

# **Interference, Diffraction and Polarization of Electromagnetic Waves**

## **Lab Report**

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## Abstract

This lab report elaborates on the EM lab conducted to analyse phenomena in wave propagation such as wave diffraction, wave interference and wave polarization.

In the first part of the report, key theory will be introduced that is required to successfully understand the results of this lab. We will specifically define what wave interference, wave diffraction and polarisation means. Then, in the same section we will look at the mathematical tools that will enable us to make more concise and accurate data analysis on our measurements, such as the Taylor-series expansion and normalization of a dataset.

The first task is the pre-lab exercise where interference was studied. We will take a look at the recreation of Young's double-slit experiment, derive equations and approximations for the electric field intensity as the function of distance parameters, and use a pre-given dataset to analyse the differences between our theoretical and approximated equations. A dielectric will introduce some phase shift into our signal and based on our shifted interference patterns, we will need to determine the relative permittivity of the material.

The second task is done in the lab where we will see how a conducting sheet will cause the waves to diffract and how a metal wire grid can be used to determine the polarization orientation of the signal. The data we measured for electric field intensity will be compared to theoretical values from the Fresnel's diffraction theory.

## Introduction

The aim of this lab is to conduct experiments that helps us further understand wave propagation, a phenomenon crucial in both physics and engineering. We will look at theories about diffraction and interference and relate these theoretical concepts to real, quantifiable measurements. Light waves, radio waves, microwaves, audio technologies, radar systems all utilize the properties of wave propagation, therefore it is essential for us to try to quantify these behaviours. The lab will mostly regard electromagnetic waves, however these derivations, results and conclusions are valid for most wave types.

## Theory

In this section of the report, I will introduce a few key theoretical concepts – both mathematical and physical - crucial for successfully completing the pre-lab and the lab task.

The first concept is interference of waves. Generally, we talk about wave interference in physics when two or more waves meet and superpose with one another forming a “field” of deformed waves where the amplitudes can be greater, lower or equal depending on what amplitudes add together. This amplitude at a given point at a given time is the vector sum of the individual waves that are incident at this point at a particular time. If the waves that meet have roughly the same frequency, phase difference and also same waveform, we call them coherent waves, which is a required property for stationary interference to happen.

Constructive interference happens when the phase difference between waves is a multiple of  $2\pi$  ( $360^\circ$ ) and crests of waves meet, forming an amplitude which is the sum of individual amplitudes. On the other hand, destructive interference is when coherent waves meet with a difference of odd multiples of  $\pi$  ( $180^\circ$ ).

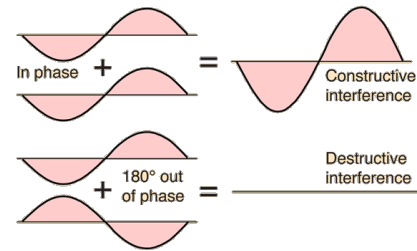


Fig.1 – Illustrations of constructive and destructive interference

Another important theoretical term is wave diffraction which is a phenomenon when the direction of propagation of waves change upon entering an obstacle, a slit or a new medium. When the slit or obstacle's size is comparable to the wavelength, we can treat the wave front as sum or interference of an infinite number of coherent point sources. When the wave front encounters any hindrance, some of these point sources can no longer contribute to this interference, whereas points at the edge of the wave that managed to pass the obstacle will “radiate” its waves after the hindrance which appears as the wave was bent. This phenomenon happens when light hits a medium with different refractive index or when sound waves travels through a medium with varying acoustic impedance.

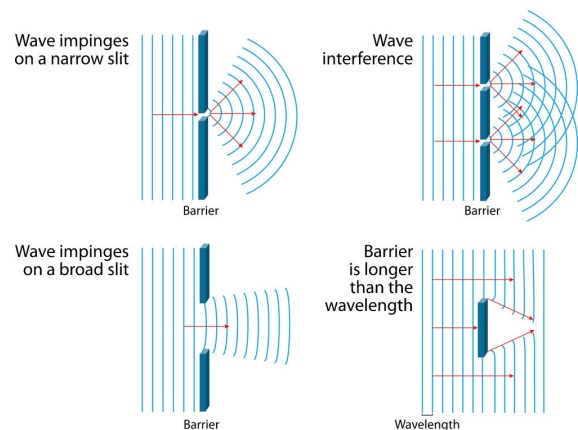


Fig.2 – Illustration of diffraction

All waves can be classified as longitudinal or transverse depending on the direction of oscillation relative to the direction of propagation. If a wave propagates in the same direction as it oscillates i.e. the two are parallel, it's a longitudinal wave (such as sound waves), whereas if the two direction is perpendicular, it is a transverse wave (electromagnetic wave). The oscillation of transverse waves, therefore is not restricted to a single dimension or plane. Polarization is done when we send a (unpolarised) wave with oscillations in many different planes through a conductor grid or filter known as a polarizer. As the grid has conductors aligned in a particular direction, it will absorb the oscillation of the wave in that particular plane. When polarizing an electromagnetic wave for example, the changing field induces an also changing current in the direction of a grid. This oscillating current induces an

oscillating field that – according to Lenz’s law – counters the original wave, thus absorbing it.

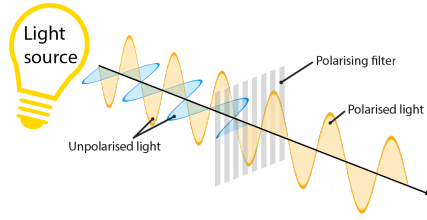


Fig3 – Wave polarization visualized

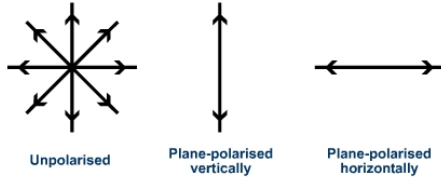


Fig.4 – Wave polarization visualized

There is also a need to introduce some mathematical concepts necessary for solving some of the questions. The Taylor series expansion states that any function of  $x$  can be expressed as a sum of terms derived from the function’s derivatives as following:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n x_o (x - x_o)^n}{n!} \quad (1)$$

This equation is extremely helpful for approximating difficult functions that cannot be solved analytically. The drawback of this method is that it is only accurate if  $x$  is really close to  $x_o$  and if we have sufficient amount of terms.

