Sample Final Exam Questions – Part 2

[Questions 1–4] Analyzing Call Center Operations:

In the bustling Bay Area, known for its tech-driven economy and fast-paced lifestyle, a busy call center operates as a critical communication hub for a major tech company. This call center is equipped to handle a substantial volume of phone calls, primarily from customers seeking technical support and information. Calls to this center follow a Poisson process, averaging 10 calls per hour. To manage this flow efficiently, the call center has a unique system: it checks for new calls at 15-minute intervals, starting at the top of each hour (i.e., at 00:00, 00:15, 00:30, 00:45, etc.). If there are any new calls in the queue, they are answered collectively. If no calls are waiting, the center holds until the next check.

Question 1. Given the constant stream of inquiries, what is the probability that exactly 4 calls are waiting to be answered at any given check interval?

Solution:

The number of calls arriving in a 15-minute interval follows a Poisson distribution with a rate of $\mu = \frac{10}{4} = 2.5$ calls per interval. The probability of getting exactly 4 calls is given by the formula:

$$P(X = k) = \frac{\mu^k e^{-\mu}}{k!}$$

Substituting $\mu = 2.5$ and $k = 4$, we get:

$$P(X = 4) = \frac{2.5^4 e^{-2.5}}{4!} = 0.1336$$

Question 2. Over a full day, which in the tech world is always a 24-hour operation, what is the expected number of 15-minute intervals where the call center doesn't find any calls to answer?

Solution:

The number of intervals in a 24-hour period is $24 \times 4 = 96$. The probability of getting no calls in a given interval is $P(X = 0) = e^{-\mu} = e^{-2.5} = 0.08$. Therefore, the expected number of such intervals is:

$$96 \times P(X = 0) = 96 \times e^{-2.5} = 7.88$$

Question 3. The management is also interested in understanding the variability of call arrivals. What is the standard deviation of the number of 15-minute intervals in a 24-hour period where no calls are found?

Solution:

The standard deviation of the number of calls is:

$$\sigma = \sqrt{96 \cdot e^{-2.5} \cdot (1 - e^{-2.5})} = 2.69$$

Note: this is the standard deviation of a binomial distribution.

Question 4. If it's known that a call was received during a specific hour, what is the probability that this call arrived within the first 15 minutes of that hour?

Solution:

Since the calls follow a Poisson process, the probability of a call arriving in a given interval is proportional to the length of the interval. Therefore, the probability of a call arriving in the first 15 minutes of an hour is

$$P(X = 1\ in\ 15 | X = 1\ in\ 1\ hr) = \frac{e^{-2.5}(2.5) \times e^{-7.5}}{e^{-10}(10)} = \frac{1}{4}$$

Question 5. Probability and Quality Control Analysis

SiliconStream Electronics, a leader in the Bay Area's thriving tech industry, is known for its cutting-edge electronic products and high-quality standards. Central to their operations is the procurement of integrated circuits (ICs), which they purchase in large quantities from a trusted supplier in the heart of Silicon Valley. Each lot consists of 3000 ICs, and under the terms of their meticulous contract, no more than 4 percent of these ICs can be defective, reflecting SiliconStream's commitment to product excellence.

Upon receipt of each IC lot, SiliconStream's quality control team swings into action with a thorough testing protocol. They start by randomly selecting a sample of $n = 15$ ICs for initial examination. If this first sample reveals at most 1 defective IC, the lot is promptly accepted, ensuring a swift supply chain process. However, if 4 or more defective ICs are discovered, the lot is rejected on the spot, upholding the company's high-quality benchmarks. In scenarios where the sample contains 2 or 3 defective ICs, the scrutiny intensifies with a second round of testing, involving another set of 15 ICs. For the lot to pass this stage and be accepted, the second sample must have 0 defective ICs. Any deviation results in rejection.

In this high-stakes environment, what is the probability that a lot from the supplier will be accepted by SiliconStream, confirming that it aligns with the stringent quality standards outlined in their contract?

Solution:

$p = 0.04, n = 15$

$$P(\text{Accept}) = P(X \leq 1) + P(2 \leq X \leq 3)P(X = 0) = 0.9441$$

[Questions 6–9] Debugging and Probability Analysis:

In the fast-paced environment of Silicon Valley, a leading tech firm is in the final stages of developing a groundbreaking new application. However, before release, the team faces a critical task: identifying and resolving software glitches that could impact user experience. These glitches are detected and resolved following a Poisson process, with an average resolution rate of $\lambda = 0.8$ glitches per hour.

The development team schedules an intensive debugging session, planning to work continuously for 3 hours. If they manage to resolve at least one glitch during this time, they will conclude the session. However, if no glitches are resolved within these 3 hours, the team will extend the session until they successfully address the first glitch. This approach ensures that the application meets the firm's high standards for quality and reliability.

Question 6. What is the probability that the debugging session will need to extend beyond the initially planned 3 hours?

Solution:

$$P(\mathrm{T} > 3) = P(X = 0) = e^{-0.8\times 3} = e^{-2.4} = 0.091$$

Question 7. What is the probability that the debugging session, if extended, will conclude within 5 hours?

Solution:

$$P(3 < \mathrm{T} < 5) = P(X = 0, \mathrm{t} = 3) \times P(X = 1, \mathrm{t} = 5) = e^{-0.8\times 3}(1 - e^{-0.8\times 2}) = 0.072$$

Question 8. Given the nature of software development and the complexity of glitches, what is the expected number of glitches the team will resolve in a single session?

