CHAPTER 7

7.1 PHstat output:

Common Data	
Mean	100
Standard Deviation	2

Probability for X <=	
X Value	95
Z Value	-2.5
P(X<=95)	0.0062097

Probability for X >	
X Value	102.2
Z Value	1.1
P(X>102.2)	0.1357

Probability for X<95 or X >102.2	
P(X<95 or X >102.2)	0.1419

Probability for a Range	
From X Value	95
To X Value	97.5
Z Value for 95	-2.5
Z Value for 97.5	-1.25
P(X<=95)	0.0062
P(X<=97.5)	0.1056
P(95<=X<=97.5)	0.0994

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	35.00%
Z Value	-0.38532
X Value	99.22936

(a) $P(\bar{X} < 95) = P(Z < -2.50) = 0.0062$

(b) $P(95 < \bar{X} < 97.5) = P(-2.50 < Z < -1.25) = 0.1056 - 0.0062 = 0.0994$

(c) $P(\bar{X} > 102.2) = P(Z > 1.10) = 1.0 - 0.8643 = 0.1357$

(d) $P(\bar{X} > A) = P(Z > -0.39) = 0.65$ $\bar{X} = 100 - 0.39(\frac{10}{\sqrt{25}}) = 99.22$

7.2 PHStat output:

Common Data	
Mean	50
Standard Deviation	0.5

Probability for X <=	
X Value	47
Z Value	-6
P(X<=47)	9.866E-10

Probability for X >	
X Value	51.5
Z Value	3
P(X>51.5)	0.0013

Probability for X<47 or X >51.5	
P(X<47 or X >51.5)	0.0013

Probability for a Range	
From X Value	47
To X Value	49.5
Z Value for 47	-6
Z Value for 49.5	-1
P(X<=47)	0.0000
P(X<=49.5)	0.1587
P(47<=X<=49.5)	0.1587

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	65.00%
Z Value	0.38532
X Value	50.19266

Probability for X >	
X Value	51.1
Z Value	2.2
P(X>51.1)	0.0139

(a) $P(\bar{X} < 47) = P(Z < -6.00) =$ virtually zero

(b) $P(47 < \bar{X} < 49.5) = P(-6.00 < Z < -1.00) = 0.1587 - 0.00 = 0.1587$

(c) $P(\bar{X} > 51.1) = P(Z > 2.20) = 1.0 - 0.9861 = 0.0139$

(d) $P(\bar{X} > A) = P(Z > 0.39) = 0.35$ $\bar{X} = 50 + 0.39(0.5) = 50.195$

7.3 (a) For samples of 25 customer receipts for a supermarket for a year, the sampling distribution of sample means is the distribution of means from all possible samples of 25 customer receipts for a supermarket for that year.

(b) For samples of 25 insurance payouts in a particular geographical area in a year, the sampling distribution of sample means is the distribution of means from all possible samples of 25 insurance payouts in that particular geographical area in that year.

(c) For samples of 25 Call Center logs of inbound calls tracking handling time for a credit card company during the year, the sampling distribution of sample means is the distribution of means from all possible samples of 25 Call Center logs of inbound calls tracking handling time for a credit card company during that year.

7.4 (a) Sampling Distribution of the Mean for $n = 2$ (without replacement)

Sample Number	Outcomes	Sample Means $\bar{X}_i$
1	1, 3	$\bar{X}_1 = 2$
2	1, 6	$\bar{X}_2 = 3.5$
3	1, 7	$\bar{X}_3 = 4$
4	1, 9	$\bar{X}_4 = 5$
5	1, 10	$\bar{X}_5 = 5.5$
6	3, 6	$\bar{X}_6 = 4.5$
7	3, 7	$\bar{X}_7 = 5$
8	3, 9	$\bar{X}_8 = 6$
9	3, 10	$\bar{X}_9 = 6.5$
10	6, 7	$\bar{X}_{10} = 6.5$
11	6, 9	$\bar{X}_{11} = 7.5$
12	6, 10	$\bar{X}_{12} = 8$
13	7, 9	$\bar{X}_{13} = 8$
14	7, 10	$\bar{X}_{14} = 8.5$
15	9, 10	$\bar{X}_{15} = 9.5$

(a) Mean of All Possible Sample Means: Mean of All Population Elements:

$$\mu_{\bar{X}} = \frac{90}{15} = 6 \qquad \mu = \frac{1+3+6+7+9+10}{6} = 6$$

Both means are equal to 6. This property is called unbiasedness.

