

# Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment

Germán Morales-España, *Student Member, IEEE*, Jesus M. Latorre, *Member, IEEE*, and Andres Ramos

**Abstract**—This paper presents a mixed-integer linear programming (MILP) formulation of start-up (SU) and shut-down (SD) power trajectories of thermal units. Multiple SU power-trajectories and costs are modeled according to how long the unit has been offline. The proposed formulation significantly reduces the computational burden in comparison with others commonly found in the literature. This is because the formulation is 1) tighter, i.e., the relaxed solution is nearer to the optimal integer solution; and 2) more compact, i.e., it needs fewer constraints, variables and nonzero elements in the constraint matrix. For illustration, the self-unit commitment problem faced by a thermal unit is employed. We provide computational results comparing the proposed formulation with others found in the literature.

**Index Terms**—Mixed-integer linear programming, start-up and shut-down ramps, thermal units, unit commitment.

## NOMENCLATURE

The main definitions and notation used are presented in this section for quick reference. Upper-case letters are used for denoting parameters and sets; and lower-case letters for variables and indexes.

**Definitions:** The following terminology is used in this paper to reference the different unit operation states, see Fig. 1.

<i>online</i>	Unit is synchronized with the system.
<i>offline</i>	Unit is not synchronized with the system.
<i>up</i>	Unit is producing above its minimum output. During the <i>up</i> state, the unit output is controllable.
<i>down</i>	Unit is producing below its minimum output, when <i>offline</i> , starting up or shutting down.

### Indexes and Sets:

$l \in L$	Start-up type, running from 1 (hottest) to $N_L$ (coldest).
$t \in T$	Hourly periods, running from 1 to $N_T$ hours.

### Variables:

$e_t$	Energy production during period $t$ [MWh].
$p_t$	Power output at the end of period $t$ , production above the minimum output [MW].

$u_t$	Commitment of the unit during period $t \in \{0, 1\}$ . Binary variable which is equal to 1 if the unit is up and 0 if it is down, see Fig. 1.
$v_t$	Start-up in period $t \in [0, 1]$ . Continuous variable which takes the value of 1 if the unit starts up in period $t$ and 0 otherwise, see Fig. 1.
$w_t$	Shut-down in period $t \in [0, 1]$ . Continuous variable which takes the value of 1 if the unit shuts down in period $t$ and 0 otherwise, see Fig. 1.
$\delta_{t,l}$	Start-up type $l$ in period $t \in [0, 1]$ . Continuous variable which takes the value of 1 in the period where the unit starts up for the start-up type $l$ and 0 otherwise.

### Parameters:

$EP_t$	Forecasted price of energy in period $t$ [\$/MWh].
$C^{LV}$	Linear variable production cost [\$/MWh].
$C^{NL}$	No-load cost [\$/h].
$C^{SD}$	Shut-down cost [\$/h].
$C_l^{SU}$	Start-up cost for the start-up type $l$ [\$/h].
$\bar{P}$	Maximum power output [MW].
$\underline{P}$	Minimum power output [MW].
$P_i^{SD}$	Power output at the beginning of the $i$ th interval of the shut-down ramp process [MW], see Fig. 1.
$P_{l,i}^{SU}$	Power output at the beginning of the $i$ th interval of the start-up ramp process type $l$ [MW], see Fig. 1.
$P_l^{syn}$	Power output at which the unit is synchronized for start-up type $l$ [MW], $P_{l,1}^{SU} = P_l^{syn}$ , see Fig. 1.
$SD^D$	Duration of the shut-down ramp process [h].
$SU_l^D$	Duration of the start-up type $l$ ramp process [h].
$RD$	Maximum ramp-down rate [MW/h].
$RU$	Maximum ramp-up rate [MW/h].
$TD$	Minimum down time [h].
$TU$	Minimum up time [h].
$T_l^{SU}$	Minimum number of periods that the unit must be down for the start-up type $l$ [h].

## I. INTRODUCTION

THE actual operation of power generation units must be considered in detail in order to rigorously model their generation schedules. Moreover, with the introduction of competi-

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The authors are with the Institute for Research in Technology (IIT), School of Engineering (ICAI), Universidad Pontificia Comillas, Madrid, Spain (e-mail: german.morales@iit.upcomillas.es; gmorales@kth.se; jesus.latorre@iit.upcomillas.es; andres.ramos@upcomillas.es).

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tion, accurate modeling and solutions for unit commitment (UC) problems are even more necessary to achieve efficiency and feasibility in energy production [1].

Most of the literature about modeling constraints of thermal units in UC problems deals with the unit operation above the minimum output [2], [3]. Units are considered to start/end their production at the minimum output while the start-up (SU) and shut-down (SD) ramps (or power trajectories) are ignored. Some papers are aware of the importance of considering these ramps in the UC optimization problem. However, they do not include these ramps because the resulting model will be considerably more complex, causing prohibitive solving times [4]–[6]. In addition, due to the increasing penetration of wind generation nowadays, thermal units are being shut down and started up more often [7]; therefore, a detailed modeling of the SU and SD processes in UC is required.

The application of direct mixed-integer linear programming (MILP) to solving UC is becoming increasingly popular due to improvements in MILP solvers. For example, PJM has switched from Lagrangian relaxation (LR) to MILP to solve the UC-based problems [8]. LR was the dominant optimization technique for solving UC problems through problem decomposition, mainly because LR does not present a high memory requirement as does MILP. However, this problem is being overcome due to the breakthrough of MILP solvers. Currently, combination of pure algorithmic speedup and the progress in computing machinery has meant that solving MILPs has become around 100 million times faster over the last 20 years [9]. Furthermore, MILP provides significant advantages over LR such as the fact that 1) there is a proven global optimal solution and 2) MILP models are easier to modify, which enhances modeling capabilities and adaptability, among other things [1], [10], [11].

