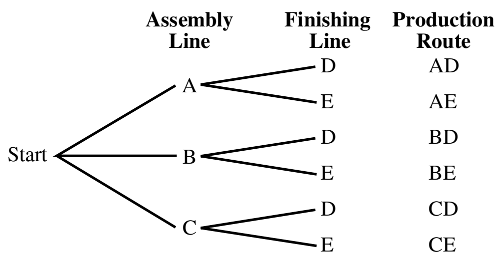
Chapter 8

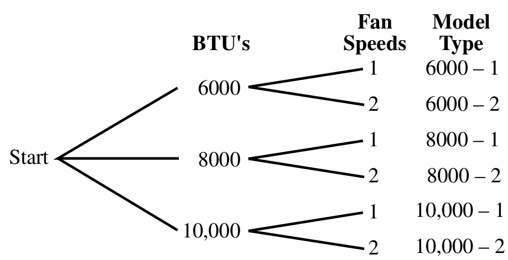
Problems 8.1

1.



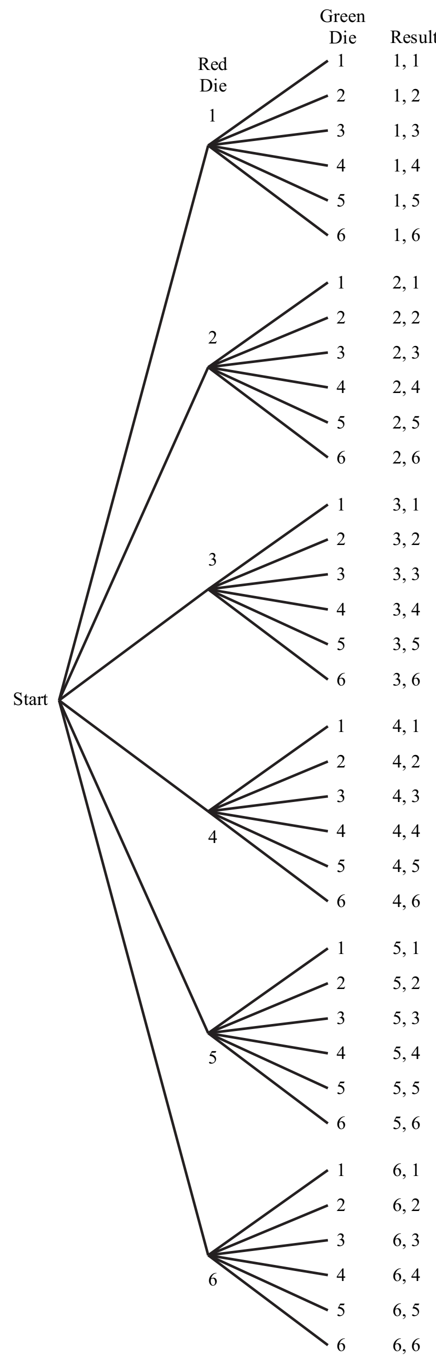
6 possible production routes

2.



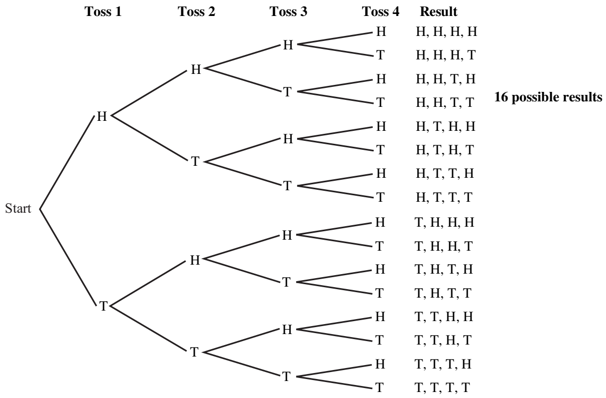
6 model types

3.



36 possible results

4.



5. There are 3 mathematics courses, 5 laboratory science courses, and 4 humanities courses. By the basic counting principle, the number of selections is $3 \cdot 5 \cdot 4 = 60$.

6. **a.** There are 5 roads from A to B, and 5 roads from B to A. By the basic counting principle, the number of possible routes for a round trip is $5 \cdot 5 = 25$.

b. There are 5 possible roads from A to B. Since a different road is to be used for the return trip, there are only 4 possible roads from B to A. By the basic counting principle, the number of possible round-trip routes is $5 \cdot 4 = 20$.

7. There are 2 appetizers, 4 entrees, 4 desserts, and 3 beverages. By the basic counting principle, the number of possible complete dinners is $2 \cdot 4 \cdot 4 \cdot 3 = 96$.

8. For each of the 6 questions, there are 4 choices. By the basic counting principle, the number of ways to answer the questions is $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^6 = 4096$.

9. For each of the 10 questions, there are 2 choices. By the basic counting principle, the number of ways to answer the examination is $2 \cdot 2 \cdot \cdots \cdot 2 = 2^{10} = 1024$.

10. Since there are 26 letters, there are 26 choices for the first, third, and fifth symbols. There are 10 possible digits (0 through 9) for the second, fourth, and sixth symbols. By the basic counting principle, the number of codes is $26 \cdot 10 \cdot 26 \cdot 10 \cdot 26 \cdot 10 = 17{,}576{,}000$.
The percentage that begins with B3H is

$$\frac{26 \cdot 10^2}{26^3 \cdot 10^3} \cdot 100\% \approx 0.015\%.$$

11. $_6P_3 = \dfrac{6!}{(6-3)!} = \dfrac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$

12. $_{95}P_1 = \dfrac{95!}{(95-1)!} = \dfrac{95!}{94!} = 95$

13. $_6P_6 = \dfrac{6!}{(6-6)!} = \dfrac{6!}{0!} = \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 720$

14. $_9P_4 = \dfrac{9!}{(9-4)!} = \dfrac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$

15. $_6P_3 \cdot {}_4P_3 = (6 \cdot 5 \cdot 4)(4 \cdot 3 \cdot 2) = 2880$

16. $\dfrac{_{99}P_5}{_{99}P_4} = \dfrac{99 \cdot 98 \cdot 97 \cdot 96 \cdot 95}{99 \cdot 98 \cdot 97 \cdot 96} = 95$

17. $\dfrac{1000!}{999!} = \dfrac{1000 \cdot 999!}{999!} = 1000$
For most calculators, attempting to evaluate $\dfrac{1000!}{999!}$ results in an error message (because of the magnitude of the numbers involved).

18. $\dfrac{_nP_r}{n!} = \dfrac{\frac{n!}{(n-r)!}}{n!} = \dfrac{1}{(n-r)!}$

19. A name for the firm is an ordered arrangement of the three last names. Thus the number of possible firm names is $_3P_3 = 3! = 3 \cdot 2 \cdot 1 = 6$.

20. The number of ways to arrange 8 teams in an order is $_8P_8 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$.

21. The number of ways of selecting 3 of 8 contestants in an order is $_8P_3 = 8 \cdot 7 \cdot 6 = 336$.

22. Six out of eight items in column 2 must be selected in an order. Thus the number of ways the matching can be done is $_8P_6 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 20{,}160$.

23. On each roll of a die, there are 6 possible outcomes. By the basic counting principle, on 4 rolls the number of possible results is $6 \cdot 6 \cdot 6 \cdot 6 = 6^4 = 1296$.

24. On each toss there are 2 possible outcomes. By the basic counting principle, the number of possible results on 8 tosses is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$.

25. The number of ways of selecting 4 of the 12 students in an order is ${}_{10}P_4 = 10 \cdot 9 \cdot 8 \cdot 7 = 5040.$

26. Three of the 26 letters must be selected (without repetition) in an order. Thus the number of possible lock combinations is ${}_{26}P_3 = 15,600$.

27. The number of ways a student can choose 4 of the 6 items in an order is ${}_6P_4 = 6 \cdot 5 \cdot 4 \cdot 3 = 360.$

28. On the second roll, there are 2 possible outcomes (a 1 or a 2). For each of the other two rolls, there are 6 possible outcomes. By the basic counting principle, the number of possible results for the three rolls is $6 \cdot 2 \cdot 6 = 72$.

29. The number of ways to select six of the six different letters in the word MEADOW in an order is ${}_6P_6 = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

30. The number of ways to select four of the five different letters in the word DISCO in an order is ${}_5P_4 = 5 \cdot 4 \cdot 3 \cdot 2 = 120.$

31. For an arrangement of books, order is important. The number of ways to arrange 5 of 7 books is ${}_7P_5 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$.
All 7 books can be arranged in ${}_7P_7 = 7! = 5040$ ways.

32. **a.** A student can enter by any of 5 doors. After a door is chosen, the student can exit by any of the 4 remaining doors. By the basic counting principle, the number of ways to enter by one door and exit by a different door is $5 \cdot 4 = 20$.

b. There are 5 doors by which to enter and 5 doors by which to exit. By the basic counting principle, the total number of ways to enter and exit is $5 \cdot 5 = 25$.

33. After a "four of a kind" hand is dealt, the cards can be arranged so that the first four have the same face value, and order is not important, There are 13 possibilities for the first four cards (all 2's, all 3's, ..., all aces). The fifth card can be any one of the 48 cards that remain. By the basic counting principle, the number of "four of a kind" hands is $13 \cdot 48 = 624$.

34. Five colors are available, and two are selected so that order is important. Thus the number of ways of placing an order is ${}_5P_2 = 5 \cdot 4 = 20.$

35. The number of ways the waitress can place six of the six different items (and order is important) is ${}_6P_6 = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$

36. Because order is important, the number of ways that the 5 people can line up is ${}_5P_5 = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$
If a woman is to be at each end, then the number of ways to place one of the two women on the left side is ${}_2P_1$. Once a woman is chosen for the left side, the other woman must be on the right side. The number of ways to line the three men in the middle is ${}_3P_3$. By the basic counting principle, the number of line ups is ${}_2P_1 \cdot {}_3P_3 = (2)(3 \cdot 2 \cdot 1) = 12$.

37. **a.** To fill the four offices by different people, 4 of 12 members must be selected, and order is important. This can be done in ${}_{12}P_4 = 12 \cdot 11 \cdot 10 \cdot 9 = 11,880$ ways.

b. If the president and vice president must be different members, then there are 12 choices for president, 11 for vice president, 12 for secretary, and 12 for treasurer. By the basic counting principle, the offices can be filled in $12 \cdot 11 \cdot 12 \cdot 12 = 19,008$ ways.

38. **a.** There are 24 possibilities for each of the three letters in a name. By the basic counting principle, the number of names is $24 \cdot 24 \cdot 24 = 24^3 = 13,824$.

b. Since the order of letters is important and no letter is used more than one time, the number of names is ${}_{24}P_3 = 24 \cdot 23 \cdot 22 = 12,144$.

39. There are 2 choices for the center position. After that choice is made, to fill the remaining four positions (and order is important), there are ${}_4P_4$ ways. By the basic counting principle, to assign positions to the five-member team there are $2 \cdot {}_4P_4 = 2(4!) = 2(24) = 48$ ways.

40. For the first letter there are three possibilities. For the second and third letters there are two possibilities; and for the number there are three possibilities. By the basic counting principle, the number of possible model names is $3 \cdot 2 \cdot 2 \cdot 3 = 36$.

41. There are ${}_3P_3$ ways to select the first three batters (order is important) and there are ${}_6P_6$ ways to select the remaining batters. By the basic counting principle, the number of possible batting orders is
${}_3P_3 \cdot {}_6P_6 = 3! \cdot 6! = 6 \cdot 720 = 4320.$

42. a. Four of four flags can be arranged (order is important) in ${}_4P_4 = 4! = 24$ ways. Thus 24 different signals are possible.

b. If only one of the four flags is used, there are ${}_4P_1$ possible signals. If exactly two flags are used, there are ${}_4P_2$ possible signals. Similarly, for exactly three and exactly four flags, there are ${}_3P_4$ and ${}_4P_4$ possible signals, respectively. Thus if at least one flag is used, the number of possible signals is
${}_4P_1 + {}_4P_2 + {}_4P_3 + {}_4P_4$
$= 4 + 4 \cdot 3 + 4 \cdot 3 \cdot 2 + 4 \cdot 3 \cdot 2 \cdot 1$
$= 4 + 12 + 24 + 24 = 64.$

Problems 8.2

1. ${}_6C_4 = \frac{6!}{4!(6-4)!} = \frac{6!}{4! \cdot 2!} = \frac{6 \cdot 5 \cdot 4!}{4!(2 \cdot 1)} = \frac{6 \cdot 5}{2 \cdot 1} = 15$

2. ${}_6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2! \cdot 4!} = \frac{6 \cdot 5 \cdot 4!}{(2 \cdot 1)4!} = \frac{6 \cdot 5}{2 \cdot 1} = 15$

3. ${}_{100}C_{100} = \frac{100!}{100!(100-100)!} = \frac{1}{0!} = \frac{1}{1} = 1$

4. ${}_{1,000,001}C_1 = \frac{1,000,001!}{1!(1,000,001-1)!}$
$= \frac{1,000,001!}{1,000,000!}$
$= 1,000,001$

5. ${}_5P_3 \cdot {}_4C_2 = 5 \cdot 4 \cdot 3 \frac{4!}{2!(4-2)!}$
$= 5 \cdot 4 \cdot 3 \frac{4 \cdot 3 \cdot 2!}{2!2!}$
$= 60 \cdot 6$
$= 360$

6. ${}_4P_2 \cdot {}_5C_3 = (4 \cdot 3)\frac{5!}{3!(5-3)!}$
$= (4 \cdot 3)\frac{5 \cdot 4 \cdot 3!}{3! \cdot 2!} = (12) \cdot 10 = 120$

7. ${}_nC_r = \frac{n!}{r!(n-r)!}$
${}_nC_{n-r} = \frac{n!}{(n-r)![n-(n-r)]!} = \frac{n!}{(n-r)!r!}.$
Thus ${}_nC_r = {}_nC_{n-r}$.

8. ${}_nC_n = \frac{n!}{n!(n-n)!} = \frac{1}{0!} = \frac{1}{1} = 1$.

9. The number of ways of selecting 5 of 19 people so that order is not important is
${}_{19}C_5 = \frac{19!}{5!(19-5)!}$
$= \frac{19!}{5!14!}$
$= \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 11,628$

10. If horses A, B, and C finish in the money, then it does not matter if A finishes in first, second, or third place. Similarly for B and C. Thus order is not important. The number of ways in which 3 of 8 horses finish in the money is the number of ways of selecting 3 of the 8 without regard to order, namely
${}_8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = 56$.

11. The number of ways of selecting 9 out of 13 questions (without regard to order) is
${}_{13}C_9 = \frac{13!}{9!(13-9)!} = \frac{13!}{9! \cdot 4!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 715.$

12. In a deck of 52 cards, 26 of the cards are red. In a four-card hand, the order is not important. Thus, the number of four-card hands from the 26 red cards is
${}_{26}C_4 = \frac{26!}{4!(26-4)!}$
$= \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22!}{4!22!}$
$= 14,950$

13. The order of selecting 10 of the 74 dresses is of no concern. Thus the number of possible samples ${}_{74}C_{10} = \frac{74!}{10!\cdot(74-10)!} = \frac{74!}{10!\cdot 64!}$.

14. This situation can be considered as a two-stage process. In the first stage, one of the 3 colors is selected. In the second stage, 3 of the 5 types of energy drink are selected (and order is not important), which can be done in ${}_5C_3$ ways. The number of different '3-paks' possible is

$$\begin{aligned} 3\cdot {}_5C_3 &= 3\cdot\frac{5!}{3!(5-3)!} \\ &= 3\cdot\frac{5!}{3!2!} \\ &= \frac{5!}{2!2!} \\ &= \frac{5\cdot 4\cdot 3}{2} \\ &= 30 \end{aligned}$$

15. To score 80, 90, or 100, exactly 8, 9, or 10 questions must be correct, respectively. The number of ways in which 8 of 10 questions can be correct is

$${}_{10}C_8 = \frac{10!}{8!(10-8)!} = \frac{10!}{8!\cdot 2!} = \frac{10\cdot 9\cdot 8!}{8!\cdot 2\cdot 1} = 45.$$

For 9 of 10 questions, the number of ways is

$${}_{10}C_9 = \frac{10!}{9!(10-9)!} = \frac{10!}{9!\cdot 1!} = \frac{10\cdot 9!}{9!\cdot 1} = 10,$$

and for 10 of 10 questions, it is

$${}_{10}C_{10} = \frac{10!}{10!(10-10)!} = \frac{10!}{10!\cdot 0!} = 1.$$

Thus the number of ways to score 80 or better is 45 + 10 + 1 = 56.

