Semester - III

Mathematics

Top Most Imp Shorts

Unit - I

Q1 Prove that in a group, the identity element is unique.

2. In a group G, for $a, b \in G$ then prove that $(ab)^{-1} = b^{-1}a^{-1}$

3. State and prove cancellation laws are hold in a group

[or]

In a group G for $a, b, c \in G \Rightarrow ab = ac$ & $b = c$ [is called left cancellation law] and $ba = ca \Rightarrow b = c$ [is called right cancellation law].

4. Prove that cube roots of unity forms an abelian group.

5. If G is a group such that $(ab)^m = a^m b^m$ for three consecutive integers $a, b \in G$. Show that $(G, \cdot)$ is an abelian group.

UNIT- II

Q1. The identity element of a sub group of a group is same as the identity of that group.

☆ 2. The intersection of two Subgroups of a group is also a Subgroup.

☆ 3. Any two left cosets (right) of a Subgroup are disjoint & identical.

4. If H is a Sub group of group G, then prove that $HH = H$

☆ 5. If H is a Sub group of group G, then prove that $H^{-1} = H$.

UNIT- III

Q1. Prove that the intersection of two normal Subgroups of a group is also a normal Sub group.

☆ 2. If G is a group and H is a Sub group of Index 2 then prove that H is a normal Subgroup of G.

☆ 3. If $(G, \cdot)$ and $(G', \cdot)$ are to be two groups and f is homomorphism from G into G' then prove that

i) $f(e) = e'$ ii) $f(a^{-1}) = [f(a)]^{-1}$

4. Prove that the Homomorphic image of a group is a group.

UNIT - IV

Q1. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 4 & 1 \end{pmatrix}$; $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$ then find fg and gf.

Q2. Check whether the following Permutation are odd or even?

i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$

UNIT - V

Q1. The intersection of two Subrings of a ring R is a Subring of R.

2. The intersection of two ideals of a ring R is an ideal of R.

3. If $\overset{A}{U_1}$ and $\overset{B}{U_2}$ are two ideals of a ring R then $\overset{A}{U_1}$ and $\overset{B}{U_2}$ is an ideal of R if and only if $\overset{A}{U_1} \subset \overset{B}{U_2}$ (or) $\overset{A}{U_2} \subset \overset{B}{U_1}$

4. A field has no zero divisors.