Chapter 12

Apply It 12.1

1. $\frac{dq}{dp} = \frac{d}{dp}\left[25 + 2\ln\left(3p^2+4\right)\right]$

$= 0 + 2\frac{d}{dp}\left[\ln\left(3p^2+4\right)\right]$

$= 2\left(\frac{1}{3p^2+4}\right)\frac{d}{dp}\left(3p^2+4\right) = \frac{2}{3p^2+4}(6p)$

$= \frac{12p}{3p^2+4}$

2. With $I_0 = 1$, $R(I) = \log I$.

$\frac{dR}{dI} = \frac{d}{dI}[\log I] = \frac{d}{dI}\left[\frac{\ln I}{\ln 10}\right]$

$= \frac{1}{\ln 10}\cdot\frac{1}{I} = \frac{1}{I\ln 10}$

Problems 12.1

1. $\frac{dy}{dx} = a\cdot\frac{d}{dx}(\ln x) = a\cdot\frac{1}{x} = \frac{a}{x}$

2. $\frac{dy}{dx} = \frac{5}{9}\left(\frac{1}{x}\right) = \frac{5}{9x}$

3. $\frac{dy}{dx} = \frac{1}{3x-7}(3) = \frac{3}{3x-7}$

4. $\frac{dy}{dx} = \frac{1}{5x-6}(5) = \frac{5}{5x-6}$

5. $y = \ln x^2 = 2\ln x$

$\frac{dy}{dx} = 2\cdot\frac{1}{x} = \frac{2}{x}$

6. $\frac{dy}{dx} = \frac{1}{5x^3+3x^2+2x+1}(15x^2+6x+2)$

$= \frac{15x^2+6x+2}{5x^3+3x^2+2x+1}$

7. $\frac{dy}{dx} = \frac{1}{1-x^2}(-2x) = -\frac{2x}{1-x^2}$

8. $\frac{dy}{dx} = \frac{1}{-x^2+6x}(-2x+6) = \frac{-2x+6}{-x^2+6x}$

$= \frac{-2(x-3)}{-x(x-6)} = \frac{2(x-3)}{x(x-6)}$

9. $f'(X) = \frac{1}{4X^6+2X^3}(24X^5+6X^2)$

$= \frac{24X^5+6X^2}{4X^6+2X^3}$

$= \frac{6X^2(4X^3+1)}{2X^3(2X^3+1)}$

$= \frac{3(4X^3+1)}{X(2X^3+1)}$

10. $f'(r) = \frac{1}{2r^4-3r^2+2r+1}\left(8r^3-6r+2\right)$

$= \frac{8r^3-6r+2}{2r^4-3r^2+2r+1}$

$= \frac{2\left(4r^3-3r+1\right)}{2r^4-3r^2+2r+1}$

11. $f'(t) = \ln t + t\left(\frac{1}{t}\right) - 1 = \ln t$

12. $\frac{dy}{dx} = x^2\left(\frac{1}{x}\right) + (\ln x)(2x) = x + 2x\ln x$

$= x(1+2\ln x)$

13. $\frac{dy}{dx} = x^3\left[\frac{1}{2x+5}(2)\right] + \ln(2x+5)\cdot 3x^2$

$= \frac{2x^3}{2x+5} + 3x^2\ln(2x+5)$

14. $\frac{dy}{dx} = (ax+b)^3\left[\frac{1}{(ax+b)}(a)\right]$

$+ [\ln(ax+b)]3(ax+b)^2(a)$

$= a(ax+b)^2 + 3a(ax+b)^2\ln(ax+b)$

$= a(ax+b)^2[1+3\ln(ax+b)]$

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15. $y = \log_3(8x-1) = \dfrac{\ln(8x-1)}{\ln 3}$

$\dfrac{dy}{dx} = \dfrac{1}{\ln 3} \cdot \dfrac{d}{dx}[\ln(8x-1)]$

$= \dfrac{1}{\ln 3} \cdot \dfrac{1}{8x-1}(8) = \dfrac{8}{(8x-1)(\ln 3)}$

16. $f(w) = \log\left(w^2+2w+1\right) = \log_{10}\left(w^2+2w+1\right)$

$= \dfrac{\ln\left(w^2+2w+1\right)}{\ln 10}$

$f'(w) = \dfrac{1}{\ln 10} \cdot \dfrac{1}{w^2+2w+1}(2w+2)$

$= \dfrac{2w+2}{(\ln 10)\left(w^2+2w+1\right)}$

17. $y = x^2 + \log_2\left(x^2+4\right) = x^2 + \dfrac{\ln\left(x^2+4\right)}{\ln 2}$

$\dfrac{dy}{dx} = 2x + \dfrac{1}{\ln 2}\left[\dfrac{1}{x^2+4}(2x)\right]$

$= 2x\left[1 + \dfrac{1}{(\ln 2)\left(x^2+4\right)}\right]$

18. $y = x^2 \log_2 x = x^2 \cdot \dfrac{\ln x}{\ln 2} = \dfrac{1}{\ln 2}(x^2 \ln x)$

$\dfrac{dy}{dx} = \dfrac{1}{\ln 2}\left[x^2\left(\dfrac{1}{x}\right) + \ln x(2x)\right]$

$= \dfrac{x}{\ln 2}(1 + 2\ln x)$

19. $f'(z) = \dfrac{z\left(\frac{1}{z}\right) - (\ln z)(1)}{z^2} = \dfrac{1-\ln z}{z^2}$

20. $\dfrac{dy}{dx} = \dfrac{(\ln x)(2x) - x^2\left(\frac{1}{x}\right)}{(\ln x)^2}$

$= \dfrac{2x\ln x - x}{\ln^2 x} = \dfrac{x[2\ln x - 1]}{\ln^2 x}$

21. $\dfrac{dy}{dx} = \dfrac{(\ln x)(4x^3+6x+1) - \frac{1}{x}(x^4+3x^2+x)}{(\ln x)^2}$

$= \dfrac{(4x^3+6x+1)\ln x - (x^3+3x+1)}{(\ln x)^2}$

22. $y = \ln x^{100} = 100\ln x$

$\dfrac{dy}{dx} = 100 \cdot \dfrac{1}{x} = \dfrac{100}{x}$

23. $y = \ln\left(x^2+4x+5\right)^3 = 3\ln\left(x^2+4x+5\right)$

$\dfrac{dy}{dx} = 3 \cdot \dfrac{1}{x^2+4x+5}(2x+4)$

$= \dfrac{3(2x+4)}{x^2+4x+5} = \dfrac{6(x+2)}{x^2+4x+5}$

24. $y = 6\ln\sqrt[3]{x} = 6 \cdot \dfrac{1}{3}\ln x = 2\ln x$

$\dfrac{dy}{dx} = 2 \cdot \dfrac{1}{x} = \dfrac{2}{x}$

25. $y = 9\ \ln\sqrt{1+x^2} = \dfrac{9}{2}\ln\left(1+x^2\right)$

$\dfrac{dy}{dx} = \dfrac{9}{2} \cdot \dfrac{1}{1+x^2}(2x) = \dfrac{9x}{1+x^2}$

26. $f(t) = \ln t^4 - \ln(1+6t+t^2)$

$f'(t) = \dfrac{1}{t^4}(4t^3) - \dfrac{1}{1+6t+t^2}(6+2t)$

$= \dfrac{4}{t} - \dfrac{6+2t}{1+6t+t^2}$

27. $f(l) = \ln\left(\dfrac{1+l}{1-l}\right) = \ln(1+l) - \ln(1-l)$

$f'(l) = \dfrac{1}{1+l} - \dfrac{1}{1-l}(-1)$

$= \dfrac{(1-l)+(1+l)}{(1+l)(1-l)} = \dfrac{2}{1-l^2}$

28. $y = \ln\left(\dfrac{2x+3}{3x-4}\right) = \ln(2x+3) - \ln(3x-4)$

$\dfrac{dy}{dx} = \dfrac{2}{2x+3} - \dfrac{3}{3x-4}$

$= \dfrac{2(3x-4) - 3(2x+3)}{(2x+3)(3x-4)} = -\dfrac{17}{(2x+3)(3x-4)}$

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29. $y = \ln\sqrt[4]{\frac{1+x^2}{1-x^2}} = \frac{1}{4}\left[\ln\left(1+x^2\right) - \ln\left(1-x^2\right)\right]$

$\frac{dy}{dx} = \frac{1}{4}\left[\frac{2x}{1+x^2} - \frac{-2x}{1-x^2}\right] = \frac{1}{4}\left[\frac{2x\left(1-x^2\right)+2x\left(1+x^2\right)}{\left(1+x^2\right)\left(1-x^2\right)}\right] = \frac{x}{1-x^4}$

30. $y = \ln\sqrt[3]{\frac{x^3-1}{x^3+1}} = \frac{1}{3}[\ln(x^3-1) - \ln(x^3+1)]$

$\frac{dy}{dx} = \frac{1}{3}\left[\frac{3x^2}{x^3-1} - \frac{3x^2}{x^3+1}\right]$

$= \frac{1}{3}\left[\frac{3x^2(x^3+1) - 3x^2(x^3-1)}{(x^3-1)(x^3+1)}\right]$

$= \frac{2x^2}{x^6-1}$

31. $y = p\ln(ax^2+bx+c) + q\ln(hx^2+kx+l)$

$\frac{dy}{dx} = \frac{p}{ax^2+bx+c}(2ax+b) + \frac{q}{hx^2+kx+l}(2hx+k)$

$= \frac{p(2ax+b)}{ax^2+bx+c} + \frac{q(2hx+k)}{hx^2+kx+l}$

32. $y = \ln\left[(5x+2)^4(8x-3)^6\right]$

$= 4\ln(5x+2) + 6\ln(8x-3)$

$\frac{dy}{dx} = 4\cdot\frac{1}{5x+2}(5) + 6\cdot\frac{1}{8x-3}(8) = \frac{20}{5x+2} + \frac{48}{8x-3}$

33. $y = 13\ln\left(x^2\sqrt[3]{5x+2}\right)$

$= 13\ln x^2 + 13\ln(5x+2)^{1/3}$

$= 26\ln x + \frac{13}{3}\ln(5x+2)$

$\frac{dy}{dx} = 26\left(\frac{1}{x}\right) + \frac{13}{3}\cdot\frac{1}{5x+2}(5) = \frac{26}{x} + \frac{65}{3(5x+2)}$

34. $y = 6\ln\frac{x}{\sqrt{2x+1}} = 6\ln x - 6\ln(2x+1)^{\frac{1}{2}}$

$= 6\ln x - 3\ln(2x+1)$

$\frac{dy}{dx} = \frac{6}{x} - 3\cdot\frac{1}{2x+1}(2) = \frac{6}{x} - \frac{6}{2x+1}$

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35. $\frac{dy}{dx}=\left(x^2+1\right)\left[\frac{1}{2x+1}(2)\right]+\ln(2x+1)\cdot(2x)$

$=\frac{2\left(x^2+1\right)}{2x+1}+2x\ln(2x+1)$

36. $\frac{dy}{dx}=(2ax+b)\ln(hx^2+kx+l)+(ax^2+bx+c)\frac{1}{hx^2+kx+l}(2hx+k)$

$=(2ax+b)\ln(hx^2+kx+l)+\frac{(ax^2+bx+c)(2hx+k)}{hx^2+kx+l}$

37. $y=\ln x^3+\ln^3 x=3\ln x+(\ln x)^3$

$\frac{dy}{dx}=3\cdot\frac{1}{x}+3(\ln x)^2\cdot\frac{1}{x}=\frac{3}{x}+\frac{3(\ln x)^2}{x}=\frac{3\left(1+\ln^2 x\right)}{x}$

38. $\frac{dy}{dx}=(\ln 2)x^{(\ln 2)-1}$

39. $y=\ln^4(ax)=[\ln(ax)]^4$

$\frac{dy}{dx}=4[\ln(ax)]^3\left(\frac{1}{ax}\cdot a\right)=\frac{4\ln^3(ax)}{x}$

40. $y=\ln^2(2x+11)=[\ln(2x+11)]^2$

$\frac{dy}{dx}=2[\ln(2x+11)]\cdot\frac{1}{2x+11}(2)=\frac{4\ln(2x+11)}{2x+11}$

41. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{2}\ln f(x)\right)=\frac{1}{2}\left(\frac{1}{f(x)}\right)f'(x)=\frac{f'(x)}{2f(x)}$

42. $y=\ln\left(x^3\sqrt[4]{2x+1}\right)=3\ln x+\frac{1}{4}\ln(2x+1)$

$\frac{dy}{dx}=3\cdot\frac{1}{x}+\frac{1}{4}\cdot\frac{1}{2x+1}(2)=\frac{3}{x}+\frac{1}{2(2x+1)}$

43. $y=\sqrt{4+3\ln x}=(4+3\ln x)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(4+3\ln x)^{-\frac{1}{2}}\cdot\frac{3}{x}=\frac{3}{2x\sqrt{4+3\ln x}}$

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44. $\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}}\left[1+\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}(2x)\right]$

$= \frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} = \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}\left(x+\sqrt{1+x^2}\right)}$

$= \frac{1}{\sqrt{1+x^2}}$

45. $y = \ln(x^2-3x-3)$

$y' = \frac{2x-3}{x^2-3x-3}$

The slope of the tangent line at $x = 4$ is

$y'(4) = \frac{8-3}{16-12-3} = 5.$ Also, if $x = 4$, then $y = \ln(16-12-3) = \ln 1 = 0$. Thus an equation of the tangent line is $y - 0 = 5(x - 4)$, or $y = 5x - 20$.

46. $y = x[\ln(x) - 1]$

$y' = x\left(\frac{1}{x}\right) + [\ln(x)-1](1) = \ln x$

When $x = 1$, $y = -1$ and $y' = 0$. The equation of the tangent line is $y - (-1) = 0(x - 1)$, or $y = -1$.

47. $y = \frac{x}{\ln x}$

$y' = \frac{(\ln x)(1) - x\left(\frac{1}{x}\right)}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$

When $x = 3$ the slope is $y'(3) = \frac{(\ln 3)-1}{\ln^2 3}$.

48. $p = \frac{25}{\ln(q+2)}$, so $r = pq = \frac{25q}{\ln(q+2)}$. Thus the marginal revenue is

$\frac{dr}{dq} = 25\cdot\frac{\ln(q+2)(1) - q\left(\frac{1}{q+2}\right)}{\ln^2(q+2)}$

$= 25\cdot\frac{(q+2)\ln(q+2)-q}{(q+2)\ln^2(q+2)}$.

49. $c = 25\ln(q + 1) + 12$

$\frac{dc}{dq} = \frac{25}{q+1}$, so $\left.\frac{dc}{dq}\right|_{q=6} = \frac{25}{7}$.

50. $\bar{c} = \frac{500}{\ln(q+20)}$

$c = \bar{c}q = \frac{500q}{\ln(q+20)}$

$\frac{dc}{dq} = 500\cdot\frac{[\ln(q+20)](1) - q\left(\frac{1}{q+20}\right)}{[\ln(q+20)]^2}$

$\left.\frac{dc}{dq}\right|_{q=50} = 500\cdot\frac{\ln 70 - \frac{50}{70}}{(\ln 70)^2} \approx \$97.90$

51. $\frac{dq}{dp} = \frac{d}{dp}[27 + 11\ln(2p+1)]$

$= 0 + 11\frac{d}{dp}[\ln(2p+1)] = 11\left(\frac{1}{2p+1}\right)\frac{d}{dp}[2p+1]$

$= \frac{11}{2p+1}(2) = \frac{22}{2p+1}$

52. With $I_0 = 17$, $L(I) = 10\log\frac{I}{17}$.

$\frac{dL}{dI} = \frac{d}{dI}\left[10\log\frac{I}{17}\right] = 10\frac{d}{dI}[\log I - \log 17]$

$= 10\frac{d}{dI}\left[\frac{\ln I}{\ln 10} - \log 17\right] = 10\left[\frac{1}{\ln 10}\cdot\frac{1}{I} - 0\right]$

$= \frac{10}{I\ln 10}$

53. $A = 6\ln\left(\frac{T}{a-T} - a\right)$. Rate of change of A with respect to T:

$\frac{dA}{dT} = 6\cdot\frac{1}{\frac{T}{a-T}-a}\left[\frac{(a-T)(1) - T(-1)}{(a-T)^2}\right]$

$= 6\cdot\frac{1}{\frac{T-a(a-T)}{a-T}}\left[\frac{a}{(a-T)^2}\right]$

$= 6\cdot\frac{a-T}{T-a^2+aT}\cdot\frac{a}{(a-T)^2}$

$= \frac{6a}{\left(T-a^2+aT\right)(a-T)}$

54. If $y = \ln f(x)$, then $\frac{dy}{dx} = \frac{1}{f(x)}f'(x) = \frac{f'(x)}{f(x)}$, which is the relative rate of change of $y = f(x)$ with respect to x.

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55. $\frac{d}{dx}(\log_b u) = \frac{d}{dx}\left(\frac{\ln u}{\ln b}\right)$

$= \frac{1}{\ln b} \cdot \frac{d}{dx}(\ln u) = \frac{1}{\ln b}\left(\frac{1}{u} \cdot \frac{du}{dx}\right)$

$= (\log_b e)\left(\frac{1}{u} \cdot \frac{du}{dx}\right) = \frac{1}{u}(\log_b e)\frac{du}{dx}$

56. $f'(x) = x^2(1 + 3\ln x)$

$f'(x) = 0$ for $x \approx 0.72$

57. Note that $f(x)$ is defined for all $x \neq 0$.

$f'(x) = \frac{x^2 \cdot \frac{1}{x^2}(2x) - \ln(x^2) \cdot 2x}{x^4} = \frac{2 - 2\ln(x^2)}{x^3}$

$f'(x) = 0$ for $x \approx -1.65, 1.65$

Apply It 12.2

3. The rate of change of temperature with respect to time is $\frac{dT}{dt}$. $T(t)$ has the form Ce^u where C is a constant and $u = kt$.

