UNIT-1 - Vector Spaces - I

CLASS NO. 1 (20m)

(4+4)

Definitions :-

① Vector space: $V(F) = ?$

$\alpha, \beta, \gamma \in V$ → vectors ; $a, b, c \in F$ → field

(i) Internal Composition :- → addition of vectors (C, A, I, I, C.)

$(V, +)$ is an abelian group - (5)

(ii) External Composition :- (multiplication of scalar)

$a \in F, \alpha \in V \Rightarrow a\alpha \in V$ only closer

(iii) Distributed laws:

$$a(\alpha + \beta) = a\alpha + a\beta$$

$$(a+b)\alpha = a\alpha + b\alpha$$

$$ab(\alpha) = a(b\alpha)$$

$$1\alpha = \alpha.$$

$\therefore$ $V(F)$ is called Vector space.

② Linear Combination:

$$\boxed{\gamma = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n}$$

$\forall$ $a_1, a_2, a_3, \ldots a_n \in F$,
$\alpha_1, \alpha_2, \ldots \alpha_n \in V$.

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③ Linear Span!

Linear span of S is Set of all Possible linear Combinations.

④ Linearly Independent!

$\gamma = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n$

$\downarrow$

$\boxed{\text{All Scalars} = 0}$

⑤ Linearly dependent!

$\gamma = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n$

$\downarrow$

$\boxed{\text{Any of Scalar} \neq 0.}$

* Closure law: for $a, b \in G \Rightarrow ab \in G$
  for $a, b \in G \rightarrow a+b \in G$

* Identity law! For $a \in G$
  $\exists e \in G \Rightarrow aoe = eoa = a.$

* Inv law! For $a \in G$
  $\exists b \in G \Rightarrow aob = boa = e.$

* Abelian law! $aob = boa.$

LONGS (10m)

① Let $V(F)$ be a Vector Space and let $W \subseteq V$. The necessary and sufficient Condition for W to be Subspace of V is $a, b \in F$ & $\alpha, \beta \in W \Rightarrow \boxed{a\alpha + b\beta \in W}$.

① ②

Proof:

Necessary Condition:

Let us Suppose that W is a Subspace.

We have to P.T. $a\alpha + b\beta \in W$

$\forall\ a, b \in F;\ \alpha, \beta \in W.$

∵ Since, W is a Subspace,

$a \in F,\ \alpha \in W \Rightarrow a\alpha \in W$

[∵ External Comp.]

$b \in F,\ \beta \in W \Rightarrow b\beta \in W.$

∴ $a\alpha \in W,\ b\beta \in W$

$\Rightarrow a\alpha + b\beta \in W$

[∵ By closure law]

[closure → సంవృతం]

Imp points

① part ① → ②

② → ①.

② $a = 1$; $b = -1$

$b = 0$

③ Distributive ✗

Sufficient Condition :-

Conversely Suppose that $a\alpha + b\beta \in \omega$

$\forall\ a, b \in F\ ;\ \alpha, \beta \in W$

Now we have to p.T. ω is a Subspace.

Taking, $a = 1$, $b = -1$ & $\alpha, \beta \in \omega$.

$\therefore\ 1(\alpha) + (-1)\beta \in \omega$ [`α-β` ఒకే element `ω` లో ఉంటే ω `ω` → ab group]

$\alpha - \beta \in \omega$

$(\omega, +)$ is an abelian group.

Again taking $b = 0$,

$\therefore\ a, 0 \in F\ ;\ \alpha, \beta \in \omega$

$\Rightarrow\ a\alpha + 0(\beta) \in \omega$

$$\boxed{a\alpha \in \omega}$$

$\therefore$ ω is closed Under Scalar Multiplication.

$\therefore$ Remaining postulates of Vector Space hold in ω as $\omega \subseteq V$.

$\therefore$ ω is Subspace of V

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CLASS NO. 2

Theorem No. ②

② A non-empty set W is a Subset of a Vector Space $V(F)$. W is a Subspace of V iff $a \in F;\ \alpha, \beta \in W \Rightarrow a\alpha + \beta \in W$.

