Chapter 11

Apply It 11.1

1. $\frac{dH}{dt} = \frac{d}{dt}\left(6+40t-16t^2\right)$

$= \lim_{h\to 0} \frac{H(t+h)-H(t)}{h}$

$= \lim_{h\to 0} \frac{\left[6+40(t+h)-16(t+h)^2\right]-\left(6+40t-16t^2\right)}{h}$

$= \lim_{h\to 0} \frac{6+40t+40h-16t^2-32th-16h^2-6-40t+16t^2}{h}$

$= \lim_{h\to 0} \frac{40h-32th-16h^2}{h} = \lim_{h\to 0}(40-32t-16h)$

$= 40 - 32t$

$\frac{dH}{dt} = 40-32t$

Problems 11.1

1. a. $f(x) = x^3+3$, $P = (-2, -5)$

To begin, if $x = -3$, then $m_{PQ} = \frac{[(-3)^3+3]-(-5)}{-3-(-2)} = 19$. If $x = -2.5$, then $m_{PQ} = \frac{[(-2.5)^3+3]-(-5)}{-2.5-(-2)} = 15.25$.

Continuing in this manner, we complete the table:

x-value of Q	−3	−2.5	−2.2	−2.1	−2.01	−2.001
m_{PQ}	19	15.25	13.24	12.61	12.0601	12.0060

b. We estimate that $m_{\tan}$ at P is 12.

2. a. $f(x) = e^x$, $P = (0, 1)$

To begin, if $x = 1$, then $m_{PQ} = \frac{e^1-1}{1-0} \approx 1.7183$. If $x = 0.5$, then $m_{PQ} = \frac{e^{0.5}-1}{0.5-0} \approx 1.2974$.

Continuing in this manner, we complete the table:

x-value of Q	1	0.5	0.2	0.1	0.01	0.001
m_{PQ}	1.7183	1.2974	1.1070	1.0517	1.0050	1.0005

b. We estimate that $m_{\tan}$ at P is 1.

3. $f(x) = x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)-x}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1$

4. $f(x) = 4x - 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[4(x+h)-1]-[4x-1]}{h}$$
$$= \lim_{h\to 0} \frac{4h}{h} = \lim_{h\to 0} 4 = 4$$

5. $y = 3x + 5$. Let $y = f(x)$.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[3(x+h)+5]-[3x+5]}{h}$$
$$= \lim_{h\to 0} \frac{3h}{h} = \lim_{h\to 0} 3 = 3$$

6. $y = -5x$. Let $y = f(x)$.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[-5(x+h)]-[-5x]}{h}$$
$$= \lim_{h\to 0} \frac{-5h}{h} = \lim_{h\to 0}(-5) = -5$$

7. Let $f(x) = 3 - 2x$.

$$\frac{d}{dx}(3-2x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[3-2(x+h)]-[3-2x]}{h}$$
$$= \lim_{h\to 0} \frac{-2h}{h} = \lim_{h\to 0}(-2) = -2$$

8. Let $f(x) = 1 - \frac{x}{2}$

$$\frac{d}{dx}\left(1-\frac{x}{2}\right) = \lim_{h\to 0} \frac{\left[1-\frac{x+h}{2}\right]-\left[1-\frac{x}{2}\right]}{h}$$
$$= \lim_{h\to 0} \frac{-\frac{h}{2}}{h} = \lim_{h\to 0}\left(-\frac{1}{2}\right) = -\frac{1}{2}$$

9. $f(x) = 3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{3-3}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0$$

10. $f(x) = 7.01$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{7.01-7.01}{h}$$
$$= \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0$$

11. Let $f(x) = x^2 + 4x - 8$.

$$\frac{d}{dx}\left(x^2+4x-8\right)$$
$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\left[(x+h)^2+4(x+h)-8\right]-\left[x^2+4x-8\right]}{h}$$
$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+4x+4h-8-x^2-4x+8}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2+4h}{h}$$
$$= \lim_{h\to 0}(2x+h+4) = 2x+0+4 = 2x+4$$

12. $y = x^2 + 3x + 2$. Let $y = f(x)$.

$$y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[(x+h)^2+3(x+h)+2]-[x^2+3x+2]}{h}$$
$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+3x+3h+2-x^2-3x-2}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2+3h}{h}$$
$$= \lim_{h\to 0}(2x+h+3) = 2x+0+3 = 2x+3$$

13. $p = f(q) = 3q^2 + 2q + 1$

$$\frac{dp}{dq} = \lim_{h\to 0} \frac{f(q+h)-f(q)}{h}$$
$$= \lim_{h\to 0} \frac{\left[3(q+h)^2+2(q+h)+1\right]-\left[3q^2+2q+1\right]}{h}$$
$$= \lim_{h\to 0} \frac{6qh+3h^2+2h}{h}$$
$$= \lim_{h\to 0}(6q+3h+2) = 6q+0+2 = 6q+2$$

14. Let $f(x) = x^2 - x - 3$.

$$\frac{d}{dx}\left(x^2 - x - 3\right)$$
$$= \lim \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\left[(x+h)^2 - (x+h) - 3\right] - \left[x^2 - x - 3\right]}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2 - h}{h} = \lim_{h\to 0}(2x + h - 1) = 2x - 1$$

15. $y = f(x) = \frac{6}{x}$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{6}{x+h} - \frac{6}{x}}{h}$$

Multiplying the numerator and denominator by $x(x + h)$ gives

$$y' = \lim_{h\to 0} \frac{6x - 6(x+h)}{hx(x+h)} = \lim_{h\to 0} \frac{-6h}{hx(x+h)}$$
$$= \lim_{h\to 0}\left[-\frac{6}{x(x+h)}\right] = -\frac{6}{x(x+0)} = -\frac{6}{x^2}$$

16. $C = f(q) = 7 + 2q - 3q^2$

$$\frac{dC}{dq} = \lim_{h\to 0} \frac{f(q+h) - f(q)}{h}$$
$$= \lim_{h\to 0} \frac{\left[7 + 2(q+h) - 3(q+h)^2\right] - \left[7 + 2q - 3q^2\right]}{h}$$
$$= \lim_{h\to 0} \frac{2h - 6qh - 3h^2}{h} = \lim_{h\to 0}(2 - 6q - 3h)$$
$$= 2 - 6q$$

17. $f(x) = \sqrt{2x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Rationalizing the numerator gives

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{(x+h)} + \sqrt{2x}}$$
$$= \lim_{h\to 0} \frac{2(x+h) - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$
$$= \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)} + \sqrt{2x}}$$
$$= \frac{1}{\sqrt{2x}}$$

18. $H(x) = \frac{3}{x-2}$

$$H'(x) = \lim_{h\to 0} \frac{H(x+h) - H(x)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{3}{x+h-2} - \frac{3}{x-2}}{h}$$

Multiplying the numerator and denominator by $(x + h - 2)(x - 2)$ gives

$$H'(x) = \lim_{h\to 0} \frac{3(x-2) - 3(x+h-2)}{h(x+h-2)(x-2)}$$
$$= \lim_{h\to 0} \frac{-3h}{h(x+h-2)(x-2)}$$
$$= \lim_{h\to 0} \frac{-3}{(x+h-2)(x-2)} = -\frac{3}{(x-2)^2}$$

19. $y = f(x) = x^2 + 4$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\left[(x+h)^2 + 4\right] - \left[x^2 + 4\right]}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0}(2x + h) = 2x + 0 = 2x$$

The slope at (–2, 8) is $y'(-2) = 2(-2) = -4$.

20. $y = f(x) = 1 - x^2$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\left[1 - (x+h)^2\right] - \left[1 - x^2\right]}{h}$$
$$= \lim_{h\to 0} \frac{-2xh - h^2}{h}$$
$$= \lim_{h\to 0}(-2x - h) = -2x$$

The slope at (1, 0) is $y'(1) = -2(1) = -2$.

21. $y = f(x) = 4x^2 - 5$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left[4(x+h)^2 - 5\right] - \left[4x^2 - 5\right]}{h}$$

$$= \lim_{h\to 0} \frac{8xh + 4h^2}{h} = \lim_{h\to 0}(8x + 4h) = 8x$$

The slope when $x = 0$ is $y'(0) = 8(0) = 0$.

22. $y = f(x) = \sqrt{2x}$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Rationalizing the numerator gives

$$y' = \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= \lim_{h\to 0} \frac{2(x+h) - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= \frac{2}{2\sqrt{2x}}$$

$$= \frac{1}{\sqrt{2x}}$$

If $x = 18$, the slope is $y'(18) = \frac{1}{\sqrt{2(18)}} = \frac{1}{6}$.

23. $y = x + 4$

$$y' = \lim_{h\to 0} \frac{[(x+h)+4] - [x+4]}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

If $x = 3$, then $y' = 1$. The tangent line at the point (3, 7) is $y - 7 = 1(x - 3)$, or $y = x + 4$.

24. $y = 3x^2 - 4$

$$y' = \lim_{h\to 0} \frac{[3(x+h)^2 - 4] - [3x^2 - 4]}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2}{h} = \lim_{h\to 0}(6x + 3h) = 6x$$

If $x = 1$, then $y' = 6(1) = 6$.

The tangent line at (1, –1) is $y + 1 = 6(x - 1)$ or $y = 6x - 7$.

25. $y = x^2 + 2x + 3$

$$y' = \lim_{h\to 0} \frac{\left[(x+h)^2 + 2(x+h) + 3\right] - \left[x^2 + 2x + 3\right]}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h\to 0}(2x + h + 2) = 2x + 2$$

If $x = 1$, then $y' = 2(1) + 2 = 4$. The tangent line at the point (1, 6) is $y - 6 = 4(x - 1)$, or $y = 4x + 2$.

26. $y = (x-7)^2 = x^2 - 14x + 49$

$$y' = \lim_{h\to 0} \frac{\left[(x+h)^2 - 14(x+h) + 49\right] - \left[x^2 - 14x + 49\right]}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 14h}{h} = \lim_{h\to 0}(2x + h - 14) = 2x - 14$$

If $x = 6$, then $y' = 2(6) - 14 = -2$. The tangent line at (6, 1) is $y - 1 = -2(x - 6)$, or $y = -2x + 13$.

27. $y = \frac{4}{x+1}$

$$y' = \lim_{h\to 0} \frac{\frac{4}{x+h+1} - \frac{4}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{4x+4-4x-4h-4}{(x+1)(x+h+1)}}{h}$$

$$= \lim_{h\to 0} \frac{-4}{(x+1)(x+h+1)}$$

$$= \frac{-4}{(x+1)^2}$$

If $x = 3$, then $y' = -\frac{4}{4^2} = -\frac{1}{4}$. The tangent line at (3, 1) is $y - 1 = -\frac{1}{4}(x - 3)$, or $y = -\frac{1}{4}x + \frac{7}{4}$.

28. $y = \frac{5}{1-3x}$

$$y' = \lim_{h\to 0} \frac{\frac{5}{1-3(x+h)} - \frac{5}{1-3x}}{h}$$

$$= \lim_{h\to 0} \frac{5(1-3x) - 5[1-3(x+h)]}{h[1-3(x+h)](1-3x)}$$

$$= \lim_{h\to 0} \frac{15h}{h[1-3(x+h)](1-3x)}$$

$$= \lim_{h\to 0} \frac{15}{[1-3(x+h)](1-3x)}$$

$$= \frac{15}{(1-3x)^2}$$

If $x = 2$, then $y' = \frac{15}{25} = \frac{3}{5}$. The tangent line at (2, –1) is $y+1 = \frac{3}{5}(x-2)$, or $y = \frac{3}{5}x - \frac{11}{5}$.

29. $r = \left(\frac{\eta}{1+\eta}\right)\left(r_L - \frac{dC}{dD}\right)$

$(1+\eta)r = \eta\left(r_L - \frac{dC}{dD}\right)$

$r + \eta r = \eta\left(r_L - \frac{dC}{dD}\right)$

$r = \eta\left(r_L - \frac{dC}{dD}\right) - \eta r$

$r = \eta\left(r_L - \frac{dC}{dD} - r\right)$

$\eta = \frac{r}{r_L - r - \frac{dC}{dD}}$

30. 1.565, 1.470

31. –3.000, 13.445

32. 0, 2.303

33. –5.120, 0.038

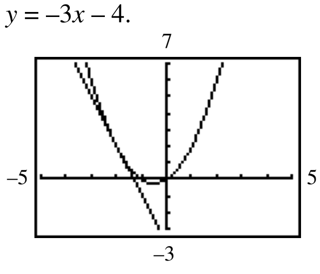
34. $y = f(x) = x^2 + x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

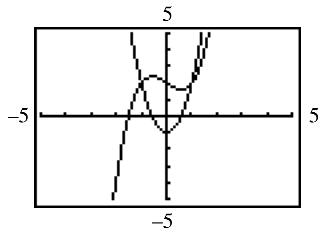
$= \lim_{h\to 0} \frac{\left[(x+h)^2 + (x+h)\right] - \left[x^2 + x\right]}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 + h}{h} = \lim_{h\to 0}(2x + h + 1) = 2x + 1$

If $x = –2$, then $f'(x) = –3$. The tangent line at the point (–2, 2) is $y – 2 = –3(x + 2)$, or $y = –3x – 4$.



35.



For the x-values of the points where the tangent to the graph of f is horizontal, the corresponding values of $f'(x)$ are 0. This is expected because the slope of a horizontal line is zero and the derivative gives the slope of the tangent line.

36. $n = 4$:
$$\begin{aligned}(z-x)\sum_{i=0}^{3} x^i z^{3-i} &= (z-x)(z^3 + xz^2 + x^2z + x^3)\\ &= z^4 - xz^3 + xz^3 - x^2z^2 + x^2z^2 - x^3z + x^3z - x^4\\ &= z^4 - x^4\end{aligned}$$

$n = 3$:
$$\begin{aligned}(z-x)\sum_{i=0}^{2} x^i z^{2-i} &= (z-x)(z^2 + xz + x^2)\\ &= z^3 - xz^2 + xz^2 - x^2z + x^2z - x^3\\ &= z^3 - x^3\end{aligned}$$

$n = 2$: $(z-x)\sum_{i=0}^{1} x^i z^{1-i} = (z-x)(z+x) = z^2 - x^2$

$f(x) = 2x^4 + x^3 - 3x^2$

$$\begin{aligned}f'(x) &= \lim_{z\to x} \frac{f(z) - f(x)}{z-x}\\ &= \lim_{z\to x} \frac{2z^4 + z^3 - 3z^2 - (2x^4 + x^3 - 3x^2)}{z-x}\\ &= \lim_{z\to x} \frac{2(z^4 - x^4) + (z^3 - x^3) - 3(z^2 - x^2)}{z-x}\\ &= \lim_{z\to x} \frac{2(z-x)(z^3 + xz^2 + x^2z + x^3) + (z-x)(z^2 + xz + x^2) - 3(z-x)(z+x)}{z-x}\\ &= \lim_{z\to x} [2(z^3 + xz^2 + x^2z + x^3) + (z^2 + xz + x^2) - 3(z+x)]\\ &= 2(4x^3) + (3x^2) - 3(2x)\\ &= 8x^3 + 3x^2 - 6x\end{aligned}$$

37. $n = 5$:
$$\begin{aligned}(z-x)\sum_{i=0}^{4} x^i z^{4-i} &= (z-x)(z^4 + xz^3 + x^2z^2 + x^3z + x^4)\\ &= z^5 - xz^4 + xz^4 - x^2z^3 + x^2z^3 - x^3z^2 + x^3z^2 - x^4z + x^4z - x^5\\ &= z^5 - x^5\end{aligned}$$

$n = 3$:
$$\begin{aligned}(z-x)\sum_{i=0}^{2} x^i z^{2-i} &= (z-x)(z^2 + xz + x^2)\\ &= z^3 - xz^2 + xz^2 - x^2z + x^2z - x^3\\ &= z^3 - x^3\end{aligned}$$

$$f(x) = 4x^5 - 3x^3$$

$$\begin{aligned}
f'(x) &= \lim_{z\to x} \frac{f(z)-f(x)}{z-x} \\
&= \lim_{z\to x} \frac{4z^5 - 3z^3 - (4x^5 - 3x^3)}{z-x} \\
&= \lim_{z\to x} \frac{4(z^5 - x^5) - 3(z^3 - x^3)}{z-x} \\
&= \lim_{z\to x} \frac{4(z-x)(z^4 + xz^3 + x^2z^2 + x^3z + x^4) - 3(z-x)(z^2 + xz + x^2)}{z-x} \\
&= \lim_{z\to x} [4(z^4 + xz^3 + x^2z^2 + x^3z + x^4) - 3(z^2 + xz + x^2)] \\
&= 4(5x^4) - 3(3x^2) \\
&= 20x^4 - 9x^2
\end{aligned}$$

Apply It 11.2

2. $r'(q) = \frac{d}{dq}(50q - 0.3q^2)$

$= \frac{d}{dq}(50q) - \frac{d}{dq}\left(0.3q^2\right)$

$= 50\frac{d}{dq}(q) - 0.3\frac{d}{dq}\left(q^2\right)$

$= 50(1) - 0.3(2q) = 50 - 0.6q$

The marginal revenue is $r'(q) = 50 - 0.6q$.

