## acoustic PCA

## January 14, 2025

```
[3]: import numpy as np
       import pandas as pd
       import matplotlib.pyplot as plt
       from sklearn.decomposition import PCA
      This notebook performs PCA on the set of acoustic features and the scaled feature differences.
      First, load the features.
[102]: orig_feat = pd.read_csv('../../data/processed/expanded/joined_with_features/
        ⇔all_features_scaled.csv')
[104]: # grab the acoustic features only
       acoustic_feat = orig_feat.iloc[:,1:21]
[106]: acoustic_feat
[106]:
                       spectral_bandwidth
                                            spectral_centroid
                                                                spectral_contrast
                 rms
       0
           -0.341878
                                 0.066554
                                                     0.440424
                                                                         0.506586
       1
           -0.434774
                                -1.315118
                                                    -1.049622
                                                                         0.290653
           -0.492169
                                                    -0.285376
       2
                                 0.172736
                                                                        -0.988950
       3
           -0.559820
                                -0.177484
                                                    -0.196093
                                                                        -1.524784
       4
            0.090652
                                -0.837952
                                                    -1.337497
                                                                         1.970131
       135 0.185126
                                -0.118947
                                                     0.339915
                                                                         0.426610
       136 0.348829
                                -0.637229
                                                    -1.384783
                                                                        -0.149211
       137 -0.548183
                                 1.734812
                                                     1.422873
                                                                        -0.461114
       138 -0.503017
                                -0.650594
                                                    -0.730586
                                                                         1.634236
       139 -0.447200
                                -0.991865
                                                    -0.915270
                                                                         2.026114
            spectral_flatness
                                spectral_rolloff
                                                   zero_crossing_rate
                                                                        mfccs_offset \
       0
                                        0.348311
                                                              0.376068
                                                                           -0.112493
                     -0.451332
       1
                     -0.480526
                                       -1.192311
                                                             -0.551984
                                                                           -0.536671
       2
                     -0.445185
                                       -0.216417
                                                             -0.802337
                                                                           -0.543731
       3
                     -0.308431
                                       -0.259075
                                                             -0.201213
                                                                           -0.912058
       4
                     -0.491282
                                       -1.457028
                                                             -1.581369
                                                                           -1.173415
```

