, :] * m_FA[0] + 4
         table_HG_HT_FG[1, :, :] * m_FA[1])  4

m_FG = table_FG[0] * m_HG[:, 0] + table_FG[1] * m_HG[:, 1]
                    2                            2
prob_HT = m_FG[HT] * table_HT[HT]
                    2
print(prob_HT)              in total 2*2 + 4*2 + 2*2 + 2 = 18
--------------
[ 0.0672  0.2528]
```

Try to eliminate *HG* after *AS*



```
m_AS = table_AS_FA_HG[1, :, :]

prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for FA in [0, 1]:
            for HG in [0, 1]:
                prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                                table_HG_HT_FG[HG, HT, FG] *
                                table_FA[FA] *
                                m_AS[FA, HG]
                               )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

Try to eliminate *HG* after *AS*

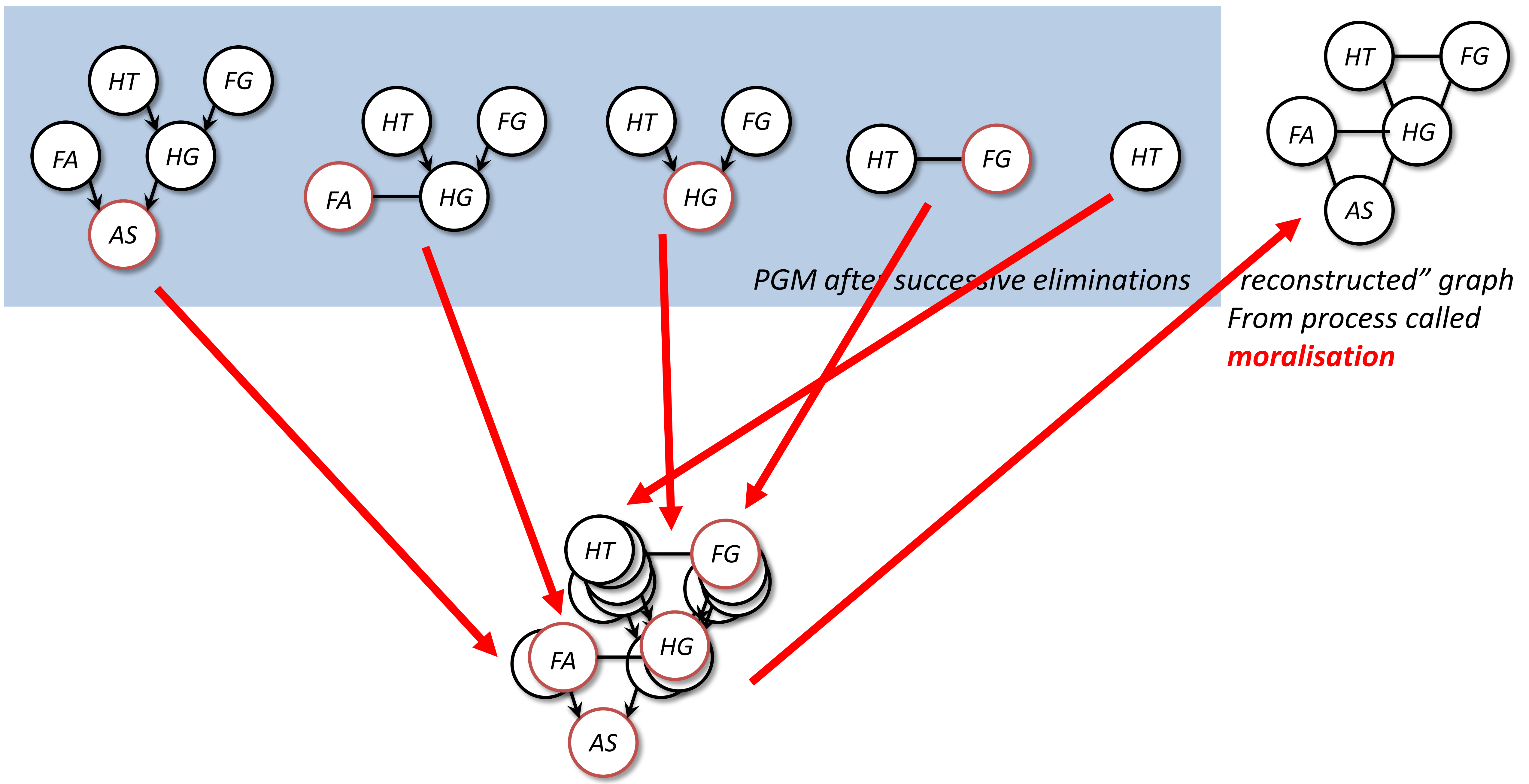
```
m_AS = table_AS_FA_HG[1, :, :]

def m_HG(FA, HT, FG):     already 8*2 = 16 multiplications
    return (m_AS[FA, 0] * table_HG_HT_FG[0, HT, FG] +
            m_AS[FA, 1] * table_HG_HT_FG[1, HT, FG])
                        because HG connected to 3 other nodes
prob_HT = np.zeros(2)
for HT in [0, 1]:
    for FG in [0, 1]:
        for FA in [0, 1]:
            prob_HT[HT] += (table_FG[FG] * table_HT[HT] *
                            table_FA[FA] *
                            m_HG(FA, HT, FG)
                           )
print(prob_HT)
--------------
[ 0.0672  0.2528]
```

