1 - Jan 9 Lecture

- Course Overview
- Systems of linear equations
- Solution sets
- 2D/3D Graphical representations of systems of linear equations
- Number of solutions of systems of linear equations in 2D/3D
- EROs
- Augmented matrices

Definitions: consistent, inconsistent, equivalent

Course Overview

Mon Jan 9 2023

Instructor email	ZacharyWelch@cmail.carleton.ca Use your Carleton email (CMAIL) for all course email correspondance
Office Hours	Herzberg Laboratories Room 5230 (confirm on Brightspace) Friday 12:30 - 1:30 PM If you cannot make it to office hours, email the instructor to arrange something!
Course Textbook	The link will be available tonight or tomorrow (Jan 10). The e-text is sold for $50. DO NOT BUY THE MYLAB VERSION! We are not using it. The textbook is sold out at the Bookstore, ask them if you want to know when it comes back in stock. The lecture content follows the course content. If you don't understand something in lecture, take a look at the textbook.
Tutorial	The tutorials are held right after the Monday lecture. Room number currently unknown. First tutorial Jan 23. No tutorials for the first two weeks. GO TO THE TUTORIALS!! Easiest marks to get.
Class Tests	Dates may be changed, but make sure you're available for the 4 dates listed in the syllabus (Jan 30, Feb 13, …) Any changes will be announced on Brightspace.
Final exam	Same exam for all 7 sections. Any class tests that you do not write will take on the mark that you receive on the final exam. For example, if you don't write a test and you get a 70% on the final, that test that you didn't write will be calculated as if you got a 70% on it. Tests will be mostly long-answer questions and there may a few multiple choice questions. DO NOT SIT NEXT TO YOUR FRIENDS DURING TESTS!! People who study together tend to have similar answers on tests. You may get flagged for cheating if your eyes wander during exams.

Ex.1 $\boxed{\frac{39}{2} = \frac{13}{2} \cdot 3}$ Ex.2 $\boxed{\frac{28}{4}}$

Calculator Use	Only non-programmable, non-graphing calculators allowed. Not necessary, though. The math required for tests can be solved without a calculator, like: <u>Ex. 1</u> $\boxed{\frac{39}{2} = \frac{13}{2} \cdot 3}$ <u>Ex. 2</u> $\boxed{\frac{28}{4}}$
Paul Menton Centre	The first class test is in three weeks! The PMC requires all accommodations to be in at least two weeks before the first test. So, get your PMC stuff sorted out in the first week!
Struggling with the course content?	You have access to many resources: - Math Tutorial Centre - Office hours with the Prof or TAs

Week 1 Content

Mon Jan 9 2023

What is a linear equation?

Any equation that can be written in the following format is a linear equation.

$$a_1 x_1 + a_2 x_2 + a_n x_n = b$$

Note that a_i, b must be real / complex numbers.
There must be a finite number of variables (x_i variables).

Examples of linear equations

Ex. 1 $\boxed{2x_1 + 3x_2 = 4}$ ✓ this is a linear equation

real numbers

Ex. 2 $\boxed{\sqrt[3]{2}\, x_1 + 0x_2 + (\pi - 3) x_3 - 7x_4 = 6}$ ✓ this is a linear equation

wrong form

Ex. 3 $\boxed{7x - 3y + 4 = 2z}$ ✓ this is a linear equation since it can be manipulated into the correct form.

correct form

$\longrightarrow 7x - 3y - 2z = -4$

Note that the variables used may not always be $x_1, x_2, \ldots, x_n$.
In Ex. 3, the variables used are x, y, and z.

Examples of non-linear equations

Ex. 4 $\boxed{x_1^2 + 2x_2 = 3}$ ✗ not a linear equation

Ex. 5 $\boxed{x_1 + \sqrt{x_2} = 4}$ ✗ not a linear equation

Ex. 6 $\boxed{x_1^3 + 2x_2 = 3}$ ✗ not a linear equation

Systems of linear equations

A collection of linear eqns that use the same variables.

Ex. $2x_1 + 3x_2 - x_3 = 4$ and $-x_1 + 2x_3 = 1$

becomes $-x_1 + 0x_2 + 2x_3 = 1$

This is ok to do!

This is a system of lin. equations :)

Ex You can *technically* do this...