One more mathematical concept that will use throughout the lab tasks mostly when plotting is normalization. When we normalise a dataset, we adjust the data measured on different scales to common one, where 1 is the maximum of the dataset and 0 is the minimum. This can be calculated by:

$$g(x)_{normalized} = \frac{g(x)}{\max(g(x))}$$

## Pre-Lab Exercise

This pre-lab exercise was a re-creation of Young’s double slit experiment using microwaves to investigate wave interference of waves with the same frequency.

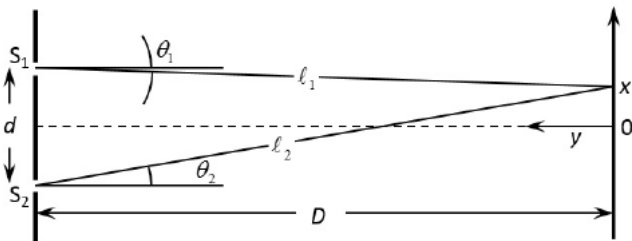


Fig.5 Experimental setup of the first pre-lab

Consider Fig.5 where there are two point sources of waves,  $S_1$  and  $S_2$  with same amplitude and frequency separated by distance  $d$  that emit spherical or cylindrical waves to a plane in an arbitrary distance  $D$  in the  $y$ -direction. We denote the point on line  $x$  (or plane in  $z$  direction) as  $x=\theta$  which is equal distance from both sources  $S_1$  and  $S_2$ . In this plane, the two waves will always be in the same phase since  $l_1$  equals  $l_2$ . When we move away

from  $x=0$ , there will be a phase difference where if  $l_1 - l_2 = n\lambda$  (difference is multiple of wavelength), there will be constructive interference, and if  $l_1 - l_2 = (2n+1)\lambda/2$ , it is destructive interference. The field at  $x$  is given by the equation:

$$E_T = \frac{e^{-jkl_1}}{l_1} + \frac{e^{-jkl_2}}{l_2} \quad (2)$$

The reduction of field with distance  $r$  is  $1/r$  and reduction of intensity  $|E_T|^2$  is  $1/r^2$ . The following relationships can also be derived from the Fig.:

$$\sin\theta_1 = (d/2 - x)/l_1 \quad \text{and} \quad \cos\theta_1 = D/l_1 \quad (3)$$

$$\sin\theta_2 = (d/2 + x)/l_2 \quad \text{and} \quad \cos\theta_2 = D/l_2 \quad (4)$$

From the identity  $\sin^2\theta + \cos^2\theta = 1$  we can express  $l_1$  and  $l_2$  the following ways:

$$l_1 = D\cos\theta_1 + (d/2 - x)\sin\theta_1 \quad (5)$$

$$l_2 = D\cos\theta_2 + (d/2 + x)\sin\theta_2 \quad (6)$$

Using  $l_2 \approx l_1 \approx D$ , we can express  $E_T$  as the following, by substituting  $l_1$  and  $l_2$  equations:

$$E_T = \frac{1}{D} (e^{-jkl_1} + e^{-jkl_2}) \quad (7)$$

$$E_T = \frac{1}{D} (e^{-jk[(d/2-x)\sin\theta_1 + D\cos\theta_1]} + e^{-jk[(d/2+x)\sin\theta_2 + D\cos\theta_2]}) \quad (8)$$

Sum further approximations arise from the fact that  $\theta_1$  and  $\theta_2$  are almost equal and very small. We can use the first two terms of the Taylor series expansion:

$$\sin\theta_1 \approx \theta_1 \approx \frac{1}{D} \left( \frac{d}{2} - x \right) \quad \text{and} \quad \sin\theta_2 \approx \theta_2 \approx \frac{1}{D} \left( \frac{d}{2} + x \right)$$

$$\cos\theta_1 \approx 1 - \frac{\theta_1^2}{2} \approx 1 - \frac{1}{2D^2} \left( \frac{d}{2} - x \right)^2 \quad (11)$$

$$\cos\theta_2 \approx 1 - \frac{\theta_2^2}{2} \approx 1 - \frac{1}{2D^2} \left( \frac{d}{2} + x \right)^2 \quad (12)$$

**Question 1:** Derive the equation for the intensity of the electric field (square of magnitude of electric field). Find the distance between two consecutive maxima and two consecutive minima.

Using the trigonometrical expressions and equations (3) and (4), we can simplify Eq.8 the following expression:

$$E_T \approx e^{-\frac{jku}{2D}} \left( \frac{1}{D} \left[ e^{-\frac{jkd x}{2D}} + e^{\frac{jkd x}{2D}} \right] \right) \quad (13)$$

This equation can be expressed as a cosine using Euler’s cosine-exponential relationship  $2\cos(x) = e^{-x} + e^x$  and rewrite Eq.13 as:

$$E_T \approx e^{-\frac{jku}{2D}} \left( \frac{2}{D} \cos\left(\frac{kdx}{2D}\right) \right) \quad (14)$$

In Eq.14, we can observe that the cosine and  $D$  term is the magnitude of the electric field and the exponential part represents the direction. The magnitude of the electric field  $|E_T|$  is therefore:

$$|E_T| \approx \left[ \frac{2}{D} \cos\left(\frac{kdx}{2D}\right) \right] \quad (15)$$

Squaring this expression, we will get the intensity of the electric field:

$$|E_T|^2 \approx \frac{4}{D^2} \cos^2\left(\frac{kdx}{2D}\right) \quad (16)$$

When the field magnitude or intensity is at maximum, the cosine term should be maximum which is 1.

$$\cos^2\left(\frac{kdx}{2D}\right) = 1 \quad (17)$$

$$\frac{kdx_n}{2D} = n\pi \quad (18)$$

$$x_n = \frac{2Dn\pi}{kd} \quad (19)$$

The difference between two maxima is the same as two minima since this is a consistent wave and can be given by the position difference of  $x_n$  and  $x_{n-1}$ :

$$\Delta x = x_n - x_{n-1} = \frac{2Dn\pi}{kd} - \frac{2D(n-1)\pi}{kd} = \frac{2\pi D}{kd} \quad (20)$$

**Question 2:** Plot the electric field versus the position  $x$  using both the actual value in Eq.2 and the approximation in Eq.16. Comment on the differences.

