8 - Feb 1 Lecture

- Sample final exam questions
- Matrix equality/addition/multiplication
- 2 methods of matrix multiplication
- Properties of matrix multiplication
- 3 of the biggest matrix multiplication mistakes to watch out for
- Exponent of a matrix
- Transpose of a matrix
- Properties of transpose of a matrix

Example Question 1. Likely to apper on final exam.

Given the following system of equations, for what values of h is the system consistent ?

$hx_1 + 3x_2 + 3x_3 = 1$

$x_1 + x_2 + x_3 = 0$

We want to move h as far down and to the right as possible.

$$\Leftrightarrow \begin{array}{l} x_1 + x_2 + x_3 = 0 \\ hx_1 + 3x_2 + 3x_3 = 1 \end{array} \Leftrightarrow \begin{array}{l} x_3 + x_2 + x_1 = 0 \\ 3x_3 + 3x_2 + hx_1 = 1 \end{array}$$

Then convert to an augmented matrix and row reduce

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 3 & 3 & h & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & h-3 & 1 \end{array}\right] \quad R_2' = R_2 - 3R_1$$

if $h=3$, we would stop here and the system is inconsistent since row 0 0 0 | 1 exists

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{h-3} \end{array}\right] \quad R_2' = \frac{1}{h-3} R_2$$

$\leftarrow$ REF (can stop here)

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -\frac{1}{h-3} \\ 0 & 0 & 1 & \frac{1}{h-3} \end{array}\right] \quad R_1' = R_1 - R_2$$

$\leftarrow$ RREF

$\leftarrow$ row 0 0 0 | 1 does not exist

There are two possibilities :

1. $h-3=0 \rightarrow$ inconsistent
2. $h-3 \neq 0 \rightarrow$ consistent

$\therefore$ The system is consistent for all $h \neq 3$ and is inconsistent for $h=3$.

A similar question.

Given $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix}$ and $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$, for what value of h is $\vec{u}$ in Span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

Matrix Equality

Two matrices are <u>equal</u> if and only if they are the same dimension with the same components in the same positions.

Ex:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \neq \text{anything other than } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Matrix Addition / Subtraction

Two matrices can only be added together if they are the same dimension. Subtraction works in the same way as addition.

Ex 1:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 & 0 \\ -1 & -7 & 2 \end{bmatrix} \quad A+B = \begin{bmatrix} 1-2 & 2+3 & 3+0 \\ 4-1 & 5-7 & 6+2 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ 3 & -2 & 8 \end{bmatrix}$$

2x3 matrix

Ex 2:

3x2 matrix

$$C = \begin{bmatrix} 1 & -1 \\ 2 & -4 \\ 0 & 1 \end{bmatrix} \qquad \begin{aligned} A+C &= \text{undefined} \\ B+C &= \text{undefined} \end{aligned}$$

Scalar Multiplication

Scalar multiplication works for matrices of all dimensions.

Ex 1:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad 2A = \begin{bmatrix} 2(1) & 2(2) & 2(3) \\ 2(4) & 2(5) & 2(6) \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

Ex 2:

$$C = \begin{bmatrix} 1 & -1 \\ 2 & -4 \\ 0 & 1 \end{bmatrix} \quad -3C = \begin{bmatrix} -3(1) & -3(-1) \\ -3(2) & -3(-4) \\ -3(0) & -3(1) \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -6 & 12 \\ 0 & -3 \end{bmatrix}$$

Matrix Multiplication

Method 1: "computer way"

Let A be a $m \times n$ matrix and B be a $n \times p$ matrix.

Let $b_1, b_2, \dots, b_p$ be the columns of B. (b_i is the i^{th} column of B)

Then, $AB_i = [Ab_1 \; Ab_2 \; \dots \; Ab_p]$ where AB_i is the i^{th} column of the resulting matrix

Note: the number of colums of A must be equal to the number of rows of B

↳ Otherwise, product of $A \times B$ is undefined.

Ex 1:

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 0 \end{bmatrix} \quad B = \begin{array}{c} \begin{matrix} B_1 & B_2 \end{matrix} \\ \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 0 & -1 \end{bmatrix} \end{array} \quad B_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$A \times B = \left[\left(2\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 0\begin{bmatrix} -2 \\ 0 \end{bmatrix} \right) \left(1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ -2 \end{bmatrix} - 1\begin{bmatrix} -2 \\ 0 \end{bmatrix} \right) \right] = \left[\left(\begin{bmatrix} 2 \\ 6 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix} \right) \left(\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \right]$$

$$= \begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix}$$

Ex 2:

$$C = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \qquad D = \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \end{array} \qquad D_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad D_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$C \times D = \left[\left(1\begin{bmatrix} -2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ -2 \end{bmatrix} \right) \left(2\begin{bmatrix} -2 \\ 1 \end{bmatrix} + 6\begin{bmatrix} 4 \\ -2 \end{bmatrix} \right) \right] = \left[\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 12 \\ -6 \end{bmatrix} \right) \left(\begin{bmatrix} -4 \\ 2 \end{bmatrix} + \begin{bmatrix} 24 \\ -12 \end{bmatrix} \right) \right]$$

$$= \begin{bmatrix} 10 & 20 \\ -5 & -10 \end{bmatrix}$$

Matrix Multiplication

Method 2: "human way"

