

Angular selectivity of thin gratings

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ABSTRACT

The angular selectivity of thin gratings is studied both experimentally and theoretically. The concepts of thick and thin gratings are analysed. Thin holographic gratings recorded in a-As-S-Se films have exhibited pronounced and oscillatory diffraction efficiency angular dependences. These results are explained by the obliquity factor in Fresnel-Kirchhof diffraction integral and by finite beam and grating sizes. It is also shown that oscillatory diffraction efficiency angular dependences, most probably, arise due to the interference of diffracted waves of different orders because dephasing can be significant for small grating strengths and large enough readout angles. Fabry-Perot resonator effect can contribute as well. Thick gratings in a-As₂S₃ and a-As-S-Se films are also studied for comparison. The conclusion is made that normally all gratings possess angular selectivity and existing criteria underestimate the angular selectivity of thin gratings because they neglect factors other than dephasing.

Keywords: holographic gratings, diffraction efficiency, angular selectivity, amorphous chalcogenide films

1. INTRODUCTION

It is widely believed that thin gratings do not possess angular selectivity, i.e., their diffraction efficiency (DE) does not significantly depend on the readout angle. Evidently, this is the reason why there are no papers on angular selectivity of thin gratings, as far as we know, although they are widely used. Thin grating theories such as amplitude transmission theory¹⁻⁴, Raman-Nath theory^{1,2,3,5} even do not consider any DE angular dependence. One of thin grating definitions⁶ states that thin gratings are gratings exhibiting relatively little angular and wavelength selectivity.

In order to distinguish between thin and thick gratings quantitatively several criteria are introduced. Let us briefly consider them in the case of the simplest unslanted amplitude-phase grating (Fig.1) described by a sinusoidal spatial modulation of intensity absorption coefficient k and refractive index n according to equations

$$k = k_0 + k_1 \cos Kx, \quad (1)$$

$$n = n_0 + n_1 \cos Kx, \quad (2)$$

where k_0 and n_0 are average absorption coefficient and refractive index, respectively, k_1 and n_1 are the spatial modulation amplitudes of absorption coefficient k and refractive index n along the x -axis, $K=2\pi/L$ is the angular spatial frequency of the grating, L - grating period.

In 1965 Klein⁷ has introduced a parameter

$$Q = \frac{2p \cdot l \cdot d}{n_0 \cdot \Lambda^2}, \quad (3)$$

where l is a free-space readout light wavelength, d - grating thickness (Fig.1). Small values of $Q < 1$ correspond to thin gratings and Raman-Nath diffraction regime. Large values of $Q > 10$ correspond to thick gratings and Bragg diffraction regime. Bragg gratings have a maximal angular selectivity^{1-4, 6-10}. The DE maximum usually is reached at $q = q_B$ where q_B is the Bragg angle (Fig.1). Gratings with $1 < Q < 10$ belong to intermediate gratings and intermediate diffraction regime takes place in them.

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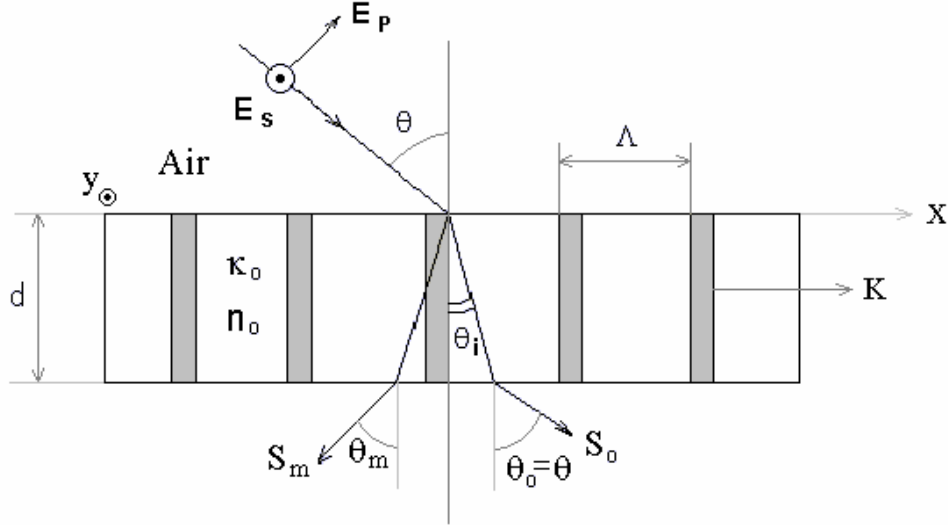


Fig.1. Transmission amplitude-phase grating with the grating vector \mathbf{K} ($K=2\pi/\Lambda$) and an incident readout light wave with the electric vector components normal to the plane of incidence (\mathbf{E}_s) and lying in the plane of incidence (\mathbf{E}_p). S_0 and S_m are the light wave amplitudes diffracted in the zeroth and in the m -th order, respectively. The light incidence and diffraction angles are counted from the normal and are assumed to be positive in counterclockwise direction. Other notations are described in the text.

