3 - Jan 16 Lecture

- Column/row vectors
- Vector equality
- Vector addition & scalar multiplication
- The zero vector
- Properties of vector addition/scalar multiplication
- Vector form/parametric vector form
- Linear combinations and spanning sets
- Properties of linear combinations and spanning sets

	Recall from previous lectures that for a matrix in RREF: 0 0 0 | 1 means that a matrix is inconsistent. 0 0 0 | 0 means that a matrix is consistent.
What is a vector?	A matrix with either 1 row or 1 column. There are two types of vectors: - Column vectors - Row vectors
What are column vectors?	A matrix with 1 column.
Exs:	Column vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} \sqrt{2} \\ -\pi \\ 3/2 \end{bmatrix}$
What are row vectors?	A matrix with 1 row.
Exs:	Row vectors $\begin{bmatrix} 1 & 2 \end{bmatrix} \quad \begin{bmatrix} \sqrt{2} & -\pi & 3/2 \end{bmatrix}$
What makes two vectors "equal"?	2 vectors are equal if they have the same dimension and all the components are the same, in the same order.
What is the "dimension" of a vector?	Dimension refers to the length and orientation of the vector.
What is the "orientation" of a vector?	Orientation refers to whether the vector is a row vector or a column vector. Row vectors can never be equal to column vectors.
Exs:	Are these vectors equal? $\underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{\text{equal } \checkmark} \neq \underbrace{[1, 2]}_{\text{wrong orientation X}} \neq \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{\text{wrong order X}} \neq \underbrace{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}_{\text{different dimension X}}$
Vector Notation	Either a v with an arrow over it or a bold lowercase v. Tests may use either. $\vec{v}$ or $\mathbf{v}$

Note: The number of entries is the dimension that the vector exists in.

Exs:

2D Vector:	3D Vector:
$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

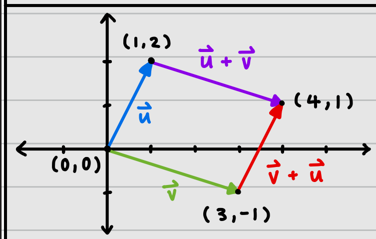
Vector Addition Formula

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \qquad \vec{u} + \vec{v} = \underbrace{\begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix} = \begin{bmatrix} v_1 + u_1 \\ v_2 + u_2 \\ \vdots \\ v_n + u_n \end{bmatrix}}_{\text{order of addition does not matter}}$$

Ex. Vector addition

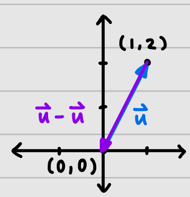
$$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \qquad \vec{u} + \vec{v} = \begin{bmatrix} 1+3 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \qquad \vec{v} + \vec{u} = \begin{bmatrix} 3+1 \\ -1+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Graphical representation of vector addition



Vectors are added head-to-tail.

Ex. Vector "subtraction"



$\vec{u} - \vec{u}$ leads to the origin.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{u} - \vec{u} = \begin{bmatrix} 1-1 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note: You can't add vectors of different dimensions/orientations together.

Exs:

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \text{undefined}$	$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \end{bmatrix} = \text{undefined}$

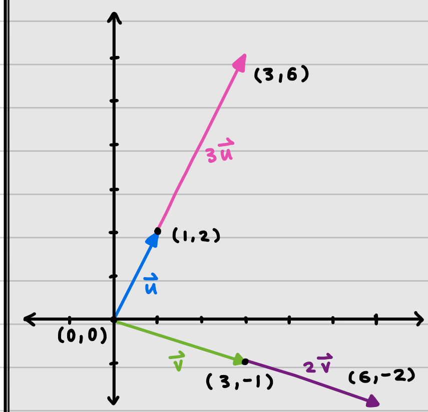
Scalar Multiplication Formula

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \qquad a\vec{u} = \begin{bmatrix} au_1 \\ au_2 \\ \vdots \\ au_n \end{bmatrix}$$

Ex: Scalar multiplication

$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ $\quad 3\vec{u} = \begin{bmatrix} 3(1) \\ 3(2) \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ $\quad 2\vec{v} = \begin{bmatrix} 2(3) \\ 2(-1) \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

Graphical representation of scalar multiplication



Formula for vector addition with scalar multiplication

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad a\vec{u} + b\vec{v} = \begin{bmatrix} a u_1 + b v_1 \\ a u_2 + b v_2 \\ \vdots \\ a u_n + b v_n \end{bmatrix}$$

Ex: Vector addition with scalar multiplication

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} -2 \\ -1 \\ 4 \\ 0 \end{bmatrix} \quad 2\vec{u} - 3\vec{v} = \begin{bmatrix} 2(1) - 3(-2) \\ 2(2) - 3(-1) \\ 2(3) - 3(4) \\ 2(4) - 3(0) \end{bmatrix} = \begin{bmatrix} 2 + 6 \\ 4 + 3 \\ 6 - 12 \\ 8 - 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -6 \\ 8 \end{bmatrix}$$

"The" zero vector

A vector consisting of only zeroes.
There is only a single zero vector in any space you're working in (2D/3D, row/col).

