Chapter 4

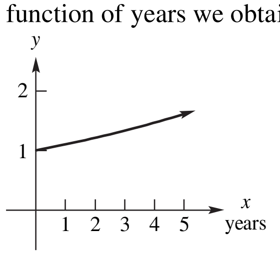
Apply It 4.1

1. The shapes of the graphs are the same. The value of A scales the value of any point by A.

2. If P = the amount of money invested and r = the annual rate at which P increases, then after 1 year, the investment has grown from P to $P + Pr = P(1 + r)$. Since $r = 0.10$, the factor by which P increases for the first year is $1 + r = 1 + 0.1 = 1.1$. Similarly, during the second year the investment grows from $P(1 + r)$ to $P(1+r)+r[P(1+r)] = P(1+r)^2$. Again, since $r = 0.10$, the multiplicative increase for the second year is $(1+0.10)^2 = (1.1)^2 = 1.21$. This pattern will continue as shown in the table.

Year	Multiplicative Increase	Expression
0	1	1.1^0
1	1.1	1.1^1
2	1.21	1.1^2
3	1.33	1.1^3
4	1.46	1.1^4

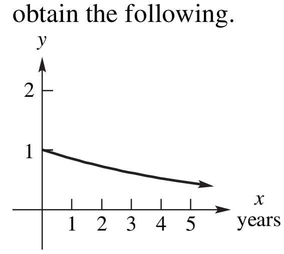
Thus, the growth of the initial investment is exponential with a base of $1 + r = 1 + 0.1 = 1.1$. If we graph the multiplicative increase as a function of years we obtain the following.



3. If V = the value of the car and r = the annual rate at which V depreciates, then after 1 year the value of the car is $V - rV = V(1 - r)$. Since $r = 0.15$, the factor by which V decreases for the first year is $1 - r = 1 - 0.15 = 0.85$. Similarly, after the second year the value of the car is $V(1-r) - r[V(1-r)] = V(1-r)^2$. Again, since $r = 0.15$, the multiplicative decrease for the second year is $(1-r)^2 = (1-0.15)^2 = 0.72$. This pattern will continue as shown in the table.

Year	Multiplicative Decrease	Expression
0	1	0.85^0
1	0.85	0.85^1
2	0.72	0.85^2
3	0.61	0.85^3

Thus, the depreciation is exponential with a base of $1 - r = 1 - 0.15 = 0.85$. If we graph the multiplicative decrease as a function of years, we obtain the following.



4. Let t = the time at which George's sister began saving, then since George is 3 years behind, $t - 3$ = the time when George began saving. Therefore, if $y = 1.08^t$ represents the multiplicative increase in George's sister's account $y = 1.08^{t-3}$ represents the multiplicative increase in George's account. A graph showing the projected increase in George's money will have the same shape as the graph of the projected increase in his sister's account, but will be shifted 3 units to the right.

5. $S = P(1+r)^n$

$S = 2000(1+0.13)^5 = 2000(1.13)^5 \approx 3684.87$

The value of the investment after 5 years will be \$3684.87. The interest earned over the first 5 years is 3684.87 – 2000 = \$1684.87.

6. Let $N(t)$ = the number of employees at time t, where t is in years. Then,

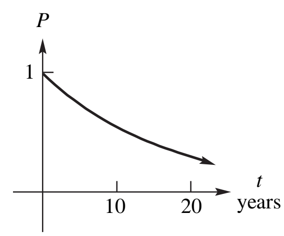
$N(4) = 5(1+1.2)^4 = 5(2.2)^4 = 117.128$

Thus, there will be 117 employees at the end of 4 years.

7. $P = e^{-0.06t} = \left(\frac{1}{e}\right)^{0.06t}$

Since $0 < \frac{1}{e} < 1$, the graph is that of an exponential function falling from left to right.

x	y
0	1
2	0.89
4	0.79
6	0.70
8	0.62
10	0.55



Problems 4.1

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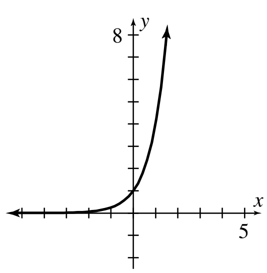

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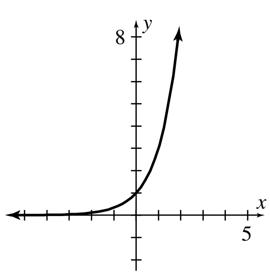

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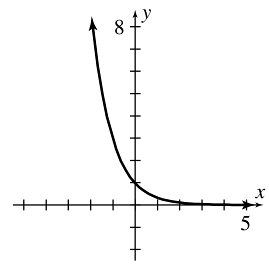

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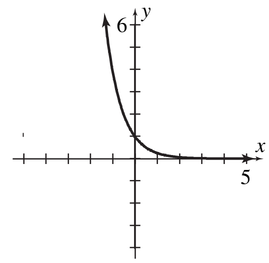

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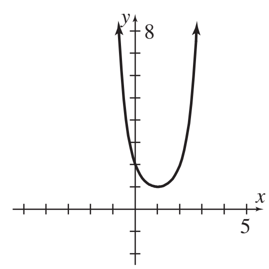

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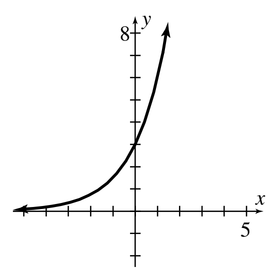

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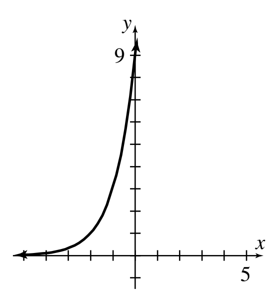

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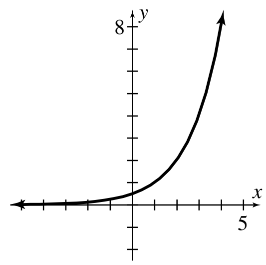

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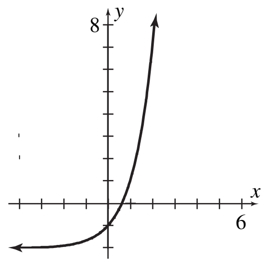

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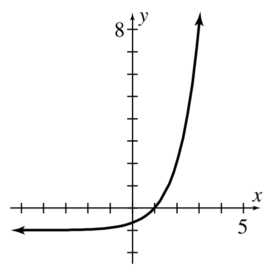

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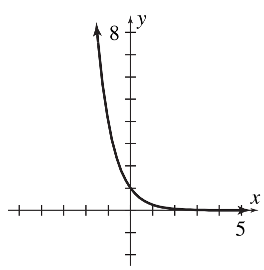

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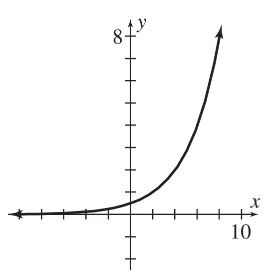


13. For the curves, the bases involved are 0.4, 2, and 5. For base 5, the curve rises from left to right, and in the first quadrant it rises faster than the curve for base 2. Thus the graph of $y = 5^x$ is *B*.

14. $y = 2^x$ has base $b = 2$ and $b > 1$, so its graph rises from left to right. Thus the graph is *C*.

15. For 2015 we have $t = 20$, so
$$P = 125,000(1.11)^{\frac{20}{20}} = 125,000(1.11)^1 = 138,750.$$

16. **a.** For 1999, $t = 1$ and
$$P = 1,527,000(1.015)^1 = 1,549,905$$

b. For 2000, $t = 2$ and
$$P = 1,527,000(1.015)^2 \approx 1,573,154$$

17. With $c = \frac{1}{2}$, $P = 1 - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = 1 - \left(\frac{1}{2}\right)^n$.

$n = 1$: $P = 1 - \left(\frac{1}{2}\right)^1 = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 2$: $P = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$n = 3$: $P = 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8}$

18. $y = 2^{3x} = \left(2^3\right)^x = 8^x$. Thus $y = 8^x$.

19. **a.** $2000(1.03)^5 \approx \$2318.55$

b. $2318.55 - 2000 = \$318.55$

20. **a.** $5000(1.05)^{20} \approx \$13,266.49$

b. $13,266.49 - 5000 = \$8266.49$

21. **a.** $700(1.035)^{30} \approx \$1964.76$

b. $1964.76 - 700 = \$1264.76$

22. **a.** $4000(1.0375)^{24} \approx \$9677.75$

b. $9677.75 - 4000 = \$5677.75$

23. **a.** $3000\left(1 + \frac{0.0875}{4}\right)^{64} \approx 11,983.37$

b. $11,983.37 - 3000 = \$8983.37$

24. **a.** $6000\left(1+\frac{0.08}{4}\right)^8 \approx \$7029.96$

b. $7029.96 - 6000 = \$1029.96$

25. **a.** $5000(1.0075)^{30} \approx \$6256.36$

b. $6256.36 - 5000 = \$1256.36$

26. **a.** $500\left(1+\frac{0.11}{2}\right)^{10} \approx \$854.07$

b. $854.07 - 500 = \$354.07$

27. **a.** $8000\left(1+\frac{0.0625}{365}\right)^{3(365)} \approx \$9649.69$

b. $9649.69 - 8000 = \$1649.69$

28. **a.** $900(1.0225)^{10} \approx \$1124.28$

b. $900(1.045)^5 \approx \$1121.56$

29. $6500\left(1+\frac{0.02}{4}\right)^{12} \approx \$6900.91$

30. **a.** $P = 5000(1.03)^t$

b. When $t = 3$, then $P = 5000(1.03)^3 \approx 5464$.

31. **a.** $N = 400(1.05)^t$

b. When $t = 1$, then $N = 400(1.05)^1 = 420$.

c. When $t = 4$, then $N = 400(1.05)^4 \approx 486$.

32. If $N = N(t)$ = the number of bacteria present at any time t, where t is in hours, and if r = the rate at which the bacteria are reduced, then, after the first hour, the number of bacteria remaining is
$N - rN = N(1 - r) = 100{,}000(1 - 0.1)$
$= 100{,}000(0.9) = 90{,}000$.
Similarly, after the second hour, the number of bacteria remaining is
$N(1 - r) - r[N(1 - r)] = N(1-r)^2$
$= 100{,}000(1-0.1)^2 = 100{,}000(0.9)^2 = 81{,}000$
This pattern will continue as shown in the table.