$\frac{dT}{dt} = \frac{d}{dt}\left[Ce^{kt}\right] = C\frac{d}{dt}\left[e^{kt}\right] = C\left(e^{kt}\right)\frac{d}{dt}[kt] = Ce^{kt}(k) = Cke^{kt}$

Problems 12.2

1. $y' = 5 \cdot \frac{d}{dx}(e^x) = 5e^x$

2. $y' = \frac{a}{b} \cdot \frac{d}{dx}(e^x) = \frac{ae^x}{b}$

3. $y' = e^{2x^2+3}(4x) = 4xe^{2x^2+3}$

4. $y' = e^{2x^2+5}(4x) = 4xe^{2x^2+5}$

5. $y' = e^{9-5x} \cdot \frac{d}{dx}(9 - 5x) = e^{9-5x}(-5) = -5e^{9-5x}$

6. $f'(q) = e^{-q^3+6q-1}\left(-3q^2 + 6\right) = -3\left(q^2 - 2\right)e^{-q^3+6q-1}$

7. $f'(r) = (12r^2 + 10r + 2)e^{4r^3+5r^2+2r+6}$

8. $y' = e^{x^2+6x^3+1}(2x + 18x^2)$

$= 2x(1 + 9x)e^{x^2+6x^3+1}$

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9. $y' = x\left(e^x\right) + e^x(1) = e^x(x+1)$

10. $y' = 3x^4\left[e^{-x}(-1)\right] + e^{-x}(12x^3) = 3x^3e^{-x}(4-x)$

11. $y' = x^2\left[e^{-x^2}(-2x)\right] + e^{-x^2}(2x)$

$= 2xe^{-x^2}\left(1-x^2\right)$

12. $y' = x\left[e^{ax}(a)\right] + e^{ax}(1) = e^{ax}(ax+1)$

13. $y = \frac{1}{3}\left(e^x + e^{-x}\right)$

$y' = \frac{1}{3}\left[e^x + e^{-x}(-1)\right] = \frac{e^x - e^{-x}}{3}$

14. $\frac{dy}{dx} = \frac{(e^x + e^{-x})[e^x - e^{-x}(-1)] - (e^x - e^{-x})[e^x + e^{-x}(-1)]}{(e^x + e^{-x})^2}$

$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2}$

15. $\frac{d}{dx}\left(5^{2x^3}\right) = \frac{d}{dx}\left[e^{(\ln 5)2x^3}\right]$

$= e^{(\ln 5)2x^3}[(\ln 5)6x^2]$

$= (6x^2)5^{2x^3}\ln 5$

16. $y = 2^x x^2 = e^{(\ln 2)x}x^2$

$y' = e^{(\ln 2)x}(2x) + x^2\left[e^{(\ln 2)x}(\ln 2)\right]$

$= 2x\left(2^x\right) + x^2\left(2^x\right)(\ln 2) = x\left(2^x\right)(2 + x\ln 2)$

17. $f'(w) = \frac{(w^2+w+1)e^{aw}(a) - (2w+1)e^{aw}}{(w^2+w+1)^2}$

$= \frac{e^{aw}[a(w^2+w+1) - (2w+1)]}{(w^2+w+1)^2}$

18. $y' = e^{x-\sqrt{x}}\left(1 - \frac{1}{2}x^{-\frac{1}{2}}\right) = e^{x-\sqrt{x}}\left(1 - \frac{1}{2\sqrt{x}}\right)$

19. $y' = e^{1+\sqrt{x}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = \frac{e^{1+\sqrt{x}}}{2\sqrt{x}}$

20. $y' = 3(e^{2x}+1)^2(e^{2x}(2)+0) = 6e^{2x}(e^{2x}+1)^2$

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21. $y = x^5 - 5^x = x^5 - e^{(\ln 5)x}$

$y' = 5x^4 - e^{(\ln 5)x}(\ln 5) = 5x^4 - 5^x \ln 5$

22. $f(z) = e^{1/z}$

$f'(z) = e^{1/z}\left(\frac{-1}{z^2}\right) = -\frac{e^{1/z}}{z^2}$

23. $\frac{dy}{dx} = \frac{\left(e^x+1\right)\left[e^x\right] - \left(e^x-1\right)\left[e^x\right]}{\left(e^x+1\right)^2}$

$= \frac{2e^x}{\left(e^x+1\right)^2}$

24. $y' = e^{2x}[1] + (x+6)\left[e^{2x}(2)\right] = e^{2x}(2x+13)$

25. $y = \ln e^x = x$ so $y' = 1$.

26. $y' = e^{-x}\cdot\frac{1}{x} + (\ln x)\left(-e^{-x}\right) = e^{-x}\left(\frac{1}{x} - \ln x\right)$

27. $y = x^x = e^{\ln x^x} = e^{x\ln x}$

$\frac{dy}{dx} = e^{x\ln x}(\ln x + 1) = x^x(\ln x + 1)$

28. $y = \ln e^{4x+1} = 4x+1$, so $\frac{dy}{dx} = 4$.

29. $f(x) = ee^xe^{x^2} = e^{1+x+x^2}$

$f'(x) = e^{1+x+x^2}(1+2x) = (1+2x)e^{1+x+x^2}$

$f'(-1) = [1+2(-1)]e^{1+(-1)+(-1)^2} = -e$

30. $f(x) = 5^{x^2\ln x} = \left(e^{\ln 5}\right)^{x^2\ln x} = e^{(\ln 5)x^2\ln x}$

$f'(x) = e^{(\ln 5)x^2\ln x}\left\{(\ln 5)\left[x^2\cdot\frac{1}{x} + (\ln x)(2x)\right]\right\}$

$= e^{(\ln 5)x^2\ln x}(\ln 5)[x + 2x\ln x]$

$f'(1) = e^0(\ln 5)[1+0] = \ln 5$

31. $y = e^x$, $y' = e^x$. When $x = -2$, then $y = e^{-2}$ and $y' = e^{-2}$. Thus an equation of the tangent line is $y - e^{-2} = e^{-2}(x+2)$, or $y = e^{-2}x + 3e^{-2}$.

32. $y' = e^x$

When $x = 1$, $y = e$ and $y' = e$. Thus an equation of the tangent line is $y - e = e(x-1)$ or $y = ex$. If $x = 0$, then $y = e(0) = 0$.

When $x = a$, then $y = e^a$ and $y' = e^a$. So the equation of the line tangent to $y = e^x$ at $x = a$ is $y - e^a = e^a(x-a)$ or $y = e^a(x-a+1)$ the y-intercept of this line is $(0, e^a(1-a))$ which is only (0, 0) when $a = 1$. So the tangent line at $(1, e)$ is the only tangent line to $y = e^x$ that passes through (0, 0).

33. $\frac{dp}{dq} = 15e^{-0.001q}(-0.001) = -0.015e^{-0.001q}$

$\left.\frac{dp}{dq}\right|_{q=500} = -0.015e^{-0.5}$

34. $\frac{dp}{dq} = 9e^{-5q/750}\left(-\frac{5}{750}\right) = -0.06e^{-5q/750}$

$\left.\frac{dp}{dq}\right|_{q=300} = -0.06e^{-2}$

35. $\bar{c} = \frac{7000e^{\frac{q}{700}}}{q}$, so $c = \bar{c}q = 7000e^{\frac{q}{700}}$. The marginal cost function is $\frac{dc}{dq} = 7000e^{\frac{q}{700}}\left(\frac{1}{700}\right)$ $= 10e^{\frac{q}{700}}$. Thus $\left.\frac{dc}{dq}\right|_{q=350} = 10e^{0.5}$ and $\left.\frac{dc}{dq}\right|_{q=700} = 10e$.

36. $\bar{c} = \frac{850}{q} + 4000\frac{e^{\frac{2q+6}{800}}}{q}$

$c = \bar{c}q = 850 + 4000e^{\frac{2q+6}{800}} = 850 + 4000e^{\frac{q+3}{400}}$

The marginal cost function is $\frac{dc}{dq} = 10e^{\frac{q+3}{400}}$.

$\left.\frac{dc}{dq}\right|_{q=97} = 10e^{0.25}$ and $\left.\frac{dc}{dq}\right|_{q=197} = 10e^{0.5}$.

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37. $w = e^{x^2}$ and $x = \frac{t+1}{t-1}$.

$$\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt}$$
$$= e^{x^2}(2x) \cdot \frac{(t-1)(1)-(1)(t+1)}{(t-1)^2}$$
$$= 2xe^{x^2} \cdot \frac{t-1-t-1}{(t-1)^2}$$
$$= -\frac{4xe^{x^2}}{(t-1)^2}$$

When $t = 2$, $x = 3$ and $\frac{dw}{dt} = -\frac{4(3)e^{3^2}}{(2-1)^2} = -12e^9$.

38. $f'(x) = x^3$ and $u = e^x$. Let $y = f(u)$. Then $\frac{d}{dx}[f(u)] = \frac{dy}{dx}$ and by the chain rule

$$\frac{d}{dx}[f(u)] = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(u)\frac{du}{dx} = u^3 \cdot e^x$$
$$= (e^x)^3 \cdot e^x = e^{3x} \cdot e^x = e^{4x}$$

39. $\frac{d}{dx}\left(c^x - x^c\right) = \frac{d}{dx}\left[\left(e^{\ln c}\right)^x - x^c\right]$

$$= \frac{d}{dx}\left[e^{(\ln c)x} - x^c\right]$$
$$= (\ln c)e^{(\ln c)x} - cx^{c-1} = (\ln c)c^x - cx^{c-1}$$
$$\left.\frac{d}{dx}(c^x - x^c)\right|_{x=1} = (\ln c)c - c$$

If this is zero, $(\ln c)c - c = 0$, or $c[\ln(c) - 1] = 0$. Since $c > 0$, we must have $\ln(c) - 1 = 0$, $\ln c = 1$, or $c = e$.

40. $f(x) = 10^{-x} + \ln(8+x) + 0.01e^{x-2}$

$$= e^{(\ln 10)(-x)} + \ln(8+x) + 0.01e^{x-2}$$
$$f'(x) = e^{(\ln 10)(-x)}(-\ln 10) + \frac{1}{8+x} + 0.01e^{x-2}$$
$$= -(\ln 10)10^{-x} + \frac{1}{8+x} + 0.01e^{x-2}$$
$$\frac{f'(2)}{f(2)} = \frac{-(\ln 10)10^{-2} + \frac{1}{10} + 0.01}{10^{-2} + \ln(10) + 0.01} \approx 0.0374$$

41. $q = 500\left(1 - e^{-0.2t}\right)$

$$\frac{dq}{dt} = 500\left(-e^{-0.2t}\right)(-0.2) = 100e^{-0.2t}$$

Thus $\left.\frac{dq}{dt}\right|_{t=10} = 100e^{-2}$.

42. $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$

$$f'(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}(-x)$$
$$f'(-1) = \frac{1}{\sqrt{2\pi}}e^{-1/2}(1) \approx 0.242$$

43. $P = 1.92e^{0.0176t}$

$$\frac{dP}{dt} = 1.92e^{0.0176t}(0.0176) = P(0.0176)$$
$$= 0.0176P = kP \text{ for } k = 0.0176.$$

44. $Y = k\alpha^{\beta^t} = ke^{(\ln \alpha)\beta^t}$

$$Y' = ke^{(\ln \alpha)\beta^t}(\ln \alpha)\frac{d}{dt}[\beta^t]$$
$$= k\alpha^{\beta^t}(\ln \alpha)\frac{d}{dt}[e^{(\ln \beta)t}]$$
$$= k\alpha^{\beta^t}(\ln \alpha)e^{(\ln \beta)t}(\ln \beta)$$
$$= k\alpha^{\beta^t}(\beta^t \ln \alpha)\ln \beta$$

45. Since $S = Pe^{rt}$, then $\frac{dS}{dt} = Pe^{rt}r = rPe^{rt}$. Thus

$$\frac{\frac{dS}{dt}}{S} = \frac{rPe^{rt}}{Pe^{rt}} = r.$$

46. $y = K\left(1 - e^{-ax}\right)$

$$\frac{dy}{dx} = K\left[-e^{-ax}(-a)\right] = aKe^{-ax}$$

Solving the original equation for e^{-ax} gives $e^{-ax} = -\frac{y}{K} + 1$. Thus substitution,

$\frac{dy}{dx} = aK\left(-\frac{y}{K} + 1\right) = a(-y + K) = a(K - y)$, as was to be shown.

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47. $N = 10^A 10^{-bM} = 10^{A-bM} = e^{(\ln 10)(A-bM)}$

$\frac{dN}{dM} = e^{(\ln 10)(A-bM)}(\ln 10)(-b)$, so

$\frac{dN}{dM} = 10^{A-bM}(\ln 10)(-b) = -b\left(10^{A-bM}\right)\ln 10$

48. $p = 0.89\left[0.01 + 0.99(0.85)^t\right]$

a. $\frac{dP}{dt} = 0.89\left[0.99(0.85)^t \ln(0.85)\right]$

$= 0.8811(0.85)^t \ln(0.85)$

This represents the rate of change of proportion of correct recalls with respect to length of recall interval.

b. If $t = 2$, then

$\frac{dp}{dt} = 0.8811(0.85)^2 \ln(0.85) \approx -0.10$

49. $C(t) = C_0 e^{-\left(\frac{r}{V}\right)t}$

$\frac{dC}{dt} = C_0 e^{-\left(\frac{r}{V}\right)t}\left(-\frac{r}{V}\right)$

$= [C(t)]\left(-\frac{r}{V}\right) = -\left(\frac{r}{V}\right)C(t)$

50. $C(t) = \frac{R}{r}\left[1 - e^{-\left(\frac{r}{V}\right)t}\right]$

a. $C(0) = \frac{R}{r}\left[1 - e^0\right] = \frac{R}{r}[1-1] = 0$

b. $\frac{dC}{dt} = \frac{R}{r}\left[\frac{r}{V}e^{-\left(\frac{r}{V}\right)t}\right] = \frac{R}{V}e^{-\left(\frac{r}{V}\right)t}$

$= \frac{R}{V}\left[1 - \left(1 - e^{-\left(\frac{r}{V}\right)t}\right)\right]$

$= \frac{R}{V}\left[1 - \frac{r}{R}\cdot\frac{R}{r}\left(1 - e^{-\left(\frac{r}{V}\right)t}\right)\right]$

$= \frac{R}{V}\left[1 - \frac{r}{R}C(t)\right] = \frac{R}{V} - \frac{r}{V}C(t)$

51. $f(t) = 1 - e^{-0.008t}$

$f'(t) = 0.008e^{-0.008t}$

$f'(100) = 0.008e^{-0.8} \approx 0.0036$

52. $S = \ln\frac{3}{2+e^{-I}} = \ln 3 - \ln(2 + e^{-I})$

a. Recall that $\frac{dC}{dI} = 1 - \frac{dS}{dI}$.

$\frac{dS}{dI} = -\frac{1}{2+e^{-I}}(e^{-I})(-1) = \frac{e^{-I}}{2+e^{-I}}$

Thus $\frac{dC}{dI} = 1 - \frac{dS}{dI} = 1 - \frac{e^{-I}}{2+e^{-I}} = \frac{2}{2+e^{-I}}$.

b. If $\frac{dS}{dI} = \frac{1}{7}$, then $\frac{e^{-I}}{2+e^{-I}} = \frac{1}{7}$.

$\frac{(e^I)e^{-I}}{(e^I)(2+e^{-I})} = \frac{1}{7}$

$\frac{1}{2e^I + 1} = \frac{1}{7}$

$2e^I + 1 = 7$

$e^I = \frac{6}{2} = 3$

$I = \ln 3 \approx \$1.099$ billion

53. $f'(x) = (6x^2 + 2x - 3)e^{2x^3 + x^2 - 3x}$

$f'(x) = 0$ for $x \approx -0.89, 0.56$

54. $f'(x) = 1 - e^{-x}$

$f'(x) = 0$ gives $e^x = 1$ or $x = 0$.

Problems 12.3

1. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-2}$.

When $q = 5$ then $p = 40 - 2(5) = 30$, so

$\eta = \frac{\frac{30}{5}}{-2} = -3$

Because $|\eta| > 1$, demand is elastic.

2. $\eta = \frac{\frac{p}{q}}{-0.04} = \frac{\frac{6}{100}}{-0.04} = -1.5$

Because $|\eta| > 1$, demand is elastic.

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3. $p = \frac{3000}{q} = 3000q^{-1}$

$\frac{dp}{dq} = -3000q^{-2} = -\frac{3000}{q^2}$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-\frac{3000}{q^2}} = \frac{\frac{(3000/q)}{q}}{-\frac{3000}{q^2}} = -1$$

Because $|\eta| = 1$, demand has unit elasticity.

4. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{\frac{-1000}{q^3}} = \frac{\frac{(500/q^2)}{q}}{-\frac{1000}{q^3}} = -\frac{1}{2}$, inelastic

5. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-\frac{500}{(q+2)^2}} = \frac{\frac{[500/(q+2)]}{q}}{-\frac{500}{(q+2)^2}} = -\frac{q+2}{q}$

When $q = 104$, then $\eta = -\frac{106}{104} = -\frac{53}{52}$. Because $|\eta| > 1$, demand is elastic.

6. $\eta = \frac{\frac{p}{q}}{-\frac{1600}{(2q+1)^2}} = \frac{\frac{800/(2q+1)}{q}}{-\frac{1600}{(2q+1)^2}} = -\frac{2q+1}{2q}$

When $q = 24$, $\eta = -\frac{49}{48}$, elastic

7. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{\frac{-e^{\frac{q}{100}}}{100}}$

When $q = 100$, then $p = 150 - e$ and

$\eta = \frac{\frac{150-e}{100}}{-\frac{e}{100}} = -\left(\frac{150}{e} - 1\right)$. Because $|\eta| > 1$, demand is elastic.

8. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{250e^{-q/50}}{q}}{-5e^{-q/50}} = -\frac{50}{q}$

When $q = 50$, $\eta = -\frac{50}{50} = -1$, so demand has unit elasticity.

9. $q = 1200 - 150p$

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp} = \frac{p}{q}(-150)$

If $p = 4$, then $q = 1200 - 150(4) = 600$, so

$\eta = \frac{4}{600}(-150) = -1$. Since $|\eta| = 1$, demand has unit elasticity.

10. $q = 100 - p$
When $p = 50$, then $q = 50$.

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$

$\frac{dq}{dp} = -1$, so $\eta = \frac{50}{50}(-1) = -1$, unit elasticity.

11. $q = \sqrt{500 - p}$

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$

$\frac{dq}{dp} = \frac{1}{2}(500 - p)^{-\frac{1}{2}}(-1) = \frac{-1}{2\sqrt{500 - p}} = -\frac{1}{2q}$

$\eta = \frac{p}{q}\left(-\frac{1}{2q}\right) = -\frac{p}{2q^2}$

If $p = 400$, then $q = \sqrt{500 - 400} = 10$, so

$\eta = -\frac{400}{200} = -2$. $|\eta| > 1$, so demand is elastic.

12. $q = \sqrt{2500 - p^2}$

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$

$\frac{dq}{dp} = \frac{1}{2}\left(2500 - p^2\right)^{-\frac{1}{2}}(-2p)$

$= \frac{-p}{\sqrt{2500 - p^2}} = -\frac{p}{q}$

$\eta = \frac{p}{q}\left(-\frac{p}{q}\right) = -\frac{p^2}{q^2}$

If $p = 20$, then $q = \sqrt{2100}$, so we have

$\eta = -\frac{400}{2100} = -\frac{4}{21}$, inelastic.

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13. $q = (p-50)^2$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$$

$\frac{dq}{dp} = 2(p-50)$, so $\eta = \frac{p}{q} \cdot 2(p-50)$.

If $p = 10$, then $q = (10-50)^2 = 1600$. Thus

$\eta = \frac{10}{1600} \cdot 2(10-50) = -\frac{1}{2}$. Demand is inelastic.

14. $q = p^2 - 50p + 850$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$$

$\frac{dq}{dp} = 2p - 50$, so $\eta = \frac{p}{q}(2p-50)$.

If $p = 20$, then $q = 250$, and

$\eta = \frac{20}{250}(40-50) = -\frac{200}{250} = -\frac{4}{5}$, inelastic.

15. $p = 13 - 0.05q$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = -\frac{p}{0.05q}$$

p	q	η	demand
10	60	$-\frac{10}{3}$	elastic
3	200	$-\frac{3}{10}$	inelastic
6.50	130	-1	unit elasticity

16. a. $p = 36 - 0.25q$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{36-0.25q}{-0.25q}$$

Setting $\frac{36-0.25q}{-0.25q} = -1$ yields $q = 72$.

b. $p = 300 - q^2$

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{300-q^2}{-2q^2} = -1$ yields $q = \pm 10$.

Since $q > 0$, we must have $q = 10$.

17. $q = 500 - 40p + p^2$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$$

$\frac{dq}{dp} = -40 + 2p$, so $\eta = \frac{p}{q}(2p-40)$.

When $p = 15$, then $q = 500 - 40(15) + 15^2 = 125$,

so $\eta\big|_{p=15} = \frac{15}{125}(30-40) = -\frac{6}{5} = -1.2$. Now,

(% change in price) $\cdot$ (η) = % change in demand. Thus if the price of 15 increases $\frac{1}{2}\%$, then the change in demand is approximately

$\left(\frac{1}{2}\%\right)(-1.2) = -0.6\%$. Thus demand decreases approximately 0.6%.

18. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q} \cdot \frac{dq}{dp}$

$q = \sqrt{3000 - p^2}$

$\frac{dq}{dp} = -\frac{p}{\sqrt{3000-p^2}} = -\frac{p}{q}$, so

$\eta = \frac{p}{q}\left(-\frac{p}{q}\right) = -\frac{p^2}{q^2}$.

Now, if $p = 40$, then $q = \sqrt{3000 - 40^2} = \sqrt{1400}$,

so $\eta\big|_{p=40} = -\frac{(40)^2}{1400} = -\frac{8}{7}$. If the price of 40 increases to 42.8, that is, it changes by

$\frac{2.8}{40} = 7\%$, then demand would change by

approximately $7\left(-\frac{8}{7}\right)\%$, or 8%. (That is, demand decreases by 8%.)

19. $p = 500 - 2q$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{500-2q}{q}}{-2} = \frac{q-250}{q}$$

If demand is elastic, then $\eta = \frac{q-250}{q} < -1$. For $q > 0$, we have $q - 250 < -q$, $2q < 250$, so $q < 125$. Thus, if $0 < q < 125$, demand is elastic.

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If demand is inelastic, then $\eta = \frac{q-250}{q} > -1$.

For $q > 0$, the inequality implies $q > 125$. Thus if $125 < q < 250$, then demand is inelastic.

Since Total Revenue $= r = pq = 500q - 2q^2$, then $r' = 500 - 4q = 4(125 - q)$. If $0 < q < 125$, then $r' > 0$, so r is increasing. If $125 < q < 250$, then $r' < 0$, so r is decreasing.

20. $p = 50 - 3q$

$r = pq = 50q - 3q^2$

$\frac{dr}{dq} = 50 - 6q$

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{50-3q}{q}}{-3} = \frac{3q-50}{3q}$

$$p\left(1+\frac{1}{\eta}\right) = (50-3q)\left(1+\frac{3q}{3q-50}\right)$$
$$= (50-3q)\left(\frac{3q-50+3q}{3q-50}\right)$$
$$= 50 - 6q = \frac{dr}{dq}$$

21. $p = \frac{1000}{q^2}$

$r = pq = \frac{1000}{q}$

$\frac{dr}{dq} = -1000q^{-2} = -\frac{1000}{q^2}$

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{1000}{q^3}}{-\frac{2000}{q^3}} = -\frac{1}{2}$

$$p\left(1+\frac{1}{\eta}\right) = \frac{1000}{q^2}(1-2) = -\frac{1000}{q^2} = \frac{dr}{dq}$$

22. $p = mq + b$

Note: $q = \frac{p-b}{m}$

a. $\lim\limits_{p\to b^-} \eta = \lim\limits_{p\to b^-} \frac{\frac{p}{q}}{\frac{dp}{dq}} = \lim\limits_{p\to b^-} \frac{\frac{p}{(p-b)/m}}{m}$

$= \lim\limits_{p\to b^-} \frac{p}{p-b} = -\infty$

b. $\eta = \frac{p}{p-b}$

Thus if $p = 0$, then $\eta = 0$.

23. a. $\frac{dq}{dp} = a\left(\frac{1}{2}\right)(b-cp^2)^{-1/2}(-2cp)$

$= -\frac{acp}{\sqrt{b-cp^2}}$

$\eta = \frac{p}{q}\cdot\frac{dq}{dp}$

$= \frac{p}{a\sqrt{b-cp^2}}\left(-\frac{acp}{\sqrt{b-cp^2}}\right)$

$= -\frac{cp^2}{b-cp^2}$

Thus η does not depend on a.

b. $\eta = \frac{1}{-\frac{b}{cp^2}+1}$

$|\eta| > 1$ if $\left|\frac{1}{1-\frac{b}{cp^2}}\right| > 1$, or $\left|1-\frac{b}{cp^2}\right| < 1$. So, $0 < \frac{b}{cp^2} < 2$, or $\left(\sqrt{\frac{b}{2c}}, \infty\right)$. Since q is undefined for $b - cp^2 < 0$, $p < \sqrt{\frac{b}{c}}$. The interval is $\left(\sqrt{\frac{b}{2c}}, \sqrt{\frac{b}{c}}\right)$.

c. If $|\eta| = 1$, then $b = 0$ (q undefined) or $b = 2cp^2$, $p = \sqrt{\frac{b}{2c}}$.

24. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\cdot\frac{dq}{dp}$

We differentiate implicitly for $\frac{dq}{dp}$.

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$$q^2(1+p)^2 = p$$

$$q^2 \cdot 2(1+p)(1) + \left(1+p^2\right)\left(2q\frac{dq}{dp}\right) = 1$$

$$2q^2(1+p) + 2q(1+p)^2\frac{dq}{dp} = 1$$

Thus $\frac{dq}{dp} = \frac{1-2q^2(1+p)}{2q(1+p)^2}$

Hence $\eta = \frac{q^2(1+p)^2}{q} \cdot \frac{1-2q^2(1+p)}{2q(1+p)^2} = \frac{1-2q^2(1+p)}{2}$

If $p = 9$, we find q from the given equation:

$$q^2(1+9)^2 = 9$$

$$q^2 = \frac{9}{100}$$

$q = \frac{3}{10}$ since $q > 0$. Thus $\eta\big|_{p=9} = \frac{1-2\left(\frac{3}{10}\right)^2(1+9)}{2} = -0.4$

25. a. $q = \frac{60}{p} + \ln\left(65 - p^3\right)$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\frac{dq}{dp} = \frac{p}{q}\left[-\frac{60}{p^2} - \frac{3p^2}{65-p^3}\right]$$

If $p = 4$, then $q = \frac{60}{4} + \ln 1 = 15$, so $\eta = \frac{4}{15}\left[-\frac{60}{16} - \frac{3(16)}{65-64}\right] = -\frac{207}{15} \approx -13.8$, and demand is elastic.

b. The percentage change in q is (–2)(–13.8) = 27.6%, so q increases by approximately 27.6%.

c. Lowering the price increases revenue because demand is elastic.

26. a. $p = 50\left[(151-q)^{0.02\sqrt{q+19}}\right]$

$$\ln p = \ln 50 + 0.02\sqrt{q+19}\ln(151-q)$$

$$\frac{1}{p}\frac{dp}{dq} = 0 + 0.02\left[\frac{\sqrt{q+19}}{151-q}(-1) + \ln(151-q) \cdot \frac{1}{2\sqrt{q+19}}\right]$$

When $q = 150$, then $p = 50$, so $\frac{dp}{dq}\bigg|_{q=150} = 0.02(50)\left[-\frac{13}{1} + \frac{0}{26}\right] = -13$

b. $\eta\big|_{q=150} = \frac{\frac{p}{q}}{\frac{dp}{dq}}\Bigg|_{q=150} = \frac{\frac{50}{150}}{-13} \approx -0.0256$

Thus demand is inelastic.

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c. (elasticity)(% change in price) = % change in demand

$(-0.0256)(\text{\% change in price}) = \frac{-10}{150} \cdot 100$

$\text{\% change in price} = -\frac{100}{15}\left(\frac{1}{-0.0256}\right) = 260\%$

Thus price per unit of \$50 changes by 2.6(50) = \$130, so it is approximately 50 + 130 = \$180.

d. The manufacturer should increase the price because demand is inelastic.

27. The percentage change in price is $\frac{-5}{80} \cdot 100 = -\frac{25}{4}\%$ and the percentage change in quantity is $\frac{50}{500} \cdot 100 = 10\%$.

Thus, since (elasticity)(% change in price) $\approx$ % change in demand,

$(\text{elasticity})\left(-\frac{25}{4}\right) \approx 10.$

$\text{elasticity } \approx -\frac{40}{25} = -\frac{8}{5} = -1.6$

To estimate $\frac{dr}{dq}$ when $p = 80$, we have

$\frac{dr}{dq} = p\left(1 + \frac{1}{\eta}\right) = 80\left(1 + \frac{1}{-\frac{8}{5}}\right) = 30.$

28. $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{2000 - q^2}{-2q^2} = \frac{1}{2} - \frac{1000}{q^2}$

For $5 \le q \le 40$, $|\eta| = \frac{1000}{q^2} - \frac{1}{2}$ and $|\eta|' = -\frac{2000}{q^3}$. Since $|\eta|' < 0$, $|\eta|$ is decreasing on [5, 40] and thus $|\eta|$ is maximum at $q = 5$ and a minimum at $q = 40$.

29. $\frac{dp}{dq} = 200(-1)(q+5)^{-2} = \frac{-200}{(q+5)^2}$

Thus $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{200}{q(q+5)}}{-\frac{200}{(q+5)^2}} = -\frac{q+5}{q}.$

For $5 \le q \le 95$, $|\eta| = \frac{q+5}{q} = 1 + \frac{5}{q}$ and $|\eta|' = -\frac{5}{q^2}$.

Since $|\eta|' < 0$, $|\eta|$ is decreasing on [5, 95], and thus $|\eta|$ is maximum at $q = 5$ and minimum at $q = 95$.

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Apply It 12.4

4. Assume that P is a function of t and differentiate both sides of $\ln\left(\frac{P}{1-P}\right) = 0.5t$ with respect to t.

$$\frac{d}{dt}\left[\ln\left(\frac{P}{1-P}\right)\right] = \frac{d}{dt}[0.5t]$$

$$\left(\frac{1}{\frac{P}{1-P}}\right)\frac{d}{dt}\left[\frac{P}{1-P}\right] = 0.5$$

$$\frac{1-P}{P}\cdot\frac{(1)(1-P)-P(-1)}{(1-P)^2}\cdot\frac{dP}{dt} = 0.5$$

$$\frac{1-P+P}{P(1-P)}\cdot\frac{dP}{dt} = 0.5$$

$$\frac{dP}{dt} = 0.5P(1-P)$$

5. $\frac{dV}{dt} = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right] = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$

When $\frac{dr}{dt} = 5$ and $r = 12$,

$\frac{dV}{dt} = 4\pi(12)^2(5) = 2880\pi$. The balloon is increasing at the rate of 2880π cubic inches/minute.

6. The hypotenuse is the length of the ladder, so $x^2 + y^2 = 100$. Differentiate both sides of the equation with respect to t.

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}[100]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

When $y = 8$, we can find x by using the Pythagorean theorem.

$$x^2 + 8^2 = 100$$

$$x^2 = 100 - 64 = 36$$

$$x = 6$$

When $x = 6$, $y = 8$, and $\frac{dx}{dt} = 3$, we have

$$2(6)(3) + 2(8)\frac{dy}{dt} = 0$$

$$36 + 16\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{36}{16} = -\frac{9}{4}$$

$\frac{dy}{dt} = -\frac{9}{4}$, thus the top of the ladder is sliding down the wall at the rate of $\frac{9}{4}$ feet/sec.

Problems 12.4

1. $2x + 8yy' = 0$

$x + 4yy' = 0$

$4yy' = -x$

$y' = -\frac{x}{4y}$

2. $6x + 12yy' = 0$

$y' = -\frac{x}{2y}$

3. $6y^2y' - 14x = 0$

$y' = \frac{14x}{6y^2} = \frac{7x}{3y^2}$

4. $10yy' - 4x = 0$

$y' = \frac{4x}{10y} = \frac{2x}{5y}$

5.
$$x^{1/3} + y^{1/3} = 3$$

$$\frac{1}{3}x^{-2/3} + \frac{1}{3}y^{-2/3}y' = 0$$

$$y^{-2/3}y' = -x^{-2/3}$$

$$y' = -\frac{x^{-2/3}}{y^{-2/3}} = -\frac{y^{2/3}}{x^{2/3}} = -\frac{\sqrt[3]{y^2}}{\sqrt[3]{x^2}} = -\sqrt[3]{\frac{y^2}{x^2}}$$

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6. $\left(\frac{1}{5}\right)x^{-\frac{4}{5}} + \left(\frac{1}{5}\right)y^{-\frac{4}{5}}y' = 0$

$$y' = -\frac{y^{\frac{4}{5}}}{x^{\frac{4}{5}}} = -\left(\frac{y}{x}\right)^{\frac{4}{5}}$$

7. $\left(\frac{3}{4}\right)x^{-\frac{1}{4}} + \left(\frac{3}{4}\right)y^{-\frac{1}{4}}y' = 0$

$$y' = -\frac{y^{\frac{1}{4}}}{x^{\frac{1}{4}}}$$

8. $3y^2y' = 4$

$$y' = \frac{4}{3y^2}$$

9. $xy' + y(1) = 0$

$$xy' = -y$$

$$y' = -\frac{y}{x}$$

10. $2x + xy' + y(1) - 4yy' = 0$

$$xy' - 4yy' = -2x - y$$

$$y' = \frac{-2x - y}{x - 4y} = \frac{2x + y}{-x + 4y}$$

11. $xy' + y(1) - y' - 11 = 0$

$$y'(x-1) = 11 - y$$

$$y' = \frac{11 - y}{x - 1}$$

12. $3x^2 - 3y^2y' = 3x^2y' + 6xy - 3x(2yy') - 3y^2$

$$y'(-3y^2 - 3x^2 + 6xy) = 6xy - 3y^2 - 3x^2$$

$$y' = 1$$

13. $6x^2 + 3y^2y' - 12\left(xy' + y\right) = 0$

$$3y^2y' - 12xy' = 12y - 6x^2$$

$$y'\left(3y^2 - 12x\right) = 12y - 6x^2$$

$$y'\left(y^2 - 4x\right) = 4y - 2x^2$$

$$y' = \frac{4y - 2x^2}{y^2 - 4x}$$

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14. $15x^2+6y+6xy'+21y^2y'=0$

$$y'(6x+21y^2)=-15x^2-6y$$

$$y'=\frac{-15x^2-6y}{6x+21y^2}=\frac{-5x^2-2y}{2x+7y^2}$$

15. $x=\sqrt{y}+\sqrt[4]{y}=y^{1/2}+y^{1/4}$

$$1=\frac{1}{2}y^{-1/2}y'+\frac{1}{4}y^{-3/4}y'$$

$$=y'\left(\frac{1}{2y^{1/2}}+\frac{1}{4y^{3/4}}\right)=y'\left(\frac{2y^{1/4}+1}{4y^{3/4}}\right)$$

$$y'=\frac{4y^{3/4}}{2y^{1/4}+1}$$

16. $x^3\left(3y^2y'\right)+y^3\left(3x^2\right)+1=0$

$$y'=-\frac{1+3x^2y^3}{3x^3y^2}$$

17. $5x^3(4y^3y')+15x^2y^4-1+2yy'=0$

$$y'(20x^3y^3+2y)=1-15x^2y^4$$

$$y'=\frac{1-15x^2y^4}{20x^3y^3+2y}$$

18. $2yy'+y'=\frac{1}{x}$

$$(2y+1)y'=\frac{1}{x}$$

$$y'=\frac{1}{x(2y+1)}$$

19. $\ln x+\ln y=e^{xy}$

$$\frac{1}{x}+\frac{1}{y}y'=e^{xy}(y+xy')$$

$$\frac{1}{y}y'-e^{xy}(xy')=ye^{xy}-\frac{1}{x}$$

$$y'\left(\frac{1}{y}-xe^{xy}\right)=ye^{xy}-\frac{1}{x}$$

$$y'=\frac{ye^{xy}-\frac{1}{x}}{\frac{1}{y}-xe^{xy}}$$

20. $\frac{xy'+y(1)}{xy}+1=0$

$$xy'+y+xy=0$$

$$xy'=-y(x+1)$$

$$y'=-\frac{y(x+1)}{x}$$

21. $\left[x\left(e^yy'\right)+e^y(1)\right]+y'=0$

$$xe^yy'+e^y+y'=0$$

$$\left(xe^y+1\right)y'=-e^y$$

$$y'=-\frac{e^y}{xe^y+1}$$

22. $8x+18yy'=0$

$$8x=-18yy'$$

$$y'=-\frac{8x}{18y}=-\frac{4x}{9y}$$

23. $2\left(1+e^{3x}\right)\left(3e^{3x}\right)=\frac{1}{x+y}(1+y')$

$$6e^{3x}\left(1+e^{3x}\right)(x+y)=1+y'$$

$$y'=6e^{3x}\left(1+e^{3x}\right)(x+y)-1$$

24. $e^{x-y}(1-y')=\frac{1}{x-y}(1-y')$, **so** $1-y'=0$ or $y'=1$.