A summary for elimination algorithms

- An efficient way to marginalize random variables

- The order of elimination affects the efficiency
  - Removing a node with many children and parents results in very large clique (message matrix)
  - Time complexity exponential in the largest clique

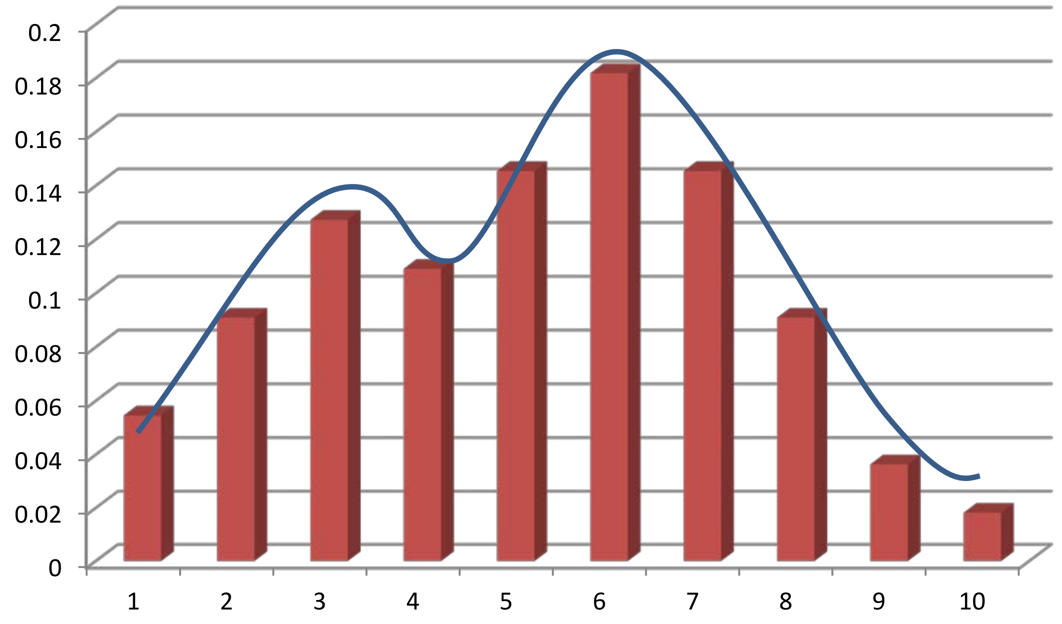
- By the way, what is the reconstructed graph?



- Put them together → the reconstructed graph

Probabilistic inference by simulation

- Exact probabilistic inference can be expensive/impossible
- Can we approximate numerically?
- Idea: sampling methods
  - Cheaply sample from desired distribution
  - Approximate **distribution** by **histogram of samples**