$x_1 + x_2 = 1 \rightarrow 0x_1 + x_2 + 0x_3 + 0x_4 = 1$

$x_3 - x_4 = 3 \rightarrow 0x_1 + 0x_2 + x_3 - x_4 = 3$

Different variables | same variables

but in practice this achieves nothing.

These are also a system of linear equations ... but doesn't achieve much.

Solution sets

The set of all values of the variables that solve the equations.
A collection of all solutions! ☺

Ex. Consider:

$2x_1 + 3x_2 - x_3 = 4$ and $-x_1 + 2x_3 = 1$

Solution 1: $x_1 = 1,\ x_2 = 1,\ x_3 = 1$
Solution 2: $x_1 = 3,\ x_2 = 0,\ x_3 = 2$

Are there exact 2 solutions? We'll come back to this!

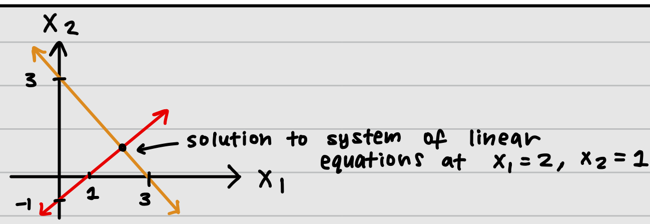
There may be more than 1 solution! Need to find ALL solutions. ☆

A graphical representation of a system of linear equations

Ex. Consider:

$x_1 + x_2 = 3$

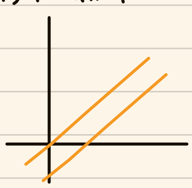
$x_1 - x_2 = 1$



Number of solutions in 2 dimensions

There are 3 possibilities:

1) Parallel

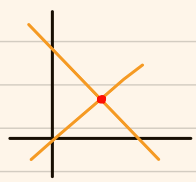


0 solutions

$x - y = 0$

$x - y = 1$

2) Intersect



1 solution

$x - y = 0$

$x + y = 2$

Solution: $x = 1,\ y = 1$

3) Coincide

Infinitely many solutions

$x - y = 0$

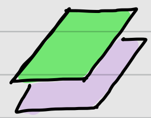
$-2x + 2y = 0$ ← same eqn, multiplied by -2.

Idea: This generalizes to higher dimensions (i.e. 3-D). (seen on next page)

Number of solutions in 3 dimensions

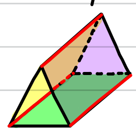
0 solutions

2 parallel planes



(imagine 2 papers, not touching but one above the other).

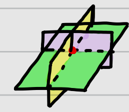
Triangular prism



3 planes form a tent shape. At every red line only two planes are touching. It will never satisfy all 3 equations, so no solutions!

1 solution

3 planes



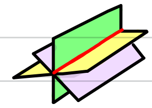
All 3 planes intersect at one point only.

Infinitely many solutions

Coincide

2 planes are the same.

"Around an axis"



3 planes intersect along an infinitely long line (the one in red). Note that these planes continue infinitely in each direction.

A system of linear equations has either: (* over reals and complex numbers)

1) No solutions. *
2) 1 solution. *
3) Infinitely many solutions. *

So.. If you are told in a problem that there are 2 solutions to a system of linear equations... that's not a system of <u>linear</u> equations! ᵕ̈

Back to the problem above.. there <u>MUST</u> be <u>infinitely many solutions!</u> We know that it doesn't have 0 or 1 solutions since we came up with 2 solutions ᵕ̈

Consistency	A system of linear equations is called **consistent** if it has at least 1 solution.
Inconsistency	A system of linear equations is called **inconsistent** if it has no solutions.

Equivalency

Two systems of linear equations are called **equivalent** if they have the same solution set.

Ex. The following two systems of linear equations are NOT equivalent. They share one solution set, but they do not share ALL of their solution sets.

$$2x_1 + 3x_2 - x_3 = 4$$
$$-x_1 + 2x_3 = 1$$

This is the example from earlier which has infinite solutions, one of which is $x_1 = x_2 = x_3 = 1$.

$$x_1 + x_2 + x_3 = 3$$
$$x_1 + x_2 - x_3 = 1$$
$$x_1 - x_2 = 0$$

The only solution is $x_1 = x_2 = x_3 = 1$

Matrices
m x n

A $m \times n$ matrix is a matrix with m rows and n columns.