25. $1+[xy'+y(1)]+2yy'=0$

$$xy'+2yy'=-1-y$$

$$(x+2y)y'=-(1+y)$$

$$y'=-\frac{1+y}{x+2y}$$

At the point (1, 2), $y'=-\frac{1+2}{1+4}=-\frac{3}{5}$.

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26. $x\left(\frac{1}{2\sqrt{y+1}}\cdot y'\right)+\sqrt{y+1}(1) = y\left(\frac{1}{2\sqrt{x+1}}\right)+\sqrt{x+1}(y')$

$\frac{x}{2\sqrt{y+1}}\cdot y'-\sqrt{x+1}\cdot y'=\frac{y}{2\sqrt{x+1}}-\sqrt{y+1}$

$\left(\frac{x}{2\sqrt{y+1}}-\sqrt{x+1}\right)y'=\frac{y}{2\sqrt{x+1}}-\sqrt{y+1}$

$y'=\frac{\frac{y}{2\sqrt{x+1}}-\sqrt{y+1}}{\frac{x}{2\sqrt{y+1}}-\sqrt{x+1}}$

At (3, 3), $\frac{dy}{dx}=\frac{\frac{3}{4}-2}{\frac{3}{4}-2}=1.$

27. $8x+18yy'=0$

$y'=-\frac{8x}{18y}=-\frac{4x}{9y}$

Thus at $\left(0, \frac{1}{3}\right)$, $y'=0$; at (x_0, y_0), $y'=-\frac{4x_0}{9y_0}$.

28. $2(x^2+y^2)(2x+2yy')=8yy'$

$(x^2+y^2)(x+yy')=2yy'$

$x^3+x^2yy'+xy^2+y^3y'=2yy'$

$(x^2y+y^3-2y)y'=-x^3-xy^2$

$y'=\frac{-x(x^2+y^2)}{y(x^2+y^2-2)}$

At (0, 2), $y'=0$.

29. $3x^2+y+xy'+3y^2y'=0$

$y'(x+3y^2)=-3x^2-y$

$y'=-\frac{3x^2+y}{x+3y^2}$

$(-1, -1)$: $y'=-\frac{2}{2}=-1$

$(-1, 0)$: $y'=-\frac{3}{-1}=3$

$(-1, 1)$: $y'=-\frac{4}{2}=-2$

$(-1, -1)$: $y+1=-1(x+1)$

$y=-x-2$

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$(-1, 0)$: $y+0=3(x+1)$
$y=3x+3$
$(-1, 1)$: $y-1=-2(x+1)$
$y=-2x-1$

30. $2yy'+[xy'+y(1)]-2x=0$

$$y'=\frac{2x-y}{2y+x}$$

At (4, 3), $y'=\frac{1}{2}$ and the tangent line is given by

$y-3=\frac{1}{2}(x-4)$, or $y=\frac{1}{2}x+1$.

31. $p=100-q^2$

$$\frac{d}{dp}(p)=\frac{d}{dp}\left(100-q^2\right)$$

$$1=-2q\cdot\frac{dq}{dp}$$

$$\frac{dq}{dp}=-\frac{1}{2q}$$

32. $p=400-\sqrt{q}$

$$\frac{d}{dp}(p)=\frac{d}{dp}\left(400-\sqrt{q}\right)$$

$$1=-\frac{1}{2\sqrt{q}}\cdot\frac{dq}{dp}$$

$$\frac{dq}{dp}=-2\sqrt{q}$$

33. $p=\frac{20}{(q+5)^2}$

$$\frac{d}{dp}(p)=\frac{d}{dp}\left[\frac{20}{(q+5)^2}\right]$$

$$\frac{d}{dp}(p)=\frac{d}{dp}\left[20(q+5)^{-2}\right]$$

$$1=-\frac{40}{(q+5)^3}\cdot\frac{dq}{dp}$$

$$\frac{dq}{dp}=-\frac{(q+5)^3}{40}$$

34. $p=\frac{3}{q^2+1}$

$$\frac{d}{dp}(p)=\frac{d}{dp}\left[\frac{3}{q^2+1}\right]$$

$$1=-\frac{6q}{(q^2+1)^2}\cdot\frac{dq}{dp}$$

$$\frac{dq}{dp}=-\frac{(q^2+1)^2}{6q}$$

From the original equation, we have $q^2+1=\frac{3}{p}$.

Thus we can write $\frac{dq}{dp}$ as

$$\frac{dq}{dp}=-\frac{\left(\frac{3}{p}\right)^2}{6q}=-\frac{\frac{9}{p^2}}{6q}=-\frac{3}{2qp^2}.$$

35. $\ln\frac{I}{I_0}=-\lambda t$

$$\ln I-\ln I_0=-\lambda t$$

$$\frac{1}{I}\frac{dI}{dt}=-\lambda$$

$$\frac{dI}{dt}=-\lambda I$$

36. $1.5M=\log\left(\frac{E}{2.5\times10^{11}}\right)$

$$1.5M=\log E-\log\left(2.5\times10^{11}\right)$$

$$\frac{d}{dM}(1.5M)=\frac{d}{dM}\left[\log E-\log\left(2.5\times10^{11}\right)\right]$$

$$\frac{d}{dM}(1.5M)=\frac{d}{dM}\left[\frac{\ln E}{\ln 10}-\log\left(2.5\times10^{11}\right)\right]$$

$$1.5=\frac{1}{\ln 10}\left(\frac{1}{E}\cdot\frac{dE}{dM}\right)$$

$$\frac{dE}{dM}=1.5E\ln 10$$

$$\frac{d}{dE}(1.5M)=\frac{d}{dE}\left[\frac{\ln E}{\ln 10}-\log(2.5\times10^{11})\right]$$

$$1.5\frac{dM}{dE}=\frac{1}{\ln 10}\cdot\frac{1}{E}$$

$$\frac{dM}{dE}=\frac{1}{1.5E\ln 10}$$

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37. $v = f\lambda$. Differentiating implicitly with respect to λ: $0 = f(1) + \lambda\frac{df}{d\lambda}$, $\frac{df}{d\lambda} = -\frac{f}{\lambda}$.

Solving $v = f\lambda$ for f and differentiating: $f = \frac{v}{\lambda}$, so $\frac{df}{d\lambda} = -\frac{v}{\lambda^2} = -\frac{f\lambda}{\lambda^2} = -\frac{f}{\lambda}$, which is the same as before.

38. $(P + a)(v + b) = k$

$$\frac{d}{dP}[(P+a)(v+b)] = \frac{d}{dP}(k)$$

$$(P+a)\frac{dv}{dP} + (v+b)(1) = 0$$

$$\frac{dv}{dP} = -\frac{v+b}{P+a}.$$

From the original equation, $v+b = \frac{k}{(P+a)}$. Thus we can write $\frac{dv}{dP}$ as $\frac{dv}{dP} = -\frac{k}{(P+a)^2}$.

39. $S^2 + \frac{1}{4}I^2 = SI + I$. Differentiating implicitly with respect to I:

$2S\frac{dS}{dI} + \frac{1}{2}I = \left[S(1) + I\frac{dS}{dI}\right] + 1$, $2S\frac{dS}{dI} - I\frac{dS}{dI} = S + 1 - \frac{I}{2}$, $(2S - I)\frac{dS}{dI} = \frac{2S+2-I}{2}$, $\frac{dS}{dI} = \frac{2S+2-I}{2(2S-I)}$. Marginal propensity to consume $= \frac{dC}{dI} = 1 - \frac{dS}{dI}$. Thus $\frac{dC}{dI} = 1 - \frac{2S+2-I}{2(2S-I)}$. When $I = 16$ and $S = 12$, $\frac{dC}{dI} = 1 - \frac{24+2-16}{2(24-16)} = 1 - \frac{10}{16} = \frac{6}{16} = \frac{3}{8}$.

40. $\ln\frac{f(t)}{1-f(t)} + \sigma\frac{1}{1-f(t)} = C_1 + C_2t$. Thus

$$\ln f(t) - \ln[1-f(t)] + \sigma[1-f(t)]^{-1} = C_1 + C_2t,$$

$$\frac{f'(t)}{f(t)} + \frac{f'(t)}{1-f(t)} + \frac{\sigma f'(t)}{[1-f(t)]^2} = C_2$$

$$f'(t)\left[\frac{1}{f(t)} + \frac{1}{1-f(t)} + \frac{\sigma}{[1-f(t)]^2}\right] = C_2$$

$$f'(t)\left[\frac{[1-f(t)]^2 + f(t)[1-f(t)] + \sigma f(t)}{f(t)[1-f(t)]^2}\right] = C_2$$

$$f'(t)\left[\frac{[1-f(t)][1-f(t)+f(t)] + \sigma f(t)}{f(t)[1-f(t)]^2}\right] = C_2$$

$$f'(t)\left[\frac{[1-f(t)] + \sigma f(t)}{f(t)[1-f(t)]^2}\right] = C_2$$

Thus $f'(t) = \frac{C_2 f(t)[1-f(t)]^2}{\sigma f(t) + [1-f(t)]}$

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Problems 12.5

1. $y = (x+1)^2(x-2)\left(x^2+3\right)$. Take natural logarithms of both sides,

$\ln y = \ln\left[(x+1)^2(x-2)\left(x^2+3\right)\right]$.

Using properties of logarithms on the right side gives

$\ln y = 2\ln(x+1) + \ln(x-2) + \ln\left(x^2+3\right)$.

Differentiating both sides with respect to *x*,

$\frac{y'}{y} = \frac{2}{x+1} + \frac{1}{x-2} + \frac{2x}{x^2+3}$.

Solving for y',

$y' = y\left[\frac{2}{x+1} + \frac{1}{x-2} + \frac{2x}{x^2+3}\right]$.

Expressing y' in terms of *x*,

$y' = (x+1)^2(x-2)\left(x^2+3\right)\left[\frac{2}{x+1} + \frac{1}{x-2} + \frac{2x}{x^2+3}\right]$

2. $\ln y = \ln\left[(3x+4)(8x-1)^2\left(3x^2+1\right)^4\right]$

$= \ln(3x+4) + 2\ln(8x-1) + 4\ln\left(3x^2+1\right)$

$\frac{y'}{y} = \frac{3}{3x+4} + 2\cdot\frac{8}{8x-1} + 4\cdot\frac{6x}{3x^2+1}$

$y' = y\left[\frac{3}{3x+4} + \frac{16}{8x-1} + \frac{24x}{3x^2+1}\right]$

$= (3x+4)(8x-1)^2\left(3x^2+1\right)^4\left[\frac{3}{3x+4} + \frac{16}{8x-1} + \frac{24x}{3x^2+1}\right]$

3. $\ln y = \ln\left[\left(3x^3-1\right)^2(2x+5)^3\right]$

$= 2\ln\left(3x^3-1\right) + 3\ln(2x+5)$

$\frac{y'}{y} = 2\cdot\frac{9x^2}{3x^3-1} + 3\cdot\frac{2}{2x+5}$

$y' = y\left[\frac{18x^2}{3x^3-1} + \frac{6}{2x+5}\right]$

$y' = \left(3x^3-1\right)^2(2x+5)^3\left[\frac{18x^2}{3x^3-1} + \frac{6}{2x+5}\right]$

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4. $y=(2x^2+1)\sqrt{8x^2-1}$

$$\ln y=\ln\left[(2x^2+1)\sqrt{8x^2-1}\right]$$
$$=\ln(2x^2+1)+\frac{1}{2}\ln(8x^2-1)$$
$$\frac{y'}{y}=\frac{4x}{2x^2+1}+\frac{1}{2}\cdot\frac{16x}{8x^2-1}$$
$$y'=y\left[\frac{4x}{2x^2+1}+\frac{8x}{8x^2-1}\right]$$
$$=(2x^2+1)\sqrt{8x^2-1}\left[\frac{4x}{2x^2+1}+\frac{8x}{8x^2-1}\right]$$

5. $y=\sqrt{x+1}\sqrt{x-1}\sqrt{x^2+1}$

$$\ln y=\ln\left(\sqrt{x+1}\sqrt{x-1}\sqrt{x^2+1}\right)$$
$$=\frac{1}{2}\ln(x+1)+\frac{1}{2}\ln(x-1)+\frac{1}{2}\ln(x^2+1)$$
$$\frac{y'}{y}=\frac{1}{2}\left[\frac{1}{x+1}+\frac{1}{x-1}+\frac{2x}{x^2+1}\right]$$
$$y'=\frac{y}{2}\left(\frac{1}{x+1}+\frac{1}{x-1}+\frac{2x}{x^2+1}\right)$$
$$y'=\frac{\sqrt{x+1}\sqrt{x-1}\sqrt{x^2+1}}{2}\left(\frac{1}{x+1}-\frac{1}{x-1}+\frac{2x}{x^2+1}\right)$$

6. $\ln y=\ln\left[(2x+1)\sqrt{x^3+2}\sqrt[3]{2x+5}\right]$

$$=\ln(2x+1)+\frac{1}{2}\ln(x^3+2)+\frac{1}{3}\ln(2x+5)$$
$$\frac{y'}{y}=\frac{2}{2x+1}+\frac{1}{2}\cdot\frac{3x^2}{x^3+2}+\frac{1}{3}\cdot\frac{2}{2x+5}$$
$$y'=y\left[\frac{2}{2x+1}+\frac{3x^2}{2(x^3+2)}+\frac{2}{3(2x+5)}\right]$$
$$=(2x+1)\sqrt{x^3+2}\sqrt[3]{2x+5}\left[\frac{2}{2x+1}+\frac{3x^2}{2(x^3+2)}+\frac{2}{3(2x+5)}\right]$$

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7. $\ln y = \ln \frac{\sqrt{1-x^2}}{1-2x} = \frac{1}{2}\ln\left(1-x^2\right) - \ln(1-2x)$

$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{-2x}{1-x^2} - \frac{-2}{1-2x}$$

$$y' = y\left[-\frac{x}{1-x^2} + \frac{2}{1-2x}\right]$$

$$y' = \frac{\sqrt{1-x^2}}{1-2x}\left[\frac{x}{x^2-1} + \frac{2}{1-2x}\right]$$

8. $\ln y = \ln\sqrt{\frac{x^2+5}{x+9}} = \frac{1}{2}\left[\ln\left(x^2+5\right) - \ln(x+9)\right]$

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{2x}{x^2+5} - \frac{1}{x+9}\right]$$

$$y' = \frac{y}{2}\left[\frac{2x}{x^2+5} - \frac{1}{x+9}\right]$$

$$y' = \frac{1}{2}\sqrt{\frac{x^2+5}{x+9}}\left[\frac{2x}{x^2+5} - \frac{1}{x+9}\right]$$

9. $y = \frac{\left(2x^2+2\right)^2}{(x+1)^2(3x+2)}$

$$\ln y = \ln\left[\frac{\left(2x^2+2\right)^2}{(x+1)^2(3x+2)}\right]$$

$$= 2\ln\left(2x^2+2\right) - 2\ln(x+1) - \ln(3x+2)$$

$$\frac{y'}{y} = 2\cdot\frac{4x}{2x^2+2} - 2\cdot\frac{1}{x+1} - \frac{3}{3x+2}$$

$$y' = y\left[\frac{8x}{2x^2+2} - \frac{2}{x+1} - \frac{3}{3x+2}\right]$$

$$= \frac{\left(2x^2+2\right)^2}{(x+1)^2(3x+2)}\left[\frac{4x}{x^2+1} - \frac{2}{x+1} - \frac{3}{3x+2}\right]$$

