Tarea 4

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1. Sea $f(x) = \frac{1}{2}(a^x + a^{-x})$ **si** $a > 0$**. Prueba que** $f(x+y) + f(x-y) = 2f(x)f(y)$**.**

2. Encuentra la inversa de $f(x) = 24(2^{-x/25})$.

Como la función es inyectiva podemos determinar su inversa

$$y = 24(2^{-x/25}) \quad f(x)$$

$$\begin{aligned}
\frac{y}{24} &= 2^{-x/25} \\
\log\frac{y}{24} &= \log 2^{-x/25} \\
\log\frac{y}{24} &= \log 2^{-x/25} \\
\log y - \log 24 &= \frac{-1}{25} x \log 2 \\
\frac{\log y - \log 24}{\log 2} &= \frac{-1}{25} x \\
(-25)(\frac{\log y - \log 24}{\log 2}) &= x \quad \blacksquare
\end{aligned}$$

3. Resolver las ecuaciones

$\ln(x+6)+\ln(x-3)=\ln\ 5+\ln\ 2$

$$
\begin{aligned}
\ln(x+6)+\ln(x-3) &= \ln\ 5+\ln\ 2 \\
\ln((x+6)(x-3)) &= \ln\ (5\cdot 2) \\
\ln((x+6)(x-3)) &= \ln\ (5\cdot 2)=\ln(10) \\
\text{Se cancela el logaritmo por el exponente en ambos términos} \\
\cancel{\ln}((x+6)(x-3)) &= \cancel{\ln}(10) \\
(x+6)(x-3) &= 10 \\
x^2+3x-18 &= 10 \\
x^2+3x &= 10+18=28 \\
\text{Completamos el binomio} \\
x^2+3x+\frac{9}{4} &= 28+\frac{9}{4} \\
\text{Factorizamos} \\
(x+\frac{3}{2})^2 &= \frac{121}{4} \\
\sqrt{(x+\frac{3}{2})^2} &= \sqrt{\frac{121}{4}} \\
x+\frac{3}{2} &= \frac{11}{2} \\
x &= \frac{11}{2}-\frac{3}{2}=\frac{8}{2}=4 \\
x &= 4 \quad \blacksquare
\end{aligned}
$$

$7e^x-e^{2x}=12$

$$< \textit{Empty Math Block} >$$

4. Calcule la derivada de las siguientes funciones.

$$f(x) = x(\sin(\ln x) - \cos(\ln x))$$

Usando la regla de cadena, $\frac{d}{dx}(f(x)) = \frac{df(u)}{du} \cdot \frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(f(u)) = f'(u)$:

$$\left(\frac{d}{dx}(x)\right)f'(x) = \frac{d}{dx}(x(-\cos(\log(x)) + \sin(\log(x))))$$

La derivada de x es 1

$$f'(x) = \frac{d}{dx}(x(-\cos(\log(x)) + \sin(\log(x))))$$

Usando la regla del producto, $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$, donde $u = x$ y $v = \sin(\log(x)) - \cos(\log(x))$:

$$f'(x) = x\left(\frac{d}{dx}\left(-\cos(\log(x)) + \sin(\log(x))\right)\right) + \left(\frac{d}{dx}(x)\right)\left(-\cos(\log(x)) + \sin(\log(x))\right)$$

Derivamos cada término de la suma

$$f'(x) = x\left(\frac{d}{dx}\left(-\cos(\log(x)) + \sin(\log(x))\right)\right) - \left(\frac{d}{dx}(\cos(\log(x)))\right) + \left(\frac{d}{dx}\sin(\log(x))\right)x$$

Usando la regla de la cade, $\frac{d}{dx}(\cos(\log(x))) = \frac{d\cos(u)}{du} \cdot \frac{du}{dx}$, donde $u = log(x)$ y $\frac{d}{du}(\cos(u)) = -\sin(u)$:

$$\begin{aligned} f'(x) &= (\frac{d}{dx}(x))(-\cos(\log(x)) + \sin(\log(x))) + x(\frac{d}{dx}(\sin(\log(x)) - (-\frac{d}{dx}(\log(x)))\sin(\log(x)))) \\ = f'(x) &= (\frac{d}{dx}(x))(-\cos(\log(x)) + \sin(\log(x))) + x(\frac{d}{dx}(\sin(\log(x)) + \frac{d}{dx}(\log(x)))\sin(\log(x))) \end{aligned}$$

Usando la regla de la cadena, $\frac{d}{dx}(\log(x)) = \frac{d\log(u)}{du} \cdot \frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(\log(u)) = 1/u$:

$$f'(x) = (\frac{d}{dx}(x))(-\cos(\log(x)) + \sin(\log(x))) + x(\frac{d}{dx}(\sin(\log(x))) + \frac{\frac{d}{dx}(x)}{x}\sin(\log(x)))$$

