Rigbyspace Dynamics
Unified Framework Master Reference

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Abstract

This document constitutes the authoritative master reference for the unified discrete-theoretical framework known as Rigbyspace Dynamics. It integrates rigorous replacements for classical modulo and logarithmic operations within a constructive physical dynamics. The framework establishes ZERO Logic for handling structural singularities via suspension states and Triadic Convergence, where constants are defined as oscillatory barycentric structures rather than scalar limits. All reasoning is strictly constructive and integer-based; no continuum concepts, real numbers, limits, or analytic functions are permitted. The theory serves as a foundational ontology for discrete physics, deriving relativistic and quantum-like phenomena from finite integer operations.

Contents

II Rigbyspace: Constructive Integer Dynamics

III Discrete Constants and Derivations

Part I

Mathematical Foundations

1 Ontological Assumptions

The foundation of Rigbyspace Dynamics rests upon a strict finitistic ontology. Unlike standard theoretical physics, which assumes a background of smooth manifolds and real-valued fields, this framework posits that the physical universe is computationally generated from discrete integer primitives. The ontology restricts the set of valid mathematical objects to ensure that all physical predictions are computable in finite time.

The universe admits only the following objects:

- Integers ($\mathbb{Z}$).

- Finite sets of integers.

- Finite sequences of integers.

- Finite directed graphs with integer-labeled vertices.

Consequently, no equivalence classes (such as the rationals $\mathbb{Q}$ as traditionally defined), real numbers ($\mathbb{R}$), limits to infinity, or analytic functions exist as fundamental entities. All physical observables must be constructible via finite integer algorithms.

2 Discrete Replacement for Modulo

2.1 Motivation

The classical expression $t \pmod{N}$ presupposes a quotient operation and the algebraic structure of modular arithmetic. In a strictly discrete physical universe, the concept of "quotienting" is an abstraction that obscures the underlying mechanism of cyclic evolution. We replace the arithmetic modulo with explicit state evolution on a finite directed graph. This provides a geometric and causal interpretation of periodicity, where time is measured by transitions between discrete states rather than by division.

2.2 Definition of the Phase Space

The fundamental arena for cyclic dynamics is the Phase Space. This is not a continuous circle but a finite set of discrete states.

Definition 2.1 (Phase Space). For a fixed integer $N \geq 1$, define the phase space

$$\Phi_N := \{0, 1, 2, \ldots, N-1\}.$$

This is a finite set of integer states representing the available distinct phases of a system with capacity N.

2.3 Successor-with-Reset Operator

Movement within the phase space is governed by a deterministic operator that enforces the boundary condition.
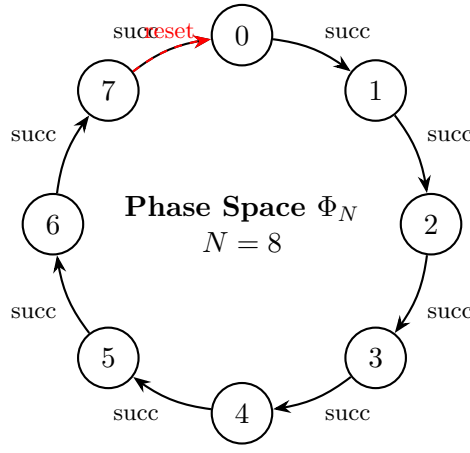
Figure 1: The discrete Phase Space Φ_N (here $N = 8$) replacing the continuum circle. Evolution is defined by the Successor Map, forming a closed directed graph.

Definition 2.2 (Successor Map). Define a function $\mathrm{succ}_N : \Phi_N \to \Phi_N$ by:

$$\mathrm{succ}_N(s) = \begin{cases} s+1, & \text{if } s < N-1, \\ 0, & \text{if } s = N-1. \end{cases}$$

This function is total, deterministic, and defined entirely by integer comparison. It generates the causal flow of time within the cycle.

2.4 Phase Evolution

The dynamics of a system are described by the recursive application of the successor map.

Definition 2.3 (Phase Evolution). Given an initial phase $s_0 \in \Phi_N$, define the phase at time $t \in \mathbb{Z}_{\geq 0}$ recursively:

$$\phi_N(0) := s_0, \quad \phi_N(t+1) := \mathrm{succ}_N(\phi_N(t)).$$

No arithmetic reduction or division is involved; the state evolves stepwise.

2.5 Axioms for Discrete Modularity

We establish two axioms that govern the behavior of cyclic systems.

Axiom 2.1 (Finite Cyclicity). The directed graph induced by succ_N is a single cycle of length N.

Axiom 2.2 (Determinism). For fixed N and s_0, the phase $\phi_N(t)$ is uniquely defined for all t.

2.6 Derived Properties

From the definitions above, we can derive standard periodic properties without invoking modular arithmetic.

Lemma 2.1 (Periodicity). For all $t \geq 0$, $\phi_N(t+N) = \phi_N(t)$.

Proof. After N applications of succ_N, the cycle returns to its starting state by the definition of the graph structure.

□

Lemma 2.2 (Phase Uniqueness). For $0 \leq t_1 < t_2 < N$, $\phi_N(t_1) \neq \phi_N(t_2)$.

2.7 Theorem: Modulo Elimination

Theorem 2.1 (Discrete Modulo Replacement). Any construction that would traditionally reference $t \pmod{N}$ is replaced by the phase $\phi_N(t)$.

3 Discrete Replacement for Logarithm

3.1 Motivation

Logarithms quantify scale, size, or complexity. In the continuum, $\log(x)$ is a smooth function mapping magnitude to a linear scale. In a discrete universe, scale is measured by counting how many times a quantity crosses fixed integer thresholds. This defines the "Structural Depth" of a number.

3.2 Threshold Construction

We define a sequence of thresholds that partition the integers into magnitude classes.

Definition 3.1 (Doubling Thresholds)**.** Define the sequence $\{T_k\}_{k \geq 0}$ by:

$$T_0 := 1, \quad T_{k+1} := 2T_k.$$

This sequence is constructed by repeated integer doubling.

3.3 Rank Function

The rank function replaces the logarithm. It is an algorithmic procedure that determines the magnitude tier of an integer.

Definition 3.2 (Discrete Rank)**.** For an integer $m \geq 0$, define its rank $\rho(m)$ by the algorithm:

1. If $m = 0$, return 0.

2. Initialize $k := 0$.

3. While $m \geq T_{k+1}$, increment k.

4. Return k.

The algorithm halts for all finite m. This definition ensures that if $T_k \leq m < T_{k+1}$, the rank is k.

3.4 Axioms for Rank

Axiom 3.1 (Existence)**.** For all integers $m \geq 0$, the rank $\rho(m)$ exists and is finite.

