Chapter 14

Problems 14.1

1. $y = ax + b$

$$dy = \frac{d}{dx}(ax+b)dx = a\ dx$$

2. $dy = y'dx = 0\ dx = 0$

3. $$d[f(x)] = f'(x)dx = \frac{1}{2}(x^4-9)^{-\frac{1}{2}}(4x^3)dx = \frac{2x^3}{\sqrt{x^4-9}}dx$$

4. $d[f(x)] = f'(x)dx$

$$= 3(8x-5)\left(4x^2-5x+2\right)^2 dx$$

5. $u = x^{-2}$

$$du = \frac{d}{dx}\left(x^{-2}\right)dx = -2x^{-3}dx = -\frac{2}{x^3}dx$$

6. $u = \sqrt{x}$

$$du = \frac{d}{dx}(x^{1/2})dx = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$$

7. $$dp = \frac{d}{dx}\left[\ln\left(x^2+7\right)\right]dx = \frac{1}{x^2+7}(2x)dx$$

$$= \frac{2x}{x^2+7}dx$$

8. $$dp = \frac{d}{dx}\left(e^{x^3+2x-5}\right)dx = (3x^2+2)e^{x^3+2x-5}dx$$

9. $dy = y'dx$

$$= \left[(9x+3)e^{2x^2+3}(4x) + e^{2x^2+3}(9)\right]dx$$

$$= 3e^{2x^2+3}[(3x+1)(4x)+3]dx$$

$$= 3e^{2x^2+3}\left(12x^2+4x+3\right)dx$$

10. $$y = \ln\sqrt{x^2+12} = \frac{1}{2}\ln(x^2+12)$$

$$dy = \frac{1}{2}\cdot\frac{1}{x^2+12}(2x)dx = \frac{x}{x^2+12}dx$$

11. $y = ax + b$; $dy = adx$

$\Delta y = adx = dy$

12. $$\Delta y = \left[5(-1.02)^2 - 5(-1)^2\right] = 0.202$$

$dy = 10x\ dx = 10(-1)(-0.02) = 0.2$

13. Δy

$$= [2(-1.9)^2+5(-1.9)-7] - [2(-2)^2+5(-2)-7]$$

$$= -0.28$$

$dy = (4x + 5)dx = [4(-2) + 5](0.1) = -0.3$

14. $$\Delta y = [3(-1.03)+2]^2 - [3(-1)+2]^2 = 0.1881$$

$dy = 6(3x + 2)\ dx = 6[3(-1) + 2](-0.03) = 0.18$

15. $$\Delta y = \sqrt{32-(3.95)^2} - \sqrt{32-(4^2)} \approx 0.049$$

$$dy = \frac{-x}{\sqrt{32-x^2}}dx = \frac{-4}{\sqrt{16}}(-0.05) = 0.050$$

16. $\Delta y = \ln 1.01 - \ln 1 \approx 0.00995$

$$dy = \frac{1}{x}dx = \frac{1}{1}(0.01) = 0.01$$

17. a. $$f(x) = \frac{x+5}{x+1}$$

$$f'(x) = \frac{(x+1)(1)-(x+5)(1)}{(x+1)^2} = \frac{-4}{(x+1)^2}$$

$$f'(1) = \frac{-4}{4} = -1$$

b. We use $f(x + dx) \approx f(x) + dy$ with $x = 1$, $dx = 0.1$.

$$f(1.1) = f(1+0.1) \approx f(1) + f'(1)dx$$

$$= \frac{6}{2} + (-1)(0.1) = 2.9$$

18. a. $y = f(x) = x^{3x}$

Using logarithmic differentiation,

$\ln y = 3x \ln x$,

$$\frac{1}{y}\cdot\frac{dy}{dx} = 3x\left(\frac{1}{x}\right) + (\ln x)(3) = 3(1+\ln x)$$

$$\frac{dy}{dx} = y[3(1+\ln x)] = 3x^{3x}(1+\ln x)$$

$$f'(1) = 3(1)(1+0) = 3$$

b. We use $f(x+dx) \approx f(x)+dy$ with $x = 1$, $dx = -0.02$

$$f(0.98) = f(1-0.02) \approx f(1)+f'(1)dx$$
$$= 1^3 + (3)(-0.02) = 0.94$$

19. Let $y = f(x) = \sqrt{x}$

$$f(x+dx) \approx f(x)+dy = \sqrt{x}+\frac{1}{2\sqrt{x}}dx$$

If $x = 289$ and $dx = -1$, then

$$\sqrt{288} = f(289-1)$$
$$\approx \sqrt{289}+\frac{1}{2\sqrt{289}}(-1)$$
$$= \frac{577}{34}$$
$$\approx 16.97$$

20. Let $y = f(x) = \sqrt{x}$

$$f(x+dx) \approx f(x)+dy = \sqrt{x}+\frac{1}{2\sqrt{x}}dx$$

If $x = 121$ and $dx = 1$, then

$$\sqrt{122} = f(121+1) \approx \sqrt{121}+\frac{1}{2\sqrt{121}}(1)$$
$$= 11\frac{1}{22}.$$

21. Let $y = f(x) = \sqrt[3]{x}$

$$f(x+dx) \approx f(x)+dy = \sqrt[3]{x}+\frac{1}{3x^{\frac{2}{3}}}dx$$

If $x = 8$ and $dx = 1$, then

$$\sqrt[3]{9} = f(8+1) \approx \sqrt[3]{8}+\frac{1}{3\left(\sqrt[3]{8}\right)^2}(1)$$
$$= 2+\frac{1}{3\cdot 2^2} = 2+\frac{1}{12} = \frac{25}{12}$$

22. Let $y = f(x) = \sqrt[4]{x}.$

$$f(x+dx) = f(x)+dy = \sqrt[4]{x}+\frac{1}{4x^{\frac{3}{4}}}dx$$

If $x = 16$ and $dx = 0.3$, then

$$\sqrt[4]{16.3} = f(16+0.3) \approx \sqrt[4]{16}+\frac{1}{4\left(\sqrt[4]{16}\right)^3}(0.3)$$
$$= 2+\frac{0.3}{2^3} = 2\frac{3}{320}$$

23. Let $y = f(x) = \ln x$

$$f(x+dx) \approx f(x)+dy = \ln(x)+\frac{1}{x}dx$$

If $x = 1$ and $dx = -0.03$, then

$$\ln(0.97) = f(1+(-0.03))$$
$$\approx \ln(1)+\frac{1}{1}(-0.03) = -0.03$$

24. Let $y = f(x) = \ln x$

$$f(x+dx) \approx f(x)+dy = \ln(x)+\frac{1}{x}dx$$

If $x = 1$ and $dx = 0.01$, then

$$\ln 1.01 = f(1+0.01) \approx \ln(1)+\frac{1}{1}(0.01) = 0.01$$

25. Let $y = f(x) = e^x$

$$f(x+dx) \approx f(x)+dy = e^x + e^x dx$$

If $x = 0$ and $dx = 0.001$, then

$$e^{0.001} = f(0+0.001) \approx e^0 + e^0(0.001) = 1.001$$

26. Let $y = f(x) = e^x$

$$f(x+dx) \approx f(x)+dy = e^x + e^x dx$$

If $x = 0$ and $dx = -0.002$, then

$$e^{-0.002} = f(0+(-0.002))$$
$$\approx e^0 + e^0(-0.002) = 0.998$$

27. $\frac{dy}{dx} = 2$, so $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{2}$

28. $\frac{dy}{dx} = 10x+3$, so $\frac{dx}{dy} = \frac{1}{10x+3}$

29. $\frac{dq}{dp} = 6p\left(p^2+5\right)^2$, so $\frac{dp}{dq} = \frac{1}{6p\left(p^2+5\right)^2}$

30. $\frac{dq}{dp} = \frac{1}{2\sqrt{p+5}}$, so $\frac{dp}{dq} = 2\sqrt{p+5}$

31. $q = p^{-2}$, $\frac{dq}{dp} = -2p^{-3} = \frac{-2}{p^3}$, so $\frac{dp}{dq} = -\frac{p^3}{2}$

32. $\frac{dq}{dp} = -2e^{4-2p}$, so $\frac{dp}{dq} = -\frac{1}{2e^{4-2p}} = -\frac{1}{2}e^{2p-4}$

33. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{14x-6}$

If $x = 3$, $\frac{dx}{dy} = \frac{1}{36}$

34. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{\frac{2}{x}} = \frac{x}{2}$

If $x = 3$, $\frac{dx}{dy} = \frac{3}{2}$.

35. $p = \frac{500}{q+2}$

$\frac{dp}{dq} = \frac{-500}{(q+2)^2}$

$\frac{dq}{dp} = -\frac{(q+2)^2}{500}$

$\left.\frac{dq}{dp}\right|_{q=18} = \left.-\frac{(q+2)^2}{500}\right|_{q=18} = -\frac{4}{5}$

36. $p = 60 - \sqrt{2q}$

$\frac{dp}{dq} = -\frac{1}{\sqrt{2q}}$

$\frac{dq}{dp} = -\sqrt{2q}$

$\left.\frac{dq}{dp}\right|_{q=50} = \left.-\sqrt{2q}\right|_{q=50} = -10$

37. $P = 397q - 2.3q^2 - 400$, q changes from 90 to 91.

$\Delta P \approx dP = P'dq = (397 - 4.6q)dq$

Choosing $q = 90$ and $dq = 1$,

$\Delta P \approx [397 - 4.6(90)](1) = -17$.

True change is

$P(91) - P(90) = 16{,}680.7 - 16{,}700 = -19.3$.

38. $r = 250q + 45q^2 - q^3$, q increases from 40 to 41.

$\Delta r \approx dr = r'dq = \left(250 + 90q - 3q^2\right)dq$

Choosing $q = 40$ and $dq = 1$,

$\Delta r \approx (-950)(1) = -950$

True change is

$r(41) - r(40) = 16{,}974 - 18{,}000 = -1026$

39. $p = \frac{10}{\sqrt{q}}$. We approximate p when $q = 24$.

$p(q+dq) \approx p + dp = \frac{10}{\sqrt{q}} - \frac{5}{\sqrt{q^3}}dq$

If $q = 25$ and $dq = -1$, then

$p(24) = p(25 + (-1)) \approx \frac{10}{\sqrt{25}} - \frac{5}{\sqrt{(25)^3}}(-1)$

$= 2 + \frac{1}{25} = \frac{51}{25} = 2.04$

40. $p = \frac{200}{\sqrt{q+8}}$

We approximate p when $q = 40$.

$p(q+dq) \approx p + dp = \frac{200}{\sqrt{q+8}} - \frac{100}{(q+8)^{\frac{3}{2}}}dq$

If $q = 41$ and $dq = 1$, then

$p(40) = p(41-1) \approx \frac{200}{\sqrt{49}} - \frac{100}{(49)^{\frac{3}{2}}}(1)$

$= \frac{200}{7} - \frac{100}{343} = \frac{9700}{343} \approx 28.28$

41. $c = f(q) = \frac{q^2}{2} + 5q + 300$

If $q = 10$ and $dq = 2$,

$\frac{\Delta c}{c} \approx \frac{dc}{c} = \frac{(q+5)dq}{\frac{q^2}{2} + 5q + 300}$

$= \frac{15(2)}{50 + 50 + 300}$

$= \frac{3}{40}$

$= 0.075 \approx 0.1$

42. $S = 20\sqrt{I}$, I decreases from 45 to $44\frac{1}{2}$.

$\Delta S \approx dS = S'dI = \frac{10}{\sqrt{I}}dI$

Choosing $I = 45$ and $dI = -\frac{1}{2}$, then

$\Delta S \approx \frac{10}{\sqrt{45}}\left(-\frac{1}{2}\right) \approx -0.745$.

43. $V = \frac{4}{3}\pi r^3$

$\Delta V \approx dV = V' dr = 4\pi r^2 dr$

$dr = \left(6.6\times10^{-4}\right) - \left(6.5\times10^{-4}\right)$

$= 0.1\times10^{-4} = 10^{-5}$

$\Delta V \approx 4\pi\left(6.5\times10^{-4}\right)^2\left(10^{-5}\right) = \left(1.69\times10^{-11}\right)\pi \text{ cm}^3$.

44. $(P + a)(v + b) = k$

$P = \frac{k}{v+b} - a$

$dP = -k(v+b)^{-2}\,dv$

45. a. We substitute $q = 40$ and $p = 20$

$2 + \frac{40^2}{200} = \frac{4000}{20^2}$

$2 + 8 = 10$

$10 = 10$

b. We differentiate implicitly with respect to p.

$0 + \frac{1}{200}\left(2q\frac{dq}{dp}\right) = -\frac{8000}{p^3}$

From part (a) $q = 40$ when $p = 20$. Substituting gives

$\frac{1}{200}\left(2\cdot 40\frac{dq}{dp}\right) = -\frac{8000}{20^3}$

$\frac{dq}{dp} = -2.5$

c. $q(p+dp) \approx q(p) + dq = q(p) + q'(p)dp$

$q(19.20) = q(20 + (-0.8))$

$\approx q(20) + q'(20)dp$

$= 40 + (-2.5)(-0.8)$

$= 42$ units

46. a. Profit $= TR - TC = pq - \bar{c}q$

$$P = \frac{1}{2}q^3 - 66q^2 + 7000q - \left(500q - q^2 + \frac{80{,}000}{2}\right) = \frac{1}{2}q^3 - 65q^2 + 6500q - 40{,}000$$

If $q = 100$, then $P = \frac{1}{2}(100)^3 - 65(100)^2 + 6500(100) - 40{,}000 = 460{,}000$

b. We use $P(q + dq) \approx P(q) + dP$ with $q = 100$ and $dq = 1$.

$$P(101) = P(100+1)$$
$$\approx P(100) + \left(\frac{3}{2}q^2 - 130q + 6500\right)dq$$
$$= 460{,}000 + \left[\frac{3}{2}(100)^2 - 130(100) + 6500\right](1)$$
$$= \$468{,}500$$

Apply It 14.2

1. $\int 28.3\, dq = 28.3q + C$

The form of the cost function is $28.3q + C$.

2. $\int 0.12t^2 dt = 0.12\frac{t^3}{3} + C = 0.04t^3 + C$

The form of the revenue function is $R(t) = 0.04t^3 + C$.

3. Let $S(t)$ = the number of subscribers t months after the competition entered the market, then $S'(t) = -\frac{480}{t^3}$.

$$S(t) = \int -\frac{480}{t^3}dt = -480\int t^{-3}dt$$
$$= -480\left(\frac{t^{-2}}{-2}\right) + C = 240t^{-2} + C = \frac{240}{t^2} + C$$

The number of subscribers is $S(t) = \frac{240}{t^2} + C$.

4. $\int\left(500 + 300\sqrt{t}\right)dt = \int\left(500 + 300t^{\frac{1}{2}}\right)dt$

$$= 500t + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$$

The population is $N(t) = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$

5. The amount of money saved is $\int \frac{dS}{dt}dt$.

$$\int\left(2.1t^2 - 65.4t + 491.6\right)dt$$
$$= 2.1\left(\frac{t^3}{3}\right) - 65.4\left(\frac{t^2}{2}\right) + 491.6t + C$$
$$= 0.7t^3 - 32.7t^2 + 491.6t + C$$

The amount of money saved is $S(t) = 0.7t^3 - 32.7t^2 + 491.6t + C$

Problems 14.2

1. $\int 7\,dx = 7x + C$

2. $\int \frac{1}{x}dx = \int \frac{1}{x}dx = \ln|x| + C$

3. $\int x^8 dx = \frac{x^{8+1}}{8+1} + C = \frac{x^9}{9} + C$

4. $\int 5x^{24}\,dx = 5\int x^{24}\,dx = 5\cdot\frac{x^{24+1}}{24+1} + C$
$= 5\cdot\frac{x^{25}}{25} + C = \frac{x^{25}}{5} + C$

5. $\int 5x^{-7}\,dx = 5\int x^{-7}dx = 5\cdot\frac{x^{-7+1}}{-7+1} + C$
$= 5\cdot\frac{x^{-6}}{-6} + C = -\frac{5}{6x^6} + C$

6. $\int \frac{z^{-3}}{3}dz = \frac{1}{3}\int z^{-3}dz = \frac{1}{3}\cdot\frac{z^{-3+1}}{-3+1} + C$
$= \frac{1}{3}\cdot\frac{z^{-2}}{-2} + C = -\frac{1}{6z^2} + C$

7. $\int \frac{5}{x^7}dx = 5\int x^{-7}dx = 5\cdot\frac{x^{-7+1}}{-7+1} + C$
$= \frac{5x^{-6}}{-6} + C = -\frac{5}{6x^6} + C$

8. $\int \frac{7}{x^4}dx = 7\int x^{-4}dx = 7\cdot\frac{x^{-4+1}}{-4+1} + C = \frac{7x^{-3}}{-3} + C$
$= -\frac{7}{3x^3} + C$

9. $\int \frac{1}{t^{7/4}}dt = \int t^{-7/4}dt = \frac{t^{-7/4+1}}{-\frac{7}{4}+1} + C = \frac{t^{-3/4}}{-\frac{3}{4}} + C$
$= -\frac{4}{3t^{3/4}} + C$

10. $\int \frac{7}{2x^{\frac{9}{4}}}dx = \frac{7}{2}\int x^{-\frac{9}{4}}dx = \frac{7}{2}\cdot\frac{x^{\frac{-9}{4}+1}}{\frac{-9}{4}+1} + C$
$= \frac{7}{2}\cdot\frac{x^{-\frac{5}{4}}}{-\frac{5}{4}} + C$
$= -\frac{14}{5x^{\frac{5}{4}}} + C$

11. $\int (4+t)dt = \int 4\,dt + \int t\,dt = 4t + \frac{t^{t+1}}{1+1} + C$
$= 4t + \frac{t^2}{2} + C$

12. $\int (7r^5 + 4r^2 + 1)dr = 7\int r^5 dr + 4\int r^2 dr + \int dr$
$= 7\cdot\frac{r^{5+1}}{5+1} + 4\cdot\frac{r^{2+1}}{2+1} + r + C$
$= \frac{7r^6}{6} + \frac{4r^3}{3} + r + C$

13. $\int \left(y^5 - 5y\right)dy = \int y^5 dy - \int 5y\;dy$
$= \frac{y^{5+1}}{5+1} - 5\cdot\frac{y^{1+1}}{1+1} + C$
$= \frac{y^6}{6} - 5\cdot\frac{y^2}{2} + C = \frac{y^6}{6} - \frac{5y^2}{2} + C$

14. $\int \left(5 - 2w - 6w^2\right)dw$
$= \int 5\;dw - 2\int w\;dw - 6\int w^2 dw$
$= 5w - 2\cdot\frac{w^2}{2} - 6\cdot\frac{w^3}{3} + C$
$= 5w - w^2 - 2w^3 + C$

15. $\int \left(3t^2 - 4t + 5\right)dt = 3\int t^2 dt - 4\int t\;dt + \int 5\;dt$
$= 3\cdot\frac{t^3}{3} - 4\cdot\frac{t^2}{2} + 5t + C = t^3 - 2t^2 + 5t + C$

16. $\int (1 + t^2 + t^4 + t^6)dt$
$= \int 1dt + \int t^2\,dt + \int t^4\,dt + \int t^6\,dt$
$= t + \frac{t^3}{3} + \frac{t^5}{5} + \frac{t^7}{7} + C$

17. Since $\sqrt{2}+e$ is a constant,
$\int\left(\sqrt{2}+e\right)dx=\left(\sqrt{2}+e\right)x+C$.

18. $\int\left(5-2^{-1}\right)dx=\int\left(5-\frac{1}{2}\right)dx=\int\frac{9}{2}dx=\frac{9}{2}x+C$

19. $\int\left(\frac{x}{7}-\frac{3}{4}x^4\right)dx=\frac{1}{7}\int x\,dx-\frac{3}{4}\int x^4dx$
$=\frac{1}{7}\cdot\frac{x^2}{2}-\frac{3}{4}\cdot\frac{x^5}{5}+C$
$=\frac{x^2}{14}-\frac{3x^5}{20}+C$

20. $\int\left(\frac{2x^2}{7}-\frac{8}{3}x^4\right)dx=\frac{2}{7}\int x^2dx-\frac{8}{3}\int x^4dx$
$=\frac{2}{7}\cdot\frac{x^3}{3}-\frac{8}{3}\cdot\frac{x^5}{5}+C$
$=\frac{2x^3}{21}-\frac{8x^5}{15}+C$

21. $\int\pi e^xdx=\pi\int e^xdx=\pi e^x+C$

22. $\int(e^x+3x^2+2x)dx=\int e^xdx+3\int x^2dx+2\int x\,dx$
$=e^x+3\cdot\frac{x^{2+1}}{2+1}+2\cdot\frac{x^{1+1}}{1+1}+C$
$=e^x+x^3+x^2+C$

23. $\int\left(x^{8.3}-9x^6+3x^{-4}+x^{-3}\right)dx$
$=\frac{x^{9.3}}{9.3}-9\cdot\frac{x^7}{7}+3\cdot\frac{x^{-3}}{-3}+\frac{x^{-2}}{-2}+C$
$=\frac{x^{9.3}}{9.3}-\frac{9x^7}{7}-\frac{1}{x^3}-\frac{1}{2x^2}+C$

24. $\int(0.7y^3+10+2y^{-3})dy$
$=0.7\cdot\frac{y^4}{4}+10y+2\cdot\frac{y^{-2}}{-2}+C$
$=0.175y^4+10y-\frac{1}{y^2}+C$

25. $\int\frac{-2\sqrt{x}}{3}dx=-\frac{2}{3}\int x^{\frac{1}{2}}dx=-\frac{2}{3}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$
$=-\frac{2}{3}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C=-\frac{4x^{\frac{3}{2}}}{9}+C$

26. $\int dz=\int 1\,dz=1\cdot z+C=z+C$

27. $\int\frac{5}{3\sqrt[3]{x^2}}dx=\frac{5}{3}\int x^{-2/3}dx$
$=\frac{5}{3}\cdot\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+C$
$=5x^{1/3}+C$

28. $\int\frac{-4}{(3x)^3}dx=\int\frac{-4}{27x^3}dx=-\frac{4}{27}\int x^{-3}\,dx$
$=-\frac{4}{27}\cdot\frac{x^{-3+1}}{-3+1}+C$
$=-\frac{4}{27}\cdot\frac{x^{-2}}{-2}+C=\frac{2}{27x^2}+C$

29. $\int\left(\frac{x^3}{3}-\frac{3}{x^3}\right)dx=\frac{1}{3}\int x^3dx-3\int x^{-3}dx$
$=\frac{1}{3}\cdot\frac{x^{3+1}}{3+1}-3\cdot\frac{x^{-3+1}}{-3+1}+C$
$=\frac{1}{3}\cdot\frac{x^4}{4}-3\cdot\frac{x^{-2}}{-2}+C=\frac{x^4}{12}+\frac{3}{2x^2}+C$

30. $\int\left(\frac{1}{2x^3}-\frac{1}{x^4}\right)dx=\frac{1}{2}\int x^{-3}dx-\int x^{-4}dx$
$=\frac{1}{2}\cdot\frac{x^{-2}}{-2}-\frac{x^{-3}}{-3}+C$
$=-\frac{1}{4x^2}+\frac{1}{3x^3}+C$

31. $\int\left(\frac{3w^2}{2}-\frac{2}{3w^2}\right)dw=\frac{3}{2}\int w^2dw-\frac{2}{3}\int w^{-2}dw$
$=\frac{3}{2}\cdot\frac{w^3}{3}-\frac{2}{3}\cdot\frac{w^{-1}}{-1}+C=\frac{w^3}{2}+\frac{2}{3w}+C$

32. $\int 7e^{-s}ds = 7\int e^{-s}ds$
$= 7\cdot e^{-s}(-1) + C$
$= -7e^{-s} + C$

33. $\int \frac{3u-4}{5}du = \frac{1}{5}\int (3u-4)du = \frac{1}{5}\left(3\int u\ du - 4\int du\right)$
$= \frac{1}{5}\left(3\frac{u^2}{2} - 4u\right) + C = \frac{3}{10}u^2 - \frac{4}{5}u + C$
$= \frac{1}{7}\left(2\int z\ dz - \int 5\ dz\right)$
$= \frac{1}{7}\left(2\cdot\frac{z^2}{2} - 5z\right) + C = \frac{1}{7}\left(z^2 - 5z\right) + C$

34. $\int \frac{1}{12}\left(\frac{1}{3}e^x\right)dx = \int \frac{1}{36}e^x dx$
$= \frac{1}{36}\int e^x dx = \frac{1}{36}e^x + C$

35. $\int (u^e + e^u)du = \int u^e\ du + \int e^u\ du$
$= \frac{u^{e+1}}{e+1} + e^u + C$

36. $\int\left(3y^3 - 2y^2 + \frac{e^y}{6}\right)dy$
$= 3\int y^3dy - 2\int y^2dy + \frac{1}{6}\int e^y dy$
$= 3\cdot\frac{y^4}{4} - 2\cdot\frac{y^3}{3} + \frac{1}{6}\cdot e^y + C$
$= \frac{3y^4}{4} - \frac{2y^3}{3} + \frac{e^y}{6} + C$

37. $\int\left(\frac{3}{\sqrt{x}} - 12\sqrt[3]{x}\right)dx = \int (3x^{-1/2} - 12x^{1/3})dx$
$= 3\int x^{-1/2}dx - 12\int x^{1/3}dx$
$= 3\cdot\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 12\cdot\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$
$= 6x^{1/2} - 9x^{4/3} + C$
$= 6\sqrt{x} - 9\sqrt[3]{x^4} + C$

38. $\int 0\ dt = 0\cdot t + C = C$

39. $\int\left(-\frac{\sqrt[3]{x^2}}{5} - \frac{7}{2\sqrt{x}} + 6x\right)dx$
$= \int\left(-\frac{x^{\frac{2}{3}}}{5} - \frac{7x^{-\frac{1}{2}}}{2} + 6x\right)dx$
$= -\frac{1}{5}\int x^{\frac{2}{3}}dx - \frac{7}{2}\int x^{-\frac{1}{2}}dx + 6\int x\ dx$
$= -\frac{1}{5}\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{7}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 6\cdot\frac{x^2}{2} + C$
$= -\frac{3x^{\frac{5}{3}}}{25} - 7x^{\frac{1}{2}} + 3x^2 + C$

40. $\int\left(\sqrt[3]{u} + \frac{1}{\sqrt{u}}\right)du = \int\left(u^{\frac{1}{3}} + u^{-\frac{1}{2}}\right)du$
$= \int u^{\frac{1}{3}}du + \int u^{-\frac{1}{2}}du$
$= \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$
$= \frac{3u^{\frac{4}{3}}}{4} + 2u^{\frac{1}{2}} + C$

41. $\int\left(x^2+5\right)(x-3)dx = \int\left(x^3 - 3x^2 + 5x - 15\right)dx$
$= \frac{x^4}{4} - 3\cdot\frac{x^3}{3} + 5\cdot\frac{x^2}{2} - 15x + C$
$= \frac{x^4}{4} - x^3 + \frac{5x^2}{2} - 15x + C$

42. $\int x^3(x^2 + 5x + 2)dx = \int (x^5 + 5x^4 + 2x^3)dx$
$= \frac{x^6}{6} + 5\cdot\frac{x^5}{5} + 2\cdot\frac{x^4}{4} + C$
$= \frac{x^6}{6} + x^5 + \frac{x^4}{2} + C$

43. $\int\sqrt{x}(x+3)dx = \int\left(x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx$
$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$
$= \frac{2x^{\frac{5}{2}}}{5} + 2x^{\frac{3}{2}} + C$

44. $\int (z+2)^2\,dz = \int \left(z^2+4z+4\right)dz$

$= \dfrac{z^3}{3} + 4\cdot\dfrac{z^2}{2} + 4z + C$

$= \dfrac{z^3}{3} + 2z^2 + 4z + C$

45. $\int (3u+2)^3\,du = \int (27u^3+54u^2+36u+8)\,du$

$= 27\cdot\dfrac{u^4}{4} + 54\cdot\dfrac{u^3}{3} + 36\cdot\dfrac{u^2}{2} + 8u + C$

$= \dfrac{27}{4}u^4 + 18u^3 + 18u^2 + 8u + C$

46. $\int \left(\dfrac{2}{\sqrt[5]{x}} - 1\right)^2 dx = \int \left(2x^{-\frac{1}{5}} - 1\right)^2 dx$

$= \int \left(4x^{-\frac{2}{5}} - 4x^{-\frac{1}{5}} + 1\right)dx$

$= 4\cdot\dfrac{x^{\frac{3}{5}}}{\frac{3}{5}} - 4\cdot\dfrac{x^{\frac{4}{5}}}{\frac{4}{5}} + x + C$

$= \dfrac{20x^{\frac{3}{5}}}{3} - 5x^{\frac{4}{5}} + x + C$

47. $\int x^{-2}(3x^4+4x^2-5)\,dx = \int (3x^2+4-5x^{-2})\,dx$

$= 3\cdot\dfrac{x^3}{3} + 4x - 5\cdot\dfrac{x^{-1}}{-1} + C$

$= x^3 + 4x + \dfrac{5}{x} + C$

48. $\int \left[6e^u - u^3\left(\sqrt{u}+1\right)\right]du = \int \left[6e^u - u^{\frac{7}{2}} - u^3\right]du$

$= 6\cdot e^u - \dfrac{u^{\frac{9}{2}}}{\frac{9}{2}} - \dfrac{u^4}{4} + C$

$= 6e^u - \dfrac{2u^{\frac{9}{2}}}{9} - \dfrac{u^4}{4} + C$

49. $\int \dfrac{z^4+10z^3}{2z^2}\,dz = \dfrac{1}{2}\int \left(\dfrac{z^4}{z^2} + \dfrac{10z^3}{z^2}\right)dz$

$= \dfrac{1}{2}\int \left(z^2+10z\right)dz$

$= \dfrac{1}{2}\left(\dfrac{z^3}{3} + 10\cdot\dfrac{z^2}{2}\right) + C$

$= \dfrac{z^3}{6} + \dfrac{5z^2}{2} + C$

50. $\int \dfrac{x^4-5x^2+2x}{5x^2}\,dx = \dfrac{1}{5}\int \left(x^2 - 5 + \dfrac{2}{x}\right)dx$

$= \dfrac{1}{5}\left(\dfrac{x^3}{3} - 5x + 2\ln|x|\right) + C$

51. $\int \dfrac{e^x+e^{2x}}{e^x}\,dx = \int \left(\dfrac{e^x}{e^x} + \dfrac{e^{2x}}{e^x}\right)dx$

$= \int \left(1+e^x\right)dx$

$= x + e^x + C$

52. $\int \dfrac{(x^2+1)^3}{x}\,dx = \int \dfrac{x^6+3x^4+3x^2+1}{x}\,dx$

$= \int (x^5+3x^3+3x+x^{-1})\,dx$

$= \dfrac{x^6}{6} + 3\cdot\dfrac{x^4}{4} + 3\cdot\dfrac{x^2}{2} + \ln|x| + C$

$= \dfrac{x^6}{6} + \dfrac{3x^4}{4} + \dfrac{3x^2}{2} + \ln|x| + C$

53. No, $F(x) - G(x)$ might be a nonzero constant.

54. **a.** $F(x) = \dfrac{d}{dx}\left(xe^x\right) = xe^x + e^x(1) = e^x(x+1)$

b. There is only one.

55. Because an antiderivative of the derivative of a function is the function itself, we have

$\int \dfrac{d}{dx}\left[\dfrac{1}{\sqrt{x^2+1}}\right]dx = \dfrac{1}{\sqrt{x^2+1}} + C$.

Apply It 14.3

6. $N(t) = \int \frac{dN}{dt}\,dt = \int \left(800 + 200e^t\right) dt$

$= 800t + 200e^t + C$

Since $N(5) = 40{,}000$, we have

$40{,}000 = 800(5) + 200e^5 + C$, so

$C = 40{,}000 - \left(4000 + 200e^5\right)$

$= 36{,}000 - 200e^5 \approx 6317.37$

$N(t) = 800t + 200e^t + 6317.37$

7. Since $y'' = \frac{d}{dt}(y') = 84t + 24$

$y' = \int (84t + 24)dt = 84\left(\frac{t^2}{2}\right) + 24t + C_1$

$= 42t^2 + 24t + C_1$

Since $y'(8) = 2891$, we have

$2891 = 42(8)^2 + 24(8) + C_1 = 2880 + C_1$, so

$C_1 = 2891 - 2880 = 11$, and $y' = 42t^2 + 24t + 11$.

$y(t) = \int y'dt = \int \left(42t^2 + 24t + 11\right) dt$

$= 42\left(\frac{t^3}{3}\right) + 24\left(\frac{t^2}{2}\right) + 11t + C_2$

$= 14t^3 + 12t^2 + 11t + C_2$

Since $y(2) = 185$, we have

$185 = 14(2)^3 + 12(2)^2 + 11(2) + C_2$

$= 182 + C_2$, so $C_2 = 185 - 182 = 3$.

$y(t) = 14t^3 + 12t^2 + 11t + 3$

Problems 14.3

1. $\frac{dy}{dx} = 3x - 4$

$y = \int (3x - 4)dx = \frac{3x^2}{2} - 4x + C$

Using $y(-1) = \frac{13}{2}$ gives

$\frac{13}{2} = \frac{3(-1)^2}{2} - 4(-1) + C$

$\frac{13}{2} = \frac{11}{2} + C$

Thus $C = 1$, so $y = \frac{3x^2}{2} - 4x + 1$.

