The School of Mathematics

Volatility Models for Safety Stock; A Simulation Based Study

by

Rija Zafar

Dissertation Presented for the Degree of
MSc in Operational Research with Computational Optimization

August 2022

Supervised by
Joe Loxham and Lars Schewe

Perfection is finally attained not when there is no longer anything to add, but when there is no longer anything to take away

- Antoine de Saint-Exupéry

Abstract

Globalization of the supply chain and proliferation of competition has transformed inventory management into a paramount research area. To mitigate the risks and uncertainties from the supply chain and to optimize inventory levels, demand forecasting and safety stock levels have been studied intensively. This study primarily focuses on the determination of safety stock levels.

Two volatility models, one based on the normal distribution and another based on a non-parametric bootstrapping technique, are used to find the inventory levels including safety stock for a given stochastic demand signal. In this research, three kinds of probability distributions are used to generate demand signals. For each demand signal, the performance of the two volatility models is studied based on the cycle service level, period service level, fill rate and inventory-demand ratio.

A total of 36 experiments were carried out with varying parameters, including the demand signal distribution, presence or not of backorders, and considerations relating to the lead time, specifically whether or not it is stochastic and what the nominal lead time is. In most cases in terms of cycle service level, the safety stock level calculated from the Normal Volatility Model performed better, whereas in terms of inventory demand ratio, the safety stock level calculated using the non-parametric model performed better. Detailed results can be found in the conclusions section.

Acknowledgments

First and foremost I would like to extend my gratitude to Joseph Loxham, who led the supervision of my dissertation. His invaluable guidance and continued support made this dissertation possible. His commitment to the project, affluence of ideas and generous provision of knowledge and expertise made this endeavour a rewarding learning experience for me.

I would also like to express my thanks to PEAK Company, for offering me the opportunity to work on an interesting and challenging project. This project impacted and inspired me and helped me grow intellectually. Additionally, I would like to acknowledge Dr Lars Schewe, who was an excellent academic supervisor and has done a great effort in terms of organizational queries and was open to an interesting thematic discussion. He played a crucial role in the success of this project.

Furthermore, I would like to extend my deepest appreciation to my friends, especially Elizabeth Simmons, Rao M. Hamza, Osaid Syed, Syed Salman Ahmed and Diego Sanchez for their unconditional support and sponsored rant sessions. Their belief in me has kept my spirits and motivation high during this process.

Lastly but not least, I would like to recognize all the women around the world who sacrifice their time, juggle life and society, handle a great deal of stress, without recognition and still are persistently kind, strong and inspiring. They inspire me to express my thoughts unabashedly and be unapologetically myself. Your decisions in life make me put my best effort forward.

Own Work Declaration

I declare that this thesis was composed by myself and that the work contained therein is my own, except where explicitly stated otherwise in the text. I understand that any false claim for this work will be penalised in accordance with the University regulations.

Rija Zafar
The University of Edinburgh
Dated: 26^{th} August 2022

Contents

List of Tables

List of Figures

1 Introduction

This chapter presents the motivation behind this project and discusses the problem statement in detail. Moreover, it also includes a critical review of previous research carried out in the same problem domain, which helped inspire the methodology for this project. Additionally, this chapter also elaborates on the plan of this thesis, showing the essential features.

1.1 Motivation and Background

The global supply chain is becoming increasingly complex. Contributing factors include intense competition, waves of product innovation [45], and vulnerability to natural and unnatural disruptions [17]. This has transformed supply chain analysis and modelling from an emerging topic into an important research area [17]. A significant contribution to the complexity of the modern supply chain is the uncertainty inherent in many of its key components, including demand fluctuation, lead time variability, disruptions to supply capacity, exchange rate volatility, and more [17, 26]. All of these factors make it challenging to predict the nature and impacts of these interacting uncertainties [23], quantify the associated risks, and communicate this information to stakeholders and decision-makers.

One of the major uncertainties in supply chain management is future demand [40]. During the second half of the 20^{th} century, product customization peaked, leading to shortened average product life cycles and increased demand-variability [45]. This demand variability is a double-edged sword: greater than expected demand leads to stock-outs, whereas lower than expected demand saddles the company withholding costs due to excess inventory.

To mitigate the risks associated with demand fluctuation, companies use two strategies: demand forecasts and safety stocks [21]. Demand forecasts, put simply, are future estimations (predictions) of customer demand [1]. Forecasting demand is one of the main challenges in supply chain management [6] but is only one component of a complete solution. Due to the uncertain nature of predicting the future, one should never assume a demand forecast to be correct. Even with a very good forecast, it should always be anticipated that the real demand could be slightly higher than the forecast, leading to shortages or unsatisfied demand; or slightly lower, leading to overstocking and holding costs. In addition to this, it is unrealistic to assume that a "true" demand model of each stock-keeping unit can exist [42], and even if such a model could exist, its implementation would still be challenging. [40]. To mitigate the effects of forecast uncertainty and other factors we will implement shortly, it is common to use safety stocks [25], which are the additional units beyond the required stock to meet lead time forecasted demand [40], used to help ensure certain measures of supply-chain performance are satisfied.

For effective end-to-end supply chain management, both strategic and tactical inventory planning using safety stocks plays a crucial role [26]. Intensive research has been carried out regarding the dimensioning, positioning, managing and placement of the safety stocks [21]. This research is solely concerned with dimensioning of safety stock.

1.2 Problem Description

When discussing models for demand, many different types of demand signals can be imagined. While these types of signals can be (and routinely are, for the purposes of selecting a forecasting method) quantitatively classified into specific categories, it suffices to say in this work that different demand signals can exhibit many different types of volatility, with the volatility being the characteristic variation of the demand about some trend. Often, a demand forecast is composed of a prediction of the trend, plus a volatility model to understand the remaining variation.

The most common way to model volatility is by using a probability distribution. Different probability distributions can be used to model different types of volatility. Since the volatility model describes anticipated variation in the demand signal, the choice of volatility model is an important consideration when designing any safety stock model.

Numerous methodologies to calculate the optimal level of safety stocks exist, of which some are theoretical (or "parametric") and others are empirical. Theoretical models assume some parametric form of volatility and are motivated by mathematical considerations (such as the central limit theorem) which means that anticipated variation in these circumstances can be expressed in a neat, mathematical way. Kernel density estimation models do not assume such mathematical structure, and instead, express the volatility in terms of past observations. Both theoretical and Kernel density estimation volatility models range from simple to complex.

The safety stock is affected by six major factors: service level, lead time, demand volatility, inventory policy, component commonality, and holding costs [21]. Nevertheless, the most reasonable model always depends upon the organization's circumstances. The most common models are based on customer service level [40]. This is because the stock-out cost calculation, which is a complex quantity to estimate, is not required when the performance of the safety stock model is evaluated on the basis of service level.

In this research, we study volatility models that can be used to calculate how much inventory is required to satisfy a certain percentage of the demand, which is a fundamental problem for many decision-makers. In such circumstances, the choice of volatility model is of critical importance, since it can strongly influence the effectiveness of the safety stock model.

The primary question of this thesis is "how does the choice of volatility model affect safety stock performance under different circumstances". In order to investigate this, we first develop a simple inventory simulator that can be used to study these questions. The performance of a given volatility model will be measured by different types of service levels, and by a dimensionless quantity referred to here as the inventory-demand ratio. Synthetic demand signals with three different types of volatility will be investigated, which will be generated from parametric distributions with known parameters in order to isolate certain effects. These parametric volatility models are the Normal distribution, the Poisson distribution, and a zero-inflated Poisson distribution ("ZIP") which is obtained by combining a Bernoulli distribution and a Poisson distribution. Attention will be restricted to the periodic review inventory policy only.

1.3 Literature Review

Numerous methodologies have been studied to evaluate the optimal safety stock level. The most common practice is the determination of safety stock based on the variation of demand. In light of the central limit theorem, the demand is often assumed to be normally distributed, though the calculation of safety stock levels based on forecasting errors is not an untouched domain. However, the most common objective in safety stock research is to answer the question: "which level of safety stock can reduce the average overall cost while providing satisfactory service level/results?" [14, 19, 28, 10].

Gallego [19] used the Monte Carlo Simulation-based methodology to drive the safety stock levels. The objective was to reduce the long-run total cost that includes transaction cost, also known as a setup cost, holding cost and amount of back-orders in a cycle while achieving the target schedule. Bahroun & Belgacern [5] also used the Monte Carlo simulation to evaluate the safety stock levels under a non-stationary demand pattern. However, the objective of Bahroun & Belgacern [5] was to propose a dynamic approach to determine safety stock levels and study the impacts of the traditional approach on non-stationary demand. They also proved that this approach simultaneously minimizes the safety stock levels and improves the service level by reducing the stock out the probability. In both cases, the demand was assumed to follow the Normal distribution. Although Gallego [19] also shows that optimal safety stock levels are unique for each product, Bahroun & Belgacern [5] shows that safety stock levels can be unique for each cycle.

Reichhart et al. [36] also used the Monte Carlo simulation process to formulate the safety stock level for multi-variant products. However, instead of assuming normal volatility, they used the variation in forecast error. The performance criteria were based on service level, inventory level and back-order cost. This study focused on multi-variant products and was focused on developing a novel approach to calculate safety stocks for multi-variant products: previous approaches dealt with this either by considering a single-product multi-tier system or a single-tier system with multiple products, despite multi-variant products having correlated demand. The simulation resulted in a generation relationship between variants, and their inventory levels which gave a novel safety stock formula for the current setting.

Dar-EL and Malmborg [14] also proposed an approach to reduce the global inventory cost while achieving the desired service level. However, this model involves rescheduling replenishment during the inventory cycle. Badinelli [28], on the other hand, evaluated the safety stock level by modelling the trade-off between holding cost and the stock out cost. They used the stochastic demand pattern with a continuous review inventory policy framework and provided an optimization technique to determine the safety stock levels iteratively in conjunction with the estimation technique.

Per J. Agrell [2] developed a multi-criteria decision-making (MCDM) model that takes batch size and safety stock decisions into account. This model is a non-linear convex optimization problem with the objective to minimize the total expected inventory cost, the number of back-orders and the frequency of stock-outs annually, This model was intended to be used in support of an operations management system, although it can be a standalone strategic inventory control system. He also assumed the demand to be normally distributed throughout. Marcello Braglia, Davide Castellano and Marco Frosolini [9] also assumed the demand to be normally distributed, and created a novel approach to manage safety stock levels in coordination with a single-vendor and single-buyer supply chain framework. The lead time was assumed to be controllable, a continuous review inventory policy was implied, and no shortages were allowed. This approach is quite useful as typically the safety stock is determined according to the safety factor value. They treated the order quantity and safety factor as functional dependence. Concerning practical application, the validity of the single vendor and single buyer assumptions is questionable.

Eppen and R. Martin [15] considered the determination of safety stock levels when both demand and lead time are random variables. They divided the study into two cases, the first case where parameters of demand and lead time distributions are known and the second case where the same parameters are unknown. In the case of known parameters, the variance of forecasting error was used by considering exponential smoothing techniques, and lead time demand was taken to be normally distributed. In this case, the result indicates that the assumption about lead time demand is misleading. In the case of unknown parameters, a Kernel density estimation approach was used to calculate the safety stock levels and to avoid the independent and normally distributed assumption about forecasting error. Trapero, Cardós and Kourentzes [40, 42] used a similar approach to Eppen and R. Martin's [15] second case. They analyzed the impacts of possible deviation from normality assumption and used a non-parametric empirical method based on a Kernel density function and a parametric method based on generalized auto-regressive conditional heteroscedasticity (GARCH) model to determine safety stock levels. They also concluded that the normality assumption for lead time demand is misleading and for shorter lead times, the normality deviation is more impactful and Kernel density estimation of safety stock is most suitable, whereas, for longer lead times, the auto-correlation of variance of forecast error is the most important deviation and therefore, GARCH model is most appropriate to determine safety stock levels in these circumstances. This was concluded based on cycle service levels, back-order levels and inventory investment volume.

Fotopoulos, Wang and Rao [18] also evaluated the safety stock level under the assumption of auto-correlated demand and arbitrary lead time. Later, the same method was used to assess the effects on safety stock level if daily demand and lead times deviate from normality. This process provides an upper bound on safety stock. They further investigated and concluded that the effects of auto-correlation are prominent in comparison to normality if demand and lead time in determining safety stock level and if auto-correlation is ignored it can lead to substantial errors in calculation.

Buffa [10] developed a programming model to determine the level of safety stock in a fixed-interval, and variable order quantity system in a multi-product environment. The main problem was limited financial and inventory resources and excess resources were required if safety stock levels were calculated based on the forecasting model alone. Hence, to overcome this problem he introduced a trade-off between potential stock out cost and the cost of additional resources, including both financial and inventory resources. He simulated the model for 46 months for 25 different products and concluded that a significant cost reduction is possible with the proposed model. Rappold and Yono [35] also worked on a model where the limitation of resources was considered. However, their approach was significantly different than Buffa [10]. Rappold and Yono [35] studied a multi-buyer, multi-vendor supply chain with limitations placed on purchase and warehouse capacities. They assumed the demand for products to be stochastic with a uniform distribution, while lead-time for receiving products varied linearly in accordance with order quantity and production rate. A fraction of the shortage was back-ordered and the rest was assumed lost. The authors determined to reorder points and safety stock and optimized for minimum supply chain costs. They showed that the problem model was of an integer nonlinear programming nature and applied a harmony search algorithm for the solution along with a genetic algorithm for validation. They demonstrated the applicability of the methodology in real-world situations.

Beutel and Minner [6] came up with an unconventional model to calculate the safety stock level. Typically, all models assumed a demand distribution and estimate the required parameters from historical data, whereas, Beutel and Minner considered the fact that demand is affected by exogenous factors including but not limited to the sales price, price variation, weather conditions, seasonal variations etc. They used two methods in their research. In the first method, the regression model was used to estimate demand and then forecast errors were used to estimate the safety stock level. Additionally, in the second method, the problem was formulated as a linear programming problem with the objective to minimize overall costs including holding costs and shortage costs subjected to the required service level. The exogenous factors were accommodated in terms of price and weather dynamics. The results from both methods were compared and they concluded that the second method with linear programming problems performs better especially when the sample size is small. It also provides more robust inventory levels than the regression method.

Kanet, Gorman and Stößlein [29] introduced a state-of-the-art methodology to estimate the safety stock while significantly reducing inventory cost. Typically the safety stocks are calculated for the complete planning horizon. However, Kanet et al. suggested that time-phased safety stocks should be calculated because in times of high uncertainty larger safety stocks are more suitable to hold, whereas when demand is the predictable a lower amount of safety stock is more appropriate. They used a linear optimization model intending to minimize the overall inventory subjected to given values of safety stock. They also used shortage frequency of products and fill rates as metrics to judge the performance of their model and applied it to a United State based industry and concluded that significant savings can be achieved by using dynamic safety stock levels.

For stochastic demand models, Van Donselaar and Broekmeulen [43] applied linear regression to modify existing approximations for the fill rate. The typical approximation of fill rates is equal to the percentage of demand immediately fulfilled by on-hand inventory in a periodic review inventory system. However, in the new approximation, they used the existing ones and applied linear regression to slightly modify them. This new approximation tested for a range of parameters and gave an average approximation error of 0.0028 for the fill rate with a standard deviation of 0.0045. Results of the study show that back ordering system approximations can lead to serious errors for fill rates of at least 95%.

Although, these errors can be larger with high uncertainty of demand and greater lead times.

Zhou and Viswanathan [48] compare their improved bootstrapping method of safety stock determination with the parametric approaches previously used. Computational experiments results show that bootstrapping works better with randomly generated data when simulated historical data is present in a large enough amount to generate a distribution. However, it is also shown that parametric methods perform better with real data sets from the industry.

Wang, Zinn and Croxton studied the situation where demand and lead time are correlated. They derive the formula for calculating the average and variance of demand and used these parameters to find the reorder point and safety stocks for periodic review inventory policy. Later, they applied this to distributions with closed-form solutions i.e., Normal distribution and Poisson Distributions and examined the effects on safety stock requirements in correlated situations. Van Ness and Stevenson [44] also derived a formula to determine the mean and variance of lead time demand but in a situation where both demand and lead time distributions were assumed to be discrete and independent. They used empirical distributions as a basis to develop formulas for computing probabilities for the lead-time demand distribution. The model serves as a valuable tool for both industry and academia. It can be used both for obtaining exact probabilities in the industry as well as for evaluating various approaches to the reorder-point problem. They deduced that the safety stock level will be the difference between reorder point and expected demand under the lead time.

Ruiz-Torres and Mahmoodi [38] developed a model based on historical data to find out possible effects of the replenishment cycle. They compared the model with the traditional approach used by Estes [16] which does not make any assumptions about the distributions of lead time and demand. They performed the analysis using data sets from an electronics manufacturer showing that the developed model is much closer to target service levels and bears lower inventory carrying costs. These results showed that the new model performs better than traditional models that assume a Normal distribution for the lead time. Chaturvedi and Martinre-de-Albeniz [12] on the other hand developed a framework for the optimization of safety stock, excess capacity, and diversifying supply sources. They used the infinite horizon periodic review setting and different diversification strategies to explore the possibility of the reduction of investments. They concluded that higher supply uncertainty results in higher safety stock along with more excess capacity and higher diversification. However, safety stock and diversification are non-monotonic in demand uncertainty.

Janssens and Ramaekers [27] explored the case where the probability distribution of demand during the lead time was not available since in such cases a single value of safety stock level cannot be determined to satisfy performance metrics. They formulated a linear optimization model which will determine the safety stock level in absence of such information. They also concluded that performance measure under worst cases of lead-time demand was guaranteed by their optimization model.

Margaretha Gansterer and Christian Almeder and Richard F. Hartl Gansterer, Almeder and Hartl [20]developed a framework for hierarchical production planning which could be used to determine planned lead-time, safety stock, and lot size. They ran a discrete event simulation imitating the production system in a make-to-order situation where demand was considered from four different markets. The authors compared different optimization methods to a systematic enumeration of different parameter combinations and found that Variable Neighborhood Search (VNS) gives the best results as a search procedure. The significant conclusion from their research was that as a matter of course the safety stock level does not need to be increased with high demand volatility.

Avci and Salim [3, 4] developed a simulation-based, multi-objective framework for inventory optimization by considering a single supplier with several plants that use periodic review order-up-to level policy and utilize premiums from the supplier in risky inventory cases. The authors solved for supplier flexibility and safety stock levels that showed optimal performance in terms of freights and holding costs, using a multi-objective differential evolution (MODE/D) algorithm. They implemented their model in a real-world automotive supply chain and compared results with a non-dominated sorting genetic algorithm II (NSGA-II) and showed that MODE/D gives better performance for both the parameters of holding cost and freight performance.

Inspired by the previous work, this research will focus on measuring the performance of safety stock levels derived from two volatility models with varying parameters. The different parameters considered in this study are demand signals generated from 3 different types of distributions, 2 types of lead time i.e. deterministic and stochastic, 3 values of lead time i.e. 2, 4 and 10 and whether backorders are permitted or not. From this information, the possible combinations of scenarios are 36. Two metrics will be used to measure the performance of volatility models, i.e. service levels (cycle service level, period service level and fill rates) and inventory-demand ratio.

1.4 Thesis Outline

This thesis is organized as follows,

Chapter 1 is consists of motivation behind the research and a detailed problem description along with an overview of previous work carried out in the same domain.

Chapter 2 explains the role of inventory management and its importance in the supply chain and discusses two widely used inventory control policies.

In Chapter 3, the Periodic Review inventory policy is discussed in detail, along with the change in parameters that comes with this control policy. It also explains the mathematical background of three different demand signals considered in this research and two (parametric and non-parametric) volatility model formulations. This chapter also includes the other concepts used in this research including cycle stocks, order up-to-level, and in-transit inventory, and elaborates on how mathematical information is utilized in this project.

Chapter 4 describes the concept of metrics used to measure the performance of two volatility models, i.e. normal volatility model and Kernel density estimation volatility model. It discusses the mathematical background of the metrics used, i.e., cycle service level, period service level, fill rate and inventory demand ratio. Additionally, it also explains how mathematical and supply chain concepts have been implemented in this research with the help of an algorithm.

In Chapter 5 the analysis of the study is discussed in detail. This chapter is divided into two sections. It contains a detailed analysis of the behaviour of cycle service level, period service level, fill rates and inventory-demand ratio in 36 different scenarios. In each scenario, the performance of the normal volatility model and Kernel density estimation volatility model is compared.

Chapter 6 states the result and conclusion of this study. The chapter 7 presents the possible prospects and direction for further research in the domain of safety stock optimization.

2 Inventory Management

This chapter details the importance of inventory optimization in supply chain management. It also elaborates on the inventory policies used to carry out the decision of when to place an order and for what quantity, and describes, in detail, the two main inventory policies most widely used for inventory management.

2.1 Importance of Inventory Management

The central question of inventory management is "how much stock is needed and when?". This question lies at the most fundamental level of supply chain analysis [45] and is critical to allowing the business to understand and solve the problem of stock [37].

Keeping inventory helps the company to achieve a certain safeguard against uncertainties while preserving client satisfaction. However, following the adage "There ain't no such thing as a free lunch", holding inventory comes at a price. It disconnects the production/procurement process from sales [45] and can be considered as "sleeping cash", which tends to depreciate over time. It also increases the risk of losing the longer it is kept.

Therefore, an optimal amount of inventory is necessary. Optimal here is context-dependent but usually means minimising the risk of a stock-out while providing an adequate service level 4.1, while keeping holding costs and other contextual risks to a manageable level. It follows that when inventory optimization is done right it can reduce overall costs while providing required service levels.

2.2 Inventory Policies

Inventory is held on hand for various reasons, including fluctuation in customer demand, variations in lead times, economies of scale offered by suppliers, and various other situations contributing to significant uncertainties [31]. To this end, inventory policies which consider such factors have been developed to assist inventory management [32].

Viewed abstractly, inventory policies determine the (best) way for product volumes to flow through the supply chain. They define when to place orders, and what quantity to order in order to achieve certain goals, such as example, minimising overall inventory costs. Considered from the point of view of the order period, that is, when to place an order for replenishment, there are two types of inventory policies that are widely used [37]:

1. Continuous Review Inventory Policy

2. Periodic Review Inventory Policy

While both policies are discussed in detail in the following section, note that this thesis only deals with Periodic Review Inventory Policy.

2.2.1 Continuous Review Inventory Policy (r, Q)

In this policy, the inventory is monitored continuously and the next order is placed, for a predetermined quantity, as soon as the current inventory reaches or goes below a given threshold. This means that the elapsed time between two orders may change, however, the order quantity will be the same. This is depicted in the graph 1. This policy is denoted by (r, Q), where r is the reorder point and Q is the predetermined order quantity.

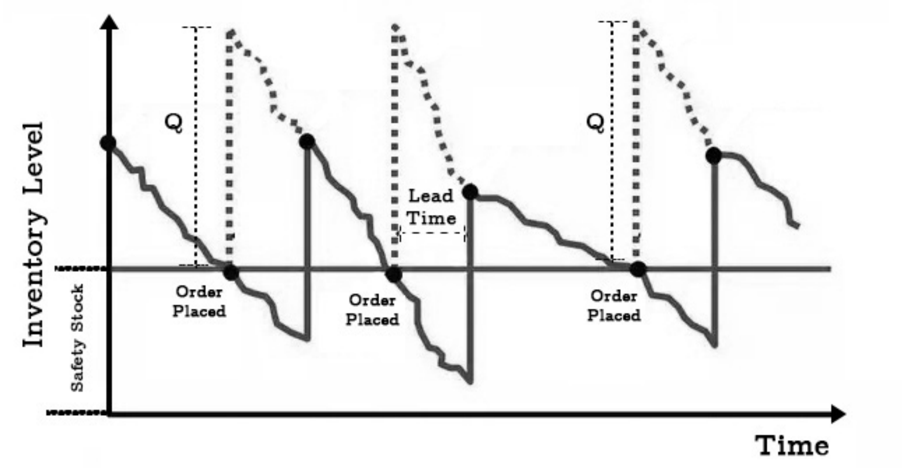


Figure 1: The continuous review inventory system where inventory level is reviewed regularly and order is placed of quantity Q as soon as the inventory level reaches or goes down below reorder point. The order quantity is the same in each order cycle however the order period varies.

This order quantity is called the "economic order quantity", which is determined by a balance between the inventory holding cost and transaction cost functions, given by 2.1:

$$Q^* = \sqrt{\frac{2DT}{h}} \tag{2.1}$$

where,

D	:	Yearly demand in units of inventory items
T	:	Transaction cost for each order
h	:	Holding cost per unit item per year
Q*	:	Optimal number of units per order (EOQ)

As the above equation indicates, the optimal (predetermined) quantity, Q, is proportional to the transaction cost and yearly demand whereas, and inversely proportional to the holding cost per unit, as shown in figure 2.

It is assumed that demand and lead times are known. Transaction cost is the only cost variable, which depends on demand [24].

2.2.2 Periodic Review Inventory Policy (R, S)

In the periodic review policy, the inventory is monitored at the start of the review period, which has a fixed schedule, and an order is placed of a quantity sufficient to bring the net inventory level to a pre-determined up-to-level (the maximum level of inventory). This means that the elapsed time between two orders will remain the same, however, the order quantity may change, as depicted by figure 3. This policy is denoted by (R, S) where R is the review period and S is the maximum inventory level that should be maintained.

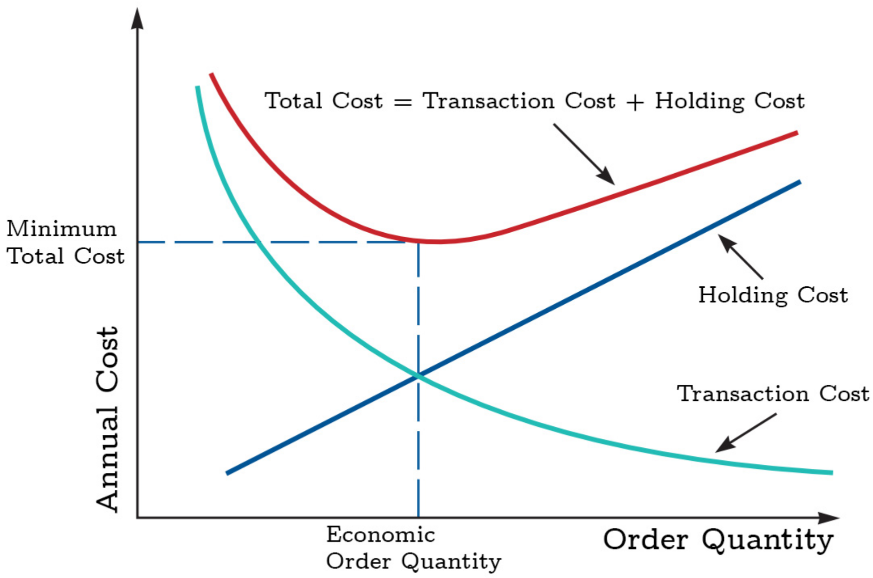


Figure 2: The graph of total inventory cost as a function of order quantity. The minimum total cost is achieved with the economic order quantity. As the order quantity increases the total number of orders placed will decrease, decreasing the transaction cost decreases however, the holding cost will increase. Similarly, if order quantity is decreased the holding cost decreases but the number of orders placed per year increases. Hence, in either case, the deviation from EOQ will result in increased total cost.

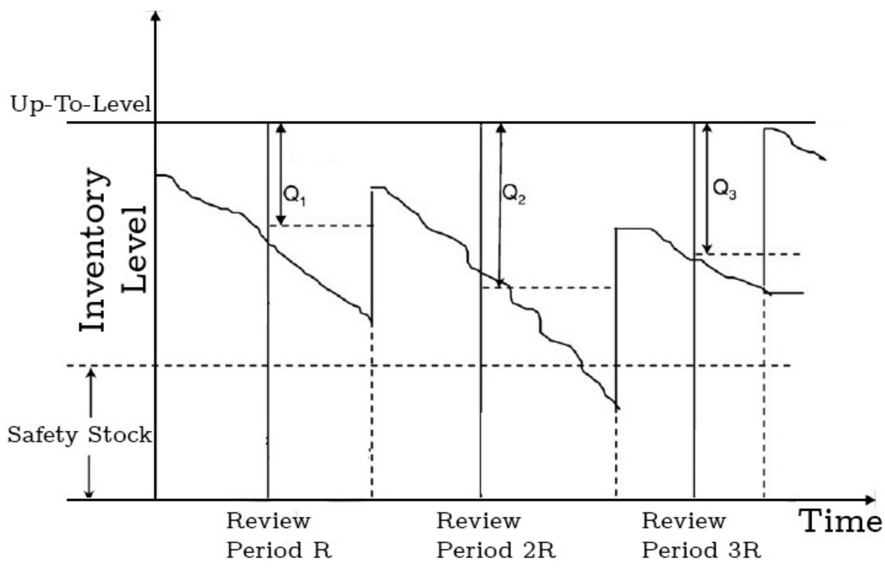


Figure 3: The periodic review inventory system where inventory level is reviewed at R and order is placed to maintain the up-to-level of inventory. The order quantity varies in each order cycle however the review period is fixed.

In comparison, the continuous review inventory policy has the advantage of an optimized order quantity, which means that in theory a lower volume of safety stock is needed. However, suppliers often prefer (or enforce) fixed order quantities [37].

There are more complex inventory policies. (R, s, S), reviews the inventory periodically, and as soon as it reaches threshold s, a new order is placed to bring the inventory level to the up-to-level S. (R, s, Q) also monitors the inventory monitored periodically, and as soon as it reaches the level s an order is placed of quantity Q. Such inventory policies have been studied widely in academia, however, even with many assumptions and simplifications, they result in complex mathematical models that are challenging to implement [33, 45].

3 Periodic Review Inventory Policy

Periodic Review Inventory Policies are commonly used in practice [39] because they allow the business to streamline their operations by grouping the orders [45, 39]. As indicated in 2.2.2, in the periodic review policy at every R unit of time, a replenishment order is placed to bring the inventory level to S.

Consequently, the two main quantities in this policy are the reorder point, R, and order-up-to-level quantity, S. In particular, it is crucial to find the value of these quantities such that the desired service level is satisfied, which is a difficult process [39].

The biggest disadvantage of this policy is that it is impossible to order in-between review periods. If a stock-out occurs before the next review period, a new order cannot (in theory) be made immediately, which means demand is either back-ordered or lost sales occur. This makes the (R, S) policy riskier than the (s, Q) policy. Another issue is that variation in order quantities can impact operation flow. Safety stocks kept using this policy are larger than the equivalent ones which would be obtained using the continuous review policy [37].

Conceptual explanations of and values for safety stock levels, order up-to-level, cycle stock, in-transit inventory and service level will be expanded on further in this thesis.

3.1 Demand Signals

As mentioned in 1.2, three different types of demand signals will be considered in this work. Each of these assumes a specific type of volatility, imposed on a known trend. By assuming the trend is known and that only the volatility is important, we make the implicit assumption that the trend is forecasted with complete accuracy and certainty. While this is of course impossible in practice, such an approach allows isolation of the effects of demand volatility specifically, without needing to worry about forecasting the trend.

The three different types of demand volatility considered are the normal (Gaussian) distribution, the Poisson distribution and a "zero-inflated" Poisson distribution obtained by combining the Bernoulli and Poisson distributions, the latter of which is a good approximation for the highly intermittent types of demand signals commonly encountered by practitioners 4.

Given a given demand volatility model, a finite number of samples from it are used to estimate the parameters for the maximum inventory level (i.e the "up-to-level") and a simulation carried out on a new (different) set of samples. This process mimics the reality encountered by practitioners, who must fit their models to finite datasets. This procedure will naturally result in sometimes overestimation and sometimes underestimation of the "true" up-to-level required, due to the uncertainty associated with estimating the value of a population parameter from a finite sample. Including this modelling challenge is interesting and a key contribution to this work, since being unable to precisely designate model parameters is a practical and critically important problem. The details of all three demand signals are discussed below.

3.1.1 Normal distribution

The demand is assumed to be normally distributed with mean $\mu_d = 100$ and standard deviation $\sigma_d = 10$, which means 3.1

$$Demand\ per\ period \sim \mathcal{N}(\mu_d, \sigma_d^2) = \mathcal{N}(100, 10) \tag{3.1}$$

The probability density function for Normal distribution expressed at value x is mathematically defined as 3.2,

$$f_{\mathcal{N}}(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}.\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{3.2}$$

The demand signals are generated through user-defined functions using the provided parameters as depicted in the algorithm below,

3.1.2 Poisson Distribution

The demand signal is assumed to have Poisson distributed with a rate of 10. The probability density function of Poisson distribution is given by 3.3,

$$f(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!} \tag{3.3}$$

Algorithm 1: Function to Generate Combination of Demand Signal

Input: *rate*, *simulation steps*, $time - steps$

Output: *demand array* /* n-dimensional array */

Function `Generate Demand Signal(`*Input*`):`

  demand array $\longleftarrow$ $poisson\_distribution(rate,\ size\ =\ time - steps)$

  return *demand array*;

End Function

3.1.3 Zero Inflated Poisson Distribution

The zero-inflated Poisson distribution is a compound distribution, obtained by combining a single Bernoulli trial (equivalently, the Binomial distribution with the number of trials, n, set to 1) with the Poisson distribution. This distribution is "zero-inflated" (through the Bernoulli trial) by introducing a "probability of demand" parameter, p which designates the probability of any demand occurring on a given timestep. If demand does occur, its volatility is Poisson with a given rate parameter. If demand does not occur, it is assumed to be zero. With $p < 1$, this causes an increased number of timesteps to not have any demand, thus "inflating" the distribution with (extra) zeros. The rate for the Poisson distribution component is the same as previously, and we set the probability of success, p to 0.3. It can be noted that increasing p to 1 would mean this demand model would be equivalent to the Poisson model with a given demand rate while decreasing p completely to zero would produce a distribution of just zeros.

