PAPER-5

4th sem

LINEAR ALGEBRA

Unit 1: Vector Spaces - I - 10+5+5
6-7T

Unit 2: Vector spaces - II - 10+5
Bases & Dimensions
3T

1T
Unit 3: Linear Transformations
Isomorphism 10+5

Unit 4: Matrix : 10+5+5
1T+10P

Unit 5: IPS - Inner product spaces. 10+5+5
4T+4P

Imp point :- 8 → Longs
5

UNIT-4

MATRIX

① **Square Matrix:** No. of rows = No. of columns.

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array} \\ \begin{array}{cc} \downarrow & \downarrow \\ C_1 & C_2 \end{array} \end{array}$$

$\boxed{2 \times 2} \longrightarrow$ Order.

RC

$\boxed{R = C}$

$R \neq C$

② **Topics:**

5 { ① Rank finding
② Simultaneous Equations

10 { ③ Characteristic Val/ Vectors
④ Cayley's - Hamilton theorem.

Defi

1. Characteristic equation:

$$\boxed{|A - \lambda I| = 0.}$$

where,

A = Square Matrix (R=C)

λ = any Constant

I = Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Cayley-Hamilton theorem:

"Every Square Matrix Satisfies its characteristic equation"

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③ Characteristic Values / Ch. Vectors:

$$|A - \lambda I| = 0$$

$\downarrow$

$\lambda = 0, 1, 2$

$\downarrow$

λ Values $\rightarrow$ Ch. Values

$\downarrow$

case i
case ii
case iii

$\rightarrow$ Vector forming

Vectors

④ Rank:

No. of Non-zero ROWS

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \\ 0 & -7 & 8 \end{bmatrix} \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{array}$$

$\rho(A) = 3$

10m V.V.Imp

1. Verify Cayley - Hamilton theorem for $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$.

Sol: The characteristic equation is given by $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{array}{ccc} (+) & (-) & (+) \end{array}$$

$$\begin{vmatrix} 3-\lambda & 1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 5-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\left[(5-\lambda)(5-\lambda) - 1\right] - 1\left[-1(5-\lambda) + 1\right] + 1\left[1 - (5-\lambda)\right] = 0$$

$$(3-\lambda)\left[25+\lambda^2-10\lambda-1\right]-1\left(-5+\lambda+1\right)+1\left(1-5+\lambda\right)=0$$

$$(3-\lambda)\left(\lambda^2-10\lambda+24\right)-1\left(\lambda-4\right)+1\left(\lambda-4\right)=0$$

$$3\lambda^2-30\lambda+72-\lambda^3+10\lambda^2-24\lambda-\cancel{\lambda}+\cancel{4}$$
$$+\cancel{\lambda}-\cancel{4}=0$$

$$-\lambda^3+13\lambda^2-54\lambda+72=0$$

$$\lambda^3-13\lambda^2+54\lambda-72=0$$

To prove that A satisfies this characteristic equation i.e

$$A^3+13A^2-54A+72I=0.$$

$$A^2=A\cdot A=\begin{bmatrix}3&1&1\\-1&5&-1\\1&-1&5\end{bmatrix}\begin{bmatrix}3&1&1\\-1&5&-1\\1&-1&5\end{bmatrix}$$

$$=\begin{bmatrix}9-\cancel{1}+\cancel{1}&3+5-1&3-1+5\\-3-5-1&-\cancel{1}+25+\cancel{1}&-1-5-5\\3+1+5&1-5-5&1+1+25\end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 7 & 7 \\ -9 & 25 & -11 \\ 9 & -9 & 27 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 9 & 7 & 7 \\ -9 & 25 & -11 \\ 9 & -9 & 27 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 27-7+7 & 9+35-7 & 9-7+35 \\ -27-25-11 & -9+125+11 & -9-25-55 \\ 27+9+27 & 9-45-27 & 9+9+135 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 27 & 37 & 37 \\ -63 & 127 & -89 \\ 63 & -63 & 153 \end{bmatrix}$$

From, $A^3 - 13A^2 + 54A - 72I$

$$\begin{bmatrix} 27 & 37 & 37 \\ -63 & 127 & -89 \\ 63 & -63 & 153 \end{bmatrix} - 13\begin{bmatrix} 9 & 7 & 7 \\ -9 & 25 & -11 \\ 9 & -9 & 27 \end{bmatrix} + 54\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix} - 72\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$\frac{13}{27}$ 2 1 $\frac{13}{28}$ 1 $\frac{13\times 9}{117}$ 2