7.4 cont. (b) Sampling Distribution of the Mean for n = 3 (without replacement)

Sample Number	Outcomes	Sample Means $\bar{X}_i$
1	1, 3, 6	$\bar{X}_1$ = 3 1/3
2	1, 3, 7	$\bar{X}_2$ = 3 2/3
3	1, 3, 9	$\bar{X}_3$ = 4 1/3
4	1, 3, 10	$\bar{X}_4$ = 4 2/3
5	1, 6, 7	$\bar{X}_5$ = 4 2/3
6	1, 6, 9	$\bar{X}_6$ = 5 1/3
7	1, 6, 10	$\bar{X}_7$ = 5 2/3
8	3, 6, 7	$\bar{X}_8$ = 5 1/3
9	3, 6, 9	$\bar{X}_9$ = 6
10	3, 6, 10	$\bar{X}_{10}$ = 6 1/3
11	6, 7, 9	$\bar{X}_{11}$ = 7 1/3
12	6, 7, 10	$\bar{X}_{12}$ = 7 2/3
13	6, 9, 10	$\bar{X}_{13}$ = 8 1/3
14	7, 9, 10	$\bar{X}_{14}$ = 8 2/3
15	1, 7, 9	$\bar{X}_{15}$ = 5 2/3
16	1, 7, 10	$\bar{X}_{16}$ = 6
17	1, 9, 10	$\bar{X}_{17}$ = 6 2/3
18	3, 7, 9	$\bar{X}_{18}$ = 6 1/3
19	3, 7, 10	$\bar{X}_{19}$ = 6 2/3
20	3, 9, 10	$\bar{X}_{20}$ = 7 1/3

$\mu_{\bar{X}} = \frac{120}{20} = 6$ This is equal to μ, the population mean.

(c) The distribution for n = 3 has less variability. The larger sample size has resulted in sample means being closer to μ.

(d) (a) Sampling Distribution of the Mean for n = 2 (with replacement)

7.4
cont.

Sample Number	Outcomes	Sample Means $\bar{X}_i$
1	1, 1	$\bar{X}_1 = 1$
2	1, 3	$\bar{X}_2 = 2$
3	1, 6	$\bar{X}_3 = 3.5$
4	1, 7	$\bar{X}_4 = 4$
5	1, 9	$\bar{X}_5 = 5$
6	1, 10	$\bar{X}_6 = 5.5$
7	3, 1	$\bar{X}_7 = 2$
8	3, 3	$\bar{X}_8 = 3$
9	3, 6	$\bar{X}_9 = 4.5$
10	3, 7	$\bar{X}_{10} = 5$
11	3, 9	$\bar{X}_{11} = 6$
12	3, 10	$\bar{X}_{12} = 6.5$
13	6, 1	$\bar{X}_{13} = 3.5$
14	6, 3	$\bar{X}_{14} = 4.5$
15	6, 6	$\bar{X}_{15} = 6$
16	6, 7	$\bar{X}_{16} = 6.5$
17	6, 9	$\bar{X}_{17} = 7.5$
18	6, 10	$\bar{X}_{18} = 8$
19	7, 1	$\bar{X}_{19} = 4$
20	7, 3	$\bar{X}_{20} = 5$
21	7, 6	$\bar{X}_{21} = 6.5$
22	7, 7	$\bar{X}_{22} = 7$
23	7, 9	$\bar{X}_{23} = 8$
24	7, 10	$\bar{X}_{24} = 8.5$
25	9, 1	$\bar{X}_{25} = 5$
26	9, 3	$\bar{X}_{26} = 6$
27	9, 6	$\bar{X}_{27} = 7.5$
28	9, 7	$\bar{X}_{28} = 8$
29	9, 9	$\bar{X}_{29} = 9$
30	9, 10	$\bar{X}_{30} = 9.5$
31	10, 1	$\bar{X}_{31} = 5.5$
32	10, 3	$\bar{X}_{32} = 6.5$
33	10, 6	$\bar{X}_{33} = 8$
34	10, 7	$\bar{X}_{34} = 8.5$
35	10, 9	$\bar{X}_{35} = 9.5$
36	10, 10	$\bar{X}_{36} = 10$

7.4 cont. (d) (a) Mean of All Possible Sample Means: $\mu_{\bar{X}} = \frac{216}{36} = 6$ Mean of All Population Elements: $\mu = \frac{1+3+6+7+7+12}{6} = 6$

Both means are equal to 6. This property is called unbiasedness.