10. $\ln y = \ln\frac{x^2(1+x^2)}{\sqrt{x^2+4}}$

$$= 2\ln x + \ln(1+x^2) + \frac{1}{2}\ln(x^2+4)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{2x}{1+x^2} + \frac{2x}{2(x^2+4)}$$

$$y' = y\left[\frac{2}{x} + \frac{2x}{1+x^2} + \frac{x}{x^2+4}\right]$$

$$y' = \frac{x^2(1+x^2)}{\sqrt{x^2+4}}\left(\frac{2}{x} + \frac{2x}{1+x^2} + \frac{x}{x^2+4}\right)$$

11. $y = \sqrt{\frac{(x+3)(x-2)}{2x-1}}$

$$\ln y = \ln\sqrt{\frac{(x+3)(x-2)}{2x-1}}$$

$$= \frac{1}{2}\ln(x+3) + \frac{1}{2}\ln(x-2) - \frac{1}{2}\ln(2x-1)$$

$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{1}{x+3} + \frac{1}{2}\cdot\frac{1}{x-2} - \frac{1}{2}\cdot\frac{2}{2x-1}$$

$$y' = \frac{y}{2}\left[\frac{1}{x+3} + \frac{1}{x-2} - \frac{2}{2x-1}\right]$$

$$= \frac{1}{2}\sqrt{\frac{(x+3)(x-2)}{2x-1}}\left[\frac{1}{x+3} + \frac{1}{x-2} - \frac{2}{2x-1}\right]$$

12. $\ln y = \ln\sqrt[3]{\frac{6\left(x^3+1\right)^2}{x^6e^{-4x}}}$

$$= \frac{1}{3}\left[\ln(6) + 2\ln\left(x^3+1\right) - 6\ln(x) - (-4x)\ln e\right]$$

$$= \frac{1}{3}\left[\ln(6) + 2\ln\left(x^3+1\right) - 6\ln(x) + 4x\right]$$

$$\frac{y'}{y} = \frac{1}{3}\left[2\cdot\frac{3x^2}{x^3+1} - \frac{6}{x} + 4\right]$$

$$y' = \frac{y}{3}\left[\frac{6x^2}{x^3+1} - \frac{6}{x} + 4\right]$$

$$y = \frac{1}{3}\sqrt[3]{\frac{6\left(x^3+1\right)^2}{x^6e^{-4x}}}\left[\frac{6x^2}{x^3+1} - \frac{6}{x} + 4\right]$$

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13. $y = x^{x^2+1}$, thus $\ln y = \ln x^{x^2+1} = (x^2+1)\ln x$.

$$\frac{y'}{y} = (x^2+1)\cdot\frac{1}{x} + (\ln x)(2x)$$

$$y' = y\left(\frac{x^2+1}{x} + 2x\ln x\right)$$

$$= x^{x^2+1}\left(\frac{x^2+1}{x} + 2x\ln x\right)$$

14. $y = (2x)^{\sqrt{x}}$. Thus

$$\ln y = \ln(2x)^{\sqrt{x}} = \sqrt{x}[\ln 2 + \ln x].$$

$$\frac{y'}{y} = \sqrt{x}\left[\frac{1}{x}\right] + [\ln 2 + \ln x]\cdot\frac{1}{2\sqrt{x}}$$

$$y' = y\left[\frac{1}{\sqrt{x}} + \frac{\ln(2x)}{2\sqrt{x}}\right]$$

$$y' = (2x)^{\sqrt{x}}\left[\frac{2+\ln(2x)}{2\sqrt{x}}\right]$$

15. $y = x^{\sqrt{x}}$. Thus $\ln y = \sqrt{x}\ln x$.

$$\frac{y'}{y} = \frac{1}{2}x^{-1/2}\ln x + \frac{\sqrt{x}}{x}$$

$$y' = y\left[\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right]$$

$$y' = \frac{x^{\sqrt{x}}(\ln x + 2)}{2\sqrt{x}}$$

16. $y = \left(\frac{3}{x^2}\right)^x$. Thus

$$\ln y = x\ln\left(\frac{3}{x^2}\right) = x[\ln 3 - 2\ln x].$$

$$\frac{y'}{y} = x\left(-\frac{2}{x}\right) + (\ln 3 - 2\ln x)(1)$$

$$= -2 + \ln\left(\frac{3}{x^2}\right)$$

$$y' = y\left[-2 + \ln\left(\frac{3}{x^2}\right)\right] = \left(\frac{3}{x^2}\right)^x\left[-2 + \ln\left(\frac{3}{x^2}\right)\right]$$

17. $y = (3x+1)^{2x}$. Thus

$$\ln y = \ln\left[(3x+1)^{2x}\right] = 2x\ln(3x+1)$$

$$\frac{y'}{y} = 2\left\{x\left(\frac{3}{3x+1}\right) + [\ln(3x+1)](1)\right\}$$

$$y' = 2y\left[\frac{3x}{3x+1} + \ln(3x+1)\right]$$

$$= 2(3x+1)^{2x}\left[\frac{3x}{3x+1} + \ln(3x+1)\right]$$

18. $y = (x^2+1)^{x+1}$, thus

$$\ln y = \ln(x^2+1)^{x+1} = (x+1)\ln(x^2+1).$$

$$\frac{y'}{y} = x+1\cdot\frac{2x}{x^2+1} + \ln(x^2+1)\cdot 1$$

$$y' = y\left[\frac{2x(x+1)}{x^2+1} + \ln(x^2+1)\right]$$

$$= (x^2+1)^{x+1}\left[\frac{2x(x+1)}{x^2+1} + \ln(x^2+1)\right]$$

19. $y = 4e^x x^{3x}$. Thus

$$\ln y = \ln 4 + \ln\left(e^x x^{3x}\right) = \ln 4 + \ln e^x + \ln x^{3x}$$

$$= \ln 4 + x + 3x\ln x.$$

$$\frac{y'}{y} = 1 + 3\left[x\left(\frac{1}{x}\right) + (\ln x)(1)\right]$$

$$y' = y(4 + 3\ln x)$$

$$y' = 4e^x x^{3x}(4 + 3\ln x)$$

20. $y = \sqrt{x}^x$. Thus $\ln y = x\ln\sqrt{x} = \frac{x}{2}\ln x$.

$$\frac{y'}{y} = \frac{1}{2}\ln x + \frac{x}{2}\left(\frac{1}{x}\right)$$

$$y' = \frac{\sqrt{x}^x}{2}(\ln x + 1)$$

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21. $y = (4x-3)^{2x+1}$

$\ln y = \ln(4x-3)^{2x+1} = (2x+1)\ln(4x-3)$

$\frac{y'}{y} = (2x+1)\left[\frac{4}{4x-3}\right] + [\ln(4x-3)](2)$

$y' = y\left[\frac{4(2x+1)}{4x-3} + 2\ln(4x-3)\right]$

When $x = 1$, then $\frac{dy}{dx} = 1\left[\frac{12}{1} + 2\ln(1)\right] = 12.$

22. $y = (\ln x)^{\ln x}$

$\ln y = \ln(\ln x)^{\ln x} = (\ln x)\ln(\ln x)$

$\frac{y'}{y} = (\ln x)\left[\frac{1}{\ln x}\cdot\frac{1}{x}\right] + [\ln(\ln x)]\left(\frac{1}{x}\right)$

$y' = y\left[\frac{1}{x} + \frac{\ln(\ln x)}{x}\right]$

$y' = (\ln x)^{\ln x}\left[\frac{1+\ln(\ln x)}{x}\right]$

When $x = e$, $\frac{dy}{dx} = 1^1\left[\frac{1+\ln(1)}{e}\right] = e^{-1}.$

23. $y = (x+1)(x+2)^2(x+3)^2$

$\ln y = \ln(x + 1) + 2\ln(x + 2) + 2\ln(x + 3)$

$\frac{y'}{y} = \frac{1}{x+1} + \frac{2}{x+2} + \frac{2}{x+3}$

$y' = y\left[\frac{1}{x+1} + \frac{2}{x+2} + \frac{2}{x+3}\right]$

When $x = 0$, then $y = 36$ and $y' = 96$. Thus an equation of the tangent line is
$y - 36 = 96(x - 0)$, or $y = 96x + 36$.

24. $y = x^x$

$\ln y = x\ln x$

$\frac{y'}{y} = x\cdot\frac{1}{x} + (\ln x)(1) = 1 + \ln x$

$y' = y(1+\ln x) = x^x(1+\ln x)$

When $x = 1$, then $y = 1$ and
$y' = 1^1(1+\ln 1) = 1(1+0) = 1$. An equation of the tangent line is $y - 1 = 1(x - 1)$ or $y = x$.

25. $\ln y = x\ln x$

$\frac{y'}{y} = \ln x + x\left(\frac{1}{x}\right) = \ln x + 1$

$y' = x^x(\ln x + 1)$

Let $x = e$.

$y' = e^e(\ln e + 1) = 2e^e$ and $y = e^e$.

Thus an equation of the tangent line is
$y - e^e = 2e^e(x-e)$, or $y = 2e^e x - 2e^{e+1} + e^e$.

26. $y = x^x$

$\ln y = x\ln x$

$\frac{y'}{y} = x\cdot\frac{1}{x} + (\ln x)(1) = 1 + \ln x$

When $x = 1$, $\frac{y'}{y} = 1 + \ln 1 = 1 + 0 = 1.$

27. $y = (3x)^{-2x}$

$\ln y = -2x\ln(3x)$

$\frac{y'}{y} = -2\left\{x\left[\frac{1}{3x}(3)\right] + [\ln(3x)](1)\right\}$

$= -2[1 + \ln(3x)]$

$\frac{y'}{y}\cdot 100$ gives the percentage rate of change.

Thus $-2[1 + \ln(3x)](100) = 60$

$1 + \ln(3x) = -0.3$

$\ln(3x) = -1.3$

$3x = e^{-1.3}$

$x = \frac{1}{3e^{1.3}}$

28. $y = [f(x)]^{g(x)}$

$\ln y = g(x)\ln[f(x)]$

$\frac{y'}{y} = g(x)\left(\frac{1}{f(x)}\cdot f'(x)\right) + \ln[f(x)]g'(x)$

$y' = y\left(f'(x)\frac{g(x)}{f(x)} + g'(x)\ln[f(x)]\right)$

$y' = [f(x)]^{g(x)}\left(f'(x)\frac{g(x)}{f(x)} + g'(x)\ln[f(x)]\right)$

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29. $\frac{r'}{r}\cdot 100\% = \frac{p'}{p}\cdot 100\% + \frac{q'}{q}\cdot 100\%$

$= (1+\eta)\frac{p'}{p}100\%$

where $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\cdot\frac{dq}{dp}.$

$\eta = \frac{p}{500-40p+p^2}\cdot(-40+2p)$

When p = 15, then $\eta = -1.2$ and a $\frac{1}{2}\%$ increase in price will result in a $(1-1.2)\left(\frac{1}{2}\%\right) = -0.1\%$ change in revenue, which is a 0.1% decrease in revenue.

30. $\frac{r'}{r}\cdot 100\% = \frac{p'}{p}\cdot 100\% + \frac{q'}{q}\cdot 100\%$

$= (1+\eta)\frac{p'}{p}100\%$

where $\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\cdot\frac{dq}{dp}.$

$\eta = \frac{p}{500-40p+p^2}\cdot(-40+2p)$

When p = 15, then $\eta = -1.2$ and a 5% decrease in price will result in a (1 – 1.2)(–5%) = 1% change in revenue, which is a 1% increase in revenue.

Apply It 12.6

7. Let $f(x) = 20x - 0.01x^2 - 850 + 3\ln x$, then $f'(x) = 20 - 0.02x + \frac{3}{x}$. $f(10) \approx -644$ and $f(50) \approx 137$, so we use 50 to be the first approximation, x_1, to find the break-even quantity between 10 and 50.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{20x_n - 0.01x_n^2 - 850 + 3\ln x_n}{20 - 0.02x_n + 3x_n^{-1}}$$

$$= x_n - \frac{20x_n^2 - 0.01x_n^3 - 850x_n + 3x_n \ln x_n}{20x_n - 0.02x_n^2 + 3}$$

$$= \frac{20x_n^2 - 0.02x_n^3 + 3x_n - \left(20x_n^2 - 0.01x_n^3 - 850x_n + 3x_n \ln x_n\right)}{20x_n - 0.02x_n^2 + 3}$$

$$= \frac{-0.01x_n^3 + 853x_n - 3x_n \ln x_n}{20x_n - 0.02x_n^2 + 3}$$

$$x_2 = 50 - \frac{f(50)}{f'(50)} \approx 42.82602$$

$$x_3 = 42.82602 - \frac{f(42.82602)}{f'(42.82602)} \approx 42.85459$$

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$$x_4 = 42.85459 - \frac{f(42.85459)}{f'(42.85459)} \approx 42.85459$$

Since the values of x_3 and x_4 differ by less than 0.0001, we take the first break-even quantity to be $x \approx 42.85459$ or 43 televisions.
$f(1900) \approx 1073$ and $f(2000) \approx -827$, so we use 2000 to be the first approximation, x_1, for the break-even quantity between 1900 and 2000.

$$x_2 = 2000 - \frac{f(2000)}{f'(2000)} \approx 1958.63703$$

$$x_3 = 1958.63703 - \frac{f(1958.63703)}{f'(1958.63703)} \approx 1957.74457$$

$$x_4 = 1957.74457 - \frac{f(1957.74457)}{f'(1957.74457)} \approx 1957.74415$$

$$x_5 = 1957.74415 - \frac{f(1957.74415)}{f'(1957.74415)} \approx 1957.74415$$

Since the values of x_4 and x_5 differ by less than 0.0001, we take the second break-even quantity to be $x \approx 1957.74415$ or 1958 televisions.

Problems 12.6

1. Let $f(x) = x^3 - 5x + 1$. $f(0) = 1$ and $f(1) = -3$ have opposite signs, so there must be a root between 0 and 1. Moreover, $f(0)$ is closer to 0 than is $f(1)$, so we select $x_1 = 0$ as our initial estimate. Since $f'(x) = 3x^2 - 5$, the recursion formula is

$$x_{n+1} = x_n - \frac{x_n^3 - 5x_n + 1}{3x_n^2 - 5}.$$

Simplifying gives $x_{n+1} = \dfrac{2x_n^3 - 1}{3x_n^2 - 5}$. Thus we obtain:

n	x_n	x_{n+1}
1	0.00000	0.20000
2	0.20000	0.20164
3	0.20164	0.20164

Because $|x_4 - x_3| < 0.0001$, the root is approximately $x_4 = 0.2016$.

2. Let $f(x) = x^3 + 2x^2 - 1$. $f\left(\frac{1}{2}\right) = -\frac{3}{8}$ and

$f(1) = 2$ (note the sign change). Since $f\left(\frac{1}{2}\right)$ is closer to 0 than is $f(1)$, we select $x_1 = \frac{1}{2}$. We have

$f'(x) = 3x^2 + 4x$, so the recursion formula is $x_{n+1} = x_n - \dfrac{x_n^3 + 2x_n^2 - 1}{3x_n^2 + 4x_n}$

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n	x_n	x_{n+1}
1	0.50000	0.63636
2	0.63636	0.61838
3	0.61838	0.61803
4	0.61803	0.61803

Because $|x_5 - x_4| < 0.0001,$ the root is approximately $x_5 = 0.61803.$
(Note that $f'(0) = 0,$ so we cannot use 0 for x_1.)

3. Let $f(x) = x^3 - x - 1.$ We have $f(1) = -1$ and $f(2) = 5$ (note the sign change). Since $f(1)$ is closer to 0 than is $f(2)$, we choose $x_1 = 1.$ We have $f'(x) = 3x^2 - 1,$ so the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$$
$$= \frac{2x_n^3 + 1}{3x_n^2 - 1}$$

n	x_n	x_{n+1}
1	1.00000	1.50000
2	1.50000	1.34783
3	1.34783	1.32520
4	1.32520	1.32472
5	1.32472	1.32472

Since $|x_6 - x_5| < 0.0001,$ the root is approximately $x_6 = 1.32472.$

4. Let $f(x) = x^3 - 9x + 6.$ We have $f(2.5) = -0.875$ and $f(3) = 6$. Since $f(2.5)$ is closer to 0 than is $f(3)$, we choose $x_1 = 2.5.$ We have

$f'(x) = 3x^2 - 9$, so $x_{n+1} = x_n - \dfrac{x_n^3 - 9x_n + 6}{3x_n^2 - 9}.$

n	x_n	x_{n+1}
1	2.50000	2.58974
2	2.58974	2.58425
3	2.58425	2.58423
4	2.58423	2.58423

Since $|x_5 - x_4| < 0.0001,$ the root is approximately $x_5 = 2.58423.$

5. Let $f(x) = x^3 + x + 1.$ We have $f(-1) = -1$ and $f(0) = 1$ (note the sign change). Choose $x_1 = -1.$ Since $f'(x) = 3x^2 + 1,$ the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 + x_n + 1}{3x_n^2 + 1}$$
$$= \frac{2x_n^3 - 1}{3x_n^2 + 1}$$

n	x_n	x_{n+1}
1	−1	−0.75000
2	−0.75000	−0.68605
3	−0.68605	−0.68234
4	−0.68234	−0.68233

Because $|x_5 - x_4| < 0.0001,$ the root is approximately $x_5 = -0.68233.$

6. Let $f(x) = x^3 - 2x - 6 = 0.$ We have $f(2) = -2$ and $f(3) = 15$. Choose $x_1 = 2.$ Since $f'(x) = 3x^2 - 2,$ the recursion formula is

$$x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 6}{3x_n^2 - 2} = \frac{2x_n^3 + 6}{3x_n^2 - 2}$$

n	x_n	x_{n+1}
1	2.00000	2.20000
2	2.20000	2.18019
3	2.18019	2.17998
4	2.17998	2.17998

Because $|x_5 - x_4| < 0.0001,$ the root is approximately $x_4 = 2.17998.$

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7. $x^4 = 3x - 1$, so use $f(x) = x^4 - 3x + 1 = 0$. Since $f(0) = 1$ and $f(1) = -1$ (note the sign change), $f(0)$ and $f(1)$ are equally close to 0. We shall choose $x_1 = 0$. Since $f'(x) = 4x^3 - 3$, the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 - 3x_n + 1}{4x_n^3 - 3}$$

$$= \frac{3x_n^4 - 1}{4x_n^3 - 3}$$

n	x_n	x_{n+1}
1	0.00000	0.33333
2	0.33333	0.33766
3	0.33766	0.33767

Because $|x_4 - x_3| < 0.0001$, the root is approximately $x_4 = 0.33767$.

