

EE 381 Homework 2 Part 2

Name, I.D. #, and Date: _____

Instructions: Attempt each exercise and show your work. You can attach pages to your submission. Submit this part of homework 2 with the additional parts of homework 2 on Monday, February 17. You may want to make copies of your work.

Definition

Let Y denote any RV. The *cumulative distribution function* of Y , denoted by $F_Y(y)$, is given by

$$F_Y(y) = P(\{Y \leq y\}) \quad \text{and} \quad -\infty < y < \infty.$$

Properties of a cumulative distribution function

If $F_Y(y)$ is a cumulative distribution function, then

1 $\lim_{y \rightarrow -\infty} F_Y(y) = 0$

2 $\lim_{y \rightarrow \infty} F_Y(y) = 1$

3 $F_Y(y)$ is a nondecreasing function of y .

(If y_1 and y_2 are any values such that $y_1 < y_2$ then $F_Y(y_1) \leq F_Y(y_2)$)

Definition

The *probability density function* for the RV Y denoted by $f_Y(y)$ is given by $f_Y(y) = \frac{dF_Y(y)}{dy}$.

Properties of a density function

If $f_Y(y)$ is a density function then

1 $f_Y(y) \geq 0$ for any value of y .

2 $\int_{-\infty}^{\infty} f_Y(y) dy = 1$

If the RV Y has a density function $f_Y(y)$ and $a \leq b$ then the probability that Y falls in the interval $[a, b]$ is

$$P(\{a \leq Y \leq b\}) = \int_a^b f_Y(y) dy$$

Suppose the RV Y possesses the density function

$$f_Y(y) = \begin{cases} cy, & 0 \leq y \leq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

a.) Find the value of c that makes $f_Y(y)$ a probability density function.

b.) Find $F_Y(y)$

c.) Graph both $f_Y(y)$ and $F_Y(y)$.

d.) Use $F_Y(y)$ to find $P(\{1 \leq Y \leq 2\})$.

e.) Use $f_Y(y)$ and geometry (the area of a triangle) to find $P(\{1 \leq Y \leq 2\})$.