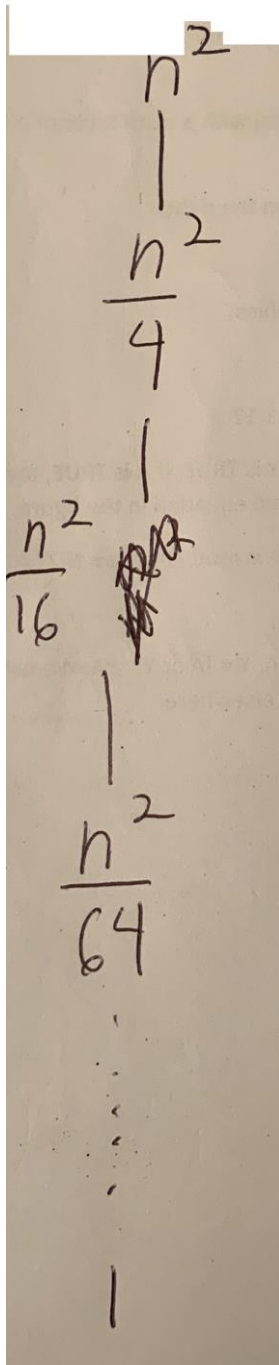


Solve the following recurrence using a recursion tree:

$$T(n) = T\left(\frac{n}{2}\right) + n^2$$

At each step, we reduce the problem size by half, then put square it.

Recursion tree:



<u>depth</u>	<u>subproblem size</u>
0	n
1	$\frac{n}{2}$
2	$\frac{n}{4}$
3	$\frac{n}{8}$
⋮	⋮
X	1

At depth i , we see that the subproblem size is $\frac{n}{2^i}$

Let X be the lowest depth, then

$$1 = \frac{n}{2^X}$$

$$2^X = n$$

$$\log 2^X = \log n$$

$$X \log 2 = \log n$$

$$X = \lg n$$

$$T(n) = n^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 + \left(\frac{n}{8}\right)^2 + \dots + \left(\frac{n}{2^{\lg n - 1}}\right)^2 + 1$$

$$= \sum_{i=0}^{\lg n - 1} \left(\frac{n}{2^i}\right)^2 + 1$$

$$= \sum_{i=0}^{\lg n - 1} \frac{n^2}{2^{2i}} + 1 = n^2 \sum_{i=0}^{\lg n - 1} \frac{1}{4^i} + 1$$

$$= n^2 \sum_{i=0}^{\lg n - 1} \left(\frac{1}{4}\right)^i + 1 = n^2 \left[\frac{\left(\frac{1}{4}\right)^{\lg n - 1 + 1} - 1}{\frac{1}{4} - 1} \right] + 1$$

$$= n^2 \left[\frac{\left(\frac{1}{4}\right)^{\lg n} - 1}{-\frac{3}{4}} \right] + 1 = n^2 \left[\frac{n^{\lg 1/4} - 1}{-3/4} \right] + 1$$

$$= -\frac{4}{3} n^2 (n^{\lg 1/4} - 1) + 1 = -\frac{4}{3} n^2 (n^{-2} - 1) + 1$$

$$= -\frac{4}{3} + \frac{4}{3} n^2 + 1 = O(n^2)$$