UNIT-4 MATHS SEM 3 LONGS

Statement: Every finite group q Isomorphic to its perbruitation	ا عا
I somorphic to its perbruitation	group.
Proof:	,00.
Let (G.) be finite group. 2000	Permis
Ret a e g for any a e g show (3) to.	Postition
that an eq [: By closure law]	Perinculat
Define: f: G -> G Such that facin:	e aa Vaeg.
Jo prove that to is well-defined:	tt
Jo prove that fa is well-defined:	19] - F
Multiply with a on both & des	denoted
extraction is staid to be an (odd)	* Event d
a as bearings = facy)	7 + P
: fato is well-defined.	Permutaki
To prove fa is one-one:	and the state of
	transfessiti
facx) = facy)	

```
and wide of the
         az= ay
 By using left cancellation law.
      x= y 1 1 1 1 1 1 1 1 (1 1 1 1
    : fa is one-one.
To prove fa is onto:
  det a e g
aff = aff [:: Inverse law]
 Fata eg such that fa (ata) = a (ata)
                     = (a ajx).
                   1 (=) e 4
    Satisfice of the control of the satisfice
     i fa is permutation.
Now we howe to snow that fi = {fataeg} is
 a group.
* Clasure law : 1 Louis Many 10 1 1. E.
   For a, beg.
fa, fb & G'

(fa. fb) (n) = fa[fb(n)]
           = fa(6x)
             = a(6x)
Similarly we can ofdigo - that do has ta.
i is the nultiplicated Tours of it
 For a, b e g = ab e g [-! By riving closureloud
                    land in A.
         fab e g'
       fa. fo G g
          ix satisfies clasure law in G.
```

```
* Associative law:
     fa, fb, fc 6 g'.
 (fa.fb) fc = fa.(fb.fc) & G'.
I.H.s = (fa. fb) fc (a).
      =(fa.fb)(En)
   = fab(cx)
    = (ab)(cm)
        = a(bc) x dan mine 6 1 1 16
     = fa (bc) x
        = fa(fb fc) x
          = R-H-S
      . " Satisfies Associative law on G'
Resistence of Identity law:
  Ret Jacq.
  I fe & G' Such that fa. fe = fe.fa = fa
 LHS : fafe
    fae (if is flowed at not)
                 a (ba)
 Similary we can prove that fifa=fa.
 : fe is the multiplicative I dentity of its ing
but racois is patisfies enistence of Identity
   taw in G:
                  foe a
                 fa. to 6 9.
    is satisfied Clasere low in G.
```

```
st Existence of Inverse law:
For tacg'
for acq => ate a
I faif q' such that faifait faite fe
 d. HS = fa. fa-1
         = fe
        = R.HS
 similarly we can prove that far.fa=fe
: far is the multiplicative house of fains
  i' is natisfies Existence of Inverse in G.
   .. (G',) is a permutation group.
 Now we have to prove that G=G.
Define: p: q -> q such that q ca) = fa & a & q
To prove of is well-defined:
  Let 0, 5 & 9
      a=b sindpanoli u
 Multiply with it on both sides
           facul = 4 b(x) and trucks
             fa= fb
             pla) = $ (b)
           : of is well-defined.
  Jo prove $ is one-one:
         $ (a) = $(b)
         fa = fb
```

 $-\int a(x) = f_b(m)$ an = ba By using right Cancellation law, a=b do the doct is bound . of is one one. To prove of is onto: het x & G' Jacq such that fcar x. to in outs outs Jo prove of is structure Preserving For a, be G & ab & G of cab) = fab and and med sand at so fa-fby first is flowed = \(\partial (a) \cdot \quad (b) : p is Structure preserving. .. p & Isomorphic. Every finite group ten Iromorphie to its Permutation group. de tot pins + plas in the is well-defined. (d) \$ - \omega) \$ \$

Theorem: (2) Error France Carl Prove that a group of prime order is Cyclic (or) If p is a prime number then Every group of order p's cyclic. 1 300cg: det P≥2 be a prime number Let (G;) be a group of prime order $O(G) \geq P$ Since, the number of elements is atleast2 One of the element is other than 'e', let the element be a'. Ret ca> be a cyclic subgroup generated by a. :. a e < a > ; < a > = {e} Ket 'h' be the order of <a>, By dagrange's theorem, o (as) in. But p'is a prime number Janus drivion = 1 Cox) h=p. Since ca> f {e} = hf' (in the contraction of south my o[ca>] = ocq) (d) .. (G, ?) is also a cyclic group.

3 Prove that every subgroup of cyclic group is cyclic. Let (G.) be a group let (H;) be a Subgroup

Since H is a subgroup of G, Every element of H is an element of G. det 'd be the least positive integer int Such that an en vne # Now we have to p.TH= <ad> Let am &H + m & # By using Division Algorithm we have to find the values of d and q. Such that m=dq+r Yos rsd. am = adatr am = (ad)a. ar -0. a € H => (ad) 9 € H + 9 € # adq EH and c H am EH, a-de EH. am adq ett. [: closure law] of his cam-day of A sulprising with WEH. This is cautradiction to our aminption i.e., de s seu least positive Integer T 20 From 0 = am = (ad) 9 . a0 10 = (adjet = H M = cad > (1-)

A = cad > 2. Every Subgroup of cyclic group is cyclic. Z ...