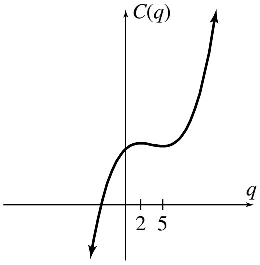
Chapter 13

Apply It 13.1

1. The graph of $c(q) = 2q^3 - 21q^2 + 60q + 500$ is shown.



There looks to be a relative maximum at $q = 2$ and a relative minimum at $q = 5$.

$c'(q) = 6q^2 - 42q + 60 = 6(q^2 - 7q + 10)$
$= 6(q - 5)(q - 2)$
$c'(q) = 0$ when $q = 2$ or $q = 5$. If $q < 2$, then $c'(q) = 6(-)(-) = +$, so $c(q)$ is increasing. If $2 < q < 5$, then $c'(q) = 6(-)(+) = -$, so $c(q)$ is decreasing. If $5 < q$, then $c'(q) = 6(+)(+) = +$, so $c(q)$ is increasing. When $q = 2$, there is a relative maximum, since $c'(q)$ changes from + to –. The relative maximum value is $2(2)^3 - 21(2)^2 + 60(2) + 500 = 552$. When $q = 5$, there is a relative minimum, since $c'(q)$ changes from – to +. The relative minimum value is $2(5)^3 - 21(5)^2 + 60(5) + 500 = 525$.

2. First, find $C'(t)$, with $C(t) = \dfrac{0.14t}{(t+2)^2}$.

$$C'(t) = \frac{0.14(t+2)^2 - 0.14t(2)(t+2)}{(t+2)^4}$$
$$= \frac{0.14(t+2) - 0.28t}{(t+2)^3} = \frac{0.28 - 0.14t}{(t+2)^3}$$
$$= \frac{0.14(2-t)}{(t+2)^3}$$

$C'(t) = 0$ when $t = 2$ and is undefined when $t = -2$. However, since t denotes the number of hours after an injection, negative values of t are not reasonable. If $0 \le t < 2$, $C'(t) = \frac{+}{+} = +$, so $C(t)$ is increasing. If $2 < t$, $C'(t) = \frac{-}{+} = -$, so $C(t)$ is decreasing. When $t = 2$, there is a relative maximum, since $C'(t)$ changes from + to –. The drug is at its greatest concentration 2 hours after the injection.

Problems 13.1

1. Decreasing on (–∞, –1) and (3, ∞); increasing on (–1, 3); relative minimum (–1, –1); relative maximum (3, 4).

2. Decreasing on (–∞, –1) and (0, 1); increasing on (–1, 0) and (1, ∞); relative minima (–1, –1) and (1, –1); relative maximum (0, 0).

3. Decreasing on (–∞, –2) and (0, 2); increasing on (–2, 0) and (2, ∞); relative minima (–2, 1) and (2, 1); no relative maximum.

4. Increasing on (–∞, 0) and decreasing on (0, ∞); no relative maximum; no relative minimum.

In the following problems, we denote the critical value by CV.

5. $f'(x) = (x+3)(x-1)(x-2)$
$f'(x) = 0$ when $x = -3, 1, 2$
CV: $x = -3, 1, 2$
$-\quad +\quad -\quad +$
$-3\quad 1\quad 2$
Increasing on (–3, 1) and (2, ∞); decreasing on (–∞, –3) and (1, 2); relative maximum when $x = 1$; relative minima when $x = -3, 2$.

6. $f'(x) = 2x(x-1)^3$
CV: $x = 0, 1$
$+\quad -\quad +$
$0\quad 1$
Increasing on (–∞, 0) and (1, ∞); decreasing on (0, 1); relative maximum when $x = 0$; relative minimum when $x = 1$.

7. $f'(x) = (x+1)(x-3)^2$
CV: $x = -1, 3$
$-\quad +\quad +$
$-1\quad 3$
Decreasing on (–∞, –1); increasing on (–1, 3) and (3, ∞); relative minimum when $x = -1$.

8. $f'(x)=\dfrac{x(x+2)}{x^2+1}$

CV: $x = 0, -2$

$+ \quad - \quad +$

$-2 \quad 0$

Increasing on (–∞, –2) and (0, ∞); decreasing on (–2, 0); relative maximum when $x = -2$; relative minimum when $x = 0$.

9. $y=-x^3-1$

$y'=-3x^2$

CV: $x = 0$

$- \quad -$

0

Decreasing on (–∞, 0); decreasing on (0, ∞); no relative maximum or minimum

10. $y=x^2+4x+3$

$y'=2x+4=2(x+2)$

CV: $x = -2$

$- \quad +$

-2

Decreasing on (–∞, –2); increasing on (–2, ∞); relative minimum when $x = -2$.

11. $y=x-x^2+2$

$y'=1-2x$

CV: $x=\dfrac{1}{2}$

$+ \quad -$

$\frac{1}{2}$

Increasing on $\left(-\infty, \dfrac{1}{2}\right)$; decreasing on $\left(\dfrac{1}{2}, \infty\right)$; relative maximum when $x=\dfrac{1}{2}$.

12. $y=x^3-\dfrac{5}{2}x^2-2x+6$

$y'=3x^2-5x-2=(3x+1)(x-2)$

CV: $x=-\dfrac{1}{3}, 2$

$+ \quad - \quad +$

$-\frac{1}{3} \quad 2$

Increasing on $\left(-\infty, -\dfrac{1}{3}\right)$ and (2, ∞); decreasing on $\left(-\dfrac{1}{3}, 2\right)$; relative maximum when $x=-\dfrac{1}{3}$; relative minimum when $x = 2$.

13. $y=-\dfrac{x^3}{3}-2x^2+5x-2$

$y'=-x^2-4x+5=-\left(x^2+4x-5\right)$

$=-(x+5)(x-1)$

CV: $x = -5, 1$

$- \quad + \quad -$

$-5 \quad 1$

Decreasing on (–∞, –5) and (1, ∞); increasing on (–5, 1); relative minimum when $x = -5$; relative maximum when $x = 1$.

14. $y=-\dfrac{x^4}{4}-x^3$

$y'=-x^3-3x^2=-x^2(x+3)$

CV: $x = -3, 0$

$+ \quad - \quad -$

$-3 \quad 0$

Increasing on (–∞, –3); decreasing on (–3, 0) and (0, ∞); relative maximum at $x = -3$.

15. $y=x^4-2x^2$

$y'=4x^3-4x=4x\left(x^2-1\right)=4x(x+1)(x-1)$

CV: $x = 0, \pm 1$

$- \quad + \quad - \quad +$

$-1 \quad 0 \quad 1$

Decreasing on (–∞, –1) and (0, 1); increasing on (–1, 0) and (1, ∞); relative maximum when $x = 0$; relative minima when $x = \pm 1$.

16. $y=-3+12x-x^3$

$y'=12-3x^2=3\left(4-x^2\right)=3(2+x)(2-x)$

CV: $x = \pm 2$

$- \quad + \quad -$

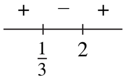
$-2 \quad 2$

Decreasing on (–∞, –2) and (2, ∞); increasing on (–2, 2); relative minimum when $x = -2$; relative maximum when $x = 2$.

17. $y=x^3-\dfrac{7}{2}x^2+2x-5$

$y'=3x^2-7x+2=(3x-1)(x-2)$

CV: $x=\dfrac{1}{3}, 2$

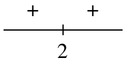


Increasing on $\left(-\infty, \frac{1}{3}\right)$ and (2, ∞); decreasing on $\left(\frac{1}{3}, 2\right)$; relative maximum when $x = \frac{1}{3}$, relative minimum when $x = 2$.

18. $y = x^3 - 6x^2 + 12x - 6$

$y' = 3x^2 - 12x + 12 = 3\left(x^2 - 4x + 4\right) = 3(x-2)^2$

CV: $x = 2$

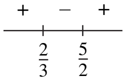


Increasing on (−∞, 2) and (2, ∞); no relative maximum or relative minimum.

19. $y = 2x^3 - \frac{19}{2}x^2 + 10x + 2$

$y' = 6x^2 - 19x + 10 = (2x-5)(3x-2)$

CV: $x = \frac{2}{3}, \frac{5}{2}$

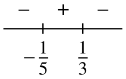


Increasing on $\left(-\infty, \frac{2}{3}\right)$ and $\left(\frac{5}{2}, \infty\right)$; decreasing on $\left(\frac{2}{3}, \frac{5}{2}\right)$; relative maximum when $x = \frac{2}{3}$; relative minimum when $x = \frac{5}{2}$.

20. $y = -5x^3 + x^2 + x - 7$

$y' = -15x^2 + 2x + 1 = -(5x+1)(3x-1)$

CV: $-\frac{1}{5}, \frac{1}{3}$



Decreasing on $\left(-\infty, -\frac{1}{5}\right)$ and $\left(\frac{1}{3}, \infty\right)$; increasing on $\left(-\frac{1}{5}, \frac{1}{3}\right)$; relative minimum when $x = -\frac{1}{5}$; relative maximum when $x = \frac{1}{3}$.

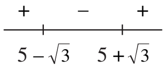
21. $y = \frac{x^3}{3} - 5x^2 + 22x + 1$

$y' = x^2 - 10x + 22$

By the quadratic formula, $y' = 0$ when

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)} \text{ or } x = 5 \pm \sqrt{3}.$$

CV: $x = 5 \pm \sqrt{3}$



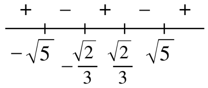
Increasing on $\left(-\infty, 5-\sqrt{3}\right)$; decreasing on $\left(5-\sqrt{3}, 5+\sqrt{3}\right)$; increasing on $\left(5+\sqrt{3}, \infty\right)$; relative maximum at $x = 5-\sqrt{3}$; relative minimum at $x = 5+\sqrt{3}$.

22. $y = \frac{9}{5}x^5 - \frac{47}{3}x^3 + 10x$

$y' = 9x^4 - 47x^2 + 10 = \left(9x^2 - 2\right)\left(x^2 - 5\right)$

$= \left(3x - \sqrt{2}\right)\left(3x + \sqrt{2}\right)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right)$

CV: $x = \pm\frac{\sqrt{2}}{3}, \pm\sqrt{5}$

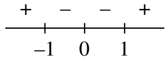


Increasing on $\left(-\infty, -\sqrt{5}\right)$, $\left(-\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)$, and $\left(\sqrt{5}, \infty\right)$; decreasing on $\left(-\sqrt{5}, -\frac{\sqrt{2}}{3}\right)$ and $\left(\frac{\sqrt{2}}{3}, \sqrt{5}\right)$; relative maxima when $x = -\sqrt{5}$, $\frac{\sqrt{2}}{3}$; relative minima when $x = -\frac{\sqrt{2}}{3}, \sqrt{5}$.

23. $y = 3x^5 - 5x^3$

$y' = 15x^4 - 15x^2 = 15x^2(x+1)(x-1)$

CV: $x = 0, \pm 1$



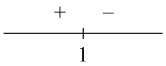
Increasing on (−∞, −1) and (1, ∞); decreasing on (−1, 0) and (0, 1); relative maximum when $x = -1$; relative minimum when $x = 1$.

24. $y = 3x - \dfrac{x^6}{2}$

$y' = 3 - 3x^5 = 3(1 - x^5)$

$= 3(1 - x)(x^4 + x^3 + x^2 + x + 1)$

CV: $x = 1$

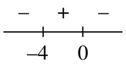


Increasing on (−∞, 1); decreasing on (1, ∞); relative maximum when $x = 1$.

25. $y = -x^5 - 5x^4 + 200$

$y' = -5x^4 - 20x^3 = -5x^3(x + 4)$

CV: $x = 0, -4$

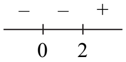


Decreasing on (−∞, −4) and (0, ∞); increasing on (−4, 0); relative minimum when $x = -4$; relative maximum when $x = 0$.

26. $y = \dfrac{3x^4}{2} - 4x^3 + 17$

$y' = 6x^3 - 12x^2 = 6x^2(x - 2)$

CV: $x = 0, 2$



Decreasing on (−∞, 0) and (0, 2); increasing on (2, ∞); relative minimum at $x = 2$.

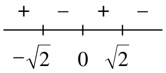
27. $y = 8x^4 - x^8$

$y' = 32x^3 - 8x^7 = 8x^3\left(4 - x^4\right)$

$= 8x^3\left(2 + x^2\right)\left(2 - x^2\right)$

$= 8x^3\left(2 + x^2\right)\left(\sqrt{2} - x\right)\left(\sqrt{2} + x\right)$

CV: $x = 0, \pm\sqrt{2}$



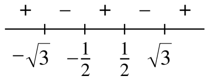
Increasing on $\left(-\infty, -\sqrt{2}\right)$ and $\left(0, \sqrt{2}\right)$; decreasing on $\left(-\sqrt{2}, 0\right)$ and $\left(\sqrt{2}, \infty\right)$; relative maxima when $x = \pm\sqrt{2}$, relative minimum when $x = 0$.

28. $y = \dfrac{4}{5}x^5 - \dfrac{13}{3}x^3 + 3x + 4$

$y' = 4x^4 - 13x^2 + 3 = \left(4x^2 - 1\right)\left(x^2 - 3\right)$

$= (2x - 1)(2x + 1)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

CV: $x = \pm\dfrac{1}{2}, \pm\sqrt{3}$

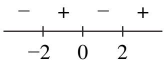


Increasing on $\left(-\infty, -\sqrt{3}\right)$, $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$, $\left(\sqrt{3}, \infty\right)$; decreasing on $\left(-\sqrt{3}, -\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2}, \sqrt{3}\right)$; relative maxima when $x = -\sqrt{3}, \dfrac{1}{2}$; relative minima when $x = -\dfrac{1}{2}, \sqrt{3}$.

29. $y = (x^2 - 4)^4$

$y' = 8x(x^2 - 4)^3 = 8x(x + 2)^3(x - 2)^3$

CV: 0, −2, 2

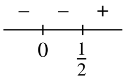


Increasing on (−2, 0) and (2, ∞); decreasing on (−∞, −2) and (0, 2); relative maximum when $x = 0$; relative minima when $x = \pm 2$.

30. $y = \sqrt[3]{x}(x - 2)$

$y' = \dfrac{2(2x - 1)}{3x^{\frac{2}{3}}}$

CV: $x = 0, \dfrac{1}{2}$



Decreasing on (−∞, 0) and $\left(0, \dfrac{1}{2}\right)$; increasing on $\left(\dfrac{1}{2}, \infty\right)$; relative minimum when $x = \dfrac{1}{2}$; no relative maximum.

31. $y=\frac{5}{x-1}=5(x-1)^{-1}$

$y'=-5(x-1)^{-2}=-\frac{5}{(x-1)^2}$

CV: None, but $x = 1$ must be included in the sign chart because it is a point of discontinuity of y.

$$\begin{array}{ccc} - & & - \\ \hline & \boxed{1} & \end{array}$$

Decreasing on $(-\infty, 1)$ and $(1, \infty)$; no relative extremum.

32. $y=\frac{3}{x}=3x^{-1}$

$y'=-3x^{-2}=-\frac{3}{x^2}$

CV: None, but $x = 0$ must be included in the sign chart because it is a point of discontinuity of y.

$$\begin{array}{ccc} - & & - \\ \hline & \boxed{0} & \end{array}$$

Decreasing on $(-\infty, 0)$ and $(0, \infty)$; no relative extremum.

33. $y=\frac{10}{\sqrt{x}}=10x^{-\frac{1}{2}}$. [Note: $x > 0$]

$y'=-5x^{-\frac{3}{2}}=-\frac{5}{\sqrt{x^3}}<0$ for $x > 0$.

Decreasing on $(0, \infty)$; no relative extremum.

34. $y=\frac{ax+b}{cx+d}$

$$y'=\frac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2}$$
$$=\frac{acx+ad-acx-bc}{(cx+d)^2}$$
$$=\frac{ad-bc}{(cx+d)^2}$$

CV: None but $x=-\frac{d}{c}$ must be included in the sign chart because it is a point of discontinuity of y.

a. For $ad - bc > 0$

$$\begin{array}{ccc} + & & + \\ \hline & \boxed{-\frac{d}{c}} & \end{array}$$

Increasing on $\left(-\infty, -\frac{d}{c}\right)$ and $\left(-\frac{d}{c}, \infty\right)$; no relative extrema.

b. For $ad - bc < 0$

$$\begin{array}{ccc} - & & - \\ \hline & \boxed{-\frac{d}{c}} & \end{array}$$

Decreasing on $\left(-\infty, -\frac{d}{c}\right)$ and $\left(-\frac{d}{c}, \infty\right)$; no relative extrema.

35. $y=\frac{x^2}{2-x}$

$y'=\frac{(2-x)(2x)-x^2(-1)}{(2-x)^2}=\frac{x(4-x)}{(2-x)^2}$

CV: $x = 0, 4$, but $x = 2$ must be included in the sign chart because it is a point of discontinuity of y.

$$\begin{array}{ccccccc} - & & + & & + & & - \\ \hline & 0 & & \boxed{2} & & 4 & \end{array}$$

Decreasing on $(-\infty, 0)$ and $(4, \infty)$; increasing on $(0, 2)$ and $(2, 4)$; relative minimum when $x = 0$; relative maximum when $x = 4$.

36. $y=4x^2+\frac{1}{x}$

$y'=8x-\frac{1}{x^2}=\frac{(2x-1)(4x^2+2x+1)}{x^2}$

CV: $x=\frac{1}{2}$, but $x = 0$ must be included in the sign chart because it is a point of discontinuity of y.

$$\begin{array}{ccccc} - & & - & & + \\ \hline & \boxed{0} & & \frac{1}{2} & \end{array}$$

Increasing on $\left(\frac{1}{2}, \infty\right)$; decreasing on $(-\infty, 0)$ and $\left(0, \frac{1}{2}\right)$; relative minimum when $x=\frac{1}{2}$.

37. $y=\frac{x^2-3}{x+2}$

$$y'=\frac{(x+2)(2x)-\left(x^2-3\right)(1)}{(x+2)^2}$$
$$=\frac{x^2+4x+3}{(x+2)^2}=\frac{(x+1)(x+3)}{(x+2)^2}$$

CV: $x = -3, -1$, but $x = -2$ must be included in the sign chart because it is a point of discontinuity of y.

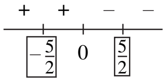
$$\begin{array}{ccccccc} + & & - & & - & & + \\ \hline & -3 & & \boxed{-2} & & -1 & \end{array}$$

Increasing on (–∞, –3) and (–1, ∞); decreasing on (–3, –2) and (–2, –1); relative maximum when $x = -3$; relative minimum when $x = -1$.

38. $y = \dfrac{2x^2}{4x^2 - 25}$

$$y' = \frac{(4x^2-25)(4x)-(2x^2)(8x)}{(4x^2-25)^2}$$
$$= -\frac{100x}{(4x^2-25)^2} = -\frac{100x}{(2x-5)^2(2x+5)^2}$$

CV: $x = 0$, but $x = \pm\frac{5}{2}$ must be included in the sign chart because they are points of discontinuity of y.



Increasing on $\left(-\infty, -\frac{5}{2}\right)$ and $\left(-\frac{5}{2}, 0\right)$; decreasing on $\left(0, \frac{5}{2}\right)$ and $\left(\frac{5}{2}, \infty\right)$; relative maximum at $x = 0$.

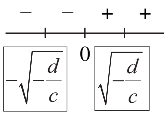
39. $y = \dfrac{ax^2 + b}{cx^2 + d}$ for $\dfrac{d}{c} < 0$.

$$y' = \frac{(cx^2+d)(2ax)-(ax^2+b)(2cx)}{(cx^2+d)^2}$$
$$= \frac{2acx^3 + 2adx - 2acx^3 - 2bcx}{(cx^2+d)^2}$$
$$= \frac{2x(ad-bc)}{(cx^2+d)^2}$$

$y' = 0$ when $x = 0$.

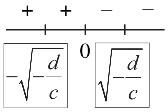
CV: $x = 0$; but $x = \pm\sqrt{-\frac{d}{c}}$ must be included in the sign chart because they are points of discontinuity of y.

a. For $ad - bc > 0$



Increasing on $\left(0, \sqrt{-\frac{d}{c}}\right)$ and $\left(\sqrt{-\frac{d}{c}}, \infty\right)$; decreasing on $\left(-\infty, -\sqrt{-\frac{d}{c}}\right)$ and $\left(-\sqrt{-\frac{d}{c}}, 0\right)$; relative minimum at $x = 0$.

b. For $ad - bc < 0$

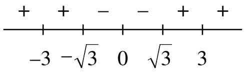


Increasing on $\left(-\infty, -\sqrt{-\frac{d}{c}}\right)$ and $\left(-\sqrt{-\frac{d}{c}}, 0\right)$; decreasing on $\left(0, \sqrt{-\frac{d}{c}}\right)$ and $\left(\sqrt{-\frac{d}{c}}, \infty\right)$; relative maximum at $x = 0$.

40. $y = \sqrt[3]{x^3 - 9x}$

$$y' = \frac{1}{3}\left(x^3 - 9x\right)^{-\frac{2}{3}}\left(3x^2 - 9\right) = \frac{\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)}{[x(x+3)(x-3)]^{\frac{2}{3}}}$$

CV: $x = \pm\sqrt{3}, 0, \pm 3$

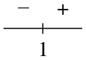


Increasing on (–∞, –3), $\left(-3, -\sqrt{3}\right)$, $\left(\sqrt{3}, 3\right)$, and (3, ∞); decreasing on $\left(-\sqrt{3}, 0\right)$ and $\left(0, \sqrt{3}\right)$; relative maximum when $x = -\sqrt{3}$; relative minimum when $x = \sqrt{3}$.

41. $y = (x-1)^{2/3}$

$$y' = \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3\sqrt[3]{x-1}}$$

CV: $x = 1$



Increasing on (1, ∞); decreasing on (–∞, 1); relative minimum when $x = 1$.

42. $y = x^2(x+3)^4$

$$y' = x^2(4)(x+3)^3 + (x+3)^4(2x)$$
$$= 2x(x+3)^3[2x + (x+3)]$$
$$= 2x(x+3)^3(3x+3) = 6x(x+3)^3(x+1)$$

CV: $x = 0, -3, -1$

$$\begin{array}{cccc} - & + & - & + \\ \hline & -3 & -1 & 0 \end{array}$$

Increasing on (–3, –1) and (0, ∞); decreasing on (–∞, –3) and (–1, 0); relative maximum when $x = -1$; relative minima when $x = -3$ and $x = 0$.

43. $y = x^3(x-6)^4$

$$y' = x^3\left[4(x-6)^3\right] + (x-6)^4\left(3x^2\right)$$
$$= x^2(x-6)^3[4x+3(x-6)]$$
$$= x^2(x-6)^3(7x-18)$$

CV: $x = 0, 6, \frac{18}{7}$

$$\begin{array}{cccc} + & + & - & + \\ \hline & 0 & \frac{18}{7} & 6 \end{array}$$

Increasing on (–∞, 0), $\left(0, \frac{18}{7}\right)$, and (6, ∞); decreasing on $\left(\frac{18}{7}, 6\right)$; relative maximum when $x = \frac{18}{7}$; relative minimum when $x = 6$.

44. $y = (1-x)^{2/3}$

$$y' = \frac{2}{3}(1-x)^{-1/3}(-1) = -\frac{2}{3(1-x)^{1/3}}$$

CV: $x = 1$

$$\begin{array}{cc} - & + \\ \hline & 1 \end{array}$$

Decreasing on (–∞, 1); increasing on (1, ∞); relative minimum when $x = 1$.

45. $y = e^{-\pi x} + \pi$

$y' = -\pi e^{-\pi x} < 0$ for all x. Thus decreasing on (–∞, ∞); no relative extremum.

46. $y = x \ln x$. (Note: $x > 0$.)

$y' = 1 + \ln x$

$y' = 0$ when $1 + \ln x = 0$, $\ln x = -1$, or

$x = e^{-1} = \frac{1}{e}$

CV: $x = \frac{1}{e}$

$$\begin{array}{ccc} & - & + \\ \hline 0 & \frac{1}{e} & \end{array}$$

Decreasing on $\left(0, \frac{1}{e}\right)$; increasing on $\left(\frac{1}{e}, \infty\right)$; relative minimum when $x = \frac{1}{e}$.

47. $y = x^2 - 9\ln x$. [Note: $x > 0$.]

$$y' = 2x - \frac{9}{x} = \frac{2x^2-9}{x}$$

CV: $x = \frac{3\sqrt{2}}{2}$

$$\begin{array}{ccc} & - & + \\ \hline 0 & \frac{3\sqrt{2}}{5} & \end{array}$$

Decreasing on $\left(0, \frac{3\sqrt{2}}{2}\right)$; increasing on $\left(\frac{3\sqrt{2}}{2}, \infty\right)$; relative minimum when $x = \frac{3\sqrt{2}}{2}$.

48. $y = x^{-1}e^x$

$$y' = x^{-1}e^x - x^{-2}e^x = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) = e^x\left(\frac{x-1}{x^2}\right)$$

CV: $x = 1$, but $x = 0$ must also be included in the sign chart because it is a point of discontinuity of y.

$$\begin{array}{ccc} - & - & + \\ \hline & \boxed{0} & 1 \end{array}$$

Increasing on (1, ∞); decreasing on (–∞, 0) and (0, 1); relative minimum when $x = 1$.

49. $y = e^x - e^{-x}$

$y' = e^x + e^{-x}$

Setting $y' = 0$ gives $e^x + e^{-x} = 0$, $e^x = -e^{-x}$,
CV: None
Increasing on (–∞, ∞); no relative extrema.

50. $y = e^{-x^2/2}$

$y' = -xe^{-x^2/2}$

CV: $x = 0$

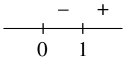
$$\begin{array}{cc} + & - \\ \hline & 0 \end{array}$$

Increasing on (–∞, 0); decreasing on (0, ∞); relative maximum at $x = 0$

51. $y = x \ln x - x$. [Note: $x > 0$.]

$y' = \left[x \cdot \frac{1}{x} + (\ln x)(1)\right] - 1 = \ln x$

CV: $x = 1$



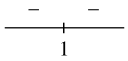
Decreasing on (0, 1); increasing on (1, ∞); relative minimum when $x = 1$; no relative maximum.

52. $y = \left(x^2 + 1\right)e^{-x}$

$y' = \left(x^2 + 1\right)\left(-e^{-x}\right) + e^{-x}(2x)$

$= -e^{-x}\left[\left(x^2 + 1\right) - 2x\right] = -e^{-x}(x - 1)^2$

CV: $x = 1$



Decreasing on (−∞, 1) and (1, ∞); never increasing; no relative extremum.

53. $y = x^2 - 3x - 10 = (x + 2)(x - 5)$

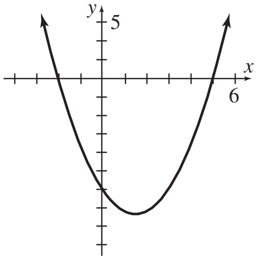
Intercepts (−2, 0), (5, 0), (0, −10)

$y' = 2x - 3$

CV: $x = \frac{3}{2}$

Decreasing on $\left(-\infty, \frac{3}{2}\right)$; increasing on $\left(\frac{3}{2}, \infty\right)$;

relative minimum when $x = \frac{3}{2}$.



54. $y = 2x^2 + x - 10 = (2x + 5)(x - 2)$

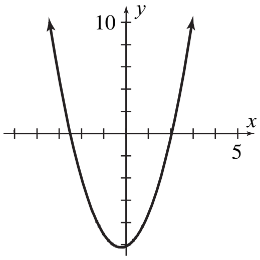
Intercepts $\left(-\frac{5}{2}, 0\right)$, (2, 0), (0, −10)

$y' = 4x + 1 = 4\left(x + \frac{1}{4}\right)$

CV: $x = -\frac{1}{4}$

Decreasing on $\left(-\infty, -\frac{1}{4}\right)$; increasing on $\left(-\frac{1}{4}, \infty\right)$; absolute minimum when $x = -\frac{1}{4}$;

symmetric about $x = -\frac{1}{4}$.



55. $y = 3x - x^3 = x\left(\sqrt{3} + x\right)\left(\sqrt{3} - x\right)$

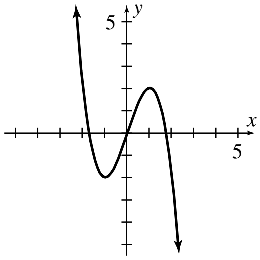
Intercepts: (0, 0), $\left(\pm\sqrt{3}, 0\right)$

Symmetric about origin.

$y' = 3 - 3x^2 = 3(1 + x)(1 - x)$

CV: $x = \pm 1$

Decreasing on (−∞, −1) and (1, ∞); increasing on (−1, 1); relative minimum when $x = -1$; relative maximum when $x = 1$.



56. $y = x^4 - 16 = \left(x^2 + 4\right)(x + 2)(x - 2)$

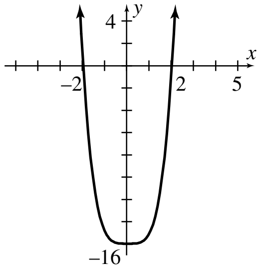
Intercepts (±2, 0), (0, −16)

Symmetric about y-axis.

$y' = 4x^3$

CV: $x = 0$

Decreasing on (−∞, 0); increasing on (0, ∞); relative minimum when $x = 0$.



57. $y = 2x^3 - 9x^2 + 12x = x\left(2x^2 - 9x + 12\right)$

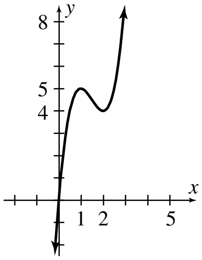
Note that $2x^2 - 9x + 12 = 0$ has no real roots. The only intercept is (0, 0).

$y' = 6x^2 - 18x + 12 = 6\left(x^2 - 3x + 2\right)$

$= 6(x - 2)(x - 1)$

CV: $x = 1, 2$

Increasing on (–∞, 1) and (2, ∞); decreasing on (1, 2); relative maximum when $x = 1$; relative minimum when $x = 2$.



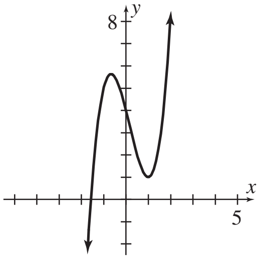
58. $y = 2x^3 - x^2 - 4x + 4$

The x-intercept is not convenient to find. y-intercept is (0, 4).

$y' = 6x^2 - 2x - 4 = 2(3x + 2)(x - 1)$

CV: $x = -\dfrac{2}{3}, 1$

Increasing on $\left(-\infty, -\dfrac{2}{3}\right)$ and (1, ∞); decreasing on $\left(-\dfrac{2}{3}, 1\right)$; relative maximum when $x = -\dfrac{2}{3}$; relative minimum when $x = 1$.



59. $y = x^4 - 2x^2$

$= x^2(x^2 - 2)$

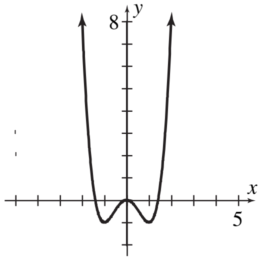
$= x^2\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

Intercepts (0, 0), $\left(-\sqrt{2}, 0\right)$, $\left(\sqrt{2}, 0\right)$

$y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x + 1)(x - 1)$

CV: $x = 0, -1, 1$

Increasing on (−1, 0) and (1, ∞); decreasing on (–∞, –1) and (0, 1); relative maximum at (0, 0); absolute minima at $x = \pm 1$; symmetric about $x = 0$.



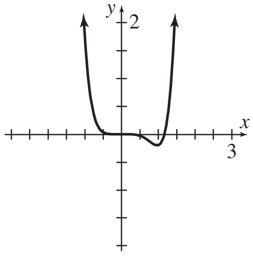
60. $y = x^6 - \dfrac{6}{5}x^5 = x^5\left(x - \dfrac{6}{5}\right)$

Intercepts (0, 0), $\left(\dfrac{6}{5}, 0\right)$

$y' = 6x^5 - 6x^4 = 6x^4(x - 1)$

CV: $x = 0, 1$

Increasing on (1, ∞); decreasing on (–∞, 0) and (0, 1); relative minimum when $x = 1$.



61. $y = (x - 1)^2(x + 2)^2$

Intercepts: (1, 0), (–2, 0), (0, 4)

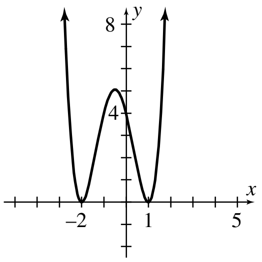
$y' = (x - 1)^2 \cdot 2(x + 2) + (x + 2)^2 \cdot 2(x - 1)$

$= 2(x - 1)(x + 2)[(x - 1) + (x + 2)]$

$= 2(x - 1)(x + 2)(2x + 1)$

CV: $x = 1, -2, -\dfrac{1}{2}$

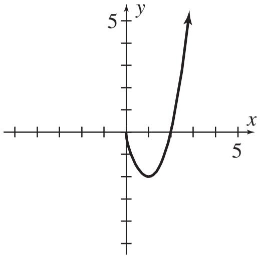
Decreasing on (–∞, –2) and $\left(-\dfrac{1}{2}, 1\right)$; increasing on $\left(-2, -\dfrac{1}{2}\right)$ and (1, ∞); relative minima when $x = -2$ or $x = 1$; relative maximum when $x = -\dfrac{1}{2}$.



62. $y = \sqrt{x}(x^2 - x - 2) = \sqrt{x}(x-2)(x+1)$
[Note $x \geq 0$.]
Intercepts (0, 0), (2, 0)
$y = x^{5/2} - x^{3/2} - 2x^{1/2}$

$$y' = \frac{5}{2}x^{3/2} - \frac{3}{2}x^{1/2} - 2 \cdot \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}}(5x^2 - 3x - 2)$$
$$= \frac{1}{2\sqrt{x}}(5x+2)(x-1)$$

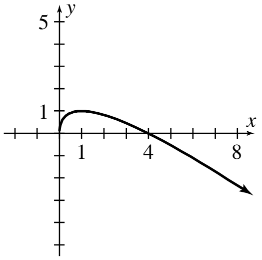
CV: $x = 0, 1$ $(x \geq 0)$
Decreasing on (0, 1); increasing on (1, ∞); relative minimum when $x = 1$.



63. $y = 2\sqrt{x} - x = \sqrt{x}\left(2 - \sqrt{x}\right)$. [Note: $x \geq 0$.]
Intercepts (0, 0), (4, 0)

$$y' = \frac{1}{\sqrt{x}} - 1 = \frac{1-\sqrt{x}}{\sqrt{x}}$$

CV: $x = 0, 1$
Increasing on (0, 1); decreasing on (1, ∞); relative maximum when $x = 1$.



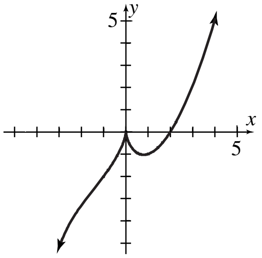
CV: $x = 0, \dfrac{4}{5}$

64. $y = x^{5/3} - 2x^{2/3} = x^{2/3}(x-2)$
Intercepts: (0, 0), (2, 0)

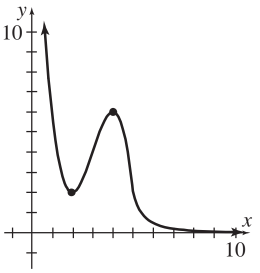
$$y' = \frac{5}{3}x^{2/3} - \frac{4}{3}x^{-1/3} = \frac{1}{3}x^{-1/3}(5x-4)$$

CV: $x = 0, \dfrac{4}{5}$

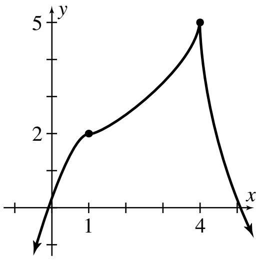
Increasing on (−∞, 0) and $\left(\dfrac{4}{5}, \infty\right)$; decreasing on $\left(0, \dfrac{4}{5}\right)$; relative maximum at (0, 0); relative minimum at $x = \dfrac{4}{5}$; no symmetry.



65.



66.



67. $c_f = 25{,}000$

$$\overline{c}_f = \frac{c_f}{q} = \frac{25{,}000}{q}$$

$\dfrac{d}{dq}\left(\overline{c}_f\right) = -\dfrac{25{,}000}{q^2} < 0$ for $q > 0$, so $\overline{c}_f$ is a decreasing function for $q > 0$.

68. $c = 3q - 3q^2 + q^3$

Marginal cost is given by $\frac{dc}{dq} = 3 - 6q + 3q^2$.

Thus $\frac{dc}{dq}$ is increasing when $\frac{d\left[\frac{dc}{dq}\right]}{dq} < 0$, that is, when –6 + 6$q$ > 0. Hence q > 1.

69. $p = 500 - 5q$
Revenue is given by
$r = pq = (500 - 5q)q = 500q - 5q^2$
Marginal revenue is $r' = 500 - 10q$. Marginal revenue is increasing when its derivative is positive. But $(r')' = -10 < 0$. Thus marginal revenue is never increasing.

70. $c = \sqrt{q}$

Marginal cost $= \frac{dc}{dq} = \frac{1}{2\sqrt{q}}$. Since $\frac{d\left[\frac{dc}{dq}\right]}{dq} = -\frac{1}{4\sqrt{q^3}} < 0$ for q > 0, marginal cost is decreasing for q > 0.

Average cost $= \overline{c} = \frac{c}{q} = \frac{1}{\sqrt{q}}$. Since $\frac{d\overline{c}}{dq} = -\frac{1}{2\sqrt{q^3}} < 0$ for q > 0, average cost is decreasing for q > 0.

71. $r = 240q + 57q^2 - q^3$
$r' = 240 + 114q - 3q^2 = 3(40 - q)(2 + q)$
Since $q \geq 0$, we have q = 40 as the only CV.
Since r is increasing on (0, 40) and decreasing on (40, ∞), r is a maximum when output is 40.

72. $z = (1 + b)w_p - bw_c$, w_p is function of w_c, and $b > 0$.

a. $\frac{dz}{dw_c} = (1 + b)\frac{dw_p}{dw_c} - b(1)$
$= (1 + b)\left[\frac{dw_p}{dw_c} - \frac{b}{1 + b}\right]$ (factoring)

b. If $\frac{dw_p}{dw_c} < \frac{b}{b+1}$, then $\frac{dw_p}{dw_c} - \frac{b}{b+1} < 0$.
Because b > 0, then 1 + b > 0. Thus from part (a), $\frac{dz}{dw_c} < 0$ so z is a decreasing function of w_c.

73. $E = 0.71\left(1 - \frac{T_c}{T_h}\right)$

$\frac{dE}{dT_h} = 0.71\left(\frac{T_c}{T_h^2}\right) > 0$, so as T_h increases, E increases.

74. $r = 2F + \left(1 - \frac{a}{b}\right)p - p^2 + \frac{a^2}{b}$

$\frac{dr}{dp} = \left(1 - \frac{a}{b}\right) - 2p = \frac{b - a}{b} - 2p = 2\left(\frac{b - a}{2b} - p\right)$

Setting $\frac{dr}{dp} = 0$ gives the critical value $p = \frac{b-a}{2b}$. If $p < \frac{b-a}{2b}$, then $\frac{dr}{dp} > 0$ and r is increasing. If $p > \frac{b-a}{2b}$, then $\frac{dr}{dp} < 0$ and r is decreasing. Thus revenue is maximum for $p = \frac{b-a}{2b}$.

75. $C(k) = 100\left[100 + 9k + \frac{144}{k}\right]$, $1 \leq k \leq 100$

a. $C(1)$ = 25,300

b. $C'(k) = 100\left[9 - \frac{144}{k^2}\right] = 100\left[\frac{9k^2 - 144}{k^2}\right]$
$= 100\left[\frac{9(k+4)(k-4)}{k^2}\right]$
Since $k \geq 1$, the only critical value is k = 4.
If $1 \leq k < 4$, then $C'(k) < 0$ and C is decreasing. If $4 < k \leq 100$, then $C'(k) > 0$ and C is increasing. Thus C has an absolute minimum for k = 4.

c. $C(4)$ = 17,200

76. $P=\dfrac{100}{1+100{,}000e^{-0.36h}}$

$$\frac{dP}{dh}=\frac{d}{dh}\left[100\left(1+100{,}000e^{-0.36h}\right)^{-1}\right]$$

$$=\frac{3{,}600{,}000}{e^{0.36h}\left(1+100{,}000e^{-0.36h}\right)^{2}}$$

Since $\dfrac{dP}{dh}>0$, P is an increasing function of h.

77. Relative minimum: (–3.83, 0.69)

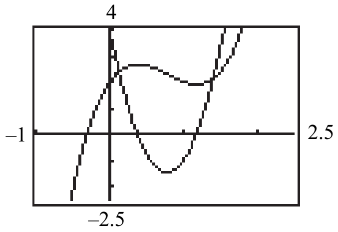
78. Relative minimum: (1.26, –5.74)

79. Relative maximum: (2.74, 3.74); relative minimum: (–2.74, –3.74)

80. Relative maximum: (0.05, 3.05)

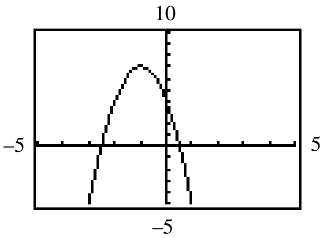
81. Relative minima: 0, 1.50, 2.00; relative maxima: 0.57, 1.77

82. *f* has relative extrema when $x\approx 0.38, 1.18$; $f'(x)=0$ when $x\approx 0.38, 1.18$.



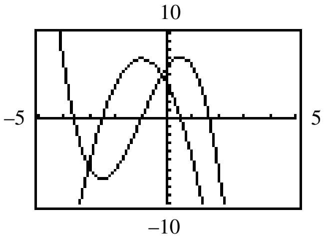
83. **a.** $f'(x)=4-6x-3x^2$

b.



c. $f'(x)>0$ on (–2.53, 0.53); $f'(x)<0$ on (–∞, –2.53), (0.53, ∞), *f* is inc. on (–2.53, 0.53); *f* is dec. on (–∞, –2.53), (0.53, ∞).

d.



84. $f'(x)=4x^3-2x-2(x+2)$
$=4x^3-4x-4$
CV: $x\approx 1.32$

Problems 13.2

1. $f(x)=x^2-2x+3$ and *f* is continuous over [0, 3].
$f'(x)=2x-2=2(x-1)$
The only critical value on (0, 3) is $x=1$. We evaluate *f* at this point and at the endpoints: $f(0)=3$, $f(1)=2$, and $f(3)=6$.
Absolute maximum: $f(3)=6$;
absolute minimum: $f(1)=2$

2. $f(x)=-2x^2-6x+5$ and *f* is continuous over [–3, 2].

$$f'(x)=-4x-6=-4\left(x+\frac{3}{2}\right)$$

The only critical value on (–3, 2) is $x=-\dfrac{3}{2}$. We have $f(-3)=5$, $f\left(-\dfrac{3}{2}\right)=\dfrac{19}{2}$, and $f(2)=-15$.

Absolute maximum: $f\left(-\dfrac{3}{2}\right)=\dfrac{19}{2}$;
absolute minimum: $f(2)=-15$

3. $f(x)=\dfrac{1}{3}x^3+\dfrac{1}{2}x^2-2x+1$ and *f* is continuous over [–1, 0].
$f'(x)=x^2+x-2=(x+2)(x-1)$
There are no critical values on (–1, 0), so we only have to evaluate *f* at the endpoints:
$f(-1)=\dfrac{19}{6}$ and $f(0)=1$.