We implement the zero-inflated Poisson distribution using the Binomial distribution, with n equal to the number of time steps. The probability density function for the Binomial distribution is given by 3.4, where n is the sample size equivalent to the number of time steps in the simulation.

$$f(k;n,p) = \binom{n}{k}\, p^k\,(1-p)^{n-k} \tag{3.4}$$

The demand signals are generated from binomial and Poisson distributions using the user-defined function explained by the algorithm below,

Algorithm 2: Function to Generate Combination of Demand Signal

```
Input: rate, success probability, simulation steps, time − steps
Output: demand array                                   /* n-dimensional array */
Function Generate Demand Signal(Input):
    demand array ⟵ array_of_zeros(size = simulation steps, time − steps);
    for i ∈ simulation steps do
        demand array[i^th − row, all columns] ⟵
         binomial_distribution(success probability, size = time − steps)
        for j ∈ time − steps do
            if i = 1 and j = 1 then
                demand array[i^th − row, j^th − column] ⟵ poisson_distribution(rate)
            end
        end
    end
    return demand array;
End Function
```

3.2 Cycle Stock

Cycle stock is the number of units ordered for each cycle, equivalent to the demand summed the over the review period (d_R). It follows that the expected cycle stock level for the periodic review inventory policy will be;

$$C_s = \frac{1}{2} d_R \tag{3.5}$$

3.3 Safety Stock Level

Safety stocks are the inventory buffer on top of cycle stock in order to safeguard against demand fluctuation and maintain a certain service level. It can be give by 3.6. Nicholas Vandeput defines the importance of safety stocks as "Buffer or Suffer" [45].

$$\begin{gathered} Inventory = Cycle\ Stock + Safety\ Stock \\ I = C_s + S_s \end{gathered} \tag{3.6}$$

3.3.1 Normal Volatility Model

We investigate one type of parametric model, in which the safety stock is calculated under the assumption that the demand is normally distributed. While other parametric assumptions are possible, we investigate the normal assumption as it is the most common. We make this assumption for both deterministic and stochastic lead times. As mentioned above, the safety stock is the inventory "buffer" on top of the cycle stock. The safety stock required to achieve a targeted service level, α, over the period, t, is noted by 3.7. Using the periodic review policy, the time period, t, is considered to be the time between two consecutive review periods plus the lead time, where the lead time is assumed to be deterministic. Therefore, for an (R, S) policy, safety stock is given by 3.10.

$$\begin{gathered} Safety\ Stock = Service\ Level\ Factor\ .\ Demand\ Deviation.\ \sqrt{time\ period} \\ S_s = z_\alpha . \sigma_d . \sqrt{t} \end{gathered} \tag{3.7}$$

Here, the demand deviation comes from a small portion of the demand signal generated in the section 3.1 using the likelihood function as described by 3.8. The maximum likelihood estimators are 3.9. For the periodic review inventory policy, the risk period is considered to be the sum of the review period, R, and the lead time, L, hence equation 3.7 becomes

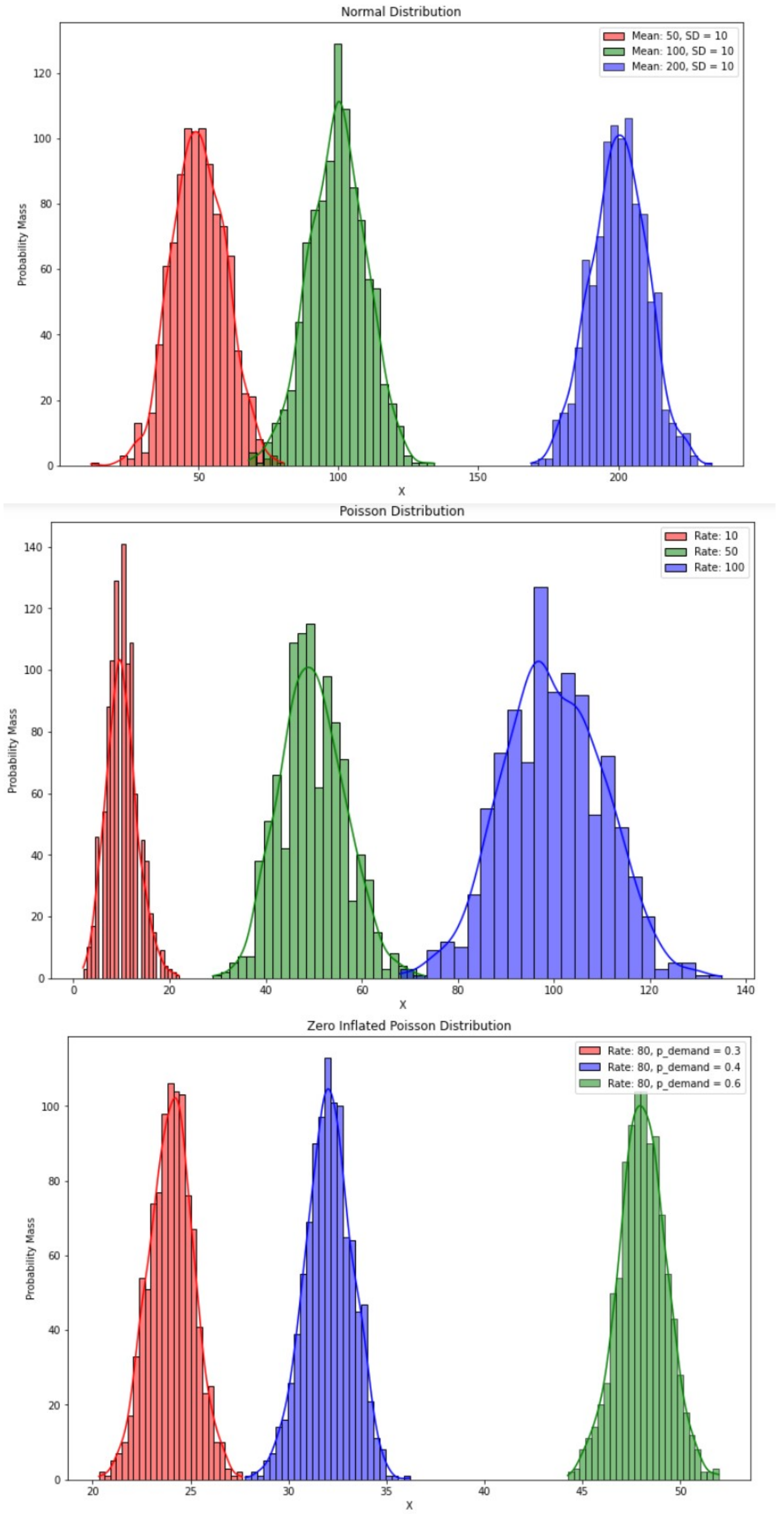


Figure 4: The three probability distributions used to generate the demand signals. From top to bottom, the first graph depicts the Normal distribution with a mean of 50, 100 and 200 and a standard deviation of 10. The middle graph depicts the Poisson distribution with rates 10, 50 and 100 and the bottom graph shows the combination of the binomial distribution and Poisson distribution with a rate 80 and a probability of success of 0.3, 0.4 and 0.6

$$l(x_1, ..., x_n; \mu, \sigma^2) = -\frac{n}{2}.\ln(2\pi) - \frac{n}{2}.\ln(\sigma^2) - \frac{1}{2\sigma^2}.\sum_{j=1}^{n}(x_j - \mu)^2 \tag{3.8}$$

$$\begin{aligned} \mu_d &= \frac{1}{n}.\sum_{j=1}^{n} x_j \\ \sigma_d^2 &= \frac{1}{n}.\sum_{j=1}^{n}(x_j - \mu_d)^2 \end{aligned} \tag{3.9}$$

$$S_s = z_\alpha.\sigma_d.\sqrt{R+L} \tag{3.10}$$

The above equation 3.10 is used when lead time is deterministic. However, the lead time is often not deterministic in practice. According to Vandeput [45], most supply chains face stochastic lead times, and the safety stock is not only a buffer against demand fluctuation but also against fluctuations in lead time, too. For stochastic demand and stochastic lead times, the safety stock for a periodic review policy is given by equation 3.11

$$S_s = z_\alpha.\sqrt{(\mu_L + R)\sigma_d^2 + \sigma_L^2\mu_d^2} \tag{3.11}$$

where

μ_L is the expected lead time and
σ_l^2 is the deviation from the expected lead time.

The second parameter used to calculate safety stock in the 3.7 is the service level factor. The service level factor is a ratio, estimated from the inverse of the cumulative standard Normal distribution function at probability α, given by 3.13.

The cumulative density function for the Normal distribution describes the probability of observing demand for a single period below some specified value and is used to compute the up-to-level (the inventory level including safety stock) at the beginning of the cycle. The inverse of the cumulative distribution function for the Normal distribution, $F_{\mathcal{N}}(\alpha)$, at the probability α is given by 3.12

$$F_{\mathcal{N}}^{-1}(\alpha;\ \mu,\ \sigma) = \mu +\ \sigma\ .\ \Phi^{-1}(\alpha) \tag{3.12}$$

The service level factor z_α is given by

$$z_\alpha = \phi^{-1}(\alpha) \tag{3.13}$$

The relationship between the cycle service level and service level factor is depicted 5

Cycle Service Level: α	50	60	70	75	80	82	85	86	88	89	90	92	95	97	99
Service Level Factor: z_α	0.00	0.25	0.52	0.67	0.84	0.92	1.04	1.08	1.17	1.23	1.28	1.41	1.64	1.88	2.33

Table 1: Service Level Factor for Cycle Service Level.

The algorithm used to calculate the safety stock level using the parametric approach is explained in 3

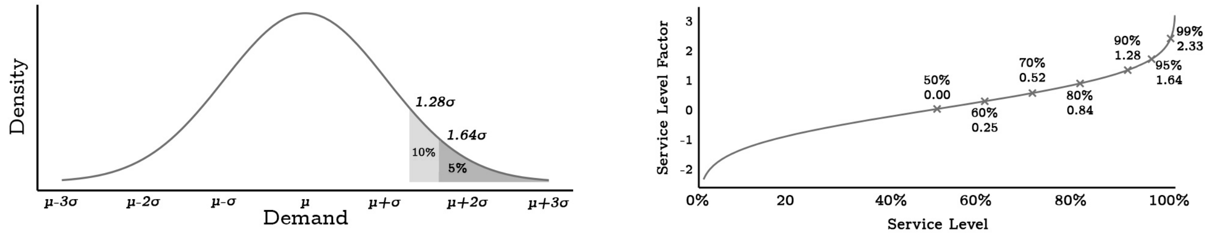


Figure 5: Typically, for a cycle service level of 95% the service level factor is 1.64. Similarly, if the inventory level at the start of the cycle is equal to the expected cycle demand and additionally 1.64 times the standard deviation of expected demand, then there is a 95% probability that stock out will not occur.

Algorithm 3: Function to Generate Safety Stock Level Using Parametric Approach

---

Input: *demand array*, *expected lead − time*, *deviation lead − time*, *target service level*, *review period*

Output: *safety stock level*

Function `normal volatility model(`*Input*`)`**:**

$demand\ mean \longleftarrow mean(demand\ array);$

$demand\ SD \longleftarrow SD(demand\ array);$

$z_\alpha \longleftarrow \phi^{-1}(\alpha);$

$x\_std \longleftarrow \sqrt{(\mu_L + R)\ \sigma_d^2 + \sigma_L^2 \mu_d^2};$

$safety\ stock\ level \longleftarrow z_\alpha * x\_std;$

return *safety stock level*;

End Function

In the above algorithm, the stochastic lead time formula is used. This caters to both deterministic and stochastic lead times because in the case of deterministic lead times the deviation of the lead time will be zero and the expected lead time will be equal to the nominal lead time value.

3.3.2 Kernel Density Estimation Volatility Model

The second approach used in this research is a non-parametric approach. This is a particularly good model for the calculation of safety stock levels when the demand volatility exhibits asymmetries, and the typical normality assumption does not hold. A well-known non-parametric method is Kernel density estimation [42].

Kernel density estimation (KDE) represents the probability density function of demand without any assumptions concerning the distribution of the data. The Kernel density formula for the array of demand X at point x is given by 3.14

$$f(x) = \frac{1}{Nh} . \sum_{i=1}^{N} K\left(\frac{(x - X_j)}{h}\right) \tag{3.14}$$

where

N is the number of time-steps in a single simulation

K(.) Kernel smoothing function with bandwidth h.

The service level quantile is then used to calculate the order up to level, which can be estimated non-parametrically using the empirical distribution fitted by the Kernel approach for the demand signal.

The algorithm used to calculate the safety stock level using a non-parametric approach is explained in 4

Algorithm 4: Function to Generate Safety Stock Level Using Non-Parametric Approach

Input: $demand\ array,\ expected\ lead - time,\ deviation\ lead - time,\ target\ service\ level, review\ period$

Output: *up_to_level*

Function `KDE volatility model(`$Input$`):`

$aggregate \longleftarrow\ expected\ lead - time + review\ period;$

$cumulative\ sum \longleftarrow cumulative\ sum(demand\ array);$

$rolling\ sum \longleftarrow cumulative\ sum[aggregate :] - cumulative\ sum[: -aggregate];$

$up\_to\_level \longleftarrow quantile_{\alpha}(rolling\ sum);$

return *up_to_level*;

End Function

3.4 Order Up-to-level

After calculating the safety stock level, the order is up-to-level (the "maximum inventory level" is calculated). Order up-to-level is the level of stock that would be needed in the risk period, which means it is the summation of demand over the lead time and review period, and the amount of safety stock kept. The basic equation for order up-to-level calculation is given by 3.15

$$
\begin{aligned}
S &= d_L + d_R + Safety\ Stock \\
S &= d_L + d_R + S_s
\end{aligned} \tag{3.15}
$$

For different volatility models, the order up-to-level changes accordingly. For the (parametric) Normal distribution approach, the order level is given by 5

Algorithm 5: Function to Calculate Order up-to-level Using Parametric Approach

Input: $demand\ mean,\ expected\ lead - time,\ safety\ stock\ level, review\ period$

Output: *up_to_level*

Function `order up_to_level parametric(`$Input$`):`

$cycle\ stock \longleftarrow\ demand\ mean * review\ period;$

$average\ in - transit \longleftarrow demand\ mean * expected\ lead - time;$

$up\_to\_level \longleftarrow safety\ stock\ level + cycle\ stock + average\ in - transit;$

return *up_to_level*;

End Function

whereas, for the non-parametric approach the order up-to-level is given by service quantile of the rolling sum of the demand as shown in the algorithm 4.

3.5 In-transit Inventory

In-transit inventory is the goods ordered from suppliers that have not yet arrived, and are hence not available "on hand" to satisfy the demand. These goods are considered to be in transit. The expected in-transit inventory is given by demand over lead time as depicted 3.16

$$
\begin{aligned}
I_s &= Q\frac{L}{T} \\
I_s &= d.L = d_L
\end{aligned} \tag{3.16}
$$

The equation 3.16 suggests that in order to decrease the average in-transit inventory, the lead time should be reduced. This is because simply reducing the number of orders will just result in the same amount of average in-transit inventory: we will just be ordering more at once to satisfy the demand. The algorithm used to calculate the in-transit inventory is given by 6

Algorithm 6: Function to Calculate In-transit Inventory

Input: *initial_intransit*, *up_to_level*, *inventory*, *timeperiod*, *review_period*, *replenishment*
Output: *intransit*
Function `Calculate In-transit`(*Input*):
  intransit = initail_intransit
  if *mod*(*timeperiod*, *review_period*) *equal to* 0 **then**
    net_inventory ⟵ *inventory* + *intransit*
    raw_order_qty ⟵ *max*(*up_to_level* − *net_inventory*, 0)
    intransit ⟵ *intransit* + *raw_order_qty*
  end
  intransit ⟵ *intransit* − *replenishment*
  return *intransit*
End Function

4 Metrics & Implementation

This chapter includes the concepts used as metrics to measure the performance of volatility models used to calculate safety stock levels. The metrics include cycle service level, period service level, fill rate and inventory-demand ratio. This chapter also includes the detailed explanation of the simulation used to carried out the experiments in form of algorithm.

4.1 Service Levels

The service level is the probability that demand will less than the supply during lead time. It is defined as the complement of the stock-out probability [24].

It can be calculated

The cost of the inventory policy increases dramatically (exponentially) with an increase in service levels

4.1.1 Cycle Service Level

Cycle service level is the complement of probability of stock-out during an order replenishment cycle, which means it is the probability that the inventory available at the beginning of review period (order cycle) will be sufficient to fulfil the demand during this cycle, conversely, there will be no stock-outs between two review periods. It is denoted by α and also known as "Type 1 service level"

$$\begin{aligned} \mathbb{P}(D \leq on-hand\ inventory) \geq \alpha \\ \mathbb{P}(total\ demand\ over\ order\ cycle \leq on-hand\ inventory) \geq \alpha \end{aligned} \tag{4.1}$$

4.1.2 Period Service Level

The period service level is similar to the cycle service level, with the difference that it describes the probability of not having stock-out in an arbitrary period (which can be identical to cycle service level, depending upon the length of the order cycle). It is denoted by α_p.

4.2 Fill Rate

Fill rate is defined as the fraction of demand that is completely served by on hand inventory. It is denoted by β and also known as "Type 2 service level".

$$\mathbb{E}\left(\frac{demand\ fulfilled\ per\ cycle}{total\ demand\ per\ order\ cycle}\right) \geq \beta \tag{4.2}$$

Demand fulfilled is calculated as the difference of total demand per cycle and back-orders. In case back-orders are not permitted, the demand fulfilled will be total demand per cycle minus the lost sales.

4.3 Inventory-Demand Ratio

Inventory-demand ratio determines how much more or less the average inventory level is kept in comparison to the average demand. For the given targeted service level it can be seen that it is possible that one volatility model results in lower inventory-demand ratio however not fulfils the required service level criterion. It is given by equation 4.3

$$inventory - demand\ ratio = \frac{avg\ inventory\ level}{avg\ demand} \tag{4.3}$$

4.4 Implementation

All experiments are carried out using an inventory simulation, programmed in the Python language.

Since these simulations needed to be fast enough to be run many thousands of times (in order to reduce the effects of the random sampling present in the experiments), we use a just-in-time code compilation strategy [30]. This approach is both fast and flexible, permitting various extensions to be added or removed as necessary.

The inventory simulation program works by sequentially updating a state variable, which holds all relevant information about a simple supply chain problem: the quantity of on-hand inventory, the number of items currently in transit, the demand on that timestep, the received goods on that timestep; along with information about the inventory policy itself, such as the order up-to-level, review period and lead time (which might be a distribution). These state variables are recorded at the end of each timestep.

While conceptually many of these quantities could be calculated at every timestep, some are pre-computed for speed: The demand on each timestep is drawn from the desired volatility distribution for the desired number of timesteps before running the simulation. If backorders are present, this array is manipulated inside the simulation function when backorders occur. The order up-to-level is pre-computed for each simulation run using a limited number of samples from the demand distribution (precisely, the first n, with n being the number of training samples), in order to mimic the process of fitting and using a given volatility model in practice as closely as possible.

A high-level overview of the logic for a single timestep, t, is as follows:

- Extract the demand at timestep t from the demand array.
- Check this timestep's demand against the current on-hand inventory:
  - If the demand is less than or equal to the on-hand inventory, record satisfied demand equivalent to the observed demand, and subtract the same amount from the on-hand inventory.
  - If the demand is greater than the on-hand inventory, record satisfied demand equivalent to the on-hand inventory, and record unsatisfied demand equivalent to the demand minus the on-hand inventory.
    - If assuming for backorders, increment the demand at $t + 1$ by the unsatisfied demand.
    - If not assuming backorders, simply record the unsatisfied demand.
- If in the review period:
  - Calculate the net inventory as the sum of the current on hand inventory, plus the currently in transit inventory.
  - If the net inventory is less than the up-to level, designate the order quantity as the difference between the up to level and the net inventory.
  - Increment the currently in transit inventory by the order quantity.
  - Sample a random lead time, l, for the order that has just been placed.
  - Increment the replenishment array at $t + l$ with the order quantity.
- Extract the replenishment at timestemp t from the replenishment array.
- Increment the on-hand inventory by the number of replenished items on this time step.
- Decrease the in-transit quantity by the number of replenished items on this time step.

Note that the above logic assumes that replenishment occurs at the end of a given timestep. Since no production (i.e. transformation of raw materials into finished goods) in this process, we implicitly assume we are dealing with demand for finished goods. The algorithm is mentioned at 7

All of the simulation code is available at [47]

Algorithm 7: Function to Generate Combination of Demand Signal

Input: $timestep, demand\ array, replenish\ array, review\ period\ , in-transit\ inventory\ , up\_to\_level\ , lead\_times\ distribution, num\_timesteps, backorders$

Output: $demand\ array, replenish\ array, on-hand\ inventory\ , in-transit\ inventory\ , satisfied\ demand\ , unsatisfied\ demand\ , replenish$

Function `r_s_policy`($Input$):

$demand \longleftarrow demand\ array[timestep]$

if $demand \leq on-hand\ inventory$ **then**

$satisfied\ demand \longleftarrow demand$

$(on-hand\ inventory) \longleftarrow (on-hand\ inventory) - satisfied\ demand$

end

if $demand > on-hand\ inventory$ **then**

$satisfied\ demand \longleftarrow on-hand\ inventory$

$unsatisfied\ demand \longleftarrow demand - (on-hand\ inventory)$

end

if $backorders$ **then**

$demand\ array[timestep+1] \longleftarrow demand\ array[timestep+1] + unsatisfied\ demand$

end

if $timestep \mod review\ period$ **then**

$net\ inventory \longleftarrow (on-hand\ inventory\ ) + (in-transit\ inventory\ )$

$raw\ order\ quantity \longleftarrow \max(net\ inventory - up\_to\_level\ , 0)$

$in-transit\ inventory \longleftarrow in-transit\ inventory + raw\ order\ quantity$

$l \longleftarrow sampleRandom(lead\_times\ distribution)$

$replenish\ array[timestep+l] \longleftarrow replenish\ array[timestep+l] + raw\ order\ quantity$

end

$replenish \longleftarrow replenish\ array[timestep]$

$on-hand\ inventory \longleftarrow (on-hand\ inventory\ ) + replenish$

$in-transit\ inventory \longleftarrow (in-transit\ inventory) - replenish$

return $demand\ array, replenish\ array, on-hand\ inventory\ , in-transit\ inventory\ , satisfied\ demand\ , unsatisfied\ demand\ , replenish$

End Function

5 Analysis

This chapter is divided into two sections. The first section presents the analysis with stochastic demand and deterministic lead time, whereas the second part presents the analysis where the stochastic lead time is considered along with stochastic demand.

5.1 Stochastic Demand and Deterministic Lead Time Analysis

This is the first section of the analysis where stochastic demand and deterministic lead time are considered. This section is divided into six sub-sections based on the possible combinations between the probability distribution used to generate the demand signal and whether the backorders were permitted or not. Since, there are three distributions used to generate demand signals namely, Normal distribution, Poisson Distribution and a Combination of Binomial and Poisson Distribution, and two possible values for backorder permission (backorders and lost sales) there is a total of six combinations. The experiment is carried out for each sub-section in a manner alike with targeted values of service level varying from 0.70 to 0.99 with a step size of 0.005. Additionally, three lead time values are considered i.e. 2, 4, and 10 which represent numbers less than, equal to and more than the value of the review period, which is 4.

5.1.1 Normal Distribution Demand With Lost Sales

Demand is generated from Normal distribution as mentioned in 3.1.1 and the backorders are not permitted.

Experiments start with lead time value 2 and review period 4, for this scenario the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 6 for both volatility models. The highest average achieved cycle and period service level and fill rate achieved by the normal volatility model are 0.94, 0.98 and 0.99 respectively which is against the target service level of 0.95, whereas for the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.94 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.95 and 0.99. A few values of above mention relation are shown in the table. 2 and 3

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.955	0.965	0.976	0.985	0.997
Achieved Cycle Service Level	0.821	0.861	0.906	0.949	0.989
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 2: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 02, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.955	0.965	0.975	0.986	0.996
Achieved Cycle Service Level	0.818	0.859	0.900	0.946	0.984
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 3: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 02, Demand Distribution: Normal

The second metric used in this thesis is an inventory-demand ratio and for this metric, the Kernel density estimation volatility model behaves slightly better in a way that for the 0.95 target service level the ratio is less than the ratio for the normal volatility model. The trend is depicted in figure 7

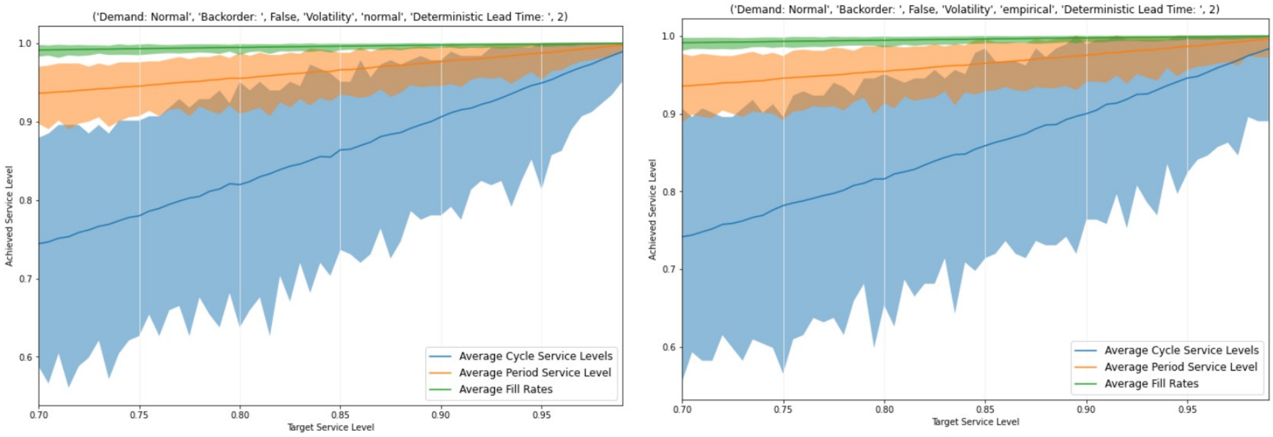


Figure 6: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend when the normal volatility model is applied to calculate safety stock levels, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock level for the demand signal generated from the Normal distribution and lead time equal to 2

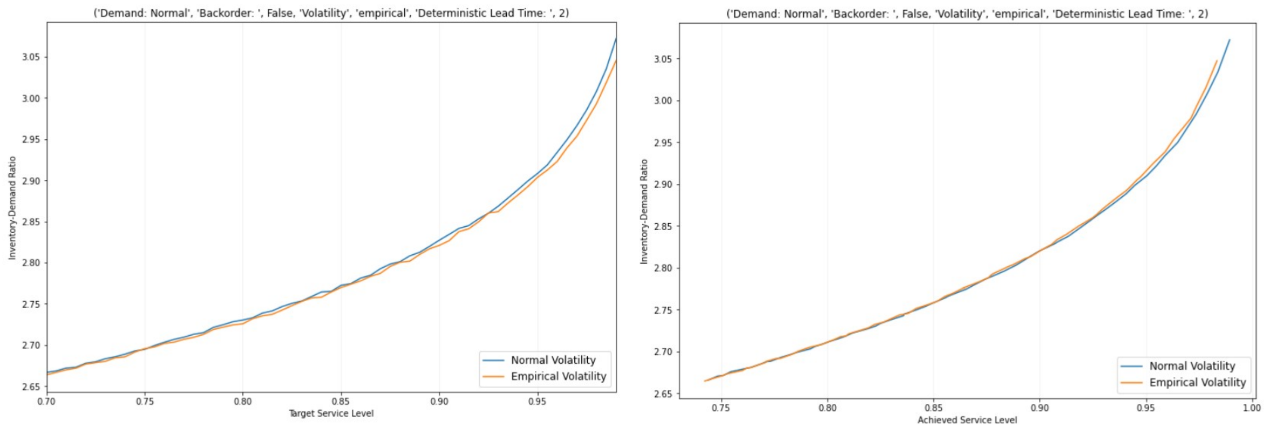


Figure 7: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 2

Next, both lead time and review period value is equal to 4 and the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 8 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.95, 0.98 and 0.99 respectively which is against the target service level of 0.95 when safety stock is calculated from the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle is 0.94 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.95 and 0.99 as depicted in the table 4 and 5

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.958	0.967	0.977	0.988	0.997
Achieved Cycle Service Level	0.831	0.868	0.910	0.951	0.990
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 4: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 04, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.956	0.966	0.976	0.987	0.996
Achieved Cycle Service Level	0.822	0.865	0.902	0.946	0.984
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 5: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 04, Demand Distribution: Normal

For the lead time value equal to 4 the inventory-demand ratio resulting from the Kernel density estimation volatility model behaves slightly better. For the 0.95 target service level, the ratio is less than the ratio for the normal volatility model. The trend is depicted in figure 9

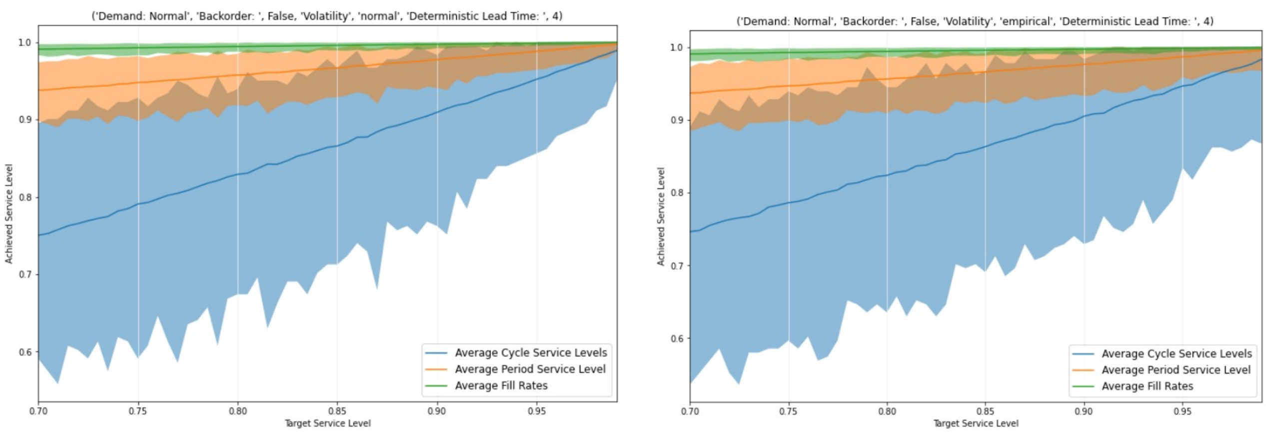


Figure 8: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 4

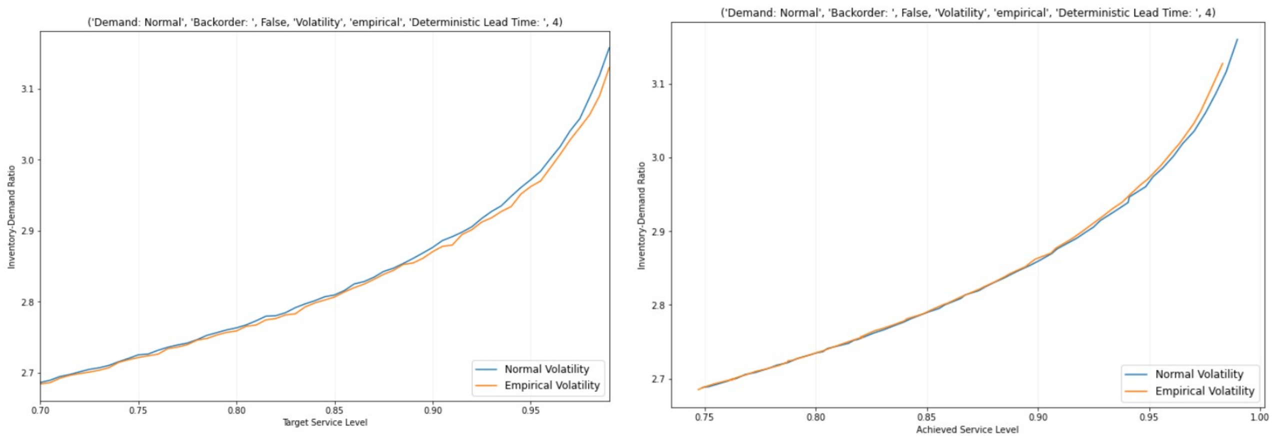


Figure 9: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 4

The last value of lead time considered is 10 and the trend of achieved period and cycle service levels and fill rates against the targeted service level, in this case, is depicted in the figure 10 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.95, 0.98 and 0.99 respectively for the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.95 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 6 and 7

