Problems for Section 6.2

LEARNING THE BASICS

6.1 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

a. Z is less than 1.57?
b. Z is greater than 1.84?
c. Z is between 1.57 and 1.84?
d. Z is less than 1.57 or greater than 1.84?

6.2 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

a. Z is between -1.57 and 1.84?
b. Z is less than -1.57 or greater than 1.84?
c. What is the value of Z if only 2.5% of all possible Z values are larger?
d. Between what two values of Z (symmetrically distributed around the mean) will 68.26% of all possible Z values be contained?

6.3 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), what is the probability that

a. Z is less than 1.08?
b. Z is greater than -0.21?
c. Z is less than -0.21 or greater than the mean?
d. Z is less than -0.21 or greater than 1.08?

6.4 Given a standardized normal distribution (with a mean of 0 and a standard deviation of 1, as in Table E.2), determine the following probabilities:

a. $P(Z > 1.08)$
b. $P(Z < -0.21)$
c. $P(-1.96 < Z < -0.21)$
d. What is the value of Z if only 15.87% of all possible Z values are larger?

6.5 Given a normal distribution with $\mu = 100$ and $\sigma = 10$, what is the probability that

a. $X > 75$?
b. $X < 70$?
c. $X < 80$ or $X > 110$?
d. Between what two X values (symmetrically distributed around the mean) are 80% of the values?

6.6 Given a normal distribution with $\mu = 50$ and $\sigma = 4$, what is the probability that

a. $X > 43$?
b. $X < 42$?
c. 5% of the values are less than what X value?
d. Between what two X values (symmetrically distributed around the mean) are 60% of the values?

APPLYING THE CONCEPTS

6.7 According to **bottledwater.org**, in 2012, the per capita consumption of bottled water in the United States was reported to be 30.8 gallons. Assume that the per capita consumption of bottled water in the United States is approximately normally distributed with a mean of 30.8 gallons and a standard deviation of 10 gallons.

a. What is the probability that someone in the United States consumed more than 32 gallons of bottled water in 2012?
b. What is the probability that someone in the United States consumed between 10 and 20 gallons of bottled water in 2012?
c. What is the probability that someone in the United States consumed less than 10 gallons of bottled water in 2012?
d. Ninety-nine percent of the people in the United States consumed less than how many gallons of bottled water?

SELF Test **6.8** Toby's Trucking Company determined that the distance traveled per truck per year is normally distributed, with a mean of 50 thousand miles and a standard deviation of 12 thousand miles.

a. What proportion of trucks can be expected to travel between 34 and 50 thousand miles in a year?
b. What percentage of trucks can be expected to travel either less than 30 or more than 60 thousand miles in a year?
c. How many miles will be traveled by at least 80% of the trucks?
d. What are your answers to (a) through (c) if the standard deviation is 10 thousand miles?

6.9 Consumers spend an average of \$13.80 on a meal at a restaurant in 2012. (Data extracted from **www.alixpartners.com**.) Assume that the amount spent on a restaurant meal is normally distributed and that the standard deviation is \$2.

a. What is the probability that a randomly selected person spent more than \$15?
b. What is the probability that a randomly selected person spent between \$10 and \$12?
c. Between what two values will the middle 95% of the amounts spent fall?

6.10 A set of final examination grades in an introductory statistics course is normally distributed, with a mean of 73 and a standard deviation of 8.

a. What is the probability that a student scored below 91 on this exam?
b. What is the probability that a student scored between 65 and 89?
c. The probability is 5% that a student taking the test scores higher than what grade?
d. If the professor grades on a curve (i.e., gives A's to the top 10% of the class, regardless of the score), are you better off with a grade of 81 on this exam or a grade of 68 on a different exam, where the mean is 62 and the standard deviation is 3? Show your answer statistically and explain.

6.11 A Nielsen study indicates that mobile subscribers between 18 and 24 years of age spend a substantial amount of time watching video on their devices, reporting a mean of 396 minutes per month. (Data extracted from **bit.ly/13f4uab**.) Assume that the amount of time watching video on a mobile device per month is normally distributed and that the standard deviation is 50 minutes.

a. What is the probability that an 18- to 24-year-old mobile subscriber spends less than 321 minutes watching video on his or her mobile device per month?
b. What is the probability that an 18- to 24-year-old mobile subscriber spends between 320 minutes and 471 minutes watching video on his or her mobile device per month?
c. What is the probability that an 18- to 24-year-old mobile subscriber spends more than 471 minutes watching video on his or her mobile device per month?
d. One percent of all 18- to 24-year-old mobile subscribers will spend less than how many minutes watching video on his or her mobile device per month?