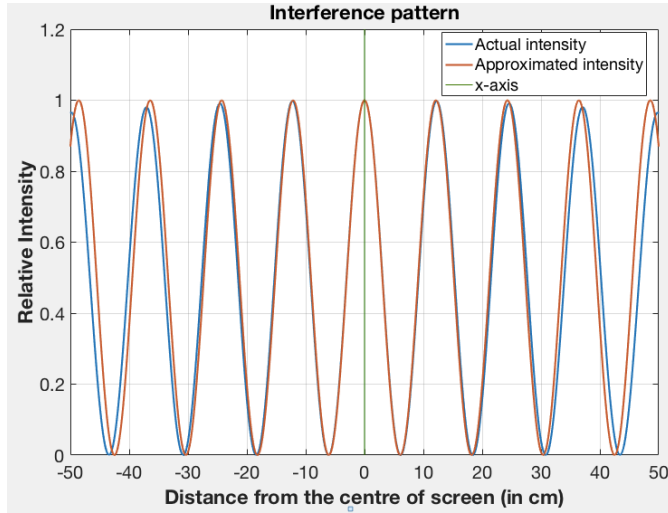


Fig.6 Interference pattern for actual and approximated intensity

The plot for intensity can be seen in Fig.6. Note that, in order to have two good comparable graphs, both were normalised and are plotted relatively to each other.

Observing the two lines, we see that as we move further away from the centre  $x=0$ , both the two amplitudes and the two phase differences will change more noticeably. The reason for these in accuracies in our estimation lies within our approximated  $\theta_1$  and  $\theta_2$ . As we move further away from the centre, these angles become larger and larger and Eq.9, 10, 11, 12 will no longer hold making  $l_1=l_2=D$  invalid. This is also explained by the Taylor series approximation which – as discussed in the theory section – only accurate when  $x$  is close to  $x_0$  so as we move apart on the  $x$ -axis, this will be less accurate.

**Experiment 1.1:** In the next part of the experiment, we will use provided data from a set-up experiment seen in Fig.7 where there are two-point wave sources with same amplitude and phase. Both of these signals are received by a horn with a microwave detector and an amplifier. As we move the horn along the  $x$ -axis, we measure the signal received by the horn. The input generates a microwave signal of 10GHz modulated in amplitude with a 5kHz

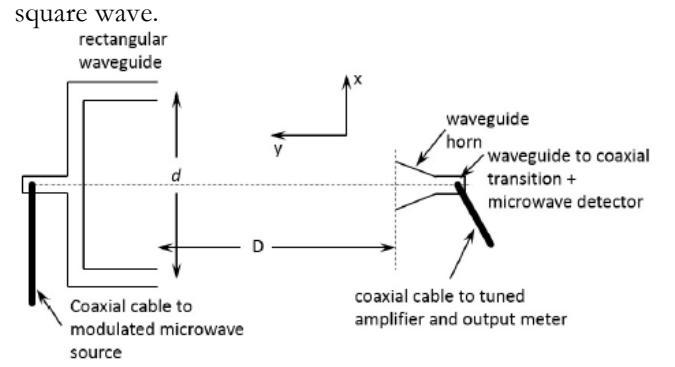


Fig.7 – Experimental setup of second pre-lab task

**Question 3:** Explain the function of the detector diode and the frequency of the signal carried by the coaxial cable. What is the relation between electric field intensity received by the horn and magnitude of the signal coming from the detector through the coaxial cable?

Since the signal received by the horn is one that was modulated, we need a demodulator which in this case is the detector diode. It detects the envelopes in the signal and converts it into an output that is useful for us. This diode is effectively a photodiode with a p-n junction operated in reverse bias. When the signal reached the junction, current will flow in proportion to the strength of the signal making it optimal for detecting microwave signals and converting RF signal to DC. The frequency carried by the coaxial cable is 5kHz. The envelope of the modulated signal represents the information signal, the 5kHz square wave and not the 10GHz signal.

The output of the detector diode is a current and expressed by the following:

$$I = I_0 \left( \frac{k^2 E^2}{2} + kE \right) \quad (21)$$

where  $k$  equals 40 at room temperature making the  $kE$  term negligible, therefore the current can be estimated as:

$$I = I_0 \left( \frac{k^2 V^2}{2} \right)$$

$$I \propto \frac{k^2 V^2}{2}$$

Magnitude of signal current is proportional to intensity.

**Question 4:** Use the data provided on Moodle to plot the received voltage versus the displacement  $x$  along with the theoretical values also and comment on the differences.

The plotted graph can be seen in Fig.8. Just like in the previous graph, here we also normalised the values to plot a intensity relative to each other. We observe that the real values do not have the same maxima as the theoretical ones. At the centre  $x=0$ , the two amplitude is the same but as we move further away, the measured data shows that the intensity decreases significantly. As the path of propagation increases, the intensity decreases inversely with the square of the distance. If we compare our collected data here with the approximations in Fig.6, we can say that those approximations stated were more or less justified. In Fig.6, although less significantly than in Fig.8, the amplitude of the approximation also decreases as the position shifts from  $x=0$ . Phase shift also arises in

both graphs as we go away from  $x=0$ , however, here the phase shift of approximation in Fig.6 is much more noticeable.

