UNIT-2

VECTOR SPACES-2

BASIS AND DIMENSIONS

* Formula's :-

Linear Combi వుంటుంది.

① Basis = L.I. + L.S.

L.I. → Linearly independent (=0)

$S = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$

② Dimension (dim) = last Value = $\underline{n}$. [Dimension వుంటే automatic Basis వుంటుంది]

③ finite dimension → finite Basis.

LONGS :- v.v.Imp ([illegible])

Theorem NO. ①: Let W_1 & W_2 are two Subspaces of finite dimensional Vector Space $V(F)$ then

P.T. $\dim(W_1+W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

PROOF:

Let $V(F)$ be a Vector space.

Let W_1 and W_2 are Subspaces of V.

W_1+W_2 & $W_1 \cap W_2$ are Sub-Spaces. [∵ Unit 1, Shorts, Th. NO. ① & ③]

Imp. points

- → RHS then LHS.
- → * $\dim(W_1 \cap W_2) = K$ → దీనికి Basis వుంటుంది.
- → B_1, B_2
- → S' Basis < L.I., L.S

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Let $\dim(W_1 \cap W_2) = K$.

L.I + L.S.

and $S = \{\gamma_1, \gamma_2, \dots \gamma_K\}$ be a basis of $W_1 \cap W_2$.

$\therefore S \subseteq W_1$ and $S \subseteq W_2$ and S is L.I.

$\therefore$ Since, S is L.I. and $S \subseteq W_1$,

S can be extended to form a basis of W_1.

Let $B_1 = \{\gamma_1, \gamma_2, \dots \gamma_K, \alpha_1, \alpha_2, \dots \alpha_m\}$

be a basis of W_1.

$\therefore \dim W_1 = K + m$.

Again, S is L.I. and $S \subseteq W_2$,

S can be extended to form a basis of W_2.

Let $B_2 = \{\gamma_1, \gamma_2 \dots \gamma_K, \beta_1, \beta_2 \dots \beta_t\}$

be a basis of W_2.

$\dim W_2 = K + t$

$RHS = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2) = K + m + \cancel{K} + t - \cancel{K}$

$= K + m + t$

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To Prove that $\dim(W_1+W_2) = K+m+t$.

Let $S' = \{\gamma_1, \gamma_2, \dots \gamma_K, \alpha_1, \alpha_2, \dots \alpha_m, \beta_1, \beta_2 \dots \beta_t\}$

is a Basis of W_1+W_2 and hence

ఎలా చూపించాలి ?

$\dim(W_1+W_2) = K+m+t$.

i, To Prove that S' is L.I :

$$\therefore c_1\gamma_1 + c_2\gamma_2 + \dots + c_K\gamma_K + a_1\alpha_1 + a_2\alpha_2 + \dots a_m\alpha_m + b_1\beta_1 + b_2\beta_2 + \dots + b_t\beta_t = 0 \longrightarrow (1)$$

$$b_1\beta_1 + b_2\beta_2 + \dots + b_t\beta_t = -c_1\gamma_1 - c_2\gamma_2 \dots - c_K\gamma_K - a_1\alpha_1 - a_2\alpha_2 \dots - a_m\alpha_m$$

$= $ L.C. of elements of B_1

$\therefore b_1\beta_1 + b_2\beta_2 + \dots + b_t\beta_t \in W_1$ [$\because B_1$ is Basis of W_1]

Again Consider,

$$0.\gamma_1 + 0.\gamma_2 + \dots + 0\gamma_K + b_1\beta_1 + b_2\beta_2 + \dots b_t\beta_t$$

$= $ l.c. of elements of B_2

$\therefore b_1\beta_1 + b_2\beta_2 + \dots + b_t\beta_t \in W_2$ [$\because B_2$ is Basis of W_2]

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$\therefore\ b_1\beta_1 + b_2\beta_2 + \cdots + b_t\beta_t \in W_1 \cap W_2$

$\therefore\ b_1\beta_1 + b_2\beta_2 + \cdots + b_t\beta_t = d_1\gamma_1 + d_2\gamma_2 + \cdots d_k\gamma_k$

$b_1\beta_1 + b_2\beta_2 + \cdots + b_t\beta_t - d_1\gamma_1 - d_2\gamma_2 - \cdots - d_k\gamma_k = 0$

$\therefore$ L.C. of elements of B_2

Since, B_2 is a Basis.

