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Homework 5: vectors

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1. For vectors $v = [-2,5,1]$ and $u = [3,0,1]$ find the following vectors:

a) $u+v$

$[3,0,1] + [-2,5,1]$

$[3+(-2), 0+5, 1+1]$

$[1,5,2]$

b) $v-u$

$[-2,5,1] - [3,0,1]$

$[-2-3, 5-0, 1-1]$

$[-5,5,0]$

c) $2v-3u$

$2[-2,5,1] - 3[3,0,1]$

$[-2\cdot2, 5\cdot2, 1\cdot2] - [3\cdot3, 3\cdot0, 3\cdot1]$

$[-4,10,2] - [9,0,3]$

$[-4-9, 10-0, 2-3]$

$[-13,10,-1]$

2. Here are six 7-vectors over GF(2):

a = 1010101 d = 0101101

b = 0101011 e = 1101011

c = 0110100 f = 1110100

For each of the following vectors u, find a subset of the above vectors whose sum is u, or report that no subset exists.

a) u = 0001111 b) u = 1000110 c) u = 1100011

no such subset exists d and e no such subset exists

a)	b)	c)
a 1010101	a 1010101	a 1010101
b 0101011	c 0110100	d 0101101
1111110	1100001	1111000
a 1010101	a 1010101	b 0101011
e 1101011	f 1110100	c 0110100
0111110	0100001	0011111

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b 0101011	b 0101011	b 0101011
d 0101101	e 1101011	f 1110100
0000110	1000000	1011111
c 0110100	c 0110100	c 0110100
d 0101101	e 1101011	f 1110100
0011001	1011111	1000000
d 0101101	d 0101101	e 1101011
e 1101011	f 1110100	f 1110100
1000110	1011001	0011111

3 show that the set of all n-vectors over GF(2) is a field.

a) closure

$x+y$

$[0_0, 0_1, 0_2, \ldots, 0_n] + [1_0, 1_1, 1_2, \ldots, 1_n]$

$[0_0+1_0, 0_1+1_1, 0_2+1_2, \ldots, 0_n+1_n] \in GF(2)$

$x*y$

$[0_0, 0_1, 0_2, \ldots, 0_n] * [1_0, 1_1, 1_2, \ldots, 1_n]$

$[0_0*1_0, 0_1*1_1, 0_2*1_2, \ldots, 0_n*1_n] \in GF(2)$

b) commutativity

$x+y = y+x$

$[0_0, 0_1, 0_2, \ldots, 0_n] + [1_0, 1_1, 1_2, \ldots, 1_n] = [1_0, 1_1, 1_2, \ldots, 1_n] + [0_0, 0_1, 0_2, \ldots, 0_n]$

$[0_0+1_0, 0_1+1_1, 0_2+1_2, \ldots, 0_n+1_n] = [1_0+0_0, 1_1+0_1, 1_2+0_2, \ldots, 1_n+0_n] \in GF(2)$

$x*y = y*x$

$[0_0, 0_1, 0_2, \ldots, 0_n] * [1_0, 1_1, 1_2, \ldots, 1_n] = [1_0, 1_1, 1_2, \ldots, 1_n] * [0_0, 0_1, 0_2, \ldots, 0_n]$

$[0_0*1_0, 0_1*1_1, 0_2*1_2, \ldots, 0_n*1_n] = [1_0*0_0, 1_1*0_1, 1_2*0_2, \ldots, 1_n*0_n] \in GF(2)$

c) associativity

$(x+y)+z = x+(y+z)$

$([0_0, 0_1, 0_2, \ldots, 0_n] + [1_0, 1_1, 1_2, \ldots, 1_n]) + [1_0, 1_1, 1_2, \ldots, 1_n] = [0_0, 0_1, 0_2, \ldots, 0_n] + ([1_0, 1_1, 1_2, \ldots, 1_n] + [1_0, 1_1, 1_2, \ldots, 1_n]) \in GF(2)$

$(x*y)*z = x*(y*z)$

$([0_0, 0_1, \ldots, 0_n] * [1_0, 1_1, \ldots, 1_n]) * [1_0, 1_1, \ldots, 1_n] = [0_0, 0_1, \ldots, 0_n] * ([1_0, 1_1, \ldots, 1_n] * [1_0, 1_1, \ldots, 1_n]) \in GF(2)$

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distributivity

$z(x+y) = zx + zy$

$[z_0, z_1, \dots, z_n]([x_0, x_1, \dots, x_n] + [y_0, y_1, y_2 \dots y_n]) = [z_0, z_1, \dots z_n]([x_0, x_1, \dots, x_n]) + [z_0, z_1, \dots, z_n]$
$([y_0, y_1, y_2 \dots y_n]) = [z_0(x_0) + z_0(y_0), z_1(x_1) + z_1(y_1), \dots, z_n(x_n) + z_n(y_n)] \in GF(2)$

Identity element

$X + I_+ = X$

$I_+ = 0n$

$[0, 0, \dots, 0n]$

$X * I_* = X$

$I_* = 1n$

$[1, 1, \dots 1n]$

Inverse Element

$X + X_+ = I_+$

$I_+ = 0n$

$[x_0, x_1, \dots, x_n] + [-x_0, -x_1, \dots, -x_n] = 0n$

$X * X_* = I_*$

$I_* = 1n$

$[x_0, x_1, \dots, x_n] * [\frac{1}{x_0}, \frac{1}{x_1}, \dots, \frac{1}{x_n}] = 1n$

Extra credit

$X * X_* = I_*$

$I_* = 1n$

$[x_0, x_1, \dots x_n] + [\frac{1}{x_0}, \frac{1}{x_1}, \dots, \frac{1}{x_n}] = 1n$

$[0_0, 0_1, \dots, 0_n] + [\frac{1}{0_0}, \frac{1}{0_1}, \dots, \frac{1}{0_n}] =$ not defined

$\therefore$ The GF(2) is not a field.