Training Problems #4 Solutions

From Sections 6.1 to 6.4:

1.

Is the array with values $\langle 23, 17, 14, 6, 13, 10, 1, 5, 7, 12\rangle$ a max-heap?

Solution:

No, it fails at element with value 6 because its child will have a value of 7 (6 is not greater than or equal to 7).

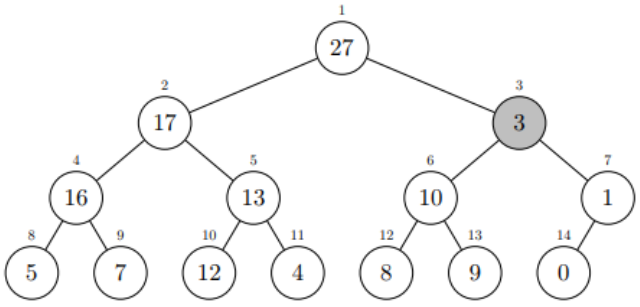
2.

Using Figure 6.2 as a model, illustrate the operation of MAX-HEAPIFY$(A, 3)$ on the array $A = \langle 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0\rangle$.

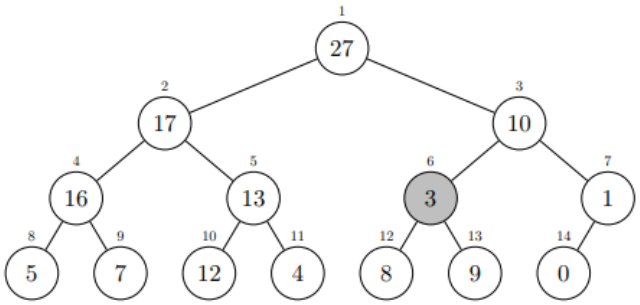
Solution:

Following the algorithm, we find that node 3 is exchanged with node 6, then node 6 is exchanged with node 13. The diagrams illustrating these operations are given below.

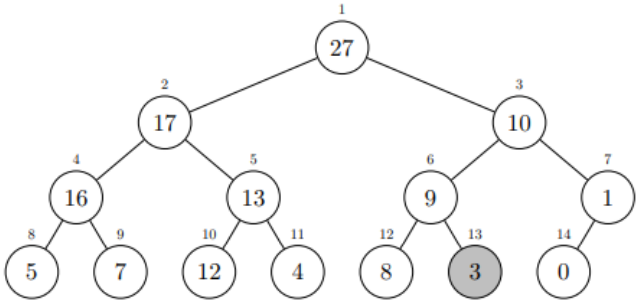
Step 0:



Step 1:



Step 2:



3.

What is the effect of calling MAX-HEAPIFY(A, i) when the element $A[i]$ is larger than its children?

Solution:

Largest will be set to i from the else condition. None of the three if conditions are met and the process terminates without modifying the heap.

4.

What is the effect of calling MAX-HEAPIFY(A, i) for $i > A.heap\text{-}size/2$?

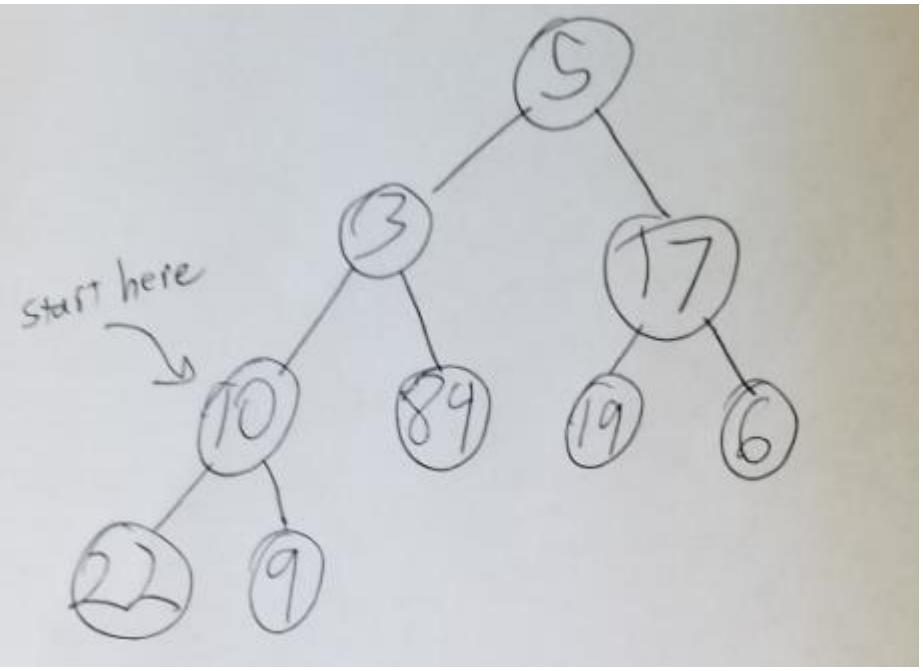
Solution:

