Theory Assignment

Answer in no more than 8 pages total
Minimum 10pt font size

October 9, 2014

1. (**Multiplier**) Consider the operational amplifier circuit in Figure 1. Draw an equivalent circuit using the model for an operational amplifier including input resistance R_i, output resistance R_o and open loop gain A (given in Figure 2.5 of the lecture notes). Analyse this circuit to obtain a relationship between the input voltage signal x and output voltage signal y. By taking limits as $R_i \to \infty$, $A \to \infty$ and $R_o \to 0$ find an expression relating x and y assuming that the operational amplifier is ideal. Obtain the same expression directly using the rules for analysing ideal operational amplifiers. Is the system that describes this circuit stable? Is it regular?

2. (**Properties of signals**) Plot each of the following signals and show whether they are: bounded, periodic, absolutely integrable, square integrable.

   (a) $x(t) = 1$

   (b) $x(t) = u(t+1)e^{-t}$ where $u(t)$ is the step function

   (c) $x(t) = \sin(2\pi t)\cos(\pi t)$

   (d) $x(t) = \frac{\sin^2(\pi t)}{\pi t}$

3. (**Properties of systems**) State whether each of the following systems are: causal, linear, time invariant, stable, regular. Plot the impulse and step response of the systems whenever they exist.

   (a) $H(x,t) = 3x(t-1) - 2x(t+1)$

   (b) $H(x,t) = \sin\big(2\pi x(t)\big)$

   (c) $H(x,t) = t^2 x(t)$

   (d) $H(x,t) = \int_{-1/2}^{1/2} \cos(\pi\tau)x(t+\tau)d\tau$

4. **(Masses, springs, and dampers)** Figure 2 depicts a mechanical system involving two masses, two springs, and a damper connected between two walls. Suppose that the spring K_2 is at rest when the mass M_2 is at position $p(t) = 0$. A force, represented by the signal f, is applied to mass M_1. Derive a differential equation relating the force f and the position p of mass M_2. Suppose that H is a linear time invariant system mapping f to p and suppose that $M_1 = K_1 = K_2 = B = 1$ and $M_2 = 2$. Find the transfer function of H. Find the poles and zeros of H and draw a pole zero plot. Determine whether H is stable and/or regular. Find and plot the impulse response and the step response of H if they exist.

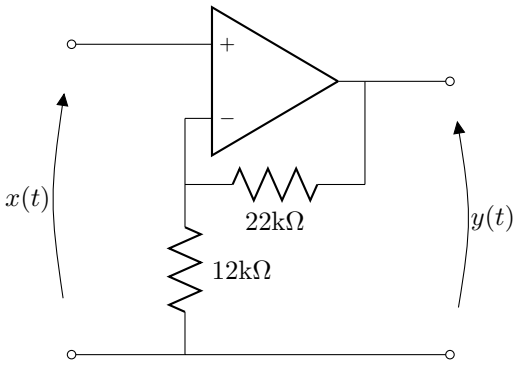

Figure 1: Operational amplifier circuit configured as a multiplier

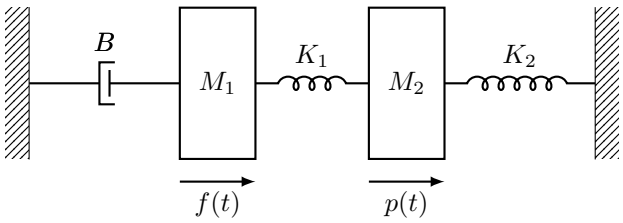

Figure 2: Two masses, a spring, and a damper