Gaylord and Moharam^{3,6} have introduced the following criterion to estimate the angular selectivity of a grating. For thin gratings with little angular selectivity the condition $d/L < 10$ should hold. Conversely, thick gratings with strong angular selectivity corresponds to $d/L > 10$.

It was also soon recognized that not only geometrical factors but also grating strength (i.e., n_1 and k_1) determines the angular selectivity. Moharam and Young⁸ have reminded that already in 1938 Nath⁵ has introduced a parameter

$$r_p = \frac{I^2}{n_0 \cdot n_1 \cdot \Lambda^2} \quad (4)$$

Thin gratings and Raman-Nath diffraction correspond to the condition $r_p \leq 1$ whereas thick gratings and Bragg diffraction correspond to $r_p \gg 1$ ^{6,8}. Analogously, for absorption gratings the parameter

$$r_a = \frac{I^2}{n_0 \cdot k_1 \cdot \Lambda^2} \quad (5)$$

was introduced by Baird, Moharam and Gaylord¹⁰ where

$$k_1 = \frac{I}{4p} \cdot k_1 \quad (5a)$$

k_1 is the absorption index modulation amplitude. Thin absorption grating condition is $r_a < 1$ but thick grating condition is $r_a \gg 1$.

We have made experimental angular selectivity studies of holographic gratings in As-S-Se amorphous chalcogenide films which were thin according to all above mentioned criterions. Nevertheless, we have found quite strong and oscillatory DE angular dependence. Does it mean that there are other reasons for angular selectivity of

gratings as well? Or are the used criterions incomplete? The aim of this paper was to try to answer these questions. Our answers are positive.

Our calculations show that monotonic angular selectivity of thin gratings can be the result of the obliquity cosine factor in Fresnel-Kirchhof diffraction integral taken in nonparaxial Fraunhofer approximation, and of a finite beam and grating sizes. We have also analysed the Fabry-Perot resonator effect in thin films as a possible origin of the observed oscillatory DE angular dependences. Multiwave interactions in the gratings according to rigorous coupled-wave theory are considered as well.

From the point of view of rigorous coupled-wave theory^{1-3, 10} the angular selectivity of gratings arises due to the dephasing of coupled waves diffracted in different orders. The above mentioned criterions allow to estimate how strong is dephasing. It is assumed that there is no dephasing in thick gratings and there is a maximal dephasing in thin gratings. As we shall see (Section 6) this is not always a case.

The experimental results are discussed taking into account the above mentioned considerations. They give a qualitative understanding of observed DE angular dependences for thin gratings in a-As-S-Se films. Thick gratings in a-As₂S₃ and a-As-S-Se films are discussed as well. For a quantitative explanation further studies are necessary, especially using rigorous diffraction theory. Exact knowledge of DE angular dependence is important for both scientific purposes¹¹ and practical applications⁴.

2. Experiments and their results

We have used amorphous chalcogenide films for holographic grating recording and their angular selectivity measurements. Thin a-As-S-Se (a-As₄₀S₁₅Se₄₅) and a-As₂S₃ films were thermally evaporated in vacuum at room temperature onto optical K-8 7×7cm² glass substrates. Their thicknesses (d) were measured interferometrically during the deposition. Films were not annealed. Transmission holographic gratings were recorded by two symmetrically incident He-Ne (in the case of As-S-Se films) or Ar⁺ (in the case of a-As₂S₃ films) laser beams. The sample holder was installed in a goniometer enabling convenient grating angular adjustment with 0.3° precision. DE was defined as a ratio of the first order diffracted light power to the incident light power (Fig.1).

In the case of a-As₂S₃ films (thicknesses 11.2 and 5.2 μm) the recording light wavelength (I_1) was 514.5 nm but the readout wavelength (I_2) was 632.8 nm. Holographic grating period (L) was 0.7 μm, light intensity I was 0.2 W/cm². In the case of a-As-S-Se films ($d=2.2\mu\text{m}$) a He-Ne laser was used both for recording and readout ($I_1=I_2=632.8\text{ nm}$). Holographic grating periods were 0.45, 4.5, 7.0 and 10 μm. Recording light intensity was 0.45 W/cm².

The measured DE angular dependences are presented in Figs. 2 – 6. Pronounced DE maxima can be seen in all cases (except $L=7.0\text{ mm}$ in a-As-S-Se which will be discussed in Section 4) at $q=q_r$ (q_r being the recording angle in the air corresponding to zero angular detuning $Dq = q - q_r$).