6.12 In 2012, the per capita consumption of soft drinks in the United States was reported to be 44 gallons. (Data extracted from **on-msn.com/XdwVlq**.) Assume that the per capita consumption of soft drinks in the United States is approximately normally distributed with a mean of 44 gallons and a standard deviation of 14 gallons.

a. What is the probability that someone in the United States consumed more than 60 gallons of soft drinks in 2012?

b. What is the probability that someone in the United States consumed between 15 and 30 gallons of soft drinks in 2012?

c. What is the probability that someone in the United States consumed less than 15 gallons of soft drinks in 2012?

d. Ninety-nine percent of the people in the United States consumed less than how many gallons of soft drinks?

6.13 Many manufacturing problems involve the matching of machine parts, such as shafts that fit into a valve hole. A particular design requires a shaft with a diameter of 22.000 mm, but shafts with diameters between 21.990 mm and 22.010 mm are acceptable. Suppose that the manufacturing process yields shafts with diameters normally distributed, with a mean of 22.002 mm and a standard deviation of 0.005 mm. For this process, what is

a. the proportion of shafts with a diameter between 21.99 mm and 22.00 mm?

b. the probability that a shaft is acceptable?

c. the diameter that will be exceeded by only 2% of the shafts?

d. What would be your answers in (a) through (c) if the standard deviation of the shaft diameters were 0.004 mm?

6.3 Evaluating Normality

As first stated in Section 6.2, the normal distribution has several important theoretical properties:

- It is symmetrical; thus, the mean and median are equal.
- It is bell-shaped; thus, the empirical rule applies.
- The interquartile range equals 1.33 standard deviations.
- The range is approximately equal to 6 standard deviations.

As Section 6.2 notes, many continuous variables used in business closely follow a normal distribution. To determine whether a set of data can be approximated by the normal distribution, you either compare the characteristics of the data with the theoretical properties of the normal distribution or construct a normal probability plot.

Comparing Data Characteristics to Theoretical Properties

Many continuous variables have characteristics that approximate theoretical properties. However, other continuous variables are often neither normally distributed nor approximately normally distributed. For such variables, the descriptive characteristics of the data are inconsistent with the properties of a normal distribution. One approach you can use to determine whether a variable follows a normal distribution is to compare the observed characteristics of the variable with what would be expected if the variable followed a normal distribution. To do so, you can

- Construct charts and observe their appearance. For small- or moderate-sized data sets, create a stem-and-leaf display or a boxplot. For large data sets, in addition, plot a histogram or polygon.
- Compute descriptive statistics and compare these statistics with the theoretical properties of the normal distribution. Compare the mean and median. Is the interquartile range approximately 1.33 times the standard deviation? Is the range approximately 6 times the standard deviation?
- Evaluate how the values are distributed. Determine whether approximately two-thirds of the values lie between the mean and ± 1 standard deviation. Determine whether approximately four-fifths of the values lie between the mean and ± 1.28 standard deviations. Determine whether approximately 19 out of every 20 values lie between the mean and ± 2 standard deviations.

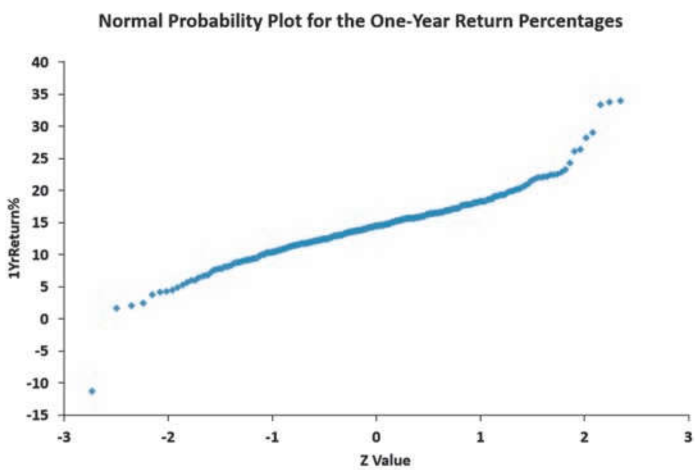
For example, you can use these techniques to determine whether the one-year returns discussed in Chapters 2 and 3 (stored in **Retirement Funds**) follow a normal distribution.

