Derivation

Bresenham Algorithm

$\Rightarrow$ $F(x, y) = mx + c$ when $\boxed{m < 1}$

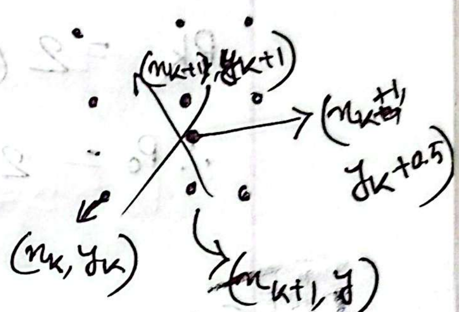
$y = mx + c$

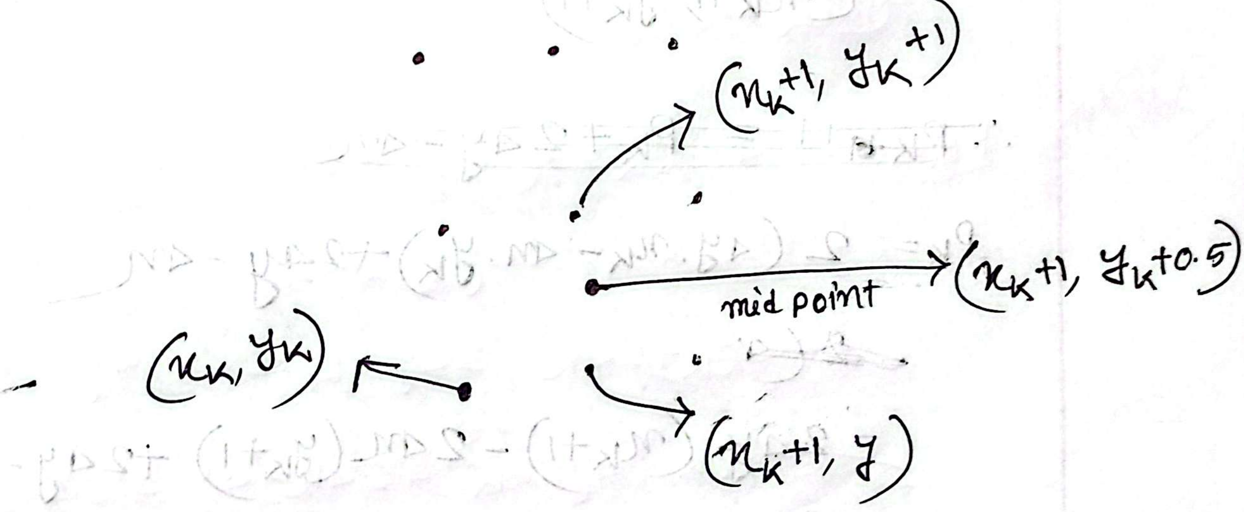
$\Rightarrow y = \frac{\Delta y}{\Delta x} \cdot x + c$

$\Rightarrow y = \frac{\Delta y . x + c . \Delta x}{\Delta x}$

$\Rightarrow y . \Delta x = x . \Delta y + c . \Delta x$

$\Rightarrow x . \Delta y - y . \Delta x + c . \Delta x = 0$





For $(m) > 0 \rightarrow$ line above the mid point choosen pixel (x_k+1, y_k+1)

For $(m) < 0 \rightarrow$ line below the midpoint choosen pixel (x_k+1, y_k)

Now, $F(x_k+1,\ y_k+0.5)$

$$= \Delta y\,(x_k+1) - \Delta x\,(y_k+0.5)$$

$$= \Delta y . x_k + \Delta y - \Delta x . y_k - 0.5\,\Delta x$$

$$= 2\Delta y . x_k + 2\Delta y - 2\Delta x\, y_k - \Delta x$$

$$P_k = 2(\Delta y . x_k - \Delta x . y_k) + 2\Delta y - \Delta x .$$

$\therefore P_0 = 2\Delta y - \Delta x \longrightarrow$ initial point.

