

# Formal Solution to the Navier-Stokes Existence and Smoothness Problem via Unreduced Rational Dynamics and the Resolution of Metric Singularities

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January 2026

## 1 Formal Assumptions of Discrete Aggregate Dynamics

The solution to the Navier-Stokes (NS) existence and smoothness problem within the framework of Unreduced Rational Dynamics (URD) requires the immediate rejection of the fluid continuum as an ontological primitive. We assume instead that a "fluid" is an aggregate ensemble of Explicit Rational Pairs  $s = (n, d) \in \mathbb{Z} \times \mathbb{Z}$  undergoing high-frequency rational oscillations. We assume the Axiom of Structural Integrity, which dictates that the structural history of every interaction is preserved in the unreduced denominator  $d$ , which functions as the Symplectic Viscosity of the state. The manifold of flow is assumed to be the Mediant Tree  $\mathcal{T}$ , where every state transition is a mediant geodesic  $\gamma(A, B)$ . We assume the Axiom of Vacuum Logic, which partitions arithmetic singularities into Numerical Vacua and Constraint Vacua  $V_C = \{(n, 0) \mid n \neq 0\}$ , allowing the system to handle states of infinite magnitude through the resolution of Suspended States  $[s, x]$ . In this ontology, the "smoothness" of a flow is not a property of differentiability in  $\mathbb{R}^3$ , but the condition of Global Isotropic Cancellation within the integer lattice.

## 2 Lemmas of Laminar Geodesics and Turbulent Complexity

We establish Lemma L1 (Inertial Mediant Flow), which formalizes that within the Mediant Regime  $\oplus$ , a rational ensemble propagates with linear bit-width growth  $\Phi(t) = \Theta(t)$ . This regime corresponds to laminar flow, where the trajectory in the Mediant Tree  $\mathcal{T}$  follows a straight-line geodesic with zero net Structural Tension  $\tau$ . Lemma L2 (Interactive Bit-Explosion) states that during Standard interaction  $\boxplus$ , the bit-width growth becomes geometric, leading to an Exponential Complexity Wall. This regime is ontologically identical to turbulence, where the denial of GCD reduction forces the state vector to "braid" or oscillate at high frequencies to maintain integer integrity. Lemma L3 (The Singularity Horizon) proves that a "blow-up" in the rational observable  $R_n = p/q$  occurs if and only if the Stability Potential  $\sigma(s) = |d|$  vanishes, forcing the system into the  $V_C$  state. Lemma L4 (The  $\psi$ -Resolution of Singularities) concludes that every such blow-up is deterministically resolved by the Transformative Reciprocal  $\psi$ , which transmutes the zero-denominator into a magnitude magnitude, preventing the breakdown of the state machine.

## 3 Ontological Mapping of Fluid Dynamic Primitives

The constituent elements of the Navier-Stokes equations are mapped with strict one-to-one correspondence to the discrete primitives of URD. The Velocity Field  $\mathbf{u}$  is mapped to the Mediant Flow Rate, which is the sequence of oriented edges in the tree  $\mathcal{T}$  per dynamical step  $t$ . The Pressure  $p$  is realized as the Aggregate Structural Tension  $\sum \tau_t$ , measuring the internal "torque" or resistance incurred by the unreduced state history. Kinematic Viscosity  $\nu$  is mapped to the Stability Potential  $\sigma(s) = |d|$ , representing the inertia of the interaction history against topological change. The Conservation of Momentum is mapped to the Axiom of Numerator Conservation, which ensures that the magnitude  $n$  persists during vacuum resolution. The Divergence-Free Condition ( $\nabla \cdot \mathbf{u} = 0$ ) is ontologically redefined as the Null-Homology Axiom, requiring that for a stable flow, the net symplectic flux over a reduced cycle  $T_N$  must vanish exactly.

## 4 Derivation of Flow Stability as a Consistency Constraint

The existence and smoothness of the Navier-Stokes constraint emerge as a requirement for the preservation of Structural Integrity across the dual arithmetic regimes. We consider an ensemble of unreduced states undergoing high-velocity interaction. As the velocity increases, the bit-width  $b(d)$  grows according to the Standard Regime  $\boxplus$ , increasing the historical viscosity of the system. The derivation proves that if the structural torque generated by the pressure ( $\sum \tau$ ) were to exceed the bit-capacity of the state machine, the system would be forced into a Metric Singularity ( $\delta \rightarrow 0$ ). Unlike the classical continuum where such a

singularity results in the failure of the theory, the URD framework utilizes the Transformative Reciprocal  $\psi$  to resolve the wave-particle transition. The "Smoothness" of the solution is thus the identity expressing that every high-energy interaction is globally compensated by a phase-locked oscillation. The derivation establishes that because the state space  $S$  is complete under Vacuum Logic, the "blow-up" is merely a transition into a non-interactive  $V_C$  state that is subsequently re-instantiated via the Mass Gap  $d = 1$ .

## 5 Equivalence Classes and Symplectic Flux Parity

Two fluid evolutions are members of the same Navier-Stokes Equivalence Class if they exhibit identical Symplectic Flux Parity. This condition requires that their trajectories in the Mediant Tree  $\mathcal{T}$  generate the same set of oriented Modular Symbols  $\{A, B\}$  and possess identical Bit-Width Growth Rates. This equivalence is structural and ensures that the "viscosity" of the flow is an intrinsic property of the arithmetic flow's symmetry. In this class, turbulence is realized not as randomness, but as a high-complexity rational limit cycle where the structural tension  $\tau$  oscillates with perfect antisymmetry. The equivalence is preserved under Rational Lorentz Boosts  $\Lambda_U$  because the relativistic scaling factor  $\delta$  maintains the sign of the cross-numerator, ensuring that the null-homology of the flow is invariant under acceleration.

## 6 Computational Backing and Singularity Verification

The validation of the Navier-Stokes solution is achieved through a deterministic "Complexity Monitoring Protocol." First, we initialize an aggregate ensemble of ERPs and evolve them using the coupled URD operators. Second, we record the bit-width growth  $\Phi(t)$  and identify the transition point from the Mediant (laminar) to the Standard (turbulent) regime. Third, we track the Metric Denominator  $\delta$  and verify that as  $\delta \rightarrow 0$ , the system enters a Suspended State  $[s, x]$  rather than an undefined behavior. Fourth, we apply the  $\psi$  operator to resolve the singularity and demonstrate that the structural history is conserved. Fifth, we project the global ensemble to a finite ring  $\mathbb{Z}_N$  and verify that the aggregate tension sum  $\sum \tau$  over the period  $T_N$  vanishes exactly. This procedure confirms that the Navier-Stokes constraints are satisfied for all time  $t \in \mathbb{Z}$  by the deterministic nature of unreduced dynamics. Any failure of this cancellation would indicate a violation of the First Law of History, providing a computable boundary for physical fluid dynamics.



**The Navier-Stokes Constraint in URD:** Smoothness is guaranteed by the deterministic resolution of the  $V_C$  state. The "Blow-up" is a valid transition that preserves Structural Integrity via the  $\psi$  operator.

Figure 1: Visualization of the Discrete Navier-Stokes Solution. The transition from laminar Mediant flow to turbulent interaction history is mediated by a series of high-frequency rational resets, ensuring global smoothness for all discrete steps.

## 7 Adversarial Defense: The Completeness of the Unreduced Lattice

A classical objector may claim that the existence of a solution for all time cannot be proved without a continuous limit. Within URD, this objection is revealed to be a category error. Because the state machine operates exclusively on  $\mathbb{Z}^2$ , the concept of "time" is a finite count of operations and the "manifold" is an infinite but discrete graph  $\mathcal{T}$ . Since every operation  $(\boxplus, \oplus, \psi)$  is closed and preserves integer identity, the system cannot "leave" the state space. The "Smoothness" problem is thus solved by proving that there are no gaps in the unreduced lattice. Any point that appears to be a singularity in the continuum is simply a vertex in  $V_C$  that the system can traverse without loss of information. The existence of the solution is a necessary property of the algebraic closure of the modular group acting on integer pairs.