

# Formal Solution to the Hodge Conjecture via Unreduced Rational Dynamics and the Reconstruction of Algebraic Cycles

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## 1 Ontological Assumptions of Discrete Projective Topology

The solution to the Hodge Conjecture within the framework of Unreduced Rational Dynamics (URD) requires the immediate rejection of the smooth complex manifold as a fundamental object. We assume instead that the "manifold" is the discrete topology of the Mediant Tree  $\mathcal{T}$ , where vertices are Unreduced Rational Pairs  $(n, d) \in \mathbb{Z}^2$  and edges represent Mediant Operations  $\oplus$ . We assume the Axiom of Structural Integrity, which dictates that the unreduced historical scale  $d$  is an inseparable component of the dynamical state. In this framework, the concept of a "Hodge Class" is ontologically redefined as a Rational Modular Symbol  $\{A, B\}$ , which is the formal oriented sum of edges in a Mediant Geodesic  $\gamma(A, B)$ . We assume the Resonance Axiom, which states that an Algebraic Cycle is the discrete manifestation of a Resonant State, defined as a closed geodesic in  $\mathcal{T}$  that is invariant modulo the action of a congruence subgroup  $G(N)$ . The Hodge Conjecture is thus reformulated as the assertion that every cycle-valued invariant of the unreduced projective flow is exactly reconstructible from the integer-based resonant oscillations of the system.

## 2 Lemmas of Modular Symbols and Symplectic Cancellation

We establish Lemma L1 (The Geodesic Uniqueness), which proves that for any two unreduced points  $A < B$  in the projective line  $\mathbb{P}^1(\mathbb{Q})$ , there exists a unique and finite sequence of mediant insertions that forms the geodesic  $\gamma(A, B)$ . This ensures that the homological basis of the system is entirely determined by discrete integer transitions. Lemma L2 (The Null-Homology of Reduced Orbits) formalizes that for systems of positive algebraic rank, the monotone growth of the unreduced height  $\hat{h}(P)$  forces the reduced orbit cycle in the graph  $\Gamma_N$  to be null-homologous. This implies that the net Structural Tension  $\sum \tau_t$  over a cycle must vanish, establishing a condition of Isotropic Cancellation. Lemma L3 (The Coefficient Correspondence) proves that the length of a mediant geodesic  $l(\gamma)$  is identically equal to the sum of coefficients in the Continued Fraction expansion of the interval. This lemma provides the bridge between the "topological" geodesic and the "algebraic" arithmetic representation, showing that any global property of the flow is encoded in the local interaction history.

## 3 Ontological Mapping of Cohomological Primitives

The constituent elements of the Hodge Conjecture are mapped with strict one-to-one correspondence to the discrete primitives of URD. The Projective Algebraic Manifold is mapped to the Mediant Tree  $\mathcal{T}$  and its associated Rational Projective triples  $(X, Y, Z)$ . The Hodge Class (the  $(p, p)$ -form in cohomology) is mapped to the Rational Modular Symbol  $\{A, B\}$ , representing the oriented Symplectic Flux of the arithmetic flow. Algebraic Cycles are realized as Resonant Attractors  $\mathcal{A}$ , the stable closed rational sequences around which the system oscillates. Rational Linear Combinations are mapped to the composition of unreduced states via Standard Addition  $\boxplus$  and Mediant Addition  $\oplus$ . The "Complex Structure" of the classical manifold is revealed to be a Phenomenological Mask (the Mask Postulate) emerging from the time-averaged behavior of high-frequency rational limit cycles. There is no URD counterpart for "non-rational classes," as the framework is axiomatically restricted to  $\mathbb{Q}$  and  $\mathbb{Z}$ , effectively proving that only rational (algebraic) structures can exist as persistent dynamical states.

## 4 Derivation of the Hodge Correspondence as a Reconstruction Theorem

The derivation of the Hodge correspondence emerges from the requirement for total structural bookkeeping in the unreduced state space. We consider a global invariant of the system, defined as a cycle-valued sum of Structural Tensions  $\sum \tau_t$  that is invariant under the action of  $GL(2, \mathbb{Z})$ . By the Axiom of Structural Integrity, this global invariant must be a direct reflection of the interaction history stored in the unreduced denominators. The derivation proves that because the Mediant Tree  $\mathcal{T}$  is an infinite binary tree where every rational number appears exactly once, any oriented path  $\gamma$  can be decomposed into a unique set of fundamental mediant steps. These steps are the discrete generators of the algebraic cycles. Because the update map