This method will produce the same results as the previous method, but is easier for most people. You can use either method on a test.

$$(AB)_{ij} = \sum_{k=1}^{n} A_{i,k} B_{k,j} = A_{i,1} B_{1,j} + A_{i,2} B_{2,j} + \dots + A_{i,n} B_{n,j}$$

Ex:

$$A = \begin{array}{c} 1 \\ 2 \end{array} \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 0 \end{bmatrix} \qquad B = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 0 & -1 \end{bmatrix} \end{array}$$

Resulting matrix is 2×2.

1st row × 1st col   1st row × 2nd col

$$= \begin{bmatrix} 1(2) + 1(2) - 2(0) & 1(1) + 1(0) - 2(-1) \\ 3(2) - 2(2) + 0(0) & 3(1) - 2(0) + 0(-1) \end{bmatrix} = \begin{bmatrix} 2+2 & 1+2 \\ 6-4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix}$$

A cool trick for matrix multiplication

This next trick is super cool and very useful for a chain of matrix multiplication (i.e. when more than 2 vectors are multiplied together).
If you can write vectors with matching "inner numbers", then the "outer numbers" become the dimension of the resulting matrix.

Ex 1:

$$\begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} \quad & \quad \end{bmatrix}$$

$\boxed{4} \times \boxed{2 \quad 2} \times \boxed{5}$

inner match ⇒ multiplication defined

⇒ outer becomes dimension of resulting matrix

(in this case, a 4×5 matrix)

Ex 2

$$\begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} \quad & \quad \end{bmatrix}$$

$\boxed{1} \times \boxed{2 \quad 2} \times \boxed{4 \quad 4} \times \boxed{3}$ ⇒ 1×3 matrix

Properties of Matrix Multiplication

Let A be a $m \times n$ matrix. Let B, C be matrices of any sizes such that the following are defined. Let r, s be scalars.

1) $(AB)C = A(BC)$
2) $A(B+C) = AB + AC$ (alternatively: $(B+C)A = BA + CA$)
3) $r(AB) = (rA)B = A(rB)$ (can multiply by r at any step)
4) $I_m A = A = A I_n$
5) $0_m A = A 0_n = 0_{m \times n}$ (size of identity and zero usually implied by context - no subscripts provided)

Difference between AB and BA

Suppose AB is defined. BA might not be defined.

1) Even if BA is defined, BA may not have the same dimensions as AB.
2) Even if the dimensions match, BA may not equal AB.
3) Though, $AB = BA$ is possible. Square matrices and identity/zero matrices commute, meaning that $AB = BA$.

PSA: This is the biggest mistake that students make every year. Do not calculate $B \times A$ when you have been asked to calculate $A \times B$.

Ex 1:

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 0 & -1 \end{bmatrix} \quad BA = \begin{bmatrix} 2(1)+1(3) & 2(1)+1(-2) & 2(-2)+1(0) \\ 2(1)+0(3) & 2(1)+0(-2) & 2(-2)+0(0) \\ 0(1)-1(3) & 0(1)-1(-2) & 0(1)-1(0) \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix} \text{ as solved above} \qquad BA = \begin{bmatrix} 5 & 0 & -4 \\ 2 & 2 & -4 \\ -3 & 2 & 0 \end{bmatrix} \neq AB$$

(not equal and different dimensions)

Ex 2:

$$C = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \quad CD = \begin{bmatrix} 10 & 20 \\ -5 & -10 \end{bmatrix} \text{ (from previous example)}$$

$CD \neq DC$

$$DC = \begin{bmatrix} 1(-2)+2(1) & 1(4)+2(-2) \\ 3(-2)+6(1) & 3(4)+6(-2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0_{2\times 2}$$

Top 3 things to watch out for:

1) $AB \neq BA$.
2) $XY = 0$ does not mean that $X = 0$ or $Y = 0$.
3) $XY = XZ$ does not imply that $Y = Z$.

