Nama : **Riofebri Prasetia**

NIM : **221911192**

Kelas : **3SI1**

Tugas Analisis Peubah Ganda pertemuan 2

link google syntax R : https://colab.research.google.com/drive/1jDJdH57-hFXrLoNN8A6OsfG4yRap9Ahz?usp=sharing

Berdasarkan data pada Exercise 1.3 Buku Jhonson & Wichern (2002),

Hitunglah:

1. Sample Covariance matrix (S) secara manual dengan
   a. memanfaatkan deviasi d_i

   hasil:

   Sample covariance matrix didapat :

   $$S_{ik} = \frac{d_i' d_k}{n-1} = \frac{d_i' d_k}{5-1} = \frac{d_i' d_k}{4}$$

   $S_{11} = 7{,}5$ $\quad S_{21} = 5$ $\quad S_{31} = -1{,}75$

   $S_{12} = 5$ $\quad S_{22} = 10$ $\quad S_{32} = 1{,}5$

   $S_{13} = -1{,}75$ $\quad S_{23} = 1{,}5$ $\quad S_{33} = 2{,}5$

   $$S_n = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix}$$

   b. menggunakan rumus S=sum(xj-x_bar)(xj-x_bar)'/(n-1)

   Matrix kovarian yang didapat :

   $$S_n = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix}$$

2. Sample Correlation matrix (R) dengan memanfaatkan matrik S

   $$R = \begin{bmatrix} 1 & 0{,}577 & -0{,}904 \\ 0{,}577 & 1 & 0{,}3 \\ -0{,}904 & 0{,}3 & 1 \end{bmatrix}$$

3. buatlah syntax R untuk nomor (1) dan (2) lalu bandingkan hasilnya dengan syntax yg sdh jadi di R: cov(data) dan cor(data)

bikin syntax no 1 dan 2

```
#buat xbar
xbar1<-mean(dataset1$x1)
xbar2<-mean(dataset1$x2)
xbar3<-mean(dataset1$x3)
xbar1
xbar2
xbar3

[1] 6

[1] 8

[1] 2
```

memanfaatkan deviasi untuk mencari matrix covariance

```
d1<-matrix(dataset1$x1)-xbar1*1
d2<-matrix(dataset1$x2)-xbar2*1
d3<-matrix(dataset1$x3)-xbar3*1
matrix(c("d1",d1,"d2",d2,"d3",d3),6,3)

     [,1] [,2] [,3]
[1,] d1   d2   d3
[2,] 3    4    1
[3,] -4   0    2
[4,] 0    -2   -2
[5,] -1   -4   0
[6,] 2    2    -1

n<-5
s11<-t(d1)%*%d1/(n-1)
s12<-t(d1)%*%d2/(n-1)
s13<-t(d1)%*%d3/(n-1)
s21<-t(d2)%*%d1/(n-1)
s22<-t(d2)%*%d2/(n-1)
s23<-t(d2)%*%d3/(n-1)
s31<-t(d3)%*%d1/(n-1)
s32<-t(d3)%*%d2/(n-1)
s33<-t(d3)%*%d3/(n-1)

#covarians matrix
cm1<-matrix(c(s11,s12,s13,s21,s22,s23,s31,s32,s33), 3, 3)
cm1

     [,1]  [,2] [,3]
[1,]  7.50  5.0 -1.75
[2,]  5.00 10.0  1.50
[3,] -1.75  1.5  2.50
```

menggunakan S=sum(xj-x_bar)(xj-x_bar)'/(n-1) untuk mencari matrix covariance

```
ss11<-sum((dataset1$x1-xbar1)*(dataset1$x1-xbar1))/(n-1)
ss12<-sum((dataset1$x1-xbar1)*(dataset1$x2-xbar2))/(n-1)
ss13<-sum((dataset1$x1-xbar1)*(dataset1$x3-xbar3))/(n-1)
ss21<-sum((dataset1$x2-xbar2)*(dataset1$x1-xbar1))/(n-1)
ss22<-sum((dataset1$x2-xbar2)*(dataset1$x2-xbar2))/(n-1)
ss23<-sum((dataset1$x2-xbar2)*(dataset1$x3-xbar3))/(n-1)
```

```
ss31<-sum((dataset1$x3-xbar3)*(dataset1$x1-xbar1))/(n-1)
ss32<-sum((dataset1$x3-xbar3)*(dataset1$x2-xbar2))/(n-1)
ss33<-sum((dataset1$x3-xbar3)*(dataset1$x3-xbar3))/(n-1)
cm2<-matrix(c(ss11,ss12,ss13,ss21,ss22,ss23,ss31,ss32,ss33),3,3)
cm2
```

```
     [,1]  [,2] [,3]
[1,]  7.50  5.0 -1.75
[2,]  5.00 10.0  1.50
[3,] -1.75  1.5  2.50
```

matrix S yang menggunakan cara memanfaatkan deviasi memiliki hasil yang sama dengan matrix S yang menggunakan cara penjumlahan biasa untuk mencari covariance

Sample Correlation matrix R dengan memanfaatkan matrix S

```
#mencari D^(-0.5)
matrixDiagonal<-matrix(c(s11^(-0.5),0,0,0,s22^(-0.5),0,0,0,s33^(-0.5)),3,3)
matrixDiagonal
```

```
     [,1]      [,2]      [,3]
[1,] 0.3651484 0.0000000 0.0000000
[2,] 0.0000000 0.3162278 0.0000000
[3,] 0.0000000 0.0000000 0.6324555
```

```
#matrix Correlation
mc <- matrixDiagonal%*%cm1%*%matrixDiagonal
mc
```

```
     [,1]       [,2]      [,3]
[1,]  1.0000000 0.5773503 -0.4041452
[2,]  0.5773503 1.0000000  0.3000000
[3,] -0.4041452 0.3000000  1.0000000
```