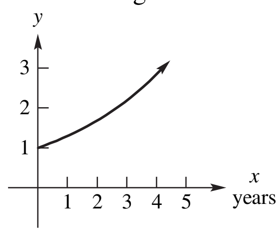
Hours	Bacteria	Expression
0	100,000	$100{,}000\left(\frac{9}{10}\right)^0$
1	90,000	$100{,}000\left(\frac{9}{10}\right)^1$
2	81,000	$100{,}000\left(\frac{9}{10}\right)^2$
3	72,900	$100{,}000\left(\frac{9}{10}\right)^3$
4	65,610	$100{,}000\left(\frac{9}{10}\right)^4$
t		$100{,}000\left(\frac{9}{10}\right)^t$

Thus, in general, the number of bacteria present after t hours is given by $N(t) = 100{,}000\left(\frac{9}{10}\right)^t$.

33. Let P = the amount of plastic recycled and let r = the rate at which P increases each year. Then after the first year, the amount of plastic recycled, increases from P to $P + rP = P(1 + r)$, since $r = 0.3$, the factor by which P increases for the first year, is $1 + r = 1 + 0.3 = 1.3$. Similarly, during the second year, the amount of plastic recycled increases from $P(1 + r)$ to $P(1 + r) + r[P(1+r)] = P(1+r)^2$. Again, since $r = 0.3$, the multiplicative increase for the second year is $(1+r)^2 = (1+0.3)^2 = (1.3)^2 = 1.69$. This pattern will continue as shown in the table.

Year	Multiplicative Increase	Expression
0	1	1.3^0
1	1.3	1.3^1
2	1.69	1.3^2
3	2.20	1.3^3

Thus, the increase in recycling is exponential with a base = 1 + r = 1 + 0.3 = 1.3. If we graph the multiplicative increase as function of years, we obtaining the following.



From the graph it appears that recycling will triple after about 4 years.

34. Population of city A after 5 years:
$270{,}000(1.06)^5 \approx 361{,}321.$
Population of city B after 5 years:
$360{,}000(1.04)^5 \approx 437{,}995.$
Difference in populations: After five years, city B has the larger population. The difference in populations is 437,995 − 361,321 = 76,675.

35. $P = 350{,}000(1-0.015)^t = 350{,}000(0.985)^t$, where P is the population after t years.
When $t = 3$, $P = 350{,}000(0.985)^3 \approx 334{,}485.$

36. $E = 14{,}000(1-0.03)^t = 14{,}000(0.97)^t$, where E is the enrollment after t years. When $t = 12$, $E = 14{,}000(0.97)^{12} \approx 9714.$

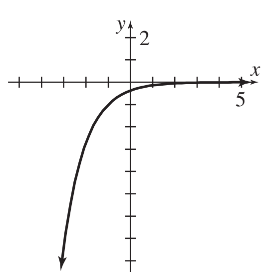
37. 4.4817

38. 29.9641

39. 0.4493

40. 0.5134

41.



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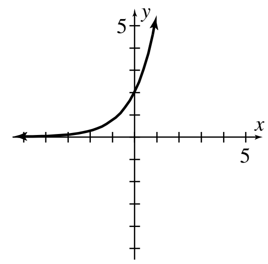


43. For $x = 3$, $P = \dfrac{e^{-3}3^3}{3!} \approx 0.2240$

44. $f(0) \approx 0.399, f(1) \approx 0.242, f(2) \approx 0.054$

45. $e^{kt} = \left(e^k\right)^t = b^t$, where $b = e^k$

46. $\dfrac{1}{e^x} = \left(\dfrac{1}{e}\right)^x = b^x$, where $b = \dfrac{1}{e}$

47. **a.** When $t = 0$, $N = 12e^{-0.031(0)} = 12 \cdot 1 = 12.$

b. When $t = 10$,
$N = 12e^{-0.031(10)} = 12e^{-0.31} = 8.8.$

c. When $t = 44$,
$N = 12e^{-0.031(44)} = 12e^{-1.364} \approx 3.1.$

d. After 44 hours, approximately $\dfrac{1}{4}$ of the initial amount remains. Because $\dfrac{1}{4} = \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)$, 44 hours corresponds to 2 half-lives. Thus the half-life is approximately 22 hours.

48. $N = 75e^{-0.045(10)} \approx 48$

49. After one half-life, $\dfrac{1}{2}$ gram remains. After two half-lives, $\dfrac{1}{2} \cdot \dfrac{1}{2} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ gram remains. Continuing in this manner, after n half-lives, $\left(\dfrac{1}{2}\right)^n$ gram remains. Because $\dfrac{1}{8} = \left(\dfrac{1}{2}\right)^3$, after 3 half-lives, $\dfrac{1}{8}$ gram remains. This corresponds to $3 \cdot 9 = 27$ years.

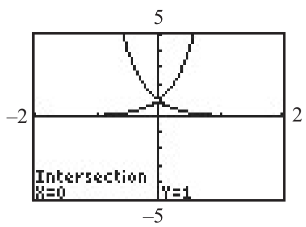
50. $f(x) = \dfrac{e^{-0.5}(0.5)^x}{x!}$

$f(2) = \dfrac{e^{-0.5}(0.5)^2}{2!} \approx 0.0758$

51. $f(x) = \dfrac{e^{-4}4^x}{x!}$

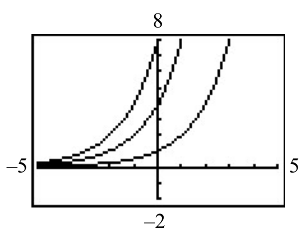
$f(2) = \dfrac{e^{-4}4^2}{2!} \approx 0.1465$

52.



The intersection point is (0, 1).

53.



If $f(x) = 2^x$, then

$y = 2^a \cdot 2^x = 2^{x+a} = f(x+a)$. Thus, the graph of $y = 2^a \cdot 2^x$ is the graph of $y = 2^x$ shifted a units to the left.

54. 0.68

55. 3.17

56. The first integer t for which the graph of $P = 1000(1.07)^t$ lies on or above the horizontal line $P = 3000$ is 17.

57. $300\left(\dfrac{4}{3}\right)^{4.1} \approx 976$

$300\left(\dfrac{4}{3}\right)^{4.2} \approx 1004$

4.2 minutes

58. **a.** When $p = 10$, then

$q = 10{,}000(0.95123)^{10} \approx 6065$.

b. Using a graphics calculator, $0.95123 = e^{-x}$ when $x \approx 0.05$. Thus, $0.95123 \approx e^{-0.05}$.

$q = 10{,}000(0.95123)^p \approx 10{,}000\left(e^{-0.05}\right)^p$.

$= 10{,}000e^{-0.05p}$

c. $q = 10{,}000e^{-0.05(10)} \approx 6065$.

59. The first integer t for which the graph of $P = 2000(1.099)^t$ lies on or above the horizontal line $P = 4000$ is 8 years.

Apply It 4.2

8. If $16 = 2^t$ is the exponential form then $t = \log_2 16$ is the logarithmic form, where t represents the number of times the bacteria have doubled.

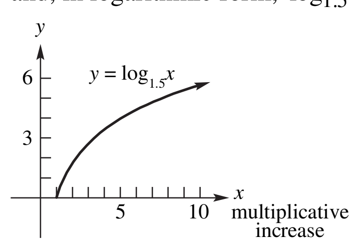
9. If $8.3 = \log_{10}\left(\dfrac{I}{I_0}\right)$ is the logarithmic form, then

$\dfrac{I}{I_0} = 10^{8.3}$ is the exponential form.

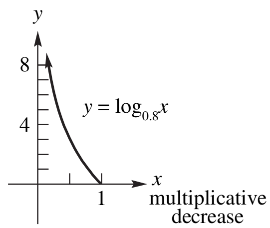
10. Let R = the amount of material recycled every year. If the amount being recycled increases by 50% every year, then the amount recycled at the end of y years is

$R(1+r)^y = R(1+0.5)^y = R(1.5)^y$ Thus, the multiplicative increase in recycling at the end of y years is $(1.5)^y$. If we let

x = the multiplicative increase, then $x = (1.5)^y$ and, in logarithmic form, $\log_{1.5} x = y$.



11. Let V = the value of the boat. If the value depreciates by 20% every year, then at the end of y years the value of the boat is $V(1-r)^y = V(1.02)^y = V(0.8)^y$. Thus, the multiplicative decrease in value at the end of y years is $(0.8)^y$. If we let x = the multiplicative decrease, then $x = (0.8)^y$ and, in logarithmic form, $\log_{0.8} x = y$



12. The equation $t(r) = \frac{\ln 4}{r}$ can be rewritten as $r = \frac{\ln 4}{t(r)}$. When this equation is graphed we find that the annual rate r needed to quadruple the investment in 10 years is approximately 13.9%. Alternatively, we can solve for r by setting $t(r) = 10$.

$r = \frac{\ln(4)}{t(r)}$

$r = \frac{\ln(4)}{10} \approx 0.139$ or $\approx 13.9\%$

13. Since $m = e^{rt}$, then $\ln m = rt$.

$\ln m = rt$

$\frac{\ln m}{t} = r$

Let $m = 3$ and $t = 12$.

$\frac{\ln 3}{12} = r$

$0.092 = r$

Thus, to triple your investment in 12 years, invest at an annual percentage rate of 9.2%.

Problems 4.2

1. $\log 10{,}000 = 4$

2. $(12)^2 = 144$

3. $2^{10} = 1024$

4. $\log_8 4 = \frac{2}{3}$

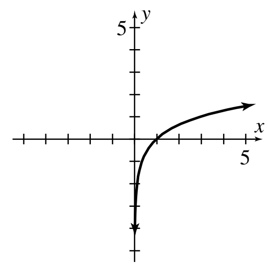
5. $\ln 20.0855 = 3$

6. $\ln 1.4 = 0.33647$

7. $e^{1.09861} = 3$

8. $10^{0.84509} \approx 7$

9.



