Chapter 5

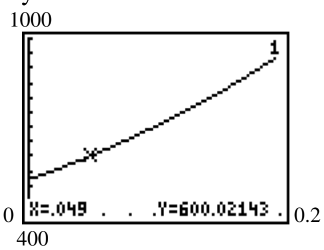
Apply It 5.1

1. Let $P = 518$ and let $n = 3(365) = 1095$.

$S = P(1+r)^n$

$$S = 518\left(1+\frac{r}{365}\right)^{1095}$$

By graphing S as a function of the nominal rate r, we find that when $r = 0.049$, $S = 600$. Thus, at the nominal rate of 4.9% compounded daily, the initial amount of \$518 will grow to \$600 after 3 years.



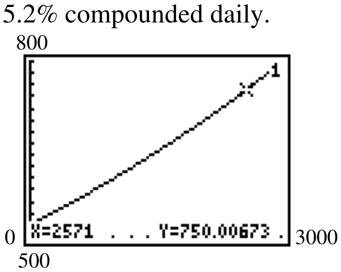
2. Let $P = 520$ and let $r = 0.052$.

$S = P(1+r)^n$

$$S = 520\left(1+\frac{0.052}{365}\right)^{n}$$

$$S = 520\left(\frac{365.052}{365}\right)^{n}$$

By graphing S as a function of n, we find that when $n = 2571$, $S = 750$. Thus, it will take $\frac{2571}{365} \approx 7.044$ years, or 7 years and 16 days for \$520 to grow to \$750 at the nominal rate of 5.2% compounded daily.

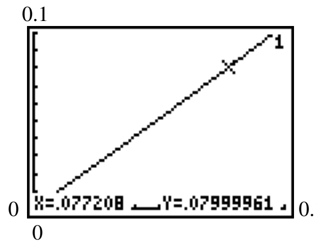


3. Let $n = 12$.

$$r_e = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$r_e = \left(1+\frac{r}{12}\right)^{12} - 1$$

By graphing r_e as a function of r, we find that, when the nominal rate $r = 0.077208$ or 7.7208%, the effective rate $r_e = 0.08$ or 8%.



4. The respective effective rates of interest are found using the formula $r_e = \left(1+\frac{r}{n}\right)^{n} - 1$.

Let $n = 12$ when $r = 0.11$:

$r_e = \left(1+\frac{0.11}{12}\right)^{12} - 1 \approx 0.1157$. Hence, when the nominal rate $r = 11\%$ is compounded monthly, the effective rate is $r_e = 11.57\%$. When

$r = 0.1125$: $r_e = \left(1+\frac{0.1125}{4}\right)^{4} - 1 \approx 0.1173$.

Hence in the second case when the nominal rate $r = 11.25\%$ is compounded quarterly, the effective rate is $r_e = 11.73\%$. This is the better effective rate of interest. To find the better investment, compare the compound amounts, S at the end of n years. With $P = 10{,}000$ and $r_e = 0.1157$,

$S_1 = P(1+r)^n = 10{,}000(1+0.1157)^n$, and, in the second case, when $P = 9700$ and $r_e = 0.1173$

$S_2 = P(1+r)^n = 9700(1+0.1173)^n$.

$S_1(20) = 10{,}000(1.1157)^{20} \approx 89{,}319.99$

$S_2(20) = 9700(1.1173)^{20} \approx 89{,}159.52$

The \$10,000 investment is slightly better over 20 years.

Problems 5.1

1. **a.** $6000(1.08)^8 \approx \$11{,}105.58$

b. $11{,}105.58 - 6000 = \$5105.58$

2. **a.** $750(1.07) = \$802.50$

b. $802.5 - 750 = \$52.50$

3. $(1.015)^2 - 1 \approx 0.030225$ or 3.023%

4. $\left(1+\frac{0.05}{4}\right)^4 - 1 = (1.0125)^4 - 1 \approx 0.05095$ or 5.095%

5. $\left(1+\frac{0.035}{365}\right)^{365} - 1 \approx 0.03562$ or 3.562%

6. $\left(1+\frac{0.06}{365}\right)^{365} - 1 \approx 0.06183$ or 6.183%

7. **a.** A nominal rate compounded yearly is the same as the effective rate, so the effective rate is 10%.

b. $\left(1+\frac{0.10}{2}\right)^2 - 1 = 0.1025$ or 10.25%

c. $\left(1+\frac{0.10}{4}\right)^4 - 1 \approx 0.10381$ or 10.381%

d. $\left(1+\frac{0.10}{12}\right)^{12} - 1 \approx 0.10471$ or 10.471%

e. $\left(1+\frac{0.10}{365}\right)^{365} - 1 \approx 0.10516$ or 10.516%

8. **a.** (i) $1000\left(1+\frac{0.07}{4}\right)^{4(5)} - 1000 \approx \$414.78$

(ii) $\left(1+\frac{0.07}{4}\right)^4 - 1 \approx 0.07186$ or 7.186%

b. (i) $1000\left(1+\frac{0.07}{12}\right)^{12(5)} - 1000 \approx \$417.63$

(ii) $\left(1+\frac{0.07}{12}\right)^{12} - 1 \approx 0.07229$ or 7.229%

c. (i) $1000\left(1+\frac{0.07}{52}\right)^{52(5)} - 1000 \approx \$418.73$

(ii) $\left(1+\frac{0.07}{52}\right)^{52} - 1 \approx 0.07246$ or 7.246%

d. (i) $1000\left(1+\frac{0.07}{365}\right)^{365(5)} - 1000 \approx \$419.02$

(ii) $\left(1+\frac{0.07}{365}\right)^{365} - 1 \approx 0.07250$ or 7.250%

9. Let r_e be the effective rate. Then

$$2000(1+r_e)^5 = 2950$$
$$(1+r_e)^5 = \frac{2950}{2000}$$
$$1+r_e = \sqrt[5]{\frac{2950}{2000}}$$
$$r_e = \sqrt[5]{\frac{2950}{2000}} - 1$$
$$r_e \approx 0.0808 \text{ or } 8.08\%.$$

10. Let r be the quarterly interest rate. Then

$$1000(1+r)^{24} = 1959$$
$$(1+r)^{24} = \frac{1959}{1000}$$
$$1+r = \sqrt[24]{\frac{1959}{1000}}$$
$$r = \sqrt[24]{\frac{1959}{1000}} - 1$$
$$r \approx 0.0284143$$

This gives a nominal rate of approximately $4(0.0284143) \approx 0.1137 = 11.37\%$ compounded quarterly.