8. Let $f(x) = x^4 + 4x - 1$. Since $f(-2) = 7$ and $f(-1) = -4$, $f(-1)$ is closer to 0 than is $f(-2)$. However, $f'(-1) = 0$, so we shall choose $x_1 = -2$. Since $f'(x) = 4x^3 + 4$, the recursion formula is

$$x_{n+1} = x_n - \frac{x_n^4 + 4x_n - 1}{4x_n^3 + 4} = \frac{3x_n^4 + 1}{4x_n^3 + 4}$$

n	x_n	x_{n+1}
1	–2.00000	–1.75000
2	–1.75000	–1.67092
3	–1.67092	–1.66332
4	–1.66332	–1.66325

Because $|x_5 - x_4| < 0.0001$, the root is approximately $x_5 = -1.66325$.

9. Let $f(x) = x^4 - 2x^3 + x^2 - 3$. $f(1) = -3$ and $f(2) = 1$ (note the sign change), so $f(2)$ is closer to 0 than is $f(1)$. We choose $x_1 = 2$. Since $f'(x) = 4x^3 - 6x^2 + 2x$, the recursion formula is

$$x_{n+1} = x_n - \frac{f(x)}{f'(x_n)} = x_n - \frac{x_n^4 - 2x_n^3 + x_n^2 - 3}{4x_n^3 - 6x_n^2 + 2x_n}$$

n	x_n	x_{n+1}
1	2.00000	1.91667
2	1.91667	1.90794
3	1.90794	1.90785

Because $|x_4 - x_3| < 0.0001$, the root is approximately $x_4 = 1.90785$.

10. Let $f(x) = x^4 - x^3 + x - 2$. $f(1) = -1$ and $f(2) = 8$, so $f(1)$ is closer to 0 than is $f(2)$. We choose $x_1 = 1$. Since $f'(x) = 4x^3 - 3x^2 + 1$, the recursion formula is

$$x_{n+1} = x_n - \frac{x_n^4 - x_n^3 + x_n - 2}{4x_n^3 - 3x_n^2 + 1}$$

n	x_n	x_{n+1}
1	1.00000	1.50000
2	1.50000	1.34677
3	1.34677	1.31040
4	1.31040	1.30858
5	1.30858	1.30857

Because $|x_6 - x_5| < 0.0001$, the root is approximately $x_6 = 1.30857$.

11. The desired number is x, where $x^3 = 73$, or $x^3 - 73 = 0$. Thus we want to find a root of $f(x) = x^3 - 73 = 0$. Since $4^3 = 64$, the solution should be close to 4, so we choose $x_1 = 4$ as our initial estimate. We have $f'(x) = 3x^2$, so the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 73}{3x_n^2} = \frac{2x_n^3 + 73}{3x_n^2}$$

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n	x_n	x_{n+1}
1	4	4.1875
2	4.1875	4.1794
3	4.1794	4.1793
4	4.1793	4.1793

Thus to three decimal places, $\sqrt[3]{73} = 4.179.$

12. The desired number is x, where $x^4 = 19,$ or $x^4 - 19 = 0.$ Thus we want to find a root of $f(x) = x^4 - 19.$ Since $2^4 = 16,$ the solution should be close to 2, so we choose $x_1 = 2$ as our initial estimate. We have $f'(x) = 4x^3,$ so the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 - 19}{4x_n^3} = \frac{3x_n^4 + 19}{4x_n^3}$$

n	x_n	x_{n+1}
1	2	2.09
2	2.09	2.09

Thus to two decimal places, $\sqrt[4]{19} = 2.09.$

13. We want real solutions to $e^x = x + 5.$ Thus we want to find roots of $f(x) = e^x - x - 5 = 0.$ A rough sketch of the exponential function $y = e^x$ and the line $y = x + 5$ shows that there are two intersection points: one when x is near –5, and the other when x is near 3. Thus we must find two roots. Since $f'(x) = e^x - 1,$ the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{e^{x_n} - x_n - 5}{e^{x_n} - 1}$$

If $x_1 = -5,$ we obtain

n	x_n	x_{n+1}
1	–5	–4.99
2	–4.99	–4.99

If $x_1 = 3,$ we obtain:

n	x_n	x_{n+1}
1	3	2.37
2	2.37	2.03
3	2.03	1.94
4	1.94	1.94

Thus the solutions are –4.99 and 1.94.

14. We must solve $\ln x = 5 - x$. That is, we must determine all roots of $f(x) = \ln(x) + x - 5 = 0$. A rough sketch shows that the graph of the logarithmic function $y = \ln x$ intersects the line $y = 5 - x$ at one point, where x is between 3 and 4. We choose $x_1 = 3.$ Since $f'(x) = \frac{1}{x} + 1,$ the recursion formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\ln(x_n) + x_n - 5}{\frac{1}{x_n} + 1}$$

n	x_n	x_{n+1}
1	3	3.676
2	2.676	3.693
3	3.693	3.693

Thus the solution is approximately 3.693.

15. The break-even quantity is the value of q when total revenue and total cost are equal: $r = c$, or $r - c = 0$. Thus we must find a root of $3q - \left(250 + 2q - 0.1q^3\right) = 0,$ or

$f(q) = q - 250 + 0.1q^3 = 0,$ so $f'(q) = 1 + 0.3q^2.$ The recursion formula is

$$q_{n+1} = q_n - \frac{f(q_n)}{f'(q_n)} = q_n - \frac{q_n - 250 + 0.1q_n{}^3}{1 + 0.3q_n{}^2}$$

We choose $q_1 = 13,$ as suggested.

n	q_n	q_{n+1}
1	13	13.33
2	13.33	13.33

Thus $q \approx 13.33$.

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16. a. The break-even quantity is the value of q when total cost = total revenue: $c = r$, $c - r = 0$. Thus we solve

$$50 + 4q + \frac{q^2}{1000} + \frac{1}{q} = 8q.$$ Multiplying both sides by q and simplifying, we see that the problem is equivalent to solving

$$f(q) = \frac{q^3}{1000} - 4q^2 + 50q + 1 = 0.$$

b. Since $f'(q) = \frac{3q^2}{1000} - 8q + 50$, the recursion formula is

$$q_{n+1} = q_n - \frac{f(q_n)}{f'(q_n)}$$
$$= q_n - \frac{\frac{q_n^3}{1000} - 4q_n{}^2 + 50q_n + 1}{\frac{3q_n^2}{1000} - 8q_n + 50}$$

We select $q_1 = 10$ as suggested.

n	q_n	q_{n+1}
1	10	13.43
2	13.43	12.61
3	12.61	12.56
4	12.56	12.56

Thus $q \approx 12.56$.

17. The equilibrium quantity is the value of q for which supply and demand are equal, that is, it is a root of $2q + 5 = \frac{100}{q^2 + 1}$, or of

$f(q) = 2q + 5 - \frac{100}{q^2 + 1} = 0$. Since

$f'(q) = 2 + \frac{200q}{\left(q^2+1\right)^2}$, the recursion formula is

$$q_{n+1} = q_n - \frac{f(q_n)}{f'(q_n)} = q_n - \frac{2q_n + 5 - \frac{100}{q_n{}^2 + 1}}{2 + \frac{200q_n}{\left(q_n{}^2+1\right)^2}}$$

A rough sketch shows that the graph of the supply equation intersects the graph of the demand equation when q is near 3. Thus we select $q_1 = 3$.

n	q_n	q_{n+1}
1	3	2.875
2	2.875	2.880
3	2.880	2.880

Thus $q \approx 2.880$.

18. In the same manner as problem 17, we must find a root of $f(q) = 0.2q^3 + 1.5q - 8 = 0$, so $f'(q) = 0.6q^2 + 1.5$. The recursion formula is

$$q_{n+1} = q_n - \frac{f(q_n)}{f'(q_n)} = q_n - \frac{0.2q_n^3 + 1.5q_n - 8}{0.6q_n^2 + 1.5}$$

We select $q_1 = 5$ as suggested.

n	q_n	q_{n+1}
1	5	3.54
2	3.54	2.85
3	2.85	2.71
4	2.71	2.70
5	2.70	2.70

Thus $q = 2.70$, so $p = 10 - 2.70 = 7.30$ (from the demand equation).

19. For a critical value of $f(x) = \frac{x^3}{3} - x^2 - 5x + 1$, we want a root of $f'(x) = x^2 - 2x - 5 = 0$. Since $\frac{d}{dx}[f'(x)] = 2x - 2$, the recursion formula is

$$x_{n+1} = x_n - \frac{x_n{}^2 - 2x_n - 5}{2x_n - 2}.$$

For the given interval [3, 4], note that $f'(3) = -2$ and $f'(4) = 3$ have opposite signs. Thus there is a root x between 3 and 4. Since 3 is closer to 0, we shall select $x_1 = 3$.

n	x_n	x_{n+1}
1	3.0	3.5
2	3.5	3.45
3	3.45	3.45

Thus $x \approx 3.45$.

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Apply It 12.7

8. $\frac{dh}{dt} = 0 - 16(2t) = -32t$ ft/sec

$\frac{d^2h}{dt^2} = \frac{d}{dt}[-32t] = -32$ feet/sec^2

The acceleration of the rock at time t is -32 feet/sec^2 or 32 feet/sec^2 downward.

9. The rate of change of the marginal cost function with respect to x is $c''(q)$.

$c'(q) = 14q + 11$

$c'' = 14$

When $x = 3$, the rate of change of the marginal cost function is 14 dollars/unit^2.

Problems 12.7

1. $y' = 12x^2 - 24x + 6$

$y'' = 24x - 24$

$y''' = 24$

2. $y' = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

$y'' = 20x^3 + 12x^2 + 6x + 2$

$y''' = 60x^2 + 24x + 6$

3. $\frac{dy}{dx} = -1$

$\frac{d^2y}{dx^2} = 0$

4. $\frac{dy}{dx} = -1 - 2x$

$\frac{d^2y}{dx^2} = -2$

5. $y' = 3x^2 + e^x$

$y'' = 6x + e^x$

$y''' = 6 + e^x$

$y^{(4)} = e^x$

6. $\frac{dF}{dq} = \frac{1}{q+1}$

$\frac{d^2F}{dq^2} = -\frac{1}{(q+1)^2}$

$\frac{d^3F}{dq^3} = \frac{2}{(q+1)^3}$

7. $f(x) = x^3 \ln x$

$f'(x) = x^3\left(\frac{1}{x}\right) + (\ln x)(3x^2) = x^2(1 + 3\ln x)$

$f''(x) = 2x(1 + 3\ln x) + x^2\left(\frac{3}{x}\right)$
$= 2x + 6x\ln x + 3x$
$= x(5 + 6\ln x)$

$f'''(x) = 1(5 + 6\ln x) + x\left(\frac{6}{x}\right)$
$= 5 + 6\ln x + 6$
$= 11 + 6\ln x$

8. $y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2}$

$y'' = 2x^{-3}$

$y''' = -6x^{-4} = -\frac{6}{x^4}$

9. $f(q) = \frac{1}{2q^4} = \frac{1}{2}q^{-4}$

$f'(q) = -2q^{-5}$

$f''(q) = 10q^{-6}$

$f'''(q) = -60q^{-7} = -\frac{60}{q^7}$

10. $f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4x^{\frac{3}{2}}}$

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11. $f(r)=\sqrt{9-r}=(9-r)^{\frac{1}{2}}$

$$f'(r)=-\frac{1}{2}(9-r)^{-\frac{1}{2}}$$

$$f''(r)=-\frac{1}{4}(9-r)^{-\frac{3}{2}}=-\frac{1}{4(9-r)^{\frac{3}{2}}}$$

12. $y=e^{ax^2}$

$$y'=e^{ax^2}(2ax)$$

$$y''=e^{ax^2}(2ax)^2+e^{ax^2}(2a)$$

$$=2ae^{ax^2}(2ax^2+1)$$

13. $y=\frac{1}{2x+3}=(2x+3)^{-1}$

$$\frac{dy}{dx}=-2(2x+3)^{-2}$$

$$\frac{d^2y}{dx^2}=8(2x+3)^{-3}=\frac{8}{(2x+3)^3}$$

14. $y=(3x+7)^5$

$$y'=15(3x+7)^4$$

$$y''=180(3x+7)^3$$

15. $y=\frac{x+1}{x-1}$

$$y'=\frac{(x-1)(1)-(x+1)(1)}{(x-1)^2}$$

$$=-\frac{2}{(x-1)^2}=-2(x-1)^{-2}$$

$$y''=4(x-1)^{-3}=\frac{4}{(x-1)^3}$$

16. $y=2x^{\frac{1}{2}}+(2x)^{\frac{1}{2}}$

$$y'=x^{-\frac{1}{2}}+\frac{1}{2}(2x)^{-\frac{1}{2}}(2)=x^{-\frac{1}{2}}+(2x)^{-\frac{1}{2}}$$

$$y''=-\frac{1}{2}x^{-\frac{3}{2}}-\frac{1}{2}(2x)^{-\frac{3}{2}}(2)=-\left[\frac{1}{2x^{\frac{3}{2}}}+\frac{1}{(2x)^{\frac{3}{2}}}\right]$$

17. $y=\ln[x(x+a)]=\ln x+\ln(x+a)$

$$y'=\frac{1}{x}+\frac{1}{x+a}$$

$$y''=-\frac{1}{x^2}-\frac{1}{(x+a)^2}$$

18. $y=\ln\frac{(2x+5)(5x-2)}{x+1}$

$$=\ln(2x+5)+\ln(5x-2)-\ln(x+1)$$

$$y'=\frac{2}{2x+5}+\frac{5}{5x-2}-\frac{1}{x+1}$$

$$y''=-\frac{4}{(2x+5)^2}-\frac{25}{(5x-2)^2}+\frac{1}{(x+1)^2}$$

19. $f(z)=z^2e^z$

$$f'(z)=z^2\left(e^z\right)+e^z(2z)=\left(ze^z\right)(z+2)$$

$$f''(z)=\left(ze^z\right)(1)+(z+2)\left[ze^z+e^z(1)\right]$$

$$=e^z\left(z^2+4z+2\right)$$

20. $y=\frac{x}{e^x}$

$$\frac{dy}{dx}=\frac{e^x(1)-x\left(e^x\right)}{\left(e^x\right)^2}=\frac{1-x}{e^x}$$

$$\frac{d^2y}{dx^2}=\frac{e^x(-1)-(1-x)e^x}{\left(e^x\right)^2}=\frac{x-2}{e^x}$$

21. $y=e^{2x}+e^{3x}$

$$\frac{dy}{dx}=2e^{2x}+3e^{3x}$$

$$\frac{d^2y}{dx^2}=4e^{2x}+9e^{3x}$$

$$\frac{d^3y}{dx^3}=8e^{2x}+27e^{3x}$$

$$\frac{d^4y}{dx^4}=16e^{2x}+81e^{3x}$$

$$\frac{d^5y}{dx^5}=32e^{2x}+243e^{3x}$$

$$\left.\frac{d^5y}{dx^5}\right|_{x=0}=32e^0+243e^0=32+243=275$$

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22. $y = e^{2\ln(x^2+1)} = e^{\ln(x^2+1)^2} = (x^2+1)^2$

$y' = 2(x^2+1)(2x) = 4(x^3+x)$

$y'' = 4(3x^2+1)$

When $x = 1$, $y'' = 4(3+1) = 16$.

