CECS 228 - HOMEWORK 2
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5

Let P(x) be the statement “x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.

a. $\exists xP(x)$
b. $\forall xP(x)$
c. $\exists x\neg P(x)$
d. $\forall x\neg P(x)$

Solution:

a. There is a student who spends more than five hours every weekday in class.
b. All students spend more than five hours every weekday in class.
c. There is a student who does not spend more than five hours every weekday in class.
d. All students do not spend more than five hours every weekday in class.

7

Translate these statements into English, where C(x) is “x is a comedian” and F(x) is “x is funny” and the domain consists of all people.

a. $\forall x(C(x)\rightarrow F(x))$
b. $\forall x(C(x) \land F(x))$
c. $\exists x(C(x)\rightarrow F(x))$
d. $\exists x(C(x) \land F(x))$

Solution:

a. Every comedian is funny.
b. Everyone is a funny comedian.
c. There exists a person such that if she/he is a comedian, then she/he is funny.
d. There exists a funny comedian.

9

Let P(x) be the statement “x can speak Russian” and let Q(x) be the statement “x knowns the computer language C++.” Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connective. The domain for quantifiers consists of all students at your school.

a. There is a student at your school who can speak Russian and who knows C++.
b. There is a student at your school who can speak Russian but who doesn’t know C++.
c. Every student at your school either can speak russian or knowns C++.
d. No student at your school can speak Russian or knows C++.

Solution:

a. $\exists x(P(x) \land Q(x))$
b. $\exists x(P(x) \land \neg Q(x))$
c. $\forall x(P(x) \lor Q(x))$
d. $\neg \exists x(P(x) \lor Q(x))$

11

Let P(x) be the statement “x=x^2.” If the domain consists of the integers, what are these truth values?

a. P(0)
b. P(1)
c. P(2)
d. P(-1)
e. $\exists xP(x)$
f. $\forall xP(x)$

Solution:

a. True
b. True
c. False
d. False
e. True
f. False

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Determine the truth value of each of these statements if the domain consists of all integers.

a. $\forall n(n+1 > n)$
b. $\exists n(2n = 3n)$
c. $\exists n(n = -n)$
d. $\forall n(3n \leq 4n)$

Solution:

a. True
b. True
c. True
d. False

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Suppose that the domain of the propositional function P(x) consists of the integers 0, 1,2,3 and 4. Write out each of these propositions using disjunctions, conjunctions and negations.

a. $\exists x P(x)$
b. $\forall x P(x)$
c. $\exists x \neg P(x)$
d. $\forall x \neg P(x)$
e. $\neg \exists x P(x)$
f. $\neg \forall x P(x)$

Solution:

a. $P(0) \lor P(1) \lor P(2) \lor P(3) \lor P(4)$
b. $P(0) \land P(1) \land P(2) \land P(3) \land P(4)$
c. $\neg P(0) \lor \neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(34)$
d. $\neg P(0) \land \neg P(1) \land \neg P(2) \land \neg P(3) \land \neg P(34)$
e. $\neg (P(0) \lor P(1) \lor P(2) \lor P(3) \lor P(4))$
f. $\neg (P(0) \land P(1) \land P(2) \land P(3) \land P(4))$

19

Suppose that the domain of the propositional function P(x) consists of the integers 1,2,3,4 and 5. Express these statements without using quantifiers, instead using only negations, disjunctions and conjunctions.

a. $\exists x P(x)$
b. $\forall x P(x)$
c. $\neg \exists x P(x)$
d. $\neg \forall x P(x)$
e. $\forall x((x \neq 3) \rightarrow P(x)) \lor \exists x \neg P(x)$

Solution:

a. $P(1) \lor P(2) \lor P(3) \lor P(4) \lor P(5)$
b. $P(1) \land P(2) \land P(3) \land P(4) \land P(5)$
c. $\neg (P(1) \lor P(2) \lor P(3) \lor P(4) \lor P(5))$
d. $\neg (P(1) \land P(2) \land P(3) \land P(4) \land P(5))$
e. $(P(1) \land P(2) \land P(4) \land P(5)) \lor (\neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(4) \lor \neg P(5))$

21

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a. Everyone is studying discrete mathematics.
b. Everyone is older than 21 years.
c. Every two people have the same mother.
d. No two different people have the same grandmother.

