Homework assignment 2

April 5, 2012

1. Define three waveforms (or functions)

$$s_1(t) = \text{rect}(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & \text{otherwise,} \end{cases}$$

$$s_2(t) = \text{rect}(t)\cos(\pi t),$$
$$s_3(t) = \text{rect}(t)\sin(2\pi t)$$

Using Matlab or otherwise, plot these three waveforms. Perform Gram-Schmidt orthonormalisation to obtain orthogonal waveforms $u_1(t), u_2(t)$, and $u_3(t)$, and specify the coefficients $c_{i,j}$ such that

$$s_1(t) = c_{1,1}u_1(t) + c_{1,2}u_2(t) + c_{1,3}u_3(t),$$
$$s_2(t) = c_{2,1}u_1(t) + c_{2,2}u_2(t) + c_{2,3}u_3(t),$$
$$s_3(t) = c_{3,1}u_1(t) + c_{3,2}u_2(t) + c_{3,3}u_3(t).$$

Using Matlab or otherwise, plot $u_1(t)$, $u_2(t)$ and $u_3(t)$

2. In this question we consider communicating with the Quadrature Phase Shift Keying (QPSK) constellation. Each QPSK symbol is of the form

$$s = A\exp\left(j\frac{\pi}{2}u + j\frac{\pi}{4}\right)$$

where u is an integer from the set $\{0, 1, 2, 3\}$. So, the constellation has four symbols in total

$$\exp\left(j\frac{\pi}{4}\right), \quad ,\exp\left(j\frac{3\pi}{4}\right), \quad \exp\left(-j\frac{\pi}{4}\right), \quad \text{and} \quad \exp\left(-j\frac{3\pi}{4}\right)$$

Consider transmitting the symbol s through an additive white Gaussian noise (AWGN) channel. The reciever obtains

$$r = s + w$$

where w is noise. The w is complex, the real and imaginary parts of w are independent and Gaussian, with variance $\sigma^2/2$. What is the signal to noise ratio (SNR)? (1 mark)

If the reciever employs Maximum-Likelihood (equivalently least squares) detection, then the estimate of the symbols at the reciever is,

$$\hat{s} = \hat{u}$$

where

$$\hat{u} = \arg \min_{u \in \{0,1,2,3\}} \left| r - A \exp\left(j\frac{\pi}{2}u + j\frac{\pi}{4} \right) \right|^2$$

Using the Q-function defined as

$$Q(f) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$$

describe the probability of error as a function of signal to noise ratio? Plot the probability of error for a range of SNR values from -5 dB to 12 dB

Using Matlab or otherwise, simulate a QPSK transmitter and reciever. Using Monte-Carlo simulation, plot the probability of error for a range of signal to noise ratios between -5 dB to 9 dB. For each value of SNR, run simulations until 1000 error events occur.