```
% #### #########
##############
#######.
% ####### ##########
% ### ##### ###### ###:
(x-x 1)*(x-x 2)*...*(x-x \{i-1\})*(x-x \{i+1\})*...*(x-x n).
##### ###
% #####:
     (x-x_1)*(x-x_2)*...*(x-x_{i-1})*(x-x_{i+1})*...*(x-x_{i+1})
x n)
   L_i =
9
          _____
       (x_i-x_1)*(x_i-x_2)*...*(x_i-x_{i-1})*(x_i-x_{i-1})
+1)*...*(x i-x n)
#######.
y = \sum_{i=1,n} y_i L_i - \#\#\#\#\#\#\#\#\#\#\#\#.
m = 2;
k = 20;
sq 2 = 1.4142; % ~sgrt(2)
sq 7 = 2.6458; % ~sqrt(7)
x \text{ val} = [-m, m*sq 2, sq 7, 2*sq 7];
y \text{ val} = [1, -k, sq 7, -2*sq 7];
n = length(x val);
L i = @(x, i) prod(...
  (x - x_val([1:(i-1), (i+1):end])) ./ ...
  (x \text{ val}(i) - x \text{ val}([1:(i-1), (i+1):end])),...
  2);
x = linspace(min(x val), max(x val)).';
v = zeros(size(x));
```

```
figure(1);
for i = 1 : n
   y i = y val(i) * L i(x, i);
   y = y + y_i;
   pl = plot(x, y_i); hold on;
   sc = scatter(x_val(i), y_val(i), 60, 'LineWidth', 1, ...
       'MarkerFaceColor', pl.Color, 'MarkerEdgeColor', 'none');
    sc. Handle Visibility = 'off'; % Don't show the scatter points in
 (future) legend
end
plot(x, y, '--k', 'LineWidth', 1.5);
hold off;
legend('L_1', 'L_2', 'L_3', 'L_4', 'L');
syms x;
for i = 1 : n
   y_i = y_val(i) * L_i(x, i);
   y = y + y_i;
end
8 ### ######### ##, # ## ###### ### #####.
assume(y,'real');
y = expand(y);
y = simplifyFraction(y);
display(y);
y =
 (49191779540368750000*x^3)/4789562232325259499
 - (98091516312036408750*x^2)/1596520744108419833
 -(16196326516388476417*x)/870829496786410818 +
 467350654950588061137/1596520744108419833
  (49191779540368750000*x^3)/4789562232325259499
 - (98091516312036408750*x^2)/1596520744108419833
 -(16196326516388476417*x)/870829496786410818 +
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 (49191779540368750000*x^3)/4789562232325259499
 - (98091516312036408750*x^2)/1596520744108419833
 - (16196326516388476417*x)/870829496786410818 +
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 (49191779540368750000*x^3)/4789562232325259499
 - (98091516312036408750*x^2)/1596520744108419833
 - (16196326516388476417*x)/870829496786410818 +
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 (49191779540368750000*x^3)/4789562232325259499
 - (98091516312036408750*x^2)/1596520744108419833
 - (16196326516388476417*x)/870829496786410818 +
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    (49191779540368750000*x^3)/4789562232325259499
 - (98091516312036408750*x^2)/1596520744108419833
```

2

```
-(16196326516388476417*x)/870829496786410818 +
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  (49191779540368750000*x^3)/4789562232325259499
- (98091516312036408750*x^2)/1596520744108419833
- (16196326516388476417*x)/870829496786410818 +
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- (98091516312036408750*x^2)/1596520744108419833
- (16196326516388476417*x)/870829496786410818 +
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- (16196326516388476417*x)/870829496786410818 +
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- (16196326516388476417*x)/870829496786410818 +
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```

```
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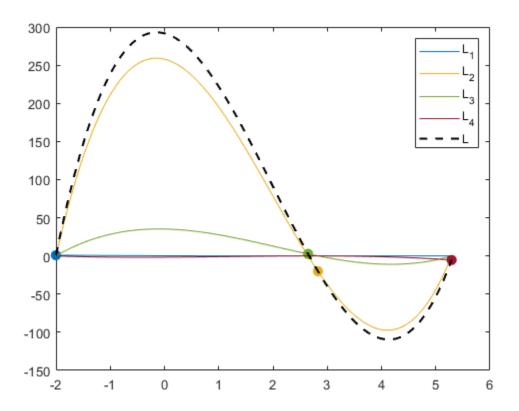
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