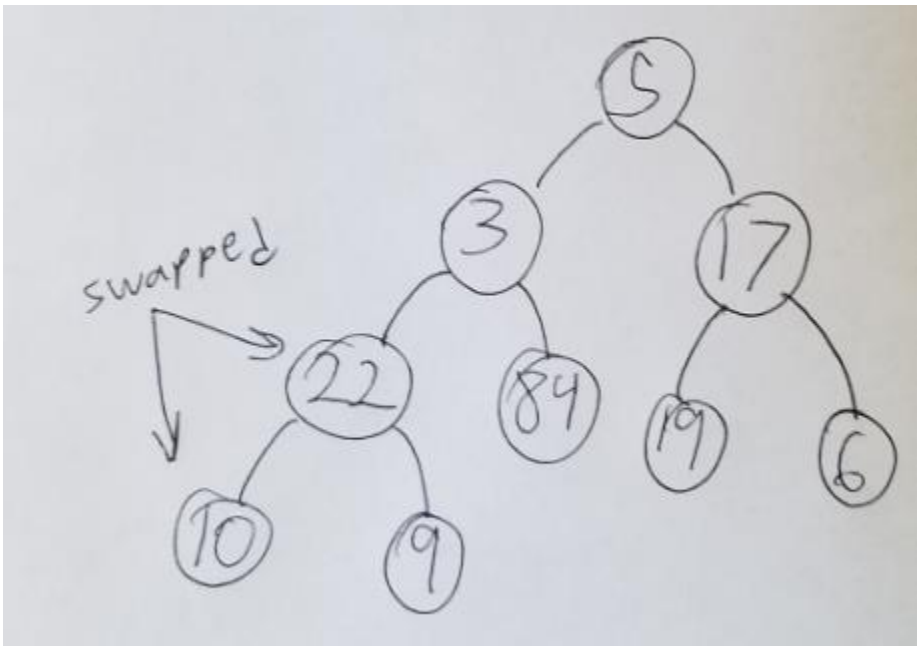
If i > A.heap-size/2, then node i has no children (and it is either at the second lowest or lowest level of the binary tree). Moreover, Left(i) and Right(i) are larger than A.heap-size, meaning that lines 3 and 6 (the first two if conditions) of the algorithm will return errors, because the array index will be out of range.