La derivada de x es 1

$$\begin{aligned} f'(x) &= x\left(\frac{d}{dx}(\sin(\log(x))) + \frac{(\frac{d}{dx}(x))\sin(\log(x))}{x}\right) + (-\cos(\log(x)) + \sin(\log(x))) \\ &= -\cos(\log(x)) + \sin(\log(x)) + x\left(\frac{d}{dx}(\sin(\log(x))) + \frac{\frac{d}{dx}(x)\sin(\log(x))}{x}\right) \end{aligned}$$

Usando la regla de la cadena, $\frac{d}{dx}(\sin(\log(x))) = \frac{d\sin(u)}{du}\frac{du}{dx}$, donde $u = \log(x)$ y $\frac{d}{du}(\sin(u)) = \cos(u)$:

$$f'(x) = -\cos(\log(x)) + \sin(\log(x)) + x\left(\frac{(\frac{d}{dx}(x))\sin(\log(x))}{x} + \cos(\log(x))(\frac{d}{dx}(\log(x)))\right)$$

Usando la regla de la cadena, $\frac{d}{dx}(\log(x)) = \frac{d\log(u)}{dx}\frac{du}{dx}$. donde $u = x$ y $\frac{d}{du}(\log(u)) = \frac{1}{u}$:

$$f'(x) = -\cos(\log(x)) + \sin(\log(x)) + x\left(\frac{(\frac{d}{dx}(x))\sin(\log(x))}{x} + \frac{\frac{d}{dx}(x)}{x}\cos(\log(x))\right)$$

La derviada de x es 1

$$
\begin{aligned}
f'(x) &= -\cos(\log(x)) + \sin(\log(x)) + x\Big(\frac{(\frac{d}{dx}(x))\sin(\log(x))}{x} + \frac{\cos(\log(x))}{x}\Big) \\
&= -\cos(\log(x)) + \sin(\log(x)) + x\Big(\frac{\sin(\log(x))}{x} + \frac{\cos(\log(x))}{x}\Big) \\
&= 2\sin(\log(x))
\end{aligned}
$$

Por lo tanto a deriva es

$$f'(x) = 2\sin(\log(x)) \quad \blacksquare$$

$f(x) = \log_x e$

Usando la regla de la cadena, $\frac{d}{dx}(f(x)) = \frac{df(u)}{du}\frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(f(u)) = f'(u)$:

$$\frac{d}{dx}(x)f'(x) = \frac{d}{dx}(\frac{1}{\log(x)})$$

La derivada de x es 1:

$$\frac{d}{dx}(x)f'(x) = \frac{d}{dx}(\frac{1}{\log(x)})$$

Usando la regla de la cadena, $\frac{d}{dx}(\frac{1}{\log(x)}) = \frac{d}{du}\frac{1}{u}\frac{du}{dx}$, donde $u = \log(x)$ y $\frac{d}{du}(\frac{1}{u}) = -\frac{1}{u^2}$:

$$f'(x) = \frac{\frac{d}{dx}(\log(x))}{\log(x)^2}$$

Usando la regla de la cadena, $\frac{d}{dx}(\log(x)) = \frac{d\log(u)}{du}\frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(\log(u)) = \frac{1}{u}$:

$$f'(x) = -\frac{1}{\log^2(x)}\frac{\frac{d}{dx}(x)}{x}$$

La derivada de x es 1

$$f'(x) = -\frac{1}{x\log^2(x)}$$

Por lo tanto la derivada es:

$$f'(x) = -\frac{1}{x\log^2(x)} \quad \blacksquare$$

$f(x) = e^{\tan x}$

Usando la regla de la cadena, $\frac{d}{dx}(e^{\tan(x)}) = \frac{de^u}{du}\frac{du}{dx}$, donde $u = \tan(x)$ y $\frac{d}{du}(e^u) = e^u$

$$e^{\tan(x)}\Big(\frac{d}{dx}(\tan(x))\Big)$$

Usando la regla de la cadena, $\frac{d}{dx}(\tan(x)) = \frac{d\tan(u)}{du}\frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(\tan(u)) = \sec^2(u)$:

$$
\begin{aligned}
&= \frac{d}{dx}(x)\sec(x)^2 e^{\tan(x)} \\
&= 1(e^{\tan(x)}\sec^2(x))
\end{aligned}
$$