Absolute maximum: $f(-1)=\dfrac{19}{6}$

Absolute minimum: $f(0)=1$

4. $f(x)=\frac{1}{4}x^4-\frac{3}{2}x^2$ and *f* is continuous over [0, 1].

$f'(x)=x^3-3x=x\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$

There are no critical values on (0, 1), so we only have to evaluate *f* at the end points: *f*(0) = 0 and $f(1)=-\frac{5}{4}$

Absolute maximum: *f*(0) = 0;

absolute minimum: $f(1)=-\frac{5}{4}$

5. $f(x)=x^3-5x^2-8x+50$ and *f* is continuous over [0, 5].

$f'(x)=3x^2-10x-8=(x-4)(3x+2)$

The only critical value on (0, 5) is *x* = 4. We evaluate *f* at this point and the endpoints: *f*(4) = 2; *f*(0) = 50; *f*(5) = 10.

Absolute maximum: *f*(0) = 50

Absolute minimum: *f*(4) = 2

6. $f(x)=x^{\frac{2}{3}}$ and *f* is continuous over [–8, 8].

$f'(x)=\frac{2}{3}x^{-\frac{1}{3}}$.

The only critical value on (–8, 8) is *x* = 0. We have *f*(–8) = 4, *f*(0) = 0, and *f*(8) = 4. Thus there is an absolute maximum when *x* = –8 or *x* = 8, and an absolute minimum when *x* = 0.

Absolute maximum: *f*(–8) = *f*(8) = 4;

absolute minimum: *f*(0) = 0

7. $f(x)=-3x^5+5x^3$ and *f* is continuous over [–2, 0].

$f'(x)=-15x^4+15x^2=15x^2\left(1-x^2\right)$

$=15x^2(1+x)(1-x)$

The only critical value on (–2, 0) is *x* = –1. We have *f*(–2) = 56, *f*(–1) = –2, and *f*(0) = 0.

Absolute maximum: *f*(–2) = 56;

absolute minimum: *f*(–1) = –2.

8. $f(x)=\frac{7}{3}x^3+2x^2-3x+1$ and *f* is continuous over [0, 3].

$f'(x)=7x^2+4x-3=(7x-3)(x+1)$

The only critical value on (0, 3) is $x=\frac{3}{7}$. We have *f*(0) = 1, $f\left(\frac{3}{7}\right)=\frac{13}{49}$, and *f*(3) = 73.

Absolute maximum: *f*(3) = 73;

absolute minimum: $f\left(\frac{3}{7}\right)=\frac{13}{49}$

9. $f(x)=3x^4-x^6$ and *f* is continuous over [–1, 2].

$f'(x)=12x^3-6x^5=6x^3\left(2-x^2\right)$

$=6x^3\left(\sqrt{2}-x\right)\left(\sqrt{2}+x\right)$

The only critical values on (–1, 2) are *x* = 0, $\sqrt{2}$. We have *f*(–1) = 2, *f*(0) = 0, $f\left(\sqrt{2}\right)=4$, and *f*(2) = –16.

Absolute maximum: $f\left(\sqrt{2}\right)=4$;

absolute minimum: *f*(2) = –16

10. $f(x)=x^4-8x^3+22x^2-24x+2$ and *f* is continuous over [0, 4].

$f'(x)=4x^3-24x^2+44x-24$

$=4(x-3)(x-2)(x-1)$

The critical values of *f* on (0, 4) are *x* = 1, 2, 3. We have *f*(0) = 2, *f*(1) = –7, *f*(2) = –6, *f*(3) = –7, and *f*(4) = 2.

Absolute maxima: *f*(0) = *f*(4) = 2

Absolute minima: *f*(1) = *f*(3) = –7

11. $f(x)=x^4-9x^2+2$ and *f* is continuous over [–1, 3].

$f'(x)=4x^3-18x=2x\left(2x^2-9\right)$

$=2x\left(\sqrt{2}x-3\right)\left(\sqrt{2}x+3\right)$

The only critical values on (–1, 3) are *x* = 0 and $x=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$. We have *f*(–1) = –6, *f*(0) = 2, $f\left(\frac{3\sqrt{2}}{2}\right)=-\frac{73}{4}$, and *f*(3) = 2.

Absolute maximum: *f*(0) = *f*(3) = 2;

absolute minimum: $f\left(\frac{3\sqrt{2}}{2}\right)=-\frac{73}{4}$

12. $f(x)=\frac{x}{x^2+1}$ and *f* is continuous over [0, 2].

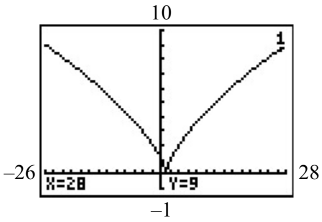
$$f'(x)=\frac{\left(x^2+1\right)-x(2x)}{\left(x^2+1\right)^2}=\frac{1-x^2}{\left(x^2+1\right)^2}$$

$$=\frac{(1+x)(1-x)}{\left(x^2+1\right)^2}$$

The only critical value on (0, 2) is $x = 1$. We have *f*(0) = 0, $f(1)=\frac{1}{2}$,and $f(2)=\frac{2}{5}$.

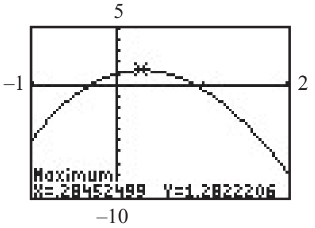
Absolute maximum: $f(1)=\frac{1}{2}$;
absolute minimum: *f*(0) = 0

13. $f(x)=(x-1)^{\frac{2}{3}}$ and *f* is continuous over [–26, 28].



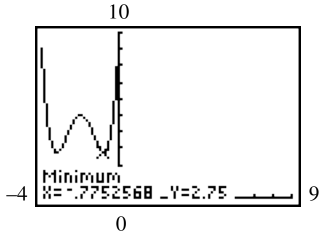
Absolute maximum: *f*(–26) = *f*(28) = 9;
absolute minimum: *f*(1) = 0

14. $f(x)=0.2x^3-3.6x^2+2x+1$ and *f* is continuous over [–1, 2].



Absolute maximum *f*(0.28) ≈ 1.28; absolute minimum *f*(2) = –7.8

15.



a. –3.22, –0.78

b. 2.75

c. 9

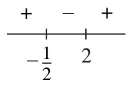
d. 14,283

Problems 13.3

1. $f(x)=x^4-3x^3-6x^2+6x+1$

$f''(x)=6(2x+1)(x-2)$

$f''(x)$ is 0 when $x=-\frac{1}{2}$, 2. Sign chart for f'':

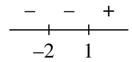


Concave up on $\left(-\infty, -\frac{1}{2}\right)$ and (2, ∞); concave down on $\left(-\frac{1}{2}, 2\right)$. Inflection points when $x=-\frac{1}{2}$, 2.

2. $f(x)=\frac{x^5}{20}+\frac{x^4}{4}-2x^2$

$f''(x)=(x-1)(x+2)^2$

$f''(x)$ is 0 when $x = 1, –2$. Sign chart for f'' :

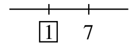


Concave down on (–∞, –2) and (–2, 1); concave up on (1, ∞). Inflection point when $x = 1$.

3. $f(x)=\frac{2+x-x^2}{x^2-2x+1}$

$f''(x)=\frac{2(7-x)}{(x-1)^4}$

$f''(x)$ is 0 when $x = 7$. Although f'' is not defined when $x = 1$, *f* is not continuous at $x = 1$. So there is no inflection point when $x = 1$, but $x = 1$ must be considered in concavity analysis. Sign chart for f'' :

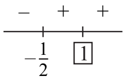


Concave up on (–∞, 1) and (1, 7); concave down on (7, ∞). Inflection point when $x = 7$.

4. $f(x)=\dfrac{x^2}{(x-1)^2}$

$f''(x)=\dfrac{2(2x+1)}{(x-1)^4}$

$f''(x)=0$ when $x=-\dfrac{1}{2}$. Although f'' is not defined when $x = 1$, f is not continuous at $x = 1$. So there is no inflection point when $x = 1$, but $x = 1$ must be considered in concavity analysis. Sign chart of f'':



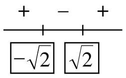
Concave up on $\left(-\dfrac{1}{2}, 1\right)$ and $(1, \infty)$; concave down on $\left(-\infty, -\dfrac{1}{2}\right)$.

Inflection point when $x=\dfrac{1}{2}$

5. $f(x)=\dfrac{x^2+1}{x^2-2}$

$f''(x)=\dfrac{6\left(3x^2+2\right)}{\left(x^2-2\right)^3}=\dfrac{6\left(3x^2+2\right)}{\left[\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)\right]^3}$

$f''(x)$ is never 0. Although f'' is not defined when $x=\pm\sqrt{2}$, f is not continuous at $x=\pm\sqrt{2}$. So there is no inflection point when $x=\pm\sqrt{2}$, but $x=\pm\sqrt{2}$ must be considered in concavity analysis. Sign chart of f'':

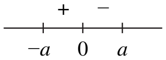


Concave up on $\left(-\infty, -\sqrt{2}\right)$ and $\left(\sqrt{2}, \infty\right)$; concave down on $\left(-\sqrt{2}, \sqrt{2}\right)$. No inflection point.

6. $f(x)=x\sqrt{a^2-x^2}$

$f''(x)=\dfrac{x(2x^2-3a^2)}{(a^2-x^2)^{3/2}}$

Note that the domain of f is $[-a, a]$. $f''(x)$ is 0 only when $x = 0$ (on the domain of f); f'' is not defined when $x = \pm a$, which are the endpoints of the domain of f. The only possible point of inflection occurs when $x = 0$. Sign chart for f'':



Concave up on $(-a, 0)$; concave down on $(0, a)$. Inflection point when $x = 0$.

7. $y=-2x^2+4x$

$y'=-4x+4$

$y''=-4<0$ for all x, so the graph is concave down for all x, that is, on $(-\infty, \infty)$.

8. $y=-74x^2+19x-37$

$y'=-148x+19$

$y''=-148<0$ for all x. Thus the graph is concave down on $(-\infty, \infty)$.

9. $y=4x^3+12x^2-12x$

$y'=12x^2+24x-12$

$y''=24x+24=24(x+1)$

Possible inflection point when $x = -1$. Concave down on $(-\infty, -1)$: concave up on $(-1, \infty)$; inflection point when $x = -1$.

10. $y=x^3-6x^2+9x+1$

$y'=3x^2-12x+9$

$y''=6x-12=6(x-2)$

Possible inflection point when $x = 2$. Concave down on $(-\infty, 2)$; concave up on $(2, \infty)$; inflection point when $x = 2$.

11. $y=ax^3+bx^2+cx+d$

$y'=3ax^2+2bx+c$

$y''=6ax+2b=6a\left(x+\dfrac{b}{3a}\right)$

Possible inflection point when $x=-\dfrac{b}{3a}$.

For $a > 0$: concave up on $\left(-\dfrac{b}{3a}, \infty\right)$; concave down on $\left(-\infty, -\dfrac{b}{3a}\right)$; inflection point when $x=-\dfrac{b}{3a}$.

For $a < 0$: concave up on $\left(-\infty, -\dfrac{b}{3a}\right)$; concave down on $\left(-\frac{b}{3a}, \infty\right)$; inflection point when $x = -\frac{b}{3a}$.

12. $y = x^4 - 8x^2 - 6$

$y' = 4x^3 - 16x$

$y'' = 12x^2 - 16 = 12\left(x^2 - \frac{4}{3}\right)$

$= 12\left(x - \frac{2\sqrt{3}}{3}\right)\left(x + \frac{2\sqrt{3}}{3}\right)$

Possible inflection points $x = \pm\frac{2\sqrt{3}}{3}$. Concave up on $\left(-\infty, -\frac{2\sqrt{3}}{3}\right)$ and $\left(\frac{2\sqrt{3}}{3}, \infty\right)$; concave down on $\left(-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$; inflection points when $x = \pm\frac{2\sqrt{3}}{3}$.

13. $y = 2x^4 - 48x^2 + 7x + 3$

$y' = 8x^3 - 96x + 7$

$y'' = 24x^2 - 96 = 24(x^2 - 4) = 24(x + 2)(x - 2)$

Possible inflection points when $x = \pm2$. Concave up on $(-\infty, -2)$ and $(2, \infty)$; concave down on $(-2, 2)$; inflection points when $x = \pm2$.

14. $y = -\frac{x^4}{4} + \frac{9x^2}{2} + 2x$

$y' = -x^3 + 9x + 2$

$y'' = -3x^2 + 9 = -3(x^2 - 3)$

$= -3\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

Possible inflection points when $x = \pm\sqrt{3}$. Concave down on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$; concave up on $\left(-\sqrt{3}, \sqrt{3}\right)$; inflection points when $x = \pm\sqrt{3}$.

15. $y = 2x^{\frac{1}{5}}$

$y' = \frac{2}{5}x^{-\frac{4}{5}}$

$y'' = -\frac{8}{25}x^{-\frac{9}{5}} = -\frac{8}{25x^{\frac{9}{5}}}$

y'' is not defined when $x = 0$ and y is continuous there. Thus there is a possible inflection point when $x = 0$. Concave up on $(-\infty, 0)$; concave down on $(0, \infty)$; inflection point when $x = 0$.

16. $y = \frac{a}{x^3} = ax^{-3}$

$y' = -\frac{3a}{x^4}$

$y'' = \frac{12a}{x^5}$

Although y'' is not defined when $x = 0$, y is not continuous there. Thus there is no possible inflection point. However, $x = 0$ must be considered in concavity analysis.
For $a > 0$: concave up on $(0, \infty)$; concave down on $(-\infty, 0)$. For $a < 0$: concave up on $(-\infty, 0)$; concave down on $(0, \infty)$.

17. $y = \frac{x^4}{2} + \frac{19x^3}{6} - \frac{7x^2}{2} + x + 5$

$y' = 2x^3 + \frac{19}{2}x^2 - 7x + 1$

$y'' = 6x^2 + 19x - 7 = (3x - 1)(2x + 7)$

Possible inflection points when $x = -\frac{7}{2}, \frac{1}{3}$.

Concave up on $\left(-\infty, -\frac{7}{2}\right)$ and $\left(\frac{1}{3}, \infty\right)$; concave down on $\left(-\frac{7}{2}, \frac{1}{3}\right)$; inflection points when $x = -\frac{7}{2}, \frac{1}{3}$.

18. $y = -\frac{5}{2}x^4 - \frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{1}{3}x - \frac{2}{5}$

$y' = -10x^3 - \frac{1}{2}x^2 + x + \frac{1}{3}$

$y'' = -30x^2 - x + 1 = -(5x + 1)(6x - 1)$

Possible inflection points when $x = -\frac{1}{5}, \frac{1}{6}$.

Concave down on $\left(-\infty, -\frac{1}{5}\right)$ and $\left(\frac{1}{6}, \infty\right)$; concave up on $\left(-\frac{1}{5}, \frac{1}{6}\right)$; inflection points when $x = -\frac{1}{5}, \frac{1}{6}$.

19. $y = \frac{1}{20}x^5 - \frac{1}{4}x^4 + \frac{1}{6}x^3 - \frac{1}{2}x - \frac{2}{3}$

$y' = \frac{1}{4}x^4 - x^3 + \frac{1}{2}x^2 - \frac{1}{2}$

$y'' = x^3 - 3x^2 + x = x\left(x^2 - 3x + 1\right)$

y'' is 0 when $x = 0$ or $x^2 - 3x + 1 = 0$. Using the quadratic formula to solve $x^2 - 3x + 1 = 0$ gives $x = \frac{3 \pm \sqrt{5}}{2}$. Thus possible inflection points occur when $x = 0, \frac{3 \pm \sqrt{5}}{2}$. Concave down on $(-\infty, 0)$ and $\left(\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$; concave up on $\left(0, \frac{3-\sqrt{5}}{2}\right)$ and $\left(\frac{3+\sqrt{5}}{2}, \infty\right)$; inflection points when $x = 0, \frac{3 \pm \sqrt{5}}{2}$.

20. $y = \frac{1}{10}x^5 - 3x^3 + 17x + 43$

$y' = \frac{1}{2}x^4 - 9x^2 + 17$

$y'' = 2x^3 - 18x = 2x(x^2 - 9)$

$= 2x(x+3)(x-3)$

Possible inflection points when $x = 0, \pm 3$. Concave down on $(-\infty, -3)$ and $(0, 3)$; concave up on $(-3, 0)$ and $(3, \infty)$; inflection points when $x = 0, \pm 3$.

21. $y = \frac{1}{30}x^6 - \frac{7}{12}x^4 + 6x^2 + 5x - 4$

$y' = \frac{1}{5}x^5 - \frac{7}{3}x^3 + 12x + 5$

$y'' = x^4 - 7x^2 + 12 = \left(x^2 - 4\right)\left(x^2 - 3\right)$

$= (x+2)(x-2)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

Possible inflection points when $x = \pm 2, \pm\sqrt{3}$. Concave up on $(-\infty, -2)$, $\left(-\sqrt{3}, \sqrt{3}\right)$, and $(2, \infty)$; concave down on $\left(-2, -\sqrt{3}\right)$ and $\left(\sqrt{3}, 2\right)$; inflection points when $x = \pm 2, \pm\sqrt{3}$.

22. $y = x^6 - 3x^4$

$y' = 6x^5 - 12x^3$

$y'' = 30x^4 - 36x^2 = 30x^2\left(x^2 - \frac{6}{5}\right)$

$= 30x^2\left(x - \sqrt{\frac{6}{5}}\right)\left(x + \sqrt{\frac{6}{5}}\right)$

Possible inflection points when $x = 0, \pm\sqrt{\frac{6}{5}}$.

Concave up on $\left(-\infty, -\sqrt{\frac{6}{5}}\right)$ and $\left(\sqrt{\frac{6}{5}}, \infty\right)$; concave down on $\left(-\sqrt{\frac{6}{5}}, 0\right)$ and $\left(0, \sqrt{\frac{6}{5}}\right)$.

Inflection points when $x = \pm\sqrt{\frac{6}{5}}$.

23. $y = \frac{x+1}{x-1}$

$y' = \frac{-2}{(x-1)^2}$

$y'' = \frac{4}{(x-1)^3}$

No possible inflection point, but we consider $x = 1$ in the concavity analysis. Concave down on $(-\infty, 1)$; concave up on $(1, \infty)$.

24. $y = 1 - \frac{1}{x^2}$

$y' = \frac{2}{x^3}$

$y'' = -\frac{6}{x^4}$

No possible inflection point, but we must consider $x = 0$ in the concavity analysis. Concave down on $(-\infty, 0)$ and $(0, \infty)$.

25. $y = \frac{x^2}{x^2+1}$

$$y' = \frac{\left(x^2+1\right)(2x) - x^2(2x)}{\left(x^2+1\right)^2} = \frac{2x}{\left(x^2+1\right)^2}$$

$$y'' = \frac{\left(x^2+1\right)^2(2) - 2x(2)\left(x^2+1\right)(2x)}{\left(x^2+1\right)^4} = \frac{\left(x^2+1\right)(2) - 8x^2}{\left(x^2+1\right)^3}$$

$$= \frac{2\left(1-3x^2\right)}{\left(x^2+1\right)^3} = \frac{2\left(1+\sqrt{3}x\right)\left(1-\sqrt{3}x\right)}{\left(x^2+1\right)^3}$$

Possible inflection points when $x = \pm\frac{1}{\sqrt{3}}$. Concave down on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$; concave up on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$; inflection points when $x = \pm\frac{1}{\sqrt{3}}$.

26. $y = \frac{ax^2}{x+b}$

$$y' = \frac{(x+b)(2ax) - (ax^2)(1)}{(x+b)^2}$$

$$= \frac{2ax^2 + 2abx - ax^2}{(x+b)^2}$$

$$= \frac{ax^2 + 2abx}{(x+b)^2}$$

$$y'' = \frac{(x+b)^2(2ax+2ab) - (ax^2+2abx)(2)(x+b)(1)}{(x+b)^4}$$

$$= \frac{2a(x+b)^3 - 2ax(x+2b)(x+b)}{(x+b)^4}$$

$$= \frac{2a(x+b)^2 - 2ax(x+2b)}{(x+b)^3}$$

$$= \frac{2ax^2 + 4abx + 2ab^2 - 2ax^2 - 4abx}{(x+b)^3}$$

$$= \frac{2ab^2}{(x+b)^3}$$

No possible inflection point, but we must include $x = -b$ in the concavity analysis. For $a > 0$: concave down on $(-\infty, -b)$; concave up on $(-b, \infty)$. For $a < 0$: concave up on $(-\infty, -b)$; concave down on $(-b, \infty)$.

27. $y=\dfrac{21x+40}{6(x+3)^2}$

$$y'=\frac{1}{6}\cdot\frac{(x+3)^2(21)-(21x+40)[2(x+3)]}{(x+3)^4}$$

$$=\frac{1}{6}\cdot\frac{(x+3)(21)-(21x+40)(2)}{(x+3)^3}$$

$$=\frac{1}{6}\cdot\frac{-21x-17}{(x+3)^3}=-\frac{1}{6}\cdot\frac{21x+17}{(x+3)^3}$$

$$y''=-\frac{1}{6}\cdot\frac{(x+3)^3(21)-(21x+17)\left[3(x+3)^2\right]}{(x+3)^6}$$

$$=-\frac{1}{6}\cdot\frac{(x+3)(21)-(21x+17)(3)}{(x+3)^4}$$

$$=-\frac{1}{6}\cdot\frac{-42x+12}{(x+3)^4}=\frac{7x-2}{(x+3)^4}$$

Possible inflection point when $x=\frac{2}{7}$ ($x=-3$ must be considered in concavity analysis). Concave down on $(-\infty, -3)$ and $\left(-3, \frac{2}{7}\right)$; concave up on $\left(\frac{2}{7}, \infty\right)$; inflection point when $x=\frac{2}{7}$.

28. $y=3(x^2-2)^2$

$$y'=12x(x^2-2)=12(x^3-2x)$$

$$y''=12(3x^2-2)=36\left(x^2-\frac{2}{3}\right)$$

$$=36\left(x-\frac{\sqrt{6}}{3}\right)\left(x+\frac{\sqrt{6}}{3}\right)$$

Possible inflection points when $x=\pm\frac{\sqrt{6}}{3}$. Concave up on $\left(-\infty, -\frac{\sqrt{6}}{3}\right)$ and $\left(\frac{\sqrt{6}}{3}, 0\right)$; concave down on $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$; inflection points when $x=\pm\frac{\sqrt{6}}{3}$.

29. $y=5e^x$

$$y'=5e^x$$

$$y''=5e^x$$

Thus $y''>0$ for all x. Concave up on $(-\infty, \infty)$.

30. $y=e^x-e^{-x}$

$$y'=e^x+e^{-x}$$

$$y''=e^x-e^{-x}$$

Setting $y''=0$ gives $e^x=e^{-x}$ or, equivalently, $x=0$. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$; inflection point when $x=0$.

31. $y=axe^x$

$$y'=axe^x+ae^x=ae^x(x+1)$$

$$y''=ae^x(1)+a(x+1)e^x=ae^x(x+2)$$

$y''=0$ if $x=-2$. For $a>0$: concave down on $(-\infty, -2)$; concave up on $(-2, \infty)$; inflection point when $x=-2$. For $a<0$, concave up on $(-\infty, -2)$; concave down on $(-2, \infty)$; inflection point when $x=-2$.

32. $y=xe^{x^2}$

$$y'=2x^2e^{x^2}+e^{x^2}=e^{x^2}(2x^2+1)$$

$$y''=e^{x^2}(4x)+2x(2x^2+1)e^{x^2}=e^{x^2}(4x^3+6x)$$

$$=2xe^{x^2}(2x^2+3)$$

$y''=0$ when $x=0$. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$; inflection point when $x=0$.

33. $y=\dfrac{\ln x}{2x}$. (Note: $x>0$.)

$$y'=\frac{2x\cdot\frac{1}{x}-(\ln x)(2)}{4x^2}=\frac{1-\ln x}{2x^2}$$

$$y''=\frac{2x^2\left(-\frac{1}{x}\right)-(1-\ln x)(4x)}{4x^4}$$

$$=\frac{-2x-(1-\ln x)(4x)}{4x^4}$$

$$=\frac{-1-(1-\ln x)(2)}{2x^3}=\frac{2\ln(x)-3}{2x^3}$$

y'' is 0 if $2\ln(x)-3=0$, $\ln x=\frac{3}{2}$, $x=e^{\frac{3}{2}}$.

Concave down on $\left(0,\ e^{\frac{3}{2}}\right)$; concave up on $\left(e^{\frac{3}{2}},\ \infty\right)$; inflection point when $x = e^{\frac{3}{2}}$.

34. $y = \dfrac{x^2+1}{3e^x}$

$$y' = \frac{3e^x(2x) - \left(x^2+1\right)3e^x}{9e^{2x}} = \frac{2x - \left(x^2+1\right)}{3e^x}$$

$$= \frac{2x - x^2 - 1}{3e^x}$$

$$y'' = \frac{3e^x(2-2x) - \left(2x - x^2 - 1\right)3e^x}{9e^{2x}}$$

$$= \frac{(2-2x) - \left(2x - x^2 - 1\right)}{3e^x}$$

$$= \frac{x^2 - 4x + 3}{3e^x} = \frac{(x-1)(x-3)}{3e^x}$$

Possible inflection points when $x = 1, 3$.
Concave up on (–∞, 1) and (3, ∞); concave down on
(1, 3); inflection point when $x = 1, 3$.

35. $y = x^2 - x - 6 = (x-3)(x+2)$

Intercepts: (0, –6), (3, 0) and (–2, 0)

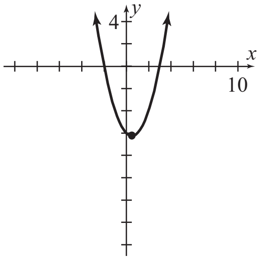
$y' = 2x - 1 = 2\left(x - \dfrac{1}{2}\right)$

CV: $x = \dfrac{1}{2}$

Decreasing on $\left(-\infty, \dfrac{1}{2}\right)$; increasing on $\left(\dfrac{1}{2}, \infty\right)$; relative minimum at $\left(\dfrac{1}{2}, -\dfrac{25}{4}\right)$.

$y'' = 2$

No possible inflection point. Concave up on (–∞, ∞).



36. $y = x^2 + a$

Intercept (0, a)

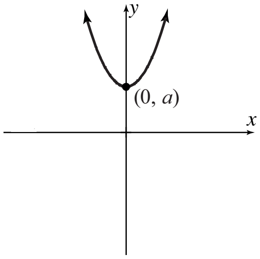
$y' = 2x$

CV: $x = 0$

Decreasing on (–∞, 0); increasing on (0, ∞); relative minimum at (0, a).

$y'' = 2$

No possible inflection point. Concave up on (–∞, ∞). Symmetric about the y-axis.



37. $y = 5x - 2x^2 = x(5-2x)$

Intercepts (0, 0) and $\left(\dfrac{5}{2}, 0\right)$

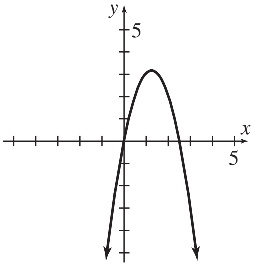
$y' = 5 - 4x$

CV: $x = \dfrac{5}{4}$

Increasing on $\left(-\infty, \dfrac{5}{4}\right)$; decreasing on $\left(\dfrac{5}{4}, \infty\right)$; relative maximum at $\left(\dfrac{5}{4}, \dfrac{25}{8}\right)$.

$y'' = -4$

No possible inflection point. Concave down on (–∞, ∞).



38. $y = x - x^2 + 2 = -(x-2)(x+1)$

Intercepts (2, 0), (–1, 0), and (0, 2)

$y' = 1 - 2x$

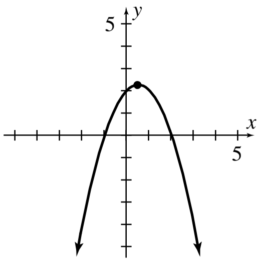
CV: $x = \dfrac{1}{2}$

Increasing on $\left(-\infty, \dfrac{1}{2}\right)$; decreasing on $\left(\dfrac{1}{2}, \infty\right)$;

relative maximum at $\left(\frac{1}{2}, \frac{9}{4}\right)$

$y'' = -2$

No possible inflection point. Concave down on (–∞, ∞).



39. $y = x^3 - 9x^2 + 24x - 19$

The x-intercepts are not convenient to find; the y-intercept is (0, –19).

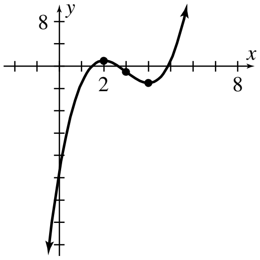
$y' = 3x^2 - 18x + 24 = 3(x-2)(x-4)$

CV: $x = 2$, $x = 4$

Increasing on (–∞, 2) and (4, ∞); decreasing on (2, 4); relative maximum at (2, 1); relative minimum at (4, –3).

$y'' = 6x - 18 = 6(x-3)$

Possible inflection point when $x = 3$. Concave down on (–∞, 3); concave up on (3, ∞); inflection point at (3, –1).



40. $y = x^3 - 25x^2 = x^2(x-25)$

Intercepts: (0, 0) and (25, 0)

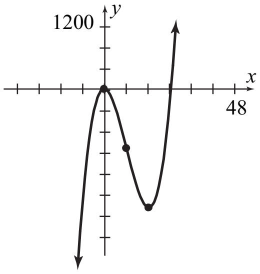
$y' = 3x^2 - 50x = 3x\left(x - \frac{50}{3}\right)$

CV: $x = 0, \frac{50}{3}$

Increasing on (–∞, 0) and $\left(\frac{50}{3}, \infty\right)$; decreasing on $\left(0, \frac{50}{3}\right)$; relative maximum at (0, 0); relative minimum at $\left(\frac{50}{3}, -\frac{62,500}{27}\right)$.

$y'' = 6x - 50 = 6\left(x - \frac{25}{3}\right)$

Possible inflection point when $x = \frac{25}{3}$. Concave down on $\left(-\infty, \frac{25}{3}\right)$; concave up on $\left(\frac{25}{3}, \infty\right)$; inflection point at $\left(\frac{25}{3}, -\frac{31,250}{27}\right)$.



41. $y = \frac{x^3}{3} - 5x = \frac{x^3 - 15x}{3}$

$= \frac{1}{3}x\left(x + \sqrt{15}\right)\left(x - \sqrt{15}\right)$

Intercepts (0, 0) and $\left(\pm\sqrt{15}, 0\right)$

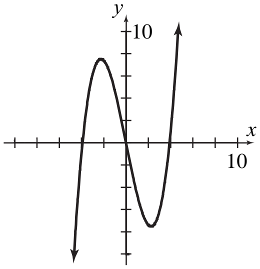
$y' = x^2 - 5 = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

CV: $x = \pm\sqrt{5}$

Increasing on $\left(-\infty, -\sqrt{5}\right)$ and $\left(\sqrt{5}, \infty\right)$; decreasing on $\left(-\sqrt{5}, \sqrt{5}\right)$; relative maximum at $\left(-\sqrt{5}, \frac{10}{3}\sqrt{5}\right)$; relative minimum at $\left(\sqrt{5}, -\frac{10}{3}\sqrt{5}\right)$.

$y'' = 2x$

Possible inflection point when $x = 0$. Concave down on (–∞, 0); concave up on (0, ∞); inflection point at (0, 0). Symmetric about the origin.



42. $y = x^3 - 6x^2 + 9x = x(x-3)^2$

Intercepts (0, 0) and (3, 0)

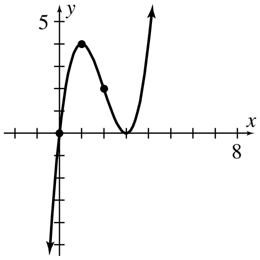
$y' = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

CV: $x = 1$ and $x = 3$

Increasing on (−∞, 1) and (3, ∞); decreasing on (1, 3); relative maximum at (1, 4); relative minimum at (3, 0).

$y'' = 6x - 12 = 6(x-2)$

Possible inflection point when $x = 2$. Concave down on (−∞, 2); concave up on (2, ∞); inflection point at (2, 2).



43. $y = x^3 - 3x^2 + 3x - 3$

Intercept (0, −3)

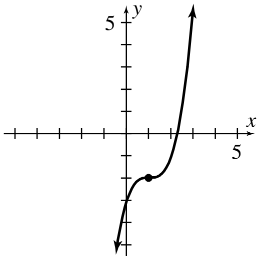
$y' = 3x^2 - 6x + 3 = 3(x-1)^2$

CV: $x = 1$

Increasing on (−∞, 1) and (1, ∞); no relative maximum or minimum

$y'' = 6(x-1)$

Possible inflection point when $x = 1$. Concave down on (−∞, 1); concave up on (1, ∞); inflection point at (1, −2).



44. $y = 2x^3 + \frac{5}{2}x^2 + 2x = x\left(2x^2 + \frac{5}{2}x + 2\right)$

Intercept (0, 0)

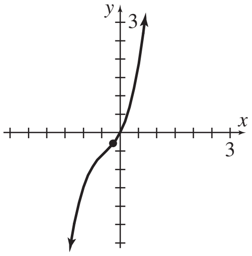
$y' = 6x^2 + 5x + 2$

CV: none

Increasing on (−∞, ∞).

$y'' = 12x + 5 = 12\left[x + \frac{5}{12}\right]$

Possible inflection point at $x = -\frac{5}{12}$. Concave down on $\left(-\infty, -\frac{5}{12}\right)$; concave up on $\left(-\frac{5}{12}, \infty\right)$; inflection point at $\left(-\frac{5}{12}, -\frac{235}{432}\right)$.



45. $y = 4x^3 - 3x^4 = x^3(4-3x)$

Intercepts (0, 0), $\left(\frac{4}{3}, 0\right)$

$y' = 12x^2 - 12x^3 = 12x^2(1-x)$

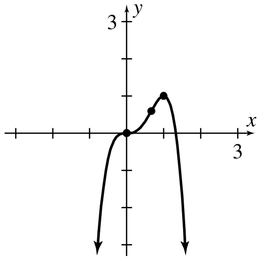
CV: $x = 0$ and $x = 1$

Increasing on (−∞, 0) and (0, 1); decreasing on (1, ∞); relative maximum at (1, 1).

$y'' = 24x - 36x^2 = 12x(2-3x)$

Possible inflection points at $x = 0$ and $x = \frac{2}{3}$.

Concave down on (−∞, 0) and $\left(\frac{2}{3}, \infty\right)$; concave up on $\left(0, \frac{2}{3}\right)$; inflection points at (0, 0) and $\left(\frac{2}{3}, \frac{16}{27}\right)$



46. $y = -x^3 + 8x^2 - 5x + 3$

Intercept (0, 3)

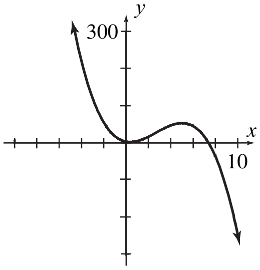
$$\begin{aligned} y' &= -3x^2 + 16x - 5 \\ &= -(3x-1)(x-5) \end{aligned}$$

CV: $\frac{1}{3}$, 5

Decreasing on $\left(-\infty, \frac{1}{3}\right)$ and (5, ∞); increasing on $\left(\frac{1}{3}, 5\right)$; relative minimum at $\left(\frac{1}{3}, \frac{59}{27}\right)$; relative maximum at (5, 53).

$y'' = -6x + 16 = -6\left(x - \frac{8}{3}\right)$

Possible inflection point when $x = \frac{8}{3}$. Concave up on $\left(-\infty, \frac{8}{3}\right)$; concave down on $\left(\frac{8}{3}, \infty\right)$; inflection point at $\left(\frac{8}{3}, \frac{745}{27}\right)$.



47. $y = -2 + 12x - x^3$

Intercept (0, –2)

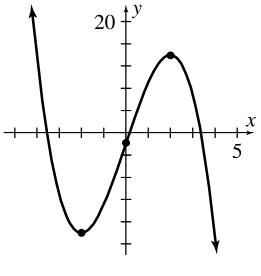
$y' = 12 - 3x^2 = 3(2 + x)(2 - x)$

CV: $x = \pm 2$

Decreasing on (–∞, –2) and (2, ∞); increasing on (–2, 2); relative minimum at (–2, –18); relative maximum at (2, 14).

$y'' = -6x$

Possible inflection point when $x = 0$. Concave up on (–∞, 0); concave down on (0, ∞); inflection point at (0, –2).



48. $y = (3 + 2x)^3$

Intercepts (0, 27), $\left(-\frac{3}{2}, 0\right)$

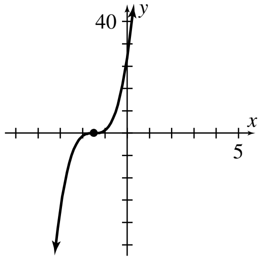
$y' = 6(3 + 2x)^2$

CV: $x = -\frac{3}{2}$

Increasing on $\left(-\infty, -\frac{3}{2}\right)$ and $\left(-\frac{3}{2}, \infty\right)$; no relative maximum or minimum.

$y'' = 24(3 + 2x)$

Possible inflection point at $x = -\frac{3}{2}$.Concave down on $\left(-\infty, -\frac{3}{2}\right)$; concave up on $\left(-\frac{3}{2}, \infty\right)$; inflection point at $\left(-\frac{3}{2}, 0\right)$.



49. $y = 2x^3 - 6x^2 + 6x - 2 = 2(x - 1)^3$

Intercepts (0, –2), (1, 0)

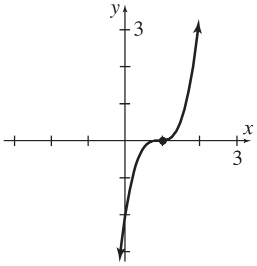
$y' = 6(x - 1)^2$

CV: $x = 1$

Increasing on (–∞, 1) and (1, ∞); no relative maximum or minimum.

$y'' = 12(x - 1)$

Possible inflection point when $x = 1$. Concave down on (–∞, 1); concave up on (1, ∞); inflection point at (1, 0).



50. $y = \frac{x^5}{100} - \frac{x^4}{20} = \frac{x^4}{100}(x - 5)$

Intercepts (0, 0), (5, 0)

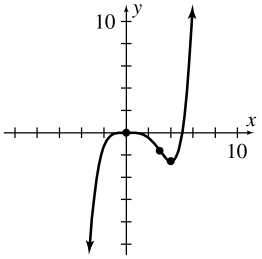
$y' = \frac{x^4}{20} - \frac{x^3}{5} = \frac{x^3}{20}(x - 4)$

CV: $x = 0$ and $x = 4$

Increasing on (–∞, 0) and (4, ∞); decreasing on (0, 4); relative maximum at (0, 0); relative minimum at (4, –2.56).

$$y'' = \frac{x^3}{5} - \frac{3x^2}{5} = \frac{x^2}{5}(x-3)$$

Possible inflection points when $x = 0$ and $x = 3$. Concave down on $(-\infty, 0)$ and $(0, 3)$; concave up on $(3, \infty)$; inflection point at $(3, -1.62)$.



51. $y = 16x - x^5 = x\left(16 - x^4\right)$

$= x(4 + x^2)(4 - x^2)$

$= x(4 + x^2)(2 + x)(2 - x)$

Intercepts (0, 0) and (±2, 0)
Symmetric about the origin.

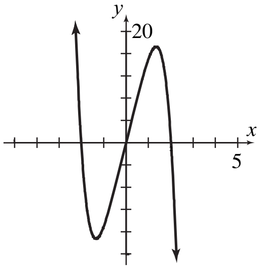
$$\begin{aligned} y' &= 16 - 5x^4 \\ &= 5\left(\frac{16}{5} - x^4\right) \\ &= 5\left(\frac{4}{\sqrt{5}} + x^2\right)\left(\frac{4}{\sqrt{5}} - x^2\right) \\ &= 5\left(\frac{4}{\sqrt{5}} + x^2\right)\left(\frac{2}{\sqrt[4]{5}} + x\right)\left(\frac{2}{\sqrt[4]{5}} - x\right) \end{aligned}$$

CV: $x = \pm\frac{2}{\sqrt[4]{5}}$

Decreasing on $\left(-\infty, -\frac{2}{\sqrt[4]{5}}\right)$ and $\left(\frac{2}{\sqrt[4]{5}}, \infty\right)$; increasing on $\left(-\frac{2}{\sqrt[4]{5}}, \frac{2}{\sqrt[4]{5}}\right)$; relative minimum at $\left(-\frac{2}{\sqrt[4]{5}}, -\frac{128}{5\sqrt[4]{5}}\right)$; relative maximum at $\left(\frac{2}{\sqrt[5]{4}}, \frac{128}{5\sqrt[4]{5}}\right)$

$y'' = -20x^3$

Possible inflection point when $x = 0$. Concave up on $(-\infty, 0)$; concave down on $(0, \infty)$; inflection point at (0, 0). Symmetric about the origin.



52. $y = x^2(x-1)^2$

Intercepts: (0, 0), (1, 0)

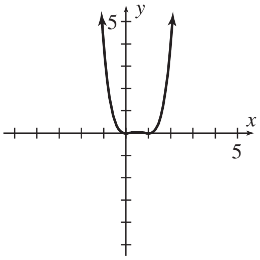
$$\begin{aligned} y' &= x^2[2(x-1)(1)] + 2x(x-1)^2 \\ &= 2x(x-1)(2x-1) \\ &= 4x^3 - 6x^2 + 2x \end{aligned}$$

CV: $x = 0$, 1 and $x = \frac{1}{2}$

Decreasing on $(-\infty, 0)$ and $\left(\frac{1}{2}, 1\right)$; increasing on $\left(0, \frac{1}{2}\right)$ and $(1, \infty)$; relative minima at (0, 0) and (1, 0); relative maximum at $\left(\frac{1}{2}, \frac{1}{16}\right)$

$y'' = 12x^2 - 12x + 2 = 2(6x^2 - 6x + 1)$

From the quadratic formula, there are possible inflection points when $x = \frac{3 \pm \sqrt{3}}{6}$. Concave up on $\left(-\infty, \frac{3-\sqrt{3}}{6}\right)$ and $\left(\frac{3+\sqrt{3}}{6}, \infty\right)$; concave down on $\left(\frac{3-\sqrt{3}}{6}, \frac{3+\sqrt{3}}{6}\right)$; inflection points at $\left(\frac{3-\sqrt{3}}{6}, \frac{1}{36}\right)$ and $\left(\frac{3+\sqrt{3}}{6}, \frac{1}{36}\right)$.



53. $y = 3x^4 - 4x^3 + 1$

Intercepts (0, 1) and (1, 0) [the latter is found by inspection of the equation]. No symmetry.

$y' = 12x^3 - 12x^2 = 12x^2(x-1)$

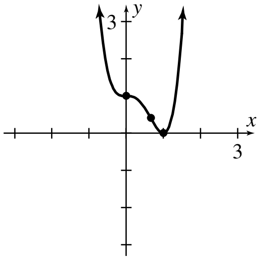
CV: $x = 0$ and $x = 1$

Decreasing on $(-\infty, 0)$ and (0, 1); increasing on $(1, \infty)$; relative minimum at (1, 0).

$y'' = 36x^2 - 24x = 12x(3x-2)$

Possible inflection points at $x = 0$ and $x = \frac{2}{3}$.

Concave up on $(-\infty, 0)$ and $\left(\frac{2}{3}, \infty\right)$; concave down on $\left(0, \frac{2}{3}\right)$; inflection points at (0, 1) and $\left(\frac{2}{3}, \frac{11}{27}\right)$.