Proof:

Necessary Condition:

Suppose that W is a Subspace

Now we have to P.T.

$$a\alpha + \beta \in W \ \forall\ a \in F,\ \alpha, \beta \in W.$$

Procedure
→ (-1) ✓
→ ① → ②
NS

Since, W is a Subspace,

$a \in F,\ \alpha \in W \Rightarrow a\alpha \in W$ [∵ External Composition]

$\therefore\ a\alpha \in W,\ \beta \in W \Rightarrow a\alpha + \beta \in W$ [∵ Closure]

Sufficient Condition:

Let $a \in F,\ \alpha, \beta \in W \Rightarrow a\alpha + \beta \in W$.

Now we have to P.T. W is a Subspace.

i, Now take, $a = -1 \in F,\ \alpha, \alpha \in W$

$$\therefore\ (-1)\alpha + \alpha \in W \Rightarrow -\alpha + \alpha \in W$$

$$\bar{0} \in W$$

$\therefore\ \bar{0}$ is the identity element.

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ii) Again take,

$$a \in F, \quad \alpha, \bar{0} \in W$$

$$\therefore a\alpha + \bar{0} \in W \quad \left[\because a \in F, \alpha, \beta \in W \Rightarrow a\alpha + \beta \in W\right]$$

$$a\alpha \in W$$

$\therefore$ W is closed w.r.t. Scalar Multiplication.

iii) $a = -1 \in F$, $\alpha, \bar{0} \in W$

$$\therefore -\alpha + \bar{0} \in W$$

$$-\alpha \in W$$

$\therefore$ Inverse exists in W.

$\therefore$ Remaining all postulates are Satisfied

$\therefore$ W is a Subspace.

V.V.Imp

Theorem NO. 3: P.T. the Union of Two Subspaces is a Subspace iff one Contained other. $[W_1 \subseteq W_2 \text{ (or) } W_2 \subseteq W_1]$

Proof: Let W_1 & W_2 are two Subspaces of V. Let us Suppose that $W_1 \cup W_2$ is a Subspace of Vectorspace $V(F)$. Now we have to P.T.

$$W_1 \subseteq W_2 \text{ (or) } W_2 \subseteq W_1$$

Imp point

If possible,

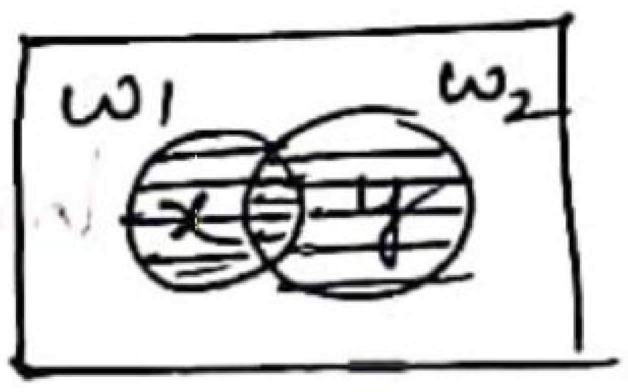
let us Suppose that $\omega_1 \nsubseteq \omega_2$, $\omega_2 \nsubseteq \omega_1$.

Let $\omega_1 \nsubseteq \omega_2 \Rightarrow x \in \omega_1 \Rightarrow x \notin \omega_2 \longrightarrow (1)$

Let $\omega_2 \nsubseteq \omega_1 \Rightarrow y \in \omega_2 \Rightarrow y \notin \omega_1 \longrightarrow (2)$

But, from (1) & (2) $\Rightarrow$

$\therefore x \in \omega_1$, $y \in \omega_2$



$x, y \in \omega_1 \cup \omega_2$

$\therefore x+y \in \omega_1 \cup \omega_2$ [$\because \omega_1 \cup \omega_2$ is a Subspace, by closure law]

$\therefore x+y \in \omega_1$, $x+y \in \omega_2$

$\therefore x+y \in \omega_1$, $x \in \omega_1$

$\therefore a = 1$, $b = -1 \in F$ [$\because a, b \in F$, $\alpha, \beta \in \omega$ $a\alpha + b\beta \in \omega$]

$\therefore 1(x+y) - 1(x) \in \omega_1$

$\cancel{x}+y - \cancel{x} \in \omega_1$

$y \in \omega_1$

$\therefore$ Which is Contradiction to (2).

$x+y \notin \omega_1$.

