Chapter 15

Apply It 15.1

1. $S(t) = \int -4te^{0.1t}\,dt$

Let $u = -4t$ and $dv = e^{0.1t}\,dt$, so $du = -4\,dt$, and

$v = \int e^{0.1t}\,dt = \frac{1}{0.1}e^{0.1t} = 10e^{0.1t}$.

$$\int -4te^{0.1t}\,dt = (-4t)\left(10e^{0.1t}\right) - \int \left(10e^{0.1t}\right)(-4)dt$$
$$= -40te^{0.1t} + \int 40e^{0.1t}\,dt$$
$$= -40te^{0.1t} + 40\frac{e^{0.1t}}{0.1} + C$$
$$= -40te^{0.1t} + 400e^{0.1t} + C$$

$S(t) = -40te^{0.1t} + 400e^{0.1t} + C$ and $S(0) = 5000$

$5000 = 0 + 400e^{0} + C$

$C = 4600$

$S(t) = -40te^{0.1t} + 400e^{0.1t} + 4600$

2. $P(t) = \int 0.1t(\ln t)^2\,dt$

Let $u = (\ln t)^2$ and $dv = 0.1t\,dt$, so

$du = 2(\ln t)\left(\frac{1}{t}\right)dt = \frac{2\ln t}{t}dt$ and

$v = \int 0.1t\,dt = 0.1\frac{t^2}{2} = 0.05t^2$

$$\int 0.1t(\ln t)^2\,dt$$
$$= 0.05t^2(\ln t)^2 - \int \left(0.05t^2\right)\left(\frac{2\ln t}{t}\right)dt$$
$$= 0.05(t\ln t)^2 - \int 0.1t\ln t\,dt$$

For $\int 0.1t\ln t\,dt$, let $u = \ln t$ and $dv = 0.1t\,dt$, so

$du = \frac{1}{t}dt$ and $v = 0.05t^2$.

$$\int 0.1t\ln t\,dt = 0.05t^2\ln t - \int \left(0.05t^2\right)\left(\frac{1}{t}\right)dt$$
$$= 0.05t^2\ln t - \int 0.05t\,dt$$
$$= 0.05t^2\ln t - 0.05\frac{t^2}{2} + C$$
$$= 0.05t^2\ln t - 0.025t^2 + C$$

Thus,

$$P(t) = 0.05(t\ln t)^2 - \left(0.05t^2\ln t - 0.025t^2\right) + C$$
$$= 0.05(t\ln t)^2 - 0.05t^2\ln t + 0.025t^2 + C$$

Problems 15.1

1. $\int f(x)dx = uv - \int v\,du$

$$= x\cdot\frac{2}{3}(x+5)^{\frac{3}{2}} - \int \frac{2}{3}(x+5)^{\frac{3}{2}}dx$$
$$= \frac{2}{3}x(x+5)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{2}{5}(x+5)^{\frac{5}{2}} + C$$
$$= \frac{2}{3}x(x+5)^{\frac{3}{2}} - \frac{4}{15}(x+5)^{\frac{5}{2}} + C$$

2. $\int xe^{3x+1}dx$

If $u = x$ and $dv = e^{3x+1}dx$, then $du = dx$ and

$v = \frac{1}{3}e^{3x+1}$.

$$\int xe^{3x+1}dx = \frac{x}{3}e^{3x+1} - \int \frac{1}{3}e^{3x+1}dx$$
$$= \frac{x}{3}e^{3x+1} - \frac{1}{3}\cdot\frac{1}{3}\int e^{3x+1}[3\,dx]$$
$$= \frac{x}{3}e^{3x+1} - \frac{1}{9}e^{3x+1} + C$$
$$= \frac{1}{9}e^{3x+1}(3x-1) + C$$

3. $\int xe^{-x}dx$

Letting $u = x$, $dv = e^{-x}dx$, then $du = dx$,

$v = -e^{-x}$.

$$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx$$
$$= -xe^{-x} - \int e^{-x}[-dx] = -xe^{-x} - e^{-x} + C$$
$$= -e^{-x}(x+1) + C$$

4. $\int xe^{ax}\,dx$

Letting $u = x,\ dv = e^{ax}\,dx,$ then $du = dx,$

$v = \frac{1}{a}e^{ax}.$

$$\int xe^{ax}dx = \frac{x}{a}e^{ax} - \int \frac{1}{a}e^{ax}dx$$
$$= \frac{x}{a}e^{ax} - \frac{1}{a^2}e^{ax} + C$$

5. $\int y^3 \ln y\ dy$

Letting $u = \ln y,\ dv = y^3 dy$, then $du = \left(\frac{1}{y}\right)dy$,

$v = \frac{y^4}{4}$

$$\int y^3 \ln y\ dy = \frac{y^4 \ln y}{4} - \int \frac{y^4}{4}\left(\frac{1}{y}dy\right)$$
$$= \frac{y^4 \ln y}{4} - \int \frac{y^3}{4}dy = \frac{y^4 \ln y}{4} - \frac{y^4}{16} + C$$
$$= \frac{y^4}{4}\left[\ln(y) - \frac{1}{4}\right] + C$$

6. $\int x^2 \ln x\ dx$

Letting $u = \ln x,\ dv = x^2 dx$, then $du = \frac{1}{x}dx$,

$v = \frac{x^3}{3}$

$$\int x^2 \ln x\ dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3}\left(\frac{1}{x}dx\right)$$
$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C = \frac{x^3}{3}\left[\ln(x) - \frac{1}{3}\right] + C$$

7. $\int \ln(4x)\ dx$

Letting $u = \ln(4x)$, $dv = dx$, then $du = \left(\frac{1}{x}\right)dx$,

$v = x.$

$$\int \ln(4x)dx = x\ln(4x) - \int x\left(\frac{1}{x}dx\right)$$
$$= x\ln(4x) - \int dx = x\ln(4x) - x + C$$
$$= x[\ln(4x) - 1] + C$$

8. $\int \left(\frac{t}{e^t}\right)dt$

Letting $u = t,\ dv = e^{-t}dt$, then $du = dt,\ v = -e^{-t}$

$$\int \left(\frac{t}{e^t}\right)dt = -te^{-t} - \int -e^{-t}dt$$
$$= -te^{-t} - e^{-t} + C = -e^{-t}(t+1) + C$$

9. $\int x\sqrt{ax+b}\,dx$

Letting $u = x,\ dv = \sqrt{ax+b}\,dx,$ then $du = dx,$

$v = \frac{1}{a}\cdot\frac{(ax+b)^{3/2}}{\frac{3}{2}} = \frac{2}{3a}(ax+b)^{3/2}.$

$$\int x\sqrt{ax+b}\,dx$$
$$= \frac{2x}{3a}(ax+b)^{3/2} - \int \frac{2}{3a}(ax+b)^{3/2}dx$$
$$= \frac{2x}{3a}(ax+b)^{3/2} - \frac{2}{3a^2}\cdot\frac{(ax+b)^{5/2}}{\frac{5}{2}} + C$$
$$= \frac{2x}{3a}(ax+b)^{3/2} - \frac{4}{15a^2}(ax+b)^{5/2} + C$$

10. $\int \frac{12x}{\sqrt{1+4x}}dx$

Letting $u = 12x,\ dv = (1+4x)^{-\frac{1}{2}}dx$,

then $du = 12dx,\ v = \frac{1}{2}(1+4x)^{\frac{1}{2}}$

$$\int \frac{12x}{\sqrt{1+4x}}dx = 12x\cdot\frac{\sqrt{1+4x}}{2} - \int \frac{(1+4x)^{\frac{1}{2}}}{2}\cdot 12dx$$
$$= 6x\sqrt{1+4x} - (1+4x)^{\frac{3}{2}} + C$$
$$= \sqrt{4x-1}[6x - (1+4x)] + C$$
$$= (2x-1)\sqrt{4x+1} + C$$

11. $\int \frac{x}{(5x+2)^3}dx$

Letting $u = x,\ dv = (5x+3)^{-3}dx,$ then $du = dx$

and $v = -\frac{1}{10}(5x+3)^{-2}.$

$$\int \frac{x}{(5x+2)^3}\,dx$$
$$= -\frac{x}{10(5x+3)^2} - \int -\frac{1}{10}(5x+3)^{-2}\,dx$$
$$= -\frac{x}{10(5x+3)^2} + \frac{1}{10}\cdot\frac{(5x+3)^{-1}}{5(-1)} + C$$
$$= -\frac{x}{10(5x+3)^2} - \frac{1}{50(5x+3)} + C$$

12. $\int \frac{\ln(x+1)}{2(x+1)}\,dx = \frac{1}{2}\int \ln(x+1)\left[\frac{1}{x+1}\,dx\right]$

(Form: $\int u^n\,du$)

$$\int \frac{\ln(x+1)}{2(x+1)}\,dx = \frac{\ln(x+1)^2}{4} + C$$

13. $\int \frac{\ln x}{x^2}\,dx$

Letting $u = \ln x$, $dv = x^{-2}dx$, then $du = \frac{1}{x}dx$, $v = -x^{-1}$.

$$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \int -x^{-1}\left(\frac{1}{x}dx\right)$$
$$= -\frac{\ln x}{x} + \int x^{-2}dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$
$$= -\frac{1}{x}(1+\ln x) + C$$

14. $\int \frac{2x+7}{3^{3x}}\,dx$

Letting $u = 2x + 7$, $dv = e^{-3x}dx$, then $du = 2\,dx$ and $v = -\frac{1}{3}e^{-3x}$.

$$\int \frac{2x+7}{e^{3x}}\,dx = -\frac{2x+7}{3e^{3x}} + \frac{2}{3}\int e^{-3x}dx$$
$$= -\frac{2x+7}{3e^{3x}} - \frac{2}{9e^{3x}} + C$$

15. $\int_1^2 4xe^{2x}\,dx$

Letting $u = 4x$, $dv = e^{2x}dx$, then $du = 4dx$, $v = \frac{1}{2}e^{2x}$

$$\int_1^2 4xe^{2x}\,dx = \left[2xe^{2x} - \int 2e^{2x}\,dx\right]\Bigg|_1^2$$
$$= \left[2xe^{2x} - e^{2x}\right]\Big|_1^2 = e^{2x}(2x-1)\Big|_1^2$$
$$= e^4(3) - e^2(1) = e^2\left(3e^2 - 1\right)$$

16. $\int_1^2 2xe^{-3x}\,dx$

Letting $u = 2x$, $dv = e^{-3x}\,dx$, then $du = 2\,dx$ and $v = -\frac{1}{3}e^{-3x}$.

$$\int_1^2 2xe^{-3x}\,dx$$
$$= \left[-\frac{2xe^{-3x}}{3} - \int -\frac{2}{3}e^{-3x}\,dx\right]\Bigg|_1^2$$
$$= \left[-\frac{2xe^{-3x}}{3} + \frac{2}{3}\cdot\frac{e^{-3x}}{-3}\right]\Bigg|_1^2$$
$$= \left[-\frac{2xe^{-3x}}{3} - \frac{2e^{-3x}}{9}\right]\Bigg|_1^2$$
$$= \left[-\frac{2e^{-3x}}{3}\left(x + \frac{1}{3}\right)\right]\Bigg|_1^2$$
$$= \left[-\frac{2e^{-6}}{3}\left(2 + \frac{1}{3}\right)\right] - \left[-\frac{2e^{-3}}{3}\left(1 + \frac{1}{3}\right)\right]$$
$$= -\frac{2e^{-6}}{3}\left[\frac{7}{3} - e^3\left(\frac{4}{3}\right)\right]$$
$$= -\frac{2}{9e^6}[7 - 4e^3]$$

17. $\int_0^1 xe^{-x^2}\,dx = -\frac{1}{2}\int_0^1 e^{-x^2}(-2x\,dx)$ (Form: $\int e^u\,du$)

$$= -\frac{1}{2}e^{-x^2}\Big|_0^1 = -\frac{1}{2}\left(e^{-1} - 1\right) = \frac{1}{2}\left(1 - e^{-1}\right)$$

18. $\int \frac{3x^3}{\sqrt{4-x^2}}dx$

Letting $u = 3x^2$, $dv = x\left(4-x^2\right)^{-\frac{1}{2}}dx$, then

$du = 6x\,dx$,

$v = -\left(4-x^2\right)^{\frac{1}{2}}$.

$$\int \frac{3x^3}{\sqrt{4-x^2}}dx$$
$$= -3x^2\left(4-x^2\right)^{\frac{1}{2}} - \int -\left(4-x^2\right)^{\frac{1}{2}}(6x\ dx)$$
$$= -3x^2\left(4-x^2\right)^{\frac{1}{2}} - 2\left(4-x^2\right)^{\frac{3}{2}} + C$$
$$= -\sqrt{4-x^2}\left[3x^2 + 2\left(4-x^2\right)\right] + C$$
$$= -\left(x^2+8\right)\sqrt{4-x^2} + C$$

19. $\int_5^8 \frac{4x}{\sqrt{9-x}}dx$

Letting $u = 4x$, $dv = (9-x)^{-1/2}dx$, then

$du = 4\ dx$ and $v = -1\cdot\frac{(9-x)^{1/2}}{\frac{1}{2}} = -2(9-x)^{1/2}$.

$$\int_5^8 \frac{4x}{\sqrt{9-x}}dx$$
$$= -8x\sqrt{9-x}\Big|_5^8 + 8\int_5^8 \sqrt{9-x}\,dx$$
$$= -8(8)\sqrt{9-8} + 8(5)\sqrt{9-5} - \frac{8(9-x)^{3/2}}{\frac{3}{2}}\Bigg|_5^8$$
$$= -64 + 80 - \frac{16}{3}[(9-8)^{3/2} - (9-5)^{3/2}]$$
$$= 16 - \frac{16}{3}(1-8)$$
$$= \frac{160}{3}$$

20. $\int (\ln x)^2 dx$

Letting $u = (\ln x)^2$, $dv = dx$, then

$du = \left[\frac{2\ln x}{x}\right]dx$, $v = x$.

$$\int (\ln x)^2 dx = x(\ln x)^2 - \int x\left[\frac{2\ln x}{x}dx\right]$$
$$= x(\ln x)^2 - 2\int \ln(x)dx$$

For $\int \ln(x)dx$, let $u = \ln x$, $dv = dx$. Then

$du\left(\frac{1}{x}\right)dx$, $v = x$, so

$$\int \ln(x)dx = x\ln x - \int x\left(\frac{1}{x}dx\right) = x[\ln(x)-1] + C_1.$$

Thus $\int (\ln x)^2 dx = x\left[(\ln x)^2 - 2\ln(x) + 2\right] + C$.

21. $\int 3(2x-2)\ln(x-2)dx$

Letting $u = 3\ln(x-2)$, $dv = (2x-2)dx$, then

$du = \frac{3}{x-2}dx$ and $v = x^2 - 2x = x(x-2)$.

$$\int 3(2x-2)\ln(x-2)dx$$
$$= 3x(x-2)\ln(x-2) - \int x(x-2)\cdot\frac{3}{x-2}dx$$
$$= 3x(x-2)\ln(x-2) - \int 3x\,dx$$
$$= 3x(x-2)\ln(x-2) - \frac{3}{2}x^2 + C$$

22. $\int \frac{xe^x}{(x+1)^2}dx$

Letting $u = xe^x$, $dv = (x+1)^{-2}dx$, then

$du = (x+1)e^x dx$, $v = -(x+1)^{-1}$.

$$\int \frac{xe^x}{(x+1)^2}dx = -\frac{xe^x}{x+1} + \int e^x dx$$
$$= -\frac{xe^x}{x+1} + e^x + C$$
$$= e^x\left(1 - \frac{x}{x+1}\right) = e^x\left(\frac{x+1-x}{x+1}\right) + C = \frac{e^x}{x+1} + C$$

23. $\int x^2e^x dx$

Letting $u = x^2$, $dv = e^x dx$, then $du = 2x\,dx$ and $v = e^x$.

$$\int x^2e^x dx = x^2e^x - \int e^x(2x\ dx)$$
$$= x^2e^x - 2\int xe^x dx$$

For $\int xe^x dx$, let $u = x$, $dv = e^x dx$. Then $du = dx$, $v = e^x$ and

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C_1$$
$$= e^x(x-1) + C_1.$$

Thus $\int x^2e^x dx = x^2e^x - 2\left[e^x(x-1)\right] + C$

$= e^x\left(x^2 - 2x + 2\right) + C$.

24. $\int_1^4 \sqrt{x}\ln(x^9)dx = 9\int_1^4 x^{1/2}\ln x\, dx$

Letting $u = \ln x$ and $dv = x^{1/2}dx,$ then $du = \frac{1}{x}dx$ and $v = \frac{2}{3}x^{3/2}.$

$$9\int_1^4 x^{1/2}\ln x\, dx$$
$$= 9\cdot \ln x \cdot \frac{2}{3}x^{3/2}\bigg|_1^4 - 9\cdot\frac{2}{3}\int_1^4 \frac{x^{3/2}}{x}dx$$
$$= 6x^{3/2}\ln x\bigg|_1^4 - 6\int_1^4 x^{1/2}dx$$
$$= 6(4^{3/2}\ln 4 - 1^{3/2}\ln 1) - 6\cdot\frac{x^{3/2}}{\frac{3}{2}}\Bigg|_1^4$$
$$= 48\ln 2^2 - 4(4^{3/2} - 1^{3/2})$$
$$= 96\ln 2 - 28$$

25. $\int\left(x - e^{-x}\right)^2 dx = \int\left(x^2 - 2xe^{-x} + e^{-2x}\right)dx$

$= \frac{x^3}{3} - \frac{e^{-2x}}{2} - 2\int xe^{-x}dx$

Using Problem 3 for $\int xe^{-x}dx$,

$\int\left(x - e^{-x}\right)^2 dx = \frac{x^3}{3} - \frac{e^{-2x}}{2} + 2e^{-x}(x+1) + C$

26. $\int x^2e^{3x}dx$

Letting $u = x^2,$ $dv = e^{3x}dx,$ then $du = 2x\, dx$ and $v = \frac{1}{3}e^{3x}.$

$$\int x^2e^{3x}dx = \frac{1}{3}x^2e^{3x} - \int\frac{1}{3}e^{3x}\cdot 2x\, dx$$
$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}dx$$

For $\int xe^{3x}dx,$ let $u = x,$ $dv = e^{3x}dx,$ then $du = dx,$ $v = \frac{1}{3}e^{3x},$ and

$$\int xe^{3x}dx = \frac{1}{3}xe^{3x} - \int\frac{1}{3}e^{3x}dx$$
$$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C_1.$$

Thus,

$$\int x^2e^{3x}dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right) + C$$
$$= \frac{1}{27}e^{3x}(9x^2 - 6x - 2) + C$$

27. $\int x^3e^{x^2}dx$

Letting $u = x^2$, $dv = xe^{x^2}dx$, then $du = 2x\, dx,$ $v = \left(\frac{1}{2}\right)e^{x^2}$.

$$\int x^3e^{x^2}dx = \frac{x^2e^{x^2}}{2} - \int\frac{e^{x^2}}{2}(2x\, dx)$$
$$= \frac{x^2e^{x^2}}{2} - \frac{e^{x^2}}{2} + C = \frac{e^{x^2}}{2}\left(x^2 - 1\right) + C$$

28. $\int x^5e^{x^2}dx$

Letting $u = x^4$ and $dv = xe^{x^2}dx,$ then $du = 4x^3\, dx$ and $v = \frac{1}{2}e^{x^2}$.

$$\int x^5e^{x^2}dx = \frac{x^4}{2}e^{x^2} - \int\frac{1}{2}e^{x^2}\cdot 4x^3\, dx$$
$$= \frac{x^4}{2}e^{x^2} - 2\int x^3e^{x^2}dx$$

Using Problem 27 for $\int x^3e^{x^2}dx,$

$$\int x^5e^{x^2}dx = \frac{x^4}{2}e^{x^2} - 2\cdot\left[\frac{1}{2}e^{x^2}(x^2-1)\right] + C$$
$$= \frac{x^4}{2}e^{x^2} - e^{x^2}(x^2-1) + C$$
$$= \frac{1}{2}e^{x^2}(x^4 - 2x^2 + 2) + C$$

29. $\int(e^x + x)^2dx = \int(e^{2x} + 2xe^x + x^2)dx$

$= \int e^{2x}dx + \int 2xe^x dx + \int x^2dx$

For $\int 2xe^x dx,$ let $u = 2x,$ $dv = e^x dx.$ Then $du = 2dx,$ $v = e^x,$ and

$\int 2xe^x dx = 2xe^x - 2\int e^x dx = 2xe^x - 2e^x + C.$

Thus $\int(e^x + x)^2dx = \frac{1}{2}e^{2x} + 2xe^x - 2e^x + \frac{x^3}{3} + C$

$= \frac{1}{2}e^{2x} + 2e^x(x-1) + \frac{x^3}{3} + C$

30. $\frac{d}{dx}\left[\ln\left(x+\sqrt{x^2+1}\right)\right]$

$$=\frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{x}{\sqrt{x^2+1}}\right)$$

$$=\frac{1}{x+\sqrt{x^2+1}}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)=\frac{1}{\sqrt{x^2+1}}$$

For $\int \ln\left(x+\sqrt{x^2+1}\right)dx$, let

$u=\ln\left(x+\sqrt{x^2+1}\right)$, $dv=dx$. Then

$du=\frac{1}{\sqrt{x^2+1}}dx$, $v=x$, and

$$\int \ln\left(x+\sqrt{x^2+1}\right)dx$$

$$=x\ln\left(x+\sqrt{x^2+1}\right)-\int x\left(x^2+1\right)^{-\frac{1}{2}}dx$$

$$=x\ln\left(x+\sqrt{x^2+1}\right)-\frac{1}{2}\int\left(x^2+1\right)^{-\frac{1}{2}}[2x\ dx]$$

$$=x\ln\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}+C$$

31. Area $=\int_1^{e^3}(\ln x)dx$. Letting $u=\ln x$, $dv=dx$,

then $du=\left(\frac{1}{x}\right)dx$, $v=x$.

$$\int_1^{e^3}(\ln x)dx=\left[(x\ln x)-\int x\cdot\frac{1}{x}dx\right]\Bigg|_1^{e^3}$$

$$=\left[(x\ln x)-\int dx\right]\Bigg|_1^{e^3}=[x\ln(x)-x]\Bigg|_1^{e^3}$$

$$=\left[e^3\cdot 3-e^3\right]-[1\cdot 0-1]=2e^3+1$$

The area is $(2e^3+1)$ sq units.