54. $y = 3x^5 - 5x^3 = 3x^3\left[x^2 - \frac{5}{3}\right]$

$= 3x^3\left(x + \sqrt{\frac{5}{3}}\right)\left(x - \sqrt{\frac{5}{3}}\right)$

Intercepts (0, 0) and $\left(\pm\sqrt{\frac{5}{3}}, 0\right)$

Symmetric about the origin.

$y' = 15x^4 - 15x^2 = 15x^2(x+1)(x-1)$

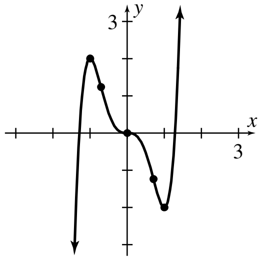
CV: $x = 0$ and $x = \pm 1$

Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on (–1, 0) and (0, 1); relative maximum at (–1, 2); relative minimum at (1, –2).

$y'' = 60x^3 - 30x = 60x\left[x + \frac{\sqrt{2}}{2}\right]\left[x - \frac{\sqrt{2}}{2}\right]$

Possible inflection points at $x = 0$ and $x = \pm\frac{\sqrt{2}}{2}$.

Concave down on $\left(-\infty, -\frac{\sqrt{2}}{2}\right)$ and $\left(0, \frac{\sqrt{2}}{2}\right)$; concave up on $\left(-\frac{\sqrt{2}}{2}, 0\right)$ and $\left(\frac{\sqrt{2}}{2}, \infty\right)$; inflection points at $\left(\frac{\sqrt{2}}{2}, -\frac{7\sqrt{2}}{8}\right)$, (0, 0), and $\left(-\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{8}\right)$.



55. $y = 4x^2 - x^4 = x^2(2+x)(2-x)$

Intercepts (0, 0) and (±2, 0)

Symmetric about the y-axis.

$y' = 8x - 4x^3 = 4x\left(2 - x^2\right)$

$= 4x\left(\sqrt{2} + x\right)\left(\sqrt{2} - x\right)$

CV: $x = 0, \pm\sqrt{2}$

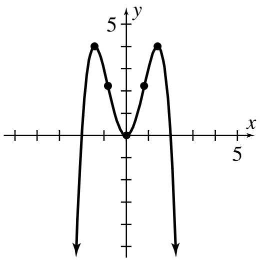
Increasing on $\left(-\infty, -\sqrt{2}\right)$ and $\left(0, \sqrt{2}\right)$; decreasing on $\left(-\sqrt{2}, 0\right)$ and $\left(\sqrt{2}, \infty\right)$; relative maxima at $\left(\pm\sqrt{2}, 4\right)$; relative minimum at (0, 0).

$y'' = 8 - 12x^2 = 12\left[\frac{2}{3} - x^2\right]$

$= 12\left(\sqrt{\frac{2}{3}} - x\right)\left(\sqrt{\frac{2}{3}} + x\right)$

Possible inflection points when $x = \pm\sqrt{\frac{2}{3}}$.

Concave down on $\left(-\infty, -\sqrt{\frac{2}{3}}\right)$ and $\left(\sqrt{\frac{2}{3}}, \infty\right)$; concave up on $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$; inflection points at $\left(\pm\sqrt{\frac{2}{3}}, \frac{20}{9}\right)$.



56. $y = x^2e^x$

Intercept (0, 0)

$y' = 2xe^x + x^2e^x = xe^x(2+x)$

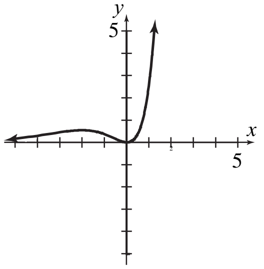
CV: $x = 0, -2$

Increasing on (–∞, –2) and (0, ∞); decreasing on (–2, 0); relative maximum at $\left(-2, \frac{4}{e^2}\right)$; relative minimum at (0, 0)

$$\begin{aligned} y'' &= 2e^x + 2xe^x + 2xe^x + x^2e^x \\ &= e^x(2+4x+x^2) \end{aligned}$$

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{16-4(1)(2)}}{2} \\ &= \frac{-4 \pm \sqrt{8}}{2} \\ &= \frac{-4 \pm 2\sqrt{2}}{2} \\ &= -2 \pm \sqrt{2} \end{aligned}$$

Possible inflection points when $x = -2 \pm \sqrt{2}$. Concave up on $\left(-\infty, -2-\sqrt{2}\right)$ and $\left(-2+\sqrt{2}, \infty\right)$; concave down on $\left(-2-\sqrt{2}, -2+\sqrt{2}\right)$; inflection points at $\left(-2-\sqrt{2}, \left(4\sqrt{2}+6\right)e^{-2-\sqrt{2}}\right)$ and $\left(-2+\sqrt{2}, \left(6-4\sqrt{2}\right)e^{-2+\sqrt{2}}\right)$



57. $y = x^{1/3}(x-8) = x^{4/3} - 8x^{1/3}$

Intercepts (0, 0) and (8, 0)

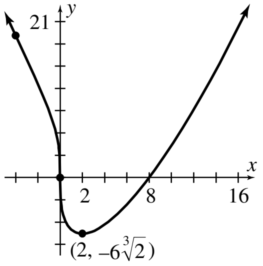
$$\begin{aligned} y' &= \frac{4}{3}x^{1/3} - \frac{8}{3}x^{-2/3} \\ &= \frac{4}{3}\left[x^{1/3} - \frac{2}{x^{2/3}}\right] = \frac{4(x-2)}{3x^{2/3}} \end{aligned}$$

CV: $x = 0, 2$

Decreasing on (–∞, 0) and (0, 2); increasing on (2, ∞); relative minimum at $\left(2, -6\sqrt[3]{2}\right) \approx \left(2, -7.56\right)$.

$$\begin{aligned} y'' &= \frac{4}{9}x^{-2/3} + \frac{16}{9}x^{-5/3} \\ &= \frac{4}{9}\left[\frac{1}{x^{2/3}} + \frac{4}{x^{5/3}}\right] = \frac{4(x+4)}{9x^{5/3}} \end{aligned}$$

Possible inflection points when $x = -4, 0$. Concave up on (–∞, –4) and (0, ∞); concave down on (–4, 0); inflection points at $\left(-4, 12\sqrt[3]{4}\right)$ and (0, 0). Observe that at the origin the tangent line exists but it is vertical.



58. $y = (x-1)^2(x+2)^2$

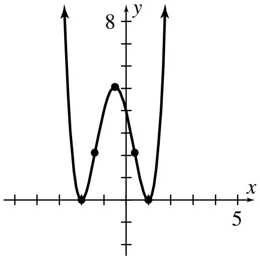
Intercepts (0, 4), (1, 0), (–2, 0)

$y' = (x-1)^2[2(x+2)] + (x+2)^2[2(x-1)]$

$= 2(x-1)(x+2)(2x+1)$

CV: $x = -2, -\frac{1}{2}, 1$

Decreasing on (–∞, –2) and $\left(-\frac{1}{2}, 1\right)$; increasing on $\left(-2, -\frac{1}{2}\right)$ and (1, ∞); relative maximum at $\left(-\frac{1}{2}, \frac{81}{16}\right)$; relative minima at (–2, 0) and (1, 0); $y' = 2(2x^3 + 3x^2 - 3x - 2)$, so $y'' = 6\left(2x^2 + 2x - 1\right)$. Setting $y'' = 0$ and using the quadratic formula gives possible inflection points at $x=\frac{-1\pm\sqrt{3}}{2}$. Concave up on $\left(-\infty, \frac{-1-\sqrt{3}}{2}\right)$ and $\left(\frac{-1+\sqrt{3}}{2}, \infty\right)$; concave down on $\left(\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right)$; inflection points when $x=\frac{-1\pm\sqrt{3}}{2}$



59. $y=4x^{1/3}+x^{4/3}=x^{1/3}(4+x)$

Intercepts (0, 0) and (–4, 0)

$$y'=\frac{4}{3}x^{-2/3}+\frac{4}{3}x^{1/3}=\frac{4}{3}\left[\frac{1}{x^{2/3}}+x^{1/3}\right]$$

$$=\frac{4(1+x)}{3x^{2/3}}$$

CV: $x = 0, -1$

Decreasing on (–∞, –1); increasing on (–1, 0) and (0, ∞); rel. min at (–1, –3)

$$y''=-\frac{8}{9}x^{-5/3}+\frac{4}{9}x^{-2/3}=\frac{4}{9}\left[\frac{1}{x^{2/3}}-\frac{2}{x^{5/3}}\right]$$

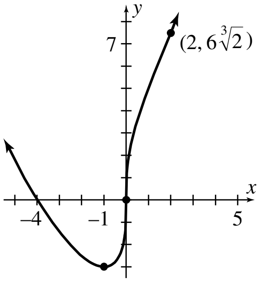
$$=\frac{4(x-2)}{9x^{5/3}}$$

Possible inflection points when $x = 0, 2$.

Concave up on (–∞, 0) and (2, ∞); concave down on

(0, 2); inflection point at (0, 0) and $\left(2, 6\sqrt[3]{2}\right)$.

Observe that at the origin the tangent line exists but it is vertical.



60. $y=(x+1)\sqrt{x+4}$ [Note: $x\ge -4$]

Intercepts (0, 2), (–1, 0) and (–4, 0)

$$y'=(x+1)\cdot\frac{1}{2\sqrt{x+4}}+\sqrt{x+4}(1)$$

$$=\frac{1}{2\sqrt{x+4}}[(x+1)+2(x+4)]$$

$$=\frac{3(x+3)}{2\sqrt{x+4}}$$

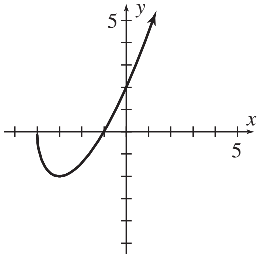
CV: $x = -3, -4$

Decreasing on (–4, –3); increasing on (–3, ∞); relative minimum at (–3, –2)

$$y''=\frac{3}{2}\cdot\frac{\sqrt{x+4}(1)-(x+3)\cdot\frac{1}{2\sqrt{x+4}}}{\left(\sqrt{x+4}\right)^2}$$

$$=\frac{3}{4}\cdot\frac{2(x+4)-(x+3)}{(x+4)^{3/2}}=\frac{3(x+5)}{4(x+4)^{3/2}}$$

No possible inflection point. Concave up on (–4, ∞).



61. $y=2x^{2/3}-x=x^{2/3}(2-x^{1/3})$

Intercepts (0, 0) and (8, 0)

$$y'=\frac{4}{3}x^{-1/3}-1$$

$y'=0$ when $x^{-1/3}=\frac{3}{4}$

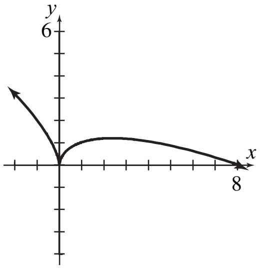
$$x=\left(\frac{3}{4}\right)^{-3}=\frac{64}{27}$$

CV: $0, \frac{64}{27}$

Increasing on $\left(0, \frac{64}{27}\right)$; decreasing on (–∞, 0) and $\left(\frac{64}{27}, \infty\right)$; relative maximum at $\left(\frac{64}{27}, \frac{32}{27}\right)$; relative minimum at (0, 0)

$$y''=-\frac{4}{9}x^{-4/3}=-\frac{4}{9x^{4/3}}$$

Possible inflection point at $x = 0$. Concave down on (–∞, 0) and (0, ∞); no inflection points; vertical tangent line at (0, 0). No symmetry.



62. $y = 5x^{2/3} - x^{5/3} = x^{2/3}(5-x)$

Intercepts (0, 0) and (5, 0)

$$y' = \frac{10}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = \frac{5}{3}\left[\frac{2}{x^{1/3}} - x^{2/3}\right]$$

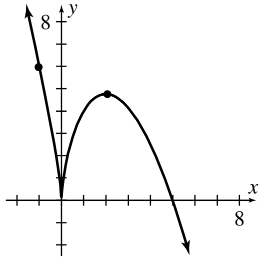
$$= \frac{5(2-x)}{3x^{1/3}}$$

CV: $x = 0, 2$

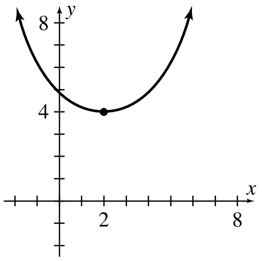
Increasing on (0, 2); decreasing on $(-\infty, 0)$ and $(2, \infty)$; relative minimum at (0, 0); relative maximum at $\left(2, 3\sqrt[3]{4}\right) \approx (2, 4.76)$

$$y'' = -\frac{10}{9}x^{-4/3} - \frac{10}{9}x^{-1/3} = -\frac{10(1+x)}{9x^{4/3}}$$

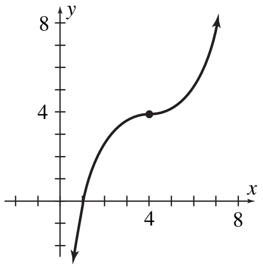
Possible inflection point when $x = 0, -1$.
Concave up on $(-\infty, -1)$; concave down on $(-1, 0)$, and $(0, \infty)$; inflection point at $(-1, 6)$.
Observe that at the origin the tangent line exists but it is vertical.



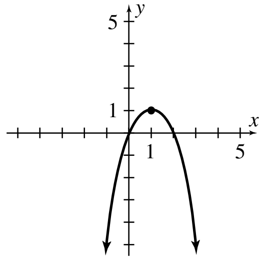
63.



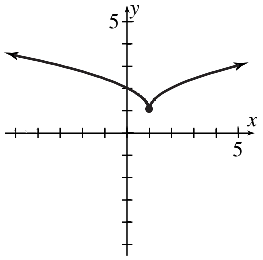
64.



65.



66.



67. $p = \frac{100}{q+2}$

$\frac{dp}{dq} = -\frac{100}{(q+2)^2} < 0$ for $q > 0$, so p is decreasing.

Since $\frac{d^2p}{dq^2} = \frac{200}{(q+2)^3} > 0$ for $q > 0$, the demand curve is concave up.

68. $c = q^2 + 2q + 1$

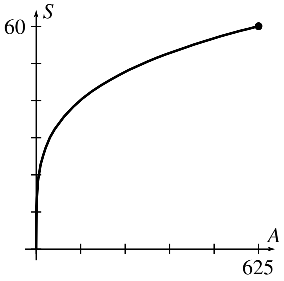
$$\bar{c} = \frac{c}{q} = q + 2 + \frac{1}{q}$$

$$\bar{c}' = 1 - \frac{1}{q^2}$$

$$\bar{c}'' = \frac{2}{q^3}$$

Since $\bar{c}'' > 0$ for $q > 0$, the graph of the average cost function is concave up for $q > 0$.

69. $S = f(A) = 12\sqrt[4]{A}$, $0 \le A \le 625$. For the given values of *A* we have $S' = 3A^{-\frac{3}{4}} > 0$ and $S'' = -\left(\frac{9}{4}\right)A^{-\frac{7}{4}} < 0$. Thus *y* is increasing and concave down.



70. $g(x) = e^{\frac{U_0}{A}} e^{-\frac{x^2}{2A}}$, *A* > 0, *x* ≥ 0 (since *x* represents quantity).

$$g'(x) = -\frac{e^{\frac{U_0}{A}}}{A}\left[xe^{-\frac{x^2}{2A}}\right]$$

$$g''(x) = -\frac{e^{\frac{U_0}{A}}}{A}\left[x \cdot e^{-\frac{x^2}{2A}}\left(-\frac{x}{A}\right) + e^{-\frac{x^2}{2A}}\right]$$

$$= \frac{e^{\frac{U_0}{A}}}{A^2} \cdot e^{-\frac{x^2}{2A}}\left(x^2 - A\right)$$

$$= \frac{e^{\frac{U_0}{A}}}{A^2} \cdot e^{-\frac{x^2}{2A}}\left(x + \sqrt{A}\right)\left(x - \sqrt{A}\right)$$

If $0 \le x < \sqrt{A}$, then $g''(x) < 0$, so the graph is concave down. If $x > \sqrt{A}$, then $g''(x) > 0$, so the graph is concave up.

71. $y = 12.5 + 5.8(0.42)^x$

$y' = 5.8(0.42)^x \ln(0.42)$

Since ln(0.42) < 0, we have $y' < 0$, so the function is decreasing.

$y'' = 5.8(0.42)^x \ln^2(0.42) > 0$, so the function is concave up.

72. $H = 1.00\left[1 - e^{-(0.0464t+0.0670}\right]$

$\frac{dH}{dt} = 0.0464e^{-(0.0464t+0.0670)} > 0$, so *H* is increasing.

$\frac{d^2H}{dt^2} = -(0.0464)^2 e^{-(0.0464t+0.0670)} < 0$, so *H* is concave down.

73. $n = f(r) = 0.1\ln(r) + \frac{7}{r} - 0.8$, $1 \le r \le 10$

a. $\frac{dn}{dr} = \frac{0.1}{r} - \frac{7}{r^2} = \frac{0.1r - 7}{r^2} = \frac{0.1(r - 70)}{r^2} < 0$

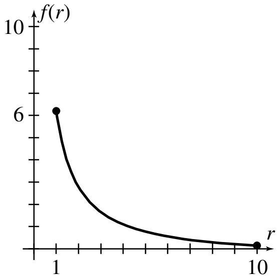
for $1 \le r \le 10$. Thus the graph of *f* is always falling. Also,

$$\frac{d^2n}{dr^2} = -\frac{0.1}{r^2} + \frac{14}{r^3} = \frac{14 - 0.1r}{r^3}$$

$$= \frac{0.1(140 - r)}{r^3} > 0$$

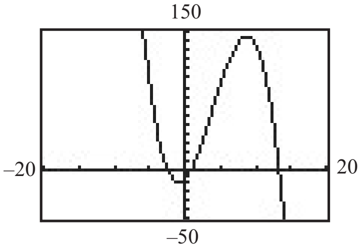
for $1 \le r \le 10$. Thus the graph is concave up.

b.



c. $\left.\frac{dn}{dr}\right|_{r=5} = -0.26$, so the rate of decrease is 0.26.

74.

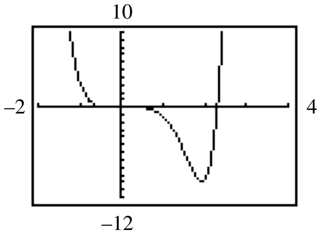


a. One relative maximum point

b. One relative minimum point

c. One inflection point

75.



Two inflection points

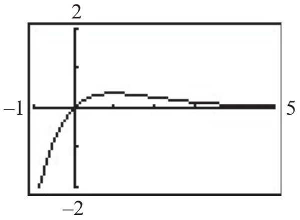
$y = x^5(x-a) = x^6 - ax^5$

$y' = 6x^5 - 5ax^4$

$y'' = 30x^4 - 20ax^3 = 10x^3(3x-2a)$

Possible inflection points when $x = 0$ and $x = \frac{2a}{3}$. If $a > 0$, y is concave up on $(-\infty, 0)$ and $\left(\frac{2a}{3}, \infty\right)$; concave down on $\left(0, \frac{2a}{3}\right)$. If $a < 0$, y is concave up on $\left(-\infty, \frac{2a}{3}\right)$ and $(0, \infty)$; concave downon $\left(\frac{2a}{3}, 0\right)$. In either case, y has two points of inflection, when $x = 0$ and $x = \frac{2a}{3}$.

76.



$y = xe^{-x}$

Intercept (0, 0)

$y' = e^{-x} - xe^{-x} = e^{-x}(1-x)$

CV: $x = 1$

Increasing on $(-\infty, 1)$; decreasing on $(1, \infty)$; relative maximum at $(1, e^{-1})$

$$\begin{aligned} y'' &= -e^{-x} - e^{-x} + xe^{-x} \\ &= -2e^{-x} + xe^{-x} \\ &= e^{-x}(x-2) \end{aligned}$$

Possible inflection point at $x = 2$.

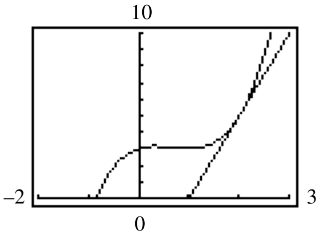
Concave down on $(-\infty, 2)$; concave up on $(2, \infty)$; inflection point at $(2, 2e^{-2})$

Answers will vary for $q(p) = Qe^{-Rp}$.

77. $y = x^3 - 2x^2 + x + 3$

$y' = 3x^2 - 4x + 1$

When $x = 2$, then $y = 5$ and $y' = 5$. Thus an equation of the tangent line at $x = 2$ is $y - 5 = 5(x - 2)$, or $y = 5x - 5$. Graphing the curve and the tangent line indicates that the curve lies above the tangent line around $x = 2$. Thus the curve is concave up at $x = 2$.



78. $f(x) = 2x^3 + 3x^2 - 6x + 1$

$f'(x) = 6x^2 + 6x - 6$

$f''(x) = 12x + 6$

The relative minimum of f' occurs at a value of x for which $(f'(x))' = f''(x) = 0$. Around this value of x, $(f'(x))'$ goes from – to +. Since $(f'(x))' = f''(x)$, the concavity of f must change from concave down to concave up.

79. $f(x) = x^6 + 3x^5 - 4x^4 + 2x^2 + 1$

$f'(x) = 6x^5 + 15x^4 - 16x^3 + 4x$

$f''(x) = 30x^4 + 60x^3 - 48x^2 + 4$

Inflection points of f when $x \approx -2.61, -0.26$.

80. $f(x) = \frac{x+1}{x^2+1}$

$f'(x) = -\frac{x^2+2x-1}{\left(x^2+1\right)^2}$

$f''(x) = \frac{2\left(x^3+3x^2-3x-1\right)}{\left(x^2+1\right)^3}$

Inflection points of f when $x \approx -3.73, -0.27, 1.00$.

Problems 13.4

1. $y = x^2 - 5x + 6$

$y' = 2x - 5$

CV: $x = \frac{5}{2}$

$y'' = 2$

$y''\left(\frac{5}{2}\right) = 2 > 0$

Thus there is a relative minimum when $x = \frac{5}{2}$.

Because there is only one relative extremum and f is continuous, the relative minimum is an absolute minimum.

2. $y = 3x^2 + 12x + 14$

$y' = 6x + 12$

CV: $x = -2$

$y'' = 6$

$y''(-2) = 6 > 0$

Thus there is a relative minimum when $x = -2$. Because there is only one relative extremum and *f* is continuous, the relative minimum is an absolute minimum.

3. $y = -4x^2 + 2x - 8$

$y' = -8x + 2$

CV: $x = \frac{1}{4}$

$y'' = -8$

$y''\left(\frac{1}{4}\right) = -8 < 0$

Thus there is a relative maximum when $x = \frac{1}{4}$.

Because there is only one relative extremum and *f* is continuous, the relative maximum is an absolute maximum.

4. $y = 3x^2 - 5x + 6$

$y' = 6x - 5$

CV: $x = \frac{5}{6}$

$y'' = 6$

$y''\left(\frac{5}{6}\right) = 6 > 0$

Thus there is a relative minimum when $x = \frac{5}{6}$.

Because there is only one relative extremum and *f* is continuous, the relative minimum is an absolute minimum.

5. $y = \frac{1}{3}x^3 + 2x^2 - 5x + 1$

$y' = x^2 + 4x - 5 = (x+5)(x-1)$

CV: $x = -5, 1$

$y'' = 2x + 4$

$y''(-5) = -6 < 0 \Rightarrow$ relative maximum when $x = -5$

$y''(1) = 6 > 0 \Rightarrow$ relative minimum when $x = 1$

6. $y = x^3 - 12x + 1$

$y' = 3x^2 - 12 = 3(x+2)(x-2)$

CV: $x = \pm 2$

$y'' = 6x$

$y''(-2) = -12 < 0 \Rightarrow$ relative maximum when $x = -2$

$y''(2) = 12 > 0 \Rightarrow$ relative minimum when $x = 2$

7. $y = 2x^3 - 3x^2 - 36x + 17$

$y' = 6x^2 - 6x - 36 = 6(x-3)(x+2)$

CV: $x = 3, -2$

$y'' = 12x - 6$

$y''(3) = 30 > 0 \Rightarrow$ relative minimum when $x = 3$

$y''(-2) = -30 < 0 \Rightarrow$ relative maximum when $x = -2$

8. $y = x^4 - 2x^2 + 4$

$y' = 4x^3 - 4x = 4x(x+1)(x-1)$

CV: $x = 0, \pm 1$

$y'' = 12x^2 - 4$

$y''(0) = -4 < 0 \Rightarrow$ relative maximum when $x = 0$

$y''(1) = 8 > 0 \Rightarrow$ relative minimum when $x = 1$

$y''(-1) = 8 > 0 \Rightarrow$ relative minimum when $x = -1$

9. $y = 7 - 2x^4$

$y' = -8x^3$

CV: $x = 0$

$y'' = -24x^2$

Since $y''(0) = 0$, the second-derivative test fails. Using the first-derivative test, we see that *f* increases for $x < 0$ and *f* decreases for $x > 0$, so there is a relative maximum when $x = 0$.

10. $y = -2x^7$

$y' = -14x^6$

CV: $x = 0$

$y'' = -84x^5$

Since $y''(0) = 0$, the second-derivative test fails. However, using the first-derivative test, we see that *f* decreases for $x < 0$ and for $x > 0$, so there is neither a relative maximum nor a relative minimum when $x = 0$.

11. $y = 81x^5 - 5x$

$y' = 81\cdot 5x^4 - 5 = 5\left(81x^4 - 1\right)$

$= 5\left(9x^2 - 1\right)\left(9x^2 + 1\right)$

$= 5(3x+1)(3x-1)\left(9x^2 + 1\right)$

CV: $x = \pm\frac{1}{3}$

$y'' = 81\cdot 5\cdot 4x^3$

$y''\left(-\frac{1}{3}\right) = -60 < 0 \Rightarrow$ relative maximum when $x = -\frac{1}{3}$

$y''\left(\frac{1}{3}\right) = 60 > 0 \Rightarrow$ relative minimum when $x = \frac{1}{3}$

12. $y = 15x^3 + x^2 - 15x + 2$

$y' = 45x^2 + 2x - 15 = (5x+3)(9x-5)$

CV: $x = -\frac{3}{5}, \frac{5}{9}$

$y'' = 90x + 2$

$y''\left(-\frac{3}{5}\right) = -52 \Rightarrow$ relative maximum when $x = -\frac{3}{5}$

$y''\left(\frac{5}{9}\right) = 52 \Rightarrow$ relative minimum when $x = \frac{5}{9}$

13. $y = \left(x^2 + 7x + 10\right)^2$

$y' = 2\left(x^2 + 7x + 10\right)(2x+7)$

$= 2(x+2)(x+5)(2x+7)$

CV: $x = -2, -5, -\frac{7}{2}$

$y'' = 2\left[\left(x^2 + 7x + 10\right)(2) + (2x+7)(2x+7)\right]$

$y''(-5) = 18 > 0 \Rightarrow$ relative minimum when $x = -5$

$y''\left(-\frac{7}{2}\right) = -9 < 0 \Rightarrow$ relative maximum when $x = -\frac{7}{2}$

$y''(-2) = 18 > 0 \Rightarrow$ relative minimum when $x = -2$

14. $y = -x^3 + 3x^2 + 9x - 2$

$y' = -3x^2 + 6x + 9 = -3(x^2 - 2x - 3)$

$= -3(x+1)(x-3)$

CV: $x = -1, 3$

$y'' = -6x + 6$

$y''(-1) = 12 > 0 \Rightarrow$ relative minimum when $x = -1$

$y''(3) = -12 < 0 \Rightarrow$ relative maximum when $x = 3$

Problems 13.5

1. $y = f(x) = \frac{x}{x-1}$

When $x = 1$ the denominator is zero but the numerator is not zero. Thus $x = 1$ is a vertical asymptote.

$\lim\limits_{x\to\infty} \frac{x}{x-1} = \lim\limits_{x\to\infty} \frac{x}{x} = \lim\limits_{x\to\infty} 1 = 1.$

Similarly $\lim\limits_{x\to-\infty} f(x) = 1$. Thus the line $y = 1$ is a horizontal asymptote.

2. $y = f(x) = \frac{x+1}{x}$

When $x = 0$ the denominator is zero but the numerator is not. Thus $x = 0$ is a vertical asymptote. $\lim\limits_{x\to\infty} \frac{x+1}{x} = \lim\limits_{x\to\infty} \frac{x}{x} = \lim\limits_{x\to\infty} 1 = 1.$

Similarly $\lim\limits_{x\to-\infty} f(x) = 1$. Thus $y = 1$ is a horizontal asymptote.

3. $f(x) = \frac{x+5}{2x+7}$

When $x = -\frac{7}{2}$ the denominator is zero but the numerator is not. Thus $x = -\frac{7}{2}$ is a vertical asymptote. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x}{2x} = \lim\limits_{x\to\infty} \frac{1}{2} = \frac{1}{2}.$

Similarly $\lim\limits_{x\to-\infty} f(x) = \frac{1}{2}$. Thus $y = \frac{1}{2}$ is a horizontal asymptote.

4. $y = f(x) = \dfrac{2x+1}{2x+1}$

Observe that both the numerator and denominator are zero for $x = -\dfrac{1}{2}$. For $x \neq -\dfrac{1}{2}$, we have $f(x) = 1$. Thus f is a constant function for $x \neq -\dfrac{1}{2}$. Hence there are no vertical or horizontal asymptotes.

5. $y = f(x) = \dfrac{4}{x}$

When $x = 0$ the denominator is zero but the numerator is not zero, so $x = 0$ is a vertical asymptote.

$\lim\limits_{x\to\infty}\left(\dfrac{4}{x}\right) = 0.$ Similarly, $\lim\limits_{x\to-\infty}\left(\dfrac{4}{x}\right) = 0,$ so $y = 0$ is a horizontal asymptote.

6. $y = f(x) = 1 - \dfrac{2}{x^2} = \dfrac{x^2-2}{x^2}$

When $x = 0$ the denominator is zero but the numerator is not. Thus $x = 0$ is a vertical asymptote. $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{x^2}\right) = 1 - 0 = 1.$ Similarly $\lim\limits_{x\to-\infty} f(x) = 1,$ so $y = 1$ is a horizontal asymptote.

7. $y = f(x) = \dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$

Vertical asymptotes are $x = 1$ and $x = -1$.

$\lim\limits_{x\to\infty}\dfrac{1}{x^2-1} = \lim\limits_{x\to\infty}\dfrac{1}{x^2} = 0$. Similarly, $\lim\limits_{x\to-\infty} f(x) = 0$. Thus $y = 0$ is a horizontal asymptote.

8. $y = f(x) = \dfrac{x}{x^2-9} = \dfrac{x}{(x-3)(x+3)}$

Vertical asymptotes: $x = 3$, $x = -3$.

$\lim\limits_{x\to\infty}\dfrac{x}{x^2-9} = \lim\limits_{x\to\infty}\dfrac{x}{x^2} = \lim\limits_{x\to\infty}\dfrac{1}{x} = 0$. Similarly, $\lim\limits_{x\to-\infty} f(x) = 0$. Thus $y = 0$ is a horizontal asymptote.

9. $y = f(x) = x^2 - 5x + 5$ is a polynomial function, so there are no horizontal or vertical asymptotes.

10. $y = f(x) = \dfrac{x^4}{x^3-4} = \dfrac{x^4}{x^3-\left(\sqrt[3]{4}\right)^3} = \dfrac{x^4}{x^3-(2^{2/3})^3}$

$= \dfrac{x^4}{(x-2^{2/3})(x^2+2^{2/3}x+2^{4/3})}$

Vertical asymptote: $x = 2^{2/3}$.

$\dfrac{x^4}{x^3-4} = x + \dfrac{4x}{x^3-4}$ so the line $y = x$ is an oblique asymptote.

11. $f(x) = \dfrac{2x^2}{x^2+x-6} = \dfrac{2x^2}{(x+3)(x-2)}$

Vertical asymptotes are $x = -3$ and $x = 2$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{2x^2}{x^2} = \lim\limits_{x\to\infty} 2 = 2$, and $\lim\limits_{x\to-\infty} f(x) = 2$. Thus $y = 2$ is a horizontal asymptote.

12. $f(x) = \dfrac{x^3}{5}$ is a polynomial function, so there are no horizontal or vertical asymptotes.

13. $y = \dfrac{15x^2+31+1}{x^2-7} = \dfrac{15x^2+31x+1}{\left(x+\sqrt{7}\right)\left(x-\sqrt{7}\right)}$

Vertical asymptotes are $x = -\sqrt{7}$ and $x = \sqrt{7}$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{15x^2}{x^2} = \lim\limits_{x\to\infty} 15 = 15$

Similarly, $\lim\limits_{x\to-\infty} = 15.$ Thus $y = 15$ is a horizontal asymptote.

14. $y = f(x) = \dfrac{2x^3+1}{3x(2x-1)(4x-3)}$

Vertical asymptotes are $x = 0,\ x = \dfrac{1}{2}$, and $x = \dfrac{3}{4}$. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{2x^3}{24x^3} = \lim\limits_{x\to\infty}\dfrac{1}{12} = \dfrac{1}{12}$.

Similarly, $\lim\limits_{x\to-\infty} f(x) = \dfrac{1}{12}$. Thus $y = \dfrac{1}{12}$ is a horizontal asymptote.

15. $y = f(x) = \frac{2}{x-3} + 5 = \frac{5x-13}{x-3}$

From the denominator, $x = 3$ is a vertical asymptote.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{5x}{x} = \lim\limits_{x\to\infty} 5 = 5,$ and

$\lim\limits_{x\to-\infty} f(x) = 5.$ Thus, $y = 5$ is a horizontal asymptote.

16. $f(x) = \frac{x^2-1}{2x^2-9x+4} = \frac{x^2-1}{(2x-1)(x-4)}$

Vertical asymptotes are $x = \frac{1}{2}$ and $x = 4$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^2}{2x^2} = \lim\limits_{x\to\infty} \frac{1}{2} = \frac{1}{2}$,and

$\lim\limits_{x\to-\infty} f(x) = \frac{1}{2}$. Thus $y = \frac{1}{2}$ is a horizontal asymptote.

17. $f(x) = \frac{3-x^4}{x^3+x^2} = \frac{3-x^4}{x^2(x+1)}$

Vertical asymptotes are $x = 0$ and $x = -1$.

$\frac{3-x^4}{x^3+x^2} = -x+1+\frac{3-x^2}{x^3+x^2}$ so the line $y = -x + 1$ is an oblique asyptote.

18. $y = f(x) = \frac{5x^2+7x^3+9x^4}{3x^2}$

Observe that both the numerator and the denominator are zero when $x = 0$. For $x \neq 0$, we have

$f(x) = \frac{x^2}{3x^2}(5+7x+9x^2) = \frac{1}{3}(5+7x+9x^2).$

Thus *f* is a polynomial function for $x \neq 0$. Hence there are neither horizontal nor vertical asymptotes.

19. $y = f(x) = \frac{x^2-3x-4}{1+4x+4x^2} = \frac{x^2-3x-4}{(1+2x)^2}$

From the denominator, $x = -\frac{1}{2}$ is a vertical asymptote.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^2}{4x^2} = \lim\limits_{x\to\infty} \frac{1}{4} = \frac{1}{4}$, and

$\lim\limits_{x\to-\infty} f(x) = \frac{1}{4}$, so $y = \frac{1}{4}$ is a horizontal asymptote.

20. $y = f(x) = \frac{x^4+1}{1-x^4} = \frac{x^4+1}{\left(1+x^2\right)(1-x)(1+x)}$

From the denominator, vertical asymptotes are $x = 1$ and $x = -1$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^4}{-x^4} = \lim\limits_{x\to\infty} -1 = -1$, and

$\lim\limits_{x\to-\infty} f(x) = -1$. Thus $y = -1$ is a horizontal asymptote.

21. $y = f(x) = \frac{9x^2-16}{2(3x+4)^2} = \frac{(3x+4)(3x-4)}{2(3x+4)^2}$

When $x = -\frac{4}{3}$, both the numerator and denominator are zero. Since

$\lim\limits_{x\to-4/3^+} f(x) = \lim\limits_{x\to-4/3^+} \frac{3x-4}{2(3x+4)} = -\infty$, the

line $x = -\frac{4}{3}$ is a vertical asymptote.

$\lim\limits_{x\to\infty} \frac{9x^2-16}{2(3x+4)^2} = \lim\limits_{x\to\infty} \frac{9x^2}{18x^2} = \lim\limits_{x\to\infty} \frac{1}{2} = \frac{1}{2}$.

Similarly, $\lim\limits_{x\to-\infty} f(x) = \frac{1}{2}$. Thus $y = \frac{1}{2}$ is a horizontal asymptote.

22. $y = f(x) = \frac{2}{5} + \frac{2x}{12x^2+5x-2} = \frac{24x^2+20x-4}{5(12x^2+5x-2)}$

$= \frac{4(x+1)(6x-1)}{5(3x+2)(4x-1)}$

When $x = -\frac{2}{3}$ or $x = \frac{1}{4}$, the denominator is 0, but the numerator is not. Thus, vertical asymptotes are $x = -\frac{2}{3}$ and

$x = \frac{1}{4}$. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{24x^2}{60x^2} = \lim\limits_{x\to\infty} \frac{2}{5} = \frac{2}{5}$.

Similarly, $\lim\limits_{x\to-\infty} f(x) = \frac{2}{5}$. Thus, $y = \frac{2}{5}$ is a horizontal asymptote.

23. $y = f(x) = 5e^{x-3} - 2$

We have $\lim\limits_{x\to\infty} f(x) = +\infty$ and

$\lim\limits_{x\to-\infty} f(x) = 5 \cdot \lim\limits_{x\to-\infty} e^{x-3} - \lim\limits_{x\to-\infty} 2$

$= 5(0) - 2 = -2$

Thus $y = -2$ is a horizontal asymptote. There is no vertical asymptote because $f(x)$ neither increases nor decreases without bound around any fixed value of x.

24. $f(x) = 12e^{-x}$

$\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = +\infty$. Thus $y = 0$ is a horizontal asymptote. There is no vertical asymptote because $f(x)$ neither increases nor decreases without bound around any fixed value of x.

25. $y = \frac{3}{x}$

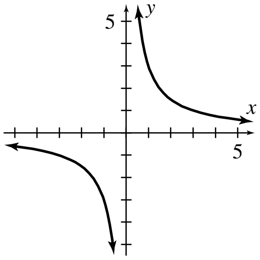
Symmetric about the origin. Vertical asymptote is $x = 0$. $\lim_{x\to\infty} \frac{3}{x} = 0 = \lim_{x\to-\infty} \frac{3}{x}$, so $y = 0$ is a horizontal asymptote.

$y' = -\frac{3}{x^2}$

CV: None, however $x = 0$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, 0)$ and $(0, \infty)$.

$y'' = \frac{6}{x^3}$

No possible inflection point, but we include $x = 0$ in the concavity analysis. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.



26. $y = \frac{2}{2x-3}$

Intercept: $\left(0, -\frac{2}{3}\right)$

Vertical asymptote is $x = \frac{3}{2}$.

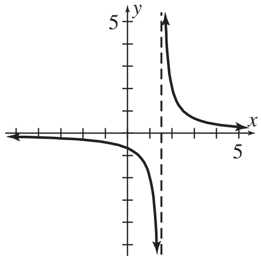
$\lim_{x\to\infty} y = 0 = \lim_{x\to-\infty} y$, so $y = 0$ is a horizontal asymptote.

$y' = -\frac{4}{(2x-3)^2}$

CV: None, but $x = \frac{3}{2}$ must be considered in the inc. dec. analysis. Decreasing on $\left(-\infty, \frac{3}{2}\right)$ and $\left(\frac{3}{2}, \infty\right)$.

$y'' = \frac{16}{(2x-3)^3}$

No possible inflection point, but $x = \frac{3}{2}$ must be considered in the concavity analysis. Concave down on $\left(-\infty, \frac{3}{2}\right)$; concave up on $\left(\frac{3}{2}, \infty\right)$.



27. $y = \frac{x}{x-1}$

Intercept (0, 0)

Vertical asymptote is $x = 1$

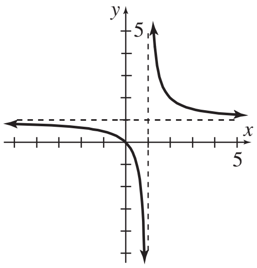
$\lim_{x\to\infty} y = 1 = \lim_{x\to-\infty} y$, so $y = 1$ is a horizontal asymptote.

$y' = \frac{(x-1)(1) - x(1)}{(x-1)^2} = -\frac{1}{(x-1)^2}$

CV: None, but $x = 1$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, 1)$ and $(1, \infty)$.

$y'' = \frac{2}{(x-1)^3}$

No possible inflection point, but $x = 1$ must be included in concavity analysis. Concave up on $(1, \infty)$, concave down on $(-\infty, 1)$.



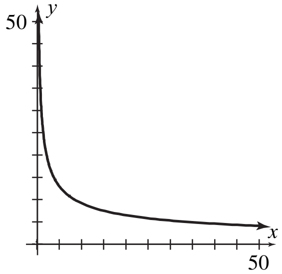
28. $y = \frac{50}{\sqrt{3x}}$ (Note: $x > 0$)

$\lim\limits_{x\to\infty} y = 0$, so $y = 0$ is a horizontal asymptote.

$\lim\limits_{x\to 0^+} y = +\infty$, so the line $x = 0$ is a vertical asymptote.

$y' = -\frac{25}{\sqrt{3x^3}} < 0$ for $x > 0$. Decreasing on $(0, \infty)$.

$y'' = \frac{75}{2\sqrt{3x^5}} > 0$ for $x > 0$. Concave up on $(0, \infty)$. No intercepts; no symmetry.



29. $y = x^2 + \frac{1}{x^2} = \frac{x^4+1}{x^2}$

$x \neq 0$, so there is no y-intercept. Setting $y = 0 \Rightarrow$ no x-intercept. Replacing x by $-x$ yields symmetry about the y-axis. Setting $x^2 = 0$ gives $x = 0$ as the only vertical asymptote. Because the degree of the numerator is greater than the degree of the denominator, no horizontal asymptote exists.

$y = x^2 + x^{-2}$

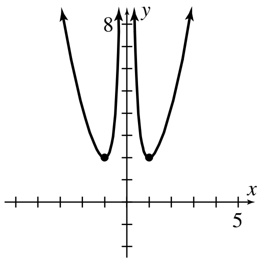
$$y' = 2x - 2x^{-3} = 2x - \frac{2}{x^3} = \frac{2x^4-2}{x^3} = \frac{2\left(x^4-1\right)}{x^3}$$

$$= \frac{2\left(x^2+1\right)(x+1)(x-1)}{x^3}.$$

CV: $x = \pm 1$, but $x = 0$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, -1)$ and $(0, 1)$; increasing on $(-1, 0)$ and $(1, \infty)$; relative minima at $(-1, 2)$ and $(1, 2)$,

$y'' = 2 + \frac{6}{x^4} > 0$

for all $x \neq 0$. Concave up on $(-\infty, 0)$ and $(0, \infty)$.



30. $y = \frac{3x^2-5x-1}{x-2}$

Intercept: $\left(0, \frac{1}{2}\right)$

Vertical asymptote is $x = 2$.