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$x+y \in W_2$ and $y \in W_2$

$a = 1, \ b = -1 \in F$

$\therefore \ 1(x+y) + (-1)y \in W_2$

$x + \cancel{y} - \cancel{y} \in W_2$

$x \in W_2$

$\therefore$ Which is Contradiction to (1).

$\therefore \ x+y \notin W_2$.

$\therefore \ x+y \notin W_1$ & $x+y \notin W_2$.

$\therefore$ Our Assumption is wrong.

$\therefore \ W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$.

Case 2: Converseley Suppose that $W_1 \subseteq W_2$, $W_2 \subseteq W_1$

Now we have to P.T. $W_1 \cup W_2$ is a Subspace

$W_1 \subseteq W_2 \Rightarrow W_1 \cup W_2 = W_2$

Since, W_2 is a Subspace,

$\therefore \ W_1 \cup W_2$ is also Subspace.

$W_2 \subseteq W_1 \Rightarrow W_1 \cup W_2 = W_1$

Since W_1 is a Subspace.

$\therefore \ W_1 \cup W_2$ is also Subspace//.

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V.V. Imp

Theorem NO. 4: If S is a Subset of a Vectorspace $V(F)$ then P.T.

(i) S is a Subspace of $V \Longleftrightarrow L(S)=S$

(ii) $L[L(S)] = L(S)$.

Proof:

(i) Suppose that 'S' is a Subspace of V.

Now we have to P.T. $L(S)=S$.

It is enough to P.T.

$$L(S) \subseteq S \ \& \ S \subseteq L(S).$$

Imp. points

→ $L(S)=S$

↓

$L(S) \subseteq S$,

$S \subseteq L(S)$

→ $L(S)$ → Linear space

Shortcut ↓

1st theorem Subspace

$x \in H, xx \in HH = H \subseteq HH$

Let $\alpha \in L(S)$

$$\alpha = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n$$

$\therefore$ S is a Subspace of V,

it is closure w.r.t. Scalar multiplication & Vecter addition.

$$\therefore a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n \in S$$

$$\alpha \in S$$

$$\therefore \alpha \in L(S), \ \alpha \in S$$

$$\Rightarrow L(S) \subseteq S \longrightarrow (1)$$

Let $\beta \in S$,

$\therefore \beta = 1 \cdot \beta$

$=$ linear Combination of S.

$\therefore \beta \in L(S)$

$\therefore \beta \in S, \beta \in L(S) \Rightarrow S \subseteq L(S)$ → (2)

from (1) & (2) $\Rightarrow$

$\therefore S \subseteq L(S), L(S) \subseteq S$

$\therefore \boxed{L(S) = S}$

$\therefore$ Conversely Suppose that $L(S) = S$

$\therefore$ Now we have to P.T. S is a Subspace of V.

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$\therefore$ We Know that,

$L(S)$ is a Subspace.

$\therefore S$ is a Subspace.

(ii) To P.T. $L[L(S)] = L(S)$.

L.H.S. $= L[L(S)] = L(S) = \text{RHS}$ [$\because$ By (1)]

UNIT-1

CLASS NO.3

V.V. Imp (10m)

Theorem NO. 5 :- If S, T are the Subsets of a Vector Space $V(F)$ then, P.T.

(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$.

Proof:

(i) let $S = \{\alpha_1, \alpha_2, \ldots \alpha_n\}$

Let $\alpha \in L(S)$

$\therefore \alpha = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n$

$\therefore S \subseteq T$ then $\{\alpha_1, \alpha_2, \ldots \alpha_n\} \subseteq T$

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$\alpha =$ L.C. of finite Subset of T

$\alpha \in L(T)$

$\therefore \alpha \in L(S)$; $\alpha \in L(T)$

$\therefore L(S) \subseteq L(T)$.

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(ii) Now we will prove $L(S\cup T) = L(S) + L(T)$.

It is enough to P.T. $L(S\cup T) \subseteq L(S)+L(T)$ &

$L(S)+L(T) \subseteq L(S\cup T)$.

If $S=\{\alpha_1,\alpha_2 \ldots \alpha_m\}$, $T=\{\beta_1,\beta_2, \ldots \beta_n\}$

Let $\alpha \in L(S\cup T)$

$$\alpha = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_m\alpha_m + b_1\beta_1 + b_2\beta_2 + \cdots + b_n\beta_n.$$

but,

$a_1\alpha_1 + a_2\alpha_2 + \cdots + a_m\alpha_m \in L(S)$ &

$b_1\beta_1 + b_2\beta_2 + \cdots + b_n\beta_n \in L(T)$.