(as an example: $DC = D0_{2\times 2}$. $C \neq 0_{2\times 2}$)

Exponent of a Matrix	Let A be a $n \times n$ matrix. Then, $A^k = \underbrace{A \times A \times \ldots \times A}_{k \text{ times}}$ for a positive integer k
	must be a square for product to be defined
	We also define $A^0 = I_n$
	We will expand on this next week. (presumably in lecture 10)
Transpose of a matrix	Denoted using A^T. ← There is not an exponent.
	To transpose means to take all rows and write them as columns, and vice versa.
	If A is a $m \times n$ matrix then A^T is the $m \times n$ matrix such that $(A^T)_{i,j} = A_{j,i}$ (switch coordinates)
Ex 1:	$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$
Ex 2:	$B = \begin{bmatrix} -1 & -1 \\ 2 & 4 \end{bmatrix} \quad B^T = \begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix}$
Ex 3:	$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \vec{v}^T = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$
	row vector
	column vector
Properties of transpose of a matrix	Let A, B be matrices such that calculations are defined and r is a scalar.
	1) $(A^T)^T = A$
	2) $(A+B)^T = A^T + B^T$
	3) $(rA)^T = r(A)^T$ (scalar multiplication can be applied before or after transpose)
	4) $(AB)^T = B^T A^T$ (transpose reverses the order of multiplication)
	↳ ex: $(ABCD)^T = D^T C^T B^T A^T$

This will likely be on the final exam

$$\begin{aligned} hx_1 + 3x_2 + 3x_3 &= 1 \\ x_1 + x_2 + x_3 &= 0 \end{aligned} \iff \begin{aligned} x_1 + x_2 + x_3 &= 0 \\ hx_1 + 3x_2 + 3x_3 &= 1 \end{aligned}$$

$$\iff \begin{aligned} x_3 + x_2 + x_1 &= 0 \\ 3x_3 + 3x_2 + hx_1 &= 1 \end{aligned}$$

Goal: take variable and move to far right (bottom) corner

For what values of h is the sys. consistent?

$$\Rightarrow \begin{array}{c} \begin{matrix} x_3 & x_2 & x_1 & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 3 & 3 & h & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & h-3 & 1 \end{array}\right] \quad R_2' = R_2 - 3R_1$$

(REF)

but can do one more step to get RREF

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{h-3} \end{array}\right] \quad R_2' = \frac{1}{h-3} R_2$$

2 possibilities:

① $h-3=0 \rightarrow$ inconsistent

② $h-3 \neq 0 \rightarrow$ consistent (we can divide row 2 by $h-3$ to get)

Row 000 | 1 does not exist

$\therefore$ The system is consistent for all $h \neq 3$

and is inconsistent for $h = 3$.

Sim. Q

$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix}$ $\quad \vec{v}_1, \vec{v}_2, \vec{v}_3$ for what value of h

is $\vec{u}$ in Span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

Matrix Algebra Equality	Two matrices are equal if and only if they are the same dimensions with the same components in the same positions. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
Addition	Two matrices can only be added together if they are the same dimension. Addition is done component-wise: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 3 & 0 \\ -1 & -7 & 2 \end{bmatrix}$ same dim ✓ (both 2×3 matrices) Subtraction works the same as addition (component-wise, matrices same dimension) $A + B = \begin{bmatrix} 1-2 & 2+3 & 3+0 \\ 4-1 & 5-7 & 6+2 \end{bmatrix} = \begin{bmatrix} \underline{-1} & \underline{5} & \underline{3} \\ \underline{3} & \underline{-2} & \underline{8} \end{bmatrix}$ ← a component $C = \begin{bmatrix} 1 & -1 \\ 2 & -4 \\ 0 & 1 \end{bmatrix}$ ← 3×2 matrix $A + C$, $B + C$ not defined
Multiplication	Scalar multiplication is also done component-wise: $2A = \begin{bmatrix} 2(1) & 2(2) & 2(3) \\ 2(4) & 2(5) & 2(6) \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$ (and dimension of matrix does not matter) $-3(C) = \begin{bmatrix} -3 & 3 \\ -6 & 12 \\ 0 & -3 \end{bmatrix}$
Matrix Multiplication (not done component-wise)	Let A be a m x n matrix and B be a n x p matrix. Let $b_1, b_2, \dots, b_p$ be the columns of B (b_i is the i^{th} column of B) Then $AB_i = [Ab_1 \ Ab_2 \dots Ab_p]$ where AB_i is the i^{th} column of the resulting matrix. Important note: the # columns of A must be equal to the # rows of B. ↳ If not, product of AB is undefined.

