

Contents

	5.4	Suffix Array	20
1	2	6	21
Number theory		Geometry	
1.1 Bigmod, Inverse Mod	2	6.1 Convex Hull	21
1.2 inclusion-exclusion - number of multiples of divs in [l, r]	2	6.2 Polar Sort	22
1.3 Linear Sieve	2	7	22
1.4 Phi Function[n]/ Phi Sieve[1 to n]	2	Miscellaneous	
1.5 Discrete Logarithm	2	7.1 C++17 Sublime Build	22
1.6 MatExpo	3	7.2 Test Case Generator with FASTIO	22
1.7 CRT	3	7.3 Output Checker Bash Script	23
1.8 Miller Rabin Primality Test	4	7.4 Custom Hash for unordered map	23
1.9 FFT	4	7.5 Release vector memory	23
2	5	7.6 Set of Structure [Operator overload]	23
Data Structure		7.7 Graph and Bitmask Operation	24
2.1 BIT	5	8	24
2.2 Sack	5	Notes	
2.3 Sparse Table	6	8.1 Stars and Bars Theorem	24
2.4 LCA with Sparse Table	6	8.2 GCD	24
2.5 PBDS/Ordered Set	7	8.3 Geometric Formula	24
2.6 Mo's Algo/ Sqrt decomposition	7	8.4 Series/Progression	24
2.7 Centroid Decomposition	8	8.5 Combinatorial formulas	25
2.8 Segment Tree	8		
2.9 Persistent Segment Tree	9		
3	9		
Graph Theory			
3.1 0-1 BFS	9		
3.2 Bellman Ford Algorithm to find Negative Cycle	10		
3.3 Kruskal MST	10		
3.4 Articulation Bridge and Point	11		
3.5 Online Articulation Bridge Finding	11		
3.6 Max Flow(Dinic's Algo)	13		
3.7 Cycle in a Directed Graph	13		
3.8 Max Flow(Dinic's Algo)	14		
3.9 Min Cost Max Flow	15		
3.10 Strongly Connected Components	16		
3.11 2 SAT	17		
4	18		
Dynamic Programming			
4.1 LIS	18		
4.2 SOS DP	18		
4.3 Digit DP	18		
5	19		
String			
5.1 Hashing	19		
5.2 Trie	19		
5.3 Z- Algo //0-based Indexing	20		

1 Number theory

1.1 Bigmod, Inverse Mod

```

1 ll bigmod(ll a, ll b, ll mod){
2     ll res = 1;
3     while (b > 0){
4         if (b & 1) res = (res * a) % mod;
5         a = (a * a) % mod;
6         b >>= 1;
7     }
8     return res;
9 }
10
11 ll inverse_mod(ll a, ll b) {
12     return 1 < a ? b - inverse_mod(b % a, a) * b / a : 1;
13 }
14 ll inv[N]; // inverse modulo pre calculate
15 void imod() {
16     inv[1] = 1;
17     for (ll i = 2; i < N; i++) inv[i] = (mod - (mod / i) * inv[mod % i]) % mod;
18 }
19 // another way to find inverse modulo of n is
20 ll inv_of_n = bigmod(n, mod - 2, mod);

```

1.2 inclusion-exclusion - number of multiples of divs in [l, r]

```

1 ll iep(int l, int r) {
2     if (l > r) return 0;
3     ll sum = 0, sz = divs.size();
4     for (ll j = 1; j < (1LL << sz); j++) {
5         ll gun = 1, one = 0;
6         for (ll i = 0; i < sz; i++)
7             if (j & (1LL << i)) {
8                 one++; gun *= divs[i];
9             }
10        ll mult = (r / gun) - ((l - 1) / gun);
11        if (one % 2 == 1) sum += mult;
12        else sum -= mult;
13    }
14    return sum;
15 }

```

1.3 Linear Sieve

```

1 const ll N = 10000000;
2 vector<ll> lp(N+1), pr;

```

```

3
4 for (ll i=2; i <= N; ++i) {
5     if (lp[i] == 0) {
6         lp[i] = i; pr.push_back(i);
7     }
8     for (ll j=0; j < (ll)pr.size() && pr[j] <= lp[i] && i*pr[j] <= N; ++j) {
9         lp[i * pr[j]] = pr[j];
10    }
11 }

```

1.4 Phi Function[n]/ Phi Sieve[1 to n]

```

1 int phi(int n) {
2     int result = n;
3     for (int i = 2; i * i <= n; i++) {
4         if (n % i == 0) {
5             while (n % i == 0) n /= i;
6             result -= result / i;
7         }
8     }
9     if (n > 1)
10        result -= result / n;
11    return result;
12 }
13
14 void phi_1_to_n(int n) {
15     vector<int> phi(n + 1);
16     for (int i = 0; i <= n; i++)
17         phi[i] = i;
18
19     for (int i = 2; i <= n; i++)
20         if (phi[i] == i)
21             for (int j = i; j <= n; j += i)
22                 phi[j] -= phi[j] / i;
23 }

```

1.5 Discrete Logarithm

```

1 int a,b,m;
2 ///this find the minimum x.
3 int discreet_log(int a,int b,int m)
4 {
5     if(a==0)
6         return b==0?1:-1;
7     a=a%m,b=b%m;
8     int n=sqrt(m)+1;
9     int res=1;
10    for(int i=0; i<n; i++)

```

```

11     res=(res*a)%m;///a^n
12     unordered_map<int,int>vals;///O(1)
13     int cur=b;
14     for(int q=0; q<n; q++)///a^q
15     {
16         vals[cur]=q;
17         cur=(cur*a)%m;
18     }
19     cur=1;
20     for(int p=1; p<=n; p++)
21     {
22         cur=(cur*res)%m;
23         if(vals.count(cur))///O(1)
24             return n*p-vals[cur];
25     }
26     return -1;
27 }
28 ///this works only if gcd(a,m)=1.
29 ///Best implementation.....O(sqrt(m)).....using hashmap.
30 /**
31 a^x=b(mod m)
32 find x
33 or, find x=loga(b)(mod m)
34 log a base b mod m
35 */
36 cin>>a>>b>>m;
37 int ans=discreet_log(a,b,m);
38 cout<<ans<<endl;

```

```

19         product.a[i][k] %= mod;
20     }
21 }
22 }
23     return product;
24 }
25 };
26 Matrix expo_power(Matrix a, long long n) {
27     Matrix res = Matrix();
28     for (ll i = 0; i < sz; i++) res.a[i][i] = 1;
29     while(n) {
30         if(n % 2) {
31             res = res * a;
32         }
33         n /= 2;
34         a = a * a;
35     }
36     return res;
37 }
38 int main() {
39     sz = 2; ll k = 7;
40     Matrix single = Matrix();
41     single.a[0][0] = 1; single.a[0][1] = 1; single.a[1][0] = 1; single.a[1][1]
↪ = 0;
42     Matrix ans = expo_power(single,k);
43 }
44
45

```

1.6 MatExpo

```

1  const ll mod = 1e9 + 7;
2  #define REP(i, n) for(int i = 0; i < (n); i++)
3  ll sz;
4  struct Matrix {
5      vector<vector<ll>> a = vector<vector<ll>>(sz,vector<ll>(sz));
6      Matrix() {
7          REP(i, sz) {
8              REP(j, sz) {
9                  a[i][j] = 0;
10             }
11         }
12     }
13     Matrix operator *(Matrix other) {
14         Matrix product = Matrix();
15         REP(i, sz) {
16             REP(j, sz) {
17                 REP(k, sz) {
18                     product.a[i][k] += a[i][j] *
↪ other.a[j][k];

```

1.7 CRT

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  #define ll long long
4
5  const int N = 20;
6  long long GCD(long long a, long long b) { return (b == 0) ? a : GCD(b, a % b); }
7  inline long long LCM(long long a, long long b) { return a / GCD(a, b) * b; }
8  inline long long normalize(long long x, long long mod) { x %= mod; if (x < 0) x
↪ += mod; return x; }
9  struct GCD_type { long long x, y, d; };
10 GCD_type ex_GCD(long long a, long long b)
11 {
12     if (b == 0) return {1, 0, a};
13     GCD_type pom = ex_GCD(b, a % b);
14     return {pom.y, pom.x - a / b * pom.y, pom.d};
15 }
16

```

```

17 pair <ll, ll> crt(vector <ll> &a, vector <ll> &n) { // 1 based indexing
18     ll ans = a[1];
19     ll lcm = n[1];
20     for (int i = 2; i < a.size(); i++)
21     {
22         auto pom = ex_GCD(lcm, n[i]);
23         int x1 = pom.x;
24         int d = pom.d;
25         if ((a[i] - ans) % d != 0) return { -1, -1}; // no solution
26
27         ans = normalize(ans + x1 * (a[i] - ans) / d % (n[i] / d) * lcm, lcm *
↪ n[i] / d);
28         lcm = LCM(lcm, n[i]); // you can save time by replacing above lcm * n[i]
↪ /d by lcm = lcm * n[i] / d
29     }
30
31     return {ans, lcm};
32 }
33
34 int main()
35 {
36     ios_base::sync_with_stdio(0); cin.tie(0);
37
38     int t; cin >> t;
39     vector <ll> a(t + 1), n (t + 1);
40     for (int i = 1; i <= t; i++) {
41         cin >> a[i] >> n[i];
42         normalize(a[i], n[i]);
43     }
44
45     return 0;
46 }

```