351 25

$$= \begin{bmatrix}27&37&37\\-63&127&-89\\63&-63&153\end{bmatrix} - \begin{bmatrix}117&91&91\\-117&325&-143\\117&-117&351\end{bmatrix} +$$

$$\begin{bmatrix}162&54&54\\-54&270&-54\\54&-54&270\end{bmatrix}$$

$$\begin{bmatrix}72&0&0\\0&72&0\\0&0&72\end{bmatrix}$$

$$= \begin{bmatrix}27-117+162-72 & 37-91+54-0 & 37-91+54-0\\ -63+117-54-0 & 127-325+270-72 & -89+143-54\\ 63-117+54-0 & -63+117-54-0 & 153-351+270-72\end{bmatrix}$$

$$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} = 0$$

$\therefore$ A satisfies its ch. equation.

$\therefore$ Cayley-Hamilton is verified

UNIT-4

CLASS NO. 2

(10m)

② If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ Verify cayley-Hamilton theorem? Hence find A^{-1}.

Sol: The characteristic equation of A is:

$$|A - \lambda I| = 0.$$

$$\left| \begin{pmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{array}{ccc} (+) & (-) & (+) \end{array}$$
$$\begin{vmatrix} 2-\lambda & 1 & 2 \\ 5 & 3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(3-\lambda)(-2-\lambda)\right] - 1\left[5(-2-\lambda)+3\right] + 2\left[3-\lambda\right] = 0$$

$$(2-\lambda)\left[-6-3\lambda+2\lambda+\lambda^2\right] - 1\left[-10-5\lambda+3\right] + 2(3-\lambda) = 0$$

$$(2-\lambda)\left[\lambda^2-\lambda-6\right] - 1\left[-5\lambda-7\right] + 6 - 2\lambda = 0$$

$$2\lambda^2 - 2\lambda - 12 - \lambda^3 + \lambda^2 + 6\lambda + 5\lambda + 7 + 6 - 2\lambda = 0$$

$$-\lambda^3 + 3\lambda^2 + 7\lambda + 1 = 0$$

$$\lambda^3 - 3\lambda^2 - 7\lambda - 1 = 0.$$

To p.T. A satisfies the ch. equation

To prove $A^3 - 3A^2 - 7A - I = 0.$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+5-2 & 2+3+0 & \cancel{4}+3-\cancel{4} \\ 10+15-3 & 5+9+0 & 10+9-6 \\ -2+0+2 & -1+0+0 & -2+0+4 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+25-3 & 7+15+0 & 14+15-6 \\ 44+70-13 & 22+42+0 & 44+42-26 \\ 0-5-2 & 0-3+0 & 0-3-4 \end{bmatrix}$$

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$$A^3 = \begin{bmatrix} 36 & 22 & 23 \\ 101 & 64 & 60 \\ -7 & -3 & -7 \end{bmatrix}$$

$$LHS = A^3 - 3A^2 - 7A - I$$

$$= \begin{bmatrix} 36 & 22 & 23 \\ 101 & 64 & 60 \\ -7 & -3 & -7 \end{bmatrix} - 3\begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix} - 7\begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 36-21-14-1 & 22-15-7-0 & 23-9-14-0 \\ 101-66-35-0 & 64-42-21-1 & 60-39-21-0 \\ -7-0+7-0 & -3+3+0-0 & -7-6+14-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = RHS.$$

Cayley's Hamilton theorem is Verified.

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Hence → [illegible] ....

$$A^2-5A+6=0$$

multiply with 'A^3'

$$A^5-5A^4+6A^3=0$$

$$A^5=-5A^4-6A^3$$

$$A^{-1}(A^2-5A+6)=0$$

$$A-5+6A^{-1}=0$$

$$6A^{-1}=-A+5I$$

$$A^{-1}=\frac{1}{6}(-A+5I)$$

Also,

$$A^3-3A^2-7A-I=0$$

Multiply with A^{-1} on b.s,

$$A^{-1}(A^3-3A^2-7A-I)=0.$$

$$A^2-3A-7-A^{-1}=0$$

$$A^2-3A-7I=A^{-1}$$

$$\therefore A^{-1}=A^2-3A-7I$$

$$=\begin{bmatrix}7&5&3\\22&14&13\\0&-1&2\end{bmatrix}-3\begin{bmatrix}2&1&2\\5&3&3\\-1&0&-2\end{bmatrix}-\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\cancel{7}-6-\cancel{7}&5-3-0&3-6-0\\22-15-0&14-9-7&13-9-0\\0+3-0&-1-0-0&2+6-7\end{bmatrix}=\begin{bmatrix}-6&2&-3\\7&-2&4\\3&-1&1\end{bmatrix}$$

LINEAR ALGEBRA

UNIT-4

CLASS NO: 3.