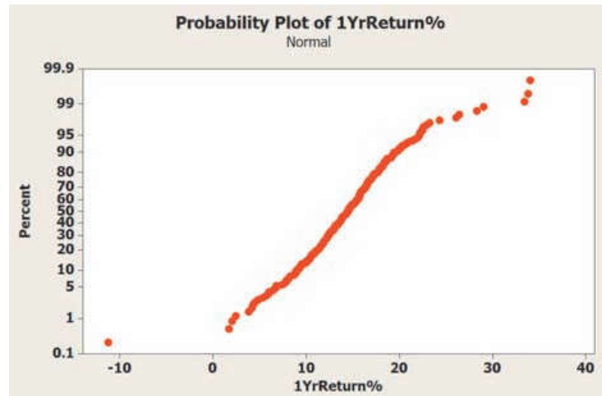
Table 6.6 presents the descriptive statistics and the five-number summary for the one-year return percentage variable. Figure 6.18 presents the Excel and Minitab boxplots for the one-year return percentages.

Figure 6.20 shows Excel (quantile–quantile) and Minitab normal probability plots for the one-year returns. The Excel quantile–quantile plot shows a single extremely low value followed by the bulk of the points that approximately follow a straight line except for a few high values.

FIGURE 6.20

Excel (quantile–quantile) and Minitab normal probability plots for the one-year returns





The Minitab normal probability plot has the one-year return percentage variable on the X axis and the cumulative percentage for a normal distribution on the Y axis. As with a quantile–quantile plot, the points will plot along an approximately straight line if the data are normally distributed. However, if the data are right-skewed, the curve will rise more rapidly at first and then level off. If the data are left-skewed, the data will rise more slowly at first and then rise at a faster rate for higher values of the variable being plotted. Observe that although the bulk of the points on the normal probability plot approximately follow a straight line, there are several high values that depart from a straight line, indicating a distribution that differs somewhat from a normal distribution.

Problems for Section 6.3

LEARNING THE BASICS

6.14 Show that for a sample of $n = 39$, the smallest and largest Z values are -1.96 and $+1.96$, and the middle (i.e., 20th) Z value is 0.00.

6.15 For a sample of $n = 6$, list the six Z values.

APPLYING THE CONCEPTS

SELF Test **6.16** The file **SUV** contains the overall miles per gallon (MPG) of 2013 small SUVs ($n = 17$):

22 23 21 22 25 26 22 22 21
19 22 22 26 23 24 21 22

Source: Data extracted from "Ratings," *Consumer Reports*, April 2013, pp. 34–35.

Decide whether the data appear to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.

6.17 As player salaries have increased, the cost of attending baseball games has increased dramatically. The file **BBCost2012** contains the cost of four tickets, two beers, four soft drinks, four hot dogs, two game programs, two baseball caps, and the parking fee for one car for each of the 30 Major League Baseball teams in 2012:

176, 337, 223, 174, 233, 185, 160, 225, 324, 187, 196, 153, 184, 217, 146, 172, 300, 166, 184, 224, 213, 242, 172, 230, 257, 152, 225, 151, 224, 198

Source: Data extracted from **fancostexperience.com/pages/fcx/fci_pdfs/8.pdf**.

Decide whether the data appear to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.

6.18 The file **Property Taxes** contains the property taxes per capita for the 50 states and the District of Columbia. Decide whether the data appear to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.

6.19 Thirty companies comprise the DJIA. How big are these companies? One common method for measuring the size of a company is to use its market capitalization, which is computed by multiplying the number of stock shares by the price of a share of stock. On March 30, 2013, the market capitalization of these companies ranged from Alcoa's \$9.1 billion to ExxonMobil's \$403.7 billion. The entire population of market capitalization values is stored in **DowMarketCap**. (Data extracted from **money.cnn.com**, March 30, 2013.) Decide whether the market capitalization of companies in the DJIA appears to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.
c. constructing a histogram.

6.20 One operation of a mill is to cut pieces of steel into parts that will later be used as the frame for front seats in an automotive plant. The steel is cut with a diamond saw, and the resulting parts must be within ± 0.005 inch of the length specified by the automobile company. The data come from a sample of 100 steel parts and are stored in **Steel**. The measurement reported is the difference, in inches, between the actual length of the steel part, as measured by a laser measurement device, and the specified length of the steel part. Determine whether the data appear to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.

6.21 The file **CD Rate** contains the yields for a one-year certificate of deposit (CD) and a five-year CD for 23 banks in the United States, as of March 20, 2013. (Data extracted from **www.Bankrate.com**, March 20, 2013.) For each type of investment, decide whether the data appear to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.