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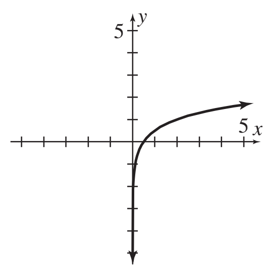

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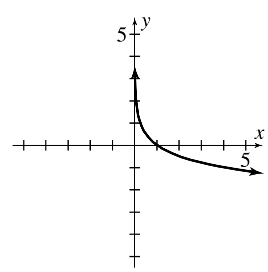

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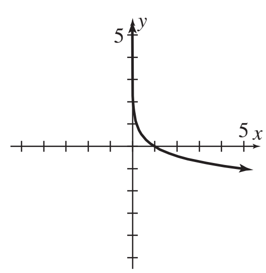

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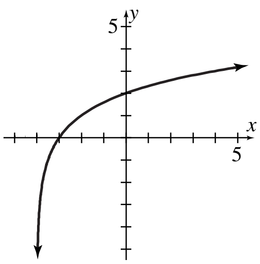

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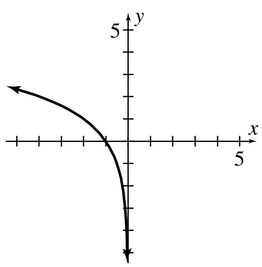

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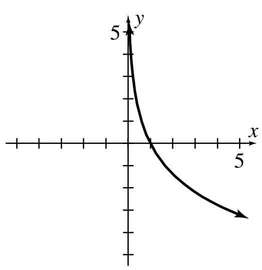

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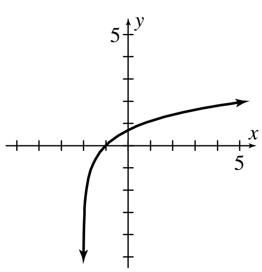


17. Because $6^2 = 36$, $\log_6 36 = 2$

18. Because $2^9 = 512$, $\log_2 512 = 9$.

19. Because $3^3 = 27$, $\log_3 27 = 3$

20. Because $16^{1/2} = 4$, $\log_{16} 4 = \frac{1}{2}$

21. Because $7^1 = 7$, $\log_7 7 = 1$

22. Because $10^4 = 10{,}000$, $\log 10{,}000 = 4$

23. Because $10^{-4} = 0.0001$, $\log 0.0001 = -4$

24. Because $2^{1/3} = \sqrt[3]{2}$, $\log_2 \sqrt[3]{2} = \frac{1}{3}$.

25. Because $5^0 = 1$, $\log_5 1 = 0$

26. Because $5^{-2} = \frac{1}{25}$, $\log_5 \frac{1}{25} = -2$

27. Because $2^{-3} = \frac{1}{8}$, $\log_2 \frac{1}{8} = -3$

28. Because $3^{1/7} = \sqrt[7]{3}$, $\log_3 \sqrt[7]{3} = \frac{1}{7}$.

29. $3^4 = x$
$x = 81$

30. $2^8 = x$
$x = 256$

31. $5^3 = x$
$x = 125$

32. $4^0 = x$
$x = 1$

33. $10^{-3} = x$
$x = \frac{1}{1000}$

34. $e^1 = x$
$x = e$

35. $e^{-3} = x$

36. $x^2 = 25$
Since $x > 0$, we choose $x = 5$.

37. $x^3 = 8$
$x = 2$

38. $x^{1/3} = 4$
$x = 64$

39. $x^{-1} = \frac{1}{6}$
$x = 6$

40. $y = x^1$
$x = y$

41. $3^{-3} = x$
$x = \frac{1}{27}$

42. $x^1 = 2x - 3$
$x = 3$

43. $12 - x = x^2$
$0 = x^2 + x - 12$
$0 = (x+4)(x-3)$
The roots of this equation are –4 and 3. But since $x > 0$, we choose $x = 3$.

44. $\log_8 64 = x - 1$
$8^{x-1} = 64$
$x - 1 = 2$
$x = 3$

45. $2 + \log_2 4 = 3x - 1$
$2 + 2 = 3x - 1$
$5 = 3x$
$x = \frac{5}{3}$

46. $3^{-2} = x + 2$
$\frac{1}{9} = x + 2$
$x = -\frac{17}{9}$

47. $x^2 = 2x + 8$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
The roots of this equation are 4 and –2. But since $x > 0$, we choose $x = 4$.

48. $16 - 4x - x^2 = x^2$
$0 = 2x^2 + 4x - 16$
$0 = x^2 + 2x - 8$
$0 = (x+4)(x-2)$
The roots of this equation are –4 and 2. But since $x > 0$, we choose $x = 2$.

49. $e^{3x} = 2$
$3x = \ln 2$
$x = \frac{\ln 2}{3}$

50. $0.1e^{0.1x} = 0.5$
$e^{0.1x} = 5$
$0.1x = \ln 5$
$x = 10 \ln 5$

51. $e^{2x-5} + 1 = 4$
$e^{2x-5} = 3$
$2x - 5 = \ln 3$
$x = \frac{5 + \ln 3}{2}$

52. $6e^{2x} - 1 = \frac{1}{2}$
$6e^{2x} = \frac{3}{2}$
$e^{2x} = \frac{1}{4}$
$2x = \ln \frac{1}{4}$
$x = \frac{1}{2} \ln \frac{1}{4}$

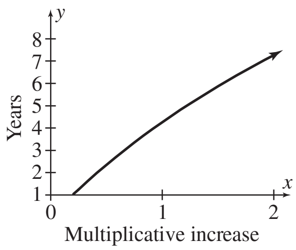
53. 2.39790

54. 1.45161

55. 2.00013

56. 2.30058

57. If V = the value of the antique. If the value appreciates by 10% every year, then at the end of y years the value of the antique is $V(1+r)^y = V(1+0.10)^y = V(1.10)^y$. Thus, the multiplicative increase in value at the end of y years is $(1.10)^y$. If we let x = the multiplicative increase, then $x = (1.10)^y$, and, in logarithm form, $\log_{1.10} x = y$.



58. $c = (5(12)\ln 12) + 15 \approx 164.09$

59. $p = \log\left[10 + \frac{1980}{2}\right] = \log[10+990] = \log 1000$
$= 3$

60. $1.5M = \log\left(\frac{E}{2.5\times 10^{11}}\right)$

$$10^{1.5M} = \frac{E}{2.5\times 10^{11}}$$
$$E = \left(2.5\times 10^{11}\right)\left(10^{1.5M}\right)$$
$$E = 2.5\times 10^{11+1.5M}$$

61. **a.** If $t = k$, then $N = N_0\left(2^1\right) = 2N_0$

b. From part (a), $N = 2N_0$ when $t = k$. Thus k is the time it takes for the population to double.

c. $N_1 = N_0 2^{\frac{t}{k}}$

$$\frac{N_1}{N_0} = 2^{\frac{t}{k}}$$
$$\frac{t}{k} = \log_2 \frac{N_1}{N_0}$$
$$t = k\log_2 \frac{N_1}{N_0}$$

62. $u_0 = A\ln\left(x_1\right) + \frac{x_2^2}{2}$

$$u_0 - \frac{x_2^2}{2} = A\ln\left(x_1\right)$$
$$\ln\left(x_1\right) = \frac{u_0 - \frac{x_2^2}{2}}{A}$$
$$x_1 = e^{\frac{u_0 - \left(x_2^2/2\right)}{A}}$$

63. $T = \frac{\ln 0.25}{-0.01920} \approx 72.2$ minutes

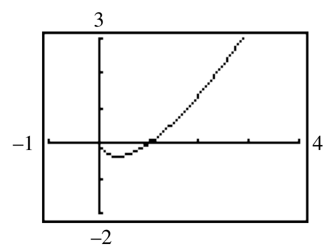
64. $T = \frac{\ln 2}{0.03194} \approx 21.7$ years

65. From $\log_y x = 3$, $y^3 = x$; from $\log_z x = 2$, $z^2 = x$. Thus $z^2 = y^3$ or $z = y^{\frac{3}{2}}$.

66. $x + 3e^{2y} - 8 = 0$

$$3e^{2y} = 8 - x$$
$$e^{2y} = \frac{8-x}{3}$$
$$\ln[e^{2y}] = \ln\left[\frac{8-x}{3}\right]$$
$$2y = \ln\left[\frac{8-x}{3}\right]$$
$$y = \frac{1}{2}\ln\left[\frac{8-x}{3}\right]$$

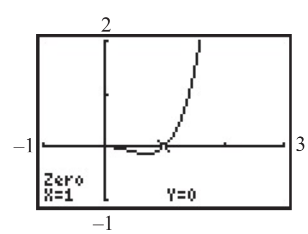
67.



a. (0, 1)

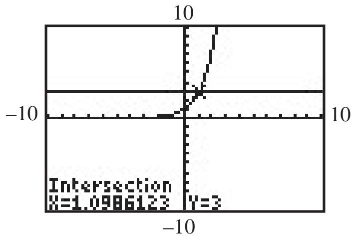
b. [−0.37, ∞)

68.



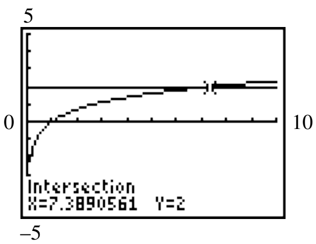
(1, 0)

69. For $y = e^x$, if $y = 3$, then $3 = e^x$ or $x = \ln 3$.



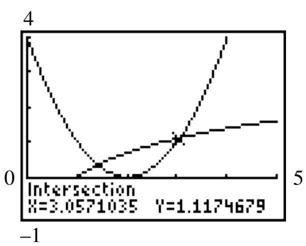
From the graph of $y = e^x$, when $y = 3$, then $x = \ln 3 \approx 1.10$.

70. For $y = \ln x$, when $y = 2$, then $2 = \ln x$ or $x = e^2$.



From the graph of $y = \ln x$, when $y = 2$, then $x = e^2 \approx 7.39$.

71.



1.41, 3.06

Apply It 4.3

14. The magnitude (Richter Scale) of an earthquake is given by $R = \log\left(\frac{I}{I_0}\right)$ where I is the intensity of the earthquake and I_0 is the intensity of a zero-level reference earthquake. $\frac{I}{I_0} =$ how many times greater the earthquake is than a zero-level earthquake. Thus, when $\frac{I}{I_0} = 900{,}000$,

$R_1 = \log(900{,}000)$

When $\frac{I}{I_0} = 9000$

$R_2 = \log(9000)$

$R_1 - R_2 = \log(900{,}000) - \log 9000$

$= \log\frac{900{,}000}{9000} = \log 100 = \log 10^2 = 2\log 10$

$= 2$

Thus, the two earthquakes differ by 2 on the Richter scale.

15. The magnitude (Richter Scale) of an earthquake is given by $R = \log\left(\frac{I}{I_0}\right)$ where I is the intensity of the earthquake and I_0 is the intensity of a zero-level reference earthquake. $\frac{I}{I_0} =$ how many times greater the earthquake is than a zero-level earthquake. Thus, if $\frac{I}{I_0} = 10{,}000$, then

$R = \log 10{,}000 = \log 10^4 = 4\log 10 = 4$

The earthquake measures 4 on the Richter scale.