Ex.

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 0 & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$ is a $\underset{\text{rows}}{3} \times \underset{\text{columns}}{4}$ real matrix.

↳ consisting of real numbers

Do not make the mistake of swapping m and n! This is NOT a 4 x 3 matrix! common mistake!!

Treat matrices like a table of numbers.

Every space needs to be filled for it to be a matrix.

Not a matrix

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$

Not a real matrix

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 2+3i & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$

not a real number!

$A_{i,j}$

$A_{i,j}$ is the element of A in the i^{th} row and j^{th} column.

Ex. $A_{2,3} = 0$

column 3

row 2 →
$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 0 & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$

A_i

Important! If there's just one subscript, like A_i, know that this does not have a standardized meaning. In this class, A_i will be defined in the question if it is used.

In this course, it is often used to represent the i^{th} column of a matrix as demonstrated below:

Ex.

A_1 is vector $[1 \ 3 \ 4]$

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 0 & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$

column 1

Augmented matrices

Useful for solving systems of linear equations.

Consider the linear system below:

$$\begin{aligned} 1x_1 + 1x_2 - 1x_3 &= 3 \\ -1x_1 - 1x_2 + 3x_3 &= -1 \\ -2x_1 + 1x_2 + 1x_3 &= 2 \end{aligned}$$

What should x_1, x_2, x_3 be? Hard to tell! There must be a way to solve this..

Take a look at the coefficients. We will use the coefficients to create both a "normal" matrix and an "augmented matrix"! :)

$$\begin{aligned} 1x_1 + 1x_2 - 1x_3 &= 3 \\ -1x_1 - 1x_2 + 3x_3 &= -1 \\ -2x_1 + 1x_2 + 1x_3 &= 2 \end{aligned} \rightarrow$$

Normal Matrix

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & -1 & 3 \\ -2 & 1 & 1 \end{bmatrix}$$

coefficients only!

☆ will not be using until lecture 3!

Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ -1 & -1 & 3 & -1 \\ -2 & 1 & 1 & 2 \end{array}\right]$$

coefficients AND constant! :)

Just learning notation for today.

has a vertical line to separate coefficients and constants

Elementary Row Operations (ERO)

Using EROs to easily solve systems of linear equations

There are 3 EROs that we can perform on matrices.

But first, some notation:

R_i' denotes the <u>new</u> row i. Ex. R_2' refers to the new row 2.

R_j denotes the <u>old</u> row j. Ex. R_2 refers to row 2 in previous matrix.

Use $\sim$, not $=$. Matrices are <u>not</u> equal, they are <u>similar</u>.
Do not use $=$, you'll lose marks! :)

Ok, time to define these operations!

1) Replacement : replace a row with the sum of itself and a multiple of another row.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ -1 & -1 & 3 & -1 \\ -2 & 1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 0 & 2 & 2 \\ -2 & 1 & 1 & 2 \end{array}\right] \quad R_2' = R_2 + 1 \cdot R_1$$

$-1 + 1(1) = 0$ | $-1 + 1(1) = 0$ | $3 + 1(-1) = 2$ | $-1 + 1(3) = 2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & -1 & 8 \end{array}\right] \quad R_3' = R_3 + 2R_1$$

changed

$-2 + 2(1) = 0$ | $1 + 2(1) = 3$ | $1 + 2(-1) = -1$ | $2 + 2(3) = 8$

2) Interchange : swap any two rows.

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 3 & -1 & 8 \\ 0 & 0 & 2 & 2 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

swapped

3) Scaling : Multiply a row by a <u>non-zero constant</u>.

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 3 & -1 & 8 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_3 = \tfrac{1}{2} R_3$$

divided in half

Only perform ONE ERO per step. Unless...! Unless the resulting matrix will be the same regardless of order of operations.

Let's convert the matrix back into equations, aka its "equivalent system".

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 3 & -1 & 8 \\ 0 & 0 & 1 & 1 \end{array}\right] \longrightarrow \begin{aligned} x_1 + x_2 - x_3 &= 3 \quad (1) \\ 3x_2 - x_3 &= 8 \quad (2) \\ x_3 &= 1 \quad (3) \end{aligned}$$

Wow, this is so much easier to solve!