6.22 The file **Utility** contains the electricity costs, in dollars, during July 2013 for a random sample of 50 one-bedroom apartments in a large city:

96	171	202	178	147	102	153	197	127	82
157	185	90	116	172	111	148	213	130	165
141	149	206	175	123	128	144	168	109	167
95	163	150	154	130	143	187	166	139	149
108	119	183	151	114	135	191	137	129	158

Decide whether the data appear to be approximately normally distributed by

a. comparing data characteristics to theoretical properties.
b. constructing a normal probability plot.

6.4 The Uniform Distribution

In the **uniform distribution**, the values are evenly distributed in the range between the smallest value, a, and the largest value, b. Because of its shape, the uniform distribution is sometimes called the **rectangular distribution** (see Panel B of Figure 6.1 on page 220). Equation (6.4) defines the probability density function for the uniform distribution.

UNIFORM PROBABILITY DENSITY FUNCTION

$$f(X) = \frac{1}{b - a} \text{ if } a \le X \le b \text{ and 0 elsewhere} \qquad \textbf{(6.4)}$$

where

a = minimum value of X
b = maximum value of X

Equation (6.5) defines the mean of the uniform distribution, and Equation (6.6) defines the variance and standard deviation of the uniform distribution.

MEAN OF THE UNIFORM DISTRIBUTION

$$\mu = \frac{a + b}{2} \qquad \textbf{(6.5)}$$

and

$$\sigma^2 = \frac{(b - a)^2}{12} = \frac{(1 - 0)^2}{12} = \frac{1}{12} = 0.0833$$

$$\sigma = \sqrt{0.0833} = 0.2887$$

Thus, the mean is 0.5, and the standard deviation is 0.2887.

Example 6.6 provides another application of the uniform distribution.

EXAMPLE 6.6
Computing Uniform Probabilities

In the Normal Downloading at MyTVLab scenario on page 219, the download time of videos was assumed to be normally distributed with a mean of 7 seconds. Suppose that the download time follows a uniform (instead of a normal) distribution between 4.5 and 9.5 seconds. What is the probability that a download time will take more than 9 seconds?

SOLUTION The download time is uniformly distributed from 4.5 to 9.5 seconds. The area between 9 and 9.5 seconds is equal to 0.5 seconds, and the total area in the distribution is $9.5 - 4.5 = 5$ seconds. Therefore, the probability of a download time between 9 and 9.5 seconds is the portion of the area greater than 9, which is equal to $0.5/5.0 = 0.10$. Because 9.5 is the maximum value in this distribution, the probability of a download time above 9 seconds is 0.10. In comparison, if the download time is normally distributed with a mean of 7 seconds and a standard deviation of 2 seconds (see Example 6.1 on page 225), the probability of a download time above 9 seconds is 0.1587.

Problems for Section 6.4

LEARNING THE BASICS

6.23 Suppose you select one value from a uniform distribution with $a = 0$ and $b = 10$. What is the probability that the value will be

a. between 5 and 7?
b. between 2 and 3?
c. What is the mean?
d. What is the standard deviation?

APPLYING THE CONCEPTS

SELF Test **6.24** The time between arrivals of customers at a bank during the noon-to-1 P.M. hour has a uniform distribution between 0 to 120 seconds. What is the probability that the time between the arrival of two customers will be

a. less than 20 seconds?
b. between 10 and 30 seconds?
c. more than 35 seconds?
d. What are the mean and standard deviation of the time between arrivals?

6.25 A study of the time spent shopping in a supermarket for a market basket of 20 specific items showed an approximately uniform distribution between 20 minutes and 40 minutes. What is the probability that the shopping time will be

a. between 25 and 30 minutes?
b. less than 35 minutes?
c. What are the mean and standard deviation of the shopping time?

6.26 How long does it take to download a two-hour movie from the iTunes store? According to Apple's technical support site, **support.apple.com/kb/ht1577**, downloading such a movie using a 5 Mbit/s broadband connection should take 18 to 24 minutes. Assume that the download times are uniformly distributed between 18 and 24 minutes. If you download a two-hour movie, what is the probability that the download time will be

a. less than 19 minutes?
b. more than 23 minutes?
c. between 20 and 22 minutes?
d. What are the mean and standard deviation of the download times?

6.27 The scheduled commuting time on the Long Island Railroad from Glen Cove to New York City is 65 minutes. Suppose that the actual commuting time is uniformly distributed between 64 and 74 minutes. What is the probability that the commuting time will be

a. less than 70 minutes?
b. between 65 and 70 minutes?
c. greater than 65 minutes?
d. What are the mean and standard deviation of the commuting time?

