n tuple, Cartesian product set operations and Set identities

Ordered n-tuples

$(a_1, a_2, a_3, \dots a_n)$ is the ordered collection. And we say that $(a_1, a_2, a_3, \dots a_n) = (b_1, b_2, b_3, \dots b_n)$ iff $a_i = b_i$ $(i = 1,2, 3..)$

And when n = 2 ----Ordered pairs (a,b), (c, d)

Cartesian Product

A X B is the set of all ordered pairs where $a \in A$ and $b \in B$

$A \times B = \{(a,b) \mid a \in A \land b \in B\}$. It is called a relation from set A to set B

- $A \times B \neq B \times A$ unless A = B
- Can be performed on more than 2 sets:
  $A_1 x A_2 x \dots A_n = \{(a_1, a_2, \dots a_n) | a_i \in A_i$ for $i=\{1,2,..,n\}$

Examples:

- A= {1,2} and B = {a, b, c} AxB = {(1,a), (1,b), (1, c), (2,a), (2, b), (2, c)}
- A={0,1}, B={1,2}, C={0,1,2)
  AxBxC ={(0,1,0),(0,1,1),(0,1,2),(0,2,0),(0,2,1),(0,2,2),(1,1,0),(1,1,1),(1,1,2),(1,2,0),(1,2,1),(1,2,2)}

Set Operations

As always there must be a universe U. All sets are assumed to be subsets of U.

Two sets can be combined in many different ways. Union, Intersection, Difference

Union A∪ B

The set that contains elements that are either in A or in B, or in both

- $A \cup B = \{x | x \in A \lor x \in B\}$
- Example A= {1,3,5} B = {1,2,3,5} A∪ B = {1,2,3,5} What is the cardinality?

Intersection A ∩ B

The set that contains elements that are in both A and in B

- $A \cap B = \{x | x \in A \land x \in B\}$
- Example A= {1,3,5} B = {1,2,3}, A ∩ B = {1,3} What is the cardinality?
  Definition: Two sets are Disjoint if their intersection is empty. $A \cap B = \emptyset$
- Example: A={2, 4, 6} B= {1, 3, 5}

Difference A-B

Set containing elements in A but not in B. In other words: Complement of B with respect to A

- $A - B = \{x | x \in A \land x \notin B\}$
- $A \cap \bar{B}$
- Example: A={1,3,5} B={1, 2, 3}, A-B = {5}, B-A ={2}

Symmetric difference A ⊕ B

- $A \oplus B = (A - B) \cup (B - A)$

Returning back to the cardinality of AUB

- The cardinality of A∪ B $|A \cup B| = |A| + |B| - |A \cap B|$

The universal set U & set complement

$\overline{A}$ is a complement of $\overline{A} = U - A$

- $\overline{A} = \{x | x \notin A\} = \{x | \neg (x \in A)\}$
  Example A = {a, e, i, o, u} vowels, $\overline{A}$ = consonant

Set Identities

Identity Laws: $A \cup \emptyset = A$, $A \cap U = A$

Domination laws $A \cup U = U$, $A \cap \emptyset = \emptyset$

Idempotent laws $A \cup A = A$, $A \cap A = A$

Complementation laws $\bar{\bar{A}} = A$

Commutative laws $A \cup B = B \cup A$, $A \cap B = B \cap A$

Associative laws $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive laws $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan's laws $\overline{A \cap B} = \bar{A} \cup \bar{B}$, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

Absorption laws $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$

Complement laws $A \cup \bar{A} = U$, $A \cap \bar{A} = \varphi$