Functions

Define Functions, Domain, codomain, Image, preimage, One-to-one functions -injection, Onto function-Surjection, Bijection, Function Inverse, Function composition

Let A and B be sets (not empty). A function (mapping, map, transformation) f from A to B (relationship), denoted:

$f : A \rightarrow B$, *is a subset of* $A \rightarrow B$ such that
$\forall x[x \in A \rightarrow \exists y[y \in B \wedge < x, y > \in f]]$
and
$[< x, y_1 > \in f \wedge < x, y_2 > \in f] \rightarrow y_1 = y_2$

Assignment of exactly one element of B to each element of A, one ordered pair (a,b) for every a $\in$ A

Note: *f associates with each x (element a) in A one and only one y in* B.

- A is called the ***domain*** *and*
- B is called the ***codomain***.

If f(a) = b

- b is called the ***image*** *of a under f*
- a is called a ***preimage*** *of b*

Note there may be **more** than **one preimage** of y but there **is only one image** of x.

Functions Equality

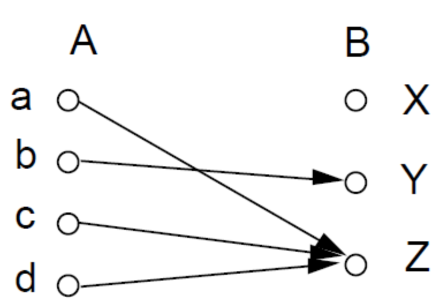
Two functions are equal when they have the same domain, the same codomain, and images and preimages (mapping) must be the same for each element.

Definition:

The ***range*** *of f is the set of all images of points in A under* f. We denote it by f(A).

- If S is a subset of A then f(S) = {f(s) | s in S}.

<u>Example:</u> Determine: f(a), image of d , domain, codomain, f(A), preimage (of y, z), preimage of z, f({c,d})



If f_1 & f_2 functions from A to R,

- $(f_1 + f_2)(x) = f_1(x) + f_2(x)$
- $(f_1 f_2)(x) = f_1(x)\ f_2(x)$

With the same domain & codomain

Example:
$(f_1)(x) = x^{2.}$ $(f_2)(x) = x - x^2$

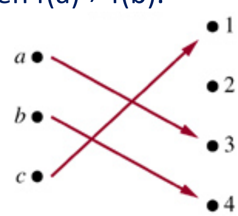
- Find $(f_1 + f_2)$, $(f_1 f_2)$

Let f be a function from A to B. We define, one-to-one (injections), onto (surjections),

Injections one-to-one

Definition: f is *one-to-one (denoted 1-1) or injective iff* preimages are ***unique***.

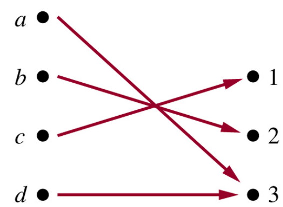
- Note: this means that if a≠b then f(a) ≠ f(b).



Surjections onto

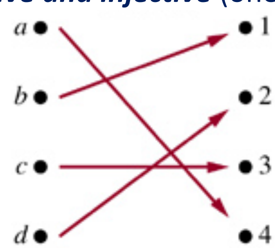
Definition: f is *onto or surjective if every y in B has a* preimage.

- Note: this means that for every y in B there must be an x in A such that f(x) = y.



Bijections

Definition: f is *bijective if it is surjective and injective* (one-to-one and onto).



- **Note**: Whenever there is a bijection from A to B, the two sets must have the same number of elements or the same *cardinality.*
- That will become our *definition, especially for infinite* sets.

Scary thought

Let E be the set of even integers {0, 2, 4, 6, ...}.
Then there is a bijection f from *N to E , the even* nonnegative integers, defined by
f(x) = 2x.
Hence, the set of even integers has the same cardinality as the set of natural numbers.
OH, NO! IT CAN'T BE....*E IS ONLY HALF AS BIG!!*

Examples:

Determine which are injections (1-1), surjections (onto), bijections:

- $f(x) = x$ ($A = B = R$)
  One-to-one and onto - bijection
- $f(x) = x^2$ ($A = B = Z$)
  Not one-to-one $f(1) = f(-1)$ and $1 \neq -1$, onto
  If the domain is Z^+ then It is one-to-one

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