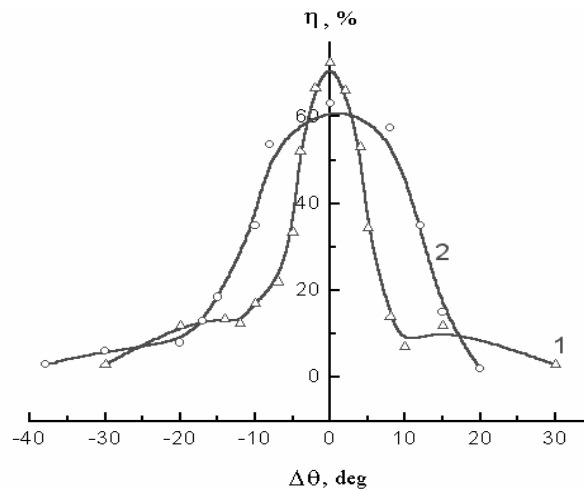


Fig.2. Angular selectivity of holographic gratings in a-As₂S₃ films: $I_1 = 514.5\text{ nm}$, $I_2 = 632.8\text{ nm}$, $L = 0.7\text{ mm}$, $I = 0.2\text{ W/cm}^2$; 1- $d=11.2\text{ μm}$, 2- $d=5.2\text{ μm}$.

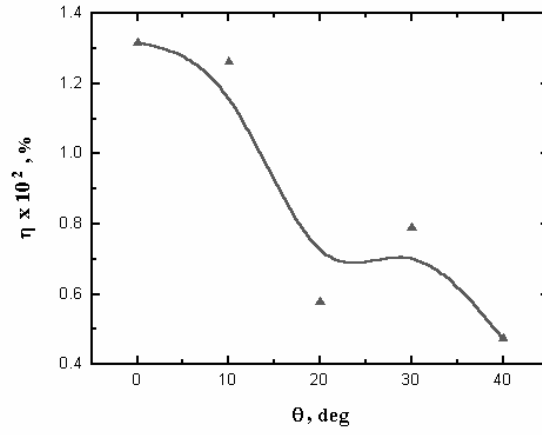


Fig. 3. Angular selectivity of the holographic grating in an a-As-S-Se film: $d=2.2\mu\text{m}$, $I_1=I_2=632.8\text{ nm}$, $L=0.45\mu\text{m}$, $I=0.45\text{ W/cm}^2$.

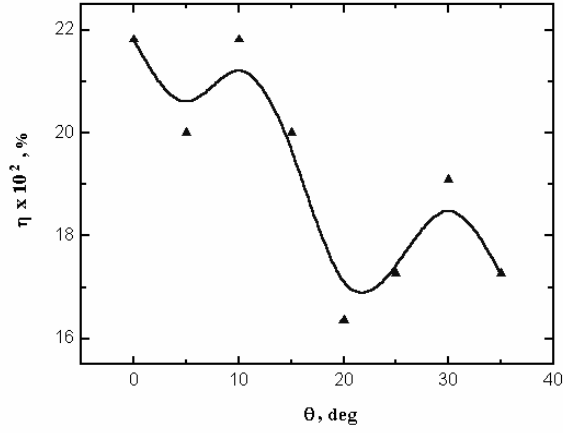


Fig. 4. Angular selectivity of the holographic grating in an a-As-S-Se film: $d=2.2\mu\text{m}$, $I_1=I_2=632.8\text{ nm}$, $L=4.5\mu\text{m}$, $I=0.45\text{ W/cm}^2$.

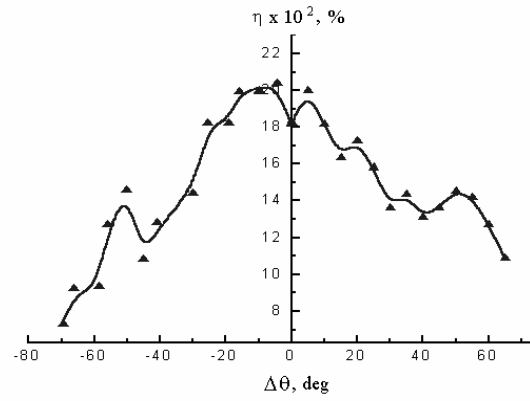


Fig 5. Angular selectivity of the holographic grating in an a-As-S-Se film: $d=2.2\mu\text{m}$, $I_1=I_2=632.8\text{ nm}$, $L=7.0\mu\text{m}$, $I=0.45\text{ W/cm}^2$.