Problems 4.3

1. $\log 30 = \log(2\cdot 3\cdot 5)$
$= \log 2 + \log 3 + \log 5$
$= a + b + c$

2. $\log 1024 = \log 2^{10} = 10\log 2 = 10a$

3. $\log\frac{2}{3} = \log 2 - \log 3 = a - b$

4. $\log\frac{5}{2} = \log 5 - \log 2 = c - a$

5. $\log\frac{8}{3} = \log 8 - \log 3 = \log 2^3 - \log 3$
$= 3\log 2 - \log 3 = 3a - b$

6. $\log\frac{6}{25} = \log\frac{2\cdot 3}{5^2}$
$= \log 2 + \log 3 - 2\log 5$
$= a + b - 2c$

7. $\log 100 = \log 10^2$
$= 2\log 10$
$= 2\log(2\cdot 5)$
$= 2(\log 2 + \log 5)$
$= 2(a + c)$

8. $\log 0.00003 = \log(3 \cdot 10^{-5})$
$= \log 3 + \log 10^{-5}$
$= \log 3 - 5\log 10$
$= \log 3 - 5\log(2 \cdot 5)$
$= \log 3 - 5(\log 2 + \log 5)$
$= b - 5(a + c)$
$= -5a + b - 5c$

9. $\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2} = \dfrac{\log 3}{\log 2} = \dfrac{b}{a}$

10. $\log_3 5 = \dfrac{\log_{10} 5}{\log_{10} 3} = \dfrac{\log 5}{\log 3} = \dfrac{c}{b}$

11. $\log_7 7^{48} = 48$

12. $\log_{11}\left(11\sqrt[3]{11}\right)^7 = 7\log_{11}(11 \cdot 11^{1/3})$
$= 7(\log_{11} 11 + \log_{11} 11^{1/3})$
$= 7\left(1 + \frac{1}{3}\log_{11} 11\right)$
$= 7\left(1 + \frac{1}{3}\right)$
$= 7\left(\frac{4}{3}\right)$
$= \frac{28}{3}$

or $\log_{11}\left(11\sqrt[3]{11}\right)^7 = \log_{11}(11 \cdot 11^{1/3})^7$
$= \log_{11}(11^{4/3})^7$
$= \log_{11} 11^{28/3}$
$= \frac{28}{3}$

13. $\log 0.0000001 = \log 10^{-7} = -7$

14. $10^{\log 3.4} = 10^{\log_{10} 3.4} = 3.4$

15. $\ln e^{5.01} = \log_e e^{5.01} = 5.01$

16. $\ln e = \log_e e = 1$

17. $\ln \dfrac{1}{\sqrt{e}} = \ln e^{-1/2} = \log_e e^{-1/2} = -\dfrac{1}{2}$

18. $\log_3 81 = \log_3 3^4 = 4$

19. $\log \dfrac{1}{10} + \ln e^3 = \log_{10} \dfrac{1}{10} + \log_e e^3 = -1 + 3 = 2$

20. $e^{\ln \pi} = e^{\log_e \pi} = \pi$

21. $\ln\left[x(x+1)^2\right] = \ln x + \ln(x+1)^2$
$= \ln x + 2\ln(x+1)$

22. $\ln \dfrac{\sqrt[5]{x}}{(x+1)^3} = \ln x^{1/5} - \ln(x+1)^3$
$= \dfrac{1}{5}\ln x - 3\ln(x+1)$

23. $\ln \dfrac{x^2}{(x+1)^3} = \ln x^2 - \ln(x+1)^3$
$= 2\ln x - 3\ln(x+1)$

24. $\ln[x(x+1)]^3 = 3\ln[x(x+1)] = 3[\ln x + \ln(x+1)]$

25. $\ln\left(\dfrac{x+1}{x+2}\right)^4 = 4\ln\dfrac{x+1}{x+2} = 4[\ln(x+1) - \ln(x+2)]$

26. $\ln\sqrt{x(x+1)(x+2)} = \ln[x(x+1)(x+2)]^{1/2}$
$= \dfrac{1}{2}[\ln x(x+1)(x+2)]$
$= \dfrac{1}{2}[\ln x + \ln(x+1)\ln(x+2)]$

27. $\ln\dfrac{x(x+1)}{x+2} = \ln[x(x+1)] - \ln(x+2)$
$= \ln x + \ln(x+1) - \ln(x+2)$

28. $\ln\dfrac{x^2(x+1)}{x+2} = \ln\left[x^2(x+1)\right] - \ln(x+2)$
$= \ln x^2 + \ln(x+1) - \ln(x+2)$
$= 2\ln x + \ln(x+1) - \ln(x+2)$

29. $\ln\dfrac{\sqrt{x}}{(x+1)^2(x+2)^3} = \ln x^{\frac{1}{2}} - \ln\left[(x+1)^2(x+2)^3\right]$
$= \dfrac{1}{2}\ln x - \left[\ln(x+1)^2 + \ln(x+2)^3\right]$
$= \dfrac{1}{2}\ln x - [2\ln(x+1) + 3\ln(x+2)]$
$= \dfrac{1}{2}\ln x - 2\ln(x+1) - 3\ln(x+2)$

30. $\ln \frac{x}{(x+1)(x+2)} = \ln x - [\ln(x+1) + \ln(x+2)]$
$= \ln x - \ln(x+1) - \ln(x+2)$

31. $\ln\left[\frac{1}{x+2}\sqrt[5]{\frac{x^2}{x+1}}\right] = \ln\left[\frac{1}{x+2}\left(\frac{x^2}{x+1}\right)^{\frac{1}{5}}\right]$

$= \ln \frac{x^{\frac{2}{5}}}{(x+2)(x+1)^{\frac{1}{5}}}$

$= \ln x^{\frac{2}{5}} - \ln\left[(x+2)(x+1)^{\frac{1}{5}}\right]$

$= \frac{2}{5}\ln x - \left[\ln(x+2) + \ln(x+1)^{\frac{1}{5}}\right]$

$= \frac{2}{5}\ln x - \ln(x+2) - \frac{1}{5}\ln(x+1)$

32. $\ln\sqrt[4]{\frac{x^2(x+2)^3}{(x+1)^5}} = \frac{1}{4}\ln\frac{x^2(x+2)^3}{(x+1)^5}$

$= \frac{1}{4}[\ln x^2 + \ln(x+2)^3 - \ln(x+1)^5]$

$= \frac{1}{2}\ln x + \frac{3}{4}\ln(x+2) - \frac{5}{4}\ln(x+1)$

33. $\log(6 \cdot 4) = \log 24$

34. $\log_3\left(\frac{10}{5}\right) = \log_3 2$

35. $\log_2 \frac{2x}{x+1}$

36. $\log x^2 - \log\sqrt{x-2} = \log\frac{x^2}{\sqrt{x-2}}$

37. $7\log_3 5 + 4\log_3 17 = \log_3 5^7 + \log_3 17^4$
$= \log_3(5^7 \cdot 17^4)$

38. $5(\log x^2 + \log y^3 - \log z^2)$

$= 5\log\left(\frac{x^2y^3}{z^2}\right)$

$= \log\left[\left(\frac{x^2y^3}{z^2}\right)^5\right]$

39. $\log 100 + \log(1.05)^{10} = \log\left[100(1.05)^{10}\right]$

40. $\frac{1}{2}\left(\log 215 + \log 6^8 - \log 169^3\right) = \frac{1}{2}\log\frac{215\left(6^8\right)}{169^3}$

$= \log\sqrt{\frac{215(6)^8}{169^3}}$

41. $e^{4\ln 3 - 3\ln 4} = e^{\ln 3^4 - \ln 4^3} = e^{\ln\left(\frac{3^4}{4^3}\right)} = \frac{3^4}{4^3} = \frac{81}{64}$

42. $\log_3\left[\ln\left(\sqrt{7+e^3}+\sqrt{7}\right) + \ln\left(\sqrt{7+e^3}-\sqrt{7}\right)\right]$

$= \log_3\left\{\ln\left[\left(\sqrt{7+e^3}+\sqrt{7}\right)\left(\sqrt{7+e^3}-\sqrt{7}\right)\right]\right\}$

$= \log_3[\ln(7+e^3-7)]$

$= \log_3(\ln e^3)$

$= \log_3(3\ln e)$

$= \log_3 3$

$= 1$

43. $\log_6 54 - \log_6 9 = \log_6\frac{54}{9} = \log_6 6 = 1$

44. $\log_3\sqrt{3} + \log_2\sqrt[3]{2} - \log_5\sqrt[4]{5}$

$= \log_3 3^{1/2} + \log_2 2^{1/3} - \log_5 5^{1/4}$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

$= \frac{7}{12}$

45. $e^{\ln(2x)} = 5$

$2x = 5$

$x = \frac{5}{2}$

46. $4^{\log_4(x)+\log_4(2)} = 3$

$4^{\log_4(2x)} = 3$

$2x = 3$

$x = \frac{3}{2}$

47. $10^{\log(x^2+2x)} = 3$

$$x^2 + 2x = 3$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x = -3, 1$$

48. $e^{3\ln x} = 8$

$$e^{\ln x^3} = 8$$
$$x^3 = 8$$
$$x = 2$$

49. From the change of base formula with $b = 2$, $m = 2x + 1$, and $a = e$, we have

$$\log_2(2x+1) = \frac{\log_e(2x+1)}{\log_e 2} = \frac{\ln(2x+1)}{\ln 2}$$

50. From the change of base formula with $b = 3$, $m = x^2 + 2x + 2$ and $a = e$,

$$\log_3(x^2+2x+2) = \frac{\log_e(x^2+2x+2)}{\log_e 3}$$
$$= \frac{\ln(x^2+2x+2)}{\ln 3}$$

51. From the change of base formula with $b = 3$, $m = x^2 + 1$, and $a = e$, we have

$$\log_3\left(x^2+1\right) = \frac{\log_e\left(x^2+1\right)}{\log_e 3} = \frac{\ln\left(x^2+1\right)}{\ln 3}.$$

52. From the change of base formula with $b = 7$, $m = x^2 + 1$, and $a = e$, we have

$$\log_7(x^2+1) = \frac{\log_e(x^2+1)}{\log_e 7} = \frac{\ln(x^2+1)}{\ln 7}$$

53. $e^{\ln z} = 7e^y$

$$z = 7e^y$$
$$\frac{z}{7} = e^y$$
$$y = \ln\frac{z}{7}$$

54. $y = ab^x$ so

$$\log y = \log(ab^x)$$
$$= \log a + \log b^x$$
$$= \log a + x\log b.$$

This is a linear expression because it is in the form $Ax + B$, where $A = \log b$ and $B = \log a$.