0.228797

-1.195560

135

136

1.652219

-0.474380

-0.126447

-0.323738

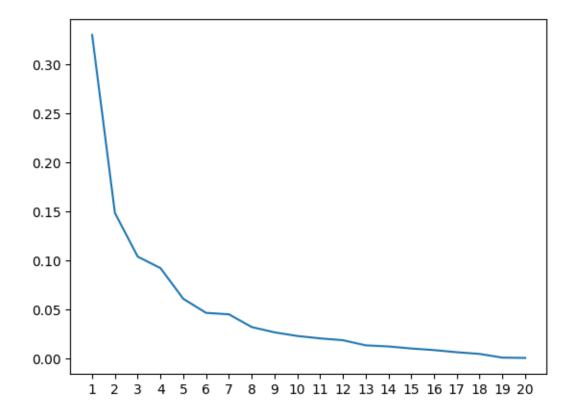
0.488865

-1.526804

```
137
            -0.379113
                               1.486074
                                                    1.051931
                                                                 -0.158671
138
             -0.462087
                              -0.742727
                                                   -0.429558
                                                                 -0.768359
139
            -0.491282
                              -0.991267
                                                   -0.528140
                                                                 -0.429696
                 mfcc2
                           mfcc3
                                     mfcc4
                                               mfcc5
                                                         mfcc6
       mfcc1
                                                                   mfcc7
0
   -0.778928 -1.015857 -0.225135 -0.330494 -0.638302
                                                      1.087056
                                                                0.617203
    0.969615 -0.115785 -1.588444 -0.826390 -2.254977
                                                      0.542361
1
                                                                 0.485861
2
   -0.006269
              0.679399 0.135850 -0.210339
                                            0.151981
                                                      1.526968
                                                                0.076963
3
   -0.862695 0.459434 -0.645434 1.070612 -0.234536
                                                      1.112880
                                                                0.882770
    0.428365
              2.255569 -0.161954 1.440183
                                           1.695354
                                                      0.449397 -1.179729
. .
                         •••
         •••
                                         •••
135 -1.586942 0.015508 -1.168933 -0.131453 -0.749538
                                                      1.346404 -1.026266
136 0.813300
              2.607361 0.570631 0.982699 -0.512595
                                                      0.539046 -0.387748
137 -0.893742 1.338935 1.328147 -1.306719 0.295250 -1.144739
                                                                0.782838
138 0.555112 0.032074 -0.971100 1.406824 -0.306330 -0.644864
                                                                1.568164
139
    1.566300 -0.290227 -0.851044 0.180735 -0.351361 -0.675017 0.457030
       mfcc8
                 mfcc9
                          mfcc10
                                    mfcc11
                                              mfcc12
0
    1.456421 0.029930 -0.097627 -0.397106 -0.374183
    0.833896 -0.633815 -0.933258 -0.018102
1
                                            0.073077
2
    0.006843 -0.047588 0.013357 -0.181287
                                            0.635608
3
    0.250759 -0.152036 0.972097 1.063965
                                            0.492936
    0.455243 -1.272529 2.203114 0.459022
                                            0.152644
135 -0.020775 -0.274373
                        1.414720 -0.118922 2.127656
136 -1.365056 -0.655223 -1.220662 -0.922210 -0.017725
137 -0.333403 0.556197
                        0.074172 0.798691 0.345922
138  0.028022  -1.310035  1.988121  0.331271  -0.285788
139 1.271847 -2.368415 0.206971 0.752087 -1.162734
```

[140 rows x 20 columns]

## 1 PCA: all features



```
[114]: print(pca.explained_variance_ratio_)
      [3.30314164e-01 1.48352958e-01 1.03767708e-01 9.19851025e-02
       6.06407776e-02 4.62850491e-02 4.48109996e-02 3.18145259e-02
       2.64058590e-02 2.26731623e-02 2.02050287e-02 1.83682637e-02
       1.31171139e-02 1.19525921e-02 9.87022726e-03 8.24868444e-03
       6.07217388e-03 4.33844904e-03 5.46481042e-04 2.30679912e-04]
      Print cumulative sums of explained variance
[117]: for i in range(len(pca.explained_variance_ratio_)):
           print("Variance explained by first %d PCs: %f"%(i+1, np.sum(pca.
        →explained_variance_ratio_[:i+1])))
      Variance explained by first 1 PCs: 0.330314
      Variance explained by first 2 PCs: 0.478667
      Variance explained by first 3 PCs: 0.582435
      Variance explained by first 4 PCs: 0.674420
      Variance explained by first 5 PCs: 0.735061
      Variance explained by first 6 PCs: 0.781346
      Variance explained by first 7 PCs: 0.826157
      Variance explained by first 8 PCs: 0.857971
      Variance explained by first 9 PCs: 0.884377
```

```
Variance explained by first 12 PCs: 0.945624
      Variance explained by first 13 PCs: 0.958741
      Variance explained by first 14 PCs: 0.970693
      Variance explained by first 15 PCs: 0.980564
      Variance explained by first 16 PCs: 0.988812
      Variance explained by first 17 PCs: 0.994884
      Variance explained by first 18 PCs: 0.999223
      Variance explained by first 19 PCs: 0.999769
      Variance explained by first 20 PCs: 1.000000
      Transform the original data and save.
[120]:
      acoustic_PCs = pca.transform(acoustic_feat)
[122]: pcs_df = pd.DataFrame(acoustic_PCs)
       pcs_df.insert(0, 'song_id', orig_feat['song_id'])
[124]:
      pcs_df
[124]:
                                                             0
                                                                                 2 \
                                            song_id
                                                                       1
       0
                AChangeIsGonnaCome_SamCooke_Cover5S 0.022937 -0.598798 -0.371011
       1
             AChangeIsGonnaCome_SamCooke_Original5S -2.891199 0.801698 -0.356564
       2
               AnotherOneBitesTheDust_Queen_Cover5S -0.913380 -1.091785 -0.395535
       3
            AnotherOneBitesTheDust_Queen_Original5S -0.794861 -1.910355 -0.635595
       4
                        BadDay_DanielPowter_Cover5S -3.152560 -0.288662 1.389626
       135
                   UptownFunk_MarkRonson_Original5S -0.232629 -0.322775
                                                                         1.001521
       136
                                   Weak_SWV_Cover5S -2.388267 1.401820
       137
                                Weak_SWV_Original5S 3.343473 -0.571673 -1.204907
                YouLightUpMyLife_DebbyBoone_Cover5S -1.767656 -0.165546
       138
             YouLightUpMyLife_DebbyBoone_Original5S -2.536229 1.136776
       139
                   3
                                                            7
       0
            0.781111
                      0.195429
                               0.796818 -0.867474 0.459830 -0.492015
       1
            0.223085 - 1.766700 \quad 0.254210 - 0.920953 - 0.823098 - 0.367784
       2
           -0.441324 0.122700 0.161476 1.209218 -1.031817 -0.000057
       3
           -1.281066 -0.055198
                                1.522063 -0.247223 -0.635801 0.812591
       4
           -1.504343
                      3.230442 -0.824530 -0.913582 -0.867813 -0.423625
       . .
                               2.997795 0.367906 -1.185539 -0.971931
       135
           1.230162
                      0.733727
       136 -1.999824
                      0.428433 -1.433496 2.149519 -0.790014
                                                               0.576768
       137 -0.446358
                      0.817755 -0.287270 0.270217 -1.468730
                                                               0.117160
       138 -0.536804
                      0.917327 -0.338145 -2.952617 -0.548051
                                                               0.838635
       139 0.181811
                      0.405505 -0.870769 -3.020261 -0.020712
                                                               0.358136
                  10
                            11
                                      12
                                                13
                                                           14
                                                                     15
                                                                               16 \
```