(b) Repeat the same process for the sampling distribution of the mean for $n = 3$ (with replacement). There will be $6^3 = 216$ different samples.

$\mu_{\bar{X}} = 6$ This is equal to μ, the population mean.

(c) The distribution for $n = 3$ has less variability. The larger sample size has resulted in more sample means being close to μ.

7.5 (a) Because the population diameter of tennis balls is approximately normally distributed, the sampling distribution of samples of 9 will also be approximately normal with a mean of

$\mu_{\bar{X}} = \mu = 2.63$ and $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 0.01$.

(b) $P(\bar{X} < 2.61) = P(Z < -2.00) = 0.0228$

Probability for X<=	
X Value	2.61
Z Value	-2
P(X<=2.61)	0.0227501

(c) $P(2.62 < \bar{X} < 2.64) = P(-1.00 < Z < 1.00) = 0.6827$

Probability for a Range	
From X Value	2.62
To X Value	2.64
Z Value for 2.62	-1
Z Value for 2.64	1
P(X<=2.62)	0.1587
P(X<=2.64)	0.8413
P(2.62<=X<=2.64)	0.6827

(d) $P(A < \bar{X} < B) = P(-1.000 < Z < 1.000) = 0.68$

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	20.00%
Z Value	-0.841621
X Value	2.621584

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	80.00%
Z Value	0.841621
X Value	2.638416

Lower bound: $\bar{X} = 2.6216$

Upper bound: $\bar{X} = 2.6384$

7.6 (a) When $n = 4$, the shape of the sampling distribution of $\overline{X}$ should closely resemble the shape of the distribution of the population from which the sample is selected. Because the mean is larger than the median, the distribution of the sales price of new houses is skewed to the right, and so is the sampling distribution of $\overline{X}$ although it will be less skewed than the population.

(b) If you select samples of $n = 100$, the shape of the sampling distribution of the sample mean will be very close to a normal distribution with a mean of \$291,200 and a standard deviation of $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$ = \$9,000.

(c) $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{90000}{\sqrt{100}} = 9000$

PHStat output:

Probability for X<=	
X Value	315000
Z Value	2.6444444
P(X<=315000)	0.9959087

$P(\overline{X} < 315{,}000) = P(Z < 2.6444) = 0.9959$

(d) PHStat output:

Probability for a Range	
From X Value	295000
To X Value	310000
Z Value for 295000	0.422222
Z Value for 310000	2.088889
P(X<=295000)	0.6636
P(X<=310000)	0.9816
P(295000<=X<=310000)	0.3181

$P(295{,}000 < \overline{X} < 310{,}000) = P(0.4222 < Z < 2.0889) = 0.3181$

7.7 (a) $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{4}{\sqrt{25}} = 0.8$

PHStat output:

Probability for a Range	
From X Value	13.6
To X Value	14.4
Z Value for 13.6	-0.5000
Z Value for 14.4	0.5000
P(X<=13.6)	0.3085
P(X<=14.4)	0.6915
P(13.6<=X<=14.4)	0.3829

$P(13.6 < \overline{X} < 14.4) = P(-0.50 < Z < 0.50) = 0.3829$

7.7 cont. (b) PHStat output:

Probability for a Range	
From X Value	13
To X Value	14
Z Value for 13	-1.2500
Z Value for 14	0.0000
P(X<=13)	0.1056
P(X<=14)	0.5000
P(13<=X<=14)	0.3944

$P(13 < \overline{X} < 14) = P(-1.25 < Z < 0) = 0.3944$

(c) $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{4}{\sqrt{100}} = 0.4$

PHStat output:

Probability for a Range	
From X Value	13.6
To X Value	14.4
Z Value for 13.6	-1.0000
Z Value for 14.4	1.0000
P(X<=13.6)	0.1587
P(X<=14.4)	0.8413
P(13.6<=X<=14.4)	0.6827

$P(13.6 < \overline{X} < 14.4) = P(-1.00 < Z < 1.00) = 0.6827$

(d) With the sample size increasing from $n = 25$ to $n = 100$, more sample means will be closer to the distribution mean. The standard error of the sampling distribution of size 100 is much smaller than that of size 25, so the likelihood that the sample mean will fall within ±0.4 minutes of the mean is much higher for samples of size 100 (probability = 0.6827) than for samples of size 25 (probability = 0. 3829).