Matriks correlation and covariance dengan syntax R yang sudah jadi: cov(), cor()

```
cov(dataset1)
```

```
   x1    x2   x3
x1  7.50  5.0 -1.75
x2  5.00 10.0  1.50
x3 -1.75  1.5  2.50
```

```
cor(dataset1)
```

```
   x1         x2         x3
x1  1.0000000 0.5773503 -0.4041452
x2  0.5773503 1.0000000  0.3000000
x3 -0.4041452 0.3000000  1.0000000
```

perbandingan:

untuk hasil covariance dan correlation untuk syntax yang di buat dengan syntax yang sudah jadi di R memiliki hasil yang sama

4. eigen value & eigen vector dari kedua matrix S dan R secara manual, lalu bandingkan hasilnya dengan output R yang menggunakan syntax eigen()

Matrix covariance :

- Secara manual

Nilai eigen (eigen value)

dengan metode sarrus :

$$\left[\begin{array}{ccc|cc} 7{,}5-\lambda & 5 & -1{,}75 & 7{,}5-\lambda & 5 \\ 5 & 10-\lambda & 1{,}5 & 5 & 10-\lambda \\ -1{,}75 & 1{,}5 & 2{,}5-\lambda & -1{,}75 & 1{,}5 \end{array}\right]$$

$$\begin{aligned} 0 &= (7{,}5-\lambda)(10-\lambda)(2{,}5-\lambda) + (5)(1{,}5)(-1{,}75) + (-1{,}75)(5)(1{,}5) \\ &\quad - (-1{,}75)(10-\lambda)(-1{,}75) - (1{,}5)(1{,}5)(7{,}5-\lambda) - (2{,}5-\lambda)(5)(5) \\ &= (7{,}5-\lambda)\left[(10-\lambda)(2{,}5-\lambda) - (1{,}5)(1{,}5)\right] + 5\left[(1{,}5)(-1{,}75) - (2{,}5-\lambda)(5)\right] \\ &\quad + (-1{,}75)\left[(5)(1{,}5) - (10-\lambda)(-1{,}75)\right] \\ &= -\lambda^3 + 20\lambda^2 - 88{,}438\lambda + 51{,}25 = 0 \end{aligned}$$

$\lambda_3 = 0{,}681$; $\lambda_2 = 5{,}414$; $\lambda_1 = 13{,}305$

Eigen vector

basis eigen vektor

$$V = \begin{bmatrix} 63{,}5053 & -1{,}006 & 0{,}618 \\ 81{,}633 & 0{,}77 & -0{,}452 \\ 1 & 1 & 1 \end{bmatrix}$$

$$L_1 = \sqrt{63{,}5053^2 + 81{,}633^2 + 1^2} = 103{,}4826 \Rightarrow \frac{1}{103{,}4826}\begin{bmatrix} 63{,}5053 \\ 81{,}633 \\ 1 \end{bmatrix} = \begin{bmatrix} 0{,}6137 \\ 0{,}7835 \\ 0{,}0097 \end{bmatrix}$$

$$L_2 = \sqrt{-1{,}006^2 + 0{,}77^2 + 1^2} = 1{,}6140 \Rightarrow \frac{1}{1{,}6140}\begin{bmatrix} -1{,}006 \\ 0{,}77 \\ 1 \end{bmatrix} = \begin{bmatrix} -0{,}6233 \\ 0{,}4771 \\ 0{,}6196 \end{bmatrix}$$

$$L_3 = \sqrt{0{,}618^2 + (-0{,}452)^2 + 1^2} = 1{,}2744 \Rightarrow \frac{1}{1{,}2744}\begin{bmatrix} 0{,}618 \\ -0{,}452 \\ 1 \end{bmatrix} = \begin{bmatrix} 0{,}4849 \\ -0{,}3861 \\ 0{,}7847 \end{bmatrix}$$

$$V = \begin{bmatrix} 0{,}6137 & -0{,}6233 & 0{,}4849 \\ 0{,}7835 & 0{,}4771 & -0{,}3861 \\ 0{,}0097 & 0{,}6196 & 0{,}7847 \end{bmatrix}$$

- Menggunakan R

```
$values
[1] 13.9049009  5.4143634  0.6807358

$vectors
             [,1]       [,2]       [,3]
[1,] -0.613679310 -0.6232975  0.4846627
[2,] -0.789496144  0.4769002 -0.3863444
[3,] -0.009671757  0.6197309  0.7847548
```

Matrix Correlation

- Secara Manual

Nilai eigen (eigen value)

dengan metode sarrus :

$$\left|\begin{array}{ccc|cc} 1-\lambda & 0,577 & -0,404 & 1-\lambda & 0,577 \\ 0,577 & 1-\lambda & 0,3 & 0,577 & 1-\lambda \\ -0,404 & 0,3 & 1-\lambda & -0,404 & 0,3 \end{array}\right|$$

$$0 = (1-\lambda)^3 + 0,577 . 0,3 . (-0,404) + (-0,404)\, 0,577 . (0,3)$$
$$- (-0,404).(1-\lambda)(-0,404) - (0,3).0,3 .(1-\lambda) - (1-\lambda). 0,577 . 0,577$$
$$= -\lambda^3 + 3\lambda^2 - 2,4138\lambda + 0,274$$
$$= -(\lambda - 0,135).(\lambda - 1,274)(\lambda - 1,591)$$
$$\lambda_1 = 1,591 \ ; \ \lambda_2 = 1,274 \ ; \ \lambda_3 = 0,135$$