Variance explained by first 10 PCs: 0.907050 Variance explained by first 11 PCs: 0.927255

```
1
          -0.486655 0.955277 0.052875 -0.697857 0.191931 -0.150907
                                                                      0.919135
      2
          -0.190678
                     3
          -0.007897
                     0.026402 - 0.315817 - 0.299160 - 0.691801 - 0.291633
                                                                      0.074297
      4
           0.216188 - 1.203085 - 0.131449 \ 0.939943 - 0.748131 \ 0.390390 - 0.278106
      135 0.984724 -0.418768 -0.996076 -0.337870 -0.218413 -0.227481
                                                                      0.121529
      136  0.226431  -0.245209  -0.176748  -0.647325  -0.636993
                                                            0.018237
                                                                      0.334956
      137 -0.671188 -0.217582 -0.325740 0.422804 0.652490 -0.070668
                                                                      0.506741
      138  0.969405  -0.958001  0.340119  -0.980307  -0.155315
                                                            0.411489
                                                                      0.162836
      139 -1.020507 -0.172352  0.841387 -0.085314 -0.347884
                                                            0.733405
                                                                      0.020393
                 17
                           18
      0
           0.013079 0.014336 -0.038065
      1
          -0.065129 0.053215 0.010242
           0.006992 -0.099736 -0.068110
      2
      3
           0.150395 -0.069100 0.028115
      4
           0.021525 -0.165151 -0.084024
      135 -0.306035 0.051720 0.056162
      136 -0.263425 -0.039031
                               0.058868
      137 -0.022114 -0.067893 -0.020132
      138 0.042291 0.037398 0.132067
      139 -0.305660 -0.004176 -0.001018
      [140 rows x 21 columns]
[126]: pcs_df.to_csv('../../data/processed/expanded/joined_with_features/acoustic_PCs.
        ⇔csv', index=False)
         PCA on scaled feature differences
 [5]: scaled_feat_diffs = pd.read_csv('../../data/processed/expanded/
        ojoined_with_features/similarity_and_feature_scaled_differences.csv')
 [9]: scaled_feat_diffs
 [9]:
                               song_id mean_sim
                                                 rms_difference
      0
           AChangeIsGonnaCome_SamCooke
                                                       -0.935007
                                        0.554166
      1
          AnotherOneBitesTheDust_Queen -0.844765
                                                       -0.998811
      2
                   BadDay_DanielPowter
                                                       -1.117945
                                        0.312090
      3
              BetteDavisEyes_KimCarnes
                                        0.351453
                                                       -0.483394
      4
              BlindingLights_TheWeeknd
                                        0.415845
                                                        1.612170
      65
            TossinAndTurnin_BobbyLewis -1.140849
                                                       0.210475
            UnbreakMyHeart_ToniBraxton -0.839904
      66
                                                       0.611743
                 UptownFunk_MarkRonson 1.742074
      67
                                                       0.164616
```