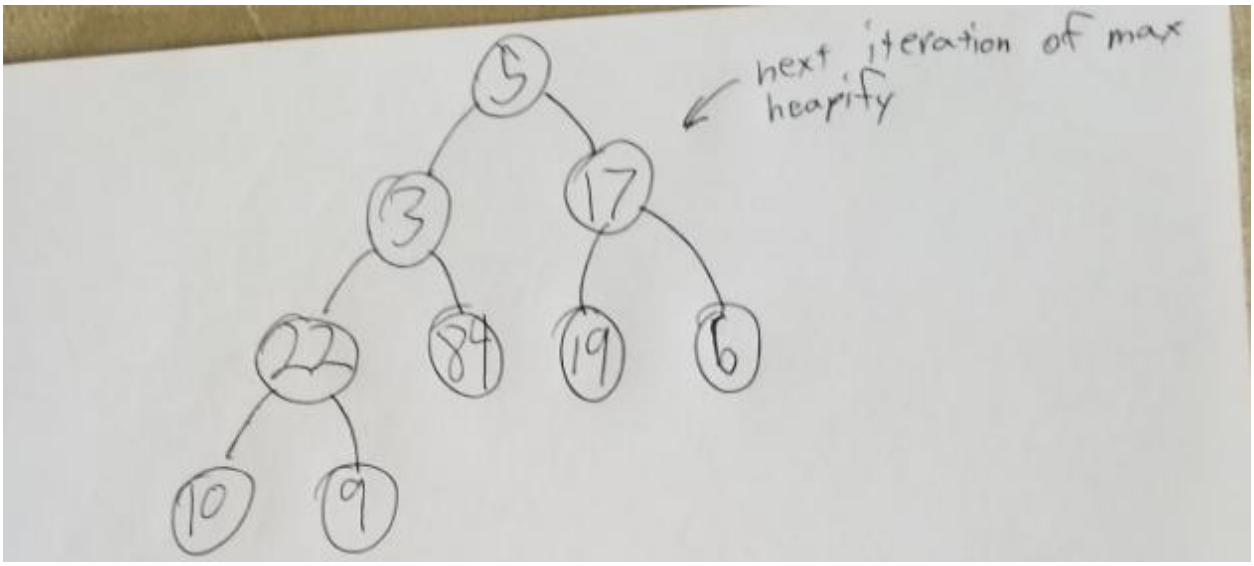
5.

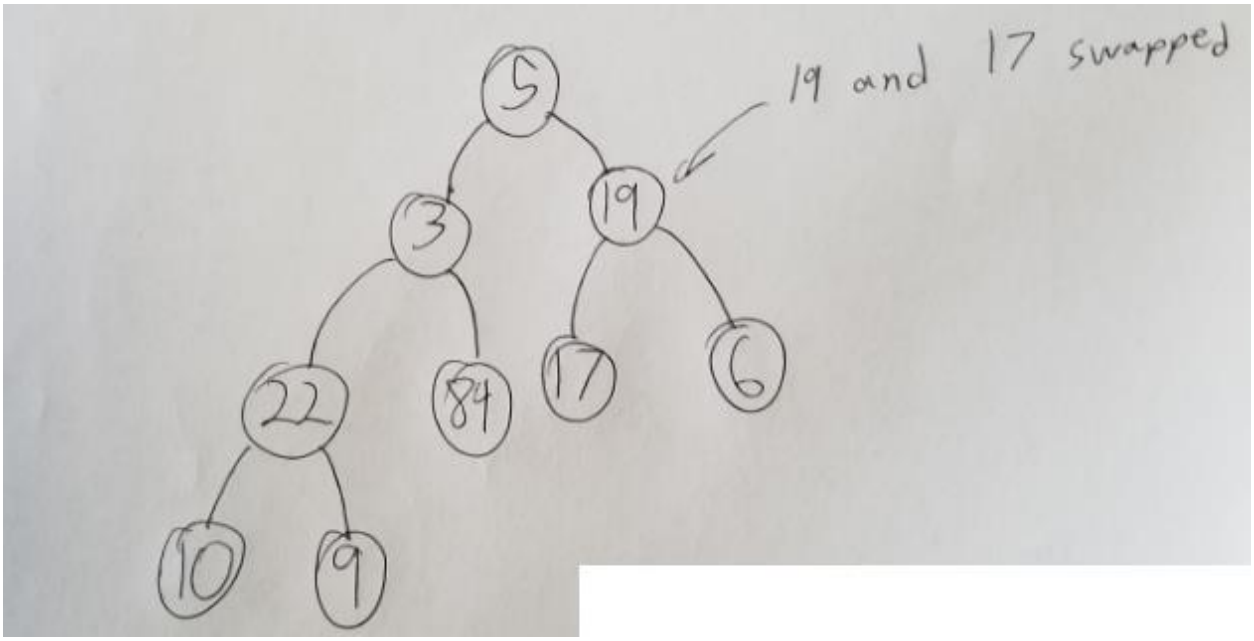
Using Figure 6.3 as a model, illustrate the operation of BUILD-MAX-HEAP on the array $A = \langle 5, 3, 17, 10, 84, 19, 6, 22, 9 \rangle$.