11. From Example 6, the number of years, n, is given by $n = \frac{\ln 2}{\ln(1.09)} \approx 8.0$ years.

12. From Example 6, the number of years, n, is given by $n = \frac{\ln 2}{\ln(1.05)} \approx 14.2$ years.

13. $6000(1.08)^7 \approx \$10,282.95$

14. $3P = P(1+r)^n$

$3 = (1+r)^n$

$\ln 3 = n \ln(1+r)$

$n = \dfrac{\ln 3}{\ln(1+r)}$

15. $25{,}500(1.03)^6 \approx \$30{,}448.33$

16. $25{,}500\left(1+\dfrac{0.02}{4}\right)^{24} \approx \$38{,}742.57$

17. **a.** $(0.015)(12) = 0.18$ or 18%

b. $(1.015)^{12} - 1 \approx 0.1956$ or 19.56%

18. $2P = P(1.01)^n$

$2 = (1.01)^n$

$\ln 2 = n \ln(1.01)$

$n = \dfrac{\ln 2}{\ln(1.01)} \approx 70$ months

19. The compound amount after the first four years is $2000(1.06)^4$. After the next four years the compound amount is $\left[2000(1.06)^4\right](1.03)^8 \approx \$3198.54$.

20. $1000 = 100\left(1+\dfrac{0.06}{12}\right)^{12t}$

$10 = 1.005^{12t}$

$\ln 10 = \ln 1.005^{12t}$

$\ln 10 = 12t \ln 1.005$

$t = \dfrac{\ln 10}{12 \ln 1.005} \approx 38.47$ years

21. 7.8% compounded semiannually is equivalent to an effective rate of $(1.039)^2 - 1 = 0.079521$ or 7.9521%. Thus 8% compounded annually, which is the effective rate, is the better rate.

22. Let r be the required nominal rate.

$\left(1+\dfrac{r}{12}\right)^{12} - 1 = 0.045$

$\left(1+\dfrac{r}{12}\right)^{12} = 1.045$

$1+\dfrac{r}{12} = \sqrt[12]{1.045}$

$\dfrac{r}{12} = \sqrt[12]{1.045} - 1$

$r = 12\left[\sqrt[12]{1.045} - 1\right] \approx 0.0441$

or 4.41%.

23. **a.** $\left(1+\dfrac{0.0475}{360}\right)^{365} - 1 \approx 0.0493$ or 4.93%

b. $\left(1+\dfrac{0.0475}{365}\right)^{365} - 1 \approx 0.0486$ or 4.86%

24. Let r be the nominal rate.

$801.06 = 700\left(1+\dfrac{r}{4}\right)^8$

$1+\dfrac{r}{4} = \sqrt[8]{\dfrac{801.06}{700}}$

$r = 4\left(\sqrt[8]{\dfrac{801.06}{700}} - 1\right) \approx 0.0680$ or 6.80%

25. Let r_e = effective rate.

$250{,}000 = 90{,}000(1+r_e)^{10}$

$(1+r_e)^{10} = \dfrac{25}{9}$

$1+r_e = \sqrt[10]{\dfrac{25}{9}}$

$r_e = \sqrt[10]{\dfrac{25}{9}} - 1 \approx 0.10757$ or 10.757%

26. Let P = average price of such a good, n = number of days.

$$2P = P\left(1+\frac{0.0725}{365}\right)^n$$

$$2 = \left(1+\frac{0.0725}{365}\right)^n$$

$$\ln 2 = n\ln\left(1+\frac{0.0725}{365}\right)$$

$$n = \frac{\ln 2}{\ln\left(1+\frac{0.0725}{365}\right)} \approx 3489.98 \text{ days}$$

or ≈ 9.56 years

27. Let r = the required nominal rate.

$$420\left(1+\frac{r}{2}\right)^{28} = 1000$$

$$\left(1+\frac{r}{2}\right)^{28} = \frac{1000}{420} = \frac{50}{21}$$

$$1+\frac{r}{2} = \sqrt[28]{\frac{50}{21}}$$

$$r = 2\left[\sqrt[28]{\frac{50}{21}}-1\right] \approx 0.0629 \text{ or } 6.29\%$$

28. $1000(1-0.01)^{20} = 1000(0.99)^{20} \approx \$817.91$

29. $S = P(1+r)^n$

Solve for P: $P = \dfrac{S}{(1+r)^n}$

Solve for r: $(1+r)^n = \dfrac{S}{P}$

$$1+r = \left(\frac{S}{P}\right)^{1/n}$$

$$r = \left(\frac{S}{P}\right)^{1/n} - 1$$

Solve for n: $(1+r)^n = \dfrac{S}{P}$

$$\ln(1+r)^n = \ln\frac{S}{P}$$

$$n = \frac{\ln\frac{S}{P}}{\ln(1+r)}$$

Problems 5.2

1. $6000(1.05)^{-20} \approx \$2261.34$

2. $3500(1.06)^{-8} \approx \$2195.94$

3. $4000(1.035)^{-24} \approx \$1751.83$

4. $1950\left(1+\frac{0.16}{12}\right)^{-36} \approx \$1210.46$

5. $9000\left(1+\frac{0.08}{4}\right)^{-22} \approx \$5821.55$

6. $6000\left(1+\frac{0.10}{2}\right)^{-13} \approx \$3181.93$

7. $8000\left(1+\frac{0.10}{12}\right)^{-60} \approx \$4862.31$

8. $500\left(1+\frac{0.0875}{4}\right)^{-12} \approx \$385.65$

9. $5000\left(1+\frac{0.075}{365}\right)^{-730} \approx \$4303.61$

10. $1250\left(1+\frac{0.135}{52}\right)^{-78} \approx \$1021.13$

11. $12{,}000\left(1+\frac{0.053}{12}\right)^{-12} \approx \$11{,}381.89$

12. $12{,}000\left(1+\frac{0.071}{2}\right)^{-2} \approx \$11{,}191.31$

13. $27{,}000(1.03)^{-22} \approx \$14{,}091.10$

14. $750\left(1+\frac{0.08}{4}\right)^{-40} + 250\left(1+\frac{0.08}{4}\right)^{-48} \approx \$436.30$

15. Let x be the payment 2 years from now. The equation of value at year 2 is

$$x = 600(1.04)^{-2} + 800(1.04)^{-4}$$

$$x \approx \$1238.58$$

16. Let x be the payment at the end of 5 years. The equation of value at year 5 is

$$3000\left(1+\frac{0.08}{12}\right)^{60}+x=7000$$

$$x=7000-3000\left(1+\frac{0.08}{12}\right)^{60}$$

$$x\approx \$2530.46$$

17. Let x be the payment at the end of 6 years. The equation of value at year 6 is

$$2000(1.025)^4+4000(1.025)^2+x=5000(1.025)+5000(1.025)^{-4}$$

$$x=5000(1.025)+5000(1.025)^{-4}-2000(1.025)^4-4000(1.025)^2$$

$x\approx$ \$3244.63.

