Exponent and Logarithm Rules

Exponent Rules	Logarithm Rules	
$a^x \cdot a^y = a^{x+y}$ $\frac{a^x}{a^y} = a^{x-y}$ and $a^{-y} = \frac{1}{a^y}$ $(a^x)^y = a^{xy}$ $(ab)^x = a^x b^x$ $(\frac{a}{b})^x = \frac{a^x}{b^x}$ $a^{x/y} = \sqrt[y]{a^x}$ $a^0 = 1$	$x = log_a y \Leftrightarrow y = a^x\ (if\ y > 0)$ $log_a(xy) = log_a x + log_a y$ $log_a(\frac{x}{y}) = log_a x - log_a y$ $ylog_a x = log_a x^y$ $log_a b = \frac{log_c b}{log_c a}$, if $c = e$ then $log_a b = \frac{ln\, b}{ln\, a}$ $log_b a = \frac{1}{log_a b}$ $a^{log_a x} = x\ (if\ x > 0)$ $log_a a^x = x$	euler's number, $e \simeq 2.71828$ $log = log_{10}$ $ln = log_e$ $log_a 1 = 0$ $log_a a = 1$

Instantaneous rate of change of f at x: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Average rate of change of f with respect to x over $[x, x + h]$: $f'(x) = \lim_{h \to h} \frac{f(x+h)-f(x)}{h}$

$\frac{dy}{dx}|_{(a,\, b)}$ denotes the value of $\frac{dy}{dx}$ at (a, b)

Equation of the tangent line to the curve $y = y(x)$ at the point (x_0, y_0) is given by:

$y - y_0 = y'(x_0)\,(x - x_0)$

Elasticity of Demand: $E(p) = -\frac{p\, f'(p)}{f(p)}$

- Demand is **elastic** at p_0 when $E(p_0) > 1$.
  - small increase in price ⇔ bigger decrease in revenue
- Demand is **unitary** at p_0 when $E(p_0) = 1$.
  - small increase in price ⇔ same increase in revenue (1:1 ratio)
- Demand is **inelastic** at p_0 when $E(p_0) < 1$.
  - small increase in price ⇔ smaller increase in revenue

Implicit differentiation:
Step ①: Replace y with $y(x)$.
Step ②: Differentiate both sides.
Step ③: Isolate for $\frac{dy}{dx}$.

What's the difference between $\frac{d}{dx}$ and $\frac{dy}{dx}$? Think of $\frac{d}{dx}$ like a verb, it is telling you to find the derivative. $\frac{dy}{dx}$ is like a noun, it is the result after taking the derivative.

Rules of Differentiation

Derivative of a Constant	$\frac{d}{dx}c = 0$
Power Rule	$\frac{d}{dx}(x^r) = r\,x^{r-1}$
Derivative of a Constant Multiple of a Function	$\frac{d}{dx}[c\,f(x)] = c\,\frac{d}{dx}f(x)$
Sum Rule	$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$
Derivative of Exponential Function	$\frac{d}{dx}(a^x) = a^x \ln a$ $\frac{d}{dx}e^x = e^x \ln e = e^x$
Derivative of Logarithmic Function	$\frac{d}{dx}\log_a \lvert x\rvert = \frac{1}{x \ln a}$ when $x \neq 0$ $\frac{d}{dx}\ln \lvert x\rvert = \frac{1}{x \ln e} = \frac{1}{x}$ when $x \neq 0$
Product Rule	$\frac{d}{dx}[f(x)g(x)] = f'(x)\,g(x) + f(x)\,g'(x)$
Quotient Rule	$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$ when $g(x) \neq 0$
Chain Rule (for composite functions) $y(x) = f(g(x))$	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ If you decompose $y(x)$ into $g(x) = u$ and $f(u)$, then: $\frac{d}{dx}[f(g(x))] = f'(u) \cdot g'(x)$

Applications of Chain Rule

Chain Rule for Power functions $y(x) = [f(x)]^n$	$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \cdot f'(x)$
Chain Rule for Exponential functions $y(x) = a^{f(x)}$ where $a > 0,\ a \neq 1$	$\frac{d}{dx}a^{f(x)} = a^{f(x)} \cdot \ln a \cdot f'(x)$ $\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$
Chain Rule for Logarithmic functions $y(x) = \log_a f(x)$ where $a > 0,\ a \neq 1$	$\frac{d}{dx}\log_a y(x) = \frac{1}{f(x) \cdot \ln a} \cdot y'(x) = \frac{f'(x)}{f(x) \cdot \ln a}$ $\frac{d}{dx}\ln f(x) = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$

Function is **increasing** when $f'(x) > 0$, and is **decreasing** when $f'(x) < 0$.

Critical points: x values where $f'(x) = 0$
x values where $f'(x)$ not defined (IF AND ONLY IF $f(x)$ IS DEFINED at x)