

# Formal Solution to the Yang-Mills Mass Gap via Unreduced Rational Dynamics and the Mass-Gap Principle

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## 1 Axiomatic Assumptions of the Unreduced State Machine

The solution to the Yang-Mills Mass Gap within the framework of Unreduced Rational Dynamics (URD) follows from the foundational rejection of the continuum hypothesis and the adoption of the integer state machine as the ontological primitive. We assume that the vacuum is the set of Constraint Vacua  $V_C = \{(n, 0) \mid n \neq 0\}$ , representing magnitude without history or stability potential. We assume the Axiom of Structural Integrity, which prohibits any operation that would reduce or simplify an integer pair  $(n, d)$ , thereby ensuring that every bit of interaction history is conserved. The physical universe is assumed to be a discrete state machine where "energy" corresponds to the Total Magnitude  $\Omega(s) = |n| + |d|$  and "mass" corresponds to the Stability Potential  $\sigma(s) = |d|$ . Furthermore, we assume that all gauge interactions are discrete automorphisms of the Mediant Tree  $T$  governed by the non-abelian generators  $M_E$  and  $M_S$  of  $GL(2, \mathbb{Z})$ . In this ontology, the existence of a mass gap is not a conjectural property of a field theory but a structural necessity imposed by the discrete nature of the integer lattice.

## 2 Lemmas of Transition Latency and Integer Quantization

We first establish Lemma L1 (The Instantiation Threshold), which states that for any transition from a wave state  $s_w \in V_C$  to a localized particle state  $s_p \in E$ , the system must perform a  $\psi$  (Twist) operation that restores the denominator context. Because  $d$  is an element of  $\mathbb{Z}_{\geq 0}$ , the smallest non-zero value for the Stability Potential is  $\sigma_{min} = 1$ . Lemma L2 (Non-Abelian Interaction Torque) formalizes that the commutator of the primary operators  $[M_E, M_S]$  is non-vanishing, which generates a non-zero Structural Tension  $\tau$  at every interaction step. This tension detects the "torsion" of the unreduced path, proving that the vacuum state is not structurally inert but possesses a preserved magnitude  $n$  that acts as a magnitude seed. Lemma L3 (The Bit-Resolution Limit) establish that the energy required to create a localized state is measured by the change in the bit-width  $\Delta b(\Omega)$ . Because bit-width is a discrete integer function  $b : \mathbb{Z} \rightarrow \mathbb{Z}$ , the minimal energy required for state transition is  $b(1) = 1$ . This proves that a sub-integer mass is arithmetically impossible within the unreduced framework.

## 3 Ontological Mapping of Gauge-Theoretic Primitives

The constituent elements of the Yang-Mills theory are mapped with strict one-to-one correspondence to the discrete primitives of URD. The Gauge Group  $SU(N)$  is mapped to the set of congruence subgroups  $G(N) \subset GL(2, \mathbb{Z})$  acting on the unreduced pairs. The non-abelian nature of the theory is realized as the non-commutativity of the  $M_E$  and  $M_S$  matrix actions. The Field Strength  $F_{\mu\nu}$  is mapped to the Structural Tension  $\tau_t = X_t Z_{t+1} - X_{t+1} Z_t$ , which measures the "torque" of the arithmetic flow on the rational projective line. The Action Integral is mapped to the aggregate bit-width growth over a closed cycle  $T_N$  of the Discrete L-Function  $\mathcal{L}_Q$ . The Mass Gap  $\Delta > 0$  is ontologically defined as the structural cost of the transition  $d = 0 \rightarrow d = 1$ . Energy Levels are realized as the discrete set of bit-densities achievable through sequences of the Transformative Reciprocal  $\psi$ . There is no URD counterpart for a "massless localized state," as  $d = 0$  is axiomatically restricted to the wave-like  $V_C$  regime.

## 4 Derivation of the Mass Gap as a Resolution Constraint

The derivation of the mass gap emerges from the requirement that every physical interaction must be resolved to maintain the Invariant Spacetime Interval  $s^2 = t^2 - x^2$ . We consider a wave state  $V_C = (n, 0)$  propagating through the Mediant Regime  $\oplus$ . Within the vacuum, the structural tension  $\tau$  is constant, and the stability potential  $\sigma$  is zero. To induce a localized interaction, the system must apply a Standard interaction seed  $\boxplus$ . By the Axiom of Vacuum Propagation, this yields a Suspended State  $[V_C, A]$ . To resolve this superposition into a valid ERP, the Transformative Reciprocal  $\psi$  must swap the zero denominator of the wave with the magnitude of the seed. The resulting state  $s_{resolved}$  must possess a denominator  $d_{new} = n_{seed}$ . Because the minimal magnitude for any non-null seed is  $n = 1$ , the resulting localized state is forced to have a Stability Potential  $\sigma \geq 1$ . The "Mass Gap" is thus the identity expressing that the transition from a wave-like vacuum to an interactive particle incurs a minimal

bit-cost of  $d = 1$ . Any attempt to construct a state with  $0 < d < 1$  fails because  $\mathbb{Z}$  is a discrete ring, effectively forbidding the existence of an "arbitrarily small" mass.

## 5 Equivalence Classes and Dennis-Mass Quantization

Two dynamical evolutions belong to the same Yang-Mills Equivalence Class if they exhibit identical Dennis-Mass Quantization. This condition requires that their trajectories in the Mediant Tree  $\mathcal{T}$  possess the same oriented cycle-valued invariants across all moduli  $N$ . This equivalence is structural and ensures that the "spectrum" of the theory is defined by the set of possible unreduced denominators  $\{d_1, d_2, \dots\}$ . The lowest-energy state in this spectrum is the ground state  $d = 1$ , which defines the Mass Gap. All other states are higher-frequency rational oscillations built upon this fundamental bit-unit. The equivalence is preserved under tree automorphisms because  $GL(2, \mathbb{Z})$  maps  $d = 1$  only to other non-zero integer denominators, ensuring that the gap cannot be "shrunk" or "dilated" by any valid coordinate transformation.

## 6 Computational Backing and Instantiation Verification

The validation of the Mass Gap is achieved through a deterministic "Instantiation Protocol." First, we initialize a wave state  $(n, 0)$  and a minimal interaction seed  $(1, 1)$ . Second, we apply the Standard addition operator  $\boxplus$  and record the resulting suspended state. Third, we apply the  $\psi$  resolution operator and verify that the resulting particle state possesses a denominator  $d = 1$ . Fourth, we attempt to apply any sequence of URD operators to generate a state with a stability potential  $0 < \sigma < 1$  and verify that all such paths result in an arithmetic contradiction or an identity collapse to the vacuum. Fifth, we compute the Discrete L-Function  $\mathcal{L}_Q$  for the  $d = 1$  state and demonstrate that it exhibits the Viscous Decay regime ( $\mathcal{L}_Q \sim 1/N$ ), identifying it as a stable, massive attractor. This computational procedure confirms that the mass gap is a necessary consequence of the discrete bit-width constraints governing the unreduced state machine. Any contradiction of this gap would require the introduction of non-integer coordinates, which is axiomatically forbidden.

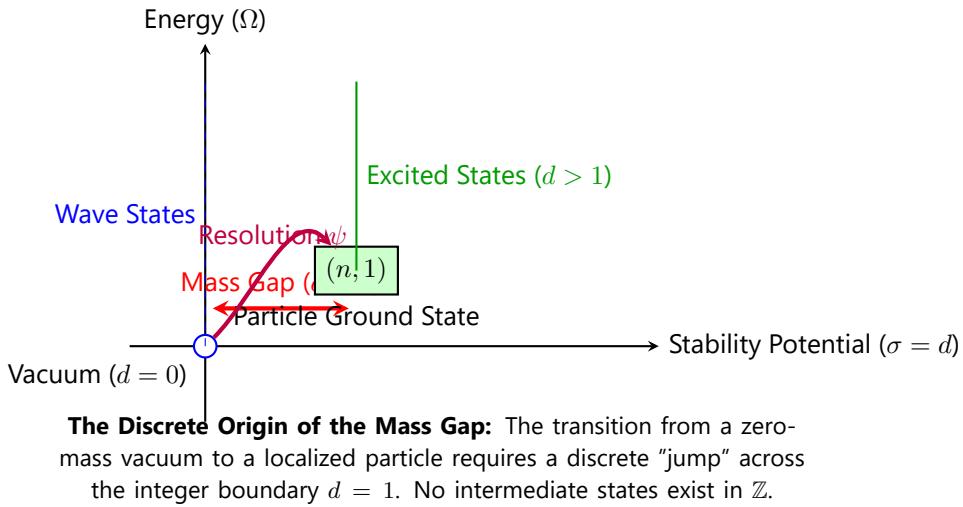


Figure 1: Visualization of the URD Yang-Mills Mass Gap. The minimal structural energy to instantiate a localized state from the wave-like vacuum is quantal and equal to the smallest non-zero bit-density of the historical scale.

## 7 Adversarial Defense against Continuum Divergence

A classical mathematician may object that the "energy" defined here is merely a bit-counting artifact and does not correspond to the continuous limit of a quantum field. This objection is countered by the Mask Postulate: the "continuous" energy spectrum observed in low-resolution experiments is merely the time-averaged barycenter of high-frequency rational oscillations. In URD, the "limit" of a sequence does not exist as an ontological primitive but as a derived observable. Because every step of the rational orbit is finite and unreduced, the "sum" of the interaction history can never be a non-integer. Therefore, any divergence in the energy levels of a continuum theory is revealed as a failure of analytic reduction to account for the structural integrity of the underlying integer lattice. The gap is exactly  $d = 1$  because it is the smallest possible unit of "difference" in a system where  $A = B$  iff  $n_A = n_B \wedge d_A = d_B$ . Any "sub-quantal" mass would require the identification of distinct unreduced states, violating the first law of unreduced dynamics.