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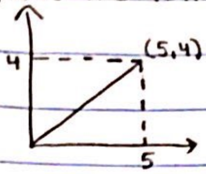
Homework 7: Vector Spaces 10/29/2019

1. Express the following line segment using a set of linear combinations:




$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{5-0} = \frac{4}{5}$ $\quad y = \frac{4}{5}x$

$y - 0 = \left(\frac{4}{5}\right)(x - 0)$

2. Let a, b be real numbers. Consider the equation $z = ax + by$. Prove that there are two 3-vectors v1, v2, such that the set of points $[x, y, z]$ satisfying the equation is exactly the set of linear combinations of v1 and v2.

If $[x, y, z]$ satisfies given equation then $z = ax + by$

∴ If $x = 1$ and $y = 0$ then $z = a$

If $x = 0$ and $y = 1$ then $z = b$

For $v_1$, $a = (1)(a) + (0)(b)$

For $v_2$, $b = (0)(a) + (1)(b)$

∴ we can see that $v_1$ and $v_2$ are solutions of $z = ax + by$

If $[x, y, z] = [x, y, ax + by]$

$= [x, 0, ax] + [0, y, by]$

$= x[1, 0, a] + y[0, 1, b]$

$= x v_1 + y v_2$

3. Let a, b, c be real numbers. Consider the equation $z = ax + by + c$. Prove that there are three 3-vectors $v_0, v_1, v_2$ such that the set of point $[x, y, z]$ satisfying the equation is exactly $\{v_0 + \alpha_1 v_1 + \alpha_2 v_2 : \alpha_1 \in \mathbb{R}, \alpha_2 \in \mathbb{R}\}$

$z = ax + by + c$ is a plane

Standard form $ax + by - z = -c$

Vector form $p + us + vt$

$N = (a, b, -1) \rightarrow$ normal vector perpendicular to the plane

$v_0 = (0, 0, c)$

$v_1 = (1, 0, a)$

$v_2 = (0, 1, b)$

4. Show that $GF(2)^n$ is a vector space

$\{1, \alpha, \alpha^2, \dots, \alpha^{n-1}\}$

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$b = b_0 + b_1\alpha + \cdots b_{n-1}\alpha^{n-1}$

$a \cdot b = (a_0 + a_1\alpha + \ldots + a_{n-1}\alpha^{n-1}) \cdot b$

$= a_0 b + a_1 \alpha b + \ldots + a_{n-1}\alpha^{n-1} b$

$$B = \begin{matrix} b_0 & = & b_{0,0} & b_{0,1} & \cdots & b_{0,n-1} \\ b_1 & = & b_{1,0} & b_{1,1} & \cdots & b_{1,n-1} \\ \vdots & : & \vdots & \vdots & \cdots & \vdots \\ b_{n-1} & = & b_{n-1,0} & b_{n-1,1} & \cdots & b_{n-1,n-1} \end{matrix}$$