$\frac{3x^2-5x-1}{x-2} = 3x+1+\frac{1}{x-2}$ so $y = 3x + 1$ is an oblique asymptote.

$$y' = \frac{(x-2)(6x-5)-(3x^2-5x-1)(1)}{(x-2)^2}$$

$$= \frac{3x^2-12x+11}{(x-2)^2}$$

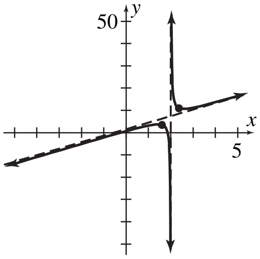
From the quadratic formula, CV: $x = \frac{6\pm\sqrt{3}}{3}$, but $x = 2$ must be included in the inc.-dec. analysis. Increasing on $\left(-\infty, \frac{6-\sqrt{3}}{3}\right)$ and $\left(\frac{6+\sqrt{3}}{3}, \infty\right)$; decreasing on $\left(\frac{6-\sqrt{3}}{3}, 2\right)$ and $\left(2, \frac{6+\sqrt{3}}{3}\right)$; relative maximum at $\left(\frac{6-\sqrt{3}}{3}, 7-2\sqrt{3}\right)$; relative minimum at $\left(\frac{6+\sqrt{3}}{3}, 7+2\sqrt{3}\right)$.

$$y'' = \frac{(x-2)^2(6x-12)-(3x^2-12x+11)2(x-2)}{(x-2)^4}$$

$$= \frac{(x-2)(6x-12)-2(3x^2-12x+11)}{(x-2)^3}$$

$$= \frac{2}{(x-2)^3}$$

No possible inflection point, but $x = 2$ must be included in the concavity analysis. Concave down on $(-\infty, 2)$; concave up on $(2, \infty)$



31. $y = \frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$

Intercept (0, –1)
Symmetric about the y-axis.
Vertical asymptotes are $x = -1$ and $x = 1$.

$\lim\limits_{x\to\infty} \frac{1}{x^2-1} = 0 = \lim\limits_{x\to-\infty} \frac{1}{x^2-1}$, so $y = 0$ is a horizontal asymptote.

$$y' = -\frac{2x}{\left(x^2-1\right)^2}$$

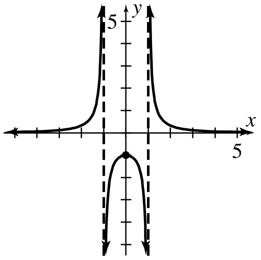
CV: $x = 0$, but $x = \pm 1$ must be included in the inc.-dec. analysis. Increasing on (–∞, –1) and (–1, 0); decreasing on (0, 1) and (1, ∞); relative maximum at (0, –1).

$$y'' = -2 \cdot \frac{\left(x^2-1\right)^2(1) - x\left[4x\left(x^2-1\right)\right]}{\left(x^2-1\right)^4}$$

$$= -2 \cdot \frac{\left(x^2-1\right)\left[\left(x^2-1\right)-4x^2\right]}{\left(x^2-1\right)^4}$$

$$= \frac{2\left(3x^2+1\right)}{\left(x^2-1\right)^3} = \frac{2\left(3x^2+1\right)}{[(x+1)(x-1)]^3}$$

No possible inflection point, but $x = \pm 1$ must be considered in the concavity analysis. Concave up on (–∞, –1) and (1, ∞); concave down on (–1, 1).



32. $y = \frac{1}{x^2+1}$

Intercept (0, 1)
Symmetric about the y-axis.

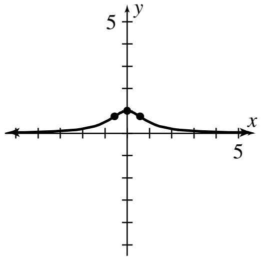
$\lim\limits_{x\to\infty} \frac{1}{x^2+1} = 0 = \lim\limits_{x\to-\infty} \frac{1}{x^2+1}$, so $y = 0$ is a horizontal asymptote.

$$y' = \frac{-2x}{\left(x^2+1\right)^2}$$

CV: $x = 0$
Increasing on (–∞, 0); decreasing on (0, ∞); relative maximum at (0, 1)

$$y'' = \frac{2\left(3x^2-1\right)}{\left(x^2+1\right)^3}$$

Possible inflection points at $x = \pm\frac{1}{\sqrt{3}}$. Concave up on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$; concave down on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$; inflection points at $\left(\pm\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$



33. $y = \frac{2+x}{3-x}$

Intercepts: $\left(0, \frac{2}{3}\right)$ and (–2, 0).

$x = \frac{2}{3}$ is the only vertical asymptote. Since

$$\lim_{x\to\infty} \frac{2+x}{3-x} = \lim_{x\to\infty} \frac{x}{-x} = \lim_{x\to\infty} -1 = -1$$
$$= \lim_{x\to-\infty} \frac{2+x}{3-x}$$

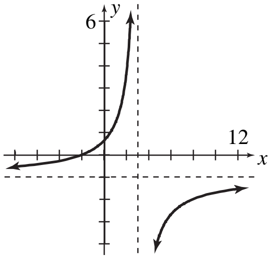
the only horizontal asymptote is $y = -1$.

$$y' = \frac{(3-x)(1)-(2+x)(-1)}{(3-x)^2} = \frac{5}{(3-x)^2}$$

No critical values, but $x = 3$ must be considered in the ind.-dec. analysis. Increasing on $(-\infty, 3)$ and $(3, \infty)$.

$$y'' = \frac{10}{(3-x)^3}$$

No possible inflection point, but $x = 3$ must be included in the concavity analysis. Concave up on $(-\infty, 3)$; concave down on $(3, \infty)$.



34. $y = \dfrac{1+x}{x^2}$

Intercept is $(-1, 0)$

Vertical asymptote is $x = 0$.

$$\lim_{x\to\infty} \frac{1+x}{x^2} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$$

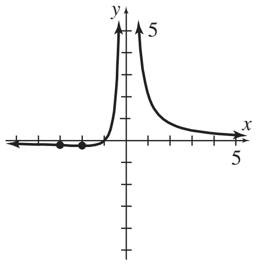
$= \displaystyle\lim_{x\to-\infty} \frac{1+x}{x^2}$, so $y = 0$ is the only horizontal asymptote.

$$y' = -\frac{x+2}{x^3}$$

CV: $x = -2$, but $x = 0$ must be included in the inc-dec. analysis. Increasing on $(-2, 0)$; decreasing on $(-\infty, -2)$ and $(0, \infty)$; relative minimum at $\left(-2, -\dfrac{1}{4}\right)$.

$$y'' = \frac{2(3+x)}{x^4}$$

Possible inflection point when $x = 3$, but $x = 0$ must be included in the concavity analysis. Concave up on $(-3, 0)$ and $(0, \infty)$; concave down on $(-\infty, -3)$; inflection point at $\left(-3, -\dfrac{2}{9}\right)$.



35. $y = \dfrac{x^2}{7x+4}$

Intercept: $(0, 0)$

Vertical asymptote is $x = -\dfrac{4}{7}$.

$\dfrac{x^2}{7x+4} = \dfrac{1}{7}x - \dfrac{4}{49} + \dfrac{16}{49(7x+4)}$ so $y = \dfrac{1}{7}x - \dfrac{4}{49}$ is an oblique asymptote.

$$y' = \frac{(7x+4)(2x) - x^2(7)}{(7x+4)^2}$$

$$= \frac{7x^2+8x}{(7x+4)^2} = \frac{x(7x+8)}{(7x+4)^2}$$

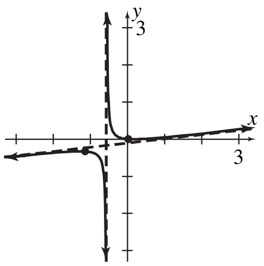
CV: $x = 0$, $-\dfrac{8}{7}$, but $x = -\dfrac{4}{7}$ must be included in the inc.-dec. analysis. Increasing on $\left(-\infty, -\dfrac{8}{7}\right)$ and $(0, \infty)$; decreasing on $\left(-\dfrac{8}{7}, -\dfrac{4}{7}\right)$ and $\left(-\dfrac{4}{7}, 0\right)$; relative maximum at $\left(-\dfrac{8}{7}, -\dfrac{16}{49}\right)$; relative minimum at $(0, 0)$.

$$y'' = \frac{\left(7x^2+4\right)^2(14x+8) - \left(7x^2+8x\right)[14(7x+4)]}{(7x+4)^4}$$

$$= \frac{(7x+4)\left[(7x+4)(14x+8) - 14\left(7x^2+8x\right)\right]}{(7x+4)^4}$$

$$= \frac{32}{(7x+4)^3}$$

No possible inflection point but $x = -\dfrac{4}{7}$ must be included in concavity analysis. Concave down on $\left(-\infty, -\dfrac{4}{7}\right)$; concave up on $\left(-\dfrac{4}{7}, \infty\right)$.



36. $y = \dfrac{x^3+1}{x}$

Intercept: (–1, 0)

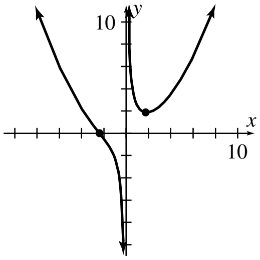
Vertical asymptote is $x = 0$. Because the degree of the numerator is greater than the degree of the denominator, no horizontal asymptote exists. Since $y = x^2 + x^{-1}$,

$$y' = 2x - x^{-2} = 2x - \frac{1}{x^2} = \frac{2x^3-1}{x^2}.$$

CV: $x = \sqrt[3]{\frac{1}{2}}$, but $x = 0$ must be included in inc.-dec. analysis. Decreasing on $(-\infty, 0)$ and $\left(0, \sqrt[3]{\frac{1}{2}}\right)$; increasing on $\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$; relative minimum at $\left(\sqrt[3]{\frac{1}{2}}, 3\sqrt[3]{\frac{1}{4}}\right)$.

$$y'' = 2 + 2x^{-3} = 2 + \frac{2}{x^3} = \frac{2\left(x^3+1\right)}{x^3}$$

Possible inflection point when $x = -1$, but $x = 0$ must be included in concavity analysis. Concave up on $(-\infty, -1)$ and $(0, \infty)$; concave down on $(-1, 0)$; inflection point at $(-1, 0)$.



37. $y = \dfrac{9}{9x^2-6x-8} = \dfrac{9}{(3x+2)(3x-4)}$

Intercept: $\left(0, -\frac{9}{8}\right)$

Vertical asymptotes: $x = -\frac{2}{3}, x = \frac{4}{3}$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{9}{9x^2} = \lim_{x\to\infty} \frac{1}{x^2} = 0 = \lim_{x\to-\infty} y$$

Thus $y = 0$ is a horizontal asymptote. Since $y = 9\left(9x^2-6x-8\right)^{-1}$,

$$y' = 9(-1)\left(9x^2-6x-8\right)^{-2}(18x-6)$$
$$= -\frac{54(3x-1)}{[(3x+2)(3x-4)]^2}$$

CV: $x = \frac{1}{3}$, but $x = -\frac{2}{3}$ and $x = \frac{4}{3}$ must be included in inc.-dec. analysis.

Increasing on $\left(-\infty, -\frac{2}{3}\right)$ and $\left(-\frac{2}{3}, \frac{1}{3}\right)$; decreasing on $\left(\frac{1}{3}, \frac{4}{3}\right)$ and $\left(\frac{4}{3}, \infty\right)$;

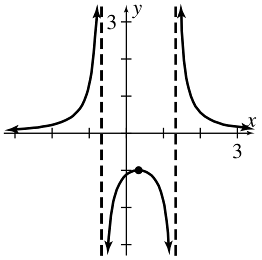
relative maximum at $\left(\frac{1}{3}, -1\right)$. Finding y'' gives:

$$y'' = -54 \cdot \frac{\left(9x^2-6x-8\right)^2(3)-(3x-1)\left[2\left(9x^2-6x-8\right)(18x-6)\right]}{\left(9x^2-6x-8\right)^4}$$

$$= -54 \cdot \frac{3\left(9x^2-6x-8\right)\left[\left(9x^2-6x-8\right)-4(3x-1)(3x-1)\right]}{\left(9x^2-6x-8\right)^4}$$

$$= \frac{-162\left(-27x^2+18x-12\right)}{\left(9x^2-6x-8\right)^3} = \frac{486\left(9x^2-6x+4\right)}{[(3x+2)(3x-4)]^3}$$

Since $9x^2-6x+4=0$ has no real roots, y'' is never zero. No possible inflection points, but $x=-\frac{2}{3}$ and $x=\frac{4}{3}$ must be included in concavity analysis. Concave up on $\left(-\infty, -\frac{2}{3}\right)$ and $\left(\frac{4}{3}, \infty\right)$; concave down on $\left(-\frac{2}{3}, \frac{4}{3}\right)$.



38. $y = \frac{4x^2+2x+1}{2x^2}$

$4x^2+2x+1$ is never 0 and x cannot be zero. Thus no intercepts. Vertical asymptote is $x = 0$.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{4x^2}{2x^2} = \lim_{x\to\infty} 2 = 2 = \lim_{x\to-\infty} y$$

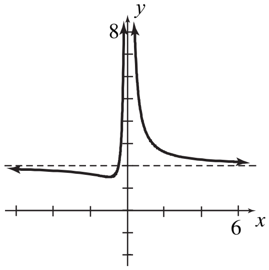
Thus $y = 2$ is a horizontal asymptote. Since $y = 2 + x^{-1} + \frac{1}{2}x^{-2}$, we have

$$y' = -x^{-2} - x^{-3} = -x^{-3}(x+1)$$

CV: $x=-1$, but $x = 0$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, -1)$ and $(0, \infty)$; increasing on $(-1, 0)$; relative minimum at $\left(-1, \frac{3}{2}\right)$.

$$y'' = 2x^{-3} + 3x^{-4} = \frac{3}{x^4}\left(\frac{2}{3}x+1\right).$$

Possible inflection point when $x=-\frac{3}{2}$, but $x = 0$ must be included in the concavity analysis. Concave down on $\left(-\infty, -\frac{3}{2}\right)$; concave up on $\left(-\frac{3}{2}, 0\right)$ and $(0, \infty)$; inflection point at $\left(-\frac{3}{2}, \frac{14}{9}\right)$. No symmetry.



39. $y = \dfrac{3x+1}{(3x-2)^2}$

Intercepts: $\left(-\dfrac{1}{3}, 0\right), \left(0, \dfrac{1}{4}\right)$

Vertical asymptote is $x = \dfrac{2}{3}$.

$\lim\limits_{x\to\infty} y = \lim\limits_{x\to\infty} \dfrac{3x}{9x^2} = \lim\limits_{x\to\infty} \dfrac{1}{3x} = 0 = \lim\limits_{x\to-\infty} y$

Thus $y = 0$ is a horizontal asymptote.

$$y' = \frac{(3x-2)^2(3) - (3x+1)(2)(3x-2)(3)}{(3x-2)^4}$$
$$= \frac{3(3x-2)[(3x-2) - 2(3x+1)]}{(3x-2)^4}$$
$$= -\frac{3(3x+4)}{(3x-2)^3}$$

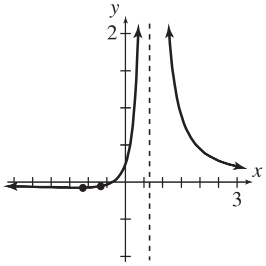
CV: $x = -\dfrac{4}{3}$, but $x = \dfrac{2}{3}$ must be included in inc.-dec. analysis.

Decreasing on $\left(-\infty, -\dfrac{4}{3}\right)$ and $\left(\dfrac{2}{3}, \infty\right)$; increasing on $\left(-\dfrac{4}{3}, \dfrac{2}{3}\right)$; relative minimum at $\left(-\dfrac{4}{3}, -\dfrac{1}{12}\right)$.

$$y'' = -3 \cdot \frac{(3x-2)^3(3) - (3x+4)(3)(3x-2)^2(3)}{(3x-2)^6}$$
$$= -3 \cdot \frac{3(3x-2)^2[(3x-2) - 3(3x+4)]}{(3x-2)^6}$$
$$= -3 \cdot \frac{3(-6x-14)}{(3x-2)^4} = \frac{18(3x+7)}{(3x-2)^4}$$

Possible inflection point when $x = -\dfrac{7}{3}$, but $x = \dfrac{2}{3}$ must be included in concavity analysis.

Concave down on $\left(-\infty, -\dfrac{7}{3}\right)$; concave up on $\left(-\dfrac{7}{3}, \dfrac{2}{3}\right)$ and $\left(\dfrac{2}{3}, \infty\right)$; inflection point at $\left(-\dfrac{7}{3}, -\dfrac{2}{27}\right)$.



40. $y = \dfrac{3x+1}{(6x+5)^2}$

Intercepts: $\left(-\dfrac{1}{3}, 0\right), \left(0, \dfrac{1}{25}\right)$

Vertical asymptote is $x = -\dfrac{5}{6}$.

$\lim\limits_{x\to\infty} y = \lim\limits_{x\to\infty} \dfrac{3x}{36x^2} = \lim\limits_{x\to\infty} \dfrac{1}{12x} = 0 = \lim\limits_{x\to-\infty} y$

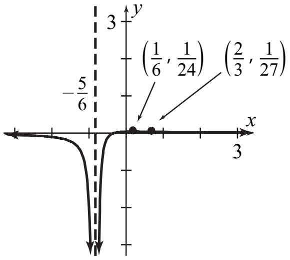
Thus $y = 0$ is horizontal asymptote.

$$y' = \frac{(6x+5)^2(3) - (3x+1)[12(6x+5)]}{(6x+5)^4}$$
$$= \frac{3(6x+5)[(6x+5) - 4(3x+1)]}{(6x+5)^4}$$
$$= \frac{3(-6x+1)}{(6x+5)^3} = \frac{-3(6x-1)}{(6x+5)^3}$$

CV: $x = \dfrac{1}{6}$, but $x = -\dfrac{5}{6}$ must be included in inc.-dec. analysis. Decreasing on $\left(-\infty, -\dfrac{5}{6}\right)$ and $\left(\dfrac{1}{6}, \infty\right)$; increasing on $\left(-\dfrac{5}{6}, \dfrac{1}{6}\right)$; relative maximum at $\left(\dfrac{1}{6}, \dfrac{1}{24}\right)$. Finding y'' gives:

$$y'' = -3\cdot\frac{(6x+5)^3(6)-(6x-1)\left[18(6x+5)^2\right]}{(6x+5)^6}$$
$$= -3\cdot\frac{6(6x+5)^2[(6x+5)-3(6x-1)]}{(6x+5)^6}$$
$$= -18\cdot\frac{-12x+8}{(6x+5)^4}$$
$$= 72\cdot\frac{3x-2}{(6x+5)^4}$$

Possible inflection point when $x=\frac{2}{3}$, but $x=-\frac{5}{6}$ must be included in concavity analysis. Concave down on $\left(-\infty, -\frac{5}{6}\right)$ and $\left(-\frac{5}{6}, \frac{2}{3}\right)$; concave up on $\left(\frac{2}{3}, \infty\right)$; inflection point at $\left(\frac{2}{3}, \frac{1}{27}\right)$.



41. $y=\frac{x^2-1}{x^3}=\frac{(x+1)(x-1)}{x^3}$

Intercepts are (–1, 0) and (1, 0).
Symmetric about the origin.
Vertical asymptote $x = 0$.

$$\lim_{x\to\infty}\frac{x^2-1}{x^3}=\lim_{x\to\infty}\frac{x^2}{x^3}=\lim_{x\to\infty}\frac{1}{x}$$

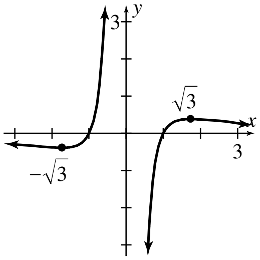
$=0=\lim_{x\to-\infty}\frac{1-x}{x^2}$, so $y = 0$ is the only horizontal asymptote. Since $y=x^{-1}-x^{-3}$, then

$$y'=-x^{-2}+3x^{-4}=x^{-4}\left(-x^2+3\right)=\frac{3-x^2}{x^4}$$

CV: $x=\pm\sqrt{3}$, but $x = 0$ must be included in the inc.-dec. analysis. Increasing on $\left(-\sqrt{3}, 0\right)$ and $\left(0, \sqrt{3}\right)$; decreasing on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$; relative maximum at $\left(\sqrt{3}, \frac{2\sqrt{3}}{9}\right)$; relative minimum at $\left(-\sqrt{3}, -\frac{2\sqrt{3}}{9}\right)$.

$$y''=2x^{-3}-12x^{-5}=2x^{-5}\left(x^2-6\right)=\frac{2\left(x^2-6\right)}{x^5}$$

Possible inflection points when $x=\pm\sqrt{6}$, but $x = 0$ must be included in the concavity analysis. Concave down on $\left(-\infty, -\sqrt{6}\right)$ and $\left(0, \sqrt{6}\right)$; concave up on $\left(-\sqrt{6}, 0\right)$ and $\left(\sqrt{6}, \infty\right)$; inflection points at $\left(\sqrt{6}, \frac{5\sqrt{6}}{36}\right)$ and $\left(-\sqrt{6}, \frac{-5\sqrt{6}}{36}\right)$.



42. $y=\frac{3x}{(x-2)^2}$

Intercept (0, 0)
Vertical asymptote at $x = 2$

$\lim_{x\to\infty}\frac{3x}{x^2-4x+4}=\lim_{x\to\infty}\frac{3x}{x^2}=\lim_{x\to\infty}\frac{3}{x}=0$ and $\lim_{x\to-\infty}\frac{3x}{x^2-4x+4}=0$, so $y = 0$ is the only horizontal asymptote.

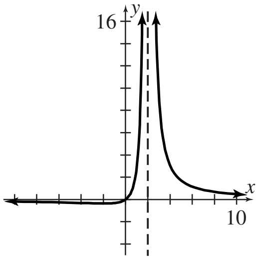
$$y'=\frac{-3(x+2)}{(x-2)^3}$$

CV: $x = -2$, but $x = 2$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, -2)$ and $(2, \infty)$; increasing on $(-2, 2)$; relative maximum at $\left(-2, -\frac{3}{8}\right)$

$$y''=\frac{6(x+4)}{(x-2)^4}$$

Possible inflection point when $x = -4$, but $x = 2$ must be included in the concavity analysis.

Concave down on (–∞, –4); concave up on (–4, 2) and (2, ∞); inflection point at $\left(-4, -\frac{1}{3}\right)$.



43. $y = 2x+1+\frac{1}{x-1} = \frac{2x^2-x}{x-1} = \frac{x(2x-1)}{x-1}$

Intercepts: (0, 0), $\left(\frac{1}{2}, 0\right)$

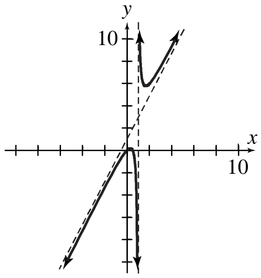
$x = 1$ is the only vertical asymptote.
$y = 2x + 1$ is an oblique asymptote.

$$y' = \frac{(x-1)(4x-1)-(1)(2x^2-x)}{(x-1)^2}$$
$$= \frac{4x^2-5x+1-2x^2+x}{(x-1)^2}$$
$$= \frac{2x^2-4x+1}{(x-1)^2}$$

CV: $\frac{2 \pm \sqrt{2}}{2} = 1 \pm \frac{\sqrt{2}}{2}$, but $x = 1$ must be included in the inc-dec. analysis. Increasing on $\left(-\infty, 1-\frac{\sqrt{2}}{2}\right)$ and $\left(1+\frac{\sqrt{2}}{2}, \infty\right)$; decreasing on $\left(1-\frac{\sqrt{2}}{2}, 1\right)$ and $\left(1, 1+\frac{\sqrt{2}}{2}\right)$; relative maximum at $\left(1-\frac{\sqrt{2}}{2}, 3-2\sqrt{2}\right)$; relative minimum at $\left(1+\frac{\sqrt{2}}{2}, 3+2\sqrt{2}\right)$

$$y'' = \frac{(x-1)^2(4x-4)-2(x-1)(2x^2-4x+1)}{(x-1)^4}$$
$$= \frac{2(x-1)}{(x-1)^4}$$
$$= \frac{2}{(x-1)^3}$$

No possible inflection point, but $x = 1$ must be included in the concavity analysis. Concave down on (–∞, 1); concave up on (1, ∞). No symmetry.



44. $y = \frac{3x^4+1}{x^3}$

No intercepts

Symmetric about the origin.

Vertical asymptote is $x = 0$. $\frac{3x^4+1}{x^3} = 3x + \frac{1}{x^3}$ so $y = 3x$ is an oblique asymptote.

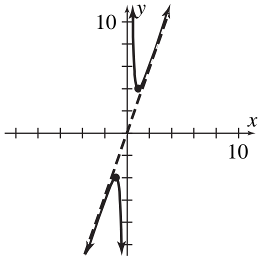
Since $y = 3x + x^{-3}$,

$$y' = 3-3x^{-4} = 3-\frac{3}{x^4} = \frac{3\left(x^2+1\right)(x+1)(x-1)}{x^4}$$

CV: ±1, but $x = 0$ must be considered in the inc.-dec. analysis. Increasing on (–∞, –1) and (1, ∞); decreasing on (–1, 0) and (0, 1); relative maximum at (–1, –4); relative minimum at (1, 4).

$$y'' = \frac{12}{x^5}$$

No possible inflection point, but $x = 0$ must be included in the concavity analysis. Concave down on (–∞, 0); concave up on (0, ∞).



45. $y = \dfrac{-3x^2+2x-5}{3x^2-2x-1} = \dfrac{-3x^2+2x-5}{(3x+1)(x-1)}$

Note that $-3x^2+2x-5$ is never zero. Intercept: (0, 5)

Vertical asymptotes are $x = -\dfrac{1}{3}$ and $x = 1$.

$\lim\limits_{x\to\infty} y = \lim\limits_{x\to\infty} \dfrac{-3x^2}{3x^2} = \lim\limits_{x\to\infty} -1 = -1 = \lim\limits_{x\to-\infty} y$

Thus $y = -1$ is horizontal asymptote.

$$y' = \frac{\left(3x^2-2x-1\right)(-6x+2) - \left(-3x^2+2x-5\right)(6x-2)}{\left(3x^2-2x-1\right)^2}$$

$$= \frac{2(3x-1)\left[\left(3x^2-2x-1\right)(-1) - \left(-3x^2+2x-5\right)\right]}{\left(3x^2-2x-1\right)^2}$$

$$= \frac{12(3x-1)}{\left(3x^2-2x-1\right)^2} = \frac{12(3x-1)}{[(3x+1)(x-1)]^2}$$

CV: $x = \dfrac{1}{3}$, but $x = -\dfrac{1}{3}$ and $x = 1$ must be included in inc.-dec. analysis.

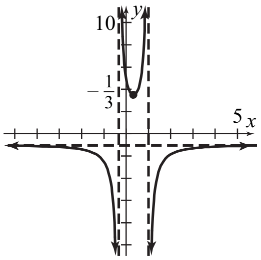
Decreasing on $\left(-\infty, -\dfrac{1}{3}\right)$ and $\left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$; increasing on $\left(\dfrac{1}{3}, 1\right)$ and $(1, \infty)$; relative minimum at $\left(\dfrac{1}{3}, \dfrac{7}{2}\right)$.

$$y'' = 12 \cdot \frac{\left(3x^2-2x-1\right)^2(3) - (3x-1)\left[2\left(3x^2-2x-1\right)(6x-2)\right]}{\left(3x^2-2x-1\right)^4}$$

$$= 12 \cdot \frac{\left(3x^2-2x-1\right)\left[3\left(3x^2-2x-1\right) - 2(3x-1)(6x-2)\right]}{\left(3x^2-2x-1\right)^4}$$

$$= 12 \cdot \frac{-27x^2+18x-7}{\left(3x^2-2x-1\right)^3} = \frac{-12\left(27x^2-18x+7\right)}{[(3x+1)(x-1)]^3}$$

Since $27x^2-18x+7$ is never zero, there is no possible inflection point, but $x = -\dfrac{1}{3}$ and $x = 1$ must be included in concavity analysis. Concave down on $\left(-\infty, -\dfrac{1}{3}\right)$ and $(1, \infty)$; concave up on $\left(-\dfrac{1}{3}, 1\right)$.



46. $y = 3x+2+\frac{1}{3x+2} = \frac{(3x+2)^2+1}{3x+2}$
$= \frac{9x^2+12x+5}{3x+2}$

Note that $9x^2+12x+5$ is never zero.

Intercept: $\left(0, \frac{5}{2}\right)$

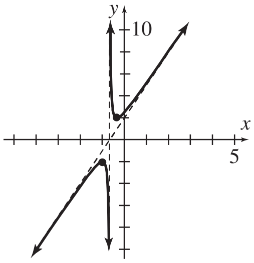
Vertical asymptote is $x = -\frac{2}{3}$; oblique asymptote is $y = 3x + 2$.

$$y' = 3-\frac{3}{(3x+2)^2} = 3\cdot\frac{(3x+2)^2-1}{(3x+2)^2}$$
$$= 3\cdot\frac{9x^2+12x+3}{(3x+2)^2} = 9\cdot\frac{(3x+1)(x+1)}{(3x+2)^2}$$

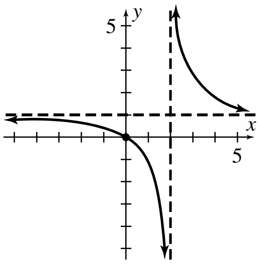
CV: $x = -\frac{1}{3}$ and $x = -1$, but $x = -\frac{2}{3}$ must be included in inc.-dec. analysis. Increasing on $(-\infty, -1)$ and $\left(-\frac{1}{3}, \infty\right)$; decreasing on $\left(-1, -\frac{2}{3}\right)$ and $\left(-\frac{2}{3}, -\frac{1}{3}\right)$; relative maximum at $(-1, -2)$; relative minimum at $\left(-\frac{1}{3}, 2\right)$.

$$y'' = -3(-2)(3x+2)^{-3}(3) = \frac{18}{(3x+2)^3}$$

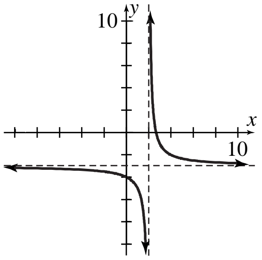
No possible inflection point, but $x = -\frac{2}{3}$ must be included in concavity analysis. Concave down on $\left(-\infty, -\frac{2}{3}\right)$; concave up on $\left(-\frac{2}{3}, \infty\right)$.



47.



48.



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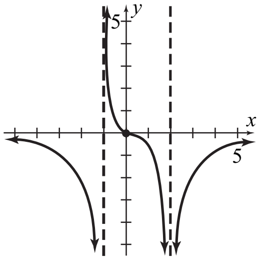
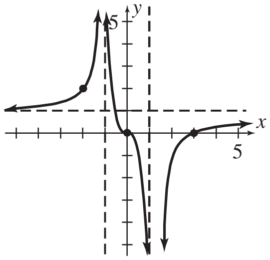


50.



51. When $x = -\frac{a}{b}$, then $a + bx = 0$ so $x = -\frac{a}{b}$ is a vertical asymptote.

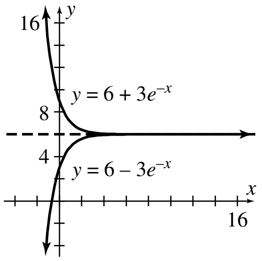
$$\lim_{x\to\infty}\frac{x}{a+bx} = \lim_{x\to\infty}\frac{x}{bx} = \lim_{x\to\infty}\frac{1}{b} = \frac{1}{b}$$

Thus $y = \frac{1}{b}$ is a horizontal asymptote.

52. For $y = 6-3e^{-x}$ we have

$$\lim_{x\to\infty}\left(6-3e^{-x}\right) = \lim_{x\to\infty}\left(6-\frac{3}{e^x}\right) = 6-3(0) = 6$$

Thus the line $y = 6$ is a horizontal asymptote for the graph of $y = 6-3e^{-x}$. For $y = 6+3e^{-x}$, we obtain $\lim\limits_{x\to\infty}\left(6+3e^{-x}\right)=6+3(0)=6$, so the line $y = 6$ is also a horizontal asymptote for the graph of $y=6+3e^{-x}$.

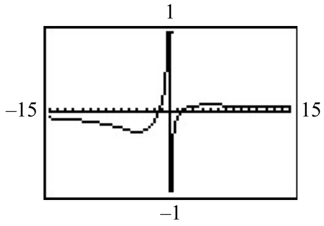


53. $\lim\limits_{t\to\infty}\left(250-83e^{-t}\right)=\lim\limits_{t\to\infty}\left(250-\frac{83}{e^t}\right)$

$= 250 - 0 = 250$

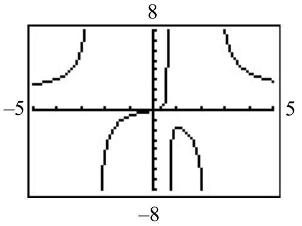
Thus $y = 250$ is a horizontal asymptote.

54.



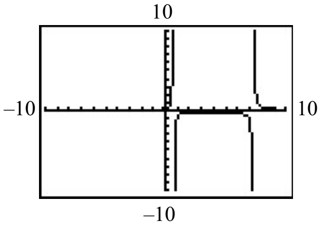
$x \approx -0.08$, $y = 0$

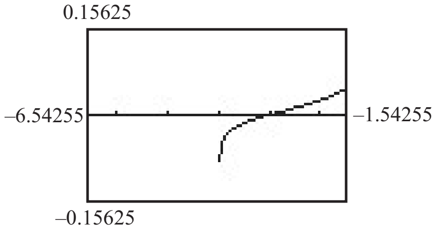
55.



$x \approx \pm 2.45$, $x \approx 0.67$, $y = 2$

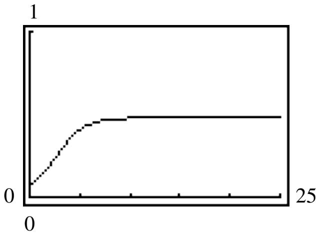
56.





In the standard window, two vertical asymptotes of the form $x = k$, where $k > 0$, are apparent ($x \approx 0.68$ and $x \approx 7.32$). By zooming around $x = -4$, another vertical asymptote is apparent ($x = -4$). Thus three vertical asymptotes exist.

57.



From the graph, it appears that $\lim\limits_{x\to\infty} y \approx 0.48$.

Thus a horizontal asymptote is $y \approx 0.48$. Algebraically, we have

$$\lim_{x\to\infty}\frac{0.34e^{0.7x}}{4.2+0.71e^{0.7x}}=\lim_{x\to\infty}\frac{\frac{0.34e^{0.7x}}{e^{0.7x}}}{\frac{4.2+0.71e^{0.7x}}{e^{0.7x}}}$$

$$=\lim_{x\to\infty}\frac{0.34}{\frac{4.2}{e^{0.7x}}+0.71}=\frac{0.34}{0+0.71}\approx 0.48$$

Problems 13.6

1. Let the numbers be x and $82 - x$. Then if $P=x(82-x)=82x-x^2$, we have $P'=82-2x$. Setting $P'=0\Rightarrow x=41$. Since $P''=-2<0$, there is a maximum when $x = 41$. Because $82 - x = 41$, the required numbers are 41 and 41.

2. Let the numbers be x and $20 - x$, where $0 \le x \le 20$. Let

$P=(2x)(20-x)^2=2x^3-80x^2+800x$.

Setting $\frac{dP}{dx}=0$ gives

$P'=6x^2-160x+800=2(3x-20)(x-20)=0$,

from which $x=\frac{20}{3}$ or $x = 20$. $P'>0$ on

$\left(0,\frac{20}{3}\right)$ and $P'<0$ on $\left(\frac{20}{3},20\right)$. Thus P has a relative and absolute maximum when $x = \frac{20}{3}$.

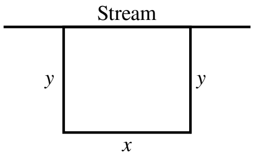
The other number is $20 - x = \frac{40}{3}$.

3. We are given that 15*x* + 9(2*y*) = 9000, or $y = \frac{9000 - 15x}{18}$. We want to maximize area *A*, where *A* = *xy*.

$$A = xy = x\left(\frac{9000 - 15x}{18}\right) = \frac{1}{18}\left(9000x - 15x^2\right)$$

$$A' = \frac{1}{18}(9000 - 30x)$$

Setting $A' = 0 \Rightarrow x = 300$. Since $A''(300) = \frac{1}{18}(-30) < 0$, we have a maximum at *x* = 300. Thus $y = \frac{9000 - 15(300)}{18} = 250$. The dimensions are 300 ft by 250 ft.

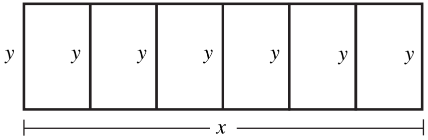


4. We are given that *xy* = 1400, or $y = \frac{1400}{x}$, and want to minimize *N* = 2*x* + 7*y*. We have

$$N = 2x + 7y = 2x + 7\left(\frac{1400}{x}\right), x > 0$$

$$N' = 2 - \frac{9800}{x^2} = \frac{2(x^2 - 4900)}{x^2}$$

Setting $N' = 0$ yields $x^2 = 4900$, so *x* = 70. We have $N'' = \frac{19,600}{x^3}$, so $N''(70) > 0$ and we have a minimum. If *x* = 70, then *y* = 20. Thus *N* = 2(70) + 7(20) = 280 ft.



5. $c = 0.05q^2 + 5q + 500$

Avg. cost per unit $= \bar{c} = \frac{c}{q} = 0.05q + 5 + \frac{500}{q}$

$\bar{c}' = 0.05 - \frac{500}{q^2}$. Setting $\bar{c}' = 0$ yields

$0.05 = \frac{500}{q^2}$, $q^2 = 10,000$, $q = \pm 100$. We exclude *q* = –100 because *q* represents the number of units. Since $\bar{c}'' = \frac{1000}{q^3} > 0$ for *q* > 0, $\bar{c}$ is an absolute minimum when *q* = 100 units.

6. $C = 0.12s - 0.0012s^2 + 0.08$, where 0 ≤ *s* ≤ 60. Setting $\frac{dC}{ds} = 0$ gives 0.12 – 0.0024*s* = 0 ⇒ *s* = 50. Since $\frac{d^2C}{ds^2} = -0.0024 < 0$, a maximum occurs when *s* = 50. Thus a minimum can occur only at an endpoint of the domain. If *s* = 0, then *C* = 0.08; if *s* = 60, then *C* = 2.96. Thus the minimum cost of $0.08 per hour occurs for *s* = 0 mi/h and might be due to depreciation, insurance, and so on.

7. *p* = –5*q* + 30
Since total revenue = (price)(quantity),

$$r = pq = (-5q + 30)q = -5q^2 + 30q$$

Setting $r' = -10q + 30 = 0 \Rightarrow q = 3$. Since $r'' = -10 < 0$, *r* is maximum at *q* = 3 units, for which the corresponding price is
p = –5(3) + 30 = $15.

8. $q = Ae^{-Bp}$

$$\text{Revenue} = r = pq = pAe^{-Bp}$$

$$\begin{aligned} r' &= A[e^{-Bp}(1) + pe^{-Bp}(-B)] \\ &= A(1 - Bp)e^{-Bp} \\ &= AB\left(\frac{1}{B} - p\right)e^{-Bp} \end{aligned}$$

Critical value: $p = \frac{1}{B}$

If $p < \frac{1}{B}$, then $r' > 0$ and *r* is increasing. If $p > \frac{1}{B}$, then $r' < 0$ and *r* is decreasing. Thus revenue is maximum when $p = \frac{1}{B}$. The answer does not depend on *A* because *A* represents the initial value of *q*, so it doesn't change *q* over time.

9. $f(p) = 170 - p - \dfrac{1600}{p+15}$, where $0 \le p \le 100$.

a. Setting $f'(p) = 0$ gives $-1 + \dfrac{1600}{(p+15)^2} = 0$,

$\dfrac{1600}{(p+15)^2} = 1$, $(p+15)^2 = 1600$,

$p + 15 = \pm 40$, from which $p = 25$.

Since $f''(p) = -\dfrac{3200}{(p+15)^3} < 0$ for $p = 25$, we have an absolute maximum of $f(25) = 105$ grams.

b. $f(0) = 63\frac{1}{3}$ and $f(100) = \dfrac{1290}{23} \approx 56.1$, so we have an absolute minimum of $f(100) = \dfrac{1290}{23} \approx 56.1$ grams.

10. $R = D^2\left(\dfrac{C}{2} - \dfrac{D}{3}\right) = \dfrac{CD^2}{2} - \dfrac{D^3}{3}$

The rate of change of R is $\dfrac{dR}{dD} = CD - D^2$. This is the function to be maximized. Setting $\dfrac{d}{dD}\left(\dfrac{dR}{dD}\right) = C - 2D = 0$ gives $D = \dfrac{C}{2}$. Since $\dfrac{d^2}{dD^2}\left(\dfrac{dR}{dD}\right) = -2 < 0$, the maximum rate of change occurs when $D = \dfrac{C}{2}$.

11. $p = 85 - 0.05q$
$c = 600 + 35q$
Profit = Total Revenue − Total Cost
$P = pq - c = (85 - 0.05q)q - (600 + 35q)$
$= -(0.05q^2 - 50q + 600)$
Setting $P' = -(0.1q - 50) = 0$ yields $q = 500$.
Since $P''(500) = -0.1 < 0$, P is a maximum when $q = 500$ units. This corresponds to a price of $p = 85 - 0.05(500) = \$60$ and a profit of $P = \$11{,}900$.

12. Cost per unit = \$3
$p = \dfrac{10}{\sqrt{q}}$
Profit = Total Revenue − Total Cost
$P = pq - c$
$P = \left(\dfrac{10}{\sqrt{q}}\right)q - (3q) = 10\sqrt{q} - 3q$
Setting $P' = \dfrac{5}{\sqrt{q}} - 3 = 0$ yields $q = \dfrac{25}{9}$.
Moreover, we have $P'' = -\dfrac{5}{2}q^{-\frac{3}{2}} < 0$ for $q > 0$, so P is maximum when $q = \dfrac{25}{9}$. The corresponding price is $p = \$6$.

13. $p = 42 - 4q$
$\overline{c} = 2 + \dfrac{80}{q}$
Total Cost $= c = \overline{c}q = 2q + 80$
Profit = Total Revenue − Total Cost
$P = pq - c = (42 - 4q)q - (2q + 80)$
$= -\left(4q^2 - 40q + 80\right)$
$P' = -(8q - 40)$
Setting $P' = -(8q - 40) = 0$ gives $q = 5$. We find that $P'' = -8 < 0$, so P has a maximum value when $q = 5$. The corresponding price p is $42 - 4(5) = \$22$.

14. $p = \dfrac{50}{\sqrt{q}}$
$\overline{c} = \dfrac{1}{4} + \dfrac{2500}{q}$
Total cost $= c = \overline{c}q = \dfrac{q}{4} + 2500$
Profit = Total Revenue − Total Cost
$P = pq - c = 50\sqrt{q} - \dfrac{q}{4} - 2500$
Setting $P' = \dfrac{25}{\sqrt{q}} - \dfrac{1}{4} = 0$ yields $q = 10{,}000$.
Since $P'' = -\dfrac{25}{2}q^{-3/2} < 0$ for $q > 0$, it follows that P is a maximum when $q = 10{,}000$. The corresponding price is $p = \dfrac{50}{100} \approx \$0.50$. Since $MR = \dfrac{25}{\sqrt{q}}$ and $MC = \dfrac{1}{4}$, then for $q = 10{,}000$ we have $MR = \dfrac{25}{100} = \dfrac{1}{4} = MC$.

15. $p = q^2 - 100q + 3200$ on [0, 120]

$\bar{c} = \frac{2}{3}q^2 - 40q + \frac{10,000}{q}$

Profit = Total Revenue – Total Cost
Since total revenue $r = pq$ and
total cost $= c = \bar{c}q$,

$$P = pq - \bar{c}q$$
$$= q^3 - 100q^2 + 3200q - \left(\frac{2}{3}q^3 - 40q^2 + 10,000\right)$$
$$= \frac{1}{3}q^3 - 60q^2 + 3200q - 10,000$$

$P' = q^2 - 120q + 3200 = (q-40)(q-80)$

Setting $P' = 0$ gives q = 40 or 80. Evaluating profit at q = 0, 40, 80, and 120 gives
$P(0) = -10,000$

$P(40) = \frac{130,000}{3} = 43,333\frac{1}{3}$

$P(80) = \frac{98,000}{3} = 32,666\frac{2}{3}$

$P(120) = 86,000$

Thus the profit maximizing output is q = 120 units, and the corresponding maximum profit is $86,000.

