CHAPTER 8

8.1
$$\overline{X} \pm Z \cdot \frac{\sigma}{\sqrt{n}} = 85 \pm 1.96 \cdot \frac{8}{\sqrt{64}}$$
 83.04 $\leq \mu \leq 86.96$

8.2
$$\overline{X} \pm Z \cdot \frac{\sigma}{\sqrt{n}} = 125 \pm 2.58 \cdot \frac{24}{\sqrt{36}}$$
 114.68 $\leq \mu \leq 135.32$

- 8.3 Since the results of only one sample are used to indicate whether something has gone wrong in the production process, the manufacturer can never know with 100% certainty that the specific interval obtained from the sample includes the true population mean. In order to have 100% confidence, the entire population (sample size N) would have to be selected.
- 8.4 Yes, it is true since 5% of intervals will not include the population mean.
- 8.5 If all possible samples of the same size n=100 are taken, 95% of them will include the true population mean time spent on the site per day. Thus you are 95 percent confident that this sample is one that does correctly estimate the true mean time spent on the site per day.
- You would compute the mean first because you need the mean to compute the standard 8.6 deviation. If you had a sample, you would compute the sample mean. If you had the population mean, you would compute the population standard deviation.
 - If you have a sample, you are computing the sample standard deviation not the (b) population standard deviation needed in Equation 8.1. If you have a population, and have computed the population mean and population standard deviation, you don't need a confidence interval estimate of the population mean since you already have computed it.
- 8.7 If the population mean time spent on the site is 36 minutes a day, the confidence interval estimate stated in Problem 8.5 is correct because it contains the value 36 minutes.
- 8.8 Equation (8.1) assumes that you know the population standard deviation. Because you are selecting a sample of 100 from the population, you are computing a sample standard deviation, not the population standard deviation.

8.9 (a)
$$\overline{X} \pm Z \cdot \frac{\sigma}{\sqrt{n}} = 0.995 \pm 2.58 \cdot \frac{0.02}{\sqrt{50}}$$
 $0.9877 \le \mu \le 1.0023$

- Since the value of 1.0 is included in the interval, there is no reason to believe that the (b) mean is different from 1.0 gallon.
- (c) No. Since σ is known and n = 50, from the Central Limit Theorem, we may assume that the sampling distribution of \overline{X} is approximately normal.

8.9 (d) The reduced confidence level narrows the width of the confidence interval.

cont.
$$\overline{X} \pm Z \cdot \frac{\sigma}{\sqrt{n}} = 0.995 \pm 1.96 \cdot \frac{0.02}{\sqrt{50}}$$

 $0.9895 \le \mu \le 1.0005$

- (b) Since the value of 1.0 is still included in the interval, there is no reason to believe that the mean is different from 1.0 gallon.
- 8.10 (a)

Confidence Interval Estimate for the Mean		
Data		
Population Standard Deviation	1000	
Sample Mean	7500	
Sample Size	64	
Confidence Level	95%	
Intermediate Calculati	ons	
Standard Error of the Mean	125	
ZValue	-1.9600	
Interval Half Width	244.9955	
Confidence Interval		
Interval Lower Limit	7255.00	
Interval Upper Limit	7745.00	

$$\overline{X} \pm Z \cdot \frac{\sigma}{\sqrt{n}} = 7500 \pm 1.96 \cdot \frac{1000}{\sqrt{64}}$$

 $7255.00 \leq \mu \leq 7745.00$

- (b) No. The manufacturer cannot support a claim that the bulbs last an average 8,000 hours with a 95% level of confidence because 8,000 does not fall inside the 95% confidence interval.
- (c) No. Since σ is known and n = 64, from the Central Limit Theorem, we may assume that the sampling distribution of \overline{X} is approximately normal.
- (d) The confidence interval is narrower based on a process standard deviation of 800 hours rather than the original assumption of 1000 hours.

(a)
$$\bar{X} \pm Z \cdot \frac{\sigma}{\sqrt{n}} = 7500 \pm 1.96 \cdot \frac{800}{\sqrt{64}}$$

 $7304.00 \le \mu \le 7696.00$

(b) No. The manufacturer cannot support a claim that the bulbs last an average 8,000 hours with a 95% level of confidence because 8,000 still does not fall inside the 95% confidence interval.

8.11
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 75 \pm 2.0301 \cdot \frac{24}{\sqrt{36}}$$

 $66.8796 \le \mu \le 83.1204$

8.12 (a)
$$df = 9$$
, $\alpha = 0.05$, $t_{\alpha/2} = 2.2622$

(b)
$$df = 9$$
, $\alpha = 0.01$, $t_{\alpha/2} = 3.2498$

(c)
$$df = 31$$
, $\alpha = 0.05$, $t_{\alpha/2} = 2.0395$

(d)
$$df = 64$$
, $\alpha = 0.05$, $t_{\alpha/2} = 1.9977$

(e)
$$df = 15$$
, $\alpha = 0.1$, $t_{\alpha/2} = 1.7531$

8.13 Set 1:
$$4.5 \pm 2.3646 \cdot \frac{3.7417}{\sqrt{8}}$$
 1.3719 $\leq \mu \leq 7.6281$
Set 2: $4.5 \pm 2.3646 \cdot \frac{2.4495}{\sqrt{8}}$ 2.4522 $\leq \mu \leq 6.5478$

The data sets have different confidence interval widths because they have different values for the standard deviation.

8.14 Original data:
$$5.8571 \pm 2.4469 \cdot \frac{6.4660}{\sqrt{7}}$$
 $-0.1229 \le \mu \le 11.8371$
Altered data: $4.00 \pm 2.4469 \cdot \frac{2.1602}{\sqrt{7}}$ $2.0022 \le \mu \le 5.9978$

The presence of an outlier in the original data increases the value of the sample mean and greatly inflates the sample standard deviation.