Problems 11.2

1. $f(x) = \pi$ is a constant function, so $f'(x) = 0$

2. $f(x) = \left(\frac{6}{7}\right)^{2/3}$ is a constant function, so $f'(x) = 0$

3. $y = x^6,\ y' = 6x^{6-1} = 6x^5$

4. $f'(x) = 21x^{21-1} = 21x^{20}$

5. $y = x^{80},\ \frac{dy}{dx} = 80x^{80-1} = 80x^{79}$

6. $y = x^{2.1},\ y' = 2.1x^{2.1-1} = 2.1x^{1.1}$

7. $f(x) = 9x^2,\ f'(x) = 9\left(2x^{2-1}\right) = 18x$

8. $y' = 4\left(3x^{3-1}\right) = 12x^2$

9. $g(w) = 8w^7,\ g'(w) = 8(7w^{7-1}) = 56w^6$

10. $v'(x) = ex^{e-1}$

11. $y = \frac{3}{5}x^6,\ y' = \frac{3}{5}(6x^{6-1}) = \frac{18}{5}x^5$

12. $f'(p) = \sqrt{3}\left(4p^{4-1}\right) = 4\sqrt{3}p^3$

13. $f(t) = \frac{t^7}{25},\ f'(t) = \frac{1}{25}(7t^{7-1}) = \frac{7}{25}t^6$

14. $y' = \frac{1}{7}\left(7x^{7-1}\right) = x^6$

15. $f(x) = x+3,\ f'(x) = 1+0 = 1$

16. $f'(x) = 5(1) - 0 = 5$

17. $f'(x) = 4(2x) - 2(1) + 0 = 8x - 2$

18. $F'(x) = 5(2x) - 9(1) = 10x - 9$

19. $g'(p) = 4p^{4-1} - 3\left(3p^{3-1}\right) - 0 = 4p^3 - 9p^2$

20. $f'(t) = -13(2t) + 14(1) + 0 = -26t + 14$

21. $y' = 4x^{4-1} - \frac{1}{3}x^{\frac{1}{3}-1} = 4x^3 - \frac{1}{3}x^{-2/3}$

22. $y' = -8\left(4x^{4-1}\right) + 0 = -32x^3$

23. $y' = -13\left(3x^{3-1}\right) + 14(2x) - 2(1) + 0$
$= -39x^2 + 28x - 2$

24. $V'(r) = 8r^{8-1} - 7\left(6r^{6-1}\right) + 3(2r) + 0 = 8r^7 - 42r^5 + 6r$

25. $f'(x) = 2\left(0 - 4x^{4-1}\right) = -8x^3$

26. $\psi'(t) = e(7t^{7-1} - 0) = 7et^6$

27. $g(x) = \frac{1}{3}\left(13 - x^4\right),$
$g'(x) = \frac{1}{3}\left(0 - 4x^{4-1}\right) = -\frac{4}{3}x^3$

28. $f(x) = \frac{5}{2}\left(x^4 - 6\right),\ f'(x) = \frac{5}{2}\left(4x^{4-1} - 0\right) = 10x^3$

29. $h(x) = 4x^4 + x^3 - \frac{9}{2}x^2 + 8x$
$h'(x) = 4\left(4x^{4-1}\right) + 3x^{3-1} - \frac{9}{2}(2x) + 8(1)$
$= 16x^3 + 3x^2 - 9x + 8$

30. $k'(x) = -2(2x) + \frac{5}{3}(1) + 0 = -4x + \frac{5}{3}$

31. $f(x) = \frac{5}{7}x^9 + \frac{3}{5}x^7$
$f'(x) = \frac{5}{7}(9x^8) + \frac{3}{5}(7x^6) = \frac{45}{7}x^8 + \frac{21}{5}x^6$

32. $p'(x) = \frac{1}{7}\left(7x^6\right) + \frac{2}{3}(1) = x^6 + \frac{2}{3}$

33. $f'(x) = \frac{3}{5}x^{\frac{3}{5}-1} = \frac{3}{5}x^{-2/5}$

34. $f'(x) = 2\left(-\frac{14}{5}\right)x^{\left(-\frac{14}{5}\right)-1} = -\frac{28}{5}x^{-\frac{19}{5}}$

35. $y' = \frac{3}{4}x^{\left(\frac{3}{4}\right)-1} + 2\left(\frac{5}{3}x^{\left(\frac{5}{3}\right)-1}\right) = \frac{3}{4}x^{-\frac{1}{4}} + \frac{10}{3}x^{\frac{2}{3}}$

36. $y' = 4(2x^1) - \left(-\frac{3}{5}\right)x^{-\frac{8}{5}} = 8x + \frac{3}{5}x^{-8/5}$

37. $f(x) = 11\sqrt{x} = 11x^{\frac{1}{2}},$
$f'(x) = 11\left(\frac{1}{2}\right)x^{\left(\frac{1}{2}\right)-1} = \frac{11}{2}x^{-\frac{1}{2}} = \frac{11}{2\sqrt{x}}$

38. $y = x^{7/2},\ y' = \frac{7}{2}x^{\frac{7}{2}-1} = \frac{7}{2}x^{5/2}$

39. $f(r) = 6r^{\frac{1}{3}},\ f'(r) = 6\left(\frac{1}{3}r^{-\frac{2}{3}}\right) = 2r^{-\frac{2}{3}}$

40. $y = 4x^{\frac{1}{4}},\ y' = 4\left(\frac{1}{4}x^{-\frac{3}{4}}\right) = x^{-\frac{3}{4}}$

41. $f(x) = x^{-6},\ f'(x) = -6x^{-6-1} = -6x^{-7}$

42. $f'(s) = 2(-3s^{-4}) = -6s^{-4}$

43. $f(x) = x^{-3} + x^{-5} - 2x^{-6}$,

$f'(x) = -3x^{-3-1} + \left(-5x^{-5-1}\right) - 2\left(-6x^{-6-1}\right)$

$= -3x^{-4} - 5x^{-6} + 12x^{-7}$

44. $f'(x) = 100\left(-3x^{-4}\right) + 10\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$= -300x^{-4} + 5x^{-\frac{1}{2}}$

45. $y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -1 \cdot x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$

46. $f(x) = \frac{3}{x^4} = 3x^{-4}$

$f'(x) = 3(-4)x^{-5} = -\frac{12}{x^5}$

47. $y = \frac{8}{x^5} = 8x^{-5}$

$y' = 8\left(-5x^{-6}\right) = -40x^{-6}$

48. $y = \frac{1}{4x^5} = \frac{1}{4}x^{-5}$

$y' = \frac{1}{4}\left(-5x^{-6}\right) = -\frac{5}{4}x^{-6}$

49. $g(x) = \frac{4}{3x^3} = \frac{4}{3}x^{-3}$

$g'(x) = \frac{4}{3}\left(-3x^{-4}\right) = -4x^{-4}$

50. $y = \frac{1}{x^2} = x^{-2},\ y' = -2x^{-3}$

51. $f(t) = \frac{3}{5t^3} = \frac{3}{5}t^{-3}$

$f'(t) = \frac{3}{5}(-3)t^{-4} = -\frac{9}{5t^4}$

52. $g(x) = \frac{7}{9}x^{-1}$

$g'(x) = \frac{7}{9}(-1x^{-2}) = -\frac{7}{9}x^{-2}$

53. $f(x) = \frac{1}{7}x + 7x^{-1}$

$f'(x) = \frac{1}{7}(1) + 7\left(-1x^{-2}\right) = \frac{1}{7} - 7x^{-2}$

54. $\Phi(x) = \frac{1}{3}x^3 - 3x^{-3}$,

$\Phi'(x) = \frac{1}{3}(3x^2) - 3(-3x^{-4}) = x^2 + 9x^{-4}$

55. $f(x) = -9x^{1/3} + 5x^{-2/5}$,

$f'(x) = -9\left(\frac{1}{3}x^{-\frac{2}{3}}\right) + 5\left(-\frac{2}{5}x^{-\frac{7}{5}}\right) = -3x^{-\frac{2}{3}} - 2x^{-\frac{7}{5}}$

56. $f(z) = 5z^{3/4} - 6^2 - 8z^{1/4}$

$f'(z) = 5\left(\frac{3}{4}\right)z^{-1/4} - 0 - 8\left(\frac{1}{4}\right)z^{-3/4}$

$= \frac{15}{4}z^{-1/4} - 2z^{-3/4}$

57. $q(x) = \frac{1}{\sqrt[3]{8}\sqrt[3]{x^2}} = \frac{1}{2x^{2/3}} = \frac{1}{2}x^{-2/3}$

$q'(x) = \frac{1}{2}\left(-\frac{2}{3}x^{-5/3}\right) = -\frac{1}{3}x^{-5/3}$

58. $f(x) = \frac{3}{\sqrt[4]{x^3}} = 3x^{-\frac{3}{4}}$

$f'(x) = 3\left(-\frac{3}{4}x^{-\frac{7}{4}}\right) = -\frac{9}{4}x^{-\frac{7}{4}}$

59. $y = \frac{2}{x^{\frac{1}{2}}} = 2x^{-\frac{1}{2}}$

$y' = 2\left(-\frac{1}{2}x^{-\frac{3}{2}}\right) = -x^{-\frac{3}{2}}$

60. $y = \frac{1}{2}x^{-\frac{1}{2}}$

$y' = -\frac{1}{4}x^{-\frac{3}{2}}$

61. $y = x^3\sqrt[3]{x} = x^{9/3} \cdot x^{1/3} = x^{10/3}$

$y' = \frac{10}{3}x^{7/3}$

62. $f(x) = (8x^5),\ f'(x) = 40x^4$

63. $f(x) = x\left(3x^2 - 10x + 7\right) = 3x^3 - 10x^2 + 7x$

$f'(x) = 9x^2 - 20x + 7$

64. $f(x) = 3x^9 - 5x^5 + 4x^3$

$f'(x) = 27x^8 - 25x^4 + 12x^2$

$= x^2\left(27x^6 - 25x^2 + 12\right)$

65. $f(x) = x^3(3x)^2 = x^3\left(9x^2\right) = 9x^5$

$f'(x) = 45x^4$

66. $s(x) = \sqrt{x}\left(\sqrt[5]{x} + 7x + 2\right)$

$= x^{1/2}(x^{1/5} + 7x^1 + 2)$

$= x^{7/10} + 7x^{3/2} + 2x^{1/2}$

$s'(x) = \frac{7}{10}x^{-3/10} + 7\left(\frac{3}{2}\right)x^{1/2} + 2\left(\frac{1}{2}\right)x^{-1/2}$

$= \frac{7}{10}x^{-3/10} + \frac{21}{2}x^{1/2} + x^{-1/2}$

67. $v(x) = x^{-\frac{2}{3}}(x+5) = x^{\frac{1}{3}} + 5x^{-\frac{2}{3}}$

$v'(x) = \frac{1}{3}x^{-\frac{2}{3}} - \frac{10}{3}x^{-\frac{5}{3}} = \frac{1}{3}x^{-\frac{5}{3}}(x-10)$

68. $f(x) = x^{\frac{3}{5}}\left(x^2 + 7x + 11\right) = x^{\frac{13}{5}} + 7x^{\frac{8}{5}} + 11x^{\frac{3}{5}}$

$f'(x) = \frac{13}{5}x^{\frac{8}{5}} + \frac{56}{5}x^{\frac{3}{5}} + \frac{33}{5}x^{-\frac{2}{5}}$

$= \frac{1}{5}x^{-\frac{2}{5}}\left(13x^2 + 56x + 33\right)$

69. $f(q) = \frac{3q^2 + 4q - 2}{q} = \frac{3q^2}{q} + \frac{4q}{q} - \frac{2}{q^2}$

$= 3q + 4 - 2q^{-1}$

$f'(q) = 3(1) + 0 - 2(-q^{-2}) = 3 + 2q^{-2} = 3 + \frac{2}{q^2}$

70. $f(w) = \frac{w-5}{w^5} = w^{-4} - 5w^{-5}$

$f'(w) = -4w^{-5} + 25w^{-6} = -w^{-6}(4w - 25)$

71. $f(x) = (x-1)(x+2) = x^2 + x - 2$

$f'(x) = 2x + 1$

72. $f(x) = x^2(x-2)(x+4) = x^4 + 2x^3 - 8x^2$

$f'(x) = 4x^3 + 6x^2 - 16x = 2x\left(2x^2 + 3x - 8\right)$

73. $w(x) = \frac{x^2 + x^3}{x^2} = \frac{x^2}{x^2} + \frac{x^3}{x^2} = 1 + x$

$w'(x) = 0 + 1 = 1$

74. $f(x) = \frac{7x^3 + x}{6\sqrt{x}}$

$= \frac{1}{6}\left(\frac{7x^3}{x^{1/2}} + \frac{x}{x^{1/2}}\right)$

$= \frac{1}{6}(7x^{5/2} + x^{1/2})$

$f'(x) = \frac{1}{6}\left(\frac{35}{2}x^{3/2} + \frac{1}{2}x^{-1/2}\right)$

$= \frac{1}{12}x^{1/2}(35x + x^{-1})$

75. $y' = 6x + 4$

$y'\big|_{x=0} = 4$

$y'\big|_{x=2} = 16$

$y'\big|_{x=-3} = -14$

76. $y' = 0 + 5 - 3(3x^2) = 5 - 9x^2$

$y'\big|_{x=0} = 5$

$y'\big|_{x=1/2} = 5 - 9\left(\frac{1}{4}\right) = \frac{11}{4}$

$y'\big|_{x=2} = 5 - 9(4) = -31$

77. y is a constant, so $y' = 0$ for all x.

78. $y' = 3 - 2x^{-1/2} = 3 - \frac{2}{\sqrt{x}}$

$y'\big|_{x=4} = 2$

$y'\big|_{x=9} = \frac{7}{3}$

$y'\big|_{x=25} = \frac{13}{5}$

79. $y = 4x^2 + 5x + 6$

$y' = 8x + 5$

$y'\big|_{x=1} = 13$

An equation of the tangent line is $y - 15 = 13(x - 1)$, or $y = 13x + 2$.

80. $y = \frac{1}{5}\left(1 - x^2\right)$

$y' = \frac{1}{5}(-2x)$

$y'\big|_{x=4} = -\frac{8}{5}$

An equation of the tangent line is $y + 3 = -\frac{8}{5}(x - 4)$, or $y = -\frac{8}{5}x + \frac{17}{5}$.

81. $y = \frac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = -\frac{2}{x^3}$

$y'\big|_{x=2} = -\frac{2}{2^3} = -\frac{1}{4}$

An equation of the tangent line is $y - \frac{1}{4} = -\frac{1}{4}(x - 2)$, or $y = -\frac{1}{4}x + \frac{3}{4}$.

82. $y = -\sqrt[3]{x} = -x^{\frac{1}{3}}$

$y' = -\frac{1}{3}x^{-\frac{2}{3}} = -\frac{1}{3x^{\frac{2}{3}}}$

$y'\big|_{x=8} = -\frac{1}{3\left(8^{\frac{2}{3}}\right)} = -\frac{1}{3 \cdot 4} = -\frac{1}{12}$

An equation of the tangent line is $y + 2 = -\frac{1}{12}(x - 8)$, or $y = -\frac{1}{12}x - \frac{4}{3}$.

83. $y = 3 + x - 5x^2 + x^4$

$y' = 1 - 10x + 4x^3$.

When $x = 0$, then $y = 3$ and $y' = 1$. Thus an equation of the tangent line is $y - 3 = 1(x - 0)$, or $y = x + 3$.

84. $y = \frac{\sqrt{x}\left(2 - x^2\right)}{x} = x^{-\frac{1}{2}}\left(2 - x^2\right) = 2x^{-\frac{1}{2}} - x^{\frac{3}{2}}$.

$y' = -x^{-\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$y'\big|_{x=4} = -\frac{1}{8} - 3 = -\frac{25}{8}$

When $x = 4$, then $y = -7$. The tangent line is $y + 7 = -\frac{25}{8}(x - 4)$, or $y = -\frac{25}{8}x + \frac{11}{2}$.

85. $y = \frac{5}{2}x^2 - x^3$

$y' = 5x - 3x^2$

A horizontal tangent line has slope 0, so we set $5x - 3x^2 = 0$. Then $x(5 - 3x) = 0$, $x = 0$ or $x = \frac{5}{3}$.

If $x = 0$, then $y = 0$. If $x = \frac{5}{3}$, $y = \frac{125}{54}$. This gives the points (0, 0) and $\left(\frac{5}{3}, \frac{125}{54}\right)$.