16. a. $c = \bar{c}q = 2q^3 - 42q^2 + 228q + 210$

$\frac{dc}{dq} = 6q^2 - 84q + 228 = 6(q^2 - 14q + 38)$

Using the quadratic formula to solve $\frac{dc}{dq} = 0$ gives $q = 7 - \sqrt{11} \approx 3.68$ or $q = 7 + \sqrt{11} \approx 10.32.$ Evaluating c at q = 3, $7 - \sqrt{11},$ $7 + \sqrt{11},$ and 12 gives 570, $434 + 44\sqrt{11} \approx 579.93,$ $434 - 44\sqrt{11} \approx 288.07,$ and 354, respectively. Thus the minimum cost is when $q = 7 + \sqrt{11} \approx 10.32.$
$c(10) = 290$ and $c(11) = 298$, so production should be fixed at q = 10 for a minimum cost of $290.

b. $c(7) = 434$, so the minimum cost still occurs when $q = 7 + \sqrt{11} \approx 10.32$ and production should again be fixed at 10 units.

17. Total fixed costs = $1200,
material-labor costs/unit = $2, and the demand equation is $p = \frac{100}{\sqrt{q}}$.

Profit = Total Revenue – Total Cost

$$P = pq - c$$
$$P = \frac{100}{\sqrt{q}} \cdot q - (2q + 1200)$$
$$= 100\sqrt{q} - 2q - 1200$$
$$= 2\left(50\sqrt{q} - q - 600\right)$$

Setting $P' = 2\left(\frac{25}{\sqrt{q}} - 1\right) = 0$ yields q = 625. We see that $P'' = -25q^{-\frac{3}{2}} < 0$ for $q > 0$, so P is maximum when q = 625. When q = 625, $\text{MR} = \frac{50}{\sqrt{625}} = 2 = \text{MC}.$ When q = 625, then p = $4.

18. Let x = number of $10 per month increases so the monthly rate is 400 + 10x and the number of rented apartments is 100 – 2x. Monthly revenue r is given by
r = (rent/apt.) (no. of apt. rented)
$r = (400 + 10x)(100 - 2x)$
$r' = (400 + 10x)(-2) + (100 - 2x)(10)$
$= 200 - 40x = 40(5 - x)$
Setting $r' = 0$ yields x = 5. Since $r'' = -40 < 0$, then r is maximum when x = 5. This results in a monthly rate for an apartment of
400 + 10(5) = $450.

19. If x = number of $0.50 decreases, where $0 \le x \le 48$, then the monthly fee for each subscriber is 24 – 0.50x, and the total number of subscribers is 6400 + 160x. Let r be the total (monthly) revenue.
revenue = (monthly rate)(number of subscribers)
$r = (24 - 0.50x)(6400 + 160x)$
$r' = (24 - 0.50x)(160) + (6400 + 160x)(-0.50)$
$= 640 - 160x = 160(4 - x)$
Setting $r' = 0$ yields x = 4.
Evaluating r when x = 0, 4, and 48, we find that r is a maximum when x = 4. This corresponds to a monthly fee of 24 – 0.50(4) = $22 and a monthly revenue r of $154,880.

20. Note that as the number of units produced and sold increases from 0 to 600, the profit increases from 0 to (600)(400) = \$24,000. Let q = number of units produced and sold beyond 600. Then the total profit P is given by

$$P = (600)(40) + (40 - 0.05q)q$$
$$= 24{,}000 + 40q - 0.05q^2$$
$$P' = 40 - 0.10q$$

Setting $P' = 0$ yields $q = 400$. Since $P'' = -0.10 < 0$, P is a maximum when $q = 400$, that is, the total number of units = 600 + 400 = 1000.

21. See the figure in the text. Given that $x^2y = 32$, we want to minimize $S = 4(xy) + x^2$. Since $y = \frac{32}{x^2}$, where $x > 0$, we have

$S = 4x\left(\frac{32}{x^2}\right) + x^2 = \frac{128}{x} + x^2$, from which

$S' = -\frac{128}{x^2} + 2x$. Setting $S' = 0$ gives

$2x^3 = 128$, $x^3 = 64$, $x = 4$. Since $S'' = \frac{256}{x^3} + 2$, we get $S''(4) > 0$, so $x = 4$ gives a minimum. If $x = 4$, then $y = \frac{32}{16} = 2$. The dimensions are 4 ft × 4 ft × 2 ft.

22. See the figure in the text. We want to maximize $V = x^2y$ given that $4xy + x^2 = 192$, or

$$y = \frac{192 - x^2}{4x}$$
$$V = x^2\left(\frac{192 - x^2}{4x}\right) = \frac{1}{4}\left(192x - x^3\right),\ x > 0$$
$$V' = \frac{1}{4}\left(192 - 3x^2\right) = \frac{3}{4}\left(64 - x^2\right)$$

Setting $V' = 0$ gives $x = 8$. Since $V'' = \left(\frac{3}{4}\right)(-2x)$, then $V''(8) < 0$, so $x = 8$ gives a maximum. If $x = 8$, then $y = 4$.
The dimensions are 8 ft × 8 ft × 4 ft.
The volume is $8^2(4) = 256\text{ ft}^3$.

23. $V = x\left(L - 2x\right)^2$

$= L^2x - 4Lx^2 + 4x^3$

where $0 < x < \frac{L}{2}$.

$$V' = L^2 - 8Lx + 12x^2$$
$$= 12x^2 - 8Lx + L^2$$
$$= (2x - L)(6x - L)$$

For $0 < x < \frac{L}{2}$, setting $V' = 0$ gives $x = \frac{L}{6}$.

Since $V' > 0$ on $\left(0, \frac{L}{6}\right)$ and $V' < 0$ on $\left(\frac{L}{6}, \frac{L}{2}\right)$, V is maximum when $x = \frac{L}{6}$. Thus the length of the side of the square must be $\frac{L}{6}$ in., which results in a volume of

$$\frac{L}{6}\left(L - \frac{L}{3}\right)^2 = \frac{2L^3}{27}\text{ in}^3.$$

24. Since $xy = 720$, then $y = \frac{720}{x}$, $x > 0$. We want to minimize A where

$$A = (x + 10)(y + 8) = (x + 10)\left(\frac{720}{x} + 8\right)$$
$$= 800 + 8x + \frac{7200}{x}$$
$$A' = 8 - \frac{7200}{x^2}$$

Setting $A' = 0$ gives $x = 30$. Since $A'' = \frac{14{,}400}{x^3} > 0$ for $x = 30$, we have a minimum. Thus $y = 24$, so the dimensions are 30 + 10 by 24 + 8, that is, 40 in. × 32 in.

25. See the figure in the text.

$V = K = \pi r^2h$ (1)

$S = 2\pi rh + \pi r^2$ (2)

From Equation (1) $h = \frac{K}{\pi r^2}$. Thus Equation (2) becomes

$$S = \frac{2K}{r} + \pi r^2$$
$$\frac{dS}{dr} = -\frac{2K}{r^2} + 2\pi r = \frac{2\left(\pi r^3 - K\right)}{r^2}.$$

If $S' = 0$, then $\pi r^3 - K = 0$, $\pi r^3 = K$,

$r = \sqrt[3]{\frac{K}{\pi}}$. Thus

$$h = \frac{K}{\pi\left(\frac{K}{\pi}\right)^{\frac{2}{3}}} = \left(\frac{K}{\pi}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{K}{\pi}}.$$

Note that since $S'' = 2\pi + \frac{4K}{r^3} > 0$ for $r > 0$, we have a minimum.

26. See the figure in the text.

$S = K = 2\pi rh + \pi r^2$ (1)

$V = \pi r^2 h$ (2)

From Equation (1), $h = \frac{K - \pi r^2}{2\pi r}$. Thus Equation (2) becomes

$$V = \frac{Kr - \pi r^3}{2}$$

$$\frac{dV}{dr} = \frac{K - 3\pi r^2}{2}.$$

Setting $V' = 0$ gives $r = \sqrt{\frac{K}{3\pi}}$. Thus

$$h = \frac{K - \pi\frac{K}{3\pi}}{2\pi\sqrt{\frac{K}{3\pi}}} = \frac{\frac{2}{3}K}{2\pi\sqrt{\frac{K}{3\pi}}}$$

$$= \frac{\frac{2}{3}K}{2\pi\sqrt{\frac{K}{3\pi}}} \cdot \frac{\sqrt{\frac{K}{3\pi}}}{\sqrt{\frac{K}{3\pi}}} = \sqrt{\frac{K}{3\pi}}$$

Note that since $V'' = -3\pi r < 0$ for $r > 0$, we have a maximum.

27. $p = 600 - 2q$

$c = 0.2q^2 + 28q + 200$

Profit = Total Revenue – Total Cost

$P = pq - c$

$P = (600 - 2q)q - \left(0.2q^2 + 28q + 200\right)$

$= -\left(2.2q^2 - 572q + 200\right)$

$P' = -(4.4q - 572)$

Setting $P' = 0$ yields $q = 130$. Since $P'' = -4.4 < 0$, P is maximum when $q = 130$ units. The corresponding price is $p = 600 - 2(130) = \$340$, and the profit is $P = \$36{,}980$. If a tax of \$22/unit is imposed on the manufacturer, then the cost equation is

$c_1 = 0.2q^2 + 28q + 200 + 22q$

$= 0.2q^2 + 50q + 200$.

The demand equation remains the same. Thus

$P_1 = pq - c_1$

$= (600 - 2q)q - \left(0.2q^2 + 50q + 200\right)$

$= -\left(2.2q^2 - 550q + 200\right)$

$P'_1 = -(4.4q - 550)$

Setting $P'_1 = 0$ yields $q = 125$. Since $P_1'' = -4.4 < 0$, P_1 is maximum when $q = 125$ units. The corresponding price is $p = \$350$ and the profit is $P_1 = \$34{,}175$.

28. Original data: $p = 600 - 2q$, $c = 0.2q^2 + 28q + 200$. Revenue, both before and after the license fee, is given by $r = pq = 600q - 2q^2$. After the license fee, the cost equation is $c_1 = c + 1000 = 0.2q^2 + 28q + 1200$ and the profit is

$P_1 = r - c_1$

$= \left(600q - 2q^2\right) - \left(0.2q^2 + 28q + 1200\right)$

As in Problem 27, we find that P_1 has a maximum when $q = 130$ units, which gives $p = \$340$. Thus the profit-maximizing price and output remain the same. Since

Profit

$= r - c_1 = r - (c + 1000) = (r - c) - 1000$, when $q = 130$ we have

Profit = 36,980 – 1000 (from Problem 27)

= \$35,980

29. Let q = number of units in a production run. Since inventory is depleted at a uniform rate, assume that the average inventory is $\frac{q}{2}$. The value of average inventory is $12\left(\frac{q}{2}\right)$, and carrying costs are $0.192\left[12\left(\frac{q}{2}\right)\right]$. The number of production runs per year is $\frac{3000}{q}$, and total set-up costs are $54\left(\frac{3000}{q}\right)$. We want to minimize the sum C of carrying costs and set-up costs.

$$C = 0.192\left[12\left(\frac{q}{2}\right)\right] + 54\left(\frac{3000}{q}\right)$$

$$= 1.152q + \frac{162{,}000}{q}$$

$$C' = 1.152 - \frac{162{,}000}{q^2}$$

Setting $C' = 0$ yields $q^2 = \frac{162{,}000}{1.152} = 140{,}625$, $q = 375$ (since $q > 0$). Since $C'' = \frac{324{,}000}{q^3} > 0$, C is minimum when $q = 375$. Thus the economic lot size is 375/lot (8 lots).

30. $c = 0.004q^3 + 20q + 5000$

$p = 450 - 4q$

Profit = Total Revenue – Total Cost

$$P = pq - c$$

$$= (450 - 4q)q - \left(0.004q^3 + 20q + 5000\right)$$

$$P = -\left(0.004q^3 + 4q^2 - 430q + 5000\right)$$

$$P' = -\left(0.012q^2 + 8q - 430\right)$$

$$= -2\left(0.006q^2 + 4q - 215\right)$$

Setting $P' = 0$ yields

$$0.006q^2 + 4q - 215 = 0$$

$$q = \frac{-4 \pm \sqrt{21.16}}{0.012} = \frac{-4 \pm 4.6}{0.012}$$

Since $q \geq 0$, choose $q = \frac{-4 + 4.6}{0.012} = 50$. Since P is increasing on [0, 50) and decreasing on (50, ∞), P is maximum when $q = 50$ units.

31. Let x = number of people over the 30.

Note: $0 \leq x \leq 10$.

Revenue = r

= (number attending)(charge/person)

= $(30 + x)(50 - 1.25x)$

$$= 1500 + 12.5x - 1.25x^2$$

$$r' = 12.5 - 2.5x$$

Setting $r' = 0$ yields $x = 5$. Since $r'' = -2.5 < 0$, r is maximum when $x = 5$, that is, when 35 attend.

32. Let N = horsepower of motor.

(Total annual cost) = C = (Annual cost to lease) + (Annual operating cost)

$$C = (200 + 0.40N) + 80{,}000\left(\frac{0.008}{N}\right)$$

$$= 200 + 0.04N + \frac{640}{N}$$

$$C' = 0.4 - \frac{640}{N^2}$$

Setting $C' = 0$ yields $N^2 = 1600$, so $N = 40$ (since $N > 0$). Since $C'' = \frac{1280}{N^3} > 0$ for $N > 0$, C is a minimum when $N = 40$ horsepower.

33. The cost per mile of operating the truck is $0.165 + \frac{s}{200}$. Driver's salary is \$18/hr. The number of hours for 700 mi trip is $\frac{700}{s}$. Driver's salary for trip is $18\left(\frac{700}{s}\right)$, or $\frac{12{,}600}{s}$. The cost of operating the truck for the trip is

$$700\left[0.165 + \frac{s}{200}\right].$$

Total cost of trip is

$$C = \frac{12{,}600}{s} + 700\left(0.165 + \frac{s}{200}\right)$$

Setting $C' = -\frac{12{,}600}{s^2} + \frac{7}{2} = 0$ yields $s^2 = 3600$, or $s = 60$ (since $s > 0$). Since $C'' = \frac{25{,}200}{s^3} > 0$ for $s > 0$, C is a minimum when $s = 60$ mi/h.

34. Let q = level of production.

$$\text{Average Cost } = \bar{c} = \frac{\text{Total Cost}}{q}$$

For $0 \le q \le 5000$, we have

$$\bar{c} = \frac{30q+10q+20{,}000}{q} = 40+\frac{20{,}000}{q}.$$

Note that total cost for 5000 units is 220,000. For
$q > 5000$,

$$\bar{c} = \frac{(\text{cost for first } 5000)+\begin{pmatrix}\text{cost for those}\\ \text{units beyond } 5000\end{pmatrix}}{q}$$

$$= \frac{220{,}000+[45(q-5000)+10(q-5000)]}{q}$$

$$\bar{c} = 55-\frac{55{,}000}{q}$$

If $0 < q \le 5000$, then $\bar{c}' = -\frac{20{,}000}{q^2} < 0$ and thus $\bar{c}$ is decreasing. If $q > 5000$, then $\bar{c}' = \frac{55{,}000}{q^2} > 0$ and thus $\bar{c}$ is increasing.
Hence c is minimum when q = 5000 units.

35. Profit P is given by
P = Total revenue – Total cost
= Total revenue – (salaries + fixed cost)
= $50q$ – $(1000m + 3000)$

$$= 50\left(m^3-15m^2+92m\right)-1000m-3000$$

$$= 50\left(m^3-15m^2+72m-60\right), \text{ where } 0 \le m \le 8$$

$$P' = 50\left(3m^2-30m+72\right)$$

$$= 150\left(m^2-10m+24\right) = 150(m-4)(m-6)$$

Setting $P' = 0$ gives the critical values 4 and 6. We now evaluate P at these critical values and also at the endpoints 0 and 8.
$P(0) = -3000$
$P(4) = 2600$
$P(6) = 2400$
$P(8) = 3400$
Thus Ms. Jones should hire 8 salespeople to obtain a maximum weekly profit of \$3400.

36. Profit P is given by
P = Total revenue – Total cost = pq – Total cost
$= 400q-50q^2 -$ Total cost. (q in hundreds)

$$\frac{dP}{dq} = 400-100q-\frac{d}{dq}(\text{Total cost})$$

$= 400 - 100q$ – Marginal cost

$$= 400-100q-\frac{800}{q+5}$$

$$= \frac{400(q+5)-100q(q+5)-800}{q+5}$$

$$= \frac{-100q^2-100q+1200}{q+5}$$

$$= \frac{-100(q+4)(q-3)}{q+5}$$

Setting $P' = 0$ gives the critical value 3 (since $q > 0$). We find that $P' > 0$ for $0 < q < 3$, and $P' < 0$ for $q > 3$. Thus there is a maximum profit when q = 3000 jackets.

37. x = tons of chemical A ($x \le 4$),
$y = \frac{24-6x}{5-x}$ = tons of chemical B, profit on A = \$2000/ton, and profit on B = \$1000/ton.

$$\text{Total Profit } = P_T = 2000x+1000\left(\frac{24-6x}{5-x}\right)$$

$$= 2000\left[x+\frac{12-3x}{5-x}\right]$$

$$P'_T = 2000\left[1+\frac{(5-x)(-3)-(12-3x)(-1)}{(5-x)^2}\right]$$

$$= 2000\left[1-\frac{3}{(5-x)^2}\right]$$

$$= 2000\left[\frac{x^2-10x+22}{(5-x)^2}\right]$$

Setting $P'_T = 0$ yields (by the quadratic formula)

$$x = \frac{10\pm 2\sqrt{3}}{2} = 5\pm\sqrt{3}$$

Because $x \le 4$, choose $x = 5-\sqrt{3}$. Since P_T is increasing on $\left[0, 5-\sqrt{3}\right)$ and decreasing on $\left(5-\sqrt{3}, 4\right]$, P_T is a maximum for $x = 5-\sqrt{3}$ tons. If profit on A is P/ton and profit on B is

$\frac{P}{2}$ /ton, then

$$P_T = Px + \frac{P}{2}\left(\frac{24-6x}{5-x}\right) = P\left[x + \frac{12-3x}{5-x}\right]$$

$$P'_T = P\left[\frac{x^2-10x+22}{(5-x)^2}\right]$$

Setting $P'_T = 0$ and using an argument similar to that above, we find that P_T is a maximum when $x = 5-\sqrt{3}$ tons.

38. x = number of floors. Let R = rate of return.

$$R = \frac{\text{Total Revenue}}{\text{Total Cost}}$$

$$= \frac{60,000x}{(10x)[120,000+3000(x-1)]+1,440,000}$$

$$= \frac{2x}{x^2+39x+48}$$

$$R' = 2\cdot\frac{48-x^2}{(x^2+39x+48)^2}$$

$R' = 0$ when $x = \sqrt{48} = 4\sqrt{3}$ $(x \geq 0)$. Since R is increasing on $\left(0,\ 4\sqrt{3}\right)$ and decreasing on $\left(4\sqrt{3},\ \infty\right)$, R is a maximum when $x = 4\sqrt{3} \approx 6.93$. The number of floors in the building must be an integer, so we evaluate R when $x = 6$ and $x = 7$: $R(6) \approx 0.0377$; $R(7) \approx 0.0378$. Thus 7 floors should be built to maximize the rate of return.

39. $P(j) = Aj\frac{L^4}{V} + B\frac{V^3L^2}{1+j}$

$$\frac{dP}{dj} = \frac{AL^4}{V} - \frac{BV^3L^2}{(1+j)^2} = 0$$

Solving for $(1+j)^2$ gives $(1+j)^2 = \frac{BV^4}{AL^2}$

40. a. $\frac{d}{dv}\left(-2at_r + v - \frac{2al}{v}\right) = 1 + \frac{2al}{v^2} = 0$ when $v = \sqrt{-2al}$. Note that

$$\frac{d^2}{dv^2}\left(-2at_r + v - \frac{2al}{v}\right) = \frac{-4al}{v^3} > 0 \text{ for}$$

$a < 0$, $l > 0$, and $v > 0$. Thus $-2at_r + v - \frac{2al}{v}$ is a minimum for $v = \sqrt{-2al}$.

b. $v = \sqrt{-2(-19.6)(20)} = \sqrt{784} = 28$ ft/s.

c. $$N = \frac{-2(-19.6)}{(-2)(-19.6)(0.5) + 28 - \frac{2(-19.6)(20)}{28}}$$

$\approx$ 0.5 cars/s = 0.5(3600) cars/h = 1800 cars/h

d. When $v = \sqrt{-2al}$, then

$$N = N(l) = \frac{-2a}{-2at_r + \sqrt{-2al} + \frac{-2al}{\sqrt{-2al}}}$$

$$= \frac{-2a}{-2at_r + 2\sqrt{-2al}} = \frac{a}{at_r - \sqrt{-2al}}$$

The relative change in N when l is reduced from 20 ft to 15 ft is $\frac{N(15)-N(20)}{N(20)}$.

With $a = -19.6$ ft/s^2 and $t_r = 0.5$ s, then

$$N(20) = \frac{-19.6}{(-19.6)(0.5) - \sqrt{-2(-19.6)(20)}}$$

≈ 0.5185

$$N(15) = \frac{-19.6}{(-19.6)(0.5) - \sqrt{-2(-19.6)(15)}}$$

≈ 0.5756

The relative change is

$$\frac{N(15)-N(20)}{N(20)} \approx \frac{0.5756-0.5158}{0.5158} \approx 0.1101$$

41. $\bar{c} = \frac{c}{q} = 3q + 50 - 18\ln(q) + \frac{120}{q}$, $q > 0$

$$\frac{d\bar{c}}{dq} = 3 - \frac{18}{q} - \frac{120}{q^2} = \frac{3q^2-18q-120}{q^2}$$

$$= \frac{3\left(q^2-6q-40\right)}{q^2}$$

$$= \frac{3(q-10)(q+4)}{q^2}$$

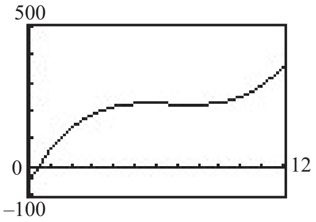
Critical value is $q = 10$ since $q \geq 0$.

Since $\frac{d\bar{c}}{dq} < 0$ for $0 < q < 10$, and $\frac{d\bar{c}}{dq} > 0$ for $q > 10$, we have a minimum when $q = 10$ cases. This minimum average cost is 3(10) + 50 – 18 ln 10 + 12 ≈ \$50.55.

42. The profit function is given by

$$P = TR - TC = q^3 - 20q^2 + 160q - (30q + 50)$$
$$= q^3 - 20q^2 + 130q - 50$$

where P is in thousands of dollars, q is in tons, and $0 \le q \le 12$. From the graph, the maximum profit occurs when $q = 12$ tons. The corresponding maximum profit is \$358,000 and the selling price per ton is \$64,000.



Chapter 13 Review Problems

1. $y = \frac{3x^2}{x^2 - 16} = \frac{3x^2}{(x+4)(x-4)}$

When $x = \pm 4$ the denominator is zero and the numerator is not zero. Thus $x = 4$ and $x = -4$ are vertical asymptotes.

$$\lim_{x\to\infty} \frac{3x^2}{x^2 - 16} = \lim_{x\to\infty} \frac{3x^2}{x^2} = \lim_{x\to\infty} 3 = 3$$

Similarly, $\lim_{x\to-\infty} y = 3$. Thus $y = 3$ is the only horizontal asymptote.

2. $y = \frac{x+3}{9x - 3x^2} = \frac{x+3}{3x(3-x)}$

When $x = 0$ or $x = 3$, the denominator is zero and the numerator is not zero. Thus $x = 0$ and $x = 3$ are vertical asymptotes.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{x}{-3x^2} = -\frac{1}{3} \lim_{x\to\infty} \frac{1}{x} = 0$$

Similarly, $\lim_{x\to-\infty} y = 0.$ Thus $y = 0$ is the only horizontal asymptote.

3. $y = \frac{5x^2 - 3}{(3x+2)^2} = \frac{5x^2 - 3}{9x^2 + 12x + 4}$

When $x = -\frac{2}{3}$, the denominator is zero and the numerator is not zero. Thus $x = -\frac{2}{3}$ is a vertical asymptote.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{5x^2}{9x^2} = \lim_{x\to\infty} \frac{5}{9} = \frac{5}{9}$$

Similarly, $\lim_{x\to-\infty} y = \frac{5}{9}$. Thus $y = \frac{5}{9}$ is the only horizontal asymptote.

4. $y = \frac{4x+1}{3x-5} - \frac{3x+1}{2x-11} = \frac{-x^2 - 30x - 6}{(3x-5)(2x-11)}$

When $x = \frac{5}{3}$ or $x = \frac{11}{2}$, the denominator is zero and the numerator is not zero. Thus $x = \frac{5}{3}$ and $x = \frac{11}{2}$ are vertical asymptotes.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{-x^2}{6x^2} = \lim_{x\to\infty} \left(-\frac{1}{6}\right) = -\frac{1}{6}$$

Similarly, $\lim_{x\to-\infty} y = -\frac{1}{6}$. Thus $y = -\frac{1}{6}$ is the only horizontal asymptote.

5. $f(x) = \frac{3x^2}{9 - x^2}$

$$f'(x) = \frac{(9 - x^2)(6x) - 3x^2(-2x)}{(9 - x^2)^2} = \frac{54x}{(9 - x^2)^2}$$

Thus $x = 0$ is the only critical value.
Note: Although $f'(3)$ is not defined, ± 3 are not critical values because ± 3 are not in the domain of *f*.

6. $f(x) = 8(x-1)^2(x+6)^4$

$$f'(x) = 8(2)(x-1)(x+6)^4 + 8(x-1)^2(4)(x+6)^3$$
$$= 16(x-1)(x+6)^3[x + 6 + 2(x-1)]$$
$$= 16(x-1)(x+6)^3(3x+4)$$

Thus $x = 1$, $x = -6$, and $x = -\frac{4}{3}$ are the critical values.

7. $f(x) = \frac{\sqrt[3]{x+1}}{3 - 4x}$

$$f'(x) = \frac{(3-4x)\left[\frac{1}{3}(x+1)^{-\frac{2}{3}}\right] - (x+1)^{\frac{1}{3}}(-4)}{(3-4x)^2}$$
$$= \frac{\frac{1}{3}(x+1)^{-\frac{2}{3}}[(3-4x) + 12(x+1)]}{(3-4x)^2}$$
$$= \frac{8x+15}{3(x+1)^{\frac{2}{3}}(3-4x)^2}$$

$f'(x)$ is zero when $x=-\frac{15}{8}$; $f'(x)$ is not defined when $x=-1$ or $x=\frac{3}{4}$. However $\frac{3}{4}$ is not in the domain of *f*. Thus $x=-\frac{15}{8}$ and $x=-1$ are critical values.

8. $f(x)=\frac{13xe^{-\frac{5x}{6}}}{6x+5}$

$$f'(x)=13\cdot\frac{(6x+5)\left[x\left(-\frac{5}{6}e^{-\frac{5x}{6}}\right)+e^{-\frac{5x}{6}}(1)\right]-xe^{-\frac{5x}{6}}(6)}{(6x+5)^2}$$

$$=\frac{13}{6}\cdot\frac{-e^{-\frac{5x}{6}}\{(6x+5)[5x-6]+36x\}}{(6x+5)^2}=\frac{13}{6}\cdot\frac{-\left\{30x^2+25x-30\right\}}{e^{\frac{5x}{6}}(6x+5)^2}$$

$$=\frac{13}{6}\cdot\frac{-5\left(6x^2+5x-6\right)}{e^{\frac{5x}{6}}(6x+5)^2}=\frac{-65(2x+3)(3x-2)}{6e^{\frac{5x}{6}}(6x+5)^2}$$

$f'(x)$ is zero when $x=-\frac{3}{2}$ or $x=\frac{2}{3}$. Although $f'(x)$ is not defined when $x=-\frac{5}{6}$, $-\frac{5}{6}$ is not in the domain of *f*. Thus $x=-\frac{3}{2}$ and $x=\frac{2}{3}$ are the only critical values.

9. $f(x)=-\frac{5}{3}x^3+15x^2+35x+10$

$$f'(x)=-5x^2+30x+35$$
$$=-5(x^2-6x-7)=-5(x-7)(x+1)$$

CV: $x=-1$ and $x=7$. Decreasing on $(-\infty, -1)$ and $(7, \infty)$; increasing on $(-1, 7)$

10. $f(x)=\frac{3x^2}{(x+2)^2}$

$$f'(x)=\frac{6x(x+2)^2-3x^2(2)(x+2)}{(x+2)^4}$$
$$=\frac{(x+2)(6x^2+12x-6x^2)}{(x+2)^4}$$
$$=\frac{12x}{(x+2)^3}$$

CV: $x=0$, but $x=-2$ is also considered in the inc.-dec. analysis. Increasing on $(-\infty, -2)$ and $(0, \infty)$; decreasing on $(-2, 0)$.

11. $f(x)=\dfrac{6x^4}{x^2-3}$

$$f'(x)=6\cdot\frac{\left(x^2-3\right)\left(4x^3\right)-x^4(2x)}{\left(x^2-3\right)^2}$$

$$=\frac{12x^3\left[2\left(x^2-3\right)-x^2\right]}{\left(x^2-3\right)^2}=\frac{12x^3\left(x^2-6\right)}{\left(x^2-3\right)^2}$$

$$=\frac{12x^3\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)}{\left[\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)\right]^2}$$

CV: $x=0,\ \pm\sqrt{6}$, but $x=\pm\sqrt{3}$ must also be considered in the inc.-dec. analysis. Decreasing on $\left(-\infty,-\sqrt{6}\right)$, $\left(0,\sqrt{3}\right)$, and $\left(\sqrt{3},\sqrt{6}\right)$; increasing on $\left(-\sqrt{6},-\sqrt{3}\right)$, $\left(-\sqrt{3},0\right)$ and $\left(\sqrt{6},\infty\right)$.

12. $f(x)=4\sqrt[3]{5x^3-7x}$

$$f'(x)=4\cdot\frac{1}{3}(5x^3-7x)^{-2/3}(15x^2-7)$$

$$=\frac{4(15x^2-7)}{3(5x^3-7x)^{2/3}}$$

$$=\frac{4\left(\sqrt{15}x+\sqrt{7}\right)\left(\sqrt{15}x-\sqrt{7}\right)}{3[x(5x^2-7)]^{2/3}}$$

$$=\frac{4\left(\sqrt{15}x+\sqrt{7}\right)\left(\sqrt{15}x-\sqrt{7}\right)}{3\left[x\left(\sqrt{5}x+\sqrt{7}\right)\left(\sqrt{5}x-\sqrt{7}\right)\right]^{2/3}}$$

CV: $x=\pm\sqrt{\dfrac{7}{15}},\ 0,\ \pm\sqrt{\dfrac{7}{5}}$

Increasing on $\left(-\infty,-\sqrt{\dfrac{7}{5}}\right)$, $\left(-\sqrt{\dfrac{7}{5}},-\sqrt{\dfrac{7}{15}}\right)$, $\left(\sqrt{\dfrac{7}{15}},\sqrt{\dfrac{7}{5}}\right)$, and $\left(\sqrt{\dfrac{7}{5}},\infty\right)$; decreasing on $\left(-\sqrt{\dfrac{7}{15}},0\right)$ and $\left(0,\sqrt{\dfrac{7}{15}}\right)$.

13. $f(x)=x^4-x^3-14$

$f'(x)=4x^3-3x^2$

$f''(x)=12x^2-6x=6x(2x-1)$

$f''(x)=0$ when $x=0$ or $x=\dfrac{1}{2}$. Concave up on $(-\infty,0)$ and $\left(\dfrac{1}{2},\infty\right)$; concave down on $\left(0,\dfrac{1}{2}\right)$.

14. $f(x)=\dfrac{x-2}{x+2}$

$$f'(x)=\frac{(x+2)(1)-(x-2)(1)}{(x+2)^2}=\frac{4}{(x+2)^2}$$

$$f''(x)=-\frac{8}{(x+2)^3}$$

$f''(x)$ is not defined when $x=-2$. Concave up on $(-\infty,-2)$; concave down on $(-2,\infty)$

15. $f(x)=\dfrac{1}{3x+2}=(3x+2)^{-1}$

$f'(x)=-(3x+2)^{-2}(3)=-3(3x+2)^{-2}$

$$f''=6(3x+2)^{-3}(3)=\frac{18}{(3x+2)^3}$$

$f''(x)$ is not defined when $x=-\dfrac{2}{3}$. Concave down on $\left(-\infty,-\dfrac{2}{3}\right)$; concave up on $\left(-\dfrac{2}{3},\infty\right)$.

16. $f(x)=x^3+2x^2-5x+2$

$f'(x)=3x^2+4x-5$

$f''(x)=6x+4=2(3x+2)$

$f''(x)=0$ when $x=-\dfrac{2}{3}$. Concave down on $\left(-\infty,-\dfrac{2}{3}\right)$; concave up on $\left(-\dfrac{2}{3},\infty\right)$.

17. $f(x)=(2x+1)^3(3x+2)$

$$\begin{aligned}f'(x)&=(2x+1)^3(3)+(3x+2)[3(2x+1)^2(2)]\\&=3(2x+1)^2(2x+1+6x+4)\\&=3(2x+1)^2(8x+5)\end{aligned}$$

$$\begin{aligned}f''(x)&=3\{(2x+1)^2(8)+(8x+5)[2(2x+1)(2)]\}\\&=12(2x+1)[2(2x+1)+8x+5]\\&=12(2x+1)(12x+7)\end{aligned}$$

$f''(x)=0$ when $x=-\frac{1}{2}$ or $x=-\frac{7}{12}$. Concave up on $\left(-\infty, -\frac{7}{12}\right)$ and $\left(-\frac{1}{2}, \infty\right)$; concave down on $\left(-\frac{7}{12}, -\frac{1}{2}\right)$.

18. $f(x)=\left(x^2-x-1\right)^2$

$$f'(x)=2\left(x^2-x-1\right)(2x-1)$$
$$=2\left(2x^3-3x^2-x+1\right)$$
$$f''(x)=2\left(6x^2-6x-1\right)$$

$f''(x)=0$ when $6x^2-6x-1=0$; by the quadratic formula $x=\frac{1}{2}\pm\frac{\sqrt{15}}{6}$. Concave up on $\left(-\infty, \frac{1}{2}-\frac{\sqrt{15}}{6}\right)$ and $\left(\frac{1}{2}+\frac{\sqrt{15}}{6}, \infty\right)$; concave down on $\left(\frac{1}{2}-\frac{\sqrt{15}}{6}, \frac{1}{2}+\frac{\sqrt{15}}{6}\right)$.

19. $f(x)=2x^3-9x^2+12x+7$

$$f'(x)=6x^2-18x+12=6\left(x^2-3x+2\right)$$
$$= 6(x-1)(x-2)$$

CV: $x = 1$ and $x = 2$

Increasing on $(-\infty, 1)$ and $(2, \infty)$; decreasing on $(1, 2)$. Relative maximum when $x = 1$; relative minimum when $x = 2$.

20. $f(x)=\frac{ax+b}{x^2}$ for $a > 0$ and $b > 0$

$$f'(x)=\frac{x^2(a)-(ax+b)(2x)}{x^4}$$
$$=\frac{ax^2-2ax^2-2bx}{x^4}$$
$$=\frac{-ax^2-2bx}{x^4}$$
$$=\frac{-ax-2b}{x^3}$$

CV: $x=-\frac{2b}{a}$, but $x = 0$ must be considered in inc.-dec. analysis. Decreasing on $\left(-\infty, -\frac{2b}{a}\right)$ and $(0, \infty)$; increasing on $\left(-\frac{2b}{a}, 0\right)$. Relative minimum when $x=-\frac{2b}{a}$.

21. $f(x)=\frac{x^{10}}{10}+\frac{x^5}{5}$

$$f'(x)=x^9+x^4=x^4(x^5+1)$$

CV: $x = 0$ and $x = -1$

Decreasing on $(-\infty, -1)$; increasing on $(-1, 0)$ and $(0, \infty)$; relative minimum when $x = -1$

22. $f(x)=\frac{x^2}{x^2-4}$

$$f'(x)=\frac{\left(x^2-4\right)(2x)-x^2(2x)}{\left(x^2-4\right)^2}$$
$$=\frac{2x\left[\left(x^2-4\right)-x^2\right]}{\left(x^2-4\right)^2}=\frac{-8x}{\left(x^2-4\right)^2}$$
$$=-\frac{8x}{[(x+2)(x-2)]^2}$$

CV: $x = 0$, but $x \pm 2$ must be considered in inc.-dec. analysis. Increasing on $(-\infty, -2)$ and $(-2, 0)$; decreasing on $(0, 2)$ and $(2, \infty)$. Relative maximum when $x = 0$.

23. $f(x)=x^{\frac{2}{3}}(x+1)=x^{\frac{5}{3}}+x^{\frac{2}{3}}$

$$f'(x)=\frac{5}{3}x^{\frac{2}{3}}+\frac{2}{3}x^{-\frac{1}{3}}=\frac{1}{3}x^{-\frac{1}{3}}(5x+2)=\frac{5x+2}{3x^{\frac{1}{3}}}$$

CV: $x = 0$ and $x=-\frac{2}{5}$

Increasing on $\left(-\infty, -\frac{2}{5}\right)$ and $(0, \infty)$; decreasing on $\left(-\frac{2}{5}, 0\right)$. Relative maximum when $x=-\frac{2}{5}$; relative minimum when $x = 0$.

24. $f(x)=x^3(x-2)^4$

$$f'(x)=x^3[4(x-2)^3(1)]+(x-2)^4(3x^2)$$
$$=x^2(x-2)^3[4x+3(x-2)]$$
$$=x^2(x-2)^3(7x-6)$$

CV: $x = 0, 2, \frac{6}{7}$

Increasing on $(-\infty, 0)$, $\left(0, \frac{6}{7}\right)$, and $(2, \infty)$; decreasing on $\left(\frac{6}{7}, 2\right)$. Relative maximum when $x = \frac{6}{7}$; relative minimum when $x = 2$.

25. $y = 3x^5 + 20x^4 - 30x^3 - 540x^2 + 2x + 3$

$$y' = 15x^4 + 80x^3 - 90x^2 - 1080x + 2$$

$$\begin{aligned} y'' &= 60x^3 + 240x^2 - 180x - 1080 \\ &= 60(x^3 + 4x^2 - 3x - 18) \\ &= 60(x-2)(x+3)^2 \end{aligned}$$

Possible inflection points occur when $x = 2$ or $x = -3$. Concave down on $(-\infty, -3)$ and $(-3, 2)$; concave up on $(2, \infty)$. Concxavity changes at $x = 2$, so there is an inflection point when $x = 2$.

26. $y = \frac{x^2+2}{5x} = \frac{1}{5}x + \frac{2}{5}x^{-1}$

$$y' = \frac{1}{5}(1 - 2x^{-2})$$

$$y'' = \frac{4}{5}x^{-3} = \frac{4}{5x^3}$$

y'' is never zero. Although y'' is not defined when $x = 0$, y is not continuous there. Thus there is no inflection point.

27. $y = 4(3x-5)\left(x^4+2\right) = 12x^5 - 20x^4 + 24x - 40$

$$y' = 60x^4 - 80x^3 + 24$$

$$y'' = 240x^3 - 240x^2 = 240x^2(x-1)$$

Possible inflection points occur when $x = 0$ or $x = 1$. Concave down on $(-\infty, 0)$ and $(0, 1)$; concave up on $(1, \infty)$. Inflection point when $x = 1$.

28. $y = x^2 + 2\ln(-x)$ (Note: $x < 0$)

$$y' = 2x + \frac{2}{x}$$

$$y'' = 2 - \frac{2}{x^2} = \frac{2x^2-2}{x^2} = \frac{2(x+1)(x-1)}{x^2}$$

Possible inflection point occurs when $x = -1$. Concave up on $(-\infty, -1)$; concave down on $(-1, 0)$. Inflection point when $x = -1$.

29. $y = \frac{x^3}{e^x} = x^3e^{-x}$

$$y' = x^3(-e^{-x}) + e^{-x}(3x^2) = -e^{-x}(x^3 - 3x^2)$$

$$\begin{aligned} y'' &= -e^{-x}(3x^2 - 6x) - (x^3 - 3x^2)(-e^{-x}) \\ &= e^{-x}(x^3 - 6x^2 + 6x) \\ &= xe^{-x}(x^2 - 6x + 6) \end{aligned}$$

y'' is defined for all x and y'' is zero only when $x = 0$ or $x^2 - 6x + 6 = 0$. Using the quadratic formula on the second equation, the possible points of inflection occur when $x = 0, 3 \pm \sqrt{3}$. Concave up on $\left(0, 3-\sqrt{3}\right)$ and $\left(3+\sqrt{3}, \infty\right)$; concave down on $(-\infty, 0)$ and $\left(3-\sqrt{3}, 3+\sqrt{3}\right)$. Inflection points when $x = 0, 3 \pm \sqrt{3}$.

30. $y = (x^2-5)^3$

$$y' = 3(x^2-5)^2(2x) = 6x(x^2-5)^2$$

$$\begin{aligned} y'' &= 6(x^2-5)^2 + 6x(2)(x^2-5)(2x) \\ &= 6(x^2-5)(x^2-5+4x^2) \\ &= 6(x^2-5)(5x^2-5) \\ &= 30(x^2-5)(x^2-1) \\ &= 30\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)(x+1)(x-1) \end{aligned}$$

Possible inflection points occur when $x = \pm\sqrt{5}$ or $x = \pm 1$. Concave up on $\left(-\infty, -\sqrt{5}\right)$, $(-1, 1)$, and $\left(\sqrt{5}, \infty\right)$; concave down on $\left(-\sqrt{5}, -1\right)$ and $\left(1, \sqrt{5}\right)$. Inflection points when $x = \pm\sqrt{5}, \pm 1$.

31. $f(x) = 3x^4 - 4x^3$ and f is continuous on $[0, 2]$.

$$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$$

The only critical value on $(0, 2)$ is $x = 1$. Evaluating f at this value and at the endpoints gives $f(0) = 0$, $f(1) = -1$, and $f(2) = 16$. Absolute maximum: $f(2) = 16$; absolute minimum: $f(1) = -1$.

32. $f(x) = 2x^3 - 15x^2 + 36x$ and f is continuous on $[0, 3]$.

$$f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$$

The only critical value on $(0, 3)$ is $x = 2$. Evaluating f at this value and at the endpoints gives $f(0) = 0$, $f(2) = 28$, $f(3) = 27$. Absolute maximum: $f(2) = 28$; absolute minimum: $f(0) = 0$.

33. $f(x)=\dfrac{x}{(5x-6)^2}$ and *f* is continuous on [–2, 0].

$$f'(x)=\frac{(5x-6)^2(1)-x[10(5x-6)]}{(5x-6)^4}$$
$$=\frac{(5x-6)[(5x-6)-10x]}{(5x-6)^4}=\frac{-5x-6}{(5x-6)^3}$$
$$=-\frac{5x+6}{(5x-6)^3}$$

The only critical value on (–2, 0) is $x=-\frac{6}{5}$. Evaluating *f* at this value and at the endpoints gives $f(-2)=-\frac{1}{128}$, $f\left(-\frac{6}{5}\right)=-\frac{1}{120}$ and *f*(0) = 0. Absolute maximum: *f*(0) = 0; absolute minimum: $f\left(-\frac{6}{5}\right)=-\frac{1}{120}$.