7.8 PHStat output:

Probability for X >	
X Value	26
Z Value	-1.0000
P(X>26)	0.8413

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	85.00%
Z Value	1.0364
X Value	28.0364

(a) $P(\overline{X} > 26) = P(Z > -1.0) = 0.8413$

(b) $P(\overline{X} < A) = P(Z < 1.0364) = 0.85$ $\overline{X} = 27 + 1.0364\ (1) = 28.0364$

(c) To be able to use the standard normal distribution as an approximation for the area under the curve, we must assume that the population is symmetrically distributed such that the central limit theorem will likely hold for samples of $n = 16$.

(d) PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	85.00%
Z Value	1.0364
X Value	27.51822

$P(\overline{X} < A) = P(Z < 1.0364) = 0.85$ $\overline{X} = 27 + 1.0364\ (0.5) = 27.5182$

7.9 (a) $p = 48/64 = 0.75$ (b) $\sigma_p = \sqrt{\dfrac{0.70(0.30)}{64}} = 0.0573$

7.10 (a) $p = 20/50 = 0.40$ (b) $\sigma_p = \sqrt{\dfrac{(0.45)(0.55)}{50}} = 0.0704$

7.11 (a) $p = 14/40 = 0.35$ (b) $\sigma_p = \sqrt{\dfrac{0.30(0.70)}{40}} = 0.0725$

7.12 (a) $\mu_p = \pi = 0.501$, $\sigma_p = \sqrt{\dfrac{\pi(1-\pi)}{n}} = \sqrt{\dfrac{0.501(1-0.501)}{100}} = 0.05$

Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	0.98
P(X>0.55)	0.1635

$P(p > 0.55) = P(Z > 0.98) = 1 – 0.8365 = 0.1635$

(b) $\mu_p = \pi = 0.60$, $\sigma_p = \sqrt{\dfrac{\pi(1-\pi)}{n}} = \sqrt{\dfrac{0.6(1-0.6)}{100}} = 0.04899$

Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	-1.020621
P(X>0.55)	0.8463

$P(p > 0.55) = P(Z > – 1.021) = 1 – 0.1539 = 0.8461$

(c) $\mu_p = \pi = 0.49$, $\sigma_p = \sqrt{\dfrac{\pi(1-\pi)}{n}} = \sqrt{\dfrac{0.49(1-0.49)}{100}} = 0.05$

Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	1.2002401
P(X>0.55)	0.1150

$P(p > 0.55) = P(Z > 1.20) = 1 – 0.8849 = 0.1151$

(d) Increasing the sample size by a factor of 4 decreases the standard error by a factor of 2.

7.12 cont. (d) (a) Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	1.9600039
P(X>0.55)	0.0250

$P(p > 0.55) = P\ (Z > 1.96) = 1 - 0.9750 = 0.0250$

(b) Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	-2.041241
P(X>0.55)	0.9794

$P(p > 0.55) = P\ (Z > -2.04) = 1 - 0.0207 = 0.9793$

(c) Partial PHstat output:

Probability for X >	
X Value	0.55
Z Value	2.4004801
P(X>0.55)	0.0082

$P(p > 0.55) = P\ (Z > 2.40) = 1 - 0.9918 = 0.0082$

If the sample size is increased to 400, the probably in (a), (b) and (c) is smaller, larger, and smaller, respectively because the standard error of the sampling distribution of the sample proportion becomes smaller and, hence, the sampling distribution is more concentrated around the true population proportion.

7.13 (a) Partial PHstat output:

Probability for a Range	
From X Value	0.5
To X Value	0.6
Z Value for 0.5	0
Z Value for 0.6	2.828427
P(X<=0.5)	0.5000
P(X<=0.6)	0.9977
P(0.5<=X<=0.6)	0.4977

$P(0.50 < p < 0.60) = P(0 < Z < 2.83) = 0.4977$

(b) Partial PHstat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	95.00%
Z Value	1.644854
X Value	0.558154

$P(-1.645 < Z < 1.645) = 0.90$

$p = .50 - 1.645(0.0354) = 0.4418$ $p = .50 + 1.645(0.0354) = 0.5582$

(c) Partial PHstat output:

Probability for X >	
X Value	0.65
Z Value	4.2426407
P(X>0.65)	0.0000

$P(p > 0.65) = P\ (Z > 4.24) =$ virtually zero

7.13 cont. (d) Partial PHstat output:

Probability for X >	
X Value	0.6
Z Value	2.8284271
P(X>0.6)	0.0023

If $n = 200$, $P(p > 0.60) = P(Z > 2.83) = 1.0 - 0.9977 = 0.0023$

Probability for X >	
X Value	0.55
Z Value	3.1622777
P(X>0.55)	0.00078

If $n = 1000$, $P(p > 0.55) = P(Z > 3.16) = 1.0 - 0.99921 = 0.00079$

More than 60% correct in a sample of 200 is more likely than more than 55% correct in a sample of 1000.