$\therefore$ $\alpha \in L(S\cup T)$

$\therefore$ $\alpha \in L(S)+L(T)$.

$\therefore$ $\alpha \in L(S\cup T)$, $\alpha \in L(S)+L(T)$

$\therefore$ $L(S\cup T) \subseteq L(S)+L(T) \longrightarrow (1)$

Let $\alpha \in L(S)+L(T)$

$\alpha \in \gamma + \delta$

where, $\gamma \in L(S)$ and $\delta \in L(T)$

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γ = L.C. of elements of S

β = L.C. of elements of T.

$\therefore$ α = L.C. of elements of SUT

$\alpha \in L(SUT)$

$\therefore$ $\alpha \in L(S) + L(T)$; $\alpha \in L(SUT)$

$\therefore$ $L(S) + L(T) \subseteq L(SUT)$ $\longrightarrow$ (2)

$\therefore$ from (1) & (2) $\Rightarrow$

$L(SUT) = L(S) + L(T)$

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UNIT-1

SHORTS Total qn's - 2

Formula's:

1. → W is a Subspace → $\boxed{a\alpha + b\beta \in W}$

→ $a\alpha + b\beta \in W$, $W = ?$ ఎవరిని పిలుస్తారు?

2. Linear Combination: S·V

Scalar × Vector

3. Linear Span: It Contains L.C.

L(S) → L.C.

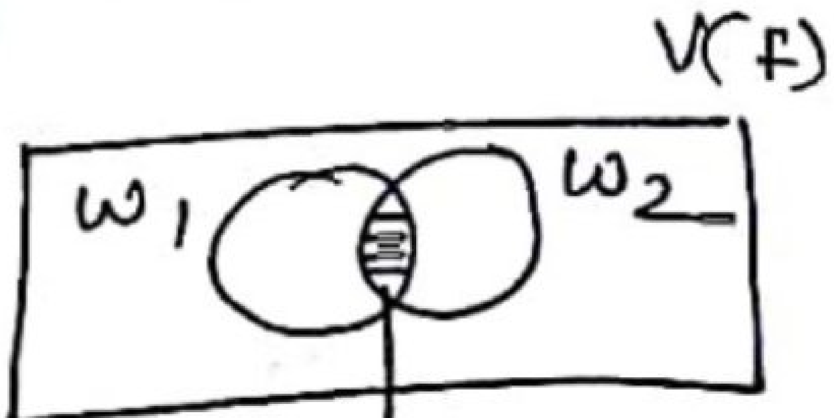
4. Linear Independent / Dep:

Not Zero's.

All Zero's

Shorts

1. P.T. the intersection of any Two Subgroups W_1 and W_2 of Vector space V(F) is a Subspace.



Subspace = ?

$\boxed{a\alpha + b\beta \in W_1 \cap W_2}$

PROOF:

Let V(F) be a Vector Space.

W_1 and W_2 are two Subspaces of V.

$\therefore$ Let $\bar{0} \in W_1$ and $\bar{0} \in W_2$

$\therefore \bar{0} \in W_1 \cap W_2$

$\therefore W_1 \cap W_2 \neq \phi$.

Process

i, $\phi \neq W_1 \cap W_2$

ii, $a\alpha + b\beta \in W_1 \cap W_2$

$\therefore a, b \in F$ and $\alpha, \beta \in W_1 \cap W_2$

Now, $a, b \in F$; $\alpha, \beta \in W_1$

Since W_1 is Subspace.

$\therefore a\alpha + b\beta \in W_1 \longrightarrow (1)$

[$\because$ By theorem (1) Longs]

Now $a, b \in F$, $\alpha, \beta \in W_2$

Since W_2 is a Subspace.

$a\alpha + b\beta \in W_2 \longrightarrow (2)$

from (1) & (2) $\Rightarrow$

$\boxed{a\alpha + b\beta \in W_1 \cap W_2}$

$\therefore W_1 \cap W_2$ is a Subspace.