0.033540 1.727875 0.501784 -0.549684 0.055635 -0.471566

0.162679

0

```
68
                         Weak_SWV -0.816959
                                                     1.097252
69
     YouLightUpMyLife_DebbyBoone 1.922587
                                                    -1.028719
    spectral_bandwidth_difference
                                    spectral_centroid_difference
0
                          0.424603
                                                          0.506157
1
                         -0.861913
                                                         -1.148024
2
                         -0.624503
                                                         -0.705878
3
                         -0.072352
                                                         -0.695101
4
                          0.384503
                                                          0.617517
. .
                         -0.904161
65
                                                         -1.084117
66
                         -0.422557
                                                         -0.395587
67
                         -0.950916
                                                         -0.996790
68
                          1.659879
                                                          2.062143
                                                         -1.035364
69
                         -0.873076
    spectral_contrast_difference
                                    spectral_flatness_difference
0
                                                        -0.479597
                        -0.862609
1
                        -0.374865
                                                        -0.284250
2
                         0.405524
                                                        -0.504713
3
                         0.503072
                                                        -0.532619
4
                        -0.033445
                                                        -0.370761
65
                         0.027523
                                                        -0.287040
                        -0.679705
                                                        -0.451690
66
67
                         0.832299
                                                         0.391093
                                                        -0.359598
68
                        -0.716286
69
                        -0.594350
                                                        -0.479597
    spectral_rolloff_difference
                                   zero_crossing_rate_difference
0
                        0.600823
                                                        -0.171949
1
                       -1.185768
                                                        -0.561438
2
                       -0.763117
                                                         0.251408
3
                       -0.584231
                                                        -0.893569
4
                        0.458411
                                                         0.520171
                       -1.043944
                                                        -0.396465
65
                       -0.188428
                                                        -0.590390
66
67
                       -0.885867
                                                        -1.266669
                        1.961684
                                                         1.794613
68
69
                       -0.940217
                                                        -1.160147
    mfccs_offset_difference
                             ... mfcc11_difference mfcc12_difference
0
                   -0.564198
                                          -0.672166
                                                              -0.697801
1
                   -0.649801
                                           0.165635
                                                              -1.075877
2
                    1.027642
                                                              -0.993376
                                           0.520546
3
                   -0.113570
                                          -0.615367
                                                              -0.090300
```

```
4
                    0.476984 ...
                                           -0.272579
                                                               -0.955178
. .
                          ... ...
                                               •••
                                                                 •••
65
                   -0.385703
                                            0.396940
                                                               -0.581084
66
                   -0.583549
                                            0.678075
                                                                0.045202
67
                   -0.575489
                                           -0.745710
                                                                0.198308
68
                   -0.961336
                                            0.625665
                                                               -0.801587
                                           -0.631727
                   -0.695267 ...
                                                               -0.164445
69
    mfccs dist
                tempo_difference key_difference
                                                     treble difference
0
      0.053461
                        -0.101062
                                         -0.136785
                                                              -0.717137
1
     -0.986748
                                          -0.136785
                                                              -0.717137
                          1.667529
2
     1.436681
                         0.606374
                                          -0.754522
                                                               1.374513
3
     -0.404296
                        -0.384037
                                          -0.754522
                                                              -0.717137
4
     -0.617530
                        -0.596268
                                          -0.754522
                                                              -0.717137
65
     -1.045328
                         0.323400
                                           2.334165
                                                              -0.717137
66
     -1.058154
                         3.436120
                                           1.098690
                                                               1.374513
67
     -1.094935
                        -0.808499
                                          -0.754522
                                                              -0.717137
68
      0.559681
                        -0.525524
                                           0.480953
                                                              -0.717137
69
     -0.741078
                        -0.596268
                                          -0.754522
                                                              -0.717137
    tbb_difference voice_type_difference mean_valence_difference \
0
         -0.672150
                                  -0.718171
                                                             -1.162158
1
         -0.672150
                                  -0.718171
                                                             -0.110879
2
          1.466509
                                   1.467567
                                                             -0.859291
3
         -0.672150
                                  -0.718171
                                                             -0.991589
         -0.672150
                                  -0.718171
                                                             -0.877836
                                      •••
                                                               •••
65
         -0.672150
                                  -0.718171
                                                              0.304665
          1.466509
                                   1.467567
66
                                                              1.637876
         -0.672150
                                  -0.718171
67
                                                             -0.610043
68
         -0.672150
                                  -0.718171
                                                             -0.375461
69
         -0.672150
                                  -0.718171
                                                             -0.366734
    mean_arousal_difference
0
                    1.051598
1
                    0.899755
2
                   -0.283486
3
                   -0.497091
4
                   -0.082950
. .
65
                   -1.058572
66
                    0.833031
67
                   -1.106173
                   -0.604696
68
69
                   -0.994633
```