7.14 (a) $\mu_p = \pi = 0.80,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.80(1-0.80)}{100}} = 0.04$

Partial PHStat output:

Probability for X<=	
X Value	0.85
Z Value	1.2500
P(X<=0.85)	0.8944

$P(p < 0.85) = P(Z < 1.2500) = 0.8944$

(b) Partial PHStat output:

Probability for a Range	
From X Value	0.75
To X Value	0.85
Z Value for 0.75	-1.2500
Z Value for 0.85	1.2500
P(X<=0.75)	0.1056
P(X<=0.85)	0.8944
P(0.75<=X<=0.85)	0.7887

$P(0.75 < p < 0.85) = P(-1.2500 < Z < 1.2500) = 0.7887$

(c) Partial PHStat output:

Probability for X >	
X Value	0.82
Z Value	0.5000
P(X>0.82)	0.3085

$P(p > 0.82) = P(Z > 0.5000) = 0.3085$

7.14 (d) $\mu_p = \pi = 0.80,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.80(1-0.80)}{400}} = 0.02$

cont. (a) Partial PHStat output:

Probability for X<=	
X Value	0.85
Z Value	2.5000
P(X<=0.85)	0.9938

$P(p < 0.85) = P(Z < 2.5000) = 0.9938$

(b) Partial PHStat output:

Probability for a Range	
From X Value	0.75
To X Value	0.85
Z Value for 0.75	-2.5000
Z Value for 0.85	2.5000
P(X<=0.75)	0.0062
P(X<=0.85)	0.9938
P(0.75<=X<=0.85)	0.9876

$P(0.75 < p < 0.85) = P(-2.5000 < Z < 2.5000) = 0.9876$

(c) Partial PHStat output:

Probability for X >	
X Value	0.82
Z Value	1.0000
P(X>0.82)	0.1587

$P(p > 0.82) = P(Z > 1.0000) = 0.1587$

7.15 (a) $\mu_p = \pi = 0.57,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.57(1-0.57)}{100}} = 0.0495$

PHStat output:

Probability for X<=	
X Value	0.57
Z Value	0.0000
P(X<=0.57)	0.5000

$P(p < 0.57) = P(Z < 0) = 0.5$

(b) Partial PHStat output:

Probability for a Range	
From X Value	0.52
To X Value	0.62
Z Value for 0.52	-1.0099
Z Value for 0.62	1.0099
P(X<=0.52)	0.1563
P(X<=0.62)	0.8437
P(0.52<=X<=0.62)	0.6875

$P(0.52 < p < 0.62) = P(-1.0099 < Z < 1.0099) = 0.6875$

7.15 (c) Partial PHStat output:

Probability for X >	
X Value	0.62
Z Value	1.0099
P(X>0.62)	0.1563

$P(p > 0.62) = P(Z > 1.0099) = 0.1563$

(d) $\mu_p = \pi = 0.57,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.57(1-0.57)}{400}} = 0.0248$

(a) PHStat output:

Probability for X<=	
X Value	0.57
Z Value	0.0000
P(X<=0.57)	0.5000

$P(p < 0.57) = P(Z < 0) = 0.5$

(b) Partial PHStat output:

Probability for a Range	
From X Value	0.52
To X Value	0.62
Z Value for 0.52	-2.0199
Z Value for 0.62	2.0199
P(X<=0.52)	0.0217
P(X<=0.62)	0.9783
P(0.52<=X<=0.62)	0.9566

$P(0.52 < p < 0.62) = P(-2.0199 < Z < 2.0199) = 0.9566$

(c) Partial PHStat output:

Probability for X >	
X Value	0.62
Z Value	2.0199
P(X>0.62)	0.0217

$P(p > 0.62) = P(Z > 2.0199) = 0.0217$

7.16 (a) PHStat output:

Probability for a Range	
From X Value	0.12
To X Value	0.18
Z Value for 0.12	-1.1882
Z Value for 0.18	1.1882
P(X<=0.12)	0.1174
P(X<=0.18)	0.8826
P(0.12<=X<=0.18)	0.7652

Since $n = 200$, which is quite large, we use the sample proportion to approximate the population proportion and, hence, $\pi = 0.15$. Also the sampling distribution of the sample proportion will be close to a normal distribution according to the central limit theorem.

