CHAPTER 5

5.1 PHStat output for Distribution A:

Probabilities & Outcomes:	P	X	Y
	0.5	0	
	0.2	1	
	0.15	2	
	0.1	3	
	0.05	4	
Statistics			
E(X)	1		
E(Y)	0		
Variance(X)	1.5		
Standard Deviation(X)	1.224745		
Variance(Y)	0		
Standard Deviation(Y)	0		
Covariance(XY)	0		
Variance(X+Y)	1.5		
Standard Deviation(X+Y)	1.224745		

PHStat output for Distribution B:

Probabilities & Outcomes:	P	X	Y
	0.05	0	
	0.1	1	
	0.15	2	
	0.2	3	
	0.5	4	
Statistics			
E(X)	3		
E(Y)	0		
Variance(X)	1.5		
Standard Deviation(X)	1.224745		
Variance(Y)	0		
Standard Deviation(Y)	0		
Covariance(XY)	0		
Variance(X+Y)	1.5		
Standard Deviation(X+Y)	1.224745		

5.1 cont.

(a)

Distribution A			Distribution B		
X	$P(X)$	$X*P(X)$	X	$P(X)$	$X*P(X)$
0	0.50	0.00	0	0.05	0.00
1	0.20	0.20	1	0.10	0.10
2	0.15	0.30	2	0.15	0.30
3	0.10	0.30	3	0.20	0.60
4	0.05	0.20	4	0.50	2.00
	1.00	1.00		1.00	3.00
	μ = 1.00			μ = 3.00	

(b) Distribution A

X	$(X-\mu)^2$	$P(X)$	$(X-\mu)^2*P(X)$
0	$(-1)^2$	0.50	0.50
1	$(0)^2$	0.20	0.00
2	$(1)^2$	0.15	0.15
3	$(2)^2$	0.10	0.40
4	$(3)^2$	0.05	0.45
		$\sigma^2=$	1.50

$\sigma = \sqrt{\sum (X-\mu)^2 \cdot P(X)} = 1.22$

(b)

Distribution B

X	$(X-\mu)^2$	$P(X)$	$(X-\mu)^2*P(X)$
0	$(-3)^2$	0.05	0.45
1	$(-2)^2$	0.10	0.40
2	$(-1)^2$	0.15	0.15
3	$(0)^2$	0.20	0.00
4	$(1)^2$	0.50	0.50
		$\sigma^2=$	1.50

$\sigma = \sqrt{\sum (X-\mu)^2 \cdot P(X)} = 1.22$

(c) The means are different but the variances are the same.

5.2 PHStat output:

Probabilities & Outcomes:	P	X
	0.1	0
	0.2	1
	0.45	2
	0.15	3
	0.05	4
	0.05	5
Statistics		
E(X)	2	
E(Y)	0	
Variance(X)	1.4	
Standard Deviation(X)	1.183216	
Variance(Y)	0	
Standard Deviation(Y)	0	
Covariance(XY)	0	
Variance(X+Y)	1.4	
Standard Deviation(X+Y)	1.183216	

(a)-(b)

X	$P(x)$	$X*P(X)$	$(X-\mu_X)^2$	$(X-\mu_X)^2*P(X)$
0	0.10	0.00	4	0.40
1	0.20	0.20	1	0.20
2	0.45	0.90	0	0.00
3	0.15	0.45	1	0.15
4	0.05	0.20	4	0.20
5	0.05	0.25	9	0.45
	(a) Mean =	2.00	Variance =	1.40
			(b) Stdev =	1.18321596

5.3 (a) Based on the fact that the odds of winning are expressed out with a base of 31,478, you will think that the automobile dealership sent out 31,478 fliers.

(b) $\mu = \Sigma_{i=1}^{N} X_i P(X_i) = \$\ 5.80$

(c) $\sigma = \sqrt{\Sigma_{i=1}^{N} [X_i - E(X_i)]^2 P(X_i)} = \$\ 140.88$

(d) The total cost of the prizes is \$25,000 + \$100 + 31,476 * \$5 = \$182,480. Assuming that the cost of producing the fliers is negligible, the cost of reaching a single customer is \$182,480/31478 = \$5.80. The effectiveness of the promotion will depend on how many customers will show up in the show room.

5.4 (a)

X	$P(X)$
\$ – 1	21/36
\$ + 1	15/36

(b)

X	$P(X)$
\$ – 1	21/36
\$ + 1	15/36

(c)

X	$P(X)$
\$ – 1	30/36
\$ + 4	6/36

(d) \$ – 0.167 for each method of play

5.5 PHstat output:

Probabilities & Outcomes:	P	X
	0.07	0
	0.155	1
	0.235	2
	0.205	3
	0.145	4
	0.105	5
	0.05	6
	0.025	7
	0.01	8
Statistics		
E(X)	2.9	
Variance(X)	3.14	
Standard Deviation(X)	1.772005	

(a) $\mu = E(X) = 2.9$

(b) $\sigma = 1.772$

5.6 PHStat output:

Probabilities & Outcomes:	P	X
	0.125	0
	0.240385	1
	0.307692	2
	0.163462	3
	0.086538	4
	0.057692	5
	0.009615	6
	0.009615	7
Statistics		
E(X)	2.105769	
Variance(X)	2.152274	
Standard Deviation(X)	1.467063	

(a) $\mu = E(X) = 2.1058$

(b) $\sigma = 1.4671$

5.7 PHstat output:

Probabilities & Outcomes:	P	X	Y
	0.4	100	200
	0.6	200	100
Weight Assigned to X	0.5		
Statistics			
E(X)	160		
E(Y)	140		
Variance(X)	2400		
Standard Deviation(X)	48.98979		
Variance(Y)	2400		
Standard Deviation(Y)	48.98979		
Covariance(XY)	-2400		
Variance(X+Y)	0		
Standard Deviation(X+Y)	0		
Portfolio Management			
Weight Assigned to X	0.5		
Weight Assigned to Y	0.5		
Portfolio Expected Return	150		
Portfolio Risk	0		

(a) $E(X) = (0.4)(\$100) + (0.6)(\$200) = \$160$
$E(Y) = (0.4)(\$200) + (0.6)(\$100) = \$140$

(b) $\sigma_X = \sqrt{(0.4)(100-160)^2 + (0.6)(200-160)^2} = \sqrt{2400} = \$48.99$

$\sigma_Y = \sqrt{(0.4)(200-140)^2 + (0.6)(100-140)^2} = \sqrt{2400} = \$48.99$

(c) $\sigma_{XY} = (0.4)(100 - 160)(200 - 140) + (0.6)(200 - 160)(100 - 140) = -2400$

(d) $E(X + Y) = E(X) + E(Y) = \$160 + \$140 = \$300$

5.8 PHStat output:

Probabilities & Outcomes:	P	X	Y
	0.2	-100	50
	0.4	50	30
	0.3	200	20
	0.1	300	20
Weight Assigned to X	0.5		
Statistics			
E(X)	90		
E(Y)	30		
Variance(X)	15900		
Standard Deviation(X)	126.0952		
Variance(Y)	120		
Standard Deviation(Y)	10.95445		
Covariance(XY)	-1300		
Variance(X+Y)	13420		
Standard Deviation(X+Y)	115.8447		
Portfolio Management			
Weight Assigned to X	0.5		
Weight Assigned to Y	0.5		
Portfolio Expected Return	60		
Portfolio Risk	57.92236		

(a) $E(X) = (0.2)(\$-100) + (0.4)(\$50) + (0.3)(\$200) + (0.1)(\$300) = \$90$

$E(Y) = (0.2)(\$50) + (0.4)(\$30) + (0.3)(\$20) + (0.1)(\$20) = \$30$

(b)

$$\sigma_X = \sqrt{(0.2)(-100-90)^2 + (0.4)(50-90)^2 + (0.3)(200-90)^2 + (0.1)(300-90)^2}$$
$$= \sqrt{15900} = 126.10$$

$$\sigma_Y = \sqrt{(0.2)(50-30)^2 + (0.4)(30-30)^2 + (0.3)(20-30)^2 + (0.1)(20-30)^2}$$
$$= \sqrt{120} = 10.95$$

(c) $\sigma_{XY} = (0.2)(-100-90)(50-30) + (0.4)(50-90)(30-30) + (0.3)(200-90)(20-30) + (0.1)(300-90)(20-30) = -1300$