$\therefore\ b_1 = b_2 = \cdots = b_t = 0;\ d_1 = d_2 = \cdots d_k = 0.$

$\therefore\ b_1 = b_2 = \cdots = b_t = 0$ sub in ① $\Rightarrow$

$c_1\gamma_1 + c_2\gamma_2 + \cdots + c_k\gamma_k + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_m\alpha_m = 0$

linear combination of elements of B_1.

Since, B_1 is Basis,

$\therefore\ c_1 = c_2 = \cdots c_k = 0;\ a_1 = a_2 = \cdots a_m = 0$

$\therefore\ S'$ is L.I.

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(ii), To P.T. $L(S') = W_1 + W_2$.

It is sufficient to P.T. $L(S') \subseteq W_1 + W_2$ and $W_1 + W_2 \subseteq L(S')$.

Every Vector of S' is a Vector of $W_1 + W_2$.

$$\therefore \quad L(S') \subseteq W_1 + W_2 \longrightarrow (2)$$

Let $\delta \in W_1 + W_2$

$\delta = \alpha + \beta$ where $\alpha \in W_1$, $\beta \in W_2$

$\delta =$ l.c. of elements B_1 + L.C. of elements of B_2.

$\delta =$ l.c. of elements γ's and α's + l.c. of elements γ's and β's

$\delta =$ l.c. of elements γ's, α's & β's
= l.c. of elements of S'.

$\delta \in L(S')$

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$\delta \in w_1 + w_2 \Rightarrow \delta \in L(S')$

$\therefore \quad w_1 + w_2 \subseteq L(S') \longrightarrow (3)$

$x \in A ; x \in B$

$\boxed{A \subseteq B}$

From (2) & (3) $\Rightarrow$

$$L(S') = w_1 + w_2$$

Hence, S' is <u>Basis</u> of $w_1 + w_2$

$$\therefore \dim(w_1 + w_2) = k + m + t.$$

$\therefore \quad LHS = RHS.$

$\therefore$ Hence the theorem is proved.

2 times

DP

LA-4 free ✓

RA-1 free ✓

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UNIT-2

CLASS NO.2

* Quotient Space:

$$\frac{G}{H}, \quad \frac{G}{K}$$

$$K+a, K+b$$

$$\frac{V}{W} = \{W+\alpha \mid \alpha \in V\}$$

$\downarrow$

It is called - Quotient space.

$\alpha, \beta \in V$

Conditions:-

i) $(W+\alpha) + (W+\beta) = W + (\alpha+\beta)$

ii) $a(W+\alpha) = aW + a\alpha$

$a[W+\alpha]$

iii) $W+\alpha = W$

$\Rightarrow \alpha \in W$.

$\boxed{\gamma \in W \Rightarrow W+\gamma = W}$

Theorem No. 2:- Let W be a subspace of a finite dimensional vector space $V(F)$ then

$$\dim\left(\frac{V}{W}\right) = \dim V - \dim W.$$

PROOF:

Since, $V(F)$ is finite dimensional vector space $\Rightarrow$ W is also finite dimensional vector space.

[W FDVC $\downarrow$ Basis]

$\therefore$ Let the set $B = \{\alpha_1, \alpha_2, \ldots \alpha_m\}$ be the Basis of W.

$\boxed{\therefore \dim W = m}$

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Since,

β is L.I., it can be expressed as to form a basis of V.