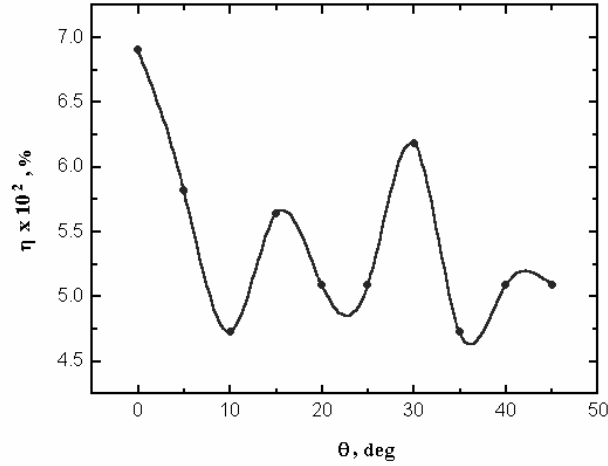


Fig 6. Angular selectivity of the holographic grating in an a-As-S-Se film: $d=2.2\mu\text{m}$, $I_1=I_2=632.8\text{ nm}$, $L=10\mu\text{m}$, $I=0.45\text{ W/cm}^2$.

One can also see that DE is not only decreasing in average (this behaviour we call large scale angular selectivity) but also oscillating (small scale angular selectivity). In thick grating case (Figs.2,3) these oscillations are due to the dephasing and interference of waves diffracted in different orders. They are described by Kogelnik's coupled-wave theory⁹. In the case of thin gratings the amplitude and angular period of oscillations do not significantly depend on L . The possible reasons for both large scale and small scale angular selectivity will be discussed in the following Sections.

The parameters Q , r and d/λ are calculated for the gratings in Figs.2-6 and shown in Table 1.

Table 1.

Parameters of recorded gratings and their angular selectivity

Grating parameters	Q	ρ	d/Λ	$\Delta\theta_{\text{exp}}$, deg	$\Delta\theta_{\text{theor}}$, deg
a – As ₂ S ₃ $d = 11.2\ \mu\text{m}$ $\Lambda = 0.7\ \mu\text{m}$	37.1	3.3	16	6	1.8
a – As ₂ S ₃ $d = 5.2\ \mu\text{m}$ $\Lambda = 0.7\ \mu\text{m}$	17.2	3.3	7.4	10	3.9
a – As-S-Se $d = 2.2\ \mu\text{m}$ $\Lambda = 0.45\ \mu\text{m}$	14.4	66	4.8	20	5.9
a – As-S-Se $d = 2.2\ \mu\text{m}$ $\Lambda = 4.5\ \mu\text{m}$	0.14	0.66	0.49	> 35	59
a – As-S-Se $d = 2.2\ \mu\text{m}$ $\Lambda = 7.0\ \mu\text{m}$	0.06	0.27	0.31	60	91
a – As-S-Se $d = 2.2\ \mu\text{m}$ $\Lambda = 10\ \mu\text{m}$	0.03	0.13	0.22	> 45	130

Measured (Dq_{exp}) and calculated (Dq_{theor}) HWHM angular selectivities (HWHM DE versus Dq widths) are shown as well. Theoretical Dq values were calculated according to Kogelnik's theory⁹ as

$$\Delta\theta_{\text{theor}} \approx \frac{\Lambda}{2d} \quad . \quad (6)$$

One can see that, as expected, thick grating angular selectivity is higher. Besides, Kogelnik's theory⁹ gives smaller Dq_{theor} values than measured in the case of thick gratings (for reasons described in Section 7) but the opposite is true for thin gratings. Thus the Kogelnik's criterion underestimates the angular selectivity of thin gratings. However, formula (6), strictly speaking, is not valid for thin gratings and can be used only as a rough estimate.

3. DIFFRACTION EFFICIENCY ANGULAR DEPENDENCE FROM FRESNEL-KIRCHHOF DIFFRACTION INTEGRAL IN NONPARAXIAL FRAUNHOFER APPROXIMATION

In this Section we show that a large scale angular selectivity of thin gratings follows from the thin amplitude-phase grating theory developed on the basis of Fresnel-Kirchhof diffraction integral in nonparaxial Fraunhofer approximation¹². From the expression (27) in paper¹² for the m -th order DE of thin cosinusoidal amplitude-phase gratings described by Eqs. (1, 2) we have derived the formula for the first-order relative DE in the case of small modulation amplitudes k_l and n_l when

$$\frac{k_l d}{\cos q_i} \ll 2, \quad \frac{2pn_l}{l \cos q_i} \ll 2 \quad . \quad (7)$$