10m

Characteristic roots & ch. Vectors:

→ ch. equation:

$$|A-\lambda I| = 0$$

$$\lambda = 2 \,\Big|\; \begin{array}{ccc} 1 & 2 & 34 \\ 0 & 5 & 25 \\ \hline & & 10 \end{array}$$

↓

λ' values → ch. roots

$\boxed{\lambda \text{ విలువలను - "ch. roots" అని పిలుస్తారు}}$

↓

Ch. Vectors → $[A-\lambda I]\,X = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \boxed{x_3 = k}$$

Variables Assign చేయాలి. k_1

k_2

$n - r$ → No. of non-zero zero.

Order

$3-2 = 1$ ✓

$3-1 = 2$ ✓

One Variable can be assm

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① find the characteristic roots and Corresponding Vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

Sol:

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{array}{ccc} (+) & (-) & (+) \end{array}$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)\left[(3-\lambda)^2 - 1\right] + 2\left[-2(3-\lambda) + 2\right] + 2\left[2 - 2(3-\lambda)\right] = 0$$

$$(6-\lambda)\left[9+\lambda^2-6\lambda-1\right]+2\left[-6+2\lambda+2\right]+2\left(2-6+2\lambda\right)=0$$

$$(6-\lambda)\left[\lambda^2-6\lambda+8\right]+2\left[2\lambda-4\right]+2\left[2\lambda-4\right]=0$$

$$6\lambda^2-36\lambda+48-\lambda^3+6\lambda^2-\cancel{8\lambda}+\cancel{4\lambda}-8+\cancel{4\lambda}-8=0$$

$$-\lambda^3+12\lambda^2-36\lambda+32=0$$

$$\lambda^3-12\lambda^2+36\lambda-32=0$$

$\lambda=2$	1	-12	36	-32
	0	2	-20	32
	1	-10	16	$\boxed{0}$

$$\lambda^2-10\lambda+16=0$$

$$\lambda^2-8\lambda-2\lambda+16=0$$

$$\lambda(\lambda-8)-2(\lambda-8)=0$$

$$(\lambda-8)(\lambda-2)=0$$

$$\lambda=2,8.$$

$\therefore$ Characteristic roots $=2,2,8.$

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Characteristic Vectors :-

Case i : If $\lambda = 2$

$\therefore$ The characteristic equation corresponding to $\lambda = 2$ is

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Put $\lambda = 2$,

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$R_1 \longrightarrow \frac{R_1}{2}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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$$2x_1 - x_2 + x_3 = 0 \longrightarrow (1)$$

$n = 3$, r = no. of non-zero rows = 1.

$\therefore$ $n - r = 3 - 1 = (2)$

$\therefore$ two Variables can be assigned.

$$\boxed{x_3 = K_1}$$

$$\boxed{x_2 = K_2}$$

$\therefore$ From (1) $\Rightarrow$

$$2x_1 - x_2 + x_3 = 0$$

$$2x_1 - K_2 + K_1 = 0$$

$$2x_1 = K_2 - K_1$$

$$\boxed{x_1 = \frac{K_2 - K_1}{2}}.$$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{K_2}{2} - \frac{K_1}{2} \\ K_2 \\ K_1 \end{bmatrix}.$$

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Case ii: If $\lambda = 8$

The characteristic vector corresponding $\lambda = 8$ is

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Put $\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$R_1 \longrightarrow \frac{R_1}{2}$$

$$\begin{bmatrix} -1 & -1 & 1 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} -1 & -1 & 1 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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$R_3 \longrightarrow R_3 - R_2$.

$$\begin{bmatrix} -1 & -1 & 1 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$-x_1 - x_2 + x_3 = 0 \longrightarrow (1)$

$-3x_2 - 3x_3 = 0 \longrightarrow (2)$

$n = 3$; $r =$ No. of non-zero rows $= 2$

$= \phi$.

$n - r = 3 - 2 = (1)$

$\therefore$ One Variable Can be assigned.