16. Each of the 11 games can be assigned to one of three cells: a win cell, a loss cell, or a tie cell. The number of ways to have 4 wins, 5 losses, and 3 ties is

$$\frac{11!}{4!\cdot 5!\cdot 2!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{4\cdot 3\cdot 2\cdot 1\cdot 5!\cdot 2\cdot 1} = 6930.$$

17. The word MISSISSAUGA has 11 letters with repetition: one M, two I's, four S's, two A's, one U, and one G. Thus the number of distinguishable arrangements is

$$\begin{aligned} \frac{11!}{1!\cdot 2!\cdot 4!\cdot 2!\cdot 1!\cdot 1!} &= \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4!}{(2)4!(2)} \\ &= 415{,}800. \end{aligned}$$

18. The word STREETSBORO has 11 letters with repetition: two S's, two T's, two R's, two E's, one B, and two O's. Thus the number of distinguishable arrangements is

$$\frac{11!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 1!\cdot 2!} = \frac{11!}{32} = 1{,}247{,}400.$$

19. The number of ways 2 heads and 4 tails can occur in 6 tosses of a coin is the same as the number of distinguishable permutations in the "word" HHTTTT, which is $\frac{6!}{2!4!} = \frac{6\cdot 5}{2} = 15.$

20. The number of ways for the given outcome to occur is the number of distinguishable permutations of six numbers such that two are 2's, three are 3's, and one is 4, which is

$$\frac{6!}{2!\cdot 3!\cdot 1!} = \frac{6\cdot 5\cdot 4\cdot 3!}{(2)3!} = 60.$$

21. Since the order in which the calls are made is important, the number of possible schedules for the 6 calls is ${}_6P_6 = 6! = 720$.

22. The number of ways to place the 12 members in three specific cars (cells), with 4 members in each car, is $\frac{12!}{4!\cdot 4!\cdot 4!} = 34{,}650$.

23. The number of ways to assign 9 scientists so 3 work on project A, 3 work on B, and 3 work on C is $\frac{9!}{3!3!3!} = 1680$.

24. There are 13 individuals: 4 quadruplets, 3 triplets, and 3 pairs of twins. They can line up in $\frac{13!}{4!3!(2!)^3} = 5{,}405{,}400$ ways.

25. A response to the true-false questions can be considered an ordered arrangement of 10 letters, 5 of which are T's and 5 of which are F's. The number of different responses is

$$\frac{10!}{5!\,5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{5!(5\cdot 4\cdot 3\cdot 2\cdot 1)} = 252.$$

26. The order in which the 7 food items are placed is important. However, there are 3 hamburgers (type 1), 2 cheeseburgers (type 2), and 2 steak sandwiches (type 3). Then the number of possible distinguishable ways of placing the items is $\frac{7!}{3!\cdot 2!\cdot 2!}=210$.

27. The number of ways to assign 15 clients to 3 caseworkers (cells) with 5 clients to each caseworker is $\frac{15!}{5!\cdot 5!\cdot 5!}=756{,}756.$

28. The number of ways of selecting 5 of the 10 remaining members so that order is important is

$$ {}_{10}P_5=\frac{10!}{(10-5)!}=\frac{10!}{5!}=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{5!}$$
$$=30{,}240.$$

29. The number of relationships between any two or more siblings is

$$\begin{aligned} {}_{12}C_2+{}_{12}C_3+\cdots+{}_{12}C_{12} &= 2^{12}-{}_{12}C_1-{}_{12}C_0\\ &=2^{12}-\frac{12!}{1!11!}-\frac{12!}{0!12!}\\ &=4096-12-1\\ &=4083 \end{aligned}$$

In a family of three siblings, we have

$$\begin{aligned} 2^3-{}_3C_1-{}_3C_0 &= 8-\frac{3!}{1!2!}-\frac{3!}{0!3!}\\ &=8-3-1\\ &=4 \text{ relationships.} \end{aligned}$$

30. Of the 10 applicants, 4 will be hired for the assembly department (cell 1), 2 for the shipping department (cell 2), and 4 will not be hired (cell 3). Thus the number of ways to fill the positions is $\frac{10!}{4!\cdot 2!\cdot 4!}=3150$.

31. The order in which the securities go into the portfolio is not important. The number of ways to select 8 of 12 stocks is ${}_{12}C_8$. The number of ways to select 4 of 7 bonds is ${}_7C_4$. By the basic counting principle, the number of ways to create the portfolio is

$$ {}_{12}C_8\cdot {}_7C_4=\frac{12!}{8!(12-8)!}\cdot\frac{7!}{4!(7-4)!}$$
$$=\frac{12!}{8!\cdot 4!}\cdot\frac{7!}{4!\cdot 3!}=\frac{12\cdot 11\cdot 10\cdot 9\cdot 8!}{8!\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{7\cdot 6\cdot 5\cdot 4!}{4!\cdot 3\cdot 2\cdot 1}$$
$$=495\cdot 35=17{,}325.$$

32. Suppose the possible games are numbered 1, 2, 3, ..., 7. The order in which four games are won is not important. The number of ways that 4 of the possible 7 games can be won is

$$ {}_7C_4=\frac{7!}{4!(7-4)!}=\frac{7!}{4!\cdot 3!}=35.$$

33. a. Selecting 3 of the 3 males can be done in only 1 way.

b. Selecting 4 of the 4 females can be done in only 1 way.

c. Selecting 2 males and 2 females can be considered as a two-stage process. In the first stage, 2 of the 3 males are selected (and order is not important), which can be done in ${}_3C_2$ ways. In the second stage, 2 of the 4 females are selected, which can be done in ${}_4C_2$ ways. By the basic counting principle, the ways of selecting the subcommittee is

$$ {}_3C_2\cdot {}_4C_2=\frac{3!}{2!(3-2)!}\cdot\frac{4!}{2!(4-2)!}$$
$$=\frac{3!}{2!\cdot 1!}\cdot\frac{4!}{2!\cdot 2!}=3\cdot 6=18$$

34. The number of ways 2 females can be selected is ${}_5C_2$. Three females can be selected ${}_5C_3$ ways. Four: ${}_5C_4$ ways. Respectively, the males can be selected ${}_3C_2$, ${}_3C_1$, and ${}_3C_0$ ways. The subcommittee of four can be selected in

$$ {}_5C_2\cdot {}_3C_2+{}_5C_3\cdot {}_3C_1+{}_5C_4\cdot {}_3C_0$$
$$=\frac{5!}{2!3!}\cdot\frac{3!}{2!1!}+\frac{5!}{3!2!}\cdot\frac{3!}{1!2!}+\frac{5!}{4!1!}\cdot\frac{3!}{0!3!}$$
$$=10\cdot 3+10\cdot 3+5\cdot 1$$
$$=65 \text{ ways.}$$

35. There are 4 cards of a given denomination and the number of ways of selecting 3 cards of that denomination is ${}_4C_3$.

Since there are 13 denominations, the number of ways of selecting 3 cards of one denomination is $13\cdot {}_4C_3$. After that selection is made, the 2 other cards must be of the same denomination (of which 12 denominations remain). Thus for the remaining 2 cards there are $12\cdot {}_4C_2$ selections. By the basic counting principle, the number of possible full-house hands is

$$13\cdot {}_4C_3\cdot 12\cdot {}_4C_2=13\cdot\frac{4!}{3!\cdot 1!}\cdot 12\cdot\frac{4!}{2!\cdot 2!}$$
$$=13\cdot 4\cdot 12\cdot 6=3744.$$

36. There are ${}_{13}C_2$ ways to choose the denominations for the two pairs, then 11 possible choices for the denomination of the remaining card. Given a denomination, there are ${}_4C_2$ ways to choose a pair and ${}_4C_1$ ways to choose a single card. Thus, the number of two-pair hands is

$${}_{13}C_2 \cdot 11 \cdot ({}_4C_2)^2 \cdot {}_4C_1$$
$$= \frac{13!}{2! \cdot 11!} \cdot 11 \cdot \left(\frac{4!}{2! \cdot 2!}\right)^2 \cdot \frac{4!}{1! \cdot 3!}$$
$$= 123{,}552$$

37. This situation can be considered as placing 18 tourists into 3 cells: 7 tourist go to the 7-passenger tram, 8 go to the 8-passenger tram, and 3 tourists remain at the bottom of the mountain. This can be done in

$\frac{18!}{7! \cdot 8! \cdot 3!}$ = 5,250,960 ways.

38. a. The 10 students are to be placed in 3 groups, with 4 in group A, 3 in group B, and 3 in group C. This can be done in

$\frac{10!}{4! \cdot 3! \cdot 3!} = 4200$ ways.

b. For a given assignment of students to the three groups, the number of ways of selecting a group leader and a secretary for group A (order is important) is ${}_4P_2$; for group B, it is ${}_3P_2$; and for group C it is ${}_3P_2$. Thus the number of ways that the instructor can split the class into 3 groups and designate a group leader and secretary in each group is

$$4200 \cdot {}_4P_2 \cdot {}_3P_2 \cdot {}_3P_2$$
$$= 4200(4 \cdot 3)(3 \cdot 2)(3 \cdot 2) = 1{,}814{,}400.$$

Apply It 8.3

1. This is a combination problem because the order in which the videos are selected is not important. The number of possible choices is the number of ways 3 videos can be selected from 400 without regard to order.

$${}_{400}C_3 = \frac{400!}{3!(400-3)!} = \frac{400!}{3!397!}$$
$$= \frac{400 \cdot 399 \cdot 398 \cdot 397!}{3!397!}$$
$$= \frac{400 \cdot 399 \cdot 398}{3 \cdot 2}$$
$$= 10{,}586{,}800$$

Problems 8.3

1. {9D, 9H, 9C, 9S]

2. {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

3. {1HH, 1HT, 1TH, 1TT, 2HH, 2HT, 2TH, 2TT, 3HH, 3HT, 3TH, 3TT, 4HH, 4HT, 4TH, 4TT, 5HH, 5HT, 5TH, 5TT, 6HH, 6HT, 6TH, 6TT}

4. {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

5. [64, 69, 60, 61, 46, 49, 40, 41, 96, 94, 90, 91, 06, 04, 09, 01, 16, 14, 19, 10]

6. {BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG}

7. a. {RR, RW, RB, WR, WW, WB, BR, BW, BB};

b. {RW, RB, WR, WB, BR, BW}

8. {AEG, AEH, AFG, AFH, BEG, BEH, BFG, BFH, CEG, CEH, CFG, CFH, DEG, DEH, DFG, DFH}

9. Sample space consists of ordered sets of six elements and each element is H or T. Since there are two possibilities for each toss (H or T), and there are six tosses, by he basic counting principle, the number of sample points is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$.

10. Sample space consists of ordered sets of five elements where each element is an integer between 1 and 6 inclusive. Since there are six possibilities for each die, and there are 5 dice, by the basic counting principle, the number of sample points is $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5 = 7776$.

11. Sample space consists of ordered pairs where the first element indicates the card drawn (52 possibilities) and the second element indicates the number on the die (6 possibilities). By the basic counting principle, the number of sample points is $52 \cdot 6 = 312$.

12. Sample space consists of ordered sets of four elements where the elements and their position indicate the rabbit selected on the respective draw. Since the rabbits are not replaced, for the first draw there are 9 possibilities, for the second draw there are 8 possibilities, and for the third and fourth there are 7 and 6 possibilities, respectively. By the basic counting principle, the number of sample points is $9 \cdot 8 \cdot 7 \cdot 6 = 3024$.

13. Sample space consists of combinations of 52 cards taken 4 at a time. Thus the number of sample points is ${}_{52}C_4 = 270,725$.

14. Sample space consists of all four letter "words." For each of the four letters there are 26 possibilities. By the basic counting principle, the number of sample points is
$26 \cdot 26 \cdot 26 \cdot 26 = 26^4 = 456,976$.

15. The sample points that are either in E, or in F, or in both E and F are 1, 3, 5, 7, and 9. Thus $E \cup F = \{1, 3, 5, 7, 9\}$.

16. The sample points in S that are not in G are 1, 3, 5, 7, 9, and 10. Thus $G' = \{1, 3, 5, 7, 9, 10\}$.

17. The sample points in S that are not in E are 2, 4, 6, 7, 8, 9, and 10. Thus $E' = \{2, 4, 6, 7, 8, 9, 10\}$.
The sample points common to both E' and F are 7 and 9. Thus $E' \cap F = \{7, 9\}$.

18. $E' = \{2, 4, 6, 7, 8, 9, 10\}$ and $G' = \{1, 3, 5, 7, 9, 10\}$, so $E' \cap G' = S$.

19. The sample points in S that are not in F are 1, 2, 4, 6, 8, and 10. Thus $F' = \{1, 2, 4, 6, 8, 10\}$.

20. $(E \cup F)' = \{1,3,5,7,9\}' = \{2,4,6,8,10\}$

21. $(F \cap G)' = \varnothing' = S$

22. $(E \cup G) \cap F'$
$= \{1,2,3,4,5,6,8\} \cap \{1,2,4,6,8,10\}$
$= \{1, 2, 4, 6, 8\}$

23. $E_1 \cap E_2 \neq \varnothing$; $E_1 \cap E_3 \neq \varnothing$; $E_1 \cap E_4 = \varnothing$;
$E_2 \cap E_3 = \varnothing$; $E_2 \cap E_4 = \varnothing$; $E_3 \cap E_4 = \varnothing$.
Thus E_1 and E_4, E_2 and E_3, E_2 and E_4, and E_3 and E_4 are mutually exclusive.

24. If both cards are jacks, then both cards can neither be clubs nor 3's. Thus $E_J \cap E_C = \varnothing$ and $E_J \cap E_3 = \varnothing$. If both cards are clubs, then both cards cannot be 3's. Thus $E_C \cap E_3 = \varnothing$.
E_J and E_C, E_J and E_3, E_C and E_3 are mutually exclusive.

25. $E \cap F \neq \varnothing$, $E \cap G = \varnothing$, $E \cap H \neq \varnothing$,
$E \cap I \neq \varnothing$, $F \cap G \neq \varnothing$, $F \cap H \neq \varnothing$
$F \cap I = \varnothing$, $G \cap H = \varnothing$, $G \cap I = \varnothing$,
$H \cap I \neq \varnothing$. Thus E and G, F and I, G and H, and G and I are mutually exclusive.

26. $E \cap F = \varnothing$, $E \cap G = \varnothing$, $E \cap H \neq \varnothing$,
$E \cap I \neq \varnothing$, $F \cap G \neq \varnothing$, $F \cap H \neq \varnothing$,
$F \cap I = \varnothing$, $G \cap H = \varnothing$, $G \cap I = \varnothing$,
$H \cap I = \varnothing$.
Thus E and F, E and G, F and I, G and H, G and I, H and I are mutually exclusive.

27. **a.** S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

b. E_1 = {HHH, HHT, HTH, HTT, THH, THT, TTH}

c. E_2 = {HHT, HTH, HTT, THH, THT, TTH, TTT}

d. $E_1 \cup E_2$ = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} = S

e. $E_1 \cap E_2$ = {HHT, HTH, HTT, THH, THT, TTH}

f. $(E_1 \cup E_2)' = S' = \varnothing$

g. $(E_1 \cap E_2)'$ = {HHT, HTH, HTT, THH, THT, TTH}$'$ = {HHH, TTT}

28. **a.** {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}

b. {BBG, BGB, BGG, GBB, GBG, GGB, GGG}

c. {BBB, BBG, BGB, BGG, GBB, GBG, GGB}

d. No; the complement is {GGG}.

29. a. {ABC, ACB, BAC, BCA, CAB, CBA}

b. {ABC, ACB}

c. {BAC, BCA, CAB, CBA}

30. a. {UUV, UUW, UUX, UUZ, UVV, UVW, UVX, UVZ, UXV, UXW, UXX, UXZ, UYV, UYW, UYX, UYZ, VUV, VUW, VUX, VUZ, VVV, VVW, VVX, VVZ, VXV, VXW, VXX, VXZ, VYV, VYW, VYX, VYZ, WUV, WUW, WUX, WUZ, WVV, WVW, WVX, WVZ, WXV, WXW, WXX, WXZ, WYV, WYW, WYX, WYZ}

b. {VVV}

c. {UUV, UUW, UUX, UUZ, UVV, UVW, UVX, UVZ, UXV, UXW, UXX, UXZ, UYV, UYW, UYX, UYZ, VUV, VUW, VUX, VUZ, VVW, VVX, VVZ, VXV, VXW, VXX, VXZ, VYV, VYW, VYX, VYZ, WUV, WUW, WUX, WUZ, WVV, WVW, WVX, WVZ, WXV, WXW, WXX, WXZ, WYV, WYW, WYX, WYZ}
More than one supplier is used.