This formula neglects a weak angular dependence of Fresnel reflection losses for p-polarized light far from Brewster's angle (71.5° in a-As-S-Se films) and reads as follows:

$$\frac{h(q)}{h(0)} = \exp \left[-k_0 d \left(\frac{1}{\cos^2 q_i} - 1 \right) \right] \frac{\cos^3 q_l}{\cos q \cdot \cos^2 q_i} \text{sinc}^2 \left[\frac{d \cdot \text{tg} q}{\Lambda} \right] \times \left[1 - \frac{l^2}{\Lambda^2} \right]^{-3/2} \quad . \quad (8)$$

Here $h(q)$ and $h(0)$ are DE at angles of incidence equal to q and 0, respectively. Other notations are given previously in Fig.1 and Section 1, except for q_l which is the first order diffraction angle (q_m at $m=1$ in Fig.1). Angle q_l can be calculated from the grating equation

$$\sin q_m = \sin q - m \frac{l}{\Lambda} \quad (9)$$

at $m=1$. The incidence angle q_i inside the the grating can be found from Snell's law

$$\frac{\sin q}{\sin q_i} = n_0 \quad . \quad (10)$$

The sinc function is defined as

$$\text{sinc}(x) = \frac{\sin(px)}{px} \quad . \quad (11)$$

It is clearly seen from Eqs. (8-11) that DE depends on readout angle q . Under the experimental conditions of Fig.5 when $d = 2.2\mu\text{m}$, $l = 0.6328\mu\text{m}$, $L = 7\mu\text{m}$, $n_0 = 3$, $k_0 = 0.39\mu\text{m}^{-1}$ the corresponding relative DE angular dependence is shown in Fig.7.

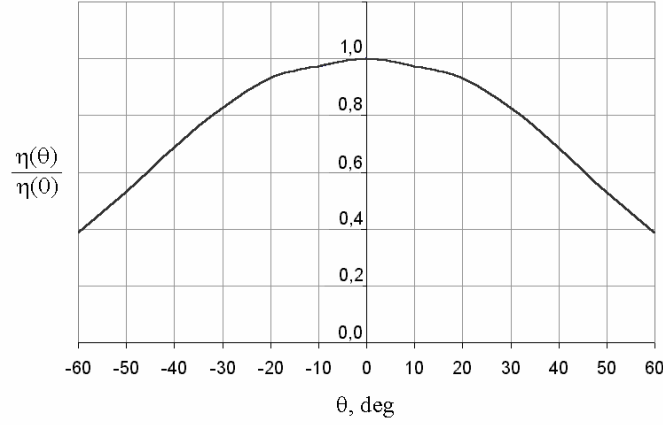


Fig.7. Relative diffraction efficiency angular dependence calculated according to thin amplitude-phase grating theory based on the Fresnel-Kirchhof diffraction integral in nonparaxial Fraunhofer approximation at $d = 2.2\mu\text{m}$, $l = 0.6328\mu\text{m}$, $L = 7\mu\text{m}$, $n_0 = 3$, $k_0 = 0.39\mu\text{m}^{-1}$ according to Eq (8).

When Fig.5 and Fig.7 are compared it is seen that large scale angular selectivity is approximately described but no DE oscillations are present. However, in Eq.(8) finite beam and grating sizes are not taken into account as well as Fabry-Perot resonator effect in thin films (see Sections 4,5).

What factors determine the large scale angular selectivity described by Eq.(8)? This is mainly the well known obliquity factor for the diffracted light wave amplitude in Fresnel-Kirchhof diffraction integral^{12, 13}. Fresnel has introduced it empirically by definition^{13, 14}. However, it also appears when scalar wave equation is solved by Green's function method in the far field¹³. Physically it can be regarded as a result of a large scale dephasing of diffracted waves outside the grating.

4. DIFFRACTION EFFICIENCY ANGULAR DEPENDENCE DUE TO FINITE BEAM AND GRATING SIZES

Until now we have assumed that a plane light wave is reading a thin grating whose size is also unrestricted. In the case of holographic gratings recorded in a photosensitive material by two interfering Gaussian laser beams (as in our experiments) the grating area S_g is limited and determined mainly by the laser beam cross section area S_l which is also limited. Further we show that finite S_g and S_l values also lead to a large scale angular selectivity of thin gratings.

First, let us assume that we have an unrestricted grating with DE h_u . Then let us take a part of this grating with area S_g and illuminate it with a laser beam at the incidence angle q (Fig.1). Then DE which is defined as a ratio of a light power diffracted in the first order P_d to the incident light power P_i can be expressed as

$$\eta = \frac{P_d}{P_i} = \frac{I_d \cdot S_{ef}}{I_i \cdot S_i} \quad , \quad (12)$$

where I_d and I_i are the diffracted and incident light intensities with respect to a horizontal xy plane in Fig.1, S_{ef} is the effective readout area, S_i is the incident beam area projection on the horizontal xy plane. Taking into account that only the overlapping area of the grating is effective and contributes to diffraction one can write

$$S_{ef} = \begin{cases} S_i, & S_i < S_g \\ S_g, & S_i \geq S_g \end{cases} \quad (13)$$

