Power Flow Equations

Buses types:

1. Load Bus (PQ)
2. Generator bus/Voltage-controlled bus (PV)
3. Slack / Swing Bus (V δ)

Steady-state analysis- under given load condition

$I_i = \sum_{j=1}^{n} Y_{ij} Vj$ ————**(1)**

$S_i = P_i + Q_i = V_i I^*$ ————-(2)

From Equations 1 and 2, we can write

$$P_i + jQ_i = V_i (\sum_{j=1}^{n} Y_{ij} Vj)^*$$

As we know that the Admittance(Y)= G+jB

Where, G is Conductance, measured in siemens

B is Susceptance measured in siemens

$$= V_i (\sum_{j=1}^{n} [Vj(G_{ij} - jB_{ij})]^*$$

$$P_i = \mathbf{R}[V_i (\sum_{j=1}^{n} [Vj(G_{ij} - jB_{ij})]^*]$$

$$= \mathbf{R}[|V_i| < \delta_i (\sum_{j=1}^{n} [|Vj| < \delta j(G_{ij} - jB_{ij})]^*]$$

$$= \mathbf{R} \sum_{j=1}^{n} |V_i||V_j| < (\delta_i - \delta j)(G_{ij} - jB_{ij})^*]]$$

$$= \sum_{j=1}^{n} |V_i||V_j| < (\delta_i - \delta j)(G_{ij} - jB_{ij})^*]]$$

$$= \sum_{j=1}^{n} |V_i||V_j|[Cos(\delta_i - \delta j) * G_{ij} + B_{ij} * Sin(\delta_i - \delta j)] \quad \text{————--(3)}$$

Similarly,

$$Q_i = \sum_{j=1}^{n} |V_i||V_j|[Sin(\delta_i - \delta j) * G_{ij} - B_{ij} * Cos(\delta_i - \delta j)] \quad \text{————-(4)}$$

Equation (3) and (4) are used to calculate the net power flow in a bus. But these two equations contain trigonometric terms so, it is a nonlinear function that requires non-linear optimization.

DC Power Flow:

The DC power flow is a linearization of the non-linear AC power flow; therefore, the reference point of the linearization has an impact.

- DCLF looks only at active power flows and neglects reactive power flows.
- In DCLF, the nonlinear model of the AC system is simplified to a linear form through these assumptions.

a. Line resistances (active power losses) are negligible (G→ 0).

b. Voltage angle differences are assumed to be small i.e. $Sin(\theta) = \theta$ and $Cos(\theta) = 1$.

c. Magnitudes of bus voltages are set to 1.0 per unit (flat voltage profile).

d. Tap settings are ignored.

Considering the above (a) and (b) assumptions, equations 3 and 4 will be

$P_i = \sum_{j=1}^{n} |V_i||V_j|[B_{ij} * (\delta_i - \delta j)]$ ————-(5)

$Q_i = \sum_{j=1}^{n} -|V_i||V_j|[B_{ij}]$ ——————————(6)

Here for the Susceptance

if j $\neq$ i, $B_{ij} = -b_{ij}$ and j=i, $B_{ij} = b_j$

So, $B_{ij} = b_i + \sum_{j=1 j\neq i}^{n}[b_{ij}]$

now equation 6 becomes

$$Q_i = -|V_i|^2 * B_{ii} - |V_i||V_j|(\sum_{j=1 j\neq i}^{n}(B_{ij})$$
$$= -|V_i|^2(b_i + \sum_{j=1 j\neq i}^{n}[b_{ij}]) - |V_i||V_j|(\sum_{j=1 j\neq i}^{n}(-b_{ij})$$
$$= -|V_i|^2 b_i - |V_i|^2 \sum_{j=1 j\neq i}^{n}[b_{ij}] + |V_i||V_j| \sum_{j=1 j\neq i}^{n}[b_{ij}]$$
$$= -|V_i|^2 b_i - \sum_{j=1 j\neq i}^{n}[b_{ij}]|V_i|(|V_i| - |V_j|) \text{————————}(7)$$

1. The first term corresponds to the reactive power supplied (if a capacitor) or consumed (if an inductor) by the shunt susceptance modeled at bus i.
2. The second term corresponds to the reactive power flowing on the circuits connected to bus i. Only these circuits will have nonzero b_{ij}. One sees that each circuit will have a per-unit reactive flow in proportion to (a) the bus i^{th} voltage magnitude and (b) the difference in per-unit voltages at the circuit's terminating buses. The flow direction will be from the higher voltage bus to the lower voltage bus.

Now considering the flat profile system,

eq (5) can be written as

$$P_i = \sum_{j=1}^{n} B_{ij}(\delta_i - \delta_j)$$

$$= \sum_{j=1 j\neq i}^{n} B_{ij}(\delta_i - \delta_j) \text{ ———————————(8)}$$

From equations 7 &8, the reactive power flow depends on voltage difference whereas active power flow depends on power angle difference. But the voltage difference is nearly zero so DC power flow did not consider reactive power. To conclude this, we will develop our optimization problem with active power only.

$$P_i = \sum_{j=1 j\neq i}^{n} B_{ij}(\delta_i - \delta_j)$$

where j $j \neq i, B_{ij} = -b_{ij}$, and $b_{ij} = -1/X_{ij}$ so

$$P_i = \sum_{j=1 j\neq i}^{n} 1/X_{ij}(\delta_i - \delta_j) \text{ ———-(9)}$$

From equation 9, The active power flow through transmission line i, between buses s and r, can be calculated

$$P_L = 1/X_{Li}(\delta_s - \delta_r)$$

DC power flow equations in the matrix form and the corresponding matrix relation for flows through branches are,

$$P_i = [B][A][\theta]$$

where,

A→ MxN Incidence matrix (That shows the relationship between nodes)

Pi→ Nx1 vector of bus active power injections for buses 1…N

PL→ Mx1 vector of bus active power flow for branches 1…M (M is the number of branches)

θ→ Nx1 vector of bus voltage angles injections for buses 1…N