The SU and SD ramps are explicitly modeled under the LR approach in [12] and under the MILP framework in [13] and [11]. In [12] and [13], only a single power trajectory for the SU process is modeled, while [11] considers different SU power trajectories depending on the unit's prior down time. Furthermore, [11] proposes a complete self-UC formulation which takes into account different constraints (e.g., power reserves and quadratic production costs) and is adapted to the Greek market rules.

References [13] and [11] made the important contribution in proposing the first MILP formulations for single and multiple SU and SD ramps, respectively. However, their main drawback is the creation of large models which greatly increase the complexity of UC problems, thereby making them unattractive for practical implementation. These models are large due to the introduction of many constraints in order to deal with the power trajectories above and below the minimum output of generating units. Apart from this, [11] needs many binary variables to model the different SU power trajectories.

The use of MILP-based UC formulations has increased significantly over the last 50 years [14]. As computational and algorithmic power increases, so does the complexity of the MILP formulations, with the addition of features such as ramping constraints, minimum up and down times and exponential SU costs [2]. The computational burden of UC problems needs to be further reduced, by improving the MILP formulations, so that even more advanced and computationally demanding problems can

be implemented, such as stochastic formulations [15], contingency-constrained models [16], and generation expansion planning [17].

Improving an MILP formulation allows a faster search for optimality by tightening (removing inefficient solutions from) the original feasible region. Tightening requires strong lower bounds for minimization problems [18]. This means formulating the problem in such a way that the associated linear programming (LP) relaxation provides a better approximation of the value of the integer optimal solution. The time required for providing optimality is often prohibitive because the gap between the integer optimal solution and its associated LP relaxation is very large. Furthermore, a poor lower bound provided by the LP relaxation will not be adequate to guide the search for good feasible solutions during the solving phase (branch-and-cut) of standard MILP solvers [19]. MILP formulations are frequently tightened by adding a huge number of constraints and (sometimes) variables. However, the resulting expanded model must close the gap enough to be worth the extra time taken to solve the LP relaxations during the solving phase [20]. In other words, usually, tightening an MILP formulation comes at the expense of expanding the model which implies extra time consumption. Therefore, creating tight and compact MILP formulations is a nontrivial task because the obvious formulations are commonly either very weak or very large.

Creating tight (or strong) MILP formulations has been widely researched [21]. In the case of UC problems, there has been work in a number of specific areas. In [22], a strong formulation of the minimum up/down time constraints is presented; in [23], a tighter linear approximation for quadratic generation costs is proposed; and [24] presents a new class of inequalities giving a tighter description of the feasible operating schedules for generators.

The main contribution of this paper is two-fold:

- 1) A tighter MILP formulation of SU and SD ramps for UC problems is proposed in order to reduce the computational burden of analogous existent MILP formulations.
- 2) This MILP formulation is also compact and hence overcoming the main disadvantage of previous models [11], [13]. If a single power trajectory is modeled for the SU and SD ramps, then there is neither a need to increase the number of constraints nor a necessity to increase the number of variables in comparison to a formulation without the SU and SD ramps. Furthermore, when considering different SU trajectories, the proposed formulation requires the introduction of merely continuous variables compared to [11].

Additionally, this formulation of SU and SD ramps is suitable for any UC problem, whether under centralized or competitive environments, and further model expansion will not require the introduction of numerous terms in the constraints in order to avoid conflicts between the up and down states, unlike [13] and [11].

In order to illustrate the effectiveness of the proposed formulation, the self-UC for a price-taker thermal generator is used. The objective of a thermal generator, in the self-UC, is to maximize the profits from selling energy in the day-ahead market, while satisfying all the technical constraints.

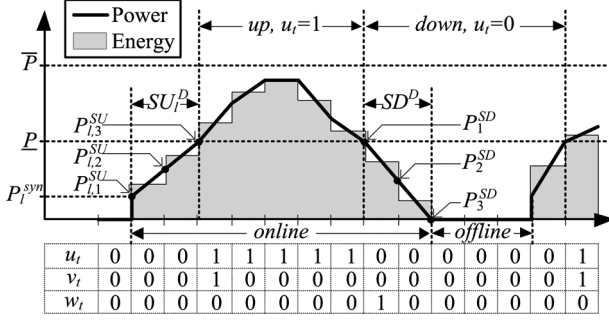


Fig. 1. Operation states of a thermal unit, including SU and SD ramps.

The remainder of this paper is organized as follows: Section II presents the formulation of the SU and SD constraints in detail. Section III provides and discusses results from several case studies, where the impact of neglecting the SU and SD ramps is shown and a comparison of the proposed formulation with those in [13] and [11] is made. Finally, some relevant conclusions are drawn in Section IV.

## II. PROPOSED APPROACH

This section presents the mathematical formulation of the SU and SD power trajectories. With the purpose of illustrating how this formulation works, the objective function is formulated for the case of a price taker self-UC problem. This section is divided into two parts: Section II-A details the mathematical formulation and Section II-B shows how the online and offline unit states can be obtained (see Fig. 1) after the optimization problem has been solved.

Hourly time intervals are considered, but it should be noted that the formulation can be easily adapted to handle shorter time periods. For the sake of simplicity, reserve constraints are not considered; however, they can be easily introduced in the model. The interested reader is referred to [5], [11], and [25].