(2) If ω_1 and ω_2 are any two Subspaces of a Vector space $V(F)$ then

i, $\omega_1+\omega_2$ is a Subspace of $V(F)$.

ii, $\omega_1 \subseteq \omega_1+\omega_2$ & $\omega_2 \subseteq \omega_1+\omega_2$.

PROOF:-

i, Let $\alpha, \beta \in \omega_1+\omega_2$

$\therefore \alpha \in \omega_1+\omega_2, \beta \in \omega_1+\omega_2$

Take, $\alpha = \alpha_1+\alpha_2$; $\beta = \beta_1+\beta_2$.

where $\alpha_1, \beta_1 \in \omega_1$
$\alpha_2, \beta_2 \in \omega_2$

$a, b \in F$

$a\alpha_1, b\beta_1 \in \omega_1$ [Scalar multi]

$a\alpha_2, b\beta_2 \in \omega_2$ ["]

$$\therefore a\alpha+b\beta = a(\alpha_1+\alpha_2)+b(\beta_1+\beta_2)$$
$$= a\alpha_1+a\alpha_2+b\beta_1+b\beta_2$$
$$= (a\alpha_1+b\beta_1)+(a\alpha_2+b\beta_2)$$
$$\in \omega_1+\omega_2$$

$\therefore a\alpha+b\beta \in \omega_1+\omega_2$

$\therefore \omega_1+\omega_2$ is a Subspace.

(ii) $\alpha_1 \in W_1$ and $\bar{0} \in W_2$

$$\therefore \alpha_1 + \bar{0} \in W_1 + W_2$$

$$\alpha_1 \in W_1 + W_2$$

$$W_1 \in W_1 + W_2.$$

$$\therefore W_1 \subseteq W_1 + W_2.$$

Unit 1
$\downarrow$
7 theorem + 3p

Similarly, $W_2 \subseteq W_1 + W_2$.

③ Express the vector $\alpha = (1,-2,5)$ as a linear combination of vectors $e_1 = (1,1,1)$, $e_2 = (1,2,3)$, $e_3 = (2,-1,1)$.

PROOF: $\alpha = a_1 e_1 + a_2 e_2 + a_3 e_3$

$$(1,-2,5) = a_1(1,1,1) + a_2(1,2,3) + a_3(2,-1,1)$$

$$(1,-2,5) = (a_1 + a_2 + 2a_3,\ a_1 + 2a_2 - a_3,\ a_1 + 3a_2 + a_3)$$

$$\therefore \quad a_1 + a_2 + 2a_3 = 1$$

$$a_1 + 2a_2 - a_3 = -2$$

$$a_1 + 3a_2 + a_3 = 5$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - R_1 ; R_3 \longrightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{bmatrix} \times 2$$

$R_3 \longrightarrow R_3 - 2R_2$

ఇక్కడ అన్నీ జీరోస్ వస్తే $\leftarrow$

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$0 - 2(0) = 0$
$2 - 2(1) = 0$
$-1 - 2(-3)$
$-1 + 6 = 5$
$4 - 2(-3)$
$4 + 6 = 10$

Can't be expressed

$\therefore \quad a_1 + a_2 + 2a_3 = 1$

$a_2 - 3a_3 = -3$

$5a_3 = 10 \Rightarrow \boxed{a_3 = 2}$

$\therefore \quad a_2 - 3(2) = -3$

$a_2 = -3 + 6$

$\boxed{a_2 = 3}$ $\quad \boxed{\therefore a_1 = -6}$

$\therefore \quad a_1 + 3 + 2(2) = 1 \Rightarrow a_1 = 1 - 3 - 4$

$\therefore \quad \alpha = -6e_1 + 3e_2 + 2e_3$

V.V. Imp

(4) P.T. the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.

$a\alpha + b\beta \in L(S)$?

PROOF:

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Let $\alpha, \beta \in L(S)$ and $a, b \in F$

$$\therefore \quad \alpha = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_m\alpha_m$$

$$\beta = b_1\beta_1 + b_2\beta_2 + \cdots\cdots + b_n\beta_n.$$

$$\therefore \quad a\alpha + b\beta = a[a_1\alpha_1 + a_2\alpha_2 + \cdots + a_m\alpha_m] + b[b_1\beta_1 + b_2\beta_2 + \cdots + b_n\beta_n]$$

$$= (aa_1)\alpha_1 + (aa_2)\alpha_2 + \cdots + (aa_m)\alpha_m + (bb_1)\beta_1 + (bb_2)\beta_2 + \cdots + (bb_n)\beta_n$$

= Linear Combination of a finite set

$$a\alpha + b\beta \in L(S).$$

$\therefore$ $L(S)$ is a Subspace.

