CECS 228 - HOMEWORK 3
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For each of the following sets, determine whether 2 is an element of that set.

a. $\{x \in R \mid x \text{ is an integer greater than } 1\}$
b. $\{x \in R \mid x \text{ is the square of an integer}\}$
c. $\{2, \{2\}\}$
d. $\{\{2\}, \{\{2\}\}\}$
e. $\{\{2\}, \{2,\{2\}\}\}$
f. $\{\{\{2\}\}\}$

Solution:

a. Yes
b. No
c. Yes
d. No
e. No
f. No

13

Determine whether each of these statements is true or false.

a. $x \in \{x\}$
b. $\{x\} \subseteq \{x\}$
c. $\{x\} \in \{x\}$
d. $\{x\} \in \{\{x\}\}$
e. $\Phi \subseteq \{x\}$
f. $\Phi \in \{x\}$

Solution:

a. True
b. True
c. False
d. True
e. True
f. False

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What is the cardinality of each of these sets?

a. $\{a\}$

b. {{a}}
c. {a, {a}}
d. {a, {a}, {a, {a}}}

Solution:

a. 1
b. 1
c. 2
d. 3

23

Find the power set of each of these sets, where a and b are distinct elements.

a. {a}
b. {a, b}
c. {Φ, {Φ}}

Solution:

a. {Φ, {a}}
b. {Φ, {a}, {b}, {a,b}}
c. {Φ, {Φ}, {{Φ}}, {Φ,{Φ}}}

29

Let A = {a, b, c, d} and B = {y, z}. Find

a. $A \times B$
b. $B \times A$

Solution:

a. {(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)}
b. {(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)}

45

Translate each of these quantifications into English and determine its truth value.

a. $\forall x \in R(x^2 \neq -1)$
b. $\exists x \in Z(x^2 = 2)$
c. $\forall x \in Z(x^2 > 0)$
d. $\exists x \in R(x^2 = x)$

Solution:

a. Every real number has its square not equal to -1.True
b. There exists at least one integer whose square is 2. False
c. The square of every integer is positive. False
d. There is at least one real number that is equal to its own square.True

47

Find the truth set of each of these predicates where the domain is the set of integers.

a. P(x): $x^2 < 3$
b. Q(x): $x^2 > x$
c. R(x): $2x + 1 = 0$

Solution:

a. $\{x \in Z \mid x^2 < 3\} = \{-1, 0, 1\}$
b. $\{x \in Z \mid x^2 > x\} = \{..., -2, -2, 2, 3, 4, ...\}$
c. $\{x \in Z \mid 2x + 1 = 0\} = \Phi$

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19

Show that is A, B, and C are sets, then A ∩ B ∩ C = !A ∪!B ∪!C …

a. by showing each side is a subset of the other side.
b. using a membership table.

Solution:

a. Let x ∈ A ∩ B ∩ C
¬(x ∈ A ∩ B ∩ C) using def of complement
¬(x ∈ A ∧ x ∈ B ∧ x ∈ C) using def. Of intersection
¬(x ∈ A) ∨ ¬(x ∈ B) ∨ ¬(x ∈ C)using De Morgan's law
x ∈ !A ∨ x ∈ !B ∨ x ∈ !C using def. Of complement
x ∈ !A∪ !B ∪ !C using def of union
A ∩ B ∩ C = !A ∪!B ∪!C

Let x ∈ !A ∪!B ∪!C
x ∈ !A ∨ x ∈ !B ∨ x ∈ !C using the def. Of union
¬(x ∈ A) ∨ ¬(x ∈ B) ∨ ¬(x ∈ C) using def. Of complement
¬(x ∈ A ∧ x ∈ B ∧ x ∈ C) using De Morgan's law
¬(x ∈ A ∩ B ∩ C) using def of intersection
x ∈ !A ∪!B ∪!C using def of complement

By doing the proofs, this shows that A ∩ B ∩ C is equal to !A ∪!B ∪!C

b.

