Genetic Algorithm Learning and the Cobweb Model

Author: Jasmina Arifovic
Presenters: Ran

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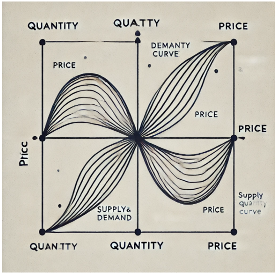
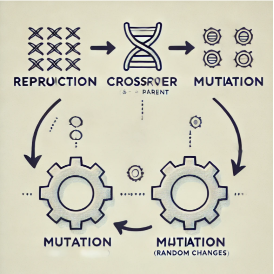
Introduction to the Cobweb Model

- The **Cobweb Model** describes a market for a single good where firms make production decisions based on expected future prices.
- Firms are **price takers**, and market price is determined by the total quantity supplied and demand.
- Depending on the **demand and supply slopes**, the market may either converge to equilibrium (stable case) or diverge (unstable case).



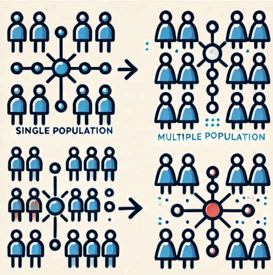
Genetic Algorithm (GA) Learning in Economics

- **Genetic Algorithms (GA)** simulate the evolution of strategies based on performance, similar to natural selection.
- GA updates firms' decision rules using three operators:
  - **Reproduction**: Selects the best-performing rules.
  - **Crossover**: Combines parts of successful rules.
  - **Mutation**: Introduces small random changes.
- **Objective**: To assess if GA can explain the behavior in experimental economies better than traditional learning algorithms.



Key Objectives of the Paper

- **Compare GA** with the behavior observed in human experimental economies and with other learning algorithms:
  - Cobweb expectations, sample average of past prices, and least squares.
- Study GA's performance in two designs:
  - **Single-population**: One set of rules representing different agents.
  - **Multiple-population**: Each agent has multiple rules, representing alternative strategies.



Cobweb Model: Market Setup

- The cobweb model describes a competitive market for a single good, where firms make production decisions before observing the market price.
- Firms are **price takers** and decide how much to produce based on expected prices.
- The market price is determined by the intersection of total supply and exogenously given demand.
- The goal is to study how market prices and quantities adjust over time based on the firm's expectations.

Cobweb Model: Cost Function

- The cost of production for firm i at time t is modeled as:

$$C_{i,t} = xq_{i,t} + \frac{1}{2}yq_{i,t}^2$$

where:

- $C_{i,t}$: Cost for firm i at time t
- $q_{i,t}$: Quantity produced at time t
- x and y: Constants representing fixed and variable costs

Cobweb Model: Expected Profit

- The expected profit for firm i at time t is given by:

$$\Pi_{i,t}^{e} = P_t^{e} q_{i,t} - x q_{i,t} - \frac{1}{2} y q_{i,t}^{2}$$

  where:
  - P_t^e: Expected market price at time t
  - $\Pi_{i,t}^e$: Profit for firm i at time t
- Firms choose production quantity $q_{i,t}$ to maximize expected profit.

Cobweb Model: Market-Clearing Price

- The market-clearing price P_t is determined by the demand curve:

$$P_t = A - B \sum_{i=1}^{n} q_{i,t}$$

where:

  - A: Maximum willingness to pay
  - B: Sensitivity of price to total quantity
  - $\sum_{i=1}^{n} q_{i,t}$: Total quantity supplied by n firms

Cobweb Model: Rational Expectations Equilibrium

- In the Rational Expectations Equilibrium, firms' expectations about future prices are correct:

$$P_t^e = P_t$$

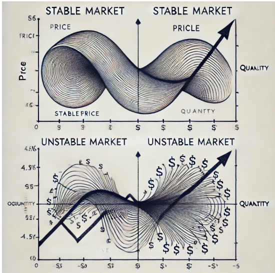
where P_t^e is the expected price and P_t is the actual market price.

- In equilibrium, the production decision $q_{i,t}$ satisfies:

$$q_{i,t} = \frac{P_t - x}{y}$$

Cobweb Model: Convergence to Equilibrium

- The goal of the model is to see whether firms' production quantities and prices converge to equilibrium values.
- If the market is stable, prices and quantities will converge to equilibrium.
- If the market is unstable, prices and quantities may diverge over time.