18. Let x be the amount of each of the equal payments. The equation of value at year 3 is

$$1500(1.07)^3+x(1.07)^2+x(1.07)+x=3500(1.07)^{-1}+5000(1.07)^{-3}$$

$$x[(1.07)^2+1.07+1]=3500(1.07)^{-1}+5000(1.07)^{-3}-1500(1.07)^3$$

$$x=\frac{3500(1.07)^{-1}+5000(1.07)^{-3}-1500(1.07)^3}{(1.07)^2+2.07}$$

$$x\approx \$1715.44$$

19. **a.** $NPV=13,000(1.01)^{-20}+14,000(1.01)^{-24}+15,000(1.01)^{-28}+16,000(1.01)^{-32}-35,000\approx \$9669.40$

b. Since $NPV>0$, the investment is profitable.

20. **a.** $NPV=8000(1.03)^{-6}+10,000(1.03)^{-8}+14,000(1.03)^{-12}-25,000\approx -\$586.72$

b. Since $NPV<0$, the investment is not profitable.

21. We consider the value of each investment at the end of eight years. The savings account has a value of $10,000(1.03)^{16}\approx \$16,047.06$.
The business investment has a value of \$16,000. Thus the better choice is the savings account.

22. The payments due B are $1000(1.07)^5$ at year 5 and $2000(1.04)^{14}$ at year 7. Let x be the payment at the end of 6 years. The equation of value at year 6 is $x=1000(1.07)^5(1.015)^4+2000(1.04)^{14}(1.015)^{-4}$ $x\approx$ \$4751.73

23. $1000\left(1+\frac{0.075}{4}\right)^{-80}\approx \$226.25$

24. $10,000\left(1+\frac{0.1}{360}\right)^{-3650}\approx \$3628.56$

25. Let r be the nominal discount rate, compounded quarterly. Then

$$4700 = 10,000\left(1+\frac{r}{4}\right)^{-32}$$

$$4700 = \frac{10,000}{\left(1+\frac{r}{4}\right)^{32}}$$

$$\left(1+\frac{r}{4}\right)^{32} = \frac{10,000}{4700} = \frac{100}{47}$$

$$1+\frac{r}{4} = \sqrt[32]{\frac{100}{47}}$$

$$r = 4\left[\sqrt[32]{\frac{100}{47}} - 1\right] \approx 0.0955 \text{ or } 9.55\%$$

26. **a.** Let r be the nominal discount rate, compounded monthly. Then

$$4700 = 10,000\left(1+\frac{r}{12}\right)^{-96}$$

$$\left(1+\frac{r}{12}\right)^{96} = \frac{100}{47}$$

$$1+\frac{r}{12} = \sqrt[96]{\frac{100}{47}}$$

$$r = 12\left(\sqrt[96]{\frac{100}{47}} - 1\right)$$

$$\approx 0.0947 \text{ or } 9.47\%$$

b. $$\left(1+\frac{r}{4}\right)^{-32} = \left(1+\frac{s}{12}\right)^{-96}$$

$$\left(1+\frac{s}{12}\right)^{96} = \left(1+\frac{r}{4}\right)^{32}$$

$$1+\frac{s}{12} = \left(1+\frac{r}{4}\right)^{1/3}$$

$$\frac{s}{12} = \sqrt[3]{1+\frac{r}{4}} - 1$$

$$s = 12\left(\sqrt[3]{1+\frac{r}{4}} - 1\right)$$

Problems 5.3

1. $S = 4000e^{0.0625(6)} \approx \$5819.97$
$5819.97 - 4000 = \$1819.97$

2. $S = 4000e^{0.09(6)} \approx \$6864.03$
$6864.03 - 4000 = \$2864.03$

3. $P = 2500e^{-0.015(8)} \approx \$2217.30$

4. $P = 2500e^{-0.08(8)} \approx \$1318.23$

5. $e^{0.04} - 1 \approx 0.0408$
Answer: 4.08%

6. $e^{0.08} - 1 \approx 0.0833$
Answer: 8.33%

7. $e^{0.03} - 1 \approx 0.0305$
Answer: 3.05%

8. $e^{0.11} - 1 \approx 0.1163$
Answer: 11.63%

9. $S = 100e^{0.045(2)} \approx \$109.42$

10. $S = 1000e^{0.03(8)} \approx \$1271.25$

11. $P = 1,000,000e^{-0.05(5)} \approx \$778,800.78$

12. $P = 50,000e^{-0.06(30)} \approx \$8264.94$

13. **a.** $21,000(1+0.035)^{21} \approx \$43,248.06$

b. $P = 43,248.06e^{-(0.035)(21)} \approx \$20,737.68$

14. With option (a), after 18 months they have $50,000(1+0.0125)^6 \approx \$53,869.16$ with option (b), they have $50,000e^{(0.045)(1.5)} \approx \$53,491.51$.

15. Effective rate $= e^r - 1$. Thus $0.05 = e^r - 1$, $e^r = 1.05$, $r = \ln 1.05 \approx 0.0488$.
Answer: 4.88%

16. If r is the annual rate compounded continuously, then at the end of 1 year the compound amount of a principal of P dollars is $Pe^{r(1)} = Pe^r$. This amount must equal the compound amount of P dollars at a nominal rate of 6% compounded semiannually, which is $P(1.03)^2$. Thus

$$Pe^r = P(1.03)^2$$

$$e^r = (1.03)^2$$

$$r = \ln(1.03)^2$$

$$r = 2 \ln 1.03 \approx 0.0591$$

Answer: 5.91%

17. $3P = Pe^{0.07t}$
$3 = e^{0.07t}$
$0.07t = \ln 3$
$t = \frac{\ln 3}{0.07} \approx 16$
Answer: 16 years

18. $2P = Pe^{r(20)}$
$2 = e^{20r}$
$20r = \ln 2$
$r = \frac{\ln 2}{20} \approx 0.0347$
Answer: 3.47%

19. The accumulated amounts under each option are:

a. $1000e^{(0.035)(2)} \approx \$1072.51$

b. $1020(1.0175)^4 \approx \$1093.30$

c. $500e^{(0.035)(2)} + 500(1.0175)^4$
$\approx 536.25 + 535.93 = \$1072.18$

20. a. On Nov. 1, 2006 the accumulated amount is $10,000e^{(0.04)(10)} \approx \$14,918.25$.
On Nov. 1, 2011 the accumulated amount is $14,918.25(1.05)^5 \approx \$19,039.89$.

b. $10,000(1.045)^{15} \approx \$19,352.82$, which is $312.93 more than the amount in part (a).