### A. Mathematical Formulation

The different operation states of a thermal unit are presented in Fig. 1. The up and down states are distinguished from the online and offline states. During the up period, the unit has the flexibility to follow any trajectory being bounded between the maximum and minimum output and by the ramping-rate limits. On the other hand, the power output when the unit is starting up or shutting down follows a predefined power trajectory. Unlike the SD ramp, the SU ramp trajectory depends on the unit's previous down time.

1) *Up/Down versus Online/Offline States*: By considering the commitment variable  $u_t$  as up/down rather than offline/online states, the generation output above and below  $\underline{P}$  can be managed independently. This characteristic makes the proposed formulation 1) compact, unlike [13] and [11], where most of the constraints involving  $p_t$  contain summations of binary variables in order to avoid conflicts between the power output above and below  $\underline{P}$ ; and 2) tight where, by considering the generation output ( $p_t$ ) above  $\underline{P}$ , the feasible region for  $p_t$  is between  $\underline{P}$  and  $\bar{P}$ , which is tighter than the region that is usually considered, between 0 and  $\bar{P}$ .

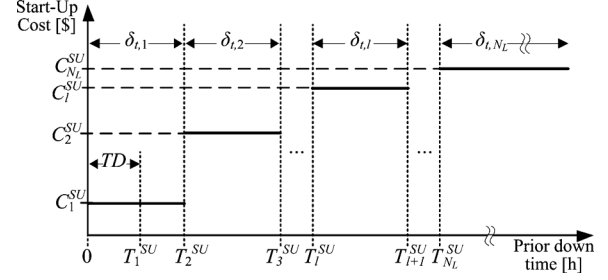


Fig. 2. Start-up costs as a function of the unit's previous down time.

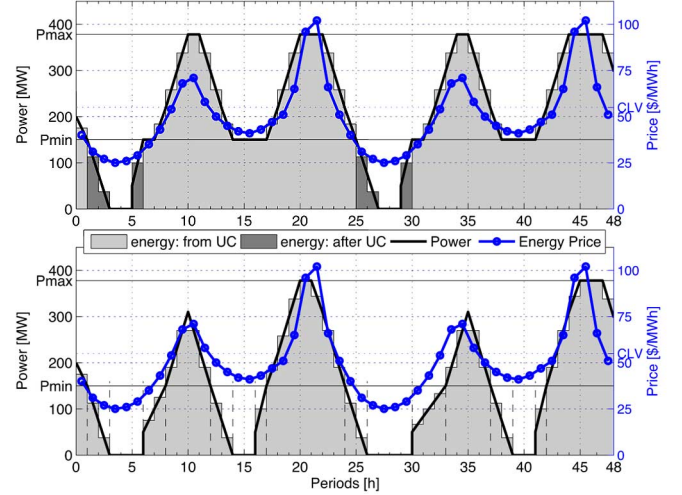


Fig. 3. Optimal generation scheduling for the *Traditional* and *Improved* formulations in the up and bottom part of the figure respectively. The darker gray area shows the SU and SD energy that the unit will produce in order to follow the optimal schedule that results from the *Traditional* formulation (this extra energy is added after solving the *Traditional* problem).

The down times are a function of the offline times (see Fig. 1). For example, the number of periods that the unit must be down to activate the SU type  $l$ ,  $T_l^{SU}$ , is equal to the SU and SD ramp durations ( $SU_l^D$  and  $SD_l^D$ ) plus the offline time required to activate the SU type  $l$ . Similarly, the down time  $TD$  is expressed as a function of the minimum offline time (see Section II-A3).

2) *Start-Up Type*: Different SU types are modeled depending on how long the unit has been down. The SU type  $\delta_{t,l}$  is selected if the unit has been previously down within the interval  $[T_l^{SU}, T_{l+1}^{SU})$ , see Fig. 2. Each SU type has a different SU power trajectory associated to it, where the colder the SU type, the longer the SU power trajectory duration (see example shown in Fig. 3 in Section III-A). As in the case of [26] and [11], new variables are introduced to select the SU type:

$$\delta_{t,l} \leq \sum_{i=T_l^{SU}}^{T_{l+1}^{SU}-1} w_{t-i} \quad \forall t \in [T_{l+1}^{SU}, N_T], l \in [1, N_L) \quad (1)$$

where the right side of (1) is equal to 1 if the unit has been down within the interval  $[T_l^{SU}, T_{l+1}^{SU})$  before hour  $t$ . Therefore,  $\delta_{t,l}$  can only be activated ( $\delta_{t,l} \leq 1$ ) if the unit has previously been down within this interval.

Note that (1) is not defined for the first hours. Appendix A details how the first SU types  $\delta_{t,l}$  are obtained depending on the initial conditions of the unit.

The following constraint ensures that just one SU type is selected when the unit starts up:

$$\sum_{l=1}^{N_L} \delta_{t,l} = v_t \quad \forall t. \quad (2)$$

Equation (1) constrains all SU types except the coldest one  $\delta_{t,N_L}$ . However, constraints (1) and (2) ensure  $\delta_{t,N_L} = 1$  when the unit starts up ( $v_t = 1$ ), and has been down for at least  $T_{N_L}^{SU}$  hours. This is because (1) makes  $\delta_{t,l} = 0$  for all  $l \neq N_L$  and then (2) forces  $\delta_{t,N_L} = 1$ . In the event that more than one SU type variable can be activated ( $\delta_{t,l} \leq 1$ ) then (2) together with the objective function ensure that the hottest, which is the cheapest, possible option is always selected. Therefore, just one of the variables is activated (equal to one). That is, these variables take binary values even though they are modeled as continuous variables. This is due to the convex (monotonically increasing) characteristic of the exponential SU costs of thermal units [2], see Fig. 2.