Eigen vector

$$\lambda_1 : \begin{bmatrix} -4,016 \\ -3,413 \\ 1 \end{bmatrix} ; \ \lambda_2 \begin{bmatrix} -0,242 \\ 0,585 \\ 1 \end{bmatrix} ; \ \lambda_3 \begin{bmatrix} 1,258 \\ -1,186 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} -4,016 & -0,242 & 1,258 \\ -3,413 & 0,585 & -1,186 \\ 1 & 1 & 1 \end{bmatrix}$$

$$L_1 = \sqrt{(-4{,}016)^2 + (-3{,}413)^2 + 1^2} \Rightarrow \frac{1}{5{,}3649}\begin{bmatrix} -4{,}016 \\ -3{,}413 \\ 1 \end{bmatrix} = \begin{bmatrix} -0{,}7486 \\ -0{,}6362 \\ 0{,}1864 \end{bmatrix}$$

$$= 5{,}3649$$

$$L_2 = \sqrt{(-0{,}242)^2 + (0{,}585)^2 + 1^2} \Rightarrow \frac{1}{1{,}1835}\begin{bmatrix} -0{,}242 \\ 0{,}585 \\ 1 \end{bmatrix} = \begin{bmatrix} -0{,}2045 \\ 0{,}4943 \\ 0{,}8449 \end{bmatrix}$$

$$= 1{,}1835$$

$$L_3 = \sqrt{1{,}258^2 + (-1{,}186)^2 + (1)^2} \Rightarrow \frac{1}{1{,}9973}\begin{bmatrix} 1{,}258 \\ -1{,}186 \\ 1 \end{bmatrix} = \begin{bmatrix} 0{,}6298 \\ -0{,}5938 \\ 0{,}5007 \end{bmatrix}$$

$$= 1{,}9973$$

$$V = \begin{bmatrix} -0{,}7486 & -0{,}2045 & 0{,}6298 \\ -0{,}6362 & 0{,}4943 & -0{,}5938 \\ 0{,}1864 & 0{,}8449 & 0{,}5007 \end{bmatrix}$$

- Menggunakan R

```
$values
[1] 1.591638 1.273514 0.134848

$vectors
           [,1]       [,2]       [,3]
[1,]  0.7490653 -0.2047782  0.6300533
[2,]  0.6347139  0.4943479 -0.5939347
[3,] -0.1898407  0.8447994  0.5002744
```

Perbandingan hasil manual dengan R:

Nilai eigen dan eigen vector pada cara manual memiliki hampir kesamaan atau mendekati dengan syntax R yang sudah jadi.

Lampiran pengerjaan secara manual

Nama : Profebri Prasetia
NIM : 221511132
Kelas : 3SI1

Tugas kedua
Analisis Peubah Ganda

- Data yang digunakan
(Buku Jhonson & Wichern 2002, exercise 1.3)

x_1	x_2	x_3
9	12	3
2	8	4
6	6	0
5	4	2
8	10	1

$$; \quad y_1 = \begin{bmatrix} 9 \\ 2 \\ 6 \\ 5 \\ 8 \end{bmatrix} ; \quad y_2 = \begin{bmatrix} 12 \\ 8 \\ 6 \\ 4 \\ 10 \end{bmatrix} ; \quad y_3 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

1.> Sampel Covariance matrik (S) secara manual
.) Memanfaatkan deviasi d_i

$n = 5$

mean :

$$\bar{x}_1 = \frac{9+2+6+5+8}{5} = 6$$

$$\bar{x}_2 = \frac{12+8+6+4+10}{5} = 8$$

$$\bar{x}_3 = \frac{3+4+0+2+1}{5} = 2$$

$$d_i = y_i - \bar{x}_i . 1$$

$$d_1 = \begin{bmatrix} 9 \\ 2 \\ 6 \\ 5 \\ 8 \end{bmatrix} - 6 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 0 \\ -1 \\ 2 \end{bmatrix}$$

$$d_2 = \begin{bmatrix} 12 \\ 8 \\ 6 \\ 4 \\ 10 \end{bmatrix} - 8 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -2 \\ -4 \\ 2 \end{bmatrix}$$

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$$d_3 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$d_i' d_k = \Sigma_{j=1}^{n} (x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k) = (n-1) S_{ik}$$

$$d_1' d_1 = \begin{bmatrix} 3 & -4 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 0 \\ -1 \\ 2 \end{bmatrix} = 30$$

$$d_1' d_2 = \begin{bmatrix} 3 & -4 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -2 \\ -4 \\ 2 \end{bmatrix} = 20$$

$$d_1' d_3 = \begin{bmatrix} 3 & -4 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 0 \\ -1 \end{bmatrix} = -7$$

$$d_2' d_1 = \begin{bmatrix} 4 & 0 & -2 & -4 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 0 \\ -1 \\ 2 \end{bmatrix} = 20$$

$$d_2' d_2 = \begin{bmatrix} 4 & 0 & -2 & -4 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -2 \\ -4 \\ 2 \end{bmatrix} = 40$$

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$$d_2'd_3 = \begin{bmatrix} 4 & 0 & -2 & -4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 0 \\ -1 \end{bmatrix} = 6$$

$$d_3'd_1 = \begin{bmatrix} 1 & 2 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 0 \\ -1 \\ 2 \end{bmatrix} = -7$$

$$d_3'd_2 = \begin{bmatrix} 1 & 2 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -2 \\ -4 \\ 2 \end{bmatrix} = 6$$

$$d_3'd_3 = \begin{bmatrix} 1 & 2 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 0 \\ -1 \end{bmatrix} = 10$$

sample covariance matrix didapat :

