The Discrete Born Rule: Constructive Probability from Integer State Superposition

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Abstract

We derive the Rigbyspace equivalent of the Born Rule ($P \propto |\psi|^2$) utilizing only the strict integer ontology of the framework. We reject the continuum postulates of complex amplitudes and real-valued probabilities in favor of a constructive approach based on **Explicit Rational States** (ERPs) and **Integer Magnitude Logic**. We demonstrate that "Interference" arises naturally from the signed addition of integer numerators in a superposition, while the "Square Law" emerges from the system's definition of Constructive Magnitude (m^2). This establishes a deterministic, integer-only pathway to quantum statistics, where probability is identified with the normalized structural intensity of the vacuum field.

1 Part I: The Algebra of Discrete Superposition

1.1 1.1 The Ontological Primitive: Signed Integer States

In standard quantum mechanics, the state is a vector in a complex Hilbert space. In Rigbyspace, the state is an **Explicit Rational Pair** (ERP) $s = (n, d) \in \mathbb{Z} \times \mathbb{Z}_{\geq 0}$ (Master Reference, Def 12.2). Crucially, the numerator n carries **sign information**, which corresponds to the chiral orientation (phase) of the state.

- **Positive Phase:** $s_+ = (1, 1)$.
- **Negative Phase:** $s_- = (-1, 1)$. (Often generated by a π-shift or chiral flip).

Unlike the denominator d (Inertial Mass), which is strictly non-negative, the numerator n (Magnitude History) tracks the orientation in the phase space.

1.2 1.2 The Principle of Linear Accumulation

When multiple histories (paths) converge at a locus (e.g., a detector), the local vacuum state is defined by the **Linear Superposition** of the arriving states. In Rigbyspace, this is governed by the **Rational Addition Rule** (consistent with Algorithm 7.1 and Case C dynamics):

Definition 1.1 (Discrete Superposition). Let two history states $s_1 = (n_1, d_1)$ and $s_2 = (n_2, d_2)$ arrive at a vertex. The resulting local state S_{local} is the unreduced rational sum:

$$S_{local} = s_1 \oplus s_2 := (n_1 d_2 + n_2 d_1, d_1 d_2) \quad (1)$$

If the denominators are identical (coherent source, $d_1 = d_2 = d$), this simplifies to:

$$S_{local} = (n_1 + n_2, d) \quad (2)$$

(Note: We retain the common denominator for coherent summation to preserve mass-level).

This addition rule allows for **Cancellation without Subtraction operators**:

$$(1, d) \oplus (-1, d) = (0, d) \tag{3}$$

The result is the **Numerical ZERO** state (Def 12.1), which physically corresponds to a non-interacting vacuum photon or node.

2 Part II: The Derivation of Quadratic Probability

2.1 2.1 Constructive Magnitude (m^2)

Why is probability quadratic ($|\psi|^2$)? In Rigbyspace, this is not a postulate but a definition of how magnitude is measured. Recall **Definition 6.10 (Constructive Magnitude)** from the Master Reference:

> "For an integer m, magnitude is defined structurally via the self-product m^2. ... The symbol $|x|$ implies $\sqrt{x^2}$ and is forbidden."

The system does not "know" the linear magnitude $|n|$. It only computes the **Structural Intensity** $I = n^2$ via the Integer Height Algorithm (Alg 7.2).

2.2 2.2 The Probability Definition

The "Probability" of an event is defined as the normalized Structural Intensity of the local state.

Definition 2.2 (The Discrete Born Rule). Let $\Omega = \{S_1, S_2, \ldots, S_k\}$ be the set of local states at mutually exclusive detectors. Let $S_i = (N_i, D_i)$. The probability $P(i)$ is the Explicit Rational Pair:

$$P(i) := \left(N_i^2, \sum_{j=1}^{k} N_j^2 \right) \tag{4}$$

This definition satisfies all RS constraints:

1. **Integer Only:** N^2 is an integer.
2. **Non-Negative:** Squares are always non-negative ($(-n)^2 = n^2$). This ensures probabilities are valid counts.
3. **Quadratic Scaling:** It scales as the square of the summed numerators (amplitudes).

2.3 2.3 Red Team Analysis: Why this works

Objection: "You just redefined probability to match the answer." **Response:** No. We used the pre-existing definition of Magnitude from the framework (Def 6.10). In a discrete system without square roots, the "size" of a signed integer n is naturally n^2 (its position in the quadratic form). If conservation laws apply to the *energy* of the wave (which goes as amplitude squared), then the counting statistics of detection must reflect that intensity.

Objection: "Does this conserve probability?" **Response:** Yes. By definition, the sum of the numerators of the probability pairs is $\sum N_i^2$, which matches the denominator. The sum is the Unit State $(1, 1)$ (or equivalent unity). Unitarity is preserved structurally.

3 Part III: Worked Example - The Mach-Zehnder Interferometer

We trace a single particle $(1, 1)$ through a split, phase shift, and recombination.

3.1 3.1 Setup

Source: $s_{in} = (1,1)$. (Unit Intensity $1^2 = 1$). **Splitter (BS1):** Generates two paths. To conserve integer mass, we map $(1,1) \to \{(1,1),(1,1)\}$ (Duplication of history potential). **Path 1:** State $h_1 = (1,1)$. **Path 2:** State $h_2 = (1,1)$.

3.2 3.2 Phase Shift (π Rotation)

A "π shift" corresponds to a sign inversion of the numerator (Chiral flip). **Path 2 Operation:** $h_2 \to h_2' = (-1,1)$.

3.3 3.3 Recombination (BS2)

The beamsplitter mixes the paths. We use the standard Hadamard-like mixing logic (integer version):

- **Port A (Constructive):** Sum of inputs. $S_A = h_1 \oplus h_2'$.
- **Port B (Destructive):** Difference of inputs. $S_B = h_1 \oplus (-h_2')$.

(Note: "Difference" means adding the sign-flipped state).

Port A Calculation:

$$S_A = (1,1) \oplus (-1,1) = (1+(-1),1) = (0,1)$$

This is the **Numerical ZERO** state. **Intensity:** $I_A = 0^2 = 0$.

Port B Calculation:

$$S_B = (1,1) \oplus (-(-1,1)) = (1,1) \oplus (1,1) = (2,1)$$

State is $(2,1)$. **Intensity:** $I_B = 2^2 = 4$.

3.4 3.4 Probabilities

Total Intensity: $\Sigma I = 0 + 4 = 4$.

$$P(A) = (0,4) \equiv 0$$

$$P(B) = (4,4) \equiv 1$$

Result: Perfect interference. The "Dark Port" (A) has zero probability not because the particle vanished, but because the local vacuum state $(0,1)$ has zero structural magnitude. The "Bright Port" (B) captures all the intensity.

3.5 3.5 Visualization