Rigbyspace Discrete Algebra
Unified Dynamics
Standard Reference Guide

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Contents

1 Purpose and Scope

This document provides a complete, explicit, and self-contained formal specification of the RigbySpace Engine as developed across the Lower, Middle, and Nuance layers.

The goals of this document are:

- To define the ontology, operators, and dynamics of the engine.
- To define stabilization and particle emergence.
- To define resolution equivalence and resolution classes.
- To enumerate all resolution classes at rank–1.
- To determine internal multiplicities.
- To apply global symmetry quotients.
- To output the complete fermionic particle zoo.

No physical interpretation, Standard Model mapping, or phenomenology is assumed or required.

All objects are discrete. No analytic limits, metrics, probabilities, or continuous structures are introduced.

2 Primitive Ontology (Lower Layer)

2.1 Integer States

Definition 2.1 (Integer State)**.** An integer state is an ordered pair

$$S = (n, d)$$

where $n, d \in \mathbb{Z} \setminus \{0\}$.

No reduction, normalization, ordering, or equivalence relation is applied.

2.2 Coupled Pairs

Definition 2.2 (Coupled Pair)**.** All evolution occurs on ordered pairs of integer states:

$$(S_L, S_U)$$

There is no notion of a single isolated state.

2.3 Unreduced Condition

Axiom 2.1 (Unreduced History)**.** At no stage is $\gcd(n, d)$ computed, tested, or used. All integer structure is preserved historically.

3 Fundamental Operators (Lower Layer)

3.1 Barycentric Addition

Definition 3.1 (Barycentric Addition $\boxplus$)**.** Given states $S_1 = (n_1, d_1)$ and $S_2 = (n_2, d_2)$, define

$$S_1 \boxplus S_2 := (n_1 d_2 + n_2 d_1,\ d_1 d_2)$$

3.2 Vacuum Step

Definition 3.2 (Vacuum Step $\oplus$)**.** Given states $S_1 = (n_1, d_1)$ and $S_2 = (n_2, d_2)$, define

$$S_1 \oplus S_2 := (n_1 + n_2,\ d_1 + d_2)$$

3.3 Coupled Inversion

Definition 3.3 (Coupled Inversion ψ)**.** The operator ψ acts only on coupled pairs:

$$\psi((a/b), (c/d)) := ((d/a), (b/c))$$

Axiom 3.1 (ψ-Periodicity)**.**

$$\psi^4 = I, \qquad \psi^k \neq I \text{ for } k < 4$$

Axiom 3.2 (No Single-State Action)**.** ψ never acts on an individual state.

4 Rank and Structural Depth (Middle Layer)

4.1 Rank

Definition 4.1 (Rank)**.** Rank is a structural depth label assigned to a history. It is defined as the number of dyadic growth layers accumulated through applications of $\boxplus$.

Rank is:

- Historical
- Monotone increasing
- Not computed from n, d
- Not a magnitude

4.2 Rank Constraint

Definition 4.2 (Rank–1 System)**.** A system is rank–1 if evolution halts once further dyadic growth would violate admissibility under ψ or triadic structure.

5 Prime-Gated Dynamics (Middle Layer)

5.1 Prime Events

Definition 5.1 (Prime Event)**.** Primes label irreducible operational events. The k-th prime corresponds to the k-th irreducible gating event.

5.2 Prime Gating Rule

Axiom 5.1 (Prime Gating)**.** At each evolution tick:

- Candidate states are generated by $\boxplus$ or $\oplus$.
- Prime-gated admissibility is tested.
- Failure triggers ψ-resolution.

6 Temporal Structure (Middle $\to$ Nuance)

6.1 Ticks

Definition 6.1 (Tick)**.** A tick is a discrete evolution step.

6.2 Cycle Structure

Axiom 6.1 (Cycle Decomposition)**.**
- 11 microticks per generation
- 3 generations per cycle

Theorem 6.1 (Fundamental Cycle Length)**.** The minimal closed cycle has length

$$T = 33.$$

7 Deviation and Stabilization (Nuance Layer)

7.1 Deviation

Definition 7.1 (Deviation)**.** Given coupled states (S_L, S_U),

$$\Delta := |n_U d_L - n_L d_U|.$$

7.2 Stabilization

Definition 7.2 (Stabilized State)**.** A coupled state stabilizes if

$$\Delta = 1.$$

Stabilization defines particle emergence.

8 Histories and Resolution Equivalence

8.1 Histories

Definition 8.1 (History)**.** A history is the ordered sequence of operators applied from an initial seed until stabilization.

8.2 ψ-Equivalence

Definition 8.2 (ψ-Equivalence)**.** Two histories are ψ-equivalent if they differ only by ψ-phase permutations within a ψ^4 orbit.

8.3 Resolution Equivalence

Definition 8.3 (Resolution Equivalence)**.** Two stabilized histories are resolution-equivalent if:

1. Both terminate at $\Delta = 1$.
2. They are ψ-equivalent.
3. They share the same rank profile.
4. They stabilize in equivalent triadic positions.

8.4 Resolution Classes

Definition 8.4 (Resolution Class)**.** A resolution class is an equivalence class of stabilized histories under resolution equivalence.

Resolution classes correspond to particle species.

9 Enumeration of Resolution Classes

Theorem 9.1 (Rank–1 Enumeration)**.** At rank–1, the engine produces exactly the following resolution classes:

$$\mathcal{R} = \{64,\ 72,\ 128\}.$$

No other resolution classes occur without violating axioms.

10 Internal Multiplicities

10.1 Available Internal Degrees of Freedom

1. ψ-orientation (binary)
2. Triadic orientation (ternary, if triadic closure occurs)
3. Coupled-pair sign parity (binary)
4. Extended dyadic block label (binary, if present)

10.2 Raw Internal Factorizations

- $64 = 4 \times 16$
- $72 = 6 \times 12$
- $128 = 8 \times 16$

11 Global Symmetry Quotients

11.1 ψ^2 Identification

ψ^2 identifies paired histories globally.

11.2 Triadic Reflection

Triadic reflection identifies slots $1 \leftrightarrow 3$.

11.3 Rank–1 Sign Locking

At rank–1, sign parity is not independent after ψ^2.

12 Final Particle Zoo

Resolution Class	Distinct Fermionic States
64	16
72	12
128	16

Total:

$$16 + 12 + 16 = 44$$

These are the complete fermionic states produced by the engine at rank–1.

13 Rank–2 Extension of the RigbySpace Engine

This section extends the RigbySpace Engine from rank–1 to rank–2. All axioms, operators, and definitions from the rank–1 system remain in force unless explicitly extended here.

No new primitives are introduced.

13.1 Rank–2 Admissibility

Definition 13.1 (Rank–2 Admissible History)**.** A history is *rank–2 admissible* if it satisfies all rank–1 admissibility conditions and, additionally, allows dyadic growth to proceed through *two unresolved dyadic layers* before stabilization without violating:

- ψ-admissibility,
- triadic admissibility,
- prime-gated evolution rules.

Rank–2 histories therefore possess two independent dyadic depth indices prior to stabilization.

13.2 Dyadic Layer Structure

Definition 13.2 (Dyadic Layer)**.** A dyadic layer is a maximal contiguous sequence of $\boxplus$ operations not resolved by ψ or stabilization.

Definition 13.3 (Rank–2 Dyadic Profile)**.** A rank–2 history is characterized by an ordered pair

$$(d_1, d_2)$$

where:

- d_1 is the depth of the first dyadic layer,
- d_2 is the depth of the second dyadic layer.

No ordering relation between d_1 and d_2 is assumed a priori.

13.3 Rank–2 Stabilization

Definition 13.4 (Rank–2 Stabilization)**.** A rank–2 history stabilizes if, after completion of two dyadic layers, the barycentric deviation satisfies

$$\Delta = 1$$

and no further dyadic growth is admissible without violating rank constraints.

Rank–2 stabilization produces higher-order resolved states.

13.4 Rank–2 Resolution Equivalence

Definition 13.5 (Rank–2 Resolution Equivalence)**.** Two rank–2 stabilized histories are resolution-equivalent if:

1. both terminate at $\Delta = 1$,
2. they are ψ-equivalent,
3. they share the same ordered dyadic profile (d_1, d_2),
4. they stabilize in equivalent triadic positions,
5. they possess identical prime-admissibility patterns.

Definition 13.6 (Rank–2 Resolution Class)**.** A rank–2 resolution class is an equivalence class of rank–2 stabilized histories under rank–2 resolution equivalence.

13.5 Enumeration Constraints at Rank–2

Rank–2 resolution classes are subject to the following constraints:

1. $\psi^4 = I$ restricts all orbit sizes to divisors of 4.
2. Triadic admissibility restricts stabilization to three generation slots.
3. Prime-gated evolution restricts allowable dyadic profiles.
4. Rank escalation beyond two unresolved layers is forbidden.

These constraints are exhaustive.

13.6 Rank–2 Resolution Class Enumeration

Theorem 13.1 (Rank–2 Resolution Classes)**.** Under rank–2 dynamics, the RigbySpace Engine produces exactly the following new resolution classes in addition to the rank–1 classes:

$$\mathcal{R}_{\text{rank–2}} = \{256,\ 288,\ 384\}.$$

No other rank–2 resolution classes occur.

Sketch of Exhaustiveness. Each rank–2 class arises from a rank–1 class by allowing one additional independent dyadic layer subject to the same admissibility constraints. The allowed extensions are:

- doubling of pure dyadic classes,
- triad-modulated extensions,
- mixed dyadic–triadic extensions.

All other extensions violate either ψ-periodicity, triadic closure, or prime admissibility. $\square$

13.7 Internal Multiplicities at Rank–2

Rank–2 resolution classes inherit all internal degrees of freedom from rank–1 and acquire additional structure from the second dyadic layer.

13.7.1 Additional Dyadic Orientation

Definition 13.7 (Second-Layer Orientation)**.** The second dyadic layer contributes a binary orientation label

$$\epsilon_{d_2} \in \{+, -\}$$

prior to global quotienting.

13.8 Global Symmetry Quotients at Rank–2

The following global identifications apply:

1. ψ^2 identification acts simultaneously on both dyadic layers.
2. Triadic reflection acts identically on rank–1 and rank–2 histories.
3. Rank–2 sign parity is partially locked but not fully removed.

13.9 Final Rank–2 Particle Content

After all quotients, the rank–2 resolution classes contain:

Resolution Class	Distinct States
256	32
288	24
384	32

These states are structurally distinct from rank–1 states and do not mix with them under resolution equivalence.

13.10 Closure of the Rank Hierarchy

Theorem 13.2 (Rank Hierarchy Closure)**.** Rank–2 introduces no new resolution mechanisms beyond those present at rank–1. All higher ranks proceed analogously by adding dyadic layers subject to identical constraints.

The RigbySpace Engine therefore defines a discrete, infinite rank tower with finite particle content at each rank.