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.965	0.937	0.981	0.989	0.997
Achieved Cycle Service Level	0.861	0.893	0.923	0.958	0.989
Fill Rate	0.995	0.996	0.997	0.999	1.000

Table 6: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 10, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.964	0.972	0.98	0.988	0.996
Achieved Cycle Service Level	0.858	0.887	0.921	0.951	0.982
Fill Rate	0.995	0.996	0.997	0.998	0.999

Table 7: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 10, Demand Distribution: Normal

With lead time equal to 10 the inventory-demand ratio resulting from the Kernel density estimation volatility model again behaves slightly better in the same way as previously. The trend is depicted in figure 11

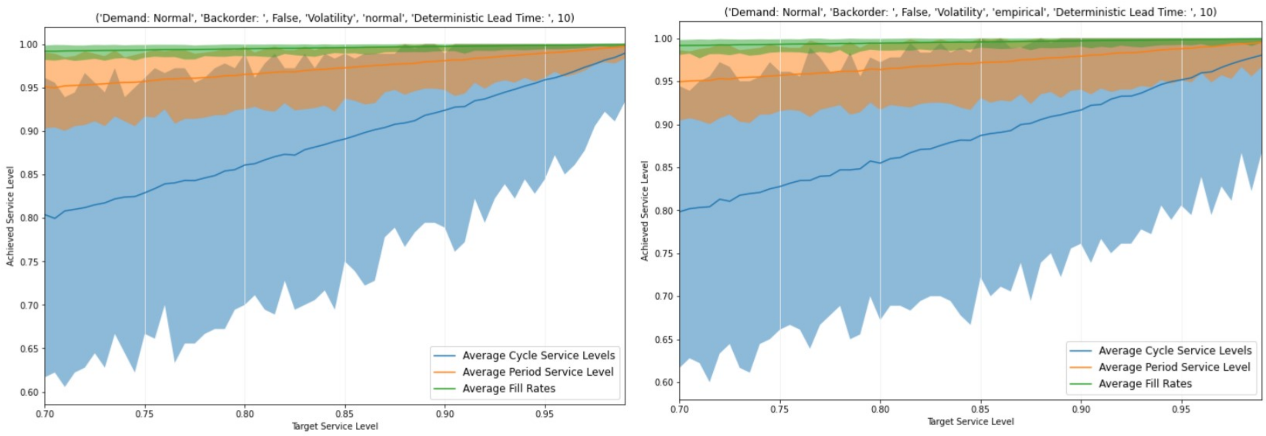


Figure 10: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 10

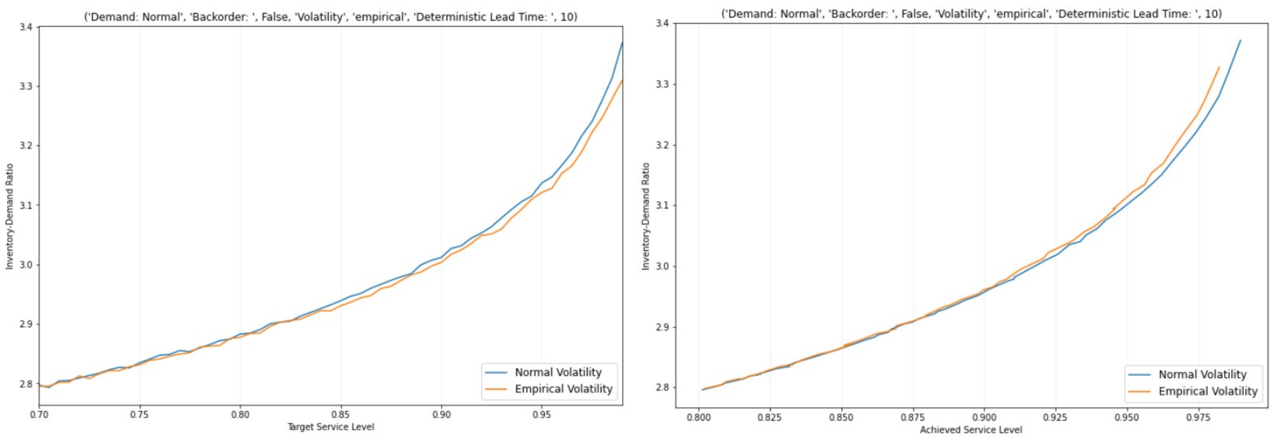


Figure 11: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 10

5.1.2 Normal Distribution Demand With Backorders

In this sub-section demand is again generated from Normal distribution as mentioned in 3.1.1 and the backorders are permitted.

For lead time value 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 12 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.94, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.94 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 8 and 9

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.948	0.961	0.973	0.985	0.997
Achieved Cycle Service Level	0.791	0.843	0.894	0.945	0.898
Fill Rate	0.993	0.995	0.997	0.999	1.000

Table 8: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 02, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.947	0.960	0.973	0.985	0.996
Achieved Cycle Service Level	0.787	0.841	0.891	0.942	0.983
Fill Rate	0.993	0.995	0.997	0.999	1.000

Table 9: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 02, Demand Distribution: Normal

For the second metric, the inventory-demand ratio, the Kernel density estimation volatility model behaves slightly better in a way that for the 0.95 target service level the ratio is less than the ratio for the normal volatility model. The trend is depicted in figure 13

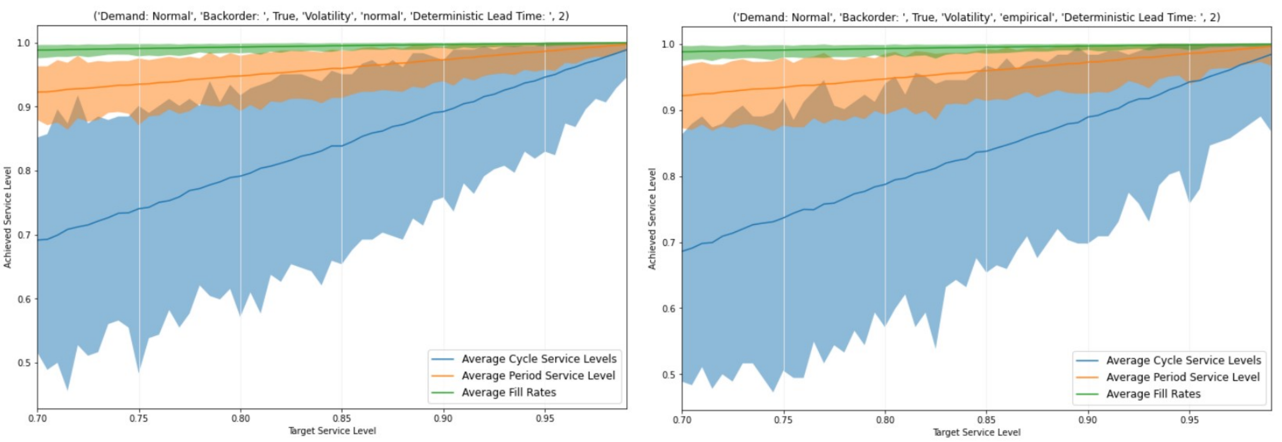


Figure 12: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 2

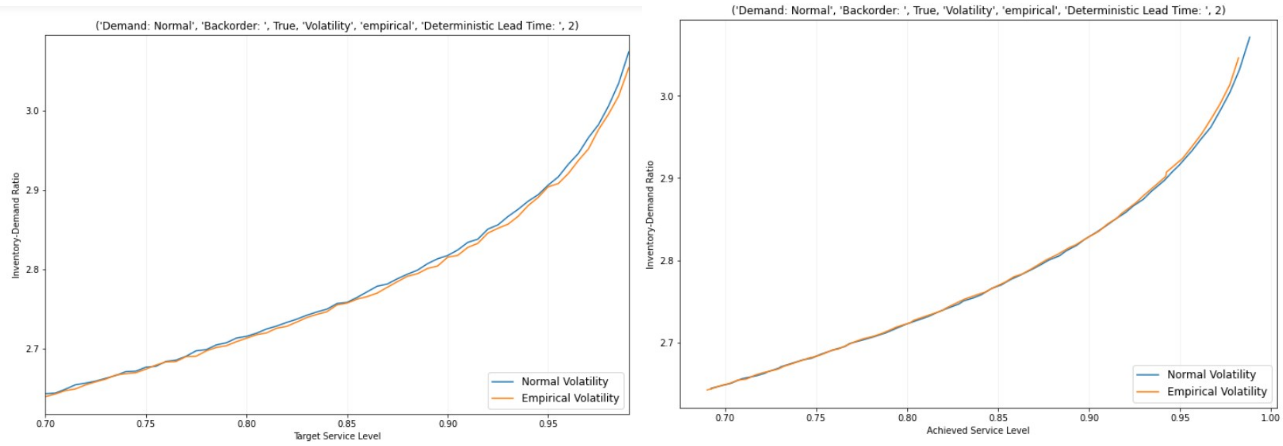


Figure 13: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 2

When both lead time value and review period are equal to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 14 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.94, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.93 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 10 and 11

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.931	0.952	0.969	0.986	0.997
Achieved Cycle Service Level	0.723	0.808	0.875	0.942	0.998
Fill Rate	0.985	0.992	0.995	0.998	1.000

Table 10: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 04, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.927	0.947	0.967	0.984	0.995
Achieved Cycle Service Level	0.710	0.789	0.869	0.934	0.980
Fill Rate	0.984	0.990	0.995	0.998	0.999

Table 11: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 04, Demand Distribution: Normal

The second metric, the inventory-demand ratio still has the same behaviour. The trend is depicted in figure 15

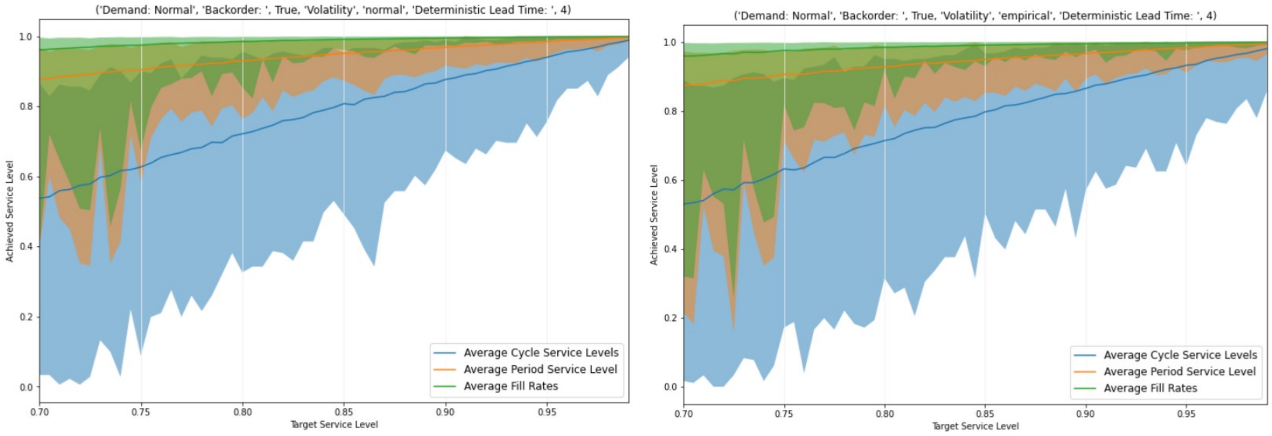


Figure 14: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 4

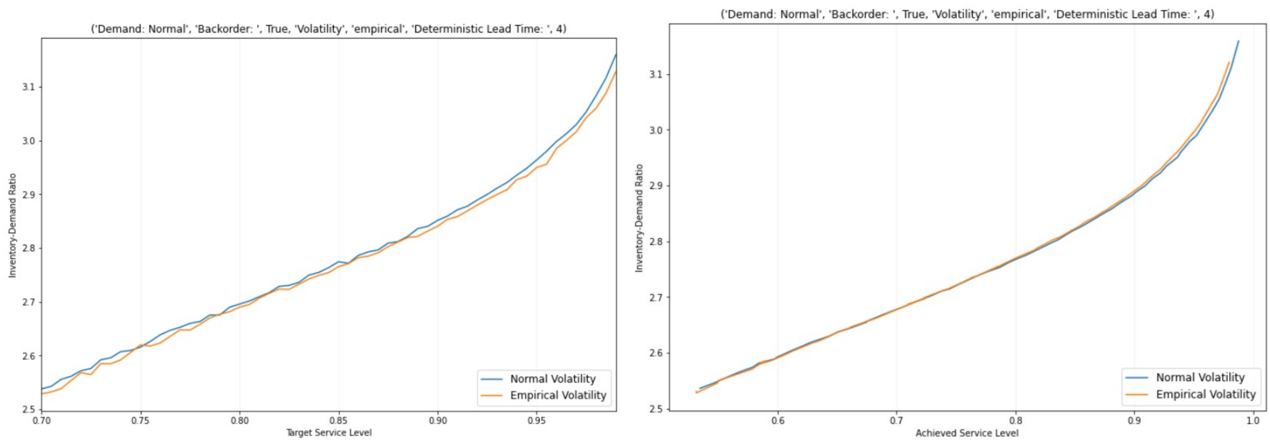


Figure 15: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 4

For lead time value 10 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 16 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.94, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.93 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 12 and 13

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.948	0.960	0.973	0.985	0.997
Achieved Cycle Service Level	0.791	0.841	0.894	0.942	0.987
Fill Rate	0.989	0.992	0.995	0.998	1.000

Table 12: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 10, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.947	0.954	0.971	0.984	0.994
Achieved Cycle Service Level	0.788	0.834	0.883	0.935	0.978
Fill Rate	0.989	0.992	0.995	0.997	0.999

Table 13: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 10, Demand Distribution: Normal

The second metric, the inventory-demand ratio resulting from the Kernel density estimation volatility model behaves slightly better than the ratio resulting from the normal volatility model, the trend is depicted in the figure 17

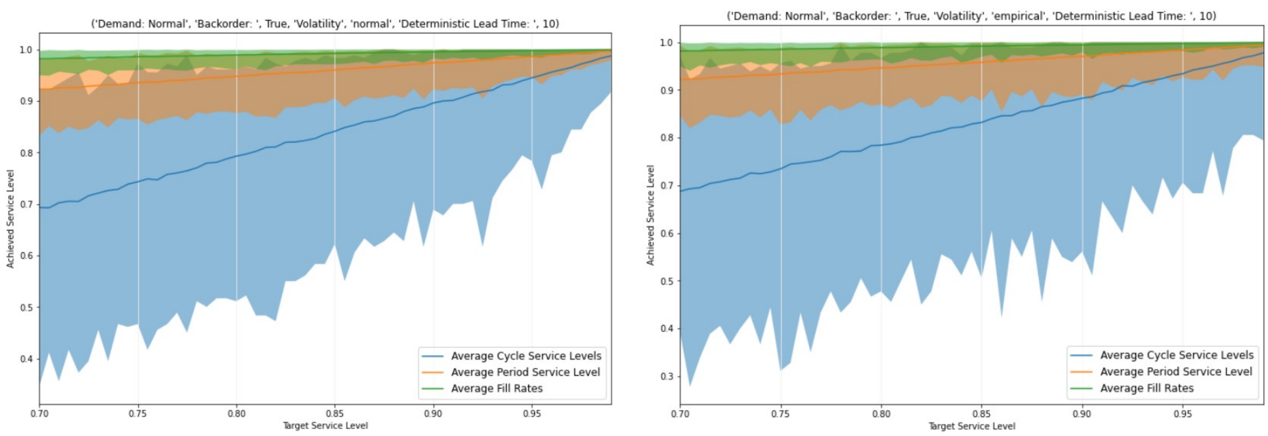


Figure 16: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 10

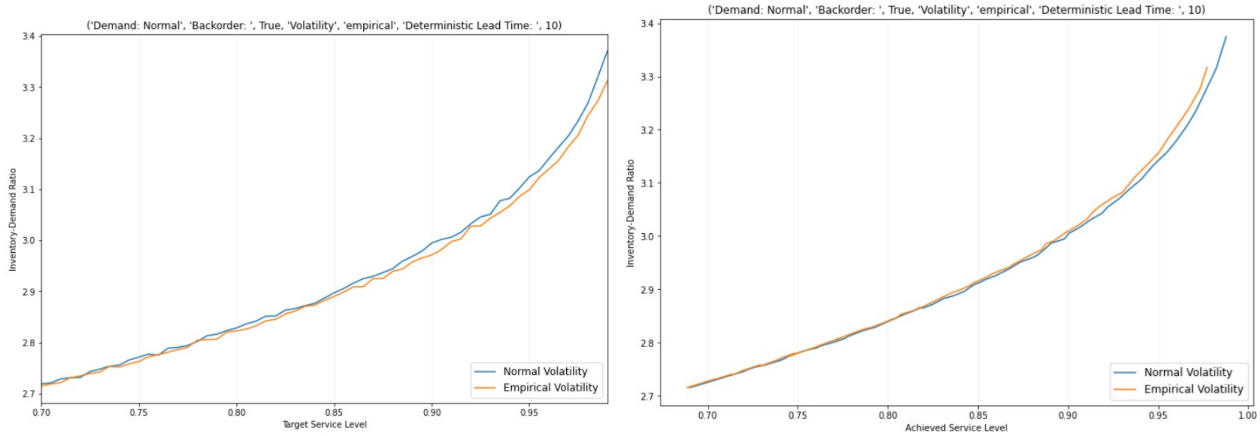


Figure 17: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 10

5.1.3 Poisson Distribution Demand With Lost Sales

In this section, the demand signal is generated from Poisson distribution as mentioned in 3.1.2 and Lost of Sales is considered.

When the lead time value is 2 and the review period is 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 18 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.95, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.95 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 14 and 15

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.956	0.966	0.976	0.987	0.997
Achieved Cycle Service Level	0.825	0.863	0.906	0.949	0.988
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 14: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 02, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.955	0.964	0.976	0.987	0.996
Achieved Cycle Service Level	0.818	0.857	0.904	0.946	0.983
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 15: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 02, Demand Distribution: Poisson

Although the second metric, the inventory-demand ratio resulting from the Kernel density estimation volatility model behaves slightly better than the ratio resulting from the normal volatility model, the trend is depicted in the figure 19

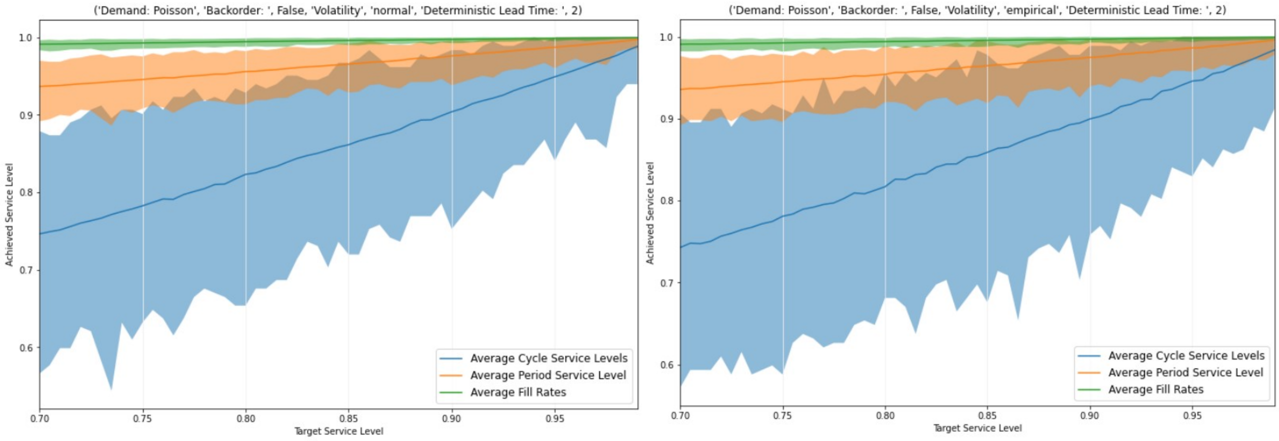


Figure 18: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 2

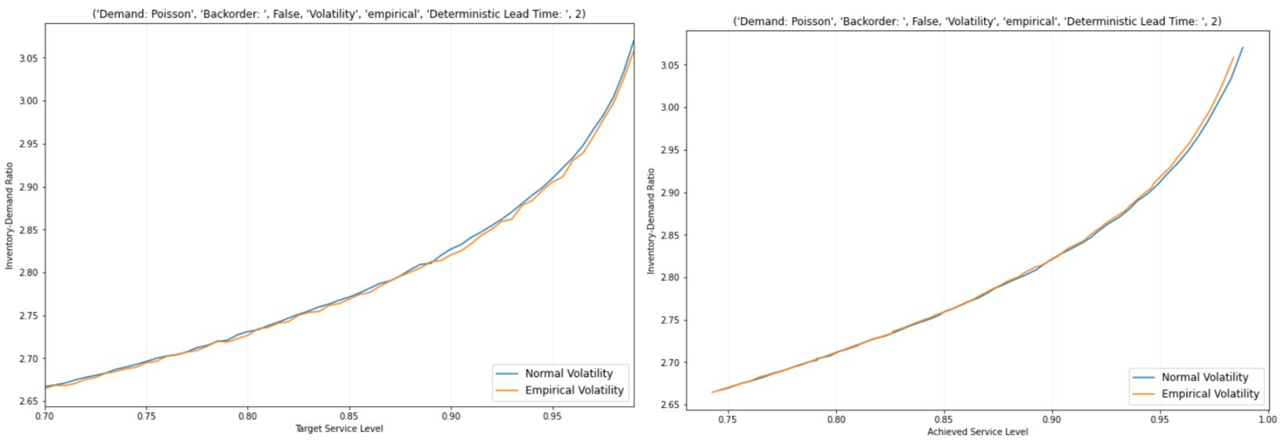


Figure 19: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 2

For both lead time value review period equals 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 20 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.95, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.95 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 16 and 17

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.957	0.966	0.977	0.988	0.997
Achieved Cycle Service Level	0.828	0.865	0.909	0.951	0.988
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 16: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 04, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.956	0.966	0.976	0.987	0.996
Achieved Cycle Service Level	0.825	0.864	0.904	0.947	0.983
Fill Rate	0.994	0.996	0.997	0.999	1.000

Table 17: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 04, Demand Distribution: Poisson

Again the inventory-demand ratio resulting from the Kernel density estimation volatility model behaves slightly better than the ratio from the normal volatility model. The trend is depicted in figure 21

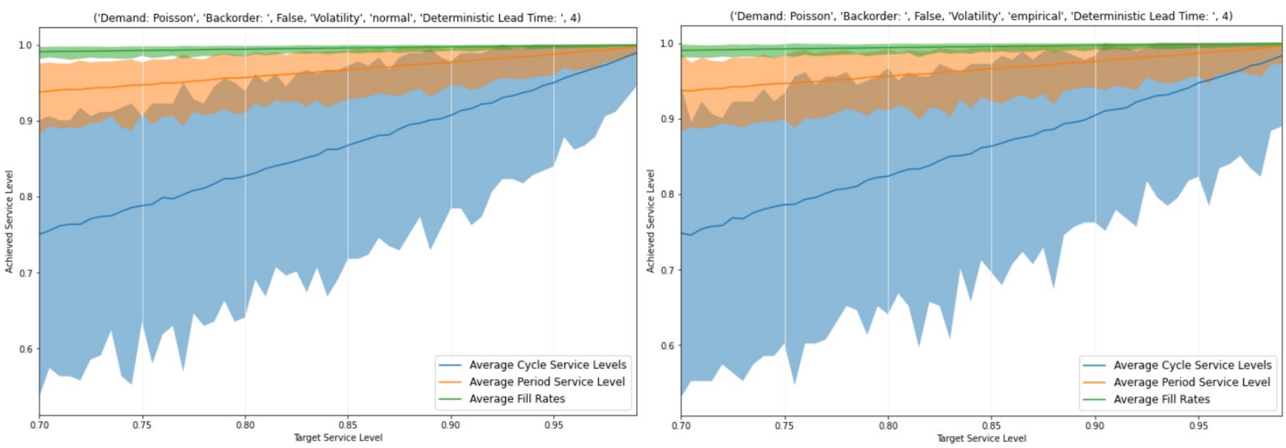


Figure 20: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 4

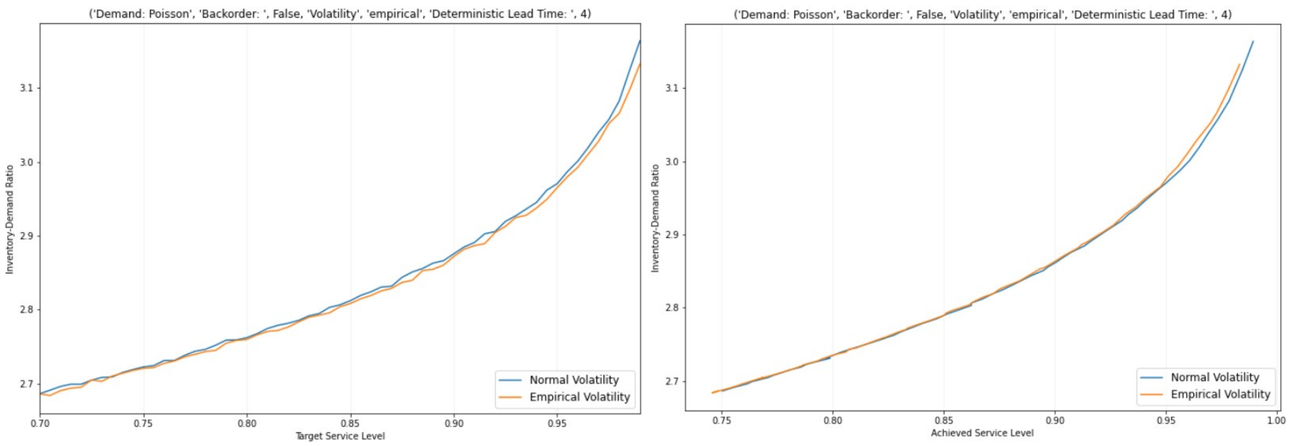


Figure 21: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 4

For lead time value equals 10 and review period equals 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 22 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.95, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.95 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 18 and 19

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.966	0.973	0.981	0.989	0.997
Achieved Cycle Service Level	0.864	0.892	0.924	0.958	0.989
Fill Rate	0.995	0.996	0.997	0.999	1.000

Table 18: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 10, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.964	0.971	0.980	0.988	0.996
Achieved Cycle Service Level	0.856	0.886	0.919	0.951	0.983
Fill Rate	0.994	0.996	0.997	0.998	0.999

Table 19: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 10, Demand Distribution: Poisson

Again inventory-demand ratio results in the same trend as previously. It is depicted in figure 23

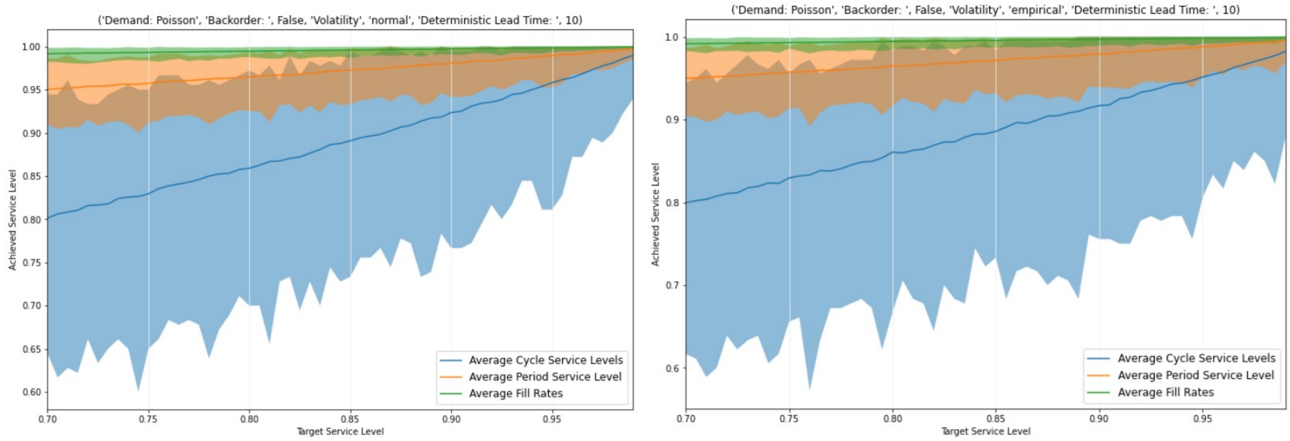


Figure 22: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 10

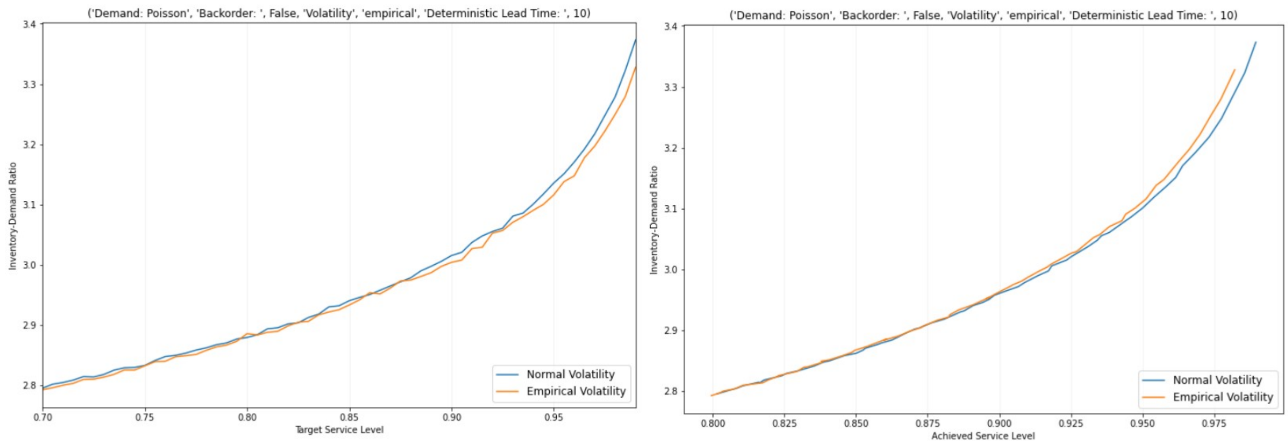


Figure 23: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 10

5.1.4 Poisson Distribution Demand With Backorders

In this section, again the demand signal is generated from Poisson distribution as mentioned in 3.1.2 and backorders are permitted.