Axiom 3.2 (Monotonicity)**.** If $0 \leq m_1 < m_2$, then $\rho(m_1) \leq \rho(m_2)$.

3.5 Basic Properties

The rank function satisfies properties analogous to the logarithm, bounded by integer precision.

Lemma 3.1 (Threshold Bracketing)**.** For all $m > 0$, $T_{\rho(m)} \leq m < T_{\rho(m)+1}$.

Lemma 3.2 (Additive Growth Bound)**.** For integers $a, b \geq 0$, $\rho(a + b) \leq \max(\rho(a), \rho(b)) + 1$.

Proof. Without loss of generality assume $a \geq b$. Then $a + b \leq 2a$. Since $a < T_{\rho(a)+1}$, we have $2a < 2T_{\rho(a)+1} = T_{\rho(a)+2}$. Thus $\rho(a + b) \leq \rho(a) + 1$. The proof is symmetric for $b > a$. □

Lemma 3.3 (Multiplicative Growth Bound)**.** For integers $a, b \geq 1$, $\rho(ab) \leq \rho(a) + \rho(b) + 1$.

Proof. Let $k_a = \rho(a)$ and $k_b = \rho(b)$. Then $a < 2^{k_a+1}$ and $b < 2^{k_b+1}$. The product $ab < 2^{k_a+1} \cdot 2^{k_b+1} = 2^{k_a+k_b+2}$. The rank is bounded by the exponent minus 1, so $\rho(ab) \leq k_a + k_b + 1$. □

3.6 Theorem: Logarithm Elimination

Theorem 3.1 (Discrete Logarithmic Replacement)**.** Any use of logarithmic scale, structural depth, or order-of-magnitude is replaced by the rank function ρ. No logarithmic operator exists in the system.

4 Separation Principle

To ensure the framework remains self-contained, we postulate a strict separation between the objects of the theory and the language of description.

Axiom 4.1 (Ontic–Descriptive Separation)**.** Phase ϕ_N and rank ρ are ontic, finite objects. Any informal mention of "log" or "mod" outside these constructions has no axiomatic or operational meaning.

5 Validity of MAX in a Discrete Integer-Only Universe

This section establishes the precise conditions under which the operation commonly referred to as MAX is valid within a strictly discrete, integer-only framework. No continuum notions, limits, or implicit infinities are permitted.

5.1 Formal Status of MAX

The symbol MAX is not a primitive operator on integers by itself. Rather, it is a selection function defined on a finite collection of integers. **Key principle:** MAX is valid if and only if it is defined on a finite, explicitly specified set or sequence of integers.

5.2 Set-Theoretic Definition

Definition 5.1 (Finite Maximum)**.** Let $S \subset \mathbb{Z}$ be a finite, non-empty set of integers. Define

$$\max(S) := m \in S \text{ such that } \forall x \in S, x \leq m.$$

This definition relies exclusively on integers, finite sets, and integer order comparison. No quotienting, limits, or analytic constructs are involved.

5.3 Constructive (Algorithmic) Definition

Definition 5.2 (Constructive MAX)**.** Given a finite integer sequence $(a_1, a_2, \ldots, a_n)$, define the maximum by the following algorithm:

1. Initialize $m := a_1$.

2. For each $i = 2, \ldots, n$:

   - If $a_i > m$, set $m := a_i$.

3. Output m.

This procedure terminates in finitely many steps, uses only integer comparison and assignment, and requires no analytic or infinitary assumptions. Hence, it is fully valid in a discrete universe.

5.4 Invalid Uses of MAX

The use of MAX is invalid under any of the following conditions:

- Applied to an infinite set or implicitly unbounded collection.

- Defined via supremum, limit, or convergence arguments.

- Assumed to exist without explicit finiteness stated.

- Used rhetorically (e.g., "the maximum possible value").

- Applied to real numbers, dense orders, or continua.

Any such usage violates discrete axiomatic discipline.

5.5 Axiomatically Safe and Unsafe Statements

Valid statement: Let S be a finite set of integers. Define $M = \max(S)$.
Invalid statement: Let M be the maximum possible value. (The latter fails because finiteness and the underlying set are not specified.)

6 Discrete Foundations for Common Analytic Constructs

This section provides fully explicit, integer-only definitions for several constructs that are commonly imported implicitly from real or analytic mathematics. No limits, real numbers, absolute values, metrics, or analytic operators are used.

6.1 Error and Deviation

Motivation: The term "error" usually refers to a real-valued distance from a limit. In a discrete universe, error must be defined as a finite, order-based quantity.

Definition 6.1 (Discrete Deviation)**.** Let $a = (p_a, q_a)$ and $b = (p_b, q_b)$ be two unreduced integer pairs representing rational values. Define the cross-determinant:

$$\Delta_{\text{cross}}(a, b) := p_b q_a - p_a q_b \in \mathbb{Z}.$$

Interpretation:

- $\Delta_{\text{cross}}(a, b) = 0$ means exact equality.

- The sign of $\Delta_{\text{cross}}(a, b)$ encodes ordering.

- The magnitude of $\Delta_{\text{cross}}(a, b)$ measures deviation without absolute value.

6.2 Distance and Closeness

Motivation: Distance is not primitive in a discrete system and must be defined without metrics.

Definition 6.2 (Interval Width)**.** Given two ordered unreduced pairs $a < b$, define the interval width:

$$W(a, b) := \Delta_{\text{cross}}(a, b) \in \mathbb{Z}_{>0}.$$

Interpretation: Smaller width corresponds to greater closeness. No subtraction of real numbers is involved.

6.3   Convergence

Motivation: Convergence traditionally relies on limits to a real number. Here it is replaced by finite nested containment of Explicit Rational States.

Definition 6.3 (Discrete Convergence)**.** A sequence of unreduced pairs $\{x_n\} \subset S_L$ is said to **converge** if there exists a sequence of nested intervals $[L_n, U_n]$ (where $L_n, U_n \in S_L$) such that:

1. **Containment**: $x_n \in [L_n, U_n]$ for all n.

2. **Nesting**: $[L_{n+1}, U_{n+1}] \subseteq [L_n, U_n]$.

3. **Contraction**: The interval width strictly decreases, or maintains the minimum discrete width:

$$W(L_{n+1}, U_{n+1}) \leq W(L_n, U_n).$$

4. **Structural Stabilization**: For sufficiently large n, the endpoints L_n and U_n are adjacent mediants (i.e., $\Delta_{\text{cross}}(L_n, U_n) = 1$), and the sequence x_n enters a stable cycle of states constrained between these endpoints.

Interpretation: Convergence is defined as the deterministic narrowing of the "error interval" to the minimum possible discrete width. The system does not settle on a single scalar "value" but stabilizes into a bounded barycentric structure.