23. $x^2 + 4y^2 - 16 = 0$

$2x + 8yy' = 0$

$8yy' = -2x$

$$y' = -\frac{x}{4y}$$

$$y'' = -\frac{4y(1) - x(4y')}{16y^2}$$

$$= -\frac{4y - 4x\left(-\frac{x}{4y}\right)}{16y^2} = -\frac{4y^2 + x^2}{16y^3}$$

$$= -\frac{16}{16y^3} = -\frac{1}{y^3}$$

24. $x^2 - y^2 = 16$

$2x - 2yy' = 0$

$$y' = \frac{x}{y}$$

$$y'' = \frac{y(1) - x(y')}{y^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2}$$

$$= \frac{y^2 - x^2}{y^3} = \frac{-16}{y^3} = -\frac{16}{y^3}$$

25. $y^2 = 4x$

$2yy' = 4$

$$y' = \frac{2}{y} = 2y^{-1}$$

$$y'' = -2y^{-2}y' = -2y^{-2}\left(2y^{-1}\right) = -\frac{4}{y^3}$$

26. $9x^2 + 16y^2 = 25$

$18x + 32yy' = 0$

$$y' = -\frac{9x}{16y}$$

$$y'' = -\frac{9}{16}\cdot\frac{y(1) - xy'}{y^2} = -\frac{9}{16}\cdot\frac{y - x\left(-\frac{9x}{16y}\right)}{y^2}$$

$$= -\frac{9}{16}\cdot\frac{16y^2 + 9x^2}{16y^3} = -\frac{225}{256y^3}$$

27. $a\sqrt{x} + b\sqrt{y} = c$

$b\sqrt{y} = c - a\sqrt{x}$

$$\sqrt{y} = \frac{c}{b} - \frac{a}{b}\sqrt{x}$$

$$\frac{y'}{2\sqrt{y}} = -\frac{a}{2b\sqrt{x}}$$

$$y' = -\frac{a}{2b\sqrt{x}}\cdot 2\left(\frac{c}{b} - \frac{a}{b}\sqrt{x}\right) = -\frac{ac}{b^2\sqrt{x}} + \frac{a^2}{b^2}$$

$$y'' = -\frac{ac}{b^2}\left(-\frac{1}{2}\right)x^{-3/2} = \frac{ac}{2b^2x^{3/2}}$$

28. $y^2 - 6xy = 4$

$2yy' - 6[xy' + y(1)] = 0$

$2yy' - 6xy' = 6y$

$(2y - 6x)y' = 6y$

$$y' = \frac{6y}{2y - 6x} = \frac{3y}{y - 3x}$$

$$y'' = 3\cdot\frac{(y-3x)y' - y(y'-3)}{(y-3x)^2} = 9\cdot\frac{y - xy'}{(y-3x)^2}$$

$$= 9\cdot\frac{y - x\left[\frac{3y}{y-3x}\right]}{(y-3x)^2} = 9\cdot\frac{y(y-3x) - 3xy}{(y-3x)^3}$$

$$= 9\cdot\frac{y^2 - 6xy}{(y-3x)^3} = 9\cdot\frac{4}{(y-3x)^3} = \frac{36}{(y-3x)^3}$$

29. $xy + y - x = 4$

$xy' + y(1) + y' - 1 = 0$

$xy' + y' = 1 - y$

$(x+1)y' = 1 - y$

$$y' = \frac{1-y}{1+x}$$

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$$y'' = \frac{(1+x)(-y') - (1-y)(1)}{(1+x)^2}$$

$$= \frac{(1+x)\left[-\frac{(1-y)}{(1+x)}\right] - (1-y)}{(1+x)^2}$$

$$= \frac{-(1-y)-(1-y)}{(1+x)^2} = \frac{-2(1-y)}{(1+x)^2} = \frac{2(y-1)}{(1+x)^2}$$

30. $x^2 + 2xy + y^2 = 1$

$2x + 2y + 2xy' + 2yy' = 0$

$(x+y)y' = -(x+y)$

$y' = -1$

$y'' = 0$

31. $y = e^{x+y}$

$y' = e^{x+y}(1+y')$

$y' - e^{x+y}y' = e^{x+y}$

$y'\left(1 - e^{x+y}\right) = e^{x+y}$

$$y' = \frac{e^{x+y}}{1-e^{x+y}}$$

$$y' = \frac{y}{1-y}$$

$$y'' = \frac{(1-y)y' - y(-y')}{(1-y)^2} = \frac{y'}{(1-y)^2}$$

$$= \frac{\frac{y}{1-y}}{(1-y)^2} = \frac{y}{(1-y)^3}$$

32. $e^x + e^y = x^2 + y^2$

$e^x + e^y y' = 2x + 2yy'$

$y'(e^y - 2y) = 2x - e^x$

$$y' = \frac{2x - e^x}{e^y - 2y}$$

$$y'' = \frac{(e^y-2y)(2-e^x) - (2x-e^x)(e^y y' - 2y')}{(e^y-2y)^2}$$

$$= \frac{(e^y-2y)(2-e^x) - (2x-e^x)(e^y-2)y'}{(e^y-2y)^2}$$

$$= \frac{(e^y-2y)^2(2-e^x) - (2x-e^x)^2(e^y-2)}{(e^y-2y)^3}$$

33. $x^2 + 3x + y^2 = 4y$

$2x + 3 + 2yy' = 4y'$

$2yy' - 4y' = -2x - 3$

$$y' = -\frac{2x+3}{2y-4} = \frac{2x+3}{4-2y}$$

$$y'' = \frac{(4-2y)(2) - (2x+3)(-2y')}{(4-2y)^2}$$

$$= \frac{2(4-2y) + 2(2x+3)\left(\frac{2x+3}{4-2y}\right)}{(4-2y)^2}$$

$$= \frac{2(4-2y)^2 + 2(2x+3)^2}{(4-2y)^3}$$

When $x = 0$ and $y = 0$, then

$$\frac{d^2y}{dx^2} = \frac{2(4)^2 + 2(3)^2}{4^3} = \frac{25}{32}.$$

34. $f(x) = (3x-5)e^{-2x}$

$f'(x) = (3x-5)\left[-2e^{-2x}\right] + e^{-2x}[3]$. Thus,

$f'(x) = e^{-2x}[-2(3x-5)+3] = (13-6x)e^{-2x}$

$f''(x) = (13-6x)\left[-2e^{-2x}\right] + e^{-2x}[-6]$

$= 2e^{-2x}[-(13-6x)-3]$

$= 4(3x-8)e^{-2x}$

$f''(x) + 4f'(x) + 4f(x)$

$= 4(3x-8)e^{-2x} + 4\left[(13-6x)e^{-2x}\right] + 4\left[(3x-5)e^{-2x}\right]$

$= [4(3x-8) + 4(13-6x) + 4(3x-5)]e^{-2x}$

$= [0]e^{-2x} = 0$, as was to be shown.

35. $f(x) = (5x-3)^4$

$f'(x) = 20(5x-3)^3$

$f''(x) = 300(5x-3)^2$

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36. $f(x) = 6x^{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{6}$

$f'(x) = 3x^{-\frac{1}{2}} - \frac{x^{-\frac{3}{2}}}{12}$

$f''(x) = -\frac{3}{2}x^{-\frac{3}{2}} + \frac{x^{-\frac{5}{2}}}{8}$

$f'''(x) = \frac{9}{4}x^{-\frac{5}{2}} - \frac{5x^{-\frac{7}{2}}}{16}$

37. $\frac{dc}{dq} = 0.4q + 2$

$\frac{d^2c}{dq^2} = 0.4$

When $q = 97.357$, $\frac{d^2c}{dq^2} = 0.4$.

38. $r = pq = 400q - 40q^2 - q^3$

$\frac{dr}{dq} = 400 - 80q - 3q^2$

$\frac{d^2r}{dq^2} = -80 - 6q$

When $q = 4$, $\frac{d^2r}{dq^2} = -104$.

39. $f(x) = x^4 - 6x^2 + 5x - 6$

$f'(x) = 4x^3 - 12x + 5$

$f''(x) = 12x^2 - 12 = 12(x+1)(x-1)$

Clearly $f''(x) = 0$ when $x = \pm 1$.

40. $e^y = y^2e^x$

a. $e^y y' = y^2\left(e^x\right) + e^x(2yy')$

$\left(e^y - 2ye^x\right)y' = y^2e^x$

$y' = \frac{y^2e^x}{e^y - 2ye^x} = \frac{y^2\left(\frac{e^y}{y^2}\right)}{e^y - 2y\left(\frac{e^y}{y^2}\right)}$

$= \frac{e^y}{e^y - \frac{2e^y}{y}} = \frac{1}{1 - \frac{2}{y}} = \frac{y}{y-2}$

b. $y'' = \frac{(y-2)(y') - y(y')}{(y-2)^2} = \frac{-2y'}{(y-2)^2}$

$= \frac{-2\left(\frac{y}{y-2}\right)}{(y-2)^2} = -\frac{2y}{(y-2)^3} = \frac{2y}{(2-y)^3}$

41. $f'(x) = 6e^x - 3x^2 - 30x$

$f''(x) = 6\left(e^x - x - 5\right)$

$f''(x) = 0$ when $x \approx -4.99$ or 1.94.

42. $f(x) = \frac{x^5}{20} + \frac{x^4}{12} + \frac{5x^3}{6} + \frac{x^2}{2}$

$f'(x) = \frac{x^4}{4} + \frac{x^3}{3} + \frac{5x^2}{2} + x$

$f''(x) = x^3 + x^2 + 5x + 1$

$f''(x) = 0$ when $x \approx -0.21$.

Chapter 12 Review Problems

1. $y' = 3e^x + 0 + e^{x^2}(2x) + (e^2)x^{e^2-1}$

$= 3e^x + 2xe^{x^2} + e^2x^{e^2-1}$

2. $f'(w) = \left(we^w + e^w\right) + 2w = we^w + e^w + 2w$

3. $f'(r) = \frac{1}{7r^2 + 4r + 5}(14r + 4) = \frac{14r + 4}{7r^2 + 4r + 5}$

4. $y = e^{\ln x} = x$. Thus $y' = 1$.

5. $y = e^{x^2+4x+5}$

$y' = e^{x^2+4x+5}(2x+4) = 2(x+2)e^{x^2+4x+5}$

6. $f(t) = \log_6\sqrt{t^2+1} = \frac{1}{2}\log_6\left(t^2+1\right)$

$= \frac{1}{2}\cdot\frac{\ln\left(t^2+1\right)}{\ln 6}$. Thus

$f'(t) = \frac{1}{2}\left(\frac{1}{\ln 6}\cdot\frac{1}{t^2+1}\cdot[2t]\right) = \frac{t}{(\ln 6)\left(t^2+1\right)}$.

7. $y' = e^x(2x) + \left(x^2+2\right)e^x = e^x\left(x^2+2x+2\right)$

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8. $y = 2^{3x^2} = e^{\ln 2^{3x^2}} = e^{(\ln 2)(3x^2)}$

$y' = e^{(\ln 2)(3x^2)}(\ln 2)(6x) = 6x\ln 2(2^{3x^2})$

9. $y = \sqrt{(x-6)(x+5)(9-x)}$

$\ln y = \ln\sqrt{(x-6)(x+5)(9-x)}$

$= \frac{1}{2}[\ln(x-6)+\ln(x+5)+\ln(9-x)]$

$\frac{y'}{y} = \frac{1}{2}\left[\frac{1}{x-6}+\frac{1}{x+5}+\frac{-1}{9-x}\right]$

$y' = \frac{y}{2}\left[\frac{1}{x-6}+\frac{1}{x+5}-\frac{1}{9-x}\right]$

$= \frac{\sqrt{(x-6)(x+5)(9-x)}}{2}\left[\frac{1}{x-6}+\frac{1}{x+5}+\frac{1}{x-9}\right]$

10. $f'(t) = e^{1/t}(-1\cdot t^{-2}) = -\frac{e^{1/t}}{t^2}$

11. $y' = \frac{e^x\left(\frac{1}{x}\right)-(\ln x)\left(e^x\right)}{e^{2x}}$

$= \frac{e^x - xe^x\ln x}{xe^{2x}} = \frac{1-x\ln x}{xe^x}$

12. $y' = \frac{x^2\left(e^x-e^{-x}\right)-\left(e^x+e^{-x}\right)(2x)}{x^4}$

$= \frac{x^2e^x - x^2e^{-x} - 2xe^x - 2xe^{-x}}{x^4}$

$= \frac{e^x(x-2)-e^{-x}(x+2)}{x^3}$

13. $f(q) = m\ln(q+a)+n\ln(q+b)$

$f'(q) = \frac{m}{q+a}+\frac{n}{q+b}$

14. $y = (x+2)^3(x+1)^4(x-2)^2$

$\ln y = 3\ln(x+2)+4\ln(x+1)+2\ln(x-2)$

$\frac{y'}{y} = \frac{3}{x+2}+\frac{4}{x+1}+\frac{2}{x-2}$

$y' = y\left[\frac{3}{x+2}+\frac{4}{x+1}+\frac{2}{x-2}\right]$

$= (x+2)^3(x+1)^4(x-2)^2\left[\frac{3}{x+2}+\frac{4}{x+1}+\frac{2}{x-2}\right]$

15. $y = e^{(2x^2+2x-5)(\ln 2)}$

$y' = e^{(2x^2+2x-5)(\ln 2)}(4x+2)(\ln 2)$

$= (4x+2)(\ln 2)2^{2x^2+2x-5}$

16. y is a constant, so $y' = 0$.

17. $y = \frac{4e^{3x}}{xe^{x-1}} = \frac{4e^{2x+1}}{x}$

$y' = 4\cdot\frac{x\left[e^{2x+1}(2)\right]-e^{2x+1}[1]}{x^2} = \frac{4e^{2x+1}(2x-1)}{x^2}$

18. $y' = \frac{\frac{1}{x}e^x - e^x(\ln x)}{e^{2x}} = \frac{\frac{1}{x}-\ln x}{e^x}$

19. $y = \log_2(8x+5)^2 = 2\log_2(8x+5)$

$= 2\cdot\frac{\ln(8x+5)}{\ln 2}$

$y' = 2\cdot\frac{1}{\ln 2}\cdot\frac{8}{8x+5} = \frac{16}{(8x+5)\ln 2}$

20. $y = \ln\left(\frac{5}{x^2}\right) = \ln 5 - 2\ln x$

$y' = 0 - 2\cdot\frac{1}{x} = -\frac{2}{x}$

21. $f(l) = \ln\left(1+l+l^2+l^3\right)$

$f'(l) = \frac{1}{1+l+l^2+l^3}\left[1+2l+3l^2\right]$

$= \frac{1+2l+3l^2}{1+l+l^2+l^3}$

22. $y = (x^2)^{x^2}$

$\ln y = x^2\ln x^2 = 2x^2\ln x$

$\frac{y'}{y} = 2x^2\left(\frac{1}{x}\right)+(\ln x)(4x)$

$y' = 2xy(1+2\ln x)$

$y' = 2x(x^2)^{x^2}(1+2\ln x)$

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23. $y = (x^2+1)^{x+1}$

$\ln y = (x+1)\ln(x^2+1)$

$\frac{y'}{y} = \frac{(x+1)(2x)}{x^2+1} + \ln(x^2+1)$

$y' = \left[\frac{2x(x+1)}{x^2+1} + \ln(x^2+1)\right](x^2+1)^{x+1}$

24. $y' = \frac{\left(1-e^x\right)e^x - \left(1+e^x\right)\left(-e^x\right)}{\left(1-e^x\right)^2} = \frac{2e^x}{\left(1-e^x\right)^2}$

25. $\phi(t) = \ln\left(t\sqrt{4-t^2}\right) = \ln t + \frac{1}{2}\ln(4-t^2)$

$\phi'(t) = \frac{1}{t} + \frac{1}{2}\cdot\frac{1}{4-t^2}\cdot(-2t) = \frac{1}{t} - \frac{t}{4-t^2}$

26. $y = (x+3)^{\ln x}$

$\ln y = [\ln x]\ \ln(x+3)$

$\frac{y'}{y} = (\ln x)\frac{1}{x+3} + \ln(x+3)\cdot\frac{1}{x}$

$y' = y\left[\frac{\ln x}{x+3} + \frac{\ln(x+3)}{x}\right]$

$= (x+3)^{\ln x}\left[\frac{\ln x}{x+3} + \frac{\ln(x+3)}{x}\right]$

27. $y = \frac{(x^2+1)^{1/2}(x^2+2)^{1/3}}{(2x^3+6x)^{2/5}}$

$\ln y = \frac{1}{2}\ln(x^2+1) + \frac{1}{3}\ln(x^2+2) - \frac{2}{5}\ln(2x^3+6x)$

$\frac{y'}{y} = \frac{1}{2}\left(\frac{1}{x^2+1}\right)(2x) + \frac{1}{3}\left(\frac{1}{x^2+2}\right)(2x) - \frac{2}{5}\left(\frac{1}{2x^3+6x}\right)(6x^2+6)$

$y' = y\left[\frac{x}{x^2+1} + \frac{2x}{3(x^2+2)} - \frac{6(x^2+1)}{5(x^3+3x)}\right]$

$= \frac{(x^2+1)^{1/2}(x^2+2)^{1/3}}{(2x^3+6x)^{2/5}}\left[\frac{x}{x^2+1} + \frac{2x}{3(x^2+2)} - \frac{6(x^2+1)}{5(x^3+3x)}\right]$

28. $y' = \frac{\sqrt{x}}{x} + (\ln x)\left(\frac{1}{2}x^{-1/2}\right)$

$= \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}$

$= \frac{2+\ln x}{2\sqrt{x}}$

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29. $y = \left(x^x\right)^x = x^{x^2}$

$\ln y = \ln x^{x^2} = x^2 \ln x$

$\frac{y'}{y} = x^2\left(\frac{1}{x}\right) + (\ln x)(2x) = x + 2x\ln x$

$y' = y(x + 2x\ln x) = \left(x^x\right)^x (x + 2x\ln x)$

30. $y = x^{\left(x^x\right)}$

$\ln y = \ln x^{\left(x^x\right)} = x^x \ln x$

$\frac{y'}{y} = x^x\left(\frac{1}{x}\right) + (\ln x)\frac{d}{dx}\left(x^x\right)$

Note: If $v = x^x$, then $\ln v = \ln x^x = x\ln x$;

$\frac{v'}{v} = x\left(\frac{1}{x}\right) + (\ln x)(1) = 1 + \ln x$

$v' = \frac{d}{dx}\left(x^x\right) = v(1 + \ln x) = x^x(1 + \ln x)$

Thus $\frac{y'}{y} = x^x\left(\frac{1}{x}\right) + (\ln x)\left[x^x(1 + \ln x)\right]$

$= x^x\left[\frac{1}{x} + (1 + \ln x)\ln x\right]$

$y' = yx^x\left[\frac{1}{x} + (1 + \ln x)\ln x\right]$

$= x^{\left(x^x\right)}x^x\left[\frac{1}{x} + (1 + \ln x)\ln x\right]$

31. $y = (x+1)\ln x^2 = 2(x+1)\ln x$

$y' = 2\left[(x+1)\left(\frac{1}{x}\right) + (\ln x)(1)\right] = 2\left[\frac{x+1}{x} + \ln x\right]$

When $x = 1$, then $y' = 2\left[\frac{2}{1} + \ln 1\right] = 4.$

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32. $y = \dfrac{e^{x^2+1}}{\sqrt{x^2+1}}$

$\ln y = \ln\left(e^{x^2+1}\right) - \dfrac{1}{2}\ln(x^2+1) = x^2+1-\dfrac{1}{2}\ln(x^2+1)$

$\dfrac{y'}{y} = 2x - \dfrac{1}{2}\cdot\dfrac{1}{x^2+1}(2x) = x\left[2-\dfrac{1}{x^2+1}\right]$

$y' = yx\left[2-\dfrac{1}{x^2+1}\right]$

$y' = \dfrac{e^{x^2+1}}{\sqrt{x^2+1}}x\left[2-\dfrac{1}{x^2+1}\right]$

When $x = 1$, then $y' = \dfrac{e^{1+1}}{\sqrt{1+1}}(1)\left[2-\dfrac{1}{1+1}\right] = \dfrac{3e^2\sqrt{2}}{4}$.