Solution:

a. Domain true: All the students in the class. Domain False: all the students in the whole school.
b. Domain true: Everyone playing in the casinos. Domain False: An elementary class.
c. Domain true:Twins in the world. Domain False:The rest of the world.
d. Domain true: Domain: A classroom with no siblings False: Siblings in the same class.

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Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consists of the students in your class and second, let it consist of all people.

a. Someone in your class can speak Hindi.
b. Everyone in your class is friendly.
c. There is a person in class who was not born is California.
d. A student in your class has been in a movie.
e. No student in your class has taken a course in logic programming.

Solution:

a. H(x): “x can speak Hindi.”
    i. $\exists x H(x)$
    ii. $\exists x(C(x) \wedge H(x))$
b. F(x): “x is friendly”
    i. $\forall x F(x)$
    ii. $\forall x(C(x) \rightarrow F(x))$
c. B(x): “x was born in California”
    i. $\exists x \neg B(x)$
    ii. $\exists x\ (C(x) \wedge \neg B(x)$
d. M(x): “x has been in a movie.”
    i. $\exists x M(x)$
    ii. $\exists x(C(x) \wedge M(x))$
e. L(x): “x has taken a course in logic programming.”
    i. $\forall x \neg L(x)$
    ii. $\forall x(C(x) \rightarrow \neg L(x))$

25

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a. No one is perfect.

b. Not everyone is perfect.
c. All your friends are perfect.
d. At least on of your friends is perfect,
e. Everyone in your friend and is perfect.
f. Not everybody is your friend or someone is not perfect.

Solution:

a. $\forall x \neg P(x)$
b. $\neg \forall x P(x)$
c. $\forall x(F(x) \rightarrow P(x))$
d. $\exists x(F(x)) \land P(x))$
e. $\forall x F(x) \land \forall x P(x)$
f. $(\neg \forall x(F(x)) \lor (\exists x \neg P(x))$

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Suppose that the domain of Q(x, y,z) consists of triples x,y,z where x = 0, 1,or2, y = 0 or 1, and z = 0 or 1. Write out these propositions using disjunctions and conjunctions.

a. $\forall y Q(0,y,0)$
b. $\exists x Q(x, 1,1)$
c. $\exists z \neg Q(0, 0, z)$
d. $\exists x \neg Q(x, 0, 1)$

Solution:

a. $Q(0, 0, 0) \land Q(0,1,0)$
b. $Q(0,1,1) \lor Q(1,1,1) \lor (2,1,1)$
c. $\neg Q(0, 0,0) \lor \neg Q(0,0,1)$
d. $\neg Q(0, 0,1) \lor \neg Q(1,0,1) \lor \neg Q(2,0,1)$

33

Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English.(Do not simply use the phrase “It is not the case that.”)

a. Some old dogs can learn new tricks.
b. No rabbit known calculus.
c. Every bird can fly.
d. There is no dog that can talk.
e. There is no one in this class who knows French and Russian.

Solution:

a. $\exists x T(x)$
    i. Negation: $\neg \exists x T(x)$
    ii. Every old dog is unable to learn new tricks

b. $\neg \exists x C(x)$
   i. Negation: $\exists x C(x)$
   ii. There is a rabbit that knows calculus.
c. $\forall x F(x)$
   i. Negation: $\exists x \neg F(x)$
   ii. There is a bird who cannot fly.
d. $\neg \exists x T(x)$
   i. $\exists x T(x)$
   ii. There is a dog that talks.
e. $\neg \exists x(F(x) \land R(x))$
   i. $\exists x(F(x) \land R(x))$
   ii. There is someone in this class who knows French and Russian.

