Sets

A ***set is an unsorted collection or group of objects, elements or*** *members.*

A set is said to *contain its elements.*

- $a \in A$
- $a \notin A$

There must be an underlying universal set U, either specifically stated or understood.

Notation Upper case for the set, lower case for the element.

Describing a set

List all elements (if possible) **between braces**:

$S = \{a, b, c, d\}=\{b, c, a, d, d\}$

- Note:
  - listing an object more than once does not change the set.
  - Ordering means nothing.
- V = { e, I, o, u, a} vowels
- O = {1, 3, 5, 7, 9 } Odd integers < 10

Use brace notation with ellipses:

- S = { . . . , -3, -2, -1} The negative integers.
- General pattern is obvious.
- L = { 1, 2, 3, …., 99} positive integers < 100
  - Infinite sets
  - Pattern

Use set builder notation

- By stating the property(ies) must have to be members
  - $O=\{x|x \text{ is a prime number} > 10\}$
- Or by specifying the universe of the elements
  - $Q^+ =\{ x \in R \mid x= p/q\}$
  - Specification by predicates: $S= \{x|\ P(x)\}$, S contains all the elements from U which make the predicate *P true.*

Common Universal Sets

R = real numbers

N = natural numbers = {0,1, 2, 3, . . . }, the *counting numbers*

Z = all integers = {. . , -3, -2, -1, 0, 1, 2, 3, 4, . .}

Z^+ is the set of positive integers

$Q^+ =\{p/q \mid p \in Z, q \in Z, \text{ and } q \neq 0\}$, rational numbers

In computer Science: Datatype, when you declare a variable.

Sets equality A = B

Two sets are equal iff they have the same elements

- $\forall x\ (x \in A \leftrightarrow x \in B)$ must be true
- Example {1, 2, 3} = {3, 1, 2} = {1, 2, 2, 3, 3, 3}

Venn Diagrams

Used to represent sets graphically, and to indicate relationship between sets.

Universal set U: contains all objects under consideration

Φ the empty set

The *void set, the null set, the empty set,* Φ, *{}, null*

Example: Set of all positive integers greater than their square = ϕ

the assertion ***$x \in \phi$ is always false***

Empty folder on your computer

The singleton set

Has one element only

example

- $\{\phi\}$ empty folder within an empty folder

Notice $\{\phi\} \neq \phi$

A Subset of B

The set A is a *subset of the set B, denoted A*$\subseteq$B, iff $\forall x\, [x \in A \rightarrow x \in B]$ must be true

- Iff every element of A is an element of B

Please Note

- *$x \in \phi$ is always **false***
- $\forall x\, [x \in \phi \rightarrow x \in B]$ is always **true**
- ϕ is a subset of **every set**.
- a set B is always a **subset of itself**.

Proper subset

If $A \subseteq B$ (A is subset of B) and $A \neq B$ then $A \subset B$

- Exist an element of B not element of A
- $\forall x\, (x \in A \rightarrow x \in B) \wedge \exists x\, (x \in B \wedge x \notin A)$
- Set of odd positive integer less than 10 is a subset of the set of all positive integers less than 10

Set notation & quantifiers

$\forall x \in S(P(x))$ is a universal quantifier of P(x) over all elements in the set S

- $\forall x \in S(P(x))$ means $\forall x \in (x \in S \rightarrow P(x))$
- $\exists\, x \in S(P(x))$ means $\exists\, x \in (x \in S \wedge P(x))$

Example:

- $\forall x \in \mathbf{R}(x^2 \geq 0)$ means Square of real ≥ 0,
- $\exists\, x \in \mathbf{Z}(x^2 = 1)$ means At least the square of one integer x =1

A = B again

To show that A = B (two sets are equal) we show that: Each set is a subset of the other

$A \subseteq B$ and $B \subseteq A$

- A = B iff $\forall x\, (x \in A \rightarrow x \in B)$ and $(x \in B \rightarrow x \in A)$
- $\forall x\, (x \in A \leftrightarrow x \in B)$

Theorem: For every set S, $\phi \subseteq S$ and $S \subseteq S$

Proof: $\forall x\, [x \in \phi \rightarrow x \in S]$ is always true

- ϕ is a subset of every set.

$\forall x\, [x \in S \rightarrow x \in S]$ is always true

- a set is always a subset of itself.

Definitions

Power set of A: P(A) The set of all subset of a set A

- Example: If A = {a, b} then P(A) = $\{\phi, \{a\}, \{b\}, \{a,b\}\}$
- sets as members

- B = P(A) = { x | x is a subset of the set {a,b}}

Note:

- P(A) = B, {a} ∈ P(A), but {a} ∉ A
- {a} ∈ B, but a ∉ B

Cardinality of set A |A|: The number of (distinct) elements in A, denoted |A|

If the cardinality is a natural number (in N), then the set is called *finite, else infinite.*

- S letters of English alphabet |S| = 26
- |∅ | = 0

If S has n elements, then P(s) has 2^n

- S= {1, 2, 3}
    - P(S) = {∅, {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}
- P(∅) = {∅}   ///empty set
- P({∅}) = {∅,{∅}}   ///set {∅}

n tuple, Cartesian product set operations and Set identities