33. $y = \dfrac{1}{x^x}$

$\ln y = \ln\left(\dfrac{1}{x^x}\right) = \ln 1 - x\ln x = -x\ln x$

$\dfrac{y'}{y} = -\ln x - x\left(\dfrac{1}{x}\right) = -\ln x - 1$

$y' = \dfrac{-(\ln x+1)}{x^x}$

When $x = e$, $y' = -\dfrac{\ln e+1}{e^e} = -\dfrac{2}{e^e}$.

34. $y = \left[\dfrac{2^{5x}(x^2-3x+5)^{1/3}}{(x^2-3x+7)^3}\right]^{-1}$

$\ln y = -1\left[5x\ln 2+\dfrac{1}{3}\ln(x^2-3x+5)-3\ln(x^2-3x+7)\right]$

$\dfrac{y'}{y} = -\left[5\ln 2+\dfrac{1}{3}\cdot\dfrac{2x-3}{x^2-3x+5}-3\cdot\dfrac{2x-3}{x^2-3x+7}\right]$

$y' = -y\left[5\ln 2+\dfrac{2x-3}{3(x^2-3x+5)}-\dfrac{3(2x-3)}{x^2-3x+7}\right]$

$y' = (-1)\left[\dfrac{2^{5x}(x^2-3x+5)^{1/3}}{(x^2-3x+7)^3}\right]^{-1}\left[5\ln 2+\dfrac{2x-3}{3(x^2-3x+5)}\dfrac{3(2x-3)}{x^2-3x+7}\right]$

When $x = 0$, then $y' = -\dfrac{343}{5^{1/3}}\left[5\ln 2-\dfrac{1}{5}+\dfrac{9}{7}\right] = -343(\ln 2)5^{2/3}-\dfrac{1862}{5^{4/3}}$.

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35. $y = 3e^x$

$y' = 3e^x$

If $x = \ln 2$, then $y = 3e^{\ln 2} = 6$ and

$y' = 3e^{\ln 2} = 6.$

An equation of the tangent line is
$y - 6 = 6(x - \ln 2)$, $y = 6x + 6 - 6\ln 2$,
$y = 6x + 6(1 - \ln 2)$. Alternatively, since
$6\ln 2 = \ln 2^6 = \ln 64$, the tangent line can be written as $y = 6x + 6 - \ln 64$.

36. $y = x + x^2 \ln x$

$$y' = 1 + \left[x^2\left(\frac{1}{x}\right) + (\ln x)(2x)\right] = 1 + x + 2x\ln x$$

When $x = 1$, then $y = 1 + 1(0) = 1$ and
$y' = 1 + 1 + 2(0) = 2$. Thus an equation of the tangent line is $y - 1 = 2(x - 1)$, or $y = 2x - 1$.

37. $y = x\left(2^{2-x^2}\right)$. To find y' we shall use logarithmic differentiation.

$$\ln y = \ln\left[x\left(2^{2-x^2}\right)\right] = \ln x + \left(2 - x^2\right)\ln 2$$

$$\frac{y'}{y} = \frac{1}{x} + (-2x)\ln 2$$

$$y' = y\left[\frac{1}{x} - 2(\ln 2)x\right]$$

When $x = 1$, then $y = 2$ and $y' = 2(1 - 2\ln 2)$. The equation of the tangent line is
$y - 2 = 2(1 - 2\ln 2)(x - 1)$. The y-intercept of the tangent line corresponds to the point where
$x = 0$: $y - 2 = 2(1 - 2\ln 2)(-1) = -2 + 4\ln 2$
Thus $y = 4\ln 2$ and the y-intercept is $(0, 4\ln 2)$.

38. $w = 2^x + \ln(1 + x^2) = e^{(\ln 2)x} + \ln(1 + x^2)$

$$\frac{dw}{dx} = (\ln 2)e^{(\ln 2)x} + \frac{2x}{1 + x^2}$$

$$\frac{dx}{dt} = \frac{2t}{1 + t^2}$$

When $t = 0$, $x = \ln(1 + 0) = 0$, and $\frac{dx}{dt} = 0$, and

$w = 2^0 + \ln(1 + 0) = 1$. Since $\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt}$ and

$\frac{dx}{dt} = 0$, $\frac{dw}{dt} = 0$.

39. $y = e^{x^2 - 2x + 1}$

$$y' = e^{x^2-2x+1}[2x - 2] = (2x - 2)e^{x^2-2x+1}$$

$$y'' = 2(x - 1)e^{x^2-2x+1}(2x - 2) + 2e^{x^2-2x+1}$$

$$= 2e^{x^2-2x+1}(2(x - 1)^2 + 1)$$

At $(1, 1)$, $y'' = 2e^0(2(0) + 1) = 2$.

40. $y = x^2 e^x$

$$y' = x^2 e^x + e^x(2x) = e^x\left(x^2 + 2x\right)$$

$$y'' = e^x(2x + 2) + \left(x^2 + 2x\right)e^x = e^x\left(x^2 + 4x + 2\right)$$

$$y''' = e^x(2x + 4) + \left(x^2 + 4x + 2\right)e^x = e^x\left(x^2 + 6x + 6\right)$$

At $(1, e)$, $y''' = e(1 + 6 + 6) = 13e$

41. $y = \ln(2x)$

$$y' = \frac{1}{2x}(2) = x^{-1}$$

$$y'' = -1 \cdot x^{-2} = -x^{-2}$$

$$y''' = -(-2)x^{-3} = \frac{2}{x^3}$$

At $(1, \ln 2)$, $y''' = \frac{2}{1^3} = 2$

42. $y = x\ln x$

$$y' = x \cdot \frac{1}{x} + (\ln x)(1) = 1 + \ln x$$

$$y'' = 0 + \frac{1}{x} = \frac{1}{x}$$

At $(1, 0)$, $y'' = \frac{1}{1} = 1$

43.

$$x^2 + 2xy + y^2 = 4$$

$$2x + 2y + 2xy' + 2yy' = 0$$

$$x + y + y'(x + y) = 0$$

$$y' = -\frac{x + y}{x + y} = -1$$

44. $3x^2y^3 + 3x^3y^2y' = 0$

$$y'(3x^3y^2) = -3x^2y^3$$

$$y' = \frac{-3x^2y^3}{3x^3y^2} = -\frac{y}{x}$$

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45. $\ln\left(xy^2\right) = xy$

$\ln x + 2\ln y = xy$

$\frac{1}{x} + \frac{2}{y}y' = xy' + y$

$y + 2xy' = x^2yy' + xy^2$

$2xy' - x^2yy' = xy^2 - y$

$\left(2x - x^2y\right)y' = xy^2 - y$

$y' = \frac{xy^2 - y}{2x - x^2y}$

46. $y^2e^{y\ln x} = e^2$

$y^2\left[e^{y\ln x}\left(y \cdot \frac{1}{x} + (\ln x)y'\right)\right] + e^{y\ln x}[2yy'] = 0$

$y^2(\ln x)y' + 2yy' = -\frac{y^3}{x}$

$y'[y(2 + y\ln x)] = -\frac{y^3}{x}$

$y' = -\frac{y^2}{x(2 + y\ln x)}$

47. $x + xy + y = 5$

$1 + xy' + y(1) + y' = 0$

$(x+1)y' = -1 - y$

$y' = -\frac{1+y}{x+1}$

$y'' = -\frac{(x+1)y' - (1+y)}{(x+1)^2}$

At (2, 1), $y' = -\frac{1+1}{2+1} = -\frac{2}{3}$ and

$y'' = -\frac{3\left(-\frac{2}{3}\right) - 2}{9} = \frac{4}{9}$

48. $x^2 + xy + y^2 = 1$

$2x + y + xy' + 2yy' = 0$

$2x + y + (x + 2y)y' = 0$

$y' = -\frac{2x+y}{x+2y}$

$y'' = -\left[\frac{(x+2y)(2+y') - (2x+y)(1+2y')}{(x+2y)^2}\right]$

At (0, −1), $y' = -\frac{0-1}{0-2} = -\frac{1}{2}$, and

$y'' = -\left[\frac{(0-2)\left(2-\frac{1}{2}\right) - (0-1)(1-1)}{(0-2)^2}\right]$

$= -\left[\frac{(-2)\left(\frac{3}{2}\right) - (-1)(0)}{(-2)^2}\right]$

$= -\left(\frac{-3}{4}\right)$

$= \frac{3}{4}.$

49. $e^y = (y+1)e^x$

$e^y y' = (y+1)e^x + e^x(y')$

$e^y y' - e^x y' = (y+1)e^x$

$\left(e^y - e^x\right)y' = (y+1)e^x$

$y' = \frac{(y+1)e^x}{e^y - e^x} = \frac{(y+1)\left(\frac{e^y}{y+1}\right)}{e^y - \left(\frac{e^y}{y+1}\right)} = \frac{e^y}{e^y - \frac{e^y}{y+1}}$

$= \frac{1}{1 - \frac{1}{y+1}} = \frac{y+1}{y}$

$y'' = \frac{y(y') - (y+1)(y')}{y^2} = \frac{-y'}{y^2} = -\frac{\frac{y+1}{y}}{y^2} = -\frac{y+1}{y^3}$

50. $x^{1/2} + y^{1/2} = 1$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

$\frac{d^2y}{dx^2} = -\frac{\left(\sqrt{x}\right)\left(\frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx}\right) - \left(\sqrt{y}\right)\left(\frac{1}{2\sqrt{x}}\right)}{\left(\sqrt{x}\right)^2}$

$= \frac{\frac{1}{2} + \frac{\sqrt{y}}{2\sqrt{x}}}{x} = \frac{\sqrt{x} + \sqrt{y}}{2x\sqrt{x}} = \frac{1}{2x\sqrt{x}}$

51. $f'(t)$

$= \left[-0.8e^{-0.01t}(-0.01) - 0.2e^{-0.0002t}(-0.0002)\right]$

$= 0.008e^{-0.01t} + 0.00004e^{-0.0002t}$

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52. $\log N = A - bM$

$$\frac{d}{dM}(\log N) = \frac{d}{dM}(A - bM)$$

$$\frac{d}{dM}\left(\frac{\ln N}{\ln 10}\right) = \frac{d}{dM}(A - bM)$$

$$\frac{1}{\ln 10}\cdot\frac{1}{N}\frac{dN}{dM} = -b$$

$$(\log e)\frac{1}{N}\frac{dN}{dM} = -b$$

$$-\frac{dN}{dM} = \frac{bN}{\log e}$$

$$\log\left(-\frac{dN}{dM}\right) = \log\left(\frac{b}{\log e}\cdot N\right)$$

$$= \log\left(\frac{b}{\log e}\right) + \log N$$

$$= \log\left(\frac{b}{q}\right) + (A - bM) = A + \log\left(\frac{b}{q}\right) - bM$$

53. $f'(x) = (4x^3 - 30x^2 + 72x - 2)e^{x^4 - 10x^3 + 36x^2 - 2x}$

$f'(x) = 0$ when $4x^3 - 30x^2 + 72x - 2 = 0$, or $x \approx 0.03$.

54. $f(x) = \dfrac{x^5}{10} + \dfrac{x^4}{6} + \dfrac{2x^3}{3} + x^2 + 1$

$$f'(x) = \frac{x^4}{2} + \frac{2x^3}{3} + 2x^2 + 2x$$

$$f''(x) = 2x^3 + 2x^2 + 4x + 2$$

$f''(x) = 0$ when $x \approx -0.57$.

55. $p = \dfrac{500}{q}$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{500/q}{q}}{-\frac{500}{q^2}} = -1$$

Since $|\eta| = 1$, demand has unit elasticity when $q = 200$.

56. $p = 900 - q^2$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{900-q^2}{q}}{-2q} = -\frac{900 - q^2}{2q^2}$$

When $q = 10$, then $\eta = -4$. Since $|\eta| > 1$, demand is elastic.

57. $p = 18 - 0.02q$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{18-0.02q}{q}}{-0.02} = -\frac{18 - 0.02q}{0.02q}$$

When $q = 600$, then $\eta = -0.5$. Because $|\eta| < 1$, demand is inelastic.

58. $p = 20 - 2\sqrt{q}$

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-\frac{1}{\sqrt{q}}} = \frac{-p}{\sqrt{q}} = \frac{-p}{10 - \frac{p}{2}} = \frac{2p}{p - 20}$$

a. When $p = 8$, then $\eta = \dfrac{2(8)}{8 - 20} = -\dfrac{4}{3}$.

b. $\eta = \dfrac{2p}{p - 20}$

If $|\eta| > 1$, demand is elastic. If $p > \dfrac{20}{3}$, $\eta < -1$. If $p = 20$, $\eta = \infty$. So, demand is elastic for $\left(\dfrac{20}{3}, 20\right)$.

59. $\eta = \dfrac{\frac{p}{q}}{\frac{dp}{dq}} = \dfrac{p}{q}\cdot\dfrac{dq}{dp}$

$$q = \sqrt{2500 - p^2}$$

$$\frac{dq}{dp} = \frac{-p}{\sqrt{2500 - p^2}} = \frac{-p}{q}, \text{ so}$$

$\eta = \dfrac{p}{q}\left(\dfrac{-p}{q}\right) = -\dfrac{p^2}{q^2}$. Now, if $p = 30$, then

$q = \sqrt{2500 - 30^2} = 40$, so

$$\eta\Big|_{p=30} = -\frac{(30)^2}{(40)^2} = -\frac{9}{16}$$

If the price of 30 decreases $\dfrac{2}{3}$%, then demand would change by approximately $\left(-\dfrac{2}{3}\right)\left(-\dfrac{9}{16}\right)$%, or $\dfrac{3}{8}$%. (That is, demand increases by approximately $\dfrac{3}{8}$%.)

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60. a. $\eta = \dfrac{\frac{p}{q}}{\frac{dp}{dq}} = \dfrac{p}{q} \cdot \dfrac{dq}{dp}$

$q = \sqrt{100-p}$, where $0 < p < 100$.

$\dfrac{dq}{dp} = \dfrac{-1}{2\sqrt{100-p}}$. Thus

$\eta = \dfrac{p}{\sqrt{100-p}} \cdot \dfrac{-1}{2\sqrt{100-p}}$

$= \dfrac{-p}{2(100-p)} = \dfrac{p}{2p-200}$

For elastic demand we want $\dfrac{p}{2p-200} < -1$.

Noting that the denominator is negative for $0 < p < 100$, we multiply both sides of the inequality by $2p - 200$ and reverse the direction of the inequality

$p > -2p + 200$, $3p > 200$, $p > \dfrac{200}{3}$

Thus $\dfrac{200}{3} < p < 100$ for elastic demand.

b. $\eta\big|_{p=40} = \dfrac{40}{80-200} = -\dfrac{1}{3}$

% change in $q \approx$ (% change in price)(η)

$= 5\left(-\dfrac{1}{3}\right)\% = -\dfrac{5}{3}\% = -1.67\%$. Thus demand decreases by approximately 1.67%.

61. We want a root of $f(x) = x^3 - 2x - 2 = 0$. We have $f(1) = -3$ and $f(2) = 2$ (note the sign change). Since $f(2)$ is closer to 0 than is $f(1)$, we choose $x_1 = 2$. We have $f'(x) = 3x^2 - 2$, so the recursion formula is

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^3 - 2x_n - 2}{3x_n^2 - 2}$

$= \dfrac{2x_n^3 + 2}{3x_n^2 - 2}$

n	x_n	x_{n+1}
1	2.00000	1.80000
2	1.80000	1.76995
3	1.76995	1.76929
4	1.76929	1.76929

Because $\left|x_5 - x_4\right| < 0.0001$, the root is approximately $x_5 = 1.7693$.

62. We want real solutions of $e^x = 3x$. Thus we want to find roots of $f(x) = e^x - 3x = 0$. A rough sketch of the exponential function $y = e^x$ and the line $y = 3x$ shows that there are two intersection points: one when x is near 0.5, and the other when x is near 1.5. Thus we must find two roots. Since $f'(x) = e^x - 3$, the recursion formula is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{e^{x_n} - 3x_n}{e^{x_n} - 3}$

If $x_1 = 0.5$, we obtain

n	x_n	x_{n+1}
1	0.5	0.610
2	0.610	0.619
3	0.619	0.619

If $x_1 = 1.5$, we obtain

n	x_n	x_{n+1}
1	1.5	1.512
2	1.512	1.512

Thus the solutions are 0.619 and 1.512.

Explore and Extend—Chapter 12

1. $F = 25$, $D = 3400$, $V = 36.5$, $R = 0.05$.

$q = \sqrt{\dfrac{2FD}{RV}} = \sqrt{\dfrac{2(25)(3400)}{(0.05)(36.5)}} \approx 305.2$

The economic order quantity is 305 units.

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2. If the number of units maintained as a safety margin is denoted by *m*, then the amount in stock at any time is increased by *m* units. The average inventory level is thus increased by *m* units, to

$m+\frac{q}{2}$ units. The carrying cost is then

$$C(q)=\frac{FD}{q}+RV\left(m+\frac{q}{2}\right)$$
$$=\frac{FD}{q}+\frac{RVq}{2}+RVm$$

Since $\frac{d}{dq}(RVm)=0,$ the maintenance of a

safetly margin does not affect the calculation of the economic order quantity.

3. Answers may vary.

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