21. a. $9000(1.0125)^4 \approx \$9458.51$

b. After one year the accumulated amount of the investment is
$10,000e^{0.055} \approx \$10,565.41$. The payoff for the loan (including interest) is
1000 + 1000(0.08) = $1080. The net return is 10,565.41 – 1080 = $9485.41.
Thus, this strategy is better by
9485.41 – 9458.51 = $26.90.

22. $2P = Pe^{0.03t}$
$\ln 2 = 0.03t$
$t = \frac{\ln 2}{0.03} \approx 23.10$ years

23. $S = Pe^{rt}$
$P = Se^{-rt}$
$\ln \frac{S}{P} = \ln e^{rt} = rt$
$r = \frac{1}{t} \ln \frac{S}{P}$
$t = \frac{1}{r} \ln \frac{S}{P}$

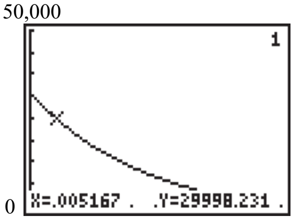
Apply It 5.4

5. Let R = 500 and let n = 72. Then, the present value A of the annuity is given by

$$A = R\left(\frac{1-(1+r)^{-n}}{r}\right) = 500\left(\frac{1-(1+r)^{-72}}{r}\right)$$

By graphing A as a function of r, we find that when $r \approx 0.005167$, A = 30,000. Thus, if the present value of the annuity is $30,000, the monthly interest rate is 0.5167%, and the nominal rate is 12(0.005167) = 0.062 or 6.2%.

50,000



0 0.05
10,000

6. Since the man pays $2000 for 6 years and $3500 for 8 years, we can consider the payments to be an annuity of $3500 for 14 years minus an annuity of $1500 for 6 years so that the first 24 payments are $2000 each. Thus, the present value is

$$3500a_{\overline{56}|0.015} - 1500a_{\overline{24}|0.015}$$
$$\approx 3500(37.705879) - 1500(20.030405)$$
$$= 101,924.97$$

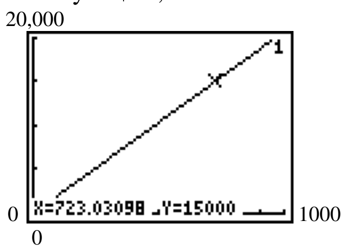
Thus, the present value of the payments is $101,925. Since the man made an initial down payment of $20,000, list price was 101,925 + 20,000 = $121,925.

7. Let $r = \frac{0.048}{4} = 0.012$, and $n = 24$.

$$A = R\left(\frac{1-(1+r)^{-n}}{r}\right)$$

$$A = R\left(\frac{1-(1+0.012)^{-24}}{0.012}\right) = R\left(\frac{1-(1.012)^{-24}}{0.012}\right)$$

By Graphing A as a function of R, we find that when $R = 723.03$, $A = 15{,}000$. Thus the monthly payment is \$723.03 if the present value of the annuity is \$15,000.



8. Find the annuity due. The man makes an initial payment of \$1200 followed by an ordinary annuity of \$1200 for 11 months. Thus, let $R = 1200$, $n = 11$, and $r = \frac{0.068}{12}$. The present value of the annuity due is

$$1200\left(1+a_{\overline{11}|\frac{0.068}{12}}\right) \approx 1200(1+10.635005)$$
$$\approx 13{,}962.01$$

Thus, he should pay \$13,962.01.

9. Let $R = 2000$ and let $r = 0.057$. Then, the value of the IRA at the end of 15 years, when $n = 15$, is given by

$$S = R\left(\frac{(1+r)^n - 1}{r}\right)$$
$$S = 2000\left(\frac{(1+0.057)^{15} - 1}{0.057}\right) \approx 45{,}502.06$$

Thus, at the end of 15 years the IRA will be worth \$45,502.06.

10. Let $R = 2000$ and let $r = 0.057$. Since the deposits are made at the beginning of each year, the value of the IRA at the end of 15 years is given by

$$S = R\left(\frac{(1+r)^{n+1} - 1}{r}\right) - R.$$

Let $n = 15$.

$$S = 2000\left(\frac{(1+0.057)^{16} - 1}{0.057}\right) - 2000 \approx 48{,}095.67$$

Thus, the IRA is worth \$48,095.67 at the end of 15 years.

Problems 5.4

1. $a_{\overline{35}|0.04} \approx 18.664613$

2. $a_{\overline{15}|0.07} \approx 9.107914$

3. $s_{\overline{8}|0.0075} \approx 8.213180$

4. $s_{\overline{12}|0.0125} \approx 12.860361$

5. $600a_{\overline{6}|0.06} \approx 600(4.917324) \approx \$2950.39$

6. $1000a_{\overline{8}|0.05} \approx 1000(6.463213) \approx 6463.21$

7. $2000a_{\overline{18}|0.02} \approx 2000(14.992031) \approx \$29{,}984.06$

8. $1500a_{\overline{15}|0.0075} \approx 1500(14.136995) \approx \$21{,}205.49$

9. $$900\left(1+a_{\overline{13}|0.04}\right) \approx 900(1+9.985648)$$
$$\approx 9887.08$$

10. $$150+\left(1+a_{\overline{59}|\frac{0.07}{12}}\right) \approx 150(1+49.796588)$$
$$\approx \$7619.49$$

11. $$2000s_{\overline{36}|0.0125} \approx 2000(45.115505)$$
$$\approx \$90{,}231.01$$

12. $600s_{\overline{16}|0.02} \approx 600(18.639285) \approx \$11{,}183.57$

13. $5000s_{\overline{20}|0.07} \approx 5000(40.995492) \approx \$204{,}977.46$

14. $$2500s_{\overline{48}|0.005} \approx 2500(54.097832)$$
$$= \$135{,}244.58$$

15. $$1200\left(s_{\overline{13}|0.08} - 1\right) \approx 1200(21.495297 - 1)$$
$$\approx \$24{,}594.36$$

16. $$600\left(s_{\overline{31}|0.025} - 1\right) \approx 600(46.000271 - 1)$$
$$\approx \$27{,}000.16$$