8.15 (a)

Confidence Interval Estimate for the Mean		
Data		
Sample Standard Deviation	55	
Sample Mean	58	
Sample Size	100	
Confidence Level	95%	
Intermediate Calcula	ations	
Standard Error of the Mean	5.5	
Degrees of Freedom	99	
t Value	1.9842	
Interval Half Width	10.9132	
Confidence Interval		
Interval Lower Limit	47.09	
Interval Upper Limit	68.91	
C		

$$\bar{X} \pm t \cdot \frac{S}{\sqrt{n}} = 58 \pm 1.9842 \cdot \frac{55}{\sqrt{100}}$$

 $47.09 \le \mu \le 68.91$

To estimate the total value of lost sales attributed to the next 1000 showroomers that (b) enter his retail store, he can multiply the 95% confidence interval for the mean by the total number of showroomers that enter his retail store, i.e., between \$47,090 and \$68,910.

8.16 (a) $\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 75 \pm 2.0049 \cdot \frac{9}{\sqrt{55}}$

 $72.57 \le \mu \le 77.43$

Confidence In	terval Estin	nate for t	he M	lear
Commutative in	tervar Estiii	iate ioi t	iie iv	ıeaı

Data	
ole Standard Deviation	9
ole Mean	75
ole Size	55
idence Level	95%
	9:

Intermediate Calculations		
Standard Error of the Mean	1.213559752	
Degrees of Freedom	54	
t Value	2.0049	
Interval Half Width	2.4330	

Confidence Interval		
Interval Lower Limit	72.57	
Interval Upper Limit	77.43	

- (b) You can be 95% confident that the population mean one-time gift donation is somewhere between \$72.57 and \$77.43.
- 8.17 (a) $\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 195.3 \pm 2.1098 \cdot \frac{21.4}{\sqrt{18}}$

 $184.6581 \le \mu \le 205.9419$

- (b) No, a grade of 200 is in the interval.
- It is not unusual to have an observed tread wear index of 210, which is outside the 95% confidence interval for the population mean tread wear index, because the standard deviation of the sample mean σ/\sqrt{n} is smaller than the standard deviation of the population σ of the tread wear index for a single observed treat wear. Hence, the value of a single observed tread wear index varies around the population mean more than a sample mean does.

8.18 PHStat output:

Confidence Interval Estimate for the Mean		
1.406031226		
7.09		
15		
95%		
Intermediate Calculations		
0.363035702		
14		
2.1448		
0.7786		
Confidence Interval		
6.31		
7.87		

(a)
$$\overline{X} \pm t \frac{S}{\sqrt{n}} = 7.09 \pm 2.1448 \frac{1.4060}{\sqrt{15}}$$

$$6.31 \leq \mu \leq 7.87$$

You can be 95% confident that the population mean amount spent for lunch (\$) at a fast-(b) food restaurant is somewhere between \$6.31 and \$7.87.

8.19 PHStat output:

Confidence Interval Estimate	for the Mean	
Data		
Sample Standard Deviation	5.027590543	
Sample Mean	27.12	
Sample Size	25	
Confidence Level	95%	
Intermediate Calcula	tions	
Standard Error of the Mean	1.005518109	
Degrees of Freedom	24	
t Value	2.0639	
Interval Half Width	2.0753	
Confidence Interval		
Interval Lower Limit	25.04	
Interval Upper Limit	29.20	

8.19 (a)
$$\overline{X} \pm t \frac{S}{\sqrt{n}} = 27.12 \pm 2.0639 \left(\frac{5.0276}{\sqrt{25}} \right)$$
 $25.04 \le \mu \le 29.20$

- cont. (b) You can be 95% confident that the population mean MPG of 2013 family sedans is somewhere between 25.04 and 29.02.
 - (c) Because the upper limit of the 95% confidence interval for population mean miles per gallon of 2013 small SUVs is lower than the lower limit of the 95% confidence interval for population mean miles per gallon of 2013 family sedans, you are able to conclude that the population mean miles per gallon of 2013 small SUVs is lower than that of 2013 family sedans.

8.20 PHStat output:

Confidence Interval Estimate for the Mean		
Data		
Sample Standard Deviation	1.841115391	
Sample Mean	22.52941176	
Sample Size	17	
Confidence Level	95%	
Intermediate Calculations		
Standard Error of the Mean	0.446536072	
Degrees of Freedom	16	
t Value	2.1199	
Interval Half Width	0.9466	
Confidence Interval		
Interval Lower Limit	21.58	
Interval Upper Limit	23.48	

(a)
$$\overline{X} \pm t \frac{S}{\sqrt{n}} = 22.5294 \pm 2.1199 \left(\frac{1.8411}{\sqrt{17}} \right)$$
 $21.58 \le \mu \le 23.48$

- (b) You can be 95% confident that the population mean MPG of 2013 small SUVs is somewhere between 21.58 and 23.48.
- (c) Because the upper limit of the 95% confidence interval for population mean miles per gallon of 2013 small SUVs is lower than the lower limit of the 95% confidence interval for population mean miles per gallon of 2013 family sedans, you are able to conclude that the population mean miles per gallon of 2013 small SUVs is lower than that of 2013 family sedans.