Por lo tanto la derivada es

$$f'(x) = e^{\tan(x)} \sec^2(x) \quad \blacksquare$$

$f(x) = e^{e^x}$

Usando la regla de la cadena, $\frac{d}{dx}(f(x)) = \frac{df(u)}{du}\frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(f(u)) = f'(u)$:

$$\frac{d}{dx}(x)f'(x) = \frac{d}{dx}(e^{e^x})$$

La derivada x es 1

Usando la regla de la cadena, $\frac{d}{dx}(e^e) = \frac{de^u}{du}\frac{du}{dx}$, donde $u = e^x$ y $\frac{d}{du}(e^u) = e^u$:

$$f'(x) = e^{e^x}\frac{d}{dx}(e^x)$$

Uando la regla de la cadena, $\frac{d}{dx}(e^x) = \frac{de^u}{du}\frac{du}{dx}$, donde $u = x$ y $\frac{d}{du}(e^u) = e^u$:

$$f'(x) = e^x\frac{d}{dx}(x)e^{e^x} = e^{e^x+x}\left(\frac{d}{dx}(x)\right)$$

La derivada de x es 1

$$f'(x) = e^{e^x+x}$$

Por lo tanto la derivada es

$$f'(x) = e^{e^x+x} \quad \blacksquare$$

5. Aplica la derivada logarítmica a:

$f(x) = (1+x)(1-x)$

Calculamos $g(x)$

$$\begin{aligned} g(x) &= \ln|f(x)| \\ &= \ln|(1+x)(1-x)| \\ &= \ln|(1+x)| + \ln|(1-x)| \end{aligned}$$

La derivada de $g(x)$ es

$$g'(x) = \frac{2x}{x^2-1}$$

Calculamos $f'(x) = f(x)g'(x)$

$$\begin{aligned} f'(x) &= f(x)g'(x) \\ &= \big((1+x)(1-x)\big)\big(\frac{2x}{x^2-1}\big) \\ &= \frac{(2x)(-1)\ \cancel{(1+x)(1-x)}}{\cancel{x^2-1}} \\ &= -2x \end{aligned}$$

Por lo tanto la derivada es

$$f'(x) = 2x \quad \blacksquare$$

$f(x) = \frac{x^2\sqrt{3-x}}{(1-x)\sqrt[3]{(3+x)^2}}$

$$\begin{aligned} g(x) &= \ln|f(x)| \\ &= \ln\left|\frac{x^2\sqrt{3-x}}{(1-x)\sqrt[3]{(3+x)^2}}\right| \\ &= \ln|x^2\sqrt{3-x}| - \ln|(1-x)\sqrt[3]{(3+x)^2}| \\ &= \ln|x^2| + \ln|\sqrt{3-x}| - \ln|(1-x)| - \ln|\sqrt[3]{(3+x)^2}| \\ &= 2\ln|x| + 1/2\ln|3-x| - \ln|(1-x)| - 2/3\ln|(3+x)| \end{aligned}$$

La derivada de $g(x)$ es

$$g'(x) = \frac{1}{1-x} - \frac{1}{2(3-x)} - \frac{2}{3(3+x)} + \frac{2}{x}$$

Calculamos $f'(x) = f(x)g'(x)$

$$\begin{aligned} f'(x) &= f(x)g'(x) \\ &= \Big(\frac{x^2\sqrt{3-x}}{(1-x)\sqrt[3]{(3+x)^2}}\Big)\Big(\frac{1}{1-x} - \frac{1}{2(3-x)} - \frac{2}{3(3+x)} + \frac{2}{x}\Big) \\ &= \frac{x^2\sqrt{3-x}}{(1-x)^2\sqrt[3]{(3+x)^2}} - \frac{x^2\sqrt{3-x}}{2(1-x)(3-x)\sqrt[3]{(3+x)^2}} - \frac{2x^2\sqrt{3-x}}{3(1-x)(3+x)\sqrt[3]{(3+x)^2}} + \frac{2x^2\sqrt{3-x}}{x(1-x)\sqrt[3]{(3+x)^2}} \\ &= \frac{x^2\sqrt{3-x}}{(1-x)^2\sqrt[3]{(3+x)^2}} - \frac{x^2}{2(1-x)\sqrt{3-x}\sqrt[3]{(3+x)^2}} - \frac{2x^2\sqrt{3-x}}{3(1-x)(3+x)\sqrt[3]{(3+x)^2}} + \frac{2x\sqrt{3-x}}{(1-x)\sqrt[3]{(3+x)^2}} \end{aligned}$$

Por lo tanto la derivada es

$$f'(x) = \frac{x^2\sqrt{3-x}}{(1-x)^2\sqrt[3]{(3+x)^2}} - \frac{x^2}{2(1-x)\sqrt{3-x}\sqrt[3]{(3+x)^2}} - \frac{2x^2\sqrt{3-x}}{3(1-x)(3+x)\sqrt[3]{(3+x)^2}} + \frac{2x\sqrt{3-x}}{(1-x)\sqrt[3]{(3+x)^2}} \quad \blacksquare$$