(d) $E(X+Y) = E(X) + E(Y) = \$90 + \$30 = \$120$

5.9 (a) $E(P) = (0.4)(\$50) + (0.6)(\$100) = \$80$

(b) $\sigma_P = \sqrt{(.4)^2(9000) + (.6)^2(15000) + 2(.4)(.6)(7500)} = 102.18$

5.10 (a) $E(\text{total time}) = E(\text{time waiting}) + E(\text{time served}) = 4 + 5.5 = 9.5$ minutes

(b) $\sigma(\text{total time}) = \sqrt{1.2^2 + 1.5^2} = 1.9209$ minutes

5.11 (a) $E(P) = 0.3(65) + 0.7(35) = \$44$

$$\sigma_P = \sqrt{(0.3)^2(37{,}525) + (0.7)^2(11{,}025) + 2(0.3)(0.7)(-19{,}275)} = \$\,26.15$$

$$CV = \frac{\sigma_P}{E(P)} = \frac{26.15}{44}(100\%) = 59.44\%$$

(b) $E(P) = 0.7(65) + 0.3(35) = \$\,56$

$$\sigma_P = \sqrt{(0.7)^2(37{,}525) + (0.3)^2(11{,}025) + 2(0.7)(0.3)(-19{,}275)} = \$106.23$$

$$CV = \frac{\sigma_P}{E(P)} = \frac{106.23}{56}(100\%) = 189.69\%$$

(c) Investing 30% in the Dow Jones index and 70% in the weak-economy fund will yield the lowest risk per unit average return at 59.44%. This will be the investment recommendation if you are a risk-averse investor.

5.12 PHStat output for (a)-(c):

Covariance Analysis			
Probabilities & Outcomes:	**P**	**X**	**Y**
	0.1	**-100**	**50**
	0.3	**0**	**150**
	0.3	**80**	**-20**
	0.3	**150**	**-100**
Statistics			
E(X)	**59**		
E(Y)	**14**		
Variance(X)	**6189**		
Standard Deviation(X)	**78.6702**		
Variance(Y)	**9924**		
Standard Deviation(Y)	**99.61928**		
Covariance(XY)	**-6306**		
Variance(X+Y)	**3501**		
Standard Deviation(X+Y)	**59.16925**		

(a) $E(X) = \sum_{i=1}^{N} x_i P(x_i) = 59$

$E(Y) = \sum_{i=1}^{N} y_i P(y_i) = 14$

(b)

$$\sigma_X = \sqrt{\sum_{i=1}^{N}\left[x_i - E(X)\right]^2 P(x_i)} = 78.6702$$

$$\sigma_Y = \sqrt{\sum_{i=1}^{N}\left[y_i - E(Y)\right]^2 P(y_i)} = 99.62$$

(c) $\sigma_{XY} = \sum_{i=1}^{N}\left[x_i - E(X)\right]\left[y_i - E(Y)\right]P(x_i, y_i) = -6306$

(d) Stock *X* gives the investor a lower standard deviation while yielding a higher expected return so the investor should select stock *X*.

5.13 (a) PHStat output:

Probabilities & Outcomes:	P	X	Y
	0.1	-100	50
	0.3	0	150
	0.3	80	-20
	0.3	150	-100
Weight Assigned to X	0.3		
Statistics			
E(X)	59		
E(Y)	14		
Variance(X)	6189		
Standard Deviation(X)	78.6702		
Variance(Y)	9924		
Standard Deviation(Y)	99.61928		
Covariance(XY)	-6306		
Variance(X+Y)	3501		
Standard Deviation(X+Y)	59.16925		
Portfolio Management			
Weight Assigned to X	0.3		
Weight Assigned to Y	0.7		
Portfolio Expected Return	27.5		
Portfolio Risk	52.64266		

$E(P) = \$27.5 \qquad \sigma_P = 52.64 \qquad CV = \dfrac{\sigma_P}{E(P)} = \dfrac{52.64}{27.5}(100\%) = 191.42\%$

5.13 cont. (b) PHStat output:

Probabilities & Outcomes:	P	X	Y
	0.1	-100	50
	0.3	0	150
	0.3	80	-20
	0.3	150	-100
Weight Assigned to X	0.5		
Statistics			
E(X)	59		
E(Y)	14		
Variance(X)	6189		
Standard Deviation(X)	78.6702		
Variance(Y)	9924		
Standard Deviation(Y)	99.61928		
Covariance(XY)	-6306		
Variance(X+Y)	3501		
Standard Deviation(X+Y)	59.16925		
Portfolio Management			
Weight Assigned to X	0.5		
Weight Assigned to Y	0.5		
Portfolio Expected Return	36.5		
Portfolio Risk	29.58462		

$E(P) = \$36.5 \qquad \sigma_P = 29.59 \qquad CV = \dfrac{\sigma_P}{E(P)} = \dfrac{29.59}{36.5}(100\%) = 81.07\%$

5.13 cont. (c) PHStat output:

Probabilities & Outcomes:	P	X	Y
	0.1	-100	50
	0.3	0	150
	0.3	80	-20
	0.3	150	-100
Weight Assigned to X	0.7		
Statistics			
E(X)	59		
E(Y)	14		
Variance(X)	6189		
Standard Deviation(X)	78.6702		
Variance(Y)	9924		
Standard Deviation(Y)	99.61928		
Covariance(XY)	-6306		
Variance(X+Y)	3501		
Standard Deviation(X+Y)	59.16925		
Portfolio Management			
Weight Assigned to X	0.7		
Weight Assigned to Y	0.3		
Portfolio Expected Return	45.5		
Portfolio Risk	35.73863		

$E(P) = \$45.5 \qquad \sigma_P = 35.74 \qquad CV = \dfrac{\sigma_P}{E(P)} = \dfrac{35.74}{45.5}(100\%) = 78.55\%$

(d) Based on the results of (a)-(c), you should recommend a portfolio with 70% of stock *X* and 30% of stock *Y* because it has the lowest risk per unit average return.

5.14 PHStat output:

Probabilities & Outcomes:	P	X	Y
	0.1	-50	-100
	0.3	20	50
	0.4	100	130
	0.2	150	200
Statistics			
E(X)	71		
E(Y)	97		
Variance(X)	3829		
Standard Deviation(X)	61.87891		
Variance(Y)	7101		
Standard Deviation(Y)	84.26743		
Covariance(XY)	5113		
Variance(X+Y)	21156		
Standard Deviation(X+Y)	145.451		

(a) $E(X)$ = \$71 $E(Y)$ = \$97

(b) $\sigma_X = 61.88$ $\sigma_Y = 84.27$

(c) $\sigma_{XY} = 5113$

(d) Stock *Y* gives the investor a higher expected return than stock *X*, but also has a higher standard deviation. Risk-averse investors would invest in stock *X*, whereas risk takers would invest in stock *Y*.

5.15 (a) PHStat output:

Probabilities & Outcomes:	P	X	Y
	0.01	400	-200
	0.09	-30	-100
	0.15	30	50
	0.35	50	90
	0.3	100	250
	0.1	100	225

Weight Assigned to X	0.3

Statistics	
E(X)	63.3
E(Y)	125.5
Variance(X)	2684.11
Standard Deviation(X)	51.8084
Variance(Y)	12572.25
Standard Deviation(Y)	112.126
Covariance(XY)	3075.85
Variance(X+Y)	21408.06
Standard Deviation(X+Y)	146.3149

Portfolio Management	
Weight Assigned to X	0.3
Weight Assigned to Y	0.7
Portfolio Expected Return	106.84
Portfolio Risk	87.71448

$E(P) = \$106.84$ $\sigma_P = \$\ 87.7145$ $CV = \dfrac{\sigma_P}{E(P)} = 82.10\%$

5.15 cont. (b) PHStat output:

Weight Assigned to X	0.5
Statistics	
E(X)	63.3
E(Y)	125.5
Variance(X)	2684.11
Standard Deviation(X)	51.8084
Variance(Y)	12572.25
Standard Deviation(Y)	112.126
Covariance(XY)	3075.85
Variance(X+Y)	21408.06
Standard Deviation(X+Y)	146.3149
Portfolio Management	
Weight Assigned to X	0.5
Weight Assigned to Y	0.5
Portfolio Expected Return	94.4
Portfolio Risk	73.15747

$E(P) = \$94.4 \quad \sigma_P = \$\ 73.1575 \qquad CV = \frac{\sigma_P}{E(P)} = 77.50\%$

(c) PHStat output:

Weight Assigned to X	0.7
Statistics	
E(X)	63.3
E(Y)	125.5
Variance(X)	2684.11
Standard Deviation(X)	51.8084
Variance(Y)	12572.25
Standard Deviation(Y)	112.126
Covariance(XY)	3075.85
Variance(X+Y)	21408.06
Standard Deviation(X+Y)	146.3149
Portfolio Management	
Weight Assigned to X	0.7
Weight Assigned to Y	0.3
Portfolio Expected Return	81.96
Portfolio Risk	61.14387

$E(P) = \$81.96 \quad \sigma_P = \$\ 61.1439 \qquad CV = \frac{\sigma_P}{E(P)} = 74.60\%$

(d) Based on the results of (a)-(c), you should recommend a portfolio with 70% of Black Swan fund and 30% of Good Times fund because it has the lowest risk per unit average return as measured by the coefficient of variation.