$\therefore x_3 = K$

$\therefore$ From (2) $\Rightarrow$

$-3x_2 - 3K = 0$

$-\cancel{3}x_2 = \cancel{3}K$

$\boxed{x_2 = -K}$

$\therefore$ From (1) $\Rightarrow -x_1 - (-K) + K = 0$

$-x_1 + 2K = 0$

$-x_1 = -2K$

$\boxed{x_1 = 2K}$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2K \\ -K \\ K \end{bmatrix}$$

$$\therefore \quad X = K \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore$ Characteristic Vectors are:

$$\begin{bmatrix} \frac{K_2}{2} - \frac{K_1}{2} \\ K_2 \\ K_1 \end{bmatrix} = K_1 \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix} + K_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$$

$$\therefore X = K \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Homework $\rightarrow$ $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

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UNIT-4

12) b)

CLASS NO.4

453 | 12

V.V.V.Imp

(10m)

① State and prove Cayley-Hamilton theorem

Statement: "Every Square matrix satisfies its characteristic Equation".

Procedure

i, $A \to n\times n$.

ii, ch. equation

$|A-\lambda I|=0$

$(-1)^n[\lambda^n + a_1\lambda^{n-1}+\dots]=0$

iii, Adj. A find.

$\cancel{A^{-1} = \frac{Adj.A}{\det A}}$

$\boxed{A.Adj\,A = |A|.I}$

iv, Compare coeff

v, Premultiplying & Adding

(.) Eq. Ans

Adj A = $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

Proof:

Let 'A' be the 'n' rowed Square Matrix.

Let $A = [a_{ij}]_{n\times n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} = 0$$

$$|A-\lambda I| = (-1)^n\left[\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n\right] = 0$$

Since all the elements of $(A-\lambda I)$ are atmost of first degree in λ.

All the elements of $Adj(A-\lambda I)$ are polynomials in λ of degee $(n-1)$ or less.

$\therefore$ $Adj(A-\lambda I)$ Can be expressed as a matrix polynomial in λ.

Let $Adj(A-\lambda I) = B_0\lambda^{n-1} + B_1\lambda^{n-2} + \cdots + B_{n-1}$

We Know that,

$$\boxed{A\,Adj.A = |A|\,I}$$

$$(A-\lambda I)\,Adj(A-\lambda I) = |A-\lambda I|\,I$$

$$(A-\lambda I)\left[B_0\lambda^{n-1} + B_1\lambda^{n-2} + \cdots + B_{n-1}\right] =$$

$$(-1)^n\left[\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdot\, a_n\right]I$$

$$AB_0\lambda^{n-1} + AB_1\lambda^{n-2} + \cdots + AB_{n-1} - IB_0\lambda^n$$

$$- B_1I\lambda^{n-1} - B_2I\lambda^{n-2} =$$

$$(-1)^n\left[\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots a_n\right].I$$

Comparing the coff of $\lambda^n, \lambda^{n-1}, \lambda^{n-2} \ldots\ldots \lambda^{n-n}$

$$-I\cancel{B_0} = (-1)^n I.$$

$$A\cancel{B_0} - I\cancel{B_1} = (-1)^n a_1 I$$

$$A\cancel{B_1} - I\cancel{B_2} = (-1)^n a_2 I$$

$$A\cancel{B_2} - I\cancel{B_3} = (-1)^n a_3 I$$

$$\vdots$$

$$A\cancel{B_{n-1}} = (-1)^n a_n I$$

Premultiplying the above equation with $A^n, A^{n-1}, A^{n-2} \ldots I$ & Adding

$\therefore$ We Obtain,

$$0 = (-1)^n \left[A^n + a_1 A^{n-1} + a_2 A^{n-2} + \cdots a_n\right] I$$

$$\therefore (-1)^n \left[A^n + a_1 A^{n-1} + a_2 A^{n-2} + \cdots a_n I\right] = 0$$

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PAPER-5

UNIT-4

SHORTS

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V.V. Imp

① Find the rank of the matrix reducible to echelon form of $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$.

