Theory Assignment

Answer in no more than 10 pages total
Minimum 10pt font size
Due Friday, 27 November 2015, 5:00 PM
Submit via learnonline

October 26, 2015

1. (**Samples and sequences**) Consider the signal

$$x(t) = \Pi(t) + \sum_{m \in \mathbb{Z} \setminus 0} \Pi\big(m^2(t-m)\big)$$

where Π is the rectangular pulse and $\mathbb{Z}\setminus 0$ is the set of all integers other than zero. Plot x and show that it is absolutely integrable and square integrable, but not periodic. Now consider the sequence of samples $c_n = x(n)$ of the signal x. Plot the sequence c and show that it is periodic, but neither absolutely summable, nor square summable. Hint:

$$\sum_{m=1}^{\infty} \frac{1}{m^2} = \frac{\pi^2}{6}.$$

2. **(Raised cosine)** Plot the signal

$$x(t) = \begin{cases} 1 & -\frac{1}{4} < t \leq \frac{1}{4} \\ \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t - \frac{\pi}{2}\right) & \frac{1}{4} < t \leq \frac{3}{4} \\ \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right) & -\frac{3}{4} < t \leq -\frac{1}{4} \\ 0 & \text{otherwise} \end{cases}$$

and find its Fourier transform $\hat{x} = \mathcal{F}x$. Plot the Fourier transform. Is the Fourier transform square integrable? Is it absolutely integrable?

3. (**Finite impulse response filter**) Design a low pass finite impulse response filter with cuttoff frequency $c = 2400\,\text{Hz}$ and sample period $P = \frac{1}{F}$ where $F = 8000\,\text{Hz}$. Ensure the filter has satisfies the following properties:

   - has no more that 81 taps,
   - affects the amplitude of frequencies in the interval $[0, 2300\,\text{Hz}]$ by no more than 10%,
   - attenuates the amplitude of frequencies in the interval $[2500\,\text{Hz}, F/2]$ by more than 90%.

   Plot the discrete impulse response and magnitude spectrum of this digital filter.