5.16 (a) PHStat output:

Covariance Analysis			
Probabilities & Outcomes:	P	X	Y
	0.01	-200	-999
	0.09	-70	-300
	0.15	30	-100
	0.35	80	100
	0.3	100	150
	0.1	120	350
Weight Assigned to X	0.5		
Statistics			
E(X)	66.2		
E(Y)	63.01		
Variance(X)	3273.56		
Standard Deviation(X)	57.21503		
Variance(Y)	38109.75		
Standard Deviation(Y)	195.2172		
Covariance(XY)	10766.74		
Variance(X+Y)	62916.79		
Standard Deviation(X+Y)	250.8322		
Portfolio Management			
Weight Assigned to X	0.5		
Weight Assigned to Y	0.5		
Portfolio Expected Return	64.605		
Portfolio Risk	125.4161		

Let X = corporate bond fund, Y = common stock fund.

(a) $E(X) = \$66.2$ $E(Y) = \$63.01$.

(b) $\sigma_X = \$57.2150$ $\sigma_Y = \$195.2172$

(c) $\sigma_{XY} = 10766.738$

(d) $CV(X) = 86.43\%$ $CV(Y) = 309.82\%$

The corporate bond fund gives the investor a slightly higher expected return than the common stock fund, and has a standard deviation about 1/3 of that of the common stock fund. An investor who does not like risk but desires a high expected return should invest in the corporate bond fund.

(e) According to the probability of 0.01, it is highly unlikely that you will lose \$999 of every \$1,000 invested.

5.17 (a) PHStat output:

Covariance Analysis			
Probabilities & Outcomes:	P	X	Y
	0.01	-200	-999
	0.09	-70	-300
	0.15	30	-100
	0.35	80	100
	0.3	100	150
	0.1	120	350
Weight Assigned to X	0.3		
Statistics			
E(X)	66.2		
E(Y)	63.01		
Variance(X)	3273.56		
Standard Deviation(X)	57.21503		
Variance(Y)	38109.75		
Standard Deviation(Y)	195.2172		
Covariance(XY)	10766.74		
Variance(X+Y)	62916.79		
Standard Deviation(X+Y)	250.8322		
Portfolio Management			
Weight Assigned to X	0.3		
Weight Assigned to Y	0.7		
Portfolio Expected Return	63.967		
Portfolio Risk	153.2659		

$E(P) = \$\ 63.967$ $\qquad \sigma_P = 153.2659$ $\qquad CV = \dfrac{\sigma_P}{E(P)} = 239.60\%$

5.17 cont. (b) PHStat output:

Covariance Analysis			
Probabilities & Outcomes:	P	X	Y
	0.01	-200	-999
	0.09	-70	-300
	0.15	30	-100
	0.35	80	100
	0.3	100	150
	0.1	120	350
Weight Assigned to X	0.5		
Statistics			
E(X)	66.2		
E(Y)	63.01		
Variance(X)	3273.56		
Standard Deviation(X)	57.21503		
Variance(Y)	38109.75		
Standard Deviation(Y)	195.2172		
Covariance(XY)	10766.74		
Variance(X+Y)	62916.79		
Standard Deviation(X+Y)	250.8322		
Portfolio Management			
Weight Assigned to X	0.5		
Weight Assigned to Y	0.5		
Portfolio Expected Return	64.605		
Portfolio Risk	125.4161		

$E(P) = \$64.61$ $\qquad \sigma_P = \$125.4161$ $\qquad CV = \dfrac{\sigma_P}{E(P)} = 194.13\%$

5.17 cont. (c) PHStat output:

Covariance Analysis			
Probabilities & Outcomes:	P	X	Y
	0.01	-200	-999
	0.09	-70	-300
	0.15	30	-100
	0.35	80	100
	0.3	100	150
	0.1	120	350
Weight Assigned to X	0.7		

Statistics	
E(X)	66.2
E(Y)	63.01
Variance(X)	3273.56
Standard Deviation(X)	57.21503
Variance(Y)	38109.75
Standard Deviation(Y)	195.2172
Covariance(XY)	10766.74
Variance(X+Y)	62916.79
Standard Deviation(X+Y)	250.8322

Portfolio Management	
Weight Assigned to X	0.7
Weight Assigned to Y	0.3
Portfolio Expected Return	65.243
Portfolio Risk	97.75455

$E(P) = \$65.24$ $\sigma_P = \$97.75455$ $CV = \dfrac{\sigma_P}{E(P)} = 149.83\%$

(d) Since investing \$700 in the corporate bond fund and \$300 in the common stock fund has the lowest coefficient of variation at 149.83%, you should recommend this portfolio.

5.18 (a) 0.5997
(b) 0.0016
(c) 0.0439
(d) 0.4018

PHstat output for part (d):

Binomial Probabilities						
Data						
Sample size	**6**					
Probability of an event of interest	**0.83**					
Statistics						
Mean	**4.98**					
Variance	**0.8466**					
Standard deviation	**0.920109**					
Binomial Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**2.41E-05**	**2.41E-05**	**0**	**0.999976**	**1**
	1	**0.000707**	**0.000731**	**2.41E-05**	**0.999269**	**0.999976**
	2	**0.008631**	**0.009362**	**0.000731**	**0.990638**	**0.999269**
	3	**0.056184**	**0.065546**	**0.009362**	**0.934454**	**0.990638**
	4	**0.205732**	**0.271277**	**0.065546**	**0.728723**	**0.934454**
	5	**0.401782**	**0.67306**	**0.271277**	**0.32694**	**0.728723**
	6	**0.32694**	**1**	**0.67306**	**0**	**0.32694**

5.19 PHstat output:

Binomial Probabilities						
Data						
Sample size	**5**					
Probability of an event of interest	**0.4**					
Statistics						
Mean	**2**					
Variance	**1.2**					
Standard deviation	**1.095445**					
Binomial Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.07776**	**0.07776**	**0**	**0.92224**	**1**
	1	**0.2592**	**0.33696**	**0.07776**	**0.66304**	**0.92224**
	2	**0.3456**	**0.68256**	**0.33696**	**0.31744**	**0.66304**
	3	**0.2304**	**0.91296**	**0.68256**	**0.08704**	**0.31744**
	4	**0.0768**	**0.98976**	**0.91296**	**0.01024**	**0.08704**
	5	**0.01024**	**1**	**0.98976**	**0**	**0.01024**

(a) $P(X = 4) = 0.0768$
(b) $P(X \leq 3) = 0.9130$
(c) $P(X < 2) = 0.3370$
(d) $P(X > 1) = 0.6630$

5.20 PHStat output for (a):

Data	
Sample size	4
Probability of an event of interest	0.1
Statistics	
Mean	0.4
Variance	0.36
Standard deviation	0.6

PHStat output for (b):

Data	
Sample size	4
Probability of an event of interest	0.4
Statistics	
Mean	1.6
Variance	0.96
Standard deviation	0.979796

PHStat output for (c):

Data	
Sample size	5
Probability of an event of interest	0.8
Statistics	
Mean	4
Variance	0.8
Standard deviation	0.894427

PHStat output for (d):

Data	
Sample size	3
Probability of an event of interest	0.5
Statistics	
Mean	1.5
Variance	0.75
Standard deviation	0.866025

	Mean	Standard Deviation
(a)	0.40	0.60
(b)	1.60	0.980
(c)	4.00	0.894
(d)	1.50	0.866

5.21 Given $\pi = 0.5$ and $n = 5$, $P(X = 5) = 0.0312$.

5.22 Partial PHStat output:

Binomial Probabilities

Data	
Sample size	6
Probability of an event of interest	0.27

Statistics	
Mean	1.62
Variance	1.1826
Standard deviation	1.087474

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	0.151334	0.151334	0	0.848666	1
1	0.335838	0.487172	0.151334	0.512828	0.848666
2	0.310535	0.797707	0.487172	0.202293	0.512828
3	0.15314	0.950847	0.797707	0.049153	0.202293
4	0.042481	0.993328	0.950847	0.006672	0.049153
5	0.006285	0.999613	0.993328	0.000387	0.006672
6	0.000387	1	0.999613	0	0.000387

Let X = number of tablets owned.