Ex: The zero vector in the scalar multiplication example at the top of this page is:

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (the origin)
← space: 2D column vector

The zero vector for a 4D row space is: $[0\ 0\ 0\ 0]$.

Zero vector notation

The zero vector is denoted $\vec{0}$. Sometimes $\vec{0}_n$ is used to denote n-dimensional $\vec{0}$.

Ex. $\vec{0}$ in regards to $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ is an n-dimensional column of all zeroes.

Properties of vector addition and scalar multiplication

1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ — Can add 2 vectors in any order
2) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ — Can add 3 vectors in any order
3) $\vec{0} + \vec{u} = \vec{u} + \vec{0} = \vec{u}$ — Zero vector + non-zero vector = non-zero vector
4) $\vec{u} - \vec{u} = -\vec{u} + \vec{u} = 0$ — Vector - equal vector = zero vector
5) $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$ — Scalar mult of sum of two vectors = scalar mult vector 1 + scalar mult of vector 2
6) $(c + d)\vec{u} = c\vec{u} + d\vec{u}$ — Distributive property of scalar multiplication
7) $c(d\vec{u}) = (cd)\vec{u}$ — Order of multiplication does not matter
8) $1\vec{u} = \vec{u}$ — Vector * 1 = original vector

Given the following solution set…

$$\begin{aligned} x_1 &= 2 - 3s + t \\ x_2 &= 4 + s + 2t \\ x_3 &= s \\ x_4 &= t \end{aligned} \quad \text{for all } s, t \in \mathbb{R}$$

Vector form

… let's write it in vector form…

Left hand side and right hand side become separate vectors.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 - 3s + t \\ 4 + s + 2t \\ s \\ t \end{bmatrix} \quad \text{for all } s, t \in \mathbb{R}$$

Parametric vector form

… and then in parametric vector form.

Split up $\underbrace{\text{constants}}$, $\underbrace{s \text{ terms}}$, $\underbrace{t \text{ terms}}$. Factor out s and t

$$= \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3s \\ s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} t \\ 2t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad \text{for all } s, t \in \mathbb{R}$$

Note: You can always write the solution set to a <u>consistent</u> linear system in <u>parametric vector form</u>.

Let $\vec{v_1}, \dots, \vec{v_k}$ be n-dimensional column vectors in $\mathbb{R}^n$.

What is a linear combination?

The vector $\vec{u} = c_1\vec{v_1} + c_2\vec{v_2} + \dots + c_k\vec{v_k}$ for $c_i \in \mathbb{R}$ is called a linear combination of $\vec{v_1}, \dots, \vec{v_k}$. ($V_1, \dots, V_k$ in $\mathbb{R}^n$, written as $c_1\vec{v_1} + \dots + c_k\vec{v_k}$ where $c_1, \dots, c_k \in \mathbb{R}$.)

What is a spanning set?

The set of all linear combinations of $\vec{v_1}, \dots, \vec{v_k}$ is called the span of $\vec{v_1}, \dots, \vec{v_k}$ and is denoted $\text{span}(\vec{v_1}, \dots, \vec{v_k})$. (A collection of vectors spans a set if every vector in the set can be expressed as a linear combination of the vectors in the collection.)

Ex. Is $\vec{u} \in \text{Span}(\vec{v_1}, \vec{v_2}, \vec{v_3})$? yes because v_1, v_2, v_3 can be expressed as a linear combination.

$$\vec{v_1} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3(-1) - 2(0) + 1 \\ 3(0) - 2(2) + 1 \\ 3(1) - 2(1) + 1 \end{bmatrix} = \boxed{3\vec{v_1} - 2\vec{v_2} + \vec{v_3}}$$