86. $y = \frac{x^6}{6} - \frac{x^2}{2} + 1$

$y' = x^5 - x$

A horizontal tangent line has slope 0, so we set $x^5 - x = 0$. Then $x(x^4 - 1) = 0$, so $x = 0$ or $x = \pm 1$. If $x = 0$, then $y = 1$; if $x = 1$, then $y = \frac{2}{3}$; if $x = -1$, then $y = \frac{2}{3}$. This gives the points (0, 1), $\left(1, \frac{2}{3}\right)$, and $\left(-1, \frac{2}{3}\right)$.

87. $y = x^2 - 5x + 3$

$y' = 2x - 5$

Setting $2x - 5 = 1$ gives $2x = 6$, $x = 3$. When $x = 3$, then $y = -3$. This gives the point $(3, -3)$.

88. $y = x^4 - 31x + 11$

$y' = 4x^3 - 31$

If $4x^3 - 31 = 1$, then $x^3 = 8$, $x = 2$. When $x = 2$, then $y = -35$. This gives the point (2, –35).

89. $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$f'(x) = \dfrac{1}{2}x^{-\frac{1}{2}} - \dfrac{1}{2}x^{-\frac{3}{2}} = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{2x\sqrt{x}} = \dfrac{x-1}{2x\sqrt{x}}$

Thus $\dfrac{x-1}{2x\sqrt{x}} - f'(x) = \dfrac{x-1}{2x\sqrt{x}} - \dfrac{x-1}{2x\sqrt{x}} = 0$.

90. $z = (1+b)w_p - bw_c$

$\dfrac{dz}{dw_c} = (1+b)\dfrac{dw_p}{dw_c} - b$

Rewriting the right side and factoring out 1 + b gives $\dfrac{dz}{dw_c} = (1+b)\dfrac{dw_p}{dw_c} - \dfrac{b(1+b)}{1+b}$,

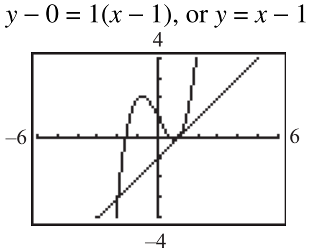
$\dfrac{dz}{dw_c} = (1+b)\left[\dfrac{dw_p}{dw_c} - \dfrac{b}{1+b}\right]$.

91. $y = x^3 - 2x + 1$

$y'(x) = 3x^2 - 2$

$y'\big|_{x=1} = 3 - 2 = 1$

The tangent line at (1, 0) is given by $y - 0 = 1(x - 1)$, or $y = x - 1$.

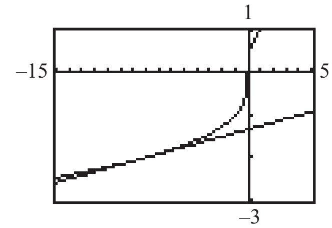


92. $y = \sqrt[3]{x} = x^{1/3}$

$y'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3\sqrt[3]{x^2}}$

$y'\big|_{x=-8} = \dfrac{1}{12}$

The tangent line at (–8, –2) is given by $y + 2 = \dfrac{1}{12}(x+8)$, or $y = \dfrac{1}{12}x - \dfrac{4}{3}$.



Apply It 11.3

3. Here $\dfrac{dP}{dp} = 5$ and $\Delta p = 25.5 - 25 = 0.5$.

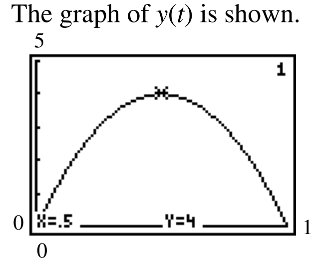
$\Delta P \approx \dfrac{dP}{dp}\Delta p = 5(0.5) = 2.5$

The profit increases by 2.5 units when the price is changed from 25 to 25.5 per unit.

4. $\dfrac{dy}{dt} = \dfrac{d}{dt}\left(16t - 16t^2\right) = 16 - 16(2t) = 16 - 32t$

$\left.\dfrac{dy}{dt}\right|_{t=0.5} = 16 - 32(0.5) = 16 - 16 = 0$

The graph of $y(t)$ is shown.



When $t = 0.5$, the object is at the peak of its flight.

5. $V'(r) = \dfrac{4}{3}\pi\left(3r^2\right) + 4\pi(2r) = 4\pi r^2 + 8\pi r$

When $r = 2$, $V'(r) = 4\pi(2)^2 + 8\pi(2) = 32\pi$ and

$V(r) = \dfrac{4}{3}\pi(2)^3 + 4\pi(2)^2 = \dfrac{32\pi}{3} + 16\pi = \dfrac{80}{3}\pi$.

The relative rate of change of the volume when $r = 2$ is $\dfrac{V'(2)}{V(2)} = \dfrac{32\pi}{\frac{80}{3}\pi} = \dfrac{6}{5} = 1.2$. Multiplying 1.2 by 100 gives the percentage rate of change: (1.2)(100) = 120%.

Problems 11.3

1. $s = f(t) = 2t^2 + 3t$

If $\Delta t = 1$, then over [1, 2] we have

$\dfrac{\Delta s}{\Delta t} = \dfrac{f(2) - f(1)}{2-1} = \dfrac{14-5}{1} = 9.$

If $\Delta t = 0.5$, then over [1, 1.5] we have $\frac{\Delta s}{\Delta t} = \frac{f(1.5) - f(1)}{1.5 - 1} = \frac{9-5}{0.5} = 8.$

Continuing this way, we obtain the following table:

Δt	1	0.5	0.2	0.1	0.01	0.001
$\frac{\Delta s}{\Delta t}$	9	8	7.4	7.2	7.02	7.002

We estimate the velocity when $t = 1$ to be 7 m/s. With differentiation we get $v = \frac{ds}{dt} = 4t + 3,$

$$\left.\frac{ds}{dt}\right|_{t=1} = 4(1) + 3 = 7 \text{ m/s.}$$

2. $y = f(x) = \sqrt{2x+5}$.

If $\Delta x = 1$, then over [3, 4] we have

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(3)}{\Delta x} = \frac{\sqrt{13} - \sqrt{11}}{1} \approx 0.2889$$

If $\Delta x = 0.5$, then over [3, 3.5] we have

$$\frac{\Delta y}{\Delta x} = \frac{f(3.5) - f(3)}{\Delta x} = \frac{\sqrt{12} - \sqrt{11}}{0.5} \approx 0.2950$$

Continuing in this way we obtain the following table:

Δx	1	0.5	0.2	0.1	0.01	0.001
$\frac{\Delta y}{\Delta x}$	0.2889	0.2950	0.2988	0.3002	0.3014	0.3015

We estimate the rate of change to be 0.3015.

$\left(\text{Note: The actual rate of change is } \frac{1}{\sqrt{11}} \approx 0.3015.\right)$

3. $s = f(t) = 2t^2 - 4t$

a. When $t = 7$, then $s = 2(7^2) - 4(7) = 70$ m.

b. $\frac{\Delta s}{\Delta t} = \frac{f(7.5) - f(7)}{0.5} = \frac{[2(7.5)^2 - 4(7.5)] - 70}{0.5} = 25$ m/s

c. $v = \frac{ds}{dt} = 4t - 4.$ If $t = 7$, then $v = 4(7) - 4 = 24$ m/s

4. $s = f(t) = \frac{1}{2}t + 1$

a. When $t = 2$, $s = \frac{1}{2}(2) + 1 = 2$ m.

b. $\frac{\Delta s}{\Delta t} = \frac{f(2.1) - f(1)}{0.1}$
$= \frac{\left[\frac{1}{2}(2.1)+1\right]-2}{0.1}$
$= 0.5$ m/s

c. $v = \frac{ds}{dt} = \frac{1}{2}$. If $t = 2$, then $v = \frac{1}{2}$ m/s

5. $s = f(t) = 5t^3 + 3t + 24$

a. When $t = 1$, $s = 5(1)^3 + 3(1) + 24 = 32$ m.

b. $\frac{\Delta s}{\Delta t} = \frac{f(1.01) - f(1)}{0.01}$
$= \frac{5(1.01)^3 + 3(1.01) + 24 - 32}{0.01}$
$= 18.1505$ m/s

c. $v = \frac{ds}{dt} = 15t^2 + 3$. If $t = 1$, then
$v = 15(1)^2 + 3 = 18$ m/s

6. $s = f(t) = -3t^2 + 2t + 1$

a. When $t = 1$, $s = -3\left(1^2\right) + 2(1) + 1 = 0$ m.

b. $\frac{\Delta s}{\Delta t} = \frac{f(1.25) - f(1)}{0.25}$
$= \frac{\left[-3(1.25)^2 + 2(1.25) + 1\right] - 0}{0.25} = -4.75$ m/s

c. $v = \frac{ds}{dt} = -6t + 2$. If $t = 1$, $v = -4$ m/s

7. $s = f(t) = t^4 - 2t^3 + t$

a. When $t = 2$, $s = 2^4 - 2\left(2^3\right) + 2 = 2$ m.

b. $\frac{\Delta s}{\Delta t} = \frac{f(2.1) - f(2)}{0.1}$
$= \frac{\left[(2.1)^4 - 2(2.1)^3 + 2.1\right] - 2}{0.1} = 10.261$ m/s

c. $v = \frac{ds}{dt} = 4t^3 - 6t^2 + 1$. If $t = 2$, then
$v = 4\left(2^3\right) - 6\left(2^2\right) + 1 = 9$ m/s

8. $s = f(t) = 3t^4 - t^{7/2}$

a. When $t = 0$, $s = 3 \cdot 0^4 = 0^{7/2} = 0$.

b. $\frac{\Delta s}{\Delta t} = \frac{f\left(\frac{1}{4}\right) - f(0)}{\frac{1}{4}} = \frac{\left[3 \cdot \left(\frac{1}{4}\right)^4 - \left(\frac{1}{4}\right)^{7/2}\right] - 0}{\frac{1}{4}}$
$= \frac{1}{64}$ m/s

c. $v = \frac{ds}{dt} = 12t^3 - \frac{7}{2}t^{5/2}$. If $t = 0$, then
$v = 12(0)^3 - \frac{7}{2}(0)^{5/2} = 0$ m/s.

9. $\frac{dy}{dx} = \frac{25}{2}x^{\frac{3}{2}}$. If $x = 9$, $\frac{dy}{dx} = \frac{25}{2}(27) = 337.50$.

10. $\frac{dV}{dr} = 4\pi r^2$. If $r = 1.5$, $\frac{dV}{dr} = 4\pi(1.5)^2 = 9\pi$.

11. $\frac{dT}{dT_e} = 0 + 0.27(1 - 0) = 0.27$

12. $\frac{dV}{dr} = 4\pi r^2$
When $r = 6.3 \times 10^{-4}$,
$\frac{dV}{dr} = 4\pi[6.3 \times 10^{-4}]^2 = 158.76\pi \times 10^{-8}$
$\approx 4.988 \times 10^{-6}$.

13. $c = 500 + 10q$, $\frac{dc}{dq} = 10$. When $q = 100$,
$\frac{dc}{dq} = 10$.

14. $c = 5000 + 6q$, $\frac{dc}{dq} = 6$. When $q = 36$, $\frac{dc}{dq} = 6$.

15. $\frac{dc}{dq} = 0.4q + 4.$ When $q = 10$,

$\frac{dc}{dq} = 0.4(10) + 4 = 8.$

16. $\frac{dc}{dq} = 0.2q + 3$. When $q = 3$, $\frac{dc}{dq} = 3.6$.

17. $\frac{dc}{dq} = 2q + 50$. Evaluating when $q = 15$, 16 and 17 gives 80, 82 and 84, respectively.

18. $\frac{dc}{dq} = 0.12q^2 - q + 4.4$

Evaluating when $q = 5$, 25, and 1000 gives 2.4, 54.4 and 119,004.4, respectively.

19. $\bar{c} = 0.01q + 5 + \frac{500}{q}$

$c = \bar{c}q = 0.01q^2 + 5q + 500$

$\frac{dc}{dq} = 0.02q + 5$

$\left.\frac{dc}{dq}\right|_{q=50} = 6$

$\left.\frac{dc}{dq}\right|_{q=100} = 7$

20. $\bar{c} = 5 + \frac{2000}{q}$

$c = \bar{c}q = 5q + 2000$

$\frac{dc}{dq} = 5$ for all q

21. $c = \bar{c}q = 0.00002q^3 - 0.01q^2 + 6q + 20{,}000$

$\frac{dc}{dq} = 0.00006q^2 - 0.02q + 6$

If $q = 100$, then $\frac{dc}{dq} = 4.6$. If $q = 500$, then

$\frac{dc}{dq} = 11$.

22. $c = \bar{c}q = 0.002q^3 - 0.5q^2 + 60q + 7000$

$\frac{dc}{dq} = 0.006q^2 - q + 60$

If $q = 15$, then $\frac{dc}{dq} = 46.35$. If $q = 25$, then

$\frac{dc}{dq} = 38.75.$

23. $r = 0.8q$

$\frac{dr}{dq} = 0.8$ for all q.

24. $r = q\left(15 - \frac{1}{30}q\right) = 15q - \frac{1}{30}q^2$

$\frac{dr}{dq} = 15 - \frac{1}{15}q$

For $q = 5$, $\frac{dr}{dq} = \frac{44}{3}$; for $q = 15$, $\frac{dr}{dq} = 14$; for

$q = 150$, $\frac{dr}{dq} = 5$.

25. $r = 240q + 40q^2 - 2q^3$

$\frac{dr}{dq} = 240 + 80q - 6q^2$. Evaluating when $q = 10$, 15, and 20 gives 440, 90, and –560, respectively.

26. $r = 60q - 0.2q^2$

$\frac{dr}{dq} = 60 - 0.4q$

Evaluating when $q = 10$ and 20 gives 56 and 52, respectively.

27. $\frac{dc}{dq} = 6.750 - 0.000328(2q) = 6.750 - 0.000656q$

$\left.\frac{dc}{dq}\right|_{q=2000} = 6.750 - 0.000656(2000) = 5.438$

$\bar{c} = \frac{c}{q} = \frac{-10{,}484.69}{q} + 6.750 - 0.000328q$

$$\bar{c}(2000) = \frac{-10{,}484.69}{2000} + 6.750 - 0.000328(2000)$$
$$= 0.851655$$

28. $\frac{dc}{dq} = -0.79 + 0.04284q - 0.0003q^2$

$\left.\frac{dc}{dq}\right|_{q=70} = 0.7388$

29. $PR^{0.93} = 5,000,000$

$P = 5,000,000R^{-0.93}$

$\frac{dP}{dR} = -4,650,000R^{-1.93}$

30. $\frac{dv}{dt} = -15,500$ for all t.

31. a. $\frac{dy}{dx} = -1.5 - x$

$\left.\frac{dy}{dx}\right|_{x=6} = -1.5 - 6 = -7.5$

b. Setting $-1.5 - x = -6$ gives $x = 4.5$.

32. $c = f(q) = 0.4q^2 + 4q + 5$

$\frac{dc}{dq} = 0.8q + 4$

If $q = 2$, then $\frac{dc}{dq} = 5.6$. Over the interval [2, 3],

$\frac{\Delta c}{\Delta q} = \frac{f(3) - f(2)}{3 - 2} = \frac{20.6 - 14.6}{1} = 6$.

33. a. $y' = 1$

b. $\frac{y'}{y} = \frac{1}{x+4}$

c. $y'(5) = 1$

d. $\frac{1}{5+4} = \frac{1}{9} \approx 0.111$

e. 11.1%

34. a. $y' = -3$

b. $\frac{y'}{y} = \frac{-3}{7-3x} = \frac{3}{3x-7}$

c. $y'(6) = -3$

d. $\frac{3}{3(6)-7} = \frac{3}{11} \approx 0.2727$

e. 27.27%

35. a. $y' = 4x$

b. $\frac{y'}{y} = \frac{4x}{2x^2+5}$

c. $y'(10) = 4(10) = 40$

d. $\frac{40}{2(10)^2+5} = \frac{40}{205} = \frac{8}{41} \approx 0.1951$

e. 19.51%

36. a. $y' = -9x^2$

b. $\frac{y'}{y} = \frac{-9x^2}{5-3x^3}$

c. $y'(1) = -9$

d. $\frac{-9}{5-3} = -\frac{9}{2} = -4.5$

e. −450%

37. a. $y' = -3x^2$

b. $\frac{y'}{y} = \frac{-3x^2}{8-x^3}$

c. $y'(1) = -3$

d. $\frac{-3}{8-1} = -\frac{3}{7} \approx -0.429$

e. −42.9%

38. a. $y' = 2x + 3$

b . $\frac{y'}{y} = \frac{2x+3}{x^2+3x-4}$

c. $y'(-1) = 2(-1) + 3 = 1$

d. $\frac{1}{1-3-4} = -\frac{1}{6} \approx -0.167$

e. -16.7%

39. $c = 0.3q^2 + 3.5q + 9$

$\frac{dc}{dq} = 0.6q + 3.5$

If $q = 10$, then $\frac{dc}{dq} = 0.6(10) + 3.5 = 9.5$. If $q = 10$, then $c = 74$ and

$\frac{\frac{dc}{dq}}{c}(100) = \frac{9.5}{74}(100) \approx 12.8\%$.