2. $\frac{dy}{dx} = x^2 - x$

$y = \int \left(x^2 - x\right) dx = \frac{x^3}{3} - \frac{x^2}{2} + C$

Using $y(3) = \frac{19}{2}$ gives $\frac{19}{2} = \frac{3^3}{3} - \frac{3^2}{2} + C$

$\frac{19}{2} = \frac{9}{2} + C$

Thus, $C = 5$, so $y = \frac{x^3}{3} - \frac{x^2}{2} + 5$.

3. $y' = \frac{9}{8\sqrt{x}}$

$y = \int \frac{9}{8\sqrt{x}}\,dx$

$= \frac{9}{8}\int x^{-1/2}dx$

$= \frac{9}{8} \cdot \frac{x^{1/2}}{\frac{1}{2}} + C$

$= \frac{9\sqrt{x}}{4} + C$

$y(16) = 10$ implies $10 = \frac{9\sqrt{16}}{4} + C$, $10 = 9 + C$,

$C = 1$. Thus $y = \frac{9\sqrt{x}}{4} + 1$.

$y(9) = \frac{9\sqrt{9}}{4} + 1 = \frac{9 \cdot 3}{4} + 1 = \frac{31}{4}$

4. $y' = -x^2 + 2x$

$y = \int \left(-x^2 + 2x\right) dx = -\frac{x^3}{3} + x^2 + C$

$y(2) = 1$ implies $1 = -\frac{8}{3} + 4 + C$, so $C = -\frac{1}{3}$.

Thus $y = -\frac{x^3}{3} + x^2 - \frac{1}{3}$.

$y(1) = -\frac{1}{3} + 1 - \frac{1}{3} = \frac{1}{3}$

5. $y'' = -3x^2 + 4x$

$y' = \int (-3x^2 + 4x)dx = -x^3 + 2x^2 + C_1$

$y'(1) = 2$ implies $2 = -1 + 2 + C_1$, so $C_1 = 1$.

$y = \int (-x^3 + 2x^2 + 1)dx = -\frac{x^4}{4} + \frac{2x^3}{3} + x + C_2$

$y(1) = 3$ implies $3 = -\frac{1}{4} + \frac{2}{3} + 1 + C_2$, so

$C_2 = \frac{19}{12}$. Thus $y = -\frac{x^4}{4} + \frac{2x^3}{3} + x + \frac{19}{12}$.

6. $y'' = x + 1$

$y' = \int (x+1)dx = \frac{x^2}{2} + x + C_1$

$y'(0) = 0$ implies $0 = 0 + 0 + C_1$, so $C_1 = 0$.

$y = \int \left[\frac{x^2}{2} + x\right]dx = \frac{x^3}{6} + \frac{x^2}{2} + C_2$.

$y(0) = 5$ implies $5 = 0 + 0 + C_2$, so $C_2 = 5$. Thus

$y = \frac{x^3}{6} + \frac{x^2}{2} + 5$.

7. $y''' = 2x$

$y'' = \int 2x\, dx = x^2 + C_1$

$y''(-1) = 3$ implies that $3 = 1 + C_1$, so $C_1 = 2$.

$y' = \int \left(x^2 + 2\right) dx = \frac{x^3}{3} + 2x + C_2$

$y'(3) = 10$ implies $10 = 9 + 6 + C_2$, so $C_2 = -5$.

$y = \int \left(\frac{x^3}{3} + 2x - 5\right) dx = \frac{x^4}{12} + x^2 - 5x + C_3$.

$y(0) = 13$ implies that $13 = 0 + 0 - 0 + C_3$, so

$C_3 = 13$. Therefore $y = \frac{x^4}{12} + x^2 - 5x + 13$.

8. $y''' = 2e^{-x} + 3$

$y'' = \int (2e^{-x} + 3)dx = -2e^{-x} + 3x + C_1$

$y''(0) = 7$ implies $7 = -2 + C_1$, so $C_1 = 9$.

$y' = \int (-2e^{-x} + 3x + 9)dx = 2e^{-x} + \frac{3x^2}{2} + 9x + C_2$

$y'(0) = 5$ implies $5 = 2 + C_2$, so $C_2 = 3$.

$y = \int \left(2e^{-x} + \frac{3x^2}{2} + 9x + 3\right)dx$

$= -2e^{-x} + \frac{x^3}{2} + \frac{9x^2}{2} + 3x + C_3$

$y(0) = 1$ implies $1 = -2 + C_3$, so $C_3 = 3$.

Thus $y = -2e^{-x} + \frac{x^3}{2} + \frac{9x^2}{2} + 3x + 3$.

9. $\frac{dr}{dq} = 0.7$

$r = \int 0.7dq = 0.7q + C$

If $q = 0$, r must be 0, so $0 = 0 + C$, $C = 0$. Thus $r = 0.7q$. Since $r = pq$, we have

$p = \frac{r}{q} = \frac{0.7q}{q} = 0.7$. The demand function is

$p = 0.7$.

10. $\frac{dr}{dq} = 10 - \frac{1}{16}q$

$r = \int \left[10 - \frac{1}{16}q\right]dq = 10q - \frac{1}{32}q^2 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 10q - \frac{1}{32}q^2$. Since $r = pq$, then

$p = \frac{r}{q} = 10 - \frac{1}{32}q$. The demand function is

$p = 10 - \frac{1}{32}q$.

11. $\frac{dr}{dq} = 275 - q - 0.3q^2$

Thus $r = \int \left(275 - q - 0.3q^2\right)dq$

$= 275q - 0.5q^2 - 0.1q^3 + C$. When $q = 0$, r must be 0, so $C = 0$ and $r = 275q - 0.5q^2 - 0.1q^3$.

Since $r = pq$, then $p = \frac{r}{q} = 275 - 0.5q - 0.1q^2$.

Thus the demand function is

$p = 275 - 0.5q - 0.1q^2$.

12. $\frac{dr}{dq} = 5000 - 3(2q + 2q^3)$, so

$r = \int (5000 - 6q - 6q^3)dq$

$= 5000q - 3q^2 - \frac{3q^4}{2} + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 5000q - 3q^2 - \frac{3q^4}{2}$. Since $r = pq$, then

$p = \frac{r}{q} = 5000 - 3q - \frac{3q^3}{2}$. Therefore the demand function is $p = 5000 - 3q - \frac{3q^3}{2}$.

13. $\frac{dc}{dq} = 2.47$

$c = \int 2.47\, dq = 2.47q + C$

When $q = 0$, then $c = 159$, so $159 = 0 + C$, or $C = 159$. Thus $c = 2.47q + 159$.

14. $\frac{dc}{dq} = 2q + 75$

$c = \int (2q + 75)dq = q^2 + 75q + C$

When $q = 0$, then $c = 2000$, so $C = 2000$. Thus the cost function is $c = q^2 + 75q + 2000$.

15. $\frac{dc}{dq} = 0.08q^2 - 1.6q + 6.5$

$c = \int \left(0.08q^2 - 1.6q + 6.5\right)dq$

$\frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + C$. If $q = 0$, then $c = 8000$, from which $C = 8000$. Hence

$c = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 8000$. If $q = 25$, substituting gives $c(25) = 8079\frac{1}{6}$ or \$8079.17.

16. $\frac{dc}{dq} = 0.000204q^2 - 0.046q + 6$

$c = \int (0.000204q^2 - 0.046q + 6)dq$

$= 0.000068q^3 - 0.023q^2 + 6q + C$

When $q = 0$, then $c = 15{,}000$, from which $C = 15{,}000$. The cost function is

$c = 0.000068q^3 - 0.023q^2 + 6q + 15{,}000$. When $q = 200$, substitution gives $c(200) = 15{,}824$.

17. $G = \int \left[-\frac{P}{25} + 2\right]dP = -\frac{P^2}{50} + 2P + C$

When $P = 10$, then $G = 38$, so $38 = -2 + 20 + C$, from which $C = 20$. Thus

$G = -\frac{1}{50}P^2 + 2P + 20$.

18. $\frac{dy}{dx} = -1.5 - x$

$y = \int (-1.5 - x)dx = -1.5x - \frac{x^2}{2} + C$

When $x = 1$, then $y = 59.6$, so $59.6 = -1.5 - 0.5 + C$, or $C = 61.6$. Thus

$y = -1.5x - 0.5x^2 + 61.6$ for $1 \le x \le 9$.

19. $v = \int -\frac{(P_1 - P_2)r}{2l\eta}dr = -\frac{(P_1 - P_2)r^2}{4l\eta} + C$

Since $v = 0$ when $r = R$, then

$0 = -\frac{(P_1 - P_2)R^2}{4l\eta} + C$, so $C = \frac{(P_1 - P_2)R^2}{4l\eta}$.

Thus

$v = -\frac{(P_1 - P_2)r^2}{4l\eta} + \frac{(P_1 - P_2)R^2}{4l\eta}$

$= \frac{(P_1 - P_2)\left(R^2 - r^2\right)}{4l\eta}$.

20. $\frac{dr}{dq} = 100 - 3q^2$

$r = \int \left(100 - 3q^2\right)dq = 100q - q^3 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 100q - q^3$. Since $r = pq$, then

$p = \frac{r}{q} = 100 - q^2$.

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-2q} = -\frac{p}{2q^2}$

When $q = 5$, then $p = 75$, so $\eta = \frac{-75}{2(25)} = -\frac{3}{2}$.

21. $\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$

$c = \int \left(0.003q^2 - 0.4q + 40\right)dq$

$= 0.001q^3 - 0.2q^2 + 40q + C$

When $q = 0$, then $c = 5000$, so

$5000 = 0 - 0 + 0 + C$, or $C = 5000$. Thus $c = 0.001q^3 - 0.2q^2 + 40q + 5000$. When $q = 100$, then $c = 8000$. Since Avg. Cost $= \overline{c} = \frac{\text{Total Cost}}{\text{Quantity}} = \frac{c}{q}$, when $q = 100$, we have $\overline{c} = \frac{8000}{100} = \$80$. (Observe that knowing $\frac{dc}{dq} = 27.50$ when $q = 50$ is not relevant to the problem.)

22. $f''(x) = 30x^4 + 12x$

$f'(x) = \int (30x^4 + 12x)dx = 6x^5 + 6x^2 + C_1$

$f'(1) = 10$, so $10 = 6 + 6 + C_1$ and $C_1 = -2$.

$f'(x) = 6x^5 + 6x^2 - 2$

$f(x) = \int (6x^5 + 6x^2 - 2)dx = x^6 + 2x^3 - 2x + C_2$

Thus

$$\begin{aligned} &f(965.335245) - f(-965.335245) \\ &= [(965.335245)^6 + 2(965.335245)^3 - 2(965.335245) + C_2] \\ &\qquad - [(-965.335245)^6 + 2(-965.335245)^3 - 2(-965.335245) + C_2] \\ &= 3{,}598{,}280{,}000 \end{aligned}$$

Apply It 14.4

8. Using the values given, $\frac{dT}{dt} = -0.5(70 - 60)e^{-0.5t} = -5e^{-0.5t}$

$$T(t) = \int \frac{dT}{dt}dt = \int -5e^{-0.5t}dt = 10e^{-0.5t} + C$$

9. The number of words memorized is $v(t)$.

$$v(t) = \int \frac{35}{t+1}dt = 35\ln|t+1| + C.$$

Problems 14.4

1. Let $u = x + 5 \Rightarrow du = 1dx = dx$

$$\int (x+5)^7[dx] = \int u^7 du = \frac{u^8}{8} + C = \frac{(x+5)^8}{8} + C$$

2. $\int 15(x+2)^4 dx = 15\int (x+2)^4[dx] = 15 \cdot \frac{(x+2)^5}{5} + C = 3(x+2)^5 + C$

3. Let $u = x^2 + 3 \Rightarrow du = 2x\ dx$

$$\int 2x\left(x^2+3\right)^5 dx = \int \left(x^2+3\right)^5 [2x\ dx] = \int u^5 du = \frac{u^6}{6} + C$$

$$= \frac{\left(x^2+3\right)^6}{6} + C$$

4. Let $u = 2x^2 + 3x + 1 \Rightarrow du = (4x+3)dx.$

$\int (4x+3)(2x^2+3x+1)dx$

$= \int (2x^2+3x+1)^1[(4x+3)dx]$

$= \int u\ du = \frac{u^2}{2} + C$

$= \frac{(2x^2+3x+1)^2}{2} + C$

5. Let $u = y^3 + 3y^2 + 1 \Rightarrow du = \left(3y^2 + 6y\right)dy$

$\int \left(3y^2+6y\right)\left(y^3+3y^2+1\right)^{\frac{2}{3}} dy$

$= \int \left(y^3+3y^2+1\right)^{\frac{2}{3}}\left[\left(3y^2+6y\right)dy\right]$

$= \int u^{\frac{2}{3}} du = \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + C$

$= \frac{3}{5}\left(y^3+3y^2+1\right)^{\frac{5}{3}} + C$

6. $\int (15t^2 - 6t + 1)(5t^3 - 3t^2 + t)^{17}\, dt$

$= \int (5t^3 - 3t^2 + t)^{17}[(15t^2 - 6t + 1)dt]$

$= \frac{(5t^3 - 3t^2 + t)^{18}}{18} + C$

7. Let $u = 3x - 1 \Rightarrow du = 3\ dx$

$\int \frac{5}{(3x-1)^3}dx = \frac{5}{3}\int \frac{1}{(3x-1)^3}[3\ dx]$

$= \frac{5}{3}\int \frac{1}{u^3}du = \frac{5}{3}\int u^{-3}du$

$= \frac{5}{3}\cdot\frac{u^{-2}}{-2} + C = -\frac{5(3x-1)^{-2}}{6} + C$

8. $\int \frac{4x}{\left(2x^2-7\right)^{10}}dx = \int \left(2x^2-7\right)^{-10}[4x\ dx]$

$= -\frac{\left(2x^2-7\right)^{-9}}{9} + C$

9. Let $u = 7x - 3 \Rightarrow du = 7\ dx$.

$\int \sqrt{7x+3}\, dx = \int (7x+3)^{\frac{1}{2}}dx$

$= \frac{1}{7}\int (7x+3)^{\frac{1}{2}}[7\ dx]$

$= \frac{1}{7}\int u^{\frac{1}{2}}du = \frac{1}{7}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{21}(7x+3)^{3/2} + C$

10. Let $u = x - 5 \Rightarrow du = dx.$

$\int \frac{1}{\sqrt{x-5}}dx = \int (x-5)^{-\frac{1}{2}}[dx]$

$\int u^{-1/2}du = \frac{u^{1/2}}{\frac{1}{2}} + C$

$= \frac{(x-5)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x-5} + C$

11. Let $u = 7x - 6 \Rightarrow du = 7\ dx$

$\int (7x-6)^4 dx = \frac{1}{7}\int (7x-6)^4[7\ dx]$

$= \frac{1}{7}\int u^4 du = \frac{1}{7}\cdot\frac{u^5}{5} + C$

$= \frac{(7x-6)^5}{35} + C$

12. $\int x^2\left(3x^3+7\right)^3 dx = \frac{1}{9}\int \left(3x^3+7\right)^3\left[9x^2dx\right]$

$= \frac{1}{9}\cdot\frac{\left(3x^3+7\right)^4}{4} + C$

$= \frac{\left(3x^3+7\right)^4}{36} + C$

13. Let $v = 5u^2 - 9 \Rightarrow dv = 10u\, du$

$\int u(5u^2-9)^{14}du = \frac{1}{10}\int (5u^2-9)^{14}[10u\, du]$

$\frac{1}{10}\int v^{14}dv = \frac{1}{10}\cdot\frac{v^{15}}{15} + C = \frac{(5u^2-9)^{15}}{150} + C$

14. Let $u = 3+5x^2 \Rightarrow du = 10x\,dx$.

$$\int x\sqrt{3+5x^2}\,dx = \frac{1}{10}\int (3+5x^2)^{1/2}[10x\,dx]$$
$$= \frac{1}{10}\int u^{1/2}du$$
$$= \frac{1}{10}\cdot\frac{u^{3/2}}{\frac{3}{2}}+C$$
$$= \frac{u^{3/2}}{15}+C$$
$$= \frac{(3+5x^2)^{3/2}}{15}+C$$

15. Let $u = 27+x^5 \Rightarrow du = 5x^4dx$

$$\int 4x^4\left(27+x^5\right)^{\frac{1}{3}}dx = \frac{4}{5}\int\left(27+x^5\right)^{\frac{1}{3}}\left[5x^4dx\right]$$
$$= \frac{4}{5}\int u^{\frac{1}{3}}du = \frac{4}{5}\cdot\frac{u^{\frac{4}{3}}}{\frac{4}{3}}+C$$
$$= \frac{3}{5}\left(27+x^5\right)^{\frac{4}{3}}+C$$

16. Let $u = 4-5x \Rightarrow du = -5dx$.

$$\int (4-5x)^9dx = -\frac{1}{5}\int (4-5x)^9[-5\,dx]$$
$$= -\frac{1}{5}\int u^9du = -\frac{1}{5}\cdot\frac{u^{10}}{10}+C = -\frac{1}{50}(4-5x)^{10}+C$$

17. Let $u = 3x \Rightarrow du = 3\,dx$

$$\int 3e^{3x}dx = \int e^{3x}[3\,dx]$$
$$= \int e^u du = e^u + C = e^{3x}+C$$

18. $\int 5e^{3t+7}dt = \frac{5}{3}\int e^{3t+7}[3\,dt] = \frac{5}{3}e^{3t+7}+C$

19. Let $u = 3t^2+2t+1 \Rightarrow du = (6t+2)dt$

$$\int (3t+1)e^{3t^2+2t+1}dt = \frac{1}{2}\int e^{3t^2+2t+1}[(6t+2)\,dt]$$
$$= \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{3t^2+2t+1}+C$$

20. $\int -3w^2e^{-w^3}dw = \int e^{-w^3}\left[-3w^2dw\right] = e^{-w^3}+C$

21. Let $u = 7x^2 \Rightarrow du = 14x\,dx$

$$\int xe^{7x^2}dx = \frac{1}{14}\int e^{7x^2}[14x\,dx] = \frac{1}{14}\int e^u du$$
$$= \frac{1}{14}e^u + C = \frac{1}{14}e^{7x^2}+C$$

22. $\int x^3e^{4x^4}dx = \frac{1}{16}\int e^{4x^4}\left[16x^3dx\right]$

$$= \frac{1}{16}\cdot e^{4x^4}+C = \frac{e^{4x^4}}{16}+C$$

23. Let $u = -3x \Rightarrow du = -3dx$.

$$\int 4e^{-3x}dx = -\frac{4}{3}\int e^{-3x}[-3\,dx]$$
$$= -\frac{4}{3}\int e^u du = -\frac{4}{3}e^u + C = -\frac{4}{3}e^{-3x}+C$$

24. $\int 24x^5e^{-2x^6+7}dx = -2\int e^{-2x^6+7}[-12x^5dx]$

$$= -2e^{-2x^6+7}+C$$

25. Let $u = x+5 \Rightarrow du = dx$

$$\int \frac{1}{x+5}[dx] = \int \frac{1}{u}du = \ln|u|+C = \ln|x+5|+C$$

26. $\int \frac{12x^2+4x+2}{x+x^2+2x^3}dx$

$$= \int \frac{2}{x+x^2+2x^3}[(1+2x+6x^2)dx]$$
$$= 2\ln\left|x+x^2+2x^3\right|+C$$
$$= \ln[(x+x^2+2x^3)^2]+C$$

27. Let $u = x^3+x^4 \Rightarrow du = (3x^2+4x^3)dx$

$$\int \frac{3x^2+4x^3}{x^3+x^4}dx = \int \frac{1}{x^3+x^4}\left[\left(3x^2+4x^3\right)dx\right]$$
$$= \int \frac{1}{u}du = \ln|u|+C$$
$$= \ln\left|x^3+x^4\right|+C$$

28. Let $u = 1 - 3x^2 + 2x^3 \Rightarrow du = (-6x + 6x^2)dx$.

$$\int \frac{6x^2 - 6x}{1 - 3x^2 + 2x^3} dx$$
$$= \int \frac{1}{1 - 3x^2 + 2x^3}[(-6x + 6x^2)dx$$
$$= \int \frac{1}{u} du = \ln|u| + C = \ln\left|1 - 3x^2 + 2x^3\right| + C$$

29. Let $u = z^2 - 5 \Rightarrow du = 2z\,dz$

$$\int \frac{8z}{(z^2 - 5)^7} dz = 4\int (z^2 - 5)^{-7}[2z\,dz]$$
$$= 4\int u^{-7} du$$
$$= 4 \cdot \frac{u^{-6}}{-6} + C$$
$$= -\frac{2}{3}(z^2 - 5)^{-6} + C$$

30. $$\int \frac{3}{(5v - 1)^4} dv = \frac{3}{5}\int (5v - 1)^{-4}[5dv]$$
$$= \frac{3}{5} \cdot \frac{(5v - 1)^{-3}}{-3} + C$$
$$= -\frac{1}{5}(5v - 1)^{-3} + C$$

31. $$\int \frac{4}{x} dx = 4\int \frac{1}{x} dx = 4\ln|x| + C$$

32. $$\int \frac{3}{1 + 2y} dy = 3 \cdot \frac{1}{2}\int \frac{1}{1 + 2y}[2\ dy]$$
$$= \frac{3}{2}\ln|1 + 2y| + C$$

33. Let $u = s^3 + 5 \Rightarrow du = 3s^2 ds$

$$\int \frac{s^2}{s^3 + 5} ds = \frac{1}{3}\int \frac{1}{s^3 + 5}\left[3s^2 ds\right]$$
$$= \frac{1}{3}\int \frac{1}{u} du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln\left|s^3 + 5\right| + C$$

34. $$\int \frac{32x^3}{4x^4 + 9} dx = 2\int \frac{1}{4x^4 + 9}[16x^3 dx]$$
$$= 2\ln\left|4x^4 + 9\right| + C$$

35. Let $u = 4 - 2x \Rightarrow du = -2\ dx$

$$\int \frac{5}{4 - 2x} dx = -\frac{5}{2}\int \frac{1}{4 - 2x}[-2dx]$$
$$= -\frac{5}{2}\int \frac{1}{u} du = -\frac{5}{2}\ln|u| + C = -\frac{5}{2}\ln|4 - 2x| + C$$

36. $$\int \frac{7t}{5t^2 - 6} dt = 7 \cdot \frac{1}{10}\int \frac{1}{5t^2 - 6}[10t\ dt]$$
$$= \frac{7}{10}\ln\left|5t^2 - 6\right| + C$$

37. $$\int \sqrt{5x}\, dx = \sqrt{5}\int x^{1/2} dx = \sqrt{5}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$
$$= \frac{2\sqrt{5}}{3} x^{\frac{3}{2}} + C$$

38. $$\int \frac{1}{(3x)^6} dx = \frac{1}{3}\int (3x)^{-6}[3dx]$$
$$= \frac{1}{3} \cdot \frac{(3x)^{-5}}{-5} + C$$
$$= -\frac{1}{15}(3x)^{-5} + C$$

39. Let $u = ax^2 + b \Rightarrow du = 2ax\ dx$

$$\int \frac{x}{\sqrt{ax^2 + b}} dx = \frac{1}{2a}\int (ax^2 + b)^{-1/2}[2ax\,dx]$$
$$= \frac{1}{2a}\int u^{-1/2} du$$
$$= \frac{1}{2a} \cdot \frac{u^{1/2}}{\frac{1}{2}} + C$$
$$= \frac{1}{a}\sqrt{ax^2 + b} + C$$

40. Let $u = 1 - 3x \Rightarrow du = -3\ dx$.

$$\int \frac{9}{1 - 3x} dx = -3\int \frac{1}{1 - 3x}[-3\,dx]$$
$$= -3\int \frac{1}{u} du = -3\ln|u| + C = -3\ln|1 - 3x| + C$$

41. Let $u = y^4 + 1 \Rightarrow du = 4y^3 dy$

$$\int 2y^3 e^{y^4 + 1} dy = 2\int y^3 e^{y^4 + 1} dy$$
$$= 2 \cdot \frac{1}{4}\int e^{y^4 + 1}\left[4y^3 dy\right]$$
$$= \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$
$$= \frac{1}{2}e^{y^4 + 1} + C$$

42. $\int 2\sqrt{2x-1}dx = \int (2x-1)^{\frac{1}{2}}[2\,dx]$

$$= \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{3}(2x-1)^{\frac{3}{2}} + C$$

43. Let $u = -2v^3 + 1 \Rightarrow du = -6v^2 dv$

$$\int v^2 e^{-2v^3+1} dv = -\frac{1}{6}\int e^{-2v^3+1}\left[-6v^2 dv\right]$$

$$= -\frac{1}{6}\int e^u du = -\frac{1}{6}e^u + C$$

$$= -\frac{1}{6}e^{-2v^3+1} + C$$

44. $\int \frac{x^2+x+1}{\sqrt[3]{x^3+\frac{3}{2}x^2+3x}}dx$

$$= \frac{1}{3}\int\left(x^3+\frac{3}{2}x^2+3x\right)^{-1/3}[(3x^2+3x+3)dx]$$

$$= \frac{1}{3}\cdot\frac{\left(x^3+\frac{3}{2}x^2+3x\right)^{2/3}}{\frac{2}{3}} + C$$

$$= \frac{\left(x^3+\frac{3}{2}x^2+3x\right)^{2/3}}{2} + C$$

45. $\int\left(e^{-5x}+2e^x\right)dx = \int e^{-5x}dx + 2\int e^x dx$

$$= -\frac{1}{5}\int e^{-5x}[-5\ dx] + 2\int e^x dx$$

$$= -\frac{1}{5}e^{-5x} + 2e^x + C$$

46. $\int 4\sqrt[3]{y+1}dy = 4\int (y+1)^{\frac{1}{3}}[dy]$

$$= 4\cdot\frac{(y+1)^{\frac{4}{3}}}{\frac{4}{3}} + C = 3(y+1)^{\frac{4}{3}} + C$$

47. $\int (8x+10)(7-2x^2-5x)^3 dx$

$$= -2\int (7-2x^2-5x)^3[(-4x-5)dx]$$

$$= -2\cdot\frac{(7-2x^2-5x)^4}{4} + C$$

$$= -\frac{1}{2}(7-2x^2-5x)^4 + C$$

48. $\int 2ye^{3y^2}dy = 2\cdot\frac{1}{6}\int e^{3y^2}[6y\ dy] = \frac{1}{3}e^{3y^2} + C$

49. $\int \frac{6x^2+8}{x^3+4x}dx = 2\int\frac{1}{x^3+4x}[(3x^2+4)dx]$

$$= 2\ln\left|x^3+4x\right| + C$$

50. $\int (e^x + 2e^{-3x} - e^{5x})dx$

$$= \int e^x\,dx - \frac{2}{3}\int e^{-3x}[(-3)dx] - \frac{1}{5}\int e^{5x}[5\ dx]$$

$$= e^x - \frac{2}{3}e^{-3x} - \frac{1}{5}e^{5x} + C$$

51. $\int \frac{16s-4}{3-2s+4s^2}dx = 2\int\frac{1}{3-2s+4s^2}[(8s-2)ds]$

$$= 2\ln\left|3-2s+4s^2\right| + C$$

52. $\int (6t^2+4t)(t^3+t^2+1)^6 dt$

$$= 2\int (t^3+t^2+1)^6[(3t^2+2t)dt]$$

$$= 2\cdot\frac{(t^3+t^2+1)^7}{7} + C$$

$$= \frac{2}{7}(t^3+t^2+1)^7 + C$$

53. $\int x\left(2x^2+1\right)^{-1}dx = \int\frac{x}{2x^2+1}dx$

$$= \frac{1}{4}\int\frac{1}{2x^2+1}[4x\ dx]$$

$$= \frac{1}{4}\ln\left(2x^2+1\right) + C$$

54. $\int (45w^4+18w^2+12)(3w^5+2w^3+4w)^{-4}dw$

$$= 3\int (3w^5+2w^3+4w)^{-4}[(15w^4+6w^2+4)dw]$$

$$= 3\cdot\frac{(3w^5+2w^3+4w)^{-3}}{-3} + C$$

$$= -(3w^5+2w^3+4w)^{-3} + C$$

55. $\int -\left(x^2-2x^5\right)\left(x^3-x^6\right)^{-10}dx$

$$= -\frac{1}{3}\int\left(x^3-x^6\right)^{-10}\left[\left(3x^2-6x^5\right)dx\right]$$

$$= -\frac{1}{3}\cdot\frac{\left(x^3-x^6\right)^{-9}}{-9} + C = \frac{1}{27}\left(x^3-x^6\right)^{-9} + C$$

56. $\int \frac{3}{5}(v-2)e^{2-4v+v^2}\,dv$
$= \frac{3}{5}\cdot\frac{1}{2}\int e^{2-4v+v^2}[(2v-4)\ dv]$
$= \frac{3}{10}e^{2-4v+v^2} + C$

57. $\int \left(2x^3+x\right)\left(x^4+x^2\right)dx$
$= \frac{1}{2}\int \left(x^4+x^2\right)^1\left[\left(4x^3+2x\right)\ dx\right]$
$= \frac{1}{2}\cdot\frac{\left(x^4+x^2\right)^2}{2} + C = \frac{1}{4}\left(x^4+x^2\right)^2 + C$

58. $\int \left(e^{3.1}\right)^2\ dx = \int e^{6.2}dx = e^{6.2}x + C$, because $e^{6.2}$ is a constant.

59. $\int \frac{9+18x}{(5-x-x^2)^4}dx$
$= -9\int (5-x-x^2)^{-4}[(-1-2x)dx]$
$= -9\cdot\frac{(5-x-x^2)^{-3}}{-3} + C$
$= 3(5-x-x^2)^{-3} + C$

60. $\int \left(e^x - e^{-x}\right)^2\ dx = \int \left(e^{2x} - 2 + e^{-2x}\right)\ dx$
$= \frac{1}{2}\int e^{2x}[2\ dx] - \int 2\ dx + \left(-\frac{1}{2}\right)\int e^{-2x}[-2\ dx]$
$= \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C$
$= \frac{1}{2}\left(e^{2x} - e^{-2x}\right) - 2x + C$

61. $u = 4x^3 + 3x^2 - 4$
$du = \left(12x^2 + 6x\right)dx = 6x(2x+1)dx$
$\int x(2x+1)e^{4x^3+3x^2-4}dx$
$= \frac{1}{6}\int e^{4x^3+3x^2-4}[6x(2x+1)dx]$
$= \frac{1}{6}\int e^u du = \frac{1}{6}e^u + C = \frac{1}{6}e^{4x^3+3x^2-4} + C$

62. $\int (u^3 - ue^{6-3u^2})du = \frac{u^4}{4} + \frac{1}{6}\int e^{6-3u^2}[-6u\,du]$
$= \frac{1}{4}u^4 + \frac{1}{6}e^{6-3u^2} + C$

63. $\int x\sqrt{\left(8-5x^2\right)^3}\,dx = -\frac{1}{10}\int \left(8-5x^2\right)^{\frac{3}{2}}[-10x\ dx]$
$= -\frac{1}{10}\cdot\frac{\left(8-5x^2\right)^{\frac{5}{2}}}{\frac{5}{2}} + C = -\frac{1}{25}\left(8-5x^2\right)^{\frac{5}{2}} + C$

64. $\int e^{ax}dx = \frac{1}{a}\int e^{ax}[a\,dx] = \frac{1}{a}e^{ax} + C$

65. $\int \left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)dx = \int \sqrt{2x}dx - \int \frac{1}{\sqrt{2x}}dx$
$= \frac{1}{2}\int (2x)^{\frac{1}{2}}[2\ dx] - \frac{1}{2}\int (2x)^{-\frac{1}{2}}[2\ dx]$
$= \frac{1}{2}\cdot\frac{(2x)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\cdot\frac{(2x)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{(2x)^{\frac{3}{2}}}{3} - \sqrt{2x} + C$
$= \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - \sqrt{2}x^{\frac{1}{2}} + C$

66. $\int 3\frac{x^4}{e^{x^5}}dx = 3\int x^4e^{-x^5}dx = -\frac{3}{5}\int e^{-x^5}[-5x^4\ dx]$
$= -\frac{3}{5}e^{-x^5} + C$

67. $\int \left(x^2+1\right)^2\ dx = \int \left(x^4 + 2x^2 + 1\right)dx$
$= \frac{x^5}{5} + \frac{2x^3}{3} + x + C$

68. $\int \left[x\left(x^2-16\right)^2 - \frac{1}{2x+5}\right]dx$
$= \frac{1}{2}\int \left(x^2-16\right)^2\ [2x\ dx] - \frac{1}{2}\int \frac{1}{2x+5}[2\ dx]$
$= \frac{1}{2}\cdot\frac{\left(x^2-16\right)^3}{3} - \frac{1}{2}\ln|2x+5| + C$
$= \frac{1}{6}\left(x^2-16\right)^3 - \frac{1}{2}\ln|2x+5| + C$

69. $\int\left(\frac{x}{x^2+1}+\frac{x}{(x^2+1)^2}\right)dx$
$= \int\frac{x}{x^2+1}\,dx+\int\frac{x}{(x^2+1)^2}\,dx$
$= \frac{1}{2}\int\frac{1}{x^2+1}[2x\,dx]+\frac{1}{2}\int(x^2+1)^{-2}[2x\,dx]$
$= \frac{1}{2}\ln\left|x^2+1\right|+\frac{1}{2}\cdot\frac{(x^2+1)^{-1}}{-1}+C$
$= \frac{1}{2}\ln\left|x^2+1\right|-\frac{1}{2(x^2+1)}+C$

70. $\int\left[\frac{3}{x-1}+\frac{1}{(x-1)^2}\right]dx = \int\frac{3}{x-1}[dx]+\int(x-1)^{-2}[dx]$
$= 3\ln|x-1|+\frac{(x-1)^{-1}}{-1}+C = 3\ln|x-1|-\frac{1}{x-1}+C$

71. $\int\left[\frac{2}{4x+1}-(4x^2-8x^5)(x^3-x^6)^{-8}\right]dx$
$= \frac{1}{2}\int\frac{1}{4x+1}[4\,dx]-\frac{4}{3}\int(x^3-x^6)^{-8}[(3x^2-6x^5)dx]$
$= \frac{1}{2}\ln|4x+1|-\frac{4}{3}\cdot\frac{(x^3-x^6)^{-7}}{-7}+C$
$= \frac{1}{2}\ln|4x+1|+\frac{4}{21}(x^3-x^6)^{-7}+C$

72. $\int\left(r^3+5\right)^2 dr = \int\left(r^6+10r^3+25\right)dr = \frac{1}{7}r^7+\frac{5}{2}r^4+25r+C$

73. $\int\left[\sqrt{3x+1}-\frac{x}{x^2+3}\right]dx = \int(3x+1)^{\frac{1}{2}}\,dx-\int\frac{x}{x^2+3}\,dx = \frac{1}{3}\int(3x+1)^{\frac{1}{2}}[3\ dx]-\frac{1}{2}\int\frac{1}{x^2+3}[2x\ dx]$
$= \frac{1}{3}\cdot\frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{2}\ln\left(x^2+3\right)+C = \frac{2}{9}(3x+1)^{\frac{3}{2}}-\ln\sqrt{x^2+3}+C$

74. $\int\left(\frac{x}{7x^2+2}-\frac{x^2}{(x^3+2)^4}\right)dx$
$= \frac{1}{14}\int\frac{1}{7x^2+2}[14x\,dx]-\frac{1}{3}\int(x^3+2)^{-4}[3x^2dx]$
$= \frac{1}{14}\ln\left|7x^2+2\right|-\frac{1}{3}\cdot\frac{(x^3+2)^{-3}}{-3}+C$
$= \frac{1}{14}\ln\left|7x^2+2\right|+\frac{1}{9(x^3+2)^3}+C$

75. Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx$.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}dx\right]$$
$$= 2\int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$$

76. $\int\left(e^5 - 3^e\right)dx = \left(e^5 - 3^e\right)x + C$, because $e^5 - 3^e$ is a constant.