32. Area $=\int_0^1 x^2e^x\,dx$.

Letting $u=x^2$, $dv=e^x dx$, then $du=2x\,dx$ and $v=e^x$.

$$\int x^2e^x=x^2e^x-2\int xe^x\,dx$$

For $\int xe^x\,dx$, let $u=x$ and $dv=e^x\,dx$, then $du=dx$ and $v=e^x$.

$$\int xe^x=xe^x-\int e^x\,dx=xe^x-e^x=e^x(x-1).$$

Thus $\int_0^1 x^2e^x\,dx=(x^2e^x-2[e^x(x-1)])\Big|_0^1$

$$=(e^x[x^2-2x+2])\Big|_0^1$$

$$=e-2$$

The area is $(e-2)$ sq units.

33. Area $=\int_1^2 x^2\ln x\,dx$.

Letting $u=\ln x$, $dv=x^2dx$, then $du=\frac{1}{x}dx$, $v=\frac{x^3}{3}$.

$$\int_1^2 x^2\ln x\,dx=\left(\frac{x^3}{3}\ln x-\int\frac{x^3}{3}\cdot\frac{1}{x}dx\right)\Bigg|_1^2$$

$$=\left(\frac{x^3}{3}\ln x-\frac{1}{3}\int x^2dx\right)\Bigg|_1^2$$

$$=\left(\frac{x^3}{3}\ln x-\frac{1}{9}x^3\right)\Bigg|_1^2$$

$$=\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\left(0-\frac{1}{9}\right)$$

$$=\frac{8}{3}\ln(2)-\frac{7}{9}$$

The area is $\left(\frac{8}{3}\ln(2)-\frac{7}{9}\right)$ sq units.

34. $p=5(q+5)e^{-(q+5)/5}=5(q+5)e^{-0.2q-1}$

When $q=7$, $p=60e^{-2.4}$.

$$\text{CS}=\int_0^7[5(q+5)e^{-0.2q-1}-60e^{-2.4}]dq$$

$$=5\int_0^7(q+5)e^{-0.2q-1}dq-60\int_0^7 e^{-2.4}dq$$

For the first integral, let $u=q+5$, $dv=e^{-0.2q-1}dq$. Then $du=dq$, $v=-5e^{-0.2q-1}$,

and $5\int_0^7(q+5)e^{-0.2q-1}dq$

$$=5\left[(q+5)(-5e^{-0.2q-1})\Big|_0^7+5\int_0^7 e^{-0.2q-1}dq\right]$$

$$=5\left[12(-5e^{-2.4})-5(-5e^{-1})+5\cdot\frac{e^{-0.2q-1}}{-0.2}\Bigg|_0^7\right]$$

$$=5[-60e^{-2.4}+25e^{-1}-25(e^{-2.4}-e^{-1})]$$

$$=-425e^{-2.4}+250e^{-1}$$

Thus, $\text{CS} = -425e^{-2.4} + 250e^{-1} - 60e^{-2.4}q\Big|_0^7$
$= -425e^{-2.4} + 250e^{-1} - 60e^{-2.4}(7)$
$\approx \$15.31$

35. a. Consider $\int p\ dq$. Letting $u = p$, $dv = dq$, then $du = \frac{dp}{dq}dq$, $v = q$. Thus

$$\int p\ dq = pq - \int q\frac{dp}{dq}dq = r - \int q\frac{dp}{dq}dq$$

(since $r = pq$).

b. From (a), $r = \int p\ dq + \int q\frac{dp}{dq}dq$.

Combining the integrals gives

$$r = \int\left(p + q\frac{dp}{dq}\right)dq.$$

c. From (b), $\frac{dr}{dq} = p + q\frac{dp}{dq}$. Thus

$$\int_0^{q_0}\left(p + q\frac{dp}{dq}\right)dq$$
$$= \int_0^{q_0}\frac{dr}{dq}dq = r(q_0) - r(0) = r(q_0)$$

[since $r(0) = 0$].

36. $\int f(x)e^x dx$

Letting $u = f(x)$, $dv = e^x dx$, then $du = f'(x)dx$, $v = e^x$. Using integration by parts,
$\int f(x)e^x dx = f(x)e^x - \int f'(x)e^x dx$. Thus
$\int f(x)e^x dx + \int f'(x)e^x dx = f(x)e^x + C$

37. f and its inverse f^{-1} satisfy the equation $f(f^{-1}(x)) = x$. Differentiating this equation using the Chain Rule we get:
$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$. Thus
$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$. Now to evaluate $\int f^{-1}(x)dx$ we will use integration by parts, letting $u = f^{-1}(x)$ and $dv = dx$. Then
$du = \frac{1}{f'(f^{-1}(x))}dx$ and $v = x$.

So $\int f^{-1}(x)dx = xf^{-1}(x) - \int \frac{x}{f'(f^{-1}(x))}dx.$

To evaluate $\int \frac{x}{f'(f^{-1}(x))}dx$ we will use the fact that $x = f(f^{-1}(x))$ and
$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$

Hence

$$\int \frac{x}{f'(f^{-1}(x))}dx = \int f(f^{-1}(x)) \cdot (f^{-1})'(x)\,dx$$
$$= F(f^{-1}(x))$$

since $F' = f$. Finally,
$\int f^{-1}(x)dx = xf^{-1}(x) - F(f^{-1}(x)) + C.$

Apply It 15.2

3. $r(q) = \int r'(q)dq = \int \frac{5(q+4)}{q^2 + 4q + 3}dq$

Express $\frac{5(q+4)}{q^2 + 4q + 3}$ as a sum of partial fractions.

$$\frac{5(q+4)}{q^2 + 4q + 3} = \frac{5(q+4)}{(q+1)(q+3)} = \frac{A}{q+1} + \frac{B}{q+3}$$

$5(q + 4) = A(q + 3) + B(q + 1)$

When $q = -3$, we get $5(1) = -2B$, so $B = -\frac{5}{2}$.

When $q = -1$, we get $5(3) = A(2)$, so $A = \frac{15}{2}$.

$$r(q) = \int \frac{5(q+4)}{q^2 + 4q + 3}dx$$
$$= \int \frac{\frac{15}{2}}{q+1}dq - \int \frac{\frac{5}{2}}{q+3}dq$$
$$= \frac{15}{2}\ln|q+1| - \frac{5}{2}\ln|q+3| + C$$
$$= \frac{5}{2}\ln\left|\frac{(q+1)^3}{q+3}\right| + C$$

Since $r(0) = 0$, $0 = \frac{5}{2}\ln\left|\frac{1}{3}\right| + C$ so $C = \frac{5}{2}\ln 3$ and

$$r(q) = \frac{5}{2}\ln\left|\frac{3(q+1)^3}{q+3}\right|.$$

4. $V(t)=\int V'(t)dt=\int \frac{300t^3}{t^2+6}dt$

Since the degree of the numerator is greater than the degree of the denominator, we first divide $300t^3$ by t^2+6 to reduce the fraction.

$\frac{300t^3}{t^2+6}=\frac{300t^3+1800t-1800t}{t^2+6}$

$=\frac{300t\left(t^2+6\right)-1800t}{t^2+6}=300t-\frac{1800t}{t^2+6}$

t^2+6 is irreducible. To integrate $\frac{1800t}{t^2+6}$, let

$u=t^2+6$, so $du=2t\,dt$

$\int\frac{300t^3}{t^2+6}dt=\int 300t\,dt-\int\frac{1800t}{t^2+6}dt$

$=150t^2-900\ln\left|t^2+6\right|+C$

$V(t)=150t^2-900\ln\left(t^2+6\right)+C$

Problems 15.2

1. $\frac{10x}{x^2+7x+6}=\frac{10x}{(x+6)(x+1)}=\frac{A}{x+6}+\frac{B}{x+1}$

$10x=A(x+1)+B(x+6)$

If $x=-1$, then $-10=5B$, or $B=-2$. If $x=-6$, then $-60=-5A$, or $A=12$.

Answer $\frac{12}{x+6}-\frac{2}{x+1}$

2. $\frac{x+5}{x^2-1}=\frac{x+5}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}$

$x+5=A(x+1)+B(x-1)$

If $x=-1$, then $4=-2B$, or $B=-2$. If $x=1$, then $6=2A$, or $A=3$.

Answer $\frac{3}{x-1}-\frac{2}{x+1}$

3. $\frac{2x^2}{x^2+5x+6}=2+\frac{-10x-12}{x^2+5x+6}$

$\frac{-10x-12}{x^2+5x+6}=\frac{-10x-12}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$

$-10x-12=A(x+3)+B(x+2)$

If $x=-3$, then $18=-B$, or $B=-18$.

If $x=-2$, then $8=A$.

Answer: $2+\frac{8}{x+2}-\frac{18}{x+3}$

4. $\frac{2x^2-15}{x^2+5x}=2+\frac{-10x-15}{x^2+5x}$ (by long division).

$\frac{-10x-15}{x^2+5x}=\frac{-10x-15}{x(x+5)}=\frac{A}{x}+\frac{B}{x+5}$

$-10x-15=A(x+5)+Bx$. If $x=0$, then $-15=5A$, or $A=-3$. If $x=-5$, then $35=-5B$, or $B=-7$.

Answer: $2-\frac{3}{x}-\frac{7}{x+5}$

5. $f(x)=\frac{3x-1}{x^2-2x+1}=\frac{3x-1}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}$

$3x-1=A(x-1)+B$

If $x=1$, then $2=B$. If $x=0$, then $-1=-A+B$, $A=1+B=1+2=3$, or $A=3$.

Answer: $\frac{3}{x-1}+\frac{2}{(x-1)^2}$

6. $\frac{2x+3}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$2x+3=Ax(x-1)+B(x-1)+Cx^2$

If $x=0$, then $3=-B$, or $B=-3$. If $x=1$, then $5=C$. If $x=-1$, then $1=2A-2B+C$, $1=2A+6+5$, or $A=-5$.

Answer: $-\frac{5}{x}-\frac{3}{x^2}+\frac{5}{x-1}$

7. $\frac{x^2+3}{x^3+x}=\frac{x^2+3}{x\left(x^2+1\right)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

$x^2+3=A\left(x^2+1\right)+(Bx+C)x$

$x^2+3=(A+B)x^2+Cx+A$

Thus $A+B=1$, $C=0$, $A=3$. This gives $A=3$, $B=-2$, $C=0$.

Answer: $\frac{3}{x}-\frac{2x}{x^2+1}$

8. $\frac{3x^2+5}{\left(x^2+4\right)^2}=\frac{Ax+B}{x^2+4}+\frac{Cx+D}{\left(x^2+4\right)^2}$

$3x^2+5=(Ax+B)\left(x^2+4\right)+(Cx+D)$

$3x^2+5=Ax^3+Bx^2+(4A+C)x+(4B+D)$

Thus $A=0$, $B=3$, $4A+C=0$, $4B+D=5$. This gives $A = 0$, $B = 3$, $C = 0$, $D = -7$.

Answer: $\frac{3}{x^2+4} - \frac{7}{\left(x^2+4\right)^2}$

9. $\frac{5x-2}{x^2-x} = \frac{5x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$5x - 2 = A(x-1) + Bx$

If $x = 1$, then $3 = B$. If $x = 0$, then $-2 = -A$, or $A = 2$.

$$\int \frac{5x-2}{x^2-x}dx = \int\left(\frac{2}{x}+\frac{3}{x-1}\right)dx$$
$$= 2\ln|x| + 3\ln|x-1| + C = \ln\left|x^2(x-1)^3\right| + C$$

10. $\frac{15x+5}{x^2+5x} = \frac{15x+5}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5}$

$15x + 5 = A(x+5) + Bx$

If $x = 0$, then $5 = 5A + 0$, or $A = 1$.

If $x = 1$, then $20 = 6A + B = 6 + B$, or $B = 14$.

$$\int \frac{15x+5}{x^2+5x}dx = \int\left(\frac{1}{x}+\frac{14}{x+5}\right)dx$$
$$= \ln|x| + 14\ln|x+5| + C$$

11. $\frac{x+10}{x^2-x-2} = \frac{x+10}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$

$x + 10 = A(x-2) + B(x+1)$

If $x = 2$, then $12 = 3B$, or $B = 4$. If $x = -1$, then $9 = -3A$, or $A = -3$.

$$\int \frac{x+10}{x^2-x-2}dx = \int\left(\frac{-3}{x+1}+\frac{4}{x-2}\right)dx$$
$$= -3\ln|x+1| + 4\ln|x-2| + C = \ln\left|\frac{(x-2)^4}{(x+1)^3}\right| + C$$

12. $\frac{2x-1}{x^2-x-12} = \frac{2x-1}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$

$2x - 1 = A(x+3) + B(x-4)$

If $x = -3$, then $-7 = -7B$, or $B = 1$. If $x = 4$, then $7 = 7A$, or $A = 1$.

$$\int \frac{2x-1}{x^2-x-12}dx = \int\left(\frac{1}{x-4}+\frac{1}{x+3}\right)dx$$
$$= \ln|x-4| + \ln|x+3| + C = \ln|(x-4)(x+3)| + C$$

13. $\frac{3x^3-3x+4}{4x^2-4} = \frac{1}{4}\cdot\frac{3x^3-3x+4}{x^2-1}$

$= \frac{1}{4}\left(3x + \frac{4}{x^2-1}\right)$

$\frac{4}{x^2-1} = \frac{4}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$4 = A(x+1) + B(x-1)$

If $x = -1$, then $4 = -2B$, or $B = -2$. If $x = 1$, then $4 = 2A$, or $A = 2$.

$$\int \frac{3x^3-3x+4}{4x^2-4}dx = \frac{1}{4}\int\left(3x + \frac{2}{x-1} + \frac{-2}{x+1}\right)dx$$
$$= \left(\frac{1}{4}\right)\left[\frac{3x^2}{2} + 2\ln|x-1| - 2\ln|x+1|\right] + C$$
$$= \left(\frac{1}{4}\right)\left[\frac{3x^2}{2} + \ln\left|\frac{x-1}{x+1}\right|^2\right] + C$$

14. $\frac{7(4-x^2)}{(x-4)(x-2)(x+3)} = \frac{7(2+x)(2-x)}{(x-4)(x-2)(x+3)}$

$= \frac{-7(x+2)}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$

$-7(x+2) = A(x+3) + B(x-4)$

If $x = -3$, then $7 = -7B$, or $B = -1$. If $x = 4$, then $-42 = 7A$, or $A = -6$.

$$\int \frac{7(4-x^2)}{(x-4)(x-2)(x+3)}dx = \int\left(\frac{-6}{x-4} + \frac{-1}{x+3}\right)dx$$
$$= -6\ln|x-4| - \ln|x+3| + C$$
$$= -\ln\left|(x-4)^6(x+3)\right| + C$$

15. $\frac{19x^2-5x-36}{2x^3-2x^2-12x} = \frac{19x^2-5x-36}{2x(x-3)(x+2)}$

$= \frac{A}{2x} + \frac{B}{x-3} + \frac{C}{x+2}$

$19x^2 - 5x - 36$

$= (x-3)(x+2)A + 2x(x+2)B + 2x(x-3)C$

If $x = 0$, then $-36 = -6A$, or $A = 6$.

If $x = 3$, then $120 = 30B$, or $B = 4$.

If $x = -2$, then $50 = 20C$, or $C = \frac{5}{2}$.

$$\int \frac{19x^2 - 5x - 36}{2x^3 - 2x^2 - 12x}\,dx$$
$$= \int \frac{6}{2x}\,dx + \int \frac{4}{x-3}\,dx + \frac{5}{2}\int \frac{1}{x+2}\,dx$$
$$= 3\ln|x| + 4\ln|x-3| + \frac{5}{2}\ln|x+2| + C$$

16. $\frac{4-x}{x^4 - x^2} = \frac{4-x}{x^2(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{x-1}$

$$4 - x = Ax(x+1)(x-1) + B(x+1)(x-1) + Cx^2(x-1) + Dx^2(x+1)$$

If $x = 0$, then $4 = -B$, or $B = -4$. If $x = -1$, then $5 = -2C$, or $C = -\frac{5}{2}$. If $x = 1$, then $3 = 2D$, or $D = \frac{3}{2}$. If $x = 2$, then $2 = 6A + 3B + 4C + 12D$, $2 = 6A - 12 - 10 + 18$, or $2 = 6A - 4$, so $A = 1$.

$$\int \frac{4-x}{x^4 - x^2}\,dx = \int\left(\frac{1}{x} - \frac{4}{x^2} + \frac{-\frac{5}{2}}{x+1} + \frac{\frac{3}{2}}{x-1}\right)dx$$
$$= \ln|x| + \frac{4}{x} - \frac{5}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C$$
$$= \frac{4}{x} + \frac{1}{2}\ln\left|\frac{x^2(x-1)^3}{(x+1)^5}\right| + C$$

17. $\int \frac{2(3x^5 + 4x^3 - x)}{x^6 + 2x^4 - x^2 - 2}\,dx = \int \frac{1}{x^6 + 2x^4 - x^2 - 2}\left[\left(6x^5 + 8x^3 - 2x\right)dx\right]$

$\left(\text{Form: } \int\left(\frac{1}{u}\right)du\right)$ (Partial fractions not required.)