17. $175a_{\overline{32}|\frac{0.04}{12}} - 25a_{\overline{8}|\frac{0.04}{12}}$

$\approx 175(30.304595) - 25(7.881321)$

$\approx \$5106.27$

18. $1500 + 1500a_{\overline{5}|0.0075} \approx 1500 + 1500(4.889440)$
$\approx \$8834.16$

19. $R = \dfrac{15{,}000}{a_{\overline{12}|0.01}} \approx \dfrac{15{,}000}{11.255077} \approx \$1332.73$

20. $3000 + 250a_{\overline{12}|0.04} \approx 3000 + 250(9.385074)$
$\approx \$5346.27$

21. **a.** $\left(50s_{\overline{48}|0.005}\right)(1.005)^{24}$
$\approx 50(54.097832)(1.005)^{24}$
$\approx \$3048.85$

b. $3048.85 - 48(50) = \$648.85$

22. Let R be the yearly payment.
$275{,}000 = R + Ra_{\overline{9}|0.035}$

$275{,}000 = R\left(1 + a_{\overline{9}|0.035}\right)$

$275{,}000 \approx R(8.607687)$,
$R \approx \$31{,}948.19$

23. $R = \dfrac{48{,}000}{s_{\overline{10}|0.07}} \approx \dfrac{48{,}000}{13.816448} \approx \$3474.12$

24. Let x be the purchase price. In the same manner as in Example 8,
$[60{,}000 - 0.06x]s_{\overline{8}|0.04} = x$

$$60{,}000 - 0.06x = \frac{x}{s_{\overline{8}|0.04}}$$

$$60{,}000 = 0.06x + \frac{x}{s_{\overline{8}|0.04}}$$

$$60{,}000 = x\left(0.06 + \frac{1}{s_{\overline{8}|0.04}}\right)$$

$$x = \frac{60{,}000}{0.06 + \frac{1}{s_{\overline{8}|0.04}}}$$

$$\approx \frac{60{,}000}{0.06 + \frac{1}{9.214226}}$$

$\approx \$356{,}000$

25. The original annual payment is $\dfrac{25{,}000}{s_{\overline{10}|0.06}}$. After six years the value of the fund is

$\dfrac{25{,}000}{s_{\overline{10}|0.06}}s_{\overline{6}|0.06}$.

This accumulates to

$\left[\dfrac{25{,}000}{s_{\overline{10}|0.06}}s_{\overline{6}|0.06}\right](1.07)^4$.

Let x be the amount of the new payment.

$$xs_{\overline{4}|0.07} = 25{,}000 - \left[\frac{25{,}000}{s_{\overline{10}|0.06}}s_{\overline{6}|0.06}(1.07)^4\right]$$

$$x = \frac{25{,}000 - \left[\frac{25{,}000}{s_{\overline{10}|0.06}}s_{\overline{6}|0.06}(1.07)^4\right]}{s_{\overline{4}|0.07}}$$

$$x \approx \frac{25{,}000 - \left[\frac{25{,}000}{13.180795}(6.975319)(1.07)^4\right]}{4.439943}$$

$x \approx \$1725$

26. Let x be the final payment.

$5000 = 1000a_{\overline{5}|0.08} + x(1.08)^{-6}$

$5000 - 1000a_{\overline{5}|0.08} = x(1.08)^{-6}$

Thus

$x = (1.08)^6\left(5000 - 1000a_{\overline{5}|0.08}\right)$

$\approx (1.08)^6[5000 - 1000(3.992710)] \approx \$1598.44$

27. $s_{\overline{60}|0.017} = \dfrac{(1.017)^{60} - 1}{0.017} \approx 102.91305$

28. $a_{\overline{9}|0.052} = \dfrac{1 - (1.052)^{-9}}{0.052} \approx 7.04494$

29. $250a_{\overline{180}|0.0235} = 250\left[\dfrac{1 - (1.0235)^{-180}}{0.0235}\right]$
$\approx 10{,}475.72$

30. $1000s_{\overline{120}|0.01} = 1000\left[\dfrac{(1.01)^{120} - 1}{0.01}\right]$
$\approx 230{,}038.69$

31. $R = \dfrac{3000}{s_{\overline{20}|0.01375}} = \dfrac{3000(0.01375)}{(1.01375)^{20} - 1} \approx \$131.34$

32. $R = \dfrac{25{,}000}{a_{\overline{60}|\frac{0.1}{12}}} = \dfrac{25{,}000\left(\frac{0.1}{12}\right)}{1 - \left(1 + \frac{0.1}{12}\right)^{-60}} \approx \$531.18$

33. $200{,}000 + 200{,}000 a_{\overline{19}|0.10}$

$= 200{,}000 + 200{,}000\left[\dfrac{1-(1.10)^{-19}}{0.10}\right]$

$\approx \$1{,}872{,}984.02$

34. **a.** 2100(20)(12) = \$504,000

b. $2100 a_{\overline{240}|0.005} = 2100\left[\dfrac{1-1.005^{-240}}{0.005}\right]$

$\approx \$293{,}120$

35. For the first situation, the compound amount is

$\left[2000\left(s_{\overline{11}|0.07} - 1\right)\right](1.07)^{30}$

$= 2000\left[\dfrac{(1.07)^{11}-1}{0.07} - 1\right](1.07)^{30}$

$\approx \$225{,}073$,
so the net earnings are
225,073 – 20,000 = \$205,073.
For the second situation, the compound amount is

$2000\left(s_{\overline{31}|0.07} - 1\right) = 2000\left[\dfrac{(1.07)^{31}-1}{0.07} - 1\right]$

$\approx \$202{,}146$,
so the net earnings are
202,146 – 60,000 = \$142,146.