So, upper pixel: $P_k \geq 0$

$$(x_k+1,\ y_k+1)$$

$\therefore$ ~~$P_{k+1} = P_k + 2\Delta y - \Delta x$~~

$$P_k = 2(\Delta y . x_k - \Delta x . y_k) + 2\Delta y - \Delta x$$

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$$= 2\Delta y\,(x_k+1) - 2\Delta x\,(y_k+1) + 2\Delta y - \Delta x$$

$$= 2\Delta y . x_k + 2\Delta y - 2\Delta x\, y_k - 2\Delta x + 2\Delta y - \Delta x$$

$$= \underline{2(\Delta y x_k - \Delta x y_k) + 2\Delta y - \Delta x} + 2\Delta y - 2\Delta x$$

$$\therefore \boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x}$$

so, lower pixel: $P_k < 0$

$(x_k+1,\ y_k)$

$$\therefore P_k = 2(\Delta y . x_k - \Delta x . y_k) + 2\Delta y - \Delta x$$

$$= 2\Delta y(x_k+1) - 2\Delta x\, y_k + 2\Delta y - \Delta x$$

$$= 2\Delta y\, x_k + 2\Delta y - 2\Delta x\, y_k + 2\Delta y - \Delta x$$

$$= 2(\Delta y\, x_k - \Delta x\, y_k) + 2\Delta y - \Delta x + 2\Delta y$$

$$\therefore P_{k+1} = P_k + 2\Delta y$$

Again

$f(x,y) = mx + c$ when $\boxed{m > 1}$

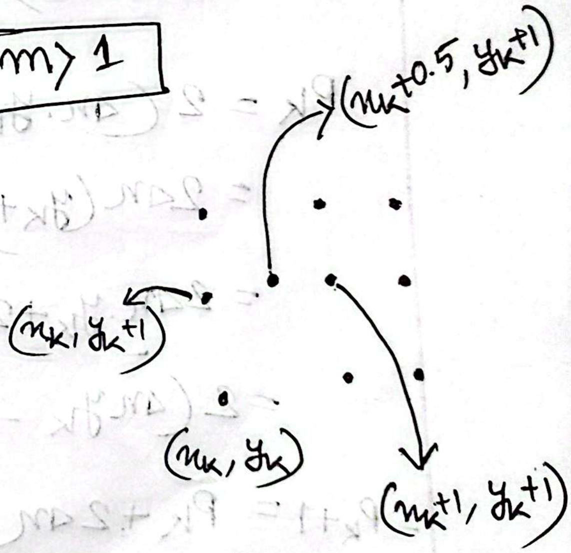
$$\Rightarrow y = mx + c$$

$$\Rightarrow y = \frac{\Delta y}{\Delta x} x + c$$

$$\Rightarrow y = \frac{\Delta y . x \cancel{\Delta x} + c.\Delta x}{\Delta x}$$

$$\Rightarrow y.\Delta x = \Delta y . x + c.\Delta x$$

$$\Rightarrow y.\Delta x - \Delta y . x \cancel{+} - c.\Delta x = 0$$



For $(m) \geq 0$ choose upper pixel $(x_k,\ y_k+1)$

For $(m) < 0$ " lower " $(x_k+1,\ y_k+1)$

For mid point: $(x_k+0.5, y_k+1)$

$$= \Delta x(y_k+1) - \Delta y(x_k+0.5) - c.\Delta x$$

$$= \Delta x . y_k + \Delta x - \Delta y . x_k - 0.5 \Delta y$$

$$= 2\Delta x . y_k + 2\Delta x - 2\Delta y . x_k - \Delta y$$

$$P_k = 2(\Delta x . y_k - \Delta y . x_k) + 2\Delta x - \Delta y$$

$\therefore P_0 = 2\Delta x - \Delta y \longrightarrow$ initial decision parameter

So, upper pixel: $P_k \geq 0$

(x_k, y_k+1)

$$P_k = 2(\Delta x . y_k - \Delta y . x_k) + 2\Delta x - \Delta y$$

$$= 2\Delta x(y_k+1) - 2\Delta y . x_k + 2\Delta x - \Delta y$$

$$= 2\Delta x . y_k + 2\Delta x - 2\Delta y . x_k + 2\Delta x - \Delta y$$

$$= 2(\Delta x y_k - \Delta y . x_k) + 2\Delta x - \Delta y + 2\Delta x$$

$$\therefore P_{k+1} = P_k + 2\Delta x$$

So, lower pixel: $P_k < 0$; (x_k+1, y_k+1)

$$P_k = 2(\Delta x \cdot y_k - \Delta y \cdot x_k) + 2\Delta x - \Delta y$$

$$= 2\Delta x(y_k+1) - 2\Delta y(x_k+1) + 2\Delta x - \Delta y$$

$$= 2\Delta x \cdot y_k + 2\Delta x - 2\Delta y \cdot x_k - 2\Delta y + 2\Delta x - \Delta y$$

$$= 2(\Delta x \cdot y_k - \Delta y \cdot x_k) + 2\Delta x - \Delta y + 2\Delta x - 2\Delta y$$

$$\therefore P_{k+1} = P_k + 2\Delta x - 2\Delta y.$$

CamScanner