$\Rightarrow \vec{u} \in \text{Span}(\vec{v_1}, \vec{v_2}, \vec{v_3})$ ✓

Properties of linear equations and spanning sets

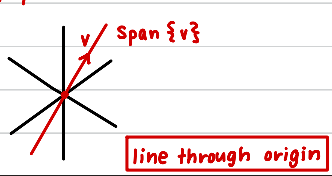
1) If $\vec{u} \in \text{Span}(\vec{v_1}, \dots, \vec{v_k})$ then $c\vec{u} \in \text{Span}(\vec{v_1}, \dots, \vec{v_k})$. Multiples of vectors that are in span are in span.
2) $\vec{0} \in \text{Span}(\vec{v_1}, \dots, \vec{v_k})$ The zero vector is always an element of span.
3) If $\vec{u_1}, \vec{u_2} \in \text{Span}(\vec{v_1}, \dots, \vec{v_k})$ then $\vec{u_1} + \vec{u_2} \in \text{Span}(\vec{v_1}, \dots, \vec{v_k})$.
4) $\vec{v_i} \in \text{Span}(\vec{v_1}, \dots, \vec{v_k})$ for all i.

"Is vector b in Span$\{v_1, \dots, v_p\}$?"

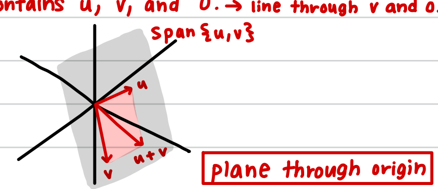
↳ Does the vector equation $x_1v_1 + x_2v_2 + \dots + x_pv_p = b$ have a solution?

↳ Does the linear system with augmented matrix $[v_1 \dots v_p \;\; b]$ have a solution?

Let $\vec{v}$ be a nonzero vector in $\mathbb{R}^3$. Then Span$\{v\}$ is set of all scalar multiples of $\vec{v}$, aka set of points on the line in $\mathbb{R}^3$ through v and 0.



If $\vec{u}$ and $\vec{v}$ are nonzero vectors in $\mathbb{R}^3$, with $\vec{v}$ not a multiple of $\vec{u}$, then Span$\{u,v\}$ is the plane in $\mathbb{R}^3$ that contains u, v, and 0.
→ line through u and 0.
→ line through v and 0.



Ex. Is $\vec{u}_1, \vec{u}_2 \in \text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$?

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \qquad \vec{u}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Transform into system of linear equations.

$$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \vec{u}_1 = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$$

$$= c_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + c_3\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 + c_2 + 0c_3 \\ c_1 - c_2 - c_3 \\ c_1 + 3c_2 + c_3 \end{bmatrix}$$

Vectors are only equal if all components are equal.

Row Reduce to see if system is consistent.

$$\begin{aligned} 2 &= c_1 + c_2 \\ 3 &= c_1 - c_2 - c_3 \\ 1 &= c_1 + 3c_2 + c_3 \end{aligned} \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & -1 & -1 & 3 \\ 1 & 3 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -2 & -1 & 1 \\ 0 & 2 & 1 & -1 \end{array}\right] \begin{array}{l} \\ R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 2 & 1 & -1 \end{array}\right] \begin{array}{l} \\ R_2' = \frac{1}{-2} R_2 \\ \\ \end{array}$$

RREF $\longrightarrow$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 5/2 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1' = R_1 - R_2 \\ \\ R_3' = R_3 - 2R_2 \end{array}$$

Is it in the span? = Is it consistent?

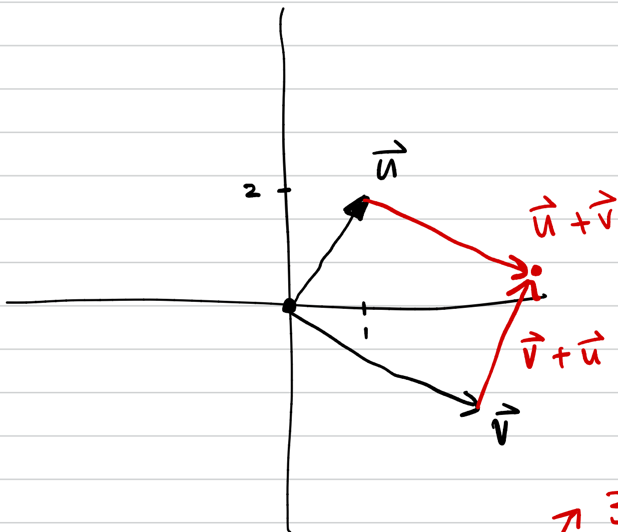
The matrix is in RREF and 000|1 is not a row.
Therefore, the system is consistent and $\vec{u}_1 \in \text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