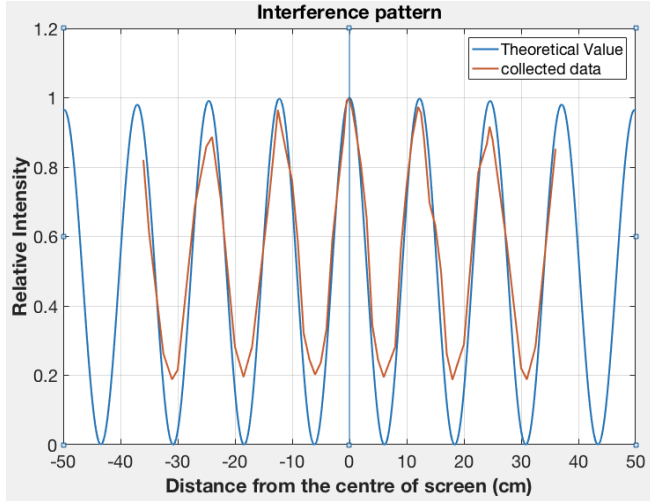


Fig.8 – Interference pattern of the theoretical and collected data

Also, unlike the theoretical data, the real ones' minima never reach zero because even when there is no signal from the point sources, the horn can still detect electromagnetic noise from the lab and there will always be a signal greater than zero.

We can say that the maxima would be a slightly better definition of position since it varies more with different distances, while the minimum are more close together and constant.

If the two point sources were to transmit electric field in the  $x$ - $y$  plane, nothing would be measured by the horn as the signal and the plane is no longer aligned and the electric field would be perpendicular to the original polarization.

Finite aperture of the horn will result in further diffraction of the signal which eventually leads to more interference affecting the final result. Therefore, a very large aperture is recommended to use.

**Experiment 1.2:** We will modify the previous experiment by placing a dielectric slab between the horn and one of the point sources that will introduce additional phase difference. We expect this to shift the interference pattern relative to the previous cases. The layout of this experiment can be seen in Fig.9.

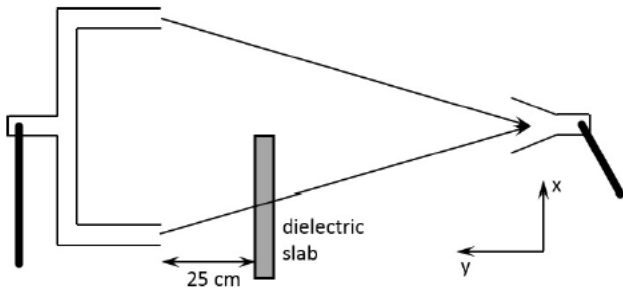


Fig.9 – Experimental data of the dielectric pre-lab task

**Question 5:** Derive the relative permittivity of the sheet.

Before we start deriving the equation we have to make some assumptions. The first one is that the relative permeability of the material is 1. The second is that the

path difference between successive minima and maxima are the same as we obtained earlier in question 1.

First let us define the phase difference from the angular frequency:

$$\Delta\theta = \omega\Delta t \quad (22)$$

We can also relate the phase difference,  $\Delta\theta$  and the path difference  $\Delta x$ .

$$\frac{\Delta\theta}{\Delta x} = \frac{2\pi}{\lambda} = \text{constant} \quad (23)$$

The speed of light is defined:

$$c = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r}} \quad (24)$$

where  $\epsilon_r$  is the relative permittivity of the material,  $\epsilon_0$  is the permittivity of free space,  $\mu_r$  is the relative permeability of the material, and  $\mu_0$  is the permeability of free space. The path difference between two successive minima has already been defined in Eq.20.

$$\Delta x = x_n - x_{n-1} = \frac{2Dn\pi}{kd} - \frac{2D(n-1)\pi}{kd} = \frac{2\pi D}{kd} \quad (20)$$

Next, we derive the angular frequency,  $\omega$  using Eq.22, 23 and 24.

$$\omega = \frac{2\pi}{\lambda} c = \frac{2\pi}{\lambda \sqrt{\epsilon_0 \mu_0}} \quad (25)$$

since here we take the angular frequency in vacuum where both  $\mu_r$  and  $\epsilon_r$  are taken as 1. The time it takes the electromagnetic wave to travel through the dielectric slab is as follows:

$$\Delta t = \frac{\delta}{\frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r}}} - \frac{\delta}{\frac{1}{\sqrt{\epsilon_0 \mu_0}}} = \delta \sqrt{\epsilon_0 \mu_0} (\sqrt{\epsilon_r} - 1) \quad (26)$$

where  $\delta$  is the thickness of the dielectric sheet. Let's continue by substituting Eq.26 and Eq.25 back into Eq.22 to derive the phase difference:

$$\Delta\theta = \omega\Delta t = \frac{2\pi\delta\sqrt{\epsilon_0\mu_0}}{\lambda\sqrt{\epsilon_0\mu_0}} (\sqrt{\epsilon_r} - 1) = \frac{2\pi\delta}{\lambda} (\sqrt{\epsilon_r} - 1)$$

Considering that the path difference between two maxima is  $2\pi D/kd$  and that the phase difference is  $2\pi$ , we can put these values into Eq.25 and Eq.26 and these back into Eq.23:

$$\frac{\Delta\theta}{\Delta x} = \frac{\frac{2\pi\delta}{\lambda} (\sqrt{\epsilon_r} - 1)}{\frac{2\pi D}{kd}} = \frac{2\pi}{\frac{2\pi D}{kd}} = \text{constant} \quad (28)$$

In Eq.28, everything is known except for the relative permittivity. Also,  $k$ , is the wave number expressed as  $k = 2\pi/\lambda$ .

$$\frac{\frac{2\pi\delta}{\lambda} (\sqrt{\epsilon_r} - 1)}{\Delta s} = \frac{2\pi}{\frac{2\pi D}{\frac{2\pi}{\lambda} d}} \quad (29)$$

After a few simplifications and arrangements, we finally managed to derive the expressions for relative permittivity.

$$(\sqrt{\epsilon_r} - 1) = \frac{d \Delta s}{D \delta}$$



$$\epsilon_r = \left(1 + \frac{d \Delta s}{D \delta}\right)^2 \quad (30)$$

This is exactly what the question asked us to prove.

