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~~Hobi~~ :

Latihan pertemuan 4 APG

1). Test H0: $\mu' = (6, 11)$ using the data

=>

$$Y = \begin{bmatrix} 3 & 10 \\ 6 & 12 \\ 5 & 14 \\ 10 & 9 \end{bmatrix}$$

n = jumlah observasi = 4
p = jumlah kolom = 2

=> # Hitung $\bar{X}$

$$\bar{X} = \frac{1}{n} X^b . 1$$

$$= \frac{1}{4} \begin{bmatrix} 3 & 6 & 5 & 10 \\ 10 & 12 & 14 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 11{,}25 \end{bmatrix}$$

$\bar{x}_1 = (3 + 6 + 5 + 10) / 4$
$= 6$

$\bar{x}_2 = (10 + 12 + 14 + 9) / 4$
$= 11{,}25$

$\mu_0 = [6, 11]$

=># Hitung diagonal untuk mencari matrix kovarian

$d_i = y_i - \bar{x}_i . 1$

$$d_1 = \begin{bmatrix} 3 \\ 6 \\ 5 \\ 10 \end{bmatrix} - 6 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ -1 \\ 4 \end{bmatrix}$$

$$d_2 = \begin{bmatrix} 10 \\ 12 \\ 14 \\ 9 \end{bmatrix} - 11{,}25 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1{,}25 \\ 0{,}75 \\ 2{,}75 \\ -2{,}25 \end{bmatrix}$$

=># Mencari matrix kovariant

$$S_{ij} = \frac{1}{n-1} d_i' d_j$$

$$S_{11} = \frac{1}{4-1} \begin{bmatrix} -3 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ -1 \\ 4 \end{bmatrix}$$

$$= 8{,}67$$

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$$S_{12} = \frac{1}{4-1}\begin{bmatrix}-3 & 0 & -1 & 4\end{bmatrix}\begin{bmatrix}-1{,}25\\ 0{,}75\\ 2{,}75\\ -2{,}25\end{bmatrix} \quad \{S_{12} = S_{21}\}$$

$$= -2{,}67$$

$$S_{22} = \frac{1}{4-1}\begin{bmatrix}-1{,}25 & 0{,}75 & 2{,}75 & -2{,}25\end{bmatrix}\begin{bmatrix}-1{,}25\\ 0{,}75\\ 2{,}75\\ -2{,}25\end{bmatrix}$$

$$= 4{,}92$$

$$S = \begin{bmatrix}S_{11} & S_{12}\\ S_{21} & S_{22}\end{bmatrix} = \begin{bmatrix}8{,}67 & -2{,}67\\ -2{,}67 & 4{,}92\end{bmatrix}$$

» # Mencari T^2

$$T^2 = n \cdot (\bar{X} - \mu_0)^t (S)^{-1} (\bar{X} - \mu_0)$$

$$= 4 \left(\begin{bmatrix}6\\ 11{,}25\end{bmatrix} - \begin{bmatrix}6\\ 11\end{bmatrix}\right)^t \left(\begin{bmatrix}8{,}67 & -2{,}67\\ -2{,}67 & 4{,}92\end{bmatrix}\right)^{-1} \left(\begin{bmatrix}6\\ 11{,}25\end{bmatrix} - \begin{bmatrix}6\\ 11\end{bmatrix}\right)$$

$$= 0{,}061$$

» # Hitung C^2

Diketahui $\alpha = 5\%$; $v_1, v_2 = 2, 2$

$F_{tabel} = 19$

$$C^2 = \frac{(n-1)\,p}{(n-p)} \cdot F_{tabel}$$

$$= \frac{(4-1)}{(4-2)} \cdot 2 \cdot 19 = 57$$

Interpretasi :

Dari hasil yang didapat : $C^2 = 57 > T^2 = 0{,}061$. Artinya belum cukup bukti untuk menolak H0.

2).