PHStat output: One-Year CD 8.21 (a)

Confidence Interval Estimate	for the Mean
Data	
Sample Standard Deviation	0.311051386
Sample Mean	0.65
Sample Size	23
Confidence Level	95%
Intermediate Calcula	ations
Standard Error of the Mean	0.064858697
Degrees of Freedom	22
t Value	2.0739
Interval Half Width	0.1345
Confidence Inten	<i>r</i> al
Interval Lower Limit	0.51
Interval Upper Limit	0.78

(a)
$$\overline{X} \pm t \frac{S}{\sqrt{n}} = 0.65 \pm 2.0739 \left(\frac{0.3111}{\sqrt{23}} \right)$$

PHStat output: Five-Year CD (b)

 $0.51 \le \mu \le 0.78$

Confidence Interval Estimate	e tor the Mear
Data	
Sample Standard Deviation	0.408998101
Sample Mean	1.28
Sample Size	23
Confidence Level	95%
Intermediate Calcula	ations
Standard Error of the Mean	0.085281999
Degrees of Freedom	22
t Value	2.0739
Interval Half Width	0.1769
Confidence Inter	<i>v</i> al
Interval Lower Limit	1.10
Interval Upper Limit	1.45

8.21 (b)
$$\overline{X} \pm t \frac{S}{\sqrt{n}} = 1.28 \pm 2.0739 \left(\frac{0.4090}{\sqrt{23}} \right)$$

cont.

 $1.10 \leq \mu \leq 1.45$

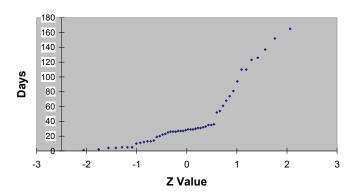
 $31.12 \le \mu \le 54.96$

(c) Because the 95% confidence interval for population mean yield of one-year certificates of deposit does not overlap with that for the population mean yield of five-year certificates of deposit, you are able to conclude that the population mean yield of one-year certificates of deposit is lower than that of five-year certificates of deposit.

8.22 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 43.04 \pm 2.0096 \cdot \frac{41.9261}{\sqrt{50}}$$

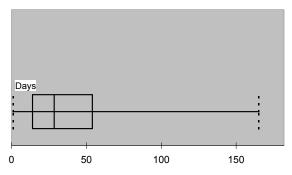
- (b) The population distribution needs to be normally distribution.
- (c)

Normal Probability Plot



(c)

Box-and-whisker Plot



Both the normal probability plot and the boxplot suggest that the distribution is skewed to the right.

(d) Even though the population distribution is not normally distributed, with a sample of 50, the *t* distribution can still be used due to the Central Limit Theorem.

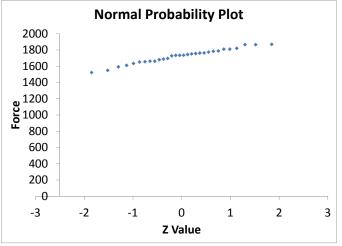
8.23 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 1723.4 \pm 2.0452 \cdot \frac{89.5508}{\sqrt{30}}$$

(c)

 $1689.96 \le \mu \le 1756.84$

Confidence Interval Estimate for the Mean		
Data		
Sample Standard Deviation	89.55083319	
Sample Mean	1723.4	
Sample Size	30	
Confidence Level	95%	
Intermediate Calculations		
Standard Error of the Mean	16.34967046	
Degrees of Freedom	29	
t Value	2.0452	
Interval Half Width	33.4388	
Confidence Interval		
Interval Lower Limit	1689.96	
Interval Upper Limit	1756.84	

The population distribution needs to be normally distributed. (b)



The normal probability plot indicates that the population distribution is normally distributed.

8.24 (a) PHStat output:

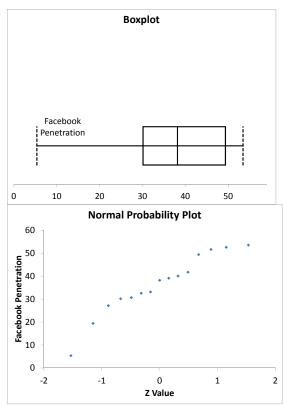
Confidence Interval Estimate for the Mean	
Data	
Sample Standard Deviation	13.19692305
Sample Mean	36.27733333
Sample Size	15
Confidence Level	95%
Intermediate Calcula	ations
Standard Error of the Mean	3.407430881
Degrees of Freedom	14
t Value	2.1448
Interval Half Width	7.3082

Confidence Interval	
Interval Lower Limit	28.97
Interval Upper Limit	43.59

$$\overline{X} \pm t \frac{S}{\sqrt{n}} = 36.2773 \pm 2.1448 \left(\frac{13.1969}{\sqrt{15}} \right)$$

 $28.97 \le \mu \le 43.59$

- (b) The population distribution needs to be normally distributed.
- (c)

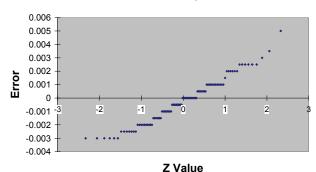


Both the normal probability plot and the boxplot show that the distribution is left-skewed, so with the small sample size, the validity of the confidence interval is in question.

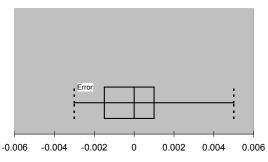
- $\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = -0.00023 \pm 1.9842 \cdot \frac{0.0017}{\sqrt{100}}$ $-0.000566 \le \mu \le 0.000106$ 8.25
 - (b) The population distribution needs to be normally distributed. However, with a sample of 100, the t distribution can still be used as a result of the Central Limit Theorem even if the population distribution is not normal.

(c)

Normal Probability Plot



Box-and-whisker Plot



Both the normal probability plot and the boxplot suggest that the distribution is skewed to the right.

(d) We are 95% confident that the mean difference between the actual length of the steel part and the specified length of the steel part is between -0.000566 and 0.000106 inch, which is narrower than the plus or minus 0.005 inch requirement. The steel mill is doing a good job at meeting the requirement. This is consistent with the finding in Problem 2.39.