$$S_{ik} = \frac{d_i'd_k}{n-1} = \frac{d_i'd_k}{5-1} = \frac{d_i'd_k}{4}$$

$S_{11} = 7,5$ $\quad S_{21} = 5$ $\quad S_{31} = -1,75$

$S_{12} = 5$ $\quad S_{22} = 10$ $\quad S_{32} = 1,5$

$S_{13} = -1,75$ $\quad S_{23} = 1,5$ $\quad S_{33} = 2,5$

$$S_n = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix}$$

•). Menggunakan rumus $S = \mathrm{sum}(x_j - \bar{x})(x_j - \bar{x})'/(n-1)$

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$$X = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{bmatrix} \end{array} = \begin{bmatrix} 9 & 12 & 3 \\ 2 & 8 & 4 \\ 6 & 6 & 0 \\ 5 & 4 & 2 \\ 8 & 10 & 1 \end{bmatrix}$$

$$S_{ik} = \frac{\sum_{j=1}^{n} (x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k)}{n-1}$$

$$S_{11} = \left(\sum_{j=1}^{5} (x_{j1} - \bar{x}_1)(x_{j1} - \bar{x}_1)\right) / (5-1)$$

$$= \left[(9-6)^2 + (2-6)^2 + \dots + (8-6)^2 \right] / 4$$

$$= 7,5$$

$$S_{12} = \left[\sum_{j=1}^{5} (x_{j1} - \bar{x}_1)(x_{j2} - \bar{x}_2) \right] / (5-1)$$

$$= \frac{\left[(9-6)(12-8) + (2-6)(8-8) + \dots + (8-6)(10-8) \right]}{4}$$

$$= 5$$

$$S_{13} = \left[\sum_{j=1}^{5} (x_{j1} - \bar{x}_1)(x_{j3} - \bar{x}_3) \right] / (5-1)$$

$$= \left[(9-6)(3-2) + (2-6)(4-2) + \dots + (8-6)(1-2) \right] / 4$$

$$= -1,75$$

$$S_{22} = \left[\sum_{j=1}^{5} (x_{j2} - \bar{x}_2)(x_{j2} - \bar{x}_2) \right] / 4$$

$$= \left[(12-8)^2 + (8-8)^2 + \dots + (10-8)^2 \right] / 4$$

$$= 10$$

$$S_{23} = \left[\sum_{j=1}^{5} (x_{j2} - \bar{x}_2)(x_{j3} - \bar{x}_3) \right] / (5-1)$$

$$= \frac{\left[(12-8)(3-2) + (8-8)(4-2) + \dots + (10-8)(1-2) \right]}{4}$$

$$= 1,5$$

$$S_{33} = \left[\Sigma_{j=1}^{5} (x_{j3} - \bar{x}_3)(x_{j3} - \bar{x}_3) \right] / (5-1)$$

$$= \left[(3-2)^2 + (4-2)^2 + \dots + (1-2)^2 \right] / 4$$

$$= 2{,}5$$

$$S_{21} = S_{12} = 5$$

$$S_{31} = S_{13} = -1{,}75$$

$$S_{32} = S_{23} = 1{,}5$$

Matrix kovarian yang didapat :

$$S_n = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix}$$

2>. Sample Correlation matrix (R) dengan memanfaatkan matrik S

$$D^{-1/2} = \operatorname{diag}\left(\frac{1}{\sqrt{S_{11}}}, \frac{1}{\sqrt{S_{22}}}, \frac{1}{\sqrt{S_{33}}} \right)$$

$$= \begin{bmatrix} S_{11}^{-1/2} & 0 & 0 \\ 0 & S_{22}^{-1/2} & 0 \\ 0 & 0 & S_{33}^{-1/2} \end{bmatrix} = \begin{bmatrix} 0{,}365 & 0 & 0 \\ 0 & 0{,}316 & 0 \\ 0 & 0 & 0{,}632 \end{bmatrix}$$

$$R = D^{-1/2} \, S \, D^{-1/2}$$

$$= \begin{bmatrix} 0{,}365 & 0 & 0 \\ 0 & 0{,}316 & 0 \\ 0 & 0 & 0{,}632 \end{bmatrix} \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix} \begin{bmatrix} 0{,}365 & 0 & 0 \\ 0 & 0{,}316 & 0 \\ 0 & 0 & 0{,}632 \end{bmatrix}$$

$$= \begin{bmatrix} 2{,}739 & 1{,}826 & -0{,}639 \\ 1{,}581 & 3{,}162 & 0{,}474 \\ -1{,}107 & 0{,}949 & 1{,}581 \end{bmatrix} \begin{bmatrix} 0{,}365 & 0 & 0 \\ 0 & 0{,}316 & 0 \\ 0 & 0 & 0{,}632 \end{bmatrix}$$

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$$R = \begin{bmatrix} 1 & 0,577 & -0,904 \\ 0,577 & 1 & 0,3 \\ -0,404 & 0,3 & 1 \end{bmatrix}$$

3>. Buat syntax R untuk nomor 1> dan 2> lalu bandingkan hasilnya dengan yang sudah jadi di R: cov(data) dan cor(data)

4>. eigen value & eigen vector secara manual, dan perbandingan dengan syntax R dengan eigen():

<matriks s> •). Cari eigen value:

$$\det(S - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = 0$$

$$\det\left(\begin{bmatrix} 7,5-\lambda & 5 & -1,75 \\ 5 & 10-\lambda & 1,5 \\ -1,75 & 1,5 & 2,5-\lambda \end{bmatrix} \right) = 0$$

dengan metode sarrus:

$$\left[\begin{array}{ccc|cc} 7,5-\lambda & 5 & -1,75 & 7,5-\lambda & 5 \\ 5 & 10-\lambda & 1,5 & 5 & 10-\lambda \\ -1,75 & 1,5 & 2,5-\lambda & -1,75 & 1,5 \end{array}\right]$$

$$\begin{aligned} 0 &= (7,5-\lambda)(10-\lambda)(2,5-\lambda) + (5)(1,5)(-1,75) + (-1,75)(5)(1,5) \\ &\quad - (-1,75)(10-\lambda)(-1,75) - (1,5)(1,5)(7,5-\lambda) - (2,5-\lambda)(5)(5) \\ &= (7,5-\lambda)[(10-\lambda)(2,5-\lambda) - (1,5)(1,5)] + 5[(1,5)(-1,75) - (2,5-\lambda)(5)] \\ &\quad + (-1,75)[(5)(1,5) - (10-\lambda)(-1,75)] \\ &= -\lambda^3 + 20\lambda^2 - 88,438\lambda + 51,25 = 0 \end{aligned}$$