For lead time value equals 2 and review period equals 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 24 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.94, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.94 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 20 and 21

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.948	0.961	0.973	0.986	0.997
Achieved Cycle Service Level	0.791	0.843	0.892	0.944	0.988
Fill Rate	0.993	0.995	0.997	0.999	1.000

Table 20: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 02, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.947	0.959	0.972	0.985	0.996
Achieved Cycle Service Level	0.788	0.837	0.890	0.940	0.983
Fill Rate	0.993	0.995	0.997	0.998	1.000

Table 21: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 02, Demand Distribution: Poisson

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model behaves slightly better in a way than the ratio resulting from the normal volatility model. The trend is depicted in figure 25

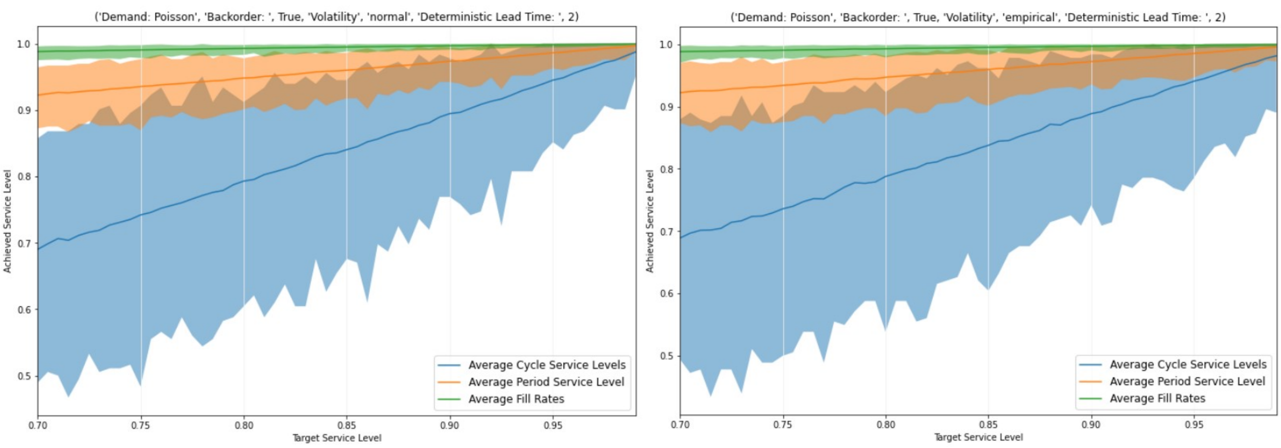


Figure 24: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 2

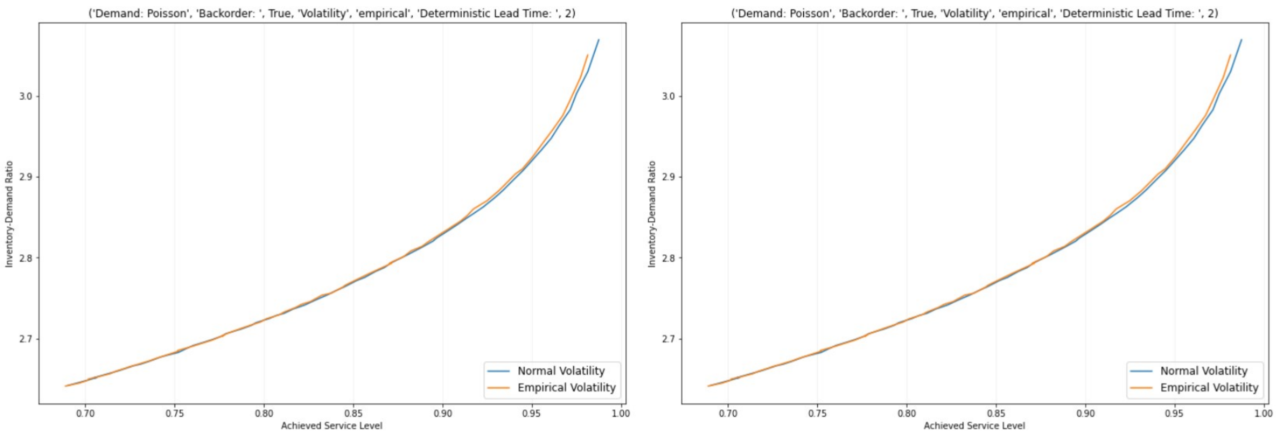


Figure 25: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 2

For both lead time value and review period equals to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 26 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.93, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.93 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 22 and 23

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.930	0.951	0.968	0.985	0.997
Achieved Cycle Service Level	0.722	0.805	0.874	0.939	0.987
Fill Rate	0.985	0.991	0.995	0.998	1.000

Table 22: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 04, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.926	0.948	0.967	0.983	0.995
Achieved Cycle Service Level	0.706	0.792	0.866	0.933	0.980
Fill Rate	0.983	0.990	0.995	0.998	0.999

Table 23: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 04, Demand Distribution: Poisson

Again the inventory-demand ratio resulting from the Kernel density estimation volatility model is lower than the ratio resulting from the normal volatility model. The trend is depicted in figure 27

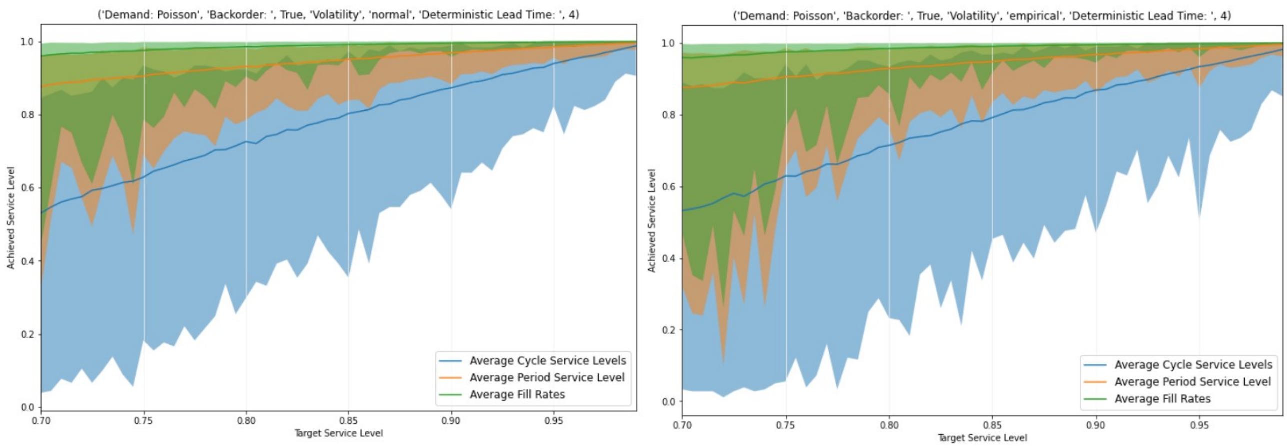


Figure 26: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 4

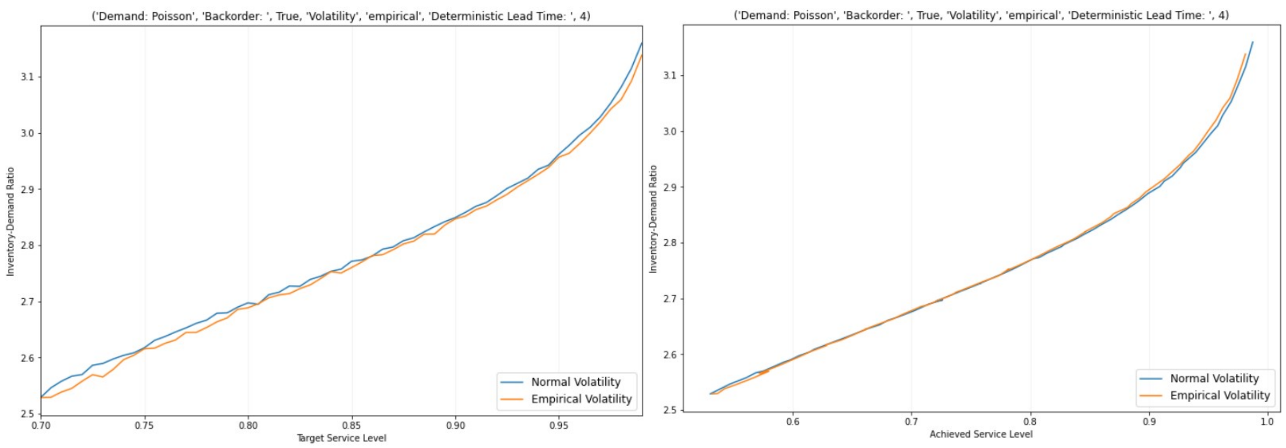


Figure 27: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 4

For lead time value 10 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 28 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.94, 0.98 and 0.99 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.93 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.98 and 0.99. A few values of above mention relation is shown in table 24 and 25

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.947	0.960	0.973	0.986	0.997
Achieved Cycle Service Level	0.790	0.842	0.892	0.943	0.987
Fill Rate	0.989	0.992	0.995	0.998	1.000

Table 24: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 10, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.946	0.958	0.972	0.984	0.994
Achieved Cycle Service Level	0.787	0.833	0.888	0.936	0.977
Fill Rate	0.989	0.992	0.995	0.997	0.999

Table 25: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 10, Demand Distribution: Poisson

The inventory-demand ratio for lead time equals to 10 resulted from both volatility models is depicted in figure 29

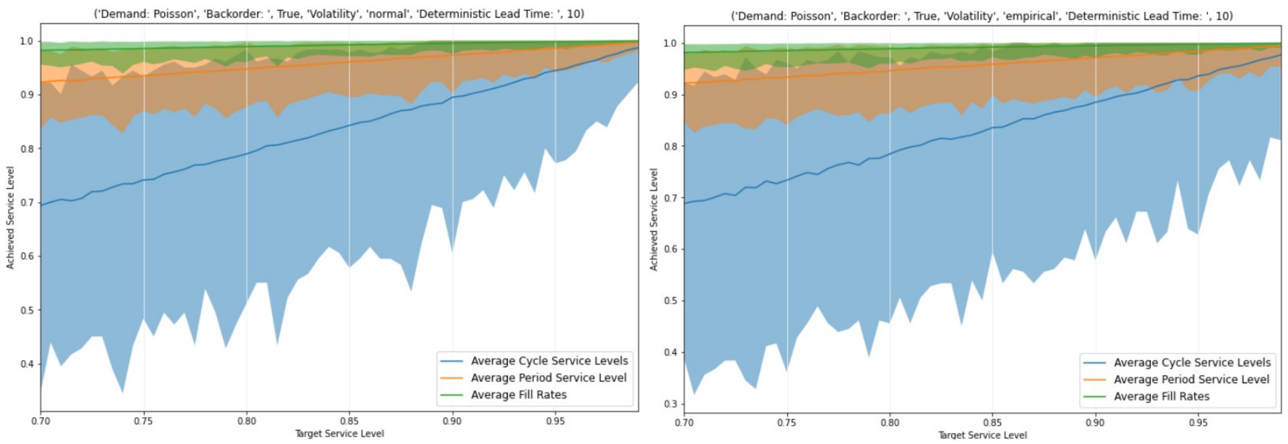


Figure 28: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 10

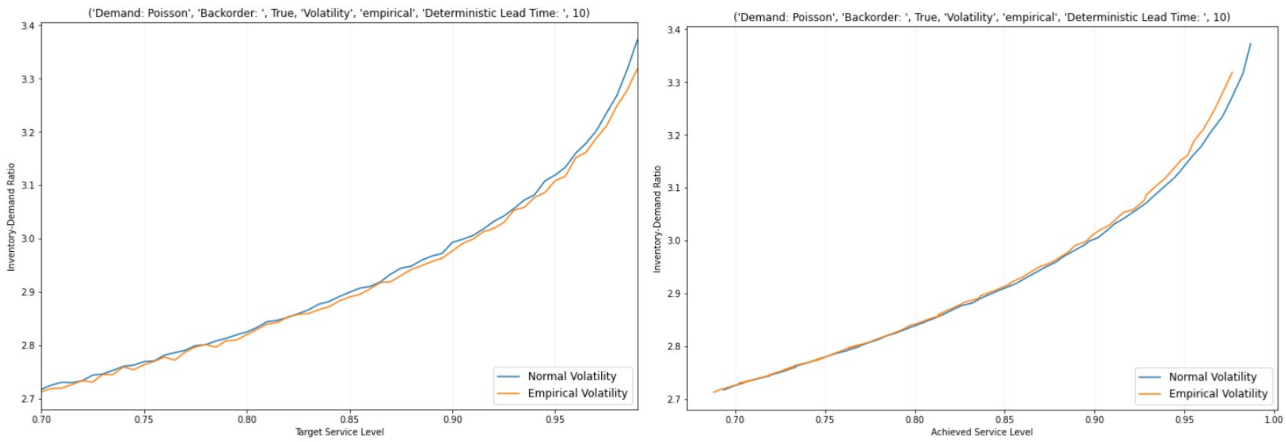


Figure 29: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 10

5.1.5 Binomial and Poisson Distribution Demand with Lost Sales

In this section, the demand signal is generated from the combination of Binomial and Poisson distribution as mentioned in 3.1.3 and backorders are not permitted.

For lead time value 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 30 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.93, 0.97 and 0.96 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.94 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.97. A few values of above mention relation is shown in table 26 and 27

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.910	0.931	0.952	0.973	0.992
Achieved Cycle Service Level	0.818	0.855	0.894	0.936	0.979
Fill Rate	0.893	0.919	0.945	0.969	0.991

Table 26: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 02, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.906	0.931	0.954	0.976	0.993
Achieved Cycle Service Level	0.811	0.855	0.898	0.943	0.982
Fill Rate	0.890	0.919	0.947	0.972	0.993

Table 27: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 02, Demand Distribution: Poisson and Binomial

Although, for the second metric, the inventory-demand ratio, the normal volatility model behaves slightly better in a way that for the 0.95 target service level the ratio is less than the ratio for the Kernel density estimation volatility model. The trend is depicted in figure 31

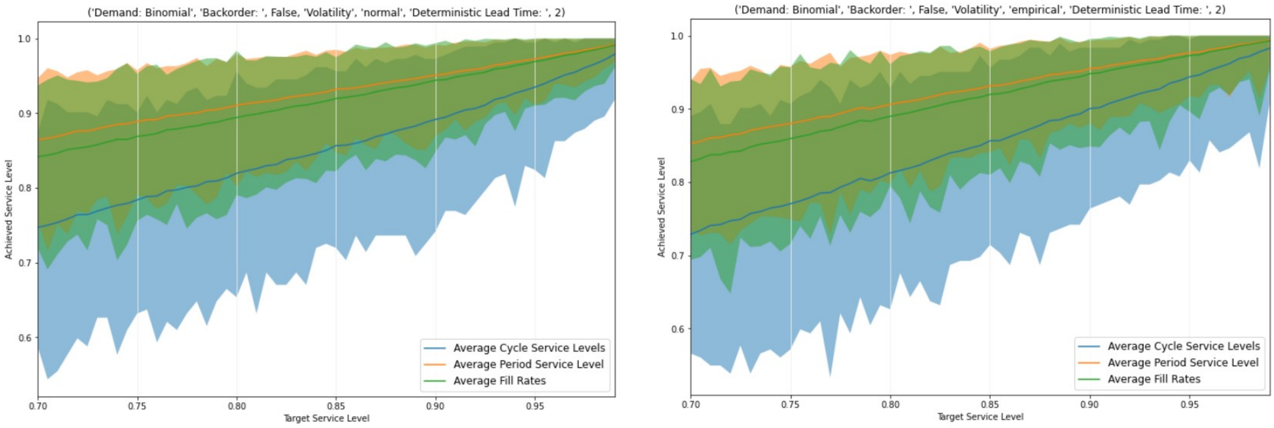


Figure 30: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 2

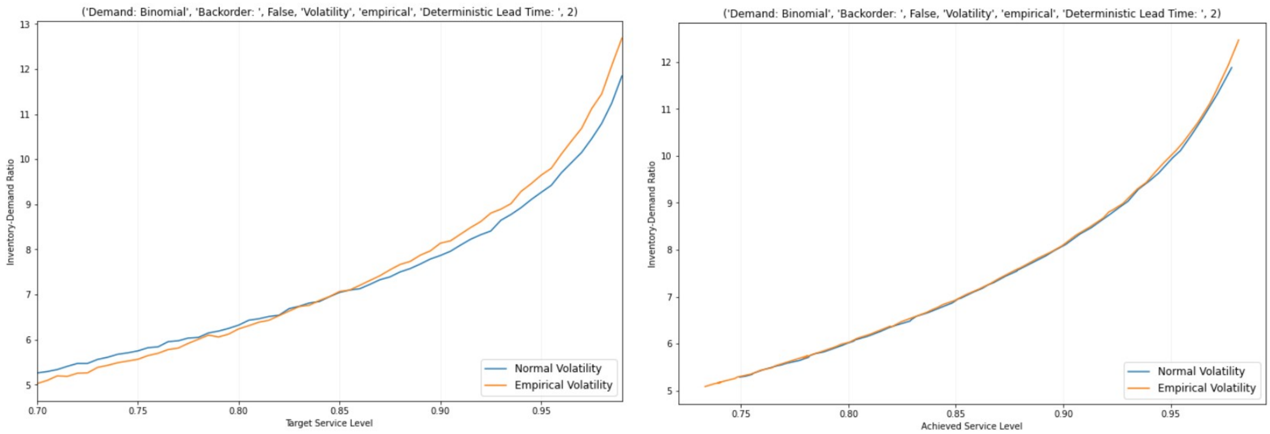


Figure 31: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 2

For both lead time value and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 32 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.92, 0.97 and 0.96 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.93 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.97. A few values of above mention relation is shown in table 28 and 29

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.909	0.930	0.951	0.972	0.992
Achieved Cycle Service Level	0.791	0.836	0.882	0.929	0.978
Fill Rate	0.895	0.919	0.944	0.968	0.991

Table 28: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 04, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.905	0.929	0.952	0.976	0.993
Achieved Cycle Service Level	0.785	0.833	0.884	0.939	0.980
Fill Rate	0.890	0.918	0.945	0.973	0.992

Table 29: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 04, Demand Distribution: Poisson and Binomial

The trend of inventory-demand ratio resulted from both models for current scenario is depicted in figure 33

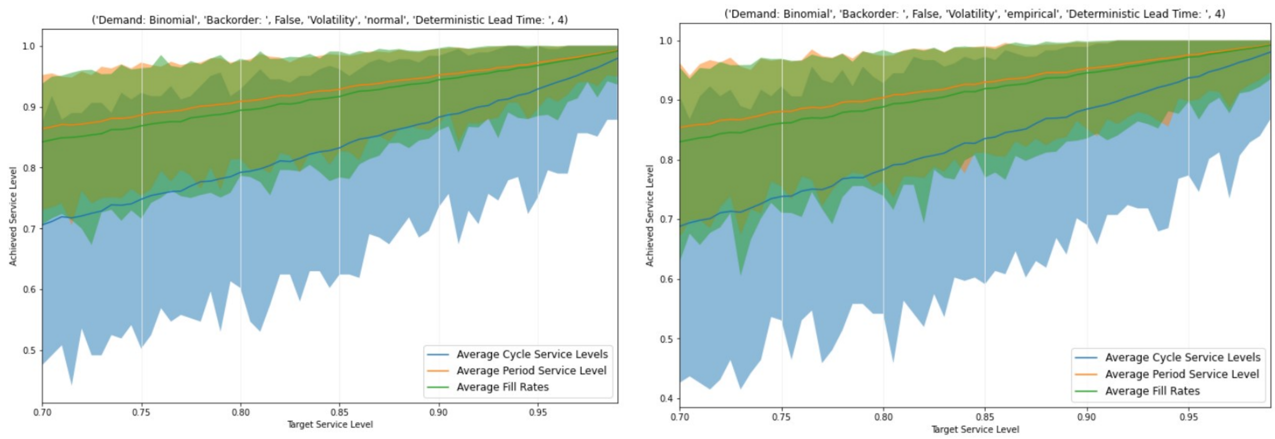


Figure 32: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 4

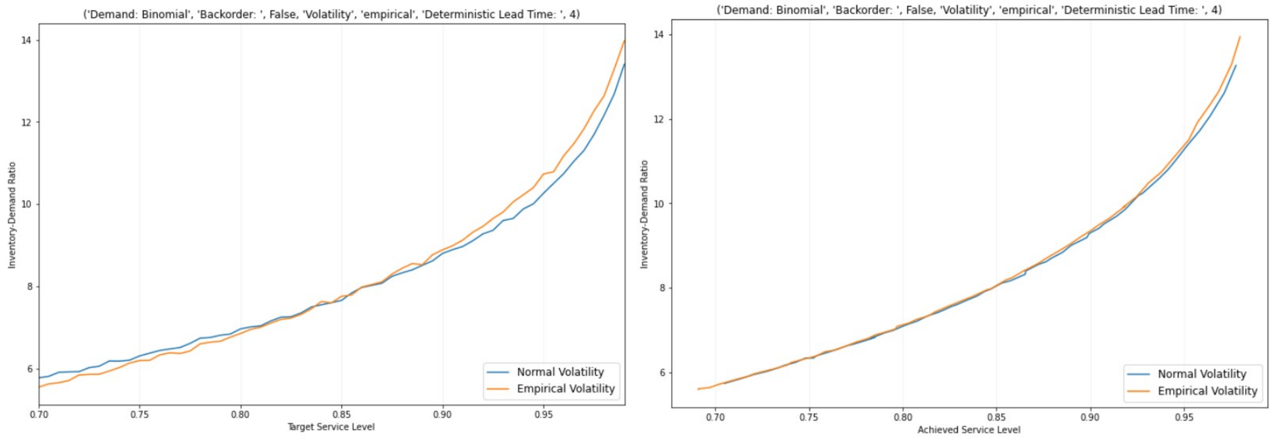


Figure 33: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 4

For lead time value equals 10 and review period equals 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 34 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.94, 0.97 and 0.96 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.95 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.97 and 0.97. A few values of above mention relation is shown in table 30 and 31

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.917	0.935	0.955	0.974	0.992
Achieved Cycle Service Level	0.855	0.884	0.916	0.950	0.984
Fill Rate	0.905	0.926	0.948	0.970	0.994

Table 30: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Deterministic Lead Time: 10, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.912	0.935	0.956	0.974	0.991
Achieved Cycle Service Level	0.849	0.885	0.919	0.950	0.981
Fill Rate	0.900	0.926	0.950	0.971	0.990

Table 31: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Deterministic Lead Time: 10, Demand Distribution: Poisson and Binomial

Inventory-demand ratio trend is again similar as previous which is depicted in figure 35

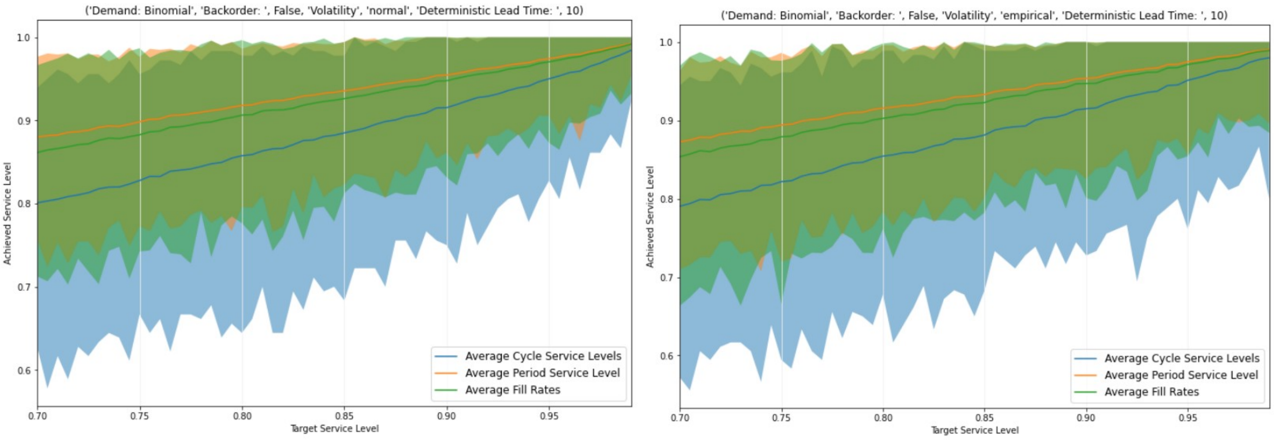


Figure 34: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 10

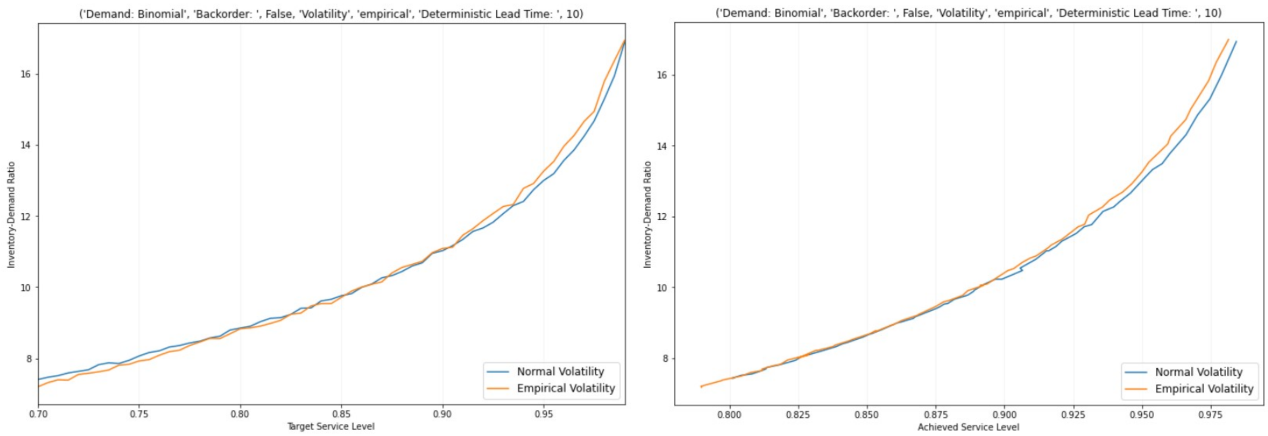


Figure 35: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 10

5.1.6 Binomial and Poisson Distribution Demand with Backorders

In this section, the demand signal is generated from a combination of Binomial and Poisson distribution as mentioned in 3.1.3 and backorders are permitted.

For lead time value 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 36 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.92, 0.96 and 0.94 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.94 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.97 and 0.95. A few values of above mention relation is shown in table 32 and 33

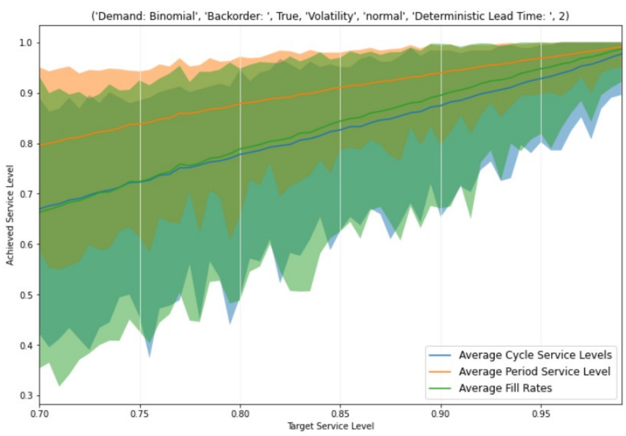
Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.876	0.909	0.940	0.969	0.992
Achieved Cycle Service Level	0.775	0.826	0.877	0.929	0.978
Fill Rate	0.784	0.840	0.896	0.949	0.987

Table 32: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 02, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.871	0.911	0.944	0.973	0.993
Achieved Cycle Service Level	0.769	0.828	0.883	0.937	0.982
Fill Rate	0.777	0.846	0.902	0.955	0.990

Table 33: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 02, Demand Distribution: Poisson and Binomial

For the second metric, the inventory-demand ratio, the normal volatility model behaves slightly better in a way that for the 0.95 target service level the ratio is less than the ratio for the Kernel density estimation volatility model. The trend is depicted in figure 37



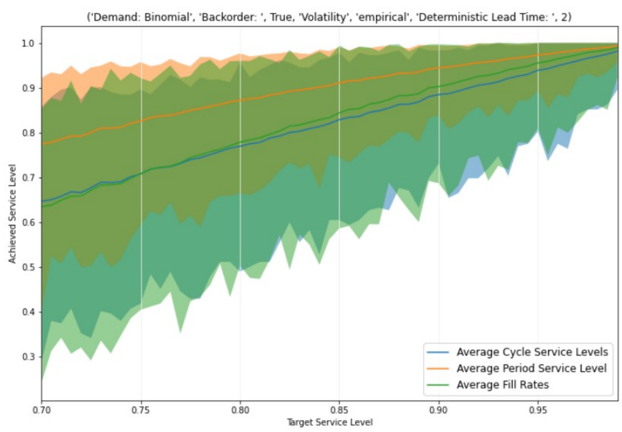


Figure 36: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 2

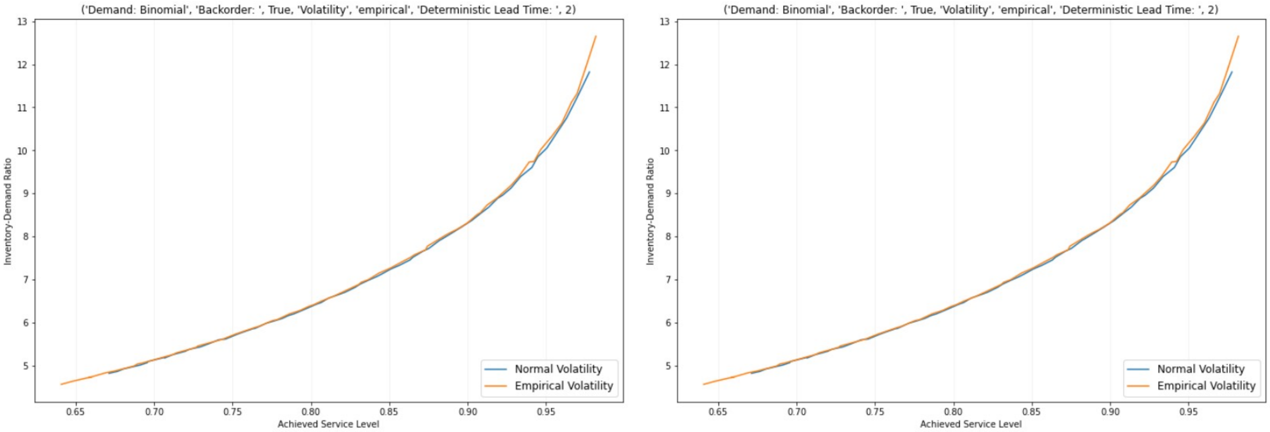


Figure 37: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 2

For both lead time value and review period equals to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 38 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.91, 0.95 and 0.91 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.91 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.96 and 0.92. A few values of above mention relation is shown in table 34 and 35

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.678	0.863	0.920	0.959	0.991
Achieved Cycle Service Level	0.618	0.759	0.843	0.911	0.976
Fill Rate	0.800	0.830	0.839	0.918	0.983

Table 34: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 04, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.784	0.861	0.919	0.964	0.990
Achieved Cycle Service Level	0.661	0.758	0.844	0.920	0.976
Fill Rate	0.602	0.727	0.838	0.929	0.983

Table 35: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 04, Demand Distribution: Poisson and Binomial

Although, for the second metric, the inventory-demand ratio, the normal volatility model behaves slightly better in a way that for the 0.95 target service level the ratio is less than the ratio for the Kernel density estimation volatility model. The trend is depicted in figure 39

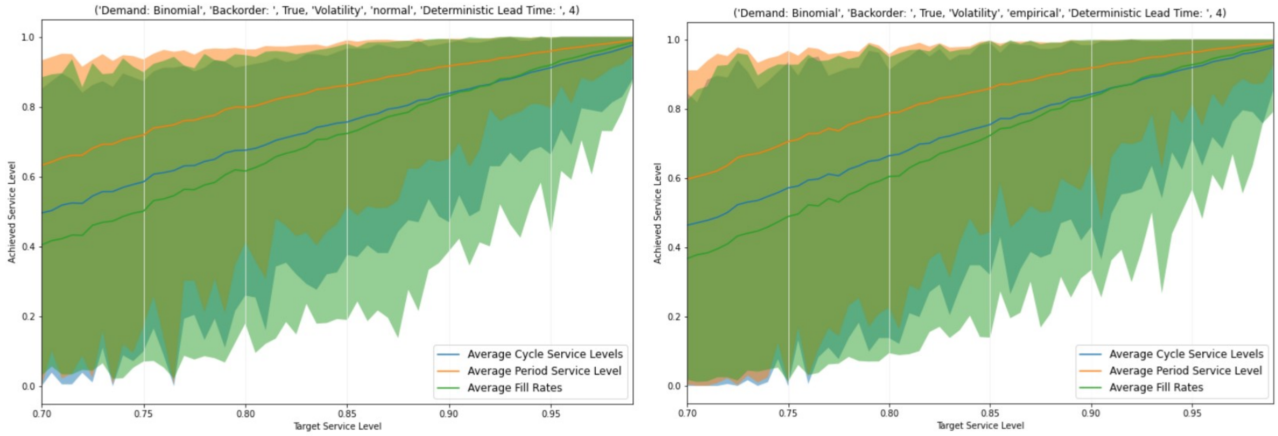


Figure 38: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 4

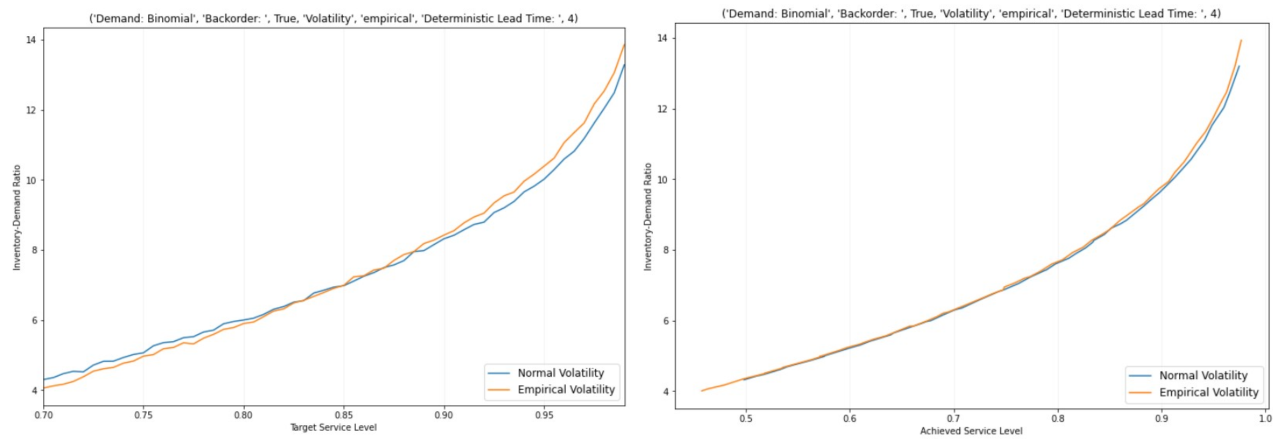


Figure 39: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 4

For lead time value 10 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 40 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.93, 0.95 and 0.90 respectively which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.93 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.95 and 0.90. A few values of above mention relation is shown in table 36 and 37