55. $C = B + E$

$$C = B\left(1 + \frac{E}{B}\right)$$
$$\ln C = \ln\left[B\left(1+\frac{E}{B}\right)\right]$$
$$\ln C = \ln B + \ln\left(1+\frac{E}{B}\right)$$

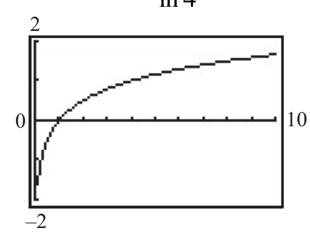
56. $M = \log(A) + 3$

a. $M = \log(10) + 3 = 1 + 3 = 4$

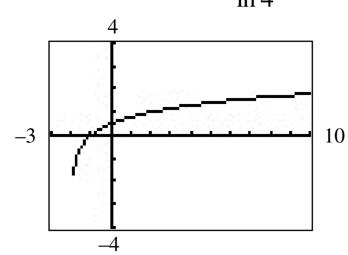
b. Given $M_1 = \log\left(A_1\right) + 3$, let

$$M = \log\left(10A_1\right) + 3$$
$$M = \log 10 + \log\left(A_1\right) + 3$$
$$M = 1 + \left[\log\left(A_1\right) + 3\right]$$
$$M = 1 + M_1$$

57. $y = \log_4 x = \dfrac{\ln x}{\ln 4}$



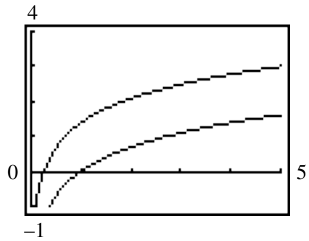
58. $y = \log_4(x+2) = \dfrac{\ln(x+2)}{\ln 4}$



59. By the change of base formula, $\log x = \dfrac{\ln x}{\ln 10}$.

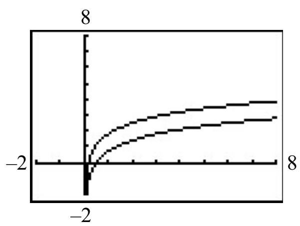
Thus the graphs of $y = \log x$ and $y = \dfrac{\ln x}{\ln 10}$ are identical.

60.



$y = \ln(4x) = \ln 4 + \ln x$. If $f(x) = \ln x$, then $y = \ln(4x) = f(x) + \ln 4$. Thus the graph of $y = \ln(4x)$ is the graph of $y = \ln x$ shifted $\ln 4$ units upward.

61.



$\ln(6x) = \ln(3 \cdot 2x) = \ln 3 + \ln(2x)$.
If $f(x) = \ln(2x)$, then $y = \ln(6x) = f(x) + \ln 3$.
Thus, the graph of $y = \ln(6x)$ is the graph of $y = \ln(2x)$ shifted $\ln 3$ units upward.

Apply It 4.4

16. Let x = the number and let y = the unknown exponent. Then

$$x \cdot 32^y = x \cdot 4^{(3y-9)}$$
$$32^y = 4^{(3y-9)}$$
$$\log 32^y = \log 4^{(3y-9)}$$
$$y \log 32 = (3y - 9) \log 4$$
$$y \log 32 = 3y \log 4 - 9 \log 4$$
$$y(\log 32 - 3 \log 4) = -9 \log 4$$
$$y = \frac{-9\log 4}{\log \frac{32}{4^3}} = \frac{-18\log 2}{\log \frac{1}{2}} = \frac{-18\log 2}{-\log 2}$$
$$y = 18$$

Thus, Greg used 32 to the power of 18.

17. Let $S = 450$.

$$S = 800\left(\frac{4}{3}\right)^{-0.1d}$$
$$450 = 800\left(\frac{4}{3}\right)^{-0.1d}$$
$$\frac{450}{800} = \left(\frac{4}{3}\right)^{-0.1d}$$
$$\log\frac{450}{800} = -0.1d\log\left(\frac{4}{3}\right)$$
$$\frac{\log\frac{450}{800}}{-0.1\log\left(\frac{4}{3}\right)} = d$$
$$20 = d$$

Thus, he should start the new campaign 20 days after the last one ends.

18. The magnitude (Richter Scale) of an earthquake is given by $R = \log\left(\frac{I}{I_0}\right)$ where I is the intensity of the earthquake and I_0 is the intensity of a zero-level reference earthquake. $\frac{I}{I_0} =$ how many times greater the earthquake is than a zero-level earthquake.

$$R_1 = \log(675{,}000)$$
$$R_2 = \log\left(\frac{I}{I_0}\right)$$

Since $R_1 - 4 = R_2$

$$\log(675{,}000) - 4 = \log\left(\frac{I}{I_0}\right)$$
$$\log\left(6.75\times 10^5\right) - 4 = \log\left(\frac{I}{I_0}\right)$$
$$\log 6.75 + 5\log 10 - 4 = \log\left(\frac{I}{I_0}\right)$$
$$1.829 = \log\left(\frac{I}{I_0}\right)$$
$$10^{1.829} = \frac{I}{I_0}$$
$$67.5 = \frac{I}{I_0}$$

Thus, the other earthquake is 67.5 times as intense as a zero-level earthquake.

Problems 4.4

1. $\log(5x+1) = \log(4x+2)$
$5x+1 = 4x+2$
$x = 1$

2. $\log x - \log 5 = \log 7$
$\log x = \log 5 + \log 7$
$\log x = \log 35$
$x = 35$

3. $\log 7 - \log(x-1) = \log 4$
$\log \frac{7}{x-1} = \log 4$
$\frac{7}{x-1} = 4$
$7 = 4x - 4$
$4x = 11$
$x = \frac{11}{4} = 2.75$

4. $\log_2 x + \log_2 2^3 = \log_2 \frac{2}{x}$
$\log_2(8x) = \log_2 \frac{2}{x}$
$8x = \frac{2}{x}$
$8x^2 = 2$
$x^2 = \frac{1}{4}$
$x = \frac{1}{2} = 0.5$ since $x > 0$

5. $\ln(-x) = \ln\left(x^2 - 6\right)$
$-x = x^2 - 6$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x = -3$ or $x = 2$
However, $x = -3$ is the only value that satisfies the original equation.
$x = -3$

6. $\ln(x+3) + \ln 4 = 2\ln x$
$\ln(4x+12) = \ln x^2$
$4x+12 = x^2$
$0 = x^2 - 4x - 12$
$0 = (x-6)(x+2)$
$x = 6$ or -2
However, $x = 6$ is the only value that satisfies the original equation.
$x = 6$

7. $e^{2x}e^{5x} = e^{14}$
$e^{7x} = e^{14}$
$7x = 14$
$x = 2$

8. $(e^{3x-2})^3 = e^3$
$e^{3(3x-2)} = e^3$
$3(3x-2) = 3$
$3x - 2 = 1$
$3x = 3$
$x = 1$

9. $(81)^{4x} = 9$
$(3^4)^{4x} = 3^2$
$3^{16x} = 3^2$
$16x = 2$
$x = \frac{2}{16} = \frac{1}{8} = 0.125$

10. $(27)^{2x+1} = 3^{-1}$
$\left(3^3\right)^{2x+1} = 3^{-1}$
$3^{6x+3} = 3^{-1}$
$6x + 3 = -1$
$6x = -4$
$x = -\frac{2}{3} \approx -0.667$

11. $e^{5x} = 7$
$5x = \ln 7$
$x = \frac{\ln 7}{5} \approx 0.389$

12. $e^{4x} = \frac{3}{4}$
$4x = \ln \frac{3}{4}$
$x = \frac{\ln\left(\frac{3}{4}\right)}{4} \approx -0.072$

13. $2e^{5x+2} = 17$

$e^{5x+2} = \frac{17}{2}$

$5x + 2 = \ln\left(\frac{17}{2}\right)$

$5x = \ln\left(\frac{17}{2}\right) - 2$

$x = \frac{1}{5}\left[\ln\left(\frac{17}{2}\right) - 2\right] \approx 0.028$

14. $5e^{2x-1} - 2 = 23$

$5e^{2x-1} = 25$

$e^{2x-1} = 5$

$2x - 1 = \ln 5$

$x = \frac{1 + \ln 5}{2} \approx 1.305$

15. $10^{\frac{4}{x}} = 6$

$\frac{4}{x} = \log 6$

$x = \frac{4}{\log 6} \approx 5.140$

16. $\frac{2(10)^{0.3x}}{7} = 5$

$10^{0.3x} = \frac{35}{2}$

$0.3x = \log\frac{35}{2}$

$x = \frac{10}{3}\log\frac{35}{2} \approx 4.143$

17. $\frac{5}{10^{2x}} = 7$

$10^{2x} = \frac{5}{7}$

$2x = \log\frac{5}{7}$

$x = \frac{\log\left(\frac{5}{7}\right)}{2} \approx -0.073$

18. $2(10)^x + (10)^{x+1} = 4$

$2(10)^x + 10(10)^x = 4$

$12(10)^x = 4$

$(10)^x = \frac{1}{3}$

$x = \log\frac{1}{3} \approx -0.477$

19. $2^x = 5$

$\ln 2^x = \ln 5$

$x \ln 2 = \ln 5$

$x = \frac{\ln 5}{\ln 2} \approx 2.322$

20. $7^{2x+3} = 9$

$\ln(7^{2x+3}) = \ln 9$

$(2x+3)\ln 7 = \ln 9$

$2x + 3 = \frac{\ln 9}{\ln 7}$

$2x = \frac{\ln 9}{\ln 7} - 3$

$x = \frac{1}{2}\left(\frac{\ln 9}{\ln 7} - 3\right) \approx -0.935$

21. $5^{7x+5} = 2$

$\ln 5^{7x+5} = \ln 2$

$(7x+5)\ln 5 = \ln 2$

$7x + 5 = \frac{\ln 2}{\ln 5}$

$7x = \frac{\ln 2}{\ln 5} - 5$

$x = \frac{1}{7}\left(\frac{\ln 2}{\ln 5} - 5\right) \approx -0.653$

22. $4^{\frac{x}{2}} = 20$

$\ln 4^{\frac{x}{2}} = \ln 20$

$\frac{x}{2}\ln 4 = \ln 20$

$\frac{x}{2} = \frac{\ln 20}{\ln 4}$

$x = \frac{2\ln 20}{\ln 4} \approx 4.322$

23. $2^{-\frac{2x}{3}} = \frac{4}{5}$

$\ln 2^{-\frac{2x}{3}} = \ln \frac{4}{5}$

$-\frac{2x}{3}\ln 2 = \ln \frac{4}{5}$

$-\frac{2x}{3} = \frac{\ln\left(\frac{4}{5}\right)}{\ln 2}$

$x = -\frac{3\ln\left(\frac{4}{5}\right)}{2\ln 2} \approx 0.483$

24. $5\left(3^x - 6\right) = 10$

$3^x - 6 = 2$

$3^x = 8$

$\ln 3^x = \ln 8$

$x \ln 3 = \ln 8$

$x = \frac{\ln 8}{\ln 3} \approx 1.893$

25. $(4)5^{3-x} - 7 = 2$

$5^{3-x} = \frac{9}{4}$

$\ln 5^{3-x} = \ln \frac{9}{4}$

$(3-x)\ln 5 = \ln \frac{9}{4}$

$3 - x = \frac{\ln\left(\frac{9}{4}\right)}{\ln 5}$

$x = 3 - \frac{\ln\left(\frac{9}{4}\right)}{\ln 5} \approx 2.496$

26. $\frac{5}{2^x} = 11$

$\frac{5}{11} = 2^x$

$\ln \frac{5}{11} = \ln 2^x$

$\ln \frac{5}{11} = x \ln 2$

$x = \frac{\ln \frac{5}{11}}{\ln 2} \approx -1.138$

27. $\log(x - 3) = 3$

$10^3 = x - 3$

$x = 10^3 + 3 = 1003$

28. $\log_2(x+1) = 4$

$2^4 = x + 1$

$x = 2^4 - 1 = 15$

29. $\log_4(9x - 4) = 2$

$4^2 = 9x - 4$

$9x = 4^2 + 4$

$x = \frac{4^2 + 4}{9} = \frac{20}{9} \approx 2.222$

30. $\log_4(2x+4) - 3 = \log_4 3$

$\log_4(2x+4) - \log_4 3 = 3$

$\log_4 \frac{2x+4}{3} = 3$

$4^3 = \frac{2x+4}{3}$

$2x + 4 = 3 \cdot 4^3$

$x = \frac{3 \cdot 4^3 - 4}{2} = \frac{188}{2} = 94$

31. $\ln(x-2) + \ln(2x+1) = 5$

$\ln[(x-2)(2x+1)] = 5$

$2x^2 - 3x - 2 = e^5$

$2x^2 - 3x - (2 + e^5) = 0$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(2+e^5)}}{2(2)}$