31. Using the properties in Table 8.1, we have

$$\begin{aligned}
&(E\cap F)\cap(E\cap F')\\
&=(E\cap F\cap E)\cap F' && \text{[property 15]}\\
&=(E\cap E\cap F)\cap F' && \text{[property 11]}\\
&=(E\cap E)\cap(F\cap F') && \text{[porperty 15]}\\
&=E\cap\varnothing && \text{[property 5]}\\
&=\varnothing && \text{[property 9]}
\end{aligned}$$

Thus
$(E\cap F)\cap(E\cap F')=\varnothing,$ so $E\cap F$ and $E\cap F'$ are mutually exclusive.

32. Using the properties in Table 8.1, we have

$$\begin{aligned}
&(E\cap F)\cup(E\cap F')\\
&=E\cap(F\cup F') && \text{[property 16]}\\
&=E\cap S && \text{[property 4]}\\
&=E && \text{[property 7]}
\end{aligned}$$

Problems 8.4

1. $4000P(E) = 4000(0.125) = 500$

2. $3000P(E) = 3000[1 - P(E')] = 3000(1 - 0.45)$
$= 3000(0.55) = 1650$

3. a. $P(E') = 1 - P(E) = 1 - 0.2 = 0.8$

b. $P(E\cup F)=P(E)+P(F)-P(E\cap F)$
$= 0.2 + 0.3 - 0.1 = 0.4$

4. a. $P(E') = 1 - P(E) = 1 - \frac{1}{4} = \frac{3}{4}$

b. $P(E\cup F)=P(E)+P(F)-P(E\cap F)$
$=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$

5. If E and F are mutually exclusive, then $E\cap F=\varnothing$.
Thus $P(E\cap F)=P(\varnothing)=0$. Since it is given that $P(E\cap F)=0.831\neq 0$, E and F are not mutually exclusive.

6. $P(E\cup F)=P(E)+P(F)-P(E\cap F)$
Thus $P(F)=P(E\cup F)+P(E\cap F)-P(E)$
$=\frac{1}{2}+\frac{1}{12}-\frac{1}{4}=\frac{1}{3}.$

7. a. $E_8 = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$
$P(E_8)=\frac{n(E_8)}{n(S)}=\frac{5}{36}$

b. $E_{2\text{ or }3} = \{(1, 1), (1, 2), (2, 1)\}$
$P(E_{2\text{ or }3})=\frac{n(E_{2\text{ or }3})}{n(S)}=\frac{3}{36}=\frac{1}{12}$

c. $E_{3,4,\text{or }5}=\{(1,2),(2,1),(1,3),(2,2),(3,1),$
$(1,4),(2,3),(3,2),(4,1)\}$
$P(E_{3,4,\text{or }5})=\frac{n(E_{3,4,\text{ or }5})}{n(S)}=\frac{9}{36}=\frac{1}{4}$

d. $E_{12\text{ or }13}=E_{12}$, since E_{13} is an impossible event.
$E_{12}=\{(6,6)\}$
$P(E_{12\text{ or }13})=\frac{n(E_{12\text{ or }13})}{n(S)}=\frac{1}{36}$

e. $E_2 = \{(1,1)\}$

$E_4 = \{(1,3),(2,2),(3,1)\}$

$E_6 = \{(1,5),(2,4),(3,3,),(4,2),(5,1)\}$

$E_8 = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$E_{10} = \{(4,6),(5,5),(6,4)\}$

$E_{12} = \{(6,6)\}$

$P(E_{\text{even}}) = P(E_2) + P(E_4)$

$+P(E_6) + P(E_8) + P(E_{10}) + P(E_{12})$

$= \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{5}{36} + \frac{3}{36} + \frac{1}{36} = \frac{18}{36} = \frac{1}{2}$

f. $P(E_{\text{odd}}) = 1 - P(E_{\text{even}}) = 1 - \frac{1}{2} = \frac{1}{2}$

g. $E'_{\text{less than 10}} = E_{10} \cup E_{11} \cup E_{12}$

$= \{(4,6),(5,5),(6,4)\} \cup \{(5,6),(6,5)\} \cup \{(6,6)\}$

$= \{(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}$.

$P(E_{\text{less than 10}}) = 1 - P(E'_{\text{less than 10}})$

$= 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$.

8. $E_{2 \text{ or } 3 \text{ shows}}$ = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 2), (4, 2), (5, 2), (6, 2), (1, 3), (4, 3), (5, 3), (6, 3)}

$P(E_{2 \text{ or } 3 \text{ shows}}) = \frac{n(E_{2 \text{ or } 3 \text{ shows}})}{n(S)} = \frac{20}{36} = \frac{5}{9}$

9. $n(S) = 52$.

a. $P(\text{king of hearts}) = \frac{n(E_{\text{king of hearts}})}{n(S)} = \frac{1}{52}$

b. $P(\text{diamond}) = \frac{n(E_{\text{diamond}})}{n(S)} = \frac{13}{52} = \frac{1}{4}$

c. $P(\text{jack}) = \frac{n(E_{\text{jack}})}{n(S)} = \frac{4}{52} = \frac{1}{13}$

d. $P(\text{red}) = \frac{n(E_{\text{red}})}{n(S)} = \frac{26}{52} = \frac{1}{2}$

e. Because a heart is not a club,
$E_{\text{heart}} \cap E_{\text{club}} = \varnothing$.
Thus

$P(E_{\text{heart or club}}) = P(E_{\text{heart}} \cup E_{\text{club}})$

$= P(E_{\text{heart}}) + P(E_{\text{club}})$

$= \frac{n(E_{\text{heart}})}{n(S)} + \frac{n(E_{\text{club}})}{n(S)} = \frac{13}{52} + \frac{13}{52}$

$= \frac{26}{52} = \frac{1}{2}$

f. $E_{\text{club and 4}} = \{4C\}$

$P(E_{\text{club and 4}}) = \frac{n(E_{\text{club and 4}})}{n(S)} = \frac{1}{52}$

g. $P(\text{club or } 4)$
$= P(\text{club}) + P(4) - P(\text{club and } 4)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

h. $E_{\text{red and king}} = \{\text{KH}, \text{KD}\}$

$P(\text{red and king}) = \frac{n(E_{\text{red and king}})}{n(S)}$

$= \frac{2}{52} = \frac{1}{26}$

i. $E_{\text{spade and heart}} = \varnothing$
Thus $P(\text{spade and heart}) = 0$

10. $n(S) = 2 \cdot 6 = 12$

a. $E_{\text{H},5} = \{\text{H5}\}$

$P(\text{head and } 5) = \frac{n(E_{\text{H},5})}{n(S)} = \frac{1}{12}$

b. $n(E_{\text{head}}) = 1 \cdot 6 = 6$.

$P(\text{head}) = \frac{n(E_{\text{head}})}{n(S)} = \frac{6}{12} = \frac{1}{2}$

c. $n(E_3) = 2 \cdot 1 = 2$

$P(3) = \frac{n(E_3)}{n(S)} = \frac{2}{12} = -\frac{1}{6}$

d. $n(E_{\text{head and even}}) = 1 \cdot 3 = 3$
$P(\text{head and even})$

$= \frac{n(E_{\text{head and even}})}{n(S)} = \frac{3}{12} = \frac{1}{4}$

11. $n(S) = 2 \cdot 6 \cdot 52 = 624$

a. $P(\text{head, 6, ace of spades})$
$= \frac{n(E_{\text{H,6,AS}})}{n(S)} = \frac{1 \cdot 1 \cdot 1}{624} = \frac{1}{624}$

b. $P(\text{head, 3, queen})$
$= \frac{n(E_{\text{H,3,Q}})}{n(S)} = \frac{1 \cdot 1 \cdot 4}{624} = \frac{1}{156}$

c. $P(\text{head, 2 or 3, queen})$
$= \frac{n(E_{\text{H,2 or 3,Q}})}{n(S)} = \frac{1 \cdot 2 \cdot 4}{624} = \frac{1}{78}$

d. $P(\text{head, odd, diamond})$
$= \frac{n(E_{\text{H,O,D}})}{n(S)} = \frac{1 \cdot 3 \cdot 13}{624} = \frac{1}{16}$

12. $n(S) = 8$

a. $E_{\text{3 heads}} = \{\text{HHH}\}$
$P(\text{3 heads}) = \frac{n(E_{\text{3 heads}})}{n(S)} = \frac{1}{8}$

b. $E_{\text{1 tail}} = \{\text{HHT,HTH,THH}\}$.
$P(\text{1 tail}) = \frac{n(E_{\text{1 tail}})}{n(S)} = \frac{3}{8}$

c. $P(\text{no more than 2 heads}) = 1 - P(\text{3 heads})$
$= 1 - \frac{1}{8} = \frac{7}{8}$

d. $E_{\text{no more than 1 tail}} = E_{\text{0 tails}} \cup E_{\text{1 tail}}$
$= \{\text{HHH}\} \cup \{\text{HHT,HTH,THH}\}$
$= \{\text{HHH, HHT, HTH, THH}\}$.
$P(\text{no more than 1 tail})$
$= \frac{n(E_{\text{no more than 1 tail}})}{n(S)} = \frac{4}{8} = \frac{1}{2}$

13. $n(S) = 52 \cdot 51 \cdot 50 = 132{,}600$

a. $P(\text{all kings}) = \frac{4 \cdot 3 \cdot 2}{132{,}600} = \frac{1}{5525}$

b. $P(\text{all hearts}) = \frac{13 \cdot 12 \cdot 11}{132{,}600} = \frac{11}{850}$

14. $n(S) = 52 \cdot 52 = 2704$

a. $P(\text{both kings}) = \frac{n(E_{\text{both kings}})}{n(S)} = \frac{4 \cdot 4}{2704}$
$= \frac{1}{169}$

b. Number of ways both cards are king of hearts: 1. Number of ways either first card is king of hearts and second card is a different heart, or vice versa: $2(1 \cdot 12) = 24$. Number of ways either first card is king of diamonds, clubs, or spades, and second card is a heart, or vice versa: $2(3 \cdot 13) = 78$. Thus, number ways one card is a king and the other is a heart is $1 + 24 + 78 = 103$, so probability of given event is $\frac{103}{2704}$.

15. $n(S) = 2 \cdot 2 \cdot 2 = 8$

a. $E_{\text{3 girls}} = \{\text{GGG}\}$
$P(\text{3 girls}) = \frac{n(E_{\text{3 girls}})}{n(S)} = \frac{1}{8}$

b. $E_{\text{1 boy}} = \{\text{BGG,GBG,GGB}\}$
$P(\text{1 boy}) = \frac{n(E_{\text{1 boy}})}{n(S)} = \frac{3}{8}$

c. $E_{\text{no girl}} = \{\text{BBB}\}$
$P(\text{no girl}) = \frac{n(E_{\text{no girl}})}{n(S)} = \frac{1}{8}$

d. $P(\text{at least 1 girl}) = 1 - \text{P(no girl)}$
$= 1 - \frac{1}{8} = \frac{7}{8}$

16. The sample space consists of 16 jelly beans. Thus $n(S) = 16$.

a. $P(\text{blue}) = \frac{n(E_{\text{blue}})}{n(S)} = \frac{2}{16} = \frac{1}{8}$

b. $P(\text{not red}) = 1 - P(\text{red})$
$= 1 - \frac{n(E_{\text{red}})}{n(S)} = 1 - \frac{5}{16} = \frac{11}{16}$

c. The events of drawing a red jelly bean and drawing a white jelly bean are mutually exclusive. Thus
$P(\text{red or white}) = P(\text{red}) + P(\text{white})$
$= \frac{5}{16} + \frac{9}{16} = \frac{14}{16} = \frac{7}{8}$

d. $P(\text{neither red nor blue}) = P(\text{white}) = \frac{9}{16}$

e. $E_{\text{yellow}} = \varnothing$. Thus $P(\text{yellow}) = 0$

f. $E_{\text{red}} \cap E_{\text{yellow}} = \varnothing$
Thus $P(\text{red or yellow}) = P(\text{red}) + P(\text{yellow})$
$= \frac{5}{16} + 0 = \frac{5}{16}$.

17. The sample space consists of 60 stocks. Thus $n(S) = 60$.

a. $P(6\% \text{ or more}) = \frac{n(E_{6\% \text{ or more}})}{n(S)}$
$= \frac{48}{60} = \frac{4}{5}$

b. $P(\text{less than } 6\%) = 1 - P(6\% \text{ or more})$
$= 1 - \frac{4}{5} = \frac{1}{5}$

18. Let N = number of ties. Then the number of pure silk ties is $0.4N$.

a. $P(100\% \text{ pure silk}) = \frac{0.4N}{N} = 0.4$

b. $P(\text{not } 100\% \text{ silk}) = 1 - P(100\% \text{ pure silk})$
$= 1 - 0.4 = 0.6$

19. $n(S) = 40$
Of the 40 students, 4 received an A, 10 a B, 14 a C, 10 a D, and 2 an F.

a. $P(\text{A}) = \frac{n(E_{\text{A}})}{n(S)} = \frac{4}{40} = \frac{1}{10} = 0.1$

b. $P(\text{A or B}) = \frac{n(E_{\text{A or B}})}{n(S)} = \frac{4+10}{40}$
$= \frac{14}{40} = 0.35$

c. $P(\text{neither D nor F}) = P(\text{A, B, or C})$
$= \frac{n(E_{\text{A,B, or C}})}{n(S)} = \frac{4+10+14}{40} = \frac{28}{40} = 0.7$

d. $P(\text{no F}) = 1 - P(\text{F}) = 1 - \frac{n(E_{\text{F}})}{n(S)}$
$= 1 - \frac{2}{40} = \frac{38}{40} = 0.95$

e. Let N = number of students. Then $n(S) = N$. Of the N students, $0.10N$ received an A, $0.25N$ a B, $0.35N$ a C, $0.25N$ a D, $0.05N$ an F.
$P(\text{A}) = \frac{0.10N}{N} = 0.1$
$P(\text{A or B}) = \frac{0.10N + 0.25N}{N}$
$= \frac{0.35N}{N} = 0.35$
$P(\text{neither D nor F}) = P(\text{A, B, or C})$
$= \frac{0.10N + 0.25N + 0.35N}{N}$
$= \frac{0.70N}{N} = 0.7$
$P(\text{no F}) = 1 - P(\text{F})$
$= 1 - \frac{0.05N}{N} = 1 - 0.05 = 0.95$

20. Bag 1 contains 5 jelly beans, and Bag 2 contains 9.
$n(S) = 5 \cdot 9 = 45$.

a. $P(\text{both red}) = \frac{n(E_{\text{R,R}})}{n(S)} = \frac{3 \cdot 4}{45} = \frac{4}{15}$

b. $P(\text{one red and other green})$
$= \frac{n(E_{R,G}) + n(E_{G,R})}{n(S)} = \frac{3 \cdot 5 + 2 \cdot 4}{45}$
$= \frac{15+8}{45} = \frac{23}{45}$

21. The sample space consists of combinations of 2 people selected from 7. Thus
$n(S) = {}_7C_2 = \frac{7!}{2!5!} = 21$. Because there are 3 women in the group, the number of possible

2-woman committees is ${}_3C_2 = \dfrac{3!}{2!1!} = 3$. Thus

$P(\text{2 women}) = \dfrac{n(E_{\text{2 women}})}{n(S)} = \dfrac{3}{21} = \dfrac{1}{7}$.

22. Because there are 3 men and 2 women, the number of possible committees consisting of a man and a woman is 3 · 2 = 6.
Thus

$P(\text{man and woman}) = \dfrac{n(E_{\text{man and woman}})}{n(S)}$.

$= \dfrac{6}{10} = \dfrac{3}{5}$.

23. Number of ways to answer exam is $2^{10} = 1024 = n(S)$.

a. There is only one way to achieve 100 points, namely to answer each question correctly. Thus

$P(\text{100 points}) = \dfrac{n(E_{\text{100 points}})}{n(S)} = \dfrac{1}{1024}$.

b. Number of ways to score 90 points = number of ways that exactly one question is answered incorrectly = 10.
Thus
P(90 or more points)
= P(90 points) + P(100 points)
$= \dfrac{10}{1024} + \dfrac{1}{1024} = \dfrac{11}{1024}$.