8.26
$$p = \frac{X}{n} = \frac{50}{200} = 0.25$$
 $p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.25 \pm 1.96 \sqrt{\frac{0.25(0.75)}{200}}$ $0.19 \le \pi \le 0.31$

8.27
$$p = \frac{X}{n} = \frac{25}{400} = 0.0625$$
 $p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.0625 \pm 2.58 \sqrt{\frac{0.0625(0.9375)}{400}}$
 $0.0313 \le \pi \le 0.0937$

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8.28 (a)

	А	В
1	1 Purchase Additional Telephone Line	
2		
3	Sample Size	500
4	Number of Successes	135
5	Confidence Level	99%
6	Sample Proportion	0.27
7	Z Value	-2.57583451
8	Standard Error of the Proportion	0.019854471
9	Interval Half Width	0.05114183
10	Interval Lower Limit	0.21885817
11	Interval Upper Limit	0.32114183

$$p = \frac{X}{n} = \frac{135}{500} = 0.27$$
 $p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.27 \pm 2.5758 \sqrt{\frac{0.27(1-0.27)}{500}}$

 $0.22 \le \pi \le 0.32$

- (b) The manager in charge of promotional programs concerning residential customers can infer that the proportion of households that would purchase a new cellphone if it were made available at a substantially reduced installation cost is between 0.22 and 0.32 with a 99% level of confidence.
- 8.29 (a) PHStat output:

Confidence Interval Estimate for the Proportion

Data	
Sample Size	3773
Number of Successes	1509
Confidence Level	95%
Intermediate Calculat	ions
Sample Proportion	0.399946992

internediate Calculations	
Sample Proportion	0.399946992
ZValue	-1.9600
Standard Error of the Proportion	0.0080
Interval Half Width	0.0156

Confidence Interval	
Interval Lower Limit	0.3843
Interval Upper Limit	0.4156

$$p = 0.40$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.40 \pm 1.96\sqrt{\frac{0.40(1-0.40)}{3773}}$

 $0.3843 \le \pi \le 0.4156$

8.29 (b) PHStat output: cont.

Confidence Interval Estimate for the Proportio	
Data	
Sample Size	3773
Number of Successes	1207
Confidence Level	95%
Intermediate Calculations	
Sample Proportion	0.319904585
ZValue	-1.9600
Standard Error of the Proportion	0.0076
Interval Half Width	0.0149
Confidence Interval	
Interval Lower Limit	0.3050
Interval Upper Limit	0.3348
$\sqrt{n(1-n)}$	0.22

$$p = 0.32$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.32 \pm 1.96\sqrt{\frac{0.32(1-0.32)}{3773}}$

 $0.3050 \le \pi \le 0.3348$

You can be 95% confident that the population proportion of travelers who said that (c) location was very important for choosing a hotel is somewhere between 0.3843 and 0.4156 while the population proportion of travelers who said that reputation was very important in choosing an airline is somewhere between 0.3050 and 0.3348.

8.30 (a) PHStat output:

Confidence Interval Estimate for the Proportio	
Data	
500	
265	
95%	
ons	
0.53	
-1.9600	
0.0223	
0.0437	
Confidence Interval	
0.4863	
0.5737	

$$p = \frac{X}{n} = \frac{265}{500} = 0.53$$
 $p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.53 \pm 1.96 \sqrt{\frac{0.53(1-0.53)}{500}}$

 $0.4863 \le \pi \le 0.5737$

(b) Since the 95% confidence interval contains 0.50, you cannot claim that more than half of all social media sports fans would likely purchase a brand mentioned by an athlete on a social media site.

(c)
$$p = \frac{X}{n} = \frac{2650}{5000} = 0.53 \quad p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.53 \pm 1.96 \sqrt{\frac{0.53(1-0.53)}{5000}}$$

 $0.5162 \le \pi \le 0.5438$

- (b) Since the lower limit of the 95% confidence interval is greater than 0.50, you can claim that more than half of all social media sports fans would likely purchase a brand mentioned by an athlete on a social media site..
- (d) The larger the sample size, the narrow is the confidence interval holding everything else constant.

8.31 PHStat output:

Confidence Interval Estimate for	the Proportio
Data	
Sample Size	280
Number of Successes	62
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.221428571
ZValue	-1.9600
Standard Error of the Proportion	0.0248
Interval Half Width	0.0486
Confidence Interval	
Interval Lower Limit	0.1728
Interval Upper Limit	0.2701

$$p = \frac{X}{n} = \frac{62}{280} = 0.2214$$

$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.2214 \pm 1.96 \sqrt{\frac{0.2214(1-0.2214)}{280}}$$

$$0.1728 \le \pi \le 0.2701$$

8.32 PHStat output: (a)

Confidence Interval Estimate for the Proportion

Data	
Sample Size	1954
Number of Successes	743
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.38024565
ZValue	-1.9600
Standard Error of the Proportion	0.0110
Interval Half Width	0.0215
Confidence Interval	
Interval Lower Limit	0.3587
Interval Upper Limit	0.4018

8.32
$$p = \frac{X}{n} = \frac{743}{1954} = 0.3802$$
 cont.
$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.3802 \pm 1.96 \sqrt{\frac{0.3802(1-0.3802)}{1954}}$$

 $0.3587 \le \pi \le 0.4018$

(b) PHStat output:

Confidence Interval Estimate for the Proportion	
Data	
Sample Size	1954
Number of Successes	430
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.220061412
ZValue	-1.9600
Standard Error of the Proportion	0.0094
Interval Half Width	0.0184
Confidence Interval	
Interval Lower Limit	0.2017
Interval Upper Limit	0.2384

$$p = \frac{X}{n} = \frac{430}{1954} = 0.2201$$
$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.2201 \pm 1.96 \sqrt{\frac{0.2201(1-0.2201)}{1954}}$$

 $0.2017 \le \pi \le 0.2384$

(c) Since the two confidence intervals do not overlap, you can conclude that with 95% confidence that the population proportion of adult cellphone owners who use their phone to keep themselves occupied during commercials or breaks in something they were watching on television is higher than the population proportion of adult cellphone owners who use their phone to check whether something they heard on television was true.