Answer: $\ln\left|x^6 + 2x^4 - x^2 - 2\right| + C$

18. $\frac{x^4 - 2x^3 + 6x^2 - 11x + 2}{x^3 - 3x^2 + 2x} = x + 1 + \frac{7x^2 - 13x + 2}{x^3 - 3x^2 + 2x}$

$$\frac{7x^2 - 13x + 2}{x^3 - 3x^2 + 2x} = \frac{7x^2 - 13x + 2}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$
$$7x^2 - 13x + 2 = A(x-1)(x-2) + Bx(x-2) + Cx(x-1)$$

If $x = 0$, then $2 = 2A$, or $A = 1$. If $x = 1$, then $-4 = -B$, or $B = 4$. If $x = 2$, then $4 = 2C$, or $C = 2$.

$$\int \frac{x^4 - 2x^3 + 6x^2 - 11x + 2}{x^3 - 3x^2 + 2x} = \int\left(x + 1 + \frac{1}{x} + \frac{4}{x-1} + \frac{2}{x-2}\right)dx$$
$$= \frac{x^2}{2} + x + \ln|x| + 4\ln|x-1| + 2\ln|x-2| + C$$
$$= \frac{x^2}{2} + x + \ln\left|x(x-1)^4(x-2)^2\right| + C$$

19. $\dfrac{2x^2-5x-2}{(x-2)^2(x-1)} = \dfrac{A}{x-1}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$

$2x^2-5x-2 = A(x-2)^2+B(x-1)(x-2)+C(x-1)$

If $x = 1$, then $-5 = A$. If $x = 2$, then $-4 = C$.
If $x = 0$, then $-2 = 4A + 2B - C$, $-2 = -20 + 2B + 4$, or $B = 7$.

$$\int \frac{2x^2-5x-2}{(x-2)^2(x-1)}\,dx = \int\left[\frac{-5}{x-1}+\frac{7}{x-2}+\frac{-4}{(x-2)^2}\right]dx$$

$$= -5\ln|x-1|+7\ln|x-2|+\frac{4}{x-2}+C = \frac{4}{x-2}+\ln\left|\frac{(x-2)^7}{(x-1)^5}\right|+C$$

20. $\dfrac{5x^3+x^2+x-3}{x^4-x^3} = \dfrac{5x^3+x^2+x-3}{x^3(x-1)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{D}{x-1}$

$5x^3+x^2+x-3 = Ax^2(x-1)+Bx(x-1)+C(x-1)+Dx^3$

If $x = 0$, $-3 = -C$, or $C = 3$.
If $x = 1$, $4 = D$.
If $x = -1$, $-8 = -2A + 2B - 2C - D$, or $2 = -2A + 2B$ and $1 = -A + B$.
If $x = 2$, $43 = 4A + 2B + C + 8D$, or $8 = 4A + 2B$ and $-4 = -2A - B$.
This gives $A = 1$ and $B = 2$.

$$\int \frac{5x^3+x^2+x-3}{x^4-x^3}\,dx = \int\frac{1}{x}\,dx+2\int\frac{1}{x^2}\,dx+3\int\frac{1}{x^3}\,dx+4\int\frac{1}{x-1}\,dx$$

$$= \ln|x|+2\cdot\frac{x^{-1}}{-1}+3\frac{x^{-2}}{-2}+4\ln|x-1|+C$$

$$= \ln\left|x(x-1)^4\right|-\frac{2}{x}-\frac{3}{2x^2}+C$$

21. $\dfrac{2(x^2+8)}{x^3+4x} = \dfrac{2x^2+16}{x\left(x^2+4\right)} = \dfrac{A}{x}+\dfrac{Bx+C}{x^2+4}$

$2x^2+16 = A\left(x^2+4\right)+(Bx+C)x$

$2x^2+16 = (A+B)x^2+Cx+4A$

Thus $A + B = 2$, $C = 0$, $4A = 16$. This gives $A = 4$, $B = -2$, $C = 0$.

$$\int\frac{2(x^2+8)}{x^3+4x}\,dx = \int\left(\frac{4}{x}+\frac{-2x}{x^2+4}\right)dx = 4\int\frac{1}{x}\,dx-\int\frac{1}{x^2+4}[2x\ dx] = 4\ln|x|-\ln\left(x^2+4\right)+C = \ln\left[\frac{x^4}{x^2+4}\right]+C$$

22. $\dfrac{4x^3-3x^2+2x-3}{(x^2+3)(x+1)(x-2)} = \dfrac{Ax+B}{x^2+3}+\dfrac{C}{x+1}+\dfrac{D}{x-2}$

$4x^3-3x^2+2x-3 = (Ax+B)(x+1)(x-2)+C(x^2+3)(x-2)+D(x^2+3)(x+1)$

If $x = -1$, then $-12 = -12C$, or $C = 1$.
If $x = 2$, then $21 = 21D$, or $D = 1$.
If $x = 0$, then $-3 = -2B - 6C + 3D$, $-3 = -2B - 6 + 3$, $0 = -2B$, or $B = 0$.

If $x = 1$, then $0 = -2(A + B) - 4C + 8D$, $0 = -2A - 4 + 8$, $-4 = -2A$, or $A = 2$.

$$\begin{aligned}\int \frac{4x^3-3x^2+2x-3}{(x^2+3)(x+1)(x-2)}dx &= \int\left(\frac{2x}{x^2+3}+\frac{1}{x+1}+\frac{1}{x-2}\right)dx\\ &= \ln(x^2+3)+\ln|x+1|+\ln|x-2|+C\\ &= \ln\left|(x^2+3)(x+1)(x-2)\right|+C\end{aligned}$$

23. $\dfrac{-x^3+8x^2-9x+2}{\left(x^2+1\right)(x-3)^2} = \dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-3}+\dfrac{D}{(x-3)^2}$

$$\begin{aligned}-x^3+8x^2-9x+2 &= (Ax+B)(x-3)^2+C(x-3)\left(x^2+1\right)+D\left(x^2+1\right)\\ &= (Ax+B)\left(x^2-6x+9\right)+C\left(x^3-3x^2+x-3\right)+D\left(x^2+1\right)\\ &= (A+C)x^3+(B-6A-3C+D)x^2+(9A-6B+C)x+(9B-3C+D)\end{aligned}$$

Thus $A + C = -1$, $B - 6A - 3C + D = 8$, $9A - 6B + C = -9$, $9B - 3C + D = 2$. This gives $A = -1$, $B = 0$, $C = 0$, $D = 2$.

$$\int\frac{-x^3+8x^2-9x+2}{\left(x^2+1\right)(x-3)^2}dx = \int\left(\frac{-x}{x^2+1}+\frac{0}{x-3}+\frac{2}{(x-3)^2}\right)dx = -\frac{1}{2}\ln\left(x^2+1\right)-\frac{2}{x-3}+C$$

24. $\dfrac{5x^4+9x^2+3}{x(x^2+1)^2} = \dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$

$$\begin{aligned}5x^4+9x^2+3 &= A(x^2+1)^2+(Bx+C)x(x^2+1)+(Dx+E)x\\ &= A(x^4+2x^2+1)+(Bx+C)(x^3+x)+Dx^2+Ex\\ &= (A+B)x^4+Cx^3+(2A+B+D)x^2+(C+E)x+A\end{aligned}$$

Thus, $A + B = 5$, $C = 0$, $2A + B + D = 9$, $C + E = 0$, and $A = 3$. This gives $A = 3$, $B = 2$, $C = 0$, $D = 1$, and $E = 0$.

$$\begin{aligned}\int\frac{5x^4+9x^2+3}{x(x^2+1)^2}dx &= \int\left(\frac{3}{x}+\frac{2x}{x^2+1}+\frac{x}{(x^2+1)^2}\right)dx\\ &= 3\ln|x|+\ln\left|x^2+1\right|-\frac{1}{2(x^2+1)}+C\\ &= \ln\left|x^3(x^2+1)\right|-\frac{1}{2(x^2+1)}+C\end{aligned}$$

25. $\dfrac{7x^3+24x}{(x^2+3)(x^2+4)} = \dfrac{Ax+B}{x^2+3}+\dfrac{Cx+D}{x^2+4}$

$$\begin{aligned}7x^3+24x &= (Ax+B)(x^2+4)+(Cx+D)(x^2+3)\\ &= (A+C)x^3+(B+D)x^2+(4A+3C)x+(4B+3D)\end{aligned}$$

Thus $A + C = 7$, $B + D = 0$, $4A + 3C = 24$, $4B + 3D = 0$.
This gives $A = 3$, $B = 0$, $C = 4$, and $D = 0$.

$$\int \frac{7x^3+24x}{(x^2+3)(x^2+4)}dx = \int \frac{3x}{x^2+3}dx + \int \frac{4x}{x^2+4}dx$$
$$= \frac{3}{2}\int \frac{2x}{x^2+3}dx + 2\int \frac{2x}{x^2+4}dx$$
$$= \frac{3}{2}\ln\left|x^2+3\right| + 2\ln\left|x^2+4\right| + C$$
$$= \ln\left|(x^2+3)^{3/2}(x^2+4)^2\right| + C$$

26. $\dfrac{12x^3+20x^2+28x+4}{3(x^2+2x+3)(x^2+1)} = \dfrac{1}{3}\left(\dfrac{Ax+B}{x^2+2x+3} + \dfrac{Cx+D}{x^2+1}\right)$

$$12x^3+20x^2+28x+4 = (Ax+B)(x^2+1)+(x^2+2x+3)(Cx+D)$$
$$= (A+C)x^3+(B+D+2C)x^2+(A+2D+3C)x+(B+3D)$$

Thus, $A + C = 12$, $B + D + 2C = 20$, $A + 2D + 3C = 28$, $B + 3D = 4$. This gives $A = 4$, $B = 4$, $C = 8$, $D = 0$.

$$\int \frac{12x^3+20x^2+28x+4}{3\left(x^2+2x+3\right)\left(x^2+1\right)}dx = \frac{1}{3}\int\left(\frac{4x+4}{x^2+2x+3} + \frac{8x}{x^2+1}\right)dx$$
$$= \frac{1}{3}\left[2\ln\left(x^2+2x+3\right) + 4\ln\left(x^2+1\right)\right] + C$$
$$= \ln\left[\left(x^2+2x+3\right)^{\frac{2}{3}}\left(x^2+1\right)^{\frac{4}{3}}\right] + C$$

27. $\dfrac{3x^3+8x}{(x^2+2)^2} = \dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{(x^2+2)^2}$

$$3x^3+8x = (Ax+B)(x^2+2)+Cx+D$$
$$= Ax^3+Bx^2+(2A+C)x+(2B+D)$$

Thus, $A = 3$, $B = 0$, $2A + C = 8$, $2B + D = 0$.
This gives $A = 3$, $B = 0$, $C = 2$, $D = 0$.

$$\int \frac{3x^3+8x}{(x^2+2)^2}dx = \int\left(\frac{3x}{x^2+2} + \frac{2x}{(x^2+2)^2}\right)dx$$
$$= \frac{3}{2}\ln(x^2+2) - \frac{1}{x^2+2} + C$$

28. $\displaystyle\int \frac{3x^2-8x+4}{x^3-4x^2+4x-6}dx$

$$= \int \frac{1}{x^3-4x^2+4x-6}\left[\left(3x^2-8x+4\right)dx\right]$$

$\left(\text{Form: } \displaystyle\int\left(\frac{1}{u}\right)du\right)$ (Partial fractions not required.)

Answer: $\ln\left|x^3-4x^2+4x-6\right| + C$

29. $\frac{2-2x}{x^2+7x+12}=\frac{2-2x}{(x+3)(x+4)}=\frac{A}{x+3}+\frac{B}{x+4}$

$2 - 2x = A(x + 4) + B(x + 3)$

If $x = -4$, then $10 = -B$, or $B = -10$. If $x = -3$, then $8 = A$.

$\int_0^1 \frac{2-2x}{x^2+7x+12}dx=\int_0^1\left(\frac{8}{x+3}+\frac{-10}{x+4}\right)dx$

$=\left[8\ln|x+3|-10\ln|x+4|\right]\Big|_0^1$

$= 8 \ln 4 - 10 \ln 5 - (8 \ln 3 - 10 \ln 4)$

$= 18 \ln(4) - 10 \ln(5) - 8 \ln(3)$

30. $\frac{x^2+5x+5}{x^2+3x+2}=1+\frac{2x+3}{x^2+3x+2}=1+\frac{2x+3}{(x+2)(x+1)}$

$\frac{2x+3}{(x+2)(x+1)}=\frac{A}{x+2}+\frac{B}{x+1}$

$2x+3=(x+1)A+(x+2)B$

If $x = -1$, $1 = B$. If $x = -2$, $-1 = -A$, or $A = 1$.

$\int_0^1 \frac{x^2+5x+5}{x^2+3x+2}dx=\int_0^1 dx+\int_0^1\frac{1}{x+2}dx+\int_0^1\frac{1}{x+1}dx$

$=x\Big|_0^1+\ln|x+2|\Big|_0^1+\ln|x+1|\Big|_0^1$

$=1+\ln 3-\ln 2+\ln 2-\ln 1$

$=1+\ln 3$

31. Note that $\frac{6\left(x^2+1\right)}{(x+2)^2}\ge 0$ on [0, 1].

Area $=\int_0^1\frac{6(x^2+1)}{(x+2)^2}dx$

$\frac{6(x^2+1)}{(x+2)^2}=6+\frac{-24x-18}{(x+2)^2}$ (by long division)

$\frac{-24x-18}{(x+2)^2}=\frac{A}{x+2}+\frac{B}{(x+2)^2}$

$-24x - 18 = A(x + 2) + B$

If $x = -2$, then $30 = B$. If $x = 0$, then $-18 = 2A + B$, $-18 = 2A + 30$, or $A = -24$.

$\int_0^1\frac{6\left(x^2+1\right)}{(x+2)^2}dx$

$=\int_0^1\left[6+\frac{-24}{x+2}+\frac{30}{(x+2)^2}\right]dx$

$=\left[6x-24\ln|x+2|-\frac{30}{x+2}\right]\Big|_0^1$

$=6-24\ln 3-10-(-24\ln 2-15)$

$=11+24\ln\frac{2}{3}$

The area is $11+24\ln\frac{2}{3}$ sq units.

32. $\text{CS}=\int_0^{10}\left[\frac{200(q+3)}{q^2+7q+6}-\frac{325}{22}\right]dq$

$\frac{200(q+3)}{q^2+7q+6}=\frac{200(q+3)}{(q+6)(q+1)}=\frac{A}{q+6}+\frac{B}{q+1}$

$200(q + 3) = A(q + 1) + B(q + 6)$

If $q = -1$, then $400 = 5B$, or $B = 80$. If $q = -6$, then $-600 = -5A$, or $A = 120$.

$\text{CS}=\int_0^{10}\left[\frac{120}{q+6}+\frac{80}{q+1}-\frac{325}{22}\right]dq$

$=\left[120\ln|q+6|-80\ln|q+1|-\frac{325}{22}q\right]\Big|_0^{10}$

$=\left[120\ln(16)+80\ln(11)-\frac{3250}{22}\right]-[120\ln(6)]$

$=120\ln\frac{8}{3}+80\ln(11)-\frac{1625}{11}\approx \$161.80$

Problems 15.3

1. Let $u = x$, $a^2=6$. Then $du = dx$.

$\int\frac{dx}{(6-x^2)^{3/2}}=\frac{x}{6\sqrt{6-x^2}}+C$

2. Let $u = 2x$, $a^2 = 25$. Then $du = 2dx$.

$$\int \frac{dx}{\left(25-4x^2\right)^{\frac{3}{2}}} = \frac{1}{2}\int \frac{(2dx)}{\left[25-(2x)^2\right]^{\frac{3}{2}}}$$

$$= \frac{1}{2}\left[\frac{(2x)}{25\sqrt{25-(2x)^2}}\right] + C$$

$$= \frac{x}{25\sqrt{25-4x^2}} + C$$

3. Let $u = 4x$, $a^2 = 3$. Then $du = 4\,dx$.

$$\int \frac{dx}{x^2\sqrt{16x^2+3}} = 4\int \frac{(4\,dx)}{(4x)^2\sqrt{(4x)^2+3}}$$

$$= 4\left[-\frac{\sqrt{(4x)^2+3}}{3(4x)}\right] + C$$

$$= -\frac{\sqrt{16x^2+3}}{3x} + C$$

4. Let $u = x^2$, $a^2 = 9$. Then $du = 2x\,dx$.

$$\int \frac{3\,dx}{x^3\sqrt{x^4-9}} = \frac{3}{2}\int \frac{(2x\,dx)}{(x^2)^2\sqrt{(x^2)^2-9}}$$

$$= \frac{3}{2}\left[-\frac{-\sqrt{(x^2)^2-9}}{9x^2} + C\right]$$

$$= \frac{\sqrt{x^4-9}}{6x^2} + C$$

5. Formula 5 with $u = x$, $a = 6$, $b = 7$. Then $du = dx$.

$$\int \frac{dx}{x(6+7x)} = \frac{1}{6}\ln\left|\frac{x}{6+7x}\right| + C$$

6. Formula 8 with $u = x$, $a = 2$, $b = 3$. Then $du = dx$.

$$\int \frac{5x^2\,dx}{(2+3x)^2}$$

$$= 5\left[\int \frac{x^2\,dx}{(2+3x)^2}\right]$$

$$= 5\left[\frac{x}{9} - \frac{4}{27(2+3x)} - \frac{4}{27}\ln|2+3x|\right] + C$$

7. Formula 28 with $u = x$, $a = 3$. Then $du = dx$.

$$\int \frac{dx}{x\sqrt{x^2+9}} = \frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}-3}{x}\right| + C$$

8. Formula 32 with $u = x$, $a^2 = 7$. Then $du = dx$.

$$\int \frac{dx}{(x^2+7)^{3/2}} = \frac{x}{7\sqrt{x^2+7}} + C$$

9. Formula 12 with $u = x$, $a = 2$, $b = 3$, $c = 4$, $k = 5$. Then $du = dx$.

$$\int \frac{x\,dx}{(2+3x)(4+5x)}$$

$$= \frac{1}{2}\left[\frac{4}{5}\ln|4+5x| - \frac{2}{3}\ln|2+3x|\right] + C$$

10. Formula 37 with $u = 5x$, $a = 2$. Then $du = 5\,dx$.

$$\int 2^{5x}\,dx = \frac{1}{5}\int 2^{5x}(5\,dx) = \frac{1}{5}\cdot\frac{2^{5x}}{\ln 2} + C$$

11. Formula 45 with $u = x$, $a = 1$, $b = 2$, $c = 3$. Then $du = dx$. $\int \frac{dx}{1+2e^{3x}} = \frac{1}{3}\left(3x - \ln\left|1+2e^{3x}\right|\right) + C$

12. Formula 14 with $u = x$, $a = 1$, $b = 1$. Then $du = dx$.

$$\int x^2\sqrt{1+x}\,dx = \frac{2\left(8-12x+15x^2\right)(1+x)^{\frac{3}{2}}}{105} + C$$

13. Formula 9 with $u = x$, $a = 5$, $b = 2$. Then $du = dx$.

$$\int \frac{7\,dx}{x(5+2x)^2} = 7\left[\int \frac{dx}{x(5+2x)^2}\right]$$

$$= 7\left[\frac{1}{5(5+2x)} + \frac{1}{25}\ln\left|\frac{x}{5+2x}\right|\right] + C$$

14. Formula 20 with $u = \sqrt{11}x$, $a = \sqrt{5}$. Then $du = \sqrt{11}\,dx$.

$$\int \frac{dx}{x\sqrt{5-11x^2}} = \int \frac{\sqrt{11}dx}{\left(\sqrt{11}x\right)\sqrt{\left(\sqrt{5}\right)^2 - \left(\sqrt{11}x\right)^2}}$$

$$= -\frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}+\sqrt{5-11x^2}}{\sqrt{11}x}\right| + C$$

15. Formula 3 with $u = x$, $a = 2$, $b = 1$. Then $du = dx$.

$$\int_0^1 \frac{x\,dx}{2+x} = \left(x - 2\ln|2+x|\right)\Big|_0^1 = 1 - 2\ln 3 + 2\ln 2$$

$$= 1 - \ln 9 + \ln 4 = 1 + \ln\left(\frac{4}{9}\right)$$

16. Formula 4 with $u = x$, $a = 2$, $b = -5$. Then $du = dx$.

$$\begin{aligned}\int \frac{-3x^2 dx}{2-5x} &= -3\int \frac{x^2 dx}{2-5x}\\ &= -3\left(\frac{x^2}{-10} - \frac{2x}{25} + \frac{4}{-125}\ln|2-5x|\right) + C\\ &= 3\left(\frac{x^2}{10} + \frac{2x}{25} + \frac{4}{125}\ln|2-5x|\right) + C\end{aligned}$$

17. Formula 23 with $u = x$, $a^2 = 3$. Then $du = dx$.

$$\int \sqrt{x^2-3}\,dx = \frac{1}{2}\left(x\sqrt{x^2-3} - 3\ln\left|x + \sqrt{x^2-3}\right|\right) + C$$

18. Formula 11 with $u = x$, $a = 1$, $b = 5$, $c = 3$, $k = 2$. Then $du = dx$.

$$\int \frac{dx}{(1+5x)(2x+3)} = \frac{1}{13}\ln\left|\frac{1+5x}{2x+3}\right| + C$$

19. Formula 38 with $u = x$, $a = 12$. Then $du = dx$.

$$\int_0^{1/12} xe^{12x} dx = \frac{e^{12x}}{144}(12x-1)\Bigg|_0^{1/12} = \frac{1}{144}[e(0) - 1(-1)] = \frac{1}{144}$$

20. Formula 46 with $u = 3x$, $a = 2$, $b = 5$.
Then $du = 3\ dx$.

$$\int \sqrt{\frac{2+3x}{5+3x}}dx = \frac{1}{3}\int \sqrt{\frac{2+3x}{5+3x}}(3\ dx) = \frac{1}{3}\left[\sqrt{(2+3x)(5+3x)} - 3\ln\left(\sqrt{2+3x} + \sqrt{5+3x}\right)\right] + C$$

21. Formula 39 with $u = x$, $n = 3$, $a = 1$. Then $du = dx$.

$$\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx$$

Applying Formula 39 to $\int x^2 e^x dx$ with $u = x$, $n = 2$, and $a = 1$ (so $du = dx$) gives $\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx$.