Let the set $S = \{\alpha_1, \alpha_2 \cdots \alpha_m, \beta_1, \beta_2 \cdots \beta_t\}$

be the basis of V.

$$\dim V = m + t$$

$$\therefore \text{RHS} = \dim V - \dim \omega$$

$$= \cancel{m} + t - \cancel{m}$$

$$= t.$$

To P.T. $\dim\left(\frac{V}{\omega}\right) = t$.

Now we shall P.T.

the set $S' = \{\omega + \beta_1, \omega + \beta_2, \cdots \omega + \beta_t\}$

is a Basis of $\frac{V}{\omega}$

S' ✓

$S' = \{t\}$

Basis of $\frac{V}{\omega}$

$d\left(\frac{V}{\omega}\right) = t$

L.I + L.S

i) To P.T. S' is L.I.:

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Consider,

$$b_1(\omega+\beta_1)+b_2(\omega+\beta_2)+\dots+b_t(\omega+\beta_t)=\omega+0$$

$$\omega+b_1\beta_1+\omega+b_2\beta_2+\dots+\omega+b_t\beta_t=\omega$$

$$\omega+(b_1\beta_1+b_2\beta_2+\dots+b_t\beta_t)=\omega$$

$$\therefore\ b_1\beta_1+b_2\beta_t+\dots+b_t\beta_t\in\omega$$

$\boxed{\begin{array}{c}\omega+\alpha=\omega\\ \alpha\in\omega\end{array}}$

$L(B)$

$$b_1\beta_1+b_2\beta_2+\dots+b_t\beta_t=a_1\alpha_1+a_2\alpha_2+\dots+a_m\alpha_m$$

$$\therefore\ b_1\beta_1+b_2\beta_2+\dots+b_t\beta_t-a_1\alpha_1-a_2\alpha_2\dots-a_m\alpha_m=0.$$

= (l.c. of the elements of S) = 0

S-Basis
↓
L.I + ~~L(S)~~

Since S is L.I.

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$\therefore\ b_1=0,\ b_2=0,\ \dots\ b_t=0$ &
$a_1=0,\ a_2=0,\ \dots a_m=0.$

$\therefore\ S'$ is L.I.

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(ii) TO P.T. $L(S') = \frac{V}{W}$.

$HH = H$
$\downarrow$
$HH \subseteq H$ &
$H \subseteq HH$

Since all the element of S' are the elements of $\frac{V}{W}$.

$\therefore \; L(S') \subseteq \frac{V}{W} \longrightarrow (1)$

$\therefore \alpha \in V$,

$W + \alpha \in \frac{V}{W}$?

Since S is a Basis of V.

$\therefore$ α can be expressed as,

$\alpha \in V \Rightarrow \alpha \in L(S)$

$$\therefore \; \alpha = \underline{c_1\alpha_1 + c_2\alpha_2 + \cdots + c_m\alpha_m} + d_1\beta_1 + d_2\beta_2 + \cdots + d_t\beta_t.$$

$$\alpha = \gamma + d_1\beta_1 + d_2\beta_2 + \cdots + d_t\beta_t$$

where, $\gamma = c_1\alpha_1 + c_2\alpha_2 + \cdots + c_m\alpha_m$

$=$ l.c. of elements of B

$\because B$ is Basis of W.

$\boxed{\therefore \gamma \in W}$

~~$\beta \alpha \in W$~~

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Practise - 2

~~for $\alpha \in \omega$~~,

for $\alpha \in V$, $\omega + \alpha \in \frac{V}{\omega}$.