From the geometrical considerations it follows that

$$S_i = \frac{S_l}{\cos q}, \quad S_g = \frac{S_l}{\cos q_r} \quad . \quad (14)$$

Besides,

$$\eta_u = \frac{I_d}{I_i} \quad (15)$$

From the Eqs.(12-15) one can easily get the expression for the relative DE

$$\frac{h(q)}{h(0)} = \begin{cases} 1, & |\theta| \leq |\theta_r| \\ \cos \theta / \cos \theta_r, & |\theta| > |\theta_r| \end{cases} \quad (16)$$

The corresponding relative DE angular dependence calculated at $I_2=632.8$ nm, $L=4.5\mu\text{m}$, $q_r=4.03^\circ$ is shown in Fig.8.

One can see that a large scale angular selectivity appears. However, the maximum DE corresponds to the incidence angle range from $-q_r$ to q_r rather than to $q=q_r$ as in the case of thick gratings. Yet, for large enough grating periods (as in given example) q_r is close to zero.

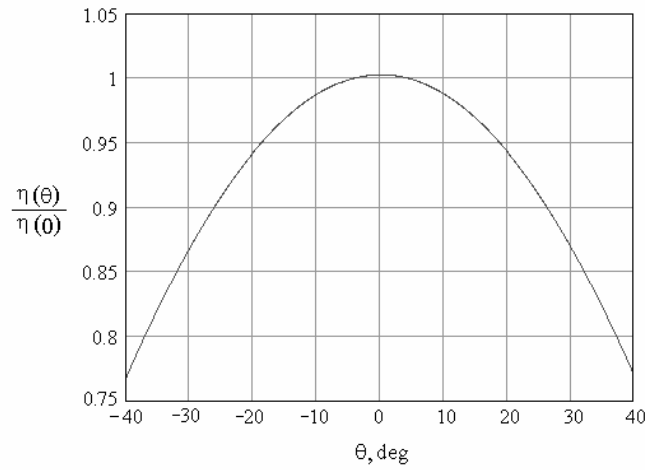


Fig.8. Relative diffraction efficiency angular dependence due to the finite grating and readout beam sizes at $I_2 = 632.8$ nm, $L=4.5\mu\text{m}$ calculated according to Eq.8.

5. FABRY-PEROT RESONATOR EFFECT

It is well known that multiple internal reflections of an incident light wave inside a thin film leads to the interference of transmitted and reflected waves which manifests itself by oscillating transmittance, reflectance and DE changes when the optical thickness is gradually changed^{15–18}. This phenomenon can be called a Fabry-Perot resonator effect^{14,17}. If the radout beam incidence angle is changed the average phase thickness of a grating¹⁵

$$\delta = \frac{4pn_0d}{I_2} \cos \theta_i = \delta_0 \cos \theta_i \quad (17)$$

is changed as well (Fig.1) and one can expect an oscillatory DE angular dependence.

As an example let us consider an absorbing phase grating with a small $n_I \gg 10^{-3}$ recorded in a-As-S-Se film on a glass substrate. This example is close to our experimental conditions because the phase modulation was dominating as shown by additional experiments. Then DE can be expressed as^{16–18}

$$h = (1 - R_1)(1 - R_2)(1 - R_3) \exp \left(-\frac{k_0 \cdot d}{\cos q_i} \right) \left[1 + 6(R_1 R_2)^{\frac{1}{2}} \cos(d_0 \cos q_i) \right] \cdot \left[\frac{(d_{10} \cos q_i)^2}{16} \right]. \quad (18)$$

Here R_1, R_2, R_3 are the Fresnel reflection coefficients of the air-film, film-substrate and substrate-air interfaces,

$$\delta_{10} = \frac{4pn_1d}{I_2} \quad (19)$$

is the phase thickness modulation amplitude at normal incidence.

In our experiments $R_1=0.25$, $R_2=0.11$, $R_3=0.04$, $k_0=0.39\mu\text{m}^{-1}$, $n_0=3$, $n_I=1 \times 10^{-3}$, $d=2.2\mu\text{m}$, $I_1=I_2=632.8$ nm. Using these parameter values and Eqs.(10, 17 – 19) we have calculated the relative DE angular dependences for different d_0 values close to $d_0 = 131.065$ which corresponds to $d=2.2\mu\text{m}$, $n_0=3$. Such slight differences in the phase thickness can arise due to the sample nonuniformity.

