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Homework 8: Vector Spaces & Matrices 11/05/2019

1. Prove or give a counterexample: $\{[x,y,z] : x,y,z \in \mathbb{R}, x+y+z+1=1\}$ is a vector space.

$V = \{[x,y,z] : x,y,z \in \mathbb{R}, x+y+z+1=1\}$

Let $[x,y,z]$ and $[u,v,w] \in V$ be arbitary vectors

$x+y+z=1$ and $u+v+w=1$

In this case x,y,z and $u,v,w \in \mathbb{R}$

$[x,y,z] + [u,v,w]$

$[x+u, y+v, z+w]$

$(x+u)+(y+v)+(z+w)$

$(x+y+z)+(u+v+w)$

Let $[x,y,z] = [1,-2,2]$

Let $[u,v,w] = [0,3,-2]$

$[x,y,z] + [u,v,w]$

$[1,-2,2] + [0,3,-2]$

$[x+u, y+v, z+w]$

$[1+0, -2+3, 2+(-2)]$

$(x+u)+(y+v)+z+w)$

$(1+0)+(-2+3)+(2+(-2))$

$(x+y+z)+(u+v+w)$

$(1+(-2)+2)+(0+3+(-2))$

$1+1=2 \neq 1$

$\therefore [x,y,z]$ and $[u,v,w] \notin V$

$\therefore$ They are not a vector space as their sum is not 1.

2. Compute the following matrix vector products:

a) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \cdot [4\ 2] = 4[2\ 1] + 2[1\ 3] = [8\ 4] + [2\ 6] = [10\ 10]$

b) $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 2 \\ 1 & 5 & 1 \end{bmatrix} \cdot [1\ 3\ 2] = 1[1\ 3\ 1] + 3[2\ 1\ 5] + 2[4\ 2\ 1]$

$= [1\ 3\ 1] + [6\ 3\ 15] + [8\ 4\ 2]$

$= [1+6+8 \quad 3+3+4 \quad 1+15+2]$

$= [15 \quad 10 \quad 18]$

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3. Compute the following matrix multiplications:

a) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2\cdot4+1\cdot2 \\ 1\cdot4+3\cdot2 \end{bmatrix} = \begin{bmatrix} 8+2 \\ 4+6 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 2 \\ 1 & 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\cdot1+2\cdot3+4\cdot2 \\ 3\cdot1+1\cdot3+2\cdot2 \\ 1\cdot1+5\cdot3+1\cdot2 \end{bmatrix} = \begin{bmatrix} 1+6+8 \\ 3+3+4 \\ 1+15+2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \\ 18 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 5 & 4 \\ 2 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\cdot3+5\cdot1+4\cdot2 \\ 2\cdot3+3\cdot1+2\cdot2 \end{bmatrix} = \begin{bmatrix} 3+5+8 \\ 6+3+4 \end{bmatrix} = \begin{bmatrix} 16 \\ 13 \end{bmatrix}$

4. Compute the following matrix multiplications using a transpose:

a) $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}^T$

$\underset{2\times3}{\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 2 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}} = \begin{bmatrix} 1\cdot1+2\cdot3+4\cdot2 \\ 3\cdot1+1\cdot3+2\cdot2 \end{bmatrix} = \begin{bmatrix} 1+6+8 \\ 3+3+4 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 1 & 5 \end{bmatrix}^T \begin{bmatrix} 2 & 0 & 5 \end{bmatrix}$

$\underset{2\times3}{\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 5 \end{bmatrix}} \underset{1\times3}{\begin{bmatrix} 2 & 0 & 5 \end{bmatrix}}$ = This will result in an ERROR as $[a_{[i,n]} \cdot b_{[j,n]}]$ can't be performed because $a_n \neq b_n$

c) $\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}^T$

$\begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\cdot1+3\cdot3+2\cdot2 \end{bmatrix} = \begin{bmatrix} 1+9+4 \end{bmatrix} = \begin{bmatrix} 14 \end{bmatrix}$