Consider,

$$\omega + \alpha = \underline{\omega + \gamma} + d_1\beta_1 + d_2\beta_2 + \cdots + d_t\beta_t$$

$$= \omega + d_1\beta_1 + d_2\beta_2 + \cdots + d_t\beta_t \quad \left[\because \gamma \in \omega \quad \underline{\omega + \gamma = \omega}\right]$$

$$= (\omega + d_1\beta_1) + (\omega + d_2\beta_2) + \cdots + (\omega + d_t\beta_t).$$ Point No ③

$$= d_1(\omega + \beta_1) + d_2(\omega + \beta_2) + \cdots + d_t(\omega + \beta_t)$$

$=$ l.c. of elements of S'

$\therefore \omega + \alpha \in L(S')$

$\therefore \omega + \alpha \in \frac{V}{\omega}$, $\omega + \alpha \in L(S')$

$$\therefore \frac{V}{\omega} \subseteq L(S') \longrightarrow ②$$

$$\therefore L(S') = \frac{V}{\omega}.$$

$\therefore S'$ is Basis of $\frac{V}{\omega}$.

$\therefore \dim\left(\frac{V}{\omega}\right) = t$. $\therefore$ LHS = RHS

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UNIT-2

CLASS NO. 3

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Theorem NO.3 :- Every finite dimensional vector space has got a Basis of V.

[or]

If $V(F)$ is a finite dimensional Vector Space then there exists a basis set of V.

PROOF:

Since $V(F)$ is finite dimensional Vector space,

There exist a finite set S such that $L(S) = V$.

Let $S = \{\alpha_1, \alpha_2, \ldots \alpha_n\}$, we may assume that S doesnot contain the $\bar{0}$ vector.

Imp. points
- → FDVS → (B)
- → Basis
  - L.I. $= 0$
  - L.S. $L(S) = V$
- → L.D. $\neq 0$
- ∃ Any Vector which is a l.c. of Preceding Vectors.
- S, S, S → L(S) Basis

If S is Linearly independent, then S is a basis of V.

If S is linearly dependent set, such that it can be

∃ a vector $\alpha_i \in S$, ~~which can be~~ expressed as linear combination of preceding vectors.

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Let $S_1 = \{\alpha_1, \alpha_2, \ldots \alpha_{i-1}, \alpha_{i+1}, \ldots \alpha_n\}$

$$L(S_1) = L(S) = V.$$

$$\Rightarrow L(S_1) = V.$$

If S_1 is linearly Independent, Then S_1 is basis of V.

If S_1 is linearly dependent, & Proceding as above for a finite no. of steps.

We will be left with linearly independent set S_K and $L(S_K) = V$. Hence S_K will be the basis of $V(F)$.

$\therefore$ Thus there exist a Basis set of $V(F)$.

Process
↓
L.D.
↓
Delete
↓
New set
↓
L(New) = L(old)

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Theorem No.4 :- State and Prove Basis Extension theorem?

(or)

Statement: Let $V(F)$ is a finite dimensional vector space and $S = \{\alpha_1, \alpha_2, \dots \alpha_n\}$ is a L.I. Subset of V. Then P.T. either S is a itself a basis of V or S can be extended to form a basis of V.

PROOF:

Let $V(F)$ be a finite dimensional Vector Space.

Let $S = \{\alpha_1, \alpha_2, \dots \alpha_n\}$ is L.I. Subset of V.

If S is Basis, Then there is nothing to prove.

Suppose S is not a basis.

Now we can prove that, S can be extended to form a basis of V.

Since $V(F)$ is FDVS, i.e. V has a Basic. [$\because$ By previous theorem]

Let it be $B = \{\beta_1, \beta_2, \dots \beta_m\}$ be a Basis of V.

Let $S' = \{\alpha_1, \alpha_2, \dots \alpha_n, \beta_1, \beta_2, \dots \beta_m\}$,

Such that, $L(S') = V$.

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If S' is L.I., Then S' is Basis of V.

Suppose S' is L.D.,

Since S is L.I., α's are not a l.c of preceeding vectors.

Suppose $\exists\ \beta_i \in S'$ such that β_i is a l.c. of its preceeding vectors & Remove that β_i from S'. &

$$S'' = \{\alpha_1, \alpha_2 \dots \alpha_n, \beta_1, \beta_2 \dots \beta_{i-1}, \beta_{i+1}, \dots \beta_m\}$$

Such that $L(S'') = L(S') = V$.

$$\Rightarrow L(S'') = V.$$

If S'' is L.I., Then S'' is Basis of V.