A summary for sampling methods

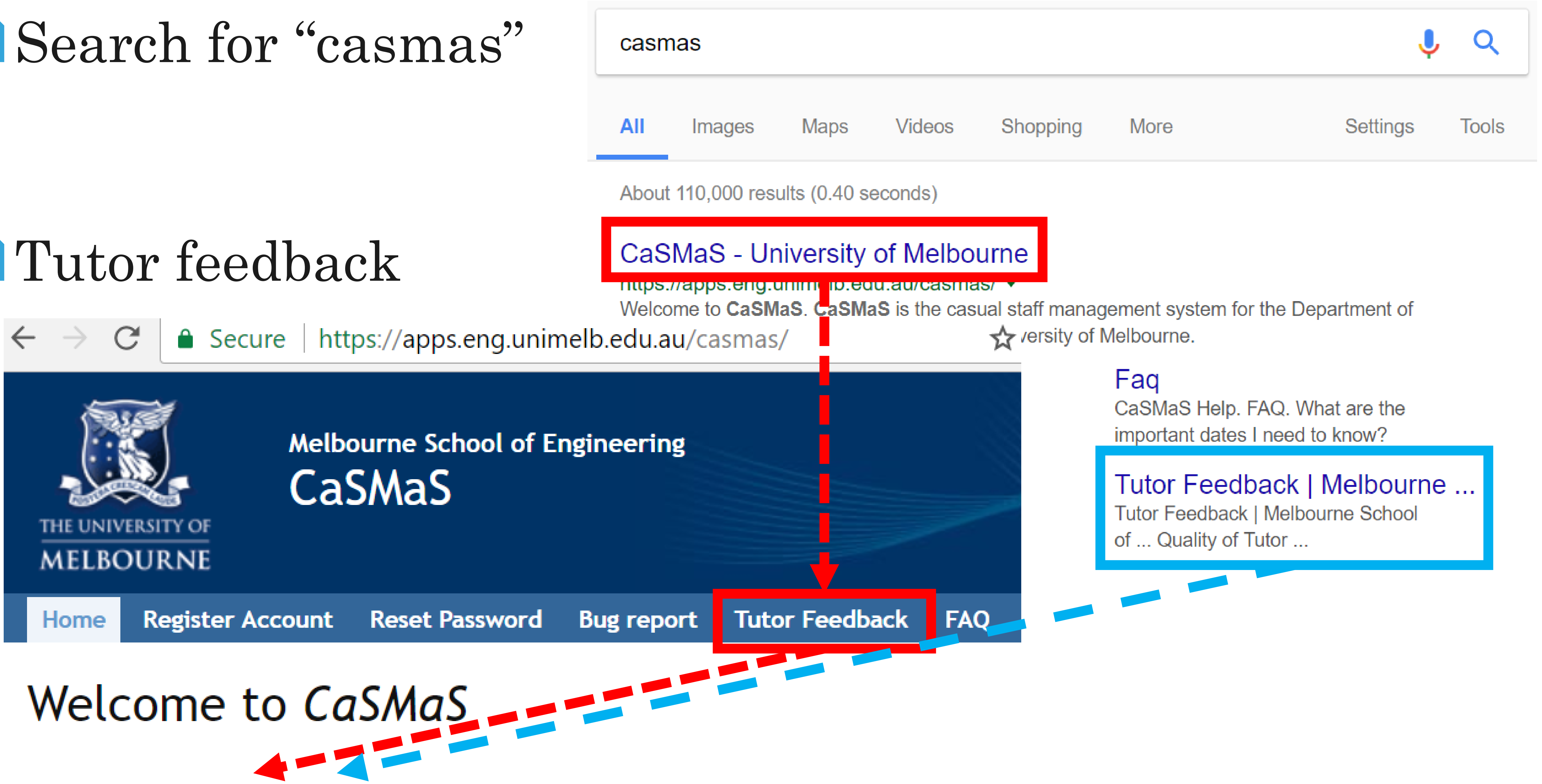
- If we get samples, we can
  - use them to calculate expectations (approximately)
  - approximate distributions by histogram of samples
- Useful when exact inference is expensive or impossible
- There are many methods can sample from unnormalized distributions
  - Very useful because normalization is the main challenge for Bayesian inference

A summary for EM algorithm

- Designed for MLE when there are latent variables
- The joint $P(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{w})$, $\boldsymbol{X}$ observed, $\boldsymbol{Z}$ unobserved, $\boldsymbol{w}$ paras
- MLE for $\boldsymbol{w}$: $\max_{\boldsymbol{w}} \log P(\boldsymbol{X}|\boldsymbol{w}) = \log \sum_{\boldsymbol{Z}} P(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{w})$
- Due to the marginalization for Z, $P(\boldsymbol{X}|\boldsymbol{w})$ is complicated
  - Gradients are often difficult to calculate
- EM can deal with $P(\boldsymbol{X}|\boldsymbol{w})$ by
  - E-step: estimate $P(\boldsymbol{Z}|\boldsymbol{X}, \boldsymbol{w})$
  - M-step: MLE for $\boldsymbol{w}$ using $P(\boldsymbol{Z}|\boldsymbol{X}, \boldsymbol{w})$, $\max_{\boldsymbol{w}} E_{\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{w}}[\log P(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{w})]$
    - Beneficial because $P(\boldsymbol{X}, \boldsymbol{Z}|\boldsymbol{w})$ can be factorized
- Sensitive to initialization, may converge to different results

Tutor Feedback

- ❑ Search for “casmas”
- ❑ Tutor feedback


casmas

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About 110,000 results (0.40 seconds)

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Tutor Feedback | Melbourne School of ... Quality of Tutor ...

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Welcome to *CaSMaS*

- ❑ COMP90051 → select a class → 5:15pm or 6:16pm → …

- Good luck on your exams!