**Question 6:** Import the measured data from Moodle on this dielectric experiment and plot the shifted interference pattern. Determine the shift and calculate the relative permittivity when the thickness of the slab is 1.2cm

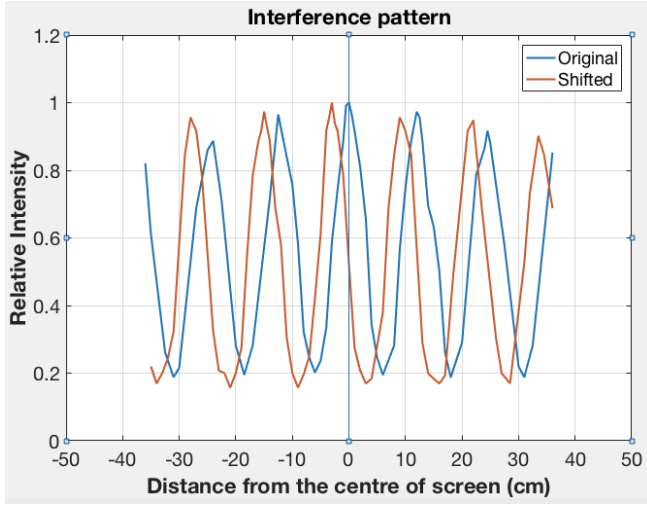


Fig.10 – interference pattern of the original and shifted data

The graph can be used to estimate the amount of shift using the MATLAB cursors. Unfortunately, the shift is measured to be different at every peak due to inaccuracies so we will settle with shift at the centre maximum which is  $\Delta s = 3\text{cm}$ .

We are given the thickness of the slab as  $\delta=1.2\text{cm}$ , the distance between the two sources  $d=63\text{cm}$  and the distance between the plane of sources and the plane of the horn  $D=255\text{cm}$ . Putting these values back into Eq.30 will give us:

$$\epsilon_r = \left(1 + \frac{63}{255} \frac{3}{1.2}\right)^2 = 2.62$$

If we rearrange Eq.30 to express the shift  $\Delta s$  in terms of all other constants, we will get:

$$\Delta s = \frac{(\sqrt{\epsilon_r} - 1)D\delta}{d} \quad (31)$$

Now we see that if the thickness of the sheet,  $\delta$ , increase while all other terms remain constant, the shift,  $\Delta s$  will also increase. Similarly, if we increase the relative permittivity  $\epsilon_r$ , the shift of the two maxima will also increase. If this almost-periodic signal is shifted by more than  $2\pi$ , it will be really hard to tell from the diagram if which was the original non-shifted peak.

Usually the position of the dielectric sheer between the horn and the source should not be important since it will introduce the same amount of extra phase difference regardless of the position. The only thing that matters is that we don't place the sheet very close to the receiver or the source, otherwise the receiver won't be able to detect any interference.

**Experiment 1.3:** In this part of the experiment we will consider the interference caused by a group of antennas (in our case, 2) that has the same frequency. We can shape

the interference pattern by positions of the antennas, the amplitude and the phase of excitation.

In Fig.11, we see the arrangement where two point antennas are separated by distance  $d$  and with currents 1 and  $e^{j\varphi}$  meaning equal magnitude but a phase difference with a value of  $\varphi$ .

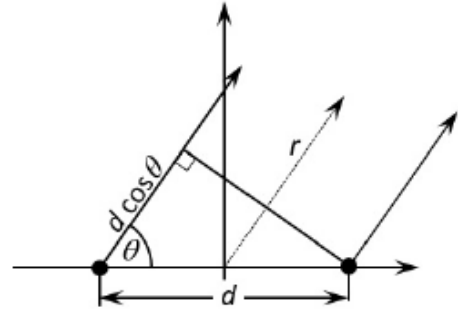


Fig. 11 – Illustration of field from two point antennas

We want to derive the equation for the field at a very far distance where we can make the assumptions that the distance is large enough for that they are parallel to each other and are equal to  $r$ . Also note that the path difference cannot be neglected when considering the phase difference. From Fig.11, we can write:

$$E(r, \theta) = \frac{e^{-jk\left(r+\frac{d\cos\theta}{2}\right)}}{r} + \frac{e^{j\varphi} e^{-jk\left(r-\frac{d\cos\theta}{2}\right)}}{r} \quad (32)$$

**Question 7:** Starting from Eq.32, find the expression for the absolute value of the far field as a function of  $\theta$ .

Let's begin with taking the absolute value of Eq.32:

$$|E(r, \theta)| = \left| \frac{e^{-jk\left(r+\frac{d\cos\theta}{2}\right)}}{r} + \frac{e^{j\varphi} e^{-jk\left(r-\frac{d\cos\theta}{2}\right)}}{r} \right|$$

Let us factorize it to separate the imaginary part from the real part:

$$\begin{aligned} |E(r, \theta)| &= \left| \frac{e^{-jk\left(r+d\cos\left(\frac{\theta}{2}\right)\right)}}{r} + \frac{e^{j\varphi} e^{-jk\left(r-d\cos\left(\frac{\theta}{2}\right)\right)} e^{jk2d\cos\left(\frac{\theta}{2}\right)}}{r} \right| \\ |E(r, \theta)| &= \left| \frac{e^{-jk\left(r+d\cos\left(\frac{\theta}{2}\right)\right)}}{r} \left( 1 + e^{j\varphi} e^{jk2d\cos\left(\frac{\theta}{2}\right)} \right) \right| \\ |E(r, \theta)| &= \frac{e^{-jk\left(r+d\cos\left(\frac{\theta}{2}\right)\right)}}{r} \left( 1 + \cos\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right) \right. \\ &\quad \left. + j\sin\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right) \right) \end{aligned}$$

To find the absolute value, we take the magnitude of the expression to get the far field at any given distance and angle:

$$|E(r, \theta)| = \frac{1}{r} \sqrt{\left(1 + \cos\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right)\right)^2 + \left(j \sin\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right)\right)^2}$$

$$|E(r, \theta)| = \frac{1}{r} \sqrt{1 + 2\cos\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right) + \cos^2\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right) + \sin^2\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right)}$$

$$|E(r, \theta)| = \frac{1}{r} \sqrt{2 + 2\cos\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right)}$$

Here we can separate the dependence on the radius  $r$  and the angle  $\theta$ , where the dependence on angle  $\theta$  is  $F(\theta)$ .

$$|E(r, \theta)| = \frac{F(\theta)}{r}$$

Where  $F(\theta)$  is called the radiation pattern of the array of or the array factor and is equal to the following expression:

$$F(\theta) = \sqrt{2 + 2\cos\left(\varphi + k2d * \cos\left(\frac{\theta}{2}\right)\right)}$$

We will also plot the normalized array factor of this array as a function of the distance between the antennas  $d$ , the phase difference between the antennas  $\varphi$ , and the angle of the incline  $\theta$ . We use the MATLAB command 'polar' to plot it in polar coordinates.

