Theory Assignment

Answer in no more than 6 pages total
Minimum 10pt font size

October 17, 2013

1. (**Multiplier**) Consider the operational amplifier circuit in Figure 1. Draw an equivalent circuit using the model for an operational amplifier including input resistance R_i, output resistance R_o and open loop gain A (given in Figure 14 of the lecture notes). Analyse this circuit to obtain a relationship between the input voltage signal x and output voltage signal y. By taking limits as $R_i \to \infty$, $A \to \infty$ and $R_o \to 0$ find an expression relating x and y assuming that the operational amplifier is ideal. Obtain the same expression directly using the rules for analysing ideal operational amplifiers. Is the system that describes this circuit stable? Is it regular?

2. (**Properties of signals**) Plot each of the following signals and show whether they are: bounded, periodic, absolutely integrable, square integrable.

   (a) $x(t) = 1$

   (b) $x(t) = u(t+1)e^{-t}$ where $u(t)$ is the step function

   (c) $x(t) = \sin(2\pi t)\cos(\pi t)$

   (d) $x(t) = \frac{\sin^2(\pi t)}{\pi t}$

3. (**Properties of systems**) State whether each of the following systems are: causal, linear, time invariant, stable, regular. Plot the impulse and step response of the systems whenever they exist.

   (a) $H(x,t) = 3x(t-1) - 2x(t+1)$

   (b) $H(x,t) = \sin\big(2\pi x(t)\big)$

   (c) $H(x,t) = t^2 x(t)$

   (d) $H(x,t) = \int_{-1/2}^{1/2} \cos(\pi\tau)x(t+\tau)d\tau$

4. **(Raised cosine)** Plot the signal

$$x(t) = \begin{cases} 1 & -\frac{1}{4} < t \leq \frac{1}{4} \\ \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t - \frac{\pi}{2}\right) & \frac{1}{4} < t \leq \frac{3}{4} \\ \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right) & -\frac{3}{4} < t \leq \frac{1}{4} \\ 0 & \text{otherwise} \end{cases}$$

   and find its Fourier transform $\hat{x} = \mathcal{F}(x)$. Plot the Fourier transform. Is the Fourier transform square integrable? Is it absolutely integrable? Now consider the signal $y = \sqrt{x}$. Without explicitly finding the Fourier transform $\hat{y} = \mathcal{F}(y)$ show that, like the sinc function, $\hat{y}$ is orthogonal to its shifts by integers. That is, show that

$$\int_{-\infty}^{\infty} \hat{y}(t)\hat{y}(t-m)dt = 0$$

   for all nonzero integers m.

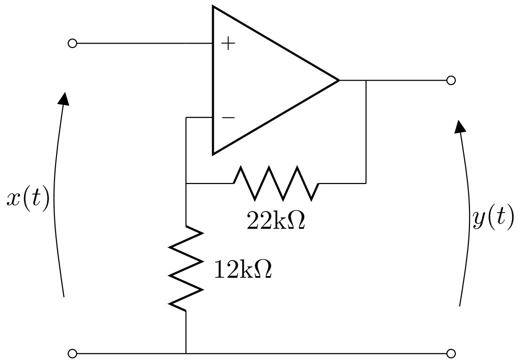

Figure 1: Operational amplifier circuit configured as a multiplier