More about Functions

Increasing & Decreasing functions

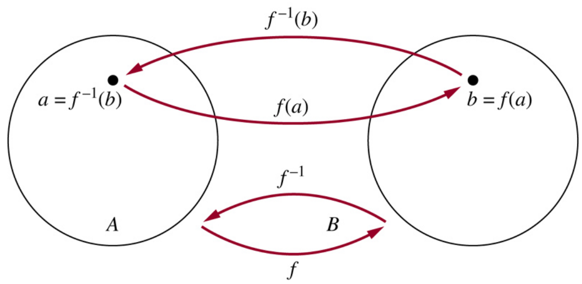
x < y (both x, y in the domain of f)

- If f(x) ≤ f(y)
  Strictly increasing f(x) < f(y)
- If f(x) ≥ f(y)
  Strictly decreasing f(x) > f(y)

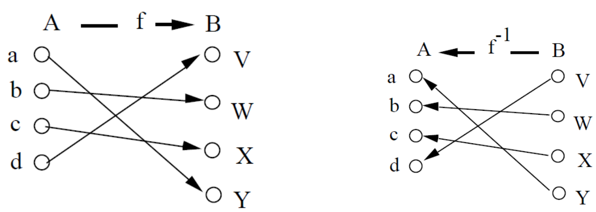
Inverse of Functions

Definition: Let f be a bijection from *A to B. Then the* *inverse of f, denoted f^{-1}, is the function from B to A* defined as

- f^{-1} (y) = x iff f(x) = y



Example:
Let f be defined by the diagram:



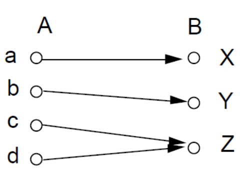
Note: No inverse exists unless f is a bijection.

Definition: Let S be a subset of B. Then

f^{-1}(S) = {x | f(x) ∈ S}

- Note: f need not be a bijection for this definition to hold.

Example:
Let f be the following function:

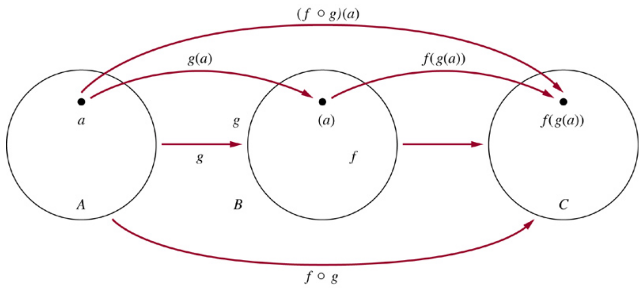


- f^{-1}({Z}) = {c, d}
- f^{-1}({X, Y}) = {a, b}

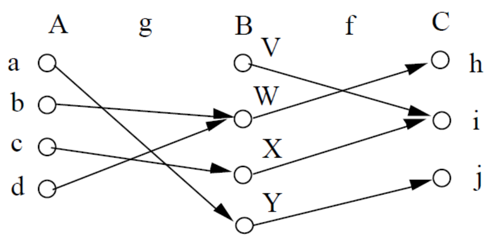
Function Compositions

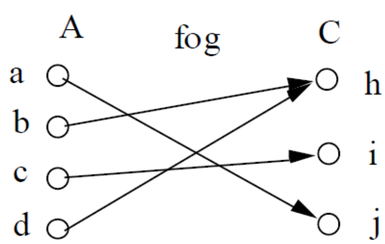
Definition: Let f: B→C, g: A → B. The *composition of* *f with g, denoted f o g, is the function from A to C defined* by

- f o g(x) = f(g(x))



Example





Example:

If $f(x) = x^2$ and $g(x) = 2x + 1$, determine f(g(x)) and g(f(x))

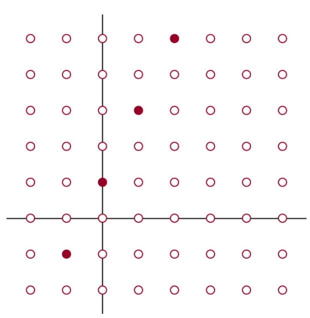
- $f(g(x)) = (2x+1)^2$
- $g(f(x)) = 2x^2 + 1$

The Graph of Functions

Graph of a function

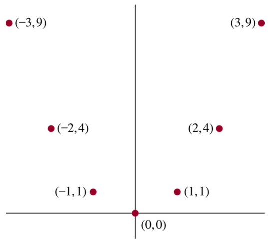
f is a function from se A to B. The graph of the function is the set of ordered pairs

{(a, b) | a ∈ A and f (a) = b}



f(n) = 2n + 1 from Z to Z

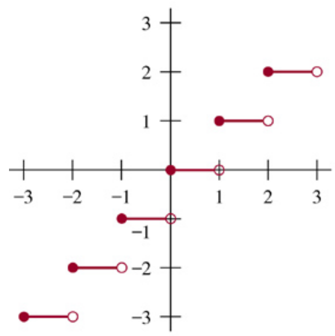
$f(x) = x^2$ from Z to Z



Special Functions

The floor function:

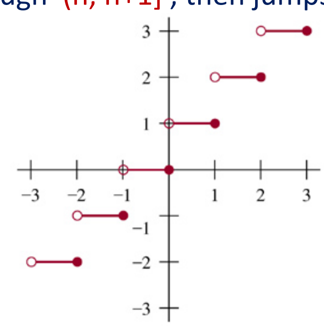
- Denoted *$f(x) = \lfloor x \rfloor$ or $f(x)$ = floor(x), is the largest integer* less than or equal to x. (real numbers)
- $\lfloor 3.5 \rfloor = 3$
- The same value "n" through [n, n+1) , then jumps to n+1
- Note: the floor function is equivalent to truncation for positive numbers.



$\lfloor 1/2 \rfloor = 0$
$\lfloor -1/2 \rfloor = -1$
$\lfloor 5 \rfloor = 5$

The ceiling function:

- Denoted *$f(x) = \lceil x \rceil$ or $f(x)$ = ceiling(x), is the smallest* integer greater than or equal to x. (real numbers)
- $\lceil 3.5 \rceil = 4$
- The same value "n + 1" through (n, n+1] , then jumps to n+2



$\lceil 1/2 \rceil = 1$

$\lceil -1/2 \rceil = 0$
$\lceil 5 \rceil = 5$

Application:

- Information stored or transmitted using strings of bytes. How many bytes needed to encode 100 bits of data?
- $100/8 = 12.5$
- $\lceil 12.5 \rceil = 13$ bytes

Floor & Ceiling properties:
N is an integer

- $\lfloor x \rfloor = n$ iff $n \leq x < n + 1$
- $\lceil x \rceil = n$ iff $n - 1 < x \leq n$
- $\lfloor x \rfloor = n$ iff $x - 1 < n \leq x$
- $\lceil x \rceil = n$ iff $n \leq x < n + 1$
- $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$
- $\lfloor -x \rfloor = - \lceil x \rceil$
- $\lceil - x \rceil = - \lfloor x \rfloor$
- $\lfloor x + n \rfloor = \lfloor x \rfloor + n$
- $\lceil x + n \rceil = \lceil x \rceil + n$

How would you calculate f^{-1} of the floor and the ceiling function
The factorial function:

- $f : N \rightarrow Z^+$
- $f(n) = n!$
- $f(n) = 1 \cdot 2 \cdot 3 \ldots (n-1)n$
- $F(0) = 0! = 1$
- $F(1) = 1! = 1$

The absolute value function