(a) $P(X = 4) = 0.0425$

(b) $P(X = 6) = 0.0004$

(c) $P(X \geq 4) = 0.0492$

(d) $E(X) = 1.62$ $\sigma_X = 1.0875$

(e) Each under-25-year-old either owns a tablet or does not own a tablet and that each person surveyed is independent of every other person.

5.23 PHStat output:

Data						
Sample size	**5**					
Probability of an event of interest	**0.25**					
Statistics						
Mean	**1.25**					
Variance	**0.9375**					
Standard deviation	**0.968246**					
Binomial Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.237305**	**0.237305**	**0**	**0.762695**	**1**
	1	**0.395508**	**0.632813**	**0.237305**	**0.367188**	**0.762695**
	2	**0.263672**	**0.896484**	**0.632813**	**0.103516**	**0.367188**
	3	**0.087891**	**0.984375**	**0.896484**	**0.015625**	**0.103516**
	4	**0.014648**	**0.999023**	**0.984375**	**0.000977**	**0.015625**
	5	**0.000977**	**1**	**0.999023**	**0**	**0.000977**

If $\pi = 0.25$ and $n = 5$,

(a) $P(X = 5) = 0.0010$

(b) $P(X \geq 4) = P(X = 4) + P(X = 5) = 0.0146 + 0.0010 = 0.0156$

(c) $P(X = 0) = 0.2373$

(d) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.2373 + 0.3955 + 0.2637 = 0.8965$

5.24 Partial PHStat output:

Data						
Sample size	10					
Probability of an event of interest	0.05					
Statistics						
Mean	0.5					
Variance	0.475					
Standard deviation	0.689202					
Binomial Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.598737	0.598737	0	0.401263	1
	1	0.315125	0.913862	0.598737	0.086138	0.401263
	2	0.074635	0.988496	0.913862	0.011504	0.086138
	3	0.010475	0.998972	0.988496	0.001028	0.011504
	4	0.000965	0.999936	0.998972	6.37E-05	0.001028
	5	6.09E-05	0.999997	0.999936	2.75E-06	6.37E-05
	6	2.67E-06	1	0.999997	8.2E-08	2.75E-06
	7	8.04E-08	1	1	1.61E-09	8.2E-08
	8	1.59E-09	1	1	1.87E-11	1.61E-09
	9	1.86E-11	1	1	9.77E-14	1.87E-11
	10	9.77E-14	1	1	0	9.77E-14

5.24 cont.
(a) $P(X = 0) = 0.5987$
(b) $P(X = 1) = 0.3151$
(c) $P(X \leq 2) = 0.9885$
(d) $P(X \geq 3) = 0.0115$

5.25 Partial PHStat output:

Binomial Probabilities						
Data						
Sample size	**20**					
Probability of an event of interest	**0.05**					
Statistics						
Mean	**1**					
Variance	**0.95**					
Standard deviation	**0.974679**					
Binomial Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.358486**	**0.358486**	**0**	**0.641514**	**1**
	1	**0.377354**	**0.73584**	**0.358486**	**0.26416**	**0.641514**
	2	**0.188677**	**0.924516**	**0.73584**	**0.075484**	**0.26416**

(a) mean = 1 standard deviation = 0.9747
(b) $P(X = 0) = 0.3585$ (c) $P(X = 1) = 0.3774$ (d) $P(X \geq 2) = 0.2642$

5.26 PHStat output:

Binomial Probabilities						
Data						
Sample size	**3**					
Probability of an event of interest	**0.83**					
Statistics						
Mean	**2.49**					
Variance	**0.4233**					
Standard deviation	**0.650615**					
Binomial Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.004913**	**0.004913**	**0**	**0.995087**	**1**
	1	**0.071961**	**0.076874**	**0.004913**	**0.923126**	**0.995087**
	2	**0.351339**	**0.428213**	**0.076874**	**0.571787**	**0.923126**
	3	**0.571787**	**1**	**0.428213**	**0**	**0.571787**

Given $\pi = 0.83$ and $n = 3$,
(a) $P(X = 3) = 0.5718$
(b) $P(X = 0) = 0.0049$

5.26 cont. (c) $P(X \geq 2) = 0.9231$

(d) $E(X) = n\pi = 2.49$

$\sigma_X = \sqrt{n\pi(1-\pi)} = 0.6506$

You can expect 2.49 orders to be filled with an average spread around the mean of 0.6506 orders.

5.27 PHStat output:

Binomial Probabilities						
Data						
Sample size	**3**					
Probability of an event of interest	**0.909**					
Statistics						
Mean	**2.727**					
Variance	**0.248157**					
Standard deviation	**0.498154**					
Binomial Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.000754**	**0.000754**	**0**	**0.999246**	**1**
	1	**0.022582**	**0.023336**	**0.000754**	**0.976664**	**0.999246**
	2	**0.225575**	**0.248911**	**0.023336**	**0.751089**	**0.976664**
	3	**0.751089**	**1**	**0.248911**	**0**	**0.751089**

Given $\pi = .909$ and $n = 3$,

(a) $P(X = 3) = 0.7511$

(b) $P(X = 0) = 0.0008$

(c) $P(X \geq 2) = 0.9767$

(d) $E(X) = n\pi = 2.727$ $\sigma_X = \sqrt{n\pi(1-\pi)} = 0.4982$

You can expect 2.727 orders to be filled with an average spread around the mean of 0.4982 orders.

(e) Because the probability of having a randomly selected order filled is the highest at McDonald's, it has the highest expected number of orders filled followed by Wendy's and finally Burger King. Because the probability of having a randomly selected order filled at Burger King is the closest to 0.5, it has the highest standard deviation, followed by Wendy's and then McDonald's.

5.28 (a) Partial PHStat output:

Poisson Probabilities						
Data						
Average/Expected number of successes:				**2.5**		
Poisson Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	2	**0.256516**	**0.543813**	**0.287297**	**0.456187**	**0.712703**

Using the equation, if $\lambda = 2.5$, $P(X = 2) = \dfrac{e^{-2.5} \cdot (2.5)^2}{2!} = 0.2565$

5.28 cont. (b) Partial PHStat output:

Poisson Probabilities					
Data					
Average/Expected number of successes:			**8**		
Poisson Probabilities Table					
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	8	**0.139587**	**0.592547**	**0.452961**	**0.407453**	**0.547039**

If $\lambda = 8.0$, $P(X = 8) = 0.1396$

(c) Partial PHStat output:

Poisson Probabilities						
Data						
Average/Expected number of successes:			**0.5**			
Poisson Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.606531**	**0.606531**	**0.000000**	**0.393469**	**1.000000**
	1	**0.303265**	**0.909796**	**0.606531**	**0.090204**	**0.393469**

If $\lambda = 0.5$, $P(X = 1) = 0.3033$

(d) Partial PHStat output:

Poisson Probabilities						
Data						
Average/Expected number of successes:			**3.7**			
Poisson Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.024724**	**0.024724**	**0.000000**	**0.975276**	**1.000000**

If $\lambda = 3.7$, $P(X = 0) = 0.0247$

5.29 (a) Partial PHStat output:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:			**2**			
Poisson Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.135335**	**0.135335**	**0.000000**	**0.864665**	**1.000000**
	1	**0.270671**	**0.406006**	**0.135335**	**0.593994**	**0.864665**
	2	**0.270671**	**0.676676**	**0.406006**	**0.323324**	**0.593994**

If $\lambda = 2.0$, $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - [0.1353 + 0.2707]$
$= 0.5940$

5.29 cont. (b) Partial PHStat output:

Poisson Probabilities					
Data					
Average/Expected number of successes:			**8**		
Poisson Probabilities Table					
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.000335**	**0.000335**	**0.000000**	**0.999665**	**1.000000**
	1	**0.002684**	**0.003019**	**0.000335**	**0.996981**	**0.999665**
	2	**0.010735**	**0.013754**	**0.003019**	**0.986246**	**0.996981**
	3	**0.028626**	**0.042380**	**0.013754**	**0.957620**	**0.986246**