8.33 (a)
$$p = \frac{94}{115} = 0.8174$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.8174 \pm 1.96\sqrt{\frac{0.8174(1-0.8174)}{115}}$

Confidence Interval Estimate for the Proportion	
Data	
Sample Size	115
Number of Successes	94
Confidence Level	95%
Intermediate Calculations	
Sample Proportion	0.817391304
ZValue	-1.9600
Standard Error of the Proportion	0.0360
Interval Half Width	0.0706
Confidence Interval	
Interval Lower Limit	0.7468
Interval Upper Limit	0.8880

(b)
$$p = \frac{40}{115} = 0.3478$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.3478 \pm 1.96\sqrt{\frac{0.3478(1-0.3478)}{115}}$ $0.2608 \le \pi \le 0.4349$

Confidence Interval Estimate for	the Proportio
Data	
Sample Size	115
Number of Successes	40
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.347826087
Z Value	-1.9600
Standard Error of the Proportion	0.0444
Interval Half Width	0.0870
Confidence Interval	
Interval Lower Limit	0.2608
Interval Upper Limit	0.4349

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8.33 (c) You can be 95% confident that the population proportion of tech CEOs who indicate customer demand as one of the reasons for making strategic change is somewhere between 0.7468 and 0.8880, and the population proportion of tech CEOs who indicate availability of talent as one of the reasons for making strategic change is somewhere between 0.2608 and 0.4349.

8.34
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot 15^2}{5^2} = 34.57$$
 Use $n = 35$

8.35
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{2.58^2 \cdot 100^2}{20^2} = 166.41$$
 Use $n = 167$

8.36
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{2.58^2 (0.5)(0.5)}{(0.04)^2} = 1,040.06$$
 Use $n = 1,041$

8.37
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.4)(0.6)}{(0.02)^2} = 2,304.96$$
 Use $n = 2,305$

8.38 (a)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot 400^2}{50^2} = 245.86$$
 Use $n = 246$

(b)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot 400^2}{25^2} = 983.41$$
 Use $n = 984$

8.39
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot (0.02)^2}{(0.004)^2} = 96.04$$
 Use $n = 97$

8.40
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot (1000)^2}{(200)^2} = 96.04$$
 Use $n = 97$

8.41
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot (0.05)^2}{(0.01)^2} = 96.04$$
 Use $n = 97$

8.42 (a)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{2.5758^2 \cdot 10^2}{3^2} = 73.7211$$
 Use $n = 74$

(b)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot 10^2}{3^2} = 42.6829$$
 Use $n = 43$

8.43 (a)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.645^2 \cdot 45^2}{5^2} = 219.19$$
 Use $n = 220$

(b)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{2.58^2 \cdot 45^2}{5^2} = 539.17$$
 Use $n = 540$

8.44 (a)
$$n = \frac{Z^2 \sigma^2}{\sigma^2} = \frac{1.96^2 \cdot 2^2}{0.25^2} = 245.85$$
 Use $n = 246$

(b)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot 2.5^2}{0.25^2} = 384.15$$
 Use $n = 385$

(c)
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{1.96^2 \cdot 3.0^2}{0.25^2} = 553.17$$
 Use $n = 554$

(d) When there is more variability in the population, a larger sample is needed to accurately estimate the mean.

8.45 (a)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.372)(1 - 0.372)}{(0.04)^2} = 560.8914$$
 Use $n = 561$

(b)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{2.5758^2 (0.372)(1 - 0.372)}{(0.04)^2} = 968.7613$$
 Use $n = 969$

(c)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.372)(1 - 0.372)}{(0.02)^2} = 2,243.5656$$
 Use $n = 2,244$

(d)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{2.5758^2 (0.372)(1 - 0.372)}{(0.02)^2} = 3,875.0450$$
 Use $n = 3,976$

(e) The higher the level of confidence desired, the larger is the sample size required. The smaller the sampling error desired, the larger is the sample size required.

8.46 (a)
$$p = 0.18$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.18 \pm 1.96\sqrt{\frac{0.18(1-0.18)}{300}}$

 $0.1365 \le \pi \le 0.2235$

(b)
$$p = 0.13$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.13 \pm 1.96\sqrt{\frac{0.13(1-0.13)}{300}}$

 $0.0919 \le \pi \le 0.1681$

(c)
$$p = 0.09$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.09 \pm 1.96\sqrt{\frac{0.09(1-0.09)}{300}}$

 $0.0576 \le \pi \le 0.1224$

(d) (a)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.18)(1 - 0.18)}{0.02^2} = 1,417.4983$$
 Use $n = 1,418$

(b)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.13)(1 - 0.13)}{0.02^2} = 1,086.1725$$
 Use $n = 1,087$

(c)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.09)(1 - 0.09)}{0.02^2} = 786.5387$$
 Use $n = 787$

8.47 (a)
$$p = 224/368 = 0.6087$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.6087 \pm 1.96\sqrt{\frac{0.6087(1-0.6087)}{368}}$

Confidence Interval Estimate for	the Proportio
_	
Data	•
Sample Size	368
Number of Successes	224
Confidence Level	95%
Intermediate Calculations	
Sample Proportion	0.608695652
Z Value	-1.9600
Standard Error of the Proportion	0.0254
Interval Half Width	0.0499
Confidence Interval	
Interval Lower Limit	0.5588
Interval Upper Limit	0.6586

(b) You are 95% confident that the proportion of San Francisco Bay Area nonprofits that collaborated with other organizations to provide services. is somewhere between 0.5588 and 0.6586

(c)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.6087)(1 - 0.6087)}{0.01^2} = 9,149.7885$$
 Use $n = 9,150$

Note: This is obtained using PHStat. If you use four decimal places of accuracy on a calculator, you will get 9150.088 which rounds up to 9151.