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.831	0.873	0.918	0.958	0.989
Achieved Cycle Service Level	0.764	0.815	0.875	0.931	0.980
Fill Rate	0.646	0.720	0.811	0.903	0.976

Table 36: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Deterministic Lead Time: 10, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.817	0.871	0.918	0.959	0.987
Achieved Cycle Service Level	0.749	0.814	0.875	0.932	0.976
Fill Rate	0.622	0.717	0.816	0.904	0.972

Table 37: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Deterministic Lead Time: 10, Demand Distribution: Poisson and Binomial

The inventory-demand ratio for a lead time equal to 10 resulting from the Kernel density estimation volatility model behaves better than the ratio resulting from the normal volatility model again. The trend is depicted in figure 41

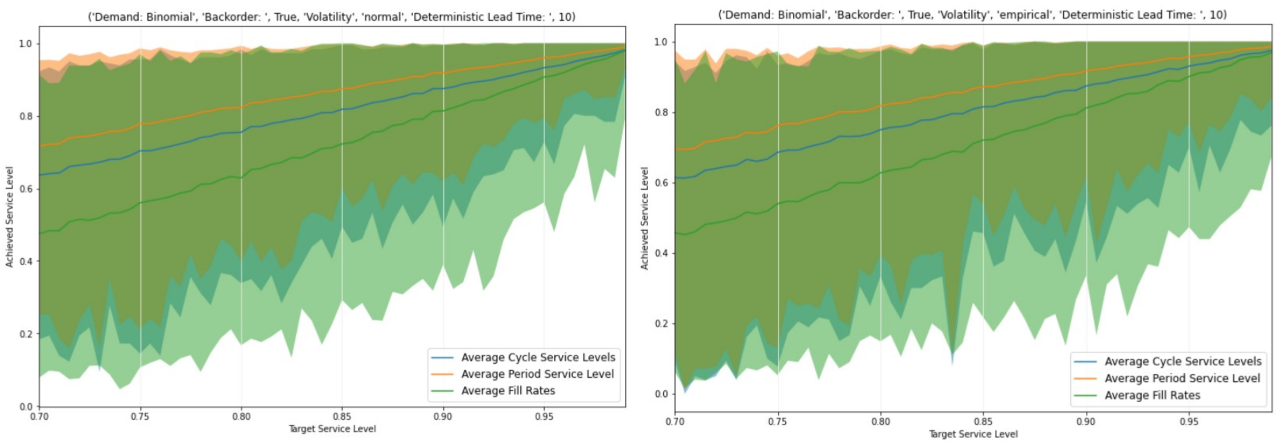


Figure 40: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 10

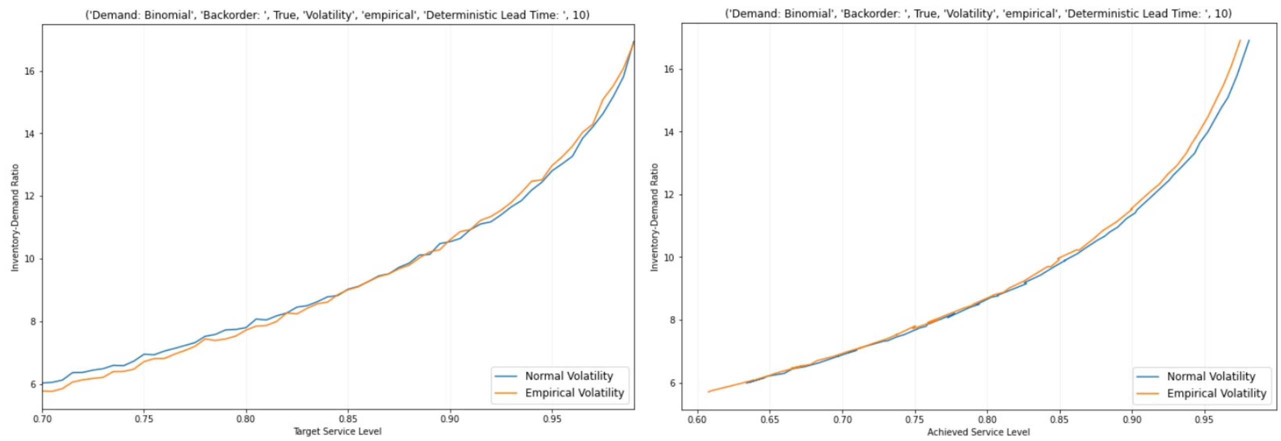


Figure 41: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 10

5.2 Stochastic Demand and Lead Time Analysis

This is the second section of the analysis where both demand and lead time are considered to be stochastic in nature. Similar to section 5.1 this section is also divided into six sub-sections. The experiment is again carried out for each sub-section in a manner alike with targeted values of service level varying from 0.70 to 0.99 with a stepsize of 0.005. Additionally, three lead time values are considered i.e. 2, 4, and 10 which represent numbers less than, equal to and more than the value of the review period, which is 4. The possible deviation (delay) in lead time is considered as 2 units.

5.2.1 Normal Distribution Demand With Lost Sales

In this section, the demand signal is generated from Normal distribution as mentioned in 3.1.1 and backorders are not permitted.

For lead time value 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 42 for both volatility models. The highest average achieved cycle and period service level and fill rate is 0.99 which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.443 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.805 and 0.875. A few values of above mention relation is shown in table 38 and 39

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.940	0.971	0.998	1.000	1.000
Achieved Cycle Service Level	0.760	0.884	0.993	1.000	1.000
Fill Rate	0.979	0.990	1.000	1.000	1.000

Table 38: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 02, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.789	0.796	0.801	0.803	0.824
Achieved Cycle Service Level	0.378	0.409	0.426	0.433	0.493
Fill Rate	0.859	0.864	0.869	0.875	0.896

Table 39: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE Stochastic Lead Time: 02, Demand Distribution: Normal

Although, for the second metric, inventory-demand ratio, the Kernel density estimation volatility model behaves better in a way that for the 0.95 target service level the ratio is less than 2.5 whereas, for the normal volatility model it is more than 4.5. The trend is depicted in figure 43

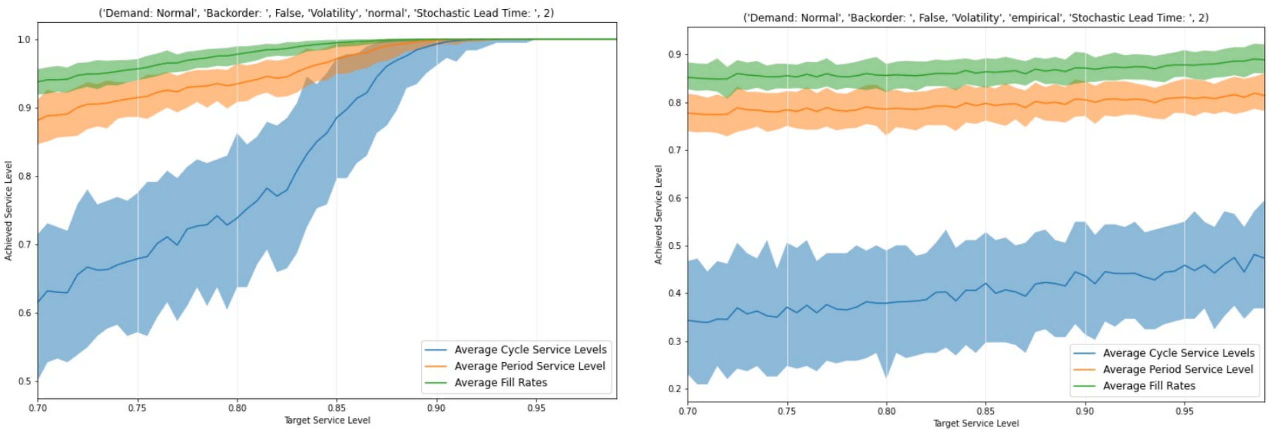


Figure 42: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend when the normal volatility model is applied to calculate safety stock levels, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock level for the demand signal generated from the Normal distribution and lead time equal to 2

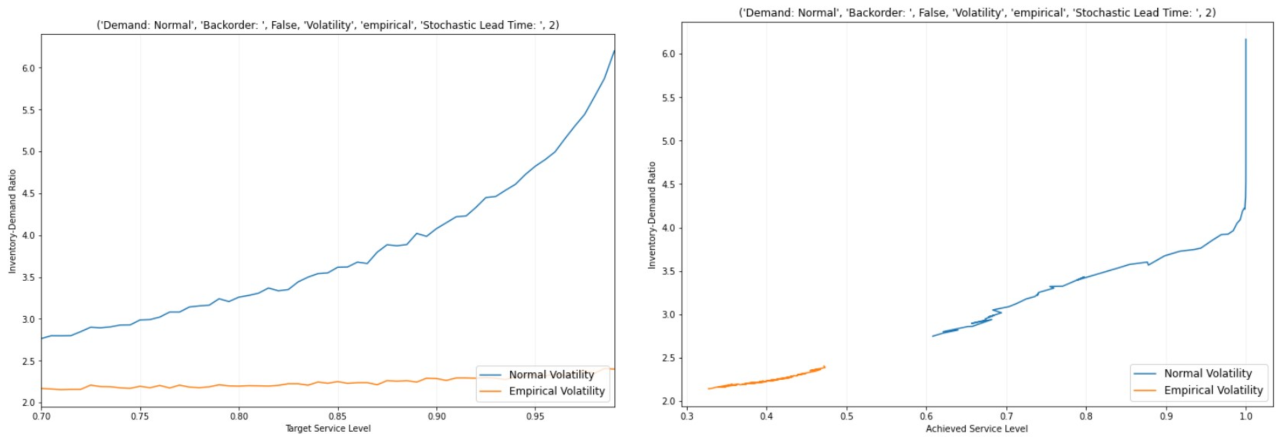


Figure 43: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 2

For lead time value and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 44 for both volatility models. In this scenario, the highest average achieved cycle and period service level and fill rate is the same as previously although for the Kernel density estimation volatility model the highest average achieved cycle and period service level and fill rate has increased in comparison to the results with lead time value 2 i.e., 0.57, 0.85, 0.92 respectively. Although, for the same lead time, the resulting service levels for normal volatility are better than the Kernel density estimation volatility model. A few values of above mention relation is shown in table 40 and 41

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.943	0.972	0.997	1.000	1.000
Achieved Cycle Service Level	0.775	0.886	0.990	1.000	1.000
Fill Rate	0.981	0.994	1.000	1.000	1.000

Table 40: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 04, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.832	0.836	0.839	0.851	0.856
Achieved Cycle Service Level	0.528	0.533	0.526	0.567	0.571
Fill Rate	0.896	0.899	0.901	0.912	0.919

Table 41: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 04, Demand Distribution: Normal

Whereas, for the second metric, inventory-demand ratio, the Kernel density estimation volatility model still behaves better in a way that for 0.95 target service level the ratio is a little more than 2.5 whereas, for the normal volatility model it is more than 4.5. The trend is depicted in figure 45

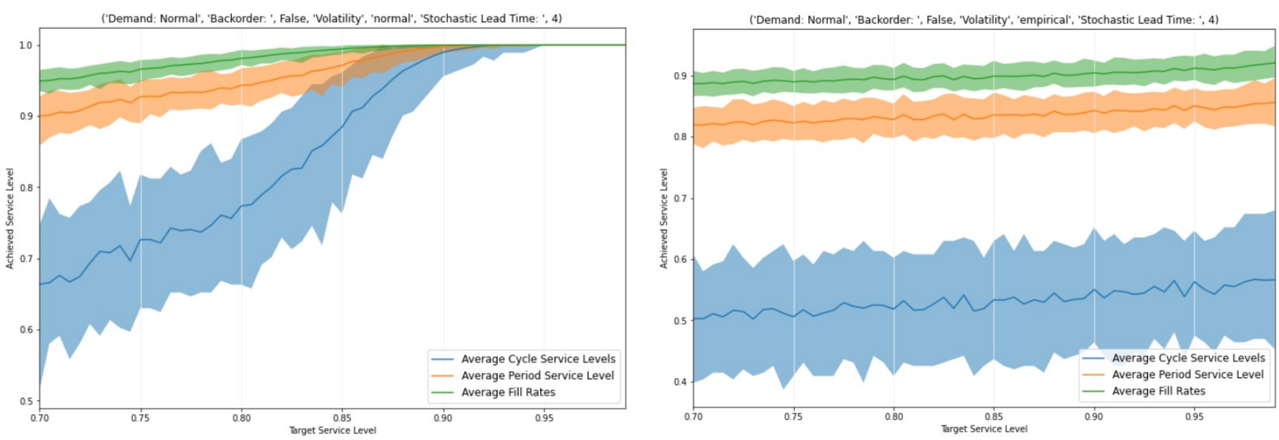


Figure 44: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 4

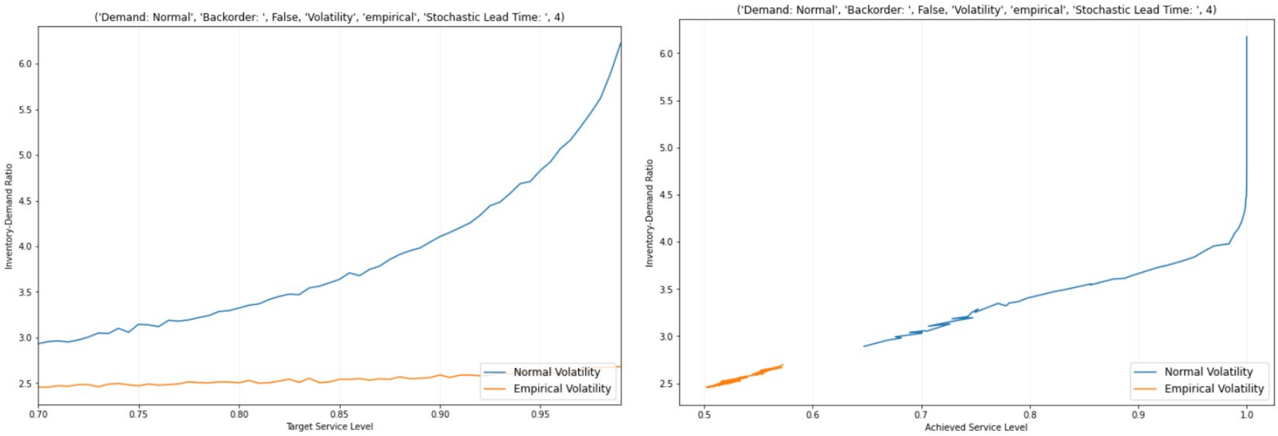


Figure 45: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 4

For lead time value 10 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 46 for both volatility models. In this scenario, the highest average achieved cycle and period service level and fill rate is again similar to the previous two scenarios with lead times 2 and 4 although for the Kernel density estimation volatility model the highest average achieved cycle and period service level and fill rate has again increased in comparison to results with lead time value 2 and 4 i.e., 0.65, 0.89, 0.94 respectively. Although, for the same lead time, the resulting service levels for normal volatility are better than the Kernel density estimation volatility model. A few values of above mention relation is shown in table 42 and 43

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.948	0.975	0.995	1.000	1.000
Achieved Cycle Service Level	0.795	0.899	0.979	1.000	1.000
Fill Rate	0.983	0.994	0.999	1.000	1.000

Table 42: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 10, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.859	0.862	0.869	0.878	0.897
Achieved Cycle Service Level	0.567	0.577	0.595	0.621	0.666
Fill Rate	0.919	0.922	0.928	0.934	0.948

Table 43: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 10, Demand Distribution: Normal

Whereas, for the second metric, inventory-demand ratio, the behaviour is identical as previously noted 45

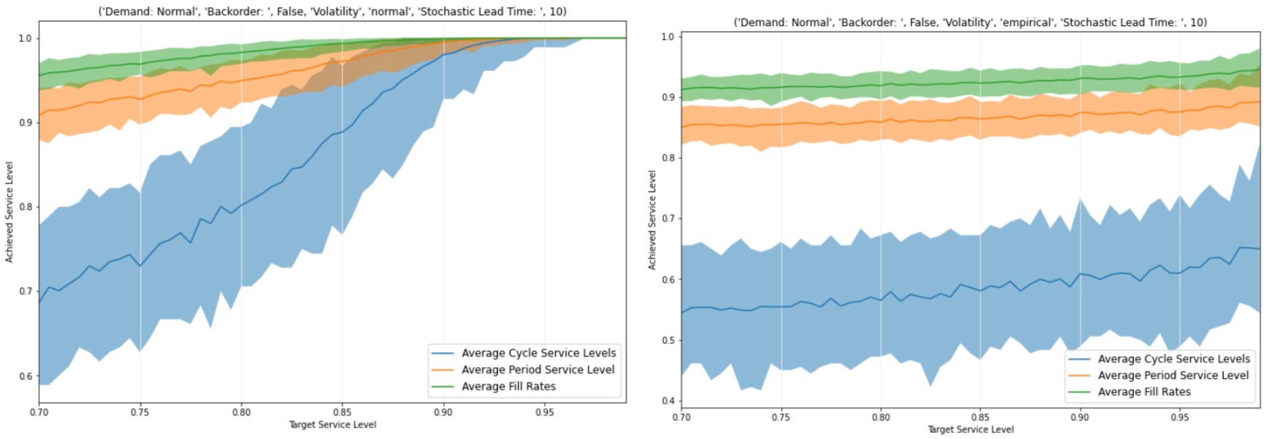


Figure 46: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 10

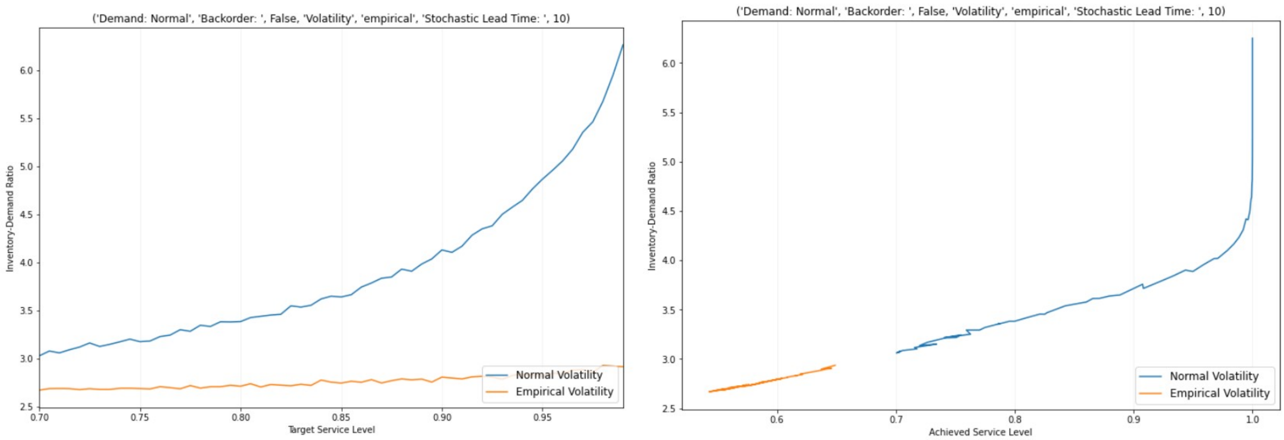


Figure 47: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 10

5.2.2 Normal Distribution Demand With Backorders

In this section, the demand signal is generated from Normal distribution as mentioned in 3.1.1 and backorders are permitted.

For lead time value 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 48 for both volatility models. The highest average achieved cycle and period service level and fill rate is 0.99 which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.198 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.56 and 0.59. A few values of above mention relation are shown in the table. 44 and 45

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.910	0.964	0.998	1.000	1.000
Achieved Cycle Service Level	0.675	0.857	0.992	1.000	1.000
Fill Rate	0.958	0.992	1.000	1.000	1.000

Table 44: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 02, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.442	0.442	0.522	0.528	0.569
Achieved Cycle Service Level	0.141	0.138	0.190	0.183	0.203
Fill Rate	0.442	0.445	0.528	0.539	0.594

Table 45: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 02, Demand Distribution: Normal

Although, for the second metric, inventory-demand ratio, the Kernel density estimation volatility model behaves better in a way that for the 0.95 target service level the ratio is less than 2.0 whereas, for the normal volatility model it is more than 4.5. The trend is depicted in figure 49

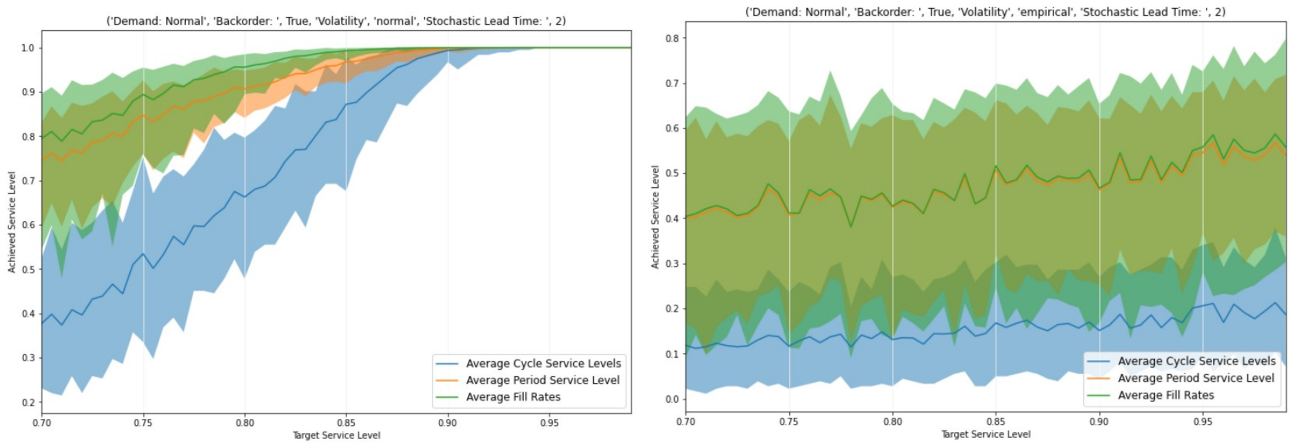


Figure 48: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 2

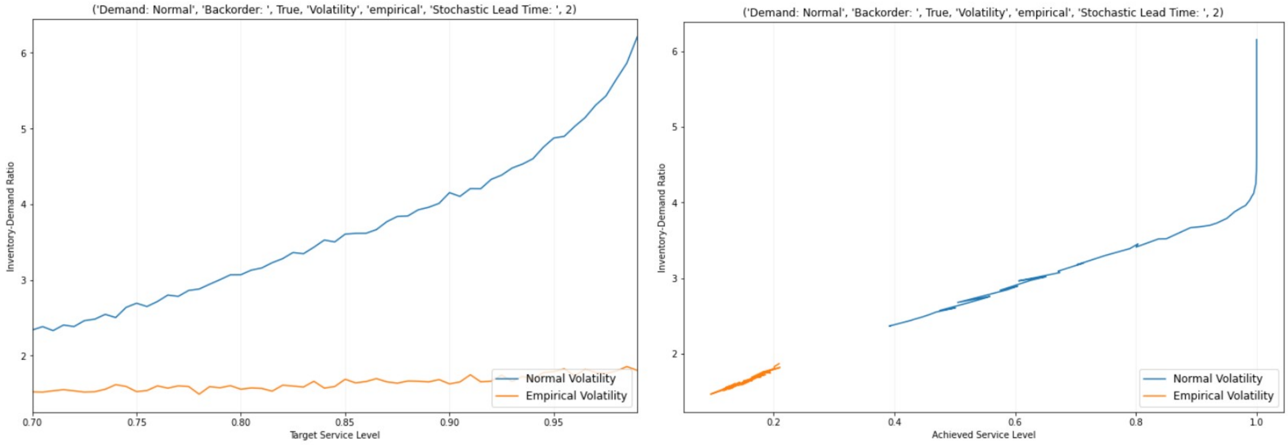


Figure 49: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 2

For lead time and review period equal to 4 units the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 50 for both volatility models. The highest average achieved cycle and period service level and fill rate are identical as for lead time value equals 2 which is against the target service level of 0.95 for the normal volatility model whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle is 0.37 against target service level of 0.95. For the same value of target service level the period service level and fill rates are 0.77 and 0.85 which are significantly higher than the results with a lead time equal to 2. A few values of above mention relation are shown in the table. 46 and 47

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.933	0.968	0.997	1.000	1.000
Achieved Cycle Service Level	0.733	0.870	0.987	1.000	1.000
Fill Rate	0.974	0.993	1.000	1.000	1.000

Table 46: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 04, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.674	0.689	0.713	0.730	0.761
Achieved Cycle Service Level	0.238	0.245	0.296	0.316	0.363
Fill Rate	0.761	0.774	0.790	0.816	0.844

Table 47: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 04, Demand Distribution: Normal

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model still behaves better in a way that for the 0.95 target service level the ratio is less than 2.0 whereas, for the normal volatility model it is more than 4.5. The trend is depicted in figure 51

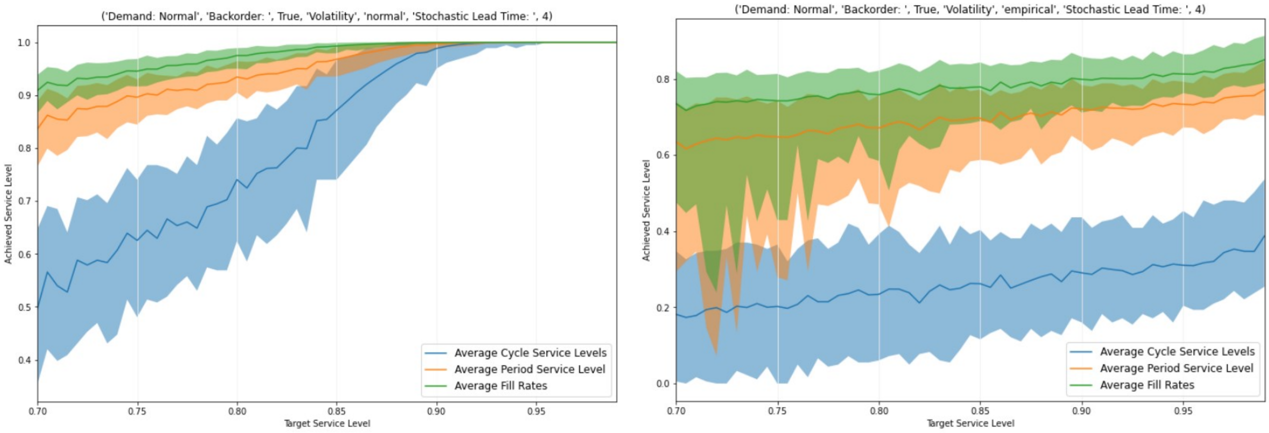


Figure 50:]
The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 4

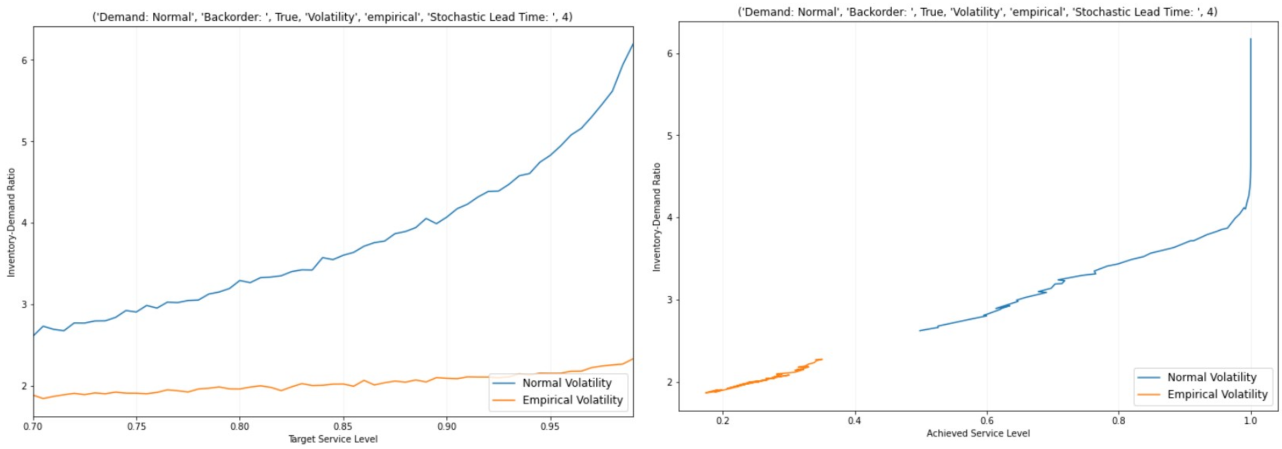


Figure 51: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 4

For lead time value 10 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 52 for both volatility models. The highest average achieved cycle and period service level and fill rate are identical to previous values for the 0.95 targeted service level whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle, period service level and fill rates are increased with values 0.29, 0.66 and 0.70 in comparison to lead times equal to 2 and 4, however, for lead time equals to 10, the normal volatility model behaves better in terms of service levels. A few values of above mention relation are shown in the table. 48 and 49

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.918	0.963	0.992	1.000	1.000
Achieved Cycle Service Level	0.713	0.856	0.969	1.000	1.000
Fill Rate	0.958	0.988	0.998	1.000	1.000

Table 48: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 10, Demand Distribution: Normal

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.491	0.508	0.548	0.579	0.644
Achieved Cycle Service Level	0.162	0.165	0.199	0.217	0.268
Fill Rate	0.504	0.526	0.567	0.601	0.677

Table 49: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 10, Demand Distribution: Normal

The second metric, inventory-demand ratio, has same relationship as previously indicated 53

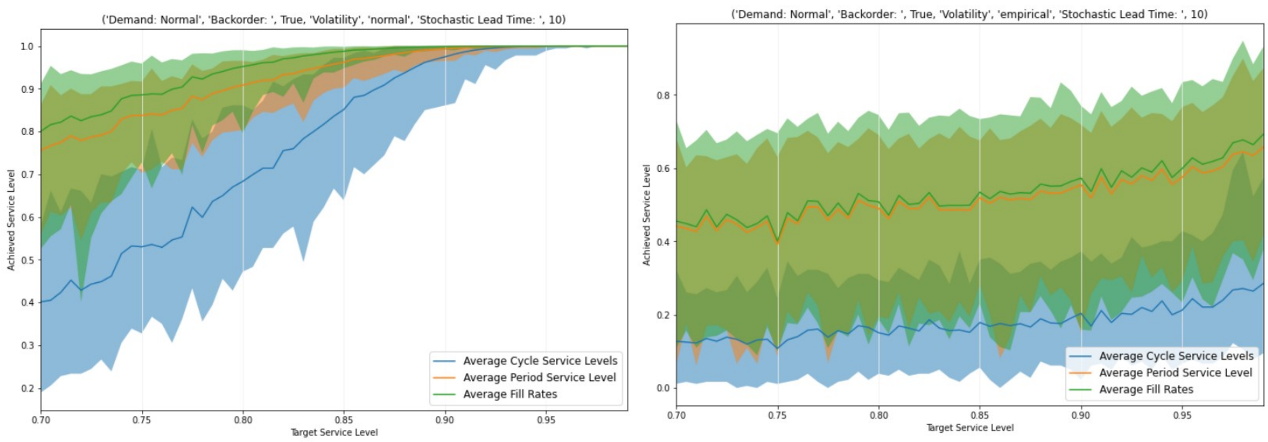


Figure 52: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Normal distribution and lead time equal to 10

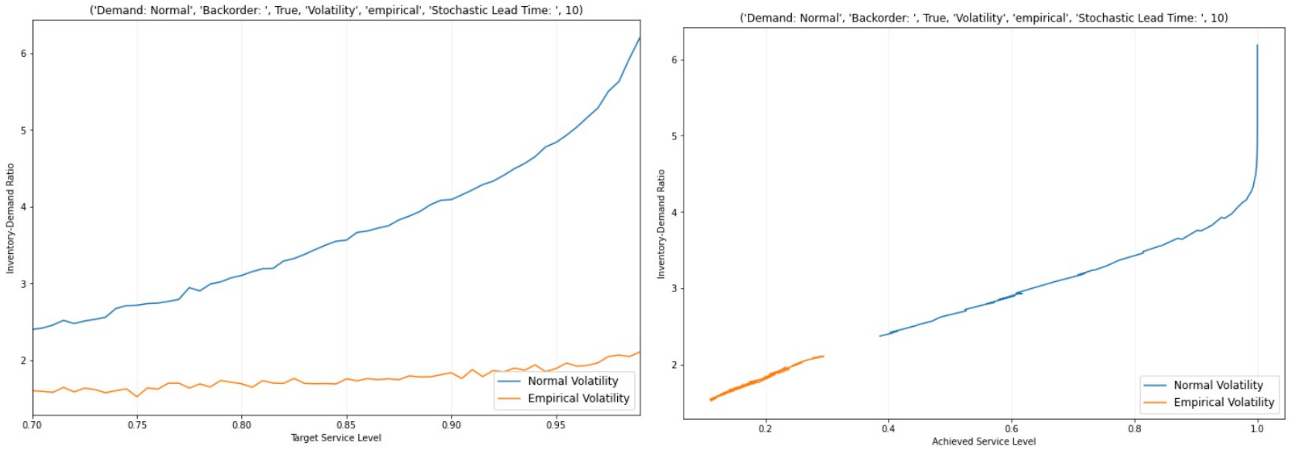


Figure 53: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Normal distribution and lead time equal to 10

5.2.3 Poisson Distribution Demand with Lost Sales

In this section, the demand signal is generated from Poisson distribution as mentioned in 3.1.2 and backorders are not permitted.