If $\lambda = 8.0$, $P(X \geq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$= 1 - [0.0003 + 0.0027 + 0.0107] = 1 - 0.0137 = 0.9863$

(c) Partial PHStat output:

Poisson Probabilities						
Data						
Average/Expected number of successes:			**0.5**			
Poisson Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.606531**	**0.606531**	**0.000000**	**0.393469**	**1.000000**
	1	**0.303265**	**0.909796**	**0.606531**	**0.090204**	**0.393469**

If $\lambda = 0.5$, $P(X \leq 1) = P(X = 0) + P(X = 1) = 0.6065 + 0.3033 = 0.9098$

(d) Partial PHStat output:

Poisson Probabilities						
Data						
Average/Expected number of successes:			**4**			
Poisson Probabilities Table						
	X	**P(X)**	**P(<=X)**	**P(<X)**	**P(>X)**	**P(>=X)**
	0	**0.018316**	**0.018316**	**0.000000**	**0.981684**	**1.000000**
	1	**0.073263**	**0.091578**	**0.018316**	**0.908422**	**0.981684**

If $\lambda = 4.0$, $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.0183 = 0.9817$

5.29 cont. (e) Partial PHStat output:

Poisson Probabilities					
Data					
Average/Expected number of successes:			5		
Poisson Probabilities Table					
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.006738	0.006738	0.000000	0.993262	1.000000
	1	0.033690	0.040428	0.006738	0.959572	0.993262
	2	0.084224	0.124652	0.040428	0.875348	0.959572
	3	0.140374	0.265026	0.124652	0.734974	0.875348

If $\lambda = 5.0$, $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
$= 0.0067 + 0.0337 + 0.0842 + 0.1404 = 0.2650$

5.30 PHStat output for (a) – (d)

Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.006738	0.006738	0.000000	0.993262	1.000000
	1	0.033690	0.040428	0.006738	0.959572	0.993262
	2	0.084224	0.124652	0.040428	0.875348	0.959572
	3	0.140374	0.265026	0.124652	0.734974	0.875348
	4	0.175467	0.440493	0.265026	0.559507	0.734974
	5	0.175467	0.615961	0.440493	0.384039	0.559507
	6	0.146223	0.762183	0.615961	0.237817	0.384039
	7	0.104445	0.866628	0.762183	0.133372	0.237817
	8	0.065278	0.931906	0.866628	0.068094	0.133372
	9	0.036266	0.968172	0.931906	0.031828	0.068094
	10	0.018133	0.986305	0.968172	0.013695	0.031828
	11	0.008242	0.994547	0.986305	0.005453	0.013695
	12	0.003434	0.997981	0.994547	0.002019	0.005453
	13	0.001321	0.999302	0.997981	0.000698	0.002019
	14	0.000472	0.999774	0.999302	0.000226	0.000698
	15	0.000157	0.999931	0.999774	0.000069	0.000226
	16	0.000049	0.999980	0.999931	0.000020	0.000069
	17	0.000014	0.999995	0.999980	0.000005	0.000020
	18	0.000004	0.999999	0.999995	0.000001	0.000005
	19	0.000001	1.000000	0.999999	0.000000	0.000001
	20	0.000000	1.000000	1.000000	0.000000	0.000000

Given $\lambda = 5.0$,

(a) $P(X = 1) = 0.0337$
(b) $P(X < 1) = 0.0067$
(c) $P(X > 1) = 0.9596$
(d) $P(X \leq 1) = 0.0404$

5.31 Partial PHStat output:

Poisson Probabilities					
Data					
Average/Expected number of successes:			2.4		
Poisson Probabilities Table					
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.090718	0.090718	0.000000	0.909282	1.000000
	1	0.217723	0.308441	0.090718	0.691559	0.909282
	2	0.261268	0.569709	0.308441	0.430291	0.691559
	3	0.209014	0.778723	0.569709	0.221277	0.430291

(a) $P(X = 0) = 0.0907$ (b) $P(X = 1) = 0.2177$

(c) $P(X \geq 2) = 0.6916$ (d) $P(X < 3) = 0.5697$

5.32 (a) – (c) Portion of PHStat output

Data						
Average/Expected number of successes:			6			
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.002479	0.002479	0.000000	0.997521	1.000000
	1	0.014873	0.017351	0.002479	0.982649	0.997521
	2	0.044618	0.061969	0.017351	0.938031	0.982649
	3	0.089235	0.151204	0.061969	0.848796	0.938031
	4	0.133853	0.285057	0.151204	0.714943	0.848796
	5	(b) 0.160623	0.445680	(a) 0.285057	0.554320	(c) 0.714943
	6	0.160623	0.606303	0.445680	0.393697	0.554320
	7	0.137677	0.743980	0.606303	0.256020	0.393697
	8	0.103258	0.847237	0.743980	0.152763	0.256020
	9	0.068838	0.916076	0.847237	0.083924	0.152763
	10	0.041303	0.957379	0.916076	0.042621	0.083924
	11	0.022529	0.979908	0.957379	0.020092	0.042621
	12	0.011264	0.991173	0.979908	0.008827	0.020092
	13	0.005199	0.996372	0.991173	0.003628	0.008827
	14	0.002228	0.998600	0.996372	0.001400	0.003628
	15	0.000891	0.999491	0.998600	0.000509	0.001400
	16	0.000334	0.999825	0.999491	0.000175	0.000509
	17	0.000118	0.999943	0.999825	0.000057	0.000175

(a) $P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$$= \frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!} + \frac{e^{-6}(6)^4}{4!}$$

$= 0.002479 + 0.014873 + 0.044618 + 0.089235 + 0.133853 = 0.2851$

(b) $P(X = 5) = \dfrac{e^{-6}(6)^5}{5!} = 0.1606$

(c) $P(X \geq 5) = 1 - P(X < 5) = 1 - 0.2851 = 0.7149$

(d) $P(X = 4 \text{ or } X = 5) = P(X = 4) + P(X = 5) = \dfrac{e^{-6}(6)^4}{4!} + \dfrac{e^{-6}(6)^5}{5!} = 0.2945$

5.33 Partial PHStat output:

Poisson Probabilities					
Data					
Average/Expected number of successes:			6		
Poisson Probabilities Table					
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	3	0.089235	0.151204	0.061969	0.848796	0.938031

If $\lambda = 6.0$, $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= 0.0025 + 0.0149 + 0.0446 + 0.0892 = 0.1512$

$n \bullet P(X \leq 3) = 100 \bullet (0.1512) = 15.12$, so 15 or 16 cookies will probably be discarded.

5.34 Partial PHStat outputu:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:			2.05			
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.128735	0.128735	0.000000	0.871265	1.000000
	1	0.263907	0.392641	0.128735	0.607359	0.871265
	2	0.270504	0.663146	0.392641	0.336854	0.607359
	3	0.184845	0.847990	0.663146	0.152010	0.336854

$\lambda = 2.05$

(a) $P(X = 0) = 0.1287$ (b) $P(X \geq 1) = 0.8713$

(c) $P(X \geq 2) = 0.6074$

5.35 Partial PHStat output:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:			0.74			
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.477114	0.477114	0.000000	0.522886	1.000000
	1	0.353064	0.830178	0.477114	0.169822	0.522886
	2	0.130634	0.960812	0.830178	0.039188	0.169822

$\lambda = 0.74$

(a) $P(X = 0) = 0.4771$ (b) $P(X \geq 1) = 0.5229$

(c) $P(X \geq 2) = 0.1698$

5.36 Partial PHStat output:

Poisson Probabilities					
Data					
Mean/Expected number of events of interest:			2.15		
Poisson Probabilities Table					
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.116484	0.116484	0.000000	0.883516	1.000000
	1	0.250441	0.366925	0.116484	0.633075	0.883516
	2	0.269224	0.636149	0.366925	0.363851	0.633075

$\lambda = 2.15$

(a) $P(X = 0) = 0.1165$

(b) $P(X = 1) = 0.2504$

(c) $P(X > 1) = 0.6331$

(d) $P(X < 2) = 0.3669$

5.37 (a) For the number of problems with 2010 model Ford to be distributed as a Poisson random variable, we need to assume that (i) the probability that a problem occurs in a given Ford is the same for any other new Ford, (ii) the number of problems that a Ford has is independent of the number of problems any other Ford has, (iii) the probability that two or more problems will occur in some area of a Ford approaches zero as the area becomes smaller. Yes, these assumptions are reasonable in this problem.

