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$\boxed{5}$

* length → 5
* {"A","T","G","C"} $\underbrace{\{``A", ``T", ``G", ``C"\}}_{4}$

* Restriction → No "AA"
* If ends "A"

$\underbrace{}_{n-2}$ C A, T, G — n

↳ must be C, T or G to don't have consecutive "A"

$3a_{n-2}$

If not ends "A"

$\underbrace{}_{n-1}$ C, $\underbrace{}_{n-1}$ T, $\underbrace{}_{n-1}$ G — n

$3a_{n-1}$

$\boxed{a_n = 3a_{n-1} + 3a_{n-2}}$ Recurrence Equation

$\boxed{n \geq 3}$ Condition

$\boxed{\begin{array}{l} a_1 = 4^1 = 4 \\ a_2 = 4^2 - 1 = 15 \end{array}}$ Initial conditions

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$$a_n = 3a_{n-1} + 3a_{n-2} \quad n \geq 3$$

$a_0 = 1$

$a_1 = 4$

$a_2 = 15$

1/ Char. poly.

$$P(x) = x^2 - 3x - 3$$

2/ Find roots

$$P(x) = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4 \cdot (-3)}}{2} \qquad \phi = \frac{3 + \sqrt{21}}{2} \qquad \bar{\phi} = \frac{3 - \sqrt{21}}{2} \Rightarrow P(x) = (x - \phi)(x - \bar{\phi})$$

3/ General form

$$a_n = (A)\phi^n + (B)\bar{\phi}^n$$

$$\begin{array}{l} a_0 = 1 \\ a_1 = 4 \end{array} \begin{cases} A\phi^0 + B\bar{\phi}^0 = 1 \rightarrow A + B = 1 \quad B = 1 - A \\ A\phi + B\bar{\phi} = 4 \rightarrow A\phi + \bar{\phi} - A\bar{\phi} = 4 \end{cases}$$

$$A\left(\frac{3+\sqrt{21}}{2}\right) + \left(\frac{3-\sqrt{21}}{2}\right) - A\left(\frac{3-\sqrt{21}}{2}\right) = 4$$

$$3A + \sqrt{21}A + 3 - \sqrt{21} - 3A + \sqrt{21}A = 8$$

$$2\sqrt{21}A + 3 - \sqrt{21} = 8$$

$$2\sqrt{21}A = 5 + \sqrt{21}$$

$$A = \frac{5}{2\sqrt{21}} + \frac{1}{2} = \boxed{\frac{5 + \sqrt{21}}{2\sqrt{21}}}$$

$$B = 1 - \frac{5 + \sqrt{21}}{2\sqrt{21}}$$

$$B = 1 - \frac{(5 + \sqrt{21})\sqrt{21}}{42}$$

$$B = 1 - \frac{5\sqrt{21} + 21}{42}$$

$$B = \frac{42 - (5\sqrt{21} + 21)}{42}$$

$$\boxed{B = \frac{21 - 5\sqrt{21}}{42}}$$

4/ Substitute

$$\boxed{a_n = \left(\frac{5 + \sqrt{21}}{2\sqrt{21}}\right)\left(\frac{3 + \sqrt{21}}{2}\right)^n + \left(\frac{21 - 5\sqrt{21}}{42}\right)\left(\frac{3 - \sqrt{21}}{2}\right)^n}$$