For lead time value equals 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 54 for both volatility models. The highest average achieved period service level and fill rate are the same for the 0.95 targeted service level which is 1.0 and the highest average achieved cycle service level is 0.99 whereas, with the Kernel density estimation volatility model we can see that the highest average achieved cycle, period service level and fill rates are 0.45, 0.80 and 0.87 respectively. A few values of above mention relation are shown in the table. 50 and 51

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.939	0.970	0.998	1.000	1.000
Achieved Cycle Service Level	0.759	0.878	0.992	1.000	1.000
Fill Rate	0.979	0.994	1.000	1.000	1.000

Table 50: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 02, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.789	0.796	0.800	0.813	0.815
Achieved Cycle Service Level	0.388	0.412	0.435	0.465	0.468
Fill Rate	0.858	0.863	0.867	0.880	0.891

Table 51: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 02, Demand Distribution: Poisson

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a lower inventory-demand ratio for the same targeted service level than the normal volatility model. For the 0.95 target service level, the inventory ratio for the Kernel density estimation volatility model is less than 2.5 whereas, for the normal volatility model same value is near 5.0 55

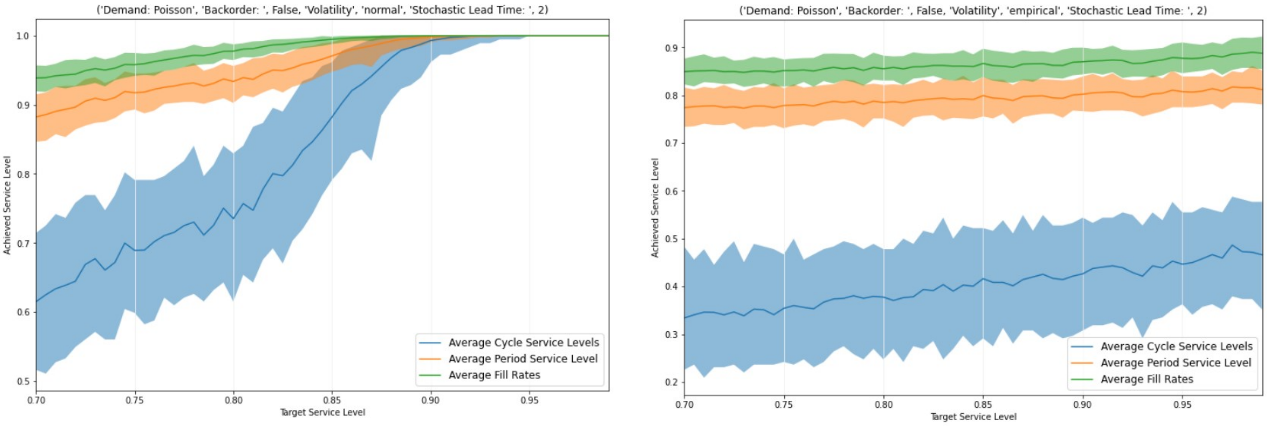


Figure 54: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 2

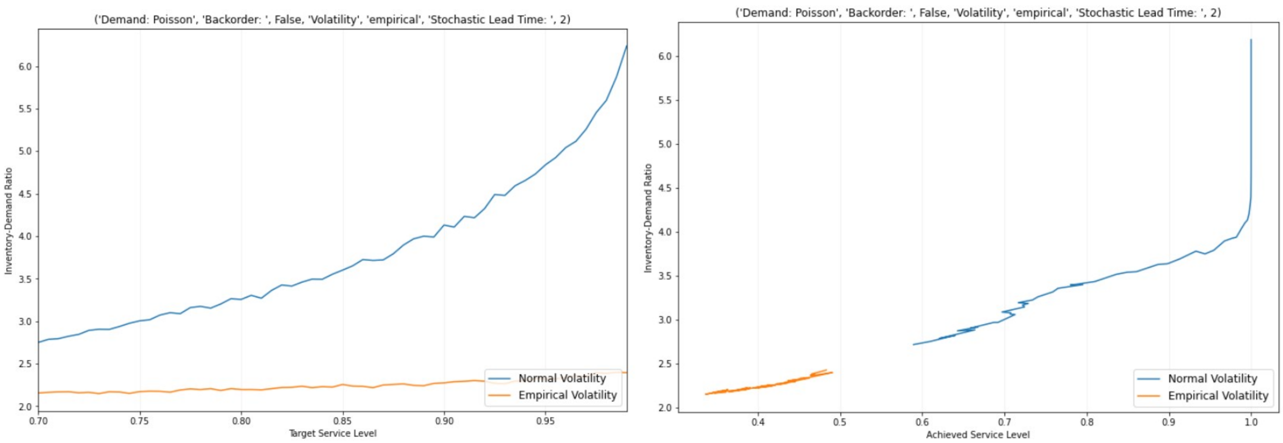


Figure 55: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 2

When both lead time to value and review period is 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 56 for both volatility models. The highest average achieved cycle and period service level and fill rate are the same for 0.95 targeted service level which is 0.99 and for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.55, 0.84 and 0.91 respectively which is more than resulted with lead time 2. A few values of above mention relation are shown in the table. 52 and 53

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.945	0.971	0.997	1.000	1.000
Achieved Cycle Service Level	0.783	0.885	0.989	1.000	1.000
Fill Rate	0.982	0.994	1.000	1.000	1.000

Table 52: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 04, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.830	0.832	0.836	0.844	0.860
Achieved Cycle Service Level	0.524	0.525	0.528	0.543	0.580
Fill Rate	0.894	0.896	0.902	0.909	0.923

Table 53: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 04, Demand Distribution: Poisson

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a lower inventory-demand ratio for the same targeted service level than the normal volatility model. For the 0.95 target service level, the inventory ratio for the Kernel density estimation volatility model is a slightly higher value than 2.5 whereas, for the normal volatility model same value is near 5.0 57

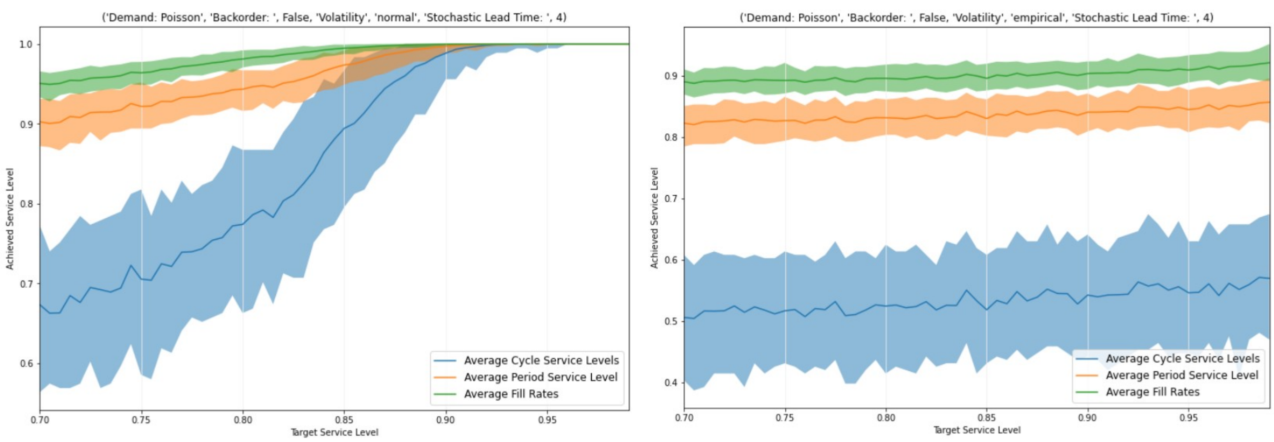


Figure 56: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 4

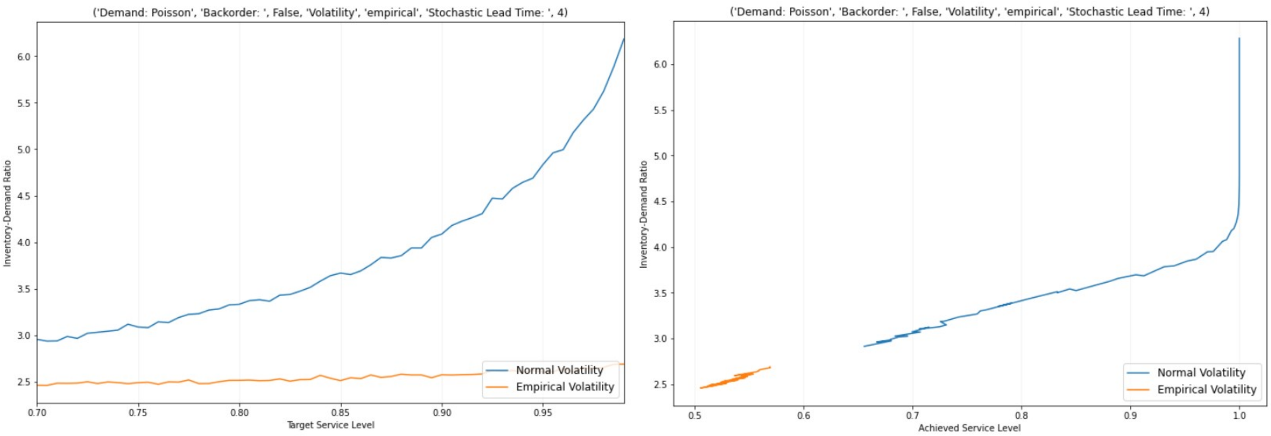


Figure 57: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 4

For lead time value equals 10 and review period equals 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 58 for both volatility models. The highest average achieved cycle and period service level and fill rate are the same for 0.95 targeted service level which is 0.99 and for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.61, 0.87 and 0.93 respectively which is again more than resulted with lead time 2 and 4. A few values of above mention relation are shown in the table. 54 and 55

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.951	0.973	0.995	1.000	1.000
Achieved Cycle Service Level	0.810	0.892	0.979	1.000	1.000
Fill Rate	0.983	0.994	0.999	1.000	1.000

Table 54: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 10, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.859	0.865	0.872	0.878	0.894
Achieved Cycle Service Level	0.569	0.582	0.604	0.615	0.657
Fill Rate	0.919	0.924	0.929	0.935	0.945

Table 55: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 10, Demand Distribution: Poisson

The second metric, inventory-demand ratio, follows the same trend as previously mentioned 59

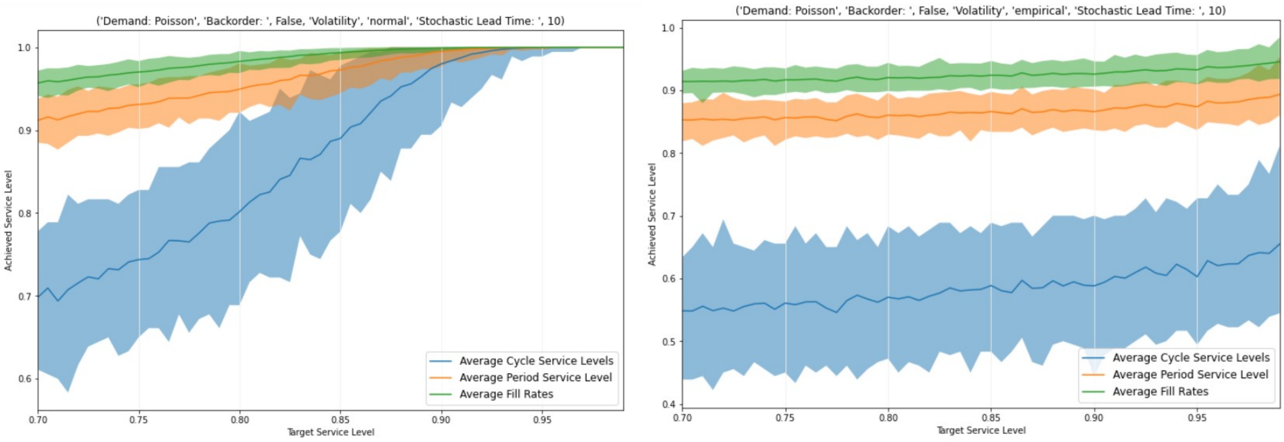


Figure 58: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 10

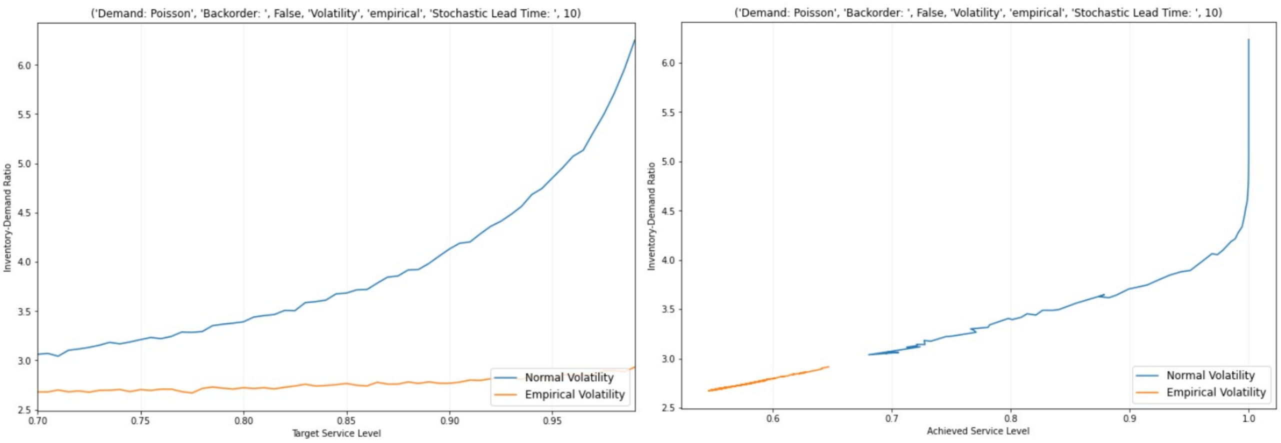


Figure 59: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 10

5.2.4 Poisson Distribution Demand with Backorders

In this section, the demand signal is generated from Poisson distribution as mentioned in 3.1.2 and backorders are permitted.

For lead time value equals 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 60 for both volatility models. The highest average achieved cycle and period service level and fill rate are the same for the 0.95 targeted service level which is 0.99 for the normal volatility model whereas, with the Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.19, 0.54 and 0.56 respectively. A few values of above mention relation are shown in the table. 56 and 57

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.909	0.963	0.998	1.000	1.000
Achieved Cycle Service Level	0.672	0.852	0.993	1.000	1.000
Fill Rate	0.957	0.992	1.000	1.000	1.000

Table 56: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 02, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.439	0.468	0.466	0.515	0.573
Achieved Cycle Service Level	0.134	0.154	0.160	0.166	0.199
Fill Rate	0.443	0.473	0.466	0.530	0.601

Table 57: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 02, Demand Distribution: Poisson

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a lower inventory-demand ratio for the same targeted service level than the normal volatility model. For 0.95 target service level the inventory ratio for Kernel density estimation volatility model is less than 2.0 whereas, for normal volatility model same value is near 5.0 61

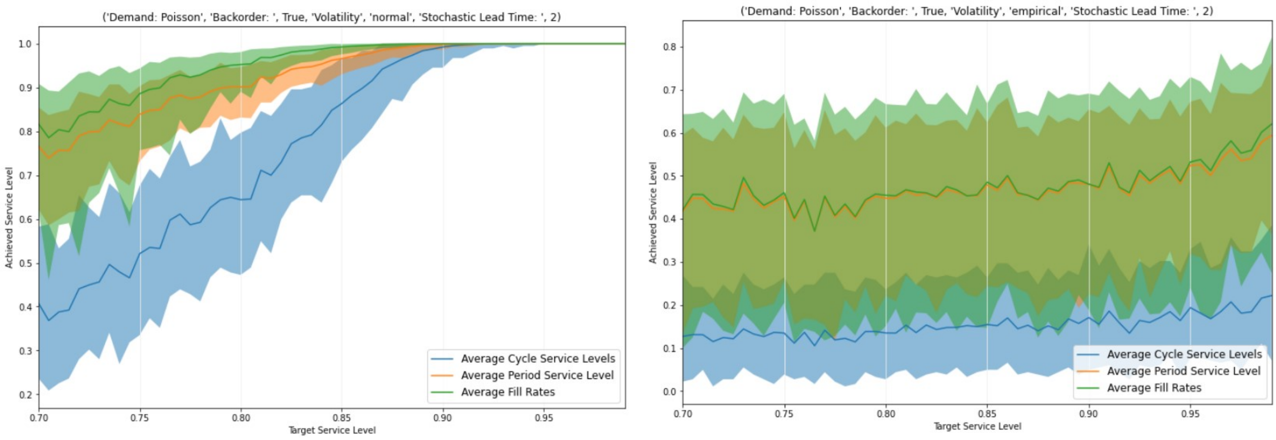


Figure 60: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 2

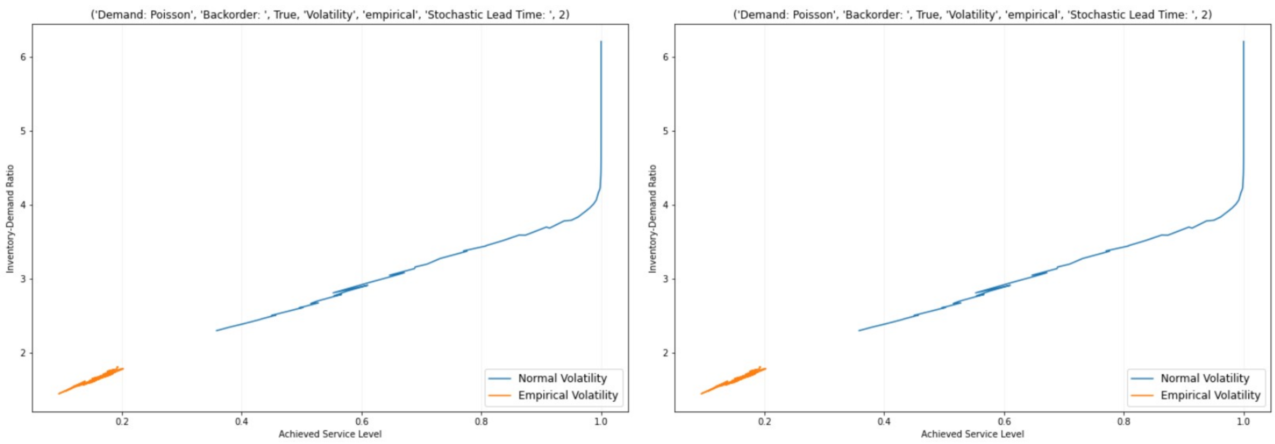


Figure 61: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 2

When both lead time and review period are equal to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 62 for both volatility models. The highest average achieved cycle and period service level and fill rate are the same for the 0.95 targeted service level which is 0.99 for the normal volatility model whereas, with the Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.31, 0.74 and 0.81 respectively, the same trend of increased values when lead time is increased. A few values of above mention relation are shown in the table. 58 and 59

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.921	0.970	0.997	1.000	1.000
Achieved Cycle Service Level	0.689	0.878	0.989	1.000	1.000
Fill Rate	0.970	0.993	1.000	0.99	1.000

Table 58: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 04, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.682	0.700	0.709	0.744	0.765
Achieved Cycle Service Level	0.240	0.272	0.274	0.329	0.357
Fill Rate	0.769	0.781	0.790	0.821	0.850

Table 59: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 04, Demand Distribution: Poisson

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model again has a lower inventory-demand ratio for the same targeted service level than the normal volatility model. For 0.95 target service level the inventory ratio for Kernel density estimation volatility model is near 2.0 whereas, for normal volatility model same value is near 5.0 63

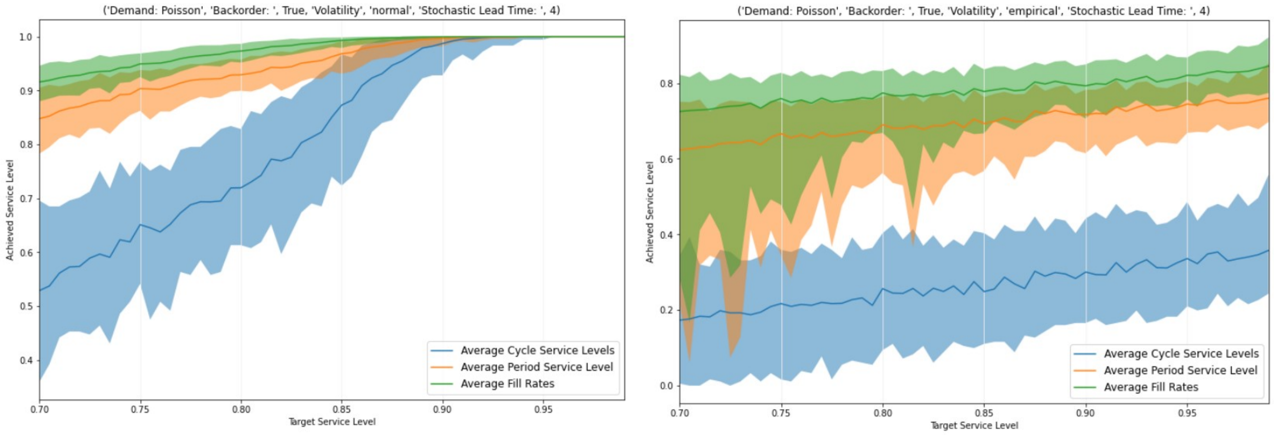


Figure 62: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 4

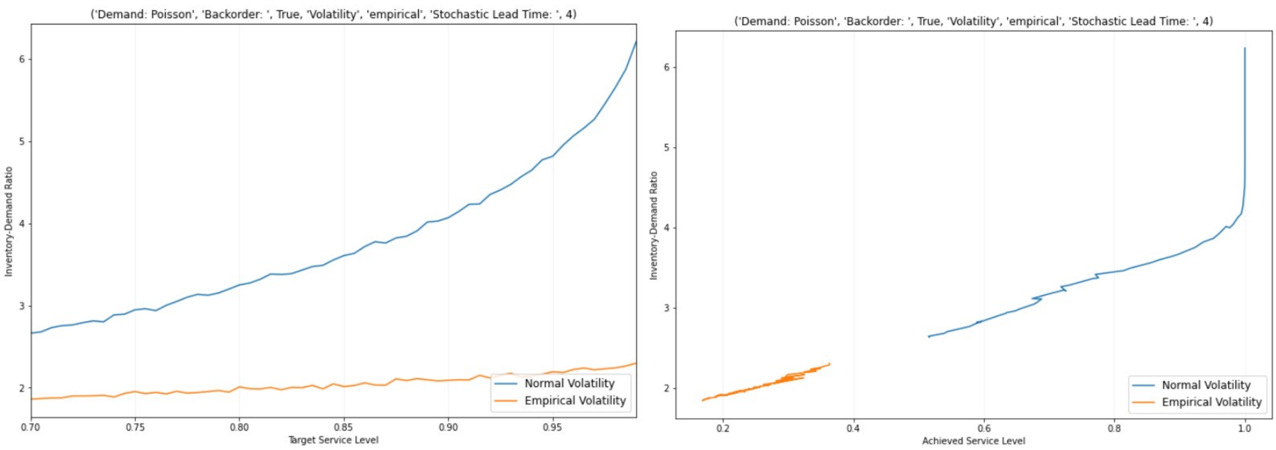


Figure 63: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 4

When both lead times are equal to 10 and the review period is equal to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 64 for both volatility models. The highest average achieved cycle and period service level and fill rate are the same for the 0.95 targeted service level which is 0.99 for the normal volatility model whereas, with the Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.23, 0.60 and 0.63 respectively, here the values have decreased with increase in lead time. A few values of above mention relation are shown in the table. 60 and 61

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.909	0.961	0.994	1.000	1.000
Achieved Cycle Service Level	0.685	0.848	0.975	1.000	1.000
Fill Rate	0.951	0.987	0.999	1.000	1.000

Table 60: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 10, Demand Distribution: Poisson

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.488	0.492	0.556	0.571	0.638
Achieved Cycle Service Level	0.159	0.159	0.201	0.214	0.264
Fill Rate	0.500	0.507	0.578	0.591	0.670

Table 61: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 10, Demand Distribution: Poisson

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model again has a lower inventory-demand ratio for the same targeted service level than the normal volatility model. For 0.95 target service level the inventory ratio for Kernel density estimation volatility model is near 2.0 whereas, for normal volatility model same value is near 5.0 51

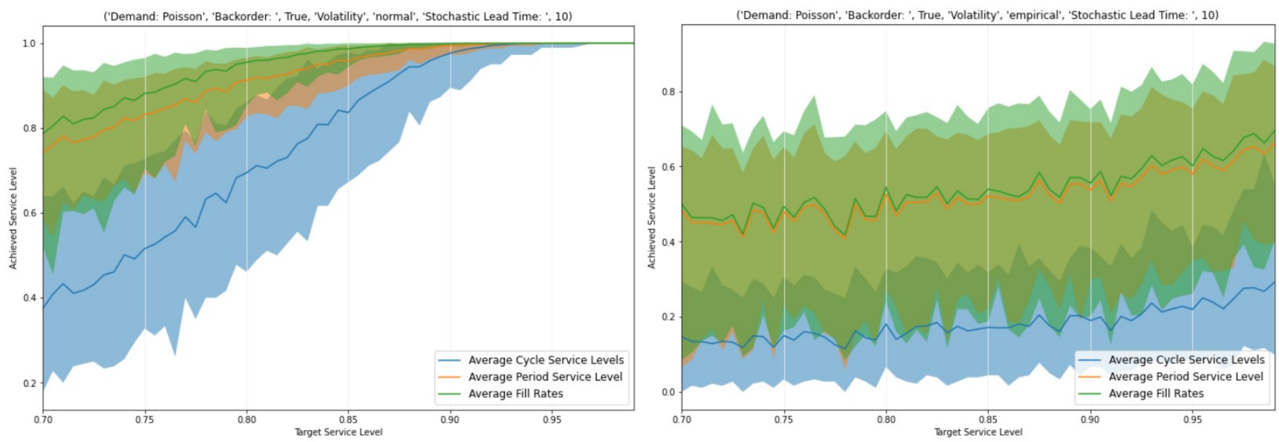


Figure 64: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from Poisson distribution and lead time equal to 10

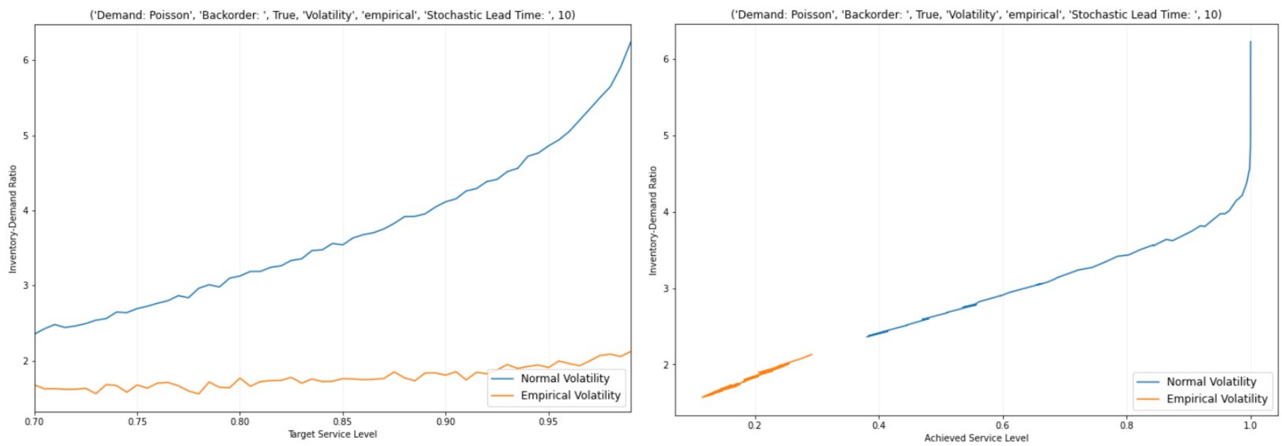


Figure 65: The graphs depicts the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from Poisson distribution and lead time equal to 10

5.2.5 Binomial and Poisson Distribution Demand with Lost Sales

In this section, the demand signal is generated from the combination of Binomial and Poisson distribution as mentioned in 3.1.3 and backorders are not permitted.