Applying Formula 38 to $\int xe^x dx$ with $u = x$, $a = 1$ (so $du = dx$) gives $\int xe^x dx = e^x(x-1) + C_1$. Thus

$$\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6e^x(x-1) + C.$$

22. Formula 6 with $u = x$, $a = 1$, $b = 1$. Then $du = dx$.

$$\int_1^2 \frac{4\,dx}{x^2(1+x)} = 4\int_1^2 \frac{dx}{x^2(1+x)} = 4\left(-\frac{1}{x} + \ln\left|\frac{1+x}{x}\right|\right)\Bigg|_1^2 = 4\left(-\frac{1}{2} + \ln\frac{3}{2}\right) - 4(-1 + \ln 2) = 2 + 4\ln\frac{3}{4}$$

23. Formula 26 with $u = \sqrt{5}x$, $a^2 = 1$. Then $du = \sqrt{5}\,dx$.

$$\begin{aligned}\int \frac{\sqrt{5x^2+1}}{2x^2}dx &= \frac{5}{2\sqrt{5}}\int \frac{\sqrt{5x^2+1}}{5x^2}\left(\sqrt{5}\,dx\right)\\ &= \frac{\sqrt{5}}{2}\left(-\frac{\sqrt{5x^2+1}}{\sqrt{5}x} + \ln\left|\sqrt{5}x + \sqrt{5x^2+1}\right|\right) + C\end{aligned}$$

24. Formula 17 with $u = x$, $a = 2$, $b = -1$. Then $du = dx$.

$$\int \frac{dx}{x\sqrt{2-x}} = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{2-x}-\sqrt{2}}{\sqrt{2-x}+\sqrt{2}}\right| + C$$

25. Formula 7 with $u = x$, $a = 1$, $b = 3$. Then $du = dx$.

$$\int \frac{x\,dx}{(1+3x)^2} = \frac{1}{9}\left(\ln|1+3x| + \frac{1}{1+3x}\right) + C$$

26. Formula 47 with $u = 2x$, $a = 1$, $b = 3$. Then $du = 2\,dx$.

$$\int \frac{2\,dx}{\sqrt{(1+2x)(3+2x)}} = \ln\left|2+2x+\sqrt{(1+2x)(3+2x)}\right| + C$$

27. Formula 34 with $u = \sqrt{5}x$, $a = \sqrt{7}$. Then $du = \sqrt{5}dx$

$$\int \frac{dx}{7-5x^2} = \frac{1}{\sqrt{5}} \int \frac{1}{\left(\sqrt{7}\right)^2 - \left(\sqrt{5}x\right)^2}\left(\sqrt{5}dx\right) = \frac{1}{\sqrt{5}}\left(\frac{1}{2\sqrt{7}} \ln\left|\frac{\sqrt{7}+\sqrt{5}x}{\sqrt{7}-\sqrt{5}x}\right|\right) + C$$

28. Formula 24 with $u = \sqrt{3}x$, $a^2 = 6$. Then $du = \sqrt{3}dx$.

$$\int 7x^2\sqrt{3x^2-6}\,dx = \frac{7}{\left(\sqrt{3}\right)^3} \int \left(\sqrt{3}x\right)^2 \sqrt{\left(\sqrt{3}x\right)^2 - 6}\left(\sqrt{3}\,dx\right)$$

$$= \frac{7}{3\sqrt{3}}\left[\frac{\sqrt{3}x}{8}(6x^2-6)\sqrt{3x^2-6} - \frac{36}{8}\ln\left|\sqrt{3}x + \sqrt{3x^2-6}\right|\right] + C$$

29. Formula 42 with $u = 3x$, $n = 5$. Then $du = 3\,dx$.

$$\int 36x^5 \ln(3x)dx = 36\int x^5 \ln(3x)dx = \frac{36}{3^6}\int (3x)^5 \ln(3x)(3\ dx)$$

$$= \frac{4}{81}\left[\frac{(3x)^6 \ln(3x)}{6} - \frac{(3x)^6}{36}\right] + C = x^6[6\ln(3x) - 1] + C$$

30. Formula 10 with $u = x$, $a = 3$, $b = 2$. Then $du = dx$.

$$\int \frac{5\,dx}{x^2(3+2x)^2} = 5\left[\int \frac{dx}{x^2(3+2x)^2}\right]$$

$$= 5\left[-\frac{3+4x}{9x(3+2x)} + \frac{4}{27}\ln\left|\frac{3+2x}{x}\right|\right] + C$$

31. Formula 13 with $u = x$, $a = 1$, $b = 2$. Then $du = dx$.

$$\int 5x\sqrt{1+2x}dx = 5\int x\sqrt{1+2x}\,dx = 5\left[\frac{2(6x-2)(1+2x)^{\frac{3}{2}}}{15\cdot 4}\right] + C$$

$$= \frac{1}{3}(3x-1)(1+2x)^{3/2} + C$$

32. Formula 42 with $u = x$, $n = 2$. Then $du = dx$.

$$\int 9x^2 \ln x\,dx = 9\int x^2 \ln x\,dx$$
$$= 9\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right) + C = 3x^3(\ln x) - x^3 + C$$

33. Formula 27 with $u = 2x$, $a^2 = 13$. Then $du = 2\,dx$.

$$\int \frac{dx}{\sqrt{4x^2 - 13}} = \frac{1}{2}\int \frac{1}{\sqrt{(2x)^2 - 13}}(2\,dx)$$
$$= \frac{1}{2}\ln\left|2x + \sqrt{4x^2 - 13}\right| + C$$

34. Formula 44 with $u = 2x$. Then $du = 2\,dx$.

$$\int \frac{dx}{x\ln(2x)} = \int \frac{(2\,dx)}{(2x)\ln(2x)}$$
$$= \ln\left|\ln(2x)\right| + C$$

35. Formula 21 with $u = 3x$, $a^2 = 16$. Then $du = 3\,dx$.

$$\int \frac{2\,dx}{x^2\sqrt{16 - 9x^2}} = 2(3)\int \frac{(3\,dx)}{(3x)^2\sqrt{16 - (3x)^2}}$$
$$= 6\left(-\frac{\sqrt{16 - 9x^2}}{16(3x)}\right) + C$$
$$= -\frac{\sqrt{16 - 9x^2}}{8x} + C$$

36. Formula 22 with $u = x$, $a = \sqrt{3}$. Then $du = dx$.

$$\int \frac{\sqrt{3 - x^2}}{x}\,dx$$
$$= \int \frac{\sqrt{\left(\sqrt{3}\right)^2 - (x)^2}}{x}\,dx$$
$$= \sqrt{3 - x^2} - \sqrt{3}\ln\left|\frac{\sqrt{3} + \sqrt{3 - x^2}}{x}\right| + C$$

37. Formula 45 with $u = \sqrt{x}$, $a = \pi$, $b = 7$, $c = 4$.

Then $du = \frac{1}{2\sqrt{x}}\,dx$

$$\int \frac{dx}{\sqrt{x}\left(\pi + 7e^{4\sqrt{x}}\right)} = 2\int \frac{1}{\pi + 7e^{4\sqrt{x}}}\left(\frac{1}{2\sqrt{x}}\,dx\right)$$
$$= 2\left[\frac{1}{4\pi}\left(4\sqrt{x} - \ln\left|\pi + 7e^{4\sqrt{x}}\right|\right)\right] + C$$
$$= \frac{1}{2\pi}\left(4\sqrt{x} - \ln\left|\pi + 7e^{4\sqrt{x}}\right|\right) + C$$

38. Formula 2 with $u = x^3$, $a = 1$, $b = 2$. Then $du = 3x^2\,dx$.

$$\int_0^1 \frac{3x^2\,dx}{1 + 2x^3} = \frac{1}{2}\ln\left|1 + 2x^3\right|\Big|_0^1$$
$$= \frac{1}{2}\ln|3| - \frac{1}{2}\ln|1| = \ln\sqrt{3}$$

39. Can be put in the form $\int \frac{1}{u}\,du$.

$$\int \frac{x\,dx}{x^2 + 1} = \frac{1}{2}\int \frac{1}{x^2 + 1}(2x\,dx)$$
$$= \frac{1}{2}\ln\left(x^2 + 1\right) + C$$

40. Can be put in the form $\int e^u\,du.$

$$\int 3x\sqrt{x}e^{x^{5/2}}\,dx = 3\cdot\frac{2}{5}\int e^{x^{5/2}}\left[\frac{5}{2}x^{3/2}dx\right]$$
$$= \frac{6}{5}e^{x^{5/2}} + C$$

41. Can be put in the form $\int u^n\,du$.

$$\int \frac{(\ln x)^3}{x}\,dx = \int (\ln x)^3\left[\frac{1}{x}\,dx\right] = \frac{1}{4}(\ln x)^4 + C$$

42. $$\int \frac{5x^3 - \sqrt{x}}{2x}\,dx = \int\left(\frac{5}{2}x^2 - \frac{1}{2}x^{-\frac{1}{2}}\right)dx$$
$$= \frac{5}{6}x^3 - \sqrt{x} + C$$

43. $\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-3)(x-2)}dx$

Formula 11 with $u = x$, $a = -3$, $b = 1$, $c = -2$, and $k = 1$. Then $du = dx$.

$$\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-3)(x-2)}dx = \ln\left|\frac{x-3}{x-2}\right| + C$$

44. Can be put in the form $\int u^n du$.

$$\int \frac{e^{2x}}{\sqrt{e^{2x}+3}}dx = \frac{1}{2}\int \left(e^{2x}+3\right)^{-\frac{1}{2}}(2e^{2x}dx) = \sqrt{e^{2x}+3} + C$$

45. Formula 42 with $u = x$ and $n = 3$. Then $du = dx$.

$$\int x^3 \ln x\, dx = \frac{x^4}{4}\left[\ln(x) - \frac{1}{4}\right] + C$$

46. Formula 38 with $u = 3x - 2$, $a = -10$. Then $du = 3\, dx$.

$$\int (9x-6)e^{-30x+20}dx$$
$$= \int 3(3x-2)e^{-10(3x-2)}dx$$
$$= \frac{e^{-30x+20}}{100}[-10(3x-2)-1] + C$$
$$= \frac{1}{100}e^{-30x+20}(-30x+19) + C$$

47. Formula 38 with $u = x^2$ and $a = 3$. Then $du = 2x\, dx$.

$$\int 4x^3e^{3x^2}dx = 2\int x^2e^{3x^2}[2x\,dx] = 2\left[\frac{e^{3x^2}}{9}(3x^2-1)\right] + C = \frac{2}{9}e^{3x^2}(3x^2-1) + C$$

48. Formula 14 with $u = x$, $a = 3$ and $b = 2$. Then $du = dx$.

$$\int_1^2 35x^2\sqrt{3+2x}dx = 35\int_1^2 x^2\sqrt{3+2x}dx$$
$$= 35 \cdot \left.\frac{2\left(72-72x+60x^2\right)(3+2x)^{\frac{3}{2}}}{840}\right|_1^2$$
$$= 98\sqrt{7} - 25\sqrt{5}$$

49. Formula 43 and then Formula 41. For Formula 43, let $u = x$, $n = 0$, and $m = 2$. Then $du = dx$.

$$\int \ln^2 x\, dx = x\ln^2 x - 2\int \ln x\, dx$$

Now we apply Formula 41 to the last integral with $u = x$ (so $du = dx$).

$$\int \ln^2 x\, dx = x(\ln x)^2 - 2x(\ln x) + 2x + C$$

50. Formula 41 with $u = x^2$. Then $du = 2x\, dx$.

$$\int_1^3 3x \ln x^2\, dx$$
$$= \frac{3}{2}\int_1^e \ln(x^2)[2x\,dx] = \frac{3}{2}[x^2\ln(x^2) - x^2]\Big|_1^e$$
$$= \frac{3}{2}[(e^2\ln(e^2) - e^2) - (1\cdot\ln 1 - 1)]$$
$$= \frac{3}{2}(e^2+1)$$

51. Formula 15 with $u = x$, $a = 3$, and $b = 1$. Then $du = dx$.

$$\int_{-2}^1 \frac{x\,dx}{\sqrt{3+x}} = \left.\frac{2(x-6)\sqrt{3+x}}{3}\right|_{-2}^1 = -\frac{10}{3}\sqrt{4} + \frac{16}{3}\sqrt{1} = -\frac{4}{3}$$

52. Formula 13 with $u = x$, $a = 2$, and $b = 3$. Then $du = dx$.

$$\int_2^3 x\sqrt{2+3x}\,dx = \left.\frac{2(9x-4)(2+3x)^{3/2}}{135}\right|_2^3$$
$$= \frac{2}{135}[23(11)^{3/2} - 14(8)^{3/2}]$$
$$= \frac{2}{135}\left(253\sqrt{11} - 224\sqrt{2}\right)$$

53. Can be put in the form $\int u^n du$.

$$\int_0^1 \frac{2x\ dx}{\sqrt{8-x^2}} = -\int_0^1 \left(8-x^2\right)^{-\frac{1}{2}}(-2x\ dx)$$

$$= -\left.\frac{\left(8-x^2\right)^{\frac{1}{2}}}{\frac{1}{2}}\right|_0^1$$

$$= -2\left(8-x^2\right)^{\frac{1}{2}}\Big|_0^1 = -2\left(\sqrt{7}-\sqrt{8}\right)$$

$$= -2\left(\sqrt{7}-2\sqrt{2}\right)$$

$$= 2\left(2\sqrt{2}-\sqrt{7}\right)$$

54. Formula 39 with $u = x$, $n = 2$, $a = 3$. Then $du = dx$.

$$\int x^2 e^{3x}\ dx = \frac{x^2e^{3x}}{3} - \frac{2}{3}\int xe^{3x}\ dx$$

For $\int xe^{3x}\ dx$, use Formula 38 with $u = x$ and $a = 3$. Then $du = dx$.

$$\int x^2 e^{3x}\ dx = \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[\frac{e^{3x}}{9}(3x-1)\right]$$

$$= \frac{e^{3x}}{27}[9x^2-6x+2]$$

$$\int_0^{\ln 2} x^2e^{3x}\ dx = \left.\left(\frac{e^{3x}}{27}[9x^2-6x+2]\right)\right|_0^{\ln 2}$$

$$= \frac{8}{27}[9(\ln 2)^2 - 6\ln 2 + 2] - \frac{1}{27}[2]$$

$$= \frac{2}{27}[36(\ln 2)^2 - 24\ln 2 + 7]$$

55. Integration by parts or Formula 42. For Formula 42, let $u = 2x$, $n = 1$. Then $du = 2\ dx$.

$$\int_1^2 x\ln(2x)dx = \frac{1}{4}\int_1^2 (2x)\ln(2x)[2\ dx]$$

$$= \frac{1}{4}\left[\frac{(2x)^2\ln(2x)}{2} - \frac{(2x)^2}{4}\right]\Bigg|_1^2$$

$$= 2\ln(4) - 1 - \frac{1}{2}\ln(2) + \frac{1}{4}$$

$$= 2\ln\left(2^2\right) - \frac{1}{2}\ln(2) - \frac{3}{4}$$

$$= 4\ln(2) - \frac{1}{2}\ln(2) - \frac{3}{4}$$

$$= \frac{7}{2}(\ln 2) - \frac{3}{4}$$

56. Can be put in the form $\int k\ dx$.

$$\int_3^5 dA = \int_3^5 1\,dA = A\Big|_3^5 = 5-3 = 2$$

57. Formula 5 with $u = q$, $a = 1$, and $b = -1$. Then $du = dq$.

$$\int_{q_0}^{q_n} \frac{dq}{q(1-q)} = \ln\left|\frac{q}{1-q}\right|\Bigg|_{q_0}^{q_n} = \ln\left|\frac{q_n}{1-q_n}\right| - \ln\left|\frac{q_0}{1-q_0}\right|$$

$$= \ln\left|\frac{q_n(1-q_0)}{q_0(1-q_n)}\right|$$

58. Formula 6 with $u = q$, $a = 1$ and $b = -1$. Then $du = dq$.

$$n = -\frac{1}{0.4}\int_{0.3}^{0.1}\frac{dq}{q^2(1-q)}$$

$$= -\frac{1}{0.4}\left[-\frac{1}{q} - \ln\left|\frac{1-q}{q}\right|\right]\Bigg|_{0.3}^{0.1}$$

$$= -\frac{1}{0.4}\left\{[-10-\ln 9] - \left[-\frac{10}{3} - \ln\frac{7}{3}\right]\right\}$$

$$= -\frac{1}{0.4}\left(-\frac{20}{3} - \ln 9 + \ln\frac{7}{3}\right) \approx 20$$

59. a. For $\int_0^9 1000e^{-0.04t}\,dt$, the form $\int e^u\,du$ can be applied.

$$\int_0^9 1000e^{-0.04t}\,dt$$
$$=\frac{1000}{-0.04}\int_0^9 e^{-0.04t}(-0.04\,dt)$$
$$=-\frac{1000}{0.04}e^{-0.04t}\Big|_0^9$$
$$=-\frac{1000}{0.04}(e^{-0.36}-1)$$
$$\approx \$7558.09$$

b. For $\int_0^{10} 500te^{-0.06t}\,dt$ use Formula 38 with $t = u$ and $a = -0.06$, so $du = dt$.

$$\int_0^{10} 500te^{-0.06t}\,dt$$
$$=500\int_0^{10} te^{-0.06t}\,dt$$
$$=500\left[\frac{e^{-0.06t}}{0.0036}(-0.06t-1)\right]\Big|_0^{10}$$
$$=\frac{500}{0.0036}[e^{-0.6}(-1.6)-(-1)]$$
$$\approx \$16,930.75$$

60. $$\int_0^T ke^{-rt}\,dt = k\left(-\frac{1}{r}\right)\int_0^T e^{-rt}(-r\ dt) = \frac{-ke^{-rt}}{r}\Big|_0^T$$
$$=-\frac{ke^{-rT}}{r}+\frac{k}{r}=k\left(\frac{1-e^{-rT}}{r}\right)$$

61. a. $$\int_0^{10} 100e^{0.02(10-t)}dt = 100\int_0^{10} e^{0.2-0.02t}\,dt$$
$$=100\int_0^{10} e^{0.2}e^{-0.02t}\,dt$$
$$=100e^{0.2}\int_0^{10} e^{-0.02t}\,dt$$
$$=100e^{0.2}\left(\frac{1}{-0.02}\right)\int_0^{10} e^{-0.02t}(-0.02\ dt)$$
$$=-5000e^{0.2}e^{-0.02t}\Big|_0^{10}$$
$$=-5000e^{0.2}\left[e^{-0.2}-1\right]$$
$$\approx \$1107.01$$

b. $$\int_0^{10} 200e^{0.01(10-t)}dt$$
$$=200\int_0^{10} e^{0.1-0.01t}\,dt$$
$$=200e^{0.1}\int_0^{10} e^{-0.01t}\,dt$$
$$=200e^{0.1}\left(\frac{1}{-0.01}\right)\int_0^{10} e^{-0.01t}(-0.01\,dt)$$
$$=-20,000e^{0.1}\cdot e^{-0.01t}\Big|_0^{10}$$
$$=-20,000e^{0.1}(e^{-0.1}-1)$$
$$\approx \$2103.42$$

62. Use Formula 38 with $u = t$ and $a = -0.07$, so $du = dt$.

$$\int_0^5 50,000te^{-0.07t}\ dt = 50,000\int_0^5 te^{-0.07t}\ dt$$
$$=50,000\left[\frac{e^{-0.07t}}{0.0049}(-0.07t-1)\right]\Big|_0^5$$
$$=\frac{50,000}{0.0049}[e^{-0.35}(-1.35)-1(-1)]$$
$$=\$496,640$$

Problems 15.4

1. $$\bar{f}=\frac{1}{3-(-1)}\int_{-1}^3 x^2dx=\frac{1}{4}\cdot\frac{x^3}{3}\Big|_{-1}^3=\frac{1}{4}\left(9+\frac{1}{3}\right)=\frac{7}{3}$$

2. $$\bar{f}=\frac{1}{1-0}\int_0^1(2x+1)dx=(x^2+x)\Big|_0^1=2-0=2$$

3. $$\bar{f}=\frac{1}{2-(-1)}\int_{-1}^2\left(2-3x^2\right)dx$$
$$=\frac{1}{3}\left(2x-x^3\right)\Big|_{-1}^2=-1$$

4. $$\bar{f}=\frac{1}{3-1}\int_1^3\left(x^2+x+1\right)dx$$
$$=\frac{1}{2}\left(\frac{x^3}{3}+\frac{x^2}{2}+x\right)\Big|_1^3=\frac{22}{3}$$

5. $\bar{f} = \frac{1}{3-(-3)}\int_{-3}^{3} 2t^5 dt$
$= \frac{1}{6}\cdot\frac{t^6}{3}\Big|_{-3}^{3}$
$= \frac{1}{18}[3^6 - (-3)^6]$
$= 0$

6. $\bar{f} = \frac{1}{4-0}\int_0^4 t\sqrt{t^2+9}dt$
$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\int_0^4 \sqrt{t^2+9}[2t\ dt]$
$= \frac{1}{8}\left[\frac{2\left(t^2+9\right)^{\frac{3}{2}}}{3}\right]\Bigg|_0^4 = \frac{49}{6}$

7. $\bar{f} = \frac{1}{1-0}\int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3}$

8. $\bar{f} = \frac{1}{3-1}\int_1^3 \frac{5}{x^2}dx = \frac{1}{2}\cdot -\frac{5}{x}\Big|_1^3 = \frac{1}{2}\left(-\frac{5}{3}+5\right)$
$= \frac{5}{3}$

9. $\bar{P} = \frac{1}{100-0}\int_0^{100}\left(369q - 2.1q^2 - 400\right)dq$
$= \frac{1}{100}\left(184.5q^2 - 0.7q^3 - 400q\right)\Big|_0^{100}$
$= \frac{1}{100}(1,845,000 - 700,000 - 40,000) - 0$
$= 11,050$
Answer: \$11,050

10. $\bar{c} = \frac{1}{500-100}\int_{100}^{500}\left(4000 + 10q + 0.1q^2\right)dq$
$= \frac{1}{400}\left(4000q + 5q^2 + \frac{0.1q^3}{3}\right)\Bigg|_{100}^{500} \approx 17,333.33$
Answer: \$17,333.33

11. $\frac{1}{2-0}\int_0^2 3000e^{0.05t}dt$
$= \frac{3000}{2}\cdot\frac{1}{0.05}\int_0^2 e^{0.05t}[0.05\ dt]$
$= 30,000e^{0.05t}\Big|_0^2 = 30,000\left(e^{0.1}-1\right) \approx 3155.13$
Answer: \$3155.13

12. $\bar{C} = \frac{1}{T-0}\int_0^T \frac{R}{F(t)}dt = \frac{1}{T}\int_0^T \frac{R(1+\alpha t)^2}{F_1}dt$

$\frac{R}{TF_1}\cdot\frac{1}{\alpha}\int_0^T (1+\alpha t)^2[\alpha\ dt] = \frac{R}{\alpha TF_1}\left[\frac{(1+\alpha t)^3}{3}\right]\Bigg|_0^T$
$= \frac{R}{\alpha TF_1}\left[\frac{(1+\alpha T)^3}{3} - \frac{1}{3}\right]$
$= \frac{R}{3\alpha TF_1}\left[1 + 3\alpha T + 3\alpha^2T^2 + \alpha^3T^3 - 1\right]$
$= \frac{R}{3\alpha TF_1}(3\alpha T)\left(1 + \alpha T + \frac{1}{3}\alpha^2T^2\right)$
$= \frac{R\left(1+\alpha T + \frac{1}{3}\alpha^2T^2\right)}{F_1}$

13. Average value $= \frac{1}{q_0 - 0}\int \frac{dr}{dq}dq$.
$= \frac{1}{q_0}\left[r(q_0) - r(0)\right]$

But $r(0) = 0$, so avg. value $= \frac{r(q_0)}{q_0}$. Since

$r(q_0)$
= [price per unit when q_0 units are sold]$\cdot q_0$,
we have

avg. value $= \frac{\begin{bmatrix}\text{price per unit}\\ \text{when } q_0 \text{ units}\\ \text{are sold}\end{bmatrix}\cdot q_0}{q_0}$
= price per unit when $\cdot q_0$ units are sold.