## [70 rows x 30 columns]

```
[17]: acoustic_feat_diffs = scaled_feat_diffs.iloc[:,2:22]
      acoustic_feat_diffs
      # don't include MFCCs distance because that's already a summary of the MFCCs
[17]:
          rms_difference spectral_bandwidth_difference
      0
               -0.935007
                                                 0.424603
               -0.998811
      1
                                                -0.861913
               -1.117945
                                                -0.624503
      3
               -0.483394
                                                -0.072352
                1.612170
                                                 0.384503
      4
                0.210475
      65
                                                -0.904161
                0.611743
      66
                                                -0.422557
                0.164616
                                                -0.950916
      67
      68
                1.097252
                                                 1.659879
               -1.028719
                                                -0.873076
          spectral_centroid_difference
                                         spectral_contrast_difference
      0
                               0.506157
                                                              -0.862609
      1
                              -1.148024
                                                              -0.374865
      2
                              -0.705878
                                                               0.405524
      3
                              -0.695101
                                                              0.503072
      4
                               0.617517
                                                              -0.033445
      . .
      65
                              -1.084117
                                                               0.027523
      66
                              -0.395587
                                                              -0.679705
      67
                              -0.996790
                                                               0.832299
      68
                               2.062143
                                                              -0.716286
      69
                              -1.035364
                                                              -0.594350
          spectral_flatness_difference
                                         spectral_rolloff_difference
      0
                              -0.479597
                                                              0.600823
                              -0.284250
                                                             -1.185768
      1
      2
                              -0.504713
                                                             -0.763117
      3
                              -0.532619
                                                            -0.584231
      4
                              -0.370761
                                                              0.458411
      65
                              -0.287040
                                                             -1.043944
      66
                              -0.451690
                                                            -0.188428
                               0.391093
                                                            -0.885867
      67
      68
                              -0.359598
                                                              1.961684
      69
                              -0.479597
                                                            -0.940217
          zero_crossing_rate_difference mfccs_offset_difference mfcc1_difference \
                               -0.171949
                                                         -0.564198
                                                                             0.943430
      0
```