24. Number of ways to answer exam is $4^8 = 65{,}536 = n(S)$.

a. $P(\text{all correct}) = \dfrac{n(E_{\text{all correct}})}{n(S)} = \dfrac{1}{65{,}536}$

b. The probability of answering one question correctly when answering in a random fashion is $\dfrac{1}{4}$ and the probability of answering incorrectly is $\dfrac{3}{4}$. Thus, the probability of answering the first four questions correctly and the last four incorrectly is $\left(\dfrac{1}{4}\right)^4\left(\dfrac{3}{4}\right)^4 = \dfrac{3^4}{4^8}$. Since there are ${}_8C_4$ distinguishable orders in which one can arrange 4 correct and 4 incorrect answers, and since each arrangement has the same overall probability of occurring, the probability of 4 correct and 4 incorrect answers is $\dfrac{3^4}{4^8}\cdot {}_8C_4 = \dfrac{3^4}{4^8}\cdot\dfrac{8!}{4!4!}$

$= \dfrac{3^4}{4^8}\cdot\dfrac{8\cdot 7\cdot 6\cdot 5\cdot 4!}{4\cdot 3\cdot 2\cdot 1\cdot 4!} = \dfrac{3^4}{4^8}\cdot\dfrac{2\cdot 7\cdot 5}{1}$

$= \dfrac{2835}{32{,}768}$.

25. A poker hand is a 5-card deal from 52 cards. Thus $n(S) = {}_{52}C_5$. In 52 cards, there are 4 cards of a particular denomination. Thus, for a four of a kind, the number of ways of selecting 4 of 4 cards of a particular denomination is ${}_4C_4$. Since there are 13 denominations, 4 cards of the same denomination can be dealt in $13\cdot {}_4C_4$ ways. For the remaining card, there are 12 denominations that are possible, and for each denomination there are ${}_4C_1$ ways of dealing a card. Thus

$$P(\text{four of a kind}) = \frac{n(E_{\text{four of a kind}})}{n(S)} = \frac{13\cdot {}_4C_4\cdot 12\cdot {}_4C_1}{{}_{52}C_5} = \frac{13\cdot 12\cdot 4}{{}_{52}C_5}$$

26. a. $P(E\cup F) = P(E) + P(F) - P(E\cap F)$

Thus $P(F) = P(E\cup F) + P(E\cap F) - P(E)$

$= \dfrac{41}{105} + \dfrac{1}{7} - \dfrac{1}{5}$

$= \dfrac{1}{3}$

b. $P(E'\cup F) = P(E') + P(F) - P(E'\cap F)$

$= \left(1 - \dfrac{1}{5}\right) + \dfrac{1}{3} - P(E'\cap F)$

$= \dfrac{17}{15} - P(E'\cap F)$

Since $F = (E\cap F)\cup(E'\cap F)$ and $E\cap F$ and $E'\cap F$ are mutually exclusive, $P(F) = P(E\cap F) + P(E'\cap F)$ so

$\dfrac{1}{3} = \dfrac{1}{7} + P(E'\cap F)$

Thus $P(E' \cap F) = \frac{1}{3} - \frac{1}{7} = \frac{4}{21}$. Hence,

$P(E' \cup F) = \frac{17}{15} - \frac{4}{21} = \frac{33}{35}$.

27. $n(S) = {}_{100}C_3 = \frac{100!}{3! \cdot 97!} = 161,700$

a. $n(E_{3\text{ females}}) = {}_{35}C_3 = \frac{35!}{3! \cdot 32!} = 6545$

$P(E_{3\text{ females}}) = \frac{n(E_{3\text{ females}})}{n(S)}$

$= \frac{6545}{161,700} \approx 0.040$

b. The number of ways of selecting one professor is 15; the number of ways of selecting two associate professors is ${}_{24}C_2$.

Thus $n(E_{1\text{ professor \& 2 associate professors}})$

$= 15 \cdot \frac{24!}{2! \cdot 22!} = 15 \cdot 276 = 4140$.

Therefore,

$P(E_{1\text{ professor \& 2 associate professers}})$

$= \frac{4140}{161,700} \approx 0.026$.

28. P(even number) = $P(2) + P(4) + P(6)$

$= \frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$

29. Shiloh needs to win 3 more rounds to win the game and Caitlin needs to win 5 more rounds. Shiloh's probability of winning is

$$\sum_{k=0}^{4} \frac{{}_7C_k}{2^7} = \frac{1}{2^7} \sum_{k=0}^{4} {}_7C_k$$
$$= \frac{1}{2^7}({}_7C_0 + {}_7C_1 + {}_7C_2 + {}_7C_3 + {}_7C_4)$$
$$= \frac{1}{2^7}(1 + 7 + 21 + 35 + 35)$$
$$= \frac{99}{128}$$

Shiloh's share of the pot is then

$\frac{99}{128}(\$25) \approx \$19.34$.

30. Here Shiloh needs to win 5 more rounds to win the game and Caitlin needs to win 8 more rounds. Shiloh's probability of winning is $\sum_{k=0}^{7} \frac{{}_{12}C_k}{2^{12}} = \frac{3302}{4096} = \frac{1651}{2048}$. Thus Shiloh's share of the pot is $\frac{1651}{2048}(\$50) \approx \$40.31$.

31. Let $p = P(1) = P(2)$. Then $2p = P(3) = P(4)$ and $3p = P(5) = P(6)$. Since $P(S) = 1$, then $p + p + 2p + 2p + 3p + 3p = 12p = 1$ and $P(1) = p = \frac{1}{12}$.

32. Let $p_1 = P(a) = P(b) = P(c) = P(d) = P(e)$, and $p_2 = P(f) = P(g)$. Then

$P(S) = 5(p_1) + 2(p_2) = 1, \quad p_2 = \frac{1}{2} - \frac{5}{2}p_1$.

Since p_1 is not known, it is not possible to determine $P(f) = p_2$. If it is also known that $P(\{a, f\}) = \frac{1}{3}$, then we have

$P(\{a, f\}) = P(a) + P(f) = p_1 + p_2 = \frac{1}{3}$.

Thus $p_1 = \frac{1}{3} - p_2$ and $p_2 = \frac{1}{2} - \frac{5}{2}\left(\frac{1}{3} - p_2\right)$.

$-\frac{3}{2}p_2 = -\frac{1}{3}$ or $p_2 = \frac{2}{9}$ and so $P(f) = \frac{2}{9}$.

33. a. Of the 100 voters, 51 favor the tax increase. Thus P(favors tax increase) = $\frac{51}{100} = 0.51$.

b. Of the 100 voters, 44 oppose the tax increase. Thus

P(opposes tax increase) = $\frac{44}{100} = 0.44$.

c. Of the 100 voters, 3 are Republican with no opinion. Thus

P(is a Republican with no opinion) $= \frac{3}{100}$

$= 0.03$.

34. a. For the chain, the total average number of sales is 170 units. For brand B, 65 units per month are sold. Thus

$P(\text{sale is for brand B}) = \frac{65}{170} = \frac{13}{34} \approx 0.38$.

b. Since 95 units per month are sold at the Exton store, and 30 are of brand C,
P(sale is for brand C given that it is at Exton store) $= \frac{30}{95} = \frac{6}{19} \approx 0.32$.

35. $\frac{P(E)}{P(E')} = \frac{P(E)}{1-P(E)} = \frac{\frac{4}{5}}{1-\left(\frac{4}{5}\right)} = \frac{\frac{4}{5}}{\frac{1}{5}} = \frac{4}{1}$

The odds are 4:1.

36. $\frac{P(E)}{P(E')} = \frac{P(E)}{1-P(E)} = \frac{\frac{2}{7}}{1-\frac{2}{7}} = \frac{\frac{2}{7}}{\frac{5}{7}} = \frac{2}{5}$

The odds are 2:5.

37. $\frac{P(E)}{P(E')} = \frac{P(E)}{1-P(E)} = \frac{0.7}{1-0.7} = \frac{0.7}{0.3} = \frac{7}{3}$

The odds are 7:3.

38. $\frac{P(E)}{P(E')} = \frac{P(E)}{1-P(E)} = \frac{0.001}{1-0.001} = \frac{0.001}{0.999} = \frac{1}{999}$

The odds are 1:999.

39. $P(E) = \frac{7}{7+5} = \frac{7}{12}$

40. $P(E) = \frac{100}{100+1} = \frac{100}{101}$

41. $P(E) = \frac{3}{3+7} = \frac{3}{10}$

42. $P(E) = \frac{a}{a+a} = \frac{a}{2a} = \frac{1}{2}$

43. Odds that it will rain tomorrow
$= \frac{P(\text{rain})}{P(\text{no rain})} = \frac{0.75}{1-0.75} = \frac{0.75}{0.25} = 3$.
The odds are 3:1.

44. The odds of E *not* occurring are the odds of event E' which is $\frac{P(E')}{P(E'')} = \frac{P(E')}{P(E)} = \frac{3}{5}$. Then $\frac{P(E)}{P(E')} = \frac{1}{\frac{P(E')}{P(E)}} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$, so the odds that E does occur are 5:3.
In general, if the odds of E *not* occurring are a:b, then the odds that E does occur are b:a.

45.
$$P(E_{24}) = \frac{23+P(E_{23})(365-23)}{365} \approx \frac{23+0.507297(342)}{365} \approx 0.538344$$
$$P(E_{25}) = \frac{24+P(E_{24})(365-24)}{365} \approx 0.568699 \approx 56.9\%$$

46.
$$P(E_{26}) = \frac{25+P(E_{25})(365-25)}{365} \approx 0.598240$$
$$P(E_{27}) = \frac{26+P(E_{26})(365-26)}{365} \approx 0.626859$$
$P(E_{28}) \approx 0.654461$
$P(E_{29}) \approx 0.680968$
$P(E_{30}) \approx 0.706316 \approx 70.6\%$

Problems 8.5

1. a. $P(E|F) = \frac{n(E\cap F)}{n(F)} = \frac{1}{5}$

b. Using the result of part (a),
$P(E' \mid F) = 1 - P(E \mid F) = 1 - \frac{1}{5} = \frac{4}{5}$.

c. $F' = \{3, 7, 8, 9\}$ so
$P(E \mid F') = \frac{n(E\cap F')}{n(F')} = \frac{1}{4}$.

d. $P(F \mid E) = \frac{n(F\cap E)}{n(E)} = \frac{1}{2}$

e. $F \cap G = \{5, 6\}$ so
$P(E \mid F\cap G) = \frac{n(E\cap(F\cap G))}{n(F\cap G)} = \frac{0}{2} = 0$.

2. a. $P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$

b. $P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{0}{2} = 0$

c. $P(E \mid G) = \frac{n(E \cap G)}{n(G)} = \frac{2}{3}$

d. $P(G \mid E) = \frac{n(G \cap E)}{n(E)} = \frac{2}{2} = 1$

e. $F' = \{1, 2, 5\}$

$P(G \mid F') = \frac{n(G \cap F')}{n(F')} = \frac{2}{3}$

f. $E' = \{3, 4, 5\}$

$P(E' \mid F') = \frac{n(E' \cap F')}{n(F')} = \frac{1}{3}$

3. $P(E \mid E) = \frac{P(E \cap E)}{P(E)} = \frac{P(E)}{P(E)} = 1$

4. $P(\varnothing \mid E) = \frac{P(\varnothing \cap E)}{P(E)} = \frac{P(\varnothing)}{P(E)} = \frac{0}{P(E)} = 0$

5. $P(E' \mid F) = 1 - P(E \mid F) = 1 - 0.62 = 0.38$

6. $P(F \mid G) = \frac{P(F \cap G)}{P(G)} = \frac{P(\varnothing)}{P(G)} = \frac{0}{P(G)} = 0$

7. a. $P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{1/3} = \frac{1}{2}$

b. $P(F \mid E) = \frac{P(F \cap E)}{P(E)} = \frac{1/6}{1/4} = \frac{2}{3}$

8. First we find $P(E \cap F)$:

$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$,

$P(E \cap F) = P\left(E \middle| F\right) P(F) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$.

Then

$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= \frac{1}{4} + \frac{1}{3} - \frac{1}{4} \\ &= \frac{1}{3}. \end{aligned}$$

9. a. $P(F \mid E) = \frac{P(F \cap E)}{P(E)} = \frac{1/6}{1/4} = \frac{2}{3}$

b. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\frac{7}{12} = \frac{1}{4} + P(F) - \frac{1}{6}$

Thus $P(F) = \frac{7}{12} - \frac{1}{4} + \frac{1}{6} = \frac{1}{2}$..

c. From part (b) $P(F) = \frac{1}{2}$.

Then $P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{1/2} = \frac{1}{3}$.

d. $P(E) = P(E \cap F) + P(E \cap F')$

$\frac{1}{4} = \frac{1}{6} + P(E \cap F')$

so $P(E \cap F') = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$.

Then $P(E \mid F') = \frac{P(E \cap F')}{P(F')}$

$= \frac{1/12}{1 - 1/2} = \frac{1/12}{1/2} = \frac{1}{6}$.

10. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$, so

$P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$= \frac{4}{5} + \frac{3}{10} - \frac{7}{10} = \frac{4}{10}$

Then $P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{4/10}{4/5} = \frac{1}{2}$.

11. a. $P(F) = \frac{125}{200} = \frac{5}{8}$

b. $P(F \mid \text{II}) = \frac{n(F \cap \text{II})}{n(\text{II})} = \frac{35}{58}$

c. $P(O \mid \text{I}) = \frac{n(O \cap \text{I})}{n(\text{I})} = \frac{22}{78} = \frac{11}{39}$

d. $P(\text{III}) = \dfrac{64}{200} = \dfrac{8}{25}$

e. $P(\text{III} \mid O) = \dfrac{n(\text{III} \cap O)}{n(O)} = \dfrac{10}{47}$

f. $P(\text{II} \mid N') = \dfrac{n(\text{II} \cap N')}{n(N')}$

$= \dfrac{35+15}{125+47} = \dfrac{50}{172} = \dfrac{25}{86}$

12. a. $P(\text{Public}|\text{Middle}) = \dfrac{n(\text{Public} \cap \text{Middle})}{n(\text{Middle})}$

$= \dfrac{55}{80} = \dfrac{11}{16}$

b. $P(\text{High}|\text{Private}) = \dfrac{n(\text{High} \cap \text{Private})}{n(\text{Private})}$

$= \dfrac{14}{49} = \dfrac{2}{7}$

c. $P(\text{Private}|\text{High}) = \dfrac{n(\text{Private} \cap \text{High})}{n(\text{High})}$

$= \dfrac{14}{25}$

d. $P(\text{Public} \cup \text{Low})$

$= P(\text{Public}) + P(\text{Low}) - P(\text{Public} \cap \text{Low})$

$= \dfrac{126}{175} + \dfrac{70}{175} - \dfrac{60}{175} = \dfrac{136}{175}$

13. a. $P(\text{A} \mid \text{B}) = \dfrac{P(\text{A} \cap \text{B})}{P(\text{B})} = \dfrac{0.20}{0.40} = \dfrac{1}{2}$

b. $P(\text{B} \mid \text{A}) = \dfrac{P(\text{B} \cap \text{A})}{P(\text{A})} = \dfrac{0.20}{0.45} = \dfrac{4}{9}$

14. $P(\text{scratched screen}|\text{def. ear pieces})$

$= \dfrac{P(\text{scratched screen} \cap \text{def. ear pieces})}{P(\text{def. ear pieces})}$

$= \dfrac{0.13}{0.19} = \dfrac{13}{19}$

15. $S = \{\text{BB, BG, GB, GG}\}$
Let E = {at least one girl} = {BG, GB, GG},
F = {at least one boy} = {BB, BG, GB}.

Thus $P(F \mid E) = \dfrac{n(E \cap F)}{n(E)} = \dfrac{2}{3}$.

16. S = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}
Let
E = {at least two girls}
= {BGG, GBG, GGB, GGG},
F = {at least one boy}
= {BBB, BBG, BGB, BGG, GBB, GBG, GGB},
G = {oldest is a girl}
= {GBB, GBG, GGB, GGG}.

a. $P(E \mid F) = \dfrac{n(E \cap F)}{n(F)} = \dfrac{3}{7}$

b. $P(E \mid G) = \dfrac{n(E \cap G)}{n(G)} = \dfrac{3}{4}$

17. S = {HHH, HHT, HTH, THH, THT, TTH, TTT}.
Let E = {exactly two tails}
= {HTT, THT, TTH},
F = {second toss is a tail}
= {HTH, HTT, TTH, TTT},
G = {second toss is a head}
= {HHH, HHT, THH, THT}.

a. $P(E \mid F) = \dfrac{n(E \cap F)}{n(F)} = \dfrac{2}{4} = \dfrac{1}{2}$

b. $P(E \mid G) = \dfrac{n(E \cap G)}{n(G)} = \dfrac{1}{4}$

18. S = {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}.
Let E = {four tails } = {TTTT}, F = {first toss is a tail} = {THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}.