```

15     while (n) {
16         if (n & 1) ret = mulmod(ret, x, MOD);
17         x = mulmod(x, x, MOD);
18         n >>= 1;
19     }
20     return ret % MOD;
21 }
22 bool isPrime(ll n) {
23     if (n == 2 || n == 3) return true;
24     if (n == 1 || !(n & 1)) return false;
25     ll d = n - 1;
26     int s = 0;
27     while (d % 2 == 0) {
28         s++;
29         d /= 2;
30     }
31
32     int a[9] = { 2, 3, 5, 7, 11, 13, 17, 19, 23 };
33     for (int i = 0; i < 9; i++) {
34         if (n == a[i]) return true;
35         bool comp = fastPow(a[i], d, n) != 1;
36         if (comp) for (int j = 0; j < s; j++) {
37             ll fp = fastPow(a[i], (1LL << (11)j) * d, n);
38             if (fp == n - 1) {
39                 comp = false;
40                 break;
41             }
42         }
43         if (comp) return false;
44     }
45     return true;
46 }

```

1.8 Miller Rabin Primality Test

```

1  /* Miller Rabin Primality Test for <= 10^18 */
2  #define ll long long
3
4  ll mulmod(ll a, ll b, ll c) {
5      ll x = 0, y = a % c;
6      while (b) {
7          if (b & 1) x = (x + y) % c;
8          y = (y << 1) % c;
9          b >>= 1;
10     }
11     return x % c;
12 }
13 ll fastPow(ll x, ll n, ll MOD) {
14     ll ret = 1;

```

1.9 FFT

```

1  /**
2   * Multiply (7x^2 + 8x^1 + 9x^0) with (6x^1 + 5x^0)
3   * ans = 42x^3 + 83x^2 + 94x^1 + 45x^0
4   * A = {9, 8, 7}
5   * B = {5, 6}
6   * V = multiply(A,B)
7   * V = {45, 94, 83, 42}
8   */
9  /** Tricks
10   * Use vector < bool > if you need to check only the status of the sum
11   * Use bigmod if the power is over same polynomial && power is big
12   * Use long double if you need more precision
13   * Use long long for overflow
14   */

```

```

15 typedef vector<int> vi;
16 const double PI = 2.0 * acos(0.0);
17 using cd = complex<double>;
18 void fft(vector<cd> &a, bool invert = 0) {
19     int n = a.size();
20     for (int i = 1, j = 0; i < n; i++) {
21         int bit = n >> 1;
22         for (; j & bit; bit >>= 1)
23             j ^= bit;
24         j ^= bit;
25
26         if (i < j)
27             swap(a[i], a[j]);
28     }
29     for (int len = 2; len <= n; len <= 1) {
30         double ang = 2 * PI / len * (invert ? -1 : 1);
31         cd wlen(cos(ang), sin(ang));
32         for (int i = 0; i < n; i += len) {
33             cd w(1);
34             for (int j = 0; j < len / 2; j++) {
35                 cd u = a[i + j], v = a[i + j + len / 2] * w;
36                 a[i + j] = u + v;
37                 a[i + j + len / 2] = u - v;
38                 w *= wlen;
39             }
40         }
41     }
42     if (invert) {
43         for (cd &x : a)
44             x /= n;
45     }
46 }
47
48 void ifft(vector<cd> &p) { fft(p, 1); }
49
50 vi multiply(vi const &a, vi const &b) {
51     vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
52     int n = 1;
53     while (n < a.size() + b.size())
54         n <= 1;
55     fa.resize(n);
56     fb.resize(n);
57
58     fft(fa);
59     fft(fb);
60     for (int i = 0; i < n; i++)
61         fa[i] *= fb[i];
62     ifft(fa);
63
64     vi result(n);
65     for (int i = 0; i < n; i++)

```

```

66         result[i] = round(fa[i].real());
67     return result;
68 }

```

2 Data Structure

2.1 BIT

```

1 11 bit[N]; //Add(0)--> RTE
2 void Add(11 x,11 v) { for(;x < N;x += x & (-x)) bit[x] = (bit[x] + v) ; }
3 11 Sum(11 x) { 11 r = 0; for(;x;x -= x & (-x)) r = (r + bit[x]) ; return r; }
4

```

2.2 Sack

```

1 int cnt[maxn];
2 bool big[maxn];
3 void add(int v, int p, int x){
4     cnt[ col[v] ] += x;
5     for(auto u: g[v])
6         if(u != p && !big[u])
7             add(u, v, x)
8 }
9 void dfs(int v, int p, bool keep){
10     int mx = -1, bigChild = -1;
11     for(auto u : g[v])
12         if(u != p && sz[u] > mx)
13             mx = sz[u], bigChild = u;
14     for(auto u : g[v])
15         if(u != p && u != bigChild)
16             dfs(u, v, 0); // run a dfs on small childs and clear them from cnt
17     if(bigChild != -1)
18         dfs(bigChild, v, 1), big[bigChild] = 1; // bigChild marked as big and
↪ not cleared from cnt
19     add(v, p, 1);
20     //now cnt[c] is the number of vertices in subtree of vertex v that has color
↪ c. You can answer the queries easily.
21     if(bigChild != -1)
22         big[bigChild] = 0;
23     if(keep == 0)
24         add(v, p, -1);
25 }

```

2.3 Sparse Table

```

1 #include <bits/stdc++.h>
2 #define ll long long
3 using namespace std;
4
5 const ll N = 2e5 + 20, M = 20;
6 ll n, q, a[N], spt[M][N], lg2[N];
7
8 ll f(int i, int j) { // do the main operation here
9     return spt[i - 1][j] + spt[i - 1][j + (1 << (i - 1))];
10 }
11
12 void build() {
13     lg2[1] = 0;
14     for (int i = 2; i < N; i++)
15         lg2[i] = lg2[i / 2] + 1;
16     for (int i = 0; i < n; i++)
17         spt[0][i] = a[i];
18     for (int i = 1; i < M; i++) {
19         for (int j = 0; j + (1 << i) <= n; j++) {
20             spt[i][j] = f(i, j);
21         }
22     }
23 }
24
25 ll querySum(int l, int r) {
26     ll sum = 0;
27     for (int i = M - 1; i >= 0; i--) {
28         if ((1 << i) <= r - l + 1) {
29             sum += spt[i][l];
30             l += (1 << i);
31         }
32     }
33     return sum;
34 }
35
36 ll queryMin(int l, int r) {
37     int k = lg2[r - l + 1];
38     return min(spt[k][l], spt[k][r - (1 << k) + 1]);
39 }
40
41 int main() {
42     ios_base::sync_with_stdio(0); cin.tie(0);
43
44     cin >> n >> q;
45     for (int i = 0; i < n; i++) cin >> a[i];
46
47     build();
48

```

```

49
50     while (q--) {
51         int l, r;
52         cin >> l >> r;
53         cout << querySum(l - 1, r - 1) << '\n';
54     }
55 }

```

2.4 LCA with Sparse Table

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 2e5, M = 20;
5 int n, level[N], par[N], root, spt[M][N];
6 vector<int> g[N];
7
8 void dfs(int u, int v, int c) {
9     par[v] = u;
10    level[v] = c;
11
12    for(int &z : g[v]) if(z != u) dfs(v, z, c + 1);
13 }
14
15 void lca_build() {
16     memset(spt, -1, sizeof spt);
17
18     for(int i = 1; i <= n; i++) spt[0][i] = par[i];
19     for(int i = 1; i < M; i++) {
20         for(int j = 1; j <= n; j++) {
21             int p = spt[i - 1][j];
22             if(p >= root) spt[i][j] = spt[i - 1][p]; // 2^i-th parent of j
23         }
24     }
25 }
26
27 int lca_of(int u, int v) {
28     int p = u, q = v;
29     if(level[p] > level[q]) swap(p, q); // lets level[p] <= level[q]
30     int d = level[q] - level[p];
31     for(int i = M - 1; i >= 0 and d; i--) {
32         if(d >= (1 << i)) {
33             d -= 1 << i;
34             q = spt[i][q];
35         }
36     }
37     // now level[p] = level[q]
38     if(p == q) return p;
39

```

```

40     for(int i = M - 1; i >= 0; i--) {
41         if(spt[i][p] >= root and spt[i][p] != spt[i][q]) {
42             p = spt[i][p];
43             q = spt[i][q];
44         }
45     }
46     return spt[0][p];
47 }
48
49 int main()
50 {
51     ios_base::sync_with_stdio(0); cin.tie(0);
52
53     root = 1;
54     cin >> n;
55     for(int i = 0; i < n - 1; i++) {
56         int u, v;
57         cin >> u >> v;
58         g[u].push_back(v);
59         g[v].push_back(u);
60     }
61
62     dfs(-1, root, 0);
63     lca_build();
64
65     /*for(int i = 0; i < n; i++) cout << i << ' ' << par[i] << endl;
66     for(int i = 0; i < 4; i++) {
67         cout << "parent level = " << (1 << i) << endl;
68         for(int j = 0; j < n; j++)
69             cout << j << ' ' << spt[i][j] << endl;
70     }*/
71
72     int q; cin >> q;
73     while(q--) {
74         int u, v;
75         cin >> u >> v;
76         cout << lca_of(u, v) << '\n';
77     }
78
79     return 0;
80 }