40. $y = \frac{100}{x} = 100x^{-1}$

$\frac{dy}{dx} = -100x^{-2} = -\frac{100}{x^2}$

If $x = 10$, $\frac{dy}{dx} = -\frac{100}{100} = -1$ and

$\frac{y'}{y}(100) = \frac{-1}{10}(100) = -10\%$.

41. a. $\frac{dr}{dq} = 30 - 0.6q$

b. If $q = 10$, $\frac{r'}{r} = \frac{30-6}{300-30} = \frac{24}{270} = \frac{4}{45} \approx 0.09$.

c. 9%

42. a. $\frac{dq}{dr} = 10 - 0.4q$

b. If $q = 25$, $\frac{r'}{r} = \frac{10-0.4(25)}{10(25)-0.2(25)^2} = 0.$

c. 0%

43. $\frac{W'}{W} = \frac{0.864t^{-0.568}}{2t^{0.432}} = \frac{0.432}{t}$

44. a. $\frac{R_1'}{R_1} = \frac{\frac{1.3I^{0.3}}{1855.24}}{\frac{I^{1.3}}{1855.24}} = \frac{1.3}{I}$

$\frac{R_2'}{R_2} = \frac{\frac{1.3I^{0.3}}{1101.29}}{\frac{I^{1.3}}{1101.29}} = \frac{1.3}{I}$

b. They are equal.

c. $\frac{f'x}{f(x)} = \frac{nC_1x^{n-1}}{C_1x^n} = \frac{n}{x}$

$\frac{g'(x)}{g(x)} = \frac{nC_2x^{n-1}}{C_2x^n} = \frac{n}{x}$

The rates are equal.

45. The cost of $q = 20$ bikes is $q\overline{c} = 20(200) = \$4000$. The marginal cost, \$150, is the approximate cost of one additional bike. Thus the approximate cost of producing 21 bikes is \$4000 + \$150 = \$4150.

46. The relative rate of change of c is $\frac{\frac{dc}{dq}}{c}$, which is given to be $\frac{1}{q}$: $\frac{\frac{dc}{dq}}{c} = \frac{1}{q}$. Thus $\frac{dc}{dq} = \frac{c}{q} = \overline{c}$, and the marginal cost function $\left(\frac{dc}{dq}\right)$ and the average cost function $(\overline{c})$ are equal.

47. \$5.07 per unit

48. 11,275 people per year

Apply It 11.4

6. $\frac{dR}{dx} = (2-0.15x)\frac{d}{dx}(225+20x) + (225+20x)\frac{d}{dx}(2-0.15x)$
$= (2 - 0.15x)(20) + (225 + 20x)(-0.15)$
$= 40 - 3x - 33.75 - 3x = 6.25 - 6x$
$\frac{dR}{dx} = 6.25 - 6x$

7. $T(x) = x^2 - \frac{1}{3}x^3$
$T'(x) = 2x - x^2$
When the dosage is 1 milligram the sensitivity is $T'(1) = 2(1) - 1^2 = 1$.

Problems 11.4

1. $f'(x) = (4x+1)(6) + (6x+3)(4) = 24x + 6 + 24x + 12 = 48x + 18 = 6(8x + 3)$

2. $f'(x) = (3x-1)(7) + (7x+2)(3) = 42x - 1$

3. $s'(t) = (5-3t)(3t^2-4t) + (t^3-2t^2)(-3) = 15t^2 - 20t - 9t^3 + 12t^2 - 3t^3 + 6t^2 = -12t^3 + 33t^2 - 20t$

4. $Q'(x) = (x^2+3x)(14x) + (2x+3)(7x^2-5)$
$= 14x^3 + 42x^2 + 14x^3 + 21x^2 - 10x - 15$
$= 28x^3 + 63x^2 - 10x - 15$

5. $f'(r) = \left(3r^2-4\right)(2r-5) + \left(r^2-5r+1\right)(6r) = 6r^3 - 15r^2 - 8r + 20 + 6r^3 - 30r^2 + 6r = 12r^3 - 45r^2 - 2r + 20$

6. $C'(I) = \left(2I^2-3\right)(6I-4) + \left(3I^2-4I+1\right)(4I) = 12I^3 - 8I^2 - 18I + 12 + 12I^3 - 16I^2 + 4I = 2\left(12I^3 - 12I^2 - 7I + 6\right)$

7. Without the product rule we have
$f(x) = x^2\left(2x^2-5\right) = 2x^4 - 5x^2$
$f'(x) = 8x^3 - 10x$

8. Without the product rule we have
$f(x) = 3x^3\left(x^2-2x+2\right) = 3x^5 - 6x^4 + 6x^3$
$f'(x) = 15x^4 - 24x^3 + 18x^2$

9. $y' = (x^2+5x-7)(12x-5) + (2x+5)(6x^2-5x+4)$
$= 12x^3 + 60x^2 - 84x - 5x^2 - 25x + 35 + 12x^3 - 10x^2 + 8x + 30x^2 - 25x + 20$
$= 24x^3 + 75x^2 - 126x + 55$

10. $\phi'(x) = (3-5x+2x^2)(1-8x)+(2+x-4x^2)(-5+4x)$
$= 3-5x+2x^2-24x+40x^2-16x^3-10-5x+20x^2+8x+4x^2-16x^3$
$= -32x^3+66x^2-26x-7$

11. $f'(w) = (w^2+3w-7)(6w^2)+(2w^3-4)(2w+3)$
$= 6w^4+18w^3-42w^2+4w^4+6w^3-8w-12$
$= 10w^4+24w^3-42w^2-8w-12$

12. $f'(x) = \left(3x-x^2\right)(-1-2x)+\left(3-x-x^2\right)(3-2x)$
$= -3x-5x^2+2x^3+9-3x-3x^2-6x+2x^2+2x^3$
$= 4x^3-6x^2-12x+9$

13. $y' = \left(x^2-1\right)\left(9x^2-6\right)+\left(3x^3-6x+5\right)(2x)-4(8x+2)$
$= 9x^4-15x^2+6+6x^4-12x^2+10x-32x-8$
$= 15x^4-27x^2-22x-2$

14. $h' = 5(7x^6)+4[(15x^2)(4x^2+7x)+(5x^3-2)(8x+7)]$
$= 35x^6+4[60x^4+105x^3+40x^4+35x^3-16x-14]$
$= 35x^6+400x^4+560x^3-64x-56$

15. $F'(p) = \frac{3}{2}\left[(5p^{1/2}-2)(3)+(3p-1)\left(5\cdot\frac{1}{2}p^{-1/2}\right)\right]$
$= \frac{3}{2}\left[15p^{1/2}-6+\frac{15}{2}p^{1/2}-\frac{5}{2}p^{-1/2}\right]$
$= \frac{3}{4}[45p^{1/2}-12-5p^{-1/2}]$

16. $g'(x) = (x^{1/2}+5x-2)\left(\frac{1}{3}x^{-2/3}-\frac{3}{2}x^{-1/2}\right)+(x^{1/3}-3x^{1/2})\left(\frac{1}{2}x^{-1/2}+5\right)$
$= \frac{1}{3}x^{-1/6}+\frac{5}{3}x^{1/3}-\frac{2}{3}x^{-2/3}-\frac{3}{2}-\frac{15}{2}x^{1/2}+3x^{-1/2}+\frac{1}{2}x^{-1/6}+5x^{1/3}-\frac{3}{2}-15x^{1/2}$
$= \frac{1}{6}(-135x^{1/2}+40x^{1/3}+5x^{-1/6}+18x^{-1/2}-4x^{-2/3}-18)$

17. $y = 7\cdot\frac{2}{3}$ is a constant function, so $y' = 0$.

18. $y = x^3-6x^2+11x-6$
$y' = 3x^2-12x+11$

19. $y = 70x^3-43x^2-276x-135$
$y' = 210x^2-86x-276$

20. $\dfrac{dy}{dx} = \dfrac{(4x+1)(2)-(2x-3)(4)}{(4x+1)^2} = \dfrac{8x+2-8x+12}{(4x+1)^2}$

$= \dfrac{14}{(4x+1)^2}$

21. $f'(x) = \dfrac{(x-1)(5)-(5x)(1)}{(x-1)^2} = \dfrac{5x-5-5x}{(x-1)^2}$

$= -\dfrac{5}{(x-1)^2}$

22. $H'(x) = \dfrac{(5-x)(-5)-(-5x)(-1)}{(5-x)^2}$

$= \dfrac{-25+5x-5x}{(5-x)^2} = -\dfrac{25}{(5-x)^2}$

23. $f(x) = \dfrac{-13}{3x^5} = -\dfrac{13}{3}x^{-5}$

$f'(x) = -\dfrac{13}{3}(-5x^{-6}) = \dfrac{65}{3x^6}$

24. $f(x) = \dfrac{3}{4}(5x^2-7)$

$f'(x) = \dfrac{3}{4}(10x) = \dfrac{15}{2}x$

25. $y' = \dfrac{(x-1)(1)-(x+2)(1)}{(x-1)^2}$

$= \dfrac{x-1-x-2}{(x-1)^2}$

$= -\dfrac{3}{(x-1)^2}$

26. $h'(w) = \dfrac{(w-3)(6w+5)-\left(3w^2+5w-1\right)(1)}{(w-3)^2}$

$= \dfrac{6w^2-13w-15-3w^2-5w+1}{(w-3)^2}$

$= \dfrac{3w^2-18w-14}{(w-3)^2}$

27. $h'(z) = \dfrac{\left(z^2-4\right)(-2)-(6-2z)(2z)}{\left(z^2-4\right)^2}$

$= \dfrac{-2z^2+8-12z+4z^2}{\left(z^2-4\right)^2} = \dfrac{2z^2-12z+8}{\left(z^2-4\right)^2}$

$= \dfrac{2\left(z^2-6z+4\right)}{\left(z^2-4\right)^2}$

28. $z' = \dfrac{(3x^2+5x+3)(4x+5)-(2x^2+5x-2)(6x+5)}{(3x^2+5x+3)^2}$

$= \dfrac{12x^3+35x^2+37x+15-(12x^3+40x^2+13x-10)}{(3x^2+5x+3)^2}$

$= \dfrac{-5x^2+24x+25}{(3x^2+5x+3)^2}$

29. $y' = \dfrac{(3x^2-2x+1)(8x+3)-(4x^2+3x+2)(6x-2)}{(3x^2-2x+1)^2}$

$= \dfrac{24x^3+9x^2-16x^2-6x+8x+3-24x^3+8x^2-18x^2+6x-12x+4}{(3x^2-2x+1)^2}$

$= \dfrac{-17x^2-4x+7}{(3x^2-2x+1)^2}$

30. $f'(x) = \dfrac{\left(x^2+1\right)\left(3x^2-2x\right)-\left(x^3-x^2+1\right)(2x)}{\left(x^2+1\right)^2}$

$= \dfrac{3x^4-2x^3+3x^2-2x-2x^4+2x^3-2x}{(x^2+1)^2}$

$= \dfrac{x\left(x^3+3x-4\right)}{\left(x^2+1\right)^2}$

31. $y' = \dfrac{\left(2x^2-3x+2\right)(2x-4)-\left(x^2-4x+3\right)(4x-3)}{\left(2x^2-3x+2\right)^2}$

$= \dfrac{4x^3-14x^2+16x-8-\left(4x^3-19x^2+24x-9\right)}{\left(2x^2-3x+2\right)^2}$

$= \dfrac{5x^2-8x+1}{\left(2x^2-3x+2\right)^2}$

32. The quotient rule can be used, or we can write

$F(z)=\dfrac{z^4+4}{3z}=\dfrac{1}{3}\left(z^3+4z^{-1}\right)$,

so $F'(z)=\dfrac{1}{3}\left(3z^2-4z^{-2}\right)=\dfrac{3z^4-4}{3z^2}$.

33. $g'(x)=\dfrac{\left(x^{100}+7\right)(0)-(1)\left(100x^{99}\right)}{\left(x^{100}+7\right)^2}=-\dfrac{100x^{99}}{\left(x^{100}+7\right)^2}$

34. $y=\dfrac{-8}{7x^6}=-\dfrac{8}{7}x^{-6}$

$y'=\dfrac{48}{7}x^{-7}$

35. $u(v)=\dfrac{v^3-8}{v}=\dfrac{v^3}{v}-\dfrac{8}{v}=v^2-8v^{-1}$

$u'(v)=2v+8v^{-2}=2\left(v+\dfrac{4}{v^2}\right)=\dfrac{2(v^3+4)}{v^2}$

36. $y=\dfrac{x-5}{8\sqrt{x}}=\dfrac{1}{8}\left(x^{\frac{1}{2}}-5x^{-\frac{1}{2}}\right)$

$y'=\dfrac{1}{8}\left(\dfrac{1}{2}x^{-\frac{1}{2}}+\dfrac{5}{2}x^{-\frac{3}{2}}\right)=\dfrac{1}{16}\left(\dfrac{1}{x^{\frac{1}{2}}}+\dfrac{5}{x^{\frac{3}{2}}}\right)=\dfrac{x+5}{16x^{\frac{3}{2}}}$

37. $y=\dfrac{3x^2-x-1}{\sqrt[3]{x}}=\dfrac{3x^2-x-1}{x^{\frac{1}{3}}}=3x^{\frac{5}{3}}-x^{\frac{2}{3}}-x^{-\frac{1}{3}}$

$y'=5x^{\frac{2}{3}}-\dfrac{2}{3}x^{-\frac{1}{3}}+\dfrac{1}{3}x^{-\frac{4}{3}}=5x^{\frac{2}{3}}-\dfrac{2}{3x^{\frac{1}{3}}}+\dfrac{1}{3x^{\frac{4}{3}}}$

$=\dfrac{15x^2-2x+1}{3x^{\frac{4}{3}}}$

38. $y'=\dfrac{\left(2x^{2.1}+1\right)\left(0.3x^{-0.7}\right)-\left(x^{0.3}-2\right)\left(4.2x^{1.1}\right)}{\left(2x^{2.1}+1\right)^2}$

$=\dfrac{0.6x^{1.4}+0.3x^{-0.7}-4.2x^{1.4}+8.4x^{1.1}}{(2x^{2.1}+1)^2}$

$=\dfrac{0.3\left(1+28x^{1.8}-12x^{2.1}\right)}{x^{0.7}\left(2x^{2.1}+1\right)^2}$

39. $y'=0-\dfrac{(2x+5)(0)-5(2)}{(2x+5)^2}+\dfrac{(3x+1)(2)-(2x)(3)}{(3x+1)^2}$

$=\dfrac{10}{(2x+5)^2}+\dfrac{6x+2-6x}{(3x+1)^2}$

$=\dfrac{10}{(2x+5)^2}+\dfrac{2}{(3x+1)^2}$

40. $q'(x)=6x^2+\dfrac{(3x-5)(5)-(5x+1)(3)}{(3x-5)^2}+6x^{-4}$

$=6x^2-\dfrac{28}{(3x-5)^2}+6x^{-4}$

41. $y'=\dfrac{[(x+2)(x-4)](1)-(x-5)(2x-2)}{[(x+2)(x-4)]^2}$

$=\dfrac{x^2-2x-8-\left(2x^2-12x+10\right)}{[(x+2)(x-4)]^2}$

$=\dfrac{-\left(x^2-10x+18\right)}{[(x+2)(x-4)]^2}$

42. $y=\dfrac{(9x-1)(3x+2)}{4-5x}=\dfrac{27x^2+15x-2}{4-5x}$

$y'=\dfrac{(4-5x)(54x+15)-\left(27x^2+15x-2\right)(-5)}{(4-5x)^2}$

$=\dfrac{-270x^2+141x+60+135x^2+75x-10}{(4-5x)^2}$

$=-\dfrac{135x-216x-50}{(4-5x)^2}$

43. $s'(t) = \dfrac{\left[\left(t^2-1\right)\left(t^3+7\right)\right](2t+3)-\left(t^2+3t\right)\left(5t^4-3t^2+14t\right)}{\left[\left(t^2-1\right)\left(t^3+7\right)\right]^2}$

$= \dfrac{-3t^6-12t^5+t^4+6t^3-21t^2-14t-21}{\left[\left(t^2-1\right)\left(t^3+7\right)\right]^2}$

44. $f(s) = \dfrac{17}{4s^4+5s^2-23s}$

$f'(s) = \dfrac{0-17(16s^3+10s-23)}{(4s^4+5s^2-23s)^2} = -\dfrac{17(16s^3+10s-23)}{(4s^4+5s^2-23s)^2}$

