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6. Let $f(x, y) = x^3 - 12xy + 8y^3$.

(a) Find the critical points of f.

(b) Determine the nature of the critical points of f.

a)

$$\nabla = \left(\frac{df(x,y)}{dx}, \frac{df(x,y)}{dy} \right) = (f_x, f_y)$$

$f_x = 3x^2 - 12y$

$f_y = -12x + 24y^2$

$$f_x = 0 \rightarrow 3x^2 - 12y = 0 \qquad x^2 - 4y = 0 \qquad x = \pm\sqrt{4y} \rightarrow \pm\sqrt{4y} = 2y^2$$

$$f_y = 0 \rightarrow -12x + 24y^2 = 0 \qquad -x + 2y^2 = 0 \qquad x = 2y^2$$

$4y = 4y^2$

$4y^2 - 4y = 0$

$4y(y-1) = 0$

$\rightarrow \boxed{y=0} \quad x = \pm\sqrt{4y} \quad \boxed{x=0}$

$\rightarrow \boxed{y=1} \quad x = \pm\sqrt{4 \cdot 1} \quad \boxed{x=\pm 2}$

* Critical Points are $(0,0), (2,1), (-2,1)$

b)

$$H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 6x & -12 \\ -12 & 48y \end{pmatrix}$$

$|H(0,0)| = (6 \cdot 0) \cdot (48 \cdot 0) - (-12)(-12) < 0 \rightarrow$ $\boxed{\text{Saddle point at } (0,0)}$, because det < 0 at $(0,0)$

$|H(2,1)| = (6 \cdot 2) \cdot (48 \cdot 1) - (-12)(-12) > 0 \rightarrow$ $\boxed{\text{Minimum at } (2,1)}$ because det > 0 & $f_{xx} > 0$ at $(2,1)$

$|H(-2,1)| = (6 \cdot (-2))(48 \cdot 1) - (-12)(-12) < 0 \rightarrow$ $\boxed{\text{Saddle point at } (-2,1)}$, because det < 0 at $(-2,1)$

$f_{xx} = 6x$

$f_{xy} = f_{yx} = -12$

$f_{yy} = 48y$