77. $$\int \frac{1+e^{2x}}{4e^x}dx = \frac{1}{4}\int\left(\frac{1}{e^x} + \frac{e^{2x}}{e^x}\right)dx$$
$$= \frac{1}{4}\int\left(e^{-x} + e^x\right)dx$$
$$= -\frac{1}{4}\int e^{-x}[-1\ dx] + \frac{1}{4}\int e^x dx$$
$$= -\frac{1}{4}e^{-x} + \frac{1}{4}e^x + C$$

78. $$\int \frac{2}{t^2}\sqrt{\frac{1}{t}+9}\ dt = -2\int\left(\frac{1}{t}+9\right)^{\frac{1}{2}}\left[-\frac{1}{t^2}dt\right]$$
$$= -2\frac{\left(\frac{1}{t}+9\right)^{\frac{3}{2}}}{\frac{3}{2}} + C$$
$$= -\frac{4}{3}\left(\frac{1}{t}+9\right)^{\frac{3}{2}} + C$$

79. Let $u = \ln(2x^2 + 3x)$

$$\Rightarrow du = \frac{1}{2x^2+3x}(4x+3)dx$$
$$\int \frac{4x+3}{2x^2+3x}\ln(2x^2+3x)\,dx$$
$$= \int \ln(2x^2+3x)\left[\frac{4x+3}{2x^2+3x}dx\right]$$
$$= \int u\,du$$
$$= \frac{u^2}{2} + C$$
$$= \frac{1}{2}[\ln(2x^2+3x)]^2 + C$$

80. Let $u = \sqrt[3]{8x^4} = 2x^{\frac{4}{3}} \Rightarrow du = \frac{8}{3}x^{\frac{1}{3}}dx$

$$\int \sqrt[3]{x}e^{\sqrt[3]{8x^4}}dx = \frac{3}{8}\int e^{2x^{\frac{4}{3}}}\left[\frac{8}{3}x^{\frac{1}{3}}dx\right] = \frac{3}{8}\int e^u du$$
$$= \frac{3}{8}e^u + C = \frac{3}{8}e^{\sqrt[3]{8x^4}} + C$$

81. $$y = \int(3-2x)^2 dx = -\frac{1}{2}\int(3-2x)^2[-2\ dx]$$
$$= -\frac{1}{2}\cdot\frac{(3-2x)^3}{3} + C = -\frac{1}{6}(3-2x)^3 + C$$

$y(0) = 1$ implies $1 = -\frac{1}{6}(27) + C$, so $C = \frac{11}{2}$.

Thus $y = -\frac{1}{6}(3-2x)^3 + \frac{11}{2}$.

82. $$y = \frac{1}{2}\int\frac{1}{x^2+6}[2x\ dx] = \frac{1}{2}\ln\left(x^2+6\right) + C$$

$y(1) = 0$ implies $0 = \frac{1}{2}\ln(7) + C$, so $C = -\frac{1}{2}\ln 7$.

Thus $y = \frac{1}{2}\left[\ln\left(x^2+6\right) - \ln 7\right]$, or

$$y = \ln\sqrt{\frac{x^2+6}{7}}$$

83. $$y'' = \frac{1}{x^2}$$
$$y' = \int x^{-2}dx = -x^{-1} + C_1$$

$y'(-2) = 3$ implies $3 = \frac{1}{2} + C_1$, so $C_1 = \frac{5}{2}$. Thus

$$y' = -x^{-1} + \frac{5}{2}.$$
$$y = \int\left(-x^{-1} + \frac{5}{2}\right)dx = -\int\frac{1}{x}dx + \int\frac{5}{2}dx$$
$$= -\ln|x| + \frac{5}{2}x + C_2$$

$y(1) = 2$ implies that $2 = 0 + \frac{5}{2} + C_2$, so

$C_2 = -\frac{1}{2}$. Thus

$$y = -\ln|x| + \frac{5}{2}x - \frac{1}{2} = \ln\left|\frac{1}{x}\right| + \frac{5}{2}x - \frac{1}{2}.$$

84. $y'' = (x+1)^{1/2}$

$y' = \int (x+1)^{1/2}\,dx = \frac{2}{3}(x+1)^{3/2} + C_1$

$y'(8) = 19 \Rightarrow 19 = \frac{2}{3}(8+1)^{3/2} + C_1 = 18 + C_1$

$\Rightarrow C_1 = 1$, so

$y = \int \left[\frac{2}{3}(x+1)^{3/2} + 1\right] dx$

$= \frac{2}{3} \cdot \frac{(x+1)^{5/2}}{\frac{5}{2}} + x + C_2$

$= \frac{4}{15}(x+1)^{5/2} + x + C_2$

$y(24) = \frac{2572}{3}$ implies that

$\frac{2572}{3} = \frac{4}{15}(25)^{5/2} + 24 + C_2 = \frac{2572}{3} + C_2$, so

$C_2 = 0$. Thus $y = \frac{4}{15}(x+1)^{5/2} + x$.

85. $V(t) = \int \frac{dV}{dt}\,dt = \int 8e^{0.05t}\,dt$

$= \frac{8}{0.05}\int e^{0.05t}[0.05\,dt]$

$= 160e^{0.05t} + C$

The house cost \$350,000 to build, so $V(0) = 350$.

$350 = 160e^0 + C = 160 + C$

$190 = C$

$V(t) = 160e^{0.05t} + 190$

86. $l(t) = \int \frac{dl}{dt}\,dt = \int \frac{12}{2t+50}\,dt$

$= 6\ln|2t+50| + C$

Since the expected life span was 63 years in 1940, $l(0) = 63$.

$63 = 6\ln|50| + C$

$C = 63 - 6\ln 50 \approx 39.53$

$l(t) = 6\ln|2t+50| + 39.53$

$l(58) = 6\ln|166| + 39.53 \approx 70.20$

The expected life span for people born in 1998 (58 years after 1940) is about 70 years.

87. Note that $r > 0$.

$C = \int \left[\frac{Rr}{2K} + \frac{B_1}{r}\right] dr = \int \frac{Rr}{2K}\,dr + \int \frac{B_1}{r}\,dr$

$= \frac{R}{2K}\int r\,dr + B_1 \int \frac{1}{r}\,dr$

$= \frac{R}{2K} \cdot \frac{r^2}{2} + B_1 \ln|r| + B_2$

Thus we obtain $C = \frac{Rr^2}{4K} + B_1 \ln|r| + B_2$.

88. $f(x) = \int (e^{3x+2} - 3x)\,dx = \frac{1}{3}e^{3x+2} - \frac{3}{2}x^2 + C$

$f\left(\frac{1}{3}\right) = 2$ implies $2 = \frac{1}{3}e^3 - \frac{1}{6} + C$, so

$C = \frac{13}{6} - \frac{1}{3}e^3$. Thus,

$f(x) = \frac{1}{3}e^{3x+2} - \frac{3}{2}x^2 + \frac{13}{6} - \frac{1}{3}e^3$,

$f(2) = \frac{1}{3}e^8 - 6 + \frac{13}{6} - \frac{1}{3}e^3$

$= \frac{1}{6}(2e^8 - 2e^3 - 23) \approx 983.12$

Problems 14.5

1. $\int \frac{2x^6 + 8x^4 - 4x}{2x^2}\,dx$

$= \int \left(\frac{2x^6}{2x^2} + \frac{8x^4}{2x^2} - \frac{4x}{2x^2}\right) dx$

$= \int x^4\,dx + 4\int x^2\,dx - 2\int \frac{1}{x}\,dx$

$= \frac{x^5}{5} + \frac{4}{3}x^3 - 2\ln|x| + C$

2. $\int \frac{9x^2 + 5}{3x}\,dx = \int \left(3x + \frac{5}{3x}\right) dx$

$= \frac{3}{2}x^2 + \frac{5}{3}\ln|x| + C$

3. $\int\left(3x^2+2\right)\sqrt{2x^3+4x+1}dx$

$=\frac{1}{2}\int\left(2x^3+4x+1\right)^{\frac{1}{2}}\left[\left(6x^2+4\right)dx\right]$

$=\frac{1}{2}\cdot\frac{\left(2x^3+4x+1\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

$=\frac{1}{3}\left(2x^3+4x+1\right)^{\frac{3}{2}}+C$

4. $\int\frac{x}{\sqrt[4]{x^2+1}}dx=\frac{1}{2}\int\left(x^2+1\right)^{-\frac{1}{4}}[2x\ dx]$

$=\frac{1}{2}\cdot\frac{\left(x^2+1\right)^{\frac{3}{4}}}{\frac{3}{4}}+C$

$=\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}+C$

5. $\int\frac{3}{\sqrt{4-5x}}dx=3\int(4-5x)^{-1/2}dx$

$=3\left(-\frac{1}{5}\right)\int(4-5x)^{-1/2}[-5dx]$

$=-\frac{3}{5}\cdot\frac{(4-5x)^{1/2}}{\frac{1}{2}}+C$

$=-\frac{6}{5}\sqrt{4-5x}+C$

6. $\int\frac{2xe^{x^2}}{e^{x^2}-2}dx=\int\frac{1}{e^{x^2}-2}\left[2xe^{x^2}dx\right]$

$=\ln\left|e^{x^2}-2\right|+C$

7. $\int 4^{7x}dx=\int\left(e^{\ln 4}\right)^{7x}dx=\int e^{(\ln 4)(7x)}dx$

$=\frac{1}{7\ln 4}\int e^{(\ln 4)(7x)}[7\ln 4\ dx]$

$=\frac{1}{7\ln 4}\cdot e^{(\ln 4)(7x)}+C$

$=\frac{1}{7\ln 4}\left(e^{\ln 4}\right)^{7x}+C=\frac{4^{7x}}{7\ln 4}+C$

8. $\int 5^t\ dt=\int(e^{\ln 5})^t\ dt=\int e^{(\ln 5)t}dt$

$=\frac{1}{\ln 5}\int e^{(\ln 5)t}[\ln 5\,dt]=\frac{1}{\ln 5}\cdot e^{(\ln 5)t}+C$

$=\frac{5^t}{\ln 5}+C$

9. $\int 2x\left(7-e^{\frac{x^2}{4}}\right)dx=\int\left(14x-2xe^{\frac{x^2}{4}}\right)dx$

$=14\int x\ dx-2\int xe^{\frac{x^2}{4}}dx$

$=14\int x\ dx-2\cdot 2\int e^{\frac{x^2}{4}}\left[\frac{1}{2}x\ dx\right]$

$=14\cdot\frac{x^2}{2}-4\cdot e^{\frac{x^2}{4}}+C=7x^2-4e^{\frac{x^2}{4}}+C$

10. $\int\frac{e^x+1}{e^x}dx=\int(1+e^{-x})dx$

$=\int dx+\int e^{-x}dx$

$=x-e^{-x}+C$

11. By long division, $\frac{6x^2-11x+5}{3x-1}=2x-3+\frac{2}{3x-1}$.

Thus $\int\frac{6x^2-11x+5}{3x-1}dx=\int\left(2x-3+\frac{2}{3x-1}\right)dx$

$=2\int x\ dx-\int 3\ dx+2\cdot\frac{1}{3}\int\frac{1}{3x-1}[3\ dx]$

$=x^2-3x+\frac{2}{3}\ln|3x-1|+C$

12. $\int\frac{(3x+2)(x-4)}{x-3}dx=\int\frac{3x^2-10x-8}{x-3}dx$

$=\int\left(3x-1-\frac{11}{x-3}\right)dx=\frac{3}{2}x^2-x-11\ln|x-3|+C$

13. $\int\frac{5e^{2x}}{7e^{2x}+4}dx=\frac{5}{14}\int\frac{1}{7e^{2x}+4}[7e^{2x}(2)dx]$

$=\frac{5}{14}\ln(7e^{2x}+4)+C$

14. $\int 6\left(e^{4-3x}\right)^2dx=-\int e^{8-6x}[-6\ dx]$

$=-e^{8-6x}+C=-\left(e^{4-3x}\right)^2+C$

15. $\int \frac{5e^{13/x}}{x^2}dx = 5\left(-\frac{1}{13}\right)\int e^{13/x}\left[-\frac{13}{x^2}dx\right]$
$= -\frac{5}{13}e^{13/x} + C$

16. By using long division on the integrand,
$\int \frac{2x^4 - 6x^3 + x - 2}{x-2}dx$
$= \int\left(2x^3 - 2x^2 - 4x - 7 - \frac{16}{x-2}\right)dx$
$= \frac{1}{2}x^4 - \frac{2}{3}x^3 - 2x^2 - 7x - 16\ln|x-2| + C$.

17. By using long division on the integrand,
$\int \frac{5x^3}{x^2+9}dx = \int\left(5x - \frac{45x}{x^2+9}\right)dx$
$= \int 5x\ dx - \frac{45}{2}\int \frac{1}{x^2+9}[2x\ dx]$
$= \frac{5}{2}x^2 - \frac{45}{2}\ln(x^2+9) + C$
Note that since $x^2 + 9 > 0$ for all values of *x*, the absolute value bars are not needed.

18. By using long division on the integrand,
$\int \frac{5-4x^2}{3+2x}dx = \int\left(-2x + 3 - \frac{4}{3+2x}\right)dx$
$= \int(-2x+3)dx - 2\int \frac{1}{3+2x}[2\ dx]$
$= -x^2 + 3x - 2\ln|3+2x| + C$

19. $\int \frac{\left(\sqrt{x}+2\right)^2}{3\sqrt{x}}dx = \frac{2}{3}\int\left(\sqrt{x}+2\right)^2\left[\frac{1}{2\sqrt{x}}dx\right]$
$= \frac{2}{3}\cdot\frac{\left(\sqrt{x}+2\right)^3}{3} + C = \frac{2}{9}\left(\sqrt{x}+2\right)^3 + C$

20. $\int \frac{5e^s}{1+3e^s}ds = \frac{5}{3}\int \frac{1}{1+3e^s}[3e^s\ ds]$
$= \frac{5}{3}\ln(1+3e^s) + C$

21. $\int \frac{5\left(x^{\frac{1}{3}}+2\right)^4}{\sqrt[3]{x^2}}dx = 3\int 5\left(x^{\frac{1}{3}}+2\right)^4\left[\frac{1}{3}x^{-\frac{2}{3}}dx\right]$
$= 3\left(x^{\frac{1}{3}}+2\right)^5 + C$

22. $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = 2\int\left(1+x^{\frac{1}{2}}\right)^{\frac{1}{2}}\left[\frac{1}{2}x^{-\frac{1}{2}}dx\right]$
$= 2\cdot\frac{\left(1+x^{\frac{1}{2}}\right)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{4}{3}\left(1+\sqrt{x}\right)^{\frac{3}{2}} + C$

23. $\int \frac{\ln x}{x}dx = \int(\ln x)\left[\frac{1}{x}dx\right] = \frac{(\ln x)^2}{2} + C$
$= \frac{1}{2}\left(\ln^2 x\right) + C$

24. $\int \sqrt{t}\left(3 - t\sqrt{t}\right)^{0.6}dt = -\frac{2}{3}\int(3-t^{3/2})^{0.6}\left[-\frac{3}{2}t^{1/2}dt\right]$
$= -\frac{2}{3}\cdot\frac{(3-t^{3/2})^{1.6}}{1.6} + C = -\frac{5}{12}\left(3-t\sqrt{t}\right)^{1.6} + C$

25. $\int \frac{r\sqrt{\ln(r^2+1)}}{r^2+1}dr = \frac{1}{2}\int\sqrt{\ln(r^2+1)}\left[\frac{2r}{r^2+1}dr\right]$
$= \frac{1}{2}\cdot\frac{[\ln(r^2+1)]^{3/2}}{\frac{3}{2}} + C$
$= \frac{1}{3}[\ln(r^2+1)]^{3/2} + C$

26. $\int \frac{9x^5 - 6x^4 - ex^3}{7x^2}dx = \int\left(\frac{9}{7}x^3 - \frac{6}{7}x^2 - \frac{e}{7}x\right)dx$
$= \frac{9}{28}x^4 - \frac{2}{7}x^3 - \frac{e}{14}x^2 + C$

27. $\int \frac{3^{\ln x}}{x}dx = \int \frac{\left(e^{\ln 3}\right)^{\ln x}}{x}dx$
$= \frac{1}{\ln 3}\int e^{(\ln 3)\ln x}\left[\frac{\ln 3}{x}dx\right]$
$= \frac{1}{\ln 3}\cdot e^{(\ln 3)\ln x} + C$
$= \frac{1}{\ln 3}\left(e^{\ln 3}\right)^{\ln x} + C = \frac{3^{\ln x}}{\ln 3} + C$

28. $\int \frac{4}{x\ln\left(2x^2\right)}dx = 2\int \frac{1}{\ln\left(2x^2\right)}\left[\frac{2}{x}dx\right]$
$= 2\ln\left|\ln\left(2x^2\right)\right| + C$

29. $\int x^2\sqrt{e^{x^3+1}}dx = \int x^2(e^{x^3+1})^{1/2}dx$
$= \frac{2}{3}\int e^{\frac{x^3+1}{2}}\left[\frac{3}{2}x^2dx\right] = \frac{2}{3}e^{\frac{x^3+1}{2}}dx$

30. By using long division on the integrand,
$\int \frac{ax+b}{cx+d}dx = \int\left[\frac{a}{c} + \frac{bc-ad}{c}\left(\frac{1}{cx+d}\right)\right]dx$
$= \frac{a}{c}\int dx + \frac{bc-ad}{c^2}\int \frac{1}{cx+d}[c\,dx]$
$= \frac{a}{c}x + \frac{bc-ad}{c^2}\ln|cx+d| + C$

31. $\int \frac{8}{(x+3)\ln(x+3)}dx = 8\int \frac{1}{\ln(x+3)}\left[\frac{1}{x+3}dx\right]$
$= 8\ln|\ln(x+3)| + C$

32. $\int\left(e^{e^2} + x^e - 2x\right)dx = e^{e^2}x + \frac{1}{e+1}x^{e+1} - x^2 + C$

33. By using long division on the integrand,
$\int \frac{x^3+x^2-x-3}{x^2-3}dx = \int\left(x+1+\frac{2x}{x^2-3}\right)dx$
$= \int (x+1)dx + \int \frac{1}{x^2-3}[2x\ dx]$
$= \frac{x^2}{2} + x + \ln\left|x^2-3\right| + C$

34. $\int \frac{4x\ln\sqrt{1+x^2}}{1+x^2}dx = \int \frac{4x\cdot\frac{1}{2}\ln\left(1+x^2\right)}{1+x^2}dx$
$= \int \ln\left(1+x^2\right)\left[\frac{2x}{1+x^2}dx\right] = \frac{\ln^2\left(1+x^2\right)}{2} + C$

35. $\int \frac{12x^3\sqrt{\ln(x^4+1)^3}}{x^4+1}dx$
$= 3\sqrt{3}\int\sqrt{\ln(x^4+1)}\left[\frac{4x^3}{x^4+1}dx\right]$
$= \frac{3\sqrt{3}[\ln(x^4+1)]^{3/2}}{\frac{3}{2}} + C$
$= 2\sqrt{3}[\ln(x^4+1)]^{3/2} + C$
$= \frac{2}{3}\cdot 3^{3/2}[\ln(x^4+1)]^{3/2} + C$
$= \frac{2}{3}[\ln(x^4+1)^3]^{3/2} + C$

36. $\int 3\left(x^2+2\right)^{-\frac{1}{2}}xe^{\sqrt{x^2+2}}dx$
$= 3\int e^{\left(x^2+2\right)^{\frac{1}{2}}}\left[x\left(x^2+2\right)^{-\frac{1}{2}}dx\right] = 3e^{\sqrt{x^2+2}} + C$

37. $\int\left(\frac{x^3-1}{\sqrt{x^4-4x}} - \ln 7\right)dx$
$= \frac{1}{4}\int (x^4-4x)^{\frac{1}{2}}[(4x^3-4)dx] - \ln 7\int dx$
$= \frac{1}{6}(x^4-4x)^{\frac{3}{2}} - (\ln 7)x + C$

38. $\int \frac{x-x^{-2}}{x^2+2x^{-1}}dx = \frac{1}{2}\int \frac{1}{x^2+2x^{-1}}\left[\left(2x-2x^{-2}\right)\right]dx$
$= \frac{1}{2}\ln\left|x^2+2x^{-1}\right| + C$

39. $\int \frac{2x^4-8x^3-6x^2+4}{x^3}dx$
$= \int\left(2x-8-\frac{6}{x}+\frac{4}{x^3}\right)dx$
$= 2\int x\ dx - \int 8\ dx - 6\int \frac{1}{x}dx + 4\int x^{-3}dx$
$= 2\cdot\frac{x^2}{2} - 8x - 6\ln|x| + 4\cdot\frac{x^{-2}}{-2} + C$
$= x^2 - 8x - 6\ln|x| - \frac{2}{x^2} + C$

40. $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx = \int \frac{1}{e^x+e^{-x}}[(e^x-e^{-x})dx]$
$= \ln\left|e^x+e^{-x}\right| + C$

41. By using long division on the integrand,

$$\int \frac{x}{x+1}\,dx = \int\left(1-\frac{1}{x+1}\right)dx = x-\ln|x+1|+C$$

42. $$\int \frac{2x}{\left(x^2+1\right)\ln\left(x^2+1\right)}\,dx$$
$$= \int \frac{1}{\ln\left(x^2+1\right)}\left[\frac{2x}{x^2+1}\,dx\right]$$
$$= \ln\left|\ln\left(x^2+1\right)\right| + C$$

43. $$\int \frac{xe^{x^2}}{\sqrt{e^{x^2}+2}}\,dx = \frac{1}{2}\int\left(e^{x^2}+2\right)^{-\frac{1}{2}}\left[2xe^{x^2}\,dx\right]$$
$$= \frac{1}{2}\cdot\frac{\left(e^{x^2}+2\right)^{\frac{1}{2}}}{\frac{1}{2}} + C = \sqrt{e^{x^2}+2}+C$$

44. $$\int \frac{5}{(3x+1)[1+\ln(3x+1)]^2}\,dx$$
$$= \frac{5}{3}\int[1+\ln(3x+1)]^{-2}\left[\frac{1}{3x+1}\cdot 3\,dx\right]$$
$$= -\frac{5}{3[1+\ln(3x+1)]}+C$$

45. $$\int \frac{\left(e^{-x}+5\right)^3}{e^x}\,dx = -\int\left(e^{-x}+5\right)^3\left[-e^{-x}dx\right]$$
$$= -\frac{\left(e^{-x}+5\right)^4}{4}+C$$

46. $$\int\left[\frac{1}{8x+1}-\frac{1}{e^x\left(8+e^{-x}\right)^2}\right]dx$$
$$= \frac{1}{8}\int\frac{1}{8x+1}[8\,dx] - (-1)\int\left(8+e^{-x}\right)^{-2}\left[-e^{-x}dx\right]$$
$$= \frac{1}{8}\ln|8x+1| + \frac{\left(8+e^{-x}\right)^{-1}}{-1}+C$$
$$= \frac{1}{8}\ln|8x+1| - \frac{1}{8+e^{-x}}+C$$

47. $$\int\left(x^3+ex\right)\sqrt{x^2+e}\,dx = \int x\left(x^2+e\right)\left(x^2+e\right)^{\frac{1}{2}}dx$$
$$= \frac{1}{2}\int\left(x^2+e\right)^{\frac{3}{2}}[2x\,dx] = \frac{1}{2}\cdot\frac{\left(x^2+e\right)^{\frac{5}{2}}}{\frac{5}{2}}+C$$
$$= \frac{1}{5}\left(x^2+e\right)^{\frac{5}{2}}+C$$

48. $$\int 3^{x\ln x}(1+\ln x)dx = \int\left(e^{\ln 3}\right)^{x\ln x}(1+\ln x)dx$$
$$= \frac{1}{\ln 3}\int e^{(\ln 3)x\ln x}[(\ln 3)(1+\ln x)dx]$$
$$= \frac{1}{\ln 3}\cdot e^{(\ln 3)x\ln x}+C = \frac{1}{\ln 3}\left(e^{\ln 3}\right)^{x\ln x}+C$$
$$= \frac{3^{x\ln x}}{\ln 3}+C$$

49. $$\int\sqrt{x}\sqrt{(8x)^{\frac{3}{2}}+3}\,dx = \int\left(8^{\frac{3}{2}}x^{\frac{3}{2}}+3\right)^{\frac{1}{2}}\cdot x^{\frac{1}{2}}dx$$
$$= \frac{2}{3\cdot 8^{\frac{3}{2}}}\int\left(8^{\frac{3}{2}}x^{\frac{3}{2}}+3\right)^{\frac{1}{2}}\left[8^{\frac{3}{2}}\cdot\frac{3}{2}\cdot x^{\frac{1}{2}}dx\right]$$
$$= \frac{2}{3\cdot 16\sqrt{2}}\cdot\frac{\left(8^{\frac{3}{2}}x^{\frac{3}{2}}+3\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$$
$$= \frac{1}{36\sqrt{2}}\left[(8x)^{\frac{3}{2}}+3\right]^{\frac{3}{2}}+C$$

50. $$\int\frac{7}{x(\ln x)^{\pi}}\,dx = 7\int(\ln x)^{-\pi}\left[\frac{1}{x}dx\right]$$
$$= 7\cdot\frac{(\ln x)^{-\pi+1}}{-\pi+1}+C$$
$$= \frac{7}{1-\pi}(\ln x)^{1-\pi}+C$$

51. $$\int\frac{\sqrt{s}}{e^{\sqrt{s^3}}}\,ds = -\frac{2}{3}\int e^{-s^{\frac{3}{2}}}\left[-\frac{3}{2}s^{\frac{1}{2}}ds\right]$$
$$= -\frac{2}{3}e^{-\sqrt{s^3}}+C$$

52. $$\int\frac{\ln^3 x}{3x}\,dx = \frac{1}{3}\int(\ln x)^3\left[\frac{1}{x}dx\right]$$
$$= \frac{1}{3}\cdot\frac{(\ln x)^4}{4}+C = \frac{1}{12}\ln^4 x+C$$

53. $e^{\ln(x^2+1)}$ is simply x^2+1. Thus

$$\int e^{\ln(x^2+1)}dx = \int (x^2+1)dx = \frac{1}{3}x^3 + x + C$$

54. $\int dx = \int 1\, dx = x + C$

55.
$$\int \frac{\ln\left(\frac{e^x}{x}\right)}{x}dx = \int \frac{\ln e^x - \ln x}{x}dx$$
$$= \int \frac{x - \ln x}{x}dx$$
$$= \int \left(1 - \frac{\ln x}{x}\right)dx$$
$$= \int dx - \int \ln x\left[\frac{1}{x}dx\right]$$
$$= x - \frac{1}{2}(\ln x)^2 + C$$

56.
$$\int e^{f(x)+\ln(f'(x))}dx = \int e^{f(x)} \cdot e^{\ln(f'(x))}dx$$
$$= \int e^{f(x)}[f'(x)\,dx]$$
$$= e^{f(x)} + C$$

57. $\frac{dr}{dq} = \frac{200}{(q+2)^2}$

$$r = \int 200(q+2)^{-2}dq = 200 \cdot \frac{(q+2)^{-1}}{-1} + C$$
$$= -\frac{200}{q+2} + C$$

When $q = 0$, then $r = 0$, so $0 = -100 + C$, or $C = 100$. Hence $r = -\frac{200}{q+2} + 100 = \frac{100q}{q+2}$. Since $r = pq$, then $p = \frac{r}{q} = \frac{100}{q+2}$.

The demand function is $p = \frac{100}{q+2}$.

58. $\frac{dr}{dq} = \frac{900}{(2q+3)^3}$

$$r = \int 900(2q+3)^{-3}dq$$
$$= 900 \cdot \frac{1}{2}\int (2q+3)^{-3}[2\ dq]$$
$$= 450 \cdot \frac{(2q+3)^{-2}}{-2} + C = -\frac{225}{(2q+3)^2} + C$$

When $q = 0$, then $r = 0$, so $0 = -25 + C$ or $C = 25$. Hence $r = -\frac{225}{(2q+3)^2} + 25$. Since $r = pq$, then $p = \frac{r}{q} = \frac{25}{q} - \frac{225}{q(2q+3)^2}$

The demand function is $p = \frac{25}{q}\left[1 - \frac{9}{(2q+3)^2}\right]$.

59. $\frac{dc}{dq} = \frac{20}{q+5}$

$$c = \int \frac{20}{q+5}dq = 20\int \frac{1}{q+5}dq = 20\ln|q+5| + C$$

When $q = 0$, then $c = 2000$, so $2000 = 20\ln(5) + C$, or $C = 2000 - 20\ln 5$.

Hence $c = 20\ln|q+5| + 2000 - 20\ln 5$

$$= 20\left(\ln|q+5| - \ln 5\right) + 2000 = 20\ln\left|\frac{q+5}{5}\right| + 2000$$

The cost function is $c = 20\ln\left|\frac{q+5}{5}\right| + 2000$.

60. $\frac{dc}{dq} = 4e^{0.005q}$

$$c = \int 4e^{0.005q}dq$$
$$= 4 \cdot \frac{1}{0.005}\int e^{0.005q}[0.005\,dq]$$
$$= 800e^{0.005q} + C$$

When $q = 0$, $c = 2000$, so $2000 = 800 + C$, or $C = 1200$.

The cost function is $c = 800e^{0.005q} + 1200$.

61. $\frac{dC}{dI} = \frac{1}{\sqrt{I}}$

$$C = \int I^{-\frac{1}{2}}dI = \frac{I^{\frac{1}{2}}}{\frac{1}{2}} + C_1 = 2\sqrt{I} + C_1$$

$C(9) = 8$ implies that $8 = 2 \cdot 3 + C_1$, or $C_1 = 2$.

Thus $C = 2\sqrt{I} + 2 = 2\left(\sqrt{I} + 1\right)$.

The consumption function is $C = 2\left(\sqrt{I} + 1\right)$.

62. $\frac{dC}{dI} = \frac{1}{2} - \frac{1}{2\sqrt{2I}}$

$C = \int\left(\frac{1}{2} - \frac{(2I)^{-\frac{1}{2}}}{2}\right)dI$

$= \frac{1}{2}\int dI - \frac{1}{4}\int (2I)^{-\frac{1}{2}}[2\,dI]$

$= \frac{1}{2}\cdot I - \frac{1}{4}\cdot\frac{(2I)^{\frac{1}{2}}}{\frac{1}{2}} + C_1$

$= \frac{I}{2} - \frac{\sqrt{2I}}{2} + C_1$

$C(2) = \frac{3}{4}$ implies $\frac{3}{4} = 1 - \frac{\sqrt{4}}{2} + C_1$, so $C_1 = \frac{3}{4}$.

The consumption function is $C = \frac{I}{2} - \frac{\sqrt{2I}}{2} + \frac{3}{4}$.

