

# Travelling Salesman Problem

**Problem Definition:** Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city? Given an undirected weighted Graph  $G$  we are to find a minimum cost Hamiltonian cycle.

**1.Exact Exponential Method:** All the permutations among the cities need to be considered.  
Time complexity  $O(n!)$ .

**2.Branch and bounding Method:** Lower bound calculation:

On a graph  $G = (V, E)$ , After partial tour  $[a, S, b]$  where  $a$  is the root  $b$  is the current node,  $S$  is the path from  $a$  to  $b$ , Lower bound on node  $b$  is the sum of the following

- Cost from  $a$  to  $b$ .
- The lightest edge from  $a$  to  $V - S$ .
- The lightest edge from  $b$  to  $V - S$ .
- The minimum spanning tree of  $V - S$ .

**3.Greedy 2-Approximation Algorithm:** MST is calculated using Prim's algorithm in  $O(n^2)$  time.  
Preorder walk needs  $O(n)$  time. So overall time  $O(n^2)$ .