(b) Partial PHStat output:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:				1.27		
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.280832	0.280832	0.000000	0.719168	1.000000
	1	0.356656	0.637488	0.280832	0.362512	0.719168
	2	0.226477	0.863964	0.637488	0.136036	0.362512

$P(X = 0) = 0.2808$

(c) $P(X \le 2) = 0.8640$

(d) An operational definition for *problem* can be "a specific feature in the car that is not performing according to its intended designed function." The operational definition is important in interpreting the initial quality score because different customers can have different expectations of what function a feature is supposed to perform.

5.38 Partial PHStat output:

Poisson Probabilities					
Data					
Mean/Expected number of events of interest:			1.12		
Poisson Probabilities Table					
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.326280	0.326280	0.000000	0.673720	1.000000
	1	0.365433	0.691713	0.326280	0.308287	0.673720
	2	0.204643	0.896356	0.691713	0.103644	0.308287

(a) $P(X = 0) = 0.3263$

(b) $P(X \leq 2) = 0.8964$

(c) Because Ford had a higher mean rate of problems per car in 2010 than Toyota, the probability of a randomly selected Ford having zero problems and the probability of no more than two problems are both lower than for Toyota.

5.39 Partial PHStat output:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:			1.24			
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.289384	0.289384	0.000000	0.710616	1.000000
	1	0.358836	0.648221	0.289384	0.351779	0.710616
	2	0.222479	0.870699	0.648221	0.129301	0.351779

(a) $P(X = 0) = 0.2894$

(b) $P(X \leq 2) = 0.8707$

(c) Because Ford had a lower mean rate of problems per car in 2009 compared to 2010, the probability of a randomly selected Ford having zero problems and the probability of no more than 2 problems are both higher in 2009 than their values in 2010.

5.40 PHStat output:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:				1.04		
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.353455	0.353455	0.000000	0.646545	1.000000
	1	0.367593	0.721048	0.353455	0.278952	0.646545
	2	0.191148	0.912196	0.721048	0.087804	0.278952

(a) $P(X = 0) = 0.3535$

(b) $P(X \leq 2) = 0.9122$

(c) Because Toyota had a lower mean rate of problems per car in 2009 compared to 2010, the probability of a randomly selected Toyota having zero problems and the probability of no more than 2 problems are both higher in 2009 than their values in 2010.

5.41 Partial PHStat output:

Poisson Probabilities						
Data						
Mean/Expected number of events of interest:				0.8		
Poisson Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.449329	0.449329	0.000000	0.550671	1.000000
	1	0.359463	0.808792	0.449329	0.191208	0.550671
	2	0.143785	0.952577	0.808792	0.047423	0.191208
	3	0.038343	0.990920	0.952577	0.009080	0.047423
	4	0.007669	0.998589	0.990920	0.001411	0.009080
	5	0.001227	0.999816	0.998589	0.000184	0.001411
	6	0.000164	0.999979	0.999816	0.000021	0.000184
	7	0.000019	0.999998	0.999979	0.000002	0.000021
	8	0.000002	1.000000	0.999998	0.000000	0.000002

(a) For the number of phone calls received in a 1-minute period to be distributed as a Poisson random variable, we need to assume that (i) the probability that a phone call is received in a given 1-minute period is the same for all the other 1-minute periods, (ii) the number of phone calls received in a given 1-minute period is independent of the number of phone calls received in any other 1-minute period, (iii) the probability that two or more phone calls received in a time period approaches zero as the length of the time period becomes smaller.

(b) $\lambda = 0.8$, $P(X = 0) = 0.4493$ (c) $\lambda = 0.8$, $P(X \geq 3) = 0.0474$

(d) $\lambda = 0.8$, $P(X \leq 6) = 0.999979$. A maximum of 6 phone calls will be received in a 1-minute period 99.99% of the time.

5.42 (a) PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	4	
No. of successes in population	5	
Population size	10	
Hypergeometric Probabilities Table		
	X	P(X)
	3	0.238095

$$P(X=3)=\frac{\binom{5}{3}\binom{10-5}{4-3}}{\binom{10}{4}}=\frac{\frac{5\cdot4\cdot3!}{3!\cdot2\cdot1}\cdot\frac{5\cdot4!}{4!\cdot1!}}{\frac{10\cdot9\cdot8\cdot7\cdot6!}{6!\cdot4\cdot3\cdot2\cdot1}}=\frac{5}{3\cdot7}=0.2381$$

(b) PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	4	
No. of successes in population	3	
Population size	6	
Hypergeometric Probabilities Table		
	X	P(X)
	1	0.2
	2	0.6
	3	0.2

$$P(X=1)=\frac{\binom{3}{1}\cdot\binom{6-3}{4-1}}{\binom{6}{4}}=\frac{\frac{3\cdot2!}{2!\cdot1}\cdot\frac{3!}{3!\cdot0!}}{\frac{6\cdot5\cdot4!}{4!\cdot2\cdot1}}=\frac{1}{5}=0.2$$

(c) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	5	
No. of successes in population	3	
Population size	12	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.159091

$$P(X=0)=\frac{\binom{3}{0}\cdot\binom{12-3}{5-0}}{\binom{12}{5}}=\frac{\frac{3!}{3!\cdot0!}\cdot\frac{9\cdot8\cdot7\cdot6\cdot5!}{5!\cdot4\cdot3\cdot2\cdot1}}{\frac{12\cdot11\cdot10\cdot9\cdot8\cdot7!}{7!\cdot5\cdot4\cdot3\cdot2\cdot1}}=\frac{7}{44}=0.1591$$

542 cont. (d) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	3	
No. of successes in population	3	
Population size	10	
Hypergeometric Probabilities Table		
	X	P(X)
	3	0.008333

$$P(X=3)=\frac{\binom{3}{3}\cdot\binom{10-3}{3-3}}{\binom{10}{3}}=\frac{\frac{3!}{3!\cdot 0!}\cdot\frac{7!}{7!\cdot 0!}}{\frac{10\cdot 9\cdot 8\cdot 7!}{7!\cdot 3\cdot 2\cdot 1}}=\frac{1}{120}=0.0083$$

5.43 (a) $\mu=\frac{nE}{N}=\frac{4\times 5}{10}=2$ $\sigma=\sqrt{\frac{nE(N-E)}{N^2}}\sqrt{\frac{N-n}{N-1}}=0.8165$

(b) $\mu=\frac{nE}{N}=\frac{4\times 3}{6}=2$ $\sigma=\sqrt{\frac{nE(N-E)}{N^2}}\sqrt{\frac{N-n}{N-1}}=0.6325$

(c) $\mu=\frac{nE}{N}=\frac{5\times 3}{12}=1.25$ $\sigma=\sqrt{\frac{nE(N-E)}{N^2}}\sqrt{\frac{N-n}{N-1}}=0.7724$

(d) $\mu=\frac{nE}{N}=\frac{3\times 3}{10}=0.9$ $\sigma=\sqrt{\frac{nE(N-E)}{N^2}}\sqrt{\frac{N-n}{N-1}}=0.7$

5.44 (a) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	6	
No. of successes in population	25	
Population size	100	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.168918
	1	0.361968
	2	0.305888
	3	0.130286
	4	0.029448
	5	0.003343
	6	0.000149

If $n = 6$, $E = 25$, and $N = 100$,

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - [\frac{\binom{25}{0}\binom{100-25}{6-0}}{\binom{100}{6}} + \frac{\binom{25}{1}\binom{100-25}{6-1}}{\binom{100}{6}}]$$

$$= 1 - [0.1689 + 0.3620] = 0.4691$$

(b) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	6	
No. of successes in population	30	
Population size	100	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.109992
	1	0.304593
	2	0.33459
	3	0.186438
	4	0.05552
	5	0.008368
	6	0.000498

If $n = 6$, $E = 30$, and $N = 100$,

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - [\frac{\binom{30}{0}\binom{100-30}{6-0}}{\binom{100}{6}} + \frac{\binom{30}{1}\binom{100-30}{6-1}}{\binom{100}{6}}]$$

$$= 1 - [0.1100 + 0.3046] = 0.5854$$

5.44 cont. (c) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	**6**	
No. of successes in population	**5**	
Population size	**100**	
Hypergeometric Probabilities Table		
	X	**P(X)**
	0	**0.729085**
	1	**0.243028**
	2	**0.026706**
	3	**0.001161**
	4	**1.87E-05**
	5	**7.97E-08**