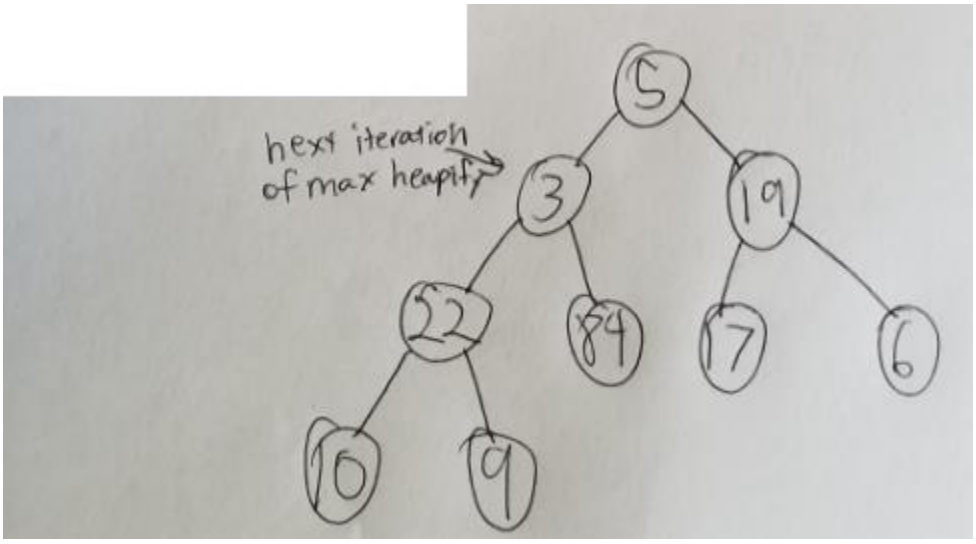
Solution:

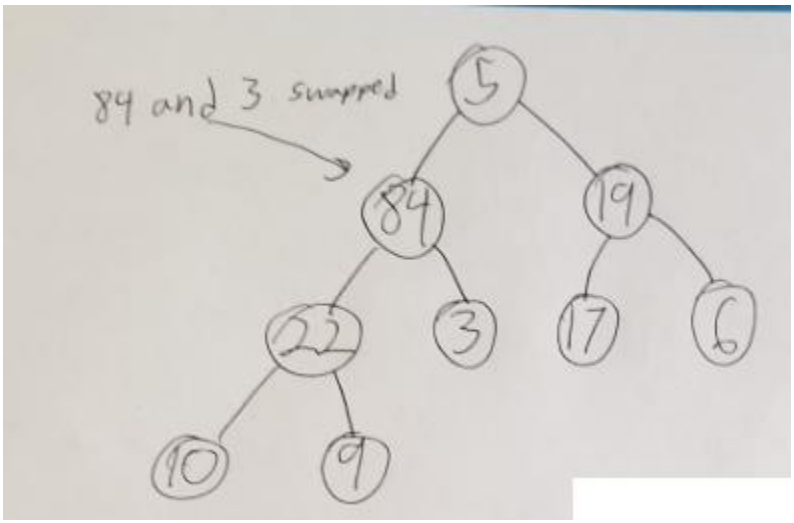


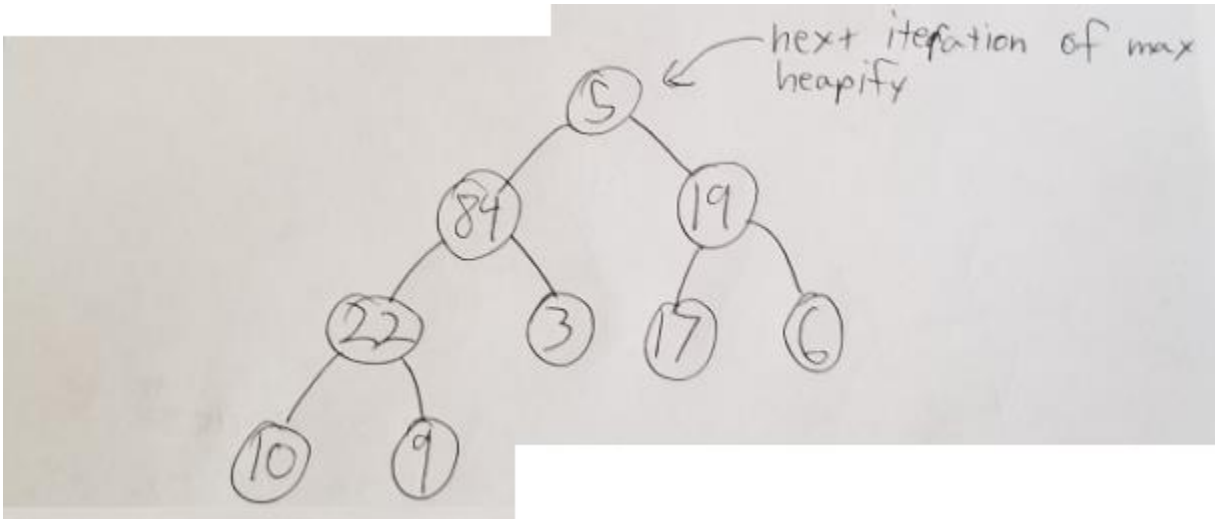

5
3
17
swapped
22
84
19
6
10
9

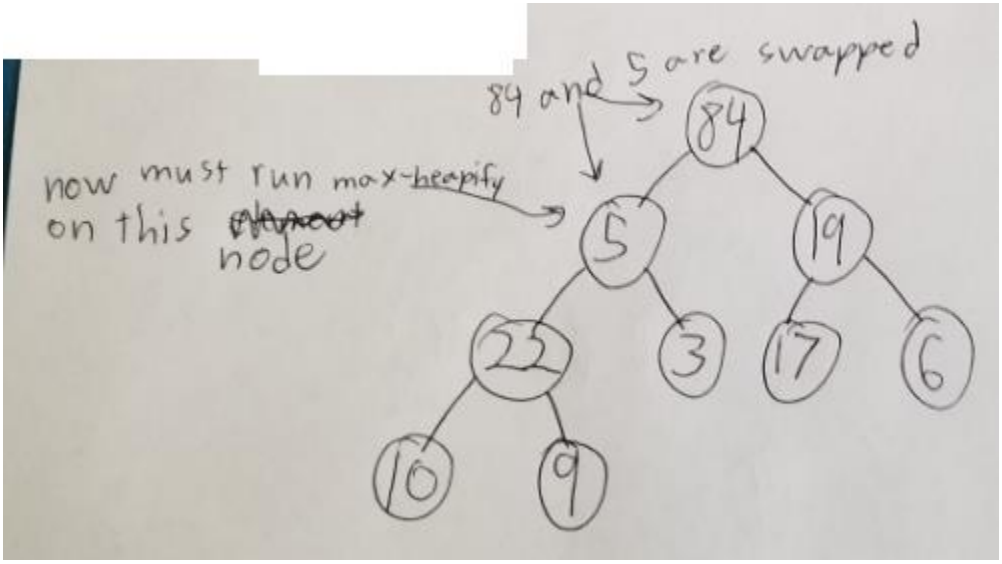

5
next iteration of max heapify
3
17
22
84
19
6
10
9

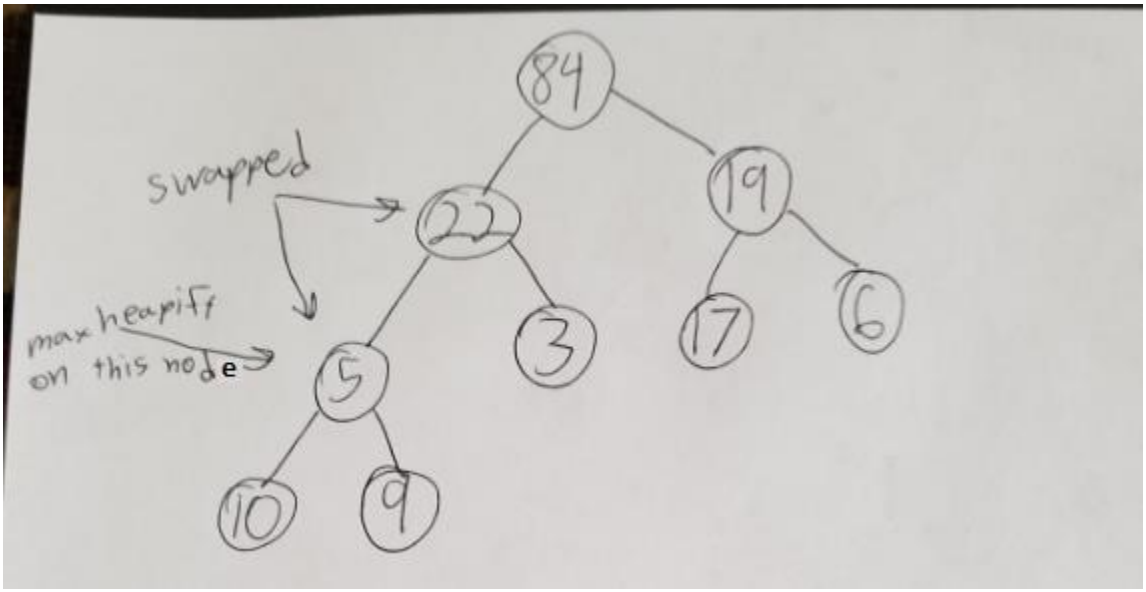

5
19 and 17 swapped
3
19
22
84