Thus, the probability that a customer will arrive within 6 minutes is 0.8647, or 86.47%. Figure 6.23 shows this probability as computed by Excel and Minitab.

FIGURE 6.23

Excel and Minitab results for computing exponential probability that a customer will arrive within six minutes

	A	B	
1	Exponential Probability		
2			
3	Data		
4	Mean	20	
5	X Value	0.1	
6			
7	Results		
8	P(<=X)	0.8647	=EXPON.DIST(B5, B4, TRUE)

Cumulative Distribution Function

```
Exponential with mean = 0.05

  x   P( X <= x )
0.1      0.864665
```

Example 6.7 illustrates the effect on the exponential probability of changing the time between arrivals.

EXAMPLE 6.7

Computing Exponential Probabilities

In the ATM example, what is the probability that the next customer will arrive within 3 minutes (i.e., 0.05 hour)?

SOLUTION For this example, $\lambda = 20$ and $X = 0.05$. Using Equation (6.10),

$$
\begin{aligned}
P(\text{arrival time} \leq 0.05) &= 1 - e^{-20(0.05)} \\
&= 1 - e^{-1} \\
&= 1 - 0.3679 = 0.6321
\end{aligned}
$$

Thus, the probability that a customer will arrive within 3 minutes is 0.6321, or 63.21%.

Problems for Section 6.5

LEARNING THE BASICS

6.28 Given an exponential distribution with $\lambda = 10$, what is the probability that the arrival time is

a. less than $X = 0.1$?
b. greater than $X = 0.1$?
c. between $X = 0.1$ and $X = 0.2$?
d. less than $X = 0.1$ or greater than $X = 0.2$?

6.29 Given an exponential distribution with $\lambda = 30$, what is the probability that the arrival time is

a. less than $X = 0.1$?
b. greater than $X = 0.1$?
c. between $X = 0.1$ and $X = 0.2$?
d. less than $X = 0.1$ or greater than $X = 0.2$?

6.30 Given an exponential distribution with $\lambda = 5$, what is the probability that the arrival time is

a. less than $X = 0.3$?
b. greater than $X = 0.3$?
c. between $X = 0.3$ and $X = 0.5$?
d. less than $X = 0.3$ or greater than $X = 0.5$?

APPLYING THE CONCEPTS

6.31 Autos arrive at a toll plaza located at the entrance to a bridge at a rate of 50 per minute during the 5:00-to-6:00 P.M. hour. If an auto has just arrived,

a. what is the probability that the next auto will arrive within 3 seconds (0.05 minute)?
b. what is the probability that the next auto will arrive within 1 second (0.0167 minute)?
c. What are your answers to (a) and (b) if the rate of arrival of autos is 60 per minute?
d. What are your answers to (a) and (b) if the rate of arrival of autos is 30 per minute?

SELF Test **6.32** Customers arrive at the drive-up window of a fast-food restaurant at a rate of 2 per minute during the lunch hour.

a. What is the probability that the next customer will arrive within 1 minute?
b. What is the probability that the next customer will arrive within 5 minutes?
c. During the dinner time period, the arrival rate is 1 per minute. What are your answers to (a) and (b) for this period?

6.33 Telephone calls arrive at the information desk of a large computer software company at a rate of 15 per hour.

a. What is the probability that the next call will arrive within 3 minutes (0.05 hour)?
b. What is the probability that the next call will arrive within 15 minutes (0.25 hour)?
c. Suppose the company has just introduced an updated version of one of its software programs, and telephone calls are now arriving at a rate of 25 per hour. Given this information, what are your answers to (a) and (b)?

6.34 Calls arrive at a call center at the rate of 12 per hour. What is the probability that the next call arrives in

a. less than 3 minutes?
b. more than 6 minutes?
c. less than 1 minute?

6.35 The time between unplanned shutdowns of a power plant has an exponential distribution with a mean of 20 days. Find the probability that the time between two unplanned shutdowns is

a. less than 14 days.
b. more than 21 days.
c. less than 7 days.

6.36 Golfers arrive at the starter's booth of a public golf course at a rate of 8 per hour during the Monday-to-Friday midweek period. If a golfer has just arrived,

a. what is the probability that the next golfer will arrive within 15 minutes (0.25 hour)?
b. what is the probability that the next golfer will arrive within 3 minutes (0.05 hour)?
c. The actual arrival rate on Fridays is 15 per hour. What are your answers to (a) and (b) for Fridays?