Since $P(E \mid F) = \dfrac{n(E \cap F)}{n(F)} = \dfrac{1}{8}$, the corresponding odds are

$\dfrac{P(E \mid F)}{P(E' \mid F)} = \dfrac{1/8}{1-(1/8)} = \dfrac{1}{7}$; that is, 1 to 7.

19. $P(<4 \mid \text{odd}) = \dfrac{n(<4 \cap \text{odd})}{n(\text{odd})} = \dfrac{n(\{1,3\})}{n(\{1,3,5\})} = \dfrac{2}{3}$

20. Let S denote a spade (which is black). There are $\frac{52}{4} = 13$ spades in a deck of 52 cards. Let B denote a black card. There are $\frac{52}{2} = 26$ black cards.

$$P(S|B) = \frac{m(S \cap B)}{m(B)} = \frac{13}{26} = \frac{1}{2}.$$

21. *Method 1.* The usual sample space has 36 outcomes, where the event "two 1's" is {(1, 1)}. Note that $\{\text{at least one } 1\}' = \{\text{no 1's}\}$, and the event "no 1's" occurs in 5 · 5 = 25 ways. Thus

P(two 1's | at least one 1)

$$= \frac{n(\text{two 1's} \cap \text{at least one 1})}{n(\text{at least one 1})} = \frac{n(\{(1,1)\})}{36-25} = \frac{1}{11}$$

Method 2. From the usual sample space, we find that the reduced sample space for "at least one 1" (which has 11 outcomes) is {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (3, 1), (4, 1), (5, 1)}.

Thus P(two 1's | at least one 1) = $\frac{1}{11}$.

22. *Method 1.* The reduced sample space, having 6 outcomes, is {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}, where, in each pair, the outcome 5 on the red die is given first. Two pairs have a sum greater than 9, namely (5, 5) and (5, 6). Thus $P(\text{sum}>9|5 \text{ on red}) = \frac{2}{6} = \frac{1}{3}$.

Method 2. The usual sample space has 36 outcomes. Let E = {5 on red}. Then $n(E) = 6$. Let F = {sum > 9}. Then $n(E \cap F) = 2$, namely (red 5, green 5) and (red 5, green 6). Thus

$$P(F \mid E) = \frac{n(E \cap F)}{n(E)} = \frac{2}{6} = \frac{1}{3}.$$

23. The usual sample space consists of ordered pairs (R, G), where R = no. on red die and G = no. on green die. Now, n(green is even) = 6 · 3 = 18, because the red die can show any of six numbers and the green any of three: 2, 4, or 6. Also,

$n(\text{total of } 7 \cap \text{green even})$

$= n(\{(5, 2), (3, 4), (1, 6)\}) = 3$.

Thus

$P(\text{total of } 7|\text{green even})$

$$= \frac{n(\text{total of } 7 \cap \text{green even})}{n(\text{green even})}$$

$$= \frac{3}{18} = \frac{1}{6}.$$

24. The usual sample space S consists of 36 ordered pairs. Let E = {sum is 6} and F = {second toss is neither 2 nor 4}. Then $n(F) = 6 \cdot 4 = 24$ and $n(E \cap F) = n\{(5, 1), (3, 3), (1, 5)\} = 3$.

a. $P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{3}{24} = \frac{1}{8}$

b. $P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

25. The usual sample space consists of 36 ordered pairs. Let E = {total > 8} and F = {first toss > 2}. Then $n(F) = 4 \cdot 6 = 24$ and

$n(E \cap F)$

$= n(\{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\})$

$= 10$

Thus $P(E|F) = \frac{n(E \cap F)}{n(F)} = \frac{10}{24} = \frac{5}{12}$.

26. Let the sample space consist of ordered pairs (c, d), where c is T or H, and d is the number showing on the die. Let E = {tails shows} and F = {die shows odd number). Then $N(F) = 2 \cdot 3 = 6$ and $n(E \cap F) = 1 \cdot 3 = 3$. Thus

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{3}{6} = \frac{1}{2}.$$

27. $P(K \mid H) = \frac{n(K \cap H)}{n(H)} = \frac{1}{13}$

28. $P(H \mid F) = \frac{n(H \cap F)}{n(F)} = \frac{3}{12} = \frac{1}{4}$

29. Let E = {second card is not a face card} and F = {first card is a face card}.

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{12 \cdot \frac{51-11}{51}}{12} = \frac{40}{51}$$

30. **a.** $P(F_1 \cap F_2) = P(F_1)P(F_2 \mid F_1) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

b. $P(F_1 \cap F_2) = P(F_1)P(F_2 \mid F_1) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$

31. $P(K_1 \cap Q_2 \cap J_3)$
$= P(K_1)P(Q_2 \mid K_1)P(J_3 \mid (K_1 \cap Q_2))$
$= \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} = \frac{8}{16,575}$

32. $P(AS_1 \cap AH_2 \cap AD_2)$
$= P(AS_1)P\left(AH_2 \middle| AS_1\right)P\left(AD_2 \middle| (AS_1 \cap AH_2)\right)$
$= \frac{1}{52} \cdot \frac{1}{51} \cdot \frac{1}{50} = \frac{1}{132,600}.$

33. $P(J_1 \cap J_2 \cap J_3)$
$= P(J_1)P(J_2 \mid J_1)P(J_3 \mid (J_1 \cap J_2))$
$= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{1}{5525}$

34. Using a probability tree, we find that there are two possible paths such that the second card is a heart, namely, a heart followed by a heart, or a nonheart followed by a heart. Thus
$P(H_2) = P(H_1 \cap H_2) + P(H_1' \cap H_2)$
$= P(H_1)P(H_2 \mid H_1) + P(H_1')P(H_2 \mid H_1')$
$= \frac{13}{52} \cdot \frac{12}{51} + \frac{39}{52} \cdot \frac{13}{51} = \frac{1}{4}.$

35. Let J = {two jacks} and F = {first card face}. We have $J \cap F$ = {two jacks} = J and $P(J) = \frac{4}{52} \cdot \frac{3}{51}.$

Thus $P\left(J \middle| F\right) = \frac{P(J \cap F)}{P(F)} = \frac{\frac{4}{52} \cdot \frac{3}{51}}{\frac{12}{52}} = \frac{1}{51}.$

36. Using a probability tree, we find that there are two possible paths such that she will be on time, namely, she gets the call and she is on time, or she doesn't get the call and she is on time.
$P(T) = P(C \cap T) + P(C' \cap T)$
$= P(C)P(T \mid C) + P(C')P(T \mid C')$
$= (0.9)(0.9) + (0.1)(0.4) = 0.85$

37. **a.** $P(U) = P(F \cap U) + P(O \cap U) + P(N \cap U)$
$= P(F)P(U|F) + P(O)P(U|O) + P(N)P(U|N)$
$= (0.60)(0.45) + (0.30)(0.55) + (0.10)(0.35)$
$= 0.47 = \frac{47}{100}$

b. $P(F \mid U) = \dfrac{P(F \cap U)}{P(U)} = \dfrac{(0.60)(0.45)}{0.47} = \dfrac{27}{47}$

38. a. $P(\text{contact} \cap \text{purchase}) = P(\text{contact})P(\text{purchase} \mid \text{contact})$
$= (0.02)(0.014) = 0.00028$

b. $100{,}000(0.00028) = 28$

39. a. After the first draw, if the rabbit drawn is red, then 4 rabbits remain, 3 of which are yellow.
$P(\text{second is yellow} \mid \text{first is red}) = \dfrac{3}{4}$

b. After red rabbit is replaced, 5 rabbits remain, 3 of which are yellow.
$P(\text{second is yellow} \mid \text{first is red}) = \dfrac{3}{5}$

40. $P(G_2) = P(G_1 \cap G_2) + P(R_1 \cap G_2) = P(G_1)P(G_2 \mid G_1) + P(R_1)P(G_2 \mid R_1) = \dfrac{5}{7} \cdot \dfrac{3}{8} + \dfrac{2}{7} \cdot \dfrac{2}{8} = \dfrac{19}{56}$

41. $P(W) = P(\text{Box } 1 \cap \text{W}) + P(\text{Box } 2 \cap \text{W}) = P(\text{Box } 1)P(W \mid \text{Box } 1) + P(\text{Box } 2)P(W \mid \text{Box } 2) = \dfrac{1}{2} \cdot \dfrac{2}{5} + \dfrac{1}{2} \cdot \dfrac{2}{4} = \dfrac{9}{20}$

42. a. $P(W) = P(B1 \cap W) + P(B2 \cap W) + P(B3 \cap W)$
$= P(B1)P(W \mid B1) + P(B2)P(W \mid B2) + P(B3)P(W \mid B3)$
$= \dfrac{1}{3} \cdot \dfrac{3}{5} + \dfrac{1}{3} \cdot \dfrac{4}{7} + \dfrac{1}{3} \cdot \dfrac{2}{6} = \dfrac{158}{315}$

b. $P(R) = P(B1 \cap R) + P(B2 \cap R) + P(B3 \cap R)$
$= P(B1)P(R \mid B1) + P(B2)P(R \mid B2) + P(B3)P(R \mid B3)$
$= \dfrac{1}{3} \cdot \dfrac{2}{5} + \dfrac{1}{3} \cdot \dfrac{3}{7} + \dfrac{1}{3} \cdot \dfrac{2}{6} = \dfrac{122}{315}$

c. $P(G) = P(B3 \cap G) = P(B3)P(G \mid B3) = \dfrac{1}{3} \cdot \dfrac{1}{3} = \dfrac{1}{9}$

43. $P(W_2) = P(B1 \cap G_1 \cap W_2) + P(B1 \cap R_1 \cap W_2) + P(B2 \cap W_1 \cap W_2)$
$= P(B1)P\left(G_1 \middle| B1\right)P\left(W_2 \middle| (G_1 \cap B1)\right) + P(B1)P\left(R_1 \middle| B1\right)P\left(W_2 \middle| (R_1 \cap B1)\right) + P(B2)P\left(W_1 \middle| B2\right)P\left(W_2 \middle| (W_1 \cap B2)\right)$
$= \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{3} + \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{3} + \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{3} = \dfrac{1}{4}$

44. $P(D_1 \cap D_2 \cap D_3 \cap D_4) = P(D_1)P\left(D_2 \middle| D_1\right)P\left(D_3 \middle| (D_1 \cap D_2)\right)P\left(D_4 \middle| (D_1 \cap D_2 \cap D_3)\right)$
$= \dfrac{5}{10} \cdot \dfrac{4}{9} \cdot \dfrac{3}{8} \cdot \dfrac{2}{7} = \dfrac{1}{42}$

45. $P(\text{Und.}) = P(\text{YC} \cap \text{Und.}) + P(\text{HT} \cap \text{Und.})$
$$= P(\text{YC})P(\text{Und.}|\text{YC}) + P(\text{HT})P(\text{Und}|\text{HT})$$
$$= \frac{36{,}000}{96{,}000} \cdot \frac{2}{100} + \frac{60{,}000}{96{,}000} \cdot \frac{1}{100}$$
$$= \frac{11}{800}$$
$$\approx 1.4\%$$

46. $P(5000) = P(B1 \cap 5000) + P(B2 \cap 5000) + P(B3 \cap 5000)$
$$= P(B1)P(5000|B1) + P(B2)P(5000|B2) + P(B3)P(5000|B3)$$
$$= \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{2}{8} + \frac{1}{3} \cdot \frac{1}{6} = \frac{11}{36}$$

47. $P(\text{Def}) = P(\text{A} \cap \text{Def}) + P(\text{B} \cap \text{Def}) + P(\text{C} \cap \text{Def})$
$$= P(\text{A})P(\text{Def} \mid \text{A}) + P(\text{B})P(\text{Def} \mid \text{B}) + P(\text{C})P(\text{Def} \mid \text{C})$$
$$= (0.10)(0.06) + (0.20)(0.04) + (0.70)(0.05) = 0.049$$

48. $P(\text{Def}) = P(\text{A} \cap \text{Def}) + P(\text{B} \cap \text{Def}) + P(\text{C} \cap \text{Def}) + P(\text{D} \cap \text{Def})$
$$= P(\text{A})P(\text{Def} \mid \text{A}) + P(\text{B})P(\text{Def} \mid \text{B}) + P(\text{C})P(\text{Def} \mid \text{C}) + P(\text{D})P(\text{Def} \mid \text{D})$$
$$= (0.30)(0.06) + (0.20)(0.03) + (0.35)(0.02) + (0.15)(0.05)$$
$$= 0.0385$$

49. a. $P(D \cap V) = P(D)P(V \mid D) = (0.40)(0.15) = 0.06$

b. $P(V) = P(D \cap V) + P(R \cap V) + P(I \cap V)$
$$= P(D)P(V \mid D) + P(R)P(V \mid R) + P(I)P(V \mid I)$$
$$= (0.40)(0.15) + (0.35)(0.20) + (0.25)(0.10)$$
$$= 0.155$$

50. Because Ellie and Richard were not hired, the number of sample points in the reduced sample space is ${}_6C_4 = 15$, of which Allison, Lesley, Tom, and Bronwen form one sample point. Thus

$P(\text{Allison, Lesley, Tom, and Bronwen were hired}) = \frac{1}{15}.$

51. $P(3 \text{ Fem}|\text{at least one Fem})$
$$= \frac{P(3 \text{ Fem} \cap \text{at least one Fem})}{P(\text{at least one Fem})}$$
$$= \frac{P(3 \text{ Fem})}{1 - P(\text{no Fem})} = \frac{\frac{{}_6C_3}{{}_{11}C_3}}{1 - \frac{{}_5C_3}{{}_{11}C_3}} = \frac{\frac{4}{33}}{1 - \frac{2}{33}} = \frac{4}{31}$$

Problems 8.6

1. a. $P(E \cap F) = P(E)P(F) = \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4}$

b. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$$= \frac{1}{3} + \frac{3}{4} - \frac{1}{4} = \frac{5}{6}$$

c. $P(E\mid F)=P(E)=\frac{1}{3}$

d. $P(E'\mid F)=1-P(E\mid F)=1-\frac{1}{3}=\frac{2}{3}$

e. $P(E\cap F')=P(E)P(F')=\frac{1}{3}\cdot\frac{1}{4}=\frac{1}{12}$

f. $P(E\cup F')=P(E)+P(F')-P(E\cap F')$
$=\frac{1}{3}+\frac{1}{4}-\frac{1}{12}=\frac{1}{2}$

g. $P(E\mid F')=\frac{P(E\cap F')}{P(F')}=\frac{1/12}{1/4}=\frac{1}{3}$

2. a. $P(E\cap F)=P(E)P(F)=(0.1)(0.3)=0.03$

b. $P(F\cap G)=P(F)P(G)=(0.3)(0.6)=0.18$

c. $P(E\cap F\cap G)=P(E)P(F)P(G)$
$=(0.1)(0.3)(0.6)=0.018$

d. $P\left(E\middle|(F\cap G)\right)=\frac{P(E\cap F\cap G)}{P(F\cap G)}$
$=\frac{0.018}{0.18}=0.1$

e. $P(E'\cap F\cap G')=P(E')P(F)P(G')$
$=(0.9)(0.3)(0.4)=0.108$

3. $P(E\cap F)=P(E)P(F)$,
$\frac{1}{9}=\frac{2}{7}\cdot P(F)$ so $P(F)=\frac{1}{9}\cdot\frac{7}{2}=\frac{7}{18}$

4. $P(E')=P(E'\mid F')=\frac{1}{4}$,
so $P(E)=1-P(E')=1-\frac{1}{4}=\frac{3}{4}$.

5. $P(E)P(F)=\frac{3}{4}\cdot\frac{8}{9}=\frac{2}{3}=P(E\cap F)$
Since $P(E)P(F)=P(E\cap F)$, events E and F are independent.

6. $P(E)P(F)=(0.28)(0.15)=0.042\neq P(E\cap F)$,
so E and F are dependent events.

7. Let F = {full service} and I = {increase in value}.
$P(F)=\frac{400}{600}=\frac{2}{3}$
and $P(F\mid I)=\frac{n(F\cap I)}{n(I)}=\frac{320}{480}=\frac{2}{3}$
Since $P(F\mid I)=P(F)$, events F and I are independent.