45. $y = 3x - \dfrac{\frac{2}{x}-\frac{3}{x-1}}{x-2} = 3x - \dfrac{\frac{2(x-1)-3x}{x(x-1)}}{x-2}$

$= 3x + \dfrac{x+2}{x(x-1)(x-2)} = 3x + \dfrac{x+2}{x^3-3x^2+2x}$

$y' = 3 + \dfrac{(x^3-3x^2+2x)(1)-(x+2)(3x^2-6x+2)}{[x(x-1)(x-2)]^2}$

$= 3 - \dfrac{2x^3+3x^2-12x+4}{[x(x-1)(x-2)]^2}$

46. $y = 3-12x^3 + \dfrac{1-\frac{5}{x^2+2}}{x^2+5} = 3-12x^3 + \dfrac{\frac{x^2+2-5}{x^2+2}}{x^2+5} = 3-12x^3 + \dfrac{x^2-3}{x^4+7x^2+10}$

$y' = -36x^2 + \dfrac{(x^4+7x^2+10)(2x)-(x^2-3)(4x^3+14x)}{(x^4+7x^2+10)^2} = -36x^2 + \dfrac{-2x^5+12x^3+62x}{[(x^2+2)(x^2+5)]^2}$

47. $f'(x) = \dfrac{(a-x)(1)-(a+x)(-1)}{(a-x)^2} = \dfrac{2a}{(a-x)^2}$

48. Simplifying, $f(x) = \dfrac{x^{-1}+a^{-1}}{x^{-1}-a^{-1}} \cdot \dfrac{ax}{ax} = \dfrac{a+x}{a-x}$

$f'(x) = \dfrac{(a-x)(1)-(a+x)(-1)}{(a-x)^2} = \dfrac{2a}{(a-x)^2}$

49. $y = (2x^2-x+3)(x^3+x+1)$

$y' = (4x-1)(x^3+x+1)+(2x^2-x+3)(3x^2+1)$

$y'(1) = (3)(3)+(4)(4) = 25$

50. $y = \frac{x^3}{x^4+1}$

$y' = \frac{(x^4+1)(3x^2)-(x^3)(4x^3)}{(x^4+1)^2}$

$y'(-1) = \frac{(2)(3)-(-1)(-4)}{(2)^2} = \frac{1}{2}$

51. $y = \frac{6}{x-1}$

$y' = \frac{(x-1)(0)-(6)(1)}{(x-1)^2} = -\frac{6}{(x-1)^2}$

$y'(3) = -\frac{6}{2^2} = -\frac{3}{2}$

The tangent line is $y-3 = -\frac{3}{2}(x-3)$, or

$y = -\frac{3}{2}x + \frac{15}{2}$.

52. $y = \frac{x+5}{x^2} = x^{-1} + 5x^{-2}$

$y' = -x^{-2} - 10x^{-3} = -\frac{1}{x^2} - \frac{10}{x^3}$

$y'(1) = -1-10 = -11$

The tangent line is $y - 6 = -11(x - 1)$ or $y = -11x + 17$.

53. $y = (2x+3)\left[2\left(x^4-5x^2+4\right)\right]$

$y' = (2x+3)\left[2\left(4x^3-10x\right)\right] + \left[2\left(x^4-5x^2+4\right)\right](2)$

$y'(0) = (3)(0) + [2(4)](2) = 16$

The tangent line is $y - 24 = 16(x - 0)$, or $y = 16x + 24$.

54. $y = \frac{x-1}{x(x^2+1)} = \frac{x-1}{x^3+x}$

$y' = \frac{(x^3+x)(1)-(x-1)(3x^2+1)}{(x^3+x)^2}$

$y'(2) = \frac{8+2-(1)(12+1)}{(8+2)^2} = \frac{10-13}{10^2} = -\frac{3}{100}$

The tangent line is $y - \frac{1}{10} = -\frac{3}{100}(x-2)$, or

$y = -\frac{3}{100}x + \frac{4}{25}$.

55. $y = \frac{x}{2x-6}$

$y' = \frac{(2x-6)(1)-x(2)}{(2x-6)^2} = \frac{-6}{(2x-6)^2}$

If $x = 1$, then $y = \frac{1}{2-6} = -\frac{1}{4}$ and

$y' = \frac{-6}{(-4)^2} = \frac{-6}{16} = -\frac{3}{8}$.

Thus $\frac{y'}{y} = \frac{-\frac{3}{8}}{-\frac{1}{4}} = \frac{3}{2} = 1.5$.

56. $y = \frac{1-x}{1+x}$

$y' = \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2} = -\frac{2}{(1+x)^2}$

When $x = 5$, then $\frac{y'}{y} = \frac{-\frac{1}{18}}{-\frac{2}{3}} = \frac{1}{12}$.

57. $s = \frac{2}{t^3+1}$. When $t = 1$, then $s = 1$ m.

$v = \frac{ds}{dt} = \frac{(t^3+1)(0)-2(3t^2)}{(t^3+1)^2} = -\frac{6t^2}{(t^3+1)^2}$

If $t = 1$, then $v = -\frac{6}{4} = -1.5$ m/s.

58. $s = \frac{t+3}{t^2+7}$

$v = \frac{ds}{dt} = \frac{(t^2+7)(1)-(t+3)(2t)}{(t^2+7)^2}$

$= \frac{7-6t-t^2}{(t^2+7)^2} = \frac{(7+t)(1-t)}{(t^2+7)^2}$

$v = 0$ when $t = -7$ or $t = 1$. Since t is positive, we choose $t = 1$.

59. $p = 80 - 0.02q$

$r = pq = 80q - 0.02q^2$

$\frac{dr}{dq} = 80 - 0.04q$

60. $p = \frac{500}{q}$

$r = pq = 500$

$\frac{dr}{dq} = 0$

61. $p = \frac{108}{q+2} - 3$

$r = pq = \frac{108q}{q+2} - 3q$

$\frac{dr}{dq} = \frac{(q+2)(108) - (108q)(1)}{(q+2)^2} - 3$

$= \frac{216}{(q+2)^2} - 3$

62. $p = \frac{q+750}{q+50}$

$r = pq = \frac{q^2 + 750q}{q+50}$

$\frac{dr}{dq} = \frac{(q+50)(2q+750) - \left(q^2 + 750q\right)(1)}{(q+50)^2}$

$= \frac{q^2 + 100q + 37{,}500}{(q+50)^2}$

63. $\frac{dC}{dI} = 0.672$

64. $\frac{dC}{dI} = 0.836$

65. $C = 3 + I^{1/2} + 2I^{1/3}$

$\frac{dC}{dI} = 0 + \frac{1}{2}I^{-1/2} + \frac{2}{3}I^{-2/3} = \frac{1}{2\sqrt{I}} + \frac{2}{3\sqrt[3]{I^2}}$

When $I = 1$, then $\frac{dC}{dI} = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$.

$\frac{dS}{dI} = 1 - \frac{dC}{dI} = 1 - \frac{1}{2\sqrt{I}} - \frac{2}{3\sqrt[3]{I^2}}$

When $I = 1$, then $1 - \frac{dC}{dI} = 1 - \frac{7}{6} = -\frac{1}{6}$.

66. $\frac{dC}{dI} = \frac{3}{4} - \frac{1}{6\sqrt{I}}$

$\left.\frac{dC}{dI}\right|_{I=25} = \frac{43}{60}$, so $\left.\frac{dS}{dI}\right|_{I=25} = 1 - \frac{43}{60} = \frac{17}{60}$

67. $\frac{dC}{dI} = \frac{\left(\sqrt{I}+4\right)\left(\frac{8}{\sqrt{I}} + 1.2\sqrt{I} - 0.2\right) - \left(16\sqrt{I} + 0.8\sqrt{I^3} - 0.2I\right)\left(\frac{1}{2\sqrt{I}}\right)}{\left(\sqrt{I}+4\right)^2}$

$\left.\frac{dC}{dI}\right|_{I=36} \approx 0.615$, so $\frac{dS}{dI} \approx 1 - 0.615 = 0.385$ when $I = 36$.

68. $\frac{dC}{dI} = \frac{\left(\sqrt{I}+5\right)\left(\frac{10}{\sqrt{I}} + 0.75\sqrt{I} - 0.4\right) - \left(20\sqrt{I} + 0.5\sqrt{I^3} - 0.4I\right)\left(\frac{1}{2\sqrt{I}}\right)}{\left(\sqrt{I}+5\right)^2}$

$\left.\frac{dC}{dI}\right|_{I=100} \approx 0.393$, so $\frac{dS}{dI} \approx 1 - 0.393 = 0.607$ when $I = 100$.

69. Simplifying gives $C = 9 + 0.8I - 0.3I^{1/2}$

a. $\frac{dC}{dI} = 0.8 - 0.15I^{-1/2}$

$\frac{dS}{dI} = 1 - \frac{dC}{dI} = 0.2 + 0.15I^{-1/2}$

$\left.\frac{dS}{dI}\right|_{I=25} = 0.2 + 0.15 \cdot 25^{-1/2} = 0.2 + \frac{0.15}{5} = 0.23$

b. $\frac{\frac{dC}{dI}}{C}$ when $I = 25$ is $\frac{0.8 - \frac{0.15}{5}}{9 + 0.8(25) - 0.3\sqrt{25}} = 0.028$

70. Simplifying S gives

$S = \frac{I - 2\sqrt{I} - 8}{\sqrt{I}+2} = \frac{\left(\sqrt{I}+2\right)\left(\sqrt{I}-4\right)}{\sqrt{I}+2} = \sqrt{I} - 4$

Thus $\frac{dS}{dI} = \frac{1}{2}I^{-1/2} = \frac{1}{2\sqrt{I}}$.

$\left.\frac{dS}{dI}\right|_{I=150} = \frac{1}{2 \cdot \sqrt{150}} \approx 0.04082$ and $\left.\frac{dC}{dI}\right|_{I=150} \approx 1 - 0.04082 \approx 0.9592$.

71. $\frac{dc}{dq} = 6 \cdot \frac{(q+2)(2q) - q^2(1)}{(q+2)^2} = 6 \cdot \frac{q^2 + 4q}{(q+2)^2} = \frac{6q(q+4)}{(q+2)^2}$

72. We assume that $\frac{d}{dq}(\overline{c})=0$. Thus $0=\frac{d\overline{c}}{dq}=\frac{d}{dq}\left(\frac{c}{q}\right)=\frac{q\cdot\frac{dc}{dq}-c(1)}{q^2}$.

This implies that $q\cdot\frac{dc}{dq}-c=0$, $q\cdot\frac{dc}{dq}=c$, $\frac{dc}{dq}=\frac{c}{q}=\overline{c}$, so the marginal cost function $\left(\frac{dc}{dq}\right)$ and the average cost function $(\overline{c})$ are equal.

73. $y=\frac{900x}{10+45x}$

$\frac{dy}{dx}=\frac{(10+45x)(900)-(900x)(45)}{(10+45x)^2}$

$\left.\frac{dy}{dx}\right|_{x=2}=\frac{(100)(900)-(1800)(45)}{(100)^2}=\frac{9}{10}$

74. $\text{RT}=\frac{0.05V}{A+xV}$

$\frac{d}{dV}(\text{RT})=\frac{(A+xV)(0.05)-(0.05V)(x)}{(A+xV)^2}$

$=\frac{0.05A}{(A+xV)^2}$

Both numerator and denominator are always positive, so $\frac{d}{dV}(\text{RT})>0$. This rate of change means that if V increases by one unit, RT increases.

75. $y=\frac{0.7355x}{1+0.02744x}$

$\frac{dy}{dx}=\frac{(1+0.02744x)(0.7355)-(0.7355x)(0.02744)}{(1+0.02744x)^2}$

$=\frac{0.7355}{(1+0.02744x)^2}$

76. $f(x)=\frac{a(1+x)-b(2+n)x}{a(2+n)(1+x)-b(2+n)x}$

For convenience let $c = 2 + n$.

Then $f(x)=\frac{a(1+x)-bcx}{ac(1+x)-bcx}=\frac{1}{c}\cdot\frac{a(1+x)-bcx}{a(1+x)-bx}$.

$f'(x)=\frac{1}{c}\cdot\frac{[a(1+x)-bx](a-bc)-[a(1+x)-bcx](a-b)}{[a(1+x)-bx]^2}$

$=\frac{1}{c}\cdot\frac{-abc+ab}{[a(1+x)-bx]^2}=\frac{1}{c}\cdot\frac{(-c+1)ab}{[a(1+x)-bx]^2}$

$=\frac{1}{2+n}\cdot\frac{[-1(2+n)+1]ab}{[a(1+x)-bx]^2}=\frac{-(1+n)ab}{[a(1+x)-bx]^2(2+n)}$

$g(x)=\frac{A+Bx}{C+Dx}$

$$g'(x) = \frac{(C+Dx)(B)-(A+Bx)(D)}{(C+Dx)^2}$$
$$= \frac{CB+BDx+AD-BDx}{(C+Dx)^2}$$
$$= \frac{BC-AD}{(C+Dx)^2}$$

Thus, $g'(x)$ has the form given. When $g'(x)$ is defined $\left(\text{for } x \neq \frac{C}{D}\right)$, its sign is constant.

77. $\frac{d\overline{c}}{dq} = \frac{d}{dq}\left(\frac{c}{q}\right) = \frac{q \cdot \frac{dc}{dq} - c(1)}{q^2}$. When $q = 20$ we have $\frac{\frac{d\overline{c}}{dq}}{\overline{c}} = \frac{\frac{q \cdot \frac{dc}{dq} - c}{q^2}}{\overline{c}} = \frac{\frac{20(125)-20(150)}{(20)^2}}{150} = -\frac{1}{120}$

78. $\frac{dy}{dx} = (3)(2x-1)(x-4)+(3x+1)(2)(x-4)+(3x+1)(2x-1)(1)$
$= 18x^2 - 50x + 3$

Apply It 11.5

8. By the chain rule,
$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{d}{dx}\left(4x^2\right) \cdot \frac{d}{dt}(6t) = (8x)(6) = 48x$.
Since $x = 6t$, $\frac{dy}{dt} = 48(6t) = 288t$.

Problems 11.5

1. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (2u-2)(2x-1) = \left[2\left(x^2-x\right)-2\right](2x-1) = \left(2x^2-2x-2\right)(2x-1) = 4x^3-6x^2-2x+2$

2. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(6u^2-8\right)\left(7-3x^2\right) = 2\left(3x^6-42x^4+147x^2-4\right)\left(7-3x^2\right)$

3. $\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dx} = \left(-\frac{1}{w^2}\right)(3) = -\frac{3}{w^2} = -\frac{3}{(3x-5)^2}$

4. $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{4}z^{-3/4}(5x^4-4x^3) = \frac{5x^4-4x^3}{4\left(\sqrt[4]{(x^5-x^4+3)^3}\right)}$

5. $\frac{dw}{dt} = \frac{dw}{du} \cdot \frac{du}{dt} = (3u^2)\left[\frac{(t+1)-(t-1)}{(t+1)^2}\right] = 3u^2\left[\frac{2}{(t+1)^2}\right]$. If $t = 1$, then $u = \frac{1-1}{1+1} = 0$, so $\left.\frac{dw}{dt}\right|_{t=1} = 3(0)^2\left[\frac{2}{4}\right] = 0$.

6. $\frac{dz}{ds} = \frac{dz}{du} \cdot \frac{du}{ds} = \left(2u + \frac{1}{2\sqrt{u}}\right)(4s)$. If $s = -1$, then $u = 1$, so $\left.\frac{dz}{ds}\right|_{s=-1} = \left(\frac{5}{2}\right)(-4) = -10$

7. $\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dx} = (6w-8)(4x)$. If $x = 0$, then $\frac{dy}{dx} = 0$.

8. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (6u^2 + 6u + 5)(3)$. If $x = 1$, then $u = 4$, so $\left.\frac{dy}{dx}\right|_{x=1} = (125)(3) = 375$

9. $y' = 6(3x+2)^5 \cdot \frac{d}{dx}(3x+2)$
$= 6(3x+2)^5(3) = 18(3x+2)^5$

10. $y' = 4\left(x^2-4\right)^3 \cdot \frac{d}{dx}\left(x^2-4\right)$
$= 4\left(x^2-4\right)^3(2x) = 8x\left(x^2-4\right)^3$

11. $y' = 5(3+2x^3)^4 \cdot \frac{d}{dx}(3+2x^3)$
$= 5(3+2x^3)^4(6x^2)$
$= 30x^2(3+2x^3)$

12. $y' = 4(x^2+x)^3 \frac{d}{dx}(x^2+x)$
$= 4(x^2+x)^3(2x+1)$
$= 4(2x+1)(x^2+x)^3$

13. $y' = 5 \cdot 100(x^3 - 3x^2 + 2x)^{99} \cdot \frac{d}{dx}(x^3 - 3x^2 + 2x)$
$= 500(x^3 - 3x^2 + 2x)^{99}(3x^2 - 6x + 2)$

14. $y = \frac{\left(2x^2+1\right)^4}{2} = \frac{1}{2}\left(2x^2+1\right)^4$
$y' = \frac{1}{2} \cdot 4\left(2x^2+1\right)^3 \frac{d}{dx}(2x^2+1)$
$= 2(2x^2+1)^3(4x) = 8x\left(2x^2+1\right)^3$

15. $y' = -3\left(x^2-2\right)^{-4} \cdot \frac{d}{dx}\left(x^2-2\right)$
$= -3\left(x^2-2\right)^{-4}(2x) = -6x\left(x^2-2\right)^{-4}$

16. $y' = -12(2x^3-8x)^{-13} \cdot \frac{d}{dx}(2x^3-8x)$
$= -12(6x^2-8)(2x^3-8x)^{-13}$

17. $y' = 2\left(-\frac{5}{7}\right)(x^2+5x-2)^{-12/7} \cdot \frac{d}{dx}(x^2+5x-2)$
$= -\frac{10}{7}(2x+5)(x^2+5x-2)^{-12/7}$

18. $y' = 3\left(-\frac{5}{3}\right)(5x-2x^3)^{-8/3}(5-6x^2)$
$= \frac{5(6x^2-5)}{(5x-2x^3)^{8/3}}$

19. $y = \sqrt{5x^2-x} = \left(5x^2-x\right)^{\frac{1}{2}}$
$y' = \frac{1}{2}\left(5x^2-x\right)^{-\frac{1}{2}}(10x-1)$
$= \frac{1}{2}(10x-1)\left(5x^2-x\right)^{-\frac{1}{2}}$

20. $y = \sqrt{3x^2-7} = \left(3x^2-7\right)^{\frac{1}{2}}$
$y' = \frac{1}{2}\left(3x^2-7\right)^{-\frac{1}{2}}(6x) = 3x\left(3x^2-7\right)^{-\frac{1}{2}}$

21. $y = \sqrt[4]{2x-1} = (2x-1)^{\frac{1}{4}}$
$y' = \frac{1}{4}(2x-1)^{-\frac{3}{4}}(2) = \frac{1}{2}(2x-1)^{-\frac{3}{4}}$

22. $y = \sqrt[3]{8x^2-1} = \left(8x^2-1\right)^{\frac{1}{3}}$
$y' = \frac{1}{3}\left(8x^2-1\right)^{-\frac{2}{3}}(16x) = \frac{16}{3}x\left(8x^2-1\right)^{-\frac{2}{3}}$

23. $y = 4\sqrt[7]{(x^2+1)^3} = 4(x^2+1)^{3/7}$
$y' = 4\left(\frac{3}{7}\right)(x^2+1)^{-4/7}(2x) = \frac{24x}{7(x^2+1)^{4/7}}$

24. $y = 7\sqrt[3]{(x^5-3)^5} = 7(x^5-3)^{5/3}$

$y' = 7\cdot\dfrac{5}{3}(x^5-3)^{2/3}(5x^4)$

$= \dfrac{175}{3}x^4(x^5-3)^{2/3}$

25. $y = \dfrac{6}{2x^2-x+1} = 6\left(2x^2-x+1\right)^{-1}$

$y' = 6(-1)\left(2x^2-x+1\right)^{-2}(4x-1)$

$= -6(4x-1)\left(2x^2-x+1\right)^{-2}$

26. $y = \dfrac{3}{x^4+2} = 3\left(x^4+2\right)^{-1}$

$y' = 3(-1)\left(x^4+2\right)^{-2}\left(4x^3\right) = -12x^3\left(x^4+2\right)^{-2}$

27. $y = \dfrac{1}{\left(x^2-3x\right)^2} = \left(x^2-3x\right)^{-2}$

$y' = -2\left(x^2-3x\right)^{-3}(2x-3)$

$= -2(2x-3)\left(x^2-3x\right)^{-3}$

28. $y = \dfrac{1}{(3+5x)^3} = (3+5x)^{-3}$

$y' = -3(3+5x)^{-4}(5) = -\dfrac{15}{(3+5x)^4}$

29. $y = \dfrac{4}{\sqrt{9x^2+1}} = 4(9x^2+1)^{-1/2}$

$y' = 4\left(-\dfrac{1}{2}\right)(9x^2+1)^{-3/2}(18x)$

$= -36x(9x^2+1)^{-3/2}$

30. $y = \dfrac{3}{\left(3x^2-x\right)^{\frac{2}{3}}} = 3\left(3x^2-x\right)^{-\frac{2}{3}}$

$y' = 3\left(-\dfrac{2}{3}\right)\left(3x^2-x\right)^{-\frac{5}{3}}(6x-1)$

$= -2(6x-1)\left(3x^2-x\right)^{-\frac{5}{3}}$

31. $y = \sqrt[3]{7x} + \sqrt[3]{7}x = (7x)^{\frac{1}{3}} + \sqrt[3]{7}x$

$y' = \dfrac{1}{3}(7x)^{-\frac{2}{3}}(7) + \sqrt[3]{7}(1) = \dfrac{7}{3}(7x)^{-\frac{2}{3}} + \sqrt[3]{7}$

32. $y = \sqrt{2x} + \dfrac{1}{\sqrt{2x}} = (2x)^{\frac{1}{2}} + (2x)^{-\frac{1}{2}}$

$y' = \left(\dfrac{1}{2}\right)(2x)^{-\frac{1}{2}}(2) + \left(-\dfrac{1}{2}\right)(2x)^{-\frac{3}{2}}(2)$

$= (2x)^{-\frac{1}{2}} - (2x)^{-\frac{3}{2}}$

33. $y' = 3x^2(2x+3)^7 + x^3(7)(2x+3)^6(2)$

$= 3x^2(2x+3)^7 + 14x^3(2x+3)^6$

$= (6x^3+9x^2)(2x+3)^6 + 14x^3(2x+3)^6$

$= (20x^3+9x^2)(2x+3)^6$

$= x^2(20x+9)(2x+3)^6$

34. $y' = x\left[4(x+4)^3(1)\right] + (x+4)^4(1)$

$= (x+4)^3(4x+x+4) = (x+4)^3(5x+4)$

35. $y = 4x^2\sqrt{5x+1} = 4x^2(5x+1)^{\frac{1}{2}}$

$y' = 4x^2\left(\dfrac{1}{2}(5x+1)^{-\frac{1}{2}}(5)\right) + \sqrt{5x+1}(8x)$

$= 10x^2(5x+1)^{-\frac{1}{2}} + 8x\sqrt{5x+1}$

36. $y = 4x^3\sqrt{1-x^2} = 4x^3(1-x^2)^{\frac{1}{2}}$

$y' = 4x^3\left[\left(\dfrac{1}{2}\right)(1-x^2)^{-1/2}(-2x)\right] + \sqrt{1-x^2}(12x^2)$

$= -\dfrac{4x^4}{\sqrt{1-x^2}} + 12x^2\sqrt{1-x^2}$

37. $y' = \left(x^2+2x-1\right)^3(5)+(5x)\left[3\left(x^2+2x-1\right)^2(2x+2)\right]$

$= 5\left(x^2+2x-1\right)^2\left[\left(x^2+2x-1\right)+3x(2x+2)\right]$

$= 5\left(x^2+2x-1\right)^2\left(7x^2+8x-1\right)$

38. $y' = 4x^3(x^4-1)^5 + x^4(5)(x^4-1)^4(4x^3)$
$= (4x^7-4x^3+20x^7)(x^4-1)^4$
$= (24x^7-4x^3)(x^4-1)$
$= 4x^3(6x^4-1)(x^4-1)^4$

39. $y' = (8x-1)^3\left[4(2x+1)^3(2)\right]+(2x+1)^4\left[3(8x-1)^2(8)\right]$

$= 8(8x-1)^2(2x+1)^3[(8x-1)+3(2x+1)]$

$= 8(8x-1)^2(2x+1)^3(14x+2)$

$= 16(8x-1)^2(2x+1)^3(7x+1)$

40. $y' = (3x+2)^5[2(4x-5)(4)]+(4x-5)^2[5(3x+2)^4(3)]$
$= (3x+2)^4(4x-5)[8(3x+2)+15(4x-5)]$
$= (3x+2)^4(4x-5)(84x-59)$

41. $y' = 12\left(\frac{x-3}{x+2}\right)^{11}\left[\frac{(x+2)(1)-(x-3)(1)}{(x+2)^2}\right]$

$= 12\left(\frac{x-3}{x+2}\right)^{11}\left[\frac{5}{(x+2)^2}\right]$

$= \frac{60(x-3)^{11}}{(x+2)^{13}}$

42. $y' = 4\left(\frac{2x}{x+2}\right)^3\left[\frac{(x+2)(2)-2x(1)}{(x+2)^2}\right] = \frac{128x^3}{(x+2)^5}$

43. $y' = \frac{1}{2}\left(\frac{x+1}{x-5}\right)^{-1/2}\left[\frac{(x-5)(1)-(x+1)(1)}{(x-5)^2}\right]$

$= \frac{1}{2}\left(\frac{x+1}{x-5}\right)^{-1/2}\cdot\frac{x-5-x-1}{(x-5)^2}$

$= -3\sqrt{\frac{x-5}{x+1}}\cdot\frac{1}{(x-5)^2}$

$= -\frac{3}{(x-5)^2}\sqrt{\frac{x-5}{x+1}}$

44. $y' = \frac{1}{3}\left(\frac{8x^2-3}{x^2+2}\right)^{-\frac{2}{3}}\left[\frac{\left(x^2+2\right)(16x)-\left(8x^2-3\right)(2x)}{\left(x^2+2\right)^2}\right]$

$= \frac{1}{3}\left(\frac{8x^2-3}{x^2+2}\right)^{-\frac{2}{3}}\frac{38x}{\left(x^2+2\right)^2}$

$= \frac{38x}{3\left(8x^2-3\right)^{\frac{2}{3}}\left(x^2+2\right)^{\frac{4}{3}}}$

45. $y' = \frac{\left(x^2+4\right)^3(2)-(2x-5)\left[3\left(x^2+4\right)^2(2x)\right]}{\left(x^2+4\right)^6}$

$= \frac{\left(x^2+4\right)^2\left\{\left(x^2+4\right)(2)-(2x-5)[3(2x)]\right\}}{\left(x^2+4\right)^6}$

$= \frac{2x^2+8-12x^2+30x}{\left(x^2+4\right)^4} = \frac{-10x^2+30x+8}{\left(x^2+4\right)^4}$

$= \frac{-2\left(5x^2-15x-4\right)}{\left(x^2+4\right)^4}$

46. $y' = \frac{(3x^2+7)[4(4x-2)^3(4)]-(4x-2)^4(6x)}{(3x^2+7)^2}$

$= \frac{(4x-2)^3[16(3x^2+7)-6x(4x-2)]}{(3x^2+7)^2}$

$= \frac{(4x-2)^3(24x^2+12x+112)}{(3x^2+7)^2}$

47. $y' = \frac{(3x-1)^3\left[5(8x-1)^4(8)\right]-(8x-1)^5\left[3(3x-1)^2(3)\right]}{(3x-1)^6}$

$= \frac{(3x-1)^2(8x-1)^4[(3x-1)(40)-(8x-1)(9)]}{(3x-1)^6}$

$= \frac{(8x-1)^4(48x-31)}{(3x-1)^4}$

48. $y = \sqrt[3]{(x-3)^3(x+5)} = (x-3)(x+5)^{1/3}$

$$y' = (1)(x+5)^{1/3} + (x-3)\left(\frac{1}{3}\right)(x+5)^{-2/3}(1)$$
$$= (x+5)^{1/3} + \frac{x-3}{3(x+5)^{2/3}}$$
$$= \frac{3x+15+x-3}{3(x+5)^{2/3}}$$
$$= \frac{4x+12}{3(x+5)^{2/3}}$$
$$= \frac{4(x+3)}{3(x+5)^{2/3}}$$

49. $y = 6\left(5x^2+2\right)\sqrt{x^4+5} = 6\left[\left(5x^2+2\right)\left(x^4+5\right)^{\frac{1}{2}}\right]$

$$y' = 6\left[\left(5x^2+2\right)\cdot\frac{1}{2}\left(x^4+5\right)^{-\frac{1}{2}}\left(4x^3\right)+\left(x^4+5\right)^{\frac{1}{2}}(10x)\right]$$
$$= 6\left[\left(5x^2+2\right)\left(x^4+5\right)^{-\frac{1}{2}}\left(2x^3\right)+\left(x^4+5\right)^{\frac{1}{2}}(10x)\right]$$
$$= 12x\left[\left(5x^2+2\right)\left(x^4+5\right)^{-\frac{1}{2}}\left(x^2\right)+\left(x^4+5\right)^{\frac{1}{2}}(5)\right]$$

Factoring out $\left(x^4+5\right)^{-\frac{1}{2}}$ gives

$$y' = 12x\left(x^4+5\right)^{-\frac{1}{2}}\left[\left(5x^2+2\right)\left(x^2\right)+\left(x^4+5\right)(5)\right]$$
$$= 12x\left(x^4+5\right)^{-\frac{1}{2}}\left(10x^4+2x^2+25\right)$$

50. $y' = 3-4\left[x(2)(7x+1)(7)+(7x+1)^2(1)\right]$

$$= 3-4\left[147x^2+28x+1\right] = -588x^2-112x-1$$

51. $y' = 8+\frac{(t+4)(1)-(t-1)(1)}{(t+4)^2}-2\left(\frac{8t-7}{4}\right)\left(\frac{1}{4}\cdot 8\right)$

$$= 8+\frac{5}{(t+4)^2}-(8t-7) = 15-8t+\frac{5}{(t+4)^2}$$

52. $y=\dfrac{(2x^3+6)(7x-5)}{(2x+4)^2}=\dfrac{14x^4-10x^3+42x-30}{(2x+4)^2}$

$$y'=\frac{(2x+4)^2(56x^3-30x^2+42)-(14x^4-10x^3+42x-30)[2(2x+4)(2)]}{(2x+4)^4}$$
$$=\frac{(2x+4)[(2x+4)(56x^3-30x^2+42)-4(14x^4-10x^3+42x-30)]}{(2x+4)^4}$$
$$=\frac{112x^4-60x^3+84x+224x^3-120x^2+168-56x^4-40x^3-168x+120}{(2x+4)^3}$$
$$=\frac{4(14x^4+51x^3-30x^2-21x+72)}{(2x+4)^3}$$

53. $y'=\dfrac{(x^2-7)^3[3(3x+2)^2(3)(x+1)^4+(3x+2)^3(4)(x+1)^4]-(3x+2)^3(x+1)^4(3)(x^2-7)^2(2x)}{(x^2-7)^6}$

54. $y'=\dfrac{(9x-3)\left[\sqrt{x+2}(2)\left(4x^2-1\right)(8x)+\left(4x^2-1\right)^2\left(\frac{1}{2}\right)(x+2)^{-\frac{1}{2}}\right]-\sqrt{x+2}\left(4x^2-1\right)^2(9)}{(9x-3)^2}$

55. $\dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}=\left[3(5u+6)^2(5)\right]\left[4\left(x^2+1\right)^3(2x)\right]$

When $x = 0$, then $\dfrac{dy}{dx}=0$.

56. $\dfrac{dz}{dt}=\dfrac{dz}{dy}\cdot\dfrac{dy}{dx}\cdot\dfrac{dx}{dt}=(4y-4)(6)(2)$

When $t = 1$, then $x = 2$ and $y = 7$. Thus $\left.\dfrac{dz}{dt}\right|_{t=1}=(24)(6)(2)=288$.

57. $y'=3\left(x^2-7x-8\right)^2(2x-7)$

If $x = 8$, then slope $=y'=3(64-56-8)^2(16-7)=0$.

58. $y=(x+2)^{\frac{1}{2}}$

$y'=\dfrac{1}{2}(x+2)^{-\frac{1}{2}}$

If $x = 7$, then slope $=y'=\dfrac{1}{6}$.

59. $y = \left(x^2 - 8\right)^{\frac{2}{3}}$

$y' = \frac{2}{3}\left(x^2 - 8\right)^{-\frac{1}{3}}(2x) = \frac{4x}{3\left(x^2-8\right)^{\frac{1}{3}}}$

If $x = 3$, then $y' = \frac{12}{3(1)} = 4$. Thus the tangent line is $y - 1 = 4(x - 3)$, or $y = 4x - 11$.

60. $y' = 3(x+3)^2(1) = 3(x+3)^2$

If $x = -1$, $y' = 3(2)^2 = 12$.
The tangent line is $y - 8 = 12(x + 1)$ or $y = 12x + 20$.