Solution:

$$\mathrm{E(X)} = P(X = 0, t = 3) \times 1 + \sum_{k=1}^{\infty} kP(x = k, t = 3) = e^{-0.8\times3} \times 1 + \mu$$

$$= 0.091 + 2.4 = 2.491$$

Question 9. What is the expected total duration of the debugging session, considering the possibility of extending the work beyond 3 hours? [2 points]

Solution:

$$\mathrm{E(T)} = P(X \geq 1, t = 3) \times 3 + P(X = 0, t = 2) \times \left(3 + \frac{1}{\lambda}\right) = 3 + e^{-0.8\times3}\left(\frac{10}{8}\right) = 3.11$$

[Questions 10–12] Performance Analysis with Normal Distribution:

In Silicon Valley, home to cutting-edge technology and innovation, a software engineering company is known for tackling some of the most challenging algorithmic problems in the tech industry. In this company, the time it takes for developers to solve these complex algorithm problems typically follows a normal distribution. On average, a developer takes 120 minutes to find a solution, with a standard deviation of 20 minutes, reflecting a range of expertise and experience levels among the team members.

Recently, a new developer named Alex joined the team. Alex brings a fresh perspective and innovative techniques to the table, claiming that his unique approach can significantly improve problem-solving efficiency. His colleagues and superiors are intrigued but skeptical; they want to understand just how Alex's performance stacks up against the established norm.

Question 10. Alex gets to work and manages to solve a particularly tough problem in 95 minutes. In the context of the company's standard, what percentile does Alex's performance fall into?

Solution:

$$z = \frac{95 - 120}{20} = -\frac{5}{4} = -1.25 \Rightarrow P(\mathrm{X} \geq 95) = 1 - P(X \leq 95) = 1 - 0.1056 = 0.8943$$

Question 11. The company recognizes and rewards exceptional talent. To be deemed "exceptionally efficient," a developer must be in the top 5% in terms of problem-solving speed. What is the maximum amount of time in which Alex must solve a problem to be categorized as exceptionally efficient according to the company's standards?

Solution:

$$-1.64 = \frac{x - 120}{20} \Rightarrow x = 120 - 1.64 \times 20 = 87.2$$

Question 12. The company also values collaboration and often schedules meetings to discuss problem-solving strategies. If they want to hold a meeting after 80% of the developers have completed their tasks, after how many minutes should this meeting be scheduled?

Solution:

$$0.84 = \frac{x - 120}{20} \Rightarrow x = 120 + 0.84 \times 20 = 136.8$$

[Questions 13–15] Team Selection and Probability Analysis:

In the dynamic and competitive world of Silicon Valley, a renowned tech company, known for its innovative software solutions and cutting-edge technology, faces the challenge of allocating its best talents to spearhead key projects. This company boasts a diverse array of talent across its three main software development teams, each with a unique mix of junior and senior developers. Team A, known for its agile development practices, consists of 5 junior and 2 senior developers. Team B, with a strong focus on AI and machine learning, comprises 3 junior and 4 senior developers. Meanwhile, Team C, specializing in cybersecurity, is formed by 2 junior and 5 senior developers.

As a new high-profile project is initiated, the company follows a process to randomly select one of these teams and then randomly choose a developer from the selected team to lead the project. This selection process is crucial as it often sets the tone for the project's direction and success.

Question 13. What is the probability that the chosen project leader will be a senior developer?

Solution:

Team A: 5 junior and 2 senior developers

Team B: 3 junior and 4 senior developers

Team C: 2 junior and 5 senior developers

Probability of choosing a senior $= \frac{1}{3} \times \frac{2}{7} + \frac{1}{3} \times \frac{4}{7} + \frac{1}{3} \times \frac{5}{7} = \frac{11}{21} = 0.52$

Once a developer is selected and assigned as the project leader, and in this case, turns out to be a senior developer, they are no longer available for future project leadership roles.

Question 14. Considering the initial selection, what is the probability that the senior developer who was chosen as the project leader came from Team B?

Solution:

Team A: 5 junior and 2 senior developers

Team B: 3 junior and 4 senior developers

Team C: 2 junior and 5 senior developers

$$P(B|\, S_1) = \frac{P(B \cap S_1)}{P(S_1)} = \frac{\frac{1}{3} \times \frac{4}{7}}{\frac{1}{3} \times \frac{2}{7} + \frac{1}{3} \times \frac{4}{7} + \frac{1}{3} \times \frac{5}{7}} = \frac{4}{11} = 0.3636$$

Question 15. If the company were to select another developer from the same team for an upcoming project, what is the probability that this next leader will be a junior developer?

Solution:

Team A: 5 junior and 2 senior developers

Team B: 3 junior and 4 senior developers

Team C: 2 junior and 5 senior developers

$$P(J_2|\, S_1) = \frac{P(J_2 \cap S_1)}{P(S_1)} = \frac{\frac{1}{3} \times \frac{2}{7} \times \frac{5}{6} + \frac{1}{3} \times \frac{4}{7} \times \frac{3}{6} + \frac{1}{3} \times \frac{5}{7} \times \frac{2}{6}}{\frac{1}{3} \times \frac{2}{7} + \frac{1}{3} \times \frac{4}{7} + \frac{1}{3} \times \frac{5}{7}} = \frac{16}{33} = 0.4848$$