If $n = 6$, $E = 5$, and $N = 100$,

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - [\frac{\binom{5}{0}\binom{100-5}{6-0}}{\binom{100}{6}} + \frac{\binom{5}{1}\binom{100-5}{6-1}}{\binom{100}{6}}]$$

$$= 1 - [0.7291 + 0.2430] = 0.0279$$

5.44 cont. (d) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	6	
No. of successes in population	10	
Population size	100	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.522305
	1	0.368686
	2	0.096458
	3	0.011826
	4	0.000706
	5	1.9E-05
	6	1.76E-07

If $n = 6$, $E = 10$, and $N = 100$,

$$P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - [\frac{\binom{10}{0}\binom{100-10}{6-0}}{\binom{100}{6}} + \frac{\binom{10}{1}\binom{100-10}{6-1}}{\binom{100}{6}}]$$

$$= 1 - [0.5223 + 0.3687] = 0.1090$$

(e) The probability that the entire group will be audited is very sensitive to the true number of improper returns in the population. If the true number is very low ($E = 5$), the probability is very low (0.0279). When the true number is increased by a factor of six ($E = 30$), the probability the group will be audited increases by a factor of almost 21 (0.5854).

5.45 Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	5	
No. of events of interest in population	8	
Population size	50	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.401493
	1	0.422625
	2	0.151711
	3	0.022757
	4	0.001388
	5	2.64E-05

5.45 cont.
(a) $P(X = 0) = 0.4015$
(b) $P(X \geq 1) = 1 - 0.4015 = 0.5985$
(c) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.9758$
(d) Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	7	
No. of events of interest in population	8	
Population size	50	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.270096
	1	0.420149
	2	0.238463
	3	0.062753
	4	0.008045
	5	0.000483
	6	1.18E-05
	7	8.01E-08

$P(X = 0) = 0.2701$

5.46 PHStat output:

Data		
Sample size	4	
No. of successes in population	4	
Population size	30	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.545521
	1	0.379493
	2	0.071155
	3	0.003795
	4	3.65E-05

(a) $P(X = 4) = 3.6490 \times 10^{-5}$ (b) $P(X = 0) = 0.5455$
(c) $P(X \geq 1) = 0.4545$
(d) $E = 6$
(a) $P(X = 4) = 0.0005$ (b) $P(X = 0) = 0.3877$
(c) $P(X \geq 1) = 0.6123$

5.47 Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	12	
No. of events of interest in population	8	
Population size	40	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.040415
	1	0.184754
	2	0.32332
	3	0.281148
	4	0.131788
	5	0.033738
	6	0.004542
	7	0.000288
	8	6.44E-06
	9	0

(a) $P(X = 0) = 0.0404$
(b) $P(X \geq 1) = 1 - P(X = 0) = 0.9596$
(c) $P(X = 4) = 0.1318$
(d) $P(X < 4) = 0.8296$

5.48 Partial PHStat output:

Hypergeometric Probabilities		
Data		
Sample size	4	
No. of events of interest in population	6	
Population size	12	
Hypergeometric Probabilities Table		
	X	P(X)
	0	0.030303
	1	0.242424
	2	0.454545
	3	0.242424
	4	0.030303

(a) $P(X = 1) = 0.2424$ (b) $P(X \geq 1) = 1 - P(X = 0) = 0.9697$
(c) $P(X = 3) = 0.2424$
(d) Because the number of events of interest in the population is a smaller fraction of the population size in (c), the probability in (c) is smaller than that in Example 5.7

5.49 The expected value is the average of a probability distribution. It is the value that can be expected to occur on the average, in the long run.

5.50 The four properties of a situation that must be present in order to use the binomial distribution are (i) the sample consists of a fixed number of observations, n, (ii) each observation can be classified into one of two mutually exclusive and collectively exhaustive categories, usually called "an event of interest" and "not an event of interest", (iii) the probability of an observation being classified as "an event of interest", π, is constant from observation to observation and (iv) the outcome (i.e., "an event of interest" or "not an event of interest") of any observation is independent of the outcome of any other observation.

5.51 The four properties of a situation that must be present in order to use the Poisson distribution are (i) you are interested in counting the number of times a particular event occurs in a given area of opportunity (defined by time, length, surface area, and so forth), (ii) the probability that an event occurs in a given area of opportunity is the same for all of the areas of opportunity, (iii) the number of events that occur in one area of opportunity is independent of the number of events that occur in other areas of opportunity and (iv) the probability that two or more events will occur in an area of opportunity approaches zero as the area of opportunity becomes smaller.

5.52 The hypergeometric distribution should be used when the probability of an event of interest of a sample containing n observations is not constant from trial to trial due to sampling without replacement from a finite population.

5.53 (a) $E(X) = 0.001(-\$1,000,000) + (0.999)(\$4,000) = \$2,996$

(b) It's always a "win" situation for the insurance company because the expected profit is \$2,996. It will also be a "win" situation for the promoter if the additional revenue generated is larger than the \$4,000 of purchasing the insurance.

5.54 Partial PHstat output:

Binomial Probabilities

Data	
Sample size	5
Probability of an event of interest	0.65

Statistics	
Mean	3.25
Variance	1.1375
Standard deviation	1.066536

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	0.005252	0.005252	0	0.994748	1
1	0.04877	0.054023	0.005252	0.945978	0.994748
2	0.181147	0.235169	0.054022	0.764831	0.945978
3	0.336416	0.571585	0.235169	0.428415	0.764831
4	0.312386	0.883971	0.571585	0.116029	0.428415
5	0.116029	1	0.883971	0	0.116029

(a) 0.65 (b) 0.65

$\pi = 0.65,\ n = 5$

(c) $P(X = 4) = 0.3124$ (d) $P(X = 0) = 0.0053$

5.54 cont. (e) Stock prices tend to rise in the years when the economy is expanding and fall in the years of recession or contraction. Hence, the probability that the price will rise in one year is not independent from year to year.

5.55 PHStat output:

Binomial Probabilities

Data	
Sample size	10
Probability of an event of interest	0.25

Statistics	
Mean	2.5
Variance	1.875
Standard deviation	1.369306

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	0.056314	0.056314	0	0.943686	1
1	0.187712	0.244025	0.056314	0.755975	0.943686
2	0.281568	0.525593	0.244025	0.474407	0.755975
3	0.250282	0.775875	0.525593	0.224125	0.474407
4	0.145998	0.921873	0.775875	0.078127	0.224125
5	0.058399	0.980272	0.921873	0.019728	0.078127
6	0.016222	0.996494	0.980272	0.003506	0.019728
7	0.00309	0.999584	0.996494	0.000416	0.003506
8	0.000386	0.99997	0.999584	2.96E-05	0.000416
9	2.86E-05	0.999999	0.99997	9.54E-07	2.96E-05
10	9.54E-07	1	0.999999	0	9.54E-07

(a) $P(X = 4) = 0.1460$
(b) $P(X \geq 4) = 0.2241$
(c) $P(X \leq 8) = 0.99997$
(d) The probability that none of the 10 respondents are "cell mostly" internet users is 0.0563, which is very low. Now that you selected the sample in a particular geographical area and found that none of the 10 respondents are "cell mostly" internet users, the 25% figure does not appear to apply to this particular area.

5.56 (a) Partial PHStat output:

Data	
Sample size	13
Probability of an event of interest	0.5
Statistics	
Mean	6.5
Variance	3.25
Standard deviation	1.802776

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
10	0.034912	0.98877	0.953857	0.01123	0.046143

If $\pi = 0.50$ and $n = 13$, $P(X \geq 10) = 0.0461$

5.56 (b) Partial PHStat output:
cont.