$= \frac{3 \pm \sqrt{9 - 8(2+e^5)}}{4}$

However, $x = \frac{3 + \sqrt{9 - 8(2+e^5)}}{4} \approx 9.455$ is the only value that satisfies the original equation.

$x \approx 9.455$

32. $\log(x-3)+\log(x-5)=1$

$$\log[(x-3)(x-5)]=1$$
$$x^2-8x+15=10$$
$$x^2-8x+5=0$$
$$x=\frac{8\pm\sqrt{(-8)^2-4(1)(5)}}{2(1)}=4\pm\sqrt{11}$$

However, $x=4+\sqrt{11}\approx 7.317$ is the only value that satisfies the original equation.

$x\approx 7.317$

33. $\log_2(5x+1)=4-\log_2(3x-2)$

$$\log_2(5x+1)+\log_2(3x-2)=4$$
$$\log[(5x+1)(3x-2)]=4$$
$$(5x+1)(3x-2)=2^4$$
$$15x^2-7x-2=16$$
$$15x^2-7x-18=0$$

$x\approx 1.353$ or $x\approx -0.887$

However, $x\approx 1.353$ is the only value that satisfies the original equation.

$x\approx 1.353$

34. $\log(x+2)^2=2$

$2\log(x+2)=2$

$\log(x+2)=1$

$10^1=x+2$

$x=8$

35. $\log_2\left(\frac{2}{x}\right)=3+\log_2 x$

$$\log_2\left(\frac{2}{x}\right)-\log_2 x=3$$
$$\log_2\frac{\frac{2}{x}}{x}=3$$
$$\log_2\frac{2}{x^2}=3$$
$$2^3=\frac{2}{x^2}$$
$$x^2=\frac{1}{4}$$
$$x=\pm\frac{1}{2}$$

However, $x=\frac{1}{2}$ is the only value that satisfies the original equation.

$x=\frac{1}{2}=0.5$

36. $$\log(x+5) = \log(3x+2)+1$$
$$\log(x+5)-\log(3x+2) = 1$$
$$\log\frac{x+5}{3x+2} = 1$$
$$\frac{x+5}{3x+2} = 10^1$$
$$x+5 = 30x+20$$
$$-29x = 15$$
$$x = -\frac{15}{29} \approx -0.517$$

37. $\log S = \log 12.4 + 0.26 \log A$
$$\log S = \log 12.4 + \log A^{0.26}$$
$$\log S = \log\left[12.4A^{0.26}\right]$$
$$S = 12.4A^{0.26}$$

38. $\log T = 1.7 + 0.2068 \log P - 0.1334(\log P)^2$
$$\log T = \log 50 + 0.2068 \log P - 0.1334(\log P)(\log P)$$
$$\log T = \log 50 + 0.2068 \log P + [-0.1334 \log P] \log P$$
$$\log T = \log 50 + \log P^{0.2068} + \log P^{[-0.1334 \log P]}$$
$$\log T = \log\left[(50)\left(P^{0.2068}\right)\left(P^{-0.1334 \log P}\right)\right]$$
$$T = 50P^{0.2068-(0.1334 \log P)}$$
$$(\log_b x)^2 = (\log_b x)(\log_b x) = \log_b(x^{\log_b x})$$

39. a. When $t = 0$, $Q = 100e^{-0.035(0)} = 100e^0 = 100\cdot 1 = 100$.

b. If $Q = 20$, then $20 = 100e^{-0.035t}$. Solving for t gives
$$\frac{20}{100} = e^{-0.035t}$$
$$\frac{1}{5} = e^{-0.035t}$$
$$\ln\frac{1}{5} = -0.035t$$
$$-\ln 5 = -0.035t$$
$$t = \frac{\ln 5}{0.035} \approx 46$$

40. $100 = 225e^{-\frac{N}{225}}$
$$e^{\frac{N}{225}} = \frac{225}{100} = \frac{9}{4}$$
$$\frac{N}{225} = \ln\frac{9}{4}$$
$$N = 225\ln\frac{9}{4} \approx 182$$

41. If P = 2,000,000, then $2,000,000 = 1,500,000(1.03)^t$. Solving for t gives

$$\frac{2,000,000}{1,500,000} = (1.02)^t$$
$$\frac{4}{3} = (1.03)^t$$
$$\ln\frac{4}{3} = \ln(1.03)^t$$
$$\ln\frac{4}{3} = t\ln 1.03$$
$$t = \frac{\ln\frac{4}{3}}{\ln 1.03} \approx 9.7 \text{ years}$$

42. If $F(0) = 0$, then $0 = \dfrac{q - pe^{-C(p+q)}}{q\left[1+e^{C(p+q)}\right]}$. Thus

$$q - pe^{-C(p+q)} = 0$$
$$-pe^{-C(p+q)} = -q$$
$$e^{-C(p+q)} = \frac{q}{p}$$
$$-C(p+q) = \ln\frac{q}{p}$$
$$C = -\frac{1}{p+q}\ln\frac{q}{p}.$$

43. $q = 80 - 2^p$

$$2^p = 80 - q$$
$$\log 2^p = \log(80-q)$$
$$p\log 2 = \log(80-q)$$
$$p = \frac{\log(80-q)}{\log 2}$$

When $q = 60$, then $p = \dfrac{\log 20}{\log 2} \approx 4.32$.

44. The investment doubles when $A = 2P$. Thus $2P = P(1.105)^t$, or $2 = (1.105)^t$. Solving for t gives

$$\ln 2 = \ln(1.105)^t$$
$$\ln 2 = t\ln 1.105$$
$$t = \frac{\ln 2}{\ln 1.105} \approx 7$$

45. $q = 1000\left(\dfrac{1}{2}\right)^{0.8^t}$

$$\log q = \log 1000 + \log\left(\frac{1}{2}\right)^{0.8^t}$$
$$\log q = 3 + 0.8^t\log\frac{1}{2}$$
$$\log q = 3 + 0.8^t(-\log 2)$$
$$\log(q) - 3 = 0.8^t(-\log 2)$$

Thus

$$0.8^t = \frac{\log(q)-3}{-\log 2} = \frac{3-\log q}{\log 2}$$
$$t\log(0.8) = \log\left(\frac{3-\log q}{\log 2}\right).$$
$$t = \frac{\log\left(\frac{3-\log q}{\log 2}\right)}{\log(0.8)}$$

$$y = Ab^{a^x}$$
$$\log y = \log A + \log b^{a^x}$$
$$\log y = \log A + a^x\log b$$
$$\log y - \log A = a^x\log b$$
$$a^x = \frac{\log y - \log A}{\log b}$$
$$\log a^x = \log\left(\frac{\log y - \log A}{\log b}\right)$$
$$x\log a = \log\left(\frac{\log y - \log A}{\log b}\right)$$
$$x = \frac{\log\left(\frac{\log y - \log A}{\log b}\right)}{\log a}$$

The previous solution was the special case $y = q$, $A = 1000$, $b = \dfrac{1}{2}$, $a = 0.8$, and $x = t$.

46. $q = 100\left(1 - e^{-0.1t}\right)$

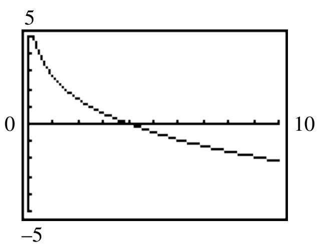
a. If $t = 1$, then $q = 100\left(1 - e^{-0.1}\right) \approx 10$.

b. If $t = 10$, then $q = 100\left(1 - e^{-1}\right) \approx 63$.

c. We solve the equation

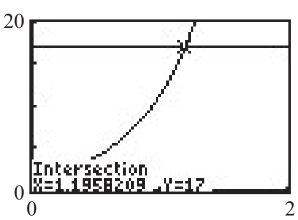
$$80 = 100\left(1 - e^{-0.1t}\right)$$
$$\frac{4}{5} = 1 - e^{-0.1t}$$
$$e^{-0.1t} = \frac{1}{5}$$
$$-0.1t = \ln\frac{1}{5} = -\ln 5$$
$$t = \frac{\ln 5}{0.1} \approx 16$$

47. $\log_2 x = 5 - \log_2(x+4)$ is equivalent to $0 = 5 - \log_2(x+4) - \log_2 x$, or $0 = 5 - \frac{\ln(x+4)}{\ln 2} - \frac{\ln x}{\ln 2}$. Thus the solutions of the original equation are the zeros of the function $y = 5 - \frac{\ln(x+4)}{\ln 2} - \frac{\ln x}{\ln 2}$.



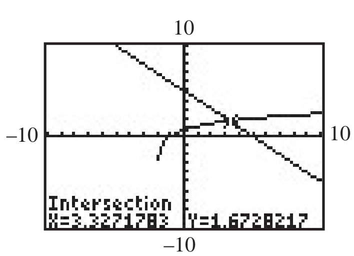
From the graph of this function, the only zero is $x = 4$. Thus 4 is the only solution of the original equation.

48.



1.20

49.

