CECS 228 HOMEWORK 7
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Prove that $\sum_{j=1}^{n} j^4 = n(n+1)(2n+1)(3n^2+3n-1)/30$ whenever n is a positive integer.

Solution:

Let P(n) be the statement that $1^4+2^4+3^4+\cdots + n^4 = n(n+1)(2n+1)(3n^2+3n-1)/30$. P(1) is true because $1 \cdot 2 \cdot 3 \cdot 5/30 = 1$. Assume that P(k) is true. Then $(1^4+2^4+3^4+\cdots + k^4) + (k+1)^4$

$= k(k+1)(2k+1)(3k2+3k-1)/30+(k+1)^4$

$= [(k+1)/30][k(2k+1)(3k2 +3k-1)+30(k+1)3]$

$= [(k+1)/30](6k4+39k3+91k2+89k+30)$

$= [(k+1)/30](k+2)(2k+3)[3(k+1)2+3(k+1)-1]$.

This demonstrates that P(k+1) is true.

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Prove that 5 divides $n^5 - n$ whenever n is a nonnegative integer.

Solution:

Let P(n) be "$n^5 - n$ is divisible by 5." Basis step: P(0) is true because $0^5 - 0 = 0$ is divisible by 5.
Inductive step: Assume that P(k) is true, that is, $k^5 - 5$ is divisible by 5.

Then $(k+1)^5-(k+1)$

$= (k^5+5k^4+10k^3+10k^2+5k+1)-(k+1)$

$= (k^5 -k)+5(k^4 +2k^3 +2k^2 +k)$

is also divisible by 5, because both terms in this sum are divisible by 5.

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Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well defined, find a formula for $f(n)$ when n is a nonnegative integer and prove that your formula is valid.

a. $f(0) = 0, f(n) = 2f(n-2)$ for $n \geq 1$
b. $f(0) = 1, f(n) = f(n-1) - 1$ for $n \geq 1$
c. $f(0) = 2, f(1) = 3, f(n) = f(n-1) - 1$ for $n \geq 2$

d. $f(0) = 1, f(1) = 2, f(n) = 2f(n-2)$ for $n \geq 2$

e. $f(0) = 1, f(n) = 3f(n-1)$ if n is odd and $n \geq 1$ and $f(n) = 9f(n-2)$ if n is even and $n \geq 2$

Solution:

a. Not valid

b. $f(n) = 1 - n$.
   Basis step: $f(0) = 1 = 1 - 0$. Inductive step: if $f(k) = 1-k$, then $f(k+1) = f(k)-1 = 1-k-1 = 1-(k+1)$.

c. $f(n) = 4 - n$ if $n > 0$, and $f(0) = 2$.
   Basis step: $f(0) = 2$ and $f(1) = 3 = 4 - 1$. Inductive step: $f(k+1) = f(k) - 1 = (4 - k) - 1 = 4 - (k + 1)$.

d. $f(n) = 2^{\text{floor}((n+1)/2)}$
   Basis step: $f(0) = 1 = 2^{\text{floor}((0+1)/2)}$ and $f(1) = 2 = 2^{\text{floor}((1+1)/2)}$
   Inductive step: $f(k+1) = 2f(k-1) = 2 \cdot 2^{\text{floor}(k/2)} = 2^{\text{floor}(k/2)+1} = 2^{\text{floor}(((k+1)+1)/2)}$

e. $f(n) = 3n$.
   Inductive step: For odd n, $f(n) = 3f(n-1) = 3 \cdot 3^{n-1} = 3^n$; and for even $n > 1$, $f(n) = 9f(n-2) = 9 \cdot 3^{n-2} = 3^n$.

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Give a recursive definition of

a. the set of even integers.

b. the set of positive integer congruent to 2 modulo 3.

c. the set of positive integers not divisible by 5.

Solution:

a. $0 \in S$, and if $x \in S$, then $x+2 \in S$ and $x-2 \in S$.

b. $2 \in S$, and if $x \in S$, then $x+3 \in S$.

c. $1 \in S, 2 \in S, 3 \in S, 4 \in S$, and if $x \in S$, then $x + 5 \in S$.

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Find these values of Ackermann’s function.

a. A(2,3)

b. A(3,3)

Solution:

a. 16

b. 65536

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Give iterative and recursive algorithms for finding the n th term of the sequence defined by $a_0 = 1$, $a_1 = 3$, $a_2 = 5$, and $a_n = a_{n-1} \cdot a_{n-2}^2 \cdot a_{n-3}^3$. Which is more efficient?

Solution:

We first give a recursive procedure and then an iterative procedure.

if n < 3 then return 2n + 1

else return r(n − 1) · (r(n − 2))^2 · (r(n − 3))^3

if n = 0 then z := 1

else if n = 1 then z := 3

else

x := 1

y := 3

z := 5

for i := 1 to n − 2

w := z · y^2 · x^3

x := y

y := z

z := w

return z {z is the nth term of the sequence}

The iterative version is more efficient.

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a. Describe the merge sort algorithm.
b. Use the merge sort algorithm to put the list 4, 10, 1, 5, 3, 8, 7, 2, 6, 9 in increasing order.
c. Give a big-O estimate for the number of comparisons used by the merge sort.

Solution:

a. A sorting algorithm sorts a list by splitting it into two sorting, each of two resulting lists and merging the results into a sorted list.

b.

First list	Second list	Merged list
4, 10, 1, 5, 3	8,7,2,6,9	
4, 10, 5 ,3	8, 7,2 ,6 9	1
4 , 10, 5 ,3	8,7,6,9	1, 2
10, 5	8, 7, 6 ,9	1,2, 3
10	8, 7,6,9	1, 2,3,4
10	8, 7,9	1,2,3,4,5
10	8,9	1,2,3,4,5,6,
10	9	1,2,3,4,5,6,7
10		1,2,3,4,5,6,7,8
10		1,2,3,4,5,6,7,8,9
		1,2,3,4,5,6,7,8,9,10

c. O(n log n) → O(10 log 10) = 10

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45 b & e

By successively using the defining rule for M (n) , find

a. M(102)
b. M(101)
c. M(99)
d. M(97)
e. M(87)
f. M(76)

Solution:

a.
b. 91
c.
d.
e. 91
f.