17
6
10
9

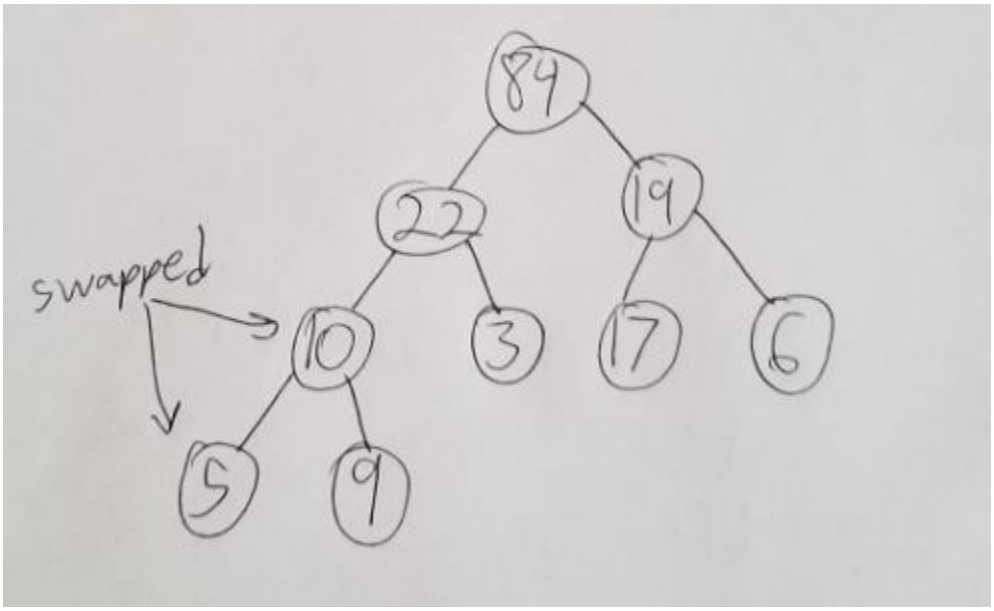

next iteration
of max heapify
5
3
19
22
84
17
6
10
9


84 and 3 swapped
5
84
19
22
3
17
6
10
9


next iteration of max heapify
5
84
19
22
3
17
6
10
9


84 and 5 are swapped
now must run max-heapify on this node
84
5
19
22
3
17
6
10
9