8.48 (a) If you conducted a follow-up study to estimate the population proportion of individuals who say that banking on their mobile device is convenient, you would use p = 0.77 in the sample size formula because it is based on past information on the proportion.

(b)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{1.96^2 (0.77)(1 - 0.77)}{0.03^2} = 755.9137$$
 Use $n = 756$

8.49 (a)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{2.5758^2 (0.46)(1 - 0.46)}{0.03^2} = 1831.2315$$
 Use $n = 1,832$

(b)
$$n = \frac{Z^2 \pi (1 - \pi)}{e^2} = \frac{2.5758^2 (0.46)(1 - 0.46)}{0.05^2} = 659.2433$$
 Use $n = 660$

(c) The higher the precision you require of your confidence interval, the larger is the sample size required.

- 8.50 The only way to have 100% confidence is to obtain the parameter of interest, rather than a sample statistic. From another perspective, the range of the normal and t distribution is infinite, so a Z or t value that contains 100% of the area cannot be obtained.
- The t distribution is used for obtaining a confidence interval for the mean when σ is unknown. 8.51
- 8.52 If the confidence level is increased, a greater area under the normal or t distribution needs to be included. This leads to an increased value of Z or t, and thus a wider interval.
- The term $\pi(1-\pi)$ reaches its largest value when the population proportion is at 0.5. Hence, the 8.53 sample size $n = \frac{Z^2 \pi (1 - \pi)}{a^2}$ needed to determine the proportion is smaller when the population proportion is 0.20 than when the population proportion is 0.50.
- 8.54 (a) Cellphone:

$$p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.8802 \pm 1.96\sqrt{\frac{0.8802(1-0.8802)}{2253}}$$

$$0.8667 \le \pi \le 0.8936$$

Desktop computer:

$$p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.5801 \pm 1.96\sqrt{\frac{0.5801(1-0.5801)}{2253}}$$

$$0.5597 \le \pi \le 0.6005$$

Laptop computer:

$$p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.6099 \pm 1.96\sqrt{\frac{0.6099(1-0.6099)}{2253}}$$

$$0.5897 \le \pi \le 0.6300$$

Ebook reader:

$$p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.1802 \pm 1.96\sqrt{\frac{0.1802(1-0.1802)}{2253}}$$
$$0.1643 \le \pi \le 0.1961$$

Tablet computer:

$$p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.1802 \pm 1.96\sqrt{\frac{0.1802(1-0.1802)}{2253}}$$

$$0.1643 \le \pi \le 0.1961$$

(b) Most adults have a cellphone. Over half of sdults have a desktop computer and over half have a laptop. Less than 20% have an ebook reader and less than 20% have a tablet computer.

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8.55 (a) Turn off lights:

$$p = 0.39 p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.39 \pm 1.96\sqrt{\frac{0.39(1-0.39)}{897}}$$

$$0.3581 \le \pi \le 0.4219$$

Turn down heat:

$$p = 0.26 p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.26 \pm 1.96\sqrt{\frac{0.26(1-0.26)}{897}}$$

$$0.2313 \le \pi \le 0.2887$$

Install more energy-saving appliances:

$$p = 0.23 p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.23 \pm 1.96\sqrt{\frac{0.23(1-0.23)}{897}}$$

$$0.2025 \le \pi \le 0.2575$$

Drive less/walk more/bicycle more:

$$p = 0.18 p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.18 \pm 1.96\sqrt{\frac{0.18(1-0.18)}{897}}$$

$$0.1549 \le \pi \le 0.2051$$

Unplug things:

$$p = 0.16$$
 $p \pm Z\sqrt{\frac{p(1-p)}{n}} = 0.16 \pm 1.96\sqrt{\frac{0.16(1-0.16)}{897}}$

 $0.1360 \le \pi \le 0.1840$

(b) The population proportion of what adults do to conserve energy, in descending order, is "turn off lights"; "turn down heat"; "install more energy-saving appliances" (which are not statistically different because the confidence intervals overlap); "drive less/walk more/bicycle more" (which are not statistically different because the confidence intervals overlap) and, finally, "unplug things" (which are not statistically different because the confidence intervals overlap).

8.56 (a) PHStat output:

Confidence Interval Estimate for the Mear	
Data	
Sample Standard Deviation	3.5
Sample Mean	41
Sample Size	40
Confidence Level	95%
Intermediate Calcula	ations
Standard Error of the Mean	0.553398591
Degrees of Freedom	39
t Value	2.0227
Interval Half Width	1.1194
Confidence Inten	<i>r</i> al
Interval Lower Limit	39.88
Interval Upper Limit	42.12

 $39.88 \le \mu \le 42.12$

(b) PHStat output:

Confidence Interval Estimate for the Proportion

Data	
Sample Size	40
Number of Successes	30
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.75
ZValue	-1.9600
Standard Error of the Proportion	0.0685
Interval Half Width	0.1342

Confidence Interval	
Interval Lower Limit	0.6158
Interval Upper Limit	0.8842

 $0.6158 \le \pi \le 0.8842$

(c)
$$n = \frac{Z^2 \cdot \sigma^2}{e^2} = \frac{1.96^2 \cdot 5^2}{2^2} = 24.01$$
 Use $n = 25$

(d)
$$n = \frac{Z^2 \cdot \pi \cdot (1 - \pi)}{e^2} = \frac{1.96^2 \cdot (0.5) \cdot (0.5)}{(0.06)^2} = 266.7680$$
 Use $n = 267$

If a single sample were to be selected for both purposes, the larger of the two sample (e) sizes (n = 267) should be used.

