Logic & Bit operations

A bit (binary digit) is a symbol with 2 possible values 0,1

A bit string is a sequence of zero or more bits.

Length of a string is the number of bits in the string

1011 1100

0101 1011

Applications of Propositional Logic

Translating English Sentences: Identify atomic propositions and represent using propositional variables. Determine appropriate logical connectives

Example You can access the Internet from campus only if you are a computer science major or you are not a freshman." $a \rightarrow (c \vee \neg f)$

System Specifications

System and Software engineers take requirements in English and express them in a precise specification language based on logic.

Example: Express in propositional logic:

"The automated reply cannot be sent when the file system is full"

p: "The automated reply can be sent"

q denote "The file system is full."

$q \rightarrow \neg p$

Consistent System Specifications

A list of propositions is consistent if it is possible to assign truth values to the proposition variables so that each proposition is true.

Example: Are these specifications consistent?

"The diagnostic message is stored in the buffer or it is retransmitted."

"The diagnostic message is not stored in the buffer."

"If the diagnostic message is stored in the buffer, then it is retransmitted."

p: "The diagnostic message is not stored in the buffer."

q: "The diagnostic message is retransmitted"

The specification can be written as: $p \vee q$, $p \rightarrow q$, $\neg p$.

When p is false and q is true all three statements are true. So the specification is consistent.

What if "The diagnostic message is not retransmitted is added."

Solution: Now we are adding $\neg q$ and there is no satisfying assignment. So the specification is not consistent.

Logic Puzzles

An island has two kinds of inhabitants, knights, who always tell the truth, and knaves, who always lie.

You go to the island and meet A and B.

A says "B is a knight."

B says "The two of us are of opposite types."

Example: What are the types of A and B?

Solution: Let p and q be the statements that A is a knight and B is a knight, respectively.

So, then $\neg$p represents the proposition that A is a knave and $\neg$q that B is a knave.

If A is a knight, then p is true. Since knights tell the truth, q must also be true.

Then $(p \land \neg q) \lor (\neg p \land q)$ would have to be true, but it is not. So, A is not a knight and therefore $\neg$p must be true.

If A is a knave, then B must not be a knight since knaves always lie. So, then both $\neg$p and $\neg$q hold since both are knaves.

Propositional Satisfiability

A compound proposition is satisfiable if there is an assignment of truth values to its variables that make it true. When no such assignments exist, the compound proposition is unsatisfiable.

A compound proposition is unsatisfiable if and only if its negation is a tautology.

Example:

Determine the satisfiability of the following compound propositions:

a. $(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$
   Satisfiable. Assign **T** to *p, q,* and *r*.

b. $(p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$
   Satisfiable. Assign **T** to *p* and ***F*** to *q*.

c. $(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p) \land (p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$
   Not satisfiable. Check each possible assignment of truth values to the propositional variables and none will make the proposition true

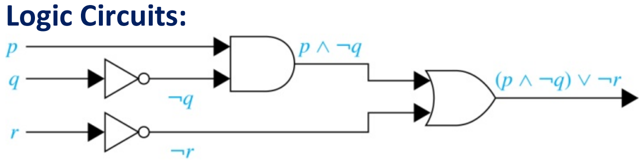
Sudoku

Notations

$\bigvee_{j=1}^{n} p_j$ is used for $p_1 \lor p_2 \lor \ldots \lor p_n$

$\bigwedge_{j=1}^{n} p_j$ is used for $p_1 \land p_2 \land \ldots \land p_n$

Logic Circuits:



Minterms & Disjunctive Normal Form (DNF)

- every variable or its negation is represented once in each conjunction (a *minterm)*
- *each minterms appears only once*
- DNF disjunctive normal form (Sum of products)
- Use the rows of the truth table where the proposition is 1 or True
  - If a zero appears under a variable, use the negation of the propositional variable in the minterm

If a one appears, use the propositional variable