14. $\bar{f} = \frac{1}{1-0}\int_0^1 \frac{1}{x^2 - 4x + 5}dx \approx 0.32$

Apply It 15.5

5. Separating variables, we have

$$\frac{dI}{dx} = -0.0085I$$

$$\frac{dI}{I} = -0.0085dx$$

$$\int \frac{1}{I} dI = -\int 0.0085\ dx$$

$$\ln|I| = -0.0085x + C_1$$

To solve for I, we convert to exponential Formula

$I = e^{-0.0085x+C_1} = Ce^{-0.0085x}$. Since $I = I_0$ when $x = 0$, $I_0 = Ce^0 = C$, so

$I(x) = I_0 e^{-0.0085x}$.

Problems 15.5

1. $y' = 2xy^2$

$$\frac{dy}{dx} = 2xy^2$$

$$\frac{dy}{y^2} = 2x\ dx$$

$$\int y^{-2} dy = \int 2x\ dx$$

$$-\frac{1}{y} = x^2 + C$$

$$y = -\frac{1}{x^2 + C}$$

2. $y' = x^2 y^2$

$$\frac{dy}{dx} = x^2 y^2$$

$$\frac{dy}{y^2} = x^2\ dx$$

$$\int \frac{dy}{y^2} = \int x^2\ dx$$

$$-\frac{1}{y} = \frac{x^3}{3} + C_1$$

$$-\frac{1}{y} = \frac{1}{3}(x^3 + 3C_1)$$

$$\frac{1}{y} = -\frac{1}{3}(x^3 + C)$$

$$y = -\frac{3}{x^3 + C}$$

3. $\frac{dy}{dx} - 2x\ln(x^2+1) = 0$

$$dy = 2x\ln(x^2+1)dx$$

$$\int dy = \int 2x\ln(x^2+1)dx$$

$$\int dy = \int \ln(x^2+1)[2x\,dx]$$

Using Formula 41 gives

$y = (x^2+1)\ln(x^2+1) - (x^2+1) + C.$

4. $\frac{dy}{dx} = \frac{x}{y}$

$$y\ dy = x\ dx$$

$$\int y\ dy = \int x\ dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C_1$$

$$y^2 = x^2 + 2C_1$$

$$y^2 = x^2 + C$$

5. $\frac{dy}{dx} = y$, where $y > 0$.

$$\frac{dy}{y} = dx$$

$$\int \frac{dy}{y} = \int dx$$

$$\ln y = x + C_1$$

$y = e^{x+C_1} = e^{C_1}e^x = Ce^x$, where $C = e^{C_1}$. Thus

$y = Ce^x$, where $C > 0$.

6. $y' = e^x y^3$

$$\frac{dy}{dx} = e^x y^3$$

$$\frac{dy}{y^3} = e^x dx$$

$$\int \frac{dy}{y^3} = \int e^x dx$$

$$-\frac{1}{2y^2} = e^x + C$$

$$y^2 = -\frac{1}{2(e^x + C)}$$

7. $y' = \frac{y}{x}$, where $x, y > 0$.

$\frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{y} = \frac{dx}{x}$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln y = \ln x + C_1$

$\ln y = \ln x + \ln C$, where $C > 0$.

$\ln y = \ln(Cx) \Rightarrow y = Cx$, where $C > 0$.

8. $\frac{dy}{dx} - x\ln x = 0$

$dy = x\ln x\, dx$

$\int dy = \int x\ln x\, dx$

Using Formula 42 gives

$y = \frac{x^2 \ln x}{2} - \frac{x^2}{2^2} + C$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C.$

9. $y' = \frac{1}{y^2}$ where $y(1) = 1$.

$\frac{dy}{dx} = \frac{1}{y^2}$

$y^2 dy = dx$

$\int y^2\, dy = \int dx$

$\frac{y^3}{3} = x + C$

Given $y(1) = 1$, we obtain $\frac{1^3}{3} = 1 + C$, so

$C = -\frac{2}{3}$. Thus $y^3 = 3\left(x - \frac{2}{3}\right) = 3x - 2$,

$y = \sqrt[3]{3x - 2}$.

10. $y' = e^{x-y}$, where $y(0) = 0$

$\frac{dy}{dx} = \frac{e^x}{e^y}$

$e^y dy = e^x dx$

$\int e^y dy = \int e^x dx$

$e^y = e^x + C$

Since $y(0) = 0$, we have $e^0 = e^0 + C$, $1 = 1 + C$, $C = 0$. Thus $e^y = e^x$, so $y = x$.

11. $e^y y' - x^2 = 0$, where $y = 0$ when $x = 0$.

$e^y \frac{dy}{dx} = x^2$

$e^y dy = x^2 dx$

$\int e^y dy = \int x^2 dx$

$e^y = \frac{x^3}{3} + C$

Given that $y(0) = 0$, we have $e^0 = 0 + C$, so $1 = C \Rightarrow e^y = \frac{x^3}{3} + 1$, $e^y = \frac{x^3 + 3}{3}$, so

$y = \ln \frac{x^3 + 3}{3}$.

12. $x^2 y' + \frac{1}{y^2} = 0$, where $y(1) = 2$

$x^2 \frac{dy}{dx} = -\frac{1}{y^2}$

$y^2 dy = -\frac{dx}{x^2}$

$\int y^2 dy = -\int \frac{dx}{x^2}$

$\frac{y^3}{3} = \frac{1}{x} + C$

Now, $y(1) = 2$ implies $C = \frac{5}{3}$. Thus

$\frac{y^3}{3} = \frac{1}{x} + \frac{5}{3}$, $y^3 = \frac{3}{x} + 5$, $y = \sqrt[3]{\frac{3}{x} + 5}$.

13. $(3x^2 + 2)^3 y' - xy^2 = 0$, where $y(0) = 2$.

$(3x^2 + 2)^3 \frac{dy}{dx} = xy^2$

$\frac{dy}{y^2} = \frac{x}{(3x^2 + 2)^3} dx$

$\int \frac{dy}{y^2} = \int \frac{x}{(3x^2 + 2)^3} dx$

$\int y^{-2} dy = \frac{1}{6} \int (3x^2 + 2)^{-3} [6x\, dx]$

$-\frac{1}{y} = -\frac{1}{12(3x^2 + 2)^2} + C$

Given that $y(0) = 2$ we have

$-\frac{1}{2} = -\frac{1}{12(0+2)^2} + C = -\frac{1}{48} + C$, so $C = -\frac{23}{48}$.

Thus,

$-\frac{1}{y} = -\frac{1}{12(3x^2+2)^2} - \frac{23}{48} = -\frac{4+23(3x^2+2)^2}{48(3x^2+2)^2}$.

Hence, $y = \frac{48(3x^2+2)^2}{4+23(3x^2+2)^2}$.

14. $y' + x^3 y = 0$ and $y = e$ when $x = 0$.

$\frac{dy}{dx} = -x^3 y$

$\frac{dy}{y} = -x^3\,dx$

$\int \frac{dy}{y} = -\int x^3\,dx$

$\ln|y| = -\frac{x^4}{4} + C$

Given $y(0) = e$, $\ln e = 0 + C$, so $C = 1$.

Thus $\ln y = -\frac{x^4}{4} + 1$, so $y = e^{-\frac{x^4}{4}+1}$.

15. $\frac{dy}{dx} = \frac{3x\sqrt{1+y^2}}{y}$, where $y > 0$ and $y(1) = \sqrt{8}$.

$\frac{y\,dy}{\sqrt{1+y^2}} = 3x\,dx$

$\frac{1}{2}\int \left(1+y^2\right)^{-\frac{1}{2}} [2y\,dy] = 3\int x\,dx$

$\left(1+y^2\right)^{\frac{1}{2}} = \frac{3x^2}{2} + C$

$y(1) = \sqrt{8} \Rightarrow (1+8)^{\frac{1}{2}} = \frac{3}{2} + C$

$C = \frac{3}{2}$

Thus

$\left(1+y^2\right)^{\frac{1}{2}} = \frac{3x^2}{2} + \frac{3}{2}$

$1+y^2 = \left[\frac{3x^2}{2} + \frac{3}{2}\right]^2$

$y^2 = \left[\frac{3x^2}{2} + \frac{3}{2}\right]^2 - 1$

Since $y > 0$, $y = \sqrt{\left[\frac{3x^2}{2} + \frac{3}{2}\right]^2 - 1}$.

16. $2y\left(x^3+2x+1\right)\frac{dy}{dx} = \frac{3x^2+2}{\sqrt{y^2+9}}$, where $y(0) = 0$.

$\int 2y\sqrt{y^2+9}\,dy = \int \frac{3x^2+2}{x^3+2x+1}\,dx$

$\int \left(y^2+9\right)^{\frac{1}{2}} [2y\,dy] = \int \frac{1}{x^3+2x+1}\left[\left(3x^2+2\right)dx\right]$

$\frac{2}{3}\left(y^2+9\right)^{\frac{3}{2}} = \ln\left|x^3+2x+1\right| + C$

Now $y(0) = 0$ implies that $\frac{2}{3}(27) = \ln(1) + C$, so $C = 18$. Thus

$\frac{2}{3}\left(y^2+9\right)^{\frac{3}{2}} = \ln\left|x^3+2x+1\right| + 18$.

17. $2\frac{dy}{dx} = \frac{xe^{-y}}{\sqrt{x^2+3}}$, where $y(1) = 0$.

$e^y\,dy = \frac{1}{2}x\left(x^2+3\right)^{-\frac{1}{2}}\,dx$

$\int e^y\,dy = \frac{1}{2}\cdot\frac{1}{2}\int \left(x^2+3\right)^{-\frac{1}{2}} [2x\,dx]$

$e^y = \frac{1}{2}\left(x^2+3\right)^{\frac{1}{2}} + C$

Now, $y(1) = 0 \Rightarrow e^0 = \frac{1}{2}(2) + C$, so $C = 0$. Thus

$e^y = \frac{1}{2}\left(x^2+3\right)^{\frac{1}{2}} \Rightarrow y = \ln\left(\frac{1}{2}\sqrt{x^2+3}\right)$.

18. $dy = 2xye^{x^2}dx$, where $y > 0$ and $y(0) = e$.

$$\frac{dy}{y} = 2xe^{x^2}dx$$

$$\int \frac{dy}{y} = \int 2xe^{x^2}dx$$

$$\int \frac{dy}{y} = \int e^{x^2}[2x\,dx]$$

$$\ln y = e^{x^2} + C$$

Now $y(0) = e$ gives $\ln e = 1 = e^0 + C = 1 + C$, so $C = 0$. Thus $\ln y = e^{x^2}$, or $e^{\ln y} = y = e^{e^{x^2}}$.

19. $(q+1)^2 \dfrac{dc}{dq} = cq$

$$\int \frac{1}{c}dc = \int \frac{q}{(q+1)^2}dq$$

Using partial fractions or Formula 7 for $\int \frac{q}{(q+1)^2}dq$, we obtain

$\ln c = \ln(q+1) + \dfrac{1}{q+1} + C$. Now, fixed cost is given to be e, which means that $c = e$ when $q = 0$. This implies $1 = 0 + 1 + C$, so $C = 0$. Thus

$\ln c = \ln(q+1) + \dfrac{1}{q+1} \Rightarrow c = e^{\ln(q+1)+\frac{1}{q+1}}$,

$c = e^{\ln(q+1)}e^{\frac{1}{q+1}}$, or $c = (q+1)e^{\frac{1}{q+1}}$.

20. $\dfrac{dy}{dx} = xe^{x-y} = \dfrac{xe^x}{e^y}$

$$\int e^y dy = \int xe^x dx$$

Using integration by parts or formula 38 gives $e^y = e^x(x-1) + C$. Now,

$f(1) = 0 \Rightarrow 1 = e(0) + C$, $1 = C$, so

$e^y = e^x(x-1) + 1$, $y = \ln\left[e^x(x-1)+1\right]$. Thus

$f(2) = \ln\left(e^2 + 1\right)$.

21. $\dfrac{dy}{dt} = -0.025y$

$$\int \frac{1}{y}dy = -0.025\int dt$$

$$\ln|y| = -0.025t + C$$

Given that $y = 1000$ when $t = 0$, we have $\ln 1000 = -0 + C = C$. Thus $\ln|y| = -0.025t + \ln 1000$. To find t when money is 95% new, we note that y would be 5%(1000) = 50. Solving $\ln 50 = -0.025t + \ln 1000$ gives

$t = \dfrac{\ln 1000 - \ln 50}{0.025} \approx 120$ weeks.

22. $\dfrac{dr}{dq} = (50-4q)e^{-\frac{r}{5}}$

$$\int e^{\frac{r}{5}}dr = \int (50-4q)dq$$

$$5e^{\frac{r}{5}} = 50q - 2q^2 + C$$

Since $r = 0$ when $q = 0$, we have $5(1) = C$, $C = 5$.

$$5e^{\frac{r}{5}} = 50q - 2q^2 + 5$$

$$e^{\frac{r}{5}} = 10q - \frac{2}{5}q^2 + 1$$

$$\frac{r}{5} = \ln\left|10q - \frac{2}{5}q^2 + 1\right|$$

$$r = 5\ln\left|10q - \frac{2}{5}q^2 + 1\right|$$

Since $r = pq$, $p = \dfrac{1}{q}r = \dfrac{5}{q}\ln\left|10q - \dfrac{2}{5}q^2 + 1\right|$.

23. Let N be the population at time t, where $t = 0$ corresponds to 1990. Since N follows exponential growth, $N = N_0e^{kt}$. Now, $N = 60{,}000$ when $t = 0$, so $N_0 = 60{,}000$. Therefore $N = 60{,}000e^{kt}$. Since $N = 64{,}000$ when $t = 10$, we have $64{,}000 = 60{,}000e^{10k}$,

$\dfrac{16}{15} = e^{10k}$, $\ln\dfrac{16}{15} = 10k$, $k = \dfrac{\ln\frac{16}{15}}{10}$

Thus
$$N = 60{,}000e^{\left(\ln\frac{16}{15}\right)\left(\frac{t}{10}\right)} = 60{,}000\left(e^{\ln\frac{16}{15}}\right)^{t/10} = 60{,}000\left(\frac{16}{15}\right)^{t/10}.$$

$$N(2010) = 60{,}000\left(\frac{16}{15}\right)^{20/10} \approx 68{,}267$$

24. Exponential growth applies, so $N = N_0e^{kt}$. When $t = 0$, then $N = 50{,}000$, So $N_0 = 50{,}000$. Thus $N = 50{,}000e^{kt}$. When $t = 50$, then

$N = 100{,}000$, or $100{,}000 = 50{,}000e^{50k}$ or

$k = \frac{\ln 2}{50}$. Thus

$N = 50{,}000e^{\frac{t \ln 2}{50}}$ (*)

$N = 50{,}000e^{\left(\frac{0.69}{50}\right)t}$

$N = 50{,}000e^{0.0138t}$ (First form)

From (*), $N = 50{,}000\left[e^{\ln 2}\right]^{\frac{t}{50}}$, so

$N = 50{,}000(2)^{\frac{t}{50}}$. (Second form)

When t = 100, then

$N = 50{,}000(2)^{\frac{100}{50}} = 50{,}000(2)^2 = 200{,}000$

25. Let N be the population (in billions) at time t, where t is the number of years past 1930. N follows exponential growth, so $N = N_0e^{kt}$. When t = 0, then N = 2, so $N_0 = 2$. Thus $N = 2e^{kt}$. Since N = 3 when t = 30, then

$3 = 2e^{30k}$

$\frac{3}{2} = e^{30k}$

$30k = \ln\frac{3}{2}$

$k = \frac{\ln\frac{3}{2}}{30}$

Thus $N = 2e^{\frac{t}{30}\ln\frac{3}{2}}$.

In 2015, t = 85 and so

$N = 2e^{\frac{85}{30}\ln\frac{3}{2}} \approx 2e^{1.14882}$ billion.

26. Let N = population at time t and N_0 = population at $t = 0$. Then $N = N_0e^{kt}$. When t = 100, then $N = 3N_0$, so

$3N_0 = N_0e^{100k}$ or $k = \frac{\ln 3}{100}$.

Setting $N = 2N_0$ and solving for t gives

$2N_0 = N_0e^{\frac{t \ln 3}{100}}$

$2 = e^{\frac{t \ln 3}{100}}$

$\ln 2 = \frac{t \ln 3}{100}$

$t = \frac{100 \ln 2}{\ln 3} \approx 63.$

The population will double in approximately 63 years.

27. Let N be amount of sample that remains after t seconds. Then $N = N_0e^{-\lambda t}$, where N_0 is the initial amount present. When t = 100, then $N = 0.3N_0$. Thus

$0.3N_0 = N_0e^{-100\lambda}$

$0.3 = e^{-100\lambda}$

$-100\lambda = \ln 0.3$

$\lambda = -\frac{\ln 0.3}{100}$

Thus $\lambda \approx 0.01204$. The half-life is

$\frac{\ln 2}{\lambda} = \frac{\ln 2}{-\frac{\ln 0.3}{100}} = -100\frac{\ln 2}{\ln 0.3} \approx 57.57$ s.

28. $N = N_0e^{-\lambda t}$

After 100 s, 80% remains.

$0.8N_0 = N_0e^{-100\lambda}$

$0.8 = e^{-100\lambda}$

$-100\lambda = \ln 0.8$

$\lambda = -\frac{\ln 0.8}{100}$

$\lambda \approx 0.0022314$

The half-life is

$\frac{\ln 2}{\lambda} = \frac{\ln 2}{-\frac{\ln 0.8}{100}} = -100\frac{\ln 2}{\ln 0.8} \approx 310.63$ s.

29. Let N be the amount of ^{14}C present in the scroll t years after it was made. Then $N = N_0e^{-\lambda t}$, where N_0 is amount of ^{14}C present when t = 0. We must find t when $N = 0.7N_0$.

$0.7N_0 = N_0e^{-\lambda t}$

$0.7 = e^{-\lambda t}$

$-\lambda t = \ln 0.7$

so $t = -\frac{\ln 0.7}{\lambda}$. By Equation 15 in the text,

$\lambda = \frac{\ln 2}{5730}$, so

$t = -\frac{\ln 0.7}{\frac{\ln 2}{5730}} = -\frac{5730 \ln 0.7}{\ln 2} \approx 2900$ years.