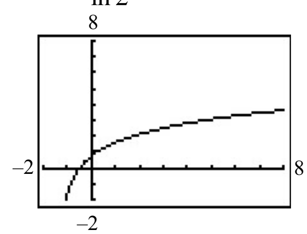


3.33

50. $(3)2^y - 4x = 5$

$$(3)2^y = 4x + 5$$
$$2^y = \frac{4x+5}{3}$$
$$\ln 2^y = \ln\left(\frac{4x+5}{3}\right)$$
$$y \ln 2 = \ln\left(\frac{4x+5}{3}\right)$$
$$y = \frac{\ln\left(\frac{4x+5}{3}\right)}{\ln 2}$$

The graph of the original equation is the graph of $y = \frac{\ln\left(\frac{4x+5}{3}\right)}{\ln 2}$.



Chapter 4 Review Problems

1. $\log_3 243 = 5$

2. $5^4 = 625$

3. $81^{\frac{1}{4}} = 3$

4. $\log 100{,}000 = 5$

5. $\ln 1096.63 \approx 7$

6. $9^1 = 9$

7. Because $5^3 = 125$, $\log_5 125 = 3$

8. Because $4^2 = 16$, $\log_4 16 = 2$

9. Because $3^{-4} = \frac{1}{81}$, $\log_3 \frac{1}{81} = -4$

10. Because $\left(\frac{1}{5}\right)^4 = \frac{1}{625}$, $\log_{1/5} \frac{1}{625} = 4.$

11. Because $\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$, $\log_{1/3} 9 = -2$

12. Because $4^{\frac{1}{2}} = 2$, $\log_4 2 = \frac{1}{2}$

13. $5^x = 625$
$x = 4$

14. $\log_x \frac{1}{81} = -4$
$$x^{-4} = \frac{1}{81}$$
$$\frac{1}{x^4} = \frac{1}{81}$$
$$x^4 = 81$$
$$x = 3$$

15. $\log_2 x = -10$
$$x = 2^{-10}$$
$$x = \frac{1}{1024}$$

16. $e^x = \frac{1}{e} = e^{-1}$
$x = -1$

17. $\ln(2x+3) = 0$
$e^0 = 2x+3$
$1 = 2x+3$
$2x = -2$
$x = -1$

18. Because $e^{\ln(x+4)} = x+4$,
$x + 4 = 7$
$x = 3$

19. $\log 8000 = \log(2 \cdot 10)^3 = 3\log(2 \cdot 10)$
$= 3(\log 2 + \log 10) = 3(a + 1)$

20. $\log \frac{1024}{\sqrt[5]{3}} = \log 1024 - \log \sqrt[5]{3}$
$$= \log 2^{10} - \log 3^{1/5}$$
$$= 10\log 2 - \frac{1}{5}\log 3$$
$$= 10a - \frac{b}{5}$$

21. $3\log 7 - 2\log 5 = \log 7^3 - \log 5^2 = \log \frac{7^3}{5^2}$

22. $5\ln x + 2\ln y + \ln z = \ln x^5 + \ln y^2 + \ln z$
$$= \ln(x^5 y^2 z)$$

23. $2\ln x + \ln y - 3\ln z = \ln x^2 + \ln y - \ln z^3$
$= \ln x^2 y - \ln z^3 = \ln \frac{x^2 y}{z^3}$

24. $\log_6 2 - \log_6 4 - 9\log_6 3$
$= \log_6 2 - \left[\log_6 4 + \log_6 3^9\right]$
$= \log_6 2 - \log_6\left(4 \cdot 3^9\right) = \log_6 \frac{2}{4 \cdot 3^9} = \log_6 \frac{1}{39{,}366}$

25. $\frac{1}{3}\ln x + 3\ln(x^2) - 2\ln(x-1) - 3\ln(x-2)$
$= \ln x^{1/3} + \ln x^6 - \ln(x-1)^2 - \ln(x-2)^3$
$= \ln \frac{x^{1/3}x^6}{(x-1)^2(x-2)^3}$
$= \ln \frac{x^{19/3}}{(x-1)^2(x-2)^3}$

26. $4\log x + 2\log y - 3(\log z + \log w)$
$= \log x^4 + \log y^2 - 3\log zw$
$= \log x^4 + \log y^2 - \log(zw)^3$
$= \log x^4 y^2 - \log z^3 w^3$
$= \log \frac{x^4 y^2}{z^3 w^3}$

27. $\ln \frac{x^3 y^2}{z^{-5}} = \ln x^3 y^2 - \ln z^{-5}$
$$= \ln x^3 + \ln y^2 - \ln z^{-5}$$
$$= 3\ln x + 2\ln y + 5\ln z$$

28. $\ln \frac{\sqrt{x}}{(yz)^2} = \ln \sqrt{x} - \ln(yz)^2 = \ln x^{\frac{1}{2}} - 2\ln(yz)$
$= \frac{1}{2}\ln x - 2(\ln y + \ln z)$

29. $\ln \sqrt[3]{xyz} = \ln(xyz)^{\frac{1}{3}} = \frac{1}{3}\ln(xyz)$
$= \frac{1}{3}(\ln x + \ln y + \ln z)$

30. $\ln\left(\frac{x^4y^3}{z^2}\right)^5 = 5\ln\frac{x^4y^3}{z^2}$
$= 5(\ln x^4 + \ln y^3 - \ln z^2)$
$= 5(4\ln x + 3\ln y - 2\ln z)$
$= 20\ln x + 15\ln y - 10\ln z$

31. $\ln\left[\frac{1}{x}\sqrt{\frac{y}{z}}\right] = \ln\frac{\left(\frac{y}{z}\right)^{1/2}}{x} = \ln\left(\frac{y}{z}\right)^{\frac{1}{2}} - \ln x$
$= \frac{1}{2}\ln\frac{y}{z} - \ln x = \frac{1}{2}(\ln y - \ln z) - \ln x$

32. $\ln\left[\left(\frac{x}{y}\right)^2\left(\frac{x}{z}\right)^3\right] = \ln\frac{x^5}{y^2z^3} = \ln x^5 - \ln y^2z^3$
$= \ln x^5 - \left(\ln y^2 + \ln z^3\right) = 5\ln x - 2\ln y - 3\ln z$

33. $\log_3(x+5) = \frac{\log_e(x+5)}{\log_e 3} = \frac{\ln(x+5)}{\ln 3}$

34. $\log_2(7x^3+5) = \frac{\log_{10}(7x^3+5)}{\log_{10} 2}$
$= \frac{\log(7x^3+5)}{\log 2}$

35. $\log_7 37 = \frac{\log_2 37}{\log_2 7} \approx \frac{5.20945}{2.80735} \approx 1.8556$

36. $\log_4 5 = \frac{\ln 5}{\ln 4} \approx 1.1610$

37. $\ln\left(16\sqrt{3}\right) = \ln 4^2 + \ln\sqrt{3} = 2\ln 4 + \frac{1}{2}\ln 3$
$= 2y + \frac{1}{2}x$

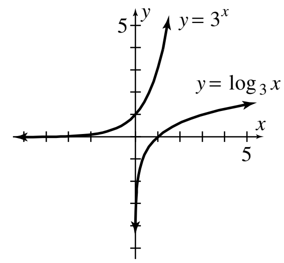
38. $\log\frac{x^3\sqrt[3]{x+1}}{\sqrt[5]{x^2+2}}$
$= \log x^3\sqrt[3]{x+1} - \log\sqrt[5]{x^2+2}$
$= \log x^3 + \log\sqrt[3]{x+1} - \log\sqrt[5]{x^2+2}$
$= 3\log x + \frac{1}{3}\log(x+1) - \frac{1}{5}\log(x^2+2)$

39. $10^{\log x} + \log 10^x + \log 10 = x + x + 1 = 2x + 1$

40. $\log\frac{1}{1000} + \log 1000 = \log 10^{-3} + \log 10^3$
$= -3 + 3$
$= 0$

41. In exponential form, $y = e^{x^2+2}$.

42.



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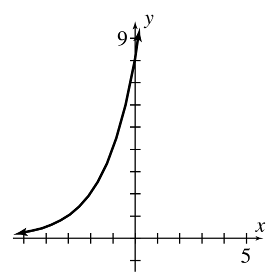

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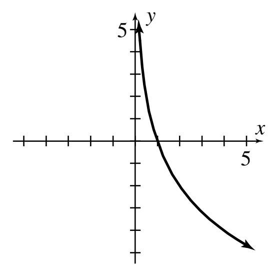


45. $\log(6x-2) = \log(8x-10)$
$6x - 2 = 8x - 10$
$-2x = -8$
$x = 4$

46. $\log 3x + \log 3 = 2$
$\log 9x = 2$
$9x = 10^2$
$9x = 100$
$x = \frac{100}{9}$

47. $3^{4x} = 9^{x+1}$

$3^{4x} = \left(3^2\right)^{x+1}$

$3^{4x} = 3^{2(x+1)}$

$4x = 2(x + 1)$

$4x = 2x + 2$

$2x = 2$

$x = 1$

48. $4^{3-x} = \dfrac{1}{16}$

$4^{3-x} = 4^{-2}$

$3 - x = -2$

$x = 5$

49. $\log x + \log(10x) = 3$

$\log x + \log 10 + \log x = 3$

$2 \log(x) + 1 = 3$

$2 \log(x) = 2$

$\log x = 1$

$x = 10^1 = 10$

50. $\ln\left(\dfrac{x-5}{x-1}\right) = \ln 6$

$\dfrac{x-5}{x-1} = 6$

$x - 5 = 6x - 6$

$-5x = -1$

$x = \dfrac{1}{5}$

51. $\ln(\log_x 3) = 2$

$\log_x 3 = e^2$

$x^{e^2} = 3$

$(x^{e^2})^{-e^2} = 3^{-e^2}$

$x^{e^2 \cdot -e^2} = 3^{-e^2}$

$x^1 = 3^{-e^2}$

$x = \dfrac{1}{3^{e^2}}$

52. $\log_2 x + \log_4 x = 3$

$\log_2 x + \dfrac{\log_2 x}{\log_2 4} = 3$

$\log_2 x + \dfrac{\log_2 x}{2} = 3$

$\dfrac{3}{2}\log_2 x = 3$

$\log_2 x = 2$

$x = 2^2$

$x = 4$

53. $e^{3x} = 14$

$3x = \ln 14$

$x = \dfrac{\ln 14}{3} \approx 0.880$

54. $10^{\frac{3x}{2}} = 5$

$\dfrac{3x}{2} = \log 5$

$x = \dfrac{2}{3}\log 5 \approx 0.466$

55. $5(e^{x+2} - 6) = 10$

$e^{x+2} - 6 = 2$

$e^{x+2} = 8$

$x + 2 = \ln 8$

$x = -2 + \ln 8 \approx 0.079$

56. $7e^{3x-1} - 2 = 1$

$7e^{3x-1} = 3$

$e^{3x-1} = \dfrac{3}{7}$

$3x - 1 = \ln\dfrac{3}{7}$

$3x = \ln\dfrac{3}{7} + 1$

$x = \dfrac{\ln\frac{3}{7} + 1}{3} \approx 0.051$

57. $4^{x+3} = 7$

$\ln 4^{x+3} = \ln 7$

$(x+3)\ln 4 = \ln 7$

$x + 3 = \frac{\ln 7}{\ln 4}$

$x = \frac{\ln 7}{\ln 4} - 3 \approx -1.596$

58. $3^{5/x} = 2$

$\frac{5}{x}\ln 3 = \ln 2$

$x = \frac{5\ln 3}{\ln 2} \approx 7.925$

59. Quarterly rate $= \frac{0.06}{4} = 0.015$

$6\frac{1}{2}$ yr = 26 quarters

a. $2600(1.015)^{26} \approx \$3829.04$

b. $3829.04 - 2600 = \$1229.04$

60. Monthly rate $= \frac{0.12}{12} = 0.01$

5 yr 4 mo = 60 + 4 = 64 months

$2000(1+0.01)^{64} \approx \$3780.92$

61. $12\left(1\frac{1}{6}\%\right) = 14\%$

62. a. $N = 600(1.05)^t$

b. When $t = 1$, $N = 600(1.05)^1 = 630$.

c. When $t = 5$, $N = 600(1.05)^5 \approx 766$.