For lead time value equals 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 66 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.89, 0.95 and 0.95 respectively for the 0.95 targeted service level for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.88, 0.95 and 0.94 respectively. A few values of above mention relation are shown in the table. 62 and 63

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.877	0.903	0.932	0.961	0.988
Achieved Cycle Service Level	0.722	0.773	0.831	0.899	0.965
Fill Rate	0.857	0.888	0.920	0.955	0.986

Table 62: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 02, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.859	0.889	0.921	0.954	0.984
Achieved Cycle Service Level	0.688	0.744	0.810	0.883	0.956
Fill Rate	0.836	0.871	0.909	0.948	0.982

Table 63: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 02, Demand Distribution: Poisson and Binomial

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a slightly lower inventory-demand ratio for the same targeted service level than the normal volatility model. 67

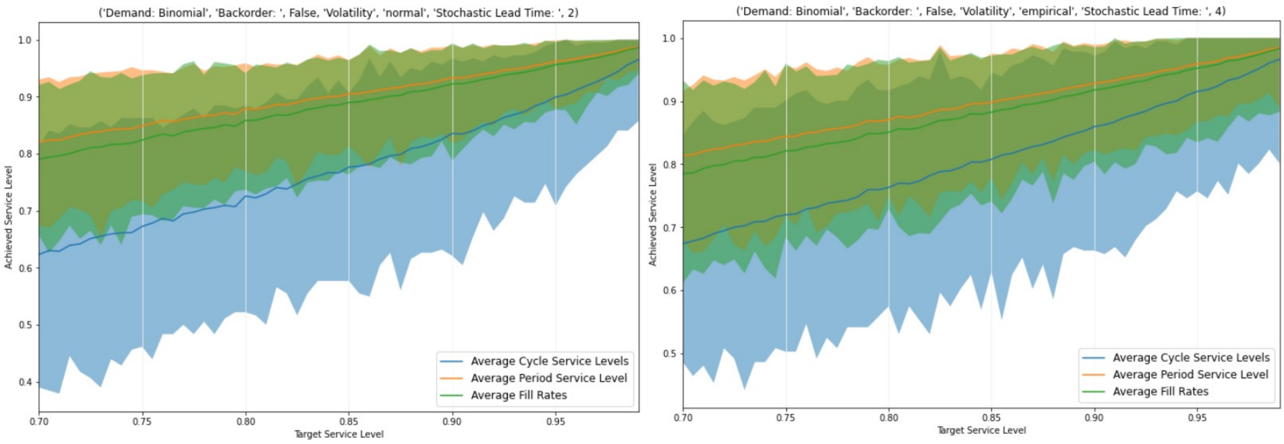


Figure 66: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 2

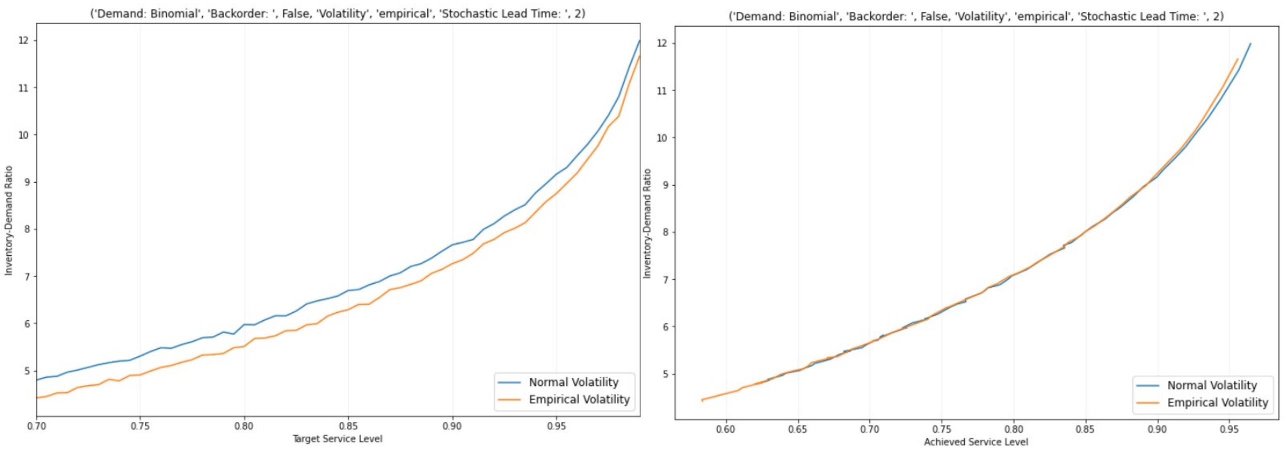


Figure 67: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 2

For both lead time value and review period equals to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 68 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.92, 0.96 and 0.96 respectively for the 0.95 targeted service level for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.91, 0.96 and 0.95 respectively. A few values of above mention relation are shown in the table. 64 and 65

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.885	0.910	0.935	0.962	0.989
Achieved Cycle Service Level	0.786	0.827	0.871	0.921	0.974
Fill Rate	0.868	0.895	0.925	0.957	0.987

Table 64: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 04, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.871	0.901	0.928	0.956	0.985
Achieved Cycle Service Level	0.762	0.812	0.859	0.910	0.967
Fill Rate	0.850	0.886	0.917	0.950	0.983

Table 65: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 04, Demand Distribution: Poisson and Binomial

For the second metric, inventory-demand ratio, again the Kernel density estimation volatility model has a slightly lower inventory-demand ratio for the same targeted service level than the normal volatility model. 69

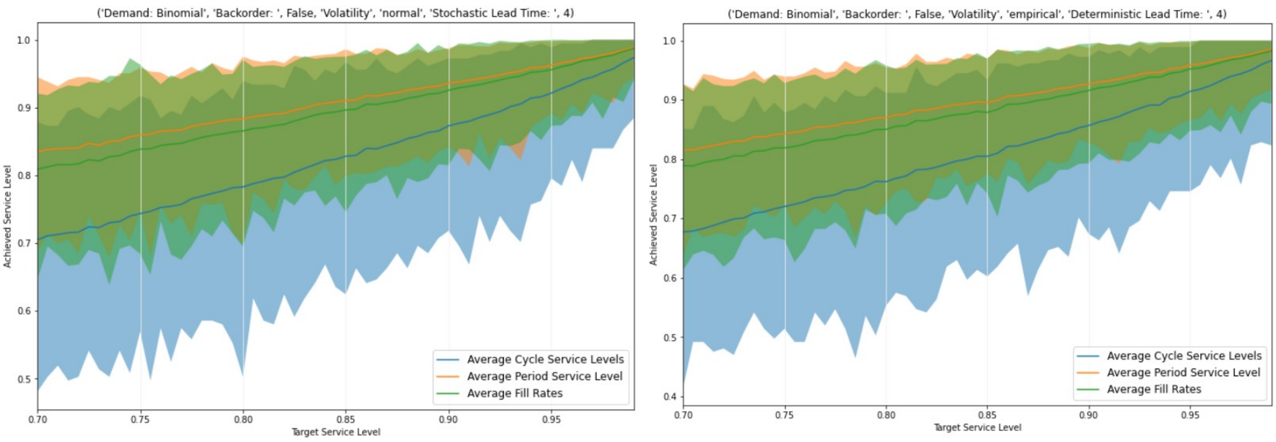


Figure 68: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 4

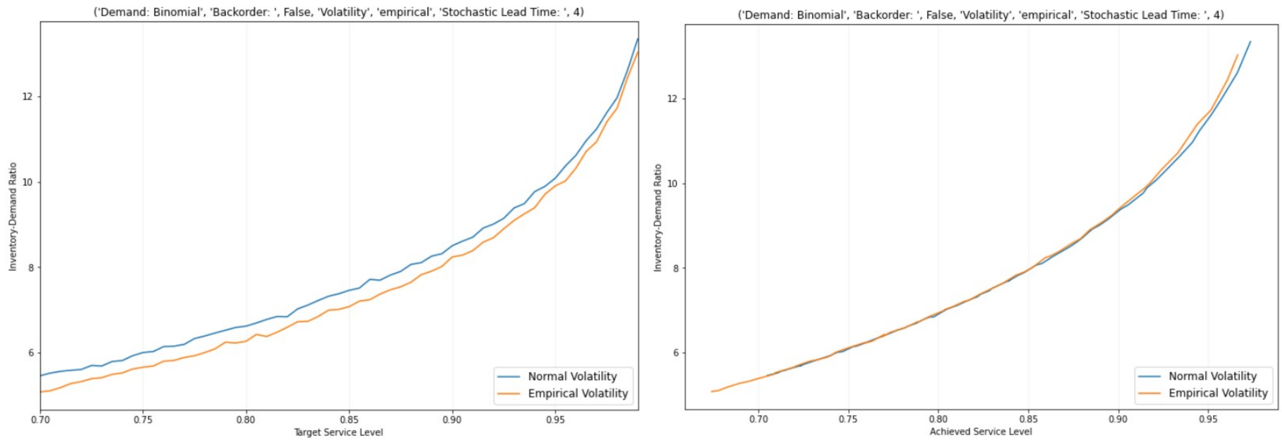


Figure 69: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 4

When the lead time value is 10 and the review period equals 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 70 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.92, 0.96 and 0.96 respectively for the 0.95 targeted service level for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.91, 0.96 and 0.95 respectively. A few values of above mention relation are shown in the table. 66 and 67

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.900	0.923	0.943	0.966	0.989
Achieved Cycle Service Level	0.798	0.838	0.877	0.922	0.974
Fill Rate	0.885	0.912	0.935	0.961	0.988

Table 66: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: Normal, Stochastic Lead Time: 10, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.892	0.913	0.938	0.962	0.986
Achieved Cycle Service Level	0.784	0.821	0.869	0.916	0.967
Fill Rate	0.876	0.901	0.930	0.957	0.984

Table 67: Targeted Service Level vs Achieved Service Levels With Lost Sales, Volatility Model: KDE, Stochastic Lead Time: 10, Demand Distribution: Poisson and Binomial

For the second metric, inventory-demand ratio, again the Kernel density estimation volatility model has a slightly lower inventory-demand ratio for the same targeted service level than the normal volatility model. 71

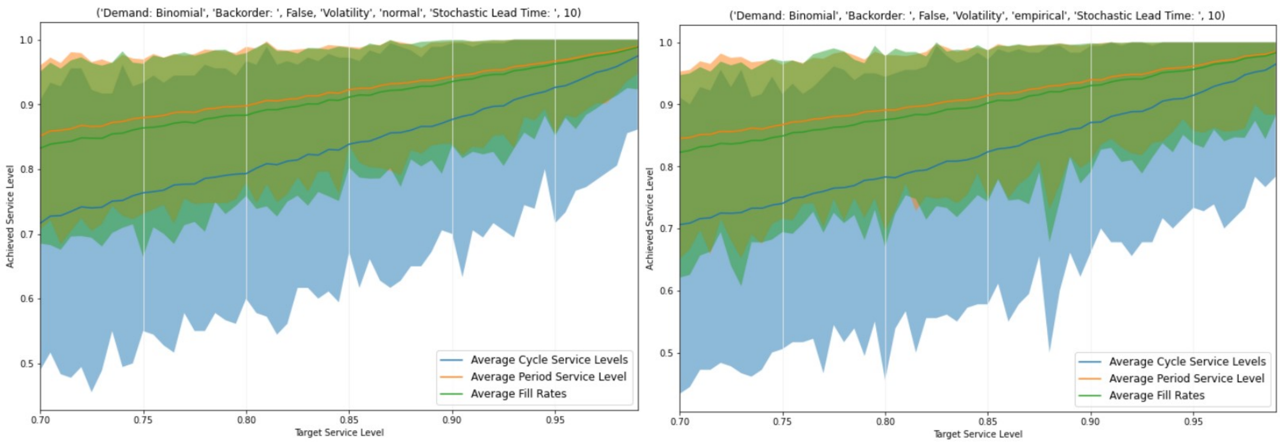


Figure 70: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 10

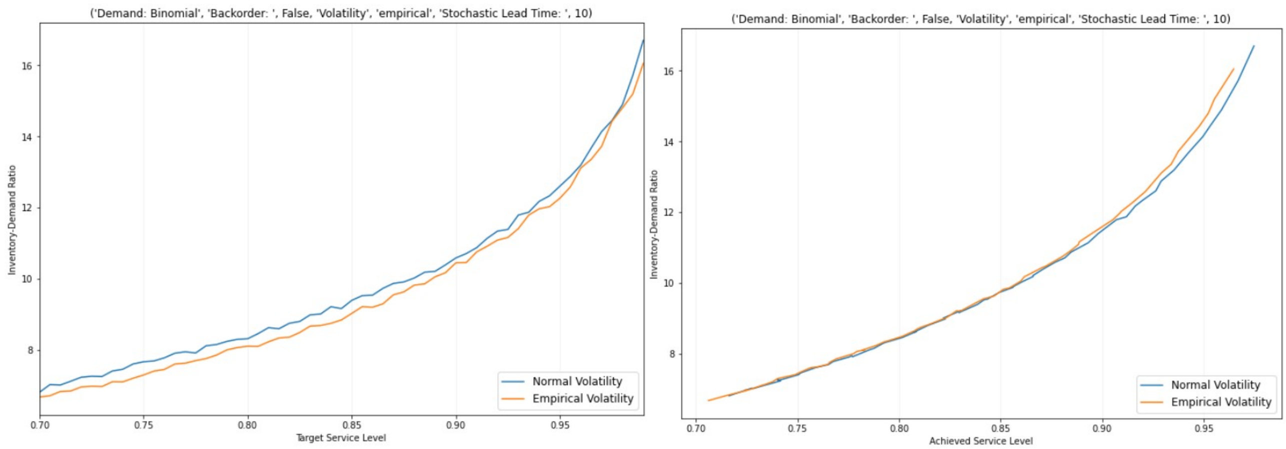


Figure 71: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 10

5.2.6 Binomial and Poisson Distribution Demand with Backorders

In this section, the demand signal is generated from the combination of Binomial and Poisson distribution as mentioned in 3.1.3 and backorders are permitted.

For lead time value equals 2 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 72 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.88, 0.95 and 0.89 respectively for the 0.95 targeted service level for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.86, 0.93 and 0.88 respectively. A few values of above mention relation are shown in the table. 68 and 69

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.779	0.839	0.893	0.947	0.986
Achieved Cycle Service Level	0.625	0.706	0.788	0.880	0.961
Fill Rate	0.593	0.692	0.790	0.895	0.974

Table 68: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 02, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.735	0.818	0.877	0.938	0.980
Achieved Cycle Service Level	0.573	0.678	0.762	0.864	0.950
Fill Rate	0.534	0.656	0.760	0.878	0.961

Table 69: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 02, Demand Distribution: Poisson and Binomial

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a slightly lower inventory-demand ratio for the same targeted service level than the normal volatility model. 73

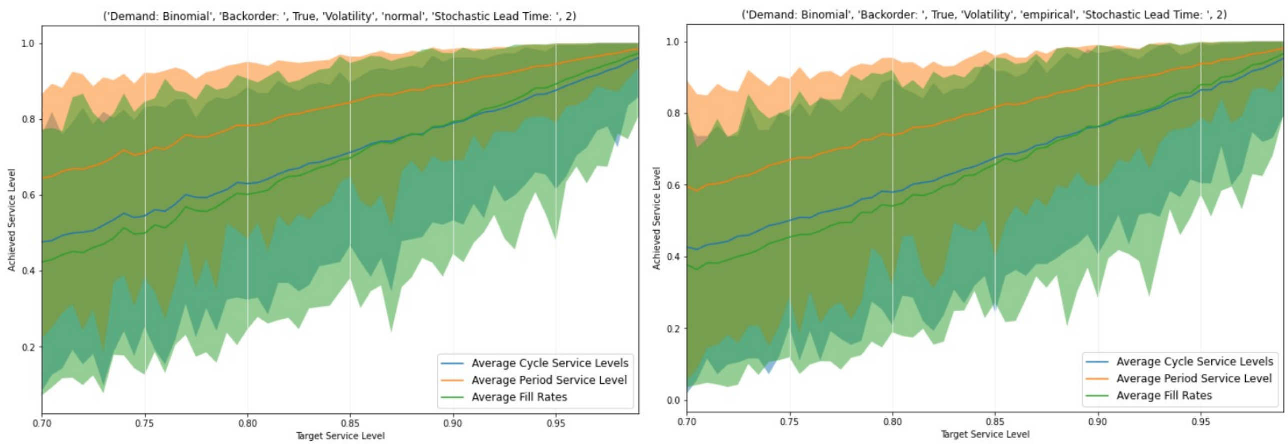


Figure 72: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 2

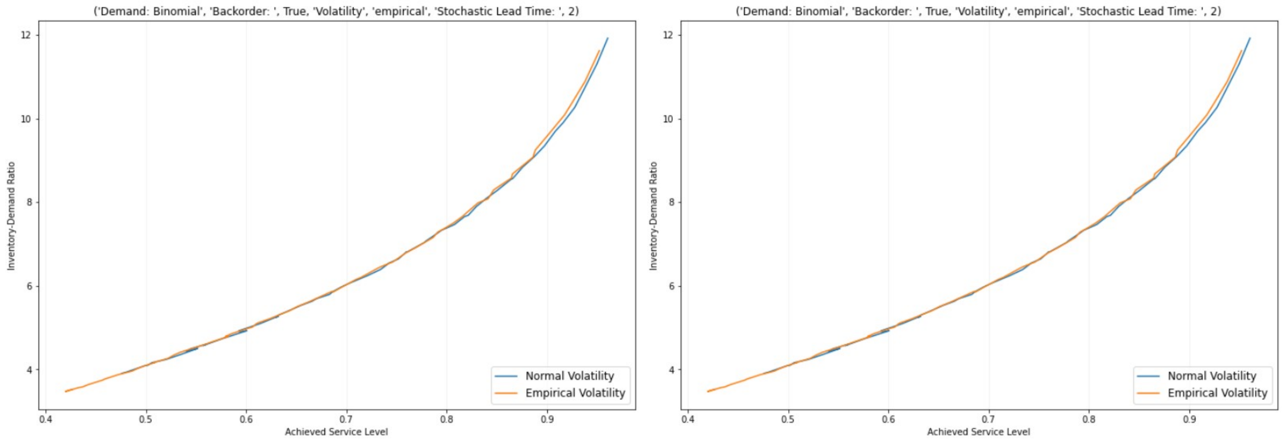


Figure 73: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 2

For both lead time value and review period equals to 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 74 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.90, 0.94 and 0.88 respectively for the 0.95 targeted service level for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.89, 0.94 and 0.87 respectively. A few values of above mention relation are shown in the table. 70 and 71

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.774	0.835	0.893	0.946	0.986
Achieved Cycle Service Level	0.666	0.742	0.820	0.900	0.97
Fill Rate	0.578	0.674	0.780	0.889	0.973

Table 70: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 04, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.723	0.807	0.877	0.939	0.980
Achieved Cycle Service Level	0.611	0.707	0.798	0.888	0.960
Fill Rate	0.511	0.630	0.748	0.874	0.960

Table 71: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 04, Demand Distribution: Poisson and Binomial

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a slightly lower inventory-demand ratio for the same targeted service level than the normal volatility model. 73

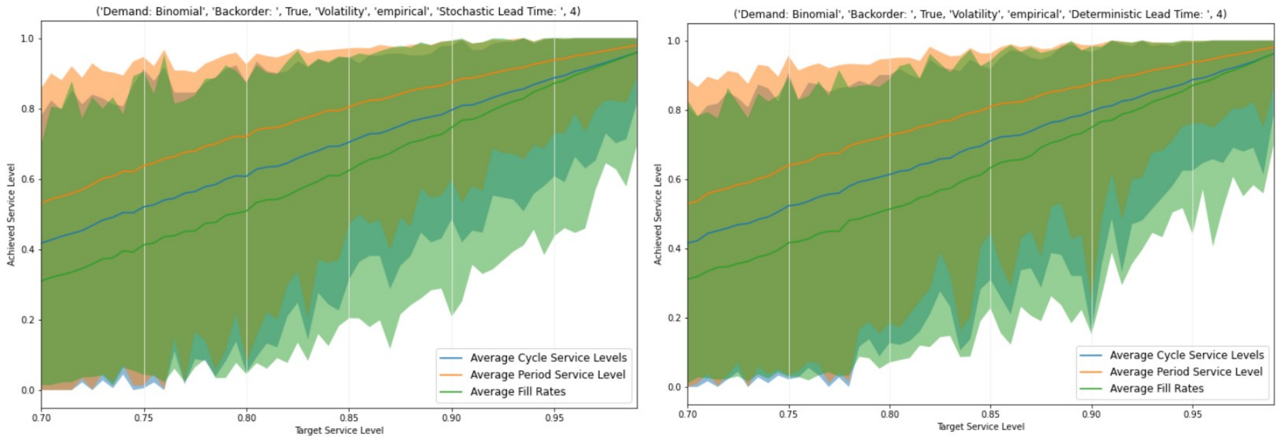


Figure 74: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 4

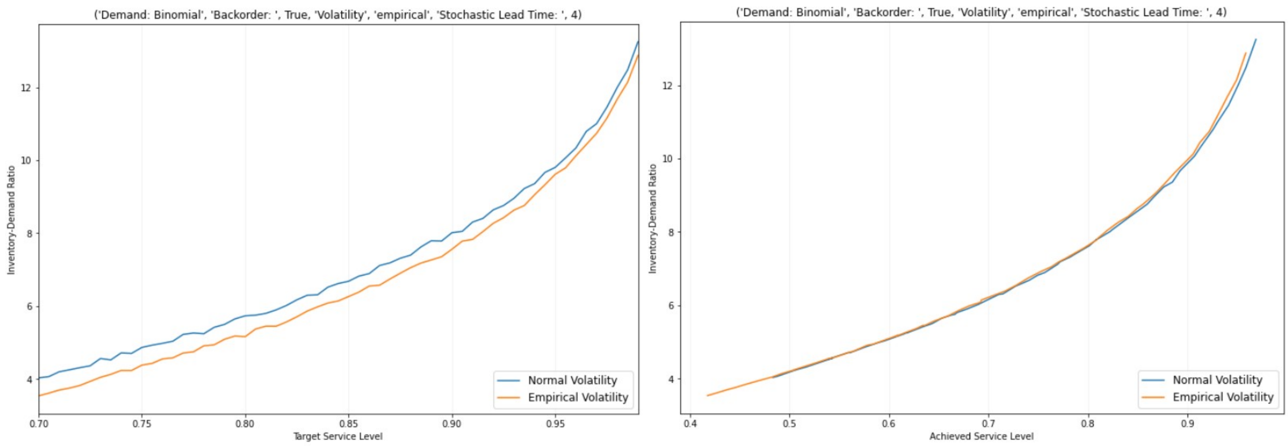


Figure 75: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 4

For lead time value equal to 10 and review period 4 the trend of achieved period and cycle service levels and fill rates against the targeted service level is depicted in the figure 76 for both volatility models. The highest average achieved cycle and period service level and fill rate are 0.89, 0.94 and 0.85 respectively for the 0.95 targeted service level for the normal volatility model whereas, with Kernel density estimation volatility model the highest average achieved cycle, period service level and fill rates are 0.87, 0.93 and 0.83 respectively. A few values of above mention relation are shown in the table. 72 and 73

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.761	0.821	0.882	0.942	0.986
Achieved Cycle Service Level	0.658	0.732	0.817	0.897	0.970
Fill Rate	0.524	0.614	0.739	0.859	0.966

Table 72: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: Normal, Stochastic Lead Time: 10, Demand Distribution: Poisson and Binomial

Targeted Service Level:	0.800	0.850	0.900	0.950	0.990
Achieved Period Service Level	0.727	0.801	0.871	0.932	0.976
Achieved Cycle Service Level	0.621	0.708	0.796	0.882	0.954
Fill Rate	0.480	0.585	0.709	0.838	0.943

Table 73: Targeted Service Level vs Achieved Service Levels With Backorders, Volatility Model: KDE, Stochastic Lead Time: 10, Demand Distribution: Poisson and Binomial

For the second metric, inventory-demand ratio, the Kernel density estimation volatility model has a slightly lower inventory-demand ratio for the same targeted service level than the normal volatility model. 77

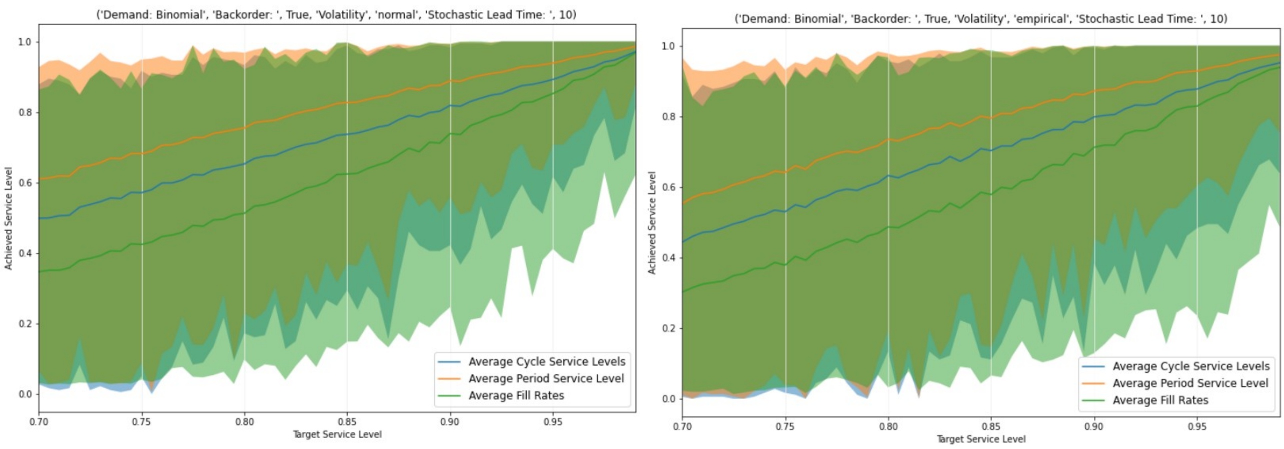


Figure 76: The graphs depict the trend and spread of cycle service level (in blue), period service level (in orange) and fill rate (in green) against the values of targeted values of service level (alpha). The graph on left shows the trend for when the normal volatility model is applied to calculate safety stock, whereas, the graph on right shows the same trend when the Kernel density estimation volatility model is used to calculate safety stock for the demand signal generated from combined Poisson and Binomial distribution and lead time equal to 10

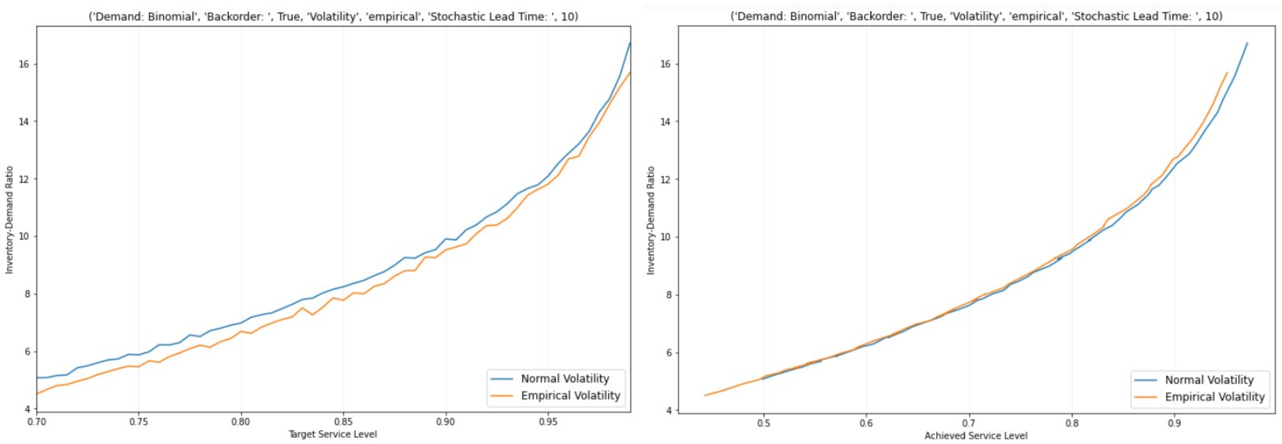


Figure 77: The graphs depict the trend of the inventory-demand ratio against service level for both normal and KDE (empirical) volatility models used to calculate safety stock levels. The graph on left shows the trend of the inventory-demand ratio against the targeted service levels whereas, the graph on the right depicts the same trend against the achieved service level for demand signal generated from combined Binomial and Poisson distribution and lead time equal to 10

6 Results and Conclusion

This chapter contains the results of the study and conclusions derived from the trends of cycle service level, period service level, fill rates and inventory-demand ratio. We also discuss which volatility model performs better with which parameters.

6.1 Stochastic Demand and Deterministic Lead Time

When demand is assumed to be stochastic and lead time was assumed to be deterministic, the simulation was carried out for three types of demand signal twice, once with lost sales and once with backorders (i.e. unsatisfied demand carried to the next timestep). When demand was generated from the Normal distribution and lost sales were assumed, the normal volatility model resulted in a better period and cycle service levels and fill rates on average than the Kernel density estimation volatility model for all lead time values. However, this difference was insignificant: all values of service level resulting from the normal volatility model were 0.001 better than the values resulting from the Kernel density estimation volatility model. Additionally, the Kernel density estimation model had a slightly lower inventory-demand ratio for the same targeted service level value as the normal volatility model in all three cases of lead time values. The same behaviour was observed when backorders were assumed instead of lost sales.

When the demand signal was generated using the Poisson distribution and loss of sales was assumed, the same conclusions could be made as those from the normal distribution. However, when considering Poisson demand with backorders, the inventory-demand ratio was the same for both volatility models at lead times of 2 and 4 but not of a lead time of 10. In this case, the Kernel density estimation volatility model had a lower inventory-demand ratio than the normal volatility model.

When the demand signal was generated using the combination of Binomial and Poisson distributions, we observed interesting results. For a targeted value less than 0.90, the normal volatility model resulted in better service levels, however, from 0.90 to 0.99 target service level, the cycle service level, period service level and fill rates resulting from the Kernel density estimation model are slightly better for all three values of lead time, though these differences were very small. When considering the inventory-demand ratio, the safety stocks derived using the normal volatility model resulted in a significantly lower inventory-demand ratio when the target service level was above 0.85 for lead times 2 and 4. Considering a lead time value of 10, the normal volatility model behaved slightly better than the Kernel density estimation volatility model in terms of inventory-demand ratio, but the difference was very small.

6.2 Stochastic Demand and Lead Time

When demand and lead time both were assumed to be stochastic, the simulation was carried out in a manner alike to previously. When demand was generated using the Normal distribution and lost sales were assumed, the achieved period and cycle service level and fill rate resulting from the normal volatility model had significantly higher values for all three lead times. This difference increased to 0.2 for the period service level and fill rate, and increased to 0.6 for the cycle service level. The inventory-demand ratio observed using the Kernel density estimation volatility model was noticeably lower than that for the normal volatlity model, at 2.5 and 4.5, respectively.

Similar, results were observed when backorders are assumed, though the largest difference between the values of period service level, cycle service level, and fill rate for both volatility models increased to 0.5 for the period service level and fill rate, and 0.8 for cycle service level.

When demand was generated from the Poisson distribution, again, the normal volatility appeared to be superior as it achieved on average higher service levels and fills rates, though the Kernel density estimation volatility model resulted in values that are at most only 0.1 units less for the period service level and less than 0.1 units less for fill rates. Cycle service levels had the highest difference at 0.6. On the other hand, the inventory-demand ratio resulting from the Kernel density estimation volatility model is significantly lower in comparison to the ratio resulting from using the normal volatility model.

When demand is generated from a combination of Poisson and Binomial distribution both volatility models yield similar results in terms of service level and fill rates, however, Kernel density estimation is slightly better because it results in higher service level and fill rate values, and produces a lower inventory-demand ratio.

6.3 Recommendations For Practitioners

It is interesting to observe that even when the volatility model matches the demand distribution identically, the non-parametric method performs similarly in all respects.

Given the fewer assumptions made by the non-parametric approach and similar performance to the normal model even under ideal conditions for the latter, it is difficult not to recommend the non-parametric model for use by practitioners given the results obtained in this study.

7 Future Work

There are avenues for future research in this study. The first and foremost is to analyze the possible impacts of the review period, i.e. how the safety stock level behaves when calculated from normal and Kernel density estimation while keeping the lead time constant and changing the review period. A preliminary experiment was carried out to understand the possible impacts which show some promising results.

For example, when demand is generated from Poisson distribution and lead stochastic lead time is considered to be equal to 4 units with a deviation of 2 units, the impact of different review periods with loss of sales is shown in figure 78 and with backorders is depicted in figure 79. This graph suggests that varying review periods which is a better option as well for companies, since this is the parameter they have control over in comparison to lead time value, can lead to some promising results.

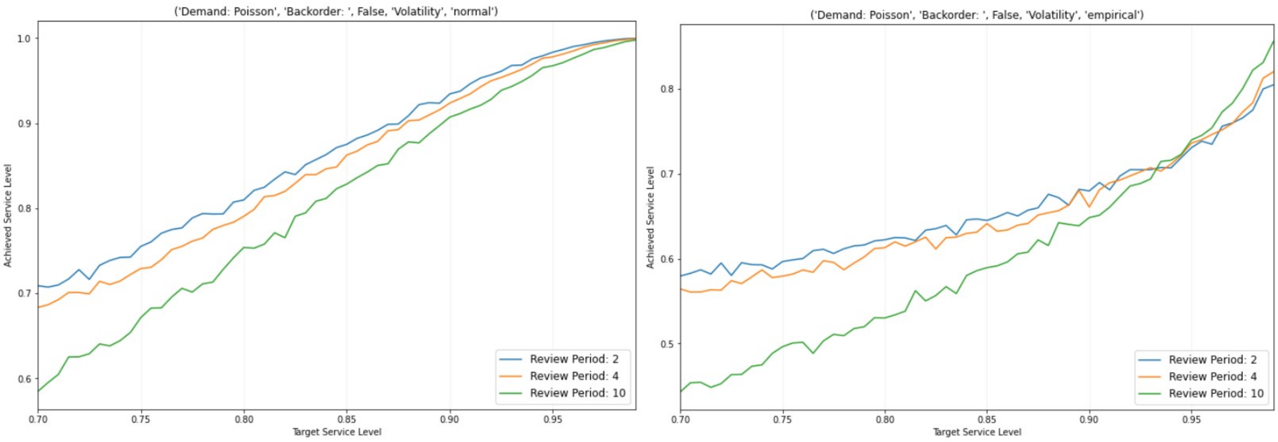


Figure 78: The graphs depict the possible achieved cycle service level against the targeted service level for different values of the review period for both volatility models. The graph on left shows the results when safety stock is calculated from the normal volatility model whereas the graph on right shows the results when the KDE volatility model is used.

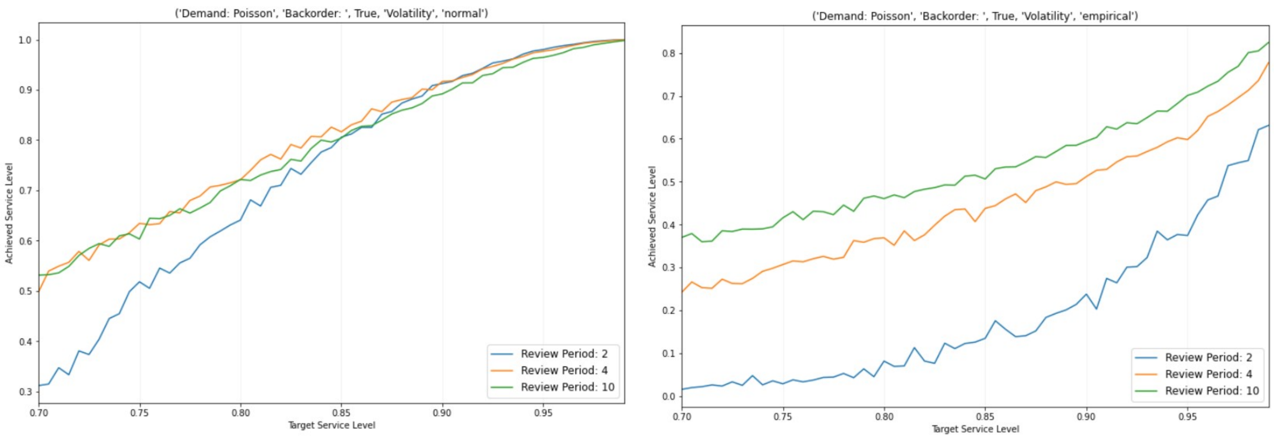


Figure 79: The graphs depict the possible achieved cycle service level against the targeted service level for different values of the review period for both volatility models. The graph on left shows the results when safety stock is calculated from the normal volatility model whereas the graph on right shows the results when the KDE volatility model is used.