Constraint (1) is made even more compact by taking into account the minimum down time constraint (see Section II-A3). The hottest SU  $\delta_{t,1}$  must be activated when the unit has been down within the interval  $[0, T_2^{SU})$ . However, the minimum down time constraint (4) ensures that the unit cannot be down for less than  $TD$  hours. Therefore, the hottest SU is only possible within the interval  $[TD, T_2^{SU})$ . By defining  $T_1^{SU} = TD$ , see Fig. 2, constraint (1) together with the minimum down time constraint (4) ensure that the hottest SU  $\delta_{t,1}$  can be activated only when the unit has been down less than  $T_2^{SU}$  hours.

3) *Minimum Up/Down Times*: Constraints (3) and (4) ensure the minimum up and down times respectively [22]. This formulation has been compared with others and has shown a better performance [22], [24]:

$$\sum_{i=t-TU+1}^t v_i \leq u_t \quad \forall t \in [TU, N_T] \quad (3)$$

$$\sum_{i=t-TD+1}^t w_i \leq 1 - u_t \quad \forall t \in [TD, N_T]. \quad (4)$$

The minimum down time  $TD$  in (4) is equal to 1) the SD ramp duration ( $SD^D$ ), plus 2) the hottest SU ramp duration ( $SU_1^D$ ), plus 3) the minimum time that the unit must be offline. Therefore, (4) is needed to avoid overlapping between the SU and SD ramps. Appendix A describes how the initial conditions force the unit to remain up/down during the first hours.

4) *Commitment, Start-Up, and Shut-Down*: The following constraint can be found in models published approximately fifty years ago [14]:

$$u_t - u_t = v_t - w_t \quad \forall t. \quad (5)$$

Once  $u_t$  is defined as a binary variable, (5) forces  $v_t$  and  $w_t$  to take binary values.

5) *Capacity Limits*: The generation level in UC problems is usually expressed as hourly energy blocks; however, it has been demonstrated that taking a generation level schedule as an energy delivery schedule may not be realizable [27], [28].

Therefore, a clear difference between power and energy is made and all technical constraints are then imposed over the power output variable. The power generation output of the unit above its minimum output is modeled as

$$0 \leq p_t \leq (\bar{P} - \underline{P})(u_t - w_{t+1}) \quad \forall t. \quad (6)$$

Constraint (6) ensures that the total power output is equal to  $\underline{P}$  ( $p_t = 0$ ) at the beginning and at the end of a continuous up period. On the other hand, the SU (SD) trajectory ends (begins), during the down period, at  $\underline{P}$  level, thereby making the connection with the up period that starts (ends) at this level, see Fig. 1.

6) *Operating Ramp Constraints*: As mentioned in Section II-A1, the proposed formulation avoids the introduction of many variables in most of the equations, unlike [13] and [11]. This is the case for the ramping constraints that only depend on the generation variables between two consecutive hours:

$$-RD \leq p_t - p_{t-1} \leq RU \quad \forall t. \quad (7)$$

7) *Energy Production*: The total unit's energy production, including the energy produced during the SU and SD processes, is presented in (8). Note that the energy is obtained for hourly periods and piecewise-linear power trajectories (see Fig. 1). However, the conversion to shorter time periods is straightforward:

$$\begin{aligned} e_t = & \underline{P}u_t + \frac{p_t + p_{t-1}}{2} \\ & + \sum_{i=1}^{SD^D} \frac{P_{i+1}^{SD} + P_i^{SD}}{2} w_{t-i+1} \\ & + \sum_{l=1}^{N_L} \sum_{i=1}^{SU_l^D} \frac{P_{l,i+1}^{SU} + P_{l,i}^{SU}}{2} \delta_{(t-i+SU_l^D+1),l} \quad \forall t. \end{aligned} \quad (8)$$

The terms of the summations in (8) include the energy produced during the SU and SD procedures.

Equation (8), together with (1) and (2), make a tight description of the SU and SD ramps in the energy output variable  $e_t$ . This could be observed from the fact that on the one hand, when the unit is starting up ( $v_t = 1$ ), (1) and (2) will choose the correct SU type ( $\delta_{t,l}$ ), and then the associated SU energy trajectory is immediately fixed in (8), while on the other hand, when the unit is not starting up ( $v_t = 0$ ) then (2) forces all SU types to be zero ( $\delta_{t,l} = 0$ ) and thus the SU energy in (8) is immediately fixed to zero. Similarly, the SD decision ( $w_t$ ) will fix the SD energy trajectory in (8). Besides, the tightness of the formulation is experimentally checked in Section III, where the integrality gap of the proposed formulation is lower than those in [13] and [11].

Note that when just a single SU power trajectory is modeled, there is no need to introduce variables  $\delta_{t,l}$ . Therefore, constraints (1) and (2) are not needed and the scheduled energy in (8) must be modified to be directly affected by the SU variable  $v_t$  instead of  $\delta_{t,l}$ .