```

2.5 PBDS/Ordered Set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 template <typename T> using ordered_set = tree<T, null_type, less<T>,
    ↪ rb_tree_tag, tree_order_statistics_node_update>;

```

```

5
6     ordered_set <pair<int,int>> st;
7     st.insert({10, 1});
8
9     cout << (st.find_by_order(2) -> first) << endl; //print element in k-th
    ↪ index
10    cout << st.order_of_key({10, 2}) << endl; //print number of items < k
11    auto print = [&] () {
12        for (auto z : st) cout << z.first << ' ';
13    };
14    print();
15    st.erase(st.lower_bound({0, 1}));
16
17    // #define ordered_set tree<int, null_type, less<int>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>
18    // #define ordered_set tree<int, null_type, less_equal<int>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>
19    //less than or equal -> use less_equal<int>
20    // #define ordered_set tree<pair<int,int>, null_type, pair<int,int>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>
21    //to use multiset -> use pair<int, int> where first element is value and second
    ↪ element is index
22

```

2.6 Mo's Algo/ Sqrt decomposition

```

1 void remove(idx); // TODO: remove value at idx from data structure
2 void add(idx);    // TODO: add value at idx from data structure
3 int get_answer(); // TODO: extract the current answer of the data structure
4 int block_size;
5 struct Query {
6     int l, r, idx;
7     bool operator<(Query other) const
8     {
9         return make_pair(l / block_size, r) <
10            make_pair(other.l / block_size, other.r);
11     }
12 };
13 vector<int> mo_s_algorithm(vector<Query> queries) {
14     vector<int> answers(queries.size());
15     sort(queries.begin(), queries.end());
16
17     // TODO: initialize data structure
18
19     int cur_l = 0;
20     int cur_r = -1;
21     // invariant: data structure will always reflect the range [cur_l, cur_r]
22     for (Query q : queries) {
23         while (cur_l > q.l) {

```

```

24         cur_l--;
25         add(cur_l);
26     }
27     while (cur_r < q.r) {
28         cur_r++;
29         add(cur_r);
30     }
31     while (cur_l < q.l) {
32         remove(cur_l);
33         cur_l++;
34     }
35     while (cur_r > q.r) {
36         remove(cur_r);
37         cur_r--;
38     }
39     answers[q.idx] = get_answer();
40 }
41 return answers;
42 }
43

```

```

27         if(nd == p || del[nd] || sz[nd] <= curSize / 2)
28             continue;
29
30         return findCentroid(nd, u);
31     }
32     return u;
33 }
34
35 void decompose(ll u, ll p)
36 {
37     dfs(u, -1);
38     curSize = sz[u];
39     ll cen = findCentroid(u, -1);
40     if(p == -1) p = cen;
41     par[cen] = p, del[cen] = 1;
42
43     for(ll i = 0; i < g[cen].size(); i++) {
44         ll nd = g[cen][i];
45
46         if(!del[nd])
47             decompose(nd, cen);
48     }
49 }
50

```

2.7 Centroid Decomposition

```

1  /// decompose(1, -1)    //For 1 rooted tree
2
3  #define ll long long
4  #define pb push_back
5  const ll MAX = 1e5;
6  vector<ll> g[MAX + 9];
7  ll del[MAX + 9], sz[MAX + 9], par[MAX + 9], curSize;
8
9  void dfs(ll u, ll p)
10 {
11     sz[u] = 1;
12     for(ll i = 0; i < g[u].size(); i++) {
13         ll nd = g[u][i];
14         if(nd == p || del[nd])
15             continue;
16         dfs(nd, u);
17         sz[u] += sz[nd];
18     }
19 }
20
21 ll findCentroid(ll u, ll p)
22 {
23     for(ll i = 0; i < g[u].size(); i++) {
24         ll nd = g[u][i];
25
26

```

2.8 Segment Tree

```

1  const int N = 1e5 + 5;
2  int n, q, a[N], sum[4 * N];
3
4  void build(int i, int l, int r) {
5      if(l == r) {
6          sum[i] = a[l];
7          return;
8      }
9
10     int mid = (l + r) / 2;
11     build(2 * i, l, mid);
12     build(2 * i + 1, mid + 1, r);
13     sum[i] = sum[2 * i] + sum[2 * i + 1];
14 }
15
16 int query(int i, int left, int right, int l, int r) {
17     if(r < left or l > right) return 0;
18     if(l >= left and r <= right) return sum[i];
19
20     int mid = (l + r) / 2;
21     return query(2 * i, left, right, l, mid) + query(2 * i + 1, left, right,
↵ mid + 1, r);

```



```

22 }
23
24 void update(int i, int p, int x, int l, int r) {
25     if(r < p or l > p) return;
26     if(l == r) {
27         sum[i] += x;
28         return;
29     }
30
31     int mid = (l + r) / 2;
32     update(2 * i, p, x, l, mid);
33     update(2 * i + 1, p, x, mid + 1, r);
34     sum[i] = sum[2 * i] + sum[2 * i + 1];
35 }
36
37 /*
38 call build(1, 1, n); // 1 based array
39 query(1, l, r, 1, n)
40 update(1, i, value, 1, n)
41 */
42

```

2.9 Persistent Segment Tree

```

1  // Count of numbers of [a,b] range in [L, R] index
2  #define nsz 100010
3  #define tsz 6000010 //take 4n + qlongn
4  ll a[nsz];
5  ll NEXT_FREE;
6  ll version[nsz];
7  ll val[tsz], Left[tsz], Right[tsz];
8  void build(ll node, ll lo, ll hi)
9  {
10     if(lo == hi) /// leaf node
11     {
12         val[node] = 0;
13         return;
14     }
15     Left[node] = NEXT_FREE++;
16     Right[node] = NEXT_FREE++;
17     ll mid = (lo + hi) >> 1;
18     build(Left[node], lo, mid);
19     build(Right[node], mid+1, hi);
20     val[node] = val[Left[node]] + val[Right[node]];
21 }
22 ll update(ll node, ll lo, ll hi, ll idx, ll v)
23 {
24     if(lo > idx || hi < idx)
25         return node; /// Out of range, use this node.

```

```

26     ll nnode = NEXT_FREE++; ///Creating a new node, as idx is in [l, r]
27
28     if (lo == hi)    /// Leaf Node, create new node and return that.
29     {
30         val[nnode] = val[node]; ///cloning current old leaf node's value to new
31         leaf node
32         val[nnode] += v; /// adding or subtracting or replacing as needed
33         return nnode;
34     }
35     ll mid = (lo + hi) >> 1;
36     /// Left[nnode] is new node's Left child, it might end up being the old one
37     too
38     /// Left[node] is current old node's Left child.
39     /// So we call to update idx in Left child of old node.
40     /// And use it's return node as new node's Left child. Same for Right.
41     Left[nnode] = update(Left[node], lo, mid, idx, v);
42     Right[nnode] = update(Right[node], mid+1, hi, idx, v);
43     val[nnode] = val[Left[nnode]] + val[Right[nnode]]; /// Update value.
44     return nnode; /// Return the new node to parent.
45 }
46 ll query(ll lnode, ll rnode, ll lo, ll hi, ll l, ll r)
47 {
48     if(lo > r || hi < l)
49         return 0;
50
51     if (lo >= l && hi <= r)
52         return val[rnode] - val[lnode];
53
54     ll mid = (lo + hi) >> 1;
55
56     return query(Left[lnode], Left[rnode], lo, mid, l, r)
57         + query(Right[lnode], Right[rnode], mid+1, hi, l, r);
58 }
59 /// NEXT_FREE = 0
60 /// version[0] = NEXT_FREE++
61 /// build(version[0], 1, n)
62 /// upd(ara[i], 1) -> increment the frequency
63 /// So, version[i] = update(version[i-1], 1, n, i, 1)
64 /// query: Count of numbers of [a,b] range in [L, R] index
65 /// So, ans = query(version[l-1], version[r], 1, n, L, R)

```

3 Graph Theory

3.1 0-1 BFS

```

1  vector<int> d(n, INF);
2  d[s] = 0;
3  deque<int> q;

```

```

4  q.push_front(s);
5  while (!q.empty()) {
6      int v = q.front();
7      q.pop_front();
8      for (auto edge : adj[v]) {
9          int u = edge.first;
10         int w = edge.second;
11         if (d[v] + w < d[u]) {
12             d[u] = d[v] + w;
13             if (w == 1)
14                 q.push_back(u);
15             else
16                 q.push_front(u);
17         }
18     }
19 }

```

3.2 Bellman Ford Algorithm to find Negative Cycle

```

1  //bellman ford with negative cycle print
2  struct edge {
3      ll v, w;
4  };
5  const ll N = 3e3 + 6, inf = 1LL << 60;
6  ll n, m, dis[N], par[N];
7  vector<edge> g[N];
8
9  int bellman_ford() {
10     lop(n + 1) dis[i] = inf;
11     dis[1] = 0;
12     int cy;
13
14     lop(n + 1) {
15         cy = -1;
16         for (int u = 1; u <= n; u++) {
17             for (auto z : g[u]) {
18                 ll v = z.v, w = z.w;
19                 if (dis[u] + w < dis[v]) {
20                     dis[v] = dis[u] + w;
21                     par[v] = u;
22                     cy = v; // if(u == n) negative cycle;
23                 }
24             }
25         }
26     }
27     return cy; //cy is a adjacent node or a node of negative cycle
28 }
29
30 int main() {

```