Plug $x_3 = 1$ into (2)

$3x_2 - (1) = 8$

$3x_2 = 9$

$x_2 = 3$

Plug $x_3 = 1$ and $x_2 = 3$ into (1)

$x_1 + x_2 - x_3 = 3$

$x_1 + (3) - (1) = 3$

$x_1 = 1$

Therefore the (only) solution is $x_1 = 1$, $x_2 = 3$, $x_3 = 1$.

Takeaway: EROs do not change the solution set of systems.

END

Email: zacharywelch@cmail.carleton.ca

Office hours HP 5230 ? Friday 12:30 – 1:30 (Check Brightspace to confirm)
alt. by email or by appt.

Course textbook link will be up tonight or tmr.
e-text: $50 DO NOT BUY MYLAB VERSION – NOT USING
sold out at Bookstore

Tutorial right after lecture, room number currently unknown
no tut today or next week
first tut Jan 23

Final exam shared by all sections

Jan 30, Feb 13, ... be avail for these dates
If moved will be announced.

Any tests not written will take on mark of final exam

Tests will be mostly long-answer w/ possible mult. choice section

Do not sit next to your friends during tests

Tutorial wkshts – easiest marks. Go to tutorials!!

Following textbook, but not in order.
If unsure about lecture, check txtbk.

Use carleton email for all course email correspondance

Calculators? $\frac{39}{2} = \frac{13}{2} \cdot 3$ $\quad \frac{28}{4}$
Not necessary,
can only use
non-programmable,
non-graphing calculators

Struggling?

Math tutorial centre

Office hours w/ prof or TAs

Get PMC stuff sorted out this week!

If covid mask requirements change, will be announced by email.

What is a linear eqn?

Any eqn. that can be written as

$$a_1 x_1 + a_2 x_2 + a_n x_n = b$$

a_i, b real / complex numbers

x_i variables

Ex. $2x_1 + 3x_2 = 4$

still real nums

$$\sqrt[3]{2}\, x_1 + 0x_2 + (\pi - 3)x_3 - 7x_4 = 6$$

might not be x_1 or x_2

$7x - 3y + 4 = 2z$ could be x, y, z

not in right form, but can be manipulated into correct form

$$7x + (-3)y + (-2)z = -4$$

Non-linear

problems if real num is neg.

$$x_1^2 + 2x_2 = 3$$

x_1^3 not valid for this course

$$x_1 + \sqrt{x_2} = 4$$

$$x_1^3 + 2x_2 = 3$$

Systems of lin. eqns.

A collection of linear eqns. that use the same variables.

Eg. $2x_1 + 3x_2 - x_3 = 4$

$-x_1 + 2x_3 = 1 \rightarrow -x_1 + \underline{0x_2} + 2x_3 = 1$ can "cheat"

can *technically* do this.

$x_1 + x_2 = 1 \rightarrow x_1 + x_2 + 0x_3 + 0x_4 = 1$

$x_3 - x_4 = 3 \rightarrow 0x_1 + 0x_2 + x_3 - x_4 = 3$

In practice this does not achieve anything.

complexity grows cubically with addition of new elts.

Solution sets

consider:

$2x_1 + 3x_2 - x_3 = 4$

$-x_1 + 2x_3 = 1$

goal: values for x_1, x_2, x_3 that will solve eqn.

sol: $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

sol: $x_1 = 3$, $x_2 = 0$, $x_3 = 2$

May be more than 1 sol!

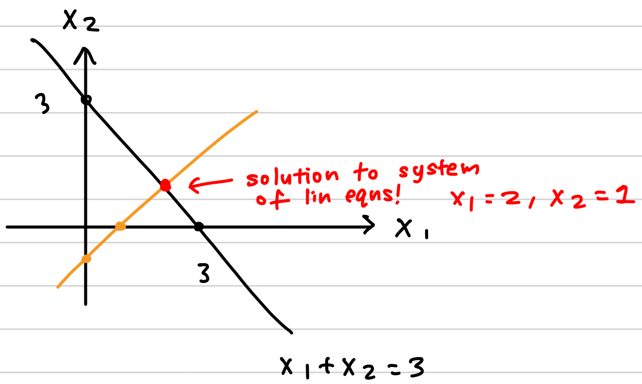
want to try to find <u>ALL</u> sol'ns.