8) *Objective Function*: The goal of a price-taker producer in a self-UC is to maximize his profit during the planning period,

which is the difference between the revenue and the total operating cost (9). For the sake of simplicity, a linear production cost is used in this paper:

$$\max \sum_{t=1}^{N_T} \left[ EP_t e_t - \left( C^{NL} u_t + C^{LV} e_t + \sum_{l=1}^{N_L} C_l^{SU'} \delta_{t,l} + C^{SD'} w_t \right) \right]. \quad (9)$$

Note that the no-load cost ( $C^{NL}$ ) considered in (9) ignores the SU and SD periods. This is because the  $C^{NL}$  only multiplies the commitment during the up state  $u_t$ . In order to consider the no-load cost during the SU and SD periods,  $C_l^{SU'}$  and  $C^{SD'}$  are introduced in (9) and defined as

$$C_l^{SU'} = C_l^{SU} + C^{NL} SU_l^D \quad \forall l \quad (9a)$$

$$C^{SD'} = C^{SD} + C^{NL} SD^D. \quad (9b)$$

### B. Final Power Schedule

The complete energy profile, including SU and SD power trajectories, was presented in (8). Nevertheless, the complete power output as well as the unit states online/offline have not yet been obtained. This information can be explicitly modeled as variables in the optimization problem, which will create a considerably larger formulation. However, these values can be obtained after the optimization problem has been solved without changing the optimal results and then with negligible computational cost. Furthermore, this also contributes to the compactness of the formulation. The total power output  $P_t$ , and on-line/offline states  $U_t^{ON}$  are presented as follows:

$$P_t = \underline{P}(u_t + v_{t+1}) + p_t + \sum_{i=2}^{SD^D+1} P_i^{SD} w_{t-i+2} + \sum_{l=1}^{N_L} \sum_{i=1}^{SU_l^D} P_i^{SU} \delta_{(t-i+SU_l^D+2),l} \quad \forall t \quad (10)$$

$$U_t^{ON} = u_t + \sum_{l=1}^{N_L} \sum_{i=1}^{SU_l^D} \delta_{(t-i+SU_l^D+1),l} + \sum_{i=1}^{SD^D} w_{t-i+1} \quad \forall t. \quad (11)$$

Furthermore, analogously to the SU and SD decisions  $v_t$  and  $w_t$  which represent the changes between the up and down states, the turn-on  $V_t^{ON}$  and turn-off  $W_t^{OFF}$  decisions representing the changes between the online and offline states are now obtained with

$$V_t^{ON} = \sum_{l=1}^{N_L} \delta_{(t+SU_l^D),l} \quad \forall t \quad (12)$$

$$W_t^{OFF} = w_{t-SD^D} \quad \forall t. \quad (13)$$

TABLE I  
THERMAL UNIT DATA

Technical Information					Cost Coefficients		
$\bar{P}$ [MW]	$\underline{P}$ [MW]	$RU/RD$ [MW/h]	$TU/TD$ [h]	$SD^D$ [h]	$C^{NL}$ [\$]	$C^{LV}$ [\$/MWh]	$C^{SD}$ [\$]
378.0	150.0	80	4	2	200	55	20
Start-Up Ramping Information							
SU Type $l$	$C_l^{SU}$ [\$]	$P_l^{syn}$ [MW]	$SU_l^D$ [h]	$T_l^{SU}$ [h]			
01	16	50	1	4			
02	28	50	2	6			
03	36	50	3	8			
04	40	50	4	11			
05	41	50	5	14			

### III. TEST RESULTS

The proposed formulation is tested for the self-UC of a price-taker producer. The technical and economic data for the thermal unit, including five different SU ramps, are presented in Table I, and the expected electricity prices for a 48-hour time span are shown in Appendix B. These data are based on information presented in [11]. The power outputs  $P_l^{SU}(P^{SD})$  for the SU (SD) power trajectories are obtained as an hourly linear change from  $P_l^{syn}(\underline{P})$  to  $\underline{P}$  (0) for a duration of  $SU_l^D(SD^D)$  hours, see Fig. 1. With respect to initial conditions, the unit has been up for 6 hours before the scheduling horizon and its initial power output is 200 MW. All tests in this paper were carried out using CPLEX 12.3 under GAMS [29] on an Intel i7 2.4-GHz personal computer with 4 GB of RAM memory. Problems are solved to optimality, more precisely to 1e-6 of relative optimality tolerance.

This section is divided into two parts. The first part shows the impact of SU and SD ramps on the unit commitment. The second part presents a comparison of the proposed formulation with those presented in [13] and [11].

#### A. Scheduling and Economic Impact

In order to illustrate how the unit operation is affected if the SU and SD ramps are considered, the case study has been solved with and without the ramp trajectories. The formulation with SU and SD ramps is labeled as *Improved* and the formulation considering just the exponential-SU and SD costs is labeled as *Traditional*.

Unlike the *Traditional* formulation, considering the energy produced during the SU and SD ramps makes the *Improved* formulation perceive revenues during these ramping processes. To make a fair comparison between both formulations, the inherent energy produced during the SU and SD ramps is introduced into the *Traditional* formulation after the problem has been solved (see darker gray area in Fig. 3). That is, even when the *Traditional* formulation ignores these ramps in the scheduling stage, they are inevitably present during the operation stage. Subsequently, this energy can be added to the solution and this extra energy can also be sold. The total revenues for the *Traditional* formulation are then obtained by adding the revenues obtained from the UC solution to the revenues obtained from the energy produced during the SU and SD processes. These latter revenues

TABLE II  
PRODUCER COSTS AND PROFITS

	Costs (\$)	Revenues (\$)	Profits (\$)
<i>Traditional</i>	576417	617252	40835
<i>Improved</i>	402201	461674	59473
% of change	-30.22	-25.20	45.64

are calculated by multiplying the electricity price by the energy produced during the SU and SD ramps.

Fig. 3 and Appendix B show the optimal power and energy schedules for the *Traditional* and *Improved* formulations. Note the different duration of the SU power trajectories for the *Improved* formulation in Fig. 3. SU durations of one, two and three can be observed (starting at hours 16, 6, and 30, respectively) as a consequence of the different unit's down (and thus offline) time durations.