61. $y' = \frac{(x+1)\left(\frac{1}{2}\right)(7x+2)^{-\frac{1}{2}}(7) - \sqrt{7x+2}(1)}{(x+1)^2}$

$= \frac{(x+1)\left(\frac{7}{2}\right)\frac{1}{\sqrt{7x+2}} - \sqrt{7x+2}}{(x+1)^2}$

If $x = 1$, then $y' = \frac{2\left(\frac{7}{2}\right)\left(\frac{1}{3}\right) - 3}{4} = -\frac{1}{6}$. The tangent line is $y - \frac{3}{2} = -\frac{1}{6}(x-1)$, or

$y = -\frac{1}{6}x + \frac{5}{3}$.

62. $y = -3\left(3x^2+1\right)^{-3}$

$y' = -3(-3)\left(3x^2+1\right)^{-4}(6x)$

If $x = 0$, then $y' = 0$. The tangent line is $y + 3 = 0(x - 0)$, or $y = -3$.

63. $y = \left(x^2+1\right)^4$ and

$y' = 4(x^2+1)^3(2x) = 8x(x^2+1)^3$. When $x = 1$, then $y = 2^4$ and $y' = 8 \cdot 2^3 = 2^6$, so

$\left(\frac{y'}{y}\right)(100) = 2^2 \cdot 100 = 400\%$.

64. $y = \frac{1}{(x^2-1)^3}$ and $y' = -\frac{6x}{(x^2-1)^4}$

When $x = 2$, $y = \frac{1}{27}$ and $y' = -\frac{12}{3^4} = -\frac{4}{27}$, so

$\left(\frac{y'}{y}\right)(100) = -\frac{4}{27} \cdot 27(100) = -400\%$

65. $q = 5m$, $p = -0.4q + 50$; $m = 6$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}$

$r = pq = -0.4q^2 + 50q$, $\frac{dr}{dq} = -0.8q + 50,$. For $m = 6$, then $q = 30$, so $\left.\frac{dr}{dq}\right|_{m=6} = -24 + 50 = 26.$

Also, $\frac{dq}{dm} = 5.$ Thus $\left.\frac{dr}{dm}\right|_{m=6} = (26)(5) = 130.$

66. $q = \frac{1}{20}\left(200m - m^2\right)$

$p = -0.1q + 70$; $m = 40$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}$

$r = pq = -0.1q^2 + 70q$, so $\frac{dr}{dq} = -0.2q + 70$. If $m = 40$, then $q = 320$, so

$\left.\frac{dr}{dq}\right|_{m=40} = -64 + 70 = 6$.

$\frac{dq}{dm} = \frac{1}{20}(200 - 2m)$. When $m = 40$, $\frac{dq}{dm} = 6$.

Thus $\left.\frac{dr}{dm}\right|_{m=40} = (6)(6) = 36$.

67. $q = \frac{10m^2}{\sqrt{m^2+9}}$

$p = \frac{525}{q+3}$; $m = 4$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}$

$r = pq = \frac{525q}{q+3}$, so

$\frac{dr}{dq} = 525 \cdot \frac{(q+3)(1) - q(1)}{(q+3)^2} = \frac{1575}{(q+3)^2}$.

If $m = 4$, then $q = 32$, so $\left.\frac{dr}{dq}\right|_{m=4} = \frac{1575}{1225} = \frac{9}{7}$.

$$\frac{dq}{dm} = \frac{\left(m^2+9\right)^{\frac{1}{2}}(20m) - 10m^2 \cdot \frac{1}{2}\left(m^2+9\right)^{-\frac{1}{2}}(2m)}{m^2+9}$$

$$= \frac{\left(m^2+9\right)^{-\frac{1}{2}}\left[20m\left(m^2+9\right) - 10m^3\right]}{m^2+9}$$

$$= \frac{10m^3+180m}{\left(m^2+9\right)^{\frac{3}{2}}}$$

When $m = 4$, then

$\frac{dq}{dm} = \frac{10(64)+180(4)}{(25)^{\frac{3}{2}}} = \frac{1360}{125} = \frac{272}{25}$. Thus

$\left.\frac{dr}{dm}\right|_{m=4} = \frac{9}{7} \cdot \frac{272}{25} \approx 13.99$.

68. $q = \frac{50m}{\sqrt{m^2+11}}$

$p = \frac{100}{q+10}$; $m = 5$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}$

$r = pq = \frac{100q}{q+10}$, so $\frac{dr}{dq} = \frac{1000}{(q+10)^2}$.

If $m = 5$, then $q = \frac{125}{3}$, so $\left.\frac{dr}{dq}\right|_{m=5} = \frac{360}{961}$.

$\frac{dq}{dm} = \frac{550}{\left(m^2+11\right)^{\frac{3}{2}}}$. When $m = 5$, then

$\frac{dq}{dm} = \frac{275}{108}$. Thus $\left.\frac{dr}{dm}\right|_{m=5} = \frac{360}{961} \cdot \frac{275}{108} = \frac{2750}{2883}$.

69. a. $\frac{dp}{dq} = 0 - \frac{1}{2}\left(q^2+20\right)^{-\frac{1}{2}}(2q) = \frac{-q}{\sqrt{q^2+20}}$

b. $$\frac{\frac{dp}{dq}}{p} = \frac{\frac{-q}{\sqrt{q^2+20}}}{100-\sqrt{q^2+20}}$$

$$= -\frac{q}{\sqrt{q^2+20}\left(100-\sqrt{q^2+20}\right)}$$

$$= -\frac{q}{100\sqrt{q^2+20} - q^2 - 20}$$

c. $r = pq = 100q - q\sqrt{q^2+20}$

$$\frac{dr}{dq} = 100 - \left[q \cdot \frac{1}{2}\left(q^2+20\right)^{-\frac{1}{2}}(2q) + \sqrt{q^2+20}(1)\right]$$

$$= 100 - \frac{q^2}{\sqrt{q^2+20}} - \sqrt{q^2+20}$$

70. $p = \frac{k}{q}$; $q = f(m)$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}$

$r = pq = k$, so $\frac{dr}{dq} = 0$. Thus $\frac{dr}{dm} = 0 \cdot \frac{dq}{dm} = 0$.

71. $\frac{dc}{dp} = \frac{dc}{dq} \cdot \frac{dq}{dp} = (12+0.4q)(-1.5)$

When $p = 85$, then $q = 772.5$, so

$\left.\frac{dc}{dp}\right|_{p=85} = -481.5$.

72. $f(t) = 1 - \left(\frac{250}{250+t}\right)^3$

$$f'(t) = -3\left(\frac{250}{250+t}\right)^2\left[-\frac{250}{(250+t)^2}\right]$$

$$f'(100) = -3\left(\frac{250}{350}\right)^2\left[-\frac{250}{350^2}\right]$$

$$= -3\left(\frac{25}{49}\right)\left(-\frac{1}{490}\right)$$

$$= \frac{15}{4802}.$$

Thus when t increases from 100 to 101, the proportion discharged increases by approximately $\frac{15}{4802}$.

73. $\dfrac{dc}{dq} = \dfrac{(q^2+2)^{1/2}(8q) - 4q^2\left(\frac{1}{2}\right)(q^2+2)^{-1/2}(2q)}{q^2+2}$

Multiplying the numerator and denominator by $\sqrt{q^2+2}$ gives

$\dfrac{dc}{dq} = \dfrac{(q^2+2)(8q)-4q^3}{(q^2+2)^{3/2}} = \dfrac{4q^3+16q}{(q^2+2)^{3/2}} = \dfrac{4q(q^2+4)}{(q^2+2)^{3/2}}.$

74. a. $\dfrac{dS}{dE} = 680E - 4360$. If $E = 16$, $\dfrac{dS}{dE} = 6520$.

b. Solving $680E - 4360 = 5000$ gives $680E = 9360, E \approx 13.8$.

75. $\dfrac{dV}{dt} = \dfrac{dV}{dr}\cdot\dfrac{dr}{dt} = \left(4\pi r^2\right)\left[10^{-8}(2t)+10^{-7}\right]$. When $t = 10$, then $r = 10^{-8}\left(10^2\right)+10^{-7}(10) = 10^{-6}+10^{-6} = 2(10)^{-6}$.
Thus

$\left.\dfrac{dV}{dt}\right|_{t=10} = 4\pi\left[2(10)^{-6}\right]^2\left[10^{-8}(2)(10)+10^{-7}\right] = 4\pi\left[4(10)^{-12}\right]\left[3\left(10^{-7}\right)\right] = 48\pi(10)^{-19}$

76. a. $\dfrac{dp}{dI} = \dfrac{1}{2}(2\rho VI)^{-\frac{1}{2}}(2\rho V) = \rho V(2\rho VI)^{-\frac{1}{2}}$

b. $\dfrac{\frac{dp}{dI}}{p} = \dfrac{\rho V(2\rho VI)^{-\frac{1}{2}}}{(2\rho VI)^{\frac{1}{2}}} = \dfrac{1}{2I}$

77. a. $\dfrac{d}{dx}(I_x) = -0.001416x^3 + 0.01356x^3 + 1.696x - 34.9$

If $x = 65$, $\dfrac{d}{dx}(I_x) = -256.238$.

b. If $x = 65$, $\dfrac{\frac{d}{dx}(I_x)}{I_x} \approx \dfrac{-256.238}{16,236.484} \approx -0.01578$

If $x = 65$, the percentage rate of change is $\dfrac{\frac{d}{dx}(I_x)}{I_x}\cdot = \dfrac{-25,623.8}{16,236.484} = -1.578\%.$

78. $(P + a)(v + b) = k$

$v+b = \dfrac{k}{P+a}$

$v = \dfrac{k}{P+a} - b$

$v = k(P+a)^{-1} - b$

$\dfrac{dv}{dP} = k(-1)(P+a)^{-2} = -\dfrac{k}{(P+a)^2}$

79. By the chain rule, $\frac{dc}{dp} = \frac{dc}{dq} \cdot \frac{dq}{dp}$. We are given that $q = \frac{100}{p} = 100p^{-1}$, so $\frac{dq}{dp} = -100p^{-2} = \frac{-100}{p^2}$. Thus $\frac{dc}{dp} = \frac{dc}{dq}\left[\frac{-100}{p^2}\right]$. When $q = 200$, then $p = \frac{100}{200} = \frac{1}{2}$ and we are given that $\frac{dc}{dq} = 0.01$. Therefore $\frac{dc}{dp} = 0.01\left[\frac{-100}{\left(\frac{1}{2}\right)^2}\right] = -4$.

80. **a.** When $m = 12$, then $q = 3000$, so $r = 1500$.
Thus $p = \frac{r}{q} = \frac{1500}{3000} = \frac{1}{2} = \$0.50$.

b. $$\frac{dr}{dq} = \frac{\sqrt{1000+3q}(50) - 50q\left(\frac{1}{2}\right)(1000+3q)^{-\frac{1}{2}}(3)}{1000+3q}$$
$$\left.\frac{dr}{dq}\right|_{q=3000} = \frac{2750}{10,000} = \frac{11}{40}$$

c. $\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}$. From part (b) we know $\frac{dr}{dq}$. Now,
$\frac{dq}{dm} = (2m)\left(\frac{3}{2}\right)(2m+1)^{\frac{1}{2}}(2) + (2m+1)^{\frac{3}{2}}(2)$, so $\left.\frac{dq}{dm}\right|_{m=12} = 610$.
Thus $\left.\frac{dr}{dm}\right|_{m=12} = \frac{11}{40} \cdot 610 = \frac{671}{4}$.

81. $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = f'(x)g'(t)$. We are given that $g(2) = 3$, so $x = 3$ when $t = 2$. Thus
$\left.\frac{dy}{dt}\right|_{t=2} = \left.\frac{dy}{dx}\right|_{x=g(2)} \cdot \left.\frac{dx}{dt}\right|_{t=2} = f'(3)g'(2) = 10(4) = 40$.

82. **a.** $$\lim_{q\to\infty} \overline{c} = \lim_{q\to\infty}\left(\frac{324}{\sqrt{q^2+35}} + \frac{5}{q} + \frac{19}{18}\right) = 0 + 0 + \frac{19}{18} = \frac{19}{18}$$

b. $$c = \overline{c}q = \frac{324q}{\sqrt{q^2+35}} + 5 + \frac{19}{18}q$$
$$\frac{dc}{dq} = \frac{\sqrt{q^2+35}(324) - 324q\left(\frac{1}{2}\right)\left(q^2+35\right)^{-\frac{1}{2}}(2q)}{q^2+35} + \frac{19}{18}$$
$$\left.\frac{dc}{dq}\right|_{q=17} = 3$$

c. From part (b) the increase in cost of the additional unit is approximately \$300. Since the corresponding revenue increases by \$275, the move should not be made.

83. 94.03

84. 5.25

Chapter 11 Review Problems

1. $f(x) = 2 - x^2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left[2-(x+h)^2\right]-\left(2-x^2\right)}{h}$$
$$= \lim_{h\to 0}\frac{\left[2-x^2-2hx-h^2\right]-\left(2-x^2\right)}{h} = \lim_{h\to 0}\frac{-2hx-h^2}{h}$$
$$= \lim_{h\to 0}\frac{-h(2x+h)}{h} = \lim_{h\to 0} -(2x+h) = -2x$$

2. $f(x) = 5x^3 - 2x + 1$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{5(x+h)^3-2(x+h)+1-(5x^3-2x+1)}{h}$$
$$= \lim_{h\to 0}\frac{5x^3+15x^2h+15xh^2+5h^3-2x-2h+1-5x^3+2x-1}{h}$$
$$= \lim_{h\to 0}\frac{15x^2h+15xh^2+5h^3-2h}{h}$$
$$= \lim_{h\to 0}(15x^2+15xh+5h^2-2)$$
$$= 15x^2-2$$

3. $f(x) = \sqrt{3x}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}$$
$$= \lim_{h\to 0}\frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}\cdot\frac{\sqrt{3(x+h)}+\sqrt{3x}}{\sqrt{3(x+h)}+\sqrt{3x}}$$
$$= \lim_{h\to 0}\frac{3(x+h)-3x}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)} = \lim_{h\to 0}\frac{3h}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)}$$
$$= \lim_{h\to 0}\frac{3}{\sqrt{3(x+h)}+\sqrt{3x}}$$
$$= \frac{3}{\sqrt{3x}+\sqrt{3x}} = \frac{3}{2\sqrt{3x}} = \frac{\sqrt{3}}{2\sqrt{x}}$$

4. $f(x)=\dfrac{2}{1+4x}$

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{\frac{2}{1+4(x+h)}-\frac{2}{1+4x}}{h}$

$=\lim\limits_{h\to 0}\dfrac{2(1+4x)-2[1+4(x+h)]}{h[1+4(x+h)](1+4x)}$

$=\lim\limits_{h\to 0}\dfrac{-8h}{h[1+4(x+h)](1+4x)}$

$=\lim\limits_{h\to 0}\dfrac{-8}{[1+4(x+h)](1+4x)}=\dfrac{-8}{[1+4(x)](1+4x)}$

$=-\dfrac{8}{(1+4x)^2}$

5. y is a constant function, so $y'=0$.

6. $y'=e(1)x^{1-1}=ex^0=e$

7. $y'=4\pi x^3-3\sqrt{2}x^2+4x$

8. $y'=4(2x+0)-7(1)=8x-7$

9. $f(s)=s^2\left(s^2+2\right)=s^4+2s^2$

$f'(s)=4s^3+2(2s)=4s^3+4s=4s\left(s^2+1\right)$

10. $y=(x+3)^{\frac{1}{2}}$

$y'=\dfrac{1}{2}(x+3)^{-\frac{1}{2}}(1)=\dfrac{1}{2}(x+3)^{-\frac{1}{2}}$

11. $y=\dfrac{1}{5}\left(x^2+1\right)$

$y'=\dfrac{1}{5}(2x)=\dfrac{2x}{5}$

12. $y=-\dfrac{1}{n}x^{-n}$, so $y'=-\dfrac{1}{n}(-n)x^{-n-1}=x^{-(n+1)}=\dfrac{1}{x^{n+1}}$.