```

31     cin >> n >> m;
32     lop(m) {
33         ll u, v, w;
34         cin >> u >> v >> w;
35         g[u].pb({v, w});
36     }
37
38     int x = bellman_ford();
39     if (x == -1) {
40         //no negative cycle
41         return 0;
42     }
43
44     //x can be not a part of cycle, so if we go through //path sometimes, x will be a
45     ↪ node of cycle
46     lop(n) x = par[x];
47     vector<int> cycle;
48     int i = x;
49     while (i != x or cycle.size() <= 1) {
50         cycle.pb(i); //retrieving cycle
51         i = par[i];
52     }
53     cycle.pb(i);
54     reverse(all(cycle));
55     for (int z : cycle)
56         cout << z << ' ';
57     return 0;
58 }

```

3.3 Kruskal MST

```

1  struct edge {
2      int u, v, w;
3      bool operator < (const edge &b) const {
4          return w > b.w;
5      }
6  };
7
8  const int N = 2e5;
9  int n, m, par[N];
10 vector<edge> eg;
11
12 int findpar(int x) {
13     return par[x] == x ? x : findpar(par[x]);
14 }
15
16 void Union(int u, int v) {
17     par[findpar(u)] = findpar(v);
18 }

```

```

19 int kruskal() {
20     sort(eg.begin(), eg.end());
21     iota(par, par + n, 0);
22
23     int cost = 0, connected = 0;
24     while (connected != n - 1) {
25         edge z = eg.back();
26         eg.pop_back();
27
28         int x = findpar(z.u), y = findpar(z.v);
29         if (x != y) {
30             connected++;
31             cost += z.w;
32             Union(x, y);
33         }
34     }
35
36     return cost;
37 }
38
39 int main()
40 {
41     ios_base::sync_with_stdio(0); cin.tie(0);
42
43     cin >> n >> m;
44     for (int i = 0; i < m; i++) {
45         int u, v, w;
46         cin >> u >> v >> w;
47         eg.push_back({u, v, w});
48     }
49
50     cout << kruskal() << '\n';
51
52     return 0;
53 }

```

```

10     d[u] = low[u] = Time;
11
12     int child = 0;
13     for (int v : g[u]) {
14         if (v == p) continue;
15
16         if (vis[v]) low[u] = min(low[u], d[v]);
17         else {
18             child++;
19             dfs(u, v);
20
21             low[u] = min(low[u], low[v]);
22             if (low[v] >= d[u] and u != root)
23                 is_arti[u] = 1;
24
25             if (d[u] < low[v])
26                 arti_bridge.push_back({min(u, v), max(u, v)});
27         }
28     }
29
30     if (u == root and child > 1)
31         is_arti[u] = 1;
32 }
33
34 int main () {
35     cin >> n >> m;
36     for (int i = 0; i < m; i++) {
37         int u, v; cin >> u >> v;
38         g[u].push_back(v);
39         g[v].push_back(u);
40     }
41
42     Time = root = 1;
43     memset(vis, 0, sizeof vis);
44     memset(is_arti, 0, sizeof is_arti);
45     dfs(root, root);
46 }

```

3.4 Articulation Bridge and Point

```

1 const int N = 1e4 + 10;
2 int n, m, root, Time, low[N], d[N];
3 bool vis[N], is_arti[N]; // is articulation point
4 vector<int> g[N];
5 vector<pair<int, int>> arti_bridge;
6
7 void dfs(int p, int u) {
8     Time++;
9     vis[u] = 1;

```

3.5 Online Articulation Bridge Finding

```

1 vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
2 int bridges;
3 int lca_iteration;
4 vector<int> last_visit;
5
6 void init(int n) {
7     par.resize(n);
8     dsu_2ecc.resize(n);
9     dsu_cc.resize(n);

```

```

10     dsu_cc_size.resize(n);
11     lca_iteration = 0;
12     last_visit.assign(n, 0);
13     for (int i = 0; i < n; ++i) {
14         dsu_2ecc[i] = i;
15         dsu_cc[i] = i;
16         dsu_cc_size[i] = 1;
17         par[i] = -1;
18     }
19     bridges = 0;
20 }
21
22 int find_2ecc(int v) {
23     if (v == -1)
24         return -1;
25     return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
26 }
27
28 int find_cc(int v) {
29     v = find_2ecc(v);
30     return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
31 }
32
33 void make_root(int v) {
34     v = find_2ecc(v);
35     int root = v;
36     int child = -1;
37     while (v != -1) {
38         int p = find_2ecc(par[v]);
39         par[v] = child;
40         dsu_cc[v] = root;
41         child = v;
42         v = p;
43     }
44     dsu_cc_size[root] = dsu_cc_size[child];
45 }
46
47 void merge_path (int a, int b) {
48     ++lca_iteration;
49     vector<int> path_a, path_b;
50     int lca = -1;
51     while (lca == -1) {
52         if (a != -1) {
53             a = find_2ecc(a);
54             path_a.push_back(a);
55             if (last_visit[a] == lca_iteration) {
56                 lca = a;
57                 break;
58             }
59             last_visit[a] = lca_iteration;
60             a = par[a];

```

```

61     }
62     if (b != -1) {
63         b = find_2ecc(b);
64         path_b.push_back(b);
65         if (last_visit[b] == lca_iteration) {
66             lca = b;
67             break;
68         }
69         last_visit[b] = lca_iteration;
70         b = par[b];
71     }
72
73 }
74
75 for (int v : path_a) {
76     dsu_2ecc[v] = lca;
77     if (v == lca)
78         break;
79     --bridges;
80 }
81 for (int v : path_b) {
82     dsu_2ecc[v] = lca;
83     if (v == lca)
84         break;
85     --bridges;
86 }
87 }
88
89 void add_edge(int a, int b) {
90     a = find_2ecc(a);
91     b = find_2ecc(b);
92     if (a == b)
93         return;
94
95     int ca = find_cc(a);
96     int cb = find_cc(b);
97
98     if (ca != cb) {
99         ++bridges;
100         if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
101             swap(a, b);
102             swap(ca, cb);
103         }
104         make_root(a);
105         par[a] = dsu_cc[a] = b;
106         dsu_cc_size[cb] += dsu_cc_size[a];
107     } else {
108         merge_path(a, b);
109     }

```

110 }

3.6 Max Flow(Dinic's Algo)

```

1
2 #define ll long long
3 const ll maxnodes = 10005;
4
5 ll nodes = maxnodes, src, dest;
6 ll dist[maxnodes], q[maxnodes], work[maxnodes];
7
8 struct Edge
9 {
10     ll to, rev;
11     ll f, cap;
12 };
13
14 vector<Edge> g[maxnodes];
15
16 void addEdge(ll s, ll t, ll cap)
17 {
18     Edge a = {t, g[t].size(), 0, cap};
19     Edge b = {s, g[s].size(), 0, 0};
20     g[s].push_back(a);
21     g[t].push_back(b);
22 }
23
24 bool dinic_bfs()
25 {
26     fill(dist, dist + nodes, -1);
27
28     dist[src] = 0;
29     ll index = 0;
30     q[index++] = src;
31
32     for (ll i = 0; i < index; i++)
33     {
34         ll u = q[i];
35         for (ll j = 0; j < (ll) g[u].size(); j++)
36         {
37             Edge &e = g[u][j];
38             if (dist[e.to] < 0 && e.f < e.cap)
39             {
40                 dist[e.to] = dist[u] + 1;
41                 q[index++] = e.to;
42             }
43         }
44     }
45     return dist[dest] >= 0;

```

```

46 }
47
48 ll dinic_dfs(ll u, ll f)
49 {
50     if (u == dest)
51         return f;
52
53     for (ll &i = work[u]; i < (ll) g[u].size(); i++)
54     {
55         Edge &e = g[u][i];
56
57         if (e.cap <= e.f) continue;
58
59         if (dist[e.to] == dist[u] + 1)
60         {
61             ll flow = dinic_dfs(e.to, min(f, e.cap - e.f));
62             if (flow > 0)
63             {
64                 e.f += flow;
65                 g[e.to][e.rev].f -= flow;
66                 return flow;
67             }
68         }
69     }
70     return 0;
71 }
72
73 ll maxFlow(ll _src, ll _dest)
74 {
75     src = _src;
76     dest = _dest;
77     ll result = 0;
78     while (dinic_bfs())
79     {
80         fill(work, work + nodes, 0);
81         while (ll delta = dinic_dfs(src, inf))
82             result += delta;
83     }
84     return result;
85 }
86
87 // addEdge(u, v, C);    edge from u to v. Capacity is C
88 // maxFlow(s, t);    max flow from s to t
89

```

3.7 Cycle in a Directed Graph