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⑤ S.T. the System of Vectors (1,3,2), (1,−7,−8), (2,1,−1) of $V_3(R)$ is Linearly dependent.

SOLUTION:

$$a_1(1,3,2)+a_2(1,-7,-8)+a_3(2,1,-1)=\bar{0}$$

$$(a_1+a_2+2a_3,\ 3a_1-7a_2+a_3,\ 2a_1-8a_2-a_3)=\bar{0}$$

$$\therefore\ a_1+a_2+2a_3=0$$
$$3a_1-7a_2+a_3=0$$
$$2a_1-8a_2-a_3=0$$

$$\begin{bmatrix}1 & 1 & 2\\ 3 & -7 & 1\\ 2 & -8 & -1\end{bmatrix}\quad \times 2 \qquad -7$$

$$R_2\to R_2-3R_1$$
$$R_3\to R_3-2R_1$$

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$$\begin{bmatrix}1 & 1 & 2\\ 0 & -10 & -5\\ 0 & -10 & -5\end{bmatrix}\qquad R_2\to\frac{R_2}{-5}\qquad R_3\to\frac{R_3}{-5}$$

DEMO

$$\begin{bmatrix}1 & 1 & 2\\ 0 & +2 & 1\\ 0 & 2 & 1\end{bmatrix}\qquad R_3\to R_3-R_2$$

$$\begin{bmatrix}1 & 1 & 2\\ 0 & 2 & 1\\ 0 & 0 & 0\end{bmatrix}$$

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$\therefore \quad a_1 + a_2 + 2a_3 = 0$

$2a_2 + a_3 = 0$

$r = 2, \quad n = 3$

$n - r = 3 - 2 = 1$ n-r

$\therefore \quad a_3 = K$

$\therefore \quad 2a_2 + K = 0$

$2a_2 = -K$

$a_2 = -K/2$

$\therefore \quad a_1 - \frac{K}{2} + 2K = 0$

$a_1 = -2K + \frac{K}{2}$

$a_1 = \frac{-4K + K}{2}$

$a_1 = \frac{-3K}{2}$

$$\therefore \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -3K/2 \\ -K/2 \\ K \end{bmatrix} = -\frac{K}{2}\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$\therefore \quad \boxed{a_1 = 3}, \quad \boxed{a_2 = 1}, \quad \boxed{a_3 = -2}$

$\therefore$ The given ~~given~~ Vectors are L.I.

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UNIT-1 - SHORTS

① Determine whether the following set of Vectors Linearly dependent or Linearly Independent $(1,2,0),(0,3,1),(-1,0,1)$.

Sol:

$$x(1,2,0)+y(0,3,1)+z(-1,0,1)=\bar{0}$$

$$(x+0-z,\ 2x+3y+0,\ 0+y+z)=\bar{0}$$

$$(x-z,\ 2x+3y,\ y+z)=(0,0,0)$$

$$x-z=0 \rightarrow (1), \quad 2x+3y=0 \rightarrow (2), \quad y+z=0 \rightarrow (3)$$

Solve (1) & (3) $\Rightarrow$

$$x-\cancel{z}=0$$
$$y+\cancel{z}=0$$
$$x+y=0 \rightarrow (4)$$

Solve (2) & (4) $\Rightarrow$

$$(2) \Rightarrow 2x+\cancel{3y}=0$$
$$(4)\times 3 \Rightarrow 3x+\cancel{3y}=0$$
$$(-) \quad (-)$$
$$-x=0 \Rightarrow \boxed{x=0}$$

$\therefore$ from (1) $\Rightarrow$

$$x-z=0$$
$$0-z=0 \Rightarrow \boxed{z=0}$$

from (4) $\Rightarrow$

$$x+y=0$$
$$0+y=0 \Rightarrow \boxed{y=0}$$

$x=y=z=0 \Rightarrow$ Hence the system is Linearly Independent.