2-D

Consider:

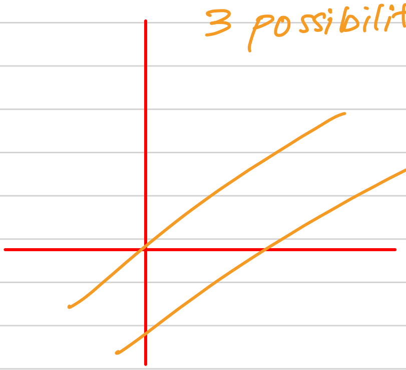
$x_1 + x_2 = 3$

$x_1 - x_2 = 1$



3 possibilities in 2 dimensions

3 possibilities:

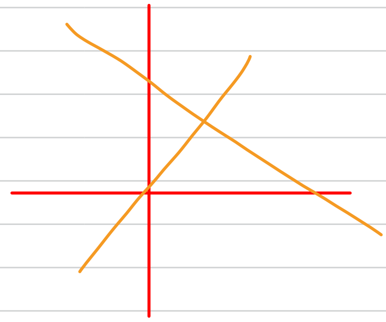


parallel

$x - y = 0$

$x - y = 1$

0 solutions



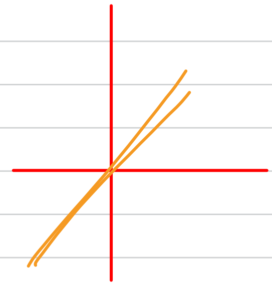
intersect (at one point)

$x - y = 0$

$x + y = 2$

sol: $x = 1,\ y = 1$

1 solution



coincide (same line)

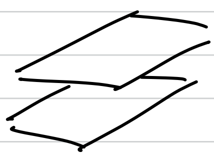
$x - y = 0$

$-2x + 2y = 0$ (same eqn. mult. by −2)

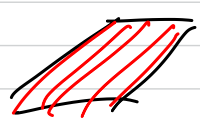
infinitely many solutions

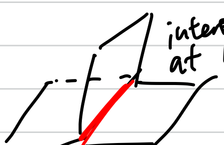
this generalizes to higher dimensions

2 parallel planes



coincide

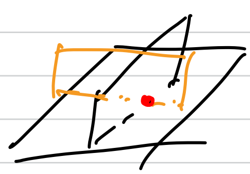




intersect at line

think of rotating around axis

intersect



need 3rd plane to intersect at 1 point!

prism

$x + y = 2$

$y + z = 2$

$x - z = 1$

satisfy 2 but never all 3

A system of linear eqns has either:

NO solutions

1 sol

∞ many sols

(only over reals and complex)

If he says there are 2 solutions... he's lying!

$$2x_1 + 3x_2 - x_3 = 4$$
$$-x_1 + 2x_3 = 1$$

we came up w/ 2 sols. (above)
thus, there are actually infinitely many solutions!

A system of lin eqns is called:

$\boxed{\text{consistent}}$ if it has at least 1 soln.

$\boxed{\text{inconsistent}}$ if it has no solns.

2 systems of linear eqns are called:

$\boxed{\text{equivalent}}$ if they have the <u>same</u> <u>solution set</u>

soln set: collection of all solutions.

$\{\{1,1,1\},\{3,0,2\} \dots\}$

A sys of lin eqns w/ soln set of JUST $\{1,1,1\}$ would be a <u>different</u> solution set and therefore the systems are <u>not</u> <u>equiv.</u>

$2x_1 + 3x_2 - x_3 = 4$

$-x_1 + 2x_3 = 1$

we know

$x_1 = x_2 = x_3 = 1$

$x_1 = 3, x_2 = 0, x_3 = 2$

share soln, but systems are non-equivalent

$x_1 + x_2 + x_3 = 3$

$x_1 + x_2 - x_3 = 1$

$x_1 - x_2 = 0$

only soln

$x_1 = x_2 = x_3 = 1$

Matrices

using as a "table of nums"

read "3 by 4"

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 0 & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$ is a 3×4 real matrix

rows cols

NOT a 4×3 matrix!!!
watch out

treat as a table of nums.

Every space needs to be filled for it to be a matrix.

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$

Not a matrix

$$A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & i & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix}$$

not a real matrix (must be filled by real nums)

A $m \times n$ matrix is a matrix with m-rows and n-columns.