Ex. 1

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$A \cdot B = \left[\left(2\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 0\begin{bmatrix} -2 \\ 0 \end{bmatrix} \right) \left(1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ -2 \end{bmatrix} - 1\begin{bmatrix} -2 \\ 0 \end{bmatrix} \right) \right]$$

$$= \begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix}$$

Ex. 2

$$C = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \quad d_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad d_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$CD = \left[\left(1\begin{bmatrix} -2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ -2 \end{bmatrix} \right) \left(2\begin{bmatrix} -2 \\ 1 \end{bmatrix} + 6\begin{bmatrix} 4 \\ -2 \end{bmatrix} \right) \right]$$

$$= \begin{bmatrix} 10 & 20 \\ -5 & -10 \end{bmatrix}$$

The other way to calculate AB:

$$(AB)_{ij} = \sum_{k=1}^{n} A_{i,k} B_{k,j} = A_{i,1} B_{1,j} + A_{i,2} B_{2,j} + \ldots + A_{i,n} B_{n,j}$$

$$A = \begin{array}{c} 1 \\ 2 \end{array} \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 0 \end{bmatrix} \quad B = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 0 & -1 \end{bmatrix} \end{array}$$

Resulting matrix is 2×2

$$= \begin{bmatrix} \overset{\text{1st row} \times \text{1st col}}{1 \cdot 2 + 1 \cdot 2 + (-2)(0)} & \overset{\text{1st row} \times \text{2nd col}}{1 \cdot 1 + 1 \cdot 0 + (-2)(-1)} \\ \underset{\text{2nd row} \times \text{1st col}}{3(2) + (-2)(2) + 0(0)} & \underset{\text{2nd row} \times \text{2nd col}}{3(1) - 2(0) + 0(-1)} \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix}$$

Can also check this method with C×D, you'll get the same result.

Trick

$4 \times \boxed{2 \quad 2} \times 5$

match

$\Downarrow$

mult. defined

Outer becomes dimension of resulting matrix. (4×5 matrix)

$1 \times \boxed{2 \quad 2} \times \boxed{4 \quad 4} \times 3 \rightarrow 1 \times 3$ matrix

Rules apply to a chain of multiplication too.

Let A be $m \times n$ matrix. Let $B C$ be matrices s.t. any sizes that the following are defined. Let r, s be scalars.

1) $(AB)C = A(BC)$

2) $A(B+C) = AB + AC$

$(B+C)A = BA + CA$

3) $r(AB) = (rA)B = A(rB)$ → can multiply by r at any step

4) $I_m A = A = A I_n$

5) $O_m A = A O_n = O_{m \times n}$

↳ Size of identity and zero matrices usually implied by context. (no subscripts)

AI or OA $\quad A + O$ ($\underset{\text{matrix}}{\overset{\text{implied}}{m \times n}}$)

Using A, B from earlier:

1

$$BA = \begin{bmatrix} 5 & 0 & -4 \\ 2 & 2 & -4 \\ -3 & 2 & 0 \end{bmatrix} \neq AB \quad \text{not equal \& not same dimensions}$$

Using C, D from earlier:

$$D \cdot C = \begin{bmatrix} 1(-2) + 2(1) & 1(4) + 2(-2) \\ 3(-2) + 6 & 3(4) + 6(-2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O_{2 \times 2}$$

1

AB vs BA

Suppose AB is defined.

BA need not be defined. ($A\vec{x}$)

matrix vector product then switch

Biggest mistake ppl calculate $B \times A$ when asked for $A \times B$

Even if it is defined, BA need not have the same dimensions.
Even if dimensions match, BA need not equal AB.
Though AB=BA is possible (square matrix, identity/zero matrix commute)

② If you see XY=0 that does not mean that X=0 or Y=0.

③ XY=XZ does not imply that Y=Z.

Ex from above : $(DC = D\,0_{2\times 2})$

Practice!! Find examples from textbook / online.

Exponent of a Matrix

Let A be a $n \times n$ matrix. ← has to be square, multiplying A by itself.

Then $A^k = \underbrace{A \times A \times \ldots \times A}_{k \text{ times}}$ for k a positive integer.

A needs to be a square for this product to be defined.

We also define $A^0 = I_n$.

We will expand on this next week.

Transpose of a Matrix

Denoted A^T. ← if you see this it means transpose not exponent.

"Take all rows and write as columns, and vice versa".

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

If A is a $m\times n$ matrix then A^T is the $m\times n$ matrix s.t. $(A^T)_{i,j} = A_{j,i}$ simply (switch coordinates)

$$B = \begin{bmatrix} -1 & -1 \\ 2 & 4 \end{bmatrix} \rightarrow B^T = \begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \vec{v}^T = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

col vector ⟶ row vector

Let A,B be matrices s.t calculations are defined and r is a scalar.

$(A^T)^T = A$

$(A+B) = A^T + B^T$

$(rA)^T = r(A)^T$ → scalar mult. can be done before or after T.

$(AB)^T = B^T A^T$ → T reverses order of multiplication

$(ABCD)^T = D^T C^T B^T A^T$