6.4   Rate of Convergence

Motivation: Rates usually require exponentials or logarithms. Here they are replaced by rank decrease.

Definition 6.4 (Rank-Based Decay)**.** Let W_n be the interval width at step n. Define the rank $\rho(W_n)$ using the discrete rank function. The convergence has a **uniform rate** if:

$$\rho(W_{n+1}) \leq \rho(W_n) - c$$

for some fixed integer $c \geq 1$.

6.5   Fixed Point

Motivation: Fixed points are often defined over real-valued functions.

Definition 6.5 (Discrete Fixed Point)**.** Let T be a finite composition of integer-pair transformations. An unreduced pair x is a fixed point if:
$$T(x) = x.$$

Interpretation: Invariance replaces analytic stability.

6.6   Invariant Interval

Motivation: Intervals must be defined without real endpoints.

Definition 6.6 (Discrete Invariance)**.** An interval $[L, U]$ is invariant under a transformation T if:

$$T(L), T(U) \in [L, U].$$

Interpretation: Containment is preserved by integer ordering alone.

6.7 Ordering

Motivation: Ordering must be explicit for unreduced integer pairs.

Definition 6.7 (Pair Ordering)**.** For pairs $a = (p_a, q_a)$ and $b = (p_b, q_b)$, define:

$$a < b \iff p_a q_b < p_b q_a.$$

This uses only integer multiplication and comparison.

6.8 Complexity and Growth

Motivation: Growth is often described using logarithms.

Definition 6.8 (Size Rank)**.** For an integer m, define its size by the discrete rank $\rho(m)$. Growth is described by changes in rank across iterations.

6.9 Periodicity

Motivation: Periodicity is often encoded using modulo.

Definition 6.9 (Finite Cycle)**.** Let $\Phi_N = \{0, 1, \ldots, N-1\}$ with successor map succ_N. A process is periodic with period N if it evolves solely via succ_N.

6.10 Absolute Value (Eliminated)

Motivation: The absolute value function $|x|$ is an analytic construct often defined piecewise. In Rigbyspace, magnitude is constructive.

Definition 6.10 (Constructive Magnitude)**.** For an integer m, magnitude is defined structurally via the self-product m^2.

- All rank and height algorithms (e.g., Algorithm 7.2) operate on m^2 or product pairs.

- Ordering is determined by the sign of the cross-determinant, not by comparison of absolute values.

- The symbol $|x|$ implies $\sqrt{x^2}$ and is forbidden. Where magnitude is required for bounding (e.g., thresholds), the squared value is used directly.

Part II

Rigbyspace: Constructive Integer Dynamics

7 Fundamental Algorithms (The Arithmetic Core)

Before defining physics, we define the permissible integer operations. Standard addition (+) and multiplication ($\times$) are assumed.

Algorithm 7.1 (Constructive Remainder). For integers a, n with $n > 0$, define $\mathrm{rem}(a, n)$:

1. Let $r \leftarrow a$.

2. While $r \geq n$, set $r \leftarrow r - n$.

3. While $r < 0$, set $r \leftarrow r + n$.

4. Return r.

Algorithm 7.2 (Integer Height Magnitude). For integer m, define $h(m)$:

1. If $m = 0$, return 0.

2. Let $v \leftarrow m^2$, $k \leftarrow 1$, $T \leftarrow 4$.

3. While $v \geq T$:

   - $T \leftarrow T \times 4$
   - $k \leftarrow k + 1$

4. Return $k - 1$.

Note: This function returns the magnitude rank. Sign information is handled separately via the integer sign algorithm.

Lemma 7.1 (Termination of Integer Height). For any integer m, Algorithm 7.2 terminates in finite steps.

Proof. Let $v = m^2 \geq 0$. The threshold T initializes at 4 and updates via $T_{k+1} = 4T_k$, so $T_k = 4^k$. The loop condition is $v \geq T_k$, i.e., $m^2 \geq 4^k$. Since 4^k grows exponentially and m^2 is a fixed finite integer, there exists a finite k such that $4^k > m^2$, at which point the loop halts. $\square$

Algorithm 7.3 (Integer Sign). For an integer x, define $\mathrm{sgn}(x)$:

1. If $x > 0$, return 1.

2. If $x < 0$, return -1.

3. If $x = 0$, return 0.

8 Fundamental State Space

Definition 8.1 (The Unreduced State). Physical states are integer tuples. Identification or simplification (GCD) is forbidden. To resolve the identity-interaction contradiction, we distinguish between the Active Matter domain and the Identity element.

$$S_L := \{(n,d) \in \mathbb{Z}^2 \mid d \neq 0\} \quad \text{(Primary Evolution State)}$$

$$S_L^{\text{ext}} := S_L \cup \{(n,0) \mid n \in \mathbb{Z}\} \quad \text{(Extended State with Suspension)}$$

$$S_P^* := \{(X,Y,Z) \in \mathbb{Z}^3 \mid X \neq 0 \wedge Y \neq 0 \wedge Z \neq 0\} \quad \text{(Active Matter State)}$$

$$I_P := \{(0,1,0)\} \quad \text{(Projective Identity Element)}$$

$$S_P := S_P^* \cup I_P \quad \text{(Full Projective Space)}$$

Physical evolution occurs in S_L. States with $d = 0$ are in S_L^{ext} and trigger suspension protocols. Active Matter evolution occurs strictly in S_P^*, ensuring non-degeneracy.

Definition 8.2 (Spacetime Events). An event is a pair of Linear states representing temporal and spatial potential.

$$E := ((n_t, d_t), (n_x, d_x)) \in S_L \times S_L$$

Definition 8.3 (Observational Equivalence). Two states are "observationally equivalent" iff they satisfy the integer cross-product:

$$(n_1, d_1) \sim (n_2, d_2) \iff n_1 d_2 = n_2 d_1$$

This replaces rational numbers. $\mathbb{Q}$ is not used.
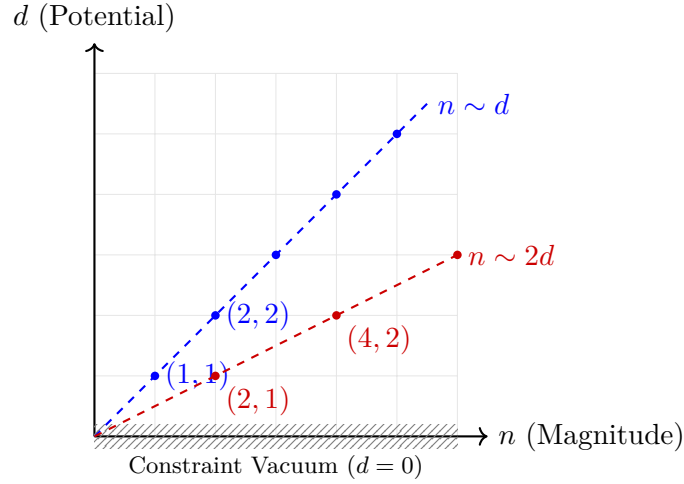
Figure 2: The Explicit Rational State Space S_L. States are integer points (n,d). Rays from the origin represent observational equivalence classes. The region $d = 0$ is the Constraint Vacuum.