6.37 Some Internet companies sell a service that will boost a website's traffic by delivering additional unique visitors. Assume that one such company claims it can deliver 1,000 visitors a day. If this amount of website traffic is experienced, then the time between visitors has a mean of 1.44 minutes (or 0.6944 per minute). Assume that your website gets 1,000 visitors a day and that the time between visitors has an exponential distribution. What is the probability that the time between two visitors is

a. less than 1 minute?
b. less than 2 minutes?
c. more than 3 minutes?
d. Do you think it is reasonable to assume that the time between visitors has an exponential distribution?

6.6 The Normal Approximation to the Binomial Distribution

In many circumstances, you can use the normal distribution to approximate the binomial distribution that is discussed in Section 5.3. The **Section 6.6 online topic** explains this approximation and illustrates its use.

USING STATISTICS

Normal Downloading at MyTVLab, Revisited

Cloki/Shutterstock

In the Normal Downloading at MyTVLab scenario, you were a project manager for an online social media and video website. You sought to ensure that a video could be downloaded quickly by visitors to the website. By running experiments in the corporate offices, you determined that the amount of time, in seconds, that passes from clicking a download link until a video is fully displayed is a bell-shaped distribution with a mean download time of 7 seconds and standard deviation of 2 seconds. Using the normal distribution, you were able to calculate that approximately 84% of the download times are 9 seconds or less, and 95% of the download times are between 3.08 and 10.92 seconds.

Now that you understand how to compute probabilities from the normal distribution, you can evaluate download times of a video using different website designs. For example, if the standard deviation remained at 2 seconds, lowering the mean to 6 seconds would shift the entire distribution lower by 1 second. Thus, approximately 84% of the download times would be 8 seconds or less, and 95% of the download times would be between 2.08 and 9.92 seconds. Another change that could reduce long download times would be reducing the variation. For example, consider the case where the mean remained at the original 7 seconds but the standard deviation was reduced to 1 second. Again, approximately 84% of the download times would be 8 seconds or less, and 95% of the download times would be between 5.04 and 8.96 seconds.

SUMMARY

In this and the previous chapter, you have learned about mathematical models called probability distributions and how they can be used to solve business problems. In Chapter 5, you used discrete probability distributions in situations where the values come from a counting process such as the number of social media sites to which you belong or the number of tagged order forms in a report generated by an accounting information system. In this chapter, you learned about continuous probability distributions where the values come from a measuring process such as your height or the download time of a video.

Continuous probability distributions come in various shapes, but the most common and most important in business is the normal distribution. The normal distribution is symmetrical; thus, its mean and median are equal. It is also bell-shaped, and approximately 68.26% of its values are within ± 1 standard deviation of the mean, approximately 95.44% of its values are within ± 2 standard deviations of the mean, and approximately 99.73% of its values are within ± 3 standard deviations of the mean. Although many variables in business are closely approximated by the normal distribution, do not think that all variables can be approximated by the normal distribution.

In Section 6.3, you learned about various methods for evaluating normality in order to determine whether the normal distribution is a reasonable mathematical model to use in specific situations. In Sections 6.4 and 6.5, you studied other continuous distributions—in particular, the uniform and exponential distributions. Chapter 7 uses the normal distribution to develop the subject of statistical inference.

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KEY EQUATIONS

Normal Probability Density Function

$$f(X) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)[(X-\mu)/\sigma]^2} \quad \textbf{(6.1)}$$

***Z* Transformation Formula**

$$Z = \frac{X - \mu}{\sigma} \quad \textbf{(6.2)}$$

Finding an *X* Value Associated with a Known Probability

$$X = \mu + Z\sigma \quad \textbf{(6.3)}$$

Uniform Probability Density Function

$$f(X) = \frac{1}{b - a} \quad \textbf{(6.4)}$$

Mean of the Uniform Distribution

$$\mu = \frac{a + b}{2} \quad \textbf{(6.5)}$$

Variance and Standard Deviation of the Uniform Distribution

$$\sigma^2 = \frac{(b - a)^2}{12} \quad \textbf{(6.6a)}$$

$$\sigma = \sqrt{\frac{(b - a)^2}{12}} \quad \textbf{(6.6b)}$$

Exponential Probability Density Function

$$f(X) = \lambda e^{-\lambda x} \text{ for } X > 0 \quad \textbf{(6.7)}$$

Mean Time Between Arrivals

$$\mu = \frac{1}{\lambda} \quad \textbf{(6.8)}$$

Standard Deviation of the Time Between Arrivals

$$\sigma = \frac{1}{\lambda} \quad \textbf{(6.9)}$$

Cumulative Exponential Probability

$$P(\text{arrival time} \le X) = 1 - e^{-\lambda x} \quad \textbf{(6.10)}$$

KEY TERMS

cumulative standardized normal distribution 223
exponential distribution 240
normal distribution 220
normal probability plot 235
probability density function 220
probability density function for the normal distribution 222
quantile–quantile plot 235
rectangular distribution 237
standardized normal variable 222
transformation formula 222
uniform distribution 237