63. $\frac{dC}{dI} = \frac{3}{4} - \frac{1}{6\sqrt{I}}$

$C = \int\left[\frac{3}{4} - \frac{I^{-\frac{1}{2}}}{6}\right]dI = \int\frac{3}{4}dI - \frac{1}{6}\int I^{-\frac{1}{2}}dI$

$= \frac{3}{4}I - \frac{1}{6}\cdot\frac{I^{\frac{1}{2}}}{\frac{1}{2}} + C_1 = \frac{3}{4}I - \frac{\sqrt{I}}{3} + C_1$

Thus $C = \frac{3}{4}I - \frac{\sqrt{I}}{3} + C_1$.

$C(25) = 23$ implies that $23 = \frac{3}{4}\cdot 25 - \frac{5}{3} + C_1$, so

$C_1 = \frac{71}{12}$.

The consumption function is

$C = \frac{3}{4}I - \frac{1}{3}\sqrt{I} + \frac{71}{12}$.

64. $\frac{dc}{dq} = 10 - \frac{100}{q+10}$

$c = \int\left(10 - \frac{100}{q+10}\right)dq = 10q - 100\ln|q+10| + C$

Avg. cost $= \frac{c}{q} = 10 - 100\frac{\ln|q+10|}{q} + \frac{C}{q}$

When $q = 100$, then avg. cost = 50, so

$50 = 10 - 100\frac{\ln(110)}{100} + \frac{C}{100}$, or

$C = 100(40 + \ln(110))$. Thus

$c = 10q - 100\ln|q+10| + 100(40 + \ln(110))$

Evaluating c when $q = 0$ gives fixed cost:

$c(0) = -100\ln(10) + 100(40 + \ln(110)) \approx 4240$.

The fixed cost is \$4240.

65. $\frac{dc}{dq} = \frac{100q^2 - 3998q + 60}{q^2 - 40q + 1}$

a. $\left.\frac{dc}{dq}\right|_{q=40} = \frac{100(40)^2 - 3998(40) + 60}{(40)^2 - 40(40) + 1}$

= \$140 per unit

b. To find c, we integrate $\frac{dc}{dq}$ by using long division:

$c = \int\frac{100q^2 - 3998q + 60}{q^2 - 40q + 1}dq$

$= \int\left(100 + \frac{2q - 40}{q^2 - 40q + 1}\right)dq$

$= \int 100\,dq + \int\frac{1}{q^2 - 40q + 1}[(2q - 40)dq]$

Thus $c = 100q + \ln\left|q^2 - 40q + 1\right| + C$. When $q = 0$, then $c = 10{,}000$, so

$10{,}000 = 0 + \ln(1) + C$, so $C = 10{,}000$.

Hence $c = 100q + \ln\left|q^2 - 40q + 1\right| + 10{,}000$.

When $q = 40$, then

$c = 4000 + \ln(1) + 10{,}000 = \$14{,}000$.

c. If $c = f(q)$, then

$f(q + dq) \approx f(q) + dc = f(q) + \frac{dc}{dq}dq$

Letting $q = 40$ and $dq = 2$, we have

$f(42) = f(40 + 2) \approx f(40) + \left.\frac{dc}{dq}\right|_{q=40}\cdot(2)$

$= 14{,}000 + 140(2) = \$14{,}280$

66. $\frac{dc}{dq} = \frac{9}{10}\sqrt{q}\sqrt{0.04q^{\frac{3}{2}} + 4}$

a. $\left.\frac{dc}{dq}\right|_{q=25} = \frac{9}{10}\sqrt{25}\sqrt{9} = \frac{9}{10}\cdot 5\cdot 3 = \frac{27}{2}$

= \$13.50 per unit

b. $c = \int \frac{9}{10}\sqrt{q}\sqrt{0.04q^{\frac{3}{2}}+4}dq$

$= \frac{0.9}{0.06}\int\left(0.04q^{\frac{3}{2}}+4\right)^{\frac{1}{2}}\left[0.06q^{\frac{1}{2}}dq\right]$

$= \frac{0.9}{0.06}\cdot\frac{\left(0.04q^{\frac{3}{2}}+4\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$

Thus $c = 10\left(0.04q^{\frac{3}{2}}+4\right)^{\frac{3}{2}}+C$. When $q = 0$, then $c = 360$, so $360 = 10(4)^{\frac{3}{2}}+C$, or $C = 280$. Hence $c = 10\left(0.04q^{\frac{3}{2}}+4\right)^{\frac{3}{2}}+280$.

When $q = 25$, then $c = 10(9)^{\frac{3}{2}}+280 = \$550$.

c. If $c = f(q)$, then $f(q + dq) \approx f(q) + dc$ $= f(q)+\frac{dc}{dq}dq$. Letting $q = 25$ and $dq = -2$, we have

$f(23) = f(25-2) \approx f(25)+\left.\frac{dc}{dq}\right|_{q=25}\cdot(-2)$

$= 550 + 13.50(-2) = \$523$

67. $\frac{dV}{dt} = \frac{8t^3}{\sqrt{0.2t^4+8000}}$

$V = \int\frac{8t^3}{\sqrt{0.2t^4+8000}}dt$

$= 10\int\left(0.2t^4+8000\right)^{-\frac{1}{2}}\left[0.8t^3\right]dt$

$= 10\frac{\left(0.2t^4+8000\right)^{\frac{1}{2}}}{\frac{1}{2}}+C$

Thus $V = 20\sqrt{0.2t^4+8000}+C$. If $t = 0$, then $V = 500$, so $500 = 20\sqrt{8000}+C$, $500 = 20\sqrt{1600\cdot 5}+C$, $500 = 800\sqrt{5}+C$, or $C = 500-800\sqrt{5}$. Hence

$V = 20\sqrt{0.2t^4+8000}+500-800\sqrt{5}$.

When $t = 10$, then

$V = 20\sqrt{10,000}+500-800\sqrt{5}$

$= 20(100)+500-800\sqrt{5} \approx \$711$ per acre.

68. $\frac{dr}{dq} = \frac{a}{e^q+b} = \frac{ae^{-q}}{(e^q+b)e^{-q}} = \frac{ae^{-q}}{1+be^{-q}}$

$r = \int\frac{ae^{-q}}{1+be^{-q}}dq = \left(-\frac{1}{b}\right)a\int\frac{1}{1+be^{-q}}[-be^{-q}dq]$

$= -\frac{a}{b}\ln(1+be^{-q})+C$

Now $r = 0$ when $q = 0$, so $0 = -\frac{a}{b}\ln(1+b)+C$, or $C = \frac{a}{b}\ln(1+b)$. Hence

$r = -\frac{a}{b}\ln(1+be^{-q})+\frac{a}{b}\ln(1+b)$

$= \frac{a}{b}\ln\frac{1+b}{1+be^{-q}}$

$p = \frac{r}{q} = \frac{a}{bq}\ln\frac{1+b}{1+be^{-q}}$

69. $S = \int\frac{dS}{dI}dI = \int\frac{5}{(I+2)^2}dI$

$= 5\int(I+2)^{-2}dI = 5\cdot\frac{(I+2)^{-1}}{-1}+C_1$

Thus $S = -\frac{5}{I+2}+C_1$. If C is the total national consumption (in billions of dollars), then $C + S = I$, or $C = I - S$. Hence $C = I+\frac{5}{I+2}-C_1$.

When $I = 8$, then $C = 7.5$, so $7.5 = 8+\frac{1}{2}-C_1$, or $C_1 = 1$. Thus $S = 1-\frac{5}{I+2}$. If $S = 0$, then

$0 = 1-\frac{5}{I+2} \Rightarrow \frac{5}{I+2} = 1 \Rightarrow 5 = I+2 \Rightarrow I = 3$

70. a. If C is total national consumption (in billions of dollars), then

$\frac{dC}{dI} = 1-\frac{dS}{dI} = 1-\left(\frac{2}{5}-\frac{1.6}{\sqrt[3]{2I^2}}\right)$. Thus

$\left.\frac{dC}{dI}\right|_{I=16} = 1-\left(\frac{2}{5}-\frac{1.6}{\sqrt[3]{2(16)^2}}\right)$

$= 1-\left(\frac{2}{5}-\frac{1.6}{8}\right) = \frac{4}{5}$.

b. $S = \int \frac{dS}{dI}\,dI$

$$= \int \left(\frac{2}{5} - \frac{1.6}{\sqrt[3]{2I^2}} \right) dT$$

$$= \int \left(\frac{2}{5} - \frac{1.6}{\sqrt[3]{2}} I^{-2/3} \right) dI$$

$$= \frac{2}{5}I - \frac{1.6}{\sqrt[3]{2}} \cdot \frac{I^{1/3}}{\frac{1}{3}} + C_1$$

$$= \frac{2}{5}I - \frac{4.8}{\sqrt[3]{2}} \sqrt[3]{I} + C_1$$

Thus $S = \frac{2}{5}I - 4.8\sqrt[3]{\frac{I}{2}} + C_1$. When $I = 54$,

$S = 10$, so $10 = \frac{2}{5}(54) - 4.8\sqrt[3]{\frac{54}{2}} + C_1$, or $C_1 = 2.8$. Thus $S = \frac{2}{5}I - 4.8\sqrt[3]{\frac{I}{2}} + 2.8$.

If C is the total national consumption (in billions of dollars), then $C + S = I$, or

$$C = I - S$$

$$= I - \left(\frac{2}{5}I - 4.8\sqrt[3]{\frac{I}{2}} + 2.8 \right)$$

$$= \frac{3}{5}I + 4.8\sqrt[3]{\frac{I}{2}} - 2.8.$$

c. From (b), when $I = 16$, then $C = \frac{3}{5}(16) + 4.8\sqrt[3]{\frac{16}{2}} - 2.8 = 16.4$

Thus consumption is \$16.4 billion when income is \$16 billion.

d. If $C = f(I)$, then

$f(I + dI) \approx f(I) + dC = f(I) + \frac{dC}{dI}dI$. Let

$I = 18$ and $dI = 2$. Then

$$f(18) = f(16+2)$$

$$\approx f(16) + \left.\frac{dC}{dI}\right|_{I=16}(2)$$

$$= 16.4 + \frac{4}{5}(2)$$

$$= 18.$$

Thus when income is \$18 billion, consumption is approximately \$18 billion.

Apply It 14.6

10. Divide the interval [0, 10] into n subintervals of equal length Δx, so $\Delta x = \frac{10}{n}$. The endpoints of the subintervals are 0, $\frac{10}{n}$, $2\left(\frac{10}{n}\right)$, $3\left(\frac{10}{n}\right)$, ... , $(n-1)\left(\frac{10}{n}\right)$, and $n\left(\frac{10}{n}\right) = 10$. Letting S_n denote the sum of the areas of the rectangles corresponding to right-hand endpoints, we have

$$S_n = \frac{10}{n}R\left(\frac{10}{n}\right)+\frac{10}{n}R\left[2\left(\frac{10}{n}\right)\right]+\ldots+\frac{10}{n}R\left[n\left(\frac{10}{n}\right)\right]$$
$$=\frac{10}{n}\left[\left\{600-0.5\left(\frac{10}{n}\right)\right\}+\left\{600-0.5(2)\left(\frac{10}{n}\right)\right\}+\ldots+\left\{600-0.5(n)\left(\frac{10}{n}\right)\right\}\right]$$
$$=\frac{10}{n}\left[600n-0.5\left(\frac{10}{n}\right)\{1+2+\ldots+n\}\right]$$
$$=\frac{10}{n}\left[600n-0.5\left(\frac{10}{n}\right)\frac{n(n+1)}{2}\right]$$
$$=\frac{10}{n}[600n-2.5(n+1)]$$
$$=6000-25\left(\frac{n+1}{n}\right)$$

Now take the limit of S_n as $n \to \infty$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[6000-25\left(\frac{n+1}{n}\right)\right] = \lim_{n\to\infty}\left[6000-25\left(1+\frac{1}{n}\right)\right] = 6000-25 = 5975$$

The total revenue for selling 10 units is $5975.

Problems 14.6

1. $f(x) = x + 1,\ y = 0,\ x = 0,\ x = 1$

$S_4,\ \Delta x = \frac{1}{4}$

$$S_4 = \frac{1}{4}f\left(\frac{1}{4}\right)+\frac{1}{4}f\left(\frac{2}{4}\right)+\frac{1}{4}f\left(\frac{3}{4}\right)\frac{1}{4}f\left(\frac{4}{4}\right)=\frac{1}{4}\left[\frac{5}{4}+\frac{6}{4}+\frac{7}{4}+\frac{8}{4}\right]=\frac{1}{4}\cdot\frac{13}{2}=\frac{13}{8}$$

The area is approximately $\frac{13}{8}$ sq units.

2. $f(x) = 3x,\ y = 0,\ x = 1$

$S_5,\ \Delta x = \frac{1}{5}$

$$S_5 = \frac{1}{5}f\left(\frac{1}{5}\right)+\frac{1}{5}f\left(\frac{2}{5}\right)+\frac{1}{5}f\left(\frac{3}{5}\right)+\frac{1}{5}f\left(\frac{4}{5}\right)+\frac{1}{5}f\left(\frac{5}{5}\right)=\frac{1}{5}\left[\frac{3}{5}+\frac{6}{5}+\frac{9}{5}+\frac{12}{5}+\frac{15}{5}\right]=\frac{1}{5}\cdot\frac{45}{5}=\frac{9}{5}$$

The area is approximately $\frac{9}{5}$ sq units.

3. $f(x) = x^2,\ y = 0,\ x = 1$

$S_4,\ \Delta x = \frac{1}{4}$

$$S_4 = \frac{1}{4}f\left(\frac{1}{4}\right)+\frac{1}{4}f\left(\frac{2}{4}\right)+\frac{1}{4}f\left(\frac{3}{4}\right)+\frac{1}{4}f\left(\frac{4}{4}\right)$$
$$=\frac{1}{4}\left[\frac{1}{16}+\frac{4}{16}+\frac{9}{16}+\frac{16}{16}\right]$$
$$=\frac{1}{4}\cdot\frac{30}{16}=\frac{15}{32}$$

The area is approximately $\frac{15}{32}$ sq units.

4. $f(x) = x^2 + 1$, $y = 0$, $x = 0$, $x = 1$

S_2, $\Delta x = \frac{1}{2}$

$$S_2 = \frac{1}{2} f\left(\frac{1}{2}\right) + \frac{1}{2} f\left(\frac{2}{2}\right) = \frac{1}{2}\left[\frac{5}{4} + \frac{8}{4}\right] = \frac{1}{2} \cdot \frac{13}{4} = \frac{13}{8}$$

The area is approximately $\frac{13}{8}$ sq units.

5. $f(x) = 4x$; [0, 1]

$\Delta x = \frac{1}{n}$

$$S_n = \frac{1}{n} f\left(\frac{1}{n}\right) + \ldots + \frac{1}{n} f\left(n \cdot \frac{1}{n}\right)$$
$$= \frac{1}{n}\left[f\left(\frac{1}{n}\right) + \ldots + f\left(n \cdot \frac{1}{n}\right)\right]$$
$$= \frac{1}{n}\left[4 \cdot \frac{1}{n} + \ldots + 4 \cdot \frac{n}{n}\right]$$
$$= \frac{4}{n^2}[1 + \ldots + n] = \frac{4}{n^2} \cdot \frac{n(n+1)}{2}$$
$$= \frac{2(n+1)}{n}$$

6. $f(x) = 2x + 1$; [0, 2]

$\Delta x = \frac{2}{n}$

$$S_n = \frac{2}{n} f\left(\frac{2}{n}\right) + \cdots + \frac{2}{n} f\left(n \cdot \frac{2}{n}\right)$$
$$= \frac{2}{n}\left[f\left(\frac{2}{n}\right) + \cdots + f\left(n \cdot \frac{2}{n}\right)\right]$$
$$= \frac{2}{n}\left\{\left[2\left(\frac{2}{n}\right) + 1\right] + \cdots + \left[2\left(n \cdot \frac{2}{n}\right) + 1\right]\right\}$$
$$= \frac{2}{n}\left\{\frac{4}{n}(1 + \cdots + n) + n\right\}$$
$$= \frac{2}{n}\left\{\frac{4}{n} \cdot \frac{n(n+1)}{2} + n\right\}$$
$$= \frac{4(n+1)}{n} + 2$$

7. a. $S_n = \frac{1}{n}\left[\left(\frac{1}{n}+1\right)+\left(\frac{2}{n}+1\right)+\ldots+\left(\frac{n}{n}+1\right)\right]$

$= \frac{1}{n}\left[\frac{1}{n}(1+2+\ldots+n)+n\right]$

$= \frac{1}{n}\left[\frac{1}{n}\cdot\frac{n(n+1)}{2}+n\right]$

$= \frac{n+1}{2n}+1$

b. $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty}\left[\frac{n+1}{2n}+1\right]$

$= \lim\limits_{n\to\infty}\left[\frac{1}{2}+\frac{1}{2n}+1\right]$

$= \frac{1}{2}+0+1 = \frac{3}{2}$

8. a. $S_n = \frac{2}{n}\left[\left(\frac{2}{n}\right)^2+\left(2\cdot\frac{2}{n}\right)^2+\ldots+\left(n\cdot\frac{2}{n}\right)^2\right]$

$= \frac{2}{n}\cdot\frac{2^2}{n^2}\left[1^2+2^2+\ldots+n^2\right] = \frac{4(n+1)(2n+1)}{3n^2}$

$= \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

b. $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty}\frac{4(n+1)(2n+1)}{3n^2} = \lim\limits_{n\to\infty}\frac{4\left[2n^2+3n+1\right]}{3n^2}$

$= \lim\limits_{n\to\infty}\left[\frac{4}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\right] = \frac{4}{3}(2+0+0) = \frac{8}{3}$

9. $f(x) = x + 1$, $y = 0$, $x = 0$, $x = 1$

$\Delta x = \frac{1}{n}$

$S_n = \frac{1}{n}f\left(\frac{1}{n}\right)+\ldots+\frac{1}{n}f\left(n\cdot\frac{1}{n}\right) = \frac{1}{n}\left[\left(\frac{1}{n}+1\right)+\cdots+\left(n\cdot\frac{1}{n}+1\right)\right]$

$= \frac{1}{n}\left[\left(\frac{1}{n}+\cdots+\frac{n}{n}\right)+n\right] = \frac{1}{n^2}(1+\cdots+n)+1 = \frac{1}{n^2}\cdot\frac{n(n+1)}{2}+1 = \frac{n+1}{2n}+1 = \frac{1}{2}+\frac{1}{2n}+1 = \frac{3}{2}+\frac{1}{2n}$

$= \frac{1}{2}\left[3+\frac{1}{n}\right]$

$\lim\limits_{n\to\infty} S_n = \frac{3}{2}$

The area is $\frac{3}{2}$ sq unit.

10. $f(x) = 3x,\ y = 0,\ x = 1$

$\Delta x = \frac{1}{n}$

$S_n = \frac{1}{n} f\left(\frac{1}{n}\right) + \ldots + \frac{1}{n} f\left(n \cdot \frac{1}{n}\right) = \frac{1}{n}\left[3 \cdot \frac{1}{n} + \ldots + 3 \cdot \frac{n}{n}\right]$

$= \frac{3}{n^2}[1 + \ldots + n] = \frac{3}{n^2} \cdot \frac{n(n+1)}{2} = \frac{3}{2} \cdot \frac{n+1}{n} = \frac{3}{2}\left[1 + \frac{1}{n}\right]$

$\lim_{n\to\infty} S_n = \frac{3}{2}$

The area is $\frac{3}{2}$ sq units.

11. $f(x) = x^2,\ y = 0,\ x = 1$

$\Delta x = \frac{1}{n}$

$S_n = \frac{1}{n} f\left(\frac{1}{n}\right) + \ldots + \frac{1}{n} f\left(n \cdot \frac{1}{n}\right) = \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \ldots + \left(n \cdot \frac{1}{n}\right)^2\right]$

$= \frac{1}{n^3}\left[1^2 + \ldots + n^2\right] = \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$

$= \frac{1}{6} \cdot \frac{2n^2 + 3n + 1}{n^2} = \frac{1}{6}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$

$\lim_{n\to\infty} S_n = \frac{1}{3}$

The area is $\frac{1}{3}$ sq unit.

12. $y = x^2,\ y = 0,\ x = 1,\ x = 2$

$$\Delta x = \frac{1}{n}$$

$$S_n = \frac{1}{n} f\left(1+\frac{1}{n}\right)+\frac{1}{n} f\left(1+2\cdot\frac{1}{n}\right)+\cdots\frac{1}{n} f\left(1+n\cdot\frac{1}{n}\right)$$

$$= \frac{1}{n}\left\{\left[1+\frac{1}{n}\right]^2+\cdots+\left[1+n\cdot\frac{1}{n}\right]^2\right\}$$

$$= \frac{1}{n}\left\{\left[1+2\cdot\frac{1}{n}+\frac{1}{n^2}\right]+\cdots+\left[1+2n\cdot\frac{1}{n}+n^2\cdot\frac{1}{n^2}\right]\right\}$$

$$= \frac{1}{n}\left\{n+\frac{2}{n}(1+\cdots+n)+\frac{1}{n^2}(1^2+\cdots+n^2)\right\}$$

$$= 1+\frac{2}{n^2}\cdot\frac{n(n+1)}{2}+\frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= 1+\frac{n+1}{n}+\frac{1}{6}\cdot\frac{2n^2+3n+1}{n^2}$$

$$= 1+\left[1+\frac{1}{n}\right]+\frac{1}{6}\left[2+\frac{3}{n}+\frac{1}{n^2}\right]$$

$$\lim_{n\to\infty} S_n = 1+1+\frac{1}{3}=\frac{7}{3}$$

The area is $\frac{7}{3}$ sq units.

13. $f(x) = 3x^2,\ y = 0,\ x = 1$

$$\Delta x = \frac{1}{n}$$

$$S_n = \frac{1}{n} f\left(\frac{1}{n}\right)+\ldots+\frac{1}{n} f\left(n\cdot\frac{1}{n}\right)=\frac{1}{n}\left[3\left(\frac{1}{n}\right)^2+\ldots+3\left(n\cdot\frac{1}{n}\right)^2\right]$$

$$= \frac{3}{n^3}\left[1^2+\ldots+n^2\right]=\frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}=\frac{1}{2}\cdot\frac{2n^2+3n+1}{n^2}=\frac{1}{2}\left[2+\frac{3}{n}+\frac{1}{n^2}\right]$$

$$\lim_{n\to\infty} S_n = 1$$

The area is 1 sq unit.

14. $f(x) = 9 - x^2,\ y = 0,\ x = 0$

$$\Delta x = \frac{3}{n}$$

$$S_n = \frac{3}{n} f\left(\frac{3}{n}\right) + \ldots + \frac{3}{n} f\left(n \cdot \frac{3}{n}\right)$$

$$= \frac{3}{n}\left\{\left[9 - \left(\frac{3}{n}\right)^2\right] + \ldots + \left[9 - \left(n \cdot \frac{3}{n}\right)^2\right]\right\}$$

$$= \frac{3}{n}\left\{9n - \left(\frac{3}{n}\right)^2 \left[1^2 + \ldots + n^2\right]\right\}$$

$$= 27 - \left[\frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= 27 - \frac{9}{2}\left[\frac{2n^2 + 3n + 1}{n^2}\right] = 27 - \frac{9}{2}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$$

$$\lim_{n \to \infty} S_n = 27 - 9 = 18$$

The area is 18 sq units.

15. $\int_1^3 5x\, dx$

Let $f(x) = 5x$.

$$\Delta x = \frac{2}{n}$$

$$S_n = \frac{2}{n} f\left(1 + \frac{2}{n}\right) + \cdots + \frac{2}{n} f\left(1 + n \cdot \frac{2}{n}\right)$$

$$= \frac{2}{n}\left[5\left(1 + \frac{2}{n}\right) + \cdots + 5\left(1 + n \cdot \frac{2}{n}\right)\right]$$

$$= \frac{10}{n}\left[(1 + \cdots + 1) + \frac{2}{n}(1 + \cdots + n)\right]$$

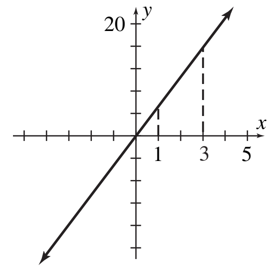
$$= \frac{10}{n}\left[n + \frac{2}{n} \cdot \frac{n(n+1)}{2}\right]$$

$$= \frac{10}{n}[n + n + 1]$$

$$= \frac{10}{n}(2n + 1)$$

$$= 20 + \frac{10}{n}$$

$$\int_1^3 5x\, dx = \lim_{n \to \infty} S_n = 20$$



16. $\int_0^a b\, dx$

Let $f(x) = b$.

$$\Delta x = \frac{a}{n}$$

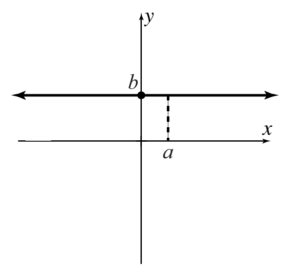
$$S_n = \frac{a}{n} f\left(\frac{a}{n}\right) + \cdots + \frac{a}{n} f\left(n \cdot \frac{a}{n}\right)$$

$$= \frac{a}{n}(b + \cdots + b)$$

$$= \frac{a}{n} \cdot nb$$

$$= ab$$

$$\int_0^a b\, dx = \lim_{n \to \infty} S_n = ab$$



17. $\int_0^3 -4x\ dx$

Let $f(x) = -4x$.

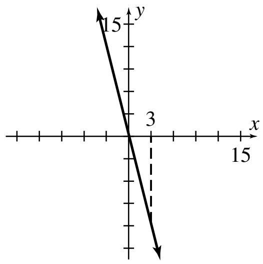
$$\Delta x = \frac{3}{n}$$

$$S_n = \frac{3}{n} f\left(\frac{3}{n}\right) + \ldots + \frac{3}{n} f\left(n \cdot \frac{3}{n}\right)$$

$$= \frac{3}{n}\left[-4\left(\frac{3}{n}\right) - \ldots - 4\left(n \cdot \frac{3}{n}\right)\right] = -\frac{36}{n^2}[1 + \ldots + n]$$

$$= -\frac{36}{n^2} \cdot \frac{n(n+1)}{2} = -18 \cdot \frac{n+1}{n} = -18\left[1 + \frac{1}{n}\right]$$

$$\int_0^3 -4x\ dx = \lim_{n \to \infty} S_n = -18$$



18. $\int_1^4 (2x+1)dx$

Let $f(x) = 2x + 1$.

$$\Delta x = \frac{4-1}{n} = \frac{3}{n}$$

$$S_n = \frac{3}{n} f\left(1+\frac{3}{n}\right)+\cdots+\frac{3}{n} f\left(1+n\cdot\frac{3}{n}\right)$$

$$= \frac{3}{n}\left\{\left[2\left(1+\frac{3}{n}\right)+1\right]+\cdots+\left[2\left(1+n\cdot\frac{3}{n}\right)+1\right]\right\}$$

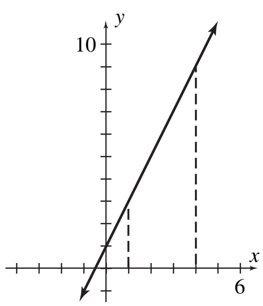
$$= \frac{3}{n}\left\{2n+\frac{6}{n}(1+2+\cdots+n)+n\right\}$$

$$= 6+\frac{18}{n^2}\cdot\frac{n(n+1)}{2}+3$$

$$= 9+9\cdot\frac{n+1}{n}$$

$$= 9+9\left(1+\frac{1}{n}\right)$$

$$\int_1^4 (2x+1)dx = \lim_{n\to\infty} S_n = 9+9 = 18$$



19. $\int_0^1 \left(x^2+x\right)dx$

Let $f(x) = x^2 + x$.

$$\Delta x = \frac{1}{n}$$

$$S_n = \frac{1}{n} f\left(\frac{1}{n}\right)+\ldots+\frac{1}{n} f\left(n\cdot\frac{1}{n}\right)$$

$$= \frac{1}{n}\left\{\left[\left(\frac{1}{n}\right)^2+\frac{1}{n}\right]+\ldots+\left[\left(n\cdot\frac{1}{n}\right)^2+n\cdot\frac{1}{n}\right]\right\}$$

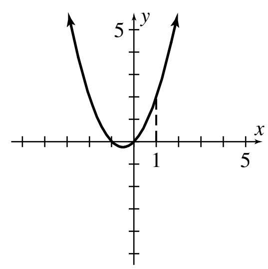
$$= \frac{1}{n}\left\{\left(\frac{1}{n}\right)^2\left[1^2+\ldots+n^2\right]+\frac{1}{n}[1+\ldots+n]\right\}$$

$$= \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{1}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= \frac{1}{6}\cdot\frac{2n^2+3n+1}{n^2}+\frac{1}{2}\cdot\frac{n+1}{n}$$

$$= \frac{1}{6}\left[2+\frac{3}{n}+\frac{1}{n^2}\right]+\frac{1}{2}\left[1+\frac{1}{n}\right]$$

$$\int_0^1 \left(x^2+x\right)dx = \lim_{n\to\infty} S_n = \frac{1}{3}+\frac{1}{2} = \frac{5}{6}$$



20. $\int_1^2 (x+2)dx$

Let $f(x) = x + 2$.

$$\Delta x = \frac{1}{n}$$

$$S_n = \frac{1}{n} f\left(1+\frac{1}{n}\right)+\ldots+\frac{1}{n} f\left(1+n\cdot\frac{1}{n}\right)$$

$$= \frac{1}{n}\left\{\left[\left(1+\frac{1}{n}\right)+2\right]+\ldots+\left[\left(1+n\cdot\frac{1}{n}\right)+2\right]\right\}$$

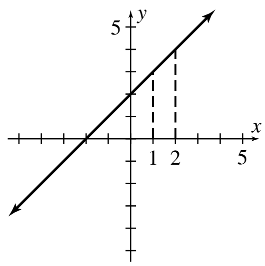
$$= \frac{1}{n}\left\{n+\frac{1}{n}(1+\cdots+n)+2n\right\}$$

$$= 1+\frac{1}{n^2}\cdot\frac{n(n+1)}{2}+2$$

$$= 3+\frac{1}{2}\cdot\frac{n+1}{n}$$

$$= 3+\frac{1}{2}\left[1+\frac{1}{n}\right]$$

$$\int_1^2 (x+2)dx = \lim_{n\to\infty} S_n = 3+\frac{1}{2} = \frac{7}{2}$$



21. $\int_0^1 \sqrt{1-x^2}\,dx$ is simply a real number. Thus

$$\frac{d}{dx}\left[\int_0^1 \sqrt{1-x^2}\,dx\right] = \frac{d}{dx}(\text{real number}) = 0.$$

22. $f(x) = \begin{cases} 2 & \text{if} \quad 0 \le x < 1 \\ 4-2x & \text{if} \quad 1 \le x < 2 \\ 5x-10 & \text{if} \quad 2 \le x \le 3 \end{cases}$

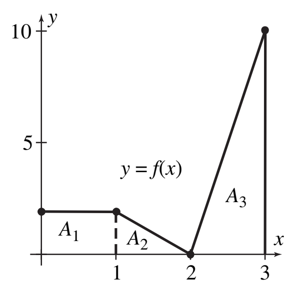
f is continuous and $f(x) \ge 0$ on [0, 3]. Thus $\int_0^3 f(x)\,dx$ gies the area *A* bounded by $y = f(x)$, $y = 0$, $x = 0$ and $x = 3$. From the diagram, this area is composed of three subareas, A_1, A_2 and A_3, and $A = A_1 + A_2 + A_3$.

A_1 = area of rectangle = (1)(2) = 2 sq unit

A_2 = area of triangle $= \frac{1}{2}(1)(2) = 1$ sq unit

A_3 = area of triangle $= \frac{1}{2}(1)(10) = 5$ sq unit

Since $A = A_1 + A_2 + A_3 = 2+1+5 = 8$ sq units, we have $\int_0^3 f(x)\,dx = 8$.



23. $f(x) = \begin{cases} 1 & \text{if} \quad x \le 1 \\ 2-x & \text{if} \quad 1 \le x \le 2 \\ -1+\frac{x}{2} & \text{if} \quad x > 2 \end{cases}$

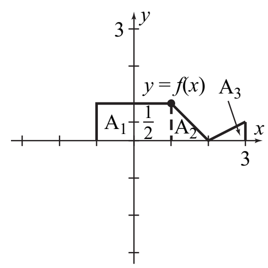
f is continuous and $f(x) \ge 0$ on [–1, 3]. Thus $\int_{-1}^3 f(x)dx$ gives the area *A* bounded by $y = f(x)$, $y = 0$, $x = -1$, and $x = 3$. From the diagram, this area is composed of three subareas, A_1, A_2, and A_3 and $A = A_1 + A_2 + A_3$.

A_1 = area of rectangle = (2)(1) = 2 sq units

A_2 = area of triangle $= \frac{1}{2}(1)(1) = \frac{1}{2}$ sq unit

A_3 = area of triangle $= \frac{1}{2}(1)\left(\frac{1}{2}\right) = \frac{1}{4}$ sq unit

Since $A = A_1 + A_2 + A_3 = 2 + \frac{1}{2} + \frac{1}{4} = \frac{11}{4}$ sq units, we have $\int_{-1}^3 f(x)dx = \frac{11}{4}$.



24. 44.6 sq units

25. 14.7 sq units

26. 0.4 sq units

27. 2.4

28. 0.7

29. –25.5

30. 0.39

Apply It 14.7

11. $\int_3^6 10{,}000e^{0.02t}dt = \left(10{,}000\frac{e^{0.02t}}{0.02}\right)\Bigg|_3^6$

$$= \left(500{,}000e^{0.02t}\right)\Big|_3^6$$
$$= 500{,}000\left(e^{0.02(6)} - e^{0.02(3)}\right)$$
$$= 500{,}000\left(e^{0.12} - e^{0.06}\right) \approx 32{,}830$$

The total income for the chain between the third and sixth years was about \$32,830.