36. $100\dfrac{1-e^{-0.05(20)}}{0.05} \approx \$1264$

37. $40{,}000\dfrac{1-e^{-0.04(5)}}{0.04} \approx \$181{,}269.25$

Problems 5.5

1. $R = \dfrac{9000}{a_{\overline{24}|\frac{0.132}{12}}} \approx \dfrac{9000}{20.992607} \approx \$428.72$

2. $A = 50 a_{\overline{36}|0.01} \approx 50(30.107505) \approx \$1505.38$

3. $R = \dfrac{8000}{a_{\overline{36}|\frac{0.04}{12}}} \approx \dfrac{8000}{33.870766} \approx \$236.19$

Finance charge = 36(236.19) – 8000 = \$502.84

4. **a.** $R = \dfrac{500}{a_{\overline{12}|0.0125}} \approx \dfrac{500}{11.079312} \approx \$45.13$

b. 12(45.13) – 500 = \$41.56

5. **a.** $R = \dfrac{7500}{a_{\overline{36}|\frac{0.04}{12}}} \approx \dfrac{7500}{33.870766} \approx \$221.43$

b. $7500\dfrac{0.04}{12} = \$25$

c. 221.43 – 25 = \$196.43

6. **a.** $R = \dfrac{65{,}000}{a_{\overline{48}|\frac{0.072}{12}}} \approx \dfrac{65{,}000}{41.59882} \approx \$1562.54$

b. $65{,}000\dfrac{0.072}{12} = \$390$

c. 1562.54 – 390 = \$1172.54

7. $R = \dfrac{5000}{a_{\overline{4}|0.07}} \approx \dfrac{5000}{3.387211} \approx \$1476.14$

The interest for the first period is (0.07)(5000) = \$350, so the principal repaid at the end of that period is 1476.14 – 350 = \$1126.14. The principal outstanding at the beginning of period 2 is 5000 – 1126.14 = \$3873.86. The interest for period 2 is (0.07)(3873.86) = \$271.17, so the principal repaid at the end of that period is 1476.14 – 271.17 = \$1204.97. The principal outstanding at beginning of period 3 is 3873.86 – 1204.97 = \$2668.89. Continuing in this manner, we construct the following amortization schedule.

Period	Prin. Outs. at Beginning	Int. for Period	Pmt. at End	Prin. Repaid at End
1	5000.00	350.00	1476.14	1126.14
2	3873.86	271.17	1476.14	1204.97
3	2668.89	186.82	1476.14	1289.32
4	1379.57	96.57	1476.14	1379.57
Total		904.56	5904.56	5000.00

8. $R = \dfrac{9000}{a_{\overline{8}|0.0475}} \approx \dfrac{9000}{6.529036} \approx \$1378.46$

The interest for the first period is (0.0475)(9000) = \$427.50, so the principal repaid at the end of that period is 1378.46 − 427.50 = \$950.96. The principal outstanding at the beginning of period 2 is 9000 − 950.96 = \$8049.04. The interest for period 2 is (0.0475)(8049.04) = \$382.33, so the principal repaid at the end of that period is 1378.46 − 382.33 = \$996.13. The principal outstanding at beginning of period 3 is 8049.04 − 996.13 = \$7052.91. Continuing in this manner, we construct the following amortization schedule. Note the adjustment in the final payment.

Period	Prin. Outs. at Beginning	Int. for Period	Pmt. at End	Prin. Repaid at End
1	9000.00	427.50	1378.46	950.96
2	8049.04	382.33	1378.46	996.13
3	7052.91	335.01	1378.46	1043.45
4	6009.46	285.45	1378.46	1093.01
5	4916.45	233.53	1378.46	1144.93
6	3771.52	179.15	1378.46	1199.31
7	2572.21	122.18	1378.46	1256.28
8	1315.93	62.51	1378.44	1315.93
Total		2027.66	11,027.66	9000.00

9. $R = \dfrac{900}{a_{\overline{5}|0.025}} \approx \dfrac{900}{4.645828} \approx \$193.72$

The interest for period 1 is (0.025)(900) = \$22.50, so the principal repaid at the end of that period is 193.72 – 22.50 = \$171.22. The principal outstanding at the beginning of period 2 is 900 – 171.22 = \$728.78. The interest for that period is (0.025)(728.78) = \$18.22, so the principal repaid at the end of that period is 193.72 – 18.22 = \$175.50. The principal outstanding at the beginning of period 3 is 728.78 – 175.50 = \$553.28. Continuing in this manner, we obtain the following amortization schedule. Note the adjustment in the final payment.

Period	Prin. Outs. at Beginning	Int. for Period	Pmt. at End	Prin. Repaid at End
1	900.00	22.50	193.72	171.22
2	728.78	18.22	193.72	175.50
3	553.28	13.83	193.72	179.89
4	313.39	9.33	193.72	184.39
5	189.00	4.73	193.73	189.00
Total		68.61	968.61	900.00

10. $R = \dfrac{10,000}{a_{\overline{5}|0.0075}} \approx \dfrac{10,000}{4.889440} \approx \$2045.22$

The interest for period 1 is (0.0075)(10,000) = \$75, so the principal repaid at the end of that period is 2045.22 – 75 = \$1970.22. The principal outstanding at the beginning of period 2 is 10,000 – 1970.22 = \$8029.78. The interest for period 2 is (0.0075)(8029.78) = \$60.22, so the principal repaid at the end of that period is 2045.22 – 60.22 = \$1985. The principal outstanding at the beginning of period 3 is 8029.78 – 1985 = \$6044.78. Continuing in this manner, we construct the following amortization schedule. Note the adjustment in the final payment.

Period	Prin. Outs. at Beginning	Int. for Period	Pmt. at End	Prin. Repaid at End
1	10,000.00	75.00	2045.22	1970.22
2	8029.78	60.22	2045.22	1985.00
3	6044.78	45.34	2045.22	1999.88
4	4044.90	30.34	2045.22	2014.88
5	2030.02	15.23	2045.25	2030.02
Total		226.13	10,226.13	10,000.00

11. From Eq. (1),

$$n = \frac{\ln(110) - \ln(110 - 1300(0.015))}{\ln(1.015)} \approx 13.106 .$$

Thus the number of full payments is 13.

12. a. $\dfrac{2000}{a_{\overline{48}|0.01}} \approx \dfrac{2000}{37.973959} \approx \$52.67$

b. $52.67a_{\overline{13}|0.01} \approx 52.67(12.133740)$

$\approx \$639.08$

c. $(639.08)(0.01) \approx \$6.39$

d. $52.67 - 6.39 = \$46.28$

e. $48(52.67) - 2000 = \$528.16$

13. Each of the original payments is $\dfrac{18{,}000}{a_{\overline{15}|0.035}}$.

After two years the value of the remaining payments is $\left[\dfrac{18{,}000}{a_{\overline{15}|0.035}}\right]a_{\overline{11}|0.035}$. Thus the new semi-annual payment is

$$\frac{18{,}000a_{\overline{11}|0.035}}{a_{\overline{15}|0.035}}\cdot\frac{1}{a_{\overline{11}|0.04}}$$

$$=\frac{18{,}000(9.001551)}{11.517411}\cdot\frac{1}{8.760477}$$

$\approx \$1606.$

14. $R = \dfrac{2000}{a_{\overline{60}|0.014}} = \dfrac{2000(0.014)}{1-(1.014)^{-60}} \approx \$49.49$

15. a. Monthly interest rate is $\dfrac{0.092}{12}$.