34. $f(x)=(x+1)^2(x-1)^{2/3}$ and *f* is continuous on [2, 3].

$$f'(x)=(x+1)^2\left[\frac{2}{3}(x-1)^{-1/3}\right]+(x-1)^{2/3}[2(x+1)]$$
$$=\frac{2}{3}(x+1)(x-1)^{-1/3}[(x+1)+3(x-1)]$$
$$=\frac{4}{3}(x+1)(x-1)^{-1/3}(2x-1)=\frac{4(x+1)(2x-1)}{3(x-1)^{1/3}}$$

There are no critical values on [2, 3]. Evaluating *f* at the endpoints gives *f*(2) = 9 and $f(3)=16(2^{2/3})\approx 25.4$.
Absolute maximum $f(3)=16(2^{2/3})\approx 25.4$; absolute minimum: *f*(2) = 9

35. *f*(*x*) = *x* ln *x*

a. $f'(x)=\ln x+x\left(\frac{1}{x}\right)=\ln x+1$

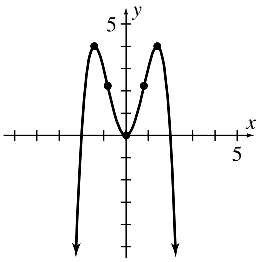
$$\ln x+1=0$$
$$\ln x=-1$$
$$x=e^{-1}=\frac{1}{e}$$

CV: $x=\frac{1}{e}$

Decreasing on $\left(0, \frac{1}{e}\right)$ and increasing on $\left(\frac{1}{e}, \infty\right)$. Relative minimum at $x=\frac{1}{e}$.

b. $f''(x)=\frac{1}{x}$

$f''>0$ for all *x* in the domain of *f*. Concave up for (0, ∞); there are no points of inflection.



56. $y = x^2e^x$

Intercept (0, 0)

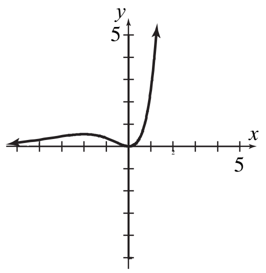
$y' = 2xe^x + x^2e^x = xe^x(2+x)$

CV: $x = 0, -2$

Increasing on (–∞, –2) and (0, ∞); decreasing on (–2, 0); relative maximum at $\left(-2, \dfrac{4}{e^2}\right)$; relative minimum at (0, 0)

$$y'' = 2e^x + 2xe^x + 2xe^x + x^2e^x$$
$$= e^x(2+4x+x^2)$$
$$x = \frac{-4 \pm \sqrt{16-4(1)(2)}}{2}$$
$$= \frac{-4 \pm \sqrt{8}}{2}$$
$$= \frac{-4 \pm 2\sqrt{2}}{2}$$
$$= -2 \pm \sqrt{2}$$

Possible inflection points when $x = -2 \pm \sqrt{2}$. Concave up on $\left(-\infty, -2-\sqrt{2}\right)$ and $\left(-2+\sqrt{2}, \infty\right)$; concave down on $\left(-2-\sqrt{2}, -2+\sqrt{2}\right)$; inflection points at $\left(-2-\sqrt{2}, \left(4\sqrt{2}+6\right)e^{-2-\sqrt{2}}\right)$ and $\left(-2+\sqrt{2}, \left(6-4\sqrt{2}\right)e^{-2+\sqrt{2}}\right)$



57. $y = x^{1/3}(x-8) = x^{4/3} - 8x^{1/3}$

Intercepts (0, 0) and (8, 0)

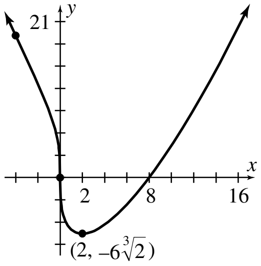
$$y' = \frac{4}{3}x^{1/3} - \frac{8}{3}x^{-2/3}$$
$$= \frac{4}{3}\left[x^{1/3} - \frac{2}{x^{2/3}}\right] = \frac{4(x-2)}{3x^{2/3}}$$

CV: $x = 0, 2$

Decreasing on (–∞, 0) and (0, 2); increasing on (2, ∞); relative minimum at $\left(2, -6\sqrt[3]{2}\right) \approx \left(2, -7.56\right)$.

$$y'' = \frac{4}{9}x^{-2/3} + \frac{16}{9}x^{-5/3}$$
$$= \frac{4}{9}\left[\frac{1}{x^{2/3}} + \frac{4}{x^{5/3}}\right] = \frac{4(x+4)}{9x^{5/3}}$$

Possible inflection points when $x = -4, 0$. Concave up on (–∞, –4) and (0, ∞); concave down on (–4, 0); inflection points at $\left(-4, 12\sqrt[3]{4}\right)$ and (0, 0). Observe that at the origin the tangent line exists but it is vertical.



58. $y = (x-1)^2(x+2)^2$

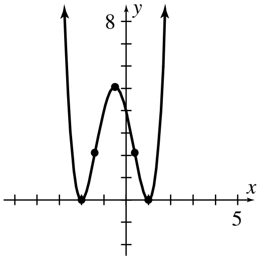
Intercepts (0, 4), (1, 0), (–2, 0)

$y' = (x-1)^2[2(x+2)] + (x+2)^2[2(x-1)]$

$= 2(x-1)(x+2)(2x+1)$

CV: $x = -2, -\dfrac{1}{2}, 1$

Decreasing on (–∞, –2) and $\left(-\dfrac{1}{2}, 1\right)$; increasing on $\left(-2, -\dfrac{1}{2}\right)$ and (1, ∞); relative maximum at $\left(-\dfrac{1}{2}, \dfrac{81}{16}\right)$; relative minima at (–2, 0) and (1, 0); $y' = 2(2x^3 + 3x^2 - 3x - 2)$, so $y'' = 6\left(2x^2 + 2x - 1\right)$. Setting $y'' = 0$ and using the quadratic formula gives possible inflection points at $x=\frac{-1\pm\sqrt{3}}{2}$. Concave up on $\left(-\infty, \frac{-1-\sqrt{3}}{2}\right)$ and $\left(\frac{-1+\sqrt{3}}{2}, \infty\right)$; concave down on $\left(\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right)$; inflection points when $x=\frac{-1\pm\sqrt{3}}{2}$



59. $y=4x^{1/3}+x^{4/3}=x^{1/3}(4+x)$

Intercepts (0, 0) and (–4, 0)

$$y'=\frac{4}{3}x^{-2/3}+\frac{4}{3}x^{1/3}=\frac{4}{3}\left[\frac{1}{x^{2/3}}+x^{1/3}\right]$$

$$=\frac{4(1+x)}{3x^{2/3}}$$

CV: $x = 0, -1$

Decreasing on (–∞, –1); increasing on (–1, 0) and (0, ∞); rel. min at (–1, –3)

$$y''=-\frac{8}{9}x^{-5/3}+\frac{4}{9}x^{-2/3}=\frac{4}{9}\left[\frac{1}{x^{2/3}}-\frac{2}{x^{5/3}}\right]$$

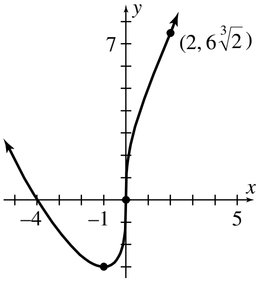
$$=\frac{4(x-2)}{9x^{5/3}}$$

Possible inflection points when $x = 0, 2$.

Concave up on (–∞, 0) and (2, ∞); concave down on

(0, 2); inflection point at (0, 0) and $\left(2, 6\sqrt[3]{2}\right)$.

Observe that at the origin the tangent line exists but it is vertical.



60. $y=(x+1)\sqrt{x+4}$ [Note: $x\ge -4$]

Intercepts (0, 2), (–1, 0) and (–4, 0)

$$y'=(x+1)\cdot\frac{1}{2\sqrt{x+4}}+\sqrt{x+4}(1)$$

$$=\frac{1}{2\sqrt{x+4}}[(x+1)+2(x+4)]$$

$$=\frac{3(x+3)}{2\sqrt{x+4}}$$

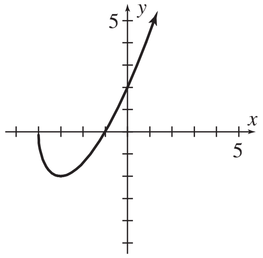
CV: $x = -3, -4$

Decreasing on (–4, –3); increasing on (–3, ∞); relative minimum at (–3, –2)

$$y''=\frac{3}{2}\cdot\frac{\sqrt{x+4}(1)-(x+3)\cdot\frac{1}{2\sqrt{x+4}}}{\left(\sqrt{x+4}\right)^2}$$

$$=\frac{3}{4}\cdot\frac{2(x+4)-(x+3)}{(x+4)^{3/2}}=\frac{3(x+5)}{4(x+4)^{3/2}}$$

No possible inflection point. Concave up on (–4, ∞).



61. $y=2x^{2/3}-x=x^{2/3}(2-x^{1/3})$

Intercepts (0, 0) and (8, 0)

$$y'=\frac{4}{3}x^{-1/3}-1$$

$$y'=0 \text{ when } x^{-1/3}=\frac{3}{4}$$

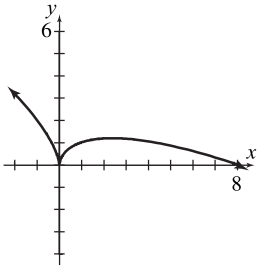
$$x=\left(\frac{3}{4}\right)^{-3}=\frac{64}{27}$$

CV: $0, \frac{64}{27}$

Increasing on $\left(0, \frac{64}{27}\right)$; decreasing on (–∞, 0) and $\left(\frac{64}{27}, \infty\right)$; relative maximum at $\left(\frac{64}{27}, \frac{32}{27}\right)$; relative minimum at (0, 0)

$$y''=-\frac{4}{9}x^{-4/3}=-\frac{4}{9x^{4/3}}$$

Possible inflection point at $x = 0$. Concave down on (–∞, 0) and (0, ∞); no inflection points; vertical tangent line at (0, 0). No symmetry.



62. $y = 5x^{2/3} - x^{5/3} = x^{2/3}(5 - x)$

Intercepts (0, 0) and (5, 0)

$$y' = \frac{10}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = \frac{5}{3}\left[\frac{2}{x^{1/3}} - x^{2/3}\right]$$

$$= \frac{5(2-x)}{3x^{1/3}}$$

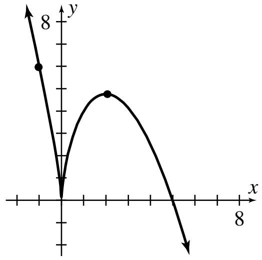
CV: $x = 0, 2$

Increasing on (0, 2); decreasing on $(-\infty, 0)$ and $(2, \infty)$; relative minimum at (0, 0); relative maximum at $\left(2, 3\sqrt[3]{4}\right) \approx (2, 4.76)$

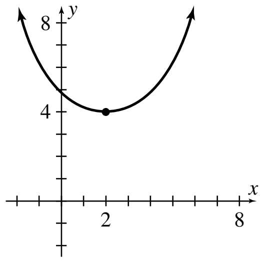
$$y'' = -\frac{10}{9}x^{-4/3} - \frac{10}{9}x^{-1/3} = -\frac{10(1+x)}{9x^{4/3}}$$

Possible inflection point when $x = 0, -1$.

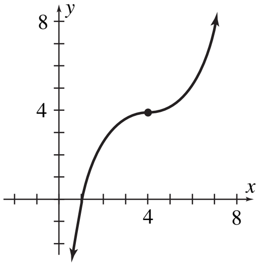
Concave up on $(-\infty, -1)$; concave down on $(-1, 0)$, and $(0, \infty)$; inflection point at $(-1, 6)$. Observe that at the origin the tangent line exists but it is vertical.



63.



64.



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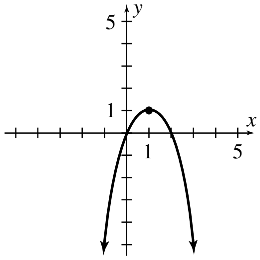
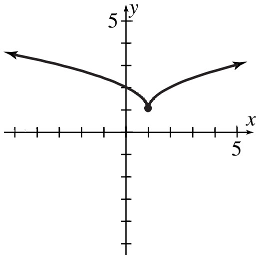


66.



67. $p = \frac{100}{q+2}$

$\frac{dp}{dq} = -\frac{100}{(q+2)^2} < 0$ for $q > 0$, so p is decreasing.

Since $\frac{d^2p}{dq^2} = \frac{200}{(q+2)^3} > 0$ for $q > 0$, the demand curve is concave up.

68. $c = q^2 + 2q + 1$

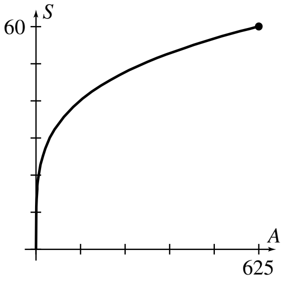
$$\bar{c} = \frac{c}{q} = q + 2 + \frac{1}{q}$$

$$\bar{c}' = 1 - \frac{1}{q^2}$$

$$\bar{c}'' = \frac{2}{q^3}$$

Since $\bar{c}'' > 0$ for $q > 0$, the graph of the average cost function is concave up for $q > 0$.

69. $S = f(A) = 12\sqrt[4]{A}$, $0 \le A \le 625$. For the given values of *A* we have $S' = 3A^{-\frac{3}{4}} > 0$ and $S'' = -\left(\frac{9}{4}\right)A^{-\frac{7}{4}} < 0$. Thus *y* is increasing and concave down.



70. $g(x) = e^{\frac{U_0}{A}}e^{-\frac{x^2}{2A}}$, *A* > 0, *x* ≥ 0 (since *x* represents quantity).

$$g'(x) = -\frac{e^{\frac{U_0}{A}}}{A}\left[xe^{-\frac{x^2}{2A}}\right]$$

$$g''(x) = -\frac{e^{\frac{U_0}{A}}}{A}\left[x\cdot e^{-\frac{x^2}{2A}}\left(-\frac{x}{A}\right)+e^{-\frac{x^2}{2A}}\right]$$

$$= \frac{e^{\frac{U_0}{A}}}{A^2}\cdot e^{-\frac{x^2}{2A}}\left(x^2-A\right)$$

$$= \frac{e^{\frac{U_0}{A}}}{A^2}\cdot e^{-\frac{x^2}{2A}}\left(x+\sqrt{A}\right)\left(x-\sqrt{A}\right)$$

If $0 \le x < \sqrt{A}$, then $g''(x) < 0$, so the graph is concave down. If $x > \sqrt{A}$, then $g''(x) > 0$, so the graph is concave up.

71. $y = 12.5 + 5.8(0.42)^x$

$y' = 5.8(0.42)^x \ln(0.42)$

Since ln(0.42) < 0, we have $y' < 0$, so the function is decreasing.

$y'' = 5.8(0.42)^x \ln^2(0.42) > 0$, so the function is concave up.

72. $H = 1.00\left[1-e^{-(0.0464t+0.0670}\right]$

$\frac{dH}{dt} = 0.0464e^{-(0.0464t+0.0670)} > 0$, so *H* is increasing.

$\frac{d^2H}{dt^2} = -(0.0464)^2 e^{-(0.0464t+0.0670)} < 0$, so *H* is concave down.

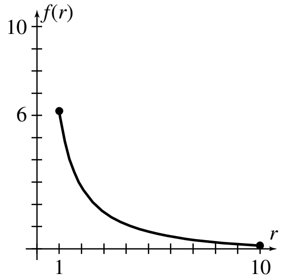
73. $n = f(r) = 0.1\ln(r)+\frac{7}{r}-0.8$, $1 \le r \le 10$

a. $\frac{dn}{dr} = \frac{0.1}{r}-\frac{7}{r^2} = \frac{0.1r-7}{r^2} = \frac{0.1(r-70)}{r^2} < 0$ for $1 \le r \le 10$. Thus the graph of *f* is always falling. Also,

$$\frac{d^2n}{dr^2} = -\frac{0.1}{r^2}+\frac{14}{r^3} = \frac{14-0.1r}{r^3}$$
$$= \frac{0.1(140-r)}{r^3} > 0$$

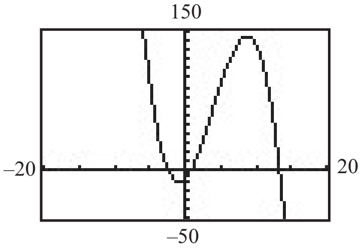
for $1 \le r \le 10$. Thus the graph is concave up.

b.



c. $\left.\frac{dn}{dr}\right|_{r=5} = -0.26$, so the rate of decrease is 0.26.

74.

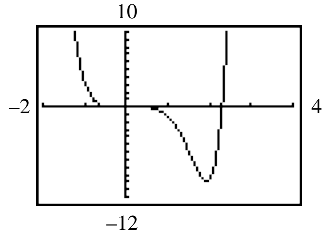


a. One relative maximum point

b. One relative minimum point

c. One inflection point

75.



Two inflection points

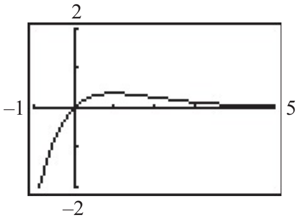
$y = x^5(x-a) = x^6 - ax^5$

$y' = 6x^5 - 5ax^4$

$y'' = 30x^4 - 20ax^3 = 10x^3(3x-2a)$

Possible inflection points when $x = 0$ and $x = \frac{2a}{3}$. If $a > 0$, y is concave up on $(-\infty, 0)$ and $\left(\frac{2a}{3}, \infty\right)$; concave down on $\left(0, \frac{2a}{3}\right)$. If $a < 0$, y is concave up on $\left(-\infty, \frac{2a}{3}\right)$ and $(0, \infty)$; concave downon $\left(\frac{2a}{3}, 0\right)$. In either case, y has two points of inflection, when $x = 0$ and $x = \frac{2a}{3}$.

76.



$y = xe^{-x}$

Intercept (0, 0)

$y' = e^{-x} - xe^{-x} = e^{-x}(1-x)$

CV: $x = 1$

Increasing on $(-\infty, 1)$; decreasing on $(1, \infty)$; relative maximum at $(1, e^{-1})$

$$y'' = -e^{-x} - e^{-x} + xe^{-x}$$
$$= -2e^{-x} + xe^{-x}$$
$$= e^{-x}(x-2)$$

Possible inflection point at $x = 2$.

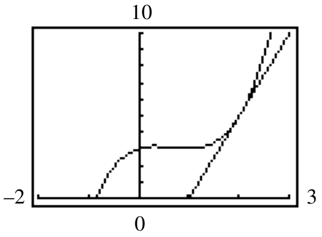
Concave down on $(-\infty, 2)$; concave up on $(2, \infty)$; inflection point at $(2, 2e^{-2})$

Answers will vary for $q(p) = Qe^{-Rp}$.

77. $y = x^3 - 2x^2 + x + 3$

$y' = 3x^2 - 4x + 1$

When $x = 2$, then $y = 5$ and $y' = 5$. Thus an equation of the tangent line at $x = 2$ is $y - 5 = 5(x - 2)$, or $y = 5x - 5$. Graphing the curve and the tangent line indicates that the curve lies above the tangent line around $x = 2$. Thus the curve is concave up at $x = 2$.



78. $f(x) = 2x^3 + 3x^2 - 6x + 1$

$f'(x) = 6x^2 + 6x - 6$

$f''(x) = 12x + 6$

The relative minimum of f' occurs at a value of x for which $(f'(x))' = f''(x) = 0$. Around this value of x, $(f'(x))'$ goes from – to +. Since $(f'(x))' = f''(x)$, the concavity of f must change from concave down to concave up.

79. $f(x) = x^6 + 3x^5 - 4x^4 + 2x^2 + 1$

$f'(x) = 6x^5 + 15x^4 - 16x^3 + 4x$

$f''(x) = 30x^4 + 60x^3 - 48x^2 + 4$

Inflection points of f when $x \approx -2.61, -0.26$.

80. $f(x) = \frac{x+1}{x^2+1}$

$$f'(x) = -\frac{x^2+2x-1}{\left(x^2+1\right)^2}$$

$$f''(x) = \frac{2\left(x^3+3x^2-3x-1\right)}{\left(x^2+1\right)^3}$$

Inflection points of f when
$x \approx -3.73, -0.27, 1.00$.

Problems 13.4

1. $y = x^2 - 5x + 6$

$y' = 2x - 5$

CV: $x = \frac{5}{2}$

$y'' = 2$

$y''\left(\frac{5}{2}\right) = 2 > 0$

Thus there is a relative minimum when $x = \frac{5}{2}$.

Because there is only one relative extremum and f is continuous, the relative minimum is an absolute minimum.

2. $y = 3x^2 + 12x + 14$

$y' = 6x + 12$

CV: $x = -2$

$y'' = 6$

$y''(-2) = 6 > 0$

Thus there is a relative minimum when $x = -2$. Because there is only one relative extremum and *f* is continuous, the relative minimum is an absolute minimum.

3. $y = -4x^2 + 2x - 8$

$y' = -8x + 2$

CV: $x = \frac{1}{4}$

$y'' = -8$

$y''\left(\frac{1}{4}\right) = -8 < 0$

Thus there is a relative maximum when $x = \frac{1}{4}$.

Because there is only one relative extremum and *f* is continuous, the relative maximum is an absolute maximum.

4. $y = 3x^2 - 5x + 6$

$y' = 6x - 5$

CV: $x = \frac{5}{6}$

$y'' = 6$

$y''\left(\frac{5}{6}\right) = 6 > 0$

Thus there is a relative minimum when $x = \frac{5}{6}$.

Because there is only one relative extremum and *f* is continuous, the relative minimum is an absolute minimum.

5. $y = \frac{1}{3}x^3 + 2x^2 - 5x + 1$

$y' = x^2 + 4x - 5 = (x+5)(x-1)$

CV: $x = -5, 1$

$y'' = 2x + 4$

$y''(-5) = -6 < 0 \Rightarrow$ relative maximum when $x = -5$

$y''(1) = 6 > 0 \Rightarrow$ relative minimum when $x = 1$

6. $y = x^3 - 12x + 1$

$y' = 3x^2 - 12 = 3(x+2)(x-2)$

CV: $x = \pm 2$

$y'' = 6x$

$y''(-2) = -12 < 0 \Rightarrow$ relative maximum when $x = -2$

$y''(2) = 12 > 0 \Rightarrow$ relative minimum when $x = 2$

7. $y = 2x^3 - 3x^2 - 36x + 17$

$y' = 6x^2 - 6x - 36 = 6(x-3)(x+2)$

CV: $x = 3, -2$

$y'' = 12x - 6$

$y''(3) = 30 > 0 \Rightarrow$ relative minimum when $x = 3$

$y''(-2) = -30 < 0 \Rightarrow$ relative maximum when $x = -2$

8. $y = x^4 - 2x^2 + 4$

$y' = 4x^3 - 4x = 4x(x+1)(x-1)$

CV: $x = 0, \pm 1$

$y'' = 12x^2 - 4$

$y''(0) = -4 < 0 \Rightarrow$ relative maximum when $x = 0$

$y''(1) = 8 > 0 \Rightarrow$ relative minimum when $x = 1$

$y''(-1) = 8 > 0 \Rightarrow$ relative minimum when $x = -1$

9. $y = 7 - 2x^4$

$y' = -8x^3$

CV: $x = 0$

$y'' = -24x^2$

Since $y''(0) = 0$, the second-derivative test fails. Using the first-derivative test, we see that *f* increases for $x < 0$ and *f* decreases for $x > 0$, so there is a relative maximum when $x = 0$.

10. $y = -2x^7$

$y' = -14x^6$

CV: $x = 0$

$y'' = -84x^5$

Since $y''(0) = 0$, the second-derivative test fails. However, using the first-derivative test, we see that *f* decreases for $x < 0$ and for $x > 0$, so there is neither a relative maximum nor a relative minimum when $x = 0$.

11. $y = 81x^5 - 5x$

$y' = 81 \cdot 5x^4 - 5 = 5\left(81x^4 - 1\right)$

$= 5\left(9x^2 - 1\right)\left(9x^2 + 1\right)$

$= 5(3x+1)(3x-1)\left(9x^2 + 1\right)$

CV: $x = \pm\frac{1}{3}$

$y'' = 81 \cdot 5 \cdot 4x^3$

$y''\left(-\frac{1}{3}\right) = -60 < 0 \Rightarrow$ relative maximum when $x = -\frac{1}{3}$

$y''\left(\frac{1}{3}\right) = 60 > 0 \Rightarrow$ relative minimum when $x = \frac{1}{3}$

12. $y = 15x^3 + x^2 - 15x + 2$

$y' = 45x^2 + 2x - 15 = (5x+3)(9x-5)$

CV: $x = -\frac{3}{5}, \frac{5}{9}$

$y'' = 90x + 2$

$y''\left(-\frac{3}{5}\right) = -52 \Rightarrow$ relative maximum when $x = -\frac{3}{5}$

$y''\left(\frac{5}{9}\right) = 52 \Rightarrow$ relative minimum when $x = \frac{5}{9}$

13. $y = \left(x^2 + 7x + 10\right)^2$

$y' = 2\left(x^2 + 7x + 10\right)(2x+7)$

$= 2(x+2)(x+5)(2x+7)$

CV: $x = -2, -5, -\frac{7}{2}$

$y'' = 2\left[\left(x^2 + 7x + 10\right)(2) + (2x+7)(2x+7)\right]$

$y''(-5) = 18 > 0 \Rightarrow$ relative minimum when $x = -5$

$y''\left(-\frac{7}{2}\right) = -9 < 0 \Rightarrow$ relative maximum when $x = -\frac{7}{2}$

$y''(-2) = 18 > 0 \Rightarrow$ relative minimum when $x = -2$

14. $y = -x^3 + 3x^2 + 9x - 2$

$y' = -3x^2 + 6x + 9 = -3(x^2 - 2x - 3)$
$= -3(x+1)(x-3)$

CV: $x = -1, 3$
$y'' = -6x + 6$

$y''(-1) = 12 > 0 \Rightarrow$ relative minimum when $x = -1$
$y''(3) = -12 < 0 \Rightarrow$ relative maximum when $x = 3$

Problems 13.5

1. $y = f(x) = \frac{x}{x-1}$

When $x = 1$ the denominator is zero but the numerator is not zero. Thus $x = 1$ is a vertical asymptote.

$\lim_{x\to\infty} \frac{x}{x-1} = \lim_{x\to\infty} \frac{x}{x} = \lim_{x\to\infty} 1 = 1$.

Similarly $\lim_{x\to-\infty} f(x) = 1$. Thus the line $y = 1$ is a horizontal asymptote.

2. $y = f(x) = \frac{x+1}{x}$

When $x = 0$ the denominator is zero but the numerator is not. Thus $x = 0$ is a vertical asymptote. $\lim_{x\to\infty} \frac{x+1}{x} = \lim_{x\to\infty} \frac{x}{x} = \lim_{x\to\infty} 1 = 1$.

Similarly $\lim_{x\to-\infty} f(x) = 1$. Thus $y = 1$ is a horizontal asymptote.

3. $f(x) = \frac{x+5}{2x+7}$

When $x = -\frac{7}{2}$ the denominator is zero but the numerator is not. Thus $x = -\frac{7}{2}$ is a vertical asymptote. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{2x} = \lim_{x\to\infty} \frac{1}{2} = \frac{1}{2}$.

Similarly $\lim_{x\to-\infty} f(x) = \frac{1}{2}$. Thus $y = \frac{1}{2}$ is a horizontal asymptote.

4. $y = f(x) = \dfrac{2x+1}{2x+1}$

Observe that both the numerator and denominator are zero for $x = -\dfrac{1}{2}$. For $x \neq -\dfrac{1}{2}$, we have *f*(*x*) = 1. Thus *f* is a constant function for $x \neq -\dfrac{1}{2}$. Hence there are no vertical or horizontal asymptotes.

5. $y = f(x) = \dfrac{4}{x}$

When *x* = 0 the denominator is zero but the numerator is not zero, so *x* = 0 is a vertical asymptote.

$\lim\limits_{x\to\infty}\left(\dfrac{4}{x}\right) = 0.$ Similarly, $\lim\limits_{x\to-\infty}\left(\dfrac{4}{x}\right) = 0,$ so *y* = 0 is a horizontal asymptote.

6. $y = f(x) = 1 - \dfrac{2}{x^2} = \dfrac{x^2-2}{x^2}$

When *x* = 0 the denominator is zero but the numerator is not. Thus *x* = 0 is a vertical asymptote. $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{x^2}\right) = 1 - 0 = 1.$ Similarly $\lim\limits_{x\to-\infty} f(x) = 1,$ so *y* = 1 is a horizontal asymptote.

7. $y = f(x) = \dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$

Vertical asymptotes are *x* = 1 and *x* = –1.

$\lim\limits_{x\to\infty}\dfrac{1}{x^2-1} = \lim\limits_{x\to\infty}\dfrac{1}{x^2} = 0$. Similarly, $\lim\limits_{x\to-\infty} f(x) = 0$. Thus *y* = 0 is a horizontal asymptote.

8. $y = f(x) = \dfrac{x}{x^2-9} = \dfrac{x}{(x-3)(x+3)}$

Vertical asymptotes: *x* = 3, *x* = –3.

$\lim\limits_{x\to\infty}\dfrac{x}{x^2-9} = \lim\limits_{x\to\infty}\dfrac{x}{x^2} = \lim\limits_{x\to\infty}\dfrac{1}{x} = 0$. Similarly, $\lim\limits_{x\to-\infty} f(x) = 0$. Thus *y* = 0 is a horizontal asymptote.

9. $y = f(x) = x^2 - 5x + 5$ is a polynomial function, so there are no horizontal or vertical asymptotes.

10. $y = f(x) = \dfrac{x^4}{x^3-4} = \dfrac{x^4}{x^3-\left(\sqrt[3]{4}\right)^3} = \dfrac{x^4}{x^3-(2^{2/3})^3}$

$$= \frac{x^4}{(x-2^{2/3})(x^2+2^{2/3}x+2^{4/3})}$$

Vertical asymptote: $x = 2^{2/3}$.

$\dfrac{x^4}{x^3-4} = x + \dfrac{4x}{x^3-4}$ so the line *y* = *x* is an oblique asymptote.

11. $f(x) = \dfrac{2x^2}{x^2+x-6} = \dfrac{2x^2}{(x+3)(x-2)}$

Vertical asymptotes are *x* = –3 and *x* = 2.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{2x^2}{x^2} = \lim\limits_{x\to\infty} 2 = 2$, and $\lim\limits_{x\to-\infty} f(x) = 2$. Thus *y* = 2 is a horizontal asymptote.

12. $f(x) = \dfrac{x^3}{5}$ is a polynomial function, so there are no horizontal or vertical asymptotes.

13. $y = \dfrac{15x^2+31+1}{x^2-7} = \dfrac{15x^2+31x+1}{\left(x+\sqrt{7}\right)\left(x-\sqrt{7}\right)}$

Vertical asymptotes are $x = -\sqrt{7}$ and $x = \sqrt{7}$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{15x^2}{x^2} = \lim\limits_{x\to\infty} 15 = 15$

Similarly, $\lim\limits_{x\to-\infty} = 15.$ Thus *y* = 15 is a horizontal asymptote.

14. $y = f(x) = \dfrac{2x^3+1}{3x(2x-1)(4x-3)}$

Vertical asymptotes are $x = 0,\ x = \dfrac{1}{2}$, and $x = \dfrac{3}{4}$. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\dfrac{2x^3}{24x^3} = \lim\limits_{x\to\infty}\dfrac{1}{12} = \dfrac{1}{12}$.

Similarly, $\lim\limits_{x\to-\infty} f(x) = \dfrac{1}{12}$. Thus $y = \dfrac{1}{12}$ is a horizontal asymptote.

15. $y = f(x) = \frac{2}{x-3} + 5 = \frac{5x-13}{x-3}$

From the denominator, $x = 3$ is a vertical asymptote.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{5x}{x} = \lim\limits_{x\to\infty} 5 = 5,$ and

$\lim\limits_{x\to-\infty} f(x) = 5.$ Thus, $y = 5$ is a horizontal asymptote.

16. $f(x) = \frac{x^2-1}{2x^2-9x+4} = \frac{x^2-1}{(2x-1)(x-4)}$

Vertical asymptotes are $x = \frac{1}{2}$ and $x = 4$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^2}{2x^2} = \lim\limits_{x\to\infty} \frac{1}{2} = \frac{1}{2}$,and

$\lim\limits_{x\to-\infty} f(x) = \frac{1}{2}$. Thus $y = \frac{1}{2}$ is a horizontal asymptote.

17. $f(x) = \frac{3-x^4}{x^3+x^2} = \frac{3-x^4}{x^2(x+1)}$

Vertical asymptotes are $x = 0$ and $x = -1$.

$\frac{3-x^4}{x^3+x^2} = -x+1+\frac{3-x^2}{x^3+x^2}$ so the line $y = -x + 1$ is an oblique asyptote.

18. $y = f(x) = \frac{5x^2+7x^3+9x^4}{3x^2}$

Observe that both the numerator and the denominator are zero when $x = 0$. For $x \neq 0$, we have

$f(x) = \frac{x^2}{3x^2}(5+7x+9x^2) = \frac{1}{3}(5+7x+9x^2).$

Thus *f* is a polynomial function for $x \neq 0$. Hence there are neither horizontal nor vertical asymptotes.

19. $y = f(x) = \frac{x^2-3x-4}{1+4x+4x^2} = \frac{x^2-3x-4}{(1+2x)^2}$

From the denominator, $x = -\frac{1}{2}$ is a vertical asymptote.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^2}{4x^2} = \lim\limits_{x\to\infty} \frac{1}{4} = \frac{1}{4}$, and

$\lim\limits_{x\to-\infty} f(x) = \frac{1}{4}$, so $y = \frac{1}{4}$ is a horizontal asymptote.

20. $y = f(x) = \frac{x^4+1}{1-x^4} = \frac{x^4+1}{\left(1+x^2\right)(1-x)(1+x)}$

From the denominator, vertical asymptotes are $x = 1$ and $x = -1$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^4}{-x^4} = \lim\limits_{x\to\infty} -1 = -1$, and

$\lim\limits_{x\to-\infty} f(x) = -1$. Thus $y = -1$ is a horizontal asymptote.

21. $y = f(x) = \frac{9x^2-16}{2(3x+4)^2} = \frac{(3x+4)(3x-4)}{2(3x+4)^2}$

When $x = -\frac{4}{3}$, both the numerator and denominator are zero. Since

$\lim\limits_{x\to-4/3^+} f(x) = \lim\limits_{x\to-4/3^+} \frac{3x-4}{2(3x+4)} = -\infty$, the

line $x = -\frac{4}{3}$ is a vertical asymptote.

$\lim\limits_{x\to\infty} \frac{9x^2-16}{2(3x+4)^2} = \lim\limits_{x\to\infty} \frac{9x^2}{18x^2} = \lim\limits_{x\to\infty} \frac{1}{2} = \frac{1}{2}$.

Similarly, $\lim\limits_{x\to-\infty} f(x) = \frac{1}{2}$. Thus $y = \frac{1}{2}$ is a horizontal asymptote.

22. $y = f(x) = \frac{2}{5} + \frac{2x}{12x^2+5x-2} = \frac{24x^2+20x-4}{5(12x^2+5x-2)}$

$= \frac{4(x+1)(6x-1)}{5(3x+2)(4x-1)}$

When $x = -\frac{2}{3}$ or $x = \frac{1}{4}$, the denominator is 0, but the numerator is not. Thus, vertical asymptotes are $x = -\frac{2}{3}$ and

$x = \frac{1}{4}$. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{24x^2}{60x^2} = \lim\limits_{x\to\infty} \frac{2}{5} = \frac{2}{5}$.

Similarly, $\lim\limits_{x\to-\infty} f(x) = \frac{2}{5}$. Thus, $y = \frac{2}{5}$ is a horizontal asymptote.

23. $y = f(x) = 5e^{x-3} - 2$

We have $\lim\limits_{x\to\infty} f(x) = +\infty$ and

$\lim\limits_{x\to-\infty} f(x) = 5 \cdot \lim\limits_{x\to-\infty} e^{x-3} - \lim\limits_{x\to-\infty} 2$

$= 5(0) - 2 = -2$

Thus $y = -2$ is a horizontal asymptote. There is no vertical asymptote because $f(x)$ neither increases nor decreases without bound around any fixed value of x.

24. $f(x) = 12e^{-x}$

$\lim\limits_{x\to\infty} f(x) = 0$ and $\lim\limits_{x\to-\infty} f(x) = +\infty$. Thus $y = 0$ is a horizontal asymptote. There is no vertical asymptote because $f(x)$ neither increases nor decreases without bound around any fixed value of x.

25. $y = \dfrac{3}{x}$

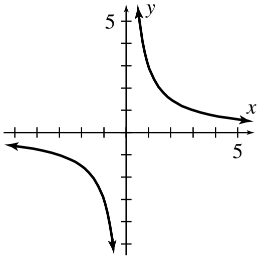
Symmetric about the origin. Vertical asymptote is $x = 0$. $\lim\limits_{x\to\infty} \dfrac{3}{x} = 0 = \lim\limits_{x\to-\infty} \dfrac{3}{x}$, so $y = 0$ is a horizontal asymptote.

$y' = -\dfrac{3}{x^2}$

CV: None, however $x = 0$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, 0)$ and $(0, \infty)$.

$y'' = \dfrac{6}{x^3}$

No possible inflection point, but we include $x = 0$ in the concavity analysis. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.



26. $y = \dfrac{2}{2x-3}$

Intercept: $\left(0, -\dfrac{2}{3}\right)$

Vertical asymptote is $x = \dfrac{3}{2}$.

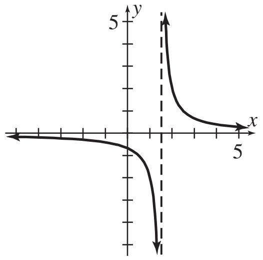
$\lim\limits_{x\to\infty} y = 0 = \lim\limits_{x\to-\infty} y$, so $y = 0$ is a horizontal asymptote.

$y' = -\dfrac{4}{(2x-3)^2}$

CV: None, but $x = \dfrac{3}{2}$ must be considered in the inc. dec. analysis. Decreasing on $\left(-\infty, \dfrac{3}{2}\right)$ and $\left(\dfrac{3}{2}, \infty\right)$.

$y'' = \dfrac{16}{(2x-3)^3}$

No possible inflection point, but $x = \dfrac{3}{2}$ must be considered in the concavity analysis. Concave down on $\left(-\infty, \dfrac{3}{2}\right)$; concave up on $\left(\dfrac{3}{2}, \infty\right)$.



27. $y = \dfrac{x}{x-1}$

Intercept $(0, 0)$

Vertical asymptote is $x = 1$

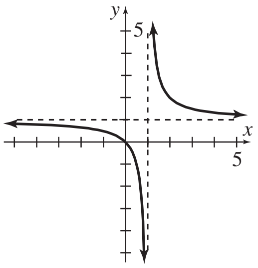
$\lim\limits_{x\to\infty} y = 1 = \lim\limits_{x\to-\infty} y$, so $y = 1$ is a horizontal asymptote.

$y' = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} = -\dfrac{1}{(x-1)^2}$

CV: None, but $x = 1$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, 1)$ and $(1, \infty)$.

$y'' = \dfrac{2}{(x-1)^3}$

No possible inflection point, but $x = 1$ must be included in concavity analysis. Concave up on $(1, \infty)$, concave down on $(-\infty, 1)$.



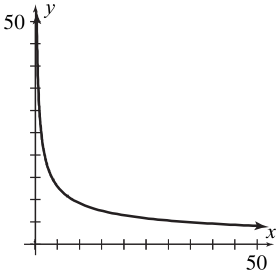
28. $y = \frac{50}{\sqrt{3x}}$ (Note: $x > 0$)

$\lim\limits_{x\to\infty} y = 0$, so $y = 0$ is a horizontal asymptote.

$\lim\limits_{x\to 0^+} y = +\infty$, so the line $x = 0$ is a vertical asymptote.

$y' = -\frac{25}{\sqrt{3x^3}} < 0$ for $x > 0$. Decreasing on $(0, \infty)$.

$y'' = \frac{75}{2\sqrt{3x^5}} > 0$ for $x > 0$. Concave up on $(0, \infty)$. No intercepts; no symmetry.



29. $y = x^2 + \frac{1}{x^2} = \frac{x^4+1}{x^2}$

$x \neq 0$, so there is no y-intercept. Setting $y = 0 \Rightarrow$ no x-intercept. Replacing x by $-x$ yields symmetry about the y-axis. Setting $x^2 = 0$ gives $x = 0$ as the only vertical asymptote. Because the degree of the numerator is greater than the degree of the denominator, no horizontal asymptote exists.

$y = x^2 + x^{-2}$

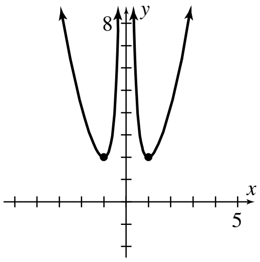
$$y' = 2x - 2x^{-3} = 2x - \frac{2}{x^3} = \frac{2x^4-2}{x^3} = \frac{2\left(x^4-1\right)}{x^3}$$

$$= \frac{2\left(x^2+1\right)(x+1)(x-1)}{x^3}.$$

CV: $x = \pm 1$, but $x = 0$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, -1)$ and $(0, 1)$; increasing on $(-1, 0)$ and $(1, \infty)$; relative minima at $(-1, 2)$ and $(1, 2)$,

$$y'' = 2 + \frac{6}{x^4} > 0$$

for all $x \neq 0$. Concave up on $(-\infty, 0)$ and $(0, \infty)$.



30. $y = \frac{3x^2-5x-1}{x-2}$

Intercept: $\left(0, \frac{1}{2}\right)$

Vertical asymptote is $x = 2$.

$\frac{3x^2-5x-1}{x-2} = 3x+1+\frac{1}{x-2}$ so $y = 3x + 1$ is an oblique asymptote.

$$y' = \frac{(x-2)(6x-5)-(3x^2-5x-1)(1)}{(x-2)^2}$$

$$= \frac{3x^2-12x+11}{(x-2)^2}$$

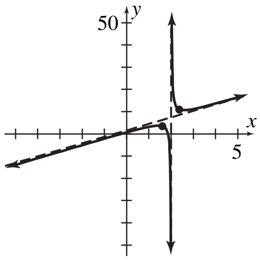
From the quadratic formula, CV: $x = \frac{6\pm\sqrt{3}}{3}$, but $x = 2$ must be included in the inc.-dec. analysis. Increasing on $\left(-\infty, \frac{6-\sqrt{3}}{3}\right)$ and $\left(\frac{6+\sqrt{3}}{3}, \infty\right)$; decreasing on $\left(\frac{6-\sqrt{3}}{3}, 2\right)$ and $\left(2, \frac{6+\sqrt{3}}{3}\right)$; relative maximum at $\left(\frac{6-\sqrt{3}}{3}, 7-2\sqrt{3}\right)$; relative minimum at $\left(\frac{6+\sqrt{3}}{3}, 7+2\sqrt{3}\right)$.

$$y'' = \frac{(x-2)^2(6x-12)-(3x^2-12x+11)2(x-2)}{(x-2)^4}$$

$$= \frac{(x-2)(6x-12)-2(3x^2-12x+11)}{(x-2)^3}$$

$$= \frac{2}{(x-2)^3}$$

No possible inflection point, but $x = 2$ must be included in the concavity analysis. Concave down on $(-\infty, 2)$; concave up on $(2, \infty)$



31. $y = \frac{1}{x^2 - 1} = \frac{1}{(x+1)(x-1)}$

Intercept (0, –1)
Symmetric about the y-axis.
Vertical asymptotes are $x = -1$ and $x = 1$.