7.16 $\mu_p = \pi = 0.15,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.15(1-0.15)}{200}} = 0.0252$

cont. $P(0.12 < p < 0.18) = P(-1.1882 < Z < 1.1882) = 0.7652$

(b)

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	5.00%
Z Value	-1.6449
X Value	0.1085

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	95.00%
Z Value	1.6449
X Value	0.1915

$P(A < p < B) = P(-1.6449 < Z < 1.6449) = 0.90$

$A = 0.1085$

$B = 0.1915$

(c) PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	2.50%
Z Value	-1.9600
X Value	0.1005

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	97.50%
Z Value	1.9600
X Value	0.1995

$P(A < p < B) = P(-1.96 < Z < 1.96) = 0.95$

$A = 0.1005$

$B = 0.1995$

7.17 (a) $\mu_p = \pi = 0.51,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.51(1-0.51)}{100}} = 0.0500$

PHStat output:

Probability for a Range	
From X Value	0.5
To X Value	0.55
Z Value for 0.5	-0.2000
Z Value for 0.55	0.8002
P(X<=0.5)	0.4207
P(X<=0.55)	0.7882
P(0.5<=X<=0.55)	0.3675

$P(0.5 < p < 0.55) = P(-0.2000 < Z < 0.8002) = 0.3675$

(b) PHStat output:

Find X and Z Given Cum. Pctage.		Find X and Z Given Cum. Pctage.	
Cumulative Percentage	5.00%	Cumulative Percentage	95.00%
Z Value	-1.6449	Z Value	1.6449
X Value	0.4278	X Value	0.5922

$P(A < p < B) = P(-1.6449 < Z < 1.6449) = 0.90$

$A = 0.4278$

$B = 0.5922$

7.17 cont. (c) Partial PHStat output:

Find X and Z Given Cum. Pctage.		Find X and Z Given Cum. Pctage.	
Cumulative Percentage	2.50%	Cumulative Percentage	97.50%
Z Value	-1.9600	Z Value	1.9600
X Value	0.4120	X Value	0.6080

$P(A < p < B) = P(-1.9600 < Z < 1.9600) = 0.95$
$A = 0.4120$
$B = 0.6080$

(d) (a) $\mu_p = \pi = 0.51,\ \sigma_p = \sqrt{\dfrac{\pi(1-\pi)}{n}} = \sqrt{\dfrac{0.51(1-0.51)}{400}} = 0.0250$

PHStat output:

Probability for a Range	
From X Value	0.5
To X Value	0.55
Z Value for 0.5	-0.4001
Z Value for 0.55	1.6003
P(X<=0.5)	0.3445
P(X<=0.55)	0.9452
P(0.5<=X<=0.55)	0.6007

$P(0.5 < p < 0.55) = P(-0.4001 < Z < 1.6003) = 0.6007$

(b) PHStat output:

Find X and Z Given Cum. Pctage.		Find X and Z Given Cum. Pctage.	
Cumulative Percentage	5.00%	Cumulative Percentage	95.00%
Z Value	-1.6449	Z Value	1.6449
X Value	0.4689	X Value	0.5511

$P(A < p < B) = P(-1.6449 < Z < 1.6449) = 0.90$
$A = 0.4689$
$B = 0.5511$

(c) Partial PHStat output:

Find X and Z Given Cum. Pctage.		Find X and Z Given Cum. Pctage.	
Cumulative Percentage	2.50%	Cumulative Percentage	97.50%
Z Value	-1.9600	Z Value	1.9600
X Value	0.4610	X Value	0.5590

$P(A < p < B) = P(-1.9600 < Z < 1.9600) = 0.95$
$A = 0.4610$
$B = 0.5590$

7.18 (a) $\mu_p = \pi = 0.39,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.39(1-0.39)}{100}} = 0.0488$

Partial PHStat output:

Probability for X<=	
X Value	0.3
Z Value	-1.8452
P(X<=0.3)	0.0325

$P(p < 0.3) = P(Z < -1.8452) = 0.0325$

(b) $\mu_p = \pi = 0.39,\ \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.39(1-0.39)}{400}} = 0.0244$

Probability for X<=	
X Value	0.3
Z Value	-3.6904
P(X<=0.3)	0.0001

$P(p < 0.3) = P(Z < -3.6904) = 0.0001$

(c) Increasing the sample size by a factor of 4 decreases the standard error by a factor of $\sqrt{4}$. The sampling distribution of the proportion becomes more concentrated around the true proportion of 0.39 and, hence, the probability in (b) becomes smaller than that in (a).

