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Homework 1

1. Find the following without using a calculator (show all steps):

a) $(5^2 \bmod 13)^3 \bmod 7$

$B = 5^2 \bmod 13 = 25 \bmod 13 = 12$ because $25 = 13 \times 1 + 12$

$B^3 \bmod 7 = 12^3 \bmod 7$

By cor. 4.1.4 $B^3 \bmod 7 = |(12 \bmod 7)(12 \bmod 7)(12 \bmod 7)| \bmod 7$

$B^3 \bmod 7 = |5 \times 5 \times 5| \bmod 7 = 125 \bmod 7 = 6$

b) $(3^3 \bmod 11)^2 \bmod 5$

$B = 3^3 \bmod 11 = 27 \bmod 11 = 5$ because $27 = 11 \times 2 + 5$

$B^2 \bmod 5 = 5^2 \bmod 5$

By cor. 4.1.4 $B^2 \bmod 5 = |(5 \bmod 5)(5 \bmod 5)| \bmod 5$

$B^2 \bmod 5 = |0 \times 0| \bmod 5 = 0 \bmod 5 = 0$

c) $(2^4 \bmod 5)^4 \bmod 3$

$B = 2^4 \bmod 5 = 16 \bmod 5 = 1$ because $16 = 5 \times 3 + 1$

$B^4 \bmod 3 = 1^4 \bmod 3$

By cor. 4.1.4 $B^4 \bmod 3 = |(1 \bmod 3)(1 \bmod 3)(1 \bmod 3)(1 \bmod 3)| \bmod 3$

$B^4 \bmod 3 = |1 \times 1 \times 1| \bmod 3 = 1 \bmod 3 = 1$

2. Show whether the following is true or false:

a) $12 \cong 5 \pmod 7$

$7 | (12-5) \rightarrow 7 | 7 \rightarrow 1 \Rightarrow$ TRUE

b) $13 \cong 2 \pmod{11}$

$11 | (13-2) \rightarrow 11 | 11 \rightarrow 1 \Rightarrow$ TRUE

c) $100 \cong 3 \pmod{13}$

$13 | (100-3) \rightarrow 13 | 97 \Rightarrow$ FALSE

3. Calculate the following:

a) $100 \bmod 5 = 0$

$100 = 5 \times 20 + 0$

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b) $12 \bmod 7 = 5$

$12 = 7 \times 1 + 5$

c) $85 \bmod 13 = 7$

$85 = 13 \times 6 + 7$

4. Give the set $\mathbb{Z}_7$

$\mathbb{Z}_7 = 0, 1, 2, 3, 4, 5, 6$