12. The total cost for the first 5 years is $M(5)$ or

$M(5) - M(0) = \int_0^5 M'(x)dx$

$\int_0^5 \left(90x^2 + 5000\right)dx = \left(90\frac{x^3}{3} + 5000x\right)\Bigg|_0^5$

$= \left(30x^3 + 5000x\right)\Big|_0^5 = 30(5)^3 + 5000(5) - 0$

= 3750 + 25,000 = 28,750

The total cost for the first 5 years is $28,750.

Problems 14.7

1. $\int_0^3 5\ dx = 5x\Big|_0^3 = 5(3) - 5(0) = 15 - 0 = 15$

2. $\int_1^5 (e + 3e)dx = \int_1^5 4e\,dx$

$= 4ex\big|_1^5$

$= 4e(5-1)$

$= 16e$

3. $\int_1^2 5x\ dx = 5 \cdot \frac{x^2}{2}\Bigg|_1^2 = 10 - \frac{5}{2} = \frac{15}{2}$

4. $\int_2^8 -5x\,dx = -5 \cdot \frac{x^2}{2}\Bigg|_2^8 = -160 - (-10) = -150$

5. $\int_{-3}^1 (2x-3)dx = \left(x^2 - 3x\right)\Big|_{-3}^1 = -2 - 18 = -20$

6. $\int_{-1}^1 (4 - 9y) = \left(4y - \frac{9y^2}{2}\right)\Bigg|_{-1}^1 = -\frac{1}{2} - \left(-\frac{17}{2}\right)$

$= \frac{16}{2} = 8$

7. $\int_1^4 (y^2 + 4y + 4)dy = \int_1^4 (y+2)^2\,dy$

$= \frac{(y+2)^3}{3}\Bigg|_1^4$

$= \frac{1}{3}[(4+2)^3 - (1+2)^3]$

$= \frac{1}{3}[216 - 27]$

$= \frac{1}{3}(189)$

$= 63$

8. $\int_4^1 (2t - 3t^2)dt = (t^2 - t^3)\Big|_4^1 = 0 - (-48) = 48$

9. $\int_{-2}^{-1}\left(3w^2 - w - 1\right)dw = \left(w^3 - \frac{w^2}{2} - w\right)\Bigg|_{-2}^{-1}$

$= -\frac{1}{2} - (-8) = \frac{15}{2}$

10. $\int_8^9 dt = \int_8^9 1\ dt = t\Big|_8^9 = 9 - 8 = 1$

11. $\int_1^3 3t^{-3}\ dt = -\frac{3}{2} \cdot t^{-2}\Big|_1^3 = -\frac{1}{6} - \left(-\frac{3}{2}\right) = \frac{4}{3}$

12. $\int_2^3 \frac{3}{x^2}dx = 3\int_2^3 x^{-2}dx$

$= 3 \cdot \frac{x^{-1}}{-1}\Bigg|_2^3$

$= -\frac{3}{x}\Bigg|_2^3$

$= \frac{-3}{3} - \left(\frac{-3}{2}\right)$

$= -1 + \frac{3}{2}$

$= \frac{1}{2}$

13. $\int_{-8}^{8} \sqrt[3]{x^4}\,dx = \int_{-8}^{8} x^{4/3}\,dx$

$$= \left.\frac{3x^{7/3}}{7}\right|_{-8}^{8}$$

$$= \frac{3\cdot 128}{7} - \frac{3(-128)}{7}$$

$$= \frac{768}{7}$$

14. $\int_{1/2}^{3/2}\left(x^2+x+1\right)dx = \left.\left(\frac{x^3}{3}+\frac{x^2}{2}+x\right)\right|_{1/2}^{3/2}$

$= \frac{15}{4} - \frac{2}{3} = \frac{37}{12}$

15. $\int_{1/2}^{3} \frac{1}{x^2}\,dx = -\left.\frac{1}{x}\right|_{1/2}^{3} = -\frac{1}{3}-(-2) = \frac{5}{3}$

16. $\int_{9}^{36}\left(\sqrt{x}-2\right)dx = \left.\left(\frac{2}{3}x^{\frac{3}{2}}-2x\right)\right|_{9}^{36} = 72-0=72$

17. $\int_{-2}^{2}(z+1)^4\,dz = \left.\frac{(z+1)^5}{5}\right|_{-2}^{2}$

$$= \frac{1}{5}[(2+1)^5-(-2+1)^5]$$

$$= \frac{1}{5}(243+1)$$

$$= \frac{244}{5}$$

18. $\int_{1}^{8}\left(x^{\frac{1}{3}}-x^{-\frac{1}{3}}\right)dx = \left.\left(\frac{3x^{\frac{4}{3}}}{4}-\frac{3x^{\frac{2}{3}}}{2}\right)\right|_{1}^{8}$

$= 6-\left(-\frac{3}{4}\right) = \frac{27}{4}$

19. $\int_{0}^{1} 2x^2\left(x^3-1\right)^3 dx = \frac{2}{3}\int_{0}^{1}\left(x^3-1\right)^3\left[3x^2dx\right]$

$= \left.\frac{1}{6}\left(x^3-1\right)^4\right|_{0}^{1} = 0-\frac{1}{6} = -\frac{1}{6}$

20. $\int_{2}^{3}(x+2)^3\,dx = \left.\frac{(x+2)^4}{4}\right|_{2}^{3} = \frac{625}{4}-64=\frac{369}{4}$

21. $\int_{1}^{8}\frac{4}{y}\,dy = 4\ln|y|\Big|_{1}^{8} = 4(\ln 8-\ln 1)$

$=4(\ln 8 - 0) = 4\ln 8$

22. $\int_{-e^{\pi}}^{-1}\frac{2}{x}\,dx = 2\ln|x|\Big|_{-e^{\pi}}^{-1}$

$$= 2\left[\ln|-1|-\ln\left|-e^{\pi}\right|\right]$$

$$= 2(0-\pi)$$

$$= -2\pi$$

23. $\int_{0}^{1} e^5\,dx = e^5x\Big|_{0}^{1} = e^5-0=e^5$

24. $\int_{2}^{e+1}\frac{1}{x-1}\,dx = \ln|x-1|\Big|_{2}^{e+1} = \ln e-\ln 1 = 1-0=1$

25. $\int_{0}^{1} 5x^2e^{x^3}\,dx = \frac{5}{3}\int_{0}^{1} e^{x^3}[3x^2dx] = \left.\frac{5}{3}e^{x^3}\right|_{0}^{1}$

$= \frac{5}{3}(e^1-e^0) = \frac{5}{3}(e-1)$

26. $\int_{0}^{1}\left(3x^2+4x\right)\left(x^3+2x^2\right)^4 dx$

$= \int_{0}^{1}\left(x^3+2x^2\right)^4\left[\left(3x^2+4x\right)dx\right]$

$= \left.\frac{\left(x^3+2x^2\right)^5}{5}\right|_{0}^{1} = \frac{243}{5}-0=\frac{243}{5}$

27. $\int_{3}^{4}\frac{3}{(x+3)^2}\,dx = 3\int_{3}^{4}(x+3)^{-2}\,dx$

$$= 3\cdot\left.\frac{(x+3)^{-1}}{-1}\right|_{3}^{4}$$

$$= -3\left[\frac{1}{4+3}-\frac{1}{3+3}\right]$$

$$= -3\left(\frac{1}{7}-\frac{1}{6}\right)$$

$$= \frac{1}{14}$$

28. $\int_{-1/3}^{20/3}\sqrt{3x+5}\,dx=\frac{1}{3}\int_{-1/3}^{20/3}(3x+5)^{\frac{1}{2}}[3\,dx]$
$$=\frac{2}{9}(3x+5)^{\frac{3}{2}}\Big|_{-\frac{1}{3}}^{\frac{20}{3}}$$
$$=\frac{2}{9}(125-8)=26$$

29. $\int_{1/3}^{2}\sqrt{10-3p}\,dp=-\frac{1}{3}\int_{1/3}^{2}(10-3p)^{\frac{1}{2}}[-3\ dp]$
$$=-\frac{2}{9}(10-3p)^{\frac{3}{2}}\Big|_{1/3}^{2}=-\frac{2}{9}(8-27)=\frac{38}{9}$$

30. $\int_{-1}^{1}q\sqrt{q^2+3}\,dq=\frac{1}{2}\int_{-1}^{1}\left(q^2+3\right)^{\frac{1}{2}}[2q\ dq]$
$$=\frac{\left(q^2+3\right)^{\frac{3}{2}}}{3}\Bigg|_{-1}^{1}=\frac{8}{3}-\frac{8}{3}=0$$

31. $\int_0^1 x^2\sqrt[3]{7x^3+1}\,dx=\frac{1}{21}\int_0^1\left(7x^3+1\right)^{\frac{1}{3}}\left[21x^2dx\right]$
$$=\frac{1}{21}\cdot\frac{\left(7x^3+1\right)^{\frac{4}{3}}}{\frac{4}{3}}\Bigg|_0^1=\frac{\left(7x^3+1\right)^{\frac{4}{3}}}{28}\Bigg|_0^1$$
$$=\frac{16}{28}-\frac{1}{28}=\frac{15}{28}$$

32. $\int_0^{\sqrt{2}}\left(2x-\frac{x}{(x^2+1)^{2/3}}\right)dx$
$$=\int_0^{\sqrt{2}}2x\,dx-\frac{1}{2}\int_0^{\sqrt{2}}(x^2+1)^{-2/3}[2x\,dx]$$
$$=2\cdot\frac{x^2}{2}\Big|_0^{\sqrt{2}}-\frac{1}{2}\frac{(x^2+1)^{1/3}}{\frac{1}{3}}\Bigg|_0^{\sqrt{2}}$$
$$=(2-0)-\frac{3}{2}[(2+1)^{1/3}-(0+1)^{1/3}]$$
$$=2-\frac{3}{2}(3^{1/3}-1)$$
$$=2-\frac{3\sqrt[3]{3}}{2}+\frac{3}{2}$$
$$=\frac{7-3\sqrt[3]{3}}{2}$$

33. $\int_0^1\frac{2x^3+x}{x^2+x^4+1}dx$
$$=\frac{1}{2}\int_0^1\frac{1}{x^4+x^2+1}\left[\left(4x^3+2x\right)dx\right]$$
$$=\frac{1}{2}\ln\left(x^4+x^2+1\right)\Big|_0^1=\frac{1}{2}[\ln 3-\ln 1]=\frac{1}{2}\ln 3$$

34. $\int_a^b(m+ny)dy=\left(my+\frac{ny^2}{2}\right)\Big|_a^b=my\Big|_a^b+\frac{n}{2}y^2\Big|_a^b$
$$=m(b-a)+\frac{n}{2}\left(b^2-a^2\right)$$

35. $\int_0^1\frac{e^x-e^{-x}}{2}dx=\frac{1}{2}(e^x+e^{-x})\Big|_0^1$
$$=\frac{1}{2}[(e+e^{-1})+(1+1)]$$
$$=\frac{1}{2}\left(e+\frac{1}{e}+2\right)$$

36. $\int_{-2}^{1}8|x|\,dx=8\left(\int_{-2}^{0}-x\ dx+\int_0^1 x\ dx\right)$
$$=8\left(-\frac{x^2}{2}\Big|_{-2}^{0}+\frac{x^2}{2}\Big|_0^1\right)=8\left\{[0-(-2)]+\left(\frac{1}{2}-0\right)\right\}$$

37. $\int_e^{\sqrt{2}}3(x^{-2}+x^{-3}-x^{-4})dx$
$$=3\left(\frac{x^{-1}}{-1}+\frac{x^{-2}}{-2}-\frac{x^{-3}}{-3}\right)\Bigg|_e^{\sqrt{2}}$$
$$=3\left(-\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{3x^3}\right)\Bigg|_e^{\sqrt{2}}$$
$$=3\left(-\frac{1}{\sqrt{2}}-\frac{1}{4}+\frac{1}{6\sqrt{2}}\right)-3\left(-\frac{1}{e}-\frac{1}{2e^2}+\frac{1}{3e^3}\right)$$
$$=3\left(-\frac{5}{6\sqrt{2}}-\frac{1}{4}+\frac{1}{e}+\frac{1}{2e^2}-\frac{1}{3e^3}\right)$$

38. $\int_1^2 \left(6\sqrt{x} - \frac{1}{\sqrt{2x}}\right) dx$
$= 6\int_1^2 x^{\frac{1}{2}} dx - \frac{1}{2}\int_1^2 (2x)^{-\frac{1}{2}}[2\,dx]$
$= \left[4x^{\frac{3}{2}} - (2x)^{\frac{1}{2}}\right]\Big|_1^2 = \left(8\sqrt{2} - 2\right) - \left(4 - \sqrt{2}\right)$
$= 9\sqrt{2} - 6$

39. $\int_1^3 (x+1)e^{x^2+2x} dx = \frac{1}{2}\int_1^3 e^{x^2+2x}[(2x+2)dx]$
$= \frac{1}{2}e^{x^2+2x}\Big|_1^3 = \frac{1}{2}\left(e^{15} - e^3\right) = \frac{e^3}{2}\left(e^{12} - 1\right)$

40. $\int_1^{95} \frac{x}{\ln e^x} dx = \int_1^{95} \frac{x}{x} dx = \int_1^{95} 1\,dx = x\Big|_1^{95}$
$= 95 - 1 = 94$

41. Using long division on the integrand
$\int_0^2 \frac{x^6 + 6x^4 + x^3 + 8x^2 + x + 5}{x^3 + 5x + 1} dx$
$= \int_0^2 \left[x^3 + x + \frac{3x^2 + 5}{x^3 + 5x + 1}\right] dx$
$= \left[\frac{x^4}{4} + \frac{x^2}{2} + \ln\left|x^3 + 5x + 1\right|\right]\Big|_0^2$
$= (6 + \ln 19) - 0 = 6 + \ln 19$

42. $\int_1^2 \frac{1}{1+e^x} dx = \int_1^2 \frac{e^{-x}}{e^{-x}+1} dx$
$= -\int_1^2 \frac{1}{e^{-x}+1}[-e^{-x} dx]$
$= -\ln\left|e^{-x} + 1\right|\Big|_1^2$
$= -\left(\ln\left|e^{-2} + 1\right| - \ln\left|e^{-1} + 1\right|\right)$
$= \ln(1 + e^{-1}) - \ln(1 + e^{-2})$

43. $\int_0^2 f(x)dx = \int_0^{1/2} 4x^2 dx + \int_{1/2}^2 \; 2x\; dx$
$= \frac{4x^3}{3}\Big|_0^{1/2} + x^2\Big|_{1/2}^2 = \left(\frac{1}{6} - 0\right) + \left(4 - \frac{1}{4}\right) = \frac{47}{12}$

44. $\left(\int_1^3 x\,dx\right)^3 - \int_1^3 x^3 dx = \left(\frac{x^2}{2}\Big|_1^3\right)^3 - \frac{x^4}{4}\Big|_1^3$
$= \left(\frac{9}{2} - \frac{1}{2}\right)^3 - \left(\frac{81}{4} - \frac{1}{4}\right)$
$= 4^3 - 20$
$= 44$

45. $f(x) = \int_1^x 3\frac{1}{t^2} dt = -\frac{3}{t}\Big|_1^x = -\frac{3}{x} + 3 = 3 - \frac{3}{x}$
$\int_e^1 f(x)dx = \int_e^1 \left(3 - \frac{3}{x}\right) dx = \left(3x - 3\ln|x|\right)\Big|_e^1$
$= (3 - 0) - (3e - 3) = 6 - 3e$

46. $\int_7^7 e^{x^2} dx + \int_0^{\sqrt{2}} \frac{1}{3\sqrt{2}} dx = 0 + \frac{1}{3\sqrt{2}}\int_0^{\sqrt{2}} 1\,dx$
$= \frac{1}{3\sqrt{2}} x\Big|_0^{\sqrt{2}}$
$= \frac{1}{3\sqrt{2}}\left(\sqrt{2} - 0\right)$
$= \frac{1}{3}$

47. $\int_2^3 f(x)dx = \int_1^3 f(x)dx - \int_1^2 f(x)dx$
$= -\int_3^1 f(x)dx - \int_1^2 f(x)dx$
$= -2 - 5$
$= -7$

48. $\int_1^4 f(x)dx = \int_1^2 f(x)dx + \int_2^4 f(x)dx$
$\int_1^4 f(x)dx = \int_1^3 f(x)dx - \int_2^3 f(x)dx + \int_2^4 f(x)dx$
$6 = 2 - \int_2^3 f(x)dx + 5$, so $\int_2^3 f(x)\,dx = 7 - 6 = 1$.

49. $\int_2^3 e^{x^3} dx$ is a constant, so $\frac{d}{dx}\int_2^3 e^{x^3} dx = 0$. Thus
$\int_2^3 \left(\frac{d}{dx}\int_2^3 e^{x^3} dx\right) dx = \int_2^3 0\,dx = C\Big|_2^3 = C - C = 0$

50. $f(x)=\int_e^x \frac{e^t-e^{-t}}{e^t+e^{-t}}dt$

$=\int_e^x \frac{1}{e^t+e^{-t}}[(e^t-e^{-t})\,dt]$

$=\ln\left|e^t+e^{-t}\right|\Big|_e^x$

$=\ln(e^x+e^{-x})-\ln(e^e-e^{-e})$

$f'(x)=\frac{1}{e^x+e^{-x}}[e^x+e^{-x}(-1)]-0$

$=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

51. $\int_0^T \alpha^{\frac{5}{2}}dt=\alpha^{\frac{5}{2}}t\Big|_0^T=\alpha^{\frac{5}{2}}T-0=\alpha^{\frac{5}{2}}T$

52. $\mu=\int_0^1 x[6(x-x^2]dx$

$=6\int_0^1(x^2-x^3)dx$

$=6\left(\frac{x^3}{3}-\frac{x^4}{4}\right)\Big|_0^1$

$=6\left(\frac{1}{3}-\frac{1}{4}\right)-6(0-0)$

$=\frac{1}{2}$

$\sigma^2=\int_0^1\left(x-\frac{1}{2}\right)^2[6(x-x^2)]dx$

$=6\int_0^1\left(-x^4+2x^3-\frac{5}{4}x^2+\frac{x}{4}\right)dx$

$=6\left(-\frac{x^5}{5}+\frac{x^4}{2}-\frac{5x^3}{12}+\frac{x^2}{8}\right)\Big|_0^1$

$=6\left(-\frac{1}{5}+\frac{1}{2}-\frac{5}{12}+\frac{1}{8}\right)-6(0)$

$=\frac{1}{20}$

Thus $\mu=\frac{1}{2};\ \sigma^2=\frac{1}{20}$.

53. The total number receiving between a and b dollars equals the number $N(a)$ receiving a or more dollars minus the number $N(b)$ receiving b or more dollars. Thus

$N(a)-N(b)=\int_b^a -Ax^{-B}dx$.

54. $\int_0^{10^{-4}} x^{-\frac{1}{2}}dx=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_0^{10^{-4}}=2\sqrt{x}\Big|_0^{10^{-4}}=2\sqrt{10^{-4}}-0$

$=2\left(10^{-2}\right)=0.02$

55. $\int_0^5 2000e^{-0.06t}dt=2000\cdot\frac{1}{-0.06}\int_0^5 e^{-0.06t}[-0.06\,dt]$

$=-\frac{2000}{0.06}e^{-0.06t}\Big|_0^5=-\frac{2000}{0.06}\left(e^{-0.03}-1\right)$

$\approx$ \$8639

56. $\int_0^t\left(e^{-a\tau}-e^{-b\tau}\right)d\tau$

$=\frac{1}{-a}\int_0^t e^{-a\tau}[-a\;d\tau]-\frac{1}{-b}\int_0^t e^{-b\tau}[-b\;d\tau]$

$=\left(-\frac{e^{-a\tau}}{a}+\frac{e^{-b\tau}}{b}\right)\Bigg|_0^t$

$=\left(-\frac{e^{-at}}{a}+\frac{e^{-bt}}{b}\right)-\left(-\frac{1}{a}+\frac{1}{b}\right)$

$=\frac{1-e^{-at}}{a}-\frac{1-e^{-bt}}{b}$

57. $\int_{10}^{29}1000\sqrt{110-t}dt$

$=-1000\int_{10}^{29}\sqrt{110-t}[-dt]$

$=-1000\frac{(110-t)^{3/2}}{\frac{3}{2}}\Bigg|_{10}^{29}$

$=-\frac{2000}{3}(110-t)^{3/2}\Big|_{10}^{29}$

$=-\frac{2000}{3}[(110-29)^{3/2}-(110-10)^{3/2}]$

$\approx 180{,}667$

For the entire population, $a = 0$ and $b = 110$.

$-\frac{2000}{3}[(110-110)^{3/2}-(110-0)^{3/2}]$

$\approx 769{,}126$

58. $\int_0^t 3000e^{0.05\tau}d\tau=3000\cdot\frac{1}{0.05}\int_0^t e^{0.05\tau}[0.05\;d\tau]$

$=60{,}000e^{0.05\tau}\Big|_0^t=60{,}000\left(e^{0.05t}-1\right)$

59. $\int_{65}^{75}(0.2q+8)dq = \left(0.1q^2+8q\right)\Big|_{65}^{75}$
$= 1162.5 - 942.5 = \$220$

60. $\int_{90}^{180}(0.004q^2-0.5q+50)\,dq$
$= \dfrac{0.004}{3}q^3 - 0.25q^2 + 50q\Big|_{90}^{180}$
$= 8676 - 3447$
$= \$5229$

61. $\int_{500}^{800}\dfrac{2000}{\sqrt{300q}}dq = \int_{500}^{800}\dfrac{2000}{10\sqrt{3q}}dq$
$= \dfrac{200}{\sqrt{3}}\int_{500}^{800}q^{-1/2}dq = \dfrac{200}{\sqrt{3}}\cdot\dfrac{q^{1/2}}{\frac{1}{2}}\Bigg|_{500}^{800}$
$= \dfrac{400}{\sqrt{3}}\sqrt{q}\Big|_{500}^{800} = \dfrac{400}{\sqrt{3}}\left(\sqrt{800}-\sqrt{500}\right) \approx \$1367.99$

62. $\int_{5}^{10}(100+50q-3q^2)dq$
$= (100q+25q^2-q^3)\Big|_{5}^{10}$
$= (1000+2500-1000)-(500+625-125)$
$= \$1500$

63. $\int_{0}^{12}(8t+10)dt = \left(4t^2+10t\right)\Big|_{0}^{12} = 696 - 0 = 696$

$\int_{6}^{12}(8t+10)dt = \left(4t^2+10t\right)\Big|_{6}^{12} = 696 - 204 = 492$

64. $\int_{0}^{700}\dfrac{81\times10^6}{(300+t)^4}dt = \left(81\times10^6\right)\int_{0}^{700}(300+t)^{-4}\,dt$
$= \left(81\times10^6\right)\dfrac{(300+t)^{-3}}{-3}\Bigg|_{0}^{700}$
$= -\left(27\times10^6\right)\dfrac{1}{(300+t)^3}\Bigg|_{0}^{700}$
$= -\left(27\times10^6\right)\left(\dfrac{1}{1000^3}-\dfrac{1}{300^3}\right)$
$= -\left(27\times10^6\right)\left(\dfrac{1}{10^9}-\dfrac{1}{27\cdot10^6}\right)$
$= -\dfrac{27}{10^3}+1 = -\dfrac{27}{1000}+1 = \dfrac{973}{1000} = 0.973$

65. $G = \int_{-R}^{R} i\ dx = ix\Big|_{-R}^{R} = iR - (-iR) = 2Ri$

66. $E = \int_{-R}^{R} \frac{i}{2}\left[e^{-k(R-x)} + e^{-k(R+x)}\right] dx$

$$= \frac{i}{2}\left[\frac{1}{k}\int_{-R}^{R} e^{-k(R-x)}[k\ dx] + \frac{1}{-k}\int_{-R}^{R} e^{-k(R+x)}[-k\ dx]\right]$$

$$= \frac{i}{2k}\left[\int_{-R}^{R} e^{-k(R-x)}[k\ dx] - \int_{-R}^{R} e^{-k(R+x)}[-k\ dx]\right]$$

$$= \frac{i}{2k}\left[e^{-k(R-x)} - e^{-k(R+x)}\right]\Big|_{-R}^{R}$$

$$= \frac{i}{2k}\left[\left(1 - e^{-k(2R)}\right) - \left(e^{-k(2R)} - 1\right)\right]$$

$$= \frac{i}{2k}\left[2 - 2e^{-2kR}\right] = \frac{i}{k}\left(1 - e^{-2kR}\right)$$

67. $A = \dfrac{\int_0^R (m+x)[1-(m+x)]dx}{\int_0^R [1-(m+x)]dx} = \dfrac{\int_0^R \left(m + x - m^2 - 2mx - x^2\right)dx}{\int_0^R (1-m-x)dx}$

$$= \frac{\left[mx + \frac{x^2}{2} - m^2x - mx^2 - \frac{x^3}{3}\right]\Big|_0^R}{\left[x - mx - \frac{x^2}{2}\right]\Big|_0^R}$$

$$= \frac{\left[mR + \frac{R^2}{2} - m^2R - mR^2 - \frac{R^3}{3}\right] - 0}{\left[R - mR - \frac{R^2}{2}\right] - 0}$$

$$= \frac{R\left[m + \frac{R}{2} - m^2 - mR - \frac{R^2}{3}\right]}{R\left[1 - m - \frac{R}{2}\right]} = \frac{m + \frac{R}{2} - m^2 - mR - \frac{R^2}{3}}{1 - m - \frac{R}{2}}$$

68. $\int_{2.5}^{3.5} (1 + 2x + 3x^2)dx = (x + x^2 + x^3)\Big|_{2.5}^{3.5}$

$$= 58.625 - 24.375$$
$$= 34.25$$

69. $\int_0^4 \frac{1}{(4x+4)^2} dx = \frac{1}{4}\int_0^4 (4x+4)^{-2}[4\ dx] = \frac{1}{4}\cdot\frac{(4x+4)^{-1}}{-1}\Bigg|_0^4$

$$= -\frac{1}{4}\cdot\frac{1}{4x+4}\Bigg|_0^4 = -\frac{1}{16}\cdot\frac{1}{x+1}\Bigg|_0^4 = -\frac{1}{16}\left(\frac{1}{5} - 1\right) = \frac{1}{20} = 0.05$$

70. $\int_0^1 e^{3t}dt = \frac{1}{3}\int_0^1 e^{3t}[3\ dt] = \frac{e^{3t}}{3}\Bigg|_0^1 = \frac{1}{3}\left(e^3 - 1\right) \approx 6.36$

71. 3.52

72. 0.23

73. 14.34

74. 3.64

Apply It 14.8

13. In this case, $f(t)=\frac{60}{\sqrt{t^2+9}}$, $n = 5$, $a = 0$, and $b = 5$. Thus $h=\frac{b-a}{n}=\frac{5-0}{5}=1$. The terms to be added are

$$f(0)=\frac{60}{\sqrt{0^2+9}}=\frac{60}{3}=20$$
$$2f(1)=\frac{2(60)}{\sqrt{1^2+9}}=\frac{120}{\sqrt{10}}\approx 37.9473$$
$$2f(2)=\frac{2(60)}{\sqrt{2^2+9}}=\frac{120}{\sqrt{13}}\approx 33.2820$$
$$2f(3)=\frac{2(60)}{\sqrt{3^2+9}}=\frac{120}{\sqrt{18}}\approx 28.2843$$
$$2f(4)=\frac{2(60)}{\sqrt{4^2+9}}=\frac{120}{5}=24$$
$$f(5)=\frac{60}{\sqrt{5^2+9}}=\frac{60}{\sqrt{34}}\approx 10.2899$$

The sum of the above terms is 153.8035. The estimate of the radius after 5 seconds is

$$\int_0^5 \frac{60}{\sqrt{t^2+9}}\,dt\approx\frac{1}{2}(153.8035)\approx 76.90 \text{ feet.}$$

14. In this case, $f(t)=0.3e^{0.2t^2}$, $n = 8$, $a = 0$, and $b = 4$. Thus, $h=\frac{b-a}{n}=\frac{4}{8}=0.5$. The terms to be added are

$$f(0)=0.3e^0=0.3$$
$$4f(0.5)=4(0.3)e^{0.05}\approx 1.2615$$
$$2f(1)=2(0.3)e^{0.2}\approx 0.7328$$
$$4f(1.5)=4(0.3)e^{0.45}\approx 1.8820$$
$$2f(2)=2(0.3)e^{0.8}\approx 1.3353$$
$$4f(2.5)=4(0.3)e^{1.25}\approx 4.1884$$
$$2f(3)=2(0.3)e^{1.8}\approx 3.6298$$
$$4f(3.5)=4(0.3)e^{2.45}\approx 13.9060$$
$$f(4)=0.3e^{3.2}\approx 7.3598$$

The sum of the above terms is 34.5956. The estimate of the amount the culture grew over the first four hours is

$$\int_0^4 0.3e^{0.2t^2}\,dt\approx\frac{0.5}{3}(34.5956)\approx 5.77 \text{ grams.}$$

Problems 14.8

1. $f(x)=\frac{170}{1+x^2}$, $n = 6$, $a = -2$, $b = 4$. Trapezoidal

$$h=\frac{b-a}{n}=\frac{4-(-2)}{6}=\frac{6}{6}=1$$

$$\begin{aligned}
f(-2) &= 34 = 34\\
2f(-1) &= 2(85) = 170\\
2f(0) &= 2(170) = 340\\
2f(1) &= 2(85) = 170\\
2f(2) &= 2(34) = 68\\
2f(3) &= 2(17) = 34\\
f(4) &= 10 = \underline{10}\\
&\ \ 826
\end{aligned}$$

$$\int_{-2}^4 \frac{170}{1+x^2}\,dx\approx\frac{1}{2}(826)=413$$

2. $f(x)=\frac{170}{1+x^2}$, $n = 6$, $a = -2$, $b = 4$

Simpson's

$$h=\frac{b-a}{n}=\frac{4-(-2)}{6}=\frac{6}{6}=1$$

$$\begin{aligned}
f(-2) &= 34 = 34\\
4f(-1) &= 4(85) = 340\\
2f(0) &= 2(170) = 340\\
4f(1) &= 4(85) = 340\\
2f(2) &= 2(34) = 68\\
4f(3) &= 4(17) = 68\\
f(4) &= 10 = \underline{10}\\
&\ \ 1200
\end{aligned}$$

$$\int_{-2}^4 \frac{170}{1+x^2}\,dx\approx\frac{1}{3}(1200)=400$$

3. $f(x)=x^3$, $n=5$, $a=0$, $b=1$

Trapezoidal

$$h=\frac{b-a}{n}=\frac{1-0}{5}=\frac{1}{5}=0.2$$

$$\begin{aligned} f(0) &= 0.0000\\ 2f(0.2) &= 0.0160\\ 2f(0.4) &= 0.1280\\ 2f(0.6) &= 0.4320\\ 2f(0.8) &= 1.0240\\ f(1) &= \underline{1.0000}\\ & \ 2.6000 \end{aligned}$$

$$\int_0^1 x^3 dx \approx \frac{0.2}{2}(2.6000)=0.260$$

Actual value: $\int_0^1 x^3 dx=\left.\frac{x^4}{4}\right|_0^1=\frac{1}{4}=0.250$

4. $f(x)=x^2$, $n=4$, $a=0$, $b=1$

Simpson's

$$h=\frac{b-a}{n}=\frac{1-0}{4}=0.25$$

$$\begin{aligned} f(0) &= 0.0000\\ 4f(0.25) &= 0.2500\\ 2f(0.50) &= 0.5000\\ 4f(0.75) &= 2.2500\\ f(1) &= \underline{1.0000}\\ & \ 4.0000 \end{aligned}$$

$$\int_0^1 x^2 dx \approx \frac{0.25}{3}(4.0000)=\frac{1}{3}\approx 0.333$$

Actual value: $\int_0^1 x^2 dx=\left.\frac{x^3}{3}\right|_0^1=\frac{1}{3}\approx 0.333$

5. $f(x)=\frac{1}{x^2}$, $n=4$, $a=1$, $b=4$

Simpson's

$$h=\frac{b-a}{n}=\frac{4-1}{4}=0.75$$

$$\begin{aligned} f(1) &= 1.0000\\ 4f(1.75) &= 1.3061\\ 2f(2.50) &= 0.3200\\ 4f(3.25) &= 0.3787\\ f(4) &= \underline{0.0625}\\ & \ 3.0673 \end{aligned}$$

$$\int_1^4 \frac{1}{x^2}dx \approx \frac{0.75}{3}(3.0673)\approx 0.767$$

Actual value:

$$\int_1^4 \frac{1}{x^2}dx=-\left.\frac{1}{x}\right|_1^4=-\frac{1}{4}-(-1)=0.750$$

6. $f(x)=\frac{1}{x}$, $n=6$, $a=1$, $b=4$

Trapezoidal

$$h=\frac{b-a}{n}=\frac{4-1}{6}=0.5$$

$$\begin{aligned} f(1) &= 1.0000\\ 2f(1.5) &= 1.3333\\ 2f(2) &= 1.0000\\ 2f(2.5) &= 0.8000\\ 2f(3) &= 0.6667\\ 2f(3.5) &= 0.5714\\ f(4) &= \underline{0.2500}\\ & \ 5.6214 \end{aligned}$$

$$\int_1^4 \frac{1}{x}dx \approx \frac{0.5}{2}(5.6214)\approx 1.405$$

Actual value:

$$\int_1^4 \frac{1}{x}dx=\ln|x|\Big|_1^4=\ln 4-\ln 1=\ln 4-0=\ln 4$$

≈ 1.386

7. $f(x)=\frac{x}{x+1}$, $n=4$, $a=0$, $b=2$

Trapezoidal

$$h=\frac{b-a}{n}=\frac{2-0}{4}=0.5$$

$$\begin{aligned} f(0) &= 0.0000\\ 2f(0.5) &= 0.6667\\ 2f(1) &= 1.0000\\ 2f(1.5) &= 1.2000\\ f(2) &= \underline{0.6667}\\ & \ 3.5334 \end{aligned}$$

Thus

$$\int_0^2 \frac{x}{x+1}dx \approx \frac{0.5}{2}(3.5334)\approx 0.883$$

8. $f(x)=\frac{1}{x}$, $n=6$, $a=1$, $b=4$

Simpson's

$$h=\frac{b-a}{n}=\frac{4-1}{6}=0.5$$

$$\begin{aligned} f(1) &= 1.0000 \\ 4f(1.5) &= 2.6667 \\ 2f(2) &= 1.0000 \\ 4f(2.5) &= 1.6000 \\ 2f(3) &= 0.6667 \\ 4f(3.5) &= 1.1429 \\ f(4) &= \underline{0.2500} \\ & \quad 8.3263 \end{aligned}$$

$$\int_1^4 \frac{dx}{x} \approx \frac{0.5}{3}(8.3263) \approx 1.388$$

9. $\int_{45}^{70} l(t)dt$, males, $n = 5$, $a = 45$, $b = 70$

$$h = \frac{70-45}{5} = 5$$

$$\begin{aligned} l(45) &= 93,717 \\ 2l(50) &= 183,232 \\ 2l(55) &= 177,292 \\ 2l(60) &= 168,376 \\ 2l(65) &= 155,094 \\ l(70) &= \underline{68,375} \\ & \quad 846,086 \end{aligned}$$

$$\int_{45}^{70} l(t)dt \approx \frac{5}{2}(846,086) = 2,115,215$$

10. $\int_{35}^{55} l(t)dt$, females, $n = 4$, $a = 35$, $b = 55$

$$h = \frac{55-35}{4} = 5$$

$$\begin{aligned} l(35) &= 97,964 \\ 2l(40) &= 194,796 \\ 2l(45) &= 193,164 \\ 2l(50) &= 190,784 \\ l(55) &= \underline{93,562} \\ & \quad 770,270 \end{aligned}$$

$$\int_{35}^{55} l(t)dt \approx \frac{5}{2}(770,270) = 1,925,675$$

11. $a = 1$, $b = 5$, $h = 1$

$$\begin{aligned} f(1) &= 0.4 &&= 0.4 \\ 4f(2) &= 4(0.6) &&= 2.4 \\ 2f(3) &= 2(1.2) &&= 2.4 \\ 4f(4) &= 4(0.8) &&= 3.2 \\ f(5) &= 0.5 &&= \underline{0.5} \\ & && \quad 8.9 \end{aligned}$$

$$\int_1^5 f(x)dx \approx \frac{1}{3}(8.9) \approx 3.0$$

The area is about 3.0 square units.