7.19 Because the average of all the possible sample means of size n is equal to the population mean.

7.20 The variation of the sample means becomes smaller as larger sample sizes are taken. This is due to the fact that an extreme observation will have a smaller effect on the mean in a larger sample than in a small sample. Thus, the sample means will tend to be closer to the population mean as the sample size increases.

7.21 As larger sample sizes are taken, the effect of extreme values on the sample mean becomes smaller and smaller. With large enough samples, even though the population is not normally distributed, the sampling distribution of the mean will be approximately normally distributed.

7.22 The population distribution is the distribution of a particular variable of interest, while the sampling distribution represents the distribution of a statistic.

7.23 When the items of interest and the items not of interest are at least 5, the normal distribution can be used to approximate the binomial distribution.

7.24 $\mu_{\bar{X}} = 0.753 \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.004}{5} = 0.0008$

PHStat output:

Common Data	
Mean	0.753
Standard Deviation	0.0008

Probability for X <=	
X Value	0.74
Z Value	-16.25
P(X<=0.74)	1.117E-59

Probability for X >	
X Value	0.76
Z Value	8.75
P(X>0.76)	0.0000

Probability for X<0.74 or X >0.76	
P(X<0.74 or X >0.76)	0.0000

Probability for a Range	
From X Value	0.74
To X Value	0.75
Z Value for 0.74	-16.25
Z Value for 0.75	-3.75
P(X<=0.74)	0.0000
P(X<=0.75)	0.0001
P(0.74<=X<=0.75)	0.00009

Probability for a Range	
From X Value	0.75
To X Value	0.753
Z Value for 0.75	-3.75
Z Value for 0.753	0
P(X<=0.75)	0.0001
P(X<=0.753)	0.5000
P(0.75<=X<=0.753)	0.4999

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	7.00%
Z Value	-1.475791
X Value	0.751819

(a) $P(0.75 < \bar{X} < 0.753) = P(-3.75 < Z < 0) = 0.5 - 0.00009 = 0.4999$

(b) $P(0.74 < \bar{X} < 0.75) = P(-16.25 < Z < -3.75) = 0.00009$

(c) $P(\bar{X} > 0.76) = P(Z > 8.75) =$ virtually zero

(d) $P(\bar{X} < 0.74) = P(Z < -16.25) =$ virtually zero

(e) $P(\bar{X} < A) = P(Z < -1.48) = 0.07 \qquad X = 0.753 - 1.48(0.0008) = 0.7518$

7.25 $\mu_{\bar{X}} = 2.0 \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.05}{5} = 0.01$

PHStat output:

Common Data	
Mean	2
Standard Deviation	0.01

Probability for X <=	
X Value	1.98
Z Value	-2
P(X<=1.98)	0.022750 1

Probability for X >	
X Value	2.01
Z Value	1
P(X>2.01)	0.1587

Probability for X<1.98 or X >2.01	
P(X<1.98 or X >2.01)	0.1814

Probability for a Range	
From X Value	1.99
To X Value	2
Z Value for 1.99	-1
Z Value for 2	0
P(X<=1.99)	0.1587
P(X<=2)	0.5000
P(1.99<=X<=2)	0.3413

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	1.00%
Z Value	-2.326348
X Value	1.976737

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	99.50%
Z Value	2.575829
X Value	2.025758

(a) $P(1.99 < \bar{X} < 2.00) = P(-1.00 < Z < 0) = 0.5 - 0.1587 = 0.3413$

(b) $P(\bar{X} < 1.98) = P(Z < -2.00) = 0.0228$

(c) $P(\bar{X} > 2.01) = P(Z > 1.00) = 1.0 - 0.8413 = 0.1587$

(d) $P(\bar{X} > A) = P(Z > -2.33) = 0.99 \qquad A = 2.00 - 2.33(0.01) = 1.9767$

(e) $P(A < X < B) = P(-2.58 < Z < 2.58) = 0.99$

$A = 2.00 - 2.58(0.01) = 1.9742 \quad B = 2.00 + 2.58(0.01) = 2.0258$

7.26 $\mu_{\bar{X}} = 4.7$ $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{0.40}{5} = 0.08$

PHstat output:

Common Data	
Mean	4.7
Standard Deviation	0.08
Probability for X >	
X Value	4.6
Z Value	-1.25
P(X>4.6)	0.8944

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	23.00%
Z Value	-0.738847
X Value	4.640892

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	15.00%
Z Value	-1.036433
X Value	4.6170853