$A_{i,j}$ is the elt of A in the i^{th} row and j^{th} col.

Ex. $A_{2,3}$

$$\begin{array}{cc} & \begin{array}{cccc} & & \text{col } 3 & \phantom{\sqrt{2}} \end{array} \\ \text{row2} & A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 0 & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix} \end{array}$$

Just 1 subscript... A_i ... no standard notation, will be defined w/ the question.

commonly used to say i^{th} col in course

Ex. A_1 is vector [1, 3, 4]

$$\begin{array}{c} A = \begin{bmatrix} 1 & -2 & 7 & \sqrt{2} \\ 3 & \pi & 0 & 0 \\ 4 & 2 & 1 & -3 \end{bmatrix} \\ \begin{array}{cccc} A_1 & & & \phantom{\sqrt{2}} \end{array} \end{array}$$

Consider the lin. sys.

$$\underline{1}\,x_1 + \underline{1}\,x_2 \underline{-1}\,x_3 = 3$$

$$\underline{-1}\,x_1 \underline{-1}\,x_2 \underline{+3}\,x_3 = -1 \rightarrow \begin{bmatrix} \underline{1} & \underline{1} & \underline{-1} \\ \underline{-1} & \underline{-1} & \underline{3} \\ \underline{-2} & \underline{1} & \underline{1} \end{bmatrix}$$

$$\underline{-2}\,x_1 + \underline{1}\,x_2 + \underline{1}\,x_3 = 2$$

normal matrix

coefficients

will not be dealing with until lecture 3

Conv. to "augmented matrix"

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ -1 & -1 & 3 & -1 \\ -2 & 1 & 1 & 2 \end{array}\right]$$

augmented matrix

Just learning the notation for today.

has vert. line

We now define the 3 "Elementary Row Operations" on matrices

1) Replacement: replace a row w/ the sum of itself and a multiple of another row

$$\begin{bmatrix} 1 & 1 & -1 & | & 3 \\ -1 & -1 & 3 & | & -1 \\ -2 & 1 & 1 & | & 2 \end{bmatrix} \sim \begin{array}{c} \begin{array}{cccc} -1+1 & -1+1 & 3+(-1) & -1+3 \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & | & 3 \\ 0 & 0 & 2 & | & 2 \\ -2 & 1 & 1 & | & 2 \end{bmatrix} \end{array}$$

new row 2   old Row 2

$R_2' = R_2 + 1 \cdot R_1$

read "row 2 prime"

each elt of row 2, add corresponding elt of row 1

$$\sim \begin{bmatrix} 1 & 1 & -1 & | & 3 \\ 0 & 0 & 2 & | & 2 \\ 0 & 3 & -1 & | & 8 \end{bmatrix} \quad R_3' = R_3 + 2R_1$$

$-2+2$ $1+2$ $1+(-2)$ $2+6$

2) Interchange: swap any 2 rows

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 3 & -1 & 8 \\ 0 & 0 & 2 & 2 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

3) Scaling: Multiply a row by a non-zero constant.

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 3 & -1 & 8 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_3' = \frac{1}{2} R_3$$

① Notation

R_i' denotes the new row i ex. R_2' is new row 2

R_j denotes the old row j R_2 refers to row 2 in prev. matrix

Use $\sim$, not $=$. matrices are not equal, they are SIMILAR.

do not use $=$, you'll lose marks

Only one elementary elt. row operation per step.
(1 ERO per step)

If all ops end w/ same matrix and order does not matter, it's ok. otherwise NO!!

can convert matrix back, into "equivalent system"

$$x_1 + x_2 - x_3 = 3$$
$$3x_2 - x_3 = 8$$
$$x_3 = 1$$

$$x_3 = 1$$
$$3x_2 - 1 = 8$$
$$x_2 = 3$$

$$x_1 + 3 - 1 = 3$$
$$x_1 = 1$$

1, 3, 1 is soln in original eqn, and only soln. to series of eqns.

takeaway: EROs do not change soln set of systems!!

$$1x_1 + 1x_2 - 1x_3 = 3$$
$$-1x_1 - 1x_2 + 3x_3 = -1 \quad \rightarrow$$
$$-2x_1 + 1x_2 + 1x_3 = 2$$