Data	
Sample size	13
Probability of an event of interest	0.75

Statistics	
Mean	9.75
Variance	2.4375
Standard deviation	1.561249

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
10	0.251651	0.667398	0.415747	0.332602	0.584253

If $\pi = 0.75$ and $n = 13$, $P(X \geq 10) = 0.5843$

5.57 Portion of the PHStat output:

Data	
Sample size	10
Probability of an event of interest	0.8

Statistics	
Mean	8
Variance	1.6
Standard deviation	1.264911

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	1.02E-07	1.02E-07	0	1	1
5	0.026424	0.032793	0.006369	0.967207	0.993631

(a) $P(X = 0) = 1.024E\text{-}07$
(b) $P(X = 5) = 0.0264$
(c) $P(X > 5) = 0.9672$
(d) $\mu = 8$, $\sigma = 1.2649$

5.58 Portion of the PHStat output:

Data	
Sample size	10
Probability of an event of interest	0.4
Statistics	
Mean	4
Variance	2.4
Standard deviation	1.549193

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	0.006047	0.006047	0	0.993953	1
5	0.200658	0.833761	0.633103	0.166239	0.366897

(a) $P(X = 0) = 0.0060$
(b) $P(X = 5) = 0.2007$
(c) $P(X > 5) = 0.1662$
(d) $\mu = 4$, $\sigma = 1.5492$
(e) Since the percentage of bills containing an error is lower in this problem, the probability is higher in (a) and (b) of this problem and lower in (c).

5.59 Partial PHStat output for Facebook:

Data	
Sample size	10
Probability of an event of interest	0.46
Statistics	
Mean	4.6
Variance	2.484
Standard deviation	1.576071

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	0.002108	0.002108	0	0.997892	1
1	0.01796	0.020068	0.002108	0.979932	0.997892
2	0.068846	0.088914	0.020068	0.911086	0.979932
3	0.156391	0.245305	0.088914	0.754695	0.911086
4	0.233138	0.478443	0.245305	0.521557	0.754695
5	0.238319	0.716762	0.478443	0.283238	0.521557

5.59 Partial PHStat output for Google:
cont.

Data	
Sample size	10
Probability of an event of interest	0.34
Statistics	
Mean	3.4
Variance	2.244
Standard deviation	1.497999

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
0	0.015683	0.015683	0	0.984317	1
1	0.080793	0.096476	0.015683	0.903524	0.984317
2	0.187293	0.28377	0.096476	0.71623	0.903524
3	0.257292	0.541061	0.28377	0.458939	0.71623
4	0.231952	0.773013	0.541061	0.226987	0.458939
5	0.143389	0.916402	0.773013	0.083598	0.226987

(a) $n = 10$, $\pi = 0.46$ $P(X > 5) = 0.2832$
(b) $n = 10$, $\pi = 0.34$ $P(X > 5) = 0.0836$
(c) $n = 10$, $\pi = 0.46$ $P(X = 0) = 0.0021$
(d) The assumptions needed are (i) there are only two mutually exclusive and collectively exhaustive outcomes – "signed in using Facebook" or "signed in using Google" and "did not sign in using Facebook" or "did not sign in using Google", (ii) the probabilities of "signed in" and "did not sign in" are constant, and (iii) the outcome of "signed in" from one log-in is independent of the outcome of "did not sign in" from any other log-ins.

5.60 Partial PHStat output:

Data	
Sample size	20
Probability of an event of interest	0.41
Statistics	
Mean	8.2
Variance	4.838
Standard deviation	2.199545

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
5	0.065636	0.107933	0.042296	0.892067	0.957704
10	0.126753	0.851976	0.725223	0.148024	0.274777

(a) $\mu = n\pi = 8.2$ (b) $\sigma = \sqrt{n\pi(1-\pi)} = 2.1995$
(c) $P(X = 10) = 0.1268$ (d) $P(X \leq 5) = 0.1079$
(e) $P(X \geq 5) = 0.9577$

5.61 Partial PHStat output:

Data						
Sample size	20					
Probability of an event of interest	0.27					
Statistics						
Mean	5.4					
Variance	3.942					
Standard deviation	1.985447					
Binomial Probabilities Table						
	X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
	0	0.001847	0.001847	0	0.998153	1
	1	0.013662	0.015509	0.001847	0.984491	0.998153
	2	0.048006	0.063515	0.015509	0.936485	0.984491
	3	0.106533	0.170048	0.063515	0.829952	0.936485

(a) $\mu = n\pi = 5.4$ (b) $\sigma = \sqrt{n\pi(1-\pi)} = 1.9854$

(c) $P(X = 0) = 0.0018$ (d) $P(X \le 2) = 0.0635$

(e) $P(X \ge 3) = 0.9365$

5.62 (a) $\pi = 0.5$, $P(X \ge 35) = 6.91\times10^{-7}$

Partial PHStat output:

Data	
Sample size	40
Probability of an event of interest	0.5
Statistics	
Mean	20
Variance	10
Standard deviation	3.162278

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
35	5.98E-07	1	0.999999	9.29E-08	6.91E-07

5.62 (b) $\pi = 0.7$, $P(X \geq 35) = 0.0086$
cont. Partial PHStat output:

Data	
Sample size	40
Probability of an event of interest	0.7
Statistics	
Mean	28
Variance	8.4
Standard deviation	2.898275

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
35	0.006057	0.997439	0.991382	0.002561	0.008618

(c) $\pi = 0.9$, $P(X \geq 35) = 0.7937$
Partial PHStat output:

Data	
Sample size	40
Probability of an event of interest	0.9
Statistics	
Mean	36
Variance	3.6
Standard deviation	1.897367

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
35	0.16471	0.370982	0.206273	0.629018	0.793727

(d) Based on the results in (a)-(c), the probability that the Standard & Poor's 500 index will increase if there is an early gain in the first five trading days of the year is very likely to be close to 0.90 because that yields a probability of 79.37% that at least 35 of the 40 years the Standard & Poor's 500 index will increase the entire year.

5.63 (a) $\pi = 0.5$, $P(X \geq 37)$ is essentially zero.

Partial PHStat output:

Binomial Probabilities

Data	
Sample size	**46**
Probability of an event of interest	**0.5**
Statistics	
Mean	**23**
Variance	**11.5**
Standard deviation	**3.391165**

Binomial Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
37	1.57E-05	0.999995	0.99998	4.62E-06	2.03E-05

(b) It will be ludicrous to believe that there is a correlation between the performance of the stock market and the winner of a Super Bowl. If the indicator is a random event, the probability of making a correct prediction 37 or more times out of 46 trials is virtually zero. The fact that the indicator has made 37 correct predictions out of 46 trials is just a reflection that an extremely unlikely event can still occur.

5.64 (a) The assumptions needed are (i) the probability of questionable insurance claims in a given interval in a day is constant, (ii) the probability of questionable insurance claims in this interval approaches zero as the interval gets smaller, (iii) the probability of questionable insurance claims is independent from interval to interval.

Partial PHStat output:

Poisson Probabilities

Data				
Mean/Expected number of events of interest:				**10**

Poisson Probabilities Table

X	P(X)	P(<=X)	P(<X)	P(>X)	P(>=X)
5	0.037833	0.067086	0.029253	0.932914	0.970747
10	0.125110	0.583040	0.457930	0.416960	0.542070
11	0.113736	0.696776	0.583040	0.303224	0.416960

$\lambda = 10.0$

(b) $P(X = 5) = 0.0378$

(c) $P(X \leq 10) = 0.5830$

(d) $P(X \geq 11) = 0.4170$

5.65 (a) $P(\text{jackpot}) = \dfrac{\binom{6}{6}\binom{47}{0}}{\binom{53}{6}} = 4.35588\text{E-}08$

(b) $P(\text{matching 5 numbers}) = \dfrac{\binom{6}{5}\binom{47}{1}}{\binom{53}{6}} = 1.22836\text{E-}05$

(c) $P(\text{matching 4 numbers}) = \dfrac{\binom{6}{4}\binom{47}{2}}{\binom{53}{6}} = 0.000706306$

(d) $P(\text{matching 3 numbers}) = \dfrac{\binom{6}{3}\binom{47}{3}}{\binom{53}{6}} = 0.014126115$

(e) $P(\text{matching 2 numbers}) = \dfrac{\binom{6}{2}\binom{47}{4}}{\binom{53}{6}} = 0.116540448$

(f) $P(\text{matching 1 numbers}) = \dfrac{\binom{6}{1}\binom{47}{5}}{\binom{53}{6}} = 0.40089914$

(g) $P(\text{matching 0 number}) = \dfrac{\binom{6}{0}\binom{47}{6}}{\binom{53}{6}} = 0.467715664$

(h) P(matching 0, 1 or 2 numbers) = 0.467715664 + 0.40089914 + 0.116540448 = 0.985155252

(i) You would expect that the expected value of the payoff and the percentage of the lottery revenue that is being used for education are also reported.