A	B	C	Ā	!B	!C	A∩ B∩ C	!A∩!B∩!C	!A U !B U !C
T	T	T	F	F	F	T	F	F
T	T	F	F	F	T	F	T	T

T	F	T	F	T	F	F	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	T	F	T	T

27(c,d)

Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}. Find

a. A ∩ B ∩ C
b. A ∪ B ∪ C
c. (A ∪ B) ∩ C
d. (A ∩ B) ∪ C

Solution:

a. {4, 6}
b. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
c. {4, 5, 6, 8, 10}
d. {0, 2, 4, 5, 6, 7, 8, 9, 10}

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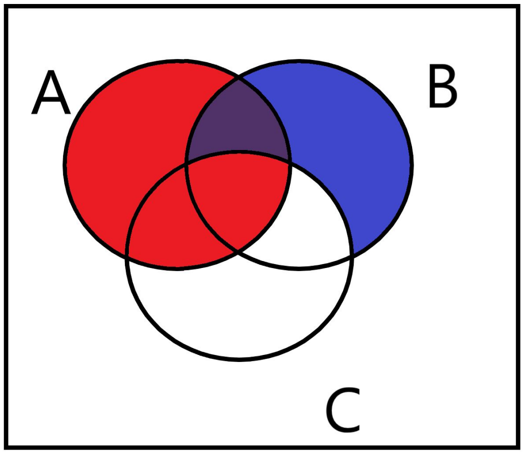
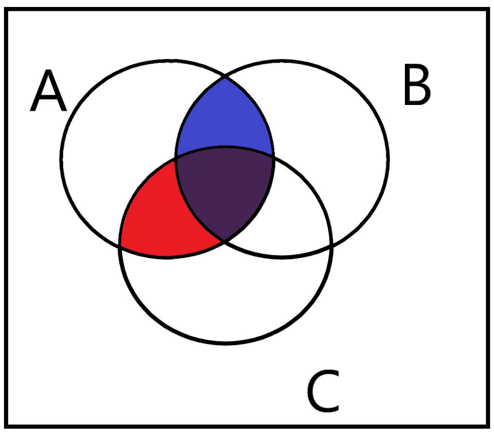
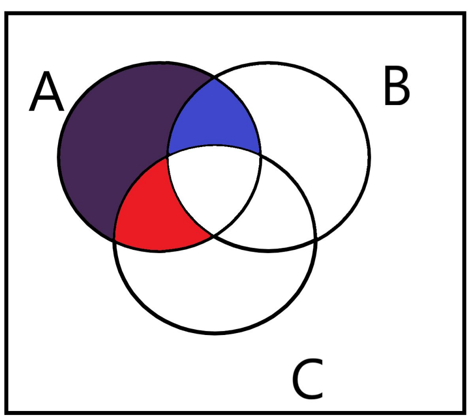
Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a. A ∩ (B - C)
b. (A ∩ B)∪ (A ∩ C)
c. (A ∩ !B) ∪ (A ∩ !C)

Solution:

a. 

b. 

c. 

31

What can you say about the sets A and B if we know that

a. $A \cup B = A$?
b. $A \cap B = A$?
c. $A - B = A$?
d. $A \cap B = B \cap A$?
e. $A - B = B - A$?

Solution:

a. $B \subseteq A$
b. $A \subseteq B$
c. $A \cap B = \Phi$
d. nothing
e. $A = B$

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Show that $A \oplus B = (A \cup B) - (A \cap B)$

Solution:

An element is in $(A \cup B) - (A \cap B)$ if it is in the union, but not in the intersection.

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Show that if A is a subset of a universal set U, then

a. $A \oplus A = \Phi$
b. $A \oplus \Phi = A$
c. $A \oplus U = \bar{A}$
d. $A \oplus \bar{A} = U$

Solution:

a. $(A - A) \cup (A - A) = \Phi \cup \Phi = \Phi$
b. $(A - \Phi) \cup (\Phi - A) = A \cup \Phi = A$
c. $(A - U) \cup (U - A) = \Phi \cup \bar{A} = \bar{A}$
d. $(A - \bar{A}) \cup (\bar{A} - A) = A \cup \bar{A} = U$