Suppose S'' is L.D,

$\exists\ \beta_j \in S''$ such that β_j is a l.c. of its preceeding vectors &

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remove that β_j from S''.

Let $S''' = \{\alpha_1, \alpha_2 \ldots \alpha_n, \beta_1, \beta_2, \ldots \beta_{i-1}, \beta_{i+1} \ldots \ldots$

$\ldots \beta_{j-1}, \beta_{j+1}, \ldots \beta_m\}$

Such that $L(S''') = L(S'') = V$.

$\Rightarrow L(S''') = V$.

$\therefore$ If S''' is L.I,

$\therefore$ S''' is Basis of V.

If S''' is L.D, Continuing the same process, after a finite no. of steps,

we will get a set $S^{(n)}$, which is a Basis of V.

Hence the theorem is proved

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UNIT-2

SHORTS

Longs
- a → $W_1 + W_2$ (or) $\frac{V}{W}$
- b → 2 theorem. X

Shorts → 3 questions ✓

① S.T. the Vectors $\{(1,1,2), (1,2,5), (5,3,4)\}$ of $R^3(R)$ do not form basis set.

Sol:

PROCESS
→ Basis < L.I., L.S.

$$a_1(1,1,2) + a_2(1,2,5) + a_3(5,3,4) = (0,0,0)$$

$$(a_1+a_2+5a_3,\ a_1+2a_2+3a_3,\ 2a_1+5a_2+4a_3) = (0,0,0)$$

$$\therefore\ a_1+a_2+5a_3 = 0$$
$$a_1+2a_2+3a_3 = 0$$
$$2a_1+5a_2+4a_3 = 0.$$

$$\therefore \begin{bmatrix} 1 & 1 & 5 \\ 1 & 2 & 3 \\ 2 & 5 & 4 \end{bmatrix}^{\times 2}$$

$R_2 \to R_2 - R_1$;

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & -2 \\ 0 & 3 & -6 \end{bmatrix}$$

$\frac{R_3}{3}$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

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$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$n = 3$, r = No. of non-zero Rows $= 2$.

$n \neq r$.

$\therefore$ The Vectors are Linearly dependent.

$\therefore$ They cannot form a Basis.

② S.T. the set $\{(1,0,0), (1,1,0), (1,1,1)\}$ is a Basis of $C^3(C)$. Hence find the Co-ordinates of the Vector $(\underset{a}{3+4i}, \underset{b}{6i}, \underset{c}{3+7i})$ in $C^3(C)$.

Sol:

$$a_1(1,0,0) + a_2(1,1,0) + a_3(1,1,1) = (0,0,0)$$

$$(a_1+a_2+a_3,\ a_2+a_3,\ a_3) = (0,0,0)$$

$$a_1+a_2+a_3 = 0 \longrightarrow ①$$

$$a_2+a_3 = 0 \longrightarrow ②$$

$$\boxed{a_3 = 0} \longrightarrow ③$$

$\therefore$ Sub ③ in ② $\Rightarrow \boxed{a_2 = 0}$

From ① $\Rightarrow$ $a_1 + 0 + 0 = 0$

$$\boxed{a_1 = 0}$$

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$\therefore$ The given Vectors are Linearly Independent.

Also,

$$p(1,0,0) + q(1,1,0) + r(1,1,1) = (a,b,c)$$

$$(p+q+r,\ q+r,\ r) = a, b, c$$

p, q, r విలువలు a, b, c లో రాయాలి.

$$p+q+r = a \longrightarrow (4)$$

$$q+r = b \longrightarrow (5)$$

$$\boxed{r = c}$$

p, q, r Values are terms of a, b, c.