$\lim\limits_{x\to\infty} \frac{1}{x^2-1} = 0 = \lim\limits_{x\to-\infty} \frac{1}{x^2-1}$, so $y = 0$ is a horizontal asymptote.

$$y' = -\frac{2x}{\left(x^2-1\right)^2}$$

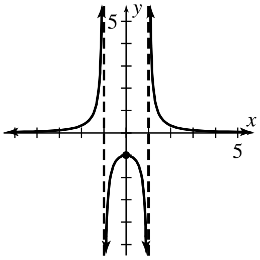
CV: $x = 0$, but $x = \pm 1$ must be included in the inc.-dec. analysis. Increasing on (–∞, –1) and (–1, 0); decreasing on (0, 1) and (1, ∞); relative maximum at (0, –1).

$$y'' = -2 \cdot \frac{\left(x^2-1\right)^2 (1) - x\left[4x\left(x^2-1\right)\right]}{\left(x^2-1\right)^4}$$

$$= -2 \cdot \frac{\left(x^2-1\right)\left[\left(x^2-1\right) - 4x^2\right]}{\left(x^2-1\right)^4}$$

$$= \frac{2\left(3x^2+1\right)}{\left(x^2-1\right)^3} = \frac{2\left(3x^2+1\right)}{[(x+1)(x-1)]^3}$$

No possible inflection point, but $x = \pm 1$ must be considered in the concavity analysis. Concave up on (–∞, –1) and (1, ∞); concave down on (–1, 1).



32. $y = \frac{1}{x^2+1}$

Intercept (0, 1)
Symmetric about the y-axis.

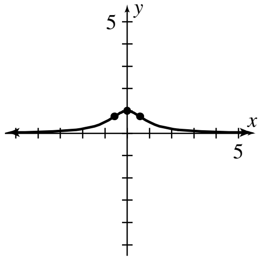
$\lim\limits_{x\to\infty} \frac{1}{x^2+1} = 0 = \lim\limits_{x\to-\infty} \frac{1}{x^2+1}$, so $y = 0$ is a horizontal asymptote.

$$y' = \frac{-2x}{\left(x^2+1\right)^2}$$

CV: $x = 0$
Increasing on (–∞, 0); decreasing on (0, ∞); relative maximum at (0, 1)

$$y'' = \frac{2\left(3x^2-1\right)}{\left(x^2+1\right)^3}$$

Possible inflection points at $x = \pm\frac{1}{\sqrt{3}}$. Concave up on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$; concave down on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$; inflection points at $\left(\pm\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$



33. $y = \frac{2+x}{3-x}$

Intercepts: $\left(0, \frac{2}{3}\right)$ and (–2, 0).

$x = \frac{2}{3}$ is the only vertical asymptote. Since

$$\lim_{x\to\infty} \frac{2+x}{3-x} = \lim_{x\to\infty} \frac{x}{-x} = \lim_{x\to\infty} -1 = -1$$
$$= \lim_{x\to-\infty} \frac{2+x}{3-x}$$

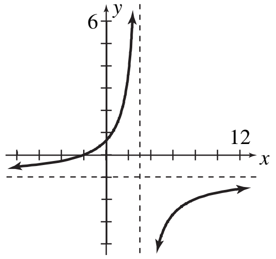
the only horizontal asymptote is $y = -1$.

$$y' = \frac{(3-x)(1)-(2+x)(-1)}{(3-x)^2} = \frac{5}{(3-x)^2}$$

No critical values, but $x = 3$ must be considered in the ind.-dec. analysis. Increasing on $(-\infty, 3)$ and $(3, \infty)$.

$$y'' = \frac{10}{(3-x)^3}$$

No possible inflection point, but $x = 3$ must be included in the concavity analysis. Concave up on $(-\infty, 3)$; concave down on $(3, \infty)$.



34. $y = \dfrac{1+x}{x^2}$

Intercept is $(-1, 0)$

Vertical asymptote is $x = 0$.

$$\lim_{x\to\infty} \frac{1+x}{x^2} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$$

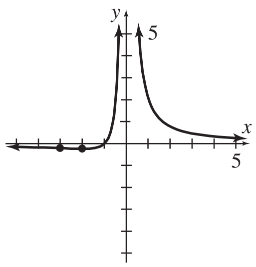
$= \displaystyle\lim_{x\to-\infty} \frac{1+x}{x^2}$, so $y = 0$ is the only horizontal asymptote.

$$y' = -\frac{x+2}{x^3}$$

CV: $x = -2$, but $x = 0$ must be included in the inc-dec. analysis. Increasing on $(-2, 0)$; decreasing on $(-\infty, -2)$ and $(0, \infty)$; relative minimum at $\left(-2, -\dfrac{1}{4}\right)$.

$$y'' = \frac{2(3+x)}{x^4}$$

Possible inflection point when $x = 3$, but $x = 0$ must be included in the concavity analysis. Concave up on $(-3, 0)$ and $(0, \infty)$; concave down on $(-\infty, -3)$; inflection point at $\left(-3, -\dfrac{2}{9}\right)$.



35. $y = \dfrac{x^2}{7x+4}$

Intercept: $(0, 0)$

Vertical asymptote is $x = -\dfrac{4}{7}$.

$\dfrac{x^2}{7x+4} = \dfrac{1}{7}x - \dfrac{4}{49} + \dfrac{16}{49(7x+4)}$ so $y = \dfrac{1}{7}x - \dfrac{4}{49}$ is an oblique asymptote.

$$y' = \frac{(7x+4)(2x) - x^2(7)}{(7x+4)^2}$$

$$= \frac{7x^2 + 8x}{(7x+4)^2} = \frac{x(7x+8)}{(7x+4)^2}$$

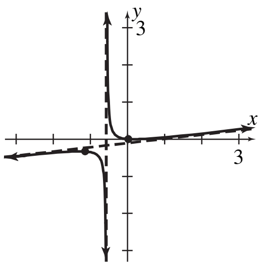
CV: $x = 0$, $-\dfrac{8}{7}$, but $x = -\dfrac{4}{7}$ must be included in the inc.-dec. analysis. Increasing on $\left(-\infty, -\dfrac{8}{7}\right)$ and $(0, \infty)$; decreasing on $\left(-\dfrac{8}{7}, -\dfrac{4}{7}\right)$ and $\left(-\dfrac{4}{7}, 0\right)$; relative maximum at $\left(-\dfrac{8}{7}, -\dfrac{16}{49}\right)$; relative minimum at $(0, 0)$.

$$y'' = \frac{\left(7x^2+4\right)^2 (14x+8) - \left(7x^2+8x\right)[14(7x+4)]}{(7x+4)^4}$$

$$= \frac{(7x+4)\left[(7x+4)(14x+8) - 14\left(7x^2+8x\right)\right]}{(7x+4)^4}$$

$$= \frac{32}{(7x+4)^3}$$

No possible inflection point but $x = -\dfrac{4}{7}$ must be included in concavity analysis. Concave down on $\left(-\infty, -\dfrac{4}{7}\right)$; concave up on $\left(-\dfrac{4}{7}, \infty\right)$.



36. $y=\frac{x^3+1}{x}$

Intercept: (–1, 0)

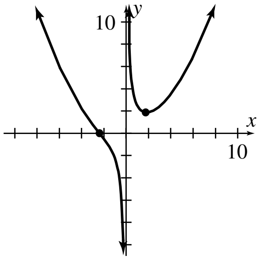
Vertical asymptote is $x = 0$. Because the degree of the numerator is greater than the degree of the denominator, no horizontal asymptote exists. Since $y=x^2+x^{-1}$,

$$y'=2x-x^{-2}=2x-\frac{1}{x^2}=\frac{2x^3-1}{x^2}.$$

CV: $x=\sqrt[3]{\frac{1}{2}}$, but $x = 0$ must be included in inc.-dec. analysis. Decreasing on (–∞, 0) and $\left(0, \sqrt[3]{\frac{1}{2}}\right)$; increasing on $\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$; relative minimum at $\left(\sqrt[3]{\frac{1}{2}}, 3\sqrt[3]{\frac{1}{4}}\right)$.

$$y''=2+2x^{-3}=2+\frac{2}{x^3}=\frac{2\left(x^3+1\right)}{x^3}$$

Possible inflection point when $x = –1$, but $x = 0$ must be included in concavity analysis. Concave up on (–∞, –1) and (0, ∞); concave down on (–1, 0); inflection point at (–1, 0).



37. $y=\frac{9}{9x^2-6x-8}=\frac{9}{(3x+2)(3x-4)}$

Intercept: $\left(0, -\frac{9}{8}\right)$

Vertical asymptotes: $x=-\frac{2}{3}$, $x=\frac{4}{3}$

$$\lim_{x\to\infty} y=\lim_{x\to\infty}\frac{9}{9x^2}=\lim_{x\to\infty}\frac{1}{x^2}=0=\lim_{x\to-\infty} y$$

Thus $y = 0$ is a horizontal asymptote. Since $y=9\left(9x^2-6x-8\right)^{-1}$,

$$y'=9(-1)\left(9x^2-6x-8\right)^{-2}(18x-6)$$
$$=-\frac{54(3x-1)}{[(3x+2)(3x-4)]^2}$$

CV: $x=\frac{1}{3}$, but $x=-\frac{2}{3}$ and $x=\frac{4}{3}$ must be included in inc.-dec. analysis.

Increasing on $\left(-\infty, -\frac{2}{3}\right)$ and $\left(-\frac{2}{3}, \frac{1}{3}\right)$; decreasing on $\left(\frac{1}{3}, \frac{4}{3}\right)$ and $\left(\frac{4}{3}, \infty\right)$;

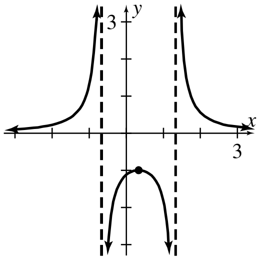
relative maximum at $\left(\frac{1}{3}, -1\right)$. Finding y'' gives:

$$y'' = -54 \cdot \frac{\left(9x^2-6x-8\right)^2(3)-(3x-1)\left[2\left(9x^2-6x-8\right)(18x-6)\right]}{\left(9x^2-6x-8\right)^4}$$

$$= -54 \cdot \frac{3\left(9x^2-6x-8\right)\left[\left(9x^2-6x-8\right)-4(3x-1)(3x-1)\right]}{\left(9x^2-6x-8\right)^4}$$

$$= \frac{-162\left(-27x^2+18x-12\right)}{\left(9x^2-6x-8\right)^3} = \frac{486\left(9x^2-6x+4\right)}{[(3x+2)(3x-4)]^3}$$

Since $9x^2-6x+4=0$ has no real roots, y'' is never zero. No possible inflection points, but $x=-\frac{2}{3}$ and $x=\frac{4}{3}$ must be included in concavity analysis. Concave up on $\left(-\infty, -\frac{2}{3}\right)$ and $\left(\frac{4}{3}, \infty\right)$; concave down on $\left(-\frac{2}{3}, \frac{4}{3}\right)$.



38. $y = \frac{4x^2+2x+1}{2x^2}$

$4x^2+2x+1$ is never 0 and x cannot be zero. Thus no intercepts. Vertical asymptote is $x = 0$.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{4x^2}{2x^2} = \lim_{x\to\infty} 2 = 2 = \lim_{x\to-\infty} y$$

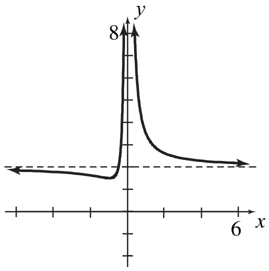
Thus $y = 2$ is a horizontal asymptote. Since $y = 2 + x^{-1} + \frac{1}{2}x^{-2}$, we have

$$y' = -x^{-2} - x^{-3} = -x^{-3}(x+1)$$

CV: $x = -1$, but $x = 0$ must be included in the inc.-dec. analysis. Decreasing on $(-\infty, -1)$ and $(0, \infty)$; increasing on $(-1, 0)$; relative minimum at $\left(-1, \frac{3}{2}\right)$.

$$y'' = 2x^{-3} + 3x^{-4} = \frac{3}{x^4}\left(\frac{2}{3}x+1\right).$$

Possible inflection point when $x = -\frac{3}{2}$, but $x = 0$ must be included in the concavity analysis. Concave down on $\left(-\infty, -\frac{3}{2}\right)$; concave up on $\left(-\frac{3}{2}, 0\right)$ and $(0, \infty)$; inflection point at $\left(-\frac{3}{2}, \frac{14}{9}\right)$. No symmetry.



39. $y = \dfrac{3x+1}{(3x-2)^2}$

Intercepts: $\left(-\dfrac{1}{3}, 0\right), \left(0, \dfrac{1}{4}\right)$

Vertical asymptote is $x = \dfrac{2}{3}.$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{3x}{9x^2} = \lim_{x\to\infty} \frac{1}{3x} = 0 = \lim_{x\to-\infty} y$$

Thus $y = 0$ is a horizontal asymptote.

$$y' = \frac{(3x-2)^2(3) - (3x+1)(2)(3x-2)(3)}{(3x-2)^4}$$
$$= \frac{3(3x-2)[(3x-2) - 2(3x+1)]}{(3x-2)^4}$$
$$= -\frac{3(3x+4)}{(3x-2)^3}$$

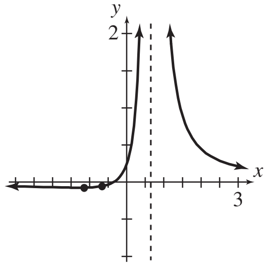
CV: $x = -\dfrac{4}{3}$, but $x = \dfrac{2}{3}$ must be included in inc.-dec. analysis.

Decreasing on $\left(-\infty, -\dfrac{4}{3}\right)$ and $\left(\dfrac{2}{3}, \infty\right)$; increasing on $\left(-\dfrac{4}{3}, \dfrac{2}{3}\right)$; relative minimum at $\left(-\dfrac{4}{3}, -\dfrac{1}{12}\right)$.

$$y'' = -3 \cdot \frac{(3x-2)^3(3) - (3x+4)(3)(3x-2)^2(3)}{(3x-2)^6}$$
$$= -3 \cdot \frac{3(3x-2)^2[(3x-2) - 3(3x+4)]}{(3x-2)^6}$$
$$= -3 \cdot \frac{3(-6x-14)}{(3x-2)^4} = \frac{18(3x+7)}{(3x-2)^4}$$

Possible inflection point when $x = -\dfrac{7}{3}$, but $x = \dfrac{2}{3}$ must be included in concavity analysis.

Concave down on $\left(-\infty, -\dfrac{7}{3}\right)$; concave up on $\left(-\dfrac{7}{3}, \dfrac{2}{3}\right)$ and $\left(\dfrac{2}{3}, \infty\right)$; inflection point at $\left(-\dfrac{7}{3}, -\dfrac{2}{27}\right)$.



40. $y = \dfrac{3x+1}{(6x+5)^2}$

Intercepts: $\left(-\dfrac{1}{3}, 0\right), \left(0, \dfrac{1}{25}\right)$

Vertical asymptote is $x = -\dfrac{5}{6}$.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{3x}{36x^2} = \lim_{x\to\infty} \frac{1}{12x} = 0 = \lim_{x\to-\infty} y$$

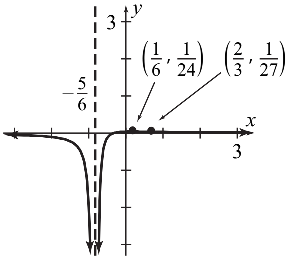
Thus $y = 0$ is horizontal asymptote.

$$y' = \frac{(6x+5)^2(3) - (3x+1)[12(6x+5)]}{(6x+5)^4}$$
$$= \frac{3(6x+5)[(6x+5) - 4(3x+1)]}{(6x+5)^4}$$
$$= \frac{3(-6x+1)}{(6x+5)^3} = \frac{-3(6x-1)}{(6x+5)^3}$$

CV: $x = \dfrac{1}{6}$, but $x = -\dfrac{5}{6}$ must be included in inc.-dec. analysis. Decreasing on $\left(-\infty, -\dfrac{5}{6}\right)$ and $\left(\dfrac{1}{6}, \infty\right)$; increasing on $\left(-\dfrac{5}{6}, \dfrac{1}{6}\right)$; relative maximum at $\left(\dfrac{1}{6}, \dfrac{1}{24}\right)$. Finding y'' gives:

$$y''=-3\cdot\frac{(6x+5)^3(6)-(6x-1)\left[18(6x+5)^2\right]}{(6x+5)^6}$$
$$=-3\cdot\frac{6(6x+5)^2[(6x+5)-3(6x-1)]}{(6x+5)^6}$$
$$=-18\cdot\frac{-12x+8}{(6x+5)^4}$$
$$=72\cdot\frac{3x-2}{(6x+5)^4}$$

Possible inflection point when $x=\frac{2}{3}$, but $x=-\frac{5}{6}$ must be included in concavity analysis. Concave down on $\left(-\infty,-\frac{5}{6}\right)$ and $\left(-\frac{5}{6},\frac{2}{3}\right)$; concave up on $\left(\frac{2}{3},\infty\right)$; inflection point at $\left(\frac{2}{3},\frac{1}{27}\right)$.



41. $y=\frac{x^2-1}{x^3}=\frac{(x+1)(x-1)}{x^3}$

Intercepts are (–1, 0) and (1, 0).
Symmetric about the origin.
Vertical asymptote $x = 0$.

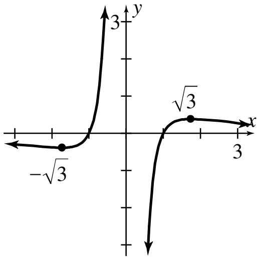
$$\lim_{x\to\infty}\frac{x^2-1}{x^3}=\lim_{x\to\infty}\frac{x^2}{x^3}=\lim_{x\to\infty}\frac{1}{x}$$
$=0=\lim\limits_{x\to-\infty}\frac{1-x}{x^2}$, so $y = 0$ is the only horizontal asymptote. Since $y=x^{-1}-x^{-3}$, then

$$y'=-x^{-2}+3x^{-4}=x^{-4}\left(-x^2+3\right)=\frac{3-x^2}{x^4}$$

CV: $x=\pm\sqrt{3}$, but $x = 0$ must be included in the inc.-dec. analysis. Increasing on $\left(-\sqrt{3},0\right)$ and $\left(0,\sqrt{3}\right)$; decreasing on $\left(-\infty,-\sqrt{3}\right)$ and $\left(\sqrt{3},\infty\right)$; relative maximum at $\left(\sqrt{3},\frac{2\sqrt{3}}{9}\right)$; relative minimum at $\left(-\sqrt{3},-\frac{2\sqrt{3}}{9}\right)$.

$$y''=2x^{-3}-12x^{-5}=2x^{-5}\left(x^2-6\right)=\frac{2\left(x^2-6\right)}{x^5}$$

Possible inflection points when $x=\pm\sqrt{6}$, but $x = 0$ must be included in the concavity analysis. Concave down on $\left(-\infty,-\sqrt{6}\right)$ and $\left(0,\sqrt{6}\right)$; concave up on $\left(-\sqrt{6},0\right)$ and $\left(\sqrt{6},\infty\right)$; inflection points at $\left(\sqrt{6},\frac{5\sqrt{6}}{36}\right)$ and $\left(-\sqrt{6},\frac{-5\sqrt{6}}{36}\right)$.



42. $y=\frac{3x}{(x-2)^2}$

Intercept (0, 0)
Vertical asymptote at $x = 2$

$\lim\limits_{x\to\infty}\frac{3x}{x^2-4x+4}=\lim\limits_{x\to\infty}\frac{3x}{x^2}=\lim\limits_{x\to\infty}\frac{3}{x}=0$ and

$\lim\limits_{x\to-\infty}\frac{3x}{x^2-4x+4}=0$, so $y = 0$ is the only horizontal asymptote.

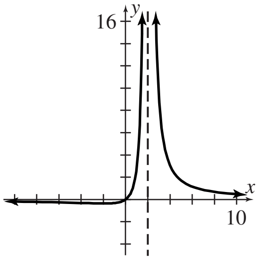
$$y'=\frac{-3(x+2)}{(x-2)^3}$$

CV: $x = -2$, but $x = 2$ must be included in the inc.-dec. analysis. Decreasing on (–∞, –2) and (2, ∞); increasing on (–2, 2); relative maximum at $\left(-2,-\frac{3}{8}\right)$

$$y''=\frac{6(x+4)}{(x-2)^4}$$

Possible inflection point when $x = -4$, but $x = 2$ must be included in the concavity analysis.

Concave down on $(-\infty, -4)$; concave up on $(-4, 2)$ and $(2, \infty)$; inflection point at $\left(-4, -\frac{1}{3}\right)$.



43. $y = 2x+1+\dfrac{1}{x-1} = \dfrac{2x^2-x}{x-1} = \dfrac{x(2x-1)}{x-1}$

Intercepts: (0, 0), $\left(\frac{1}{2}, 0\right)$

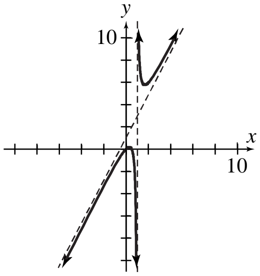
$x = 1$ is the only vertical asymptote.
$y = 2x + 1$ is an oblique asymptote.

$$y' = \frac{(x-1)(4x-1)-(1)(2x^2-x)}{(x-1)^2}$$
$$= \frac{4x^2-5x+1-2x^2+x}{(x-1)^2}$$
$$= \frac{2x^2-4x+1}{(x-1)^2}$$

CV: $\dfrac{2\pm\sqrt{2}}{2} = 1\pm\dfrac{\sqrt{2}}{2}$, but $x = 1$ must be included in the inc-dec. analysis. Increasing on $\left(-\infty, 1-\frac{\sqrt{2}}{2}\right)$ and $\left(1+\frac{\sqrt{2}}{2}, \infty\right)$; decreasing on $\left(1-\frac{\sqrt{2}}{2}, 1\right)$ and $\left(1, 1+\frac{\sqrt{2}}{2}\right)$; relative maximum at $\left(1-\frac{\sqrt{2}}{2}, 3-2\sqrt{2}\right)$; relative minimum at $\left(1+\frac{\sqrt{2}}{2}, 3+2\sqrt{2}\right)$

$$y'' = \frac{(x-1)^2(4x-4)-2(x-1)(2x^2-4x+1)}{(x-1)^4}$$
$$= \frac{2(x-1)}{(x-1)^4}$$
$$= \frac{2}{(x-1)^3}$$

No possible inflection point, but $x = 1$ must be included in the concavity analysis. Concave down on $(-\infty, 1)$; concave up on $(1, \infty)$. No symmetry.



44. $y = \dfrac{3x^4+1}{x^3}$

No intercepts
Symmetric about the origin.

Vertical asymptote is $x = 0$. $\dfrac{3x^4+1}{x^3} = 3x+\dfrac{1}{x^3}$ so $y = 3x$ is an oblique asymptote.

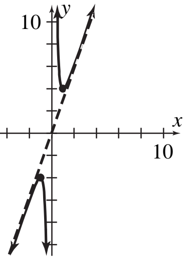
Since $y = 3x+x^{-3}$,

$$y' = 3-3x^{-4} = 3-\frac{3}{x^4} = \frac{3\left(x^2+1\right)(x+1)(x-1)}{x^4}$$

CV: ±1, but $x = 0$ must be considered in the inc.-dec. analysis. Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 0)$ and $(0, 1)$; relative maximum at $(-1, -4)$; relative minimum at $(1, 4)$.

$$y'' = \frac{12}{x^5}$$

No possible inflection point, but $x = 0$ must be included in the concavity analysis. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.



45. $y=\frac{-3x^2+2x-5}{3x^2-2x-1}=\frac{-3x^2+2x-5}{(3x+1)(x-1)}$

Note that $-3x^2+2x-5$ is never zero. Intercept: (0, 5)

Vertical asymptotes are $x=-\frac{1}{3}$ and $x = 1$.

$\lim_{x\to\infty} y=\lim_{x\to\infty}\frac{-3x^2}{3x^2}=\lim_{x\to\infty}-1=-1=\lim_{x\to-\infty} y$

Thus $y = -1$ is horizontal asymptote.

$$y'=\frac{\left(3x^2-2x-1\right)(-6x+2)-\left(-3x^2+2x-5\right)(6x-2)}{\left(3x^2-2x-1\right)^2}$$

$$=\frac{2(3x-1)\left[\left(3x^2-2x-1\right)(-1)-\left(-3x^2+2x-5\right)\right]}{\left(3x^2-2x-1\right)^2}$$

$$=\frac{12(3x-1)}{\left(3x^2-2x-1\right)^2}=\frac{12(3x-1)}{[(3x+1)(x-1)]^2}$$

CV: $x=\frac{1}{3}$, but $x=-\frac{1}{3}$ and $x = 1$ must be included in inc.-dec. analysis.

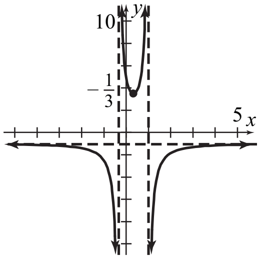
Decreasing on $\left(-\infty,-\frac{1}{3}\right)$ and $\left(-\frac{1}{3},\frac{1}{3}\right)$; increasing on $\left(\frac{1}{3},1\right)$ and (1, ∞); relative minimum at $\left(\frac{1}{3},\frac{7}{2}\right)$.

$$y''=12\cdot\frac{\left(3x^2-2x-1\right)^2(3)-(3x-1)\left[2\left(3x^2-2x-1\right)(6x-2)\right]}{\left(3x^2-2x-1\right)^4}$$

$$=12\cdot\frac{\left(3x^2-2x-1\right)\left[3\left(3x^2-2x-1\right)-2(3x-1)(6x-2)\right]}{\left(3x^2-2x-1\right)^4}$$

$$=12\cdot\frac{-27x^2+18x-7}{\left(3x^2-2x-1\right)^3}=\frac{-12\left(27x^2-18x+7\right)}{[(3x+1)(x-1)]^3}$$

Since $27x^2-18x+7$ is never zero, there is no possible inflection point, but $x=-\frac{1}{3}$ and $x = 1$ must be included in concavity analysis. Concave down on $\left(-\infty,-\frac{1}{3}\right)$ and (1, ∞); concave up on $\left(-\frac{1}{3},1\right)$.



46. $y=3x+2+\frac{1}{3x+2}=\frac{(3x+2)^2+1}{3x+2}$

$=\frac{9x^2+12x+5}{3x+2}$

Note that $9x^2+12x+5$ is never zero.

Intercept: $\left(0, \frac{5}{2}\right)$

Vertical asymptote is $x=-\frac{2}{3}$; oblique asymptote is $y = 3x + 2$.

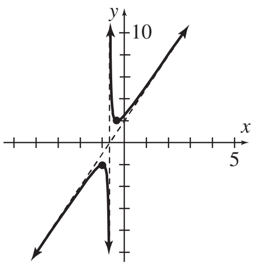
$$y'=3-\frac{3}{(3x+2)^2}=3\cdot\frac{(3x+2)^2-1}{(3x+2)^2}$$

$$=3\cdot\frac{9x^2+12x+3}{(3x+2)^2}=9\cdot\frac{(3x+1)(x+1)}{(3x+2)^2}$$

CV: $x=-\frac{1}{3}$ and $x = -1$, but $x=-\frac{2}{3}$ must be included in inc.-dec. analysis. Increasing on $(-\infty, -1)$ and $\left(-\frac{1}{3}, \infty\right)$; decreasing on $\left(-1, -\frac{2}{3}\right)$ and $\left(-\frac{2}{3}, -\frac{1}{3}\right)$; relative maximum at $(-1, -2)$; relative minimum at $\left(-\frac{1}{3}, 2\right)$.

$$y''=-3(-2)(3x+2)^{-3}(3)=\frac{18}{(3x+2)^3}$$

No possible inflection point, but $x=-\frac{2}{3}$ must be included in concavity analysis. Concave down on $\left(-\infty, -\frac{2}{3}\right)$; concave up on $\left(-\frac{2}{3}, \infty\right)$.



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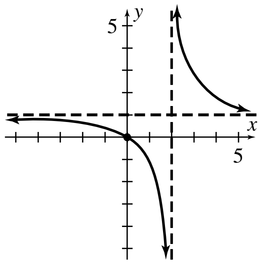

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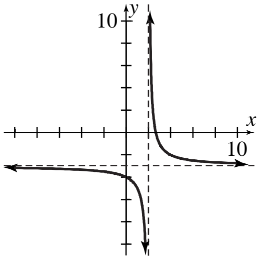

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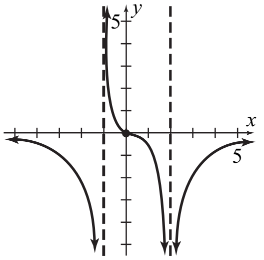

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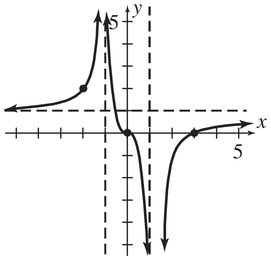


51. When $x=-\frac{a}{b}$, then $a + bx = 0$ so $x=-\frac{a}{b}$ is a vertical asymptote.

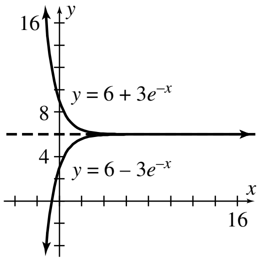
$$\lim_{x\to\infty}\frac{x}{a+bx}=\lim_{x\to\infty}\frac{x}{bx}=\lim_{x\to\infty}\frac{1}{b}=\frac{1}{b}$$

Thus $y=\frac{1}{b}$ is a horizontal asymptote.

52. For $y=6-3e^{-x}$ we have

$$\lim_{x\to\infty}\left(6-3e^{-x}\right)=\lim_{x\to\infty}\left(6-\frac{3}{e^x}\right)=6-3(0)=6$$

Thus the line $y = 6$ is a horizontal asymptote for the graph of $y=6-3e^{-x}$. For $y=6+3e^{-x}$, we obtain $\lim\limits_{x\to\infty}\left(6+3e^{-x}\right)=6+3(0)=6$, so the line $y = 6$ is also a horizontal asymptote for the graph of $y=6+3e^{-x}$.

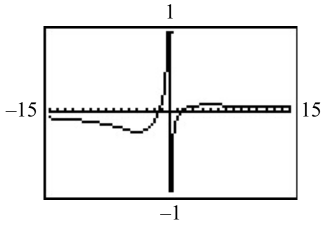


53. $\lim\limits_{t\to\infty}\left(250-83e^{-t}\right)=\lim\limits_{t\to\infty}\left(250-\dfrac{83}{e^t}\right)$

$= 250 - 0 = 250$

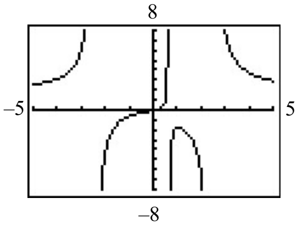
Thus $y = 250$ is a horizontal asymptote.

54.



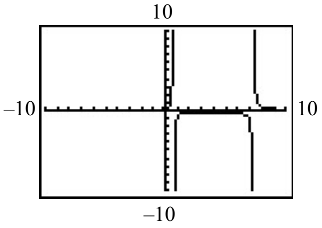
$x \approx -0.08$, $y = 0$

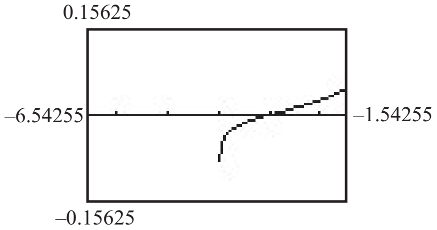
55.



$x \approx \pm 2.45$, $x \approx 0.67$, $y = 2$

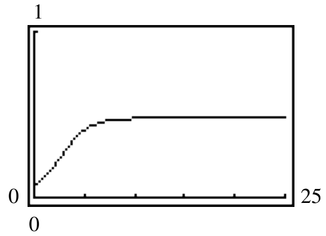
56.





In the standard window, two vertical asymptotes of the form $x = k$, where $k > 0$, are apparent ($x \approx 0.68$ and $x \approx 7.32$). By zooming around $x = -4$, another vertical asymptote is apparent ($x = -4$). Thus three vertical asymptotes exist.

57.



From the graph, it appears that $\lim\limits_{x\to\infty} y \approx 0.48$.

Thus a horizontal asymptote is $y \approx 0.48$. Algebraically, we have

$$\lim_{x\to\infty}\frac{0.34e^{0.7x}}{4.2+0.71e^{0.7x}}=\lim_{x\to\infty}\frac{\frac{0.34e^{0.7x}}{e^{0.7x}}}{\frac{4.2+0.71e^{0.7x}}{e^{0.7x}}}$$

$$=\lim_{x\to\infty}\frac{0.34}{\frac{4.2}{e^{0.7x}}+0.71}=\frac{0.34}{0+0.71}\approx 0.48$$

Problems 13.6

1. Let the numbers be x and $82 - x$. Then if $P = x(82-x) = 82x - x^2$, we have $P' = 82 - 2x$. Setting $P' = 0 \Rightarrow x = 41$. Since $P'' = -2 < 0$, there is a maximum when $x = 41$. Because $82 - x = 41$, the required numbers are 41 and 41.

2. Let the numbers be x and $20 - x$, where $0 \le x \le 20$. Let

$P=(2x)(20-x)^2=2x^3-80x^2+800x$.

Setting $\dfrac{dP}{dx}=0$ gives

$P'=6x^2-160x+800=2(3x-20)(x-20)=0$,

from which $x=\dfrac{20}{3}$ or $x = 20$. $P' > 0$ on

$\left(0, \dfrac{20}{3}\right)$ and $P' < 0$ on $\left(\dfrac{20}{3}, 20\right)$. Thus P has a relative and absolute maximum when $x = \frac{20}{3}$.

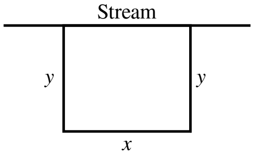
The other number is $20 - x = \frac{40}{3}$.

3. We are given that 15x + 9(2y) = 9000, or $y = \frac{9000 - 15x}{18}$. We want to maximize area A, where $A = xy$.

$$A = xy = x\left(\frac{9000 - 15x}{18}\right) = \frac{1}{18}\left(9000x - 15x^2\right)$$

$$A' = \frac{1}{18}(9000 - 30x)$$

Setting $A' = 0 \Rightarrow x = 300$. Since $A''(300) = \frac{1}{18}(-30) < 0$, we have a maximum at x = 300. Thus $y = \frac{9000 - 15(300)}{18} = 250$. The dimensions are 300 ft by 250 ft.

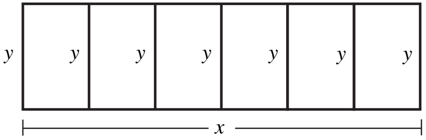


4. We are given that xy = 1400, or $y = \frac{1400}{x}$, and want to minimize N = 2x + 7y. We have

$$N = 2x + 7y = 2x + 7\left(\frac{1400}{x}\right), x > 0$$

$$N' = 2 - \frac{9800}{x^2} = \frac{2(x^2 - 4900)}{x^2}$$

Setting $N' = 0$ yields $x^2 = 4900$, so x = 70. We have $N'' = \frac{19,600}{x^3}$, so $N''(70) > 0$ and we have a minimum. If x = 70, then y = 20. Thus N = 2(70) + 7(20) = 280 ft.



5. $c = 0.05q^2 + 5q + 500$

Avg. cost per unit $= \overline{c} = \frac{c}{q} = 0.05q + 5 + \frac{500}{q}$

$\overline{c}' = 0.05 - \frac{500}{q^2}$. Setting $\overline{c}' = 0$ yields

$0.05 = \frac{500}{q^2}$, $q^2 = 10,000$, $q = \pm 100$. We exclude q = –100 because q represents the number of units. Since $\overline{c}'' = \frac{1000}{q^3} > 0$ for q > 0, $\overline{c}$ is an absolute minimum when q = 100 units.

6. $C = 0.12s - 0.0012s^2 + 0.08$, where 0 ≤ s ≤ 60. Setting $\frac{dC}{ds} = 0$ gives 0.12 – 0.0024s = 0 ⇒ s = 50. Since $\frac{d^2C}{ds^2} = -0.0024 < 0$, a maximum occurs when s = 50. Thus a minimum can occur only at an endpoint of the domain. If s = 0, then C = 0.08; if s = 60, then C = 2.96. Thus the minimum cost of $0.08 per hour occurs for s = 0 mi/h and might be due to depreciation, insurance, and so on.

7. p = –5q + 30

Since total revenue = (price)(quantity),

$r = pq = (-5q + 30)q = -5q^2 + 30q$

Setting $r' = -10q + 30 = 0 \Rightarrow q = 3$. Since $r'' = -10 < 0$, r is maximum at q = 3 units, for which the corresponding price is p = –5(3) + 30 = $15.

8. $q = Ae^{-Bp}$

Revenue = $r = pq = pAe^{-Bp}$

$$\begin{aligned} r' &= A[e^{-Bp}(1) + pe^{-Bp}(-B)] \\ &= A(1 - Bp)e^{-Bp} \\ &= AB\left(\frac{1}{B} - p\right)e^{-Bp} \end{aligned}$$

Critical value: $p = \frac{1}{B}$

If $p < \frac{1}{B}$, then $r' > 0$ and r is increasing. If $p > \frac{1}{B}$, then $r' < 0$ and r is decreasing. Thus revenue is maximum when $p = \frac{1}{B}$. The answer does not depend on A because A represents the initial value of q, so it doesn't change q over time.

9. $f(p)=170-p-\frac{1600}{p+15}$, where $0 \le p \le 100$.

a. Setting $f'(p)=0$ gives $-1+\frac{1600}{(p+15)^2}=0$,

$\frac{1600}{(p+15)^2}=1$, $(p+15)^2=1600$,

$p + 15 = \pm 40$, from which $p = 25$.

Since $f''(p)=-\frac{3200}{(p+15)^3}<0$ for $p = 25$,

we have an absolute maximum of

$f(25) = 105$ grams.

b. $f(0)=63\frac{1}{3}$ and $f(100)=\frac{1290}{23}\approx 56.1$, so

we have an absolute minimum of

$f(100)=\frac{1290}{23}\approx 56.1$ grams.

10. $R=D^2\left(\frac{C}{2}-\frac{D}{3}\right)=\frac{CD^2}{2}-\frac{D^3}{3}$

The rate of change of R is $\frac{dR}{dD}=CD-D^2$. This is the function to be maximized. Setting $\frac{d}{dD}\left(\frac{dR}{dD}\right)=C-2D=0$ gives $D=\frac{C}{2}$. Since $\frac{d^2}{dD^2}\left(\frac{dR}{dD}\right)=-2<0$, the maximum rate of change occurs when $D=\frac{C}{2}$.

11. $p = 85 - 0.05q$

$c = 600 + 35q$

Profit = Total Revenue – Total Cost

$P=pq-c=(85-0.05q)q-(600+35q)$

$=-(0.05q^2-50q+600)$

Setting $P'=-(0.1q-50)=0$ yields $q = 500$.

Since $P''(500)=-0.1<0$, P is a maximum when $q = 500$ units. This corresponds to a price of $p = 85 - 0.05(500) = \$60$ and a profit of $P = \$11,900$.

12. Cost per unit = \$3

$p=\frac{10}{\sqrt{q}}$

Profit = Total Revenue – Total Cost

$P = pq - c$

$P=\left(\frac{10}{\sqrt{q}}\right)q-(3q)=10\sqrt{q}-3q$

Setting $P'=\frac{5}{\sqrt{q}}-3=0$ yields $q=\frac{25}{9}$.

Moreover, we have $P''=-\frac{5}{2}q^{-\frac{3}{2}}<0$ for $q > 0$,

so P is maximum when $q=\frac{25}{9}$. The corresponding price is $p = \$6$.

13. $p = 42 - 4q$

$\bar{c}=2+\frac{80}{q}$

Total Cost $=c=\bar{c}q=2q+80$

Profit = Total Revenue – Total Cost

$P = pq - c = (42 - 4q)q - (2q + 80)$

$=-\left(4q^2-40q+80\right)$

$P'=-(8q-40)$

Setting $P'=-(8q-40)=0$ gives $q = 5$. We find that $P''=-8<0$, so P has a maximum value when $q = 5$. The corresponding price p is $42 - 4(5) = \$22$.

14. $p=\frac{50}{\sqrt{q}}$

$\bar{c}=\frac{1}{4}+\frac{2500}{q}$

Total cost $=c=\bar{c}q=\frac{q}{4}+2500$

Profit = Total Revenue – Total Cost

$P=pq-c=50\sqrt{q}-\frac{q}{4}-2500$

Setting $P'=\frac{25}{\sqrt{q}}-\frac{1}{4}=0$ yields $q = 10,000$.

Since $P''=-\frac{25}{2}q^{-3/2}<0$ for $q > 0$, it follows that P is a maximum when $q = 10,000$. The corresponding price is $p=\frac{50}{100}\approx \$0.50$. Since $MR=\frac{25}{\sqrt{q}}$ and $MC=\frac{1}{4}$, then for $q = 10,000$ we have $MR=\frac{25}{100}=\frac{1}{4}=MC$.

15. $p = q^2 - 100q + 3200$ on [0, 120]

$\bar{c} = \frac{2}{3}q^2 - 40q + \frac{10,000}{q}$

Profit = Total Revenue – Total Cost
Since total revenue $r = pq$ and
total cost $= c = \bar{c}q$,

$$P = pq - \bar{c}q$$

$$= q^3 - 100q^2 + 3200q - \left(\frac{2}{3}q^3 - 40q^2 + 10,000\right)$$

$$= \frac{1}{3}q^3 - 60q^2 + 3200q - 10,000$$

$P' = q^2 - 120q + 3200 = (q - 40)(q - 80)$

Setting $P' = 0$ gives $q = 40$ or 80. Evaluating profit at $q = 0$, 40, 80, and 120 gives

$P(0) = -10,000$

$P(40) = \frac{130,000}{3} = 43,333\frac{1}{3}$

$P(80) = \frac{98,000}{3} = 32,666\frac{2}{3}$

$P(120) = 86,000$

Thus the profit maximizing output is $q = 120$ units, and the corresponding maximum profit is $86,000.

16. a. $c = \bar{c}q = 2q^3 - 42q^2 + 228q + 210$

$\frac{dc}{dq} = 6q^2 - 84q + 228 = 6(q^2 - 14q + 38)$

Using the quadratic formula to solve $\frac{dc}{dq} = 0$ gives $q = 7 - \sqrt{11} \approx 3.68$ or $q = 7 + \sqrt{11} \approx 10.32$. Evaluating c at $q = 3$, $7 - \sqrt{11}$, $7 + \sqrt{11}$, and 12 gives 570, $434 + 44\sqrt{11} \approx 579.93$, $434 - 44\sqrt{11} \approx 288.07$, and 354, respectively. Thus the minimum cost is when $q = 7 + \sqrt{11} \approx 10.32$.

$c(10) = 290$ and $c(11) = 298$, so production should be fixed at $q = 10$ for a minimum cost of $290.

b. $c(7) = 434$, so the minimum cost still occurs when $q = 7 + \sqrt{11} \approx 10.32$ and production should again be fixed at 10 units.