63. a. $P = 6000[1 + (-0.005)]^t$ or

$P = 6000(0.995)^t$

b. When $t = 10$, then

$P = 6000(0.995)^{10} \approx 5707$.

64. If $t = 2$, $R = 200{,}000e^{-0.4} \approx 134{,}064$

If $t = 3$, $R = 200{,}000e^{-0.6} \approx 109{,}762$

65. $N = 10e^{-0.41t}$

a. When $t = 0$, then $N = 10e^0 = 10 \cdot 1 = 10$ mg

b. When $t = 1$, then $N = 10e^{-0.41} \approx 6.6$ mg

c. When $t = 5$, then $N = 10e^{-2.05} \approx 1.3$ mg

d. $\frac{\ln 2}{0.41} \approx 1.7$ hours

e. If $N = 0.1$, then $0.1 = 10e^{-0.41t}$. Solving for t gives

$\frac{1}{100} = e^{-0.41t}$

$-0.41t = \ln\frac{1}{100} = -\ln 100$

$t = \frac{\ln 100}{0.41} \approx 11.2$ hours

66. Because $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, it will take 3 · 10 = 30 days for $\frac{1}{8}$ of the initial amount to be present.

67. $R = 10e^{-\frac{t}{40}}$

a. If $t = 20$, $R = 10e^{-\frac{20}{40}} = 10e^{-\frac{1}{2}} \approx 6$.

b. $5 = 10e^{-\frac{t}{40}}$, $\frac{1}{2} = e^{-\frac{t}{40}}$. Thus

$-\frac{t}{40} = \ln\frac{1}{2} = -\ln 2$

$t = 40 \ln 2 \approx 28$.

68. Let d = depth in centimeters.

$(0.9)^{\frac{d}{20}} = 0.0017$

$\ln(0.9)^{\frac{d}{20}} = \ln 0.0017$

$\frac{d}{20}\ln 0.9 = \ln 0.0017$

$d = \frac{20\ln 0.0017}{\ln 0.9} \approx 1210$ cm

69. $T_t - T_e = (T_t - T_e)_o e^{-at}$

$$e^{-at} = \frac{T_t - T_e}{(T_t - T_e)_o}$$

$$-at = \ln \frac{T_t - T_e}{(T_t - T_e)_o}$$

$$a = -\frac{1}{t} \ln \frac{T_t - T_e}{(T_t - T_e)_o}$$

$$a = \frac{1}{t} \ln \frac{(T_t - T_e)_o}{T_t - T_e}$$

70. For double-declining balance depreciation, the equation is $V = C\left(1 - \frac{2}{N}\right)^n$.

$$500 = 1500\left(1 - \frac{2}{36}\right)^n$$

$$\frac{500}{1500} = \left(\frac{34}{36}\right)^n$$

$$\frac{1}{3} = \left(\frac{17}{18}\right)^n$$

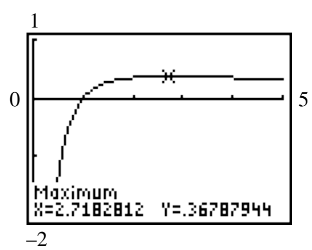
$$\ln \frac{1}{3} = \ln\left(\frac{17}{18}\right)^n$$

$$\ln \frac{1}{3} = n \ln \frac{17}{18}$$

$$n = \frac{\ln \frac{1}{3}}{\ln \frac{17}{18}} \approx 19.22$$

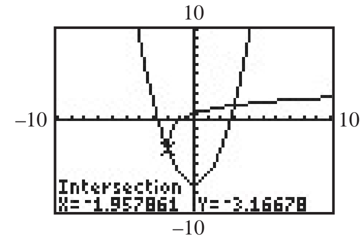
The value drops below \$500 at about 19 months.

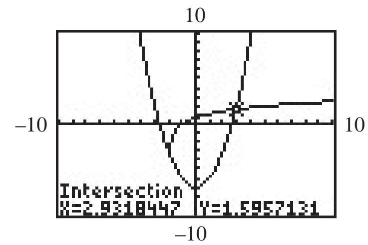
71.



(–∞, 0.37]

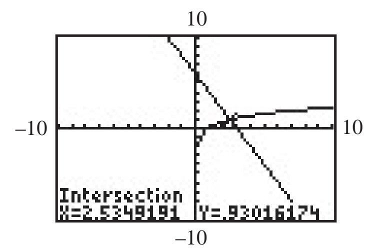
72.





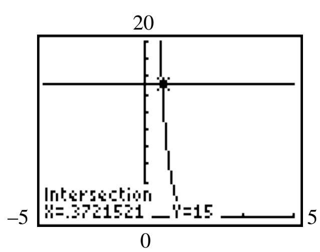
(–1.96, –3.17), (2.93, 1.60)

73.



2.53

74.



0.37

75. f: (0, 1) → (1, ∞) where $f(x) = \frac{1}{x}$ with domain all of (0, 1) has range all of (1, ∞) and has an inverse g defined by the same formula. That is: $g(x) = \frac{1}{x}$ as well. This establishes a one-to-one correspondence between the two kinds of bases.

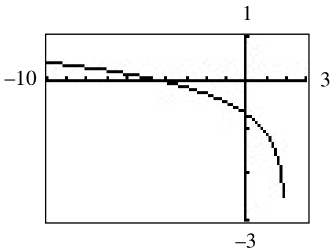
76. $(6)5^y + x = 2$

$$5^y = \frac{2-x}{6}$$

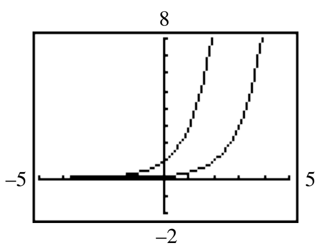
$$\ln 5^y = \ln \frac{2-x}{6}$$

$$y \ln 5 = \ln \frac{2-x}{6}$$

$$y = \frac{\ln \frac{2-x}{6}}{\ln 5}$$



77.



$$y = \frac{3^x}{9} = \frac{3^x}{3^2} = 3^{x-2}.$$

If $f(x) = 3^x$, then we have $y = 3^{x-2} = f(x-2)$.

Thus the graph of $y = \frac{3^x}{9}$ is the graph of $y = 3^x$ shifted 2 units to the right.

Explore and Extend—Chapter 4

1. $T = \dfrac{P\left(1-e^{-dkI}\right)}{e^{kI}-1}$

a. $T\left(e^{kI}-1\right) = P\left(1-e^{-dkI}\right)$

$$\frac{T\left(e^{kI}-1\right)}{1-e^{-dkI}} = P \text{ or } P = \frac{T\left(e^{kI}-1\right)}{1-e^{-dkI}}$$

b. $T\left(e^{kI}-1\right) = P - Pe^{-dkI}$

$$Pe^{-dkI} = P - T\left(e^{kI}-1\right)$$

$$e^{-dkI} = \frac{P - T\left(e^{kI}-1\right)}{P}$$

$$-dkI = \ln\left[\frac{P - T\left(e^{kI}-1\right)}{P}\right]$$

$$d = -\frac{1}{kI}\ln\left[\frac{P - T\left(e^{kI}-1\right)}{P}\right]$$

$$d = \frac{1}{kI}\ln\left[\frac{P}{P - T\left(e^{kI}-1\right)}\right]$$

2. From the text, the half-life H is given by $H = \frac{\ln 2}{k}$ or, equivalently, $k = \frac{\ln 2}{H}$. If $H = I$, then $k = \frac{\ln 2}{I}$. Thus

$$T = \frac{P\left(1-e^{-dkI}\right)}{e^{kI}-1} = \frac{P\left(1-e^{-d\cdot\frac{\ln 2}{I}\cdot I}\right)}{e^{\frac{\ln 2}{I}\cdot I}-1}$$

$$= \frac{P\left(1-\left[e^{\ln 2}\right]^{-d}\right)}{e^{\ln 2}-1} = \frac{P\left(1-2^{-d}\right)}{2-1}$$

$$= P\left(1-2^{-d}\right) = \left(1-\frac{1}{2^d}\right)P.$$

3. $P = 100$, $I = 4$, $d = 3$, $H = 8$, $k = \frac{\ln 2}{H} = \frac{\ln 2}{8}$

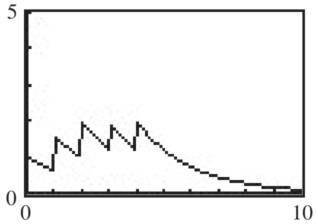
a. $T = \dfrac{P\left(1-e^{-dkI}\right)}{e^{kI}-1} = \dfrac{100\left(1-e^{-3\cdot\frac{\ln 2}{8}\cdot 4}\right)}{e^{\frac{\ln 2}{8}\cdot 4}-1}$

$$= \frac{100\left(1-\left[e^{\ln 2}\right]^{-\frac{3}{2}}\right)}{\left[e^{\ln 2}\right]^{\frac{1}{2}}-1} = \frac{100\left(1-2^{-\frac{3}{2}}\right)}{2^{\frac{1}{2}}-1} \approx 156$$

b. $R = P\left(1 - e^{-dkI}\right)$. From part (a),

$P\left(1 - e^{-dkI}\right) = 100\left(1 - 2^{-\frac{3}{2}}\right)$. Thus

$R = 100\left(1 - 2^{-\frac{3}{2}}\right) \approx 65.$

4. 

As d changes, some of the coefficients need to change from P to Y1 or vice versa.