The graphs for the other two demand signal distributions with both lost sales and backorders are mentioned in appendix A

Another important area to dive into is the impact of sample size. As mentioned in 3.1 a small sample size was used to estimate the parameters from the demand signal which was used to calculate the inventory level including the safety stock level. It is an interesting exploration to see how the performance of safety stock level changes if the sample size is varied.

Additionally, various parameters in this study were kept constant such as lead time deviation, and the review period initial parameters for the demand signal generation, which can be explored in detail for further analysis.

References

[1] Demand forecasting: An industry guide, Nov 2020.

[2] P. J. Agrell. A multicriteria framework for inventory control. *International Journal of Production Economics*, 41(1):59–70, 1995. Proceedings of the 12th International Conference on Production Research.

[3] M. G. Avci and H. Selim. A multi-objective, simulation-based optimization framework for supply chains with premium freights. *Expert Systems with Applications*, 67:95–106, 2017.

[4] M. G. Avci and H. Selim. A multi-objective simulation-based optimization approach for inventory replenishment problem with premium freights in convergent supply chains. *Omega*, 80:153–165, 2018.

[5] Z. Bahroun and N. Belgacem. Determination of dynamic safety stocks for cyclic production schedules. *Operations Management Research*, 12(1-2):62–93, 06 2019. Copyright - Operations Management Research is a copyright of Springer, (2019). All Rights Reserved; Last updated - 2020-11-17.

[6] A.-L. Beutel and S. Minner. Safety stock planning under causal demand forecasting. *International Journal of Production Economics*, 140(2):637–645, 2012. Sixteenth internationalworkingseminaronproductioneconomics, Innsbruck, 2010.

[7] B. Bouslah, A. Gharbi, and R. Pellerin. Joint production and quality control of unreliable batch manufacturing systems with rectifying inspection. *International Journal of Production Research*, 52(14):4103–4117, 2014.

[8] R. N. Boute, S. M. Disney, M. R. Lambrecht, and B. V. Houdt. Coordinating lead times and safety stocks under autocorrelated demand. *European Journal of Operational Research*, 232(1):52–63, 2014.

[9] M. Braglia, D. Castellano, and M. Frosolini. A novel approach to safety stock management in a coordinated supply chain with controllable lead time using present value. *Applied Stochastic Models in Business and Industry*, 32(1):99–112, 2016.

[10] F. P. Buffa. A model for allocating limited resources when making safety-stock decisions. *Decision Sciences*, 8(2):415–426, 1977.

[11] T. Chakraborty and B. Giri. Joint determination of optimal safety stocks and production policy for an imperfect production system. *Applied Mathematical Modelling*, 36(2):712–722, 2012.

[12] A. Chaturvedi and V. Martínez-de Albéniz. Safety stock, excess capacity or diversification: Trade-offs under supply and demand uncertainty. *Production and Operations Management*, 25(1):77–95, 2016.

[13] F. Cheng and S. P. Sethi. A periodic review inventory model with demand influenced by promotion decisions. *Management Science*, 45(11):1510–1523, 1999.

[14] E. M. DAR-EL and C. J. MALMBORG. Improved strategy for service level-based safety stock determination. *Production Planning & Control*, 2(2):116–121, 1991.

[15] G. D. Eppen and R. K. Martin. Determining safety stock in the presence of stochastic lead time and demand. *Management Science*, 34(11):1380–1390, 1988.

[16] R. Estes. The joint probability approach and reorder point determination. *Journal of Production and Inventory Management*, 14, 1973.

[17] B. Fahimnia, C. S. Tang, H. Davarzani, and J. Sarkis. Quantitative models for managing supply chain risks: A review. *European Journal of Operational Research*, 247(1):1–15, 2015.

[18] S. Fotopoulos, M.-C. Wang, and S. Rao. Safety stock determination with correlated demands and arbitrary lead times. *European Journal of Operational Research*, 35(2):172–181, 1988.

[19] G. Gallego. Scheduling the production of several items with random demands in a single facility. *Management Science*, 36(12):1579, 12 1990. Copyright - Copyright Institute for Operations Research and the Management Sciences Dec 1990; Last updated - 2021-09-09; CODEN - MNSCDI.

[20] M. Gansterer, C. Almeder, and R. F. Hartl. Simulation-based optimization methods for setting production planning parameters. *International Journal of Production Economics*, 151:206–213, 2014.

[21] J. N. Gonçalves, M. Sameiro Carvalho, and P. Cortez. Operations research models and methods for safety stock determination: A review. *Operations Research Perspectives*, 7:100164, 2020.

[22] R. W. Grubbström. A net present value approach to safety stocks in planned production. *International Journal of Production Economics*, 56-57:213–229, 1998. Production Economics: The Link Between Technology And Management.

[23] I. Heckmann, T. Comes, and S. Nickel. A critical review on supply chain risk – definition, measure and modeling. *Omega*, 52:119–132, 2015.

[24] J. Heizer, B. Render, and C. Munson. *Operations management: Sustainability and Supply Chain Management*. Pearson, 12 edition, 2022.

[25] T. Hosoda and S. M. Disney. Impact of market demand mis-specification on a two-level supply chain. *International Journal of Production Economics*, 121(2):739–751, 2009.

[26] S. Humair, J. D. Ruark, B. Tomlin, and S. P. Willems. Incorporating stochastic lead times into the guaranteed service model of safety stock optimization. *Interfaces*, 43(5):421–434, 2013. Copyright - Copyright Institute for Operations Research and the Management Sciences Sep/Oct 2013; Document feature - Graphs; Tables; Diagrams; Equations; ; Last updated - 2021-09-10; CODEN - INFAC4; SubjectsTermNotLitGenreText - United States–US.

[27] G. K. Janssens and K. M. Ramaekers. A linear programming formulation for an inventory management decision problem with a service constraint. *Expert Systems with Applications*, 38(7):7929–7934, 2011.

[28] H. Jodlbauer and S. Reitner. Optimizing service-level and relevant cost for a stochastic multi-item cyclic production system. *International Journal of Production Economics*, 136(2):306–317, 2012.

[29] J. Kanet, M. Gorman, and M. Stößlein. Dynamic planned safety stocks in supply networks. *International Journal of Production Research*, 48(22):6859–6880, 2010.

[30] S. K. Lam, A. Pitrou, and S. Seibert. Numba: A llvm-based python jit compiler. In *Proceedings of the Second Workshop on the LLVM Compiler Infrastructure in HPC*, pages 1–6, 2015.

[31] P. Larson. Designing and managing the supply chain: Concepts, strategies, and case studies, david simchi-levi philip kaminsky edith simchi-levi. *Journal of Business Logistics*, 22, 03 2001.

[32] F. Longo and G. Mirabelli. An advanced supply chain management tool based on modeling and simulation. *Comput. Ind. Eng.*, 54(3):570–588, apr 2008.

[33] J. Moors and L. Strijbosch. Exact fill rates for (r, s, s) inventory control with gamma distributed demand. *Journal of the Operational Research Society*, 53, 02 2001.

[34] A. Persona, D. Battini, R. Manzini, and A. Pareschi. Optimal safety stock levels of subassemblies and manufacturing components. *International Journal of Production Economics*, 110(1):147–159, 2007. Recent Developments in Operations Strategy, Supply Chain Design and Production Systems.

[35] J. A. Rappold and K. D. Yoho. Setting safety stocks for stable rotation cycle schedules. *International Journal of Production Economics*, 156:146–158, 2014.

[36] A. Reichhart, J. M. Framiñán, and M. Holweg. On the link between inventory and responsiveness in multi-product supply chains. *International Journal of Systems Science*, 39(7):677–688, 2008.

[37] I. Rizkya, K. Syahputri, R. M. Sari, Anizar, I. Siregar, and E. Ginting. Comparison of periodic review policy and continuous review policy for the automotive industry inventory system. *IOP Conference Series: Materials Science and Engineering*, 288:12–85, jan 2018.

[38] A. J. Ruiz-Torres and F. Mahmoodi. Safety stock determination based on parametric lead time and demand information. *International Journal of Production Research*, 48(10):2841–2857, 2010.

[39] E. Silver, H. Naseraldin, and D. Bischak. Determining the reorder point and order-up-to-level in a periodic review system so as to achieve a desired fill rate and a desired average time between replenishments. *JORS*, 60:1244–1253, 09 2009.

[40] J. R. Trapero, M. Cardós, and N. Kourentzes. Empirical safety stock estimation based on kernel and garch models. *Omega*, 84:199–211, 2019.

[41] J. R. Trapero, M. Cardós, and N. Kourentzes. Quantile forecast optimal combination to enhance safety stock estimation. *International Journal of Forecasting*, 35(1):239–250, 2019. Special Section: Supply Chain Forecasting.

[42] J. R. Trapero, D. J. Pedregal, R. Fildes, and N. Kourentzes. Analysis of judgmental adjustments in the presence of promotions. *International Journal of Forecasting*, 29(2):234–243, 2013.

[43] K. H. van Donselaar and R. A. Broekmeulen. Determination of safety stocks in a lost sales inventory system with periodic review, positive lead-time, lot-sizing and a target fill rate. *International Journal of Production Economics*, 143(2):440–448, 2013. Focusing on Inventories: Research and Applications.

[44] P. D. Van Ness and W. J. Stevenson. Reorder-point models with discrete probability distributions. *Decision Sciences*, 14(3):363–369, 1983.

[45] N. Vandeput. *Inventory Optimization: Models and Simulations*. Business & economics. De Gruyter, 2020.

[46] P. Wang, W. Zinn, and K. L. Croxton. Sizing inventory when lead time and demand are correlated. *Production and Operations Management*, 19(4):480–484, 2010. Cited By :11.

[47] R. Zafar. Volatility models for safety stock; a simulation based study. https://github.com/rija-zafar/MSc_dissertation_2022, 2022.

[48] C. Zhou and S. Viswanathan. Comparison of a new bootstrapping method with parametric approaches for safety stock determination in service parts inventory systems. *International Journal of Production Economics*, 133(1):481–485, 2011. Leading Edge of Inventory Research.

Appendices

A Review Period Preliminary Analysis

A.1 Target Service Level vs Achieved Service Level with varying Review Period

These graphs depicts the possible achieved cycle service level against targeted service level for different values of review period for both volatility models. The graph on left shows the results when safety stock is calculated from normal volatility model whereas the graph on right shows the results when KDE volatility model is used.

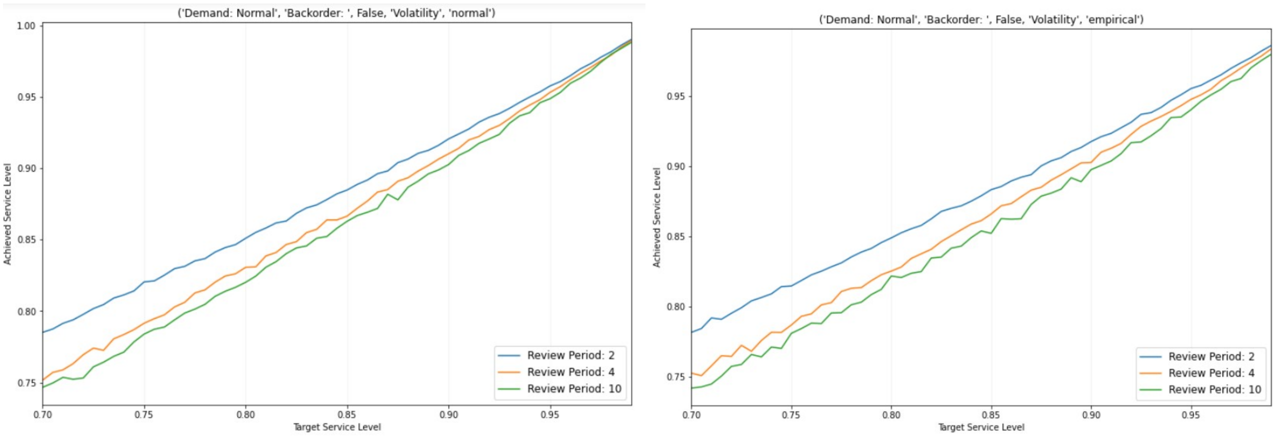


Figure 80: Demand Signal: Normal Distribution, Lead Time Type: Deterministic, With Lost Sales

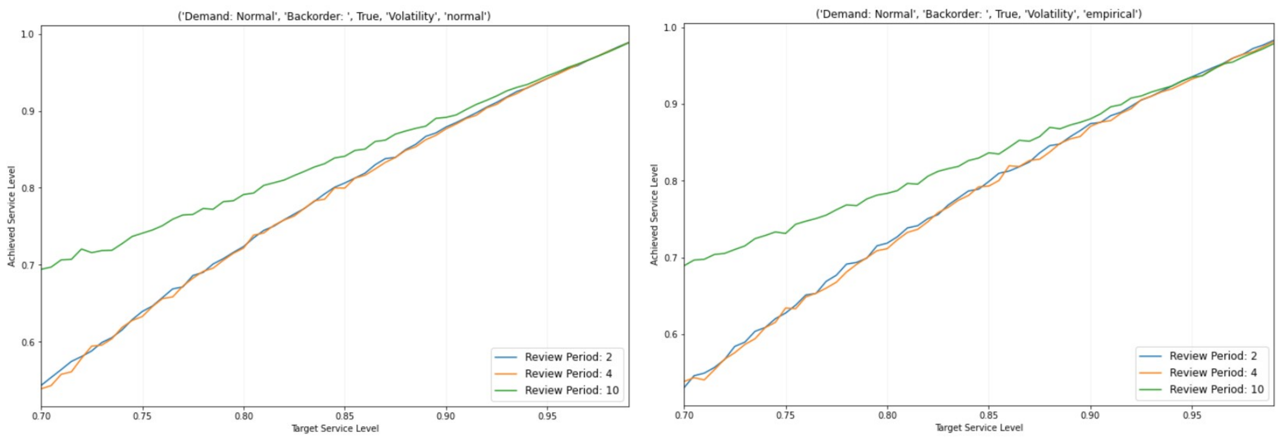


Figure 81: Demand Signal: Normal Distribution, Lead Time Type: Deterministic, With Backorders

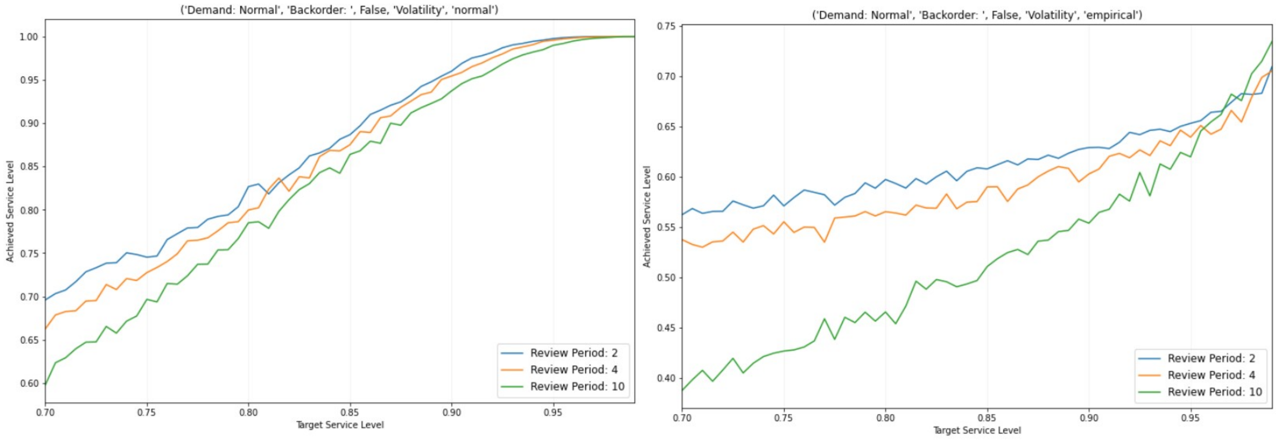


Figure 82: Demand Signal: Normal Distribution, Lead Time Type: Stochastic, With Lost Sales

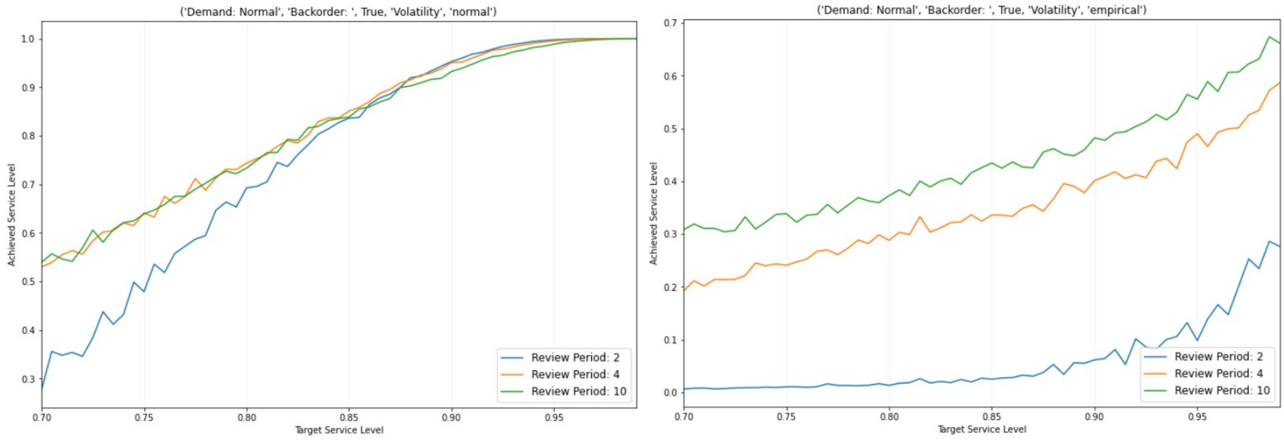


Figure 83: Demand Signal: Normal Distribution, Lead Time Type: Stochastic, With Backorders

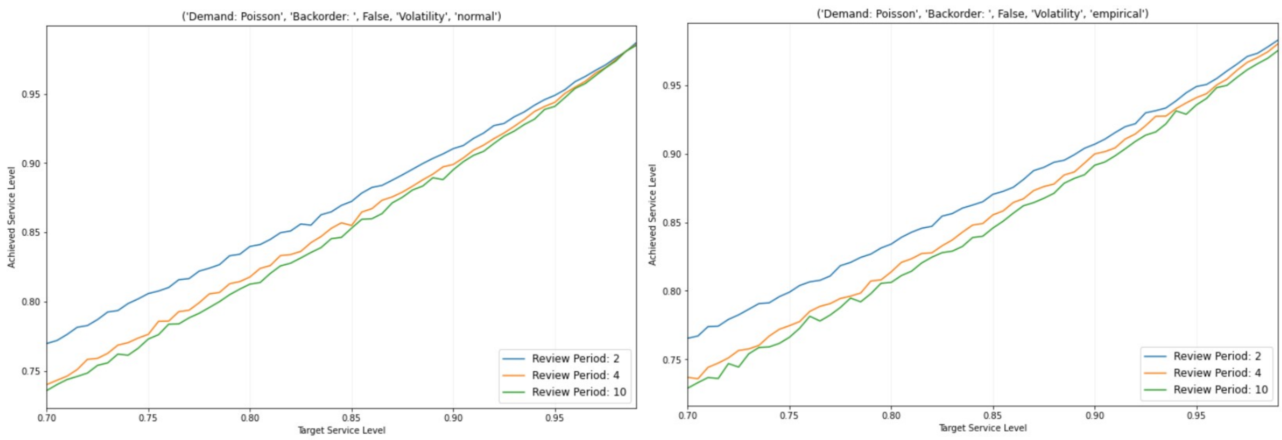


Figure 84: Demand Signal: Poisson Distribution, Lead Time Type: Deterministic, With Lost Sales

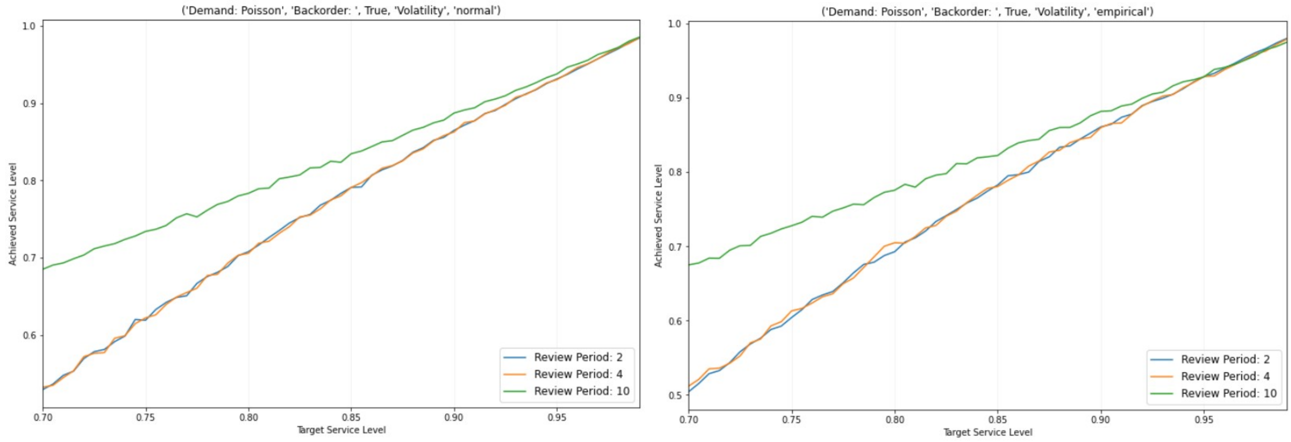


Figure 85: Demand Signal: Poisson Distribution, Lead Time Type: Deterministic, With Backorders

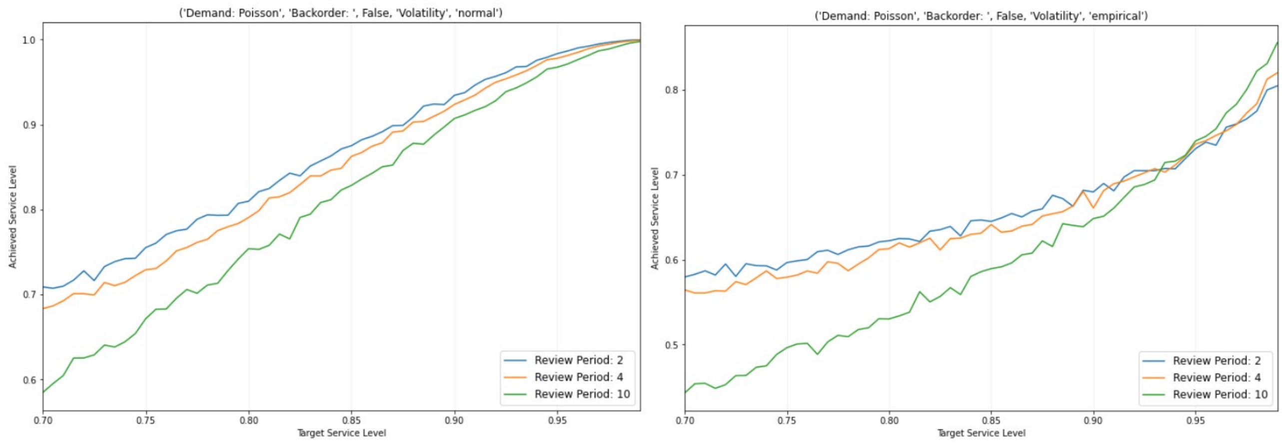


Figure 86: Demand Signal: Poisson Distribution, Lead Time Type: Stochastic, With Lost Sales

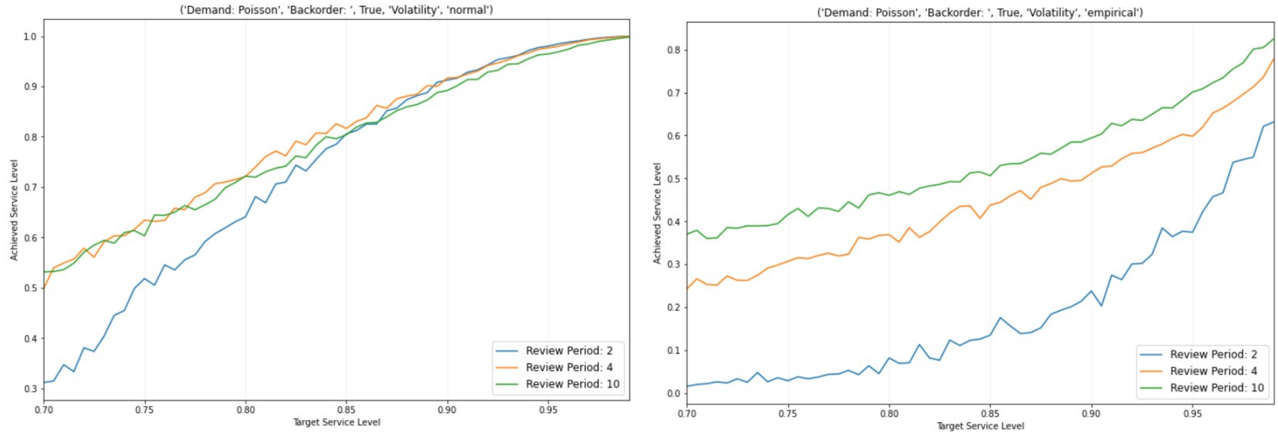


Figure 87: Demand Signal: Poisson Distribution, Lead Time Type: Stochastic, With Backorders

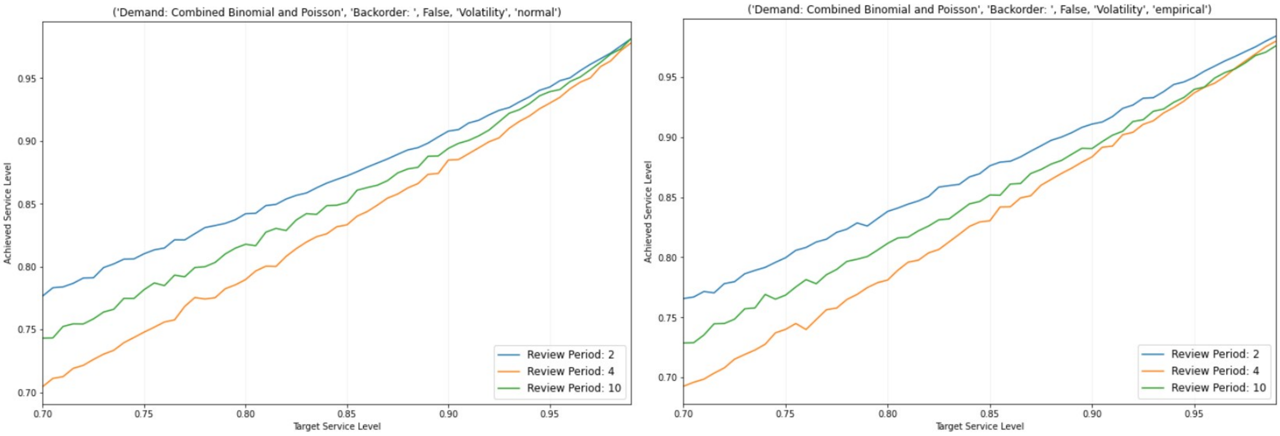


Figure 88: Demand Signal: Combined Binomial and Poisson Distribution, Lead Time Type: Deterministic, With Lost Sales

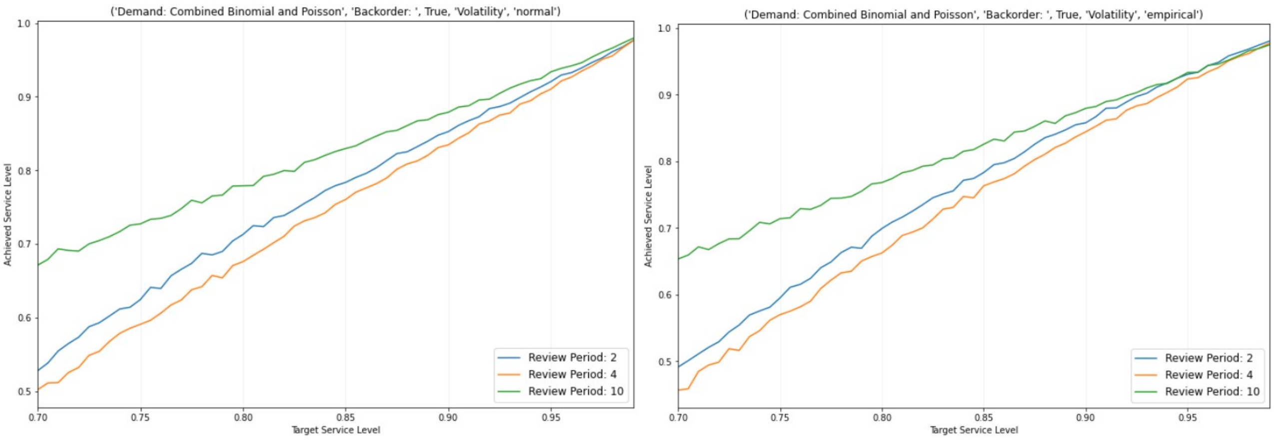


Figure 89: Demand Signal: Combined Binomial and Poisson Distribution, Lead Time Type: Deterministic, With Backorders

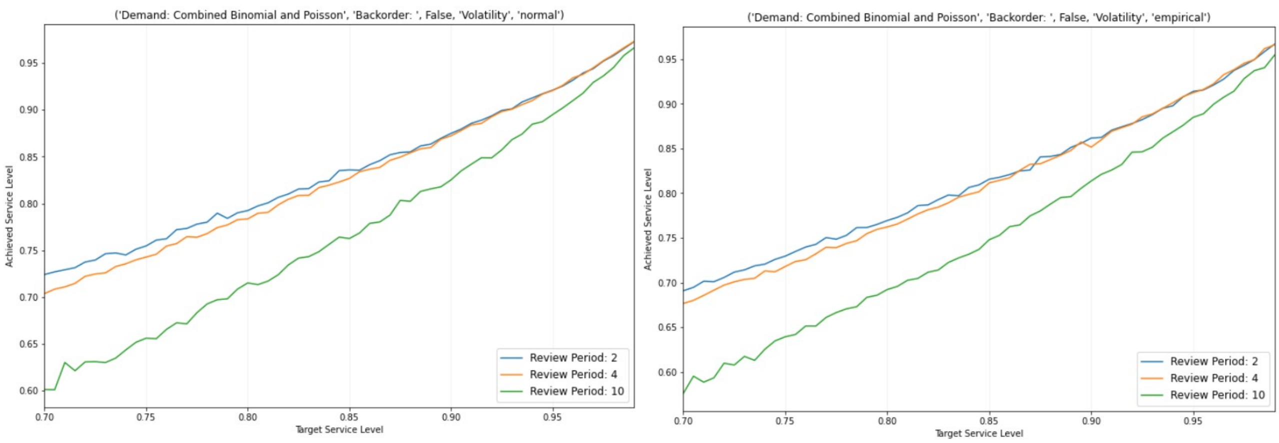


Figure 90: Demand Signal: Combined Binomial and Poisson Distribution, Lead Time Type: Stochastic, With Lost Sales

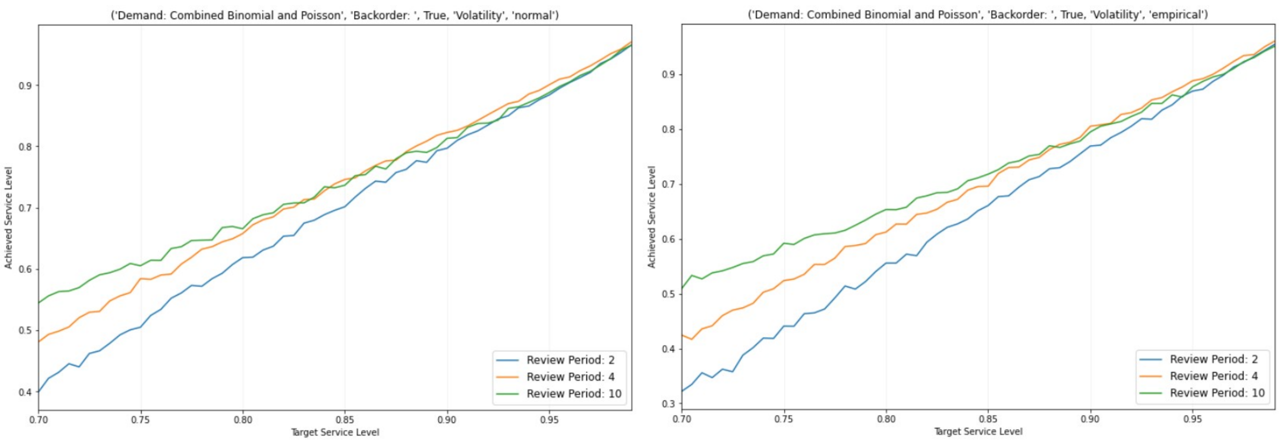


Figure 91: Demand Signal: Combined Binomial and Poisson Distribution, Lead Time Type: Stochastic, With Backorders