30. $N = N_0 e^{-\lambda t}$

$0.1N_0 = N_0 e^{-\lambda t}$

$0.1 = e^{-\lambda t}$

$-\lambda t = \ln(0.1)$

$t = -\dfrac{\ln 0.1}{\lambda}$

By Equation 15 in the text, $\lambda = \dfrac{\ln 2}{5730}$, so

$t = -\dfrac{\ln 0.1}{\frac{\ln 2}{5730}} = -\dfrac{5730 \ln 0.1}{\ln 2} \approx 19{,}000$ years.

31. $\dfrac{dN}{dt} = kN$

$N = Ae^{kt}$

$N_0 = Ae^{kt_0}$

$A = \dfrac{N_0}{e^{kt_0}}$

Thus $N = \dfrac{N_0}{e^{kt_0}}\left(e^{kt}\right) = N_0 e^{kt - kt_0}$, or

$N = N_0 e^{k(t-t_0)}$, where $t \ge t_0$.

32. **a.** From Equation 15 in the text, $140 = \dfrac{\ln 2}{\lambda}$.

Thus $\lambda = \dfrac{\ln 2}{140}$.

b. $N = N_0 e^{-\lambda t} = N_0 e^{-\frac{t \ln 2}{140}} = N_0 e^{-\frac{365 \ln 2}{140}}$

$\dfrac{N}{N_0} = e^{-\frac{365 \ln 2}{140}} \approx 0.164$

33. $N = N_0 e^{-\lambda t}$

When $t = 2$, then $N = 12$. Thus $12 = N_0 e^{-2\lambda}$,

$N_0 = 12e^{2\lambda}$. By Equation 15 in the text,

$6 = \dfrac{\ln 2}{\lambda}$

$\lambda = \dfrac{\ln 2}{6}$

Thus

$N_0 = 12e^{2\frac{\ln 2}{6}} = 12e^{\frac{\ln 2}{3}} = 12 \cdot 2^{1/3} \approx 15.1$ units.

34. $N = N_0 e^{-\lambda t}$

We want to find t when

$N = \left(\dfrac{3}{5}\right) N_0$

$\left(\dfrac{3}{5}\right) N_0 = N_0 e^{-\lambda t}$

$\dfrac{3}{5} = e^{-\lambda t}$

$-\lambda t = \ln\left(\dfrac{3}{5}\right)$

$t = -\dfrac{\ln \frac{3}{5}}{\lambda}$

By Equation 15 in the text, $8 = \dfrac{\ln 2}{\lambda}$, $\lambda = \dfrac{\ln 2}{8}$.

Thus $t = -\dfrac{\ln \frac{3}{5}}{\frac{\ln 2}{8}} = -\dfrac{8 \ln \frac{3}{5}}{\ln 2} \approx 5.9$ days.

35. $\dfrac{dA}{dt} = 200 - 0.50A$

$\displaystyle\int \frac{dA}{200 - 0.50A} = \int dt$

$-\dfrac{1}{0.50} \ln(200 - 0.50A) = t + C_1$

$\ln(200 - 0.50A) = -0.50t - 0.50C_1$

$= -0.50t + C_2$

Thus

$200 - 0.50A = e^{-0.50t + C_2} = e^{-0.50t} e^{C_2}$

$200 - \dfrac{A}{2} = Ce^{-0.50t}$

Given that $A = 0$ when $t = 0$, we have $C = 200$,

so $200 - \dfrac{A}{2} = 200e^{-\frac{t}{2}}$

$200 - 200e^{-\frac{t}{2}} = \dfrac{A}{2}$

$200\left(1 - e^{-\frac{t}{2}}\right) = \dfrac{A}{2}$

Thus $A = 400\left(1 - e^{-\frac{t}{2}}\right)$. If $t = 1$,

$A = 400\left(1 - e^{-\frac{1}{2}}\right) \approx 157$ grams per square meter.

36. $$\frac{dP}{dx} = k(150{,}000 - 2P)$$

$$\int \frac{dP}{150{,}000 - 2P} = \int k\,dx$$

$$-\frac{1}{2}\ln[150{,}000 - 2P] = kx + C$$

Since $P(0) = 15{,}000$, we have

$-\frac{1}{2}\ln[150{,}000 - 30{,}000] = C$, so

$$-\frac{1}{2}\ln[150{,}000 - 2P] = kx - \frac{1}{2}\ln[120{,}000].$$

Since $P(1000) = 70{,}000$,

$$-\frac{1}{2}\ln[150{,}000 - 140{,}000] = 1000k - \frac{1}{2}\ln[120{,}000]$$

$$k = \frac{1}{2}\cdot\frac{\ln[120{,}000] - \ln[10{,}000]}{1000} = \frac{\ln 12}{2000}$$

Thus

$$-\frac{1}{2}\ln[150{,}000 - 2P] = \frac{\ln 12}{2000}x - \frac{1}{2}\ln[120{,}000]$$

$$\ln[150{,}000 - 2P] = -\frac{\ln 12}{1000}x + \ln[120{,}000]$$

$$150{,}000 - 2P = e^{-\frac{\ln 12}{1000}x}e^{\ln[120{,}000]}$$

$$150{,}000 - 2P = 120{,}000e^{-\frac{\ln 12}{1000}x}$$

$$P = \frac{1}{2}\left(150{,}000 - 120{,}000e^{-\frac{\ln 12}{1000}x}\right)$$

$$= 75{,}000 - 60{,}000\left(12^{-\frac{x}{1000}}\right)$$

If $x = 2000$, then

$$P = 75{,}000 - 60{,}000(12^{-2}) \approx \$74{,}583.$$

37. a. $$\frac{dV}{dt} = kV$$

$$\int \frac{1}{V}dV = \int k\ dt$$

$$\ln V = kt + C_1$$

$$V = e^{kt}e^{C_1}$$

or $V = Ce^{kt}$. Now $t = 0$ corresponds to July 1, 1996 where $V = 0.75 \cdot 80{,}000 = 60{,}000$, so $60{,}000 = C(1)$. Thus $V = 60{,}000e^{kt}$. Also $V = 38{,}900$ for $t = 9.5$, so

$$38{,}900 = 60{,}000e^{9.5k}$$

$$\frac{389}{600} = e^{9.5k}$$

$$9.5k = \ln\left(\frac{389}{600}\right)$$

$$k = \frac{1}{9.5}\ln\left(\frac{389}{600}\right)$$

Thus $V = 60{,}000e^{\frac{t}{9.5}\ln\left(\frac{389}{600}\right)}$.

b. $$14{,}000 = 60{,}000e^{\frac{t}{9.5}\ln\left(\frac{389}{600}\right)}$$

$$\frac{7}{30} = e^{\frac{t}{9.5}\ln\left(\frac{389}{600}\right)}$$

$$\ln\left(\frac{7}{30}\right) = \frac{t}{9.5}\ln\left(\frac{389}{600}\right)$$

$$t = \frac{9.5\ln\left(\frac{7}{30}\right)}{\ln\left(\frac{389}{600}\right)} \approx 31.903$$

This corresponds to about 31 years and 11 months after July 1, 1996 ⇒ June 2028.

Problems 15.6

1. $$N = \frac{M}{1 + be^{-ct}}$$

$M = 100{,}000$

Since $N = 50{,}000$ at $t = 0$ (1995), we have

$50{,}000 = \frac{100{,}000}{1+b}$, so $1 + b = \frac{100{,}000}{50{,}000} = 2$, or $b = 1$.

Hence, $N = \frac{100{,}000}{1 + e^{-ct}}$. If $t = 5$, then $N = 60{,}000$, so

$$60{,}000 = \frac{100{,}000}{1 + e^{-5c}}$$

$$1 + e^{-5c} = \frac{100{,}000}{60{,}000} = \frac{5}{3}$$

$$e^{-5c} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$e^{-c} = \left(\frac{2}{3}\right)^{1/5}$$

Hence, $N = \frac{100{,}000}{1 + \left(\frac{2}{3}\right)^{t/5}}$. In 2005, $t = 10$, so

$$N = \frac{100{,}000}{1 + \left(\frac{2}{3}\right)^2} \approx 69{,}200.$$

2. $N = \dfrac{M}{1+be^{-ct}}$

Since $M = 500$, and $N = 200$ when $t = 0$, we have

$$200 = \frac{500}{1+b}$$

$$1+b = \frac{500}{200} = \frac{5}{2} \Rightarrow b = \frac{3}{2}.$$

Hence $N = \dfrac{500}{1+\frac{3}{2}e^{-ct}}$. When $t = 1$ we are given $N = 300$. Thus

$$300 = \frac{500}{1+\frac{3}{2}e^{-c}}$$

$$1+\frac{3}{2}e^{-c} = \frac{500}{300} = \frac{5}{3}$$

$$\frac{3}{2}e^{-c} = \frac{2}{3}$$

$$e^{-c} = \frac{4}{9}$$

Hence $N = \dfrac{500}{1+\frac{3}{2}\left(\frac{4}{9}\right)^t}$. When $t = 2$, then

$$N = \frac{500}{1+\frac{3}{2}\left(\frac{4}{9}\right)^2} \approx 386.$$

3. $N = \dfrac{M}{1+be^{-ct}}$

$M = 40{,}000$, and $N = 20$ when $t = 0$, so

$$20 = \frac{40{,}000}{1+b}$$

$$1+b = \frac{40{,}000}{20} = 2000$$

$$b = 1999$$

Hence $N = \dfrac{40{,}000}{1+1999e^{-ct}}$.

Since $N = 100$ when $t = 1$, $100 = \dfrac{40{,}000}{1+1999e^{-c}}$,

$$1+1999e^{-c} = \frac{40{,}000}{100} = 400$$

$$e^{-c} = \frac{399}{1999}$$

Hence $N = \dfrac{40{,}000}{1+1999\left(\frac{399}{1999}\right)^t}$.

If $t = 2$, then $N = \dfrac{40{,}000}{1+1999\left(\frac{399}{1999}\right)^2} \approx 500.$

4. $N = \dfrac{M}{1+be^{-ct}}$

Since $M = 50{,}000$, and $N = 500$ when $t = 0$, we have

$$500 = \frac{50{,}000}{1+b}$$

$$1+b = \frac{50{,}000}{500} = 100$$

$$b = 99$$

Hence $N = \dfrac{50{,}000}{1+99e^{-ct}}$. If $t = 1$, then $N = 1500$.

Thus

$$1500 = \frac{50{,}000}{1+99e^{-c}}$$

$$1+99e^{-c} = \frac{50{,}000}{1500} = \frac{100}{3}$$

$$99e^{-c} = \frac{97}{3}$$

$$e^{-c} = \frac{97}{297}$$

Hence $N = \dfrac{50{,}000}{1+99\left(\frac{97}{297}\right)^t}$.

5. $N = \dfrac{M}{1+be^{-ct}}$

$M = 100{,}000$, and since $N = 500$ when $t = 0$, we have

$$500 = \frac{100{,}000}{1+b}$$

$$1+b = \frac{100{,}000}{500} = 200$$

$$b = 199$$

Hence $N = \dfrac{100{,}000}{1+199e^{-ct}}$. If $t = 1$, then $N = 1000$. Thus

$$1000 = \frac{100,000}{1+199e^{-c}}$$
$$1+199e^{-c} = \frac{100,000}{1000} = 100$$
$$199e^{-c} = 99$$
$$e^{-c} = \frac{99}{199}$$

Hence $N = \frac{100,000}{1+199\left(\frac{99}{199}\right)^t}$. If $t = 2$, then

$$N = \frac{100,000}{1+199\left(\frac{99}{199}\right)^2} \approx 1990.$$

6. a. $$\frac{dN}{dt} = N(1-N)$$
$$\frac{dN}{N(1-N)} = dt$$
$$\int \frac{dN}{N(1-N)} = \int dt$$

Using Formula 5 in the Table of Integrals, for $\int \frac{dN}{N(1-N)}$, we get $\ln\left|\frac{N}{1-N}\right| = t + C.$

Since $N(0) = \frac{1}{2}$, $\ln\left|\frac{\frac{1}{2}}{1-\frac{1}{2}}\right| = \ln 1 = 0 = C.$

Also, $N > 0$, and since $M = 1$, $N < 1$. Thus

$$\ln\left(\frac{N}{1-N}\right) = t.$$
$$\frac{N}{1-N} = e^t$$
$$N = (1-N)e^t$$
$$N(e^t+1) = e^t$$
$$N = \frac{e^t}{e^t+1} = \frac{1}{1+e^{-t}}$$

b. $$\frac{dN}{dt} = N(1-N) = N - N^2$$
$$\frac{d^2N}{dt^2} = 1-2N$$

$\frac{d^2N}{dt^2} = 0$ when $N = \frac{1}{2}$.

$1 - 2N > 0$ when $N < \frac{1}{2}$ and $1 - 2N < 0$ when $N > \frac{1}{2}$, so there is an inflection point when $N = \frac{1}{2}$.

$$\frac{1}{2} = \frac{1}{1+e^{-t}}$$
$$1+e^{-t} = 2$$
$$e^{-t} = 1$$
$$t = 0$$

Thus the point $\left(0, \frac{1}{2}\right)$ is an inflection point on the graph.

c. $$f(t) = \frac{1}{1+e^{-t}} - \frac{1}{2} = \frac{2-(1+e^{-t})}{2(1+e^{-t})} = \frac{1-e^{-t}}{2(1+e^{-t})} = \frac{e^t-1}{2(e^t+1)}$$

Replace t by $-t$ then multiply numerator and denominator by e^t.

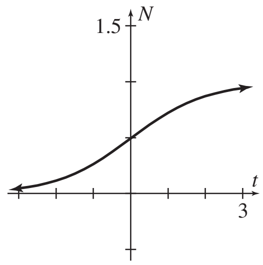
$$\frac{e^{-t}-1}{2(e^{-t}+1)} = \frac{1-e^t}{2(1+e^t)} = -\frac{e^t-1}{2(e^t+1)} = -f(t)$$

Thus, $f(t)$ is symmetric about the origin.

d. The graph of $N(t)$ is the graph of $f(t)$ shifted $\frac{1}{2}$ unit upward. Thus, since $f(t)$ is symmetric about (0, 0), $N(t)$ is symmetric about $\left(0, \frac{1}{2}\right)$.

$$N(t) = f(t) + \frac{1}{2}$$
$$N(-t) = f(-t) + \frac{1}{2} = -f(t) + \frac{1}{2} = -\left[f(t) + \frac{1}{2}\right] + 1 = 1 - N(t)$$

e.



7. a. $N = \dfrac{375}{1+e^{5.2-2.3t}} = \dfrac{375}{1+e^{5.2}e^{-2.3t}}$

$\approx \dfrac{375}{1+181.27e^{-2.3t}}$

b. $\lim\limits_{t\to\infty} N = \dfrac{375}{1+181.27(0)} = 375$

8. a. $N = \dfrac{0.2524}{e^{-2.128x}+0.005125}$

$= \dfrac{\frac{0.2524}{0.005125}}{\frac{e^{-2.128x}+0.005125}{0.005125}}$

$\approx \dfrac{49.25}{\frac{e^{-2.128x}}{0.005125}+1} \approx \dfrac{49.25}{1+195.1e^{-2.128x}}$

b. If $x = 0$, then $N \approx \dfrac{49.25}{1+195.1} \approx 0.2511 \text{ cm}^2$.

9. $\dfrac{dT}{dt} = k(T-a)$ where $a = -5$.

$\dfrac{dT}{T+5} = k\,dt$

$\int \dfrac{dT}{T+5} = \int k\,dt$

Thus $\ln(T+5) = kt + C$. At $t = 0$, we have $T = 27$, so $\ln(27+5) = 0 + C$, $C = \ln 32$, and $\ln(T+5) = kt + \ln 32$.

$\ln(T+5) - \ln 32 = kt$

Hence $\ln\left(\dfrac{T+5}{32}\right) = kt$.

If $t = 1$, then $T = 19$. Thus $\ln\left(\dfrac{19+5}{32}\right) = k\cdot 1$, so

$k = \ln\dfrac{24}{32} = \ln\dfrac{3}{4}$. Hence $\ln\left(\dfrac{T+5}{32}\right) = \left(\ln\dfrac{3}{4}\right)t$.

If $T = 37$, then $\ln\left(\dfrac{42}{32}\right) = \left(\ln\dfrac{3}{4}\right)t$

$t = \dfrac{\ln\frac{42}{32}}{\ln\frac{3}{4}} \approx -0.945 \text{ hr}$

which corresponds to 57 minutes. Time of murder: 3:17 A.M. – 57 min = 2:20 A.M.

10. $\dfrac{dp}{dt} = kp(I-p)$

This is logistic growth, so the maximum rate of formation (growth) occurs when $p = \dfrac{I}{2}$, which is when there are equal amounts of both enzymes.

11. $\dfrac{dx}{dt} = k(200,000-x)$

$\int \dfrac{dx}{200,000-x} = \int k\ dt$

$-\ln(200,000-x) = kt + C$

$\ln(200,000-x) = -kt - C$

$200,000 - x = e^{-kt-C} = e^{-C}e^{-kt} = Ae^{-kt}$, where $A = e^{-C}$. Thus $x = 200,000 - Ae^{-kt}$. If $t = 0$, then $x = 50,000$, so $50,000 = 200,000 - A \Rightarrow A = 150,000$. Thus $x = 200,000 - 150,000e^{-kt}$. If $t = 1$, then $x = 100,000$, so

$100,000 = 200,000 - 150,000e^{-k}$

$150,000e^{-k} = 100,000$

$e^{-k} = \dfrac{100,000}{150,000} = \dfrac{2}{3}$

Thus $x = 200,000 - 150,000\left(\dfrac{2}{3}\right)^t$. If $t = 3$, then

$x = 200,000 - 150,000\left(\dfrac{8}{27}\right) \approx \$155,555.56.$

12. $\dfrac{dN}{dt} = kN^2$

$\int \dfrac{dN}{N^2} = \int k\ dt$

$-\dfrac{1}{N} = kt + C$

If $t = 0$, then $N = N_0$. Thus $-\dfrac{1}{N_0} = C$, so

$$-\frac{1}{N} = kt - \frac{1}{N_0}$$

$$\frac{1}{N} = \frac{-kN_0t+1}{N_0}$$

$$N = \frac{N_0}{1-kN_0t}$$

As $t \to \left(\frac{1}{kN_0}\right)^-$, then $1-kN_0t \to 0^+$, so $N \to \infty$.

13. $\frac{dN}{dt} = k(M-N)$

$$\int \frac{dN}{M-N} = \int k\ dt$$

$-\ln(M-N) = kt + C$

If $t = 0$, then $N = N_0$, so $-\ln(M-N_0) = C$.

Thus we have

$$-\ln(M-N) = kt - \ln(M-N_0)$$

$$\ln(M-N_0) - \ln(M-N) = kt$$

$$\ln\frac{M-N_0}{M-N} = kt$$

$$\ln\frac{M-N}{M-N_0} = -kt$$

$$\frac{M-N}{M-N_0} = e^{-kt}$$

$$M-N = (M-N_0)e^{-kt}$$

$$N = M-(M-N_0)e^{-kt}$$

Apply It 15.7

6. $\int_0^\infty \left(3e^{-0.1t} - 3e^{-0.3t}\right) dt$

$$= \lim_{r\to\infty} \int_0^r \left(3e^{-0.1t} - 3e^{-0.3t}\right) dt$$

$$= \lim_{r\to\infty} \left(-30e^{-0.1t} + 10e^{-0.3t}\right)\Big|_0^r$$

$$= \lim_{r\to\infty} \left[-\frac{30}{e^{-0.1r}} + \frac{10}{e^{0.3r}} - \left(-30e^0 + 10e^0\right)\right]$$

$$= \lim_{r\to\infty} \left[-\frac{30}{e^{0.1r}} + \frac{10}{e^{0.3r}} - (-20)\right]$$

$= 0 + 0 + 20 = 20$

The total amount of the drug that is eliminated is approximately 20 milliliters.

