CECS 228 - HOMEWORK 9
PAGE 608: 1 (d,e,f), 7 (d,e,g,h), 27, 47

1 (d, e, f)

List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}. Where (a,b) ∈ R if and only if

    a. $a = b$

    b. $a + b = 4$

    c. $a > b$

    d. $a \mid b$

    e. $\gcd(a,b) = 1 \geq$

    f. $\operatorname{lcm}(a,b) = 2$

Solution:

    a.

    b.

    c.

    d. {(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 0), (3, 3), (4, 0)}

    e. {(0, 1), (1, 0), (1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (4, 1), (4, 3)}

    f. {(1, 2), (2, 1), (2, 2)}

7 (d, e, g, h)

Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where (x,y) ∈ R if and only if

    a. $x \neq y$

    b. $xy \geq 1$

    c. $x = y + 1$ or $x = y - 1$

    d. $x \equiv y \pmod 7$

    e. x is a multiple of y

    f. x and y are both negative or both nonnegative

    g. $x = y^2$

    h. $x \geq y^2$

Solution:

    a.

    b.

    c.

    d. reflexive, symmetric, transitive

    e. reflexive, transitive

    f.

    g. antisymmetric

    h. antisymmetric, transitive

27

Let R be the relation R = {(a,b) | a divides b} on the set of positive integers. Find

    a. R^{-1}

    b. $\overline{R}$

Solution:

    a. {(a, b) | b divides a}

    b. {(a, b) | a does not divide b}

47

    a. How many relations are there on the set {a, b, c, d}?

    b. How many relations are there on the set {a, b, c, d} that contain the pair (a, a)?

Solution:

    a. 65,536

    b. 32,768

PAGE 619: 1, 7

1

List the triples in the relation {{a, b, c) | a, b, and c are integers with $0 < a < b < c < 5$}

Solution:

{(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)}

7

The 3-tuples in a 3-ary relation represent the following attributes of a student database: student ID number, name, phone number.

    a. Is student ID number likely to be a primary key?

    b. Is name likely to be a primary key?

    c. Is phone number likely to be a primary key?

Solution:

    a. Yes

    b. No

    c. No

PAGE 626: 13, 9, 27

13

Let R be the relation represented by the matrix

$M_R =$

0	1	1

1	1	0
1	0	1

Find the matrix representing

    a. R^{-1}

    b. $\overline{R}$

    c. R^2

Solution:

    a.

0	1	1
1	1	0
1	0	1

    b.

1	0	0
0	0	1
0	1	0

    c.

1	1	1
1	1	1
1	1	1

9

How many nonzero entries does the matrix representing the relation R on A = {1, 2, 3, …, 100} consisting of the first 100 positive integers have if R is

    a. {(a, b) | a > b}?

    b. {(a, b) | a ≠ b}?

    c. {(a, b) | a = b + 1}?

    d. {(a, b) | a = 1}?
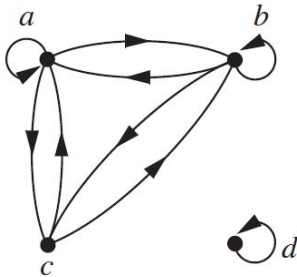
    e. {(a, b) | ab = 1}?

Solution:

    a. 4950

    b. 9900

    c. 99

d. 100

e. 1

27

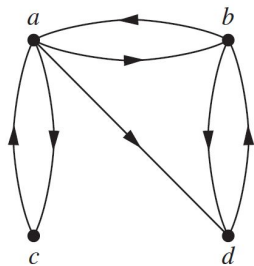List the ordered pairs in the relations represented by the directed graphs.

Solution:

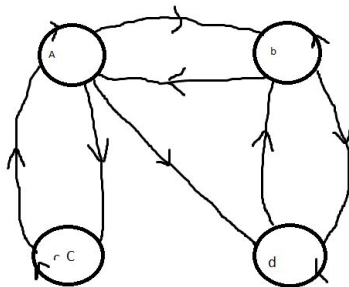{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (d, d)}

PAGE 637: 7, 25 (c,d)

7

Draw the directed graph of the reflexive closure of the relations with the directed graph shown.

Solution:

25 (c, d)

Use Algorithm 1 to find the transitive closures of these relations on {1, 2, 3, 4}.

a. {(1, 2) , (2, 1) , (2, 3) , (3, 4) , (4, 1)}
b. {(2, 1) , (2, 3) , (3, 1) , (3, 4) , (4, 1) , (4, 3)}
c. {(1, 2) , (1, 3) , (1, 4) , (2, 3) , (2, 4) , (3, 4)}
d. {(1, 1) , (1, 4) , (2, 1) , (2, 3) , (3, 1) , (3, 2) , (3, 4) , (4, 2)}

Solution:

a.

b.

c.

0	1	1	1
0	0	1	1
0	0	0	1
0	0	0	0

d.