37

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variable consists of all integers.

a. $\forall x(x^2\ x)$
b. $\forall x(x > 0 \lor x < 0)$
c. $\forall x(x = 1)$

Solution:

a. True, no counterexample.
b. Since 0 is neither greater than nor less than 0, this is a counterexample.
c. This proposition says that 1 is the only integer - that every integer equals 1. If is obviously false, and any other integer, such as -1000000, provides a counterexample

41

Translate these specification into English, where F(p) is "Printer p is out of service," B(p) is "Printer p is busy," L(j) is "Print job j is lost," and Q(j) is "Prin job j is queued."

a. $\exists p(F(p) \land B(p)) \rightarrow \exists j L(j)$
b. $\forall x B(p) \rightarrow \exists j Q(j)$
c. $\exists j(Q(j) \land L(j)) \rightarrow \exists j L(j)$
d. $(\forall p B(p) \land \forall j Q(j)) \rightarrow \exists j L(j)$

Solution:

a. If there is a printer that is both out of service and busy, then some job has been lost.
b. If every printer is busy, then there is a job in queue.
c. If there is a job that is both queued and lost, then some printer is out of service.
d. If every printer is busy and every job is queued, then some job is lost.

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Express each of these systems specifications using predicates, quantifiers, and logical connectives.

a. At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space.
b. Whenever there is an active alert, all queued messages are transmitted.
c. The diagnostic monitor tracks the status of all systems except the main console.
d. Each participant on the conference call whom the host of the call did not put on a special list was billed.

Solution:

a. $(\exists x F(x,10)) \to \exists x S(x)$, where F(x,y) is “Disk x has more than y kilobytes of free space,” and S(x) is “Mail message x can be saved”
b. $(\exists x A(x)) \to \forall x(Q(x) \to T(x))$, where A(x) is “Alert x is active,” Q(x) is “Message x is queued,” and T(x) is “Message x is transmitted”
c. $\forall x((x \neq \text{main console}) \to T(x))$, where T(x) is “The diagnostic monitor tracks the status of system x”
d. $\forall x(\neg L(x) \to B(x))$, where L(x) is “The host of the conference call put participant x on a special list” and B(x) is “Participant x was billed”

55

what are the truth values of these statements?

a. $\exists !x P(x) \to \exists x P(x)$
b. $\forall x P(x) \to \exists !x P(j)$
c. $\exists !x \neg P(x) \to \neg \forall x P(x)$

Solution:

a. True
b. True
c. True

63

Let P(x), Q(x), R(x) and S(x) be the statements “x is a baby,” “x is logical,” “x is able to manage a crocodile,” and “x is despised,” respectively. Suppose that the domain consists of all people.Express each of these statements using quantifiers;logical connectives; and P(x), Q(x), R(x), and S(x).

a. Babies are illogical.
b. Nobody is despised who can manage a crocodile.
c. Illogical persons are despised.
d. Babied cannot manage crocodiles.
e. Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

Solution:

a. $\forall x(P(x) \to \neg Q(x))$
b. $\forall x(R(x) \to \neg S(x))$
c. $\forall x(\neg Q(x) \to S(x))$
d. $\forall x(P(x) \to \neg R(x))$

e. Suppose that x is a baby. Then by a) x is illogical and by c) x is despised. b) says that if x could manage a crocodile, then x would not be despised. Therefore x cannot manage a crocodile. And therefore we can say that babies cannot manage crocodiles.

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11

Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," and A(x,y,z) the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.

a. Lois has asked professor Michaels a question.
b. Every student has asked Professor Gross a question.
c. Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
d. Some students has not asked any faculty member a question.
e. There is a faculty member who has never been asked a question by a student.
f. Some students has asked every faculty member a question.
g. There is a faculty member who has asked every other faculty member a question.
h. Some student has never been asked a question by a faculty member.