CHECKING YOUR UNDERSTANDING

6.38 Why is only one normal distribution table such as Table E.2 needed to find any probability under the normal curve?

6.39 How do you find the area between two values under the normal curve?

6.40 How do you find the *X* value that corresponds to a given percentile of the normal distribution?

6.41 What are some of the distinguishing properties of a normal distribution?

6.42 How does the shape of the normal distribution differ from the shapes of the uniform and exponential distributions?

6.43 How can you use the normal probability plot to evaluate whether a set of data is normally distributed?

6.44 Under what circumstances can you use the exponential distribution?

CHAPTER REVIEW PROBLEMS

6.45 An industrial sewing machine uses ball bearings that are targeted to have a diameter of 0.75 inch. The lower and upper specification limits under which the ball bearings can operate are 0.74 inch and 0.76 inch, respectively. Past experience has indicated that the actual diameter of the ball bearings is approximately normally distributed, with a mean of 0.753 inch and a standard deviation of 0.004 inch. What is the probability that a ball bearing is

a. between the target and the actual mean?
b. between the lower specification limit and the target?
c. above the upper specification limit?
d. below the lower specification limit?
e. Of all the ball bearings, 93% of the diameters are greater than what value?

6.46 The fill amount in 2-liter soft drink bottles is normally distributed, with a mean of 2.0 liters and a standard deviation of 0.05 liter. If bottles contain less than 95% of the listed net content (1.90 liters, in this case), the manufacturer may be subject to penalty by the state office of consumer affairs. Bottles that have a net content above 2.10 liters may cause excess spillage upon opening. What proportion of the bottles will contain

a. between 1.90 and 2.0 liters?
b. between 1.90 and 2.10 liters?
c. below 1.90 liters or above 2.10 liters?
d. At least how much soft drink is contained in 99% of the bottles?
e. Ninety-nine percent of the bottles contain an amount that is between which two values (symmetrically distributed) around the mean?

6.47 In an effort to reduce the number of bottles that contain less than 1.90 liters, the bottler in Problem 6.46 sets the filling machine so that the mean is 2.02 liters. Under these circumstances, what are your answers in Problem 6.46 (a) through (e)?

6.48 An Ipsos MediaCT study indicates that mobile device owners who used their mobile device while shopping for consumer electronics spent an average of $1,539 on consumer electronics in the past six months. (Data extracted from **iab.net/showrooming**.) Assume that the amount spent on consumer electronics in the last six months is normally distributed and that the standard deviation is $500.

a. What is the probability that a mobile device owner who used his or her mobile device while shopping for consumer electronics spent less than $1,000 on consumer electronics?
b. What is the probability that a mobile device owner who used his or her mobile device while shopping for consumer electronics spent between $2,500 and $3,000 on consumer electronics?
c. Ninety percent of the amounts spent on consumer electronics by mobile device owners who used their mobile device while shopping for consumer electronics are less than what value?
d. Eighty percent of the amounts spent on consumer electronics by mobile device owners who used their mobile device while shopping for consumer electronics are between what two values symmetrically distributed around the mean?

6.49 The file **DomesticBeer** contains the percentage alcohol, number of calories per 12 ounces, and number of carbohydrates (in grams) per 12 ounces for 152 of the best-selling domestic beers in the United States. Determine whether each of these variables appears to be approximately normally distributed. Support your decision through the use of appropriate statistics and graphs. (Data extracted from **www.Beer100.com**, March 20, 2013.)

6.50 The evening manager of a restaurant was very concerned about the length of time some customers were waiting in line to be seated. She also had some concern about the seating times—that is, the length of time between when a customer is seated and the time he or she leaves the restaurant. Over the course of one week, 100 customers (no more than 1 per party) were randomly selected, and their waiting and seating times (in minutes) were recorded in **Wait**.

a. Think about your favorite restaurant. Do you think waiting times more closely resemble a uniform, an exponential, or a normal distribution?
b. Again, think about your favorite restaurant. Do you think seating times more closely resemble a uniform, an exponential, or a normal distribution?
c. Construct a histogram and a normal probability plot of the waiting times. Do you think these waiting times more closely resemble a uniform, an exponential, or a normal distribution?
d. Construct a histogram and a normal probability plot of the seating times. Do you think these seating times more closely resemble a uniform, an exponential, or a normal distribution?