```

1 //Complexity : O(V+E).
2 //Find only one cycle and print it.

```

```

3  vector<int>adj[1005];
4  vector<char> color;
5  vector<int> parent;
6  int cycle_start, cycle_end;
7  int n,m;
8  bool dfs(int v) {
9      color[v] = 1;
10     for (int u : adj[v]) {
11         if (color[u] == 0) {
12             parent[u] = v;
13             if (dfs(u))
14                 return true;
15         } else if (color[u] == 1) {
16             cycle_end = v;
17             cycle_start = u;
18             return true;
19         }
20     }
21     color[v] = 2;
22     return false;
23 }
24
25 void find_cycle() {
26     color.assign(n, 0);
27     parent.assign(n, -1);
28     cycle_start = -1;
29
30     for (int v = 0; v < n; v++) {
31         if (color[v] == 0 && dfs(v))
32             break;
33     }
34
35     if (cycle_start == -1) {
36         cout<<-1<<endl;
37     } else {
38         vector<int> cycle;
39         cycle.push_back(cycle_start);
40         for (int v = cycle_end; v != cycle_start; v = parent[v])
41             cycle.push_back(v);
42         // cycle.push_back(cycle_start);
43         reverse(cycle.begin(), cycle.end());
44
45         cout<<cycle.size()<<endl;
46         for (int v : cycle)
47             cout << v+1 <<endl;
48     }
49 }

```

3.8 Max Flow(Dinic's Algo)

```

1
2  #define ll long long
3  const ll maxnodes = 10005;
4
5  ll nodes = maxnodes, src, dest;
6  ll dist[maxnodes], q[maxnodes], work[maxnodes];
7
8  struct Edge
9  {
10     ll to, rev;
11     ll f, cap;
12 };
13
14 vector<Edge> g[maxnodes];
15
16 void addEdge(ll s, ll t, ll cap)
17 {
18     Edge a = {t, g[t].size(), 0, cap};
19     Edge b = {s, g[s].size(), 0, 0};
20     g[s].push_back(a);
21     g[t].push_back(b);
22 }
23
24 bool dinic_bfs()
25 {
26     fill(dist, dist + nodes, -1);
27
28     dist[src] = 0;
29     ll index = 0;
30     q[index++] = src;
31
32     for (ll i = 0; i < index; i++)
33     {
34         ll u = q[i];
35         for (ll j = 0; j < (ll) g[u].size(); j++)
36         {
37             Edge &e = g[u][j];
38             if (dist[e.to] < 0 && e.f < e.cap)
39             {
40                 dist[e.to] = dist[u] + 1;
41                 q[index++] = e.to;
42             }
43         }
44     }
45     return dist[dest] >= 0;
46 }
47
48 ll dinic_dfs(ll u, ll f)

```

```

49 {
50     if (u == dest)
51         return f;
52
53     for (ll &i = work[u]; i < (ll) g[u].size(); i++)
54     {
55         Edge &e = g[u][i];
56
57         if (e.cap <= e.f) continue;
58
59         if (dist[e.to] == dist[u] + 1)
60         {
61             ll flow = dinic_dfs(e.to, min(f, e.cap - e.f));
62             if (flow > 0)
63             {
64                 e.f += flow;
65                 g[e.to][e.rev].f -= flow;
66                 return flow;
67             }
68         }
69     }
70     return 0;
71 }
72
73 ll maxFlow(ll _src, ll _dest)
74 {
75     src = _src;
76     dest = _dest;
77     ll result = 0;
78     while (dinic_bfs())
79     {
80         fill(work, work + nodes, 0);
81         while (ll delta = dinic_dfs(src, inf))
82             result += delta;
83     }
84     return result;
85 }
86
87 // addEdge(u, v, C);    edge from u to v. Capacity is C
88 // maxFlow(s, t);    max flow from s to t

```

3.9 Min Cost Max Flow

```

1  const ll maxnodes = 10005;
2
3  ll nodes = maxnodes, src, dest;
4  ll dist[maxnodes], exist[maxnodes];
5  pll par[maxnodes];

```

```

6
7  struct Edge
8  {
9      ll to, rev;
10     ll f, cap, cost;
11 };
12
13 vector<Edge> g[maxnodes];
14
15 void addEdge(ll s, ll t, ll cap, ll cost)
16 {
17     Edge a = {t, g[t].size(), 0, cap, cost};
18     Edge b = {s, g[s].size(), 0, 0, -cost};
19     g[s].push_back(a);
20     g[t].push_back(b);
21 }
22
23 bool spfa()
24 {
25     fill(dist, dist + nodes, inf);
26     fill(exist, exist + nodes, 0);
27
28     dist[src] = 0, exist[src] = 1;
29     queue <ll> q;
30     q.push(src);
31
32     while(!q.empty()) {
33         ll u = q.front();
34         q.pop();
35         exist[u] = 0;
36
37         for(ll i = 0; i < g[u].size(); i++) {
38
39             Edge e = g[u][i];
40
41             if(dist[e.to] > dist[u] + e.cost && e.f < e.cap) {
42
43                 dist[e.to] = dist[u] + e.cost;
44                 par[e.to] = mp(u, i);
45
46                 if(!exist[e.to]) {
47                     q.push(e.to);
48                     exist[e.to] = 1;
49                 }
50             }
51         }
52     }
53
54     return dist[dest] != inf;
55 }

```

```

56 }
57
58 pll minCostMaxFlow(ll _src, ll _dest)
59 {
60     src = _src;
61     dest = _dest;
62     pll result = mp(0,0);
63     while (spfa())
64     {
65         ll cur = _dest, flow = inf;
66         while(cur != _src) {
67             ll p = par[cur].first;
68             Edge e = g[p][ par[cur].second ];
69             flow = min(flow, e.cap - e.f);
70
71             cur = p;
72         }
73         result.first += flow;
74
75         cur = _dest;
76         while(cur != _src) {
77             ll p = par[cur].first;
78             Edge &e = g[p][ par[cur].second ];
79
80             e.f += flow;
81             g[e.to][e.rev].f -= flow;
82
83             result.second += flow * e.cost;
84
85             cur = p;
86         }
87     }
88     return result;
89 }
90 // addEdge(u, v, C, cst); edge from u to v. Capacity=C, Cost=cst.
91 // minCostMaxFlow(s, t); min cost max flow from s to t

```

3.10 Strongly Connected Components

```

1  const int N = 3e5 + 9;
2
3  // given a directed graph return the minimum number of edges to be added so that
4  // → the whole graph become an SCC
5  bool vis[N];
6  vector<int> g[N], r[N], G[N], vec; //G is the condensed graph
7  void dfs1(int u) {
8      vis[u] = 1;
9      for (auto v : g[u]) if (!vis[v]) dfs1(v);
10     vec.push_back(u);

```

```

10 }
11
12 vector<int> comp;
13 void dfs2(int u) {
14     comp.push_back(u);
15     vis[u] = 1;
16     for (auto v : r[u]) if (!vis[v]) dfs2(v);
17 }
18
19 int idx[N], in[N], out[N];
20 int main() {
21     ios_base::sync_with_stdio(0);
22     cin.tie(0);
23
24     int n, m;
25     cin >> n >> m;
26     for (int i = 1; i <= m; i++) {
27         int u, v;
28         cin >> u >> v;
29         g[u].push_back(v);
30         r[v].push_back(u);
31     }
32     for (int i = 1; i <= n; i++) if (!vis[i]) dfs1(i);
33     reverse(vec.begin(), vec.end());
34     memset(vis, 0, sizeof vis);
35     int scc = 0;
36     for (auto u : vec) {
37         if (!vis[u]) {
38             comp.clear();
39             dfs2(u);
40             scc++;
41             for (auto x : comp) idx[x] = scc;
42         }
43     }
44     for (int u = 1; u <= n; u++) {
45         for (auto v : g[u]) {
46             if (idx[u] != idx[v]) {
47                 in[idx[v]]++, out[idx[u]]++;
48                 G[idx[u]].push_back(idx[v]);
49             }
50         }
51     }
52     int needed_in = 0, needed_out = 0;
53     for (int i = 1; i <= scc; i++) {
54         if (!in[i]) needed_in++;
55         if (!out[i]) needed_out++;
56     }
57     int ans = max(needed_in, needed_out);
58     if (scc == 1) ans = 0;
59     cout << ans << '\n';

```



```

60     return 0;
61 }

```

3.11 2 SAT

```

1  const int N = 3e5 + 9;
2
3  /*
4  zero Indexed
5  we have vars variables
6  F=(x_0 XXX y_0) and (x_1 XXX y_1) and ... (x_{vars-1} XXX y_{vars-1})
7  here {x_i,y_i} are variables
8  and XXX belongs to {OR,XOR}
9  is there any assignment of variables such that F=true
10 */
11 struct twosat {
12     int n; // total size combining +, -. must be even.
13     vector< vector<int> > g, gt;
14     vector<bool> vis, res;
15     vector<int> comp;
16     stack<int> ts;
17     twosat(int vars = 0) {
18         n = vars << 1;
19         g.resize(n);
20         gt.resize(n);
21     }
22
23     //zero indexed, be careful
24     //if you want to force variable a to be true in OR or XOR combination
25     //add addOR (a,1,a,1);
26     //if you want to force variable a to be false in OR or XOR combination
27     //add addOR (a,0,a,0);
28
29     //(x_a or (not x_b))-> af=1,bf=0
30     void addOR(int a, bool af, int b, bool bf) {
31         a += a + (af ^ 1);
32         b += b + (bf ^ 1);
33         g[a ^ 1].push_back(b); // !a => b
34         g[b ^ 1].push_back(a); // !b => a
35         gt[b].push_back(a ^ 1);
36         gt[a].push_back(b ^ 1);
37     }
38     //(!x_a xor !x_b)-> af=0, bf=0
39     void addXOR(int a, bool af, int b, bool bf) {
40         addOR(a, af, b, bf);
41         addOR(a, !af, b, !bf);
42     }
43     //add this type of condition->
44     //add(a,af,b,bf) means if a is af then b must need to be bf

```

