CECS228
Lecture 1

Goal: What is a correct mathematical argument, Tools to construct those arguments. Validate an argument

Rules of logic give a precise meaning to mathematical statements

Propositions

- Basic Building blocks of logic (Statement)
- A (declarative) sentence that declares a fact, could be True 1 or False 0 , not both. (1,0 binary logic)

Propositional Statements

- I love Banana
- Toronto is the capital of Canada
- 4 - 7 = 8

I love Banana A. True B. False C. Don't know D. Is not an option E. I don't know my alphabet F. Give me zero for this question

Toronto is the capital of Canada A. True B. False C. Don't know D. Is not an option E. I don't know my alphabet F. Give me zero for this question

Non propositional Statements

- How are you? imperative
- Read chapter 5 interrogative
  - Non declarative statements (question, order)
- x + y = z
- The rule is:
  - A none propositional statements are neither true nor false

But….

- Today is Friday

- It is raining in Miami now

We need a fixed time, place, person…. And we will assume so

Propositional variables Like the Letters represent numerical variables
- We use variables (*p,q,r,s..*) to denote propositions.

Atomic propositions: Propositions that cannot be expressed in terms of simpler propositions

Compound propositions

We can produce new propositions from what we already have (George Boole)
- Many mathematical statements are constructed by combining one or more propositions using **logical operators** (unary, binary)
- We use truth tables to evaluate our proposition. There is A row for *each* possible truth values of a proposition

Let us assume we have proposition p, q

the **negation of p** is represented by

$\sim p, \neg p, \bar{p}$, $!p, p'$, Np, not p Unary.

it is not the case that p (p and $\neg p$ will always have opposite truth value)
- Today is Friday
- It is not the case that today is Friday (Don't keep it like this, write in proper English)
- Today is not Friday

The conjunction of p, q:

p and q, p Λ q, pq, p.q

The new proposition is True when both p and q are true, false otherwise.

But, sometimes is used instead of and
- I earned A in the exam, but my I got B in the class.

Not every and is a conjunction

The disjunction of p,q:

p or q, p V q, p + q

The compound proposition is False when both p and q are false, True otherwise.
- Inclusive
  - Prerequisite CECS 174 or CECS 175
- Exclusive p XOR q, p ⊕ q True when exactly one of p and q is true, false otherwise
  - Soup or salad comes with an entrée
  - I can drive my red or blue car

The Conditional statements- implication. p → q
  - p hypothesis, premise, antecedent.
  - q conclusion, consequent.
- It is false only when p is true and q is false, true otherwise. (obligation, contract)

- Asserts the q is true in the condition that p holds…… more about implication next lecture
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Bidirectional $p \leftrightarrow q$, p iff q

Bi-implication, p if and only if q

- True when p and q have the same truth value, false otherwise
  $(p \rightarrow q) \wedge (q \rightarrow p)$
  Bidirectional in English:
- p is necessary and sufficient for q
  - if p then q and conversely
  - p iff q
  - Not always explicit in natural language
    - If you finish your meal, then you can have dessert.
    - Precision is essential in math and logic, we will distinguish between $p \rightarrow q$ and $p \leftrightarrow q$

Other logical operations:

- NAND (|) Sheffer Stroke
- NOR (↓) Peirce Arrow
- Breaking assertions into component propositions

Compound propositions

- Logical connectives + negation → more complicated compound propositions.
- In truth tables: we use a separate column to find the truth value of each compound expression that occurs in the compound proposition.