No	y_1	y_2	y_3	y_4	y_5
1	51	36	50	35	42
2	27	20	24	17	27
3	37	22	41	37	30
4	42	36	32	34	27
5	27	18	33	14	29
6	43	32	43	35	40
7	41	22	36	25	38
8	38	21	31	20	16
9	36	23	27	25	28
10	26	31	31	32	36
11	29	20	25	26	25

Ditanyakan :

a). Tes $H_0: M = (30, 25, 40, 25, 30)$

b). jika tolak H_0, Lakukan tes beberapa variabel secara univariate

$$M_0 = \begin{bmatrix} 30 \\ 25 \\ 40 \\ 25 \\ 30 \end{bmatrix} \quad n = 11 \quad p = 5$$

a) ⇒ Hitung $\bar{X}$:

$$\bar{X} = \frac{1}{n} X^t \cdot 1$$

$$= \frac{1}{14} \begin{bmatrix} 51 & 27 & 37 & 42 & 27 & 43 & 41 & 38 & 36 & 26 & 29 \\ 36 & 20 & 22 & 36 & 18 & 32 & 22 & 21 & 23 & 31 & 20 \\ 50 & 26 & 41 & 32 & 33 & 43 & 36 & 31 & 27 & 31 & 25 \\ 35 & 17 & 37 & 34 & 14 & 35 & 25 & 20 & 25 & 32 & 26 \\ 42 & 27 & 30 & 27 & 29 & 40 & 38 & 16 & 28 & 36 & 25 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} 36,091 \\ 25,545 \\ 34,091 \\ 28,273 \\ 30,727 \end{bmatrix}$$

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>> Hitung diagonal untuk mencari matrix kovarians

$d_i = y_i - \bar{x}_i \cdot 1$

$$d_1 = \begin{bmatrix} 51 \\ 27 \\ 37 \\ 42 \\ 27 \\ 43 \\ 41 \\ 38 \\ 36 \\ 26 \\ 29 \end{bmatrix} - 32{,}031 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 18{,}969 \\ -5{,}031 \\ 4{,}969 \\ 9{,}969 \\ -5{,}031 \\ 10{,}969 \\ 8{,}969 \\ 5{,}969 \\ 3{,}969 \\ -6{,}031 \\ -3{,}031 \end{bmatrix}$$

$$d_2 = \begin{bmatrix} -0{,}031 \\ -16{,}031 \\ -14{,}031 \\ -0{,}031 \\ -18{,}031 \\ -9{,}031 \\ -19{,}031 \\ -15{,}031 \\ -13{,}031 \\ -5{,}031 \\ -16{,}031 \end{bmatrix} \quad d_3 = \begin{bmatrix} 13{,}969 \\ -10{,}031 \\ 9{,}969 \\ -4{,}031 \\ -3{,}031 \\ 6{,}969 \\ -0{,}031 \\ -5{,}031 \\ -3{,}031 \\ -5{,}031 \\ 11{,}031 \end{bmatrix} \quad d_4 = \begin{bmatrix} -1{,}031 \\ -13{,}031 \\ 0{,}969 \\ -4{,}031 \\ -22{,}031 \\ -1{,}031 \\ -11{,}031 \\ -16{,}031 \\ -11{,}031 \\ -9{,}031 \\ -10{,}031 \end{bmatrix} \quad d_5 = \begin{bmatrix} 5{,}969 \\ -3{,}031 \\ -6{,}031 \\ -3{,}031 \\ -7{,}031 \\ 3{,}969 \\ 1{,}969 \\ -20{,}031 \\ -8{,}031 \\ -0{,}031 \\ -11{,}031 \end{bmatrix}$$