8. Let M = {male} and C = {cruncher}.
$P(M)=\frac{130}{175}=\frac{26}{35}$ and
$P(M\mid C)=\frac{n(M\cap C)}{n(C)}=\frac{55}{80}=\frac{11}{16}$
Since $P(M\mid C)\neq P(M)$, events M and C are dependent.

9. Let S be the usual sample space consisting of ordered pairs of the form (R, G), where the first component of each pair represents the number showing on the red die, and the second component represents the number on the green die. Then $n(S)=6\cdot 6=36.$ For E, any number greater than three can occur on the red die, and any number on the green die. Thus $n(E)=3\cdot 6=18.$ For F we have F = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}, so $n(F)=6$.
Also, $E\cap F=\{(4,3),(5,2),(6,1)\}$, so $n(E\cap F)=3.$ Thus $P(E)P(F)=\frac{18}{36}\cdot\frac{6}{36}=\frac{1}{12}$
and $P(E\cap F)=\frac{3}{36}=\frac{1}{12}$. Since $P(E)P(F)=P(E\cap F)$, events E and F are independent.

10. $P(E)=\frac{26}{52}=\frac{1}{2}$
$P(F)=\frac{12}{52}=\frac{3}{13}$, and $P(E\cap F)=\frac{6}{52}=\frac{3}{26}$.
Because $P(E)P(F)=\frac{1}{2}\cdot\frac{3}{13}=\frac{3}{26}=P(E\cap F)$, events E and F are independent.

11. $S = \{\text{HH, HT, TH, TT}\}$,
$E = \{\text{HT, TH, TT}\}$,
$F = \{\text{HT, TH}\}$, and $E \cap F = \{\text{HT}, \text{TH}\}$.

Thus $P(E) = \frac{3}{4}$

$P(F) = \frac{2}{4} = \frac{1}{2}$, and

$P(E \cap F) = \frac{2}{4} = \frac{1}{2}$. We have

$P(E)P(F) = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8} \neq P(E \cap F)$, so events E and F are dependent.

12. $S = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$ and $n(S) = 8$.
$E = \{\text{HTT, THT, TTH, TTT}\}$ and $n(E) = 4$.
$F = \{\text{HHT, HTH, THH, HTT, THT, TTH}\}$ and $n(F) = 6$.
$E \cap F = \{\text{HTT}, \text{THT}, \text{TTH}\}$ and $n(E \cap F) = 3$.

Thus $P(E)P(F) = \frac{4}{8} \cdot \frac{6}{8} = \frac{3}{8} = P(E \cap F)$, so E and F are independent.

13. Let S be the set of ordered pairs whose first (*second*) component represents the number on the first (*second*) chip. Then $n(S) = 7 \cdot 7 = 49$, $n(E) = 1 \cdot 7 = 7$, and $n(F) = 7 \cdot 1 = 7$. For G, if the first chip is 1, 3, 5 or 7, then the second chip must be 2, 4 or 6; if the first chip is 2, 4 or 6, the second must be 1, 3, 5 or 7. Thus $n(G) = 4 \cdot 3 + 3 \cdot 4 = 24$.

a. $E \cap F = \{(3,3)\}$, so $P(E \cap F) = \frac{1}{49}$. Since

$P(E)P(F) = \frac{7}{49} \cdot \frac{7}{49} = \frac{1}{49} = P(E \cap F)$,

events E and F are independent.

b. $E \cap G = \{(3,2), (3,4), (3,6)\}$,

so $P(E \cap G) = \frac{3}{49}$. Since

$P(E)P(G) = \frac{7}{49} \cdot \frac{24}{49} = \frac{24}{343} \neq P(E \cap G)$,

events E and G are dependent.

c. $F \cap G = \{(2,3), (4,3), (6,3)\}$

so $P(F \cap G) = \frac{3}{49}$.

Since

$P(F)P(G) = \frac{7}{49} \cdot \frac{24}{49} = \frac{24}{343} \neq P(F \cap G)$.

Events F and G are dependent.

d. $E \cap F \cap G = \varnothing$, so $P(E \cap F \cap G) = 0$.
However,
$P(E)P(F)P(G) \neq 0 = P(E \cap F \cap G)$,
so events E, F and G are not independent.

14. a. $E = \{\text{pairs of even numbers}\}$
$F = \{\text{pairs of odd numbers}\}$
$E \cap F = \varnothing$, so E and F are mutually exclusive.

b. $P(E) = P(F) = \frac{3}{6} \cdot \frac{2}{5} = \frac{1}{5}$

$P(E \cap F) = 0$

$P(E)P(F) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25} \neq P(E \cap F)$

Thus E and F are not independent.

15. $P(E \cap F) = P(E)P(F \mid E)$, thus

$P(E) = \frac{P(E \cap F)}{P(F \mid E)} = \frac{0.3}{0.4} = 0.75$

Since $P(E) = 0.75 \neq 0.5 = P(E \mid F)$, E and F are dependent.

16. $P(E \cap F) = P(F)P(E \mid F)$,

thus $P(F) = \frac{P(E \cap F)}{P(E \mid F)} = \frac{\frac{5}{9}}{\frac{2}{3}} = \frac{5}{6}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$, so

$P(E) = P(E \cup F) - P(F) + P(E \cap F)$

$= \frac{17}{18} - \frac{5}{6} + \frac{5}{9} = \frac{2}{3}$

Since $P(E) = \frac{2}{3} = P(E \mid F)$, events E and F are independent.

17. Let $E = \{\text{red } 4\}$ and $F = \{\text{green} > 4\}$. Assume E and F are independent.

$P(E \cap F) = P(E)P(F) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$

18. E_i = {2 or 3 shows on *i*th roll}, where $i = 1, 2, 3$. Assume the E_i's are independent.

$$P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3) = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{27}$$

19. Let F = {first person attends regularly} and S = {second person attends regularly}.
Then $P(F \cap S) = P(F)P(S) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$.

20. $P(\text{double on any throw}) = \frac{6}{36} = \frac{1}{6}$

Assume that the throws are independent.
P(double on all three throws) = *P*(double on 1st) · *P*(double on 2nd) · *P*(double on 3rd)
$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$.

21. Because of replacement, assume the cards selected on the draws are independent events.
P(ace, then face card, then spade) = *P*(ace) · *P*(face card) · *P*(spade)
$= \frac{4}{52} \cdot \frac{12}{52} \cdot \frac{13}{52} = \frac{3}{676}$

22. Assume the outcomes on the rolls are independent events.

a. $P(>4, >4, >4, >4, >4, >4, >4) = \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{2187}$

b. $P(<4, <4, <4, <4, <4, <4, <4) = \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{128}$

23. **a.** *P*(Bill gets A $\cap$ Jim gets A $\cap$ Linda gets A)
= *P*(Bill gets A) · *P*(Jim gets A) · *P*(Linda gets A)
$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{4}{5} = \frac{3}{10}$.

b. *P*(Bill no A $\cap$ Jim no A $\cap$ Linda no A)
= *P*(Bill no A) · *P*(Jim no A) · *P*(Linda no A)
$= \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{40}$

c. *P*(Bill no A $\cap$ Jim no A $\cap$ Linda gets A)
= *P*(Bill no A) · *P*(Jim no A) · *P*(Linda gets A)
$= \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{4}{5} = \frac{1}{10}$

24. Assume independence of rolls.
P(at least one 1) = 1 – *P*(no 1's) $= 1 - \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) = \frac{671}{1296}$

25. Let A = {A survives 15 more years},
B = {B survives 15 more years}.

a. $P(A \cap B) = P(A)P(B) = \frac{2}{3} \cdot \frac{3}{5} = \frac{2}{5}$

b. $P(A' \cap B) = P(A')P(B) = \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$

c. $A \cap B'$ and $A' \cap B$ are mutually exclusive.

$P[(A \cap B') \cup (A' \cap B)] = P(A)P(B') + P(A')P(B) = \frac{2}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{3}{5} = \frac{7}{15}$

d. P(at least one survives) = P(exactly one survives) + P(both survive) $= \frac{7}{15} + \frac{2}{5} = \frac{13}{15}$.

e. P(neither survives) = 1 – P(at least one survives) $= 1 - \frac{13}{15} = \frac{2}{15}$.

26. Assume that drawing a particular size of paper and a particular size of envelope are independent events.
$P(\text{paper A} \cap \text{envelope A}) + P(\text{paper B} \cap \text{envelope B}) = (0.63)(0.57) + (0.37)(0.43) \approx 0.52$

27. Assume the colors selected on the draws are independent events.

a. $P(W_1 \cap G_2) = P(W_1)P(G_2) = \frac{7}{18} \cdot \frac{6}{18} = \frac{7}{54}$

b. $P[(R_1 \cap W_2) \cup (W_1 \cap R_2)] = P(R_1)P(W_2) + P(W_1)P(R_2) = \frac{5}{18} \cdot \frac{7}{18} + \frac{7}{18} \cdot \frac{5}{18} = \frac{35}{162}$

28. Assume the rolls are independent.

$P(\text{7 on a roll}) = P\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} = \frac{6}{36} = \frac{1}{6}$

$P(\text{12 on a roll}) = P\{(6, 6)\} = \frac{1}{36}$

$P(\text{7 on one roll and 12 on the other}) = \frac{1}{6} \cdot \frac{1}{36} + \frac{1}{36} \cdot \frac{1}{6} = \frac{1}{108}$

29. Assume that the selections are independent.

$P(\text{both red} \cup \text{both white} \cup \text{both blue} \cup \text{both green}) = \frac{3}{11} \cdot \frac{3}{11} + \frac{2}{11} \cdot \frac{2}{11} + \frac{4}{11} \cdot \frac{4}{11} + \frac{2}{11} \cdot \frac{2}{11} = \frac{3}{11}$

30. Assume the throws are independent. For a particular number,

$P(\text{particular number on three throws}) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^3$.

Since the particular number can be any of 6 numbers,

$P(\text{same number in 3 throws}) = 6\left(\frac{1}{6}\right)^3 = \frac{1}{36}$.

31. Assume that the draws are independent.
P(particular 1st ticket $\cap$ particular 2nd ticket)
$= \frac{1}{20} \cdot \frac{1}{20} = \frac{1}{400}$
P(sum is 35) = P\{(20, 15), (19, 16), (18, 17), (17, 18), (16, 19), (15, 20)\}
$= 6\left(\frac{1}{400}\right) = \frac{3}{200}$

32. a. $P(\{TT33\}) = P(\text{T on 1st coin})\, P(\text{T on 2nd coin})\, P(\text{3 on 1st die})\, P(\text{3 on 2nd die})$
$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{144}$

b. P(two heads, one 4 and one 6)
= P(H on 1st coin) P(H on 2nd coin) P(4 on 1st die) P(6 on 2nd die)
+ P(H on 1st coin)P(H on 2nd coin) P(6 on 1st die) P(4 on 2nd die)
$= \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{6}\right) \cdot 2 = \frac{1}{72}$

33. a. $\frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$

b. To get exactly one even, there are ${}_3C_1 = 3$ ways.
P(one even and two odd) = 3[P(even 1st spin) · P(odd 2nd spin)·P(odd 3rd spin)]
$= 3\left(\frac{6}{12} \cdot \frac{6}{12} \cdot \frac{6}{12}\right) = \frac{3}{8}$.

34. a. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{26} = \frac{1}{416}$

b. $\frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{2197}$

c. The queen, spade, and black ace can be drawn in any order, so there are 3! = 6 orders, thus
$6 \cdot \frac{2}{52} \cdot \frac{13}{52} \cdot \frac{2}{52} = \frac{3}{1352}$.

d. The ace can come first, second, or third, so $3 \cdot \frac{4}{52} \cdot \frac{48}{52} \cdot \frac{48}{52} = \frac{432}{2197}$.

35. a. The number of ways of getting exactly four correct answers out of five is ${}_5C_4 = 5$. Each of these ways has a probability of $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{1024}$. Thus
$P(\text{exactly 4 correct}) = 5 \cdot \frac{3}{1024} = \frac{15}{1024}$.

b. P(at least 4 correct) = P(exactly 4) + P(exactly 5)
$= \frac{15}{1024} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$

c. The number of ways of getting exactly three correct answers out of five is $_5C_3 = 10$. Each of these ways has a probability of $\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{9}{1024}$, so

$P(\text{exactly 3 correct}) = 10\cdot\frac{9}{1024} = \frac{45}{512}$. Thus

$P(\text{3 or more correct}) = P(\text{exactly 3}) + P(\text{at least 4})$

$= \frac{45}{512} + \frac{1}{64} = \frac{53}{512}$.

36. a. $P(\text{none hit}) = (0.5)(0.6)(0.3) = 0.09$

b. $P(\text{only Linda hits}) = (0.5)(0.6)(0.7) = 0.21$

c. $P(\text{exactly one hits target}) = P(\text{only Bill}) + P(\text{only Jim}) + P(\text{only Linda})$
$= (0.5)(0.6)(0.3) + (0.5)(0.4)(0.3) + (0.5)(0.6)(0.7) = 0.36$

d. $P(\text{exactly 2}) = P(\text{not Bill}) + P(\text{not Jim}) + P(\text{not Linda})$
$= (0.5)(0.4)(0.7) + (0.5)(0.6)(0.7) + (0.5)(0.4)(0.3) = 0.41$

e. $P(\text{all hit}) = (0.5)(0.4)(0.7) = 0.14$

37. A wrong majority decision can occur in one of two mutually exclusive ways: exactly two wrong recommendations, or three wrong recommendations. Exactly two wrong recommendations can occur in $_3C_2 = 3$ mutually exclusive ways. Thus
$P(\text{wrong majority decision})$
$= [(0.04)(0.05)(0.9) + (0.04)(0.95)(0.1) + (0.96)(0.05)(0.1)] + (0.04)(0.05)(0.1)$
$= 0.0106$.

Problems 8.7

1. $$P(E\mid D) = \frac{P(E)P(D\mid E)}{P(E)P(D\mid E)+P(F)P(D\mid F)} = \frac{\frac{2}{5}\cdot\frac{1}{10}}{\frac{2}{5}\cdot\frac{1}{10}+\frac{3}{5}\cdot\frac{1}{5}} = \frac{1}{4}$$

For the second part, $P(D'\mid F) = 1 - P(D\mid F) = 1-\frac{1}{5} = \frac{4}{5}$, and

$P(D'\mid E) = 1 - P(D\mid E) = 1 - \frac{1}{10} = \frac{9}{10}$. Then

$$P(F\mid D') = \frac{P(F)P(D'\mid F)}{P(E)P(D'\mid E)+P(F)P(D'\mid F)} = \frac{\frac{3}{5}\cdot\frac{4}{5}}{\frac{2}{5}\cdot\frac{9}{10}+\frac{3}{5}\cdot\frac{4}{5}} = \frac{4}{7}.$$

2. $$P(E_1\mid S) = \frac{P(E_1)P(S\mid E_1)}{P(E_1)P(S\mid E_1)+P(E_2)P(S\mid E_2)+P(E_3)P(S\mid E_3)} = \frac{\frac{1}{5}\cdot\frac{2}{5}}{\frac{1}{5}\cdot\frac{2}{5}+\frac{3}{10}\cdot\frac{7}{10}+\frac{1}{2}\cdot\frac{1}{2}} = \frac{4}{27}.$$

$$P(E_3\mid S') = \frac{P(E_3)P(S'\mid E_3)}{P(E_1)P(S'\mid E_1)+P(E_2)P(S'\mid E_2)+P(E_3)P(S'\mid E_3)} = \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{5}\cdot\frac{3}{5}+\frac{3}{10}\cdot\frac{3}{10}+\frac{1}{2}\cdot\frac{1}{2}} = \frac{25}{46}.$$

3. $D = \{\text{is Democrat}\}$,
$R = \{\text{is Republican}\}$,
$I = \{\text{is Independent}\}$,
$V = \{\text{voted}\}$.

$$P(D \mid V) = \frac{P(D)P(V \mid D)}{P(D)P(V \mid D) + P(R)P(V \mid R) + P(I)P(V \mid I)}$$
$$= \frac{(0.42)(0.45)}{(0.42)(0.45) + (0.33)(0.37) + (0.25)(0.35)}$$
$$= \frac{945}{1993} \approx 47\%$$

4. $D = \{\text{tire is domestic}\}$
$I = \{\text{tire is imported}\}$
$S = \{\text{tire is all-season}\}$

$P(D) = \frac{2000}{3000} = \frac{2}{3}$ and $P(I) = \frac{1000}{3000} = \frac{1}{3}$.

