UNIT-3 MATHS - SEM3 LONGS

	(DA Suboroup H OL O GOTHER CO. 1)
	The subgroup H of a group G is Normal
	iff the each left coset of H in g is a right coset of H in a word
	right cool of H in q.
	Let (G,.) be a group. House
	let (H,.) be a Subgroup. of G.
	The condition is necessary
	Let us suppose that His Normal.
	Now we have to prove that each left
	coset of H in G is a right coset of H in G.
	XHa = Hlonool of Theorem QJ
	Multiply 'x' on both sides.
,	MHX (M) = (HXO) Dang cont on x'H.
	AH (2=2)= HA port [= By Associative]
	TH(e)=Hn
	ALL HOLLOSSIC SOHOLAND
	The condition is sufficient.
The state of	Conversely suppose that each left coset of
	Hen Gusispan right coset of Him G.
	Now we have to prove that H is normal !
	For NEG, 0- HO KNIS
	2H = Hy Y yeq. X 3 d 10 3 10
	Since k is a Normal Hubbrate Dar
	2= xe. O . se = r
	NEXH [= e is identity element of ti]

[: cosets, theorem 3] Facility -then Ha=HB HX= 2H Multiply with Haat = XHat [: theorem @] H = aHar

Theorem : (2) A Subgroup H of groupg is normal iff the product of two right caseds of A for G is a right caset of 41 in q. Proof : Let (G.) be a group. Let (H,) be a subgroup of G. The condition is necessary. Let us suppose that I is normal. Now we have to prove that the product of two right cosets of Hin G is also a right coset of H in G. a, beg 3 abe & [: By closure law] Ha, Hb, Hab are the right cosets of Hin G. ar(H) per a suplaint of when I Ha. Hb = H(aH)b. = H(Ha) b do [- By theorem Q] = (HH) ab = Habar [: By th. (3 in S.G.] the condition is sufficient 51401314

Conversely Suppose that the product of two right cosets of H in G is also a right cosets of H in G is also a right coset of H in G is also a right coset of H in G.

Novo we have to prove that H is Normal.

neg, he H.

xhat = (ea)hat

E(Ha)(Ha-1) [: e is the identity
in H]

E H(xx-1)

EHE properties

Shate H. grongolde K. Miller

H & Normal

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If f is a homomorphic from a group G Into G' then Prove that Kernal of a normal subgroup. Homomorphism is Dept - 7 7 p. ii) kerf-18-6(08-1) Let (G,.) (G',.) to be two groups 11 KNEGUEG Let e, e' are to be two identity elements of G, G'. Ket I be Homonorphism from G into G By the definition of temal f, Kert = { f(n) = e' /269}. Now much travely to more that Kert 18 a Normal Subgroup. we know that fresh the office of the word of man want fresh = I see I that the or all the order of the order have to prove that the get. / : Kerf # \$. 10: 10) + 6 1 9 0 10) CONDINE K f(a) = e' + f(b) = e' = 0)1f (abt) or fla) inf(b) or bit is home] = f(a). [f(b)] [= g =] [: By +60]

= e (e) -1 f(a61) = e' ab E Kerf. .. kerf is a Subgroup. f(nax') = f(x) f(a) f (x') = f(a). e'[f(x)] + $= f(x) \cdot [f(x)]^{-1}$ fixaxi) = e1. nant e kerf l'. Kerf is a Normal Sub group.

Theorem : 7 12) b) 4) State and prove Fundamental theorem of Homomorphism of Groups. Statement: Every Homomorphic Image of a group q is Isomorphic to Its quotient. e) 1. 1 (bin) welldeport group. Proof: Let (G,:), (G',.) are to be two groups. Let e, e' are two identity elements of G, 5. Let of is a Homomorphism from G into G By the definition of Homonworphic Image, f(g) = Im(f) = { f(a) = a'/a & g ; f(g) & g'. We know that Homomorphic Image of a group is a group. By the definition of kerf, Kerf = k = { f(n) = c'/nle6 } We know that kerf is a Normal Subgroup. : (G, .) is a quotient group Define ! $\phi: G \rightarrow G'$ such that $\phi(ka) = f(a); Va, b \in G$ Now we have to Prove that 4 2 4.10 F Jo prove of is well-defined: For a, be q, ka, kb eng. did ka=Kb. [2" banks about cock] [-: By the of cosets] φ(κα.τь) = φ(κα) · φ(κμ)= (+d1) · (ω)} Multiply with of the mobioth sides & f(a)[f(b)] "rof(b)q-a)f(b)[11

1 10m

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f(a) = f(b)
           o (Ka) = o (Kb)
             of is well dofined
    To prove of is one one!
       For a,b∈ G → ka, kb∈G
                $ (ka) = $ (kb)
                -s(a) = -s(b)
            Multiply with [f(b)] on both sides
             f(a) . [f(b)] = f(b)[f(b)]
             f(a). f(b-1) = e'
               f(ab1) = e1
                  abi e k
                 ka & Kb, cont.
    trapped to make by is lone-one;
     To prove puis orthograp a in a
such that Police that yabes
    Facq such that fla)=2
    Jaeg such that p(ka) = fla)
         , 6 ε 6, ka, kb . cotro 2 d.
    To prove $ 15 Homomorphism:
    (c) ka, Fib (f f 18 ) [: Coset mutic defined in []
                        P: Since (fds) Hamo]
               f (a), f(b)
     φ(κα.κω) = φ(κω) - ('d) + (ω) +
         of assign that locate mediplacism judituit
          of is Ishlatorphingent (13) +7(10) +
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