```

45     void add(int a, bool af, int b, bool bf) {
46         a += a + (af ^ 1);
47         b += b + (bf ^ 1);
48         g[a].push_back(b);
49         gt[b].push_back(a);
50     }
51     void dfs1(int u) {
52         vis[u] = true;
53         for (int v : g[u]) if (!vis[v]) dfs1(v);
54         ts.push(u);
55     }
56     void dfs2(int u, int c) {
57         comp[u] = c;
58         for (int v : gt[u]) if (comp[v] == -1) dfs2(v, c);
59     }
60     bool ok() {
61         vis.resize(n, false);
62         for (int i = 0; i < n; ++i) if (!vis[i]) dfs1(i);
63         int scc = 0;
64         comp.resize(n, -1);
65         while (!ts.empty()) {
66             int u = ts.top();
67             ts.pop();
68             if (comp[u] == -1) dfs2(u, scc++);
69         }
70         res.resize(n / 2);
71         for (int i = 0; i < n; i += 2) {
72             if (comp[i] == comp[i + 1]) return false;
73             res[i / 2] = (comp[i] > comp[i + 1]);
74         }
75         return true;
76     }
77 };
78
79 int main() {
80     int n, m; cin >> n >> m;
81     twosat ts(n);
82     for (int i = 0; i < m; i++) {
83         int u, v, k; cin >> u >> v >> k;
84         --u; --v;
85         if (k) ts.add(u, 0, v, 0), ts.add(u, 1, v, 1), ts.add(v, 0, u, 0), ts.add(v,
↪ 1, u, 1);
86         else ts.add(u, 0, v, 1), ts.add(u, 1, v, 0), ts.add(v, 0, u, 1), ts.add(v, 1,
↪ u, 0);
87     }
88     int k = ts.ok();
89     if (!k) cout << "Impossible\n";
90     else {
91         vector<int> v;
92         for (int i = 0; i < n; i++) if (ts.res[i]) v.push_back(i);

```

```

93     cout << (int)v.size() << '\n';
94     for (auto x : v) cout << x + 1 << ' ';
95     cout << '\n';
96 }
97 return 0;
98 }

```

4 Dynamic Programming

4.1 LIS

```

1 int lis(vector<int> const& a) {
2     int n = a.size();
3     const int INF = 1e9; //INF must be > max(a)
4     vector<int> d(n + 1, INF);
5     d[0] = -INF;
6     for (int i = 0; i < n; i++) {
7         int j = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
8         if (d[j - 1] <= a[i] && a[i] <= d[j]) d[j] = a[i];
9     }
10    int ans = 0;
11    for (int i = 0; i <= n; i++) {
12        if (d[i] < INF) ans = i;
13    }
14    return ans;
15 }
16

```

4.2 SOS DP

```

1 // iterate over all the masks
2 for (int mask = 0; mask < (1<<n); mask++)
3 {
4     F[mask] = A[0];
5     // iterate over all the subsets of the mask
6     for(int i = mask; i > 0; i = (i-1) & mask)
7     {
8         F[mask] += A[i];
9     }
10 }
11 //memory optimized, super easy to code.
12 for(int i = 0; i<(1<<N); ++i)
13     F[i] = A[i];
14 for(int i = 0; i < N; ++i)
15     for(int mask = 0; mask < (1<<N); ++mask)
16     {
17         if(mask & (1<<i))

```

```

18         F[mask] += F[mask^(1<<i)];
19     }

```

4.3 Digit DP

```

1 // How many numbers x are there in the range a to b, where the digit d occurs
2 // exactly k times in x?
3 vector<int> num;
4 int a, b, d, k;
5 int DP[12][12][2];
6 // DP[p][c][f] = Number of valid numbers <= b from this state
7 // p = current position from left side (zero based)
8 // c = number of times we have placed the digit d so far
9 // f = the number we are building has already become smaller than b? [0 = no, 1
10 //    = yes]
11
12 int call(int pos, int cnt, int f){
13     if(cnt > k) return 0;
14     if(pos == num.size()){
15         if(cnt == k) return 1;
16         return 0;
17     }
18     if(DP[pos][cnt][f] != -1) return DP[pos][cnt][f];
19     int res = 0; int LMT;
20     if(f == 0) LMT = num[pos]; else LMT = 9;
21     for(int dgt = 0; dgt<=LMT; dgt++){
22         int nf = f; int ncnt = cnt;
23         if(f == 0 && dgt < LMT) nf = 1; // The number is getting smaller at this
24         // position
25         if(dgt == d) ncnt++;
26         if(ncnt <= k) res += call(pos+1, ncnt, nf);
27     }
28     return DP[pos][cnt][f] = res;
29 }
30
31 int solve(int b){
32     num.clear();
33     while(b>0){
34         num.push_back(b%10);
35         b/=10;
36     }
37     reverse(num.begin(), num.end());
38     memset(DP, -1, sizeof(DP));
39     int res = call(0, 0, 0);
40     return res;
41 }
42
43 int main () {
44     cin >> a >> b >> d >> k;

```

```

42     int res = solve(b) - solve(a-1);
43     cout << res << endl;
44 }

```

```

42     return x == y;
43 }

```

5 String

5.1 Hashing

```

1  const ll N = 1e6 + 10, mod = 2e9 + 63, base1 = 1e9 + 21, base2 = 1e9 + 181;
2  ll pw1[N], pw2[N], hp1[N], hp2[N], hs1[N], hs2[N], n, q;
3  string s;
4
5  void pw_cal() {
6      pw1[0] = pw2[0] = 1;
7      for (int i = 1; i < N; i++) {
8          pw1[i] = (pw1[i - 1] * base1) % mod;
9          pw2[i] = (pw2[i - 1] * base2) % mod;
10     }
11 }
12 void init() {
13     hp1[0] = hp2[0] = hs1[n + 1] = hs2[n + 1] = 0;
14     for (int i = 1; i <= n; i++) {
15         hp1[i] = (hp1[i - 1] * base1 + s[i - 1]) % mod;
16         hp2[i] = (hp2[i - 1] * base2 + s[i - 1]) % mod;
17     }
18     for (int i = n; i > 0; i--) {
19         hs1[i] = (hs1[i + 1] * base1 + s[i - 1]) % mod;
20         hs2[i] = (hs2[i + 1] * base2 + s[i - 1]) % mod;
21     }
22 }
23
24 ll hashp(int l, int r) {
25     ll hash1 = (hp1[r] - hp1[l - 1] * pw1[r - l + 1]) % mod;
26     if (hash1 < 0) hash1 += mod;
27     ll hash2 = (hp2[r] - hp2[l - 1] * pw2[r - l + 1]) % mod;
28     if (hash2 < 0) hash2 += mod;
29     return (hash1 << 32) | hash2;
30 }
31
32 ll hashs(int l, int r) {
33     ll hash1 = (hs1[l] - hs1[r + 1] * pw1[r - l + 1]) % mod;
34     if (hash1 < 0) hash1 += mod;
35     ll hash2 = (hs2[l] - hs2[r + 1] * pw2[r - l + 1]) % mod;
36     if (hash2 < 0) hash2 += mod;
37     return (hash1 << 32) | hash2;
38 }
39
40 bool ispal(int l, int r) {
41     int mid = (r + l) / 2;
42     ll x = hashp(l, mid), y = hashs(mid, r);

```

5.2 Trie

```

1  const int N = 1e5+10, M = 26;
2  int trie[N][M], nnode;
3  bool isword[N];
4
5  void reset(int k) {
6      for (int i = 0; i < M; i++)
7          trie[k][i] = -1;
8  }
9
10 void Insert(string &s) {
11     int n = s.size(), node = 0;
12     for (int i = 0; i < n; i++) {
13         if (trie[node][s[i] - 'a'] == -1) {
14             trie[node][s[i] - 'a'] = ++nnode;
15             reset(nnode);
16         }
17         node = trie[node][s[i] - 'a'];
18     }
19     isword[node] = 1;
20 }
21
22 bool Search(string &s) {
23     int n = s.size(), node = 0;
24     for (int i = 0; i < s.size(); i++) {
25         if (trie[node][s[i] - 'a'] == -1) return 0;
26         node = trie[node][s[i] - 'a'];
27     }
28     return isword[node];
29 }
30
31 //find maximum subarray xor sum
32 int doxor(int s) {
33     int nw = 0, t = 0;
34     for (int i = 31; i >= 0; i--) {
35         bool p = (1 << i) & s;
36         if (node[nw][p ^ 1] != -1) {
37             t |= 1 << i;
38             nw = node[nw][p ^ 1];
39         } else
40             nw = node[nw][p];
41     }
42     return t;
43 }
44 //minimum subarray xor sum

```