Note: $40\% = \frac{2}{5}$ and $10\% = \frac{1}{10}$.

$$P(I|S) = \frac{P(I)P(S \mid I)}{P(I)P(S \mid I) + P(D)P(S \mid D)}$$
$$= \frac{\frac{1}{3} \cdot \frac{1}{10}}{\frac{1}{3} \cdot \frac{1}{10} + \frac{2}{3} \cdot \frac{2}{5}} = \frac{1}{9}$$

5. $D = \{\text{has the disease}\}$
$D' = \{\text{does not have the disease}\}$
$R = \{\text{positive reaction}\}$
$N = \{\text{negative reaction}\} = R'$

a. $$P(D \mid R) = \frac{P(D)P(R \mid D)}{P(D)P(R \mid D) + P(D')P(R \mid D')} = \frac{(0.03)(0.86)}{(0.03)(0.86) + (0.97)(0.07)} = \frac{258}{937} \approx 0.275$$

b. $$P(D \mid N) = \frac{P(D)P(N \mid D)}{P(D)P(N \mid D) + P(D')P(N \mid D')} = \frac{(0.03)(0.14)}{(0.03)(0.14) + (0.97)(0.93)} = \frac{14}{3021} \approx 0.005$$

6. $I = \{\text{increase in earnings}\}$
$D = \{\text{declare a dividend}\}$

Note: $60\% = \frac{3}{5}$ and $10\% = \frac{1}{10}$.

$$P(I \mid D) = \frac{P(I)P(D \mid I)}{P(I)P(D \mid I) + P(I')P(D \mid I')} = \frac{\frac{1}{3} \cdot \frac{3}{5}}{\frac{1}{3} \cdot \frac{3}{5} + \frac{2}{3} \cdot \frac{1}{10}} = \frac{3}{4} = 75\%$$

7. $B_1 = \{\text{first bag selected}\}$
$B_2 = \{\text{second bag selected}\}$
$R = \{\text{red jelly bean drawn}\}$

$P(B_1) = P(B_2) = \frac{1}{2}$.

$$P(B_1 \mid R) = \frac{P(B_1)P(R \mid B_1)}{P(B_1)P(R \mid B_1) + P(B_2)P(R \mid B_2)} = \frac{\frac{1}{2}\cdot\frac{4}{6}}{\frac{1}{2}\cdot\frac{4}{6}+\frac{1}{2}\cdot\frac{2}{5}} = \frac{5}{8}.$$

8. $B_1 = \{\text{Bowl I selected}\}$
$B_2 = \{\text{Bowl II selected}\}$
$B_3 = \{\text{Bowl III selected}\}$
$R = \{\text{red ball selected}\}$

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

$$P(B_2 \mid R) = \frac{P(B_2)P(R \mid B_2)}{P(B_1)P(R \mid B_1) + P(B_2)P(R \mid B_2) + P(B_3)P(R \mid B_3)} = \frac{\frac{1}{3}\cdot\frac{1}{6}}{\frac{1}{3}\cdot\frac{3}{10}+\frac{1}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{3}{5}} = \frac{5}{32}$$

9. $A = \{\text{unit from line A}\}$
$B = \{\text{unit from line B}\}$
$D = \{\text{defective unit}\}$.

$P(A) = \frac{300}{800} = \frac{3}{8}$

$P(B) = \frac{500}{800} = \frac{5}{8}$

$$P(A \mid D) = \frac{P(A)P(D \mid A)}{P(A)P(D \mid A) + P(B)P(D \mid B)} = \frac{\frac{3}{8}\cdot\frac{2}{100}}{\frac{3}{8}\cdot\frac{2}{100}+\frac{5}{8}\cdot\frac{5}{100}} = \frac{6}{31}$$

10. $A = \{\text{unit from line A}\}$
$B = \{\text{unit from line B}\}$
$C = \{\text{unit from line C}\}$
$D = \{\text{unit from line D}\}$
$F = \{\text{defective unit}\}$

a. $$P(A \mid F) = \frac{P(A)P(F \mid A)}{P(A)P(F \mid A) + P(B)P(F \mid B) + P(C)P(F \mid C) + P(D)P(F \mid D)}$$
$$= \frac{(0.35)(0.02)}{(0.35)(0.02) + (0.20)(0.05) + (0.30)(0.03) + (0.15)(0.04)} = \frac{7}{32}$$

Parts (b), (c), and (d) are similarly determined.

b. $\frac{10}{32} = \frac{5}{16}$

c. $\frac{9}{32}$

d. $\frac{6}{32} = \frac{3}{16}$

11. $C = \{\text{call made}\}$
$T = \{\text{on time for meeting}\}$

$$P(C \mid T) = \frac{P(C)P(T \mid C)}{P(C)P(T \mid C) + P(C')P(T \mid C')}$$

$$= \frac{(0.95)(0.9)}{(0.95)(0.9) + (0.05)(0.75)} = \frac{114}{119} \approx 0.958$$

12. $J_D = \{\text{jar with dark chocolate only selected}\}$
$J_M = \{\text{jar with dark and milk chocolates selected}\}$
$D = \{\text{dark chocolate selected}\}$

$$P(J_D) = P(J_M) = \frac{1}{2}$$

$$P(J_D \mid D) = \frac{P(J_D)P(D \mid J_D)}{P(J_D)P(D \mid J_D) + P(J_M)P(D \mid J_M)} = \frac{\frac{1}{2} \cdot \frac{50}{50}}{\frac{1}{2} \cdot \frac{50}{50} + \frac{1}{2} \cdot \frac{20}{50}} = \frac{5}{7}$$

13. $W = \{\text{walking reported}\}$
$B = \{\text{cycling reported}\}$
$R = \{\text{running reported}\}$
$C = \{\text{completed requirement}\}$

$$P(W \mid C) = \frac{P(W)P(C \mid W)}{P(W)P(C \mid W) + P(B)P(C \mid B) + P(R)P(C \mid R)} = \frac{\frac{1}{3} \cdot \frac{9}{10}}{\frac{1}{3} \cdot \frac{9}{10} + \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{1}{3}} = \frac{27}{62} \approx 43.5\%$$

43.5% would be expected to report walking.

14. $C = \{\text{charges battery}\}$
$S = \{\text{car starts}\}$

$$P(C' \mid S') = \frac{P(C')P(S' \mid C')}{P(C')P(S' \mid C') + P(C)P(S' \mid C)} = \frac{\frac{1}{10} \cdot \frac{4}{5}}{\frac{1}{10} \cdot \frac{4}{5} + \frac{9}{10} \cdot \frac{1}{8}} = \frac{32}{77} \approx 0.416$$

15. $J = \{\text{had Japanese-made car}\}$
$E = \{\text{had European-made car}\}$
$A = \{\text{had American-made car}\}$
$B = \{\text{buy same make again}\}$

$$P(J \mid B) = \frac{P(J)P(B \mid J)}{P(J)P(B \mid J) + P(E)P(B \mid E) + P(A)P(B \mid A)} = \frac{\frac{3}{5} \cdot \frac{85}{100}}{\frac{3}{5} \cdot \frac{85}{100} + \frac{1}{10} \cdot \frac{50}{100} + \frac{3}{10} \cdot \frac{40}{100}} = \frac{3}{4}$$

16. $D = \{\text{dalhousium is present}\}$
$P = \{\text{positive test}\}$
$N = \{\text{negative test}\} = P'$

a. $$P(D \mid P) = \frac{P(D)P(P \mid D)}{P(D)P(P \mid D) + P(D')P(P \mid D')} = \frac{(0.005)(0.80)}{(0.005)(0.80) + (0.995)(0.15)} = \frac{400}{15{,}325} \approx 0.0261$$

b. $$P(D \mid N) = \frac{P(D)P(N \mid D)}{P(D)P(N \mid D) + P(D')P(N \mid D')} = \frac{(0.005)(0.20)}{(0.005)(0.20) + (0.995)(0.85)} = \frac{100}{84{,}675} \approx 0.0012$$

17. $P = \{\text{pass the exam}\}$
$A = \{\text{answer every question}\}$

$$P(A \mid P) = \frac{P(A)P(P \mid A)}{P(A)P(P \mid A) + P(A')P(P \mid A')} = \frac{(0.75)(0.8)}{(0.75)(0.8) + (0.25)(0.50)} = \frac{24}{29} \approx 0.828$$

18. $P = \{\text{predicted smoking}\}$
$S = \{\text{smoking now}\}$

$$P(P \mid S') = \frac{P(P)P(S' \mid P)}{P(P)P(S' \mid P) + P(P')P(S' \mid P')} = \frac{(0.50)(0.8)}{(0.50)(0.8) + (0.50)(0.95)} = \frac{16}{35} \approx 46\%$$

19. $S = \{\text{signals sent}\}$
$D = \{\text{signals detected}\}$

$$P(S \mid D) = \frac{P(S)P(D \mid S)}{P(S)P(D \mid S) + P(S')P(D \mid S')} = \frac{\frac{2}{5} \cdot \frac{3}{5}}{\frac{2}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{1}{10}} = \frac{4}{5}$$

20. $A_M = \{\text{A average at midterm}\}$
$A = \{\text{A for course}\}$

$$P(A'_M \mid A) = \frac{P(A'_M)P(A \mid A'_M)}{P(A'_M)P(A \mid A'_M) + P(A_M)P(A \mid A_M)} = \frac{(0.4)(0.6)}{(0.4)(0.6) + (0.6)(0.7)} = \frac{4}{11} \approx 0.364$$

21. $S = \{\text{movie is a success}\}$
$U = \{\text{“Two Thumbs Up”}\}$

$$P(S \mid U) = \frac{P(S)P(U \mid S)}{P(S)P(U \mid S) + P(S')P(U \mid S')} = \frac{\frac{8}{10} \cdot \frac{70}{100}}{\frac{8}{10} \cdot \frac{70}{100} + \frac{2}{10} \cdot \frac{20}{100}} = \frac{14}{15} \approx 0.933$$

22. $G_1 = \{\text{green ball drawn from Bowl 1}\}$
$R_1 = \{\text{red ball drawn from Bowl 1}\}$
$G_2 = \{\text{green ball drawn from Bowl 2}\}$

$$P(G_1 \mid G_2) = \frac{P(G_1)P(G_2 \mid G_1)}{P(G_1)P(G_2 \mid G_1) + P(R_1)P(G_2 \mid R_1)} = \frac{\frac{5}{9} \cdot \frac{4}{8}}{\frac{5}{9} \cdot \frac{4}{8} + \frac{4}{9} \cdot \frac{3}{8}} = \frac{5}{8}$$

23. $S = \{\text{is substandard request}\}$
$C = \{\text{is considered substandard request by Blackwell}\}$

a. $P(C) = P(S)P(C \mid S) + P(S')P(C \mid S') = (0.25)(0.85) + (0.75)(0.10) = 0.2875 = \dfrac{23}{80}$

b. $P(S \mid C) = \dfrac{P(S)P(C \mid S)}{P(S)P(C \mid S) + P(S')P(C \mid S')} = \dfrac{(0.25)(0.85)}{0.2875} = \dfrac{17}{23}$

c.
$$\begin{aligned} P(\text{Error}) &= P(C' \cap S) + P(C \cap S') \\ &= P(S)P(C' \mid S) + P(S')P(C \mid S') \\ &= (0.25)(0.15) + (0.75)(0.10) = 0.1125 = \frac{9}{80} \end{aligned}$$

24. I = {first chest selected}
II = {second chest selected}
III = {third chest selected}
G = {gold coin found}.
For the coin in the other drawer to be silver, we want the probability that the third chest was selected given that a gold coin was found.

$$P(III \mid G) = \frac{P(III)P(G \mid III)}{P(I)P(G \mid I) + P(II)P(G \mid II) + P(III)P(G \mid III)} = \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot\frac{1}{2}} = \frac{1}{3}$$

25. a. $$P(L \mid E) = \frac{P(L)P(E \mid L)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.25)(0.49)}{(0.25)(0.49) + (0.25)(0.64) + (0.5)(0.81)} \approx 0.18$$

b. $$P(M \mid E) = \frac{P(M)P(E \mid M)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.25)(0.64)}{(0.25)(0.49) + (0.25)(0.64) + (0.5)(0.81)} \approx 0.23$$

c. $$P(H \mid E) = \frac{P(H)P(E \mid H)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.5)(0.81)}{(0.25)(0.49) + (0.25)(0.64) + (0.5)(0.81)} \approx 0.59$$

d. High quality

26. a. (a) $$P(L \mid E) = \frac{P(L)P(E \mid L)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.25)(0.44)}{(0.25)(0.44) + (0.25)(0.32) + (0.5)(0.18)} \approx 0.39$$

(b) $$P(M \mid E) = \frac{P(M)P(E \mid M)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.25)(0.32)}{(0.25)(0.44) + (0.25)(0.32) + (0.5)(0.18)} \approx 0.29$$

(c) $$P(H \mid E) = \frac{P(H)P(E \mid H)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.5)(0.18)}{(0.25)(0.44) + (0.25)(0.32) + (0.5)(0.18)} \approx 0.32.$$

(d) Low quality

b. (a) $$P(L \mid E) = \frac{P(L)P(E \mid L)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$$
$$= \frac{(0.25)(0.07)}{(0.25)(0.07) + (0.25)(0.04) + (0.5)(0.01)} \approx 0.54$$

(b) $P(M \mid E) = \dfrac{P(M)P(E \mid M)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$

$= \dfrac{(0.25)(0.04)}{(0.25)(0.07) + (0.25)(0.04) + (0.5)(0.01)} \approx 0.31$

(c) $P(H \mid E) = \dfrac{P(H)P(E \mid H)}{P(L)P(E \mid L) + P(M)P(E \mid M) + P(H)P(E \mid H)}$

$= \dfrac{(0.5)(0.01)}{(0.25)(0.07) + (0.25)(0.04) + (0.5)(0.01)} \approx 0.15$

(d) Low quality

27. F = {fair weather}
I = {inclement weather}
W = {predict fair weather}.

$$P(F \mid W) = \frac{P(F)P(W \mid F)}{P(F)P(W \mid F) + P(I)P(W \mid I)} = \frac{(0.6)(0.7)}{(0.6)(0.7) + (0.4)(0.3)} = \frac{7}{9} \approx 0.78$$

Chapter 8 Review Problems

1. ${}_8P_3 = 8 \cdot 7 \cdot 6 = 336$

2. ${}_nP_1 = \dfrac{n!}{(n-1)!} = \dfrac{n(n-1)!}{(n-1)!} = n$

3. ${}_9C_7 = \dfrac{9!}{7!(9-7)!} = \dfrac{9!}{7! \cdot 2!} = \dfrac{9 \cdot 8 \cdot 7!}{7! \cdot 2 \cdot 1} = \dfrac{9 \cdot 8}{2} = 36$

4. ${}_{12}C_5 = \dfrac{12!}{5!(12-5)!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7!} = 792$

5. For each of the first 3 characters there are 26 choices, while for each of the last 3 characters there are 10 choices. By the basic counting principle, the number of license plates that are possible is
$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17{,}576{,}000$.

6. The number of choices for appetizers is 2, for the entrée it is 4, and for the dessert it is 3. By the basic counting principle, the number of complete dinners that are possible is $2 \cdot 4 \cdot 3 = 24$.

7. Each of the six switches has 2 possible positions. By the basic counting principle, the number of different codes is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$.

8. A batting order consists of nine names selected from nine names such that order is important. The number of such selections is ${}_9P_9 = 9! = 362{,}880$.

9. A possibility for first, second, and third place is a selection of three of the seven teams so that order is important. Thus the number of ways the season can end is ${}_7P_3 = 7 \cdot 6 \cdot 5 = 210$.

10. Nine of the nine trophies can be arranged so that order is important. The first two can be placed on the top shelf, the next three on the middle shelf, and the last four on the bottom shelf. The number of such arrangements is ${}_9P_9 = 9! = 362{,}880$.