17. Total fixed costs = $1200, material-labor costs/unit = $2, and the demand equation is $p = \frac{100}{\sqrt{q}}$.

Profit = Total Revenue – Total Cost

$P = pq - c$

$$P = \frac{100}{\sqrt{q}} \cdot q - (2q + 1200)$$

$$= 100\sqrt{q} - 2q - 1200$$

$$= 2\left(50\sqrt{q} - q - 600\right)$$

Setting $P' = 2\left(\frac{25}{\sqrt{q}} - 1\right) = 0$ yields $q = 625$. We see that $P'' = -25q^{-\frac{3}{2}} < 0$ for $q > 0$, so P is maximum when $q = 625$. When $q = 625$, $\text{MR} = \frac{50}{\sqrt{625}} = 2 = \text{MC}$. When $q = 625$, then $p = \$4$.

18. Let x = number of $10 per month increases so the monthly rate is $400 + 10x$ and the number of rented apartments is $100 - 2x$. Monthly revenue r is given by

r = (rent/apt.) (no. of apt. rented)

$r = (400 + 10x)(100 - 2x)$

$r' = (400 + 10x)(-2) + (100 - 2x)(10)$

$= 200 - 40x = 40(5 - x)$

Setting $r' = 0$ yields $x = 5$. Since $r'' = -40 < 0$, then r is maximum when $x = 5$. This results in a monthly rate for an apartment of $400 + 10(5) = \$450$.

19. If x = number of $0.50 decreases, where $0 \le x \le 48$, then the monthly fee for each subscriber is $24 - 0.50x$, and the total number of subscribers is $6400 + 160x$. Let r be the total (monthly) revenue.

revenue = (monthly rate)(number of subscribers)

$r = (24 - 0.50x)(6400 + 160x)$

$r' = (24 - 0.50x)(160) + (6400 + 160x)(-0.50)$

$= 640 - 160x = 160(4 - x)$

Setting $r' = 0$ yields $x = 4$.

Evaluating r when $x = 0$, 4, and 48, we find that r is a maximum when $x = 4$. This corresponds to a monthly fee of $24 - 0.50(4) = \$22$ and a monthly revenue r of $154,880.

20. Note that as the number of units produced and sold increases from 0 to 600, the profit increases from 0 to (600)(400) = \$24,000. Let q = number of units produced and sold beyond 600. Then the total profit P is given by

$$P = (600)(40) + (40 - 0.05q)q$$
$$= 24,000 + 40q - 0.05q^2$$
$$P' = 40 - 0.10q$$

Setting $P' = 0$ yields $q = 400$. Since $P'' = -0.10 < 0$, P is a maximum when $q = 400$, that is, the total number of units = 600 + 400 = 1000.

21. See the figure in the text. Given that $x^2y = 32$, we want to minimize $S = 4(xy) + x^2$. Since $y = \frac{32}{x^2}$, where $x > 0$, we have

$S = 4x\left(\frac{32}{x^2}\right) + x^2 = \frac{128}{x} + x^2$, from which

$S' = -\frac{128}{x^2} + 2x$. Setting $S' = 0$ gives

$2x^3 = 128$, $x^3 = 64$, $x = 4$. Since $S'' = \frac{256}{x^3} + 2$, we get $S''(4) > 0$, so $x = 4$ gives a minimum. If $x = 4$, then $y = \frac{32}{16} = 2$. The dimensions are 4 ft × 4 ft × 2 ft.

22. See the figure in the text. We want to maximize $V = x^2y$ given that $4xy + x^2 = 192$, or

$$y = \frac{192 - x^2}{4x}$$
$$V = x^2\left(\frac{192 - x^2}{4x}\right) = \frac{1}{4}\left(192x - x^3\right), x > 0$$
$$V' = \frac{1}{4}\left(192 - 3x^2\right) = \frac{3}{4}\left(64 - x^2\right)$$

Setting $V' = 0$ gives $x = 8$. Since $V'' = \left(\frac{3}{4}\right)(-2x)$, then $V''(8) < 0$, so $x = 8$ gives a maximum. If $x = 8$, then $y = 4$.
The dimensions are 8 ft × 8 ft × 4 ft.
The volume is $8^2(4) = 256 \text{ ft}^3$.

23. $V = x\left(L - 2x\right)^2$

$= L^2x - 4Lx^2 + 4x^3$

where $0 < x < \frac{L}{2}$.

$$V' = L^2 - 8Lx + 12x^2$$
$$= 12x^2 - 8Lx + L^2$$
$$= (2x - L)(6x - L)$$

For $0 < x < \frac{L}{2}$, setting $V' = 0$ gives $x = \frac{L}{6}$.

Since $V' > 0$ on $\left(0, \frac{L}{6}\right)$ and $V' < 0$ on $\left(\frac{L}{6}, \frac{L}{2}\right)$, V is maximum when $x = \frac{L}{6}$. Thus the length of the side of the square must be $\frac{L}{6}$ in., which results in a volume of

$$\frac{L}{6}\left(L - \frac{L}{3}\right)^2 = \frac{2L^3}{27} \text{ in}^3.$$

24. Since $xy = 720$, then $y = \frac{720}{x}$, $x > 0$. We want to minimize A where

$$A = (x + 10)(y + 8) = (x + 10)\left(\frac{720}{x} + 8\right)$$
$$= 800 + 8x + \frac{7200}{x}$$
$$A' = 8 - \frac{7200}{x^2}$$

Setting $A' = 0$ gives $x = 30$. Since $A'' = \frac{14,400}{x^3} > 0$ for $x = 30$, we have a minimum. Thus $y = 24$, so the dimensions are 30 + 10 by 24 + 8, that is, 40 in. × 32 in.

25. See the figure in the text.

$V = K = \pi r^2 h$ (1)

$S = 2\pi rh + \pi r^2$ (2)

From Equation (1) $h = \frac{K}{\pi r^2}$. Thus Equation (2) becomes

$$S = \frac{2K}{r} + \pi r^2$$
$$\frac{dS}{dr} = -\frac{2K}{r^2} + 2\pi r = \frac{2\left(\pi r^3 - K\right)}{r^2}.$$

If $S' = 0$, then $\pi r^3 - K = 0$, $\pi r^3 = K$, $r = \sqrt[3]{\frac{K}{\pi}}$. Thus

$$h = \frac{K}{\pi\left(\frac{K}{\pi}\right)^{\frac{2}{3}}} = \left(\frac{K}{\pi}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{K}{\pi}}.$$

Note that since $S'' = 2\pi + \frac{4K}{r^3} > 0$ for $r > 0$, we have a minimum.

26. See the figure in the text.

$S = K = 2\pi rh + \pi r^2$ (1)

$V = \pi r^2 h$ (2)

From Equation (1), $h = \frac{K - \pi r^2}{2\pi r}$. Thus Equation (2) becomes

$$V = \frac{Kr - \pi r^3}{2}$$

$$\frac{dV}{dr} = \frac{K - 3\pi r^2}{2}.$$

Setting $V' = 0$ gives $r = \sqrt{\frac{K}{3\pi}}$. Thus

$$h = \frac{K - \pi\frac{K}{3\pi}}{2\pi\sqrt{\frac{K}{3\pi}}} = \frac{\frac{2}{3}K}{2\pi\sqrt{\frac{K}{3\pi}}}$$

$$= \frac{\frac{2}{3}K}{2\pi\sqrt{\frac{K}{3\pi}}} \cdot \frac{\sqrt{\frac{K}{3\pi}}}{\sqrt{\frac{K}{3\pi}}} = \sqrt{\frac{K}{3\pi}}$$

Note that since $V'' = -3\pi r < 0$ for $r > 0$, we have a maximum.

27. $p = 600 - 2q$

$c = 0.2q^2 + 28q + 200$

Profit = Total Revenue – Total Cost

$P = pq - c$

$P = (600 - 2q)q - \left(0.2q^2 + 28q + 200\right)$

$= -\left(2.2q^2 - 572q + 200\right)$

$P' = -(4.4q - 572)$

Setting $P' = 0$ yields $q = 130$. Since $P'' = -4.4 < 0$, P is maximum when $q = 130$ units. The corresponding price is $p = 600 - 2(130) = \$340$, and the profit is $P = \$36{,}980$. If a tax of \$22/unit is imposed on the manufacturer, then the cost equation is

$c_1 = 0.2q^2 + 28q + 200 + 22q$

$= 0.2q^2 + 50q + 200$.

The demand equation remains the same. Thus

$P_1 = pq - c_1$

$= (600 - 2q)q - \left(0.2q^2 + 50q + 200\right)$

$= -\left(2.2q^2 - 550q + 200\right)$

$P'_1 = -(4.4q - 550)$

Setting $P'_1 = 0$ yields $q = 125$. Since $P_1'' = -4.4 < 0$, P_1 is maximum when $q = 125$ units. The corresponding price is $p = \$350$ and the profit is $P_1 = \$34{,}175$.

28. Original data: $p = 600 - 2q$, $c = 0.2q^2 + 28q + 200$. Revenue, both before and after the license fee, is given by $r = pq = 600q - 2q^2$. After the license fee, the cost equation is $c_1 = c + 1000 = 0.2q^2 + 28q + 1200$ and the profit is

$P_1 = r - c_1$

$= \left(600q - 2q^2\right) - \left(0.2q^2 + 28q + 1200\right)$

As in Problem 27, we find that P_1 has a maximum when $q = 130$ units, which gives $p = \$340$. Thus the profit-maximizing price and output remain the same. Since

Profit

$= r - c_1 = r - (c + 1000) = (r - c) - 1000$, when $q = 130$ we have

Profit = 36,980 – 1000 (from Problem 27)

= \$35,980

29. Let q = number of units in a production run. Since inventory is depleted at a uniform rate, assume that the average inventory is $\frac{q}{2}$. The value of average inventory is $12\left(\frac{q}{2}\right)$, and carrying costs are $0.192\left[12\left(\frac{q}{2}\right)\right]$. The number of production runs per year is $\frac{3000}{q}$, and total set-up costs are $54\left(\frac{3000}{q}\right)$. We want to minimize the sum C of carrying costs and set-up costs.

$$C = 0.192\left[12\left(\frac{q}{2}\right)\right] + 54\left(\frac{3000}{q}\right)$$

$$= 1.152q + \frac{162,000}{q}$$

$$C' = 1.152 - \frac{162,000}{q^2}$$

Setting $C' = 0$ yields $q^2 = \frac{162,000}{1.152} = 140,625,$

$q = 375$ (since $q > 0$). Since $C'' = \frac{324,000}{q^3} > 0$,

C is minimum when $q = 375$. Thus the economic lot size is 375/lot (8 lots).

30. $c = 0.004q^3 + 20q + 5000$

$p = 450 - 4q$

Profit = Total Revenue – Total Cost

$P = pq - c$

$= (450 - 4q)q - \left(0.004q^3 + 20q + 5000\right)$

$P = -\left(0.004q^3 + 4q^2 - 430q + 5000\right)$

$P' = -\left(0.012q^2 + 8q - 430\right)$

$= -2\left(0.006q^2 + 4q - 215\right)$

Setting $P' = 0$ yields

$0.006q^2 + 4q - 215 = 0$

$$q = \frac{-4 \pm \sqrt{21.16}}{0.012} = \frac{-4 \pm 4.6}{0.012}$$

Since $q \geq 0$, choose $q = \frac{-4 + 4.6}{0.012} = 50$. Since P is increasing on [0, 50) and decreasing on (50, ∞), P is maximum when $q = 50$ units.

31. Let x = number of people over the 30.

Note: $0 \leq x \leq 10$.

Revenue = r

= (number attending)(charge/person)

= $(30 + x)(50 - 1.25x)$

$= 1500 + 12.5x - 1.25x^2$

$r' = 12.5 - 2.5x$

Setting $r' = 0$ yields $x = 5$. Since $r'' = -2.5 < 0$, r is maximum when $x = 5$, that is, when 35 attend.

32. Let N = horsepower of motor.

(Total annual cost) = C = (Annual cost to lease) + (Annual operating cost)

$$C = (200 + 0.40N) + 80,000\left(\frac{0.008}{N}\right)$$

$$= 200 + 0.04N + \frac{640}{N}$$

$$C' = 0.4 - \frac{640}{N^2}$$

Setting $C' = 0$ yields $N^2 = 1600,$ so $N = 40$ (since $N > 0$). Since $C'' = \frac{1280}{N^3} > 0$ for $N > 0$, C is a minimum when $N = 40$ horsepower.

33. The cost per mile of operating the truck is $0.165 + \frac{s}{200}$. Driver's salary is \$18/hr. The number of hours for 700 mi trip is $\frac{700}{s}$. Driver's salary for trip is $18\left(\frac{700}{s}\right)$, or $\frac{12,600}{s}$. The cost of operating the truck for the trip is

$$700\left[0.165 + \frac{s}{200}\right].$$

Total cost of trip is

$$C = \frac{12,600}{s} + 700\left(0.165 + \frac{s}{200}\right)$$

Setting $C' = -\frac{12,600}{s^2} + \frac{7}{2} = 0$ yields $s^2 = 3600$, or $s = 60$ (since $s > 0$). Since $C'' = \frac{25,200}{s^3} > 0$ for $s > 0$, C is a minimum when $s = 60$ mi/h.

34. Let q = level of production.

$$\text{Average Cost } = \bar{c} = \frac{\text{Total Cost}}{q}$$

For $0 \le q \le 5000$, we have

$$\bar{c} = \frac{30q+10q+20{,}000}{q} = 40+\frac{20{,}000}{q}.$$

Note that total cost for 5000 units is 220,000. For $q > 5000$,

$$\bar{c} = \frac{(\text{cost for first 5000})+\begin{pmatrix}\text{cost for those}\\ \text{units beyond 5000}\end{pmatrix}}{q}$$

$$= \frac{220{,}000+[45(q-5000)+10(q-5000)]}{q}$$

$$\bar{c} = 55-\frac{55{,}000}{q}$$

If $0 < q \le 5000$, then $\bar{c}' = -\frac{20{,}000}{q^2} < 0$ and thus $\bar{c}$ is decreasing. If $q > 5000$, then $\bar{c}' = \frac{55{,}000}{q^2} > 0$ and thus $\bar{c}$ is increasing.

Hence c is minimum when $q = 5000$ units.

35. Profit P is given by

P = Total revenue – Total cost

= Total revenue – (salaries + fixed cost)

= $50q - (1000m + 3000)$

$$= 50\left(m^3-15m^2+92m\right)-1000m-3000$$

$$= 50\left(m^3-15m^2+72m-60\right), \text{ where } 0 \le m \le 8$$

$$P' = 50\left(3m^2-30m+72\right)$$

$$= 150\left(m^2-10m+24\right) = 150(m-4)(m-6)$$

Setting $P' = 0$ gives the critical values 4 and 6. We now evaluate P at these critical values and also at the endpoints 0 and 8.

$P(0) = -3000$

$P(4) = 2600$

$P(6) = 2400$

$P(8) = 3400$

Thus Ms. Jones should hire 8 salespeople to obtain a maximum weekly profit of $3400.

36. Profit P is given by

P = Total revenue – Total cost = pq – Total cost

$= 400q - 50q^2 -$ Total cost. (q in hundreds)

$$\frac{dP}{dq} = 400-100q-\frac{d}{dq}(\text{Total cost})$$

$$= 400 - 100q - \text{Marginal cost}$$

$$= 400-100q-\frac{800}{q+5}$$

$$= \frac{400(q+5)-100q(q+5)-800}{q+5}$$

$$= \frac{-100q^2-100q+1200}{q+5}$$

$$= \frac{-100(q+4)(q-3)}{q+5}$$

Setting $P' = 0$ gives the critical value 3 (since $q > 0$). We find that $P' > 0$ for $0 < q < 3$, and $P' < 0$ for $q > 3$. Thus there is a maximum profit when q = 3000 jackets.

37. x = tons of chemical A ($x \le 4$), $y = \frac{24-6x}{5-x}$ = tons of chemical B, profit on A = \$2000/ton, and profit on B = \$1000/ton.

$$\text{Total Profit } = P_T = 2000x+1000\left(\frac{24-6x}{5-x}\right)$$

$$= 2000\left[x+\frac{12-3x}{5-x}\right]$$

$$P'_T = 2000\left[1+\frac{(5-x)(-3)-(12-3x)(-1)}{(5-x)^2}\right]$$

$$= 2000\left[1-\frac{3}{(5-x)^2}\right]$$

$$= 2000\left[\frac{x^2-10x+22}{(5-x)^2}\right]$$

Setting $P'_T = 0$ yields (by the quadratic formula)

$$x = \frac{10\pm 2\sqrt{3}}{2} = 5\pm\sqrt{3}$$

Because $x \le 4$, choose $x = 5-\sqrt{3}$. Since P_T is increasing on $\left[0, 5-\sqrt{3}\right)$ and decreasing on $\left(5-\sqrt{3}, 4\right]$, P_T is a maximum for $x = 5-\sqrt{3}$ tons. If profit on A is P/ton and profit on B is

$\frac{P}{2}$ /ton, then

$$P_T = Px + \frac{P}{2}\left(\frac{24-6x}{5-x}\right) = P\left[x + \frac{12-3x}{5-x}\right]$$

$$P'_T = P\left[\frac{x^2-10x+22}{(5-x)^2}\right]$$

Setting $P'_T = 0$ and using an argument similar to that above, we find that P_T is a maximum when $x = 5-\sqrt{3}$ tons.

38. x = number of floors. Let R = rate of return.

$$R = \frac{\text{Total Revenue}}{\text{Total Cost}}$$
$$= \frac{60{,}000x}{(10x)[120{,}000+3000(x-1)]+1{,}440{,}000}$$
$$= \frac{2x}{x^2+39x+48}$$

$$R' = 2\cdot\frac{48-x^2}{(x^2+39x+48)^2}$$

$R' = 0$ when $x = \sqrt{48} = 4\sqrt{3}$ $(x \ge 0)$. Since R is increasing on $\left(0,\ 4\sqrt{3}\right)$ and decreasing on $\left(4\sqrt{3},\ \infty\right)$, R is a maximum when $x = 4\sqrt{3} \approx 6.93$. The number of floors in the building must be an integer, so we evaluate R when $x = 6$ and $x = 7$: $R(6) \approx 0.0377$; $R(7) \approx 0.0378$. Thus 7 floors should be built to maximize the rate of return.

39. $P(j) = Aj\frac{L^4}{V} + B\frac{V^3L^2}{1+j}$

$$\frac{dP}{dj} = \frac{AL^4}{V} - \frac{BV^3L^2}{(1+j)^2} = 0$$

Solving for $(1+j)^2$ gives $(1+j)^2 = \frac{BV^4}{AL^2}$

40. a. $\frac{d}{dv}\left(-2at_r + v - \frac{2al}{v}\right) = 1 + \frac{2al}{v^2} = 0$ when $v = \sqrt{-2al}$. Note that

$$\frac{d^2}{dv^2}\left(-2at_r + v - \frac{2al}{v}\right) = \frac{-4al}{v^3} > 0 \text{ for}$$

$a < 0$, $l > 0$, and $v > 0$. Thus $-2at_r + v - \frac{2al}{v}$ is a minimum for $v = \sqrt{-2al}$.

b. $v = \sqrt{-2(-19.6)(20)} = \sqrt{784} = 28$ ft/s.

c. $N = \frac{-2(-19.6)}{(-2)(-19.6)(0.5)+28-\frac{2(-19.6)(20)}{28}}$

$\approx$ 0.5 cars/s = 0.5(3600) cars/h = 1800 cars/h

d. When $v = \sqrt{-2al}$, then

$$N = N(l) = \frac{-2a}{-2at_r + \sqrt{-2al} + \frac{-2al}{\sqrt{-2al}}}$$
$$= \frac{-2a}{-2at_r + 2\sqrt{-2al}} = \frac{a}{at_r - \sqrt{-2al}}$$

The relative change in N when l is reduced from 20 ft to 15 ft is $\frac{N(15)-N(20)}{N(20)}$.

With $a = -19.6$ ft/s^2 and $t_r = 0.5$ s, then

$$N(20) = \frac{-19.6}{(-19.6)(0.5)-\sqrt{-2(-19.6)(20)}}$$
$$\approx 0.5185$$

$$N(15) = \frac{-19.6}{(-19.6)(0.5)-\sqrt{-2(-19.6)(15)}}$$
$$\approx 0.5756$$

The relative change is

$$\frac{N(15)-N(20)}{N(20)} \approx \frac{0.5756-0.5158}{0.5158} \approx 0.1101$$

41. $\bar{c} = \frac{c}{q} = 3q + 50 - 18\ln(q) + \frac{120}{q},\ q > 0$

$$\frac{d\bar{c}}{dq} = 3 - \frac{18}{q} - \frac{120}{q^2} = \frac{3q^2-18q-120}{q^2}$$
$$= \frac{3\left(q^2-6q-40\right)}{q^2}$$
$$= \frac{3(q-10)(q+4)}{q^2}$$

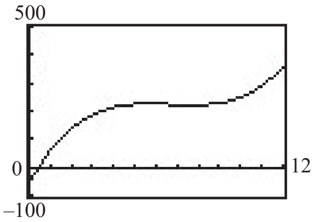
Critical value is $q = 10$ since $q \ge 0$.

Since $\frac{d\bar{c}}{dq} < 0$ for $0 < q < 10$, and $\frac{d\bar{c}}{dq} > 0$ for $q > 10$, we have a minimum when $q = 10$ cases. This minimum average cost is
3(10) + 50 – 18 ln 10 + 12 ≈ \$50.55.

42. The profit function is given by

$$\begin{aligned} P = TR - TC &= q^3 - 20q^2 + 160q - (30q + 50) \\ &= q^3 - 20q^2 + 130q - 50 \end{aligned}$$

where P is in thousands of dollars, q is in tons, and $0 \le q \le 12$. From the graph, the maximum profit occurs when $q = 12$ tons. The corresponding maximum profit is $358,000 and the selling price per ton is $64,000.



Chapter 13 Review Problems

1. $y = \dfrac{3x^2}{x^2 - 16} = \dfrac{3x^2}{(x+4)(x-4)}$

When $x = \pm 4$ the denominator is zero and the numerator is not zero. Thus $x = 4$ and $x = -4$ are vertical asymptotes.

$$\lim_{x\to\infty} \frac{3x^2}{x^2 - 16} = \lim_{x\to\infty} \frac{3x^2}{x^2} = \lim_{x\to\infty} 3 = 3$$

Similarly, $\lim\limits_{x\to-\infty} y = 3$. Thus $y = 3$ is the only horizontal asymptote.

2. $y = \dfrac{x+3}{9x - 3x^2} = \dfrac{x+3}{3x(3-x)}$

When $x = 0$ or $x = 3$, the denominator is zero and the numerator is not zero. Thus $x = 0$ and $x = 3$ are vertical asymptotes.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{x}{-3x^2} = -\frac{1}{3} \lim_{x\to\infty} \frac{1}{x} = 0$$

Similarly, $\lim\limits_{x\to-\infty} y = 0.$ Thus $y = 0$ is the only horizontal asymptote.

3. $y = \dfrac{5x^2 - 3}{(3x+2)^2} = \dfrac{5x^2 - 3}{9x^2 + 12x + 4}$

When $x = -\dfrac{2}{3}$, the denominator is zero and the numerator is not zero. Thus $x = -\dfrac{2}{3}$ is a vertical asymptote.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{5x^2}{9x^2} = \lim_{x\to\infty} \frac{5}{9} = \frac{5}{9}$$

Similarly, $\lim\limits_{x\to-\infty} y = \dfrac{5}{9}$. Thus $y = \dfrac{5}{9}$ is the only horizontal asymptote.

4. $y = \dfrac{4x+1}{3x-5} - \dfrac{3x+1}{2x-11} = \dfrac{-x^2 - 30x - 6}{(3x-5)(2x-11)}$

When $x = \dfrac{5}{3}$ or $x = \dfrac{11}{2}$, the denominator is zero and the numerator is not zero. Thus $x = \dfrac{5}{3}$ and $x = \dfrac{11}{2}$ are vertical asymptotes.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{-x^2}{6x^2} = \lim_{x\to\infty} \left(-\frac{1}{6}\right) = -\frac{1}{6}$$

Similarly, $\lim\limits_{x\to-\infty} y = -\dfrac{1}{6}$. Thus $y = -\dfrac{1}{6}$ is the only horizontal asymptote.

5. $f(x) = \dfrac{3x^2}{9 - x^2}$

$$f'(x) = \frac{(9 - x^2)(6x) - 3x^2(-2x)}{(9 - x^2)^2} = \frac{54x}{(9 - x^2)^2}$$

Thus $x = 0$ is the only critical value.
Note: Although $f'(3)$ is not defined, ±3 are not critical values because ±3 are not in the domain of *f*.

6. $f(x) = 8(x-1)^2(x+6)^4$

$$\begin{aligned} f'(x) &= 8(2)(x-1)(x+6)^4 + 8(x-1)^2(4)(x+6)^3 \\ &= 16(x-1)(x+6)^3[x + 6 + 2(x-1)] \\ &= 16(x-1)(x+6)^3(3x+4) \end{aligned}$$

Thus $x = 1$, $x = -6$, and $x = -\dfrac{4}{3}$ are the critical values.

7. $f(x) = \dfrac{\sqrt[3]{x+1}}{3 - 4x}$

$$f'(x) = \frac{(3-4x)\left[\frac{1}{3}(x+1)^{-\frac{2}{3}}\right] - (x+1)^{\frac{1}{3}}(-4)}{(3-4x)^2}$$

$$= \frac{\frac{1}{3}(x+1)^{-\frac{2}{3}}[(3-4x) + 12(x+1)]}{(3-4x)^2}$$

$$= \frac{8x + 15}{3(x+1)^{\frac{2}{3}}(3-4x)^2}$$

$f'(x)$ is zero when $x=-\frac{15}{8}$; $f'(x)$ is not defined when $x = -1$ or $x=\frac{3}{4}$. However $\frac{3}{4}$ is not in the domain of *f*. Thus $x=-\frac{15}{8}$ and $x = -1$ are critical values.

8. $f(x)=\frac{13xe^{-\frac{5x}{6}}}{6x+5}$

$$f'(x)=13\cdot\frac{(6x+5)\left[x\left(-\frac{5}{6}e^{-\frac{5x}{6}}\right)+e^{-\frac{5x}{6}}(1)\right]-xe^{-\frac{5x}{6}}(6)}{(6x+5)^2}$$

$$=\frac{13}{6}\cdot\frac{-e^{-\frac{5x}{6}}\{(6x+5)[5x-6]+36x\}}{(6x+5)^2}=\frac{13}{6}\cdot\frac{-\left\{30x^2+25x-30\right\}}{e^{\frac{5x}{6}}(6x+5)^2}$$

$$=\frac{13}{6}\cdot\frac{-5\left(6x^2+5x-6\right)}{e^{\frac{5x}{6}}(6x+5)^2}=\frac{-65(2x+3)(3x-2)}{6e^{\frac{5x}{6}}(6x+5)^2}$$

$f'(x)$ is zero when $x=-\frac{3}{2}$ or $x=\frac{2}{3}$. Although $f'(x)$ is not defined when $x=-\frac{5}{6}$, $-\frac{5}{6}$ is not in the domain of *f*. Thus $x=-\frac{3}{2}$ and $x=\frac{2}{3}$ are the only critical values.

9. $f(x)=-\frac{5}{3}x^3+15x^2+35x+10$

$$f'(x)=-5x^2+30x+35$$
$$=-5(x^2-6x-7)=-5(x-7)(x+1)$$

CV: $x = -1$ and $x = 7$. Decreasing on (−∞, −1) and (7, ∞); increasing on (−1, 7)

10. $f(x)=\frac{3x^2}{(x+2)^2}$

$$f'(x)=\frac{6x(x+2)^2-3x^2(2)(x+2)}{(x+2)^4}$$
$$=\frac{(x+2)(6x^2+12x-6x^2)}{(x+2)^4}$$
$$=\frac{12x}{(x+2)^3}$$

CV: $x = 0$, but $x = –2$ is also considered in the inc.-dec. analysis. Increasing on (–∞, –2) and (0, ∞); decreasing on (–2, 0).

11. $f(x)=\dfrac{6x^4}{x^2-3}$

$$f'(x)=6\cdot\frac{\left(x^2-3\right)\left(4x^3\right)-x^4(2x)}{\left(x^2-3\right)^2}$$

$$=\frac{12x^3\left[2\left(x^2-3\right)-x^2\right]}{\left(x^2-3\right)^2}=\frac{12x^3\left(x^2-6\right)}{\left(x^2-3\right)^2}$$

$$=\frac{12x^3\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)}{\left[\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)\right]^2}$$

CV: $x=0,\ \pm\sqrt{6}$, but $x=\pm\sqrt{3}$ must also be considered in the inc.-dec. analysis. Decreasing on $\left(-\infty,-\sqrt{6}\right)$, $\left(0,\sqrt{3}\right)$, and $\left(\sqrt{3},\sqrt{6}\right)$; increasing on $\left(-\sqrt{6},-\sqrt{3}\right)$, $\left(-\sqrt{3},0\right)$ and $\left(\sqrt{6},\infty\right)$.

12. $f(x)=4\sqrt[3]{5x^3-7x}$

$$f'(x)=4\cdot\frac{1}{3}(5x^3-7x)^{-2/3}(15x^2-7)$$

$$=\frac{4(15x^2-7)}{3(5x^3-7x)^{2/3}}$$

$$=\frac{4\left(\sqrt{15}x+\sqrt{7}\right)\left(\sqrt{15}x-\sqrt{7}\right)}{3[x(5x^2-7)]^{2/3}}$$

$$=\frac{4\left(\sqrt{15}x+\sqrt{7}\right)\left(\sqrt{15}x-\sqrt{7}\right)}{3\left[x\left(\sqrt{5}x+\sqrt{7}\right)\left(\sqrt{5}x-\sqrt{7}\right)\right]^{2/3}}$$

CV: $x=\pm\sqrt{\dfrac{7}{15}},\ 0,\ \pm\sqrt{\dfrac{7}{5}}$

Increasing on $\left(-\infty,-\sqrt{\dfrac{7}{5}}\right)$, $\left(-\sqrt{\dfrac{7}{5}},-\sqrt{\dfrac{7}{15}}\right)$, $\left(\sqrt{\dfrac{7}{15}},\sqrt{\dfrac{7}{5}}\right)$, and $\left(\sqrt{\dfrac{7}{5}},\infty\right)$; decreasing on $\left(-\sqrt{\dfrac{7}{15}},0\right)$ and $\left(0,\sqrt{\dfrac{7}{15}}\right)$.

13. $f(x)=x^4-x^3-14$

$f'(x)=4x^3-3x^2$

$f''(x)=12x^2-6x=6x(2x-1)$

$f''(x)=0$ when $x=0$ or $x=\dfrac{1}{2}$. Concave up on $(-\infty,0)$ and $\left(\dfrac{1}{2},\infty\right)$; concave down on $\left(0,\dfrac{1}{2}\right)$.

14. $f(x)=\dfrac{x-2}{x+2}$

$$f'(x)=\frac{(x+2)(1)-(x-2)(1)}{(x+2)^2}=\frac{4}{(x+2)^2}$$

$$f''(x)=-\frac{8}{(x+2)^3}$$

$f''(x)$ is not defined when $x=-2$. Concave up on $(-\infty,-2)$; concave down on $(-2,\infty)$

15. $f(x)=\dfrac{1}{3x+2}=(3x+2)^{-1}$

$f'(x)=-(3x+2)^{-2}(3)=-3(3x+2)^{-2}$

$$f''=6(3x+2)^{-3}(3)=\frac{18}{(3x+2)^3}$$

$f''(x)$ is not defined when $x=-\dfrac{2}{3}$. Concave down on $\left(-\infty,-\dfrac{2}{3}\right)$; concave up on $\left(-\dfrac{2}{3},\infty\right)$.

16. $f(x)=x^3+2x^2-5x+2$

$f'(x)=3x^2+4x-5$

$f''(x)=6x+4=2(3x+2)$

$f''(x)=0$ when $x=-\dfrac{2}{3}$. Concave down on $\left(-\infty,-\dfrac{2}{3}\right)$; concave up on $\left(-\dfrac{2}{3},\infty\right)$.

17. $f(x)=(2x+1)^3(3x+2)$

$$\begin{aligned}f'(x)&=(2x+1)^3(3)+(3x+2)[3(2x+1)^2(2)]\\&=3(2x+1)^2(2x+1+6x+4)\\&=3(2x+1)^2(8x+5)\end{aligned}$$

$$\begin{aligned}f''(x)&=3\{(2x+1)^2(8)+(8x+5)[2(2x+1)(2)]\}\\&=12(2x+1)[2(2x+1)+8x+5]\\&=12(2x+1)(12x+7)\end{aligned}$$

$f''(x)=0$ when $x=-\frac{1}{2}$ or $x=-\frac{7}{12}$. Concave up on $\left(-\infty, -\frac{7}{12}\right)$ and $\left(-\frac{1}{2}, \infty\right)$; concave down on $\left(-\frac{7}{12}, -\frac{1}{2}\right)$.

18. $f(x)=\left(x^2-x-1\right)^2$

$f'(x)=2\left(x^2-x-1\right)(2x-1)$

$=2\left(2x^3-3x^2-x+1\right)$

$f''(x)=2\left(6x^2-6x-1\right)$

$f''(x)=0$ when $6x^2-6x-1=0$; by the quadratic formula $x=\frac{1}{2}\pm\frac{\sqrt{15}}{6}$. Concave up on $\left(-\infty, \frac{1}{2}-\frac{\sqrt{15}}{6}\right)$ and $\left(\frac{1}{2}+\frac{\sqrt{15}}{6}, \infty\right)$; concave down on $\left(\frac{1}{2}-\frac{\sqrt{15}}{6}, \frac{1}{2}+\frac{\sqrt{15}}{6}\right)$.

19. $f(x)=2x^3-9x^2+12x+7$

$f'(x)=6x^2-18x+12=6\left(x^2-3x+2\right)$

$= 6(x-1)(x-2)$

CV: $x = 1$ and $x = 2$

Increasing on $(-\infty, 1)$ and $(2, \infty)$; decreasing on $(1, 2)$. Relative maximum when $x = 1$; relative minimum when $x = 2$.

20. $f(x)=\frac{ax+b}{x^2}$ for $a > 0$ and $b > 0$

$$f'(x)=\frac{x^2(a)-(ax+b)(2x)}{x^4}$$

$$=\frac{ax^2-2ax^2-2bx}{x^4}$$

$$=\frac{-ax^2-2bx}{x^4}$$

$$=\frac{-ax-2b}{x^3}$$

CV: $x=-\frac{2b}{a}$, but $x = 0$ must be considered in inc.-dec. analysis. Decreasing on $\left(-\infty, -\frac{2b}{a}\right)$ and $(0, \infty)$; increasing on $\left(-\frac{2b}{a}, 0\right)$. Relative minimum when $x=-\frac{2b}{a}$.

21. $f(x)=\frac{x^{10}}{10}+\frac{x^5}{5}$

$f'(x)=x^9+x^4=x^4(x^5+1)$

CV: $x = 0$ and $x = -1$

Decreasing on $(-\infty, -1)$; increasing on $(-1, 0)$ and $(0, \infty)$; relative minimum when $x = -1$

22. $f(x)=\frac{x^2}{x^2-4}$

$$f'(x)=\frac{\left(x^2-4\right)(2x)-x^2(2x)}{\left(x^2-4\right)^2}$$

$$=\frac{2x\left[\left(x^2-4\right)-x^2\right]}{\left(x^2-4\right)^2}=\frac{-8x}{\left(x^2-4\right)^2}$$

$$=-\frac{8x}{[(x+2)(x-2)]^2}$$

CV: $x = 0$, but x ±2 must be considered in inc.-dec. analysis. Increasing on $(-\infty, -2)$ and $(-2, 0)$; decreasing on $(0, 2)$ and $(2, \infty)$. Relative maximum when $x = 0$.

23. $f(x)=x^{\frac{2}{3}}(x+1)=x^{\frac{5}{3}}+x^{\frac{2}{3}}$

$$f'(x)=\frac{5}{3}x^{\frac{2}{3}}+\frac{2}{3}x^{-\frac{1}{3}}=\frac{1}{3}x^{-\frac{1}{3}}(5x+2)=\frac{5x+2}{3x^{\frac{1}{3}}}$$

CV: $x = 0$ and $x=-\frac{2}{5}$

Increasing on $\left(-\infty, -\frac{2}{5}\right)$ and $(0, \infty)$; decreasing on $\left(-\frac{2}{5}, 0\right)$. Relative maximum when $x=-\frac{2}{5}$; relative minimum when $x = 0$.

24. $f(x)=x^3(x-2)^4$

$f'(x)=x^3[4(x-2)^3(1)]+(x-2)^4(3x^2)$

$=x^2(x-2)^3[4x+3(x-2)]$

$=x^2(x-2)^3(7x-6)$

CV: $x = 0, 2, \frac{6}{7}$

Increasing on $(-\infty, 0)$, $\left(0, \frac{6}{7}\right)$, and $(2, \infty)$; decreasing on $\left(\frac{6}{7}, 2\right)$. Relative maximum when $x = \frac{6}{7}$; relative minimum when $x = 2$.

25. $y = 3x^5 + 20x^4 - 30x^3 - 540x^2 + 2x + 3$

$y' = 15x^4 + 80x^3 - 90x^2 - 1080x + 2$

$$\begin{aligned} y'' &= 60x^3 + 240x^2 - 180x - 1080 \\ &= 60(x^3 + 4x^2 - 3x - 18) \\ &= 60(x-2)(x+3)^2 \end{aligned}$$

Possible inflection points occur when $x = 2$ or $x = -3$. Concave down on $(-\infty, -3)$ and $(-3, 2)$; concave up on $(2, \infty)$. Concxavity changes at $x = 2$, so there is an inflection point when $x = 2$.

26. $y = \frac{x^2+2}{5x} = \frac{1}{5}x + \frac{2}{5}x^{-1}$

$y' = \frac{1}{5}(1 - 2x^{-2})$

$y'' = \frac{4}{5}x^{-3} = \frac{4}{5x^3}$

y'' is never zero. Although y'' is not defined when $x = 0$, y is not continuous there. Thus there is no inflection point.

27. $y = 4(3x-5)\left(x^4+2\right) = 12x^5 - 20x^4 + 24x - 40$

$y' = 60x^4 - 80x^3 + 24$

$y'' = 240x^3 - 240x^2 = 240x^2(x-1)$

Possible inflection points occur when $x = 0$ or $x = 1$. Concave down on $(-\infty, 0)$ and $(0, 1)$; concave up on $(1, \infty)$. Inflection point when $x = 1$.

28. $y = x^2 + 2\ln(-x)$ (Note: $x < 0$)

$y' = 2x + \frac{2}{x}$

$y'' = 2 - \frac{2}{x^2} = \frac{2x^2-2}{x^2} = \frac{2(x+1)(x-1)}{x^2}$

Possible inflection point occurs when $x = -1$. Concave up on $(-\infty, -1)$; concave down on $(-1, 0)$. Inflection point when $x = -1$.

29. $y = \frac{x^3}{e^x} = x^3e^{-x}$

$y' = x^3(-e^{-x}) + e^{-x}(3x^2) = -e^{-x}(x^3 - 3x^2)$

$$\begin{aligned} y'' &= -e^{-x}(3x^2 - 6x) - (x^3 - 3x^2)(-e^{-x}) \\ &= e^{-x}(x^3 - 6x^2 + 6x) \\ &= xe^{-x}(x^2 - 6x + 6) \end{aligned}$$

y'' is defined for all x and y'' is zero only when $x = 0$ or $x^2 - 6x + 6 = 0$. Using the quadratic formula on the second equation, the possible points of inflection occur when $x = 0, 3 \pm \sqrt{3}$. Concave up on $\left(0, 3-\sqrt{3}\right)$ and $\left(3+\sqrt{3}, \infty\right)$; concave down on $(-\infty, 0)$ and $\left(3-\sqrt{3}, 3+\sqrt{3}\right)$. Inflection points when $x = 0, 3 \pm \sqrt{3}$.

30. $y = (x^2-5)^3$

$y' = 3(x^2-5)^2(2x) = 6x(x^2-5)^2$

$$\begin{aligned} y'' &= 6(x^2-5)^2 + 6x(2)(x^2-5)(2x) \\ &= 6(x^2-5)(x^2-5+4x^2) \\ &= 6(x^2-5)(5x^2-5) \\ &= 30(x^2-5)(x^2-1) \\ &= 30\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)(x+1)(x-1) \end{aligned}$$

Possible inflection points occur when $x = \pm\sqrt{5}$ or $x = \pm 1$. Concave up on $\left(-\infty, -\sqrt{5}\right)$, $(-1, 1)$, and $\left(\sqrt{5}, \infty\right)$; concave down on $\left(-\sqrt{5}, -1\right)$ and $\left(1, \sqrt{5}\right)$. Inflection points when $x = \pm\sqrt{5}, \pm 1$.

31. $f(x) = 3x^4 - 4x^3$ and f is continuous on $[0, 2]$.

$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

The only critical value on $(0, 2)$ is $x = 1$. Evaluating f at this value and at the endpoints gives $f(0) = 0$, $f(1) = -1$, and $f(2) = 16$. Absolute maximum: $f(2) = 16$; absolute minimum: $f(1) = -1$.

32. $f(x) = 2x^3 - 15x^2 + 36x$ and f is continuous on $[0, 3]$.

$f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$

The only critical value on $(0, 3)$ is $x = 2$. Evaluating f at this value and at the endpoints gives $f(0) = 0$, $f(2) = 28$, $f(3) = 27$. Absolute maximum: $f(2) = 28$; absolute minimum: $f(0) = 0$.

33. $f(x) = \dfrac{x}{(5x-6)^2}$ and f is continuous on [–2, 0].

$$f'(x) = \frac{(5x-6)^2(1) - x[10(5x-6)]}{(5x-6)^4}$$
$$= \frac{(5x-6)[(5x-6)-10x]}{(5x-6)^4} = \frac{-5x-6}{(5x-6)^3}$$
$$= -\frac{5x+6}{(5x-6)^3}$$

The only critical value on (–2, 0) is $x = -\frac{6}{5}$. Evaluating f at this value and at the endpoints gives $f(-2) = -\frac{1}{128}$, $f\left(-\frac{6}{5}\right) = -\frac{1}{120}$ and $f(0) = 0$. Absolute maximum: $f(0) = 0$; absolute minimum: $f\left(-\frac{6}{5}\right) = -\frac{1}{120}$.

34. $f(x) = (x+1)^2(x-1)^{2/3}$ and f is continuous on [2, 3].

$$f'(x) = (x+1)^2\left[\frac{2}{3}(x-1)^{-1/3}\right] + (x-1)^{2/3}[2(x+1)]$$
$$= \frac{2}{3}(x+1)(x-1)^{-1/3}[(x+1) + 3(x-1)]$$
$$= \frac{4}{3}(x+1)(x-1)^{-1/3}(2x-1) = \frac{4(x+1)(2x-1)}{3(x-1)^{1/3}}$$

There are no critical values on [2, 3]. Evaluating f at the endpoints gives $f(2) = 9$ and $f(3) = 16(2^{2/3}) \approx 25.4$. Absolute maximum $f(3) = 16(2^{2/3}) \approx 25.4$; absolute minimum: $f(2) = 9$

35. $f(x) = x \ln x$

a. $f'(x) = \ln x + x\left(\frac{1}{x}\right) = \ln x + 1$

$$\ln x + 1 = 0$$
$$\ln x = -1$$
$$x = e^{-1} = \frac{1}{e}$$

CV: $x = \frac{1}{e}$

Decreasing on $\left(0, \frac{1}{e}\right)$ and increasing on $\left(\frac{1}{e}, \infty\right)$. Relative minimum at $x = \frac{1}{e}$.

b. $f''(x) = \frac{1}{x}$

$f'' > 0$ for all x in the domain of f. Concave up for $(0, \infty)$; there are no points of inflection.