2.1: Application of Learning Algorithms

- **Cobweb Expectations** (Ezekiel, 1938): Agents expect price at time t to be equal to price at $t-1$

$$P_t^e = P_{t-1}$$

  - Stable case: $B/y < 1 \rightarrow$ prices converge to equilibrium.
  - Unstable case: $B/y > 1 \rightarrow$ prices diverge.
- **Sample Average of Past Prices** (Carlson, 1969): Agents use the average of past prices to form expectations.

$$P_t^e = \frac{1}{t} \sum_{s=0}^{t-1} P_s$$

  - Both stable and unstable cases follow similar patterns to cobweb expectations.
- **Least Squares**: Agents run a regression on past prices to estimate the price:

$$P_t^e = \beta_t P_{t-1}$$

Bray and Savin (1986) Analysis

- In the cobweb model with stochastic demand and supply shocks, firms observe these shocks before making decisions.
- Bray and Savin derived conditions for the least squares estimate(β_t) to converge to equilibrium ($\beta^* = 1$) in the stable case.
- In the unstable case, their simulations showed that the estimate diverges away from equilibrium.
  - Stable case Estimate converges to equilibrium.
  - Unstable case Estimate diverges.

2.2: Wellford's Cobweb Experiments

- Wellford conducted twelve cobweb experiments, simulating both stable and unstable market conditions with 5 participants over 30 periods.
- Sellers decided how much to offer, and prices were set based on supply and demand.
- In unstable cases, price fluctuations remained within a range between the competitive and Cournot prices.
- The hypothesis that price variance is the same in stable and unstable cases was rejected—unstable cases had significantly greater price fluctuations.

Single-population genetic algorithm

- Firms' decision-making is based on a genetic algorithm, where each firm's production decision is represented by a string of binary digits.
- Each firm calculates how much to produce using a decoded binary string:

$$X_{i,t} = \sum_{k=1}^{l} a_{k,t} \cdot 2^{k-1}$$

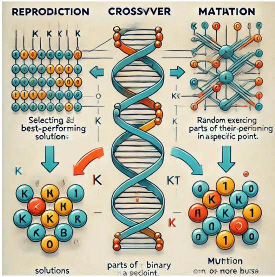
- Firms earn profits based on market price P_t and quantity $q_{i,t}$, with profit determined as:

$$\Pi_{i,t} = P_t q_{i,t} - C_{i,t}$$

- The goal is for firms to learn production strategies that maximize profit.

Three Genetic Operators

- Reproduction: Copies decision rules (strings) based on their fitness, with more profitable strings being copied more often.
- Crossover: Swaps parts of two selected strings to create new combinations of decision rules.
- Mutation: Randomly changes some parts of a string, introducing new variations and ideas into the population.



Summary of the Main Points

- Firms' decision-making relies on genetic algorithms, where their strategies evolve through reproduction, crossover, and mutation.
- Well-performing strings (high-profit decisions) are copied more often, while low-profit strings are replaced.
- Crossover and mutation introduce new ideas by combining existing strategies or introducing randomness.
- Over time, firms learn how to maximize profits and predict equilibrium prices without using traditional methods like first-order conditions.

Simulation Results Summary

- Price and Production Stabilization: Market prices stabilize over time as firms optimize production.
- Profit Maximization: Firms with higher profits dominate future generations.
- Convergence: Prices and production quantities converge toward equilibrium.
- Genetic Operators' Role: Reproduction, crossover, and mutation drive learning and optimization.

Parameter Values of the Cobweb Model (Table 1)

- The table presents parameter values used in genetic algorithm simulations for the cobweb model.
- Parameters include:
  - A: Maximum willingness to pay.
  - B: Slope of the demand curve.
  - x: Fixed cost of production.
  - y: Variable cost of production.
- Set 6 represents the stable cobweb case, and Set 7 represents the unstable cobweb case used in Wellford's experiments.

Set	A	B	x	y
1	100	0.02	3	1
2	10	0.03	2	1
3	100	0.02	1	1
4	7	0.003	2	1
5	1000	0.02	200	1
6	2.184	0.0152	0	0.016
7	2.296	0.0168	0	0.016

Crossover and Mutation Rates (Table 2)

- This table shows crossover (‘pcross‘) and mutation (‘pmut‘) rates used in genetic algorithm simulations.
- Crossover rate: Probability of swapping parts of two strings.
- Mutation rate: Probability of changing bits in a string to introduce variation.
- Different sets (1-8) represent various configurations for these rates.