11. The order of the group is not important. Thus the number of groups that can board is $_{11}C_6 = \frac{11!}{6!\cdot 5!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!} = 462.$

12. There are four cards with each denomination. There are $_4C_3$ ways of selecting three of them, and $_4C_2$ ways of selecting two of them. The number of ways to select two different denominations of cards is $_{13}C_2$. Then there are two ways of selecting which denomination will have three cards. The total number of ways of selecting a full house is

$$2\cdot {_{13}C_2}\cdot {_4C_3}\cdot {_4C_2} = 2\cdot\frac{13!}{2!11!}\cdot\frac{4!}{3!1!}\cdot\frac{4!}{2!2!} = 3744.$$

13. **a.** Three bulbs are selected from 24, and the order of selection is not important. Thus the number of possible selections is

$$_{24}C_3 = \frac{24!}{3!(24-3)!} = \frac{24!}{3!\cdot 21!}$$
$$= \frac{24\cdot 23\cdot 22\cdot 21!}{3\cdot 2\cdot 1\cdot 21!} = \frac{24\cdot 23\cdot 22}{3\cdot 2\cdot 1} = 2024 .$$

b. Only one bulb is defective and that bulb must be included in the selection. The other two bulbs must be selected from the 23 remaining bulbs and there are $_{23}C_2$ such selections possible. Thus the number of ways of selecting three bulbs such that one is defective is

$$1\cdot {_{23}C_2} = {_{23}C_2} = \frac{23!}{2!(23-2)!} = \frac{23!}{2!\cdot 21!}$$
$$\frac{23\cdot 22\cdot 21!}{2\cdot 1\cdot 21!} = \frac{23\cdot 22}{2\cdot 1} = 253.$$

14. To score 90, exactly nine questions must be correct; to score 100, all ten questions must be correct. If exactly nine questions are answered correctly, there are three ways of answering the tenth question incorrectly. But the number of ways of selecting nine of ten items is $_{10}C_9$. Thus the number of ways to score 90 is $3\cdot {_{10}C_9}$. The number of ways to answer all ten questions correctly is $_{10}C_{10}$, or more simply, 1. Thus the number of ways to score 90 or better is

$$3\cdot {_{10}C_9} + 1 = 3\cdot\frac{10!}{9!\cdot 1!} + 1$$
$$= 3\cdot 10 + 1 = 31.$$

15. In the word MISSISSIPPI, there are 11 letters with repetition: 1 M, 4 I's, 4 S's, and 2 P's. Thus the number of distinguishable permutations is $\frac{11!}{1!\cdot 4!\cdot 4!\cdot 2!} = 34,650$.

16. Nine flags must be arranged: two are red (type 1), three are green (type 2) and four are white (type 3). Thus the number of distinguishable permutations is $\frac{9!}{2!\cdot 3!\cdot 4!} = 1260.$

17. Of the 13 professors, 5 go to Dalhousie University (Cell A), 6 go to St. Mary's (Cell B), and two are not assigned (Cell C). The number of possible assignments is $\frac{13!}{5!6!2!} = 36,036.$

18. Two of the three vans can be selected in $_3C_2$ ways. After two vans are chosen, the operator must assign 14 people so that 7 go to one van (cell 1) and 7 go to the other van (cell 2). This can be done in $\frac{14!}{7!\cdot 7!}$ ways. By the basic counting principle, the number of ways to assign the people to two vans is

$$_3C_2\cdot\frac{14!}{7!\cdot 7!} = \frac{3!}{2!\cdot 1!}\cdot\frac{14!}{7!\cdot 7!} = 10,296 .$$

19. **a.** $E_1 \cup E_2 = \{1, 2, 3, 4, 5, 6, 7\}$

b. $E_1 \cap E_2 = \{4, 5, 6\}$

c. $E_1' \cup E_2 = \{7,8\} \cup \{4,5,6,7\} = \{4,5,6,7,8\}$

d. The intersection of any event and its complement is $\varnothing$.

e. $(E_1 \cap E_2')' = (\{1,2,3,4,5,6\} \cap \{1,2,3,8\})'$
$= \{1, 2, 3\}' = \{4, 5, 6, 7, 8\}$

f. From (b), $E_1 \cap E_2 \neq \varnothing$, so E_1 and E_2 are not mutually exclusive.

20. **a.** {1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

b. {2H, 2T}

c. {2H, 4H, 6H}

21. a. $\{R_1R_2R_3, R_1R_2G_3, R_1G_2R_3, R_1G_2G_3, G_1R_2R_3, G_1R_2G_3, G_1G_2R_3, G_1G_2G_3\}$

b. $\{R_1R_2G_3, R_1G_2R_3, G_1R_2R_3\}$

c. $\{R_1R_2R_3, G_1G_2G_3\}$

22. $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$0.6 = 0.5 + P(E_2) - 0.2$
$P(E_2) = 0.3$, so $P(E_2') = 1 - 0.3 = 0.7$.

23. $n(S) = {}_{10}C_2 = \frac{10!}{2!\cdot 8!} = \frac{10\cdot 9\cdot 8!}{2\cdot 1\cdot 8!} = \frac{10\cdot 9}{2\cdot 1} = 45$
Let E be the event that box is rejected. If box is rejected, the one defective chip must be in the two-chip sample and there are nine possibilities for the other chip. Thus $n(E) = 9$
and $P(E) = \frac{n(E)}{n(S)} = \frac{9}{45} = \frac{1}{5} = 0.2$.

24. Percentage of rats given drug
D = 100 – (35 + 25 + 15) = 25%.
Number of rats given C = 100(0.15) = 15.
Number of rats given D = 100(0.25) = 25.
If E = event that rat was injected with C or D, then $P(E) = \frac{n(E)}{n(S)} = \frac{15+25}{100} = 0.40$.
If the experiment is repeated on a larger group of 300 rats but with the drugs given in the same proportion, then the number of rats given drug C is 300(0.15) = 45 and the number of rats given drug D is 300(0.25) = 75 and
$P(E) = \frac{n(E)}{n(S)} = \frac{45+75}{300} = 0.40.$ Thus there is no effect on the previous probability.

25. Number of ways to answer exam is $4^5 = 1024 = n(S)$. Let E = {exactly two questions are incorrect). The number of ways of selecting two of the five questions that are incorrect is ${}_5C_2 = \frac{5!}{2!\cdot 3!} = 10$. However, there are three ways to answer a question incorrectly. Since two questions are incorrect $n(E) = 10 \cdot 3 \cdot 3 = 90$. Thus
$P(E) = \frac{n(E)}{n(S)} = \frac{90}{1024} = \frac{45}{512}$.

26. a. Of the 200 cola drinkers, 35 like both A and B. Thus
$P(\text{likes both A and B}) = \frac{35}{200} = \frac{7}{40}$.

b. If a person likes A but not B, then the person likes A only, and conversely. Thus
$P(\text{likes A, but not B}) = \frac{70}{200} = \frac{7}{20}$.

27. a. There are 12 jelly beans in the bag.
$n(S) = 12 \cdot 12 = 144$
$n(E_{\text{both red}}) = 5 \cdot 5 = 25$
Thus $P(E_{\text{both red}}) = \frac{n(E_{\text{both red}})}{n(S)} = \frac{25}{144}$.

b. $n(S) = 12 \cdot 11 = 132$
$n(E_{\text{both red}}) = 5 \cdot 4 = 20$
Thus $P(E_{\text{both red}}) = \dfrac{20}{132} = \dfrac{5}{33}$.

28. $n(S) = 6 \cdot 6 = 36$

a. $E_{2 \text{ or } 7} = \{(1, 1), (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$
$P(E_{2 \text{ or } 7}) = \dfrac{n(E_{2 \text{ or } 7})}{n(S)} = \dfrac{7}{36}$

b. $E_{\text{multiple of 3}} = E_{3,\, 6,\, 9 \text{ or } 12}$
$= \{(1, 2), (2, 1), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)\}$
$P(E_{\text{multiple of 3}}) = \dfrac{n(E_{\text{multiple of 3}})}{n(S)} = \dfrac{12}{36} = \dfrac{1}{3}$

c. $E_{\text{no less than 7}} = E_{7,\, 8,\, 9,\, 10,\, 11, \text{ or } 12} = \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4), (3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$
$P(E_{\text{no less than 7}}) = \dfrac{n(E_{7,\, 8,\, 9,\, 10,\, 11, \text{ or } 12})}{n(S)} = \dfrac{21}{36} = \dfrac{7}{12}$

29. $n(S) = 52 \cdot 52 \cdot 52$.

a. There are 26 black cards in a deck. Thus $n(E_{\text{all black}}) = 26 \cdot 26 \cdot 26$ and
$P(E_{\text{all black}}) = \dfrac{26 \cdot 26 \cdot 26}{52 \cdot 52 \cdot 52} = \dfrac{1}{8}$.

b. There are 13 diamonds in a deck, none of which are black. If E = event that two cards are black and the other is a diamond, then E occurs if the diamond is the first, second, or third card. Thus
$n(E) = 13 \cdot 26 \cdot 26 + 26 \cdot 13 \cdot 26 + 26 \cdot 26 \cdot 13 = 3 \cdot 13 \cdot 26 \cdot 26$ and $P(E) = \dfrac{3 \cdot 13 \cdot 26 \cdot 26}{52 \cdot 52 \cdot 52} = \dfrac{3}{16}$.

30. $n(S) = {}_{52}C_2 = \dfrac{52!}{2! \cdot 50!} = 1326$

a. There are 13 hearts in a deck. Thus $n(E_{\text{both hearts}}) = {}_{13}C_2 = \dfrac{13!}{2! \cdot 11!} = 78$ and $P(E_{\text{both hearts}}) = \dfrac{78}{1326} = \dfrac{1}{17}$.

b. There are four aces and two red kings, and no red king is an ace. If E = event that one card is an ace and the other is a red king, then $n(E) = 4 \cdot 2 = 8$ and
$P(E) = \dfrac{8}{1326} = \dfrac{4}{663} \approx 0.006$.

31. $\dfrac{P(E)}{P(E')} = \dfrac{\frac{3}{8}}{1-\left(\frac{3}{8}\right)} = \dfrac{\frac{3}{8}}{\frac{5}{8}} = \dfrac{3}{5}$ or 3:5

32. $\dfrac{P(E)}{P(E')} = \dfrac{0.84}{1-0.84} = \dfrac{0.84}{0.16} = \dfrac{84}{16} = \dfrac{21}{4}$ or 21:4

33. $P(E) = \frac{6}{6+1} = \frac{6}{7}$

34. $P(E) = \frac{3}{3+4} = \frac{3}{7}$

35. $P\left(F'\middle|H\right) = \frac{P(F' \cap H)}{P(H)} = \frac{\frac{10}{52}}{\frac{1}{4}} = \frac{10}{13}$

36. The reduced sample space consists of {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)}.
In none of these 11 points, is the sum less than 7. Thus $P(\text{sum} < 7 \mid \text{a 6 shows}) = 0$.

37. $P(S \cap M) = P(S)P(M \mid S) = (0.65)(0.80) = 0.52$

38. $P(Q \cap H \cap AC) = P(Q)P(H)P(AC)$
$= \frac{4}{52} \cdot \frac{13}{52} \cdot \frac{1}{52} = \frac{1}{2704}$

39. a. The reduced sample space consists of {(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)}.
In two of these 11 points, the sum of the components is 7. Thus
$P(\text{sum} = 7 \mid \text{a 4 shows}) = \frac{2}{11}$.

b. Out of 36 sample points, the event {getting a total of 7 and having a 4 show} is {(4, 3), (3, 4)}. Thus the probability of this event is $\frac{2}{36} = \frac{1}{18}$.

40. The reduced sample space consists of {(3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)}. Out of these 10 points, only one has a first toss that is less than 4. Thus the conditional probability is $\frac{1}{10}$.

41. The second number must be a 1 or 2, so the reduced sample space has $6 \cdot 2 = 12$ sample points. Of these, the event {first number ≤ second number} consists of (1, 1), (1, 2), and (2, 2). Thus the conditional probability is $\frac{3}{12} = \frac{1}{4}$.

42. It does not matter whether the first three cards are drawn or are left in place. Thus, imagine that they are merely lifted high enough for the fourth card to be drawn. The probability that this card is a heart is $\frac{1}{4}$.

43. a. $P(L' \mid F) = \frac{n(L' \cap F)}{n(F)} = \frac{160}{480} = \frac{1}{3}$

b. $P(L) = \frac{400}{600} = \frac{2}{3}$ and
$P(L|M) = \frac{n(L \cap M)}{n(M)} = \frac{80}{120} = \frac{2}{3}$.
Since $P(L|M) = P(L)$, events L and M are independent.

44. $E = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$
$F = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)\}$

a. Since $E \cap F = \{(4,4)\} \neq \varnothing$, E and F are not mutually exclusive.

b. $P(E) = \frac{6}{36} = \frac{1}{6}$ and
$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$.
Since $P(E) = P(E \mid F)$, events E and F are independent.

45. P = {attend public college}
M = {from middle-class family}
$P(P) = \frac{125}{175} = \frac{5}{7}$
$P(P \mid M) = \frac{n(P \cap M)}{n(M)} = \frac{55}{80} = \frac{11}{16}$
Since $P(P \mid M) \neq P(P)$, events P and M are dependent.

46. $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

so $P(E \cap F) = P\left(E\middle|F\right)P(F) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$, thus
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= \frac{1}{4} + \frac{1}{3} - \frac{1}{18} = \frac{19}{36}$.

47. a. $P(\text{all take root}) = 0.8^5 = 0.32768$

b. The probability that a particular three shrubs take root and the remaining two do not is $0.8^3 \cdot 0.2^2$. The number of ways the three that take root can be chosen from the five shrubs is ${}_5C_3$. Thus

$P(\text{exactly three take root}) = {}_5C_3(0.8)^3(0.2)^2 = 0.2048.$

c. For at least three shrubs to take root, either exactly three, four, or five do.

$P(3) + P(4) + P(5) = 0.2048 + {}_5C_4(0.8)^4(0.2) + 0.32768 = 0.94208$

48. Being effective for at least three of the persons means that it is effective for exactly three of them or for all four of them. Thus

P(exactly three) + P(all four)

$= {}_4C_3(0.75)(0.75)(0.75)(0.25) + (0.75)(0.75)(0.75)(0.75)$

≈ 0.738

49. $P(R_{\text{II}}) = P(G_{\text{I}})P(R_{\text{II}} \mid G_{\text{I}}) + P(R_{\text{I}})P(R_{\text{II}} \mid R_{\text{I}})$

$= \frac{3}{5} \cdot \frac{4}{9} + \frac{2}{5} \cdot \frac{5}{9} = \frac{22}{45}.$

50. a. $P(W) = P(B_{\text{I}})P(W \mid B_{\text{I}}) + P(B_{\text{II}})P(W \mid B_{\text{II}})$

$= \frac{1}{2} \cdot \frac{2}{6} + \frac{1}{2} \cdot \frac{3}{5} = \frac{1}{6} + \frac{3}{10} = \frac{7}{15}$

b. $P(B_{\text{II}} \mid W) = \dfrac{P(B_{\text{II}})P(W \mid B_{\text{II}})}{P(B_{\text{I}})P(W \mid B_{\text{I}}) + P(B_{\text{II}})P(W \mid B_{\text{II}})} = \dfrac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{7}{15}} = \dfrac{9}{14}$

51. $P(G \mid A) = \dfrac{P(G \cap A)}{P(A)} = \dfrac{0.1}{0.4} = \dfrac{1}{4}$

52. S = {live within the state} and
F = {first time attending}.

$$P(S' \mid F') = \frac{P(S')P(F' \mid S')}{P(S)P(F' \mid S) + P(S')P(F' \mid S')}$$

$$= \frac{\frac{114}{723} \cdot \frac{22}{100}}{\frac{609}{723} \cdot \frac{56}{100} + \frac{114}{723} \cdot \frac{22}{100}} = \frac{209}{3051} \approx 0.0685$$

53. a. F = {produced by first shift}
S = {produced by second shift}
D = {scratched}

$P(D) = P(F)P(D \mid F) + P(S)P(D \mid S)$

$= \frac{3000}{8000} \cdot (0.01) + \frac{5000}{8000} \cdot (0.02)$

$= 0.00375 + 0.0125 = 0.01625$

b. $$P(F \mid D) = \frac{P(F)P(D \mid F)}{P(F)P(D \mid F) + P(S)P(D \mid S)}$$
$$= \frac{0.00375}{0.01625} = \frac{3}{13} \approx 0.23$$

54. E = {passed the exam}
S = {satisfactory performance}.
$$P(E \mid S) = \frac{P(E)P(S \mid E)}{P(E)P(S \mid E) + P(E')P(S \mid E')}$$
$$= \frac{(0.35)(0.8)}{(0.35)(0.8) + (0.65)(0.3)} = \frac{0.28}{0.475} = \frac{56}{95} \approx 0.59$$

Explore and Extend—Chapter 8

1. Trial and error should yield a critical value of around 0.645.

2. Possible answers: One could use cellular automata to model disease spread. The rules would be similar to the fad model, since a person who recovers from a disease is generally immune for some time afterward. One could also use cellular automata to model the formation of political opinion blocks. Each cell could be in one of three of four states, and a cell could be influenced by its neighbors. Some cells could be highly subject to neighbor influence while others were relatively immune.