$\lambda_3 = 0,681$; $\lambda_2 = 5,419$; $\lambda_1 = 13,905$

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•). Cari eigen vector

untuk $\lambda_3 = 0{,}681$

$(\lambda_3 I - S)\, V = 0$

$$\left(\begin{bmatrix} 0{,}681 & 0 & 0 \\ 0 & 0{,}681 & 0 \\ 0 & 0 & 0{,}681 \end{bmatrix} - \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix} \right) V = 0$$

$$\begin{bmatrix} -6{,}819 & -5 & 1{,}75 \\ -5 & -9{,}319 & -1{,}5 \\ 1{,}75 & -1{,}5 & -1{,}819 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -6{,}819 V_1 - 5V_2 + 1{,}75 V_3 \\ -5V_1 - 9{,}319 V_2 - 1{,}5 V_3 \\ 1{,}75 V_1 - 1{,}5 V_2 - 1{,}819 V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solusi penyederhanaan matrix:

$$\begin{bmatrix} -6{,}819 & -5 & 1{,}75 \\ -5 & -9{,}319 & -1{,}5 \\ 1{,}75 & -1{,}5 & -1{,}819 \end{bmatrix} \xrightarrow{(*(-1))} \begin{bmatrix} 6{,}819 & 5 & -1{,}75 \\ 5 & 9{,}319 & 1{,}5 \\ -1{,}75 & 1{,}5 & 1{,}819 \end{bmatrix}$$

$$\xrightarrow{R_1/6{,}819} \begin{bmatrix} 1 & 0{,}733 & -0{,}257 \\ 5 & 9{,}319 & 1{,}5 \\ -1{,}75 & 1{,}5 & 1{,}819 \end{bmatrix} \xrightarrow[R_3+1{,}75R_1]{R_2-5R_1} \begin{bmatrix} 1 & 0{,}733 & -0{,}257 \\ 0 & 5{,}653 & 2{,}783 \\ 0 & 2{,}783 & 1{,}370 \end{bmatrix}$$

$$\xrightarrow{R_2/5{,}653} \begin{bmatrix} 1 & 0{,}733 & -0{,}257 \\ 0 & 1 & 0{,}492 \\ 0 & 2{,}783 & 1{,}370 \end{bmatrix} \xrightarrow[R_3-2{,}783R_2]{R_1-0{,}733R_2} \begin{bmatrix} 1 & 0 & -0{,}618 \\ 0 & 1 & 0{,}492 \\ 0 & 0 & -0{,}0004 \end{bmatrix}$$

$$\xrightarrow{R_3/-0{,}0004} \begin{bmatrix} 1 & 0 & -0{,}618 \\ 0 & 1 & 0{,}492 \\ 0 & 0 & 1 \end{bmatrix}$$

menjadi:

$$\begin{bmatrix} 1 & 0 & -0{,}618 \\ 0 & 1 & 0{,}492 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} V_1 - 0{,}618 V_3 \\ V_2 + 0{,}492 V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$V_1 = 0{,}618 V_3$

$V_2 = -0{,}492 V_3$

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$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0{,}618\, V_3 \\ -0{,}452\, V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} 0{,}618 \\ -0{,}452 \\ 1 \end{bmatrix}$$

untuk $\lambda_2 = 5{,}414$:

$$(\lambda_2 I - S)\, V = 0$$

$$\left(\begin{bmatrix} 5{,}414 & 0 & 0 \\ 0 & 5{,}414 & 0 \\ 0 & 0 & 5{,}414 \end{bmatrix} - \begin{bmatrix} 7{,}5 & 5 & -1{,}75 \\ 5 & 10 & 1{,}5 \\ -1{,}75 & 1{,}5 & 2{,}5 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2{,}086 & -5 & 1{,}75 \\ -5 & -4{,}586 & -1{,}5 \\ 1{,}75 & -1{,}5 & 2{,}914 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

Menggunakan reduksi matriks :

$R_1/(-2{,}086)$
$R_2/(-5)$
$R_3/1{,}75$

$$\xrightarrow{} \begin{bmatrix} 1 & 2{,}397 & -0{,}839 \\ 1 & 0{,}917 & 0{,}3 \\ 1 & -0{,}857 & 1{,}665 \end{bmatrix} \xrightarrow{R_2 - R_1,\ R_3 - R_1} \begin{bmatrix} 1 & 2{,}397 & -0{,}839 \\ 0 & -1{,}48 & 1{,}139 \\ 0 & -1{,}774 & 1{,}365 \end{bmatrix}$$

$R_2/(-1{,}48)$
$R_3/(-1{,}774)$

$$\xrightarrow{} \begin{bmatrix} 1 & 2{,}397 & -0{,}839 \\ 0 & 1 & -0{,}77 \\ 0 & 1 & -0{,}77 \end{bmatrix} \xrightarrow{R_1 - 2{,}397R_2,\ R_3 - R_2} \begin{bmatrix} 1 & 0 & 1{,}006 \\ 0 & 1 & -0{,}77 \\ 0 & 0 & 0{,}0003 \end{bmatrix}$$