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	85.00%
Z Value	1.036433
X Value	4.782915

(a) $P(4.60 < \bar{X}) = P(-1.25 < Z) = 1 - 0.1056 = 0.8944$

(b) $P(A < \bar{X} < B) = P(-1.04 < Z < 1.04) = 0.70$

$A = 4.70 - 1.04(0.08) = 4.6168$ ounces $X = 4.70 + 1.04(0.08) = 4.7832$ ounces

(c) $P(\bar{X} > A) = P(Z > -0.74) = 0.77$ $A = 4.70 - 0.74(0.08) = 4.6408$

7.27 $\mu_{\bar{X}} = 5.0$ $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{0.40}{5} = 0.08$

(a) Partial PHStat output:

Probability for X >	
X Value	4.6
Z Value	-5
P(X>4.6)	1.0000

$P(4.60 < \bar{X}) = P(-5 < Z) =$ essentially 1.0

(b) Partial PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	15.00%
Z Value	-1.036433
X Value	4.917085

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	85.00%
Z Value	1.036433
X Value	5.082915

$P(A < \bar{X} < B) = P(-1.0364 < Z < 1.0364) = 0.70$

$A = 5.0 - 1.0364(0.08) = 4.9171$ ounces

$X = 5.0 + 1.0364(0.08) = 5.0829$ ounces

7.27 cont. (c) Partial PHStat output:

Find X and Z Given Cum. Pctage.	
Cumulative Percentage	23.00%
Z Value	-0.738847
X Value	4.940892

$P(\bar{X} > A) = P(Z > -0.7388) = 0.77$ $A = 5.0 - 0.7388(0.08) = 4.9409$

7.28 $\mu_{\bar{X}} = \mu = 17.87,\ \sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{20}{\sqrt{16}} = 5$

(a) PHStat output:

Probability for X<=	
X Value	0
Z Value	-3.5740
P(X<=0)	0.00017578

$P(\bar{X} < 0) = P(Z < -3.5740) = 0.0002$

(b) PHStat output:

Probability for a Range	
From X Value	-10
To X Value	10
Z Value for -10	-5.5740
Z Value for 10	-1.5740
P(X<=-10)	0.0000
P(X<=10)	0.0577
P(-10<=X<=10)	0.0577

$P(-10 < \bar{X} < 10) = P(0 < Z < 0.0577) = 0.0577$

(c) PHStat output:

Probability for X >	
X Value	10
Z Value	-1.5740
P(X>10)	0.9423

$P(\bar{X} > 10) = P(Z > -1.5740) = 0.9423$

7.29 (a) $\mu = 1.54$ $\sigma = 10$

Partiel PHStat output:

Probability for X<=	
X Value	0
Z Value	-0.1540
P(X<=0)	0.4388

$P(X < 0) = P(Z < -0.1540) = 0.4388$

7.29 cont. (b) Partial PHStat output:

Probability for a Range	
From X Value	-20
To X Value	-10
Z Value for -20	-2.1540
Z Value for -10	-1.1540
P(X<=-20)	0.0156
P(X<=-10)	0.1243
P(-20<=X<=-10)	0.1086

$P(-20 < X < -10) = P(-2.1540 < Z < -1.1540) = 0.1086$

(c) Partial PHStat output:

Probability for X >	
X Value	-5
Z Value	-0.6540
P(X>-5)	0.7434

$P(X > -5) = P(Z > -0.6540) = 0.7434$

(d) $\mu_{\bar{X}} = \mu = 1.54,\ \sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{4}} = 5$

Partiel PHStat output:

Probability for X<=	
X Value	0
Z Value	-0.3080
P(X<=0)	0.3790

$P(X < 0) = P(Z < -0.3080) = 0.3790$

(e) Partial PHStat output:

Probability for a Range	
From X Value	-20
To X Value	-10
Z Value for -20	-4.3080
Z Value for -10	-2.3080
P(X<=-20)	0.0000
P(X<=-10)	0.0105
P(-20<=X<=-10)	0.0105

$P(-20 < X < -10) = P(-4.3080 < Z < -2.3080) = 0.0105$

(f) Partial PHStat output:

Probability for X >	
X Value	-5
Z Value	-1.3080
P(X>-5)	0.9046

$P(X > -5) = P(Z > -1.3080) = 0.9046$

(g) Since the sample mean of returns of a sample of stocks is distributed closer to the population mean than the return of a single stock, the probabilities in (a) and (b) are higher than those in (d) and (e) while the probability in (c) is lower than that in (f).