Problems 15.7

1. $\int_3^\infty \frac{1}{x^3} dx = \lim_{r\to\infty} \int_3^r x^{-3} dx$

$$= \lim_{r\to\infty} \frac{x^{-2}}{-2}\Big|_3^r = -\frac{1}{2}\lim_{r\to\infty} \frac{1}{x^2}\Big|_3^r$$

$$= -\frac{1}{2}\lim_{r\to\infty}\left(\frac{1}{r^2} - \frac{1}{9}\right) = -\frac{1}{2}\left(0 - \frac{1}{9}\right) = \frac{1}{18}$$

2. $\int_1^\infty \frac{1}{(3x-1)^2} dx = \lim_{r\to\infty} \frac{1}{3}\int_1^r (3x-1)^{-2}[3\,dx]$

$$= \lim_{r\to\infty}\left[-\frac{1}{3(3x-1)}\right]\Bigg|_1^r$$

$$= \lim_{r\to\infty}\left[-\frac{1}{3(3r-1)} + \frac{1}{6}\right]$$

$$= 0 + \frac{1}{6}$$

$$= \frac{1}{6}$$

3. $\int_1^\infty \frac{1}{x} dx = \lim_{r\to\infty}\int_1^r \frac{1}{x} dx = \lim_{r\to\infty} \ln|x|\Big|_1^r$

$$= \lim_{r\to\infty}\left(\ln|r| - 0\right)$$

$$= \lim_{r\to\infty} \ln|r| = \infty \Rightarrow \text{ diverges}$$

4. $\int_2^\infty \frac{1}{\sqrt[3]{(x+2)^2}} dx = \lim_{r\to\infty}\int_2^r (x+2)^{-\frac{2}{3}} dx$

$$= \lim_{r\to\infty} \frac{(x+2)^{\frac{1}{3}}}{\frac{1}{3}}\Bigg|_2^r$$

$$= \lim_{r\to\infty} 3\left[\sqrt[3]{r+2} - \sqrt[3]{(4)}\right]$$

$= \infty \Rightarrow$ diverges

5. $\int_{37}^\infty e^{-x} dx = \lim_{r\to\infty} -\int_{37}^r e^{-x}[-dx] = \lim_{r\to\infty}(-e^{-x})\Big|_{37}^r$

$$= \lim_{r\to\infty}\left(-e^{-r} + e^{-37}\right) = \lim_{r\to\infty}\left(-\frac{1}{e^r} + \frac{1}{e^{37}}\right)$$

$$= 0 + \frac{1}{e^{37}} = \frac{1}{e^{37}} = e^{-37}$$

6. $\int_0^\infty \left(5+e^{-x}\right)dx = \lim_{r\to\infty}\int_0^r \left(5+e^{-x}\right)dx$

$= \lim_{r\to\infty}\left(5x-e^{-x}\right)\Big|_0^r = \lim_{r\to\infty}\left[\left(5r-e^{-r}\right)-(0-1)\right]$

$= \lim_{r\to\infty}\left(5r-\frac{1}{e^r}+1\right) = \infty \Rightarrow$ diverges

7. $\int_1^\infty \frac{1}{\sqrt{x}}dx = \lim_{r\to\infty}\int_1^r x^{-\frac{1}{2}}dx = \lim_{r\to\infty} 2x^{\frac{1}{2}}\Big|_1^r$

$= \lim_{r\to\infty}\left(2\sqrt{r}-2\right) = \infty \Rightarrow$ diverges

8. $\int_4^\infty \frac{x}{\sqrt{\left(x^2+9\right)^3}}dx = \lim_{r\to\infty}\frac{1}{2}\int_4^r \left(x^2+9\right)^{-\frac{3}{2}}[2x\ dx]$

$= \lim_{r\to\infty}\left[-\left(x^2+9\right)^{-\frac{1}{2}}\right]\Big|_4^r = \lim_{r\to\infty}\left[-\frac{1}{\sqrt{r^2+9}}+\frac{1}{5}\right]$

$= 0+\frac{1}{5} = \frac{1}{5}$

9. $\int_{-\infty}^{-3}\frac{1}{(x+1)^2}dx = \lim_{r\to-\infty}\int_r^{-3}(x+1)^{-2}dx$

$= \lim_{r\to-\infty} -\frac{1}{x+1}\Big|_r^{-3}$

$= \lim_{r\to-\infty}\left[\frac{1}{2}+\frac{1}{r+1}\right]$

$= \frac{1}{2}+0$

$= \frac{1}{2}$

10. $\int_1^\infty \frac{1}{\sqrt[3]{x-1}}dx = \lim_{r\to\infty}\int_1^r (x-1)^{-1/3}dx$

$= \lim_{r\to\infty}\frac{3}{2}(x-1)^{2/3}\Big|_1^r$

$= \lim_{r\to\infty}\frac{3}{2}[(r-1)^{2/3}-0]$

$= \infty \Rightarrow$ diverges

11. $\int_{-\infty}^\infty 2xe^{-x^2}dx = \int_{-\infty}^0 2xe^{-x^2}dx + \int_0^\infty 2xe^{-x^2}dx$

$\int_{-\infty}^0 2xe^{-x^2}dx = \lim_{r\to-\infty} -\int_r^0 e^{-x^2}[-2x\,dx]$

$= \lim_{r\to-\infty} -e^{-x^2}\Big|_r^0$

$= \lim_{r\to-\infty}\left[-1+\frac{1}{e^{r^2}}\right] = -1+0 = -1$

$\int_0^\infty 2xe^{-x^2}dx = \lim_{r\to\infty} -\int_0^r e^{-x^2}[-2x\,dx]$

$= \lim_{r\to\infty} -e^{x^2}\Big|_0^r$

$= \lim_{r\to\infty}\left[-\frac{1}{e^{r^2}}+1\right] = 0+1 = 1$

Thus $\int_{-\infty}^\infty 2xe^{-x^2}dx = -1+1 = 0.$

12. $\int_{-\infty}^\infty (5-3x)dx = \int_{-\infty}^0 (5-3x)dx + \int_0^\infty (5-3x)dx$

$\int_{-\infty}^0 (5-3x)dx = \lim_{r\to-\infty}\int_r^0 (5-3x)dx$

$= \lim_{r\to-\infty}\left(5x-\frac{3}{2}x^2\right)\Big|_r^0$

$= \lim_{r\to-\infty}\left[(0-0)-\left(5r-\frac{3}{2}r^2\right)\right]$

$= \lim_{r\to-\infty}\left(-5r+\frac{3}{2}r^2\right) = \infty$

Thus $\int_{-\infty}^\infty (5-3x)dx$ diverges

13. a. $\int_{800}^\infty \frac{k}{x^2}dx = 1$

$\lim_{r\to\infty} k\int_{800}^r x^{-2}dx = 1$

$\lim_{r\to\infty} -\frac{k}{x}\Big|_{800}^r = 1$

$\lim_{r\to\infty}\left(-\frac{k}{r}+\frac{k}{800}\right) = 1$

$0+\frac{k}{800} = 1$

$k = 800$

b. $\int_{1200}^{\infty} \frac{800}{x^2} dx = \lim_{r\to\infty} 800 \int_{1200}^{r} x^{-2} dx$

$= \lim_{r\to\infty} -\frac{800}{x}\Big|_{1200}^{r}$

$= \lim_{r\to\infty}\left(-\frac{800}{r} + \frac{800}{1200}\right) = 0 + \frac{2}{3} = \frac{2}{3}$

14. $\int_1^{\infty} ke^{-2x} dx = 1$

$\lim_{r\to\infty} -\frac{k}{2}\int_1^r e^{-2x}[-2\,dx] = 1$

$\lim_{r\to\infty} -\frac{ke^{-2x}}{2}\Big|_1^r = 1$

$\lim_{r\to\infty}\left(-\frac{k}{2e^{2r}} + \frac{k}{2e^2}\right) = 1$

$0 + \frac{k}{2e^2} = 1$

Thus $k = 2e^2$.

15. $\int_0^{\infty} 500{,}000e^{-0.02t} dt$

$= \lim_{r\to\infty} \frac{500{,}000}{-0.02}\int_0^r e^{-0.02t}[-0.02\ dt]$

$= \lim_{r\to\infty} -\frac{500{,}000}{0.02} e^{-0.02t}\Big|_0^r$

$= \lim_{r\to\infty} -\frac{500{,}000}{0.02}\left(\frac{1}{e^{0.02r}} - 1\right)$

$= -\frac{500{,}000}{0.02}(-1) = 25{,}000{,}000$

16. $\alpha = \int_{x_c}^{\infty} e^{-x} dx = \lim_{r\to\infty} -\int_{x_c}^{r} e^{-x}[-dx]$

$= \lim_{r\to\infty} -e^{-x}\Big|_{x_c}^{r}$

$= \lim_{r\to\infty}\left(-\frac{1}{e^r} + e^{-x_c}\right) = 0 + e^{-x_c} = e^{-x_c}$

$\beta = \int_{x_c}^{\infty} \frac{1}{8} e^{-\frac{x}{8}} dx = \lim_{r\to\infty} -\int_{x_c}^{r} e^{-\frac{x}{8}}\left[-\frac{1}{8}dx\right]$

$= \lim_{r\to\infty} -e^{-\frac{x}{8}}\Big|_{x_c}^{r}$

$= \lim_{r\to\infty}\left[-\frac{1}{e^{\frac{r}{8}}} + e^{-\left(\frac{1}{8}\right)x_c}\right]$

$= 0 + e^{-\left(\frac{1}{8}\right)x_c}$

$= e^{-\left(\frac{1}{8}\right)x_c}$

17. Area $= -\int_{-\infty}^{0} -e^{3x}\, dx = \lim_{r\to-\infty} \frac{1}{3}\int_r^0 e^{3x}[3dx]$

$= \lim_{r\to-\infty} \frac{1}{3}\cdot e^{3x}\Big|_r^0 = \lim_{r\to-\infty}\left[\frac{1}{3} - \frac{1}{3}e^r\right]$

$= \frac{1}{3} - 0 = \frac{1}{3}$ sq units

18. $V = \pi_0 \int_0^{\infty} e^{\theta t} e^{-\rho t} dt = \pi_0 \lim_{r\to\infty}\int_0^r e^{(\theta-\rho)t} dt$

$= \pi_0 \lim_{r\to\infty} \frac{1}{\theta-\rho}\int_0^r e^{(\theta-\rho)t}[(\theta-\rho)dt]$

$= \lim_{r\to\infty} \frac{\pi_0}{\theta-\rho} e^{(\theta-\rho)t}\Big|_0^r = \lim_{r\to\infty}\frac{\pi_0}{\theta-\rho}\left[e^{(\theta-\rho)r} - 1\right]$

$= \frac{\pi_0}{\theta-\rho}[0-1]$ (since $\theta-\rho<0$)

$= -\frac{\pi_0}{\theta-\rho}$

Thus $V = -\frac{\pi_0}{\theta-\rho} = \frac{\pi_0}{\rho-\theta}$.

19. $\int_0^{\infty}\frac{40{,}000}{(t+2)^2} dt = \lim_{r\to\infty}\int_0^r \frac{40{,}000}{(t+2)^2} dt$

$= \lim_{r\to\infty} -\frac{40{,}000}{t+2}\Big|_0^r$

$= \lim_{r\to\infty}\left[-\frac{40{,}000}{r+2} + \frac{40{,}000}{2}\right]$

$= 0 + \frac{40{,}000}{2} = 20{,}000$ increase

Chapter 15 Review Problems

1. Use Formula 42 with $u = x$ and $n = 2$. Then $du = dx$.

$$\int x^2 \ln x\,dx = \frac{x^3 \ln x}{3} - \frac{x^3}{3^2} + C$$
$$= \frac{1}{3}x^3\left(\ln x - \frac{1}{3}\right) + C$$

2. Use Formula 27 with $u = 2x$, $a^2 = 1$. Then $du = 2\,dx$.

$$\int \frac{1}{\sqrt{4x^2+1}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{(2x)^2+1}}(2\,dx)$$
$$= \frac{1}{2}\ln\left|2x + \sqrt{4x^2+1}\right| + C$$

3. Use Formula 23 with $u = 3x$, $a^2 = 16$. Then $du = 3\,dx$.

$$\int_0^2 \sqrt{9x^2+16}\,dx = \frac{1}{3}\int_0^2 \sqrt{(3x)^2+16}(3\,dx)$$
$$= \frac{1}{3}\left[\frac{1}{2}\left((3x)\sqrt{9x^2+16} + 16\ln\left|3x+\sqrt{9x^2+16}\right|\right)\right]_0^2$$
$$= \left(2\sqrt{13} + \frac{8}{3}\ln\left(6+2\sqrt{13}\right)\right) - \left(0 + \frac{8}{3}\ln 4\right)$$
$$= 2\sqrt{13} + \frac{8}{3}\ln\left(\frac{6+2\sqrt{13}}{4}\right)$$
$$= 2\sqrt{13} + \frac{8}{3}\ln\left(\frac{3+\sqrt{13}}{2}\right)$$

4. By long division, $\int \frac{16x}{3-4x}\,dx = \int\left(-4 + \frac{12}{3-4x}\right)dx = -4x - 3\ln|3-4x| + C$

Or, by Formula 3 with $u = x$, $a = 3$, and $b = -4$. Then $du = dx$.

$$\int \frac{16x}{3-4x}\,dx = 16\int \frac{x}{3-4x}\,dx = 16\left[\frac{x}{-4} - \frac{3}{16}\ln|3-4x|\right] + C = -4x - 3\ln|3-4x| + C$$

5. $\int \frac{15x-2}{(3x+1)(x-2)}\,dx = \int\left(\frac{15x}{(3x+1)(x-2)} - \frac{2}{(3x+1)(x-2)}\right)dx$

For $\int \frac{15x}{(3x+1)(x-2)}\,dx$, use Formula 12 with $u = x$, $a = 1$, $b = 3$, $c = -2$, and $k = 1$. Then $du = dx$.

$$\int \frac{15x}{(3x+1)(x-2)}\,dx = 15\int \frac{x}{(3x+1)(x-2)}\,dx = 15\left[\frac{1}{-7}\left(-2\ln|x-2| - \frac{1}{3}\ln|3x+1|\right)\right] + C$$

For $\int \frac{2}{(3x+1)(x-2)}\,dx$, use Formula 11 with

$u = x$, $a = 1$, $b = 3$, $c = -2$, and $k = 1$. Then $du = dx$.

$$\int \frac{2}{(3x+1)(x-2)}\,dx = 2\int \frac{dx}{(3x+1)(x-2)} = 2\left(\frac{1}{-7}\ln\left|\frac{3x+1}{x-2}\right|\right) + C$$

Thus, $\int \frac{15x-2}{(3x+1)(x-2)}dx$

$$= \frac{30}{7}\ln|x-2| + \frac{5}{7}\ln|3x+1| + \frac{2}{7}\ln\left|\frac{3x+1}{x-2}\right| + C$$
$$= \frac{30}{7}\ln|x-2| + \frac{5}{7}\ln|3x+1| + \frac{2}{7}\ln|3x+1| - \frac{2}{7}\ln|x-2| + C$$
$$= 4\ln|x-2| + \ln|3x+1| + C$$

6. The integral can be put in the form $\int \frac{1}{u}du$ with $u = \ln x$.

$$\int_{e^a}^{e^b} \frac{1}{x\ln x}dx = \int_{e^a}^{e^b} \frac{1}{\ln x}\left[\frac{1}{x}dx\right] = \ln|\ln x|\Big|_{e^a}^{e^b}$$
$$= \ln\left|\ln e^b\right| - \ln\left|\ln e^a\right| = \ln|b| - \ln|a|$$

7. Use Formula 9 with $u = x$, $a = 2$, and $b = 1$. Then $du = dx$.

$$\int \frac{dx}{x(x+2)^2} = \frac{1}{2(x+2)} + \frac{1}{4}\ln\left|\frac{x}{x+2}\right| + C$$

8. Use Formula 35 with $u = x$ and $a = 1$. Then $du = dx$.

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

9. Use Formula 21 with $u = 4x$ and $a^2 = 9$. Then $du = 4\ dx$.

$$\int \frac{dx}{x^2\sqrt{9-16x^2}} = 4\int \frac{(4\ dx)}{(4x)^2\sqrt{9-(4x)^2}} = 4\left[-\frac{\sqrt{9-16x^2}}{9(4x)}\right] + C$$
$$= -\frac{\sqrt{9-16x^2}}{9x} + C$$

10. Use Formula 42 with $u = x^2$ and $n = 1$. Then $du = 2x\,dx$.

$$\int x^3 \ln x^2 dx = \frac{1}{2}\int (x^2)\ln(x^2)[2x\,dx]$$
$$= \frac{1}{2}\left(\frac{(x^2)^2 \ln x^2}{2} - \frac{(x^2)^2}{2^2}\right) + C$$
$$= \frac{1}{4}x^4 \ln x^2 - \frac{1}{8}x^4 + C$$

11. Use Formula 35 with $u = x$ and $a = a$. Then $du = dx$.

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

12. Use Formula 15 with $u = x$, $a = 2$, and $b = 5$. Then $du = dx$.

$$\int \frac{x}{\sqrt{2+5x}}dx = \frac{2(5x-4)\sqrt{2+5x}}{75} + C$$

13. Use Formula 38 with $u = x$ and $a = 7$. Then $du = dx$.

$$\int 49xe^{7x}\,dx = 49\int xe^{7x}\,du$$
$$= 49\left[\frac{e^{7x}}{49}(7x-1)\right]+C = e^{7x}(7x-1)+C$$

14. Use Formula 45 with $u = x$, $a = 2$, $b = 3$, and $c = 4$. Then $du = dx$.

$$\int \frac{dx}{2+3e^{4x}} = \frac{1}{8}\left[4x-\ln\left(2+3e^{4x}\right)\right]+C$$

15. The integral has the form $\int \frac{1}{u}\,du$.

$$\int \frac{dx}{2x\ln x^2} = \frac{1}{4}\int \frac{1}{\ln x^2}\left[\frac{2}{x}dx\right] = \frac{1}{4}\ln\left|\ln x^2\right|+C$$

16. Use Formula 5 with $u = x$, $a = a$, and $b = 1$. Then $du = dx$.

$$\int \frac{dx}{x(x+a)} = \frac{1}{a}\ln\left|\frac{x}{x+a}\right|+C$$

17. Long division or Formula 3. For long division,

$$\int \frac{2x}{3+2x}dx = \int\left[1-\frac{3}{3+2x}\right]dx$$
$$= x-3\cdot\frac{1}{2}\int\frac{1}{3+2x}[2\,dx]$$
$$= x-\frac{3}{2}\ln|3+2x|+C.$$

For Formula 3, use $u = x$, $a = 3$, and $b = 2$. Then $du = dx$.

$$\int \frac{2x}{3+2x}dx = 2\int\frac{x}{3+2x}dx$$
$$= 2\left(\frac{x}{2}-\frac{3}{4}\ln|3+2x|\right)+C = x-\frac{3}{2}\ln|3+2x|+C$$

18. Use Formula 30 with $u = 2x$ and $a^2 = 9$. Then $du = 2\,dx$.

$$\int\frac{dx}{x^2\sqrt{4x^2-9}} = 2\int\frac{(2\,dx)}{(2x)^2\sqrt{(2x)^2-9}}$$
$$= 2\left(-\frac{-\sqrt{4x^2-9}}{9(2x)}\right)+C = \frac{\sqrt{4x^2-9}}{9}+C$$

19. Partial fractions

$$\frac{5x^2+2}{x^3+x} = \frac{5x^2+2}{x(x^2+1)} = \frac{A}{x}+\frac{Bx+C}{x^2+1}$$
$$5x^2+2 = A(x^2+1)+(Bx+C)x$$
$$= (A+B)x^2+Cx+A$$

Thus, $A + B = 5$, $C = 0$, $A = 2$. This gives $A = 2$, $B = 3$, $C = 0$.

$$\int\frac{5x^2+2}{x^3+x}dx = \int\left[\frac{2}{x}+\frac{3x}{x^2+1}\right]dx$$
$$= 2\ln|x|+\frac{3}{2}\ln\left(x^2+1\right)+C$$

20. Partial fractions

$$\frac{3x^3+5x^2+4x+3}{x^4+x^3+x^2} = \frac{3x^3+5x^2+4x+3}{x^2(x^2+x+1)}$$
$$= \frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+x+1}$$

$$3x^3+5x^2+4x+3$$
$$= Ax(x^2+x+1)+B(x^2+x+1)+(Cx+D)x^2$$
$$= (A+C)x^3+(A+B+D)x^2+(A+B)x+B$$

Thus, $A + C = 3$, $A + B + D = 5$, $A + B = 4$, $B = 3$.

This gives $A = 1$, $B = 3$, $C = 2$, $D = 1$.

$$\int\frac{3x^3+5x^2+4x+3}{x^4+x^3+x^2}dx$$
$$= \int\left[\frac{1}{x}+\frac{3}{x^2}+\frac{2x+1}{x^2+x+1}\right]dx$$
$$= \ln|x|-\frac{3}{x}+\ln\left(x^2+x+1\right)+C$$

21. Integration by parts

$u = \ln(x + 1)$

$dv = (x+1)^{1/2}dx$

Then $du = \frac{1}{x+1}dx$ and $v = \frac{2}{3}(x+1)^{3/2}$.

$$\int\ln(x+1)\sqrt{x+1}\,dx$$
$$= \frac{2}{3}(x+1)^{3/2}\ln(x+1)-\int\frac{2}{3}(x+1)^{3/2}\frac{1}{x+1}dx$$
$$= \frac{2}{3}(x+1)^{3/2}\ln(x+1)-\frac{2}{3}\int(x+1)^{1/2}dx$$
$$= \frac{2}{3}(x+1)^{3/2}\ln(x+1)-\frac{2}{3}\cdot\frac{(x+1)^{3/2}}{\frac{3}{2}}+C$$
$$= \frac{2}{3}(x+1)^{3/2}\left[\ln(x+1)-\frac{2}{3}\right]+C$$

22. Integration by parts

$u = x^2$

$dv = e^x dx$

Then $du = 2x\,dx$ and $v = e^x$.