$0 \ 0 \ 0 \ldots 0 \mid 1$

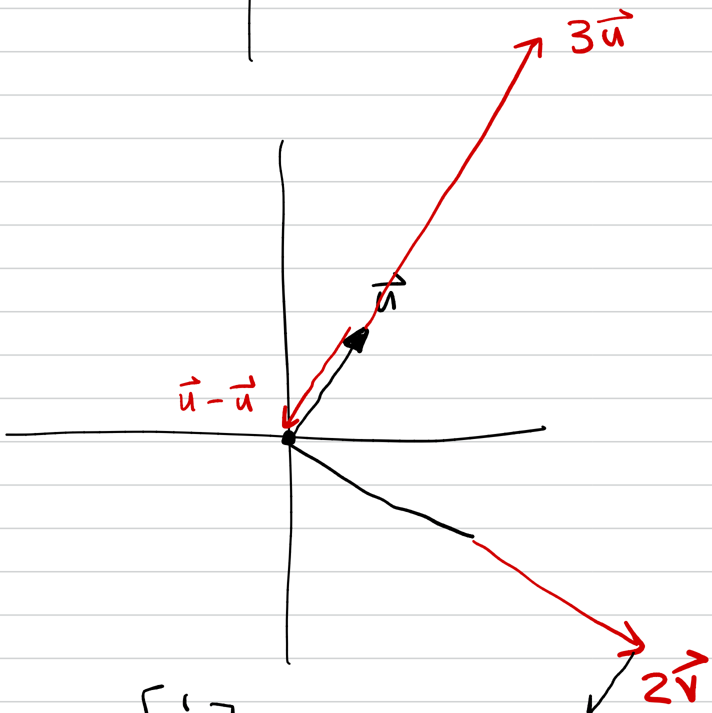
$0 \ 0 \ 0 \ldots 0 \mid x \qquad x = 0$

	A vector is a material with either 1 row or 1 column. A matrix with 1 column is called a column vector. A matrix with 1 row is called a row vector.
Ex: column vectors	$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 2 \\ -\pi \\ 3/2 \end{bmatrix} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ } column vectors all reals
Ex: row vectors	$[1 \ \ 2] \quad [\sqrt{2} \ \ -\pi \ \ 3/2]$ } row vectors
Equal vectors	2 vectors are equal if they have the same dimension and all the components are the same, in the same order. length + orientation row vector ≠ column vector $\underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{\text{equal}} \neq [1, 2]$ wrong orientation $\neq \begin{bmatrix} 2 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ wrong order extended dimension
Notation	$\vec{v}$ $\mathbf{v}$ ← textbook the one used in class tests might use both. usually will be defined. Note: the number of entries is the dimension the vector exists in. * ex *

Vector Addition



Scalar Multiplication



$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$3\vec{u} = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 12 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$a\vec{u} + b\vec{v} = \begin{bmatrix} au_1 + bv_1 \\ au_2 + bv_2 \\ \vdots \\ au_n + bv_n \end{bmatrix}$$

Ex. $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$$\underline{\vec{u} + \vec{v}} = \begin{bmatrix} 1+3 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

same!

$$\underline{\vec{v} + \vec{u}} = \begin{bmatrix} 3+1 \\ -1+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Ex.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} -2 \\ -1 \\ 4 \\ 0 \end{bmatrix}$$

$$2\vec{u} - 3\vec{v} = \begin{bmatrix} 2(1)+(-3)(-2) \\ 2(2)+(-3)(-1) \\ 2(3)+(-3)(4) \\ 2(4)+(-3)(0) \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -6 \\ 8 \end{bmatrix}$$

You can't add vectors of different dimensions together (in this course).

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is undefined.

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \end{bmatrix}$ = undefined

"The" zero vector

A vector consisting of only zeroes.

Only a single zero vector in any space you're working in.

The zero vector in our earlier drawing would be $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (the origin)

space: 2D column vector

4D: $\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$

rows

Notation

The zero vector is denoted $\vec{0}$.

Ex. $\vec{0}$ in regards to $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ is n-dimensional column vector of all zeroes.

Sometimes you might see $\vec{0}_n$ to denote n-dimensional $\vec{0}$.

Properties of vector addition and scalar multiplication:

1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

2) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ can calculate add. of 3 vectors in any order.