Monthly payment is

$$\frac{245{,}000}{a_{\overline{300}|\frac{0.092}{12}}} = 245{,}000\left[\frac{\frac{0.092}{12}}{1-\left(1+\frac{0.092}{12}\right)^{-300}}\right]$$

$\approx \$2089.69$

b. $245{,}000\left(\dfrac{0.092}{12}\right) = \$1878.33$

c. $2089.69 - 1878.33 = \$211.36$

d. $300(2089.69) - 245{,}000 = \$381{,}907$

16. a. Monthly interest rate is $\dfrac{0.072}{12} = 0.006$.

Monthly payment is

$$\frac{23{,}500}{a_{\overline{60}|0.006}} = 23{,}500\left[\frac{0.006}{1-(1.006)^{-60}}\right]$$

$\approx \$467.55$

b. $60(467.55) - 23{,}500 = \$4553$

17. $n = \dfrac{\ln\left[\frac{100}{100-2000(0.015)}\right]}{\ln 1.015} \approx 23.956$. Thus the number of full payments is 23.

18. $R = \dfrac{9500}{a_{\overline{60}|0.0077}} = 9500\left[\dfrac{0.0077}{1-(1.0077)^{-60}}\right]$

$\approx \$198.31$

19. Present value of mortgage payments is

$$600a_{\overline{360}|\frac{0.076}{12}} = 600\left[\frac{1-\left(1+\frac{0.076}{12}\right)^{-360}}{\frac{0.076}{12}}\right]$$

$\approx \$84{,}976.84$

This amount is 75% of the purchase price x.

$0.75x = 84{,}976.84$

$x = \$113{,}302.45 \approx \$113{,}302$

20. For the 15-year mortgage, the monthly payment is

$$\frac{240{,}000}{a_{\overline{180}|0.005}} = 240{,}000\left[\frac{0.005}{1-(1+0.005)^{-180}}\right]$$

$\approx \$2025.26$

The finance charge is

$180(2025.26) - 240{,}000 = \$124{,}546.80$

For the 25-year mortgage, the monthly payment is

$$\frac{240{,}000}{a_{\overline{300}|0.005}} = 240{,}000\left[\frac{0.005}{1-(1+0.005)^{-300}}\right]$$

$\approx \$1546.32$

The finance charge is

$300(1546.32) - 240{,}000 = 223{,}896.00$

Thus the savings is

$223{,}896.00 - 124{,}546.80 = \$99{,}349.20$

21. $\dfrac{45{,}000}{a_{\overline{48}|0.008}} - \dfrac{45{,}000}{a_{\overline{48}|0.007}}$

$$= 45{,}000\left[\frac{1}{a_{\overline{48}|0.008}} - \frac{1}{a_{\overline{48}|0.007}}\right]$$

$$= 45{,}000\left[\frac{0.008}{1-(1.008)^{-48}} - \frac{0.007}{1-(1.007)^{-48}}\right]$$

$\approx \$25.64$

22. The government's payment is

$$(y-x)a_{\overline{60}|\frac{0.0925}{12}}$$

$$= \left[\frac{5000}{a_{\overline{60}|\frac{0.0925}{12}}} - \frac{5000}{a_{\overline{60}|\frac{0.04}{12}}}\right] a_{\overline{60}|\frac{0.0925}{12}}$$

$$= 5000\left[1 - \frac{a_{\overline{60}|\frac{0.0925}{12}}}{a_{\overline{60}|\frac{0.04}{12}}}\right]$$

$$= 5000\left[1 - \frac{\frac{1-\left(1+\frac{0.0925}{12}\right)^{-60}}{\frac{0.0925}{12}}}{\frac{1-\left(1+\frac{0.04}{12}\right)^{-60}}{\frac{0.04}{12}}}\right]$$

$$= 5000\left[1 - \frac{1-\left(1+\frac{0.0925}{12}\right)^{-60}}{1-\left(1+\frac{0.04}{12}\right)^{-60}} \cdot \frac{0.04}{0.0925}\right]$$

$\approx$ \$589.89

Problems 5.6

1. $A = \frac{R}{r} = \frac{60}{0.015} = \$4000$

2. $A = \frac{R}{r} = \frac{5000}{0.005} = \$1,000,000$

3. $A = \frac{R}{r} = \frac{60,000}{0.08} = \$750,000$

4. $A = \frac{R}{r} = \frac{4000}{0.1} = \$40,000$

5. $A = \frac{R}{r} = \frac{120}{0.025} = \$4800$

6. a. Pierre needs $\frac{\$30,000}{0.05} = \$600,000$ to withdraw \$30,000 in perpetuity. Pierre will make 10 payments R earning 8% per year, which will amount to $Rs_{\overline{10}|0.08}$ when the interest rate changes. The interest at 5% on this amount must be \$30,000.

$$0.05 \cdot Rs_{\overline{10}|0.08} = 30,000$$

$$R = \frac{600,000}{s_{\overline{10}|0.08}}$$

$$= 600,000\left(\frac{0.08}{1.08^{10}-1}\right)$$

$$\approx \$41,417.69$$

b. A perpetuity maintains its principal, so the Princeton Mathematics Department will inherit \$600,000.

7. $\lim\limits_{n\to\infty} \frac{n^2+3n-6}{n^2+4} = \lim\limits_{n\to\infty} \frac{1+\frac{3}{n}-\frac{6}{n^2}}{1+\frac{4}{n^2}} = \frac{1}{1} = 1$

8. $\lim\limits_{n\to\infty} \frac{n+5}{3n^2+2n-7} = \lim\limits_{n\to\infty} \frac{\frac{1}{n}+\frac{5}{n^2}}{3+\frac{2}{n}-\frac{7}{n^2}} = \frac{0}{3} = 0$

9. $\lim\limits_{k\to\infty}\left(\frac{k+1}{k}\right)^{2k} = \left[\lim\limits_{k\to\infty}\left(\frac{k+1}{k}\right)^{k}\right]^2$

Recall that $\lim\limits_{k\to\infty}\left(\frac{k+1}{k}\right)^{k} = e,$ so

$\lim\limits_{k\to\infty}\left(\frac{k+1}{k}\right)^{2k} = e^2.$

10. $\lim\limits_{n\to\infty}\left(\frac{n}{n+1}\right)^{n} = \lim\limits_{n\to\infty}\left(\frac{n+1}{n}\right)^{-n}$

$$= \left[\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{n}\right]^{-1}$$

Recall that $\lim\limits_{n\to\infty}\left(\frac{n+1}{n}\right)^{n} = e,$ so

$\lim\limits_{k\to\infty}\left(\frac{n}{n+1}\right)^{n} = \frac{1}{e}.$

Chapter 5 Review Problems

1. $2P = P(1+r)^n$

$\ln 2 = \ln(1+r)^n = n\ln(1+r)$

$n = \frac{\ln 2}{\ln(1+r)}$

2. $\left(1+\frac{0.05}{12}\right)^{12}-1\approx 0.0512$ or 5.12%

3. 8.2% compounded semiannually corresponds to an effective rate of $(1.041)^2-1=0.083681$ or 8.37%. Thus the better choice is 8.5% compounded annually.