$\int x^2 e^x dx = x^2 e^x - \int 2xe^x dx$

For $\int 2xe^x dx$, use integration by parts again.

$u = 2x$

$dv = e^x dx$

Then $du = 2\,dx$ and $v = e^x$.

$\int 2xe^x dx = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x + C$

$\int x^2 e^x dx = x^2 e^x - (2xe^x - 2e^x) + C$

$= e^x (x^2 - 2x + 2) + C$

23. $\bar{f} = \frac{1}{4-2}\int_2^4 \left(3x^2 + 2x\right) dx = \frac{1}{2}\left(x^3 + x^2\right)\Big|_2^4$

$= \frac{1}{2}[(64+16) - (8+4)] = 34$

24. $\bar{f} = \frac{1}{1-0}\int_0^1 t^2 e^t\,dt$

For $\int t^2 e^t\,dt$, use Formula 39 with $u = t$, $n = 2$, and $a = 1$. Then $du = dt$.

$\int t^2 e^t\,dt = t^2 e^t - 2\int te^t\,dt$

For $\int te^t\,dt$, use Formula 38 with $u = t$ and $a =$ 1. Then $du = dt$.

$\int t^2 e^t\,dt = t^2 e^t - 2[e^t (t-1)] + C$

$= e^t (t^2 - 2t + 2) + C$

Thus,

$\bar{f} = \int_0^1 t^2 e^t\,dt = e^t (t^2 - 2t + 2)\Big|_0^1 = e(1) - 1(2)$

$= e - 2.$

25. $y' = 3x^2 y + 2xy$, $y > 0$

$\frac{dy}{y} = \left(3x^2 + 2x\right) dx$

$\int \frac{dy}{y} = \int \left(3x^2 + 2x\right) dx$

$\ln y = x^3 + x^2 + C_1$, from which $y = e^{x^3 + x^2 + C_1}$,

$y = Ce^{x^3 + x^2}$, where $C > 0$.

26. $y' - f'(x)e^{f(x) - y} = 0,\ y(0) = f(0)$

$\frac{dy}{dx} - f'(x)e^{f(x)}e^{-y} = 0$

$dy = f'(x)e^{f(x)}e^{-y}dx$

$e^y dy = f'(x)e^{f(x)}dx$

$\int e^y dy = \int e^{f(x)}[f'(x)\,dx]$

$e^y = e^{f(x)} + C$

$y(0) = f(0)$ implies $e^{f(0)} = e^{f(0)} + C,\ C = 0.$

Thus $e^y = e^{f(x)}$ or $y = f(x)$.

27. $\int_1^\infty \frac{1}{x^{2.5}} dx = \lim_{r\to\infty} \int_1^r x^{-2.5} dx$

$= \lim_{r\to\infty} \frac{x^{-1.5}}{-1.5}\Big|_1^r$

$= \lim_{r\to\infty} -\frac{2}{3x^{1.5}}\Big|_1^r$

$= \lim_{r\to\infty} \left(-\frac{2}{3r^{1.5}} + \frac{2}{3}\right)$

$= 0 + \frac{2}{3}$

$= \frac{2}{3}$

28. $\int_{-\infty}^0 e^{2x}\,dx = \lim_{r\to-\infty} \int_r^0 e^{2x}\,dx = \lim_{r\to-\infty} \frac{e^{2x}}{2}\Big|_r^0$

$= \lim_{r\to-\infty} \left[\frac{1}{2} - \frac{1}{2}e^{2r}\right] = \frac{1}{2} - 0 = \frac{1}{2}$

29. $\int_1^\infty \frac{1}{2x} dx = \lim_{r\to\infty} \int_1^r \frac{1}{2x} dx = \lim_{r\to\infty} \frac{1}{2}\ln|x|\Big|_1^r$

$= \lim_{r\to\infty} \left[\frac{1}{2}\ln|r| - 0\right] = \infty \Rightarrow$ diverges

30. $\int_{-\infty}^{\infty} xe^{1-x^2}\,dx = \int_{-\infty}^{0} xe^{1-x^2}\,dx + \int_{0}^{\infty} xe^{1-x^2}\,dx$

$\int_{-\infty}^{0} xe^{1-x^2}\,dx = \lim_{r\to-\infty} -\frac{1}{2}\int_{r}^{0} e^{1-x^2}[-2x\,dx]$

$= \lim_{r\to-\infty} -\frac{1}{2}e^{1-x^2}\Big|_{r}^{0}$

$= \lim_{r\to-\infty}\left[-\frac{1}{2}e+\frac{1}{2}e^{1-r^2}\right] = -\frac{1}{2}e - 0 = -\frac{1}{2}e$

$\int_{0}^{\infty} xe^{1-x^2}\,dx = \lim_{r\to\infty} -\frac{1}{2}\int_{0}^{r} e^{1-x^2}[-2x\,dx]$

$= \lim_{r\to\infty} -\frac{1}{2}e^{1-x^2}\Big|_{0}^{r}$

$= \lim_{r\to\infty}\left[-\frac{1}{2}e^{1-r^2}+\frac{1}{2}e\right] = 0+\frac{1}{2}e = \frac{1}{2}e$

Thus $\int_{-\infty}^{\infty} xe^{1-x^2} = -\frac{1}{2}e+\frac{1}{2}e = 0$

31. $N = N_0e^{kt}$

Since $N = 500{,}000$ when $t = 0$ (1980),

$N_0 = 500{,}000$. Thus $N = 500{,}000e^{kt}$.

Since $N = 1{,}000{,}000$ when $t = 20$, then

$1{,}000{,}000 = 500{,}000e^{20k}$

$2 = e^{20k}$

$\ln 2 = 20k$, or $k = \frac{\ln 2}{20}$.

Thus $N = 500{,}000e^{t\frac{\ln 2}{20}}$

$= 500{,}000(e^{\ln 2})^{t/20}$

$= 500{,}000(2)^{t/20}$

For the year 2020, we have $t = 40$ and

$N = 500{,}000(2)^{40/20}$

$= 500{,}000(2)^2$

$= 2{,}000{,}000.$

32. $N = N_0e^{kt}$

When $t = 0$, then $N = 40{,}000$. Thus $N_0 = 40{,}000$ and $N = 40{,}000e^{kt}$. When $t = 10$, then $N = 80{,}000$, so

$80{,}000 = 40{,}000e^{10k}$

$2 = e^{10k}$

$10k = \ln 2$, or $k = \frac{\ln 2}{10}$. Thus $N = 40{,}000e^{\frac{t\ln 2}{10}}$.

33. $N = N_0e^{-\lambda t}$, where N_0 is the original amount present. When $t = 100$, then $N = 0.95N_0$, so we have

$0.95N_0 = N_0e^{-100\lambda}$

$0.95 = e^{-100\lambda}$

$-100\lambda = \ln 0.95$

$\lambda = -\frac{\ln 0.95}{100} \approx 0.0005$ (decay constant). After 200 years, $N = N_0e^{-200\lambda}$. Thus

$\frac{N}{N_0} = e^{-200\lambda} = e^{-200\left[-\frac{\ln 0.95}{100}\right]} = e^{2\ln 0.95}$

$\approx 0.90 = 90\%$

34. $\frac{dq}{dt} = -kq$

$\frac{dq}{q} = -k\,dt$

$\int \frac{dq}{q} = \int -k\,dt$

$\ln q = -kt + C$

When $t = 0$, $q = q_0$, so $\ln q_0 = 0 + C = C$. Thus

$\ln q = -kt + \ln q_0$

$q = e^{-kt}e^{\ln q_0} = q_0e^{-kt}$

When $t = \frac{7}{k}$, $\frac{q}{q_0} = e^{-7} \approx 0.09\%$.

35. $N = \frac{450}{1+be^{-ct}}$

If $t = 0$, then $N = 2$. Thus $2 = \frac{450}{1+b}$,

$1+b = \frac{450}{2} = 225$, $b = 224$, so $N = \frac{450}{1+224e^{-ct}}$.

If $t = 6$, then

$N = 300 \Rightarrow 300 = \dfrac{450}{1+224e^{-6c}}$

$1+224e^{-6c} = \dfrac{450}{300} = \dfrac{3}{2}$

$224e^{-6c} = \dfrac{3}{2} - 1 = \dfrac{1}{2}$

$e^{-6c} = \dfrac{1}{448}$

$e^{6c} = 448$

$6c = \ln 448$

$c = \dfrac{\ln 448}{6} \approx 1.02$

Thus $N \approx \dfrac{450}{1+224e^{-1.02t}}$.

36. $N = \dfrac{20{,}000}{1+be^{-ct}}$

When $t = 0$ (last year), then $N = 10{,}000$. Thus

$10{,}000 = \dfrac{20{,}000}{1+b}$

$1+b = \dfrac{20{,}000}{10{,}000} = 2$

$b = 1$

So $N = \dfrac{20{,}000}{1+e^{-ct}}$. When $t = 1$ then $N = 11{,}000$.

Thus

$11{,}000 = \dfrac{20{,}000}{1+e^{-c}}$

$1+e^{-c} = \dfrac{20{,}000}{11{,}000} = \dfrac{20}{11}$

$e^{-c} = \dfrac{9}{11}$

Hence $N = \dfrac{20{,}000}{1+\left(\frac{9}{11}\right)^t}$. When $t = 2$, then

$N = \dfrac{20{,}000}{1+\left(\frac{9}{11}\right)^2} \approx 11{,}980$.

37. $\dfrac{dT}{dt} = k(T-25)$

$\dfrac{dT}{T-25} = k\ dt$

$\displaystyle\int \frac{dT}{T-25} = \int k\ dt$

$\ln(T-25) = kt + C$

If $t = 0$, then $T = 35$. Thus $\ln 10 = C$, so $\ln(T-25) = kt + \ln 10$, or $\ln\left(\dfrac{T-25}{10}\right) = kt$. If $t = 1$, then $T = 34$ and $\ln\left(\dfrac{9}{10}\right) = k$. Thus

$\ln\left(\dfrac{T-25}{10}\right) = (\ln 0.9)t$. If

$T = 37$,

$\ln\dfrac{12}{10} = (\ln 0.9)t$

$\ln 1.2 = (\ln 0.9)t$,

$t = \dfrac{\ln 1.2}{\ln 0.9} \approx -1.73$

Note that 1.73 hr corresponds approximately to 1 hr 44 min. Thus

6:00 P.M. – 1 hr 44 min = 4:16 P.M.

38. Use Formula 38 with $u = t$, and $a = -0.06$, so $du = dt$.

$\displaystyle\int_0^{12} 10te^{-0.06t}\,dt = 10\left[\frac{e^{-0.06t}}{0.0036}(-0.06t-1)\right]\Bigg|_0^{12}$

$= \dfrac{10}{0.0036}[e^{-0.72}(-0.72-1)-(-1)]$

$\approx \$452$

39. $\displaystyle\int_0^{\infty} f(x)\,dx$

$\displaystyle= \lim_{r\to\infty}\int_0^r (0.007e^{-0.01x} + 0.00005e^{-0.0002x})dx$

$\displaystyle= \lim_{r\to\infty}(-0.7e^{-0.01x} - 0.25e^{-0.0002x})\Big|_0^r$

$\displaystyle= \lim_{r\to\infty}\left[-\frac{0.7}{e^{0.01r}} - \frac{0.25}{e^{-0.0002r}} - (-0.7-0.25)\right]$

$= 0 - 0 + 0.7 + 0.25$

$= 0.95$

40. $\displaystyle\int_{-\infty}^{t_1} A_0e^{kt}\,dt = \lim_{r\to-\infty}\int_r^{t_1} A_0e^{kt}\,dt$

$\displaystyle= \lim_{r\to-\infty} A_0\cdot\frac{1}{k}\int_r^{t_1} e^{kt}(k\ dt)$

$\displaystyle= \lim_{r\to-\infty}\frac{A_0e^{kt}}{k}\Bigg|_r^{t_1} = \lim_{r\to-\infty}\frac{A_0}{k}\left(e^{kt_1} - e^{kr}\right)$

$= \dfrac{A_0}{k}e^{kt_1}$ since $e^{kr} \to 0$ as $r \to -\infty$ for $k > 0$.

$$\int_{t_1}^{t_2} A_0 e^{kt}\,dt = \left.\frac{A_0 e^{kt}}{k}\right|_{t_1}^{t_2} = \frac{A_0}{k}\left(e^{kt_2} - e^{kt_1}\right)$$

$$= \frac{A_0}{k} e^{kt_1}\left(e^{kt_2} e^{-kt_1} - 1\right)$$

$$= \frac{A_0}{k} e^{kt_1}\left[e^{k(t_2-t_1)} - 1\right]. \qquad (1)$$

If $A_0 e^{kt_2} = 2A_0 e^{kt_1}$, then $e^{kt_2} = 2e^{kt_1}$,

$2 = \frac{e^{kt_2}}{e^{kt_1}} = e^{k(t_2-t_1)}$. Substituting into (1) gives

$\frac{A_0}{k} e^{kt_1}[2-1] = \frac{A_0}{k} e^{kt_1}$.

41. a. Total revenue $= r(12) - r(0) = \int_0^{12} \frac{dr}{dq}\,dq$.

$f(q) = \frac{dr}{dq}$

$n = 4$, $a = 0$, $b = 12$

$h = \frac{b-a}{n} = \frac{12-0}{4} = 3$

Trapezoidal

$f(0) =$	25
$2f(3) =$	44
$2f(6) =$	36
$2f(9) =$	26
$f(12) =$	7
	138

$TR \approx \frac{3}{2}(138) = 207$

Simpson's

$f(0) =$	25
$4f(3) =$	88
$2f(6) =$	36
$4f(9) =$	52
$f(12) =$	7
	208

$TR \approx \frac{3}{3}(208) = 208$

b. Total variable cost $c(12) - c(0) = \int_0^{12} \frac{dc}{dq}\,dq$

$f(q) = \frac{dc}{dq}$

$a = 0$, $b = 12$

Using as few data values as possible, we choose $n = 1$ for Trapezoidal and $n = 2$ for Simpson's (n must be even).

Trapezoidal ($n = 1$)

$h = \frac{b-a}{n} = \frac{12-0}{1} = 12$

$f(0) =$	15
$f(12) =$	7
	22

$VC \approx \frac{12}{2}(22) = 132$

Simpson's ($n = 2$)

$h = \frac{b-a}{n} = \frac{12-0}{2} = 6$

$f(0) =$	15
$4f(6) =$	48
$f(12) =$	7
	70

$VC \approx \frac{6}{3}(70) = 140$

To each of our results we must add on the fixed cost of 25 to obtain total cost. Thus for trapezoidal we get $TC \approx 132 + 25 = 157$, and for Simpson's we have $TC \approx 140 + 25 = 165$.

c. We use the relation $P(12) = \int_0^{12}\left[\frac{dr}{dq} - \frac{dc}{dq}\right]dq - 25$. First we determine variable cost for each rule with $n = 4$ and $h = \frac{b-a}{n} = \frac{12-0}{4} = 3$.

Trapezoidal

$f(0) =$	15
$2f(3) =$	28
$2f(6) =$	24
$2f(9) =$	20
$f(12) =$	7
	94

$VC \approx \frac{3}{2}(94) = 141$

Simpson's

$f(0) =$	15
$4f(3) =$	56
$2f(6) =$	24
$4f(12) =$	40
$f(12) =$	7
	142

$VC \approx \frac{3}{3}(142) = 142$

Using these results and those of part (a), we have:

Trapezoidal
$P(12) \approx 207 - 141 - 25 = 41$
Simpson's
$P(12) \approx 208 - 142 - 25 = 41$

Explore and Extend—Chapter 15

1. $C = 2000,\ w_0 = 200$

$$w_{\text{eq}} = \frac{C}{17.5} = \frac{2000}{17.5} \approx 114$$

$$w(t) = \frac{C}{17.5} + \left(w_0 - \frac{C}{17.5}\right)e^{-0.005t}$$

$$= \frac{2000}{17.5} + \left(200 - \frac{2000}{17.5}\right)e^{-0.005t}$$

Letting $w(t) = 175$ and solving for t gives

$$175 = \frac{2000}{17.5} + \left(200 - \frac{2000}{17.5}\right)e^{-0.005t}$$

$$175 - \frac{2000}{17.5} = \left(200 - \frac{2000}{17.5}\right)e^{-0.005t}$$

$$\frac{175 - \frac{2000}{17.5}}{200 - \frac{2000}{17.5}} = e^{-0.005t}$$

$$-0.005t = \ln\left[\frac{175 - \frac{2000}{17.5}}{200 - \frac{2000}{17.5}}\right]$$

$$t = \frac{\ln\left[\frac{175 - \frac{2000}{17.5}}{200 - \frac{2000}{17.5}}\right]}{-0.005} \approx 69$$

Thus $w_{\text{eq}} = 114$ and $t = 69$ days.

2. $$\frac{dw}{dt} = \frac{1}{3500}(C - 17.5w)$$

$$\frac{dw}{C - 17.5w} = \frac{1}{3500}dt$$

$$\int \frac{dw}{C - 17.5w} = \int \frac{1}{3500}dt$$

$$-\frac{1}{17.5}\ln|C - 17.5w| = \frac{1}{3500}t + C_1$$

$$\ln|C - 17.5w| = -\frac{17.5}{3500}t - 17.5C_1 = -0.005t + C_2$$

$$|C - 17.5w| = e^{-0.005t + C_2}$$

$$= e^{C_2}e^{-0.005t} = C_3e^{-0.005t}$$

Thus $C - 17.5w = C_4e^{-0.005t}$, where C_4 is a constant and $C_4 = \pm C_3$. When $t = 0$, then $w = w_0$, so $C - 17.5w_0 = C_4$. Thus

$$C - 17.5w = (C - 17.5w_0)e^{-0.005t}$$

$$-17.5w = -C + (C - 17.5w_0)e^{-0.005t}$$

$$w = \frac{C}{17.5} + \left(-\frac{C}{17.5} + \frac{17.5}{17.5}w_0\right)e^{-0.005t}$$

$$w = \frac{C}{17.5} + \left(w_0 - \frac{C}{17.5}\right)e^{-0.005t}$$

which is Equation 2.

3. $$w(t) = \frac{C}{17.5} + \left(w_0 - \frac{C}{17.5}\right)e^{-0.005t}$$

Since $\frac{C}{17.5} = w_{\text{eq}}$, we have $w(t) = w_{\text{eq}} + (w_0 - w_{\text{eq}})e^{-0.005t}$. Simplifying the equation $w(t+d) = w(t) - \frac{1}{2}\left[w(t) - w_{\text{eq}}\right]$ gives $w(t+d) = \frac{1}{2}\left[w(t) + w_{\text{eq}}\right]$. Thus

$$w_{\text{eq}} + (w_0 - w_{\text{eq}})e^{-0.005(t+d)}$$

$$= \frac{1}{2}\left[w_{\text{eq}} + (w_0 - w_{\text{eq}})e^{-0.005t} + w_{\text{eq}}\right], \text{ or}$$

$$w_{\text{eq}} + (w_0 - w_{\text{eq}})e^{-0.005(t+d)}$$

$$= w_{\text{eq}} + \frac{1}{2}(w_0 - w_{\text{eq}})e^{-0.005t}$$

Solving for d gives

$$e^{-0.005(t+d)} = \frac{1}{2}e^{-0.005t}$$

$$e^{-0.005t}e^{-0.005d} = \frac{1}{2}e^{-0.005t}$$

$$e^{-0.005d} = \frac{1}{2}$$

$$-0.005d = \ln\frac{1}{2} = -\ln 2$$

$$d = \frac{\ln 2}{0.005}$$

as was to be shown.

4. BMI $= \frac{w}{h^2}$, so $w = \text{BMI} \cdot h^2$ with w in kilograms and h in meters. 5 feet, 8 inches equals 68 inches, or
1.7272 meters. The upper BMI limit then corresponds to a weight of
$24.9(1.7272)^2 \approx 74.28$ kilograms, or about
163 pounds. So the woman would need to lose

27 pounds. On a 2200 calorie-per-day diet,
$w_{\text{eq}} = \frac{2200}{17.5} \approx 125.71$ lb and the weight function is
$w(t) = 125.71 + (190 - 125.71)e^{-0.005t}$
$= 125.71 + 64.29e^{-0.005t}$.
The solution of the equation
$163 = 125.71 + 64.29e^{-0.005t}$ is $t \approx 109$. It would take about 109 days.

5. Answers may vary.