The optimal scheduling decision taken by the *Traditional* formulation around hours 15–17 and 39–41 was to produce at minimum output  $\underline{P}$  ( $P_{min}$  in Fig. 3) even when electricity prices were lower than the unit's linear variable production cost  $C^{LV}$  (CLV in Fig. 3). This is a very common behavior, where producing at  $\underline{P}$  generates fewer losses than shutting down and starting up the unit within a short period. On the other hand, when SU and SD ramps are considered, the optimal scheduling decision is to turn off the unit during these hours. The reason is that the SU and SD costs are offset by the revenues received from the energy produced during the SU and SD ramping processes. In short, unlike the *Improved* formulation, in the *Traditional* model, the SU and SD processes are perceived as pure losses. Therefore, the optimal decision of the *Traditional* UC formulation is to not turn off the unit for short periods to avoid these losses.

The main problem affecting the *Traditional* formulation is that revenues during the SU and SD processes are not considered in the optimization problem. Therefore, there is a tendency to produce at least at minimum output, even when electricity prices are lower than  $C^{LV}$ , and thus obtain some revenues that compensate for the losses. This drawback is overcome by considering the SU and SD power production in the formulation.

Table II shows the difference between costs, revenues and profits for the solutions of both formulations. As mentioned before, the total revenue for the *Traditional* model is obtained by adding the revenues due to the ramping process (\$14 800) to the revenues obtained from the optimal solution (\$602 452). For this illustrative case, the profits when considering the SU and SD ramps are around 46% higher than when these ramps are not taken into account.

### B. Comparing Different Formulations

The proposed formulation is compared with those available in [13] and [11]. Reference [13] proposes a formulation to deal with a single SU and SD ramp trajectory, whereas [11] deals with different SU trajectories depending on the unit's prior down time. The different SU types, their associated costs and power-trajectories are inherent characteristics of thermal units and these data are provided by the manufacturer. However, in order to compare the different formulations, the proposed formulation is implemented considering one, three and five

different SU ramp types, and these models are labeled as R1, R3, and R5, respectively. The single ramp type model R1 can be directly compared with the single-ramp formulation proposed in [13]. The model presented in [11] was implemented considering three SU ramp types, and hence it can be directly compared with R3. Model R5 is presented in order to observe the extra computational burden which results from considering extra SU ramp trajectories.

In order to assess the impact of the problem size on the computational performance of the models, several case studies of different sizes were solved. The price profile of one day (see Appendix B) has been replicated over different time spans from 4 to 256 days (each case is solved in one step for the complete time span). The unit data are presented in Table I, where the information for the single-ramp models (R1 and [13]) is the SU ramp type 02, the three-ramp models (R3 and [11]) are the first three SU types, and the five-ramp model (R5) are the five SU types.

1) *Assumptions for the Formulations:* In order to compare all the formulations, [13] and [11] were implemented using the same objective function and the same set of constraints as the formulation presented in Section II. Therefore, all the models are characterizing the same problem; the difference between them is how the constraints are formulated. In other words, two models considering the same SU types (R3 and [11], or R1 and [13]) obtain the same optimal results, e.g., commitments, generating outputs and profits.

The distinction between power and energy was made when implementing [13] and [11]. Additionally, as modeled here, the (usual) power variable is considered to be the power at the end of the period; and the energy is obtained by applying a piecewise-linear power profile. [11] was implemented with the same minimum up/down constraints presented in Section II-A3 as those are the constraints they also use. The synchronization time was set to zero in [11] as we believe this time does not need to be explicitly modeled, thus making the formulation simpler. That is to say, the synchronization time can be considered as a part of the offline time and obtained after the problem has been solved, without changing the optimal results (similar to the turn-on state presented in Section II-B). Finally, the other constraints presented in [11], which are not related to the SU and SD ramps, were not implemented (e.g., quadratic production costs and different power reserves).

Table III presents the optimal solutions for all the models for different time spans. As expected, the optimal solution for models R1 and [13] are the same, as well as the solution for models R3 and [11]. Interestingly, model R5, which considers five ramp types, also presents the same solution as R3 and [11]. This is because, in R5, ramp types 04 and 05 were never activated for this example case because the unit was not down for long enough. As in the case of the difference between R1 and R3 (see Table III), if the five different ramps had been activated in R5, this would have decreased the operational costs in comparison with R3 because more flexibility is possible (more SU ramp types).

1) *Problem Size:* Table III shows the dimension of all the models for the different case studies. Models R1 and [13] have the same number of variables, but [13] presents three times as many binary variables as R1. This is because [13] defines the



TABLE III  
COMPARISON OF DIFFERENT FORMULATIONS

	Problem Size																		
Case	# of Constraints					# of Real Variables					# of Binary Variables				Nonzero elements				
(# of days)	R1	[13]	R3	[11]	R5	R1	[13]	R3	[11]	R5	R1	[13]	R3&R5	[11]	R1	[13]	R3	[11]	R5
4	668	1444	944	2395	1113	384	192	657	382	824	96	288	96	1540	3131	7515	4576	15050	6352
16	2684	5764	3824	9595	4569	1536	768	2673	1534	3416	384	1152	384	6148	12635	30267	18688	75808	26512
64	10748	23044	15344	38395	18393	6144	3072	10737	6142	13784	1536	4608	1536	24580	50651	121275	75136	318880	107152
256	43004	92164	61424	153595	73689	24576	12288	42993	24574	55256	6144	18432	6144	98308	202715	485307	300928	1291168	429712

	Computational Performance																
Case	Optimal Solution (\$)		Integrality Gap (%)					# of Nodes					CPU Time (s)				
(# of days)	R1&[13]	R3,R5&[11]	R1	[13]	R3	[11]	R5	R1	[13]	R3	[11]	R5	R1	[13]	R3	[11]	R5
4	120250.5	118899.5	15.61	29.06	16.18	27.56	15.97	0	64	14	453	21	0.047	0.234	0.125	1.264	0.109
16	486382.5	475459.4	14.76	28.71	16.53	29.78	16.48	0	550	477	473	604	0.156	3.494	1.545	6.474	2.013
64	1950910.5	1901699.2	14.56	28.62	16.62	30.33	16.60	0	496	508	498	488	0.998	16.255	7.956	37.908	8.565
256	7809022.5	7606658.5	14.50	28.60	16.64	30.47	16.63	0	483	467	490	494	10.405	84.599	48.422	268.665	53.181

SU and SD variables as binary; however, they can be considered as continuous variables (see Section II-A4). The formulation in [13] also requires more than twice the quantity of constraints and nonzero elements than the proposed formulation R1.