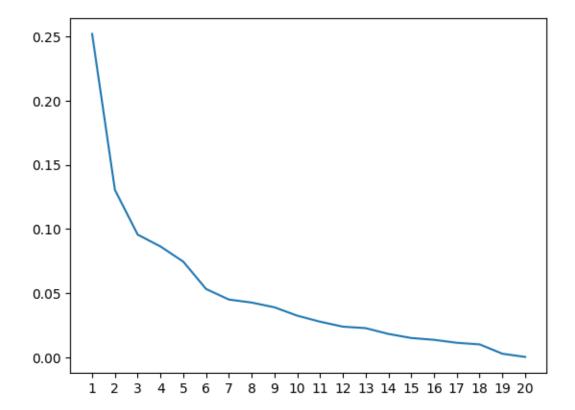
```
1
                         -0.561438
                                                    -0.649801
                                                                        -0.223937
2
                          0.251408
                                                     1.027642
                                                                        0.066898
3
                         -0.893569
                                                    -0.113570
                                                                        0.330067
4
                          0.520171
                                                     0.476984
                                                                        0.250896
65
                         -0.396465
                                                    -0.385703
                                                                        -1.244793
                                                                        -1.246677
66
                         -0.590390
                                                    -0.583549
67
                         -1.266669
                                                    -0.575489
                                                                       -1.120696
68
                          1.794613
                                                    -0.961336
                                                                        0.889125
69
                         -1.160147
                                                    -0.695267
                                                                        -0.021426
    mfcc2_difference mfcc3_difference
                                          mfcc4_difference
                                                             mfcc5_difference
0
            -0.125612
                                0.381175
                                                  -0.792648
                                                                      0.970694
1
           -0.831154
                               -0.417782
                                                   0.334562
                                                                     -0.789615
2
             1.979177
                                0.153676
                                                   1.665942
                                                                      2.695823
3
            0.156865
                               -0.401805
                                                  -1.467522
                                                                     -0.568214
4
           -0.926032
                               -1.152439
                                                  -1.152338
                                                                      0.439341
. .
           -0.476234
                               -0.048247
                                                  -0.734066
                                                                     -0.326068
            -1.030751
                               -0.655654
                                                                     -0.074347
66
                                                   0.126229
67
           -0.689092
                               -1.416031
                                                   0.365155
                                                                     -0.395500
68
            0.256517
                               -0.450408
                                                   1.782553
                                                                     -0.186711
           -0.724992
                               -1.325462
                                                   0.255788
                                                                     -1.278267
69
    mfcc6_difference mfcc7_difference
                                          mfcc8 difference
                                                              mfcc9 difference \
0
           -0.581842
                               -0.983287
                                                  -0.565536
                                                                     -0.474225
                               -0.244787
                                                  -1.068977
1
           -0.766987
                                                                     -1.141754
2
            0.744638
                                                  -0.954179
                                                                     -0.463830
                               -0.755445
                               -0.141675
3
           -1.027181
                                                  -0.487904
                                                                     -0.515454
                                                                     -0.562841
4
            0.953000
                               -0.877507
                                                   0.863111
            -0.477608
65
                               -0.519190
                                                   0.607264
                                                                      0.598843
66
           -0.584739
                                0.887475
                                                  -0.252086
                                                                     -1.112414
            0.927327
                               -0.642599
                                                  -0.804383
                                                                     -1.043626
                                0.154626
                                                  -0.021512
                                                                      0.179435
68
            1.032908
69
           -1.311245
                                0.089530
                                                   0.260616
                                                                     -0.003220
                        mfcc11_difference
                                            mfcc12 difference
    mfcc10_difference
0
            -0.161547
                                 -0.672166
                                                     -0.697801
1
              0.007680
                                  0.165635
                                                     -1.075877
2
              3.358993
                                  0.520546
                                                     -0.993376
3
              0.173161
                                 -0.615367
                                                     -0.090300
             -0.217041
                                                     -0.955178
4
                                 -0.272579
              0.087054
                                  0.396940
65
                                                     -0.581084
             -0.612336
66
                                  0.678075
                                                      0.045202
67
              1.473048
                                 -0.745710
                                                      0.198308
```

```
68 0.469681 0.625665 -0.801587
69 1.138178 -0.631727 -0.164445
```

[70 rows x 20 columns]

```
[21]: pca = PCA()
pca.fit(acoustic_feat_diffs)
```

[21]: PCA()