6.51 The major stock market indexes had strong results in 2012. The mean one-year return for stocks in the S&P 500, a group of 500 very large companies, was +13.41%. The mean one-year return for the NASDAQ, a group of 3,200 small and medium-sized companies, was +15.91%. Historically, the one-year returns are approximately normally distributed, the standard deviation in the S&P 500 is approximately 20%, and the standard deviation in the NASDAQ is approximately 30%.
a. What is the probability that a stock in the S&P 500 gained value in 2012?
b. What is the probability that a stock in the S&P 500 gained 10% or more in 2012?
c. What is the probability that a stock in the S&P 500 lost 20% or more in 2012?
d. What is the probability that a stock in the S&P 500 lost 30% or more in 2012?
e. Repeat (a) through (d) for a stock in the NASDAQ.
f. Write a short summary on your findings. Be sure to include a discussion of the risks associated with a large standard deviation.

6.52 The speed at which you can log into a website through a smartphone is an important quality characteristic of that website. In a recent test, the mean time to log into the JetBlue Airways website through a smartphone was 4.237 seconds. (Data extracted from N. Trejos, "Travelers Have No Patience for Slow Mobile Sites," *USA Today*, April 4, 2012, p. 3B.) Suppose that the download time is normally distributed, with a standard deviation of 1.3 seconds. What is the probability that a download time is
a. less than 2 seconds?
b. between 1.5 and 2.5 seconds?
c. above 1.8 seconds?
d. Ninety-nine percent of the download times are slower (higher) than how many seconds?
e. Ninety-five percent of the download times are between what two values, symmetrically distributed around the mean?
f. Suppose that the download times are uniformly distributed between 1 and 9 seconds. What are your answers to (a) through (c)?

6.53 The speed at which you can log into a website through a smartphone is an important quality characteristic of that website. In a recent test, the mean time to log into the Hertz website through a smartphone was 7.524 seconds. (Data extracted from N. Trejos, "Travelers Have No Patience for Slow Mobile Sites," *USA Today*, April 4, 2012, p. 3B.) Suppose that the download time is normally distributed, with a standard deviation of 1.7 seconds. What is the probability that a download time is
a. less than 2 seconds?
b. between 1.5 and 2.5 seconds?
c. above 1.8 seconds?
d. Ninety-nine percent of the download times are slower (higher) than how many seconds?
e. Ninety-five percent of the download times are between what two values, symmetrically distributed around the mean?
f. Suppose that the download times are uniformly distributed between 1 and 14 seconds. What are your answers to (a) through (d)?
g. Compare the results for the JetBlue Airways site computed in Problem 6.52 to those of the Hertz website.

6.54 (Class Project) One theory about the daily changes in the closing price of stock is that these changes follow a *random walk*—that is, these daily events are independent of each other and move upward or downward in a random manner—and can be approximated by a normal distribution. To test this theory, use either a newspaper or the Internet to select one company traded on the NYSE, one company traded on the American Stock Exchange, and one company traded on the NASDAQ and then do the following:
1. Record the daily closing stock price of each of these companies for six consecutive weeks (so that you have 30 values per company).
2. Compute the daily changes in the closing stock price of each of these companies for six consecutive weeks (so that you have 30 values per company).

Note: The random-walk theory pertains to the daily changes in the closing stock price, not the daily closing stock price.

For each of your six data sets, decide whether the data are approximately normally distributed by
a. constructing the stem-and-leaf display, histogram or polygon, and boxplot.
b. comparing data characteristics to theoretical properties.
c. constructing a normal probability plot.
d. Discuss the results of (a) through (c). What can you say about your three stocks with respect to daily closing prices and daily changes in closing prices? Which, if any, of the data sets are approximately normally distributed?

CASES FOR CHAPTER 6

Managing Ashland MultiComm Services

The AMS technical services department has embarked on a quality improvement effort. Its first project relates to maintaining the target upload speed for its Internet service subscribers. Upload speeds are measured on a standard scale in which the target value is 1.0. Data collected over the past year indicate that the upload speed is approximately normally distributed, with a mean of 1.005 and a standard deviation of 0.10. Each day, one upload speed is measured. The upload speed is considered acceptable if the measurement on the standard scale is between 0.95 and 1.05.