As shown in Table III, [11] presents about 16 times more binary variables than the proposed formulation R3. Models R3 and R5 have more real variables than [11]. However, the total number of variables in R3 and R5 is smaller than the number of binary variables in [11]. R3 and R5 also present less than half the number of constraints than [11]. Furthermore, [11] presents up to 4.3 and 3 times more nonzero elements than R3 and R5 respectively. Similarly to [11], [13] needs these extra variables and nonzero elements to deal with the different SU ramp types and to avoid conflicts between the up and down states.

Note that model R5 is slightly larger than R3, with respect to the number of variables and constraints, because R5 considers two more ramp types than R3. This also shows that the compact formulation does not increase considerably when considering more SU types.

2) *Computational Performance:* Apart from the compactness of the proposed MILP formulation, the tightness has a significant impact on the computational performance, as mentioned in the Introduction. In fact, a compact formulation usually presents a weak LP relaxation that can dramatically increase the MILP resolution time. The tightness of an MILP can be measured with the integrality gap [24]. The integrality gap, for a maximization problem, is defined as  $(Z_{LP} - Z_{MILP})/Z_{MILP}$ , where  $Z_{LP}$  is the optimal value of the relaxed LP problem, and  $Z_{MILP}$  is the best integer solution found after the MILP problem is solved.

Table III shows the integrality gaps for the different formulations. Compared to [13], the proposed single-ramp formulation R1 has improved (reduced) the integrality gap between 46% and 49%. Similarly, with respect to [11], R3 improves the integrality gap between 41% and 45%. Table III also shows the nodes explored during the branch-and-cut phase; these are usually decreased with tighter formulations. Note that, for all the different cases, CPLEX was able to solve R1 with the required optimality tolerance without needing to branch, because the nodes were pruned earlier by the initial heuristics and cuts applied. Apart from the number of nodes, the performance of an MILP formu-

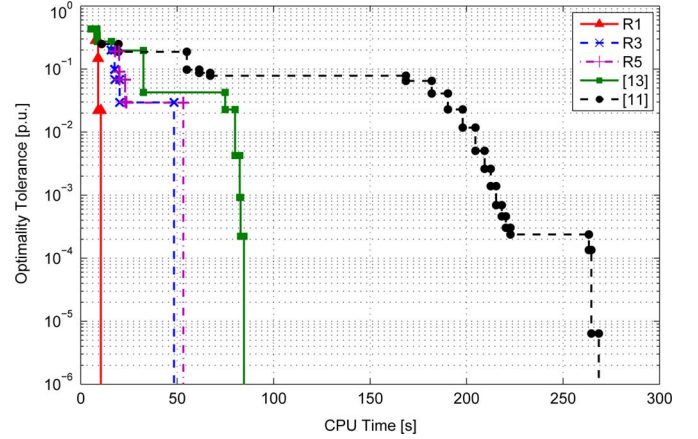


Fig. 4. Convergence evolution until low optimality tolerances for the different formulations.

lation is dramatically affected by the use of heuristics and cuts, and all of these are influenced by the tightness of the formulation [19]. Therefore, we will only comment about CPU times which offer a more complete view of the model's performance.

The CPU times for the different case studies are presented in Table III, where R1 and R3 are up to 22.4 and 10.1 times faster than [13] and [11], respectively. This significant speed up is due to the simultaneous tightness and compactness of the proposed formulation. It is interesting to note that the formulation in [13], which models a single ramp trajectory and does not take into account exponential SU costs, is a larger model (presents more constraints and nonzero elements) and also requires more time to solve the problem than R5, which considers five different SU ramps and also exponential SU costs.

Finally, Fig. 4 shows the convergence evolution for the different formulations to small optimality tolerances for the case study of 256 days. The proposed formulation converges significantly faster than [13] and [11]. This is mainly due to its tightness.

#### IV. CONCLUSIONS

This paper presented an MILP formulation of the SU and SD power trajectories of thermal units. This formulation is si-