From (5) $\Rightarrow$ $q + c = b$

$$\boxed{q = b-c}$$

From (4) $\Rightarrow$ $p + b - \cancel{c} + \cancel{c} = a$

$$\boxed{p = a-b}$$

$$\therefore (a-b)(1,0,0) + (b-c)(1,1,0) + c(1,1,1) = (a,b,c)$$

$\therefore$ It forms a Basis.

$$\therefore \quad p = a-b = 3+4i - 6i = 3-2i$$

$$q = b-c = 6i-(3+7i)$$
$$= 6i-3-7i = -3-i$$

$$r = c = 3+7i$$

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③ Find the Co-ordinates of $\alpha = (4,5,6)$ w.r.t. basis set $\{x, y, z\}$ where $x = (1,1,1)$, $y = (-1,1,1)$, $z = (1,0,-1)$.

SOL:

$$\alpha = px + qy + rz$$

$$\alpha = p(1,1,1) + q(-1,1,1) + r(1,0,-1)$$

$$(4,5,6) = (p-q+r,\ p+q,\ p+q-r)$$

$$\therefore \quad p-q+r = 4 \longrightarrow ①$$

$$p+q = 5 \longrightarrow ②$$

$$p+q-r = 6 \longrightarrow ③$$

Some ① & ③ ⇒

$$p - \cancel{q} + \cancel{r} = 4$$

$$p + \cancel{q} - \cancel{r} = 6$$

$$2p = 10 \Rightarrow \boxed{p=5}$$

from ② ⇒

$$5+q = 5 \Rightarrow \boxed{q=0}$$

from ① ⇒

$$5 - 0 + r = 4$$

$$r = 4-5$$

$$\boxed{r = -1}$$

∴ Co-ordinates of α are: $\{5, 0, -1\}$//

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Long (or) Short v.v.v. Imp

(4) Let w_1 and w_2 are two subspaces of R^4 given by $w_1 = \{(a,b,c,d) : b-2c+d=0\}$, $w_2 = \{(a,b,c,d) ; a=d ; b=2c\}$ find dimensions of

i) w_1

ii) w_2

iii) $w_1 \cap w_2$

iv) Hence find $w_1 + w_2$

SOL:

i) $w_1 = \{(a,b,c,d) ; b-2c+d=0\}$

$b = 2c-d$

Imp. point

→ Basis ✓

$S = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$ of w_2

$\boxed{\dim w_2 = n}$

$\therefore (a,b,c,d) = (a, 2c-d, c, d)$

$= a(1,0,0,0) + c(0,2,1,0) + d(0,-1,0,1)$

= Linear Combination of Linearly Independent Set.

$\therefore \{(1,0,0,0), (0,2,1,0), (0,-1,0,1)\}$ is a Basis of w.

$\therefore \dim w = 3$.//

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ii) $W_2 = \{(a,b,c,d) \,;\, a = d,\ b = 2c\}$

$\therefore (a,b,c,d) = (d, 2c, c, d)$

$= c(0,2,1,0) + d(1,0,0,1)$

= linear Combination of linearly Independent set of W_2.

$\{(1,2,1,0), (1,0,0,1)\}$ is a <u>Basis</u> of W_2.

$\therefore \dim W_2 = 2$. ([illegible] of vectors [illegible])

iii) $W_1 \cap W_2 = \{(a,b,c,d) \,;\, b - 2c + d = 0;\ b = 2c,\ a = d\}$

$\therefore b - 2c + d = 0$

$\cancel{2c} - \cancel{2c} + d = 0$

$\boxed{d = 0}$

$\therefore a = d \Rightarrow \boxed{a = 0}$

$\therefore W_1 \cap W_2 = (a,b,c,d) \,;\, b = 2c, a = 0, d = 0.$

$= (a,b,c,d) = (0, 2c, c, 0) = c(0,2,1,0)$

$\therefore \dim(W_1 \cap W_2) = \underline{1}.$

iv) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

$= 3 + 2 - 1 = 5 - 1 = 4$

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