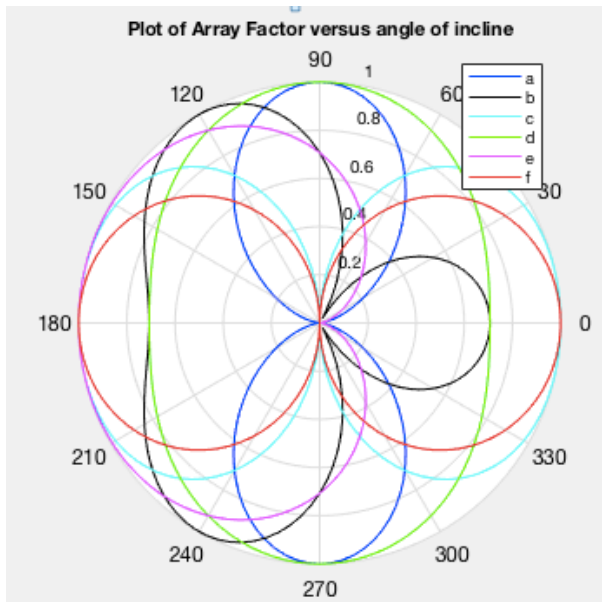


Fig. 12 – The array factor of the two antenna sources

## Lab Experiment Part 2

**Question 8:** Consult the references for indicated for Fresnel's refraction theory using your own words.

In the 'Theory' section, the concept of diffraction has already been touched and can use Fresnel's diffraction theory to derive some form of expectation for the experiment in the next part. As mentioned before, diffraction is just a propagating wave that 'bends' when

encountering or passing through an obstacle comparable to the size of its wave length. When we look at the difference between diffraction and interference in physical terms there is really no difference according to the 'Feynman Lectures'. When there are only a few wave sources interfering with each other, we usually call it interference. In this case we can usually see an interference pattern with points of constructive and destructive interference. However, when we consider an infinite (or very large) number of sources, the result is usually regarded as diffraction. The phenomenon of waves 'bending' when passing through a slit can be explained by treating the wave fronts as an infinite number of sources interfering with each other, thus creating an interference pattern that resembles wave 'bending'. We call the phenomenon in the next experiment diffraction since in this case the wave passes by an obstacle.

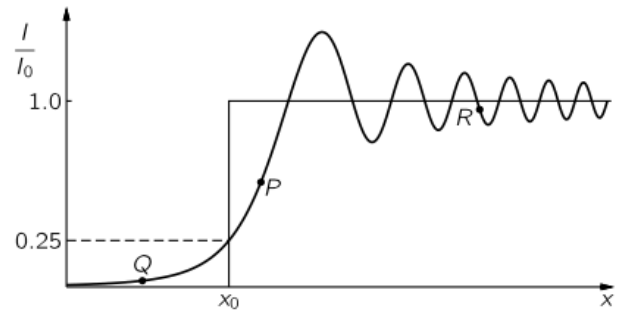


Fig.12 – Diagram of expected results from the 'Feynman Lectures'

Fresnel's theory of diffraction is represented in an equation that describes propagation of fields in the near-field. It can be used to derive the interference pattern when a wave passes by a close object. This equation is a very tedious integral that cannot be solved analytically in most cases:

$$E(x, y, z) = \frac{1}{j\lambda} \iint_{-\infty}^{\infty} E(x', y', 0) \frac{e^{jkr}}{r} \cos(\theta) dx' dy'$$

where we can get the field diffraction at point  $(x, y, z)$ .  $E(x', y', 0)$  is the aperture of the object in the  $xy$ -plane and  $k$  is the wavenumber. The integral can be calculated using the Taylor-series expansion allowing us to know the diffraction anywhere in space. In Fig.12, we can see what interference pattern we might expect from a diffraction by a screen in the  $x$ -dimension.

**Experiment 2.1:** Just as in the previous experiment, we have a horn receiver that receives the signal of only one source in this case. We have a rail with a conductor sheet perpendicular to the line between the receiver and the source as seen in Fig.13. We slide the rail along the  $x$ -axis and record the horn receiver output in cases when the horn is fully blocked, partially blocked and not blocked by the conductor sheet. From Fresnel's theory, we should already expect a result similar to the one in Fig.12.