$R_3/0{,}0003$

$$\xrightarrow{} \begin{bmatrix} 1 & 0 & 1{,}006 \\ 0 & 1 & -0{,}77 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 + 1{,}006\, V_3 \\ V_2 - 0{,}77\, V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$V_1 = -1{,}006\, V_3$
$V_2 = 0{,}77\, V_3$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1{,}006\, V_3 \\ 0{,}77\, V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} -1{,}006 \\ 0{,}77 \\ 1 \end{bmatrix}$$

Untuk $\lambda_1 = 13,9049$:

$(\lambda_1 I - S)\, v = 0$

$$\left(\begin{bmatrix} 13,9049 & 0 & 0 \\ 0 & 13,9049 & 0 \\ 0 & 0 & 13,9049 \end{bmatrix} - \begin{bmatrix} 7,5 & 5 & -1,75 \\ 5 & 10 & 1,5 \\ -1,75 & 1,5 & 2,5 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6,4049 & -5 & 1,75 \\ -5 & 3,9049 & -1,5 \\ 1,75 & -1,5 & 11,4049 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Menggunakan reduksi matriks

$$\xrightarrow{\substack{R_1/6,4049 \\ R_2/(-5) \\ R_3/1,75}} \begin{bmatrix} 1 & -0,781 & 0,273 \\ 1 & -0,781 & 0,3 \\ 1 & -0,857 & 6,517 \end{bmatrix} \xrightarrow{\substack{R_2 - R_1 \\ R_3 - R_1}} \begin{bmatrix} 1 & -0,781 & 0,273 \\ 0 & -0,0003 & 0,027 \\ 0 & -0,076 & 6,244 \end{bmatrix}$$

$$\xrightarrow{\substack{R_2/(-0,0003) \\ R_3/(-0,076)}} \begin{bmatrix} 1 & -0,781 & 0,273 \\ 0 & 1 & -81,699 \\ 0 & 1 & -81,629 \end{bmatrix} \xrightarrow{\substack{R_1 + 0,781 R_2 \\ R_3 - R_2}} \begin{bmatrix} 1 & 0 & -63,5053 \\ 0 & 1 & -81,699 \\ 0 & 0 & 0,07 \end{bmatrix}$$

$$\xrightarrow{R_3/0,07} \begin{bmatrix} 1 & 0 & -63,5053 \\ 0 & 1 & -81,699 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 - 63,5053 v_3 \\ v_2 - 81,699 v_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = 63,5053\, v_3 \qquad v_2 = 81,699\, v_3 \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 63,5053\, v_3 \\ 81,699\, v_3 \\ v_3 \end{bmatrix} = v_3 \begin{bmatrix} 63,5053 \\ 81,699 \\ 1 \end{bmatrix}$$

eigen vector :

$$\lambda_1 : \begin{bmatrix} 63,5053 \\ 81,699 \\ 1 \end{bmatrix} ; \quad \lambda_2 : \begin{bmatrix} -1,006 \\ 0,77 \\ 1 \end{bmatrix} ; \quad \lambda_3 : \begin{bmatrix} 0,618 \\ -0,492 \\ 1 \end{bmatrix}$$

baris eigen vektor

$$V=\begin{bmatrix} 63,5053 & -1,006 & 0,618 \\ 81,693 & 0,77 & -0,492 \\ 1 & 1 & 1 \end{bmatrix}$$

<matriks R> •). Cari eigen value

$\det(R - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 1 & 0,577 & -0,404 \\ 0,577 & 1 & 0,3 \\ -0,404 & 0,3 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 1-\lambda & 0,577 & -0,404 \\ 0,577 & 1-\lambda & 0,3 \\ -0,404 & 0,3 & 1-\lambda \end{bmatrix}\right) = 0$$

dengan metode sarrus:

$$\left|\begin{array}{ccc|cc} 1-\lambda & 0,577 & -0,404 & 1-\lambda & 0,577 \\ 0,577 & 1-\lambda & 0,3 & 0,577 & 1-\lambda \\ -0,404 & 0,3 & 1-\lambda & -0,404 & 0,3 \end{array}\right|$$

$$\begin{aligned} 0 &= (1-\lambda)^3 + 0,577 . 0,3 . (-0,404) + (-0,404)\, 0,577 . (0,3) \\ &\quad - (-0,404) . (1-\lambda)(-0,404) - (0,3) . 0,3 . (1-\lambda) - (1,\lambda) . 0,577 . 0,577 \\ &= -\lambda^3 + 3\lambda^2 - 2,4138\lambda + 0,274 \\ &= -(\lambda - 0,135) . (\lambda - 1,274)(\lambda - 1,591) \end{aligned}$$

$\lambda_1 = 1,591$; $\lambda_2 = 1,274$; $\lambda_3 = 0,135$

•). Cari eigen vector

untuk $\lambda_1 = 1,591$:

$(\lambda_1 I - R)\, V = 0$

$$\left(\begin{bmatrix} 1,591 & 0 & 0 \\ 0 & 1,591 & 0 \\ 0 & 0 & 1,591 \end{bmatrix} - \begin{bmatrix} 1 & 0,577 & -0,404 \\ 0,577 & 1 & 0,3 \\ -0,404 & 0,3 & 1 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

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$$\begin{bmatrix} 0,591 & -0,577 & 0,404 \\ -0,577 & 0,591 & -0,3 \\ 0,404 & -0,3 & 0,591 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