Sol: Given that

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix} \times 3$$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & -4 & -7 \end{bmatrix}$$

$\boxed{\text{Same sign} \to \ominus}$

$\boxed{\text{Opp} \to \oplus}$

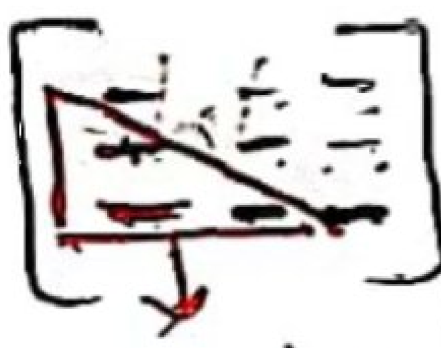
$R_3 \longrightarrow 3R_3 - 4R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & -17 \end{bmatrix}_{3\times 3}$$

$\therefore$ Rank of $A = \rho(A) = 3$ //

Rank
- Echelon [Row oper.]
- Normal [Row + column]

Process

1) 
3x3
zeros

2) check the 9th element. $\neq 0$

Zero – ఉండకూడదు.

$\rho(A) = 3$ //

Zero వస్తే → 2

② Find the rank of the matrix of $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 7 & -7 \end{bmatrix}$.

Sol: Given that,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 7 & -7 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -4 \\ 0 & 6 & -8 \end{bmatrix}$$

$R_3 \longrightarrow \frac{R_3}{2}$

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$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -4 \\ 0 & 3 & -4 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & -4 \\ 0 & 0 & 0 \end{bmatrix} \quad 3\times 2$$

$$\boxed{\rho(A) = 2}$$ //

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③ Find the rank of the matrix reducible to normal form $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$

Sol: Given that,

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{array}{c} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & -4 & -7 \end{bmatrix} \end{array} \begin{array}{l} \\ \times 4 \\ \times 3 \end{array}$$

$-21+4$

$R_3 \to 3R_3 - 4R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & -17 \end{bmatrix}$$

$C_2 \to C_2 - C_1$; $C_3 \to C_3 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & -1 \\ 0 & 0 & -17 \end{bmatrix} \qquad \frac{C_2}{-3}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & -1 \\ 0 & 0 & -17 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -17 \end{bmatrix}$$

$C_3 \to C_3 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -17 \end{bmatrix}$$

$\dfrac{C_3}{-17}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \text{Identity matrix}$$

$$\boxed{\rho(A) = 3}$$

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④ Solve the equations $x+y+z=4$, $2x+5y-2z=3$, $x+7y-7z=5$.

Sol:

$$A=\begin{bmatrix}1&1&1\\2&5&-2\\1&7&-7\end{bmatrix},\quad B=\begin{bmatrix}4\\3\\5\end{bmatrix}$$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

Augmented Matrix $=[A|B]$

$$\begin{bmatrix}1&1&1&4\\2&5&-2&3\\1&7&-7&5\end{bmatrix}$$

$R_2 \to R_2-2R_1$
$R_3 \to R_3-R_1$

$$\begin{bmatrix}1&1&1&4\\0&3&-4&-5\\0&6&-8&1\end{bmatrix}\times 2$$

$R_3 \to R_3-2R_2$

$$\begin{bmatrix}1&1&1&4\\0&3&-4&-5\\0&0&0&11\end{bmatrix}$$

$\therefore\ \rho(A|B)=3$, but, $\rho(A)=2$

$\therefore\ \rho(A|B)\neq\rho(A)$ $\therefore$ Inconsistent.

Procedure

i) 3 eqn's < C / N-C.

ii) Augmented matrix $[A|B]$

iii) Rank ✓

iv) $\rho(A|B)=\rho(A)$ Consisten.

$\rho(A|B)\neq\rho(A)$ Non-Consistent.

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UNIT-4

① Find the rank of the Matrix reducible to normal form of matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$

Sol:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & -4 & -7 \end{bmatrix}$$

$R_3 \to 3R_3 - 4R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & -17 \end{bmatrix}$$

Rank of $A = \rho(A) = 3$

UNIT-4

(3) Find the rank of the Matrix of $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 4 & 8 \\ 6 & 7 & 8 & 9 \\ 2 & 4 & 0 & 0 \end{bmatrix}$

SOL: Given Matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 4 & 8 \\ 6 & 7 & 8 & 9 \\ 2 & 4 & 0 & 0 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - R_1$, $R_3 \longrightarrow R_3 - 6R_1$, $R_4 \longrightarrow R_4 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 1 & 4 \\ 0 & -5 & -10 & -15 \\ 0 & 0 & -6 & -8 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 5R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -15 & -35 \\ 0 & 0 & -6 & -8 \end{bmatrix}$$

$R_3/-5$; $R_4/-2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 3 & 4 \end{bmatrix}$$

$R_4 \longrightarrow R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$\therefore$ Rank of $A = \rho(A) = 4$ //..