Solve the following recurrence using a recursion tree:

$$T(n) = T\left(\frac{n}{2}\right) + n^2$$

At each step, we reduce the problem size by half, then put square it.

Recursion tree:

$$n^2$$

$$|$$

$$\frac{n^2}{4}$$

$$|$$

$$\frac{n^2}{16}$$

$$|$$

$$\frac{n^2}{64}$$

$$\vdots$$

$$1$$

depth	subproblem size
0	n
1	$\frac{n}{2}$
2	$\frac{n}{4}$
3	$\frac{n}{8}$
$\vdots$	$\vdots$
x	1

At depth i, we see that the subproblem size is $\frac{n}{2^i}$

Let X be the lowest depth, then

$$1 = \frac{n}{2^x}$$

$$2^x = n$$

$$\log 2^x = \log n$$

$$x \log 2 = \log n$$

$$x = \lg n$$

$$T(n) = n^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 + \left(\frac{n}{8}\right)^2 + \dots + \left(\frac{n}{2^{\lg n-1}}\right)^2 + 1$$

$$= \sum_{i=0}^{\lg n-1} \left(\frac{n}{2^i}\right)^2 + 1$$

$$= \sum_{i=0}^{\lg n-1} \frac{n^2}{2^{2i}} + 1 = n^2 \sum_{i=0}^{\lg n-1} \frac{1}{4^i} + 1$$

$$= n^2 \sum_{i=0}^{\lg n-1} \left(\frac{1}{4}\right)^i + 1 = n^2 \left[\frac{\left(\frac{1}{4}\right)^{\lg n-1+1} - 1}{1/4 - 1}\right] + 1$$

$$= n^2 \left[\frac{\left(\frac{1}{4}\right)^{\lg n} - 1}{-3/4}\right] + 1 = n^2 \left[\frac{n^{\lg 1/4} - 1}{-3/4}\right] + 1$$

$$= -\frac{4}{3} n^2 \left(n^{\lg 1/4} - 1\right) + 1 = -\frac{4}{3} n^2 \left(n^{-2} - 1\right) + 1$$

$$= -\frac{4}{3} + \frac{4}{3} n^2 + 1 = O(n^2)$$