3) $\vec{0} + \vec{u} = \vec{u} + \vec{0} = \vec{u}$

4) $\vec{u} - \vec{u} = -\vec{u} + \vec{u} = 0$

5) $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$ scalar mult of sum of two vectors = scalar mult of each vector, added together

6) $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

7) $c(d\vec{u}) = (cd)\vec{u}$

8) $1\vec{u} = \vec{u}$

Linear system

$x_1 = 2 - 3s + t$

$x_2 = 4 + s + 2t$ for all $s, t \in \mathbb{R}$

$x_3 = s$

$x_4 = t$

Vector Form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 - 3s + t \\ 4 + s + 2t \\ s \\ t \end{bmatrix}$$ for all $s, t \in \mathbb{R}$

Parametric Vector Form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2-3s+t \\ 4+s+2t \\ s \\ t \end{bmatrix} = \underbrace{\begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}}_{\text{constant}} + \underbrace{\begin{bmatrix} -3s \\ s \\ s \\ 0 \end{bmatrix}}_{s \text{ terms}} + \underbrace{\begin{bmatrix} t \\ 2t \\ 0 \\ t \end{bmatrix}}_{t \text{ terms}}$$

factor out s and t

$$= \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \text{ for all } s,t \in \mathbb{R}$$

You can always write the solution set to a consistent linear system in parametric vector form.

Linear combinations and spanning sets

Let $\vec{v}_1 \dots \vec{v}_k$ be n-dimensional col vectors in $\mathbb{R}^n$

The vector $\vec{u} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k$ for $c \in \mathbb{R}$ is called a linear combination of $\vec{v}_1 \dots \vec{v}_k$

The set of all linear combos of $\vec{v}_1, \dots, \vec{v}_k$ is called the span of $\vec{v}_1, \dots, \vec{v}_k$ and is denoted $\text{Span}(\vec{v}_1, \dots, \vec{v}_k)$

Ex) $\vec{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\vec{u} = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} = 3\vec{v}_1 - 2\vec{v}_2 + \vec{v}_3 \Rightarrow \vec{u} \in \text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ $\vec{u}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Is $\vec{u}_1, \vec{u}_2 \in \text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$?

$$\begin{bmatrix}2\\3\\1\end{bmatrix} = \vec{u}_1 = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$$

$$= c_1\begin{bmatrix}1\\1\\1\end{bmatrix} + c_2\begin{bmatrix}1\\-1\\3\end{bmatrix} + c_3\begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$= \begin{bmatrix}c_1 + c_2\\ c_1 - c_2 - c_3\\ c_1 + 3c_2 + c_3\end{bmatrix}$$

Vectors are only equal if all components are equal.

$$\begin{aligned}2 &= c_1 + c_2\\ 3 &= c_1 - c_2 - c_3\\ 1 &= c_1 + 3c_2 + c_3\end{aligned} \rightarrow \left[\begin{array}{ccc|c}1 & 1 & 0 & 2\\ 1 & -1 & -1 & 3\\ 1 & 3 & 1 & 1\end{array}\right] \checkmark$$

$$\sim \left[\begin{array}{ccc|c}1 & 1 & 0 & 2\\ 0 & -2 & -1 & 1\\ 0 & 2 & 1 & -1\end{array}\right] \begin{array}{l}R_2' = R_2 - R_1\\ R_3' = R_3 - R_1\end{array} \checkmark$$

$$\sim \left[\begin{array}{ccc|c}1 & 1 & 0 & 2\\ 0 & 1 & \frac{1}{2} & -\frac{1}{2}\\ 0 & 2 & 1 & -1\end{array}\right] R_2' = -\frac{1}{2}R_2 \checkmark$$

RREF $\rightarrow$

$$\begin{array}{cccc}B & B & F & F\end{array}$$
$$\sim \left[\begin{array}{ccc|c}\boxed{1} & 0 & -\frac{1}{2} & \frac{5}{2}\\ 0 & \boxed{1} & \frac{1}{2} & -\frac{1}{2}\\ 0 & 0 & 0 & 0\end{array}\right] \begin{array}{l}R_1' = R_1 - R_2\\ \\ R_3' = R_3 - 2R_2\end{array} \checkmark$$

Is it in the span? ↯

Is it consistent?

If there is no 0 0 0 | 1 then it is consistent.

The matrix is in RREF and 0 0 0 | 1 is not a row

$\therefore$ the system is consistent. $\Rightarrow \vec{u}_1 \in \text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

Properties multiples of ~~things~~ vectors that are in span are in span.

1) If $\vec{u} \in \text{span}(\vec{v}_1, \dots, \vec{v}_k)$ then $c\vec{u} \in \text{span}(\vec{v}_1, \dots \vec{v}_k)$

2) $\vec{0} \in \text{span}(\vec{v}_1, \dots, \vec{v}_k)$ 0 vector always elt of span.

3) If $\vec{u}_1, \vec{u}_2 \in \text{Span}(\vec{v}_1, \dots, \vec{v}_k)$ then $\vec{u}_1 + \vec{u}_2 \in \text{Span}(\vec{v}_1, \dots, \vec{v}_k)$

4) $\vec{v}_i \in \text{Span}(\vec{v}_1, \dots, \vec{v}_k)$ for all i