Menggunakan reduksi matriks :

$$\xrightarrow[R_3/0,404]{R_1/0,591 \quad R_2/(-0,577)} \begin{bmatrix} 1 & -0,976 & 0,684 \\ 1 & -1,024 & 0,519 \\ 1 & -0,743 & 1,463 \end{bmatrix} \xrightarrow[R_3-R_1]{R_2-R_1} \begin{bmatrix} 1 & -0,976 & 0,684 \\ 0 & -0,048 & -0,164 \\ 0 & 0,234 & 0,779 \end{bmatrix}$$

$$\xrightarrow[R_3/0,234]{R_2/(-0,048)} \begin{bmatrix} 1 & -0,976 & 0,684 \\ 0 & 1 & 3,413 \\ 0 & 1 & 3,334 \end{bmatrix} \xrightarrow[R_3-R_2]{R_1+0,976R_2} \begin{bmatrix} 1 & 0 & 4,016 \\ 0 & 1 & 3,413 \\ 0 & 0 & -0,079 \end{bmatrix}$$

$$\xrightarrow{R_3/(-0,079)} \begin{bmatrix} 1 & 0 & 4,016 \\ 0 & 1 & 3,413 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 + 4016 V_3 \\ V_2 + 3,413 V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 = -4,016 V_3 \qquad V_2 = -3,413 V_3 \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -4,016 V_3 \\ -3,413 V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} -4,016 \\ -3,413 \\ 1 \end{bmatrix}$$

Untuk $\lambda_2 = 1,274$:

$(\lambda_2 I - R) V = 0$

$$\left(\begin{bmatrix} 1,274 & 0 & 0 \\ 0 & 1,274 & 0 \\ 0 & 0 & 1,274 \end{bmatrix} - \begin{bmatrix} 1 & 0,577 & -0,404 \\ 0,577 & 1 & 0,3 \\ -0,404 & 0,3 & 1 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0,274 & -0,577 & 0,404 \\ -0,577 & 0,274 & -0,3 \\ 0,404 & -0,3 & 0,274 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Menggunakan reduksi matriks

$$\xrightarrow[R_3/0,404]{R_1/0,274 \;\; R_2/(-0,577)} \begin{bmatrix} 1 & -2,106 & 1,474 \\ 1 & -0,475 & 0,52 \\ 1 & -0,743 & 0,678 \end{bmatrix} \xrightarrow{R_2-R_1 \;\; R_3-R_1} \begin{bmatrix} 1 & -2,106 & 1,474 \\ 0 & 1,631 & -0,954 \\ 0 & -0,268 & 0,158 \end{bmatrix}$$

$$\xrightarrow{R_2/1,631 \;\; R_3/(-0,268)} \begin{bmatrix} 1 & -2,106 & 1,474 \\ 0 & 1 & -0,585 \\ 0 & 1 & -0,591 \end{bmatrix} \xrightarrow{R_1+2,106R_2 \;\; R_3-R_2} \begin{bmatrix} 1 & 0 & 0,242 \\ 0 & 1 & -0,585 \\ 0 & 0 & -0,006 \end{bmatrix}$$

$$\xrightarrow{R_3/(-0,006)} \begin{bmatrix} 1 & 0 & 0,242 \\ 0 & 1 & -0,585 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 + 0,242V_3 \\ V_2 - 0,585V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 = -0,242V_3 \qquad V_2 = 0,585V_3 \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -0,242V_3 \\ 0,585V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} -0,242 \\ 0,585 \\ 1 \end{bmatrix}$$

untuk $\lambda_3 = 0,135$

$(\lambda_3 I - R) V = 0$

$$\left(\begin{bmatrix} 0,135 & 0 & 0 \\ 0 & 0,135 & 0 \\ 0 & 0 & 0,135 \end{bmatrix} - \begin{bmatrix} 1 & 0,577 & -0,404 \\ 0,577 & 1 & 0,3 \\ -0,404 & 0,3 & 1 \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0,865 & -0,577 & 0,904 \\ -0,577 & -0,865 & -0,3 \\ 0,909 & -0,3 & -0,865 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Menggunakan reduksi matriks

$$\begin{matrix} R_1/(-0,865) \\ R_2/(-0,577) \\ R_3/0,909 \end{matrix} \longrightarrow \begin{bmatrix} 1 & 0,667 & -0,967 \\ 1 & 1,499 & 0,52 \\ 1 & -0,793 & -2,141 \end{bmatrix} \xrightarrow{\begin{matrix} R_2-R_1 \\ R_3-R_1 \end{matrix}} \begin{bmatrix} 1 & 0,667 & -0,967 \\ 0 & 0,832 & 0,987 \\ 0 & -1,91 & -1,679 \end{bmatrix}$$

$$\begin{matrix} R_2/0,832 \\ R_3/(-1,91) \end{matrix} \longrightarrow \begin{bmatrix} 1 & 0,667 & -0,467 \\ 0 & 1 & 1,186 \\ 0 & 1 & 1,188 \end{bmatrix} \xrightarrow{\begin{matrix} R_1-0,667R_2 \\ R_3-R_2 \end{matrix}} \begin{bmatrix} 1 & 0 & -1,258 \\ 0 & 1 & 1,186 \\ 0 & 0 & 0,001 \end{bmatrix}$$

$$R_3/0,001 \quad \begin{bmatrix} 1 & 0 & -1,258 \\ 0 & 1 & 1,186 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 - 1,258V_3 \\ V_2 + 1,186V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} V_1 = 1,258V_3 \\ V_2 = -1,186V_3 \end{matrix} \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1,258V_3 \\ -1,186V_3 \\ V_3 \end{bmatrix} = V_3 \begin{bmatrix} 1,258 \\ -1,186 \\ 1 \end{bmatrix}$$

basis eigen vector yang didapat

$$\lambda_1 : \begin{bmatrix} -4,016 \\ -3,413 \\ 1 \end{bmatrix} ; \lambda_2 \begin{bmatrix} -0,242 \\ 0,585 \\ 1 \end{bmatrix} ; \lambda_3 \begin{bmatrix} 1,258 \\ -1,186 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} -4,016 & -0,242 & 1,258 \\ -3,413 & 0,585 & -1,186 \\ 1 & 1 & 1 \end{bmatrix}$$