12. $a = 2$, $b = 5$, $h = 0.5$

$$\begin{aligned} f(2) &= 0 \\ 4f(2.5) &= 24 \\ 2f(3) &= 20 \\ 4f(3.5) &= 44 \\ 2f(4) &= 28 \\ 4f(4.5) &= 60 \\ f(5) &= \underline{16} \\ & \quad 192 \end{aligned}$$

$$\int_2^5 f(x)dx \approx \frac{0.5}{3}(192) = 32$$

The area is about 32 square units.

13. $\int_1^3 f(x)dx$, $n = 4$, $a = 1$, $b = 3$

$$h = \frac{3-1}{4} = 0.5$$

$$\begin{aligned} f(1) &= 1 &&= 1 \\ 4f(1.5) &= 4(2) &&= 8 \\ 2f(2) &= 2(2) &&= 4 \\ 4f(2.5) &= 4(0.5) &&= 2 \\ f(3) &= 1 &&= \underline{1} \\ & && \quad 16 \end{aligned}$$

$$\int_1^3 f(x)dx \approx \frac{0.5}{3}(16) = \frac{8}{3}$$

14. $f(x) = \dfrac{2}{\sqrt{1+x}}$, $a = 1$, $b = 3$, $n = 4$

$$h = \frac{3-1}{4} = 0.5$$

Simpson's

$$\begin{aligned} f(1) &\approx 1.4142 \\ 4f(1.5) &\approx 5.0596 \\ 2f(2) &\approx 2.3094 \\ 4f(2.5) &\approx 4.2762 \\ f(3) &= \underline{1.0000} \\ & \quad 14.0594 \end{aligned}$$

$$\int_1^3 \frac{2}{\sqrt{1+x}}dx \approx \frac{0.5}{3}(14.0594) \approx 2.343$$

For the actual value, we have

$$\int_1^3 \frac{2}{\sqrt{1+x}}dx = 2\int_1^3 (1+x)^{-1/2}dx$$

$$= 2[2(1+x)^{1/2}]\Big|_1^3 = 4\left(2-\sqrt{2}\right) \approx 2.343$$

15. $f(x)=\sqrt{1-x^2}$, $a = 0$, $b = 1$, $n = 4$

$h=\frac{1-0}{4}=0.25$

Simpson's

$$\begin{aligned} f(0) &= 1.0000 \\ 4f(0.25) &= 3.8730 \\ 2f(0.50) &= 1.7321 \\ 4f(0.75) &= 2.6458 \\ f(1) &= \underline{0.0000} \\ & 9.2509 \end{aligned}$$

$$\int_0^1 \sqrt{1-x^2}\,dx \approx \frac{0.25}{3}(9.2509) \approx 0.771$$

16. $\int_0^{80} \frac{dr}{dq}dq = r(80)-r(0)=r(80)$

[since $r(0) = 0$]

Using Simpson's rule with $h = 10$ and $f(q)=\frac{dr}{dq}$:

$$\begin{aligned} f(0) &= 10 &&= 10 \\ 4f(10) &= 4(9) &&= 36 \\ 2f(20) &= 2(8.5) &&= 17 \\ 4f(30) &= 4(8) &&= 32 \\ 2f(40) &= 2(8.5) &&= 17 \\ 4f(50) &= 4(7.5) &&= 30 \\ 2f(60) &= 2(7) &&= 14 \\ 4f(70) &= 4(6.5) &&= 26 \\ f(80) &= 7 &&= \underline{7} \\ & && 189 \end{aligned}$$

$$\int_0^{80} \frac{dr}{dq}dq \approx \frac{10}{3}(189) = 630$$

The total revenue is about \$630.

17. The distance along the fence is x.
The distance across the pool is $f(x)$.
$a = 0$, $b = 8$, and $n = 8$.

$h=\frac{8-0}{8}=1$

$$\begin{aligned} \text{Area} &\approx \frac{h}{3}[4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+2f(x_6)+4f(x_7)] \\ &= \frac{1}{3}[4(3)+2(4)+4(3)+2(3)+4(2)+2(2)+4(2)] \\ &= \frac{58}{3} \end{aligned}$$

Yes; Lesley's calculation is correct.

18. a. $MC = \frac{dc}{dq}$

$\int_0^{100} \frac{dc}{dq}\,dq$
$= c(100) - c(0)$
$= (\text{total cost of 100 units}) - (\text{fixed costs})$
$= \text{total variable costs of 100 units}$
Using the trapezoidal rule with $h = 20$ and $f(q) = \frac{dc}{dq}$ to estimate the integral:

$$\begin{aligned} f(0) &= 260 \\ 2f(20) &= 500 \\ 2f(40) &= 480 \\ 2f(60) &= 400 \\ 2f(80) &= 480 \\ f(100) &= \underline{250} \\ & \quad 2370 \end{aligned}$$

$\int_0^{100} \frac{dc}{dq}\,dq \approx \frac{20}{2}(2370) = \$23,700$

b. $MR = \frac{dr}{dq}$

$\int_0^{100} \frac{dr}{dq}\,dq = r(100) - r(0) = r(100)$
[since $r(0) = 0$]
= total revenue from sale of 100 units
Using the trapezoidal rule with $h = 20$ and $g(q) = \frac{dr}{dq}$ to estimate the integral:

$$\begin{aligned} g(0) &= 410 \\ 2g(20) &= 700 \\ 2g(40) &= 600 \\ 2g(60) &= 500 \\ 2g(80) &= 540 \\ g(100) &= \underline{250} \\ & \quad 3000 \end{aligned}$$

$\int_0^{100} \frac{dr}{dq}\,dq \approx \frac{20}{2}(3000) = \$30,000$

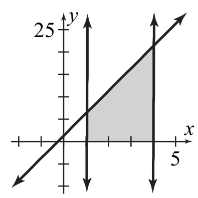
c. At $q = 100$: total revenue = 30,000
total cost = (total var. costs) + (fixed costs)
$= 23,700 + 2000 = 25,700$
Thus maximum profit
= (total revenue) − (total costs)
= 30,000 − 25,700 = \$4300.

Problems 14.9

In Problems 1–24, answers are assumed to be expressed in square units.

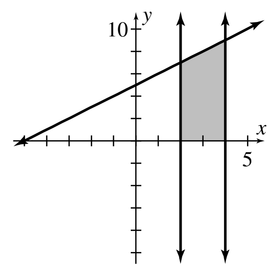
1. $y = 5x + 2$, $x = 1$, $x = 4$

$\text{Area} = \int_1^4 (5x+2)dx$
$= \left(\frac{5x^2}{2} + 2x\right)\Big|_1^4 = 48 - \frac{9}{2} = \frac{87}{2}$



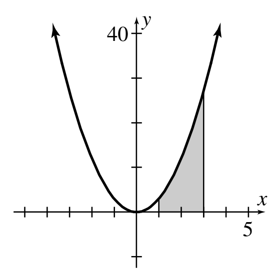
2. $y = x + 5$, $x = 2$, $x = 4$

$\text{Area} = \int_2^4 (x+5)dx = \left(\frac{x^2}{2} + 5x\right)\Big|_2^4$
= 28 − 12 = 16



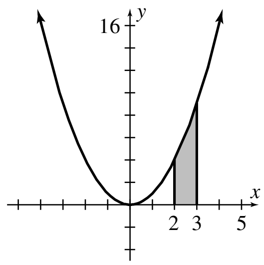
3. $y = 3x^2$, $x = 1$, $x = 3$

$\text{Area} = \int_1^3 3x^2\,dx = x^3\Big|_1^3 = 27 - 1 = 26$



4. $y = x^2,\ x = 2,\ x = 3$

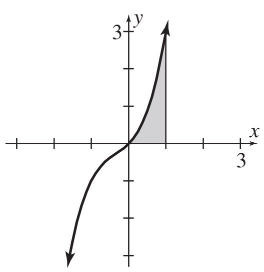
$$\text{Area } = \int_2^3 x^2 dx = \left.\frac{x^3}{3}\right|_2^3 = 9 - \frac{8}{3} = \frac{19}{3}$$



5. $y = x + x^2 + x^3,\ x = 1$

$$\text{Area } = \int_0^1 (x + x^2 + x^3)dx = \left.\left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}\right)\right|_0^1$$

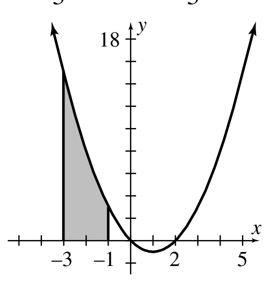
$$= \frac{13}{12} - 0 = \frac{13}{12}$$



6. $y = x^2 - 2x$, $x = -3$, $x = -1$

$$\text{Area } = \int_{-3}^{-1}\left(x^2 - 2x\right)dx = \left.\left(\frac{x^3}{3} - x^2\right)\right|_{-3}^{-1}$$

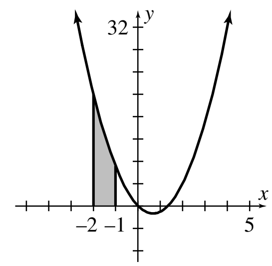
$$= -\frac{4}{3} - (-18) = \frac{50}{3}$$



7. $y = 3x^2 - 4x$, $x = -2$, $x = -1$

$$\text{Area } = \int_{-2}^{-1}\left(3x^2 - 4x\right)dx = \left.\left(x^3 - 2x^2\right)\right|_{-2}^{-1}$$

$= -3 - (-16) = 13$

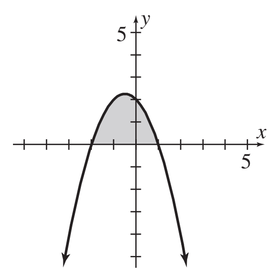


8. $y = 2 - x - x^2$

$$\text{Area} = \int_{-2}^{1}(2 - x - x^2)dx = \left.\left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\right|_{-2}^{1}$$

$$= \frac{7}{6} - \left(-\frac{10}{3}\right)$$

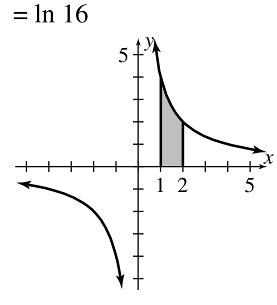
$$= \frac{9}{2}$$



9. $y = \frac{4}{x}$, $x = 1$, $x = 2$

$$\text{Area } = \int_1^2 \frac{4}{x}dx = 4\ln|x|\Big|_1^2 = 4\ln(2) - 0 = 4\ln 2$$

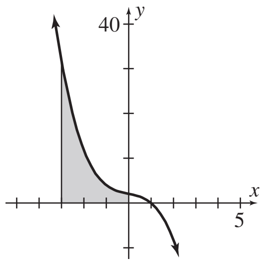
$= \ln 16$



10. $y = 2 - x - x^3$, $x = -3$, $x = 0$

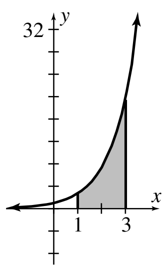
$$\text{Area} = \int_{-3}^{0} \left(2 - x - x^3\right) dx = \left(2x - \frac{x^2}{2} - \frac{x^4}{4}\right)\Bigg|_{-3}^{0}$$

$$= 0 - \left(-\frac{123}{4}\right) = \frac{123}{4}$$



11. $y = e^x$, $x = 1$, $x = 3$

$$\text{Area} = \int_1^3 e^x dx = e^x \Big|_1^3 = e^3 - e$$

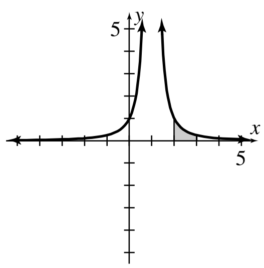


12. $y = \dfrac{1}{(x-1)^2}$, $x = 2$, $x = 3$

$$\text{Area} = \int_2^3 \frac{1}{(x-1)^2} dx = \int_2^3 (x-1)^{-2} dx$$

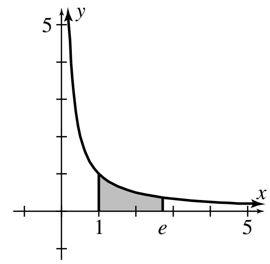
$$= \frac{(x-1)^{-1}}{-1}\Bigg|_2^3 = \left(-\frac{1}{x-1}\right)\Bigg|_2^3$$

$$= -\frac{1}{2} - (-1) = \frac{1}{2}$$



13. $y = \dfrac{1}{x}$, $x = 1$, $x = e$

$$\text{Area} = \int_1^e \frac{1}{x} dx = \ln|x|\Big|_1^e = \ln e - \ln 1 = 1 - 0 = 1$$

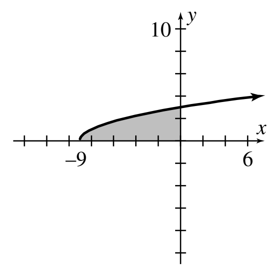


14. $y = \sqrt{x+9}$, $x = -9$, $x = 0$

$$\text{Area} = \int_{-9}^{0} \sqrt{x+9}\, dx = \int_{-9}^{0} (x+9)^{\frac{1}{2}} dx$$

$$= \frac{(x+9)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_{-9}^{0} = \frac{2(x+9)^{\frac{3}{2}}}{3}\Bigg|_{-9}^{0}$$

$= 18 - 0 = 18$

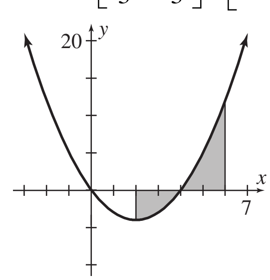


15. $y = x^2 - 4x$, $x = 2$, $x = 6$

$$\text{Area} = \int_2^4 -(x^2 - 4x) dx + \int_4^6 (x^2 - 4x) dx$$

$$= \left(-\frac{x^3}{3} + 2x^2\right)\Bigg|_2^4 + \left(\frac{x^3}{3} - 2x^2\right)\Bigg|_4^6$$

$$= \left[\frac{32}{3} - \frac{16}{3}\right] + \left[0 - \left(-\frac{32}{3}\right)\right] = 16$$

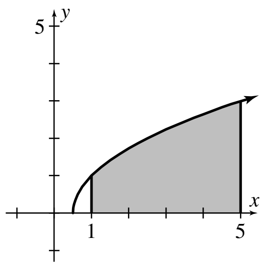


16. $y=\sqrt{2x-1}$, $x=1$, $x=5$

Area $=\int_1^5 \sqrt{2x-1}\,dx$

$=\frac{1}{2}\int_1^5 (2x-1)^{\frac{1}{2}}[2\ dx]$

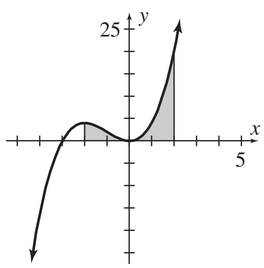
$=\left.\frac{(2x-1)^{\frac{3}{2}}}{3}\right|_1^5 = 9-\frac{1}{3}=\frac{26}{3}$



17. $y=x^3+3x^2$, $x=-2$, $x=2$

Area $=\int_{-2}^{2}\left(x^3+3x^2\right)dx = \left.\left(\frac{x^4}{4}+x^3\right)\right|_{-2}^{2}$

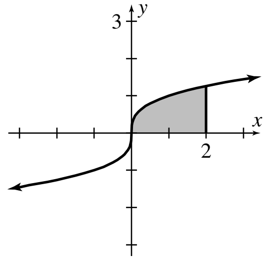
= 12 – (–4) = 16



18. $y=\sqrt[3]{x}$, $x=2$

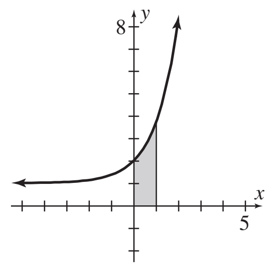
Area $=\int_0^2 \sqrt[3]{x}dx = \int_0^2 x^{\frac{1}{3}}dx = \left.\frac{3x^{\frac{4}{3}}}{4}\right|_0^2 = \frac{3(2)^{\frac{4}{3}}}{4}-0$

$=\frac{3\left(2\sqrt[3]{2}\right)}{4}=\frac{3}{2}\sqrt[3]{2}$



19. $y=e^x+1$, $x=0$, $x=1$

Area $=\int_0^1 (e^x+1)dx = (e^x+x)\Big|_0^1 = (e^1+1)-1=e$

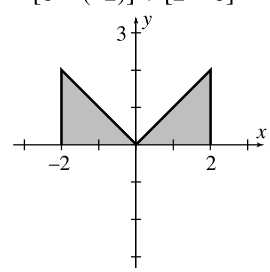


20. $y=|x|$, $x=-2$, $x=2$

Area $=\int_{-2}^{2}|x|\,dx = \int_{-2}^{0}(-x)dx+\int_0^2 x\ dx$

$=-\left.\frac{x^2}{2}\right|_{-2}^{0}+\left.\frac{x^2}{2}\right|_0^2$

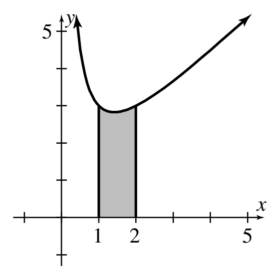
= [0 – (–2)] + [2 – 0] = 4



21. $y=x+\frac{2}{x}$, $x=1$, $x=2$

Area $=\int_1^2\left(x+\frac{2}{x}\right)dx = \left.\left(\frac{x^2}{2}+2\ln|x|\right)\right|_1^2$

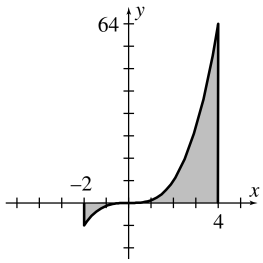
$=(2+2\ln 2)-\frac{1}{2}=\frac{3}{2}+2\ln 2=\frac{3}{2}+\ln 4$



22. $y = x^3$, $x = -2$, $x = 4$

$\text{Area} = \int_{-2}^{0} -x^3 dx + \int_0^4 x^3 dx = -\left.\frac{x^4}{4}\right|_{-2}^{0} + \left.\frac{x^4}{4}\right|_0^4$

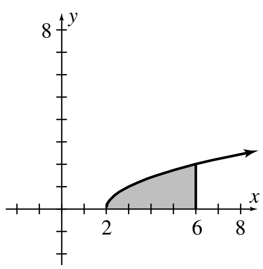
$= [0 - (-4)] + [64 - 0] = 68$



23. $y = \sqrt{x-2}$, $x = 2$, $x = 6$

$\text{Area} = \int_2^6 \sqrt{x-2}\, dx = \left.\frac{2(x-2)^{\frac{3}{2}}}{3}\right|_2^6$

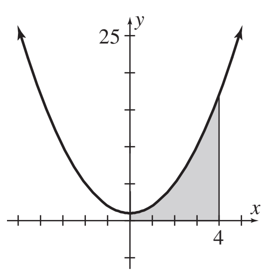
$= \frac{16}{3} - 0 = \frac{16}{3}$



24. $y = x^2 + 1$, $x = 0$, $x = 4$

$\text{Area} = \int_0^4 \left(x^2 + 1\right) dx = \left.\left(\frac{x^3}{3} + x\right)\right|_0^4$

$= \frac{76}{3} - 0 = \frac{76}{3}$

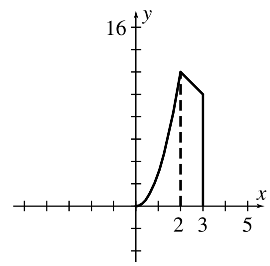


25. $f(x) = \begin{cases} 3x^2 & \text{if } \ 0 \le x < 2 \\ 16 - 2x & \text{if } \ x \ge 2 \end{cases}$

$\text{Area} = \int_0^3 f(x)dx = \int_0^2 3x^2 dx + \int_2^3 (16 - 2x)dx$

$= \left.x^3\right|_0^2 + \left.\left(16x - x^2\right)\right|_2^3$

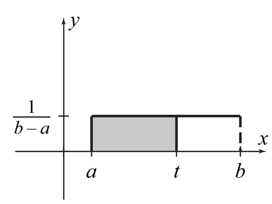
$= [8 - 0] + [39 - 28] = 19$ sq units



26. $y = \frac{1}{b-a}$

$\text{Area} = \int_a^t \frac{1}{b-a} dx = \left.\frac{x}{b-a}\right|_a^t$

$= \frac{t}{b-a} - \frac{a}{b-a} = \frac{t-a}{b-a}$ sq units



27. a. $P(0 \le x \le 1) = \int_0^1 \frac{1}{8} x\, dx = \left.\frac{x^2}{16}\right|_0^1 = \frac{1}{16} - 0$

$= \frac{1}{16}$

b. $P(2 \le x \le 4) = \int_2^4 \frac{1}{8} x\, dx = \left.\frac{x^2}{16}\right|_2^4 = 1 - \frac{1}{4} = \frac{3}{4}$

c. $P(x \ge 3) = \int_3^4 \frac{1}{8} x\, dx = \left.\frac{x^2}{16}\right|_3^4 = 1 - \frac{9}{16} = \frac{7}{16}$

28. **a.** $P(1 \le x \le 2) = \int_1^2 \frac{1}{3}(1-x)^2\,dx$

$= \frac{1}{3}(-1)\int_1^2 (1-x)^2[-dx] = -\frac{1}{3} \cdot \left.\frac{(1-x)^3}{3}\right|_1^2$

$= -\frac{1}{9}(-1-0) = \frac{1}{9}$

b. $P\left(1 \le x \le \frac{5}{2}\right) = \int_1^{5/2} \frac{1}{3}(1-x)^2\,dx$

$= -\frac{1}{9}(1-x)^3\Big|_1^{5/2} = -\frac{1}{9}\left(-\frac{27}{8} - 0\right) = \frac{3}{8}$

c. $P(x \le 1) = \int_0^1 \frac{1}{3}(1-x)^2\,dx = -\frac{1}{9}(1-x)^3\Big|_0^1$

$= -\frac{1}{9}(0-1) = \frac{1}{9}$

d. $\int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx$

$1 = \frac{1}{9} + P(x \ge 1)$

Thus, $P(x \ge 1) = \frac{8}{9}$.

29. **a.** $P(3 \le x \le 7) = \int_3^7 \frac{1}{x}\,dx = \ln|x|\Big|_3^7$

$= \ln 7 - \ln 3 = \ln\frac{7}{3}$

b. $P(x \le 5) = \int_e^5 \frac{1}{x}\,dx = \ln|x|\Big|_e^5$

$= \ln(5) - \ln e = \ln(5) - 1$

c. $P(x \ge 4) = \int_4^{e^2} \frac{1}{x}\,dx = \ln|x|\Big|_4^{e^2}$

$= \ln e^2 - \ln 4 = 2 - \ln 4$

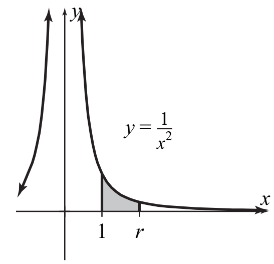
d. $P\left(e \le x \le e^2\right) = \int_e^{e^2} \frac{1}{x}\,dx$

$= \ln|x|\Big|_e^{e^2} = \ln e^2 - \ln e$

$= 2 - 1 = 1$

30. **a.** $\int_1^r \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_1^r = -\frac{1}{r} + 1 = 1 - \frac{1}{r}$

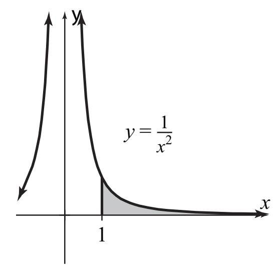
b.



c. $\lim\limits_{r\to\infty} \int_1^r \frac{1}{x^2}\,dx = \lim\limits_{r\to\infty}\left(1 - \frac{1}{r}\right)$ [from part (a)]

$= 1 - 0 = 1$

d.



31. 1.89 sq units

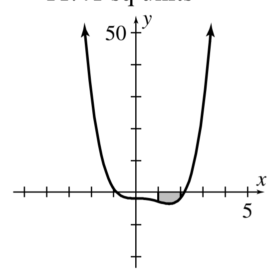
32. 7.18 sq units

33. The x-intercept on [1, 3] is $A \approx 2.190327947$

Area

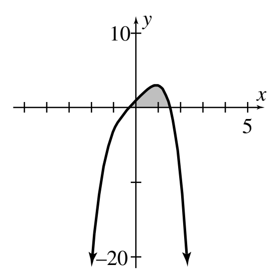
$= \int_1^A -\left(x^4 - 2x^3 - 2\right)dx + \int_A^3 \left(x^4 - 2x^3 - 2\right)dx$

$\approx$ 11.41 sq units



34. The x-intercepts are $A \approx -0.3294085282$ and $B \approx 1.539613346$

Area $= \int_A^B \left(1 + 3x - x^4\right)dx \approx 3.53$ sq units



35. Intersection points:
$x^2 - x = 2x,\ x^2 - 3x = 0,\ x(x-3) = 0 \Rightarrow x = 0$ or $x = 3$

$$\text{Area} = \int_0^3 (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx + \int_3^4 (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx$$
$$= \int_0^3 \left[2x - \left(x^2 - x\right)\right]dx + \int_3^4 \left[\left(x^2 - x\right) - 2x\right]dx$$

36. Intersection points: $x(x-3)^2 = 2x,\ \ x(x-3)^2 - 2x = 0,\ \ x\left[(x-3)^2 - 2\right] = 0,\ \ x(x^2 - 6x + 7) = 0 \Rightarrow x = 0,\ 3 \pm \sqrt{2}$ (from the quadratic formula)

$$\text{Area} = \int_0^{3-\sqrt{2}} (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx + \int_{3-\sqrt{2}}^{3+\sqrt{2}} (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx$$
$$= \int_0^{3-\sqrt{2}} \left[x(x-3)^2 - 2x\right]dx + \int_{3-\sqrt{2}}^{3+\sqrt{2}} \left[2x - x(x-3)^2\right]dx$$

37. The graphs of $y = 1 - x^2$ and $y = x - 1$ intersect when $1 - x^2 = x - 1$, $0 = x^2 + x - 2$, $0 = (x-1)(x+2) \Rightarrow x = 1$ or $x = -2$. When $x = 1$, then $y = 0$. We use horizontal elements, where y ranges from 0 to 1. Solving $y = x - 1$ for x gives $x = y + 1$, and solving $y = 1 - x^2$ for x gives $x^2 = 1 - y$, $x = \pm\sqrt{1-y}$. We must choose $x = \sqrt{1-y}$ because x is not negative over the given region.

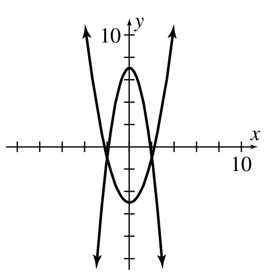
$$\text{Area} = \int_0^1 (x_{\text{RIGHT}} - x_{\text{LEFT}})\,dy = \int_0^1 \left[(y+1) - \sqrt{1-y}\right]dy$$

38. The graphs of $y = 2x$ and $y = -2x - 8$ intersect when $2x = -2x - 8$, $4x = -8$, $x = -2$. When $x = -2$, then $y = -4$. We use horizontal elements, where y ranges from -4 to 4. Solving $y = 2x$ for x gives $x = \frac{y}{2}$; solving $y = -2x - 8$ for x gives $2x = -y - 8$, $x = \frac{-y-8}{2}$.

$$\text{Area} = \int_{-4}^4 (x_{\text{RIGHT}} - x_{\text{LEFT}})\,dy = \int_{-4}^4 \left[\frac{y}{2} - \left(\frac{-y-8}{2}\right)\right]dy$$

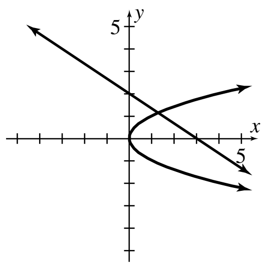
39. The graphs of $y = x^2 - 5$ and $y = 7 - 2x^2$ intersect when $x^2 - 5 = 7 - 2x^2$, $3x^2 = 12$, $x^2 = 4$, so $x = \pm\sqrt{4} = \pm 2$. We use vertical elements.

$$\text{Area} = \int_1^2 (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx$$
$$= \int_1^2 [(7 - 2x^2) - (x^2 - 5)]\,dx$$



40. The curves $y^2 = x$ and $2y = 3 - x$ (or $x = 3 - 2y$) intersect when $y^2 = 3 - 2y$, $y^2 + 2y - 3 = 0$, $(y + 3)(y - 1) = 0 \Rightarrow y = -3$ or 1. We use horizontal elements.

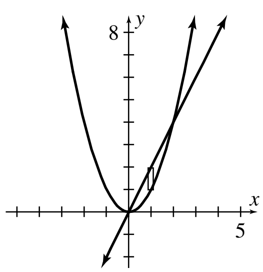
$$\text{Area} = \int_0^1 (x_{\text{RIGHT}} - x_{\text{LEFT}})\,dy$$
$$= \int_0^1 [(3 - 2y) - y^2]\,dy$$



In Problems 41–58, the answers are assumed to be expressed in square units.