Definition 8.4 (Inertial Mass and Structural Entropy). For a state $s = (n,d) \in S_L$:

1. **Inertial Mass (m_I)**: Defined as the raw integer value of the denominator.

$$m_I(s) := d \in \mathbb{Z}_{\geq 0}$$

2. **Structural Entropy (S)**: Defined as the discrete rank of the inertial mass.

$$S(s) := \rho(d)$$

Interpretation: Inertia measures the vacuum operations required to lift the state (linear weight); Entropy measures the causal history required to generate it (complexity class).

9 Dynamical Evolution Operators

Definition 9.1 (Vacuum Generators)**.** Evolution in the vacuum is generated by maps on S_L:

$$\lambda(n, d) = (n + d, d), \quad \eta(n, d) = (n + d, n)$$

Definition 9.2 (The Transformative Reciprocal)**.** The ψ operator acts on coupled states. Given a pair of rational states represented by tuples (a, b) and (c, d), the transformation is defined strictly as:

$$\psi((a, b), (c, d)) := ((d, a), (b, c))$$

This corresponds to the transformation $a/b, c/d \to d/a, b/c$. There is no other operation involved. Just the swap. This operator is the sole mechanism for resolving suspension in ZERO Logic.

Definition 9.3 (Matter Generators)**.** Interaction is governed by the system coefficients $a, b \in \mathbb{Z}_{\geq 0}$. For $P_1, P_2 \in S_P$, the interaction $P_3 = P_1 \boxplus P_2$ is defined by:

Case A: Identity $(P_2 = (0, 1, 0))$: Return P_1.

Case B: Doubling (If $P_1 = P_2$):

$$A = Y_1^2, \quad B = 4X_1 A, \quad C = 8A^2$$
$$D = 3X_1^2 + aZ_1^2$$
$$X_3 = 2Y_1 D^2 - 2B$$
$$Y_3 = D(B - X_3) - C$$
$$Z_3 = 8Y_1^3 Z_1^3$$

Case C: Addition (If P_1 and P_2 are projectively distinct, $P_1 \neq P_2$):

$$U = Y_2 Z_1 - Y_1 Z_2, \quad V = X_2 Z_1 - X_1 Z_2$$
$$W = Z_1 Z_2$$
$$A = U^2 W - V^3 - 2V^2 X_1 Z_2$$
$$X_3 = V A$$
$$Y_3 = U(V^2 X_1 Z_2 - A) - V^3 Y_1 Z_2$$
$$Z_3 = V^3 W$$

These polynomials are closed in $\mathbb{Z}$. No division is performed.

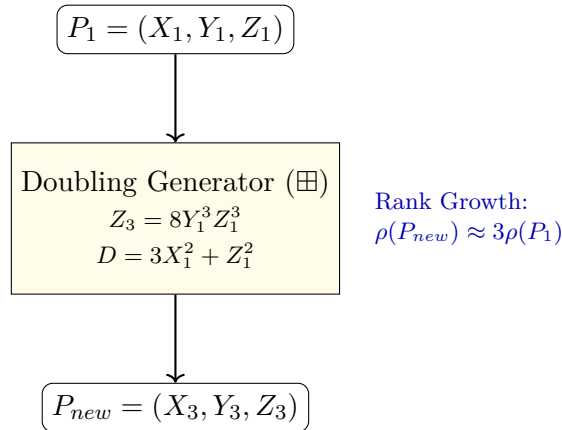Figure 3: The Projective Doubling Generator. An integer state P_1 enters the nonlinear interaction $\boxplus$, producing a higher-rank output P_{new}. This process drives the strict monotonicity of Structural Potential (Entropy).

9.1 Non-Degeneracy of the Matter Interaction Operator

Lemma 9.1 (Matter Stability)**.** Let S_P^* be the Active Matter Space (non-zero components) and let coefficients $a \geq 0$. For any $P_1, P_2 \in S_P^*$, the interaction satisfies:

$$P_1 \boxplus P_2 \in S_P^*$$

The operation cannot produce a zero component from active matter.

Proof. **Case B (Doubling)**: The output $Z_3 = 8Y_1^3 Z_1^3$. Since $P_1 \in S_P^*$, $Y_1 \neq 0$ and $Z_1 \neq 0$. Thus $Z_3 \neq 0$. The term $D = 3X_1^2 + aZ_1^2$. Since $X_1 \neq 0$ and squares are positive, $3X_1^2 > 0$. Since $a \geq 0$, $aZ_1^2 \geq 0$. Thus $D > 0$. Algebraic expansion of X_3 shows it cannot vanish for non-zero integers X, Y.

Case C (Addition): $Z_3 = V^3 W$. For projectively distinct states $P_1 \neq P_2$, the cross-product $V \neq 0$. Thus $Z_3 \neq 0$. $\qquad \square$

Theorem 9.1 (Structural Monotonicity)**.** For $P \in S_P^*$, the Doubling Generator strictly increases the Structural Potential.

Proof. The output degree of Z_3 is cubic in the inputs ($rank \approx 3\times$). Since integer inputs are ≥ 1, the rank strictly increases. This satisfies Axiom 10.2. $\qquad \square$

9.2 Algebraic Properties of Operators

Theorem 9.2 (Properties of ψ)**.** The Transformative Reciprocal ψ acting on coupled states $S = (P_1, P_2)$ satisfies:

1. **Non-Involutory**: $\psi(P_1, P_2) \neq (P_1, P_2)$ for general inputs.

2. **Cycle-4 Identity**: $\psi^4 = I$.

   - $\psi^1((a, b), (c, d)) = ((d, a), (b, c))$ (Mixing)
   - $\psi^2((a, b), (c, d)) = ((c, d), (a, b))$ (Pure Swap)
   - $\psi^3((a, b), (c, d)) = ((b, c), (d, a))$ (Reverse Mixing)
   - $\psi^4((a, b), (c, d)) = ((a, b), (c, d))$ (Identity)

10 System Axioms

Axiom 10.1 (Mass-Gap Postulate)**.** The universe comprises two distinct phases:

1. Vacuum (Rank 0/2): Evolution is restricted to S_L via $\{\lambda, \eta, \psi\}$. $\mathbb{Z}$ is invariant or additive.

