

5

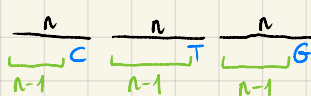
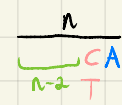
\* length  $\rightarrow 5$

\* Restriction  $\rightarrow$  No "AA"

\*  $\{A, T, G, C\}$   
4

\* If ends "A"

If not ends "A"



$\rightarrow$  must be C, T or G to don't have consecutive "A"

$$3a_{n-2}$$

$$3a_{n-1}$$

$$a_n = 3a_{n-1} + 3a_{n-2}$$

$$n \geq 3$$

$$a_1 = 4^1 = 4$$

$$a_2 = 4^2 - 1 = 15$$

Recurrence Equation

Condition

Initial conditions

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$$a_n = 3a_{n-1} + 3a_{n-2} \quad n \geq 3$$

$$a_0 = 1$$

$$a_1 = 4$$

$$a_2 = 15$$

1/ Char. poly.

$$P(x) = x^2 - 3x - 3$$

2/ Find roots

$$P(x) = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(-3)}}{2}$$

$$\phi = \frac{3 + \sqrt{21}}{2}$$

$$\bar{\phi} = \frac{3 - \sqrt{21}}{2}$$

$$\Rightarrow P(x) = (x - \phi)(x - \bar{\phi})$$

3/ General form

$$a_n = (A)\phi^n + (B)\bar{\phi}^n$$

$$a_0 = 1 \begin{cases} A\phi^0 + B\bar{\phi}^0 = 1 \end{cases} \rightarrow A + B = 1 \quad B = 1 - A$$

$$a_1 = 4 \begin{cases} A\phi + B\bar{\phi} = 4 \end{cases} \rightarrow A\phi + \bar{\phi} - A\bar{\phi} = 4$$

$$A\left(\frac{3 + \sqrt{21}}{2}\right) + \left(\frac{3 - \sqrt{21}}{2}\right) - A\left(\frac{3 - \sqrt{21}}{2}\right) = 4$$

$$3A + \sqrt{21}A + 3 - \sqrt{21} - 3A + \sqrt{21}A = 8$$

$$2\sqrt{21}A + 3 - \sqrt{21} = 8$$

$$2\sqrt{21}A = 5 + \sqrt{21}$$

$$A = \frac{5}{2\sqrt{21}} + \frac{1}{2} = \frac{5 + \sqrt{21}}{2\sqrt{21}}$$

$$B = 1 - \frac{5 + \sqrt{21}}{2\sqrt{21}}$$

$$B = 1 - \frac{(5 + \sqrt{21})\sqrt{21}}{42}$$

$$B = 1 - \frac{5\sqrt{21} + 21}{42}$$

$$B = \frac{42 - (5\sqrt{21} + 21)}{42}$$

$$B = \frac{21 - 5\sqrt{21}}{42}$$

4/ Substitute

$$a_n = \left(\frac{5 + \sqrt{21}}{2\sqrt{21}}\right)\left(\frac{3 + \sqrt{21}}{2}\right)^n + \left(\frac{21 - 5\sqrt{21}}{42}\right)\left(\frac{3 - \sqrt{21}}{2}\right)^n$$