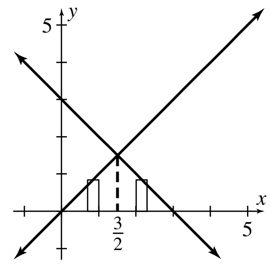
41. $y = x^2$, $y = 2x$
Region appears below.
Intersection: $x^2 = 2x$, $x^2 - 2x = 0$, $x(x - 2) = 0$, so $x = 0$ or 2.

$$\text{Area} = \int_0^2 \left(2x - x^2\right) dx = \left(x^2 - \frac{x^3}{3}\right)\Bigg|_0^2$$
$$= \left(4 - \frac{8}{3}\right) - 0 = \frac{4}{3}$$



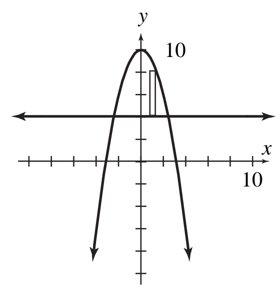
42. $y = x$, $y = -x + 3$, $y = 0$. Region appears below.
Intersection: $x = -x + 3$, $2x = 3$, $x = \frac{3}{2}$

$$\text{Area} = \int_0^{3/2} x\,dx + \int_{3/2}^{3} (-x + 3)dx$$
$$= \frac{x^2}{2}\Bigg|_0^{3/2} + \left(-\frac{x^2}{2} + 3x\right)\Bigg|_{3/2}^{3}$$
$$= \left[\frac{9}{8} - 0\right] + \left[\left(-\frac{9}{2} + 9\right) - \left(-\frac{9}{8} + \frac{9}{2}\right)\right] = \frac{9}{4}$$



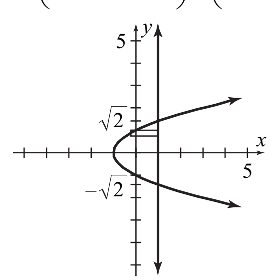
43. $y = 10 - x^2$, $y = 4$. Region appears below.
Intersection: $10 - x^2 = 4$, $x^2 = 6$, so $x = \pm\sqrt{6}$

$$\text{Area} = \int_{-\sqrt{6}}^{\sqrt{6}} [(10 - x^2) - 4]\,dx$$
$$= \int_{-\sqrt{6}}^{\sqrt{6}} (6 - x^2)\,dx$$
$$= \left(6x - \frac{x^3}{3}\right)\Bigg|_{-\sqrt{6}}^{\sqrt{6}}$$
$$= \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-6\sqrt{6} + \frac{6\sqrt{6}}{3}\right)$$
$$= 8\sqrt{6}$$



44. $y^2 = x + 1$, $x = 1$. Region appears below.
Intersection: $y^2 = 2$, $y = \pm\sqrt{2}$

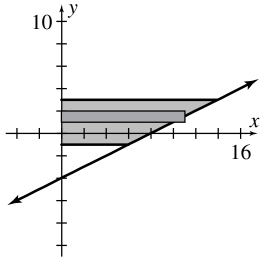
$$\text{Area} = \int_{-\sqrt{2}}^{\sqrt{2}} \left[1 - \left(y^2 - 1\right)\right] dy = \left(2y - \frac{y^3}{3}\right)\Bigg|_{-\sqrt{2}}^{\sqrt{2}}$$
$$= \left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right) - \left(-2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) = \frac{8\sqrt{2}}{3}$$



45. $x = 8 + 2y$, $x = 0$, $y = -1$, $y = 3$. Region appears below.

Area $= \int_{-1}^{3}(8+2y)dy = \left(8y + y^2\right)\Big|_{-1}^{3}$

$= (24 + 9) - (-8 + 1) = 40$



46. $y = x - 6$, $y^2 = x$. Region appears below.

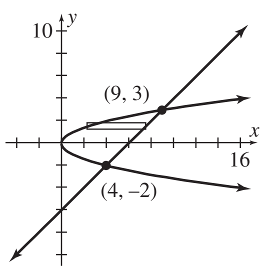
Intersection: $y^2 = y + 6$, $y^2 - y - 6 = 0$,

$(y + 2)(y - 3) = 0$, so $y = -2, 3$.

Area $= \int_{-2}^{3}\left[(y+6) - (y^2)\right]dy$

$= \left(\frac{y^2}{2} + 6y - \frac{y^3}{3}\right)\Bigg|_{-2}^{3}$

$= \left(\frac{9}{2} + 18 - 9\right) - \left(2 - 12 + \frac{8}{3}\right) = \frac{125}{6}$



47. $y^2 = 4x$, $y = 2x - 4$. Region appears below.

Intersection: $y^2 = 4\left(\frac{y}{2} + 2\right)$, $y^2 - 2y - 8 = 0$,

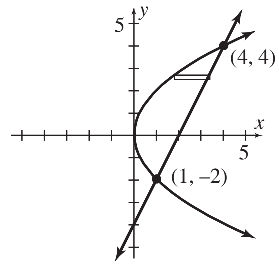
$(y + 2)(y - 4) = 0$, so $y = -2$ or 4.

Area $= \int_{-2}^{4}\left[\left(\frac{y}{2} + 2\right) - \frac{y^2}{4}\right]dy$

$= \left(\frac{y^2}{4} + 2y - \frac{y^3}{12}\right)\Bigg|_{-2}^{4}$

$= \left(4 + 8 - \frac{16}{3}\right) - \left(1 - 4 + \frac{2}{3}\right)$

$= 9$



48. $y = x^3$, $y = x + 6$, $x = 0$

Region appears below.

Intersection: $x^3 = x + 6$, $x^3 - x - 6 = 0$,

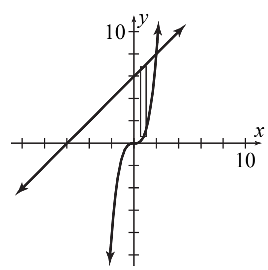
$(x-2)(x^2 + 2x + 3) = 0 \Rightarrow x = 2$

$x^3 = 0 \Rightarrow x = 0$

Area $= \int_{0}^{2}[(x+6) - x^3]\,dx$

$= \left(\frac{x^2}{2} + 6x - \frac{x^4}{4}\right)\Bigg|_{0}^{2}$

$= (2 + 12 - 4) - (0) = 10$



49. $2y = 4x - x^2$, $2y = x - 4$. Region appears below.

Intersection: $x - 4 = 4x - x^2$, $x^2 - 3x - 4 = 0$, $(x + 1)(x - 4) = 0$, so $x = -1$ or 4. Note that the y-values of the curves are given by $y = \frac{4x - x^2}{2}$ and $y = \frac{x-4}{2}$.

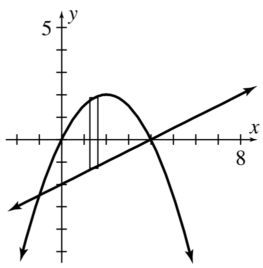
$$\text{Area} = \int_{-1}^{4}\left[\left(\frac{4x-x^2}{2}\right)-\left(\frac{x-4}{2}\right)\right]dx$$

$$= \int_{-1}^{4}\left(\frac{3}{2}x-\frac{x^2}{2}+2\right)dx$$

$$= \left(\frac{3x^2}{4}-\frac{x^3}{6}+2x\right)\Bigg|_{-1}^{4}$$

$$= \left(12-\frac{64}{6}+8\right)-\left(\frac{3}{4}+\frac{1}{6}-2\right)$$

$$= \frac{125}{12}$$



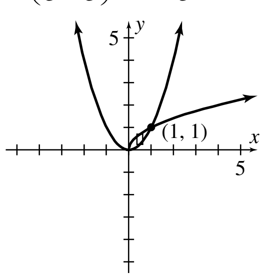
50. $y=\sqrt{x},\ y=x^2$. Region appears below.

Intersection: $x^2=\sqrt{x},\ x^4=x,\ x^4-x=0$,

$x\left(x^3-1\right)=0$, so $x = 0, 1$.

$$\text{Area} = \int_0^1\left(\sqrt{x}-x^2\right)dx = \left(\frac{2x^{\frac{3}{2}}}{3}-\frac{x^3}{3}\right)\Bigg|_0^1$$

$$= \left(\frac{2}{3}-\frac{1}{3}\right)-0=\frac{1}{3}$$

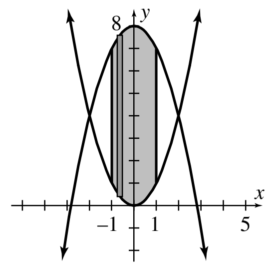


51. $y=8-x^2,\ y=x^2,\ x=-1,\ x=1$. Region appears below.

Intersection: $x^2=8-x^2,\ 2x^2=8,\ x^2=4$, so $x = \pm 2$.

$$\text{Area} = \int_{-1}^{1}\left[\left(8-x^2\right)-x^2\right]dx = \int_{-1}^{1}\left(8-2x^2\right)dx$$

$$= \left(8x-\frac{2x^3}{3}\right)\Bigg|_{-1}^{1} = \left(8-\frac{2}{3}\right)-\left(-8+\frac{2}{3}\right)=\frac{44}{3}$$



52. $y=x^3+x,\ y=0$ (x-axis), $x=-1,\ x=2$

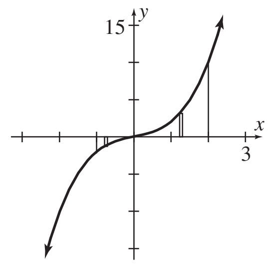
Region appears below.

$$\text{Area} = \int_{-1}^{0}-(x^3+x)dx+\int_0^2(x^3+x)dx$$

$$= \left(-\frac{x^4}{4}-\frac{x^2}{2}\right)\Bigg|_{-1}^{0}+\left(\frac{x^4}{4}+\frac{x^2}{2}\right)\Bigg|_0^2$$

$$= \left[0-\left(-\frac{1}{4}-\frac{1}{2}\right)\right]+[(4+2)-0]$$

$$= \frac{27}{4}$$



53. $y=x^3-1,\ y=x-1$. Region appears below.

Intersection: $x^3-1=x-1,\ x^3-x=0$,

$x\left(x^2-1\right)=0$,

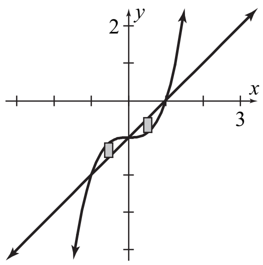
$x(x+1)(x-1)=0$, so $x = 0$ or $x = \pm 1$.

Area $= \int_{-1}^{0}[x^3-1-(x-1)]\,dx+\int_{0}^{1}[x-1-(x^3-1)]\,dx$

$= \int_{-1}^{0}\left(x^3-x\right)dx+\int_{0}^{1}\left(x-x^3\right)dx$

$= \left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Bigg|_{-1}^{0}+\left(\frac{x^2}{2}-\frac{x^4}{4}\right)\Bigg|_{0}^{1}$

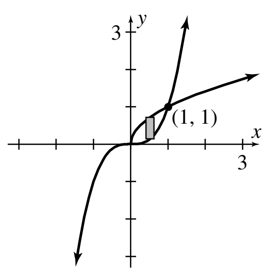
$= \left[0-\left(\frac{1}{4}-\frac{1}{2}\right)\right]+\left[\left(\frac{1}{2}-\frac{1}{4}\right)-0\right]=\frac{1}{2}$



54. $y=x^3,\ y=\sqrt{x}$. Region appears below. Intersection: $x^3=\sqrt{x},\ x^6=x,\ x^6-x=0$, $x\left(x^5-1\right)=0$, $x=0, 1$

Area $= \int_{0}^{1}\left(\sqrt{x}-x^3\right)dx=\left(\frac{2x^{\frac{3}{2}}}{3}-\frac{x^4}{4}\right)\Bigg|_{0}^{1}$

$= \left(\frac{2}{3}-\frac{1}{4}\right)-0=\frac{5}{12}$

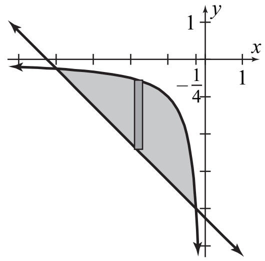


55. $4x+4y+17=0,\ y=\frac{1}{x}$. Region appears below. Intersection: $\frac{-17-4x}{4}=\frac{1}{x}$, $-17x-4x^2=4$, $4x^2+17x+4=0$,

$(4x+1)(x+4)=0$, so $x=-\frac{1}{4}$ or -4.

Area $= \int_{-4}^{-1/4}\left[\frac{1}{x}-\left(\frac{-17-4x}{4}\right)\right]dx=\left(\ln|x|+\frac{17}{4}x+\frac{x^2}{2}\right)\Bigg|_{-4}^{-1/4}$

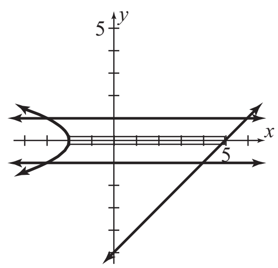
$= \left(\ln\frac{1}{4}-\frac{17}{16}+\frac{1}{32}\right)-\left(\ln 4-17+8\right)=\frac{255}{32}-4\ln 2$



56. $y^2 = -x-2,\ x-y=5,\ y=-1,\ y=1.$
Region appears below.
Intersection: $y^2 = -x-2$ intersects $y = \pm 1$ when $x = -3$; $x - y = 5$ intersects $y = 1$ when $x = 6$; $x - y = 5$ intersects $y = -1$ when $x = 4$

$$\text{Area} = \int_{-1}^{1}[(y+5)-(-y^2-2)]\,dy = \int_{-1}^{1}(y+7+y^2)\,dy = \left(\frac{y^2}{2}+7y+\frac{y^3}{3}\right)\Bigg|_{-1}^{1}$$

$$= \left(\frac{1}{2}+7+\frac{1}{3}\right)-\left(\frac{1}{2}-7-\frac{1}{3}\right) = \frac{44}{3}$$

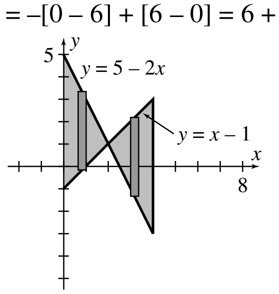


57. $y = x-1$, $y = 5-2x$. Region appears below.
Intersection: $x - 1 = 5 - 2x$, $3x = 6$, so $x = 2$.

$$\text{Area} = \int_0^2 [(5-2x)-(x-1)]dx + \int_2^4 [(x-1)-(5-2x)]dx = \int_0^2 (6-3x)dx + \int_2^4 (3x-6)dx$$

$$= -\frac{1}{3}\int_0^2 (6-3x)[-3\ dx] + \frac{1}{3}\int_2^4 (3x-6)[3\ dx] = -\frac{(6-3x)^2}{6}\Bigg|_0^2 + \frac{(3x-6)^2}{6}\Bigg|_2^4$$

$$= -[0-6]+[6-0] = 6+6 = 12$$



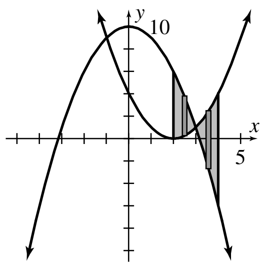
58. $y = x^2 - 4x + 4,\ y = 10 - x^2$. Region appears below.

Intersection: $x^2 - 4x + 4 = 10 - x^2$, $2x^2 - 4x - 6 = 0$, $x^2 - 2x - 3 = 0$, $(x - 3)(x + 1) = 0$, so $x = 3, -1$.

$$\text{Area } = \int_2^3\left[\left(10-x^2\right)-\left(x^2-4x+4\right)\right]dx + \int_3^4\left[\left(x^2-4x+4\right)-\left(10-x^2\right)\right]dx$$

$$= \int_2^3\left(6+4x-2x^2\right)dx + \int_3^4\left(2x^2-4x-6\right)dx = 2\left\{\int_2^3\left(3+2x-x^2\right)dx + \int_3^4\left(x^2-2x-3\right)dx\right\}$$

$$= 2\left\{\left(3x+x^2-\frac{x^3}{3}\right)\Bigg|_2^3 + \left(\frac{x^3}{3}-x^2-3x\right)\Bigg|_3^4\right\} = 2\left\{\left[9-\frac{22}{3}\right]+\left[-\frac{20}{3}-(-9)\right]\right\} = 2\{4\} = 8$$



59. $$\frac{\text{Area between curve and diag.}}{\text{Area under diagonal}} = \frac{\int_0^1\left[x-\left(\frac{14}{15}x^2+\frac{1}{15}x\right)\right]dx}{\int_0^1 x\,dx}$$

$$\text{Numerator } = \int_0^1\left[\frac{14}{15}x-\frac{14}{15}x^2\right]dx = \frac{14}{15}\int_0^1\left(x-x^2\right)dx = \frac{14}{15}\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Bigg|_0^1 = \frac{14}{15}\left[\left(\frac{1}{2}-\frac{1}{3}\right)-0\right] = \frac{14}{15}\cdot\frac{1}{6} = \frac{7}{45}$$

$$\text{Denominator } = \int_0^1 x\,dx = \frac{x^2}{2}\Bigg|_0^1 = \frac{1}{2}$$

$$\text{Coefficient of inequality } = \frac{\frac{7}{45}}{\frac{1}{2}} = \frac{14}{45}$$

60. $$\frac{\text{Area between curve and diag.}}{\text{Area under diagonal}} = \frac{\int_0^1\left[x-\left(\frac{11}{12}x^2+\frac{1}{12}x\right)\right]dx}{\int_0^1 x\,dx}$$

$$\text{Numerator } = \frac{11}{12}\int_0^1\left(x-x^2\right)dx = \frac{11}{12}\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Bigg|_0^1 = \frac{11}{12}\left[\left(\frac{1}{2}-\frac{1}{3}\right)-0\right] = \frac{11}{12}\cdot\frac{1}{6} = \frac{11}{72}$$

Denominator $= \frac{1}{2}$ (see Problem 35).

$$\text{Coefficient of inequality } = \frac{\frac{11}{72}}{\frac{1}{2}} = \frac{11}{36}$$

61. $y^2 = 3x,\ y = mx$

Intersection: $(mx)^2 = 3x,\ m^2x^2 = 3x$

$m^2x^2 - 3x = 0,\ x(m^2x - 3) = 0,\ x = 0$ or

$x = \dfrac{3}{m^2}.$

If $x = 0$, then $y = 0$; if $x = \dfrac{3}{m^2}$, then $y = \dfrac{3}{m}.$

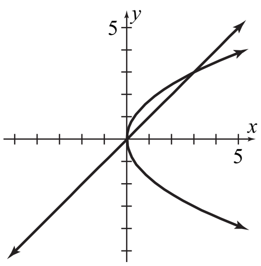
With horizontal elements,

$$\text{Area} = \int_0^{3/m}\left(\frac{y}{m} - \frac{y^2}{3}\right)dy = \left(\frac{y^2}{2m} - \frac{y^3}{9}\right)\Bigg|_0^{\frac{3}{m}}$$

$$= \frac{9}{2m^3} - \frac{3}{m^3} = \frac{3}{2m^3} \text{ square units}$$

Note: With vertical elements,

$$\text{Area} = \int_0^{3/m^2}\left(\sqrt{3}\sqrt{x} - mx\right)dx.$$



62. a. $y = x^2 - 1$, $y = 2x + 2$

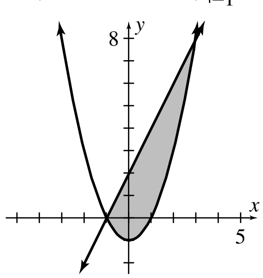
Intersection: $x^2 - 1 = 2x + 2$,

$x^2 - 2x - 3 = 0$, $(x - 3)(x + 1)$, so $x = 3$ and -1. The area is

$$\int_{-1}^{3}\left[2x + 2 - \left(x^2 - 1\right)\right]dx$$

$$= \int_{-1}^{3}\left(-x^2 + 2x + 3\right)dx$$

$$= \left(-\frac{x^3}{3} + x^2 + 3x\right)\Bigg|_{-1}^{3} = \frac{32}{3}$$



b. The area below is $\int_{-1}^{1}\left(1 - x^2\right)dx = \dfrac{4}{3}$. Thus the area above is $\dfrac{32}{3} - \dfrac{4}{3} = \dfrac{28}{3}$. Hence the percentage above the x-axis is

$$\frac{\frac{28}{3}}{\frac{32}{3}} \cdot 100 = 87.5\%$$

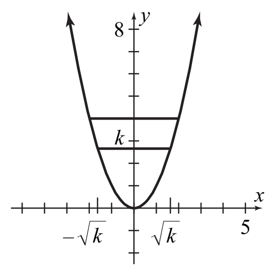
63. $y = x^2$ and $y = k$ intersect when $x^2 = k,\ x = \pm\sqrt{k}$. Equating areas gives

$$\int_{-\sqrt{k}}^{\sqrt{k}}\left(k - x^2\right)dx = \frac{1}{2}\int_{-2}^{2}\left(4 - x^2\right)dx$$

$$\left(kx - \frac{x^3}{3}\right)\Bigg|_{-\sqrt{k}}^{\sqrt{k}} = \frac{1}{2}\left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{2}$$

$$\frac{4}{3}k^{\frac{3}{2}} = \frac{16}{3}$$

$$k^{\frac{3}{2}} = 4 \Rightarrow k = 4^{\frac{2}{3}} = \left(2^2\right)^{\frac{2}{3}} = 2^{\frac{4}{3}} \approx 2.52$$



64. 0.23 sq units

65. 4.76 sq units

66. Two integrals are involved.
Answer: 36.65 sq units

67. Two integrals are involved.
Answer: 7.26 sq units

68. Three integrals are involved.
Answer: 358.18 sq units

Problems 14.10

1. $\left.\begin{matrix} D: p = 22 - 0.8q \\ S: p = 6 + 1.2q \end{matrix}\right\}$

Equilibrium pt. $= (q_0, p_0) = (8, 15.6)$

$$\text{CS} = \int_0^{q_0} [f(q) - p_0]\,dq$$
$$= \int_0^8 [(22 - 0.8q) - 15.6] = \int_0^8 (6.4 - 0.8q)dq$$
$$= \left(6.4q - 0.4q^2\right)\Big|_0^8 = (51.2 - 25.6) - 0 = 25.6$$
$$\text{PS} = \int_0^{q_0} [p_0 - g(q)]\,dq$$
$$= \int_0^8 [15.6 - (6 + 1.2q)]dq = \int_0^8 (9.6 - 1.2q)dq$$
$$= \left(9.6q - 0.6q^2\right)\Big|_0^8 = (76.8 - 38.4) - 0 = 38.4$$

2. $\left.\begin{matrix} D: p = 2200 - q^2 \\ S: p = 400 + q^2 \end{matrix}\right\}$

Equilibrium point $= (q_0, p_0) = (30, 1300)$

$$\text{CS} = \int_0^{30} [(2200 - q^2) - 1300]dq$$
$$= \int_0^{30} (900 - q^2)\,dq = \left(900q - \frac{q^3}{3}\right)\Bigg|_0^{30}$$
$$= (27{,}000 - 9000) - 0 = 18{,}000$$
$$\text{PS} = \int_0^{30} [1300 - (400 + q^2)]\,dq$$
$$= \int_0^{30} (900 - q^2)\,dq$$
$$= \left(900q - \frac{q^3}{3}\right)\Bigg|_0^{30}$$
$$= (27{,}000 - 9000) - 0$$
$$= 18{,}000$$

3. $\left.\begin{matrix} D: p = \frac{50}{q+5} \\ S: p = \frac{q}{10} + 4.5 \end{matrix}\right\}$

Equilibrium pt. $= (q_0, p_0) = (5, 5)$

$$\text{CS} = \int_0^{q_0} [f(q) - p_0]\,dq$$
$$= \int_0^5 \left[\frac{50}{q+5} - 5\right] dq = \left(50 \ln|q+5| - 5q\right)\Big|_0^5$$
$$= [50 \ln(10) - 25] - [50 \ln(5)]$$
$$= 50[\ln(10) - \ln(5)] - 25 = 50 \ln(2) - 25$$

$$\text{PS} = \int_0^{q_0} [p_0 - g(q)]\,dq$$
$$= \int_0^5 \left[5 - \left(\frac{q}{10} + 4.5\right)\right] dq = \int_0^5 \left(0.5 - \frac{q}{10}\right) dq$$
$$= \left(0.5q - \frac{q^2}{20}\right)\Bigg|_0^5 = (2.5 - 1.25) - 0 = 1.25$$

4. $\left.\begin{matrix} D: p = 900 - q^2 \\ S: p = 10q + 300 \end{matrix}\right\}$

Equilibrium pt. $(q_0, p_0) = (20, 500)$

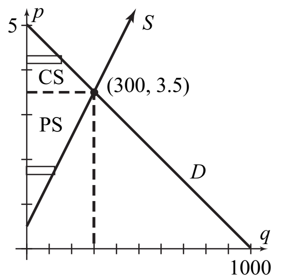
$$\text{CS} = \int_0^{20} [(900 - q^2) - 500]dq$$
$$= \int_0^{20} (400 - q^2)dq$$
$$= \left(400q - \frac{q^3}{3}\right)\Bigg|_0^{20}$$
$$= \left(8000 - \frac{8000}{3}\right) - 0$$
$$= \frac{16{,}000}{3}$$
$$\text{PS} = \int_0^{20} [500 - (10q + 300)]dq$$
$$= \int_0^{20} (200 - 10q)dq$$
$$= (200q - 5q^2)\Big|_0^{20}$$
$$= (4000 - 2000) - 0$$
$$= 2000$$

5. $\left.\begin{matrix} D: q = 100(10 - 2p) \\ S: q = 50(2p - 1) \end{matrix}\right\}$

Equilibrium pt. $= (q_0, p_0) = (300, 3.5)$

We use horizontal strips and integrate with respect to p.

$$\text{CS} = \int_{3.5}^5 100(10 - 2p)dp = 100(10p - p^2)\Big|_{3.5}^5$$
$$= 100[(50 - 25) - (35 - 12.25)]$$
$$= 225$$
$$\text{PS} = \int_{0.5}^{3.5} 50(2p - 1)dp = 50(p^2 - p)\Big|_{0.5}^{3.5}$$
$$= 50[(12.25 - 3.5) - (0.25 - 0.5)]$$
$$= 450$$



6. $\left.\begin{aligned} D&: q=\sqrt{100-p} \\ S&: q=\frac{p}{2}-10 \end{aligned}\right\}$

Equilibrium pt. $=(q_0,\ p_0)=(8,\ 36)$

Integrating with respect to *p*,

$$\text{CS}=\int_{36}^{100}\sqrt{100-p}\,dp$$

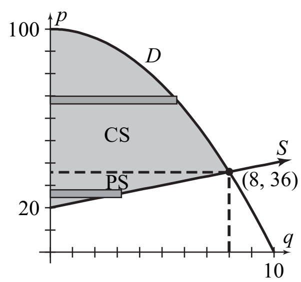
$$=-\frac{2}{3}(100-p)^{\frac{3}{2}}\Big|_{36}^{100}$$

$$=0-\left(-\frac{2}{3}\cdot 512\right)=\frac{1024}{3}$$

$$\text{PS}=\int_{20}^{36}\left[\frac{p}{2}-10\right]dp$$

$$=\left(\frac{p^2}{4}-10p\right)\Big|_{20}^{36}=(324-360)-(100-200)$$

$$=64$$



7. We integrate with respect to *p*. From the demand equation, when $q = 0$, then $p = 100$.

$$\text{CS}=\int_{84}^{100}10\sqrt{100-p}dp$$

$$=\int_{84}^{100}-10(100-p)^{\frac{1}{2}}[-dp]$$

$$=-\frac{20}{3}(100-p)^{\frac{3}{2}}\Big|_{84}^{100}$$

$$=-\frac{20}{3}\left[0-(16)^{\frac{3}{2}}\right]=-\frac{20}{3}(-64)$$

$$=426\frac{2}{3}\approx \$426.67$$

8. At equilibrium, $p=\dfrac{400-p^2}{60}+5$,

$60p=400-p^2+300$, $p^2+60p-700=0$,
$(p+70)(p-10)=0\Rightarrow p=10$ and
$q=400-10^2=300$, so equilibrium pt. is
$(q_0,\ p_0)=(300,\ 10)$.

$$\text{PS}=\int_0^{300}\left[10-\left(\frac{q}{60}+5\right)\right]dq$$

$$=\left(5q-\frac{q^2}{120}\right)\Big|_0^{300}=(1500-750)-0=750$$

For CS we integrate with respect to *p*. From the demand equation, $q=0\Rightarrow p=20$.

$$\text{CS}=\int_{10}^{20}\left(400-p^2\right)dp=\left(400p-\frac{p^3}{3}\right)\Big|_{10}^{20}$$

$$=\left(8000-\frac{8000}{3}\right)-\left(4000-\frac{1000}{3}\right)=1666\frac{2}{3}$$

9. At equilibrium,
$2^{10-q}=2^{q+2}\Rightarrow 10-q=q+2\Rightarrow q=4,$ so
$p=2^{10-4}=64$

$$\text{CS}=\int_0^4(2^{10-q}-64)dq$$

$$=\left(-\frac{2^{10-q}}{\ln 2}-64q\right)\Big|_0^4$$

$$=\left(-\frac{2^6}{\ln 2}-256\right)-\left(-\frac{2^{10}}{\ln 2}-0\right)$$

≈ 1128.987 hundred
$\approx \$113{,}000$

10. a. (10 + 10)(30 + 20) = 1000, (20)(50) = 1000, 1000 = 1000
30 − 4(10) + 10 = 0, 30 − 40 + 10 = 0, 0 = 0

b. $(p + 10)(q + 20) = 1000,\ p+10=\dfrac{1000}{q+20},$

$$p=\frac{1000}{q+20}-10$$

$$\begin{aligned}\text{CS} &= \int_0^{30}\left[\left(\frac{1000}{q+20}-10\right)-10\right]dq \\ &= [1000\ln(q+20)-20q]\Big|_0^{30} \\ &= 1000\ln(50)-600-[1000\ln(20)] \\ &= 1000\ln\left(\frac{50}{20}\right)-600 \\ &= 1000\ln\left(\frac{5}{2}\right)-600\end{aligned}$$

11. CS $\approx$ 1197; PS $\approx$ 477

12. Let $p = f(q)$.

$$\begin{aligned}\text{PS} &= \int_0^{40}[80-f(q)]\,dq \\ &= \int_0^{40}80\ dq-\int_0^{40}f(q)\,dq \\ &= 3200-\int_0^{40}f(q)\,dq\end{aligned}$$

Use the trapezoid rule with $h = 10$ to estimate $\int_0^{40} f(q)\,dq$:

$f(0)$	=	25	=	25
$2f(10)$	=	2(49)	=	98
$2f(20)$	=	2(59)	=	118
$2f(30)$	=	2(71)	=	142
$f(40)$	=	80	=	80
				463

$$\int_0^{40} f(q)\,dq \approx \frac{10}{2}(463) = 2315$$

Thus PS = 3200 − 2315 = \$885.