```

45 int doxor2(int s) {
46     int nw = 0, t = 0;
47     for (int i = 31; i >= 0; i--) {
48         bool p = (1 << i) & s;
49         if (node[nw][p] != -1) nw = node[nw][p];
50         else {
51             t |= 1 << i;
52             nw = node[nw][p ^ 1];
53         }
54     }
55     return t;
56 }
57 //at first insert(0), then calculate xor before inserting each element of the
58   ↪ array
59 //calculate number of subarray having xor>=k
60 int doxor(int s) {
61     int nw = 0, t = 0;
62     for (int i = 31; i >= 0; i--) {
63         bool p = (1 << i) & s;
64         bool q = (1 << i) & k;
65         if (!q) {
66             t += (node[nw][p ^ 1] != -1 ? word[node[nw][p ^ 1]] : 0);
67             nw = node[nw][p];
68         } else {
69             nw = node[nw][p ^ 1];
70             if (nw == -1)
71                 break;
72         }
73         if (nw != -1)
74             t += word[nw];
75     }
76     return t;
77 }
78 //insert(0), sum returned value, insert prefix xor
79 int main() {
80     reset(0);
81     int n; cin >> n;
82     for (int i = 0; i < n; i++) {
83         string s;
84         cin >> s;
85         Insert(s);
86     }
87     int q; cin >> q;
88     while (q--) {
89         string s;
90         cin >> s;
91         cout << Search(s) << endl;
92     }
93 }

```

5.3 Z- Algo //0-based Indexing

```

1 vector<int> z_function(string s) {
2     int n = (int) s.length();
3     vector<int> z(n);
4     for (int i = 1, l = 0, r = 0; i < n; ++i) {
5         if (i <= r)
6             z[i] = min (r - i + 1, z[i - l]);
7         while (i + z[i] < n && s[z[i]] == s[i + z[i]])
8             ++z[i];
9         if (i + z[i] - 1 > r)
10             l = i, r = i + z[i] - 1;
11     }
12     return z;
13 }

```

- The Z-function for this string s is an array of length n where the i -th element is equal to the greatest number of characters starting from the position i that coincides with the first characters of s .

5.4 Suffix Array

```

1 #define MAX_N 1000020
2 int n, t;
3 // char s[500099];
4 string s;
5 int SA[MAX_N], LCP[MAX_N];
6 int RA[MAX_N], tempRA[MAX_N];
7 int tempSA[MAX_N];
8 int c[MAX_N];
9 int Phi[MAX_N], PLCP[MAX_N];
10 // second approach: O(n log n)
11 // the input string, up to 100K characters
12 // the length of input string
13 // rank array and temporary rank array
14 // suffix array and temporary suffix array
15 // for counting/radix sort
16 void countingSort(int k) { // O(n)
17     int i, sum, maxi = max(300, n);
18     // up to 255 ASCII chars or length of n
19     memset(c, 0, sizeof c);
20     // clear frequency table
21     for (i = 0; i < n; i++)
22         // count the frequency of each integer rank
23         c[i + k < n ? RA[i + k] : 0]++;
24     for (i = sum = 0; i < maxi; i++) {
25         int t = c[i]; c[i] = sum; sum += t;
26     }

```

```

27     for (i = 0; i < n; i++)
28         // shuffle the suffix array if necessary
29         tempSA[c[SA[i] + k < n ? RA[SA[i] + k] : 0]++] = SA[i];
30
31     for (i = 0; i < n; i++)
32         // update the suffix array SA
33         SA[i] = tempSA[i];
34 }
35
36 void buildSA() {
37     int i, k, r;
38     for (i = 0; i < n; i++) RA[i] = s[i];
39     // initial rankings
40     for (i = 0; i < n; i++) SA[i] = i;
41     // initial SA: {0, 1, 2, ..., n-1}
42     for (k = 1; k < n; k <= 1) {
43         // repeat sorting process log n times
44         countingSort(k); // actually radix sort: sort based on the second item
45         countingSort(0);
46         // then (stable) sort based on the first item
47         tempRA[SA[0]] = r = 0;
48         // re-ranking; start from rank r = 0
49         for (i = 1; i < n; i++)
50             // compare adjacent suffixes
51             tempRA[SA[i]] = // if same pair => same rank r; otherwise, increase r
52                 (RA[SA[i]] == RA[SA[i - 1]] && RA[SA[i] + k] == RA[SA[i - 1] +
53 → k]) ? r : ++r;
54         for (i = 0; i < n; i++)
55             // update the rank array RA
56             RA[i] = tempRA[i];
57
58         if (RA[SA[n - 1]] == n - 1) break;
59         // nice optimization trick
60     }
61 }
62
63 void buildLCP() {
64     int i, L;
65     Phi[SA[0]] = -1;
66     // default value
67     for (i = 1; i < n; i++)
68         // compute Phi in O(n)
69         Phi[SA[i]] = SA[i - 1];
70     // remember which suffix is behind this suffix
71     for (i = L = 0; i < n; i++) {
72         // compute Permuted LCP in O(n)
73         if (Phi[i] == -1) { PLCP[i] = 0; continue; }
74         // special case
75         while (s[i + L] == s[Phi[i] + L]) L++;
76         // L increased max n times
77         PLCP[i] = L;

```

```

77         L = max(L - 1, 0);
78         // L decreased max n times
79     }
80     for (i = 0; i < n; i++)
81         // compute LCP in O(n)
82         LCP[i] = PLCP[SA[i]];
83     // put the permuted LCP to the correct position
84 }
85 // n = string length + 1
86 // s = the string
87 // memset(LCP, 0, sizeof(LCP)); setting all index of LCP to zero
88 // buildSA(); for building suffix array
89 // buildLCP(); for building LCP array
90 // LCP is the longest common prefix with the previous suffix here
91 // SA[0] holds the empty suffix "\0".
92
93 int main()
94 {
95     s = "banana";
96     s += "$";
97     n = s.size();
98
99     memset(LCP, 0, sizeof(LCP));
100    buildSA();
101    buildLCP();
102
103    for (int i = 0; i < n; i++) cout << SA[i] << ' ' << s.substr(SA[i], n -
104 → SA[i]) << endl;;
105    printf("\n");
106    for (int i = 0; i < n; i++) printf("%d ", LCP[i]);
107    printf("\n");
108
109    return 0;
110 }

```

6 Geometry

6.1 Convex Hull

```

1 struct pt {
2     double x, y;
3 };
4
5 int orientation(pt a, pt b, pt c) {
6     double v = a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y);
7     if (v < 0) return -1; // clockwise
8     if (v > 0) return +1; // counter-clockwise
9     return 0;

```

```

10 }
11
12 bool cw(pt a, pt b, pt c, bool include_collinear) {
13     int o = orientation(a, b, c);
14     return o < 0 || (include_collinear && o == 0);
15 }
16 bool ccw(pt a, pt b, pt c, bool include_collinear) {
17     int o = orientation(a, b, c);
18     return o > 0 || (include_collinear && o == 0);
19 }
20
21 void convex_hull(vector<pt>& a, bool include_collinear = false) {
22     if (a.size() == 1)
23         return;
24
25     sort(a.begin(), a.end(), [](pt a, pt b) {
26         return make_pair(a.x, a.y) < make_pair(b.x, b.y);
27     });
28     pt p1 = a[0], p2 = a.back();
29     vector<pt> up, down;
30     up.push_back(p1);
31     down.push_back(p2);
32     for (int i = 1; i < (int)a.size(); i++) {
33         if (i == a.size() - 1 || cw(p1, a[i], p2, include_collinear)) {
34             while (up.size() >= 2 && !cw(up[up.size()-2], up[up.size()-1], a[i],
35         ↪ include_collinear))
36                 up.pop_back();
37             up.push_back(a[i]);
38         }
39         if (i == a.size() - 1 || ccw(p1, a[i], p2, include_collinear)) {
40             while (down.size() >= 2 && !ccw(down[down.size()-2],
41         ↪ down[down.size()-1], a[i], include_collinear))
42                 down.pop_back();
43             down.push_back(a[i]);
44         }
45     }
46     if (include_collinear && up.size() == a.size()) {
47         reverse(a.begin(), a.end());
48         return;
49     }
50     a.clear();
51     for (int i = 0; i < (int)up.size(); i++)
52         a.push_back(up[i]);
53     for (int i = down.size() - 2; i > 0; i--)
54         a.push_back(down[i]);
55 }

```

6.2 Polar Sort

```

1 inline bool up (point p) {
2     return p.y > 0 or (p.y == 0 and p.x >= 0);
3 }
4
5 sort(v.begin(), v.end(), [] (point a, point b) {
6     return up(a) == up(b) ? a.x * b.y > a.y * b.x : up(a) < up(b);
7 });

```

7 Miscellaneous

7.1 C++17 Sublime Build

```

1 {
2     "cmd" : ["g++ -std=c++17 -Wall -fsanitize=address $file_name -o
3         $file_base_name && timeout 10s ./$file_base_name <in>out"],
4     "selector" : "source.cpp",
5     "shell" : true,
6     "working_dir" : "$file_path"
7 }

```

7.2 Test Case Generator with FASTIO

```

1 #pragma GCC optimize("Ofast,unroll-loops")
2 #pragma GCC target("avx,avx2,fma")
3
4 //generator to generate testcase
5 #include <bits/stdc++.h>
6 using namespace std;
7 #define ll long long
8 mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
9 // return a random number in [l, r] range
10 ll rand(ll l, ll r) {
11     return uniform_int_distribution<ll>(l, r)(rng);
12 }
13
14 void tree() {
15     int n = rand(1, 10); // number of nodes
16     int t = rand(1, 5); // random parameter
17
18     vector<int> p(n);
19     for (int i = 1; i < n; i++) {
20         for (int j = 0; j <= t; j++)
21             p[i] = max(p[i], (int)rand(0, i - 1));
22     }
23
24     vector<int> perm(n);

```

