

# Insights through asymptotics:

## Benchmarking and analysis of climate effects on flood models

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## I Background

There is currently unmistakable evidence that the climate of the planet is changing. We currently do not understand what the potential outcomes of these changes may be. One route we have to understand these changes takes the form of models these models then can be used to help us predict the future impact of climate change.

In the United Kingdom one of the largest potential effects of climate change we will take the form of river flooding. The government cost the 2015/16 floods at 1.6 billion pounds. This number does not take into account the human toll of this event. Additionally, it appears that the number and severity of flooding is increasing.

This has motivated the hydrological community in increasing the understanding of river flooding. This has been done by developing a 'zoo' of potential flooding models. However, there are two problems regarding this large group of models. One since there is so many it is hard to separate them REF. Secondly we believe there is still information which is not being extracted from these models that can help us predict how flooding will change. We believe that we can solve both of these problems by examining models using asymptotic analysis.

### How do we model river flooding

There are currently three main approaches to flooding model. We have statistical models this rely on using statistical method to fit models to data and use statistical models to make inferences about the future. We have conceptual models, these rely in creating a set of states from which water can move between this is best represented by a FIGURE. Finally we have physical models these take a continuime mechanics approach and set up a set of conservation laws and use these to drive a set PDE sytems that can be used to mdoel the flooding event.

### Conceptual models

As briefly mentioned in the previous section conceptual models attempt to capture a complex system such as river flooding and simplify it down to a set of possible states/locations the variable of interest may reside in. Travel between these states is then governed by a simple mathematical expression.

Figure 1 gives a diagrammatic example of a conceptual model this model show how rain water might flow through a catchment area. It shows rain entering the system precipitation and how the path it might take, either evaporating immediately, or by flowing into surface or groundwater storage, before being eventually discharged.



## Reduced PDE model

Along with conceptual models we are also intrested in analysing a PDE of flooding. The paritucaler model we are intresed in is a reduced PDE model that has been developed by Piotro as part of his PhD thesis. The model reduces the problem by

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## II Problem formulation and method

### A new approach to benchmarking

The new approach we have to Benchmarking flooding models will allows us to derive asymptotic scaling laws. Such laws will allow us additional insight by telling us how the output of the model will change with respect to some perturbation of input data  $\epsilon$ .

For example in the G2G model used by Bell we would be able to derevie a scaling law which will tell us how the flood response time vary with perturbations to the rainfall. Similarly in for the models shown in the benchmarking paper we would be able to derive scaling laws that may help to further separate the 13 different models.

The new approach is two pronged first we use a numerically derive a scaling law, then use this to inform a search analytical scaling law. Note that it might not always be able to derive analytical scaling laws but it will always be possible to derive numerical ones.

The process used to derive the numerical scaling laws is best demonstrated with the use of diagrams.

### Extending the model with periodic forcing

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## III Objectives and strategy

The work outlined above is designed to be undertaken as PhD project. A potential student undertaken this project would require a strong background in fluid mechanics, asymptotic analysis and, would require a passion for environmental mathematics. Additionally, it would be beneficial if a potential student had some experience with time series analysis, and numerical PDEs.

### Objectives

Below is a list of initial targets which shall form the bases of project.

1. **Numerical derive scaling laws**, from conceptual models. This would require conducting a literature review to ascertain influential models that can be analyse to draw more inference from.
2. **Analytically derive scaling laws**, this shall be accomplished by using numerical laws derived to inform a discretion of the conceptual models. To conduct this integration we shall use asymptotic analysis.
3. **Apply periodic rainfall to the reduced PDE model**. To this end we will need to conduct a time series analysis of rainfall data. This shall be used to find appropriate time scales to inform the analysis of the reduced PDE model.

## Potential future objectives

There are a number of directions this project may be extended dependent upon the interests of the student. A couple of potential avenues of extension are outlined below to show the scope of where this project.

The reduced PDE model has a number of other extension which can be explored. One approach would be to use the asymptotic method of homogenisation, this would allow us to explore how a more complex soil structure will effect flooding effects. Another approach we could use to examine the water flow in soil would be to replace the equation which governs the flow through the soil with a Darcy's law equation this would allow us to explore more complex soil dynamics.

We could also take a dynamical systems approach to analysing the reduced PDE model. For example it is very natural to assume that there are two possible states in which a catchment area could be in, a saturated state this would be equivalent to a wetland, and a dry/drought state. We then assume that as the land dries out it becomes less porous so more water flows directly overland. We can then explore how much rainfall would have to change to transition between these states.

Another promising avenue we could explore would be attempting to match the conceptual models with the reduced PDE model. This would be of great interest as the different methodologies for building flooding models are metaphorically exist on separated islands. By matching the PDE model to a conceptual model we hope to be able to connect these models together.

These few possible research directions are not meant to be taken as a demonstration of the scope of this project and its potential.

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## IV Impact

### Mathematical impact

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## V References