Chapter 14 Review Problems

1. $\int\left(x^3+2x-7\right)dx = \dfrac{x^4}{4}+2\cdot\dfrac{x^2}{2}-7x+C$

$$=\frac{x^4}{4}+x^2-7x+C$$

2. $\int dx = \int 1\ dx = 1\cdot x + C = x + C$

3.
$$\begin{aligned}\int_0^{12}\left(9\sqrt{3x}+3x^2\right)dx &= \int_0^{12}\left(9\sqrt{3}x^{1/2}+3x^2\right)dx \\ &= \left(9\sqrt{3}\frac{x^{3/2}}{\frac{3}{2}}+x^3\right)\Bigg|_0^{12} \\ &= \left(6\sqrt{3}x^{3/2}+x^3\right)\Big|_0^{12} \\ &= \left(6\sqrt{3}(12)^{3/2}+12^3\right)-0 \\ &= 2160\end{aligned}$$

4.
$$\begin{aligned}\int\frac{4}{5-3x}dx &= 4\left(-\frac{1}{3}\right)\int\frac{1}{5-3x}[-3\ dx] \\ &= -\frac{4}{3}\ln|5-3x|+C\end{aligned}$$

5.
$$\begin{aligned}\int\frac{6}{(x+5)^3}dx &= 6\int(x+5)^{-3}dx \\ &= \frac{6(x+5)^{-2}}{-2}+C \\ &= -3(x+5)^{-2}+C\end{aligned}$$

6.
$$\begin{aligned}\int_3^9(y-6)^{301}dy &= \frac{(y-6)^{302}}{302}\Bigg|_3^9 \\ &= \frac{3^{302}}{302}-\frac{(-3)^{302}}{302}=0\end{aligned}$$

7.
$$\begin{aligned}\int\frac{6x^2-12}{x^3-6x+1}dx &= 2\int\frac{1}{x^3-6x+1}\left[\left(3x^2-6\right)dx\right] \\ &= 2\ln\left|x^3-6x+1\right|+C\end{aligned}$$

8.
$$\begin{aligned}\int_0^3 2xe^{5-x^2}dx &= -\int_0^3 e^{5-x^2}[-2x\,dx] \\ &= -e^{5-x^2}\Big|_0^3 \\ &= -e^{5-9}+e^{5-0} \\ &= -e^{-4}+e^5\end{aligned}$$

9. $\int_0^1 \sqrt[3]{3t+8}dt = \frac{1}{3}\int_0^1 (3t+8)^{\frac{1}{3}}[3\ dt]$

$= \frac{1}{3}\cdot\frac{(3t+8)^{\frac{4}{3}}}{\frac{4}{3}}\Bigg|_0^1$

$= \frac{(3t+8)^{\frac{4}{3}}}{4}\Bigg|_0^1 = \frac{11\sqrt[3]{11}}{4} - 4$

10. $\int \frac{4-2x}{7}dx = \int\left(\frac{4}{7}-\frac{2}{7}x\right)dx = \frac{4}{7}x - \frac{1}{7}x^2 + C$

11. $\int y(y+1)^2 dy = \int\left(y^3 + 2y^2 + y\right)dy$

$= \frac{y^4}{4} + \frac{2y^3}{3} + \frac{y^2}{2} + C$

12. $\int_0^1 10^{-8}dx = 10^{-8}x\Big|_0^1 = 10^{-8} - 0 = 10^{-8}$

13. $\int \frac{\sqrt[7]{t}-\sqrt{t}}{\sqrt[3]{t}}dt = \int\left(\frac{\sqrt[7]{t}}{\sqrt[3]{t}} - \frac{\sqrt{t}}{\sqrt[3]{t}}\right)dt$

$= \int (t^{-4/21} - t^{1/6})dt$

$= \frac{t^{17/21}}{\frac{17}{21}} - \frac{t^{7/6}}{\frac{7}{6}} + C$

$= \frac{21}{17}t^{17/21} - \frac{6}{7}t^{7/6} + C$

14. $\int \frac{(0.5x-0.1)^4}{0.4}dx$

$= \frac{1}{0.4}\cdot\frac{1}{0.5}\int (0.5x-0.1)^4\,[0.5\ dx]$

$= \frac{1}{0.2}\cdot\frac{(0.5x-0.1)^5}{5} + C = (0.5x-0.1)^5 + C$

15. $\int_1^3 \frac{2t^2}{3+2t^3}dt = \frac{1}{3}\int_1^3 \frac{1}{3+2t^3}[6t^2\ dt]$

$= \frac{1}{3}\ln(3+2t^3)\Big|_1^3$

$= \frac{1}{3}[\ln(57) - \ln(5)] = \frac{1}{3}\ln\left(\frac{57}{5}\right)$

16. $\int \frac{4x^2-x}{x}dx = \int (4x-1)dx = 2x^2 - x + C$

17. $\int x^2\sqrt{3x^3+2}dx = \frac{1}{9}\int\left(3x^3+2\right)^{\frac{1}{2}}\left[9x^2dx\right]$

$= \frac{1}{9}\cdot\frac{\left(3x^3+2\right)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{27}\left(3x^3+2\right)^{\frac{3}{2}} + C$

18. $\int (6x^2+4x)(x^3+x^2)^{3/2}dx$

$= 2\int (x^3+x^2)^{3/2}[(3x^2+2x)dx]$

$= 2\cdot\frac{(x^3+x^2)^{5/2}}{\frac{5}{2}} + C$

$= \frac{4}{5}(x^3+x^2)^{5/2} + C$

19. $\int\left(e^{2y} - e^{-2y}\right)dy$

$= \frac{1}{2}\int e^{2y}[2\ dy] - \left(-\frac{1}{2}\right)\int e^{-2y}[-2\ dy]$

$= \frac{1}{2}e^{2y} + \frac{1}{2}e^{-2y} + C = \frac{1}{2}\left(e^{2y} + e^{-2y}\right) + C$

20. $\int \frac{8x}{3\sqrt[3]{7-2x^2}}dx = \frac{8}{3}\left(-\frac{1}{4}\right)\int\left(7-2x^2\right)^{-\frac{1}{3}}[-4x\ dx]$

$= -\frac{2}{3}\cdot\frac{3\left(7-2x^2\right)^{\frac{2}{3}}}{2} + C = -\left(7-2x^2\right)^{\frac{2}{3}} + C$

21. $\int\left(\frac{1}{x}+\frac{2}{x^2}\right)dx = \int\frac{1}{x}dx + 2\int x^{-2}dx$

$= \ln|x| + 2\cdot\frac{x^{-1}}{-1} + C$

$= \ln|x| - \frac{2}{x} + C$

22. $\int_0^2 \frac{3e^{3x}}{1+e^{3x}}dx = \int_0^2 \frac{1}{1+e^{3x}}[3e^{3x}\ dx]$

$= \ln(1+e^{3x})\Big|_0^2$

$= \ln(1+e^6) - \ln(1+1)$

$= \ln\left(\frac{1+e^6}{2}\right)$

23. $\int_{-2}^{2}(y^4+y^3+y^2+y)dy$

$=\left(\frac{y^5}{5}+\frac{y^4}{4}+\frac{y^3}{3}+\frac{y^2}{2}\right)\Bigg|_{-2}^{2}$

$=\left(\frac{32}{5}+\frac{16}{4}+\frac{8}{3}+\frac{4}{2}\right)-\left(-\frac{32}{5}+\frac{16}{4}-\frac{8}{3}+\frac{4}{2}\right)$

$=\frac{272}{15}$

24. $\int_7^{70} dx = x\Big|_7^{70}=70-7=63$

25. $\int_1^2 5x\sqrt{5-x^2}\,dx=-\frac{5}{2}\int_1^2(5-x^2)^{1/2}[-2x\,dx]$

$=-\frac{5}{2}\cdot\frac{(5-x^2)^{3/2}}{\frac{3}{2}}\Bigg|_1^2=-\frac{5}{3}(5-x^2)^{3/2}\Big|_1^2$

$=-\frac{5}{3}(1^{3/2}-4^{3/2})=-\frac{5}{3}(1-8)=\frac{35}{3}$

26. $\int_0^1(2x+1)\left(x^2+x\right)^4 dx$

$=\int_0^1\left(x^2+x\right)^4[(2x+1)\ dx]=\frac{\left(x^2+x\right)^5}{5}\Bigg|_0^1$

$=\frac{2^5}{5}-0=\frac{32}{5}$

27. $\int_0^1\left[2x-\frac{1}{(x+1)^{\frac{2}{3}}}\right]dx=2\int_0^1 x\ dx-\int_0^1(x+1)^{-\frac{2}{3}}[dx]$

$=\left[2\cdot\frac{x^2}{2}-\frac{(x+1)^{\frac{1}{3}}}{\frac{1}{3}}\right]\Bigg|_0^1=\left[x^2-3(x+1)^{\frac{1}{3}}\right]\Bigg|_0^1$

$=\left[1-3\sqrt[3]{2}\right]-[0-3]=4-3\sqrt[3]{2}$

28. $\int_0^{18}\left(2x-3\sqrt{2x}+1\right)dx$

$=\int_0^{18}\left(2x-3\sqrt{2}x^{1/2}+1\right)dx$

$=\left(\frac{2x^2}{2}-3\sqrt{2}\frac{x^{3/2}}{\frac{3}{2}}+x\right)\Bigg|_0^{18}$

$=\left(x^2-2\sqrt{2}x^{3/2}+x\right)\Big|_0^{18}$

$=\left(18^2-2\sqrt{2}(18)^{3/2}+18\right)-0$

$=126$

29. $\int\frac{\sqrt{t}-3}{t^2}dt=\int\left[\frac{t^{\frac{1}{2}}}{t^2}-\frac{3}{t^2}\right]dt=\int\left(t^{-\frac{3}{2}}-3t^{-2}\right)dt$

$=\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}-3\cdot\frac{t^{-1}}{-1}+C=-2t^{-\frac{1}{2}}+3t^{-1}+C$

$=\frac{3}{t}-\frac{2}{\sqrt{t}}+C$

30. $\int\frac{3z^3}{z-1}dz=3\int\left(z^2+z+1+\frac{1}{z-1}\right)dz$

$=3\left(\frac{z^3}{3}+\frac{z^2}{2}+z+\ln|z-1|\right)+C$

31. $\int_{-1}^{0}\frac{x^2+4x-1}{x+2}dx=\int_{-1}^{0}\left(x+2-\frac{5}{x+2}\right)dx$

$=\left(\frac{x^2}{2}+2x-5\ln|x+2|\right)\Bigg|_{-1}^{0}$

$=(-5\ln 2)-\left(\frac{1}{2}-2-0\right)=\frac{3}{2}-5\ln 2$

32. $\int\frac{\left(x^2+4\right)^2}{x^2}dx=\int\frac{x^4+8x^2+16}{x^2}dx$

$=\int\left(x^2+8+16x^{-2}\right)dx$

$=\frac{x^3}{3}+8x+16\frac{x^{-1}}{-1}+C=\frac{x^3}{3}+8x-\frac{16}{x}+C$

33. $\int \frac{e^{\sqrt{x}}+x}{2\sqrt{x}}dx = \int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx + \int \frac{\sqrt{x}}{2}dx$
$= \int e^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}dx\right] + \frac{1}{2}\int x^{1/2}dx$
$= e^{\sqrt{x}} + \frac{1}{2}\cdot\frac{x^{3/2}}{\frac{3}{2}} + C$
$= e^{\sqrt{x}} + \frac{1}{3}x^{3/2} + C$

34. $\int \frac{e^{\sqrt{5x}}}{\sqrt{3x}}dx = \frac{1}{\sqrt{3}}\int \frac{e^{\sqrt{5}x^{\frac{1}{2}}}}{x^{\frac{1}{2}}}dx$
$= \frac{2}{\sqrt{3}\cdot\sqrt{5}}1\int e^{\sqrt{5}x^{\frac{1}{2}}}\left[\frac{\sqrt{5}}{2}x^{-\frac{1}{2}}\,dx\right]$
$= \frac{2}{\sqrt{15}}\left(e^{\sqrt{5}x^{\frac{1}{2}}}\right) + C$
$= \frac{2}{\sqrt{15}}e^{\sqrt{5x}} + C$

35. $\int_1^e \frac{e^{\ln x}}{x^2}dx = \int_1^e \frac{x}{x^2}dx = \int_1^e \frac{1}{x}dx = \ln|x|\Big|_1^e$
$= \ln e - \ln 1$
$= 1 - 0 = 1$

36. $\int \frac{6x^2+4}{e^{x^3+2x}}dx = -2\int e^{-\left(x^3+2x\right)}\left[-\left(3x^2+2\right)dx\right]$
$= -2e^{-\left(x^3+2x\right)} + C = \frac{-2}{e^{x^3+2x}} + C$

37. $\int \frac{(1+e^{2x})^3}{e^{-2x}}dx = \frac{1}{2}\int (1+e^{2x})^3[2e^{2x}dx]$
$= \frac{(1+e^{2x})^3}{8} + C$

38. $\int \frac{c}{e^{bx}(a+e^{-bx})^n}dx$ for $n \neq 1$ and $b \neq 0$
$= -\frac{c}{b}\int (a+e^{-bx})^{-n}[-be^{-bx}dx]$
$= -\frac{c}{b}\cdot\frac{(a+e^{-bx})^{-n+1}}{-n+1} + C$
$= \frac{c}{b(n-1)}(a+e^{-bx})^{1-n} + C$

39. $\int 3\sqrt{10^{3x}}dx = 3\int e^{\frac{3x}{2}\ln 10}dx$
$= 3\cdot\frac{2}{3\ln 10}\int e^{\frac{3x}{2}\ln 10}\left[\frac{3\ln 10}{2}dx\right]$
$= \frac{2}{\ln 10}e^{\frac{3x}{2}\ln 10} + C = \frac{2}{\ln 10}10^{\frac{3x}{2}} + C$
$= \frac{2\sqrt{10^{3x}}}{\ln 10} + C$

40. $\int \frac{5x^3+15x^2+37x+3}{x^2+3x+7}dx$
$= \int\left(\frac{5x^3+15x^2+35x}{x^2+3x+7} + \frac{2x+3}{x^2+3x+7}\right)dx$
$= \int 5x\,dx + \int \frac{1}{x^2+3x+7}[(2x+3)\,dx]$
$= \frac{5x^2}{2} + \ln\left|x^2+3x+7\right| + C$

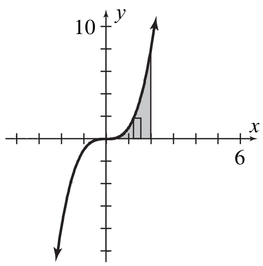
41. $y = \int\left(e^{2x}+3\right)dx = \int e^{2x}dx + \int 3\,dx$
$= \frac{1}{2}\int e^{2x}[2\,dx] + \int 3\,dx$
$= \frac{1}{2}e^{2x} + 3x + C$
$y(0) = -\frac{1}{2}$ implies that $-\frac{1}{2} = \frac{1}{2} + 0 + C$, so
$C = -1$. Thus $y = \frac{1}{2}e^{2x} + 3x - 1$

42. $y = \int \frac{x+5}{x}dx = \int\left(1+\frac{5}{x}\right)dx = x + 5\ln|x| + C$
$y(1) = 3$ implies $3 = 1 + 0 + C$, so $C = 2$. Thus
$y = x + 5\ln|x| + 2$

In Problems 43–58, answers are assumed to be expressed in square units.

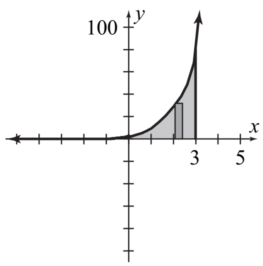
43. $y = x^3$, $x = 0$, $x = 2$. Region appears below.

Area $= \int_0^2 x^3dx = \frac{x^4}{4}\Big|_0^2 = \frac{16}{4} - 0 = 4$



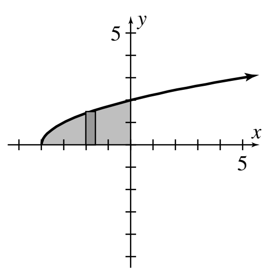
44. $y = 4e^x$, $x = 0$, $x = 3$. Region appears below.

$$\text{Area} = \int_0^3 4e^x\,dx = 4e^x\Big|_0^3 = 4(e^3 - 1)$$



45. $y = \sqrt{x+4}$, $x = 0$. Region appears below.

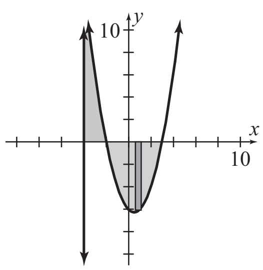
$$\text{Area} = \int_{-4}^0 \sqrt{x+4}\,dx = \int_{-4}^0 (x+4)^{\frac{1}{2}}[dx] = \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_{-4}^0 = \frac{2(x+4)^{\frac{3}{2}}}{3}\Bigg|_{-4}^0 = \frac{16}{3} - 0 = \frac{16}{3}$$



46. $y = x^2 - x - 6$, $x = -4$, $x = 3$. Region appears below.

$$\text{Area} = \int_{-4}^{-2} (x^2 - x - 6)\,dx + \int_{-2}^{3} -(x^2 - x - 6)\,dx = \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\Bigg|_{-4}^{-2} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\Bigg|_{-2}^{3}$$

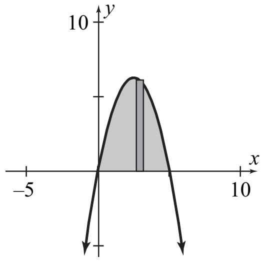
$$= \left[\left(-\frac{8}{3} - 2 + 12\right) - \left(-\frac{64}{3} - 8 + 24\right)\right] - \left[\left(9 - \frac{9}{2} - 18\right) - \left(-\frac{8}{3} - 2 + 12\right)\right] = \frac{67}{2}$$



47. $y = 5x - x^2$. Region appears below.

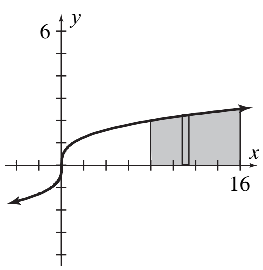
$$\text{Area } = \int_0^5 \left(5x - x^2\right) dx = \left(\frac{5x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^5$$

$$= \left(\frac{125}{2} - \frac{125}{3}\right) - 0 = \frac{125}{6}$$



48. $y = \sqrt[3]{x},\ x = 8,\ x = 16$. Region appears below.

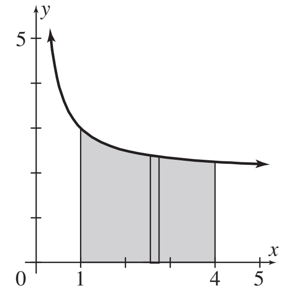
$$\begin{aligned}\text{Area} &= \int_8^{16} \sqrt[3]{x}\,dx \\ &= \int_8^{16} x^{1/3}\,dx \\ &= \frac{x^{4/3}}{\frac{4}{3}}\Bigg|_8^{16} \\ &= \frac{3}{4}x^{4/3}\Bigg|_8^{16} \\ &= \frac{3}{4}(16^{4/3} - 8^{4/3}) \\ &\approx 18.24\end{aligned}$$



49. $y = \frac{1}{x} + 2,\ x = 1,\ x = 4$. Region appears below.

$$\text{Area } = \int_1^4 \left(\frac{1}{x} + 2\right) dx = \left(\ln|x| + 2x\right)\Big|_1^4$$

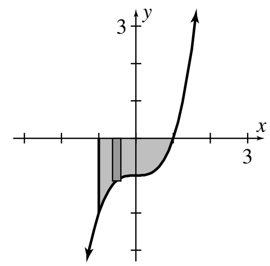
$$= [\ln(4) + 8] - [0 + 2] = 6 + \ln 4$$



50. $y = x^3 - 1$, $x = -1$. Region appears below.

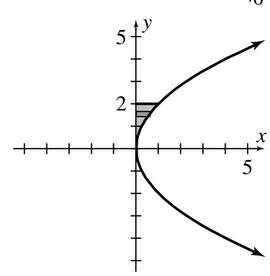
$$\text{Area } = \int_{-1}^1 -\left(x^3 - 1\right) dx = -\left(\frac{x^4}{4} - x\right)\Bigg|_{-1}^1$$

$$= -\left(-\frac{3}{4}\right) + \left(\frac{5}{4}\right) = 2$$



51. $y^2 = 4x$, $x = 0$, $y = 2$. Region appears below.

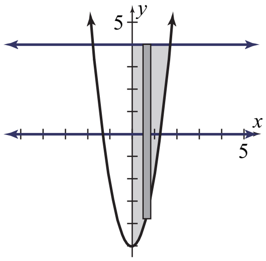
$$\text{Area } = \int_0^2 \frac{y^2}{4}\,dy = \frac{y^3}{12}\Bigg|_0^2 = \frac{8}{12} - 0 = \frac{2}{3}$$



52. $y = 3x^2 - 5$, $x = 0$, $y = 4$. Region appears below.

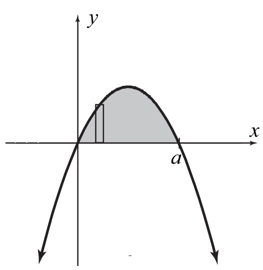
$3x^2 - 5 = 4,\ 3x^2 = 9,\ x^2 = 3,$ so $x = \pm\sqrt{3}$.

$$\begin{aligned}\text{Area} &= \int_0^{\sqrt{3}} [4 - (3x^2 - 5)]\,dx \\ &= \int_0^{\sqrt{3}} [9 - 3x^2]\,dx = (9x - x^3)\Big|_0^{\sqrt{3}} \\ &= \left(9\sqrt{3} - 3\sqrt{3}\right) - 0 = 6\sqrt{3}\end{aligned}$$



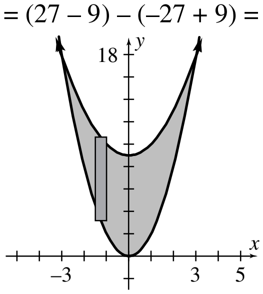
53. $y = -x(x - a)$, $y = 0$ for $0 < a$. Region appears below.
$-x(x - a) = 0$, so $x = 0$, a.

$$\text{Area} = \int_0^a [-x(x-a)]dx$$
$$= \int_0^a (ax - x^2)dx$$
$$= \left(\frac{a}{2}x^2 - \frac{x^3}{3}\right)\Bigg|_0^a$$
$$= \left(\frac{a^3}{2} - \frac{a^3}{3}\right) - 0$$
$$= \frac{a^3}{6}$$



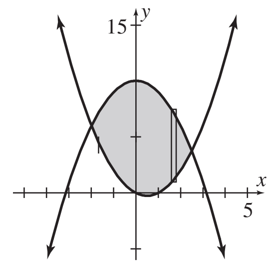
54. $y = 2x^2$, $y = x^2 + 9$. Region appears below.
$2x^2 = x^2 + 9$, $x^2 = 9$, so $x = \pm 3$

$$\text{Area} = \int_{-3}^{3}\left[\left(x^2+9\right) - \left(2x^2\right)\right]dx$$
$$= \int_{-3}^{3}\left(9 - x^2\right)dx = \left(9x - \frac{x^3}{3}\right)\Bigg|_{-3}^{3}$$
$$= (27 - 9) - (-27 + 9) = 36$$



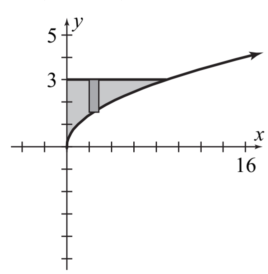
55. $y = x^2 - x$, $y = 10 - x^2$. Region appears below.
$x^2 - x = 10 - x^2$, $2x^2 - x - 10 = 0$,
$(x + 2)(2x - 5) = 0$, so $x = -2$ or $\frac{5}{2}$.

$$\text{Area} = \int_{-2}^{5/2}[(10 - x^2) - (x^2 - x)]dx$$
$$= \int_{-2}^{5/2}(10 + x - 2x^2)dx$$
$$= \left(10x + \frac{x^2}{2} - \frac{2x^3}{3}\right)\Bigg|_{-2}^{5/2}$$
$$= \left(25 + \frac{25}{8} - \frac{125}{12}\right) - \left(-20 + 2 + \frac{16}{3}\right)$$
$$= \frac{243}{8}$$



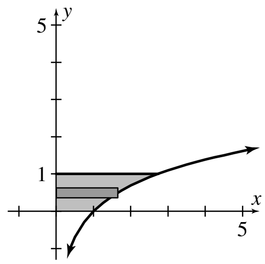
56. $y = \sqrt{x}$, $x = 0$, $y = 3$. Region appears below.
$\sqrt{x} = 3$, so $x = 9$.

$$\text{Area} = \int_0^9\left(3 - \sqrt{x}\right)dx = \left(3x - \frac{2x^{\frac{3}{2}}}{3}\right)\Bigg|_0^9$$
$$= (27 - 18) - 0 = 9$$



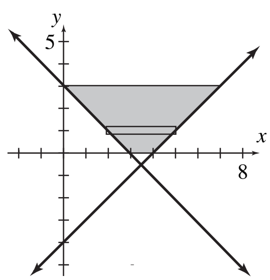
57. $y = \ln x$, $x = 0$, $y = 0$, $y = 1$. Region appears below.
$y = \ln x \Rightarrow x = e^y$

$$\text{Area} = \int_0^1 e^y\,dy = e^y\Big|_0^1 = e - 1$$



58. $y = 3 - x$, $y = x - 4$, $y = 0$, $y = 3$. Region appears below.

$$\text{Area} = \int_0^3 [(y+4)-(3-y)]dy$$
$$= \int_0^3 (2y+1)dy$$
$$= (y^2 + y)\Big|_0^3$$
$$= (9+3)-0$$
$$= 12$$



59. $r = \int\left(100 - \frac{3}{2}\sqrt{2q}\right)dq = \int 100dq - \frac{3}{2}\sqrt{2}\int q^{\frac{1}{2}}dq$

$= 100q - \frac{3}{2}\sqrt{2}\cdot\frac{q^{\frac{3}{2}}}{\frac{3}{2}} + C = 100q - \sqrt{2}q^{\frac{3}{2}} + C$

When $q = 0$, then $r = 0$. Thus $0 = 0 - 0 + C$, so $C = 0$. Hence $r = 100q - \sqrt{2}q^{\frac{3}{2}}$. Since $r = pq$, then $p = \frac{r}{q} = 100 - \sqrt{2}q^{\frac{1}{2}} = 100 - \sqrt{2q}$. Thus $p = 100 - \sqrt{2q}$.

60. $c = \int\left(q^2 + 7q + 6\right)dq = \frac{q^3}{3} + \frac{7}{2}q^2 + 6q + C$

When $q = 0$, then $c = 2500$. Thus $2500 = 0 + 0 + 0 + C$, so $C = 2500$. Hence $c = \frac{q^3}{3} + \frac{7}{2}q^2 + 6q + 2500$. When $q = 6$, then $c = \$2734$.

61. $\int_{15}^{25}(250 - q - 0.2q^2)dq$

$$= \left(250q - \frac{q^2}{2} - \frac{0.2q^3}{3}\right)\Bigg|_{15}^{25}$$
$$= \left(6250 - 312.5 - \frac{3125}{3}\right) - (3750 - 112.5 - 225)$$
$$\approx \$1483.33$$

62. $\int_{10}^{33}\frac{1000}{\sqrt{3q+70}}dq = 1000\cdot\frac{1}{3}\int_{10}^{33}(3q+70)^{-\frac{1}{2}}[3\,dq]$

$$= \frac{1000}{3}\cdot\frac{(3q+70)^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_{10}^{33}$$
$$= \frac{2000}{3}\sqrt{3q+70}\Bigg|_{10}^{33}$$
$$= \frac{2000}{3}[13-10] = \$2000$$

63. $\int_0^{100} 0.007e^{-0.007t}dt = -\int_0^{100} e^{-0.007t}[-0.007dt]$

$$= -e^{-0.007t}\Big|_0^{100}$$
$$= -e^{-0.7} + 1$$
$$\approx 0.5034$$

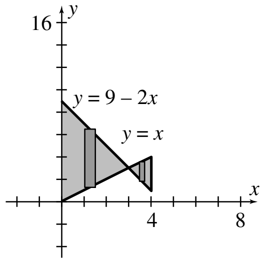
64. $\int_0^5 4000e^{0.05t}dt = 4000\cdot\frac{1}{0.05}\int_0^5 e^{0.05t}[0.05\ dt]$

$$= \frac{4000}{0.05}e^{0.05t}\Bigg|_0^5 = \frac{4000}{0.05}\left[e^{0.25} - 1\right] \approx \$22{,}722$$

65. $y = 9 - 2x$, $y = x$; from $x = 0$ to $x = 4$. Region appears below. Intersection: $x = 9 - 2x$, $3x = 9$, so $x = 3$.

$$\text{Area} = \int_0^3 [(9-2x)-x]dx + \int_3^4 [x-(9-2x)]dx$$
$$= \int_0^3 (9-3x)dx + \int_3^4 (3x-9)dx$$
$$= \left(9x - \frac{3x^2}{2}\right)\Bigg|_0^3 + \left(\frac{3x^2}{2} - 9x\right)\Bigg|_3^4$$
$$= \left[\left(27 - \frac{27}{2}\right) - 0\right] + \left[(24-36) - \left(\frac{27}{2} - 27\right)\right]$$

= 15 square units



66. $y = 2x^2$, $y = 2 - 5x$; from $x = -1$ to $x = \frac{1}{3}$.

Region appears below.

$2x^2 = 2 - 5x$, $2x^2 + 5x - 2 = 0$,

$x = \frac{-5 \pm \sqrt{41}}{4}$ (from the quadratic formula),

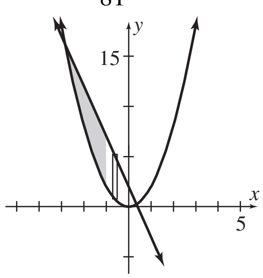
$x \approx -2.85$ or 0.35.

$$\text{Area} = \int_{-1}^{1/3} [(2-5x) - 2x^2]dx$$

$$= \left(2x - \frac{5x^2}{2} - \frac{2x^3}{3} \right)\Bigg|_{-1}^{1/3}$$

$$= \left(\frac{2}{3} - \frac{5}{18} - \frac{2}{81} \right) - \left(-2 - \frac{5}{2} + \frac{2}{3} \right)$$

$$= \frac{340}{81} \text{ square units}$$



67. $D: p = 0.01q^2 - 1.1q + 30$
$S: p = 0.01q^2 + 8$

Equilibrium pt. $= (q_0, p_0) = (20, 12)$

$$\text{CS} = \int_0^{q_0} [f(q) - p_0]dq$$

$$= \int_0^{20} \left[\left(0.01q^2 - 1.1q + 30\right) - 12 \right] dq$$

$$= \int_0^{20} \left(0.01q^2 - 1.1q + 18 \right) dq$$

$$= \left(\frac{0.01q^3}{3} - \frac{1.1q^2}{2} + 18q \right)\Bigg|_0^{20}$$

$$= \left(\frac{80}{3} - 220 + 360 \right) - 0 = 166\frac{2}{3}$$

$$\text{PS} = \int_0^{q_0} [p_0 - g(q)]dq = \int_0^{20} \left[12 - \left(0.01q^2 + 8\right) \right] dq$$

$$= \int_0^{20} \left(4 - 0.01q^2 \right) dq = \left(4q - \frac{0.01q^3}{3} \right)\Bigg|_0^{20}$$

$$= \left(80 - \frac{80}{3} \right) - 0 = 53\frac{1}{3}$$

68. $D: p = (q-4)^2$
$S: p = q^2 + q + 7$

Equilibrium pt. $= (q_0, p_0) = (1, 9)$

$$\text{CS} = \int_0^1 \left[(q-4)^2 - 9 \right] dq = \left[\frac{(q-4)^3}{3} - 9q \right]\Bigg|_0^1$$

$$= \left(-\frac{27}{3} - 9 \right) - \left(-\frac{64}{3} - 0 \right)$$

$$= \frac{10}{3} \text{ thousands} \approx \$3333$$

69. $\int_{q_0}^{q_n} \frac{dq}{q - \hat{q}} = -(u+v)\int_0^n dt$

$$\ln|q - \hat{q}|\Big|_{q_0}^{q_n} = -(u+v)t\Big|_0^n$$

$$\ln|q_n - \hat{q}| - \ln|q_0 - \hat{q}| = -(u+v)n$$

$$\ln|q_0 - \hat{q}| - \ln|q_n - \hat{q}| = (u+v)n$$

$$\ln\left| \frac{q_0 - \hat{q}}{q_n - \hat{q}} \right| = (u+v)n$$

$$n = \frac{1}{u+v} \ln\left| \frac{q_0 - \hat{q}}{q_n - \hat{q}} \right|$$

as was to be shown.

70. $Q = \int_0^R 2\pi rv\, dr = 2\pi \int r \cdot \frac{(P_1 - P_2)(R^2 - r^2)}{4\eta l} dr$

$= \frac{\pi(P_1 - P_2)}{2\eta l} \int_0^R r(R^2 - r^2)\, dr$

$= \frac{\pi(P_1 - P_2)}{2\eta l} \int_0^R (R^2 r - r^3)\, dr$

$= \frac{\pi(P_1 - P_2)}{2\eta l} \left(\frac{R^2 r^2}{2} - \frac{r^4}{4} \right)\Bigg|_0^R$

$= \frac{\pi(P_1 - P_2)}{2\eta l} \left[\left(\frac{R^4}{2} - \frac{R^4}{4} \right) - 0 \right]$

$= \frac{\pi(P_1 - P_2)}{2\eta l} \left(\frac{R^4}{4} \right) = \frac{\pi R^4 (P_1 - P_2)}{8\eta l}$

As was to be shown.

71. Case 1. $r \neq -1$

$g(x) = \frac{1}{k} \int_1^{1/x} ku^r\, du = \int_1^{1/x} u^r\, du = \frac{u^{r+1}}{r+1}\Bigg|_1^{1/x}$

$= \frac{1}{r+1}\left(x^{-r-1} - 1\right)$

$g'(x) = \frac{1}{r+1}\left[-(r+1)x^{-r-2}\right] = -\frac{1}{x^{r+2}}$

Case 2. $r = -1$

$g(x) = \frac{1}{k} \int_1^{1/x} ku^{-1} du = \int_1^{1/x} \frac{1}{u} du$

$= \ln|u|\Big|_1^{1/x} = \ln\left(\frac{1}{x}\right) - 0 = -\ln x$

$g'(x) = -\frac{1}{x} = -\frac{1}{x^{r+2}}$

72. Two integrals are needed.
Answer: 101.75 sq units

73. Two integrals are involved.
Answer: 0.50 sq units

74. Two integrals are needed.
Answer: 32.75

75. CS $\approx$ 1148; PS $\approx$ 251

Explore and Extend—Chapter 14

1. a. $\int_0^5 f(t)dt = \int_0^5 (100 - 2t)dt = \left(100t - t^2\right)\Big|_0^5$

$= (500 - 25) - 0 = 475$

b. $\int_{20}^{25} f(t)dt = \int_{20}^{25} (100 - 2t)dt = (100t - t^2)\Big|_{20}^{25}$

$= (2500 - 625) - (2000 - 400) = 275$

2. a. Total revenue $= \int_0^R (m + st) f(t) dt$

$= \int_0^{80} (50 + 0.2t) \cdot (40 - 0.5t) dt$

$= \int_0^{80} \left(2000 - 17t - 0.1t^2\right) dt$

$= \left(2000t - \frac{17}{2}t^2 - \frac{1}{30}t^3\right)\Bigg|_0^{80}$

$= 160{,}000 - 54{,}400 - \frac{51{,}200}{3} \approx \$88{,}533.33$

b. Total number of units sold

$= \int_0^R f(t)dt = \int_0^{80} (40 - 0.5t)dt$

$= \left(40t - 0.25t^2\right)\Big|_0^{80} = 3200 - 1600 = 1600$

c. Average delivered price

$= \frac{\text{total revenue}}{\text{total number of units sold}}$

$\approx \frac{88{,}533.33}{1600} \approx \$55.33$

3. a. Total revenue

$= \int_0^R (m + st) f(t) dt = \int_0^{30} (100 + t)(900 - t^2) dt$

$= \int_0^{30} \left(90{,}000 + 900t - 100t^2 - t^3\right) dt$

$= 90{,}000t + 450t^2 - \frac{100}{3}t^3 - \frac{1}{4}t^4 \Bigg|_0^{30}$

$= 2{,}700{,}000 + 405{,}000 - 900{,}000 - 202{,}500$
$= \$2{,}002{,}500$

b. Total number of units sold

$= \int_0^R f(t)dt = \int_0^{30} \left(900 - t^2\right) dt$

$= \left(900t - \frac{1}{3}t^3\right)\Bigg|_0^{30} = 27{,}000 - 9000 = 18{,}000$

c. Average delivered price

$= \frac{\text{total revenue}}{\text{total number of units sold}}$

$= \frac{2{,}002{,}500}{18{,}000} = \$111.25$

4. Answers may vary.