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

PAGE 646: 3, 21, 23, 31, 35 (c,d)

3

Which of these relations on the set of all functions from Z to Z are equivalence relations?
Determine the properties of an equivalence relation that the others lack.
   a. $\{(f, g) \mid f(1) = g(1)\}$
   b. $\{(f, g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$
   c. $\{(f, g) \mid f(x) - g(x) = 1 \text{ for all } x \in Z\}$
   d. $\{(f, g) \mid \text{for some } C \in Z, \text{ for all } x \in Z, f(x) - g(x) = C\}$
   e. $\{(f, g) \mid f(0) = g(1) \text{ and } f(1) = g(0)\}$

Solution:
   a. equivalence relation
   b. not transitive
   c. not reflexive, not symmetric, not transitive
   d. equivalence relation
   e. not reflexive, not transitive

21

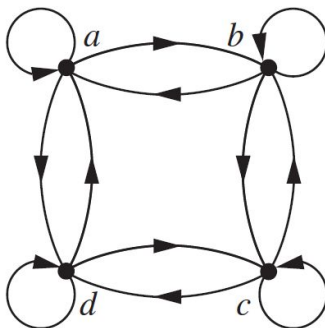Determine whether the relation with the directed graph shown is an equivalence relation.
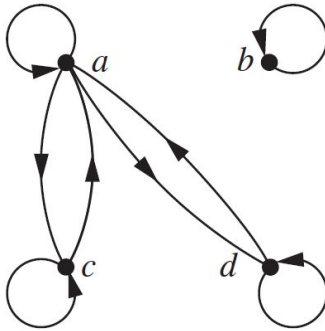
Solution:

No

23

Determine whether the relation with the directed graph shown is an equivalence relation.

Solution:

No

31

What are the equivalence classes of the bit strings in Exercise 30 for the equivalence relation from Exercise 12?

Solution:

a. The set of all bit strings of length 3
b. The set of all bit strings of length 4 that end with a 1
c. The set of all bit strings of length 5 that end 11
d. The set of all bit strings of length 8 that end with 10101

35 (c, d)

What is the congruence class $[n]_5$ (that is, the equivalence class of n with respect to congruence modulo 5) when n is

a. 2?
   b. 3?
   c. 6?
   d. -3?

Solution:

   a.

   b.

   c. $[6]_5 = \{i \mid i \equiv 6 \pmod 5)\} = \{\ldots,-9,-4, 1, 6, 11,\ldots\}$

   d. $[-3]_5 = \{i \mid i \equiv -3 \pmod 5)\} = \{\ldots,-8,-3, 2, 7, 12,\ldots\}$

PAGE 662: 7, 9, 11, 23 (c,d), 27, 33

7

Determine whether the relations represented by these zero-one matrices are partial orders

   a.

1	1	1
1	1	0
0	0	1

   b.

1	1	1
0	1	0
0	0	1

   c.

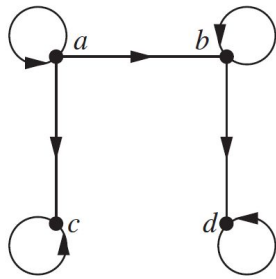|			
1	1	1	0
0	1	1	0
0	0	1	1
1	1	0	1

Solution:

   a. No

   b. Yes

   c. No
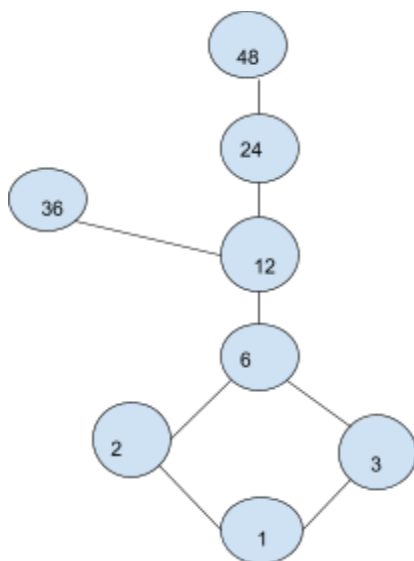
9

Solution:

No

11

Solution:

Yes

23 (c, d)

Draw the Hasse diagram for divisibility on the set

   a. {1, 2, 3, 4, 5, 6, 7, 8}
   b. {1, 2, 3, 5, 7, 11, 13}
   c. {1, 2, 3, 6, 12, 24, 36, 48}
   d. {1, 2, 4, 8, 16, 32, 64}.

Solution:

C.

D.

64
|
32
|
16
|
8
|
4
|
2
|
1

27

d   e   f

g

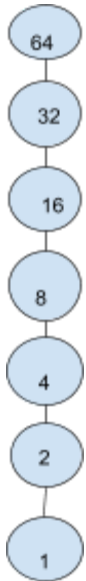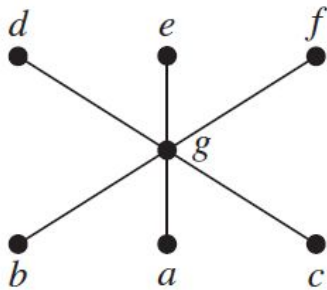b   a   c

Solution:
(a, a), (a, g), (a, d), (a, e), (a, f), (b, b), (b, g), (b, d), (b, e), (b, f), (c, c), (c, g), (c, d), (c, e), (c, f), (g, d), (g, e), (g, f), (g, g), (d, d), (e, e), (f, f)

33
Answer these questions for the poset ($\{3, 5, 9, 15, 24, 45\}$, |)
   a. Find the maximal elements
   b. Find the minimal elements
   c. Is there a greatest element?
   d. Is there a least element?
   e. Find all upper bounds of $\{3, 5\}$
   f. Find the least upper bound of $\{3, 5\}$, if it exists
   g. Find all lower bounds of $\{15, 45\}$
   h. Find the greatest lower bound of $\{15,45\}$, if it exits

Solution:

a. 24, 45
b. 3, 5
c. No
d. No
e. 15, 45
f. 15
g. 15, 5, 3
h. 15