8.57 (a) PHStat output:

Confidence Interval Estimate for the Mean	
Data	
Sample Standard Deviation	10
Sample Mean	45
Sample Size	70
Confidence Level	99%
Intermediate Calculations	
Standard Error of the Mean	1.195228609
Degrees of Freedom	69
t Value	2.6490
Interval Half Width	3.1661
Confidence Interval	
Interval Lower Limit	41.83
Interval Upper Limit	48.17

 $41.83 \le \mu \le 48.17$

(b) PHStat output:

Confidence Interval Estimate for the Proportion	
D. L.	
Data	1
Sample Size	70
Number of Successes	36
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.514285714
ZValue	-1.9600
Standard Error of the Proportion	0.0597
Interval Half Width	0.1171
Confidence Interval	
Interval Lower Limit	0.3972
Interval Upper Limit	0.6314

 $0.3972 \le \pi \le 0.6314$

8.58 (a) PHStat output:

Confidence Interval Estimate	for the Mean
Data	
Sample Standard Deviation	7.3
Sample Mean	6.2
Sample Size	25
Confidence Level	95%
Intermediate Calcula	ations
Standard Error of the Mean	1.46
Degrees of Freedom	24
t Value	2.0639
Interval Half Width	3.0133
Confidence Interval	
Interval Lower Limit	3.19
Interval Upper Limit	9.21

 $3.19 \le \mu \le 9.21$

(b) PHStat output:

Confidence Interval Estimate for the Proportio	
Data	
Sample Size	25
Number of Successes	13
Confidence Level	95%
Intermediate Calculations	
Sample Proportion	0.52
ZValue	-1.9600
Standard Error of the Proportion	0.0999
Interval Half Width	0.1958
Confidence Interval	
Interval Lower Limit	0.3242
Interval Upper Limit	0.7158

 $0.3241 \le \pi \le 0.7158$

(c)
$$n = \frac{Z^2 \cdot \sigma^2}{e^2} = \frac{1.96^2 \cdot 8^2}{1.5^2} = 109.2682$$
 Use $n = 110$

(d)
$$n = \frac{Z^2 \cdot \pi \cdot (1 - \pi)}{e^2} = \frac{1.645^2 \cdot (0.5) \cdot (0.5)}{(0.075)^2} = 120.268$$
 Use $n = 121$

If a single sample were to be selected for both purposes, the larger of the two sample (e) sizes (n = 121) should be used.

8.59 (a) PHStat output:

Confidence Interval Estimate for the Mean	
Data	
Sample Standard Deviation	1.5
Sample Mean	8.1
Sample Size	100
Confidence Level	95%
Intermediate Calculations	
Standard Error of the Mean	0.15
Degrees of Freedom	99
t Value	1.9842
Interval Half Width	0.2976
Confidence Interval	
Interval Lower Limit	7.80
Interval Upper Limit	8.40

 $7.8 \le \mu \le 8.4$

(b) PHStat output:

Confidence Interval Estimate for the Proportion

Data	
Sample Size	100
Number of Successes	30
Confidence Level	95%
Intermediate Calculation	ons
Sample Proportion	0.3
ZValue	-1.9600
Standard Error of the Proportion	0.0458
Interval Half Width	0.0898
Confidence Interval	
Interval Lower Limit	0.2102
Interval Upper Limit	0.3898

 $0.2102 \le \pi \le 0.3898$

(c)
$$n = \frac{Z^2 \cdot \sigma^2}{e^2} = \frac{2.58^2 \cdot 1.5^2}{1.5^2} = 6.6349$$
 Use $n = 7$

(d)
$$n = \frac{Z^2 \cdot \pi \cdot (1 - \pi)}{e^2} = \frac{1.645^2 \cdot (0.5) \cdot (0.5)}{(0.045)^2} = 334.07$$
 Use $n = 335$

If a single sample were to be selected for both purposes, the larger of the two sample sizes (n = 335) should be used.

8.60 (a)
$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.31 \pm 1.645 \cdot \sqrt{\frac{0.31(0.69)}{200}}$$
 $0.2562 \le \pi \le 0.3638$

(b)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 3.5 \pm 1.9720 \cdot \frac{2}{\sqrt{200}}$$
 $3.22 \le \mu \le 3.78$

(c)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 18000 \pm 1.9720 \cdot \frac{3000}{\sqrt{200}}$$
 $\$17,581.68 \le \mu \le \$18,418.32$

8.61 (a)
$$\bar{X} \pm t \cdot \frac{S}{\sqrt{n}} = \$21.34 \pm 1.9949 \cdot \frac{\$9.22}{\sqrt{70}}$$
 $\$19.14 \le \mu \le \23.54

(b)
$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.3714 \pm 1.645 \cdot \sqrt{\frac{0.3714(0.6286)}{70}}$$

 $0.2764 \le \pi \le 0.4664$

(c)
$$n = \frac{Z^2 \cdot \sigma^2}{e^2} = \frac{1.96^2 \cdot 10^2}{1.5^2} = 170.74$$
 Use $n = 171$

(d)
$$n = \frac{Z^2 \cdot \pi \cdot (1 - \pi)}{e^2} = \frac{1.645^2 \cdot (0.5) \cdot (0.5)}{(0.045)^2} = 334.08$$
 Use $n = 335$

(e) If a single sample were to be selected for both purposes, the larger of the two sample sizes (n = 335) should be used.