4. NPV $=3400(1.035)^{-4}+3500(1.035)^{-8}-7000\approx -\$1379.16$

5. Let x be the payment at the end of 3 years. The equation of value at the end of year 3 is

$$2000(1.03)^3+x=1500(1.03)^{-2}+2000(1.03)^{-4}$$
$$x=1500(1.03)^{-2}+2000(1.03)^{-4}-2000(1.03)^3$$
$$\approx \$1005.41$$

6. $250a_{\overline{48}|0.005}\approx 250(42.580318)\approx \$10,645.08$

7. **a.** $A=200a_{\overline{13}|0.04}\approx 200(9.985648)$
$\approx \$1997.13$

b. $S=200s_{\overline{13}|0.04}\approx 200(16.626838)$
$\approx \$3325.37$

8. $150s_{\overline{14}|0.04}-150=150(18.291911)-150$
≈ 2593.79

9. $200s_{\overline{13}|\frac{0.08}{12}}-200\approx 200(13.532926)-200$
$\approx \$2506.59$

10. $350a_{\overline{30}|0.01}\approx 350(25.807708)\approx \$9032.70$

11. $\dfrac{5000}{s_{\overline{5}|0.06}}\approx\dfrac{5000}{5.637093}\approx \$886.98$

12. **a.** $\dfrac{7000}{a_{\overline{36}|\frac{0.04}{12}}}\approx\dfrac{7000}{33.870766}\approx \$206.67$

b. $36(206.67)-7000=\$440.12$

13. Let x be the first payment. The equation of value now is

$$x+2x(1.07)^{-3}=500(1.05)^{-3}+500(1.03)^{-8}$$
$$x\left[1+2(1.07)^{-3}\right]=500(1.05)^{-3}+500(1.03)^{-8}$$
$$x=\frac{500(1.05)^{-3}+500(1.03)^{-8}}{1+2(1.07)^{-3}}$$
$$x\approx \$314.00$$

14. $R = \dfrac{3500}{a_{\overline{3}|0.01375}} = 3500\left[\dfrac{0.01375}{1-(1.01375)^{-3}}\right]$

$\approx \$1198.90$

The interest for the first period is (0.01375)(3500) = \$48.13, so the principal repaid at the end of that period is 1198.90 – 48.13 = \$1150.77. The principal outstanding at the beginning of period 2 is 3500 – 1150.77 = \$2349.23. The interest for that period is (0.01375)(2349.23) = \$32.30. The principal repaid at the end of that period is 1198.90 – 32.30 = \$1166.60. The principal outstanding at the beginning of period 3 is 2349.23 – 1166.60 = \$1182.63. Continuing, we obtain the following amortization schedule. Note the adjustment in the final payment.

Period	Prin. Outs. at Beginning	Int. for Period	Pmt. at End	Prin. Repaid at End
1	3500.00	48.13	1198.90	1150.77
2	2349.23	32.30	1198.90	1166.60
3	1182.63	16.26	1198.89	1182.63
Total		96.69	3596.69	3500.00

15. $R = \dfrac{15{,}000}{a_{\overline{5}|0.0075}} \approx \dfrac{15{,}000}{4.889440} \approx \$3067.84$

The interest for period 1 is (0.0075)(15,000) = \$112.50, so the principal repaid at the end of that period is 3067.84 – 112.50 = \$2955.34. The principal outstanding at beginning of period 2 is 15,000 – 2955.34 = \$12,044.66. The interest for period 2 is 0.0075(12,044.66) = \$90.33, so the principal repaid at the end of that period is 3067.84 – 90.33 = \$2977.51. Principal outstanding at the beginning of period 3 is 12,044.66 – 2977.51 = \$9067.15. Continuing, we obtain the following amortization schedule. Note the adjustment in the final payment.

Period	Prin. Outs. at Beginning	Int. for Period	Pmt. at End	Prin. Repaid at End
1	15,000	112.50	3067.84	2955.34
2	12,044.66	90.33	3067.84	2977.51
3	9067.15	68.00	3067.84	2999.84
4	6067.31	45.50	3067.84	3022.34
5	3044.97	22.84	3067.81	3044.97
Total		339.17	15,339.17	15,000.00

16. $460a_{\overline{108}|\frac{0.06}{12}} = 460\left[\dfrac{1-\left(1+\frac{0.06}{12}\right)^{-108}}{\frac{0.06}{12}}\right]$

$\approx \$38{,}314.98$

17. The monthly payment is

$$\frac{11{,}000}{a_{\overline{48}|\frac{0.055}{12}}} = 11{,}000\left[\frac{\frac{0.055}{12}}{1-\left(1+\frac{0.055}{12}\right)^{-48}}\right] \approx \$255.82$$

The finance charge is 48(255.82) – 11,000 = \$1279.36

Explore and Extend—Chapter 5

1. $\frac{0.085}{2} = 0.0425$, thus $R = 0.0425(25{,}000) = 1062.50$.

$$P = 25{,}000(1.0825)^{-25} + 1062.50 \cdot \frac{1-(1.0825)^{-25}}{\sqrt{1.0825}-1}$$
$$\approx \$26{,}102.13$$

2. $\frac{0.065}{2} = 0.0325$, thus $R = 0.0325(10{,}000) = 325$.

On a graphics calculator, let Y_1 = 10,389 and Y_2 = 10,000(1 + x)^ – 7 + 325(1 – (1 + x)^ – 7)/$\left(\sqrt{\ }(1+x)-1\right)$.

The curves intersect at 0.0590. The yield is 5.9%.

3. The normal yield curve assumes a stable economic climate. By contrast, if investors are expecting a drop in interest rates, and with it a drop in yields from future investments, they will gladly give up liquidity for long-term investment at current, more favorable, interest rates. T-bills, which force the investor to find a new investment in a short time, are correspondingly less attractive, and so prices drop and yields rise.