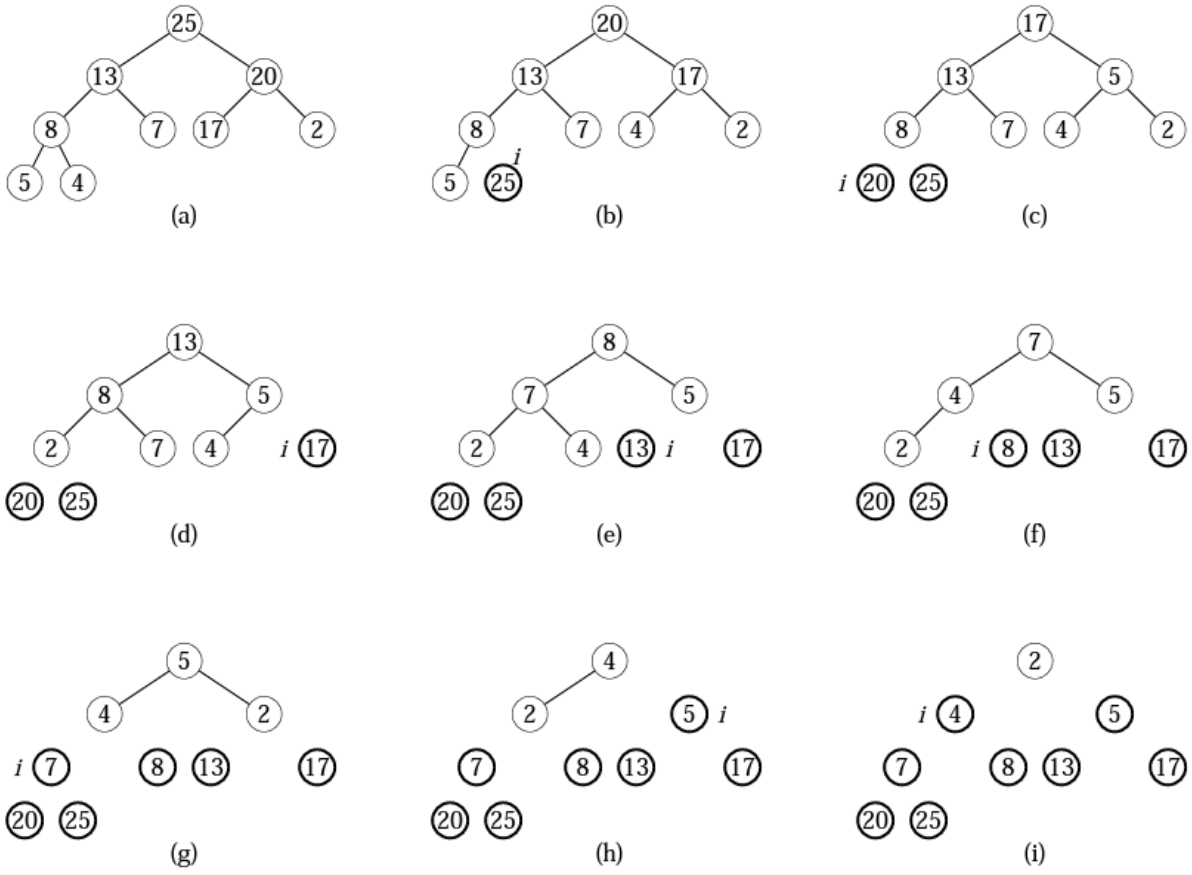

84
swapped
22
19
max heapify
on this node
5
3
17
6
10
9


84
22
19
swapped
10
3
17
6
5
9

6.

Using Figure 6.4 as a model, illustrate the operation of HEAPSORT on the array $A = \langle 5, 13, 2, 25, 7, 17, 20, 8, 4 \rangle$.

Solution:



A	2	4	5	7	8	13	17	20	25

7.

Illustrate the operation of MAX-HEAP-INSERT$(A, 10)$ on the heap $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$.

Solution: