CECS 228 - HOMEWORK 6
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PAGE 177 #15, 17, 29, 33, 35, 37, 39

15

Show that the sequence { a_n } is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if

a. $a_n = -n + 2$
b. $a_n = 5(-1)^n - n + 2$
c. $a_n = 3(-1)^n + 2^n - n + 2$
d. $a_n = 7 \cdot 2^n - n + 2$

Solution:

a. $a_n = a_{n-1} + 2a_{n-2} + 2n - 9 = -(n-1) + 2 + 2[-(n - 2) + 2] + 2n - 9 = -n + 2$
b. $a_n = a_{n-1} + 2a_{n-2} + 2n - 9 = 5(-1)^{n-1} - (n - 1) + 2 + 2[5(-1)^{n-2} - (n - 2) + 2] + 2n - 9 = 5(-1)^{n-2} (-1+2) - n + 2$
c. $a_n = a_{n-1} + 2a_{n-2} + 2n - 9 = 3(-1)^{n-1} + 2^{n-1} - (n - 1) + 2 + 2[3(-1)^{n-2} + 2^{n-2} - (n - 2) + 2] + 2n - 9 = 3(-1)^{n-2} (-1 + 2) + 2^{n-2} (2 + 2) - n + 2$
d. $a^{n-1} + 2a^{n-2} + 2n - 9 = 7 \cdot 2^{n-1} - (n - 1) + 2 + 2[7 \cdot 2^{n-2} - (n - 2) + 2] + 2n - 9 = 2^{n-2}(7 \cdot 2 + 2 \cdot 7) - n + 2$

17

Find the solution to each of these recurrence relations and initial conditions. Use an iterative approach such as that used in Example 10.

a. $a_n = 3a_{n-1}$, $a_0 = 2$
b. $a_n = a_{n-1} + 2$, $a_0 = 3$
c. $a_n = a_{n-1} + n$, $a_0 = 1$
d. $a_n = a_{n-1} + 2n + 3$, $a_0 = 4$
e. $a_n = 2a_{n-1} - 1$, $a_0 = 1$
f. $a_n = 3a_{n-1} + 1$, $a_0 = 1$
g. $a_n = n a_{n-1}$, $a_0 = 5$
h. $a_n = 2n a_{n-1}$, $a_0 = 1$

Solution:

a. $a_n = 2 \cdot 3^n$
b. $a_n = 2n + 3$
c. $a_n = 1 + n(n + 1) / 2$
d. $a_n = n^2 + 4n + 4$
e. $a_n = 1$
f. $a_n = (3^{n+1} - 1) / 2$
g. $a_n = 5n!$

h. $a_n = 2^n n!$

29

What are the values of these sums?

a. $\sum_{k=1}^{5} (k+1)$

b. $\sum_{j=0}^{4} (-2)^j$

c. $\sum_{i=1}^{10} 3$

d. $\sum_{j=0}^{8} (2^{j+1} - 2^j)$

Solution:

a. 20

b. 11

c. 30

d. 511

33

Compute each of these double sums.

a. $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$

b. $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i+3j)$

c. $\sum_{i=1}^{3} \sum_{j=0}^{2} i$

d. $\sum_{i=0}^{2} \sum_{j=0}^{2} ij$

Solution:

a. 21

b. 78

c. 18

d. 18

35

Show that $\sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0$, where a_0, a_1, …, a_n is a sequence of real numbers. This type of sum is called telescoping.

Solution:

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = (a_1 - a_0) + (a_2 - a_1)(a_3 - a_2) + ... + (a_{n-1} - a_{n-2}) + (a_n - a_{n-1})$$

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_1 - a_0 + a_2 - a_1 + a_3 - a_2 + ... + a_{n-1} - a_{n-2} + a_n - a_{n-1}$$

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = - a_0 + 0 + 0... + 0 + 0 + a_n$$

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$$

Therefore, $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$

37

Sum both sides of the identity $k^2 - (k-1)^2 = 2k - 1$ from $k = 1$ to $k = n$ and use Exercise 35 to find

a. a formula for $\sum_{k=1}^{n}(2k-1)$ (the sum of the first n odd natural numbers).

b. a formula for $\sum_{k=1}^{n} k$.

Solution:

a. n^2

b. $n(n + 1) / 2$

39

Find $\sum_{k=100}^{200} 100^k$

Solution:

15150

PAGE 197 #1, 3, 5, 27, 29

1

Let A be the set of English words that contain the letter x, and let B be the set of English words that contain the letter q. Express each of these sets as a combination of A and B.

a. The set of English words that do not contain the letter x.

b. The set of English words that contain both an x and a q.

c. The set of English words that contain an x but not a q.

d. The set of English words that do not contain either an x or a q.

e. The set of English words that contain an x or a q, but not both.

Solution:

a. not A
b. $A \cap B$
c. $A - B$
d. not $A \cap$ not B
e. $A \oplus B$

3

Suppose that A and B are sets such that the power set of A is a subset of the power set of B. Does it follow that A is a subset of B?

Solution:

Yes

5

Show that if A and B are sets, then $A - (A - B) = A \cap B$.

Solution:

A - (A - B) = A - (A ∩ not B) = A ∩ not (A ∩ not B) = A ∩ (not A ∪ B) = (A ∩ not A) ∪ (A ∩ B) = ∅ ∪ (A ∩ B) = A ∩ B.

27

Prove that if m is a positive integer and x is a real number, then

floor[mx] = floor[x] + floor[x+ $\frac{1}{m}$] + floor[x+ $\frac{2}{m}$] + … + floor[x+ $\frac{m-1}{m}$] .

Solution:

Let $x = n + (r/m) + \epsilon$, where n is an integer, r is a nonnegative integer less than m, and ϵ is a real number with $0 \le \epsilon < 1/m$. The left-hand side is floor[$nm + r + m\epsilon$] = nm + r. On the right hand side, the terms floor[x] through floor[$x + (m + r - 1)/m$] are all just n and the terms from floor[$x + (m - r)/m$] on are all n + 1. Therefore, the right-hand side is $(m - r)n + r(n + 1) = nm + r$, as well.

29

Determine the value of $\prod_{k=1}^{100} \frac{k+1}{k}$.

Solution:

101