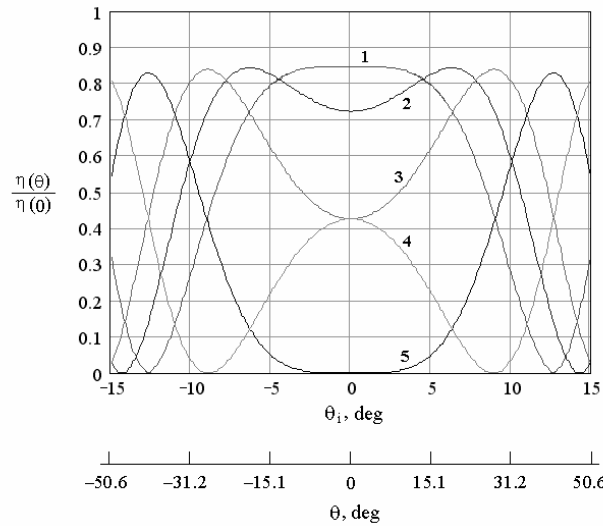


Fig. 9. Relative diffraction efficiency angular dependences of a thin absorbing phase grating due to the Fabry-Perot resonator effect calculated according to Eqs. (17 – 19) for different values of average phase thickness d_0 : 1 – 125.66, 2 – 127.33, 3 – 128.81, 4 – 130.38, 5 – 131.95. Other parameters: $d = 2.2\mu\text{m}$, $I=0.6328 \mu\text{m}$, $n_0=3$, $k_0=0.39\mu\text{m}^{-1}$.

The results are presented in Fig.9. One can see that even small optical thickness $n_0 \cdot d$ changes can significantly change $h(q)$ dependence. As expected, these dependences indeed are oscillatory. However, the angular periods of these oscillations are in the $40^\circ - 60^\circ$ range which is much less than the experimental values around 10° in Figs. 4-6. Thus Fabry-Perot resonator effect cannot explain the oscillatory DE angular dependences observed in our experiments. Other reasons should be found.

6. MULTIWAVE INTERACTIONS IN THE GRATING

In this Section we analyse the possible reasons for oscillatory DE angular dependences basing on rigorous coupled-wave theory of unrestricted gratings at plane wave incidence^{2,3,10}. This theory is based on Floquet theorem which is analogous to Bloch theorem in solid state physics. According to Floquet theorem the diffracted wavevectors inside the grating may be represented for the infinite periodic medium case by

$$\mathbf{k}_m = \mathbf{k}_0 - m\mathbf{K}, \quad (20)$$

where \mathbf{k}_m is the wavevector of the m -th order ($m=0, \pm 1, \pm 2, \dots$) wave while \mathbf{k}_0 is the wavevector of the incident wave which coincides with the zeroth-order wave wavevector. Using coupled-wave approach and Floquet theorem one can get the following expression for the s-polarized light electric field inside the phase grating^{2,3} (Fig.1):

$$E(x, z) = \sum_{j=-\infty}^{\infty} S_j(z) \cdot e^{-i \left(\vec{k}_0 - j\vec{K} \right) \cdot \vec{r}} \quad (21)$$

where $S_j(z)$ are the amplitudes of j -order coupled waves, \vec{r} is a radiusvector. They should obey the following coupled-wave equations^{2,3}:

$$\frac{1}{2p^2} \frac{d^2 S_j(z)}{dz^2} - i \frac{2n_0 \cos q_i}{p \cdot l} \frac{dS_j(z)}{dz} + \frac{2j}{\Lambda^2} (m-j) S_j(z) + \frac{2n_0 n_1}{l^2} [S_{j+1}(z) + S_{j-1}(z)] = 0, \quad (22)$$

where $j = \dots -2, -1, 0, +1, +2, \dots$ and

$$m = \frac{2 \cdot \Lambda \cdot n_0 \cdot \sin q_i}{l}. \quad (23)$$

By inspection of Eq.(22) it is seen that the wave (space harmonic) corresponding to each value of j is coupled to its adjacent ($j+1$ and $j-1$) space harmonics. The quantity m may have any value in general. For the case when m is an integer Eq.(23) becomes the well known Bragg condition:

$$2L \sin q_{Bm} = ml. \quad (24)$$

For $q = q_{Bm}$ all diffracted order waves are in phase. For $q \neq q_{Bm}$ the dephasing and destructive interference takes place.