13. $y'=(x^3+7x^2)(3x^2-2x)+(x^3-x^2+5)(3x^2+14x)$
$=3x^5+19x^4-14x^3+3x^5+11x^4-14x^3+15x^2+70x$
$=6x^5+30x^4-28x^3+15x^2+70x$

14. $y'=\left(x^2+1\right)^{100}(1)+(x-6)(100)\left(x^2+1\right)^{99}(2x)=(x^2+1)^{99}[x^2+1+200x(x-6)]=\left(x^2+1\right)^{99}\left(201x^2-1200x+1\right)$

15. $f'(x) = 100\left(2x^2+4x\right)^{99}(4x+4) = 400(x+1)[(2x)(x+2)]^{99}$

16. $f(w) = w\sqrt{w} + w^2 = w^{\frac{3}{2}} + w^2$

$f'(w) = \frac{3}{2}w^{\frac{1}{2}} + 2w$

17. $y = \frac{c}{ax+b} = c(ax+b)^{-1}$

$y' = -c(ax+b)^{-2}(a) = -\frac{ac}{(ax+b)^2}$

18. $y = \frac{5x^2-8x}{2x} = \frac{5}{2}x - 4$

$y' = \frac{5}{2}$

19. $y' = (8+2x)\left[(4)\left(x^2+1\right)^3(2x)\right] + \left(x^2+1\right)^4(2)$

$= 2\left(x^2+1\right)^3\left[4x(8+2x)+\left(x^2+1\right)\right]$

$= 2\left(x^2+1\right)^3\left(32x+8x^2+x^2+1\right)$

$= 2\left(x^2+1\right)^3\left(9x^2+32x+1\right)$

20. $g'(z) = \left(\frac{3}{5}\right)(2z)^{-\frac{2}{5}}(2) + 0 = \frac{6}{5}(2z)^{-\frac{2}{5}}$

21. $f'(z) = \frac{\left(z^2+4\right)(2z) - \left(z^2-1\right)(2z)}{\left(z^2+4\right)^2} = \frac{10z}{\left(z^2+4\right)^2}$

22. $y' = \frac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2} = \frac{acx+ad-acx-bc}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$

23. $y = (4x-1)^{\frac{1}{3}}$

$y' = \frac{1}{3}(4x-1)^{-\frac{2}{3}}(4) = \frac{4}{3}(4x-1)^{-\frac{2}{3}}$

24. f is a constant function, so $f'(x) = 0$.

25. $y = (1-x^2)^{-\frac{1}{2}}$

$y' = \left(-\frac{1}{2}\right)(1-x^2)^{-\frac{3}{2}}(-2x) = x(1-x^2)^{-\frac{3}{2}}$

26. $y = \dfrac{x^2 + x}{2x^2 + 3}$

$$y' = \frac{\left(2x^2+3\right)(2x+1) - \left(x^2+x\right)(4x)}{\left(2x^2+3\right)^2} = \frac{-2x^2+6x+3}{\left(2x^2+3\right)^2}$$

27. $h'(x) = m(ax+b)^{m-1}(a)(cx+d)^n + n(cx+d)^{n-1}(c)(ax+b)^m$

$= am(ax+b)^{m-1}(cx+d)^n + cn(ax+b)^m(cx+d)^{n-1}$

28. $y' = \dfrac{x(5)(x+3)^4 - (x+3)^5(1)}{x^2} = \dfrac{(x+3)^4(4x-3)}{x^2}$

29. $y' = \dfrac{(x+6)(5) - (5x-4)(1)}{(x+6)^2} = \dfrac{34}{(x+6)^2}$

30. $f(x) = 5x^3\sqrt{3+2x^4} = 5x^3(3+2x^4)^{1/2}$

$f'(x) = (3+2x^4)^{1/2}(15x^2) + 5x^3\left[\dfrac{1}{2}(3+2x^4)^{-1/2}(8x^3)\right]$

$= 15x^2(3+2x^4)^{1/2} + 20x^6(3+2x^4)^{-1/2}$

31. $y' = 2\left(-\dfrac{3}{8}\right)x^{-\frac{11}{8}} + \left(-\dfrac{3}{8}\right)(2x)^{-\frac{11}{8}}(2) = -\dfrac{3}{4}x^{-\frac{11}{8}} - \dfrac{3}{4}\left(2^{-\frac{11}{8}}\right)x^{-\frac{11}{8}}$

$= -\dfrac{3}{4}x^{-\frac{11}{8}}\left(1+2^{-\frac{11}{8}}\right) = -\dfrac{3}{4}\left(1+2^{-\frac{11}{8}}\right)x^{-\frac{11}{8}}$

32. $y' = \dfrac{1}{2}\left(\dfrac{x}{a}\right)^{-1/2}\left(\dfrac{1}{a}\right) + \dfrac{1}{2}\left(\dfrac{a}{x}\right)^{-1/2}\left(\dfrac{-a}{x^2}\right) = \dfrac{1}{2a}\sqrt{\dfrac{a}{x}} - \dfrac{a}{2x^2}\sqrt{\dfrac{x}{a}} = \dfrac{1}{2\sqrt{ax}} - \dfrac{\sqrt{ax}}{2x^2}$

33. $y' = \dfrac{\left(x^2+5\right)^{\frac{1}{2}}(2x) - \left(x^2+6\right)\left(\frac{1}{2}\right)\left(x^2+5\right)^{-\frac{1}{2}}(2x)}{x^2+5}$

Multiplying the numerator and denominator by $\left(x^2+5\right)^{\frac{1}{2}}$ gives

$$y' = \frac{\left(x^2+5\right)(2x) - x\left(x^2+6\right)}{\left(x^2+5\right)^{\frac{3}{2}}} = \frac{x^3+4x}{\left(x^2+5\right)^{\frac{3}{2}}} = \frac{x\left(x^2+4\right)}{\left(x^2+5\right)^{\frac{3}{2}}}$$

34. $y = \left(7-3x^2\right)^{\frac{2}{3}}$

$y' = \dfrac{2}{3}\left(7-3x^2\right)^{-\frac{1}{3}}(-6x) = -4x\left(7-3x^2\right)^{-\frac{1}{3}}$

35. $y' = \frac{3}{5}\left(x^3+6x^2+9\right)^{-\frac{2}{5}}\left(3x^2+12x\right)$

$= \frac{3}{5}\left(x^3+6x^2+9\right)^{-\frac{2}{5}}(3x)(x+4)$

$= \frac{9}{5}x(x+4)\left(x^3+6x^2+9\right)^{-\frac{2}{5}}$

36. $z' = 0.4[x^2(-3)(x+1)^{-4}(1)+(x+1)^{-3}(2x)]+0$

$= 0.4(x+1)^{-4}[-3x^2+(x+1)(2x)]$

$= 0.4(x+1)^{-4}(-x^2+2x)$

37. $g(z) = -3z(z-2)^3$

$g'(z) = -3[(z-2)^3+z(3)(z-2)^2]$

$= -3(z-2)^2[(z-2)+3z]$

$= -3(z-2)^2(4z-2)$

$= -6(z-2)^2(2z-1)$

38. $g(z) = -\frac{3}{4}\left(z^5+2z-5\right)^{-4}$

$g'(z) = -\frac{3}{4}(-4)\left(z^5+2z-5\right)^{-5}\left(5z^4+2\right)$

$= \frac{3\left(5z^4+2\right)}{\left(z^5+2z-5\right)^5}$

39. $y = x^2-6x+4$

$y' = 2x-6$

When $x = 1$, then $y = -1$ and $y' = -4$. An equation of the tangent line is $y - (-1) = -4(x - 1)$, or $y = -4x + 3$.

40. $y = -2x^3+6x+1$

$y' = -6x^2+6$

When $x = 2$, then $y = -3$ and $y' = -18$. An equation of the tangent line is $y - (-3) = -18(x - 2)$, or $y = -18x + 33$.

41. $y = x^{\frac{1}{3}}$

$y' = \frac{1}{3}x^{-\frac{2}{3}}$

When $x = 8$, then $y = 2$ and $y' = \frac{1}{12}$. An equation of the tangent line is $y-2 = \frac{1}{12}(x-8)$, or $y = \frac{1}{12}x+\frac{4}{3}$.

42. $y = \frac{x^2}{x-10}$

$y' = \frac{(x-10)(2x)-x^2(1)}{(x-10)^2} = \frac{x^2-20x}{(x-10)^2}$

When $x = 11$, then $y = 121$ and $y' = -99$. An equation of the tangent line is $y - 121 = -99(x - 11)$ or $y = -99x + 1210$.

43. $f(x) = 4x^2+2x+8$

$f'(x) = 8x+2$

$f(1) = 14$ and $f'(1) = 10$. The relative rate of change is $\frac{f'(1)}{f(1)} = \frac{10}{14} = \frac{5}{7} \approx 0.714$, so the percentage rate of change is 71.4%.

44. $f(x) = \frac{x}{x+4}$

$f'(x) = \frac{(x+4)(1)-x(1)}{(x+4)^2} = \frac{4}{(x+4)^2}$

$f(1) = \frac{1}{5}$ and $f'(1) = \frac{4}{25}$. The relative rate of change is $\frac{f'(1)}{f(1)} = \frac{4}{5} = 0.8$, so the percentage rate of change is 80%.

45. $r = q(20-0.1q) = 20q-0.1q^2$

$\frac{dr}{dq} = 20-0.2q$

46. $\frac{dc}{dq} = 0.0003q^2-0.04q+3$

$\left.\frac{dc}{dq}\right|_{q=100} = 2$

47. $\frac{dC}{dI} = 0.7 - 0.2\left(\frac{1}{2}\right)I^{-1/2} = 0.7 - \frac{0.1}{\sqrt{I}}$

$\left.\frac{dC}{dI}\right|_{I=25} = 0.7 - \frac{0.1}{\sqrt{25}} = 0.68$

Thus the marginal propensity to consume is 0.68, so the marginal propensity to save is 1 − 0.68 = 0.32.

48. $\frac{dp}{dq} = \frac{(q+5)(1)-(q+12)(1)}{(q+5)^2} = -\frac{7}{(q+5)^2}$

49. Since $p = -0.1q + 500$, then

$r = pq = -0.1q^2 + 500q$. Thus $\frac{dr}{dq} = 500 - 0.2q$.

50. Since $\bar{c} = 0.03q + 1.2 + \frac{3}{q}$, then

$c = q\bar{c} = 0.03q^2 + 1.2q + 3$. Thus

$\frac{dc}{dq} = 0.06q + 1.2$, so $\left.\frac{dc}{dq}\right|_{q=100} = 7.2$.

51. $\frac{dc}{dq} = 0.125 + 0.00878q$

$\left.\frac{dc}{dq}\right|_{q=70} = 0.7396$

52. $q = 60m - m^2$

$p = -0.02q + 12$; $m = 10$

$\frac{dr}{dm} = \frac{dr}{dq}\cdot\frac{dq}{dm}$

$r = pq = -0.02q^2 + 12q$, so $\frac{dr}{dq} = -0.04q + 12$.

If $m = 10$, then $q = 500$, so $\left.\frac{dr}{dq}\right|_{m=12} = -8$.

$\frac{dq}{dm} = 60 - 2m$. When $m = 10$, $\frac{dq}{dm} = 40$.

Thus $\left.\frac{dr}{dm}\right|_{m=10} = -8(40) = -320$.

53. $\frac{dy}{dx} = 42x^2 - 34x - 16$

$\left.\frac{dy}{dx}\right|_{x=2} = 84$ eggs/mm

54. $y = 12 - \frac{12}{1+3x}$

$\frac{dy}{dx} = -12(-1)(1+3x)^{-2}(3) = \frac{36}{(1+3x)^2}$

Setting $\frac{36}{(1+3x)^2} = \frac{1}{3}$ gives $(1+3x)^2 = 108$,

$1+3x = \pm 6\sqrt{3}$, $x = \frac{-1\pm 6\sqrt{3}}{3}$, $x \approx 3.13$ or $x \approx -3.80$.

Because we must have $x \ge 0$, then $x \approx 3.13$.

55. a. $\frac{dt}{dT}$ when $T = 38$ is

$\left.\frac{d}{dT}\left[\frac{4}{3}T - \frac{175}{4}\right]\right|_{T=38} = \left.\frac{4}{3}\right|_{T=38} = \frac{4}{3}$.

b. $\frac{dt}{dT}$ when $T = 35$ is

$\left.\frac{d}{dT}\left[\frac{1}{24}T + \frac{11}{4}\right]\right|_{T=35} = \left.\frac{1}{24}\right|_{T=35} = \frac{1}{24}$.

56. $s = 9\left(2t^2+3\right)^{-1}$

$v = \frac{ds}{dt} = -9\left(2t^2+3\right)^{-2}(4t) = \frac{-36t}{\left(2t^2+3\right)^2}$

If $t = 1$, then $v = -\frac{36}{25}$ m/s.

57. $V' = \frac{1}{2}\pi d^2$. If $d = 2$ ft, then

$V' = \frac{1}{2}\pi(4) = 2\pi\ \frac{\text{ft}^3}{\text{ft}}$.

58. $v = 128 - 32t$. Set $128 - 32t = 64$ to get $t = 2$.

59. $c = \bar{c}q = 2q^2 + \frac{10,000}{q} = 2q^2 + 10,000q^{-1}$

$\frac{dc}{dq} = 4q - 10,000q^{-2} = 4q - \frac{10,000}{q^2}$

60. $y = \frac{(x^3+2)\sqrt{x+1}}{x^4+2x} = \frac{(x^3+2)\sqrt{x+1}}{x(x^3+2)} = \frac{\sqrt{x+1}}{x}$

$\frac{dy}{dx} = \frac{x\left(\frac{1}{2}(x+1)^{-\frac{1}{2}}(1)\right) - \sqrt{x+1}(1)}{x^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{3}{4}\sqrt{2}$ and $y = \sqrt{2}$ when $x = 1$. An equation of the tangent line is

$y - \sqrt{2} = -\frac{3}{4}\sqrt{2}(x-1)$ or $y = -\frac{3}{4}\sqrt{2}x + \frac{7}{4}\sqrt{2}.$

61. a. $q = 10\sqrt{m^2+4900} - 700$

$p = \sqrt{19,300-8q};\ m = 240$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}.$

$r = pq = q\sqrt{19,300-8q}$, so

$\frac{dr}{dq} = q\left(\frac{1}{2}\right)(19,300-8q)^{-\frac{1}{2}}(-8) + \sqrt{19,300-8q}(1).$

If $m = 240$, then $q = 1800$, so

$\left.\frac{dr}{dq}\right|_{m=240} = -\frac{230}{7} \approx -32.86.$

$\frac{dq}{dm} = 10 \cdot \frac{1}{2}\left(m^2+4900\right)^{-\frac{1}{2}}(2m).$

$\left.\frac{dq}{dm}\right|_{m=240} = 9.6.$ Thus

$\left.\frac{dr}{dm}\right|_{m=240} \approx (-32.86)(9.6) = -315.456$

b. $\left.\frac{\frac{dr}{dm}}{r}\right|_{m=240} = \left.\frac{-315.456}{r}\right|_{q=1800}$

$= \frac{-315.456}{1800\sqrt{4900}}$

$= -0.0025$

c. No. Since $\frac{dr}{dm} < 0$, there would be no additional revenue generated to offset the cost of $400.

62. 0.000

63. 0.305

64. $5.05

65. Basic Rule 0: $\frac{d}{dx}(c) = 0$ (c is constant)

Basic Rule 1: $\frac{d}{dx}(x^a) = ax^{a-1}$ (a is a real number)

Combining Rule 1: $\frac{d}{dx}(cf(x)) = cf'(x)$

Let $f(x) = x^a$; then

$\frac{d}{dx}(cf(x)) = \frac{d}{dx}(cx^a) = cax^{a-1}.$ With $a = 0$, we have

$\frac{d}{dx}(cf(x)) = \frac{d}{dx}(cx^0) = \frac{d}{dx}(c) = c(0)x^{0-1} = 0.$

Therefore, $\frac{d}{dx}(c) = 0.$

66. Basic Rule 1: $\frac{d}{dx}(x^a) = ax^{a-1}$ (a is a real number)

Combining Rule 3: If f and g are differentiable functions, then the product fg is differentiable and $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$

The $a = 1$ case of Basic Rule 1 is

$\frac{d}{dx}(x) = 1x^{1-1} = 1.$ Then by Combining Rule 3,

$$\begin{aligned}\frac{d}{dx}(x^2) &= \frac{d}{dx}(x \cdot x)\\ &= \frac{d}{dx}(x)\cdot x + x \cdot \frac{d}{dx}(x)\\ &= 1 \cdot x + x \cdot 1\\ &= 2x.\end{aligned}$$

$$\begin{aligned}\frac{d}{dx}(x^3) &= \frac{d}{dx}(x^2 \cdot x)\\ &= \frac{d}{dx}(x^2)\cdot x + x^2\frac{d}{dx}(x)\\ &= 2x \cdot x + x^2 \cdot 1\\ &= 2x^2 + x^2\\ &= 3x^2\end{aligned}$$

Continuing in this manner, Basic Rule 1 can be shown to be true for all positive integers a.

Explore and Extend—Chapter 11

1. In Problems 63 and 64 of Sec. 11.4, the slope is less than the slope in Fig. 11.15, which is above 0.9. More is spent; less is saved.

2. In the lowest quintile, the average family spends more than it earns, thus accumulating debt.

3. The slope of the family consumption curve is $\frac{112{,}040}{\sqrt{1.9667\times 10^{10}+224{,}080x}}$, which for $x = 25{,}000$ equals about 0.705. You would expect the family to spend \$705 and save \$295.

4. For $x = 90{,}000$, the slope of the consumption curve is 0.561. You would expect the family to spend \$561 and save \$439.

5. Answers may vary.