Lampiran syntax R

Dataset yang digunakan

```
url<-"https://raw.githubusercontent.com/rii92/tugas-APG/main/28%20februari%202022/tugas%20kedua%20APG.csv"
dataset1 <- read.csv(url, sep = ",")
dataset1
```

```
  x1 x2 x3
1 9  12 3
2 2   8 4
3 6   6 0
4 5   4 2
5 8  10 1
```

bikin syntax no 1 dan 2

```
#buat xbar
xbar1<-mean(dataset1$x1)
xbar2<-mean(dataset1$x2)
xbar3<-mean(dataset1$x3)
xbar1
xbar2
xbar3
```

```
[1] 6

[1] 8

[1] 2
```

memanfaatkan deviasi untuk mencari matrix covariance

```
d1<-matrix(dataset1$x1)-xbar1*1
d2<-matrix(dataset1$x2)-xbar2*1
d3<-matrix(dataset1$x3)-xbar3*1
matrix(c("d1",d1,"d2",d2,"d3",d3),6,3)
```

```
     [,1] [,2] [,3]
[1,] d1   d2   d3
[2,] 3    4    1
[3,] -4   0    2
[4,] 0    -2   -2
[5,] -1   -4   0
[6,] 2    2    -1
```

```
n<-5
s11<-t(d1)%*%d1/(n-1)
s12<-t(d1)%*%d2/(n-1)
s13<-t(d1)%*%d3/(n-1)
s21<-t(d2)%*%d1/(n-1)
s22<-t(d2)%*%d2/(n-1)
s23<-t(d2)%*%d3/(n-1)
s31<-t(d3)%*%d1/(n-1)
s32<-t(d3)%*%d2/(n-1)
s33<-t(d3)%*%d3/(n-1)
```

```
#covarians matrix
cm1<-matrix(c(s11,s12,s13,s21,s22,s23,s31,s32,s33), 3, 3)
cm1
```

```
     [,1]  [,2] [,3]
[1,]  7.50  5.0 -1.75
[2,]  5.00 10.0  1.50
[3,] -1.75  1.5  2.50
```

menggunakan S=sum(xj-x_bar)(xj-x_bar)'/(n-1) untuk mencari matrix covariance

```
ss11<-sum((dataset1$x1-xbar1)*(dataset1$x1-xbar1))/(n-1)
ss12<-sum((dataset1$x1-xbar1)*(dataset1$x2-xbar2))/(n-1)
ss13<-sum((dataset1$x1-xbar1)*(dataset1$x3-xbar3))/(n-1)
ss21<-sum((dataset1$x2-xbar2)*(dataset1$x1-xbar1))/(n-1)
ss22<-sum((dataset1$x2-xbar2)*(dataset1$x2-xbar2))/(n-1)
ss23<-sum((dataset1$x2-xbar2)*(dataset1$x3-xbar3))/(n-1)
ss31<-sum((dataset1$x3-xbar3)*(dataset1$x1-xbar1))/(n-1)
ss32<-sum((dataset1$x3-xbar3)*(dataset1$x2-xbar2))/(n-1)
ss33<-sum((dataset1$x3-xbar3)*(dataset1$x3-xbar3))/(n-1)
cm2<-matrix(c(ss11,ss12,ss13,ss21,ss22,ss23,ss31,ss32,ss33),3,3)
cm2
```

```
     [,1]  [,2] [,3]
[1,]  7.50  5.0 -1.75
[2,]  5.00 10.0  1.50
[3,] -1.75  1.5  2.50
```

matrix S yang menggunakan cara memanfaatkan deviasi memiliki hasil yang sama dengan matrix S yang menggunakan cara penjumlahan biasa untuk mencari covariance

Sample Correlation matrix R dengan memanfaatkan matrix S

```
#mencari D^(-0.5)
matrixDiagonal<-matrix(c(s11^(-0.5),0,0,0,s22^(-0.5),0,0,0,s33^(-0.5)),3,3)
matrixDiagonal
```

```
     [,1]      [,2]      [,3]
[1,] 0.3651484 0.0000000 0.0000000
[2,] 0.0000000 0.3162278 0.0000000
[3,] 0.0000000 0.0000000 0.6324555
```

```
#matrix Correlation
mc <- matrixDiagonal%*%cm1%*%matrixDiagonal
mc
```

```
     [,1]       [,2]      [,3]
[1,]  1.0000000 0.5773503 -0.4041452
[2,]  0.5773503 1.0000000  0.3000000
[3,] -0.4041452 0.3000000  1.0000000
```

Matriks correlation and covariance dengan syntax R yang sudah jadi: cov(), cor()

```
cov(dataset1)
```

```
   x1   x2   x3
x1  7.50  5.0 -1.75
x2  5.00 10.0  1.50
x3 -1.75  1.5  2.50
```

```
cor(dataset1)
```

```
   x1         x2         x3
x1  1.0000000 0.5773503 -0.4041452
x2  0.5773503 1.0000000  0.3000000
x3 -0.4041452 0.3000000  1.0000000
```

Eigen value and eigen vector

```
eigen(cov(dataset1))
```

```
eigen() decomposition
$values
[1] 13.9049009  5.4143634  0.6807358

$vectors
             [,1]       [,2]       [,3]
[1,] -0.613679310 -0.6232975  0.4846627
[2,] -0.789496144  0.4769002 -0.3863444
[3,] -0.009671757  0.6197309  0.7847548
```

```
eigen(cor(dataset1))
```

```
eigen() decomposition
$values
[1] 1.591638 1.273514 0.134848

$vectors
           [,1]       [,2]       [,3]
[1,]  0.7490653 -0.2047782  0.6300533
[2,]  0.6347139  0.4943479 -0.5939347
[3,] -0.1898407  0.8447994  0.5002744
```