```

25     for (int i = 0; i < n; i++) perm[i] = i;
26     shuffle(perm.begin(), perm.end(), rng);
27
28     vector<pair<int, int> > edges;
29     for (int i = 1; i < n; i++)
30         if (rand(0, 1))
31             edges.push_back(make_pair(perm[i], perm[p[i]]));
32         else
33             edges.push_back(make_pair(perm[p[i]], perm[i]));
34     shuffle(edges.begin(), edges.end(), rng);
35
36     printf("%d\n", n);
37     for (int i = 0; i + 1 < n; i++)
38         printf("%d %d\n", edges[i].first + 1, edges[i].second + 1);
39 }
40
41 int main(int argc, char* argv[]) {
42     ios_base::sync_with_stdio(0); cin.tie(0);
43
44     ll t = rand(1, 1);
45     cout << t << endl;
46     while (t--) {
47         ll n = rand(1, 15);
48         cout << n << endl;
49     }
50     return 0;
51 }

```

7.3 Output Checker Bash Script

```

1 //Bash script to auto check output
2 for((i = 1; ; ++i)); do
3     echo $i
4     ./gen $i > in
5     # ./a < in > out1
6     # ./brute < in > out2
7     # diff -w out1 out2 || break
8     diff -w <./sol < in> <./brute < in> || break
9 done
10 echo case
11 cat in
12 #create and build a brute force code named brute.cpp, main solution code sol.cpp
   → and a random test case generator gen.cpp. To make this script runnable, run
   → this command chmod +x s.sh (s.sh is this bash script name). Then run the
   → script by ./s.sh or bash s.sh

```

7.4 Custom Hash for unordered_map

```

1 // To prevent collision in unordered_map
2 #include <bits/stdc++.h>
3 using namespace std;
4 struct custom_hash {
5     static uint64_t splitmix64(uint64_t x) {
6         // http://xorshift.di.unimi.it/splitmix64.c
7         x += 0x9e3779b97f4a7c15;
8         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
9         x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
10        return x ^ (x >> 31);
11    }
12    size_t operator()(uint64_t x) const {
13        static const uint64_t FIXED_RANDOM =
14            chrono::steady_clock::now().time_since_epoch().
15            count();
16        return splitmix64(x + FIXED_RANDOM);
17    }
18 };
19 /// Declaration: unordered_map <int, int, custom_hash > numbers;
20 /// Usage: same as normal unordered_map
21 /// Ex: numbers[5] = 2;
22 /// *** To use gp_hash_table (faster than unordered_map) **** ///
23 /// Add these extra two lines:
24 /// #include <ext/pb_ds/assoc_container.hpp>
25 /// using namespace __gnu_pbds;
26 /// Declaration: gp_hash_table<int, int, custom_hash > numbers;
27 /// Usage: Same as unordered_map

```

7.5 Release vector memory

```

1 vector<int> Elements ;
2 // fill the vector up
3 vector<int>().swap(Elements);

```

7.6 Set of Structure [Operator overload]

```

1 struct seg{
2     ll l , r , c;
3     bool operator < (const seg &S) const {
4         return l < S.l ;
5     }
6 };
7 set<seg> s ; s.insert({0,0,0}) ;

```

7.7 Graph and Bitmask Operation

```

1 #pragma GCC optimize("Ofast,unroll-loops")
2 #pragma GCC target("avx,avx2,fma")
3 /* Infos */
4 ~ 4 Direction
5 int dr[] = {1, -1, 0, 0};
6 int dc[] = {0, 0, 1, -1};
7 ~ 8 Direction
8 int dr[] = {1, -1, 0, 0, 1, 1, -1, -1};
9 int dc[] = {0, 0, 1, -1, 1, -1, 1, -1};
10 ~ Knight Direction
11 int dr[] = {1, -1, 1, -1, 2, 2, -2, -2};
12 int dc[] = {2, 2, -2, -2, 1, -1, 1, -1};
13 ~ Hexagonal Direction
14 int dr[] = {2, -2, 1, 1, -1, -1};
15 int dc[] = {0, 0, 1, -1, 1, -1};
16 ~ bitmask operations
17 int Set(int n, int pos) { return n = n | (1 << pos); }
18 int reset(int n, int pos) { return n = n & ~(1 << pos); }
19 bool check(int n, int pos) { return (bool)(n & (1 << pos)); }
20 int toggle(int n, int pos) { return n = (n ^ (1 << pos)); }
21 bool isPower2(int x) { return (x && !(x & (x - 1))); }
22 ll LargestPower2LessEqualX(ll x) { for (int i = 1; i <= x / 2; i *= 2) x = x | (x
  ↳ >> i); return (x + 1) / 2; }
23 // for unlimited stack, run the command in terminal and run the code in terminal
24 ulimit -s unlimited

```

8 Notes

8.1 Stars and Bars Theorem

1. The number of ways to put n identical objects into k labeled boxes is $\binom{n+k-1}{n}$
2. Suppose, there are n objects to be placed into k bins, ways = $\binom{n-1}{k-1}$
3. Statement of 1no. and empty bins are valid, ways = $\binom{n+k-1}{k-1}$

8.2 GCD

1. $\gcd(a, b) = \gcd(a, a - b)$ [$a > b$]
2. $\gcd(F(a), F(b)) = F(\gcd(a, b))$ [F =fibonacci]
3. $\gcd(a, b) = \sum \phi(k)$ where k are all common divisors of a and b

8.3 Geometric Formula

1. Point Slope Form: $y - y_1 = m \cdot (x - x_1)$

2. Slope, $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

3. Slope from line, $m = -\frac{A}{B}$

4. Angle, $\tan \theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$

5. Distance from a Point (x_0, y_0) to a Line $(Ax + By + C = 0) = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$

6. Area of segment in radian angle: $A = \frac{1}{2} \cdot r^2 (\theta - \sin \theta)$

7. The sum of interior angles of a polygon with n sides = $180 \cdot (n - 2)$

8. Number of diagonals of a n -sided polygon = $\frac{n(n-3)}{2}$

9. The measure of interior angles of a regular n -sided polygon = $\frac{180(n-2)}{n}$

10. The measure of exterior angles of a regular n -sided polygon = $\frac{360}{n}$

11. Picks theorem: $A = I + \frac{B}{2} - 1$ where A = Area of Polygon, B = Number of integral points on edges of polygon, I = Number of integral points strictly inside the polygon.

12. Sine rule of a Triangle: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

13. Cosine Rule of a Triangle: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

14. Surface Area (SA) and Volumes(V):

- Sphere: $SA = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$
- Cone: $SA = \pi r^2 + \pi rs$, $V = \frac{1}{3}\pi r^2 h$, side, $s = \sqrt{h^2 + r^2}$
- Cylinder: $SA = 2\pi r^2 + 2\pi rh$, $V = \pi r^2 h$
- Cuboid: $SA = 2(wh + lw + lh)$, $V = lwh$
- Trapezoid: Area = $\frac{1}{2}(b_1 + b_2)h$

8.4 Series/Progression

1. Sum of first n positive number = $\frac{n(n+1)}{2}$

2. Sum of first n odd number = n^2

3. Sum of first n even number = $n \cdot (n + 1)$

4. Arithmetic Progression: n -th term = $a + (n - 1) \cdot d$, sum = $\frac{n}{2} \{2a + (n - 1) \cdot d\}$; a = first element, d = difference between two elements

5. Geometric Progression: n -th term = $a \cdot r^{n-1}$, sum = $a \cdot \frac{r^n - 1}{r - 1}$; a = first element, r = ratio of two elements

6. Catalan Numbers: $1, 1, 2, 5, 14, 42, 132 \dots C_n = \frac{(2n)!}{n! \cdot (n+1)!}; n \geq 0$

8.5 Combinatorial formulas

1. $\sum_{k=0}^n k^2 = n(n+1)(2n+1)/6$
2. $\sum_{k=0}^n k^3 = n^2(n+1)^2/4$
3. $\sum_{k=0}^n k^4 = (6n^5 + 15n^4 + 10n^3 - n)/30$
4. $\sum_{k=0}^n k^5 = (2n^6 + 6n^5 + 5n^4 - n^2)/12$
5. $\sum_{k=0}^n x^k = (x^{n+1} - 1)/(x - 1)$
6. $\sum_{k=0}^n kx^k = (x - (n+1)x^{n+1} + nx^{n+2})/(x - 1)^2$
7. $\binom{n}{k} = \frac{n!}{(n-k)!k!}$
8. $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
9. $\binom{n}{k} = \frac{n}{n-k} \binom{n-1}{k}$
10. $\binom{n}{k} = \frac{n-k+1}{k} \binom{n}{k-1}$
11. $\binom{n+1}{k} = \frac{n+1}{n-k+1} \binom{n}{k}$
12. $\binom{n}{k+1} = \frac{n-k}{k+1} \binom{n}{k}$
13. $\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$
14. $\sum_{k=1}^n k^2 \binom{n}{k} = (n + n^2)2^{n-2}$
15. $\binom{m+n}{r} = \sum_{k=0}^r \binom{m}{k} \binom{n}{r-k}$
16. $\binom{n}{k} = \prod_{i=1}^k \frac{n-k+i}{i}$
17. $a^{\phi(n)} \equiv 1 \pmod{n}$ where $\phi(n)$ is Euler Totient Function.
18. $a^b \pmod{m} = a^{b \pmod{\phi(m)}} \pmod{m}$ where a and m are coprime.