TABLE IV  
PRICE DATA AND OPTIMAL GENERATION SCHEDULE

$t$	$EP_t$	$p_t^I$	$e_t^I$	$p_t^T$	$e_t^T$	$t$	$EP_t$	$p_t^I$	$e_t^I$	$p_t^T$	$e_t^T$	$t$	$EP_t$	$p_t^I$	$e_t^I$	$p_t^T$	$e_t^T$	$t$	$EP_t$	$p_t^I$	$e_t^I$	$p_t^T$	$e_t^T$
	\$/MWh	MW	MWh	MW	MWh		\$/MWh	MW	MWh	MW	MWh		\$/MWh	MW	MWh	MW	MWh		\$/MWh	MW	MWh	MW	MWh
0	—	200	—	200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1	40	150	175	150	175	13	50	75	112.5	218	258	25	40	75	112.5	150	184	37	50	150	190	218	258
2	31	75	112.5	(75)	(112.5)	14	45	0	37.5	150	184	26	31	0	37.5	(75)	(112.5)	38	45	75	112.5	150	184
3	27	0	37.5	(0)	(37.5)	15	42	0	0	150	150	27	27	0	0	(0)	(37.5)	39	42	0	37.5	150	150
4	25	0	0	0	0	16	41	50	0	150	150	28	25	0	0	0	0	40	41	0	0	150	150
5	26	0	0	(50)	0	17	43	150	100	150	150	29	26	0	0	(50)	0	41	43	50	0	150	150
6	29	50	0	150	(100)	18	47	218	184	218	184	30	29	50	0	150	(100)	42	47	150	100	218	184
7	35	100	75	150	150	19	51	298	258	298	258	31	35	83.33	66.67	150	150	43	51	230	190	298	258
8	43	150	125	218	184	20	65	378	338	378	338	32	43	116.67	100	218	184	44	65	310	270	378	338
9	54	230	190	298	258	21	96	378	378	378	378	33	54	150	133.33	298	258	45	96	378	344	378	378
10	68	310	270	378	338	22	102	310	344	378	378	34	68	230	190	378	338	46	102	378	378	378	378
11	71	230	270	378	378	23	66	230	270	298	338	35	71	310	270	378	378	47	66	378	378	378	378
12	58	150	190	298	338	24	51	150	190	218	258	36	58	230	270	298	338	48	51	298	338	298	338

multaneously tighter and more compact than equivalent formulations found in the literature. Consequently, the computation time is dramatically reduced. The proposed MILP formulation was analyzed in the context of a price taker self-unit commitment problem. However, its application to any unit commitment problem is straightforward, either under centralized or competitive environments. Several case studies were analyzed to show the improvements of this formulation with respect to others available in the literature. Although SU and SD ramps are usually not considered, mainly because of the computation complexity, simulation results showed that ignoring them changes the commitment decisions causing a negative economic impact.

## APPENDIX

### A. Initial Conditions

The following parameters are needed to deal with the unit state during the first periods:

- $u_0$  Initial commitment state  $\{0,1\}$ .
- $TU_0$  Number of hours that the unit has been up before the scheduling horizon.
- $TD_0$  Number of hours that the unit has been down before the scheduling horizon.

1) *Initial Minimum Up/Down Times*: The following condition must be satisfied if  $(TU_R + TD_R) \geq 1$ :

$$u_t = u_0 \quad \forall t \in [1, TU_R + TD_R] \quad (14)$$

where  $TU_R$  and  $TD_R$  are the number of initial hours during which the unit must remain up or down at the beginning of the scheduling horizon.  $TU_R$  and  $TD_R$  are defined as follows:

$$TU_R = \max \{0, (TU - TU_0) u_0\} \quad (14a)$$

$$TD_R = \max \{0, (TD - TD_0) (1 - u_0)\}. \quad (14b)$$

2) *Initial Start-Up Type*: Equation (15a) complements (1) taking into account the initial conditions if  $TD_0 \geq 2$ :

$$\delta_{t,l} = 0 \quad \forall l \in [1, N_L], t \in (T_{l+1}^{SU} - TD_0, T_{l+1}^{SU}). \quad (15a)$$

Finally, the following equation guarantees that the SU  $\delta_{t,l}$  is not activated before the SU ramp duration  $SU_l^D$ :

$$\delta_{t,l} = 0 \quad \forall l, t \in [1, SU_l^D]. \quad (15b)$$

In other words, this condition ensures that if the unit is turned on in the first hour, the power output above  $\underline{P}$  is produced after the ramp duration  $SU_l^D$ .

### B. Generator-Schedules and Price-Data

The expected electricity prices and optimal power schedules mentioned in Section III are shown in Table IV, where super-indexes  $T$  and  $I$  refer to the *Traditional* and *Improved* formulations. The numbers in parentheses are the power and energy that were included after the UC problem was solved.

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**Germán Morales-España** (S'10) received the B.Sc. degree in electrical engineering from the Universidad Industrial de Santander, Bucaramanga, Colombia, in 2007 and the M.Sc. degree from the Delft University of Technology, Delft, The Netherlands, in 2010. He is now pursuing the (Ph.D.) Erasmus Mundus Joint Doctorate in sustainable energy technologies and strategies (SETS) hosted by the Universidad Pontificia Comillas, Madrid, Spain; the Royal Institute of Technology, Stockholm, Sweden; and Delft University of Technology, Delft, The Netherlands.

He is currently an assistant researcher at the Institute for Research in Technology (IIT) at the Universidad Pontificia Comillas, and he is also a member of the Research Group on Electric Power Systems (GISEL) at the Universidad Industrial de Santander. His areas of interest are power systems operation, economics and reliability, as well as power quality and protective relaying.



**Jesus M. Latorre** (S'00–M'07) was born in Madrid, Spain, in 1977. He received the degree of Electronic Engineer in 2001 and the Ph.D. degree in November 2007, from Comillas Pontifical University, Madrid, Spain.

He is currently a postdoctoral researcher at the Institute for Research in Technology, of the Comillas Pontifical University. His main interest areas include operations research and mathematical modeling, stochastic programming, parallel and distributed computing, algorithms, and numerical methods.



**Andres Ramos** received the degree of Electrical Engineering from Universidad Pontificia Comillas, Madrid, Spain, in 1982 and the Ph.D. degree in Electrical Engineering from Universidad Politécnica de Madrid, Madrid, Spain, in 1990.

He is a Research Fellow at Instituto de Investigación Tecnológica, Madrid, Spain, and a Full Professor at Comillas' School of Engineering, Madrid, Spain, where he has been the Head of the Department of Industrial Organization. His areas of interest include the operation, planning, and economy of power systems and the application of operations research to industrial organization.