

# Theory Assignment

Answer in no more than 10 pages total  
Minimum 10pt font size

October 26, 2014

1. **(Multiplier)** Consider the operational amplifier circuit in Figure 1. Draw an equivalent circuit using the model for an operational amplifier including input resistance  $R_i$ , output resistance  $R_o$  and open loop gain  $A$  (given in Figure 2.5 of the lecture notes). Analyse this circuit to obtain a relationship between the input voltage signal  $x$  and output voltage signal  $y$ . By taking limits as  $R_i \rightarrow \infty$ ,  $A \rightarrow \infty$  and  $R_o \rightarrow 0$  find an expression relating  $x$  and  $y$  assuming that the operational amplifier is ideal. Obtain the same expression directly using the rules for analysing ideal operational amplifiers. Is the system that describes this circuit stable? Is it regular?
2. **(Properties of signals)** Plot each of the following signals and show whether they are: bounded, periodic, absolutely integrable, square integrable.
  - (a)  $x(t) = 1$
  - (b)  $x(t) = u(t+1)e^{-t}$  where  $u(t)$  is the step function
  - (c)  $x(t) = \sin(2\pi t)\cos(\pi t)$
  - (d)  $x(t) = \frac{\sin^2(\pi t)}{\pi t}$
3. **(Properties of systems)** State whether each of the following systems are: causal, linear, time invariant, stable, regular. Plot the impulse and step response of the systems whenever they exist.
  - (a)  $H(x, t) = 3x(t-1) - 2x(t+1)$
  - (b)  $H(x, t) = \sin(2\pi x(t))$
  - (c)  $H(x, t) = t^2 x(t)$
  - (d)  $H(x, t) = \int_{-1/2}^{1/2} \cos(\pi\tau)x(t+\tau)d\tau$
4. **(Masses, springs, and dampers)** Figure 2 depicts a mechanical system involving two masses, two springs, and a damper connected between two walls. Suppose that the spring  $K_2$  is at rest when the mass  $M_2$  is at position  $p(t) = 0$ . A force, represented by the signal  $f$ , is applied to mass  $M_1$ . Derive a differential equation relating the force  $f$  and the position  $p$  of mass  $M_2$ . Suppose that  $H$  is a linear time invariant system mapping  $f$  to  $p$  and suppose that  $M_1 = K_1 = K_2 = B = 1$  and  $M_2 = 2$ . Find the transfer function of  $H$ . Find the poles and zeros of  $H$  and draw a pole zero plot. Determine whether  $H$  is stable and/or regular. Find and plot the impulse response and the step response of  $H$  if they exist.
5. **(Properties of sequences)** Plot each of the following sequences and show whether they are: bounded, periodic, absolutely summable.
  - (a)  $x_n = 2^{-n}u_n$  where  $u_n$  is the step sequence
  - (b)  $x_n = \sin(\pi n/4)\cos(\pi n/6)$
  - (c)  $x_n = \sin(n)$
  - (d)  $x_n = u_{n-1}/n$

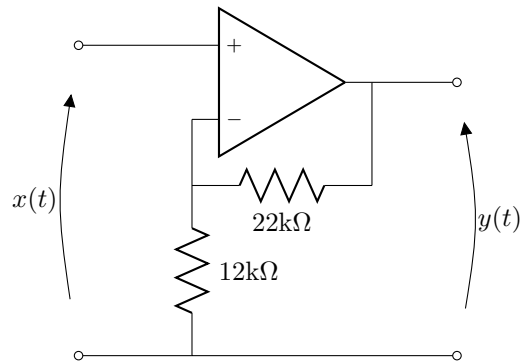


Figure 1: Operational amplifier circuit configured as a multiplier

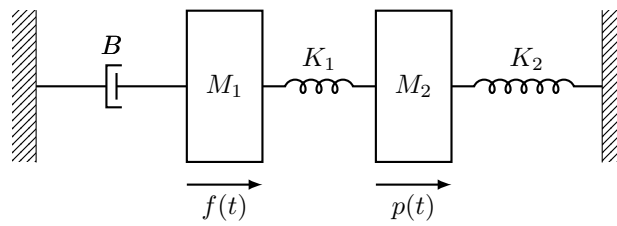


Figure 2: Two masses, a spring, and a damper