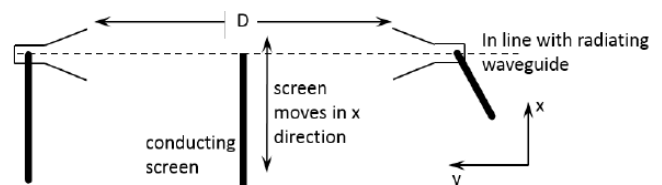


Fig.13 – Experimental set up of lab task 2.1

**Question 9:** Plot the received intensity against the position  $x$  of the conductor sheet.

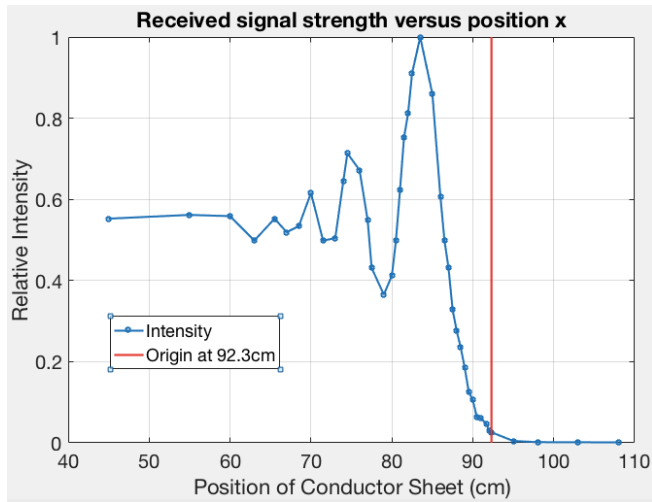


Fig.14 - Received intensity versus the position of conductive sheet

We convert our measurements into intensity and normalize them to get relative intensity. It can be observed that our result is very similar to the theoretical graph in Fig.12. The reason it is flipped is that we used the given trail ruler to measure and moved the sheet into the negative  $x$ -direction. The origin, when the edge of the sheet, the receiver and the horn is aligned is at 92.3cm and the sheet was moved sufficiently in both directions to get data. We can see the exponential increase as we move the sheet away and also the damped oscillation. An unnoticeable but important difference between our graph and the published results of Fresnel's diffraction theory is that when we move into the positive direction of  $x$ , the intensity doesn't reach zero. The reason is the sheet is not large enough to shadow the receiver entirely and there is also some amount of electromagnetic noise from the lab equipment. The set up can be improved by a less noisy environment, a larger conductor sheet and a more precise display that shows the output.

A conductive screen does not necessarily correspond to an opaque screen. Conductive materials are more likely to refract the waves whereas opaque screen is more likely to absorb them. They would most likely give different results. The experimental results would also differ significantly if we used an orthogonal polarization incident on the screen. In theory, the wave can still pass through however the drop in intensity would never be visible as it does not block the wave.

If we fix the screen and instead move the receiving horn, the amplitude of the received signal would be affected because previously the intensity only depended on how much the screen blocked the wave, but in this case it would also depend on the distance between the horn and the source.

'Near' and 'far' fields are usually referring to the regions of electric or magnetic field around an object. Usually, at 'near-field', non-radiative behaviours of electromagnetic fields dominate, close to the source, while electromagnetic radiation 'far-field' behaviours dominate at greater distances.

**Experiment 2.2 polarization:** In this experiment the set up will be similar to the previous one. Here the source emits a linearly polarized electromagnetic field and we will need to determine in which direction by rotating a wire grid and a perforated mesh in between the receiver and the source.

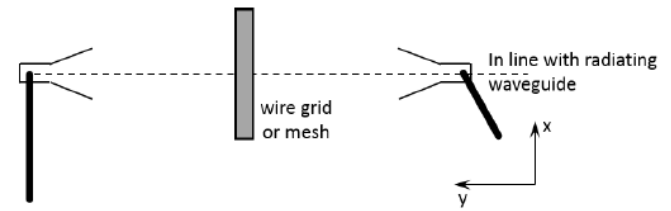


Fig.15 – Experimental set up of the last lab task

**Question 10:** Explain briefly what is polarization.

The concept of polarization has been defined earlier in the 'Theory' section but briefly, it just defines the geometrical orientation of the wave oscillations. A polarizer can be thought of as a filter that only allows wave that pass through it to oscillate in a certain direction and absorbing all others. Polarizers can be used in many use cases such as anti-glare sunglasses, where the glasses polarize some of the oscillation planes of light that were formed when they are reflected from surfaces such as lakes, shiny windows, making sunlight more 'bearable'.

When we put a polarizer in between the source and receiver, the polarizer will absorb some oscillation planes of the unpolarized wave and the receiver will receive the remaining. When we interrupt a polarized wave from a source with a polarizer, if the orientation of our polarizer matches the orientation of oscillation of the wave, the receiver output is reduced to 0. If the polarizer absorbs another orientation, the output remains unchanged. Initially when they match, the output is 0, when we start rotating the polarizer, we will see an increasing intensity up to a maximum point.

**Question 11:** What do you think the polarisation of the emitted wave should be and why? Now, from the observed orientation of the grid and the corresponding received intensity, what is the polarisation of the source?

Since rectangular waveguides are used, we expect the emitted wave to be restricted to ensure it is linearly polarized which is parallel to the vertical axis. First we put the polarizer with straight wires vertically first and see that the intensity the horn detected was around 0.15. When we rotated the grid by 90 degrees, the intensity changed to 3.5. This confirmed our initial hypothesis that the source polarizes the emitted wave vertically.

When we used the perforated mesh in vertical direction our intensity was around 0.35 while horizontal orientation resulted in an intensity of 3.1. The reason for this is that this grid consists of wires of angle 30 and 60 degree to the planes of oscillation of the waves which can neither block an entire oscillation direction nor let an entire oscillation direction through. We can therefore say that the grid with the straight wires were a better polarizer.

The physical process of polarization is also explained in the 'Theory' section. Briefly, the incoming oscillating magnetic or electric field induces an oscillating current in



the wire, but since these are either horizontal or vertical, only one of the two planes of oscillation induces this. This induced current induces another EM wave that counteracts the original one, thus cancelling out this particular orientation. This occurs only if the incoming wave is parallel to the wires.

## Conclusion

In conclusion, the pre-lab and the lab exercise helped us understand the concepts of wave interference and diffraction through visually representative experiments and tasks. Several equations in connection with this topic was derived and proved, furthermore, many theoretical concepts from either Feynman or Fresnel was confirmed. There were some inaccuracies due to lack of precise equipment, presence of background noise, inaccurate values read, however all errors were identified and explained. Fortunately, these errors didn't invalidate our results in any way which makes this lab task successful. If there was more time to take more data, an overall average calculation for multiple datasets would have made these results more similar to the theoretical ones.

