The Conditional statements- implication. p → q

p hypothesis, premise, antecedent.
q conclusion, consequent.

- It is false only when p is true and q is false, true otherwise. (obligation, contract)
- Asserts the q is true in the condition that p holds.
- Implication: p & q are true and when p is false no matter what q is
- **No connection between** p, q. Mathematical concept is independent of a cause-and-effect relationship between. (it depends on the truth value and not the English language)--Propositional language is an artificial language.

  - 'If the moon is made of green cheese then I have more than Bill Gates' (T)
  - 'If the moon is made of green cheese then I'm on welfare' (T)

  - If it is sunny today, then we will go to the beach.??
  - If today is Friday, then 2 + 3 = 5
    - the truth value of the hypothesis does not matter
  - If today is Friday, then 2 + 3 = 8
    - The whole statement is true every day except Friday

Implication in English

if *p*, **then** *q*	*p* **implies** *q*
if *p*, *q*	*p* **only if** *q*
q **unless** ¬*p*	*q* **when** *p*
q **if** *p*	*q* **when** *p*
q **whenever** *p*	*p* **is sufficient for** *q*
q **follows from** *p*	*q* **is necessary for** *p*

a necessary condition for *p* **is** *q*
a sufficient condition for *q* **is** *p*

If p then q in Programming

q executed only if p is true.
x = 0
If 2 + 2 = 4 then x := x + 1

From $p \to q$ we can form new conditional statements: (Write the truth table for each)

- Converse: $q \to p$
- Contrapositive: $\neg q \to \neg p$ equivalent $p \to q$
- Inverse: $\neg p \to \neg q$

Example 'Raining tomorrow is a sufficient condition for my not going to town.'

- Assign propositional variables to component propositions
- Symbolize the assertion
- Symbolize the converse
- Convert the symbols back into words

If I don't go to town then it will rain tomorrow'
'Raining tomorrow is a *necessary condition for my* not going to town.'
'My not going to town is a *sufficient condition for it* raining tomorrow.'

Bi-implication, p if and only if q

True when p and q have the same truth value, false otherwise $(p \to q) \wedge (q \to p)$

Bidirectional in English:

p is necessary and sufficient for q
if p then q and conversely
piffq
However, it is not always explicit in natural language

example If you finish your meal, then you can have dessert.

Precision is essential in math and logic, we will distinguish between $p \to q$ and $p \leftrightarrow q$

p	q	conjunction	disjunction	implication	bidirectional	Exclusive or