Solution:

a. A(Lois, Professor Michaels)
b. $\forall x(S(x) \rightarrow A\ (x, \text{Professor Gross}))$
c. $\forall x(F(x) \rightarrow (A(x,\text{Professor Miller}) \vee A(\text{Professor Miller}, x)))$
d. $\exists x(S(x) \wedge \forall y(F(y) \rightarrow \neg A(x,y)))$
e. $\exists x(F(x) \wedge \forall y(S(y) \rightarrow \neg A(y,x)))$
f. $\forall y(F(y) \rightarrow \exists x(S(x) \wedge A(x,y)))$
g. $\exists x(F(x) \wedge \forall y\ ((F(y) \wedge y \neq x) \rightarrow A(x,y)))$
h. $\exists x(S(x) \wedge \forall y(F(y) \rightarrow \neg A(y,x)))$

13

Let M(x,y,z) be "x has sent y an e-mail message" and T(x,y,z) be "x has telephoned y," where the domain consists of all students in your class.Use quantifiers to express each of these statements.(Assume that all e-mail messages that were sent are received, which is not the way things often work.)

a. Chou has never sent an e-mail message to Koko.
b. Alrelene has never sent an e-mail message to or telephoned Sarah.
c. Jose has never received an email message form Deborah.
d. Every student in your class has sent an e-mail message to ken.
e. No one in your class has telephoned Nina.
f. Everyone in your class has either telephoned Avi or sent him an e-mail message.
g. There is a student in your class who has sent everyone else in your class an e-mail message.

h. There is a student in your class who has either sent an e-mail message or telephoned everyone else in your class.

Solution:

a. $\neg M$ (Chou, Koko)
b. $\neg$(M(Arlene,Sarah) $\vee$ T(Arlene, Sarah))
c. $\neg$M(Deborah, Jose)
d. $\forall x$M(x,Ken)
e. $\neg \exists x$T(x,Nina)
f. $\forall x$(T(x,Avi) $\vee$ M(x,Avi))
g. $\exists x \forall y(y \neq x \rightarrow M(x,y))$
h. $\exists x \forall y(y \neq x \rightarrow M(x,y) \vee T(x,y)))$

25

Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

a. $\exists x \forall y(xy=y)$
b. $\forall x \forall y(((x<0) \wedge (y<0)) \rightarrow (xy >0))$
c. $\exists x \exists y((x^2 > y) \wedge (x<y))$
d. $\forall x \forall y \exists z(x+y=z)$

Solution:

a. True
b. True
c. True
d. True

27

Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a. $\forall n \exists m(n^2 < m)$
b. $\exists n \forall m(n<m^2)$
c. $\forall n \exists m(n+m=0)$
d. $\exists n \forall m(nm = m)$
e. $\exists n \exists m(n^2 + m^2 = 5)$
f. $\exists n \exists m(n^2 + m^2 = 6)$
g. $\exists n \exists m(n + m = 4 \wedge n - m = 1)$
h. $\exists n \exists m(n + m = 4 \wedge n - m = 2)$
i. $\forall n \forall m \exists p(p = (m + n) / 2)$

Solution:

a. True
b. True
c. True
d. True
e. True

f. False
g. False
h. True
i. False

33
Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a. $\neg\forall x\forall y P(x,y)$
b. $\neg\forall y\exists x P(x,y)$
c. $\neg\forall y\forall x(P(x,y)\vee Q(x,y))$
d. $\neg(\exists x\exists y\neg P(x,y)\wedge\forall x\forall y Q(x,y))$
e. $\neg\forall x(\exists y\forall z P(x,y,z)\wedge\exists z\forall y P(x,y,z))$

Solution:

a. $\exists x\exists y\neg P(x,y)$
b. $\exists y\forall x\neg P(x,y)$
c. $\exists y\exists x\neg(P(x,y)\ \vee\ Q(x,y))$ - De Morgan's Law - $\exists y\exists x(\neg P(x,y)\ \wedge\neg\ Q(x,y))$
d. $(\neg\exists x\exists y\neg P(x,y))\ \vee\ (\neg\forall x\forall y Q(x,y))$ - $(\neg\neg P(x,y)\equiv P(x,y))$ - $(\forall x\forall y P(x,y))\ \vee$ $(\exists x\exists y\neg Q(x,y))$
e. $\exists x\neg(\exists y\forall z P(x,y,z)\ \wedge\ \exists z\forall y P(x,y,z))\equiv\exists x(\neg\exists y\forall z P(x,y,z)\ \vee\ \neg\exists z\forall y P(x,y,z))\equiv$ $\exists x(\forall y\exists z\neg P(x,y,z)\ \vee\ \forall z\exists y\neg P(x,y,z))$