$s_{11} = d_1' d_1 = 65{,}09$

$s_{12} = s_{21} = d_1' d_2 = 33{,}19$

$s_{13} = s_{31} = d_1' d_3 = 97{,}53$

$s_{14} = s_{41} = d_1' d_4 = 31{,}77$

$s_{15} = s_{51} = d_1' d_5 = 25{,}93$

$s_{22} = d_2' d_2 = 46{,}07$

$s_{23} = s_{32} = d_2' d_3 = 28{,}34$

$s_{24} = s_{42} = d_2' d_4 = 40{,}34$

$s_{25} = s_{52} = d_2' d_5 = 28{,}36$

$s_{33} = d_3' d_3 = 60{,}69$

$s_{34} = d_3' d_4 = s_{43} = 37{,}37$

$s_{35} = s_{53} = d_3' d_5 = 41{,}13$

$s_{44} = 62{,}82$

$s_{45} = s_{54} = d_4' d_5 = 31{,}68$

$s_{55} = d_5' d_5 = 58{,}22$

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» Matriks kovarian

$$S = \begin{bmatrix} 65,03 & 33,64 & 47,59 & 36,77 & 25,43 \\ 33,64 & 46,07 & 28,39 & 40,34 & 28,32 \\ 47,59 & 28,39 & 60,63 & 37,37 & 41,13 \\ 36,77 & 40,34 & 37,37 & 62,82 & 31,68 \\ 25,43 & 28,32 & 41,13 & 31,68 & 58,22 \end{bmatrix}$$

» Hitung T^2

$$T^2 = n \cdot (\bar{X} - \mu_0)^t \, S^{-1} (\bar{X} - \mu_0)$$

$$= 11 \cdot \left(\begin{bmatrix} 36,091 \\ 25,545 \\ 34,091 \\ 27,273 \\ 30,727 \end{bmatrix} - \begin{bmatrix} 30 \\ 25 \\ 40 \\ 25 \\ 30 \end{bmatrix} \right)^t \cdots S^{-1} \left(\begin{bmatrix} 36,091 \\ 25,545 \\ 34,091 \\ 27,273 \\ 30,727 \end{bmatrix} - \begin{bmatrix} 30 \\ 25 \\ 40 \\ 25 \\ 30 \end{bmatrix} \right)$$

$$= 85,33$$

» Hitung C^2:

$$C^2 = \frac{(n-1)p}{(n-p)} F_{(p,n-p)} = \frac{(11-1)}{(11-5)} \cdot 5 \, F_{(5,6)}$$

$$= 36,58$$

Kesimpulan:

$T^2 = 85,33 > C^2 = 36,58$. Maka kesimpulan tolak H0. dengan tingkat signifikan = 5% dari sampel yang ada maka belum cukup bukti untuk menyatakan bahwa rata-rata tidak seperti μ_0

b). $a_1 = [1 \; 0 \; 0 \; 0 \; 0]$

$a_2 = [0 \; 1 \; 0 \; 0 \; 0]$

$a_3 = [0 \; 0 \; 1 \; 0 \; 0]$

$a_4 = [0 \; 0 \; 0 \; 1 \; 0]$

$a_5 = [0 \; 0 \; 0 \; 0 \; 1]$

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$$a_i \bar{x} - \sqrt{\frac{c^2}{n} \cdot a_i' S a_i} \leq a_i' \bar{x} \leq a_i \bar{x} + \sqrt{\frac{c^2}{n} a_i' S a_i}$$

Confidence interval:

>>. $y_1 \Rightarrow 21{,}38 \leq \mu \leq 50{,}8$

$y_2 \Rightarrow 15{,}17 \leq \mu \leq 37{,}32$

$y_3 \Rightarrow 13{,}89 \leq \mu \leq 98{,}23$

$y_4 \Rightarrow 12{,}82 \leq \mu \leq 91{,}72$

$y_5 \Rightarrow 16{,}02 \leq \mu \leq 99{,}69$

Interpretasi:

bisa dilihat confidence intervalnya hanya pada nilai positif saja (tidak melewati), sehingga dapat disimpulkan nilai ~~x tidak~~ μ tidak sama dengan μ_0 untuk setiap variabel y_1, y_2, y_3, y_4, y_5.