Set	**Crossover rate (p_{cross})**	**Mutation rate (p_{mut})**
1	0.6	0.0033
2	0.6	0.033
3	0.75	0.0033
4	0.75	0.033
5	0.9	0.0033
6	0.9	0.033
7	0.3	0.0033
8	0.3	0.033

Table: Crossover and mutation rates used in simulations.

Simulation Results for Single-Population GA (Table 3)

- This table shows the results of single-population GA for the cobweb model.
- Metrics include:
  - $\bar{P}$: Average price over 200 periods.
  - δ: Standard deviation of the average price.
  - δ_R: Standard deviation relative to the equilibrium price.
- The results vary for basic (stable and unstable) and augmented models.

GA Set	Average Price ($\bar{P}$)	Std. Dev. (δ)	Std. Dev. to Equilibrium (δ_R)
1 (Basic - Stable)	1.174	0.081	0.098
2 (Basic - Unstable)	1.200	0.075	0.108
3 (Augment - Stable)	1.119	0.005	0.005
4 (Augment - Unstable)	1.121	0.007	0.008

Table: Results for single-population GA based on different parameter sets.

Alternative Representation of Multiple-Population GA

- Each agent has a whole population of decision rules (strings).
- Agents select one string per period based on its performance.
- All strings are evaluated based on new information.
- Agents update their beliefs using genetic operators, assigning higher reproduction probability to better-performing strings.
- Applied to the cobweb model to simulate firms with competing ideas about production quantities.
- Allows firms to choose from multiple strategies without assuming profit maximization.
- Provides richer decision-making than single-population GA, while maintaining simplicity.

Description of the Model (Part 1)

- Each firm j has a population A_t^j of n decision rules (strings) that represent possible production quantities.
- The decoded and normalized value of each string $q_{i,t}$ represents a possible production value for the next period.
- A fitness is assigned to each string based on profit from the previous period, $t-1$.
- Selection Probability:

$$\pi_{i,j,t} = \frac{P_{t-1}q_{i,t} - C(q_{i,t})}{\sum_{i=1}^{n}\left(P_{t-1}q_{i,t} - C(q_{i,t})\right)}$$

  where $\pi_{i,j,t}$ is the probability of string i being chosen by firm j, and $C(q_{i,t})$ is the cost of production.
- The selected string determines firm j's production $q_{j,t}$ for the next period.

Description of the Model (Part 2)

- Market Clearing Price:

$$P_t = A - B\sum_{j=1}^{m} q_{j,t}$$

where P_t is the price at time t, and $\sum q_{j,t}$ is the total production of all firms.

- Cost Function for firm j:

$$C_j = xq_t^j + \frac{1}{2}y(q_t^j)^2$$

where x and y are cost parameters.

- Profit Function:

$$\Pi_t^j = P_t q_t^j - C_j(q_t^j)$$

where Π_t^j is the profit of firm j at time t.

- Firms update their strategies using genetic operators (reproduction, crossover, mutation) to form a new set of decision rules for the next period.

Election Operator (Augmented Multiple-Population GA)

- In the augmented version, the election operator is added.
- Before a string enters the competition for the next period's production, it must pass a qualifying test by the election operator.
- Only strings that pass this test are allowed to compete in the production decision.
- This operator is applied similarly to the single-population GA, helping filter out unfit strategies before the next generation.

Simulation Results for Multiple-Population GA (Table 4)

- Basic GA shows continuous price oscillations, influenced by mutation, without convergence to the rational expectations equilibrium.
- Augmented GA converged to the rational expectations equilibrium for all sets of cobweb model parameters (stable and unstable cases).
- Despite mutations, the entire population of strings in the augmented GA converged to the same value as the rational expectations equilibrium.
- Basic GA populations did not fully converge, with variations persisting among strings.

GA Set	**Avg Price ($\bar{P}$)**	**Std. Dev (δ)**	**Std. Dev from Equilibrium (δ_R)**
Basic Stable	1.109 - 1.139	0.060 - 0.113	0.060 - 0.115
Basic Unstable	1.119 - 1.168	0.055 - 0.112	0.061 - 0.113
Augment Stable	1.113 - 1.121	0.006 - 0.024	0.007 - 0.025
Augment Unstable	1.110 - 1.121	0.007 - 0.025	0.007 - 0.027

Table: Simulation results for sets 6 and 7 of cobweb model parameters.