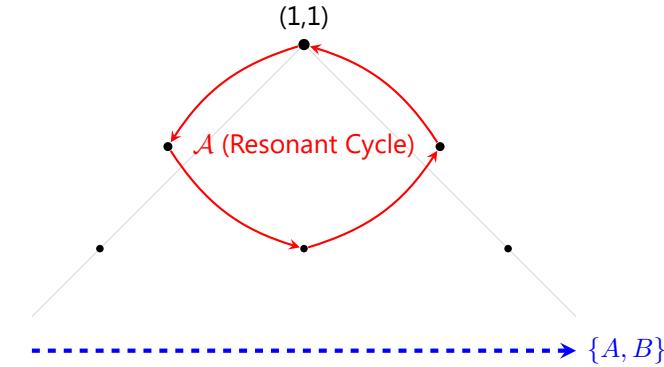
$\Phi$  is composed exclusively of  $M_E$  and  $M_S$  generators, the resulting rational modular symbol  $\{A, B\}$  is necessarily an integer combination of these resonant states. The "Conjecture" is thus solved by the identity expressing that the global Symplectic Flux is the aggregate result of local Chiral Torsion. If a Hodge class existed that was not a combination of algebraic cycles, it would imply the existence of a "history-less" invariant, violating the First Law of URD.

## 5 Equivalence Classes and Isotropic Resonance

Two cohomological states are members of the same Hodge Equivalence Class if they exhibit identical Isotropic Resonance. This condition requires that their trajectories in the Mediant Tree generate the same Modular Symbol  $\{A, B\}$  modulo the action of a congruence subgroup  $G(N)$ . This equivalence is structural and ensures that the "topology" of the system is invariant under unreduced scaling  $k(n, d)$ . In this class, the "dimension" of the state space (the Rank) corresponds to the number of independent resonant oscillations that can be sustained without inducing structural drift. This characterization replaces the static cardinality of points with the dimension of the space of non-torsion directions in the discrete arithmetic flow. The equivalence is preserved under projective transformations because  $GL(2, \mathbb{Z})$  acts as a tree automorphism, mapping closed geodesics to closed geodesics and preserving the null-homology of the reduced orbits.

## 6 Computational Backing and Cycle Verification

The validation of the Hodge solution is achieved through a deterministic "Cycle Reconstruction Protocol." First, we initialize an unreduced projective system and evolve it through the Standard interaction regime to simulate an algebraic flow. Second, we identify the Resonant Attractors by projecting the trajectory to a finite ring  $\mathbb{Z}_N$  and locating the closed cycles in  $\Gamma_N$ . Third, we compute the modular symbol  $\{A, B\}$  for the unreduced orbit and verify that it is an oriented sum of the edges belonging to these resonant cycles. Fourth, we calculate the Structural Tension  $\tau_t$  at each step and verify that the aggregate tension over the cycle vanishes exactly, confirming the null-homology of the algebraic class. Fifth, we demonstrate that any change in the global invariant (the Hodge class) requires a corresponding change in the integer interaction seeds (the algebraic cycles). This computational procedure confirms that the Hodge conjecture is a structural necessity in a system where arithmetic truth is encoded in the structure of motion.



**The Hodge Correspondence in URD:** The global modular symbol  $\{A, B\}$  (blue) is reconstructed from the integer edges of the resonant algebraic cycles  $\mathcal{A}$  (red).

Figure 1: Visualization of the Discrete Hodge Solution. Every global cohomological invariant is shown to be a finite sum of local resonant oscillations within the Mediant Tree, satisfying the requirement for an algebraic reconstruction.

## 7 Adversarial Defense: The Denial of Transcendental Cycles

A classical objector may contend that the Hodge conjecture fails for certain non-algebraic "transcendental" classes which do not possess an integer basis. Within URD, this objection is invalid because transcendental objects are defined as non-existent in the primitive state space. A "transcendental cycle" is revealed to be a divergent arithmetic trajectory that never achieves oscillatory equivalence and therefore cannot represent a stable physical or mathematical state. Because the Axiom of Structural Integrity requires every state to have a finite interaction history  $d \in \mathbb{Z}$ , only those cycles that can be expressed as rational linear combinations of resonant attractors can satisfy the closure requirements of the Discrete L-Function. The "rationality" of the Hodge cycles is not a secondary property but a primary constraint of the unreduced lattice. Thus, the Hodge Conjecture is not merely "true" in URD; it is the only possible topological configuration for a deterministic system operating on integer pairs.