8.62 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = \$38.54 \pm 2.0010 \cdot \frac{\$7.26}{\sqrt{60}}$$
 $\$36.66 \le \mu \le \40.42

(b)
$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.30 \pm 1.645 \cdot \sqrt{\frac{0.30(0.70)}{60}}$$
 $0.2027 \le \pi \le 0.3973$

(c)
$$n = \frac{Z^2 \cdot \sigma^2}{e^2} = \frac{1.96^2 \cdot 8^2}{1.5^2} = 109.27$$
 Use $n = 110$

(d)
$$n = \frac{Z^2 \cdot \pi \cdot (1 - \pi)}{e^2} = \frac{1.645^2 \cdot (0.5) \cdot (0.5)}{(0.04)^2} = 422.82$$
 Use $n = 423$

(e) If a single sample were to be selected for both purposes, the larger of the two sample sizes (n = 423) should be used.

8.63 (a)
$$n = \frac{Z^2 \cdot \pi \cdot (1 - \pi)}{e^2} = \frac{1.96^2 \cdot (0.5) \cdot (0.5)}{(0.05)^2} = 384.16$$
 Use $n = 385$

If we assume that the population proportion is only 0.50, then a sample of 385 would be required. If the population proportion is 0.90, the sample size required is cut to 103.

(b)
$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.84 \pm 1.96 \cdot \sqrt{\frac{0.84(0.16)}{50}}$$

$$0.7384 \le \pi \le 0.9416$$

- (c) The representative can be 95% confidence that the actual proportion of bags that will do the job is between 74.5% and 93.5%. He/she can accordingly perform a cost-benefit analysis to decide if he/she wants to sell the Ice Melt product.
- 8.64 (a)

Confidence Interval Estimate for	the Proportio
Data	
Sample Size	90
· ·	
Number of Successes	51
Confidence Level	95%
Intermediate Calculations	
Sample Proportion	0.566666667
ZValue	-1.9600
Standard Error of the Proportion	0.0522
Interval Half Width	0.1024
Confidence Interval	
Interval Lower Limit	0.4643
Interval Upper Limit	0.6690

$$p \pm Z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.5667 \pm 1.96 \cdot \sqrt{\frac{0.5667(1-0.5667)}{90}}$$

 $0.4643 \le \pi \le 0.6690$

8.64 (b) cont.

Confidence Interval Estimate for the Mean	
Data	
Sample Standard Deviation	1103.6491
Sample Mean	563.38
Sample Size	51
Confidence Level	95%
Intermediate Calculations	
Standard Error of the Mean	154.5417855
Degrees of Freedom	50
t Value	2.0086
Interval Half Width	310.4063
Confidence Interval	
Interval Lower Limit	252.97
Interval Upper Limit	873.79

$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 563.38 \pm 2.0086 \left(\frac{1103.6491}{\sqrt{51}} \right)$$
 \$252.97 \le \mu \le \mathbb{\$\frac{8}{873.79}}\$

8.65 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 5.5014 \pm 2.6800 \cdot \frac{0.1058}{\sqrt{50}}$$
 $5.46 \le \mu \le 5.54$

- Since 5.5 grams is within the 99% confidence interval, the company can claim that the mean (b) weight of tea in a bag is 5.5 grams with a 99% level of confidence.
- (c) The assumption is valid as the weight of the tea bags is approximately normally distributed.

8.66 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 8.4209 \pm 2.0106 \cdot \frac{0.0461}{\sqrt{49}}$$
 $8.41 \le \mu \le 8.43$

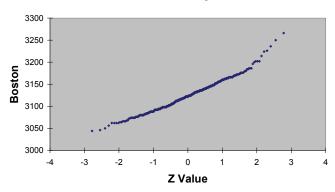
- With 95% confidence, the population mean width of troughs is somewhere between 8.41 (b) and 8.43 inches. Hence, the company's requirement of troughs being between 8.31 and 8.61 is being met with a 95% level of confidence.
- The assumption is valid as the width of the troughs is approximately normally distributed. (c)

8.67 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 3124.2147 \pm 1.9665 \cdot \frac{34.713}{\sqrt{368}}$$
 $3120.66 \le \mu \le 3127.77$

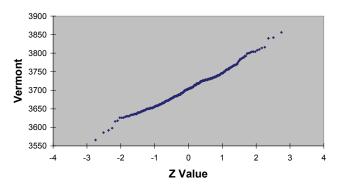
(b)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 3704.0424 \pm 1.9672 \cdot \frac{46.7443}{\sqrt{330}}$$
 $3698.98 < \mu < 3709.10$

(c)

Normal Probability Plot



Normal Probability Plot



The weight for Boston shingles is slightly skewed to the right while the weight for Vermont shingles appears to be slightly skewed to the left.

(d) Since the two confidence intervals do not overlap, the mean weight of Vermont shingles is greater than the mean weight of Boston shingles.

8.68 (a)
$$\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 0.2641 \pm 1.9741 \cdot \frac{0.1424}{\sqrt{170}}$$

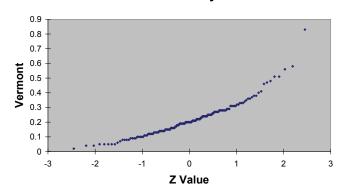
$$0.1975 \le \mu \le 0.2385$$

 $0.2425 \le \mu \le 0.2856$

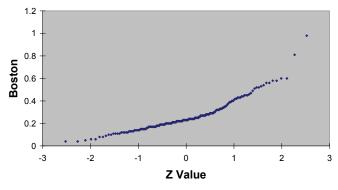
 $\overline{X} \pm t \cdot \frac{S}{\sqrt{n}} = 0.218 \pm 1.9772 \cdot \frac{0.1227}{\sqrt{140}}$

(c)

Normal Probability Plot



Normal Probability Plot



The amount of granule loss for both brands are skewed to the right but the sample sizes are large enough so the violation of the normality assumption is not critical.

(d) Because the two confidence intervals do not overlap, you can conclude that the mean granule loss of Boston shingles is higher than that of Vermont shingles