37
Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a. Every student in this class has taken exactly two mathematics classes at this school.
b. Someone has visited every country in the world except Libya.
c. No one has climbed every mountain in the Himalayas.
d. Every movie actor has either been in a movie with Kevin Bacon or has either been in a movie with someone who has been in a movie with someone who has been in a movie with Kevin Bacon.

Solution:

a. Let $T(x,y)$ be the predicate that has taken y, where x ranges over students in this class and y ranges over mathematics classes at this school. Then our original statement is $\forall x\exists y\exists z(y\neq z\ \wedge\ T(x,y)\ \wedge\ T(x,z)\ \wedge\ \forall w T(x,w)\rightarrow(w=y\ \vee\ w=z)))$. Then y and z are the two math classes that x has taken, and our statement says that these are different and that if x has taken any math class w, then w is one of these two. $\exists x\forall y\forall z(y=z\ \vee$ $\neg T(x,y)\ \vee\neg T(x,z)\ \vee\ \exists w(T(x,w)\ \wedge\ w\neq y\ \wedge\ w\neq z))$. In simple English, this statement reads "There is someone in this class for whom no matter which two distinct math courses you consider, these are not the two and only two math courses this person has taken."

b. Let $V(x,y)$ be the predicar that x has visited y, where x ranges over people and y ranges over countries. The statement seems to be asserting that the person identified here has visited country y if and only if y is not Libya. So we can write this symbolically as $\exists x \forall y(V(x,y) \leftrightarrow y \neq \text{Libya})$. One way to form the negation of $P \leftrightarrow Q$ is to write $P \leftrightarrow \neg Q$; this can be seen by looking at truth tables. Thus the negation is $\forall x \exists y(V(x,y) \leftrightarrow y = \text{Libya})$. Note that there are two ways for a biconditional to be true; therefore in English this reads “For every person there is a country such that either that country is Libya and the person has visited it, or else that country is not Libya and the person has not visited it.” More simply, “For every person, either that person has visited Libya or else the person has failed to visit some country other than Libya.”

c. Let $C(x,y)$ be the predicate that x has climbed y, where x ranges over people and y ranges over mountains in the Himalayas. Our statement is $\neg \exists x \forall y C(x,y)$. Its negation is, of course, simply $\exists x \forall y C(x,y)$. In English this reads “Someone has climbed every mountain in the Himalayas.”

d. There are different ways to approach this, depending on how many variables we want to introduce. Let $M(x,y,z)$ be the predicate that x has been in movie z with y, where the domains for x and y are movie actors, and for z is movies. The statement then reads: $\forall x((\exists z M(x,\text{Kevin Bacon},z)) \vee (\exists y \exists z1 \exists z2\ (M(x,y,z1) \wedge (M(y, \text{Kevin Bacon}, z2)))$. The negation is formed in the usual manner: $\exists x((\forall z \neg M(x,\text{Kevin Bacon},z)) \wedge (\forall y \forall z1 \forall z2(\neg M(x,y,z1) \vee \neg M(y,\text{Kevin Bacon},z2))))$. In simple English this means that there is someone who has neither been in a movie with Kevin Bacon nor been in a movie with someone who has been in a movie with Kevin Bacon.

47

Show that the two statements $\neg \exists x \forall y P(x,y)$ and $\forall x \exists y \neg P(x,y)$, where both quantifiers over the first variable in $P(x,y)$ have the same domain, and both quantifiers over the second variable in $P(x,y)$ have the same domain, are logically equivalent.

Solution:

$\neg \exists x \forall y P(x,y) \equiv \forall x \neg \forall y P(x,y) \equiv \forall x \exists y \neg P(x,y)$