Variance explained by first 1 PCs: 0.251840 Variance explained by first 2 PCs: 0.382331 Variance explained by first 3 PCs: 0.478004 Variance explained by first 4 PCs: 0.564475

```
Variance explained by first 5 PCs: 0.639120
     Variance explained by first 6 PCs: 0.692495
     Variance explained by first 7 PCs: 0.737659
     Variance explained by first 8 PCs: 0.780484
     Variance explained by first 9 PCs: 0.819606
     Variance explained by first 10 PCs: 0.852239
     Variance explained by first 11 PCs: 0.880167
     Variance explained by first 12 PCs: 0.904239
     Variance explained by first 13 PCs: 0.927165
     Variance explained by first 14 PCs: 0.945586
     Variance explained by first 15 PCs: 0.960831
     Variance explained by first 16 PCs: 0.974669
     Variance explained by first 17 PCs: 0.986200
     Variance explained by first 18 PCs: 0.996472
     Variance explained by first 19 PCs: 0.999488
     Variance explained by first 20 PCs: 1.000000
[27]: acoustic_diffs_PCs = pca.transform(acoustic_feat_diffs)
[29]: pcs_df = pd.DataFrame(acoustic_diffs_PCs)
      pcs_df.insert(0, 'song_id', scaled_feat_diffs['song_id'])
[31]: pcs_df
[31]:
                               song_id
                                               0
           AChangeIsGonnaCome_SamCooke -0.374017 -2.093954 -0.983404
      0
                                                                      0.172342
          AnotherOneBitesTheDust Queen -2.767174 -0.222172 -0.021951
      1
      2
                   BadDay_DanielPowter 0.083418 0.683486 1.049502 -2.687584
      3
             BetteDavisEyes_KimCarnes -1.655406 -0.250530 0.153637
      4
             BlindingLights_TheWeeknd 0.494351 -0.741291 -0.991394
                                                                      0.650119
            TossinAndTurnin BobbyLewis -1.657682
      65
                                                 1.330511 -0.453783 -0.011227
            UnbreakMyHeart_ToniBraxton -1.428879 0.381009 -0.340910 1.321582
      66
      67
                 UptownFunk MarkRonson -2.232949 1.453336 -0.411766 -0.308380
      68
                              Weak_SWV 3.328566 -1.946731 -0.381569 -0.002573
           YouLightUpMyLife_DebbyBoone -2.353917 0.429639 -0.659606 -0.257131
      69
                 4
                           5
                                     6
                                               7
                                                         8
                                                                     10
                                                                               11
      0
          0.229117 0.727443 0.842296 -0.291432
                                                  0.131026
                                                            ... 0.184890 -0.173028
      1
          0.673920 -0.697974 -0.130476 -0.713310
                                                 0.314995
                                                           ... -0.335039
                                                                         0.569564
      2
          3.171424 1.838529 1.520556 0.872181
                                                  0.569732
                                                           ... -0.365611
                                                                         0.555498
      3 -1.378321 -0.141657 0.381310
                                        0.411631
                                                  0.251046
                                                            ... -0.074047
                                                                         0.193762
       -0.141926  0.821419 -1.611815  1.600517 -0.736378 ... -0.577032
                                                                         0.358192
      65 -0.084075 -0.638924 -0.019345 0.239198 -1.161473 ... 0.131338 -0.182919
         0.514653 -1.018259 0.043578 -0.183472 -0.326499
                                                            ... -0.450417
                                                                         0.599480
      67
          0.387419 0.618492 -0.857742 1.006386 1.240431 ... -1.420942 0.428414
         1.934744 -1.082110 -1.249410 0.070914 0.263362 ... -1.174703 -0.004119
```

```
69 -0.165968 -1.593396 -0.583401 -0.026247 1.026941 ... 0.838128 0.300880
         12
                   13
                             14
                                      15
                                                16
                                                          17
                                                                    18 \
   0.401422 \quad 0.907841 \quad 0.414219 \ -0.082112 \quad 0.273074 \ -0.314486 \ -0.218664
   0.269630 - 0.375228 \ 0.386080 \ 0.102849 \ 0.746984 \ 0.251434 \ 0.227936
1
2
   0.710201 \quad 0.129054 \quad -0.514092 \quad 0.171567 \quad 0.877684 \quad -1.008613 \quad 0.184720
   0.595595 0.794270 0.248803 -0.354313 0.259602 -0.176120 0.075333
3
4 -1.276384 0.744934 0.187789 0.148970 0.253568 -0.346319 -0.075830
66 -0.911863 -0.066470 -0.909736 0.116486 -0.521595 0.596628 -0.271913
67 0.444618 -0.733999 0.043695 0.305817 -0.719720 0.003388 -0.178033
68 -0.131696 -0.113355 -0.077465 -0.308537 -0.598912 -0.012273 0.127976
69 0.712892 -0.196497 -0.001033 0.051079 -0.052375 -0.680431 -0.346798
         19
   0.011600
0
   0.046095
1
2 0.059770
   0.059403
3
4 -0.101702
. .
65 0.038593
66 0.006634
67 0.010751
68 0.035059
69 -0.035687
[70 rows x 21 columns]
```