We suppose that dephasing can be responsible for oscillatory DE angular dependence not only for thick gratings⁹ but also for thin ones. Let us find a condition of negligible dephasing. From Eq.(22) it is seen that for this the third term describing the dephasing should be much less than the fourth. The second and the fourth terms appear also in Raman-Nath theory^{2, 3} which does not predict angular selectivity but the first term does not depend on angle. Thus at $j=1$ one gets a condition

$$n_1 \gg \frac{2l \sin q_i}{\Lambda} - \frac{l^2}{n_0 \Lambda^2}. \quad (25)$$

Clearly, it is met at $q_i=0$ for any n_i . However, for large enough q_i and q at fixed n_i inequality (25) does not hold any more. This is the case for our thin gratings in a-As-S-Se films (Figs.4-6) when $L \geq 4.5\mu\text{m}$, $n_0=3$, $n_i \approx 1 \times 10^{-3}$, $l = 632.8$ starting from $q \approx 10^0$. Thus indeed multiwave interactions also in thin gratings can be responsible for oscillatory DE angular dependences. Similiar coupled-wave equations are derived for cosinusoidal asorption gratings¹⁰, too.

7. DISCUSSION

The observed large scale angular selectivity of thin gratings (Figs. 4-6) cannot be explained by angle-dependent Fresnel reflection losses because we have used p-polarized beams both for recording and readout (Fig.1) far from the Brewster's angle of 71.5° . Therefore, they decrease rather than increase angular selectivity.

If we smooth $h(q)$ curves in Figs.4-6 they are qualitatively well described by the product of functions of Eqs. (8) and (16) yet giving higher large scale selectivity than observed. This discrepancy can be explained by a nonuniformity of real gratings both in x,y and z directions (Fig.1) caused by the Gaussian laser beam profile and absorption, respectively. These factors are known to decrease the angular selectivity^{19, 20}. Thus obliquity factor and finite grating and beam size effect allow to explain the large scale angular selectivity of thin gratings. The lower experimental angular selectivity of thick gratings (Figs.2,3 and Table 1) compared to the predictions of Kogelnik's theory⁹ can also be explained by the real grating nonuniformity.

Fabry-Perot resonator effect can contribute both to large and small scale angular selectivity (Fig.9). Although this effect cannot explain the oscillatory DE angular dependence in our experiments for larger d_0 values it will be able to do that. The DE minimum at $Dq=0$ in Fig.5, obviously, is the result of Fabry-Perot resonator effect.

Gaussian recording and readout beams can lead to oscillatory angular selectivity but only when the grating thickness is much larger than the beam diameter. This was not the case for our experiments with a-As-S-Se films since $d=2.2\mu\text{m}$ was much less than 2.1 mm of He-Ne laser $1/e^2$ beam diameter.

In our opinion, the reason for the observed oscillatory DE angular dependence is the interference of different order diffracted waves whose dephasing depends on the readout angle, as for thick gratings. The dephasing can be efficient also in thin gratings (Section 6) if the readout angle is large enough but the grating strength is small enough. It influences also the large scale angular selectivity.

For quantitative theoretical description of a thin grating angular selectivity the solution of rigorous coupled-wave equations is necessary taking into account multiple internal reflections, Gaussian laser beam profiles and grating's nonuniformity. This is a formidable mathematical problem. In this case the large scale and the small scale angular selectivity will be described simultaneously.

Thus thin grating studies have led us to the conclusion that there are also other reasons for angular selectivity than dephasing – obliquity factor, finite grating and beam sizes, multiple internal reflections. The proposed criterions (Section 1,2) do not take them into account, and therefore work most properly for the determination of Bragg diffraction regime when dephasing is the dominant factor. Every grating has some angular selectivity if no special measures are taken to avoid it.

8. CONCLUSIONS

The angular selectivity of thin gratings is studied both experimentally and theoretically. Thin (according to all known criterions) holographic gratings have exhibited appreciable and oscillatory DE angular dependences. (For comparison, thick gratings in a-As₂S₃ and a-As-S-Se films are studied as well.) To explain these results two kinds of angular selectivity have been introduced – a large scale selectivity which corresponds to smoothed (excluding oscillations) DE angular dependences, and a small scale selectivity which corresponds to details of angular dependences, mainly oscillations.

It is shown by calculations that the obliquity factor in Fresnel-Kirchhof diffraction integral and finite beam and grating sizes can explain the observed large scale selectivity of thin gratings.

The small scale selectivity of thin gratings manifesting itself mainly in DE oscillations is due to the interference of diffracted waves of different orders with dephasing which depends on the readout angle. Dephasing can influence also the large scale selectivity.

Fabry-Perot resonator effect can contribute both to large scale and small scale selectivity, however, in our experiments we have not observed a strong Fabry-Perot resonator effect, evidently, because of absorption and relatively small phase thickness of the studied thin gratings.

Thus it is established that factors other than dephasing can also determine the angular selectivity of gratings. This means that practically every grating exhibits some angular selectivity not only those corresponding to the known thin grating criterions which are based on dephasing.

In order to explain the experimental results quantitatively further theoretical studies are necessary basing on rigorous theory and taking into account multiple internal reflections, Gaussian laser beam profile and grating nonuniformity. More detailed experimental studies are necessary as well.

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