Compound Propositions

Logical connectives + negation $\rightarrow$ more complicated compound propositions.
In truth tables: we use a separate column to find the truth value of each compound expression that occurs in the compound proposition.

<u>Example:</u> 'If I go to Harry's or go to the country I will not go shopping.'

P: I go to Harry's
Q: I go to the country
R: I will go shopping
If......P......or.....Q.....then....not.....R
$(P \vee Q) \rightarrow \neg R$

Construct the truth table

Precedence of logical operators

Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

$\neg p \wedge q \equiv (\neg p) \wedge q$
$p \wedge q \vee r \equiv (p \wedge q) \vee r$ and not $p \wedge (q \vee r)$
$p \vee q \rightarrow r \equiv (p \vee q) \rightarrow r$ and not $p \vee (q \rightarrow r)$

Tautologies, Contradictions, and Contingencies

A **tautology** is a proposition which is always true. Example: $p \vee \neg p$
A **contradiction** is a proposition which is always false. Example: $p \wedge \neg p$
A **contingency** is a proposition which is neither a tautology nor a contradiction. Example p

$\Leftrightarrow$ Logical Equivalences $\equiv$

Compound propositions (p, q) that have the **same truth values in all possible cases**
$p \equiv q$, if $p \leftrightarrow q$ is tautology.
We can show it By using the truth table, or by developing a series of logically equivalent statements.

De Morgan's laws. $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Identity Laws $p \wedge T \equiv p$, $p \vee F \equiv p$
Domination Laws $p \vee T \equiv T$, $p \wedge F \equiv F$
Idempotent laws $p \vee p \equiv p$, $p \wedge p \equiv p$
Double negation law $\neg(\neg p) \equiv p$
Commutative law $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$

Associative Laws $(p \vee q) \vee r \equiv p \vee (q \vee r)$, $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive Laws $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Absorption laws. $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$
Negation laws. $p \vee \neg p \equiv T$, $p \wedge \neg p \equiv F$

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg (p \to \neg q)$
$\neg (p \to q) \equiv p \wedge \neg q$
$p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$
$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Show that
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
$\neg(p \vee(\neg \wedge q)) \equiv \neg p \wedge \neg q$
$(p \wedge q) \to (p \vee q)$ is a tautology