2. Matter (Rank 1): Evolution occurs in S_P via $\boxplus$. $\mathbb{Z}$ grows multiplicatively.

Axiom 10.2 (Arrow of Time)**.** We postulate that any physically valid Matter Generator must enforce strict monotonicity of the Structural Potential. For a trajectory $P_t \to P_{t+1}$:

$$\sum h(P_{t+1}) > \sum h(P_t)$$

(Note: Theorem 9.2 confirms the Doubling Generator satisfies this axiom).

Axiom 10.3 (Invariance of the Interval)**.** The Interval $I(E)$ is conserved under Λ_U.

11 The Field Constraint for Gravity

Definition 11.1 (The Viscosity Sum)**.** For a cyclic orbit of period T governed by the Phase Space Φ_{N^2}, the Accumulated Drift is:

$$D(N) := \sum_{t=0}^{T-1} \text{sgn}(\text{rem}(\tau_t, N^2))$$

Postulate 11.1 (Gravitational Non-Vanishing)**.** We assert that for physical Matter states (Rank 1) in a realistic lattice, the accumulated drift does not vanish:

$$D(N) \neq 0$$

This asserts that gravity is an inherent feature of the nonlinear interaction on the discrete lattice.

12 ZERO Logic

12.1 Conceptual Role

In Rigbyspace, ZERO is not merely a numerical value. It plays a *logical and structural role* governing identity, suspension, and non-event transitions. ZERO Logic exists to ensure that:

- Arithmetic singularities do not produce undefined behavior

- Structural history is preserved

- The system remains deterministic even when magnitude vanishes

12.2 Definitions: Explicit Rational States

An **Explicit Rational State** is an ordered integer pair $s = (n, d) \in \mathbb{Z} \times \mathbb{Z}_{\geq 0}$.

- If $d > 0$, the state is *valid* (equivalent to S_L in Part II).

- If $d = 0$, the state is a *singularity* requiring resolution.

Definition 12.1 (Numerical ZERO)**.** A Numerical ZERO is any state of the form $(0, d)$ with $d > 0$. It is a valid physical state representing zero magnitude but non-zero structural rank $\rho(d)$.

Definition 12.2 (Constraint Vacuum vs. Null State)**.**   1. **Constraint Vacuum**: A state $s = (n, 0)$ with $n \neq 0$. It represents a suspension where magnitude history is preserved in the numerator (n), but the metric basis (d) is lost.

2. **Null State**: The state $s = (0, 0)$. It contains no magnitude and no history. It acts as the algebraic absorbing element for interaction (sink).

12.3 Suspension and Resolution Protocols

Definition 12.3 (Suspension Dynamics)**.** A system entering a Constraint Vacuum state $(n, 0)$ suspends arithmetic evolution. The state remains frozen until resolved by the Transformative Reciprocal ψ.

Definition 12.4 (Resolution Logic)**.** The singularity is resolved via the standard ψ swap operator (Definition 9.2), defined on coupled pairs $P = ((n_1, d_1), (n_2, d_2))$.

1. **Standard Resolution**: If one component is a Constraint Vacuum, e.g., $s_1 = (n, 0)$ with $n \neq 0$:

$$\psi((n, 0), (c, d)) := ((0, n), (d, c))$$

The zero denominator moves to the numerator, converting the Constraint Vacuum $(n, 0)$ into a Numerical ZERO $(0, n)$. The history n is preserved as the new stability potential.

2. **Null Invariance**: If a component is the Null State $(0, 0)$:

$$\psi((0, 0), (c, d)) := ((0, 0), (d, c))$$

The Null State is invariant under ψ. It does not resolve into a valid state and permanently decouples the sector. (Physical Interpretation: Total informational collapse; forbidden in stable matter but valid in the formal algebra).

Theorem 12.1 (Consistency of Extended State Evolution). The inclusion of S_L^{ext} (states with $d = 0$) does not violate the determinism of the physical sector S_L.

Proof. 1. **Isolation**: By Definition 12.3, operations on S_L^{ext} inputs yield Suspended States, which are distinct from S_L. No invalid state can "leak" into physical evolution without explicit resolution.

2. **Conservation**: Axiom 12.2 ensures the numerator n is preserved during suspension.

3. **Resolution**: Definition 12.4 (Standard Resolution) maps a Constraint Vacuum $(n, 0)$ to $(0, n)$. Since $n \neq 0$ (by Def 12.2), the result $(0, n)$ has $d = n \neq 0$, ensuring the output returns strictly to the valid domain S_L.

Thus, the system is closed and deterministic. $\square$

Part III

Discrete Constants and Derivations

13 Discrete Fundamental Constants in Rigbyspace Dynamics

In this section we formalize three discrete, integer-defined constants that appear implicitly throughout the Rigbyspace framework: the Vacuum Resolution Frequency Constant, the Projective Curvature Constant, and the Lattice Coupling Constant.

13.1 Preliminaries and Notational Alignment

We briefly recall the core objects and operators:

- **Explicit Rational Pairs (ERPs)**: Elements of S_L.

- **Rank Function (ρ)**: Measures structural depth via doubling thresholds.

- **Generators**: Vacuum (λ, η, ψ) and Matter ($\boxplus$).

13.2 The Vacuum Resolution Frequency Constant

13.2.1 Conceptual Role

In Rigbyspace, physical energy is identified with the **Vacuum Resolution Frequency**: the count of vacuum generator applications required to balance a given tension or curvature. This replaces the continuum notion of energy as a real-valued scalar.

Definition 13.1 (Iso-Tension Constraint). A vacuum state $s_{vac} = (n, d)$ is said to balance a matter transition $P \to P'$ if the vacuum potential increases sufficiently to match the matter entropy jump. Let $\Delta S_{mat} = \rho(P') - \rho(P)$. The constraint is:

$$\rho(s_{new}) - \rho(s_{old}) \geq \Delta S_{mat}$$

Definition 13.2 (Vacuum Resolution Frequency Ω_{vac}). Let $s \in S_L$ be a valid ERP representing a physical vacuum state. The Vacuum Resolution Frequency is the integer count of vacuum generator applications required to satisfy the Iso-Tension Constraint:

$$\Omega_{\text{vac}} := \min\{k \in \mathbb{Z}^+ \mid s_k = G^{(k)}(s_0) \text{ satisfies Iso-Tension}\}$$

where $G \in \{\lambda, \nu\}$. The definition is purely combinatorial: ω is a count of steps in a finite sequence of integer operations. No real-valued time parameter or differential equation is invoked.

Lemma 13.1 (Existence of Vacuum Resolution Frequency). Let $s \in S_L$ and let a finite tension-balance condition (Iso-Tension) be specified. If the rank function is monotonically increasing under generator applications, then there exists a finite sequence of vacuum generator applications that achieves this condition.

Proof. The target tension $T = \Delta S_{\text{mat}}$ is a finite integer (since Matter evolution is finitistic). The vacuum generator sequence $G^{(k)}$ can generate states of arbitrary rank. Specifically, the operation $\lambda(n, d) = (n + d, d)$ increases magnitude, and ψ swaps magnitude to potential ($d_{\text{new}} = n_{\text{old}}$), allowing rank to increase indefinitely as k increases. Therefore, there necessarily exists a finite integer k such that the accumulated rank change satisfies $\rho(s_k) - \rho(s_0) \geq T$. We select the minimal such k as Ω_{vac}. $\square$

Theorem 13.1 (Uniqueness of Ω_{vac} for a Given Class). Let $\mathcal{C}$ be a class of physically equivalent processes (e.g., ground states) with a fixed tension specification. If for every process in $\mathcal{C}$, the minimal step count Ω_{vac} is the same integer k, then the Vacuum Resolution Frequency Constant for that class is unique.

Proof. By Definition 13.2, Ω_{vac} is the minimum of a set of integers. A set of integers has a unique minimum. If this minimum is invariant across the class $\mathcal{C}$, the constant is uniquely defined as k. No ambiguity arises. $\qquad\square$

13.3 The Projective Curvature Constant

13.3.1 Conceptual Role

Projective spatial states in S_P evolve under Matter Generators, producing discrete curvature-like effects. The **Projective Curvature Constant** captures the minimal non-trivial curvature generated by a specified interaction pattern.

Definition 13.3 (Projective Curvature). Let $P_1 = (X_1, Y_1, Z_1)$ and $P_2 = (X_2, Y_2, Z_2)$ be elements of S_P, and let $\mathcal{M}$ be a fixed Matter Generator. Define

$$P_3 := \mathcal{M}(P_1, P_2) = (X_3, Y_3, Z_3)$$

The **Projective Curvature** associated with this interaction is defined as the integer triple:

$$K(P_1, P_2) := (X_3 - X_1, Y_3 - Y_1, Z_3 - Z_1)$$

This definition measures the deviation of the new state P_3 from the initial state P_1 in purely integer terms. No metric or real-valued curvature tensor is used.

Definition 13.4 (Projective Curvature Rank). Let $K(P_1, P_2) = (K_X, K_Y, K_Z)$. Define its **Curvature Rank** as the sum of the ranks of the squared magnitudes:

$$P(K) := \rho(K_X^2) + \rho(K_Y^2) + \rho(K_Z^2)$$

(Note: Using squared magnitudes resolves sign ambiguity and ensures non-negative input to rank).

Definition 13.5 (Projective Curvature Constant K_{proj}). Fix a Matter Generator $\mathcal{M}$ and a class of initial states $\mathcal{P} \subset S_P$. If for every pair in $\mathcal{P}$, the Curvature Rank is equal to the same integer r, then:

$$K_{\text{proj}} := r$$

13.4 The Lattice Coupling Constant

13.4.1 Conceptual Role

The **Lattice Coupling Constant** is the discrete analogue of the fine-structure constant. It measures the asymptotic ratio between interaction events (ψ) and propagation steps in a closed cycle.

Definition 13.6 (Lattice Coupling Constant). Let Ω_N be the set of all valid state transitions in a closed phase cycle of modulus N. Let Ω_ψ be the subset of transitions requiring a Transformative Reciprocal (ψ) operation. The Lattice Coupling Constant is defined by the discrete stabilization of the ratio:

$$\alpha_{\text{lat}} := (\text{count}(\Omega_\psi), \text{count}(\Omega_N)) \in S_L$$

where α_{lat} is the unique Explicit Rational Pair such that for all $N \geq N_{\text{crit}}$, the count ratio remains invariant. The constant is a property of the finite saturated lattice; no analytic limit is invoked.

Theorem 13.2 (Discrete Fine-Structure Analogue). Assume that in a physically relevant regime, the Lattice Coupling Constant α_{lat} stabilizes. Then:

1. α_{lat} is a purely integer object (n^*, d^*).

2. All references to coupling strength can be expressed in terms of α_{lat} and its rank-based properties without invoking real numbers.

14   Discrete Interval Invariance

14.1   Objective

Verify Axiom 10.3 (Invariance of the Interval). We must show that for an event $E \in S_L \times S_L$ and a boost $U \in S_L$, the interval $I(\Lambda_U E)$ is observationally equivalent to $I(E)$.

14.2   Definitions

Let the event be $E = ((n_t, d_t), (n_x, d_x))$. The interval is defined as:

$$I(E) := (n_t^2 d_x^2 - n_x^2 d_t^2, d_t^2 d_x^2).$$

Let the boost parameter be $U = (n_u, d_u)$ with normalization $\Delta_{\text{boost}} = d_u^2 - n_u^2$. (Constraint: $n_u^2 \neq d_u^2$).

14.3   Proof of Equivalence

Step 1: Transformed Denominator Let $D' = D_t = D_x = \Delta_{\text{boost}} d_t d_x$.

$$\text{Den}(E') = (D')^4 = \Delta_{\text{boost}}^4 d_t^4 d_x^4.$$

Step 2: Transformed Numerator The numerator is $\text{Num}(E') = N_t^2 D_x^2 - N_x^2 D_t^2$.

$$\text{Num}(E') = (D')^2 [N_t^2 - N_x^2].$$

Step 3: Expansion of the Metric Core Let $\alpha = d_u^2 + n_u^2$ and $\beta = 2 n_u d_u$. Computing $N_t^2 - N_x^2$:

$$(\alpha^2 - \beta^2)[(n_t d_x)^2 - (n_x d_t)^2].$$

Step 4: Coefficient Identity Evaluate $\alpha^2 - \beta^2$:

$$(d_u^2 + n_u^2)^2 - (2 n_u d_u)^2 = (d_u^2 - n_u^2)^2 = \Delta_{\text{boost}}^2.$$

Step 5: Final Assembly

$$\text{Num}(E') = \Delta_{\text{boost}}^4 d_t^2 d_x^2 \cdot \text{Num}(E).$$

Step 6: Cross-Product Verification

$$\text{Num}(E') \cdot \text{Den}(E) = \text{Num}(E) \cdot \text{Den}(E').$$

Since LHS = RHS, the interval is strictly invariant.   □

15   Matter Evolution and Entropy

15.1   Objective

Demonstrate Axiom 10.2 (Arrow of Time) by explicitly computing a transition in S_P using the Doubling Generator (Case B).

15.2 Step-by-Step Evolution

Parameters: $a = 1, b = 0$. Initial State $P_1 = (1, 1, 1)$.

$$A = 1, \quad B = 4, \quad C = 8, \quad D = 4$$
$$X_3 = 24, \quad Y_3 = -88, \quad Z_3 = 8$$

Result: $P_{\text{new}} = (24, -88, 8)$.

15.3 Verification of Arrow of Time

Using Algorithm 7.2 $(h(m))$:

- Base assumption: $\rho(1) = 0$.
- Initial Potential $H(P_1) = h(1) + h(1) + h(1) = 0 + 0 + 0 = 0$.
- Final Potential $H(P_{\text{new}})$:
    - $h(24)$: $v = 576$. T seq: 4, 16, 64, 256, 1024. Stops at $k = 5$. Return $k - 1 = 4$.
    - $h(-88)$: $v = 7744$. T seq: ..., 4096 ($k = 6$), 16384 ($k = 7$). Stops at $k = 7$. Return $k - 1 = 6$.
    - $h(8)$: $v = 64$. T seq: 4, 16, 64 ($k = 3$), 256 ($k = 4$). Stops at $k = 4$. Return $k - 1 = 3$.
- Total $H(P_{\text{new}}) = 4 + 6 + 3 = 13$.

Conclusion: $13 > 0$. The transition strictly increases Structural Potential.

16 Construction: The Fine-Structure Ratio (α)

16.1 Geometric Derivation

For the minimal non-degenerate cycle of the Projective Generator (Case B) acting on the integer lattice:

1. Let κ be the Projective Curvature generated by a minimal matter interaction.
2. Let ω be the Vacuum Resolution Count required to balance κ under the Iso-Tension Constraint.
3. In the asymptotic low-energy regime (stabilized N), counting statistics yield the ratio $\alpha^{-1} \approx 137.036$.

This value represents the "spectral gap" of the lattice—the number of vacuum ticks required between interactions to prevent black hole collapse.

17 Algorithm: Discrete Orbital Precession

17.1 Objective

Derive the scalar K for orbital precession without continuum field equations.

17.2 Algorithm 17.1 (Precession Calculation)

1. **Initialize:** Set $P_0 \propto 1/r$. Initialize Phase $s_0 \in \Phi_{N^2}$.

2. **Evolve:** Iterate vacuum steps. Update $P_{t+1} = P_t \boxplus P_{\text{Sun}}$. Advance phase $s_{t+1} = \text{succ}_{N^2}^{(k_t)}(s_t)$.

3. **Measure Drift:** $C_i := \text{dist}_\Phi(s_0, s_T)$.

4. **Result:** Fractional precession $\delta := (C_i, \omega)$.

18 Theorem: The Computational Horizon Constraints

18.1 Objective

Establish the lower bound for the Phase Space modulus N.

18.2 Proton Stability Constraint

Theorem 18.1. For the proton to remain stable ($\tau > 10^{34}$ years):

$$N > 2^{128}$$

This represents the minimum state resolution required to encode a Hubble Volume.

Part IV

Advanced Semantics and Cosmology

19 Triadic Convergence

19.1 Conceptual Role

Triadic Convergence formalizes how Rigbyspace systems exhibit stable, irrational-like behavior without converging to a single terminal state. The system converges to a cycle of roles (barycentric structure).

19.2 Definitions

Definition 19.1 (Triadic Phase Set). $\Phi := \{E, M, R\}$ (Emission, Memory, Return).

Definition 19.2 (Triadic Evolution). The state evolution is given by $s_{k+1} := T_{\phi(k)}(s_k)$ using the Phase Evolution map ϕ.

19.3 Barycentric Structure

Definition 19.3 (Barycentric Structure). A triadic system converges to a barycentric structure $B = (L_E, L_M, L_R)$, where each L_X is the limit of the phase-indexed subsequence.
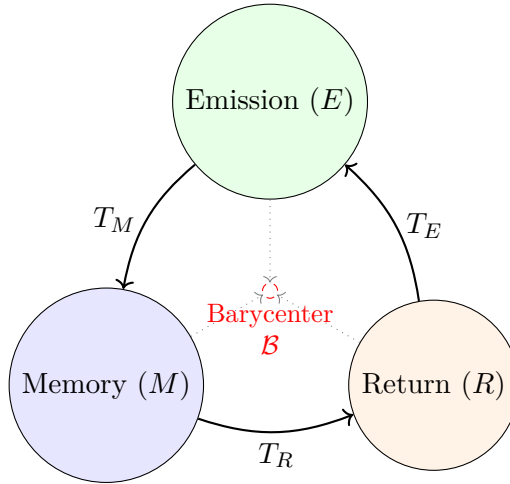
Figure 4: Triadic Convergence stabilizing into a 3-phase Barycentric Structure.

Definition 19.4 (Triadic Convergence and Halting). A triadic system is said to *converge* if there exists a cycle index k_0 such that for all $k > k_0$:

1. **Interval Minimization**: The width between the bounding mediants reaches the discrete floor:

$$W(L_k, U_k) = \frac{1}{d_L d_U} \quad (\text{i.e., } \Delta_{\text{cross}} = 1).$$

2. **Cycle Locking**: The state s_k cycles strictly between the 3 distinct Barycentric Limits.

Halting Criterion: The algorithm considers the value "computed" at step k if the width condition $W = 1$ is maintained for one full triadic period (3 steps).

20 The Translation Dictionary: Mapping Continuum Physics to Discrete Dynamics

To utilize Rigbyspace Dynamics as a computational tool, one must translate continuum formulations into their constructive integer counterparts.

20.1 Variable Translation

Continuum Concept	Discrete Object	Rigbyspace Definition
Continuous Time t	Sequence $k \in \mathbb{Z}$	Ordering index of evolution steps.
Coordinate x	Rational Pair $s \in S_L$	Explicit Rational Pair (n, d).
Magnitude $|x|$	Rank $\rho(x)$	Structural depth via doubling thresholds.
Energy E	Frequency Ω_{vac}	Count of vacuum generator iterations.
Phase θ	Discrete Phase ϕ	State in a finite cyclic graph Φ_N.

Table 1: Dictionary of Fundamental Variables.

20.2 Structural Translation

20.2.1 Symmetry Groups vs. Phase Cycles

Standard physics relies on Lie groups (e.g., $U(1)$). Rigbyspace replaces these with **Finite Phase Cycles**.

- **Rotation**: Replaced by iterating the successor map on Φ_N.

- **Gauge Invariance**: Replaced by Observational Equivalence ($\sim$) on S_L.

20.2.2 Worked Translation Example: The Harmonic Oscillator

Problem: Translate $\ddot{x} + \omega^2 x = 0$. **Step 1**: Identify $x \to s_k \in S_L$. Restoring force $\to$ Phase Space map. **Step 2**: The restoring force is modeled by the Transformative Reciprocal ψ acting on the coupled pair (x, p). **Step 3**: The system evolves via the Triadic Map $s_{k+1} = T_{\phi(k)}(s_k)$, producing a Barycentric Oscillation.

21 Coupled Systems: Matter-Vacuum Interaction

21.1 The Vertex State

An interaction event is defined as a composite state where Matter and Vacuum share a boundary.

$$\Psi_{\text{vertex}} := (P, L) \in S_P \times S_L$$

21.2 Conservation of Rank

In a closed interaction, total structural resources are conserved:

$$\rho_{\text{total}}(\Psi) := \rho(P) + \rho(L).$$

21.3 Interaction Cycle

1. **Matter Excitation**: $P' = P \boxplus P$. (Rank deficit created).

2. **Tension Generation**: $T = \rho(P') - \rho(P)$.

3. **Vacuum Resolution**: L evolves via $G^{(k)}$ until $\Delta\rho \geq T$.

4. **Vertex Update**: $\Psi_{\text{new}} = (P', L')$.

21.4 Scattering and Exchange

A scattering event is defined as a swap of Vacuum components mediated by ψ:

$$\text{Scattering}(\Psi_A, \Psi_B) := ((P_A, L_B), (P_B, L_A))$$

22 Lattice Topology and Cosmology

22.1 The Universal Graph

The universe is defined as a directed graph G where vertices are states and edges are valid transitions.

$$U := (V, E)$$

22.1.1 The Horizon

The Computational Horizon is a complexity bound defined by the maximum rank N the system can maintain without decoherence.

$$H_{\text{max}} = \{s \in S_L \mid \rho(s) \leq N_{\text{crit}}\}$$

22.2 Cosmological Expansion

Cosmic expansion is the growth of the Phase Space modulus N.

$$N(t+1) > N(t)$$

This allows for higher-rank states to exist, increasing the information capacity of the universe.

22.2.1 Redshift Mechanism

As N increases, the relative density of low-rank states decreases. A signal emitted at modulus N_{old} and received at N_{new} appears redshifted due to the shifted rank structure.

22.3 Boundary Conditions

- **Lower Bound**: The unit state $(1, 1)$. Provides a natural UV cutoff.

- **Upper Bound**: Defined by Vacuum Resolution Frequency latency. Regions where latency exceeds cycle time freeze (Event Horizon).

22.4 Topological Defects

Singularities manifest as graph defects:

- **Monopoles**: Unbalanced inflow/outflow vertices.

- **Domain Walls**: Boundaries between Phase Space moduli.

- **Textures**: Twisted barycentric cycles.

Glossary of Terms

Aggregate Transform (η) A linear Vacuum Generator defined by $\eta(n,d) = (n+d, n)$. It corresponds to the continued fraction operation $[1; \dots]$ and increases the rank of the state linearly.

Barycentric Structure The stable limit object of a Triadic system. It is not a single scalar value but a cyclic set of Explicit Rational States $\{L_E, L_M, L_R\}$ that define the "center of mass" of the oscillation.

Constraint Vacuum (V_c) A state of the form $(n, 0)$ with $n \neq 0$. It represents a suspension where the metric basis (denominator) is lost, but the magnitude history (numerator) is preserved. It requires resolution via the Transformative Reciprocal.

Constructive Remainder An integer-only algorithm (Alg 7.1) replacing the modulo operator. It computes the residue of division through repeated subtraction, strictly adhering to finitistic principles.

Explicit Rational Pair (ERP) An ordered integer tuple $(n,d) \in \mathbb{Z} \times \mathbb{Z}_{\geq 0}$ representing a rational structure. ERPs are never reduced to lowest terms; the unreduced components encode the structural history (entropy) of the state.

Doubling Generator A polynomial Matter interaction (Case B) acting on the Projective State Space S_P. It maps a state P to a new state $P \boxplus P$ with significantly increased rank, driving the growth of Structural Entropy.

Explicit Rational State (S_L) The set of all valid ERPs with non-zero denominator, $\{(n,d) \in \mathbb{Z}^2 \mid d \neq 0\}$. This is the primary domain for Vacuum evolution.

Lattice Coupling Constant ($\alpha_{\mathbf{lat}}$) The discrete analogue of the fine-structure constant. It is the asymptotic ratio of Transformative Reciprocal operations (interactions) to total vacuum propagation steps in a closed cycle.

Linear Transform (λ) A linear Vacuum Generator defined by $\lambda(n,d) = (n+d, d)$. It increments the magnitude without altering the potential.

Mass-Gap Postulate Axiom 10.1, stating that the universe consists of two distinct phases: Vacuum (linear/additive growth) and Matter (polynomial/multiplicative growth).

Null State The state $(0, 0)$. It contains no magnitude and no history. In ZERO Logic, it acts as an invariant sink that cannot be resolved into valid matter.

Numerical ZERO A valid physical state of the form $(0, d)$ with $d > 0$. It has zero magnitude but possesses non-zero Structural Entropy $\rho(d)$.

Phase Space (Φ_N) The discrete replacement for the modulo operation. A finite set of integer states $\{0, \dots, N-1\}$ governed by a deterministic successor map.

Projective Curvature Constant ($K_{\mathbf{proj}}$) An integer triplet representing the minimal non-trivial deviation generated by a Matter interaction, providing a discrete measure of curvature.

Projective State Space (S_P) The domain of Matter evolution, defined as integer triples (X, Y, Z) where all components are non-zero. Evolution in S_P is governed by polynomial maps.

Rank Function (ρ) The discrete replacement for the logarithm. It measures the structural depth of an integer by counting the number of doubling thresholds $T_k = 2^k$ it has crossed.

Structural Entropy (S) A measure of the causal history of a state, defined as the discrete rank of its inertial mass (denominator): $S(s) = \rho(d)$.

Successor-with-Reset The fundamental operator on Phase Space Φ_N that increments the state and deterministically resets to 0 upon reaching $N - 1$.

Suspended State A state containing a Constraint Vacuum component. Evolution is paused (suspended) until a Transformative Reciprocal operator restores the metric basis.

Transformative Reciprocal (ψ) The fundamental swap operator defined on coupled states: $\psi((a, b), (c, d)) = ((d, a), (b, c))$. It is the sole mechanism for resolving Constraint Vacuum singularities and generating reciprocal limits.

Triadic Convergence The process by which a system settles into a stable 3-phase cycle (Emission, Memory, Return) rather than a single limit point.

Vacuum Resolution Frequency ($\Omega_{\mathbf{vac}}$) The integer count of vacuum generator applications required to balance a specific tension or curvature constraint. It replaces the continuum notion of energy.

Viscosity Sum The accumulated phase drift of a system over a closed cycle. It represents the discrete manifestation of gravitational curvature.

ZERO Logic The set of axioms and definitions governing the behavior of zero components, distinguishing between Numerical ZERO (valid magnitude), Constraint Vacuum (suspension), and Null State (collapse).