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<http://www.hep.manchester.ac.uk/u/xiaguowaveoptics/Fresnel%20diffraction.pdf>

# Appendix

## Question 2

```
% Script to model the interference between 2 point sources
% separated by a distance d and on a line parallel to a screen
% at a distance D.
% variables names as in Fig. 1 in the lab. script
%
% CHANGE THESE VALUES ACCORDING TO YOUR EXPERIMENTAL SETUP:
d=0.63;           % separation between the sources (in m)
D=2.55;           % distance from sources to screen (in m)
lambda=0.03;      % wavelength (in m)
k=2*pi/lambda;
%
x=-0.5:0.001:0.5; % to cover 50 cm at either side of the centre
theta1=atan((d/2-x)/D);
theta2=atan((d/2+x)/D);
l1=D./cos(theta1);
l2=D./cos(theta2);
j=0+i;
Et=exp(-j*k*l1)./l1+exp(-j*k*l2)./l2;
Et=Et.*conj(Et)/(max(Et)^2);
Et2=(2/D).*cos((k*d*x)/(2*D));
Et2=Et2.*conj(Et2)/(max(Et2)^2);
%
y=x*100;          % converting to cm
plot(y,abs(Et))
hold on
plot(y, abs(Et2))
axis([-50 50 0 1.2])
set(gca,'XTick',[-50:10:50])
title('{\bfInterference pattern}','FontSize',14)
xlabel('{\bfDistance from the centre of screen (in cm)}')
ylabel('{\bfRelative Intensity}')
legend('Actual intensity','Approximated intensity')
line([0 0],[0 1.2])
hold off;
grid on;
% line([-0.5 0.5],[1 1],'linestyle',':')
% text(-0.48, 1.1,'drawn by {\bf <...insert your name here...> }')
```

#### Question 6

```
data1=importdata('Int1.txt');
x1=data1(:,1);
y1=data1(:,2);
y1=y1/max(y1); plot(x1,abs(y1),'linewidth', 2);
hold on
data2=importdata('Int2.txt');
x2=data2(:,1);
y2=data2(:,2);
y2=y2/max(y2);
plot(x2,abs(y2),'linewidth', 2);
axis([-50 50 0 1.2]);
set(gca,'XTick',[-50:10:50])
title('{\bfInterference pattern}','FontSize',14);
xlabel('{\bfDistance from the centre of screen (in cm)}');
ylabel('{\bfRelative Intensity}')
grid on
line([0 0],[0 1.2])
line([-0.5 0.5],[1 1],'linestyle',':')
legend('actual','shifted');
hold off
```

#### Question 4

```
0.63; % separation between the sources (in m)
D=2.55; % distance from sources to screen (in m)
lambda=0.03; % wavelength (in m)
k=2*pi/lambda;
%
x=-0.5:0.001:0.5; % to cover 50 cm at either side of the centre
theta1=atan((d/2-x)/D);
theta2=atan((d/2+x)/D);
l1=D./cos(theta1);
l2=D./cos(theta2);
j=0+i;
Et=exp(-j*k*l1)./l1+exp(-j*k*l2)./l2; Et=Et.*conj(Et)/(max(Et)^2);
y=x*100; % converting to cm
plot(y,abs(Et),'linewidth', 2)
hold on
data=importdata('Int1.txt');
x1=data(:,1);
y1=data(:,2);
y1=y1/(max(y1));
plot(x1,abs(y1),'linewidth', 2);
axis([-50 50 0 1.2]);
set(gca,'XTick',[-50:10:50])
title('{\bfInterference pattern}','FontSize',14);
xlabel('{\bfDistance from the centre of screen (in cm)}');
ylabel('{\bfRelative Intensity}')
grid on
line([0 0],[0 1.2])
line([-0.5 0.5],[1 1],'linestyle',':')
legend('actual','collected data');
hold off
```

### Question 7

```
lambda = 1;
k = 2*pi/lambda;
Angle = 0: pi/64: 2*pi;
d1 = lambda/2;
d2 = lambda/4;
phase1 = 0;
phase2 = pi/2;
phase3 = pi;
F1 = (2 + 2 * cos( k*d1*cos(Angle) + phase1)).^(1/2);
F1 = F1/max(F1);
F2 = (2 + 2 * cos( k*d1*cos(Angle) + phase2)).^(1/2);
F2 = F2/max(F2);
F3 = (2 + 2 * cos( k*d1*cos(Angle) + phase3)).^(1/2);
F3 = F3/max(F3);
F4 = (2 + 2 * cos( k*d2*cos(Angle) + phase1)).^(1/2);
F4 = F4/max(F4);
F5 = (2 + 2 * cos( k*d2*cos(Angle) + phase2)).^(1/2);
F5 = F5/max(F5);
F6 = (2 + 2 * cos( k*d2*cos(Angle) + phase3)).^(1/2);
F6 = F6/max(F6);
polar(iAngle, F1, 'b');
hold on;
polar(iAngle, F2, 'k');
polar(iAngle, F3, 'c');
polar(iAngle, F4, 'g');
polar(iAngle, F5, 'm');
polar(iAngle, F6, 'r');
legend( 'a', 'b', 'c', 'd', 'e', 'f');
```

### Question 9

```
x = [108 103 98 95 92.3 92 91.7 91 90.5 90 89.5 89 88.5 88 87.5 87 86.5 86
85 83.5 82.5 82 81.5 81 80.5 80 79 77.5 77 76 74.5 74 73 71.5 70 68.5 67
65.5 63 60 55 45];
current = [0.17 0.18 0.2 0.33 0.78 0.85 1.03 1.19 1.21 1.57 1.71 2.08 2.34
2.54 2.77 3.17 3.41 3.76 4.48 4.83 4.61 4.35 4.19 3.81 3.41 3.1 2.92 3.17
3.58 3.96 4.08 3.88 3.43 3.41 3.79 3.53 3.48 3.59 3.41 3.61 3.62 3.59];
xflip = fliplr(x);
currentflip = fliplr(current);
current = current.^2/max(current).^2;

%center is 92.3cm
plot(x, abs(current))
hold on
line([92.3 92.3],[0 1])
title("Received signal strength versus position x")
grid on
hold off
```