COURSE - 7B

* Integral transforms with applications *

Syllabus :-

U1 - Laplace transform I → 3 theorems

[3-5 Longs + 3-6 shorts]

U2 - Laplace transforms - II → 2 theorems

U3 - Inverse Laplace Transforms → 4 theorems

U4 - Applications of Laplace transforms → Problems

U5 - Fourier transforms → 3 theorems

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Problem ⟶ ✓

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Procedure → ✓

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UNIT-1

Laplace Transform - 1

Integration

↓

$$\int x^2\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \frac{x^3}{3} + C$$

Formula's :-

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★ Imp

$$L[f(t)] = \int_0^\infty e^{-pt} f(t)\,dt.$$

① $L(1) = \frac{1}{p}$

2. $L(e^{at}) = \frac{1}{p-a}$

Ex. $e^{2t} = \frac{1}{p-2}$

3. $L(e^{-at}) = \frac{1}{p+a}$

4. $L(t^n) = \frac{n!}{p^{n+1}} \Big/ \frac{\Gamma(n+1)}{p^{n+1}}$

$n = 1, 2, 3, \ldots$ $\qquad$ $n =$ fraction.

$$L(t^3) = \frac{3!}{p^4}.$$

5. $L(\sin at) = \dfrac{a}{p^2+a^2}$

6. $L(\cos at) = \dfrac{p}{p^2+a^2}$

7. $L(\sinh at) = \dfrac{a}{p^2+a^2}$; $p > |a|$

8. $L(\cosh at) = \dfrac{p}{p^2-a^2}$; $p > |a|$

9. $L\left(e^{-at}\cos bt\right) = \dfrac{p+a}{(p+a)^2+b^2}$

10. $L\left(e^{-at}\sin bt\right) = \dfrac{b}{(p+a)^2+b^2}$.

11. $L\left(e^{-at}\,t^n\right) = \dfrac{n!}{(p+a)^{n+1}}$.

12. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

13. $\Gamma\left(-\frac{1}{2}\right) = 2\sqrt{\pi}$

1) $L[f(t)] = \int_0^\infty e^{-pt} f(t)\, dt$

$$L(1) = \int_0^\infty e^{-pt} (1)\, dt$$

$$= \int_0^\infty e^{-pt}\, dt$$

$$= \left[\frac{e^{-pt}}{-p}\right]_0^\infty$$

$\left[\int e^{ax} = \frac{e^{ax}}{a}\right]$

(Upper limit - lower limit)

$$\frac{e^{-p(\infty)}}{-p} - \frac{e^{-p(0)}}{-p}$$

$$0 + \frac{e^0}{p}$$

Any value $\to \infty$

0

$$= 0 + \frac{1}{p}$$

$$\boxed{L(1) = \frac{1}{p}}$$

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PROBLEMS :-

① Find the laplace transforms of

(i) $e^{2t}-4t^3-2\sin 3t+3\cos 3t$

(ii) t^3+2t^2-4t+6

SOL:

(i) $L\left(e^{2t}-4t^3-2\sin 3t+3\cos 3t\right)$

$$= L\left(e^{2t}\right)-4L\left(t^3\right)-2L\left(\sin 3t\right)+3L\left(\cos 3t\right)$$

$$= \frac{1}{p-2}-4\cdot\frac{3!}{p^{3+1}}-2\frac{3}{p^2+3^2}+3\frac{p}{p^2+3^2}$$

$\left[L\left(e^{at}\right)=\frac{1}{p-a}\right]$

$\left[L\left(t^n\right)=\frac{n!}{p^{n+1}}\right]$

$\left[L\left(\sin at\right)=\frac{a}{p^2+a^2}\right]$

$$= \frac{1}{p-2}-4\cdot\frac{3\times 2\times 1}{p^4}-2\frac{3}{p^2+9}+\frac{3p}{p^2+9}$$

$\left[L\left(\cos at\right)=\frac{p}{p^2+a^2}\right]$

$n! = n(n-1)(n-2)\cdots$

$$= \frac{1}{p-2}-\frac{24}{p^4}-\frac{6}{p^2+9}+\frac{3p}{p^2+9}.$$

(ii) $L(t^3+2t^2-4t+6)$

$= L(t^3)+2L(t^2)-4L(t^1)+L(6)$

$= \dfrac{3!}{p^{3+1}} + 2\dfrac{2!}{p^{2+1}} - 4.\dfrac{1!}{p^{1+1}} + 6\dfrac{1}{p}$

$= \dfrac{3\times2\times1}{p^4} + 2\dfrac{2\times1}{p^3} - 4\dfrac{1}{p^2} + \dfrac{6}{p}$

$= \dfrac{6}{p^4} + \dfrac{4}{p^3} - \dfrac{4}{p^2} + \dfrac{6}{p}$ //..

☆ ② Find the Laplace transforms of

(i) $(5e^{2t}-3)^3$

(ii) $\cos(at+b)$

(iii) $(\sin t - \cos t)^2$.

$\boxed{(a-b)^3 = a^3-b^3-3a^2b^1+3ab^2}$

↓ Alg + Trig + Inte + deri

SOL: (i) $L(5e^{2t}-3)^3$

$= L\left[(5e^{2t})^3 - 3^3 - 3\times(5e^{2t})^2 3 + 3(5e^{2t})3^2\right]$

$$= L\left[125e^{6t} - 27 - 225e^{4t} + 135e^{2t}\right]$$

$$= L\left(125e^{6t}\right) - 27\left(L(1)\right) - 225\,L\left(e^{4t}\right) + 135\,L\left(e^{2t}\right).$$

$$= 125 \times \frac{1}{p-6} - 27 \times \frac{1}{p} - 225\frac{1}{p-4} + 135\frac{1}{p-2}$$

$$= \frac{125}{p-6} - \frac{27}{p} - \frac{225}{p-4} + \frac{135}{p-2} ..$$

ii) $L\left[\cos(at+b)\right]$

$$= L\left[\cos at \cos b - \sin at \sin b\right]$$

$[\because \cos(a+b) = \cos a \cos b - \sin a \sin b]$

$$= L\left(\cos at \cos b\right) - L\left(\sin at \sin b\right)$$

$$= \cos b \, L\left(\cos at\right) - \sin b \, L\left(\sin at\right)$$

$$= \cos b \frac{p}{p^2+a^2} - \sin b \frac{a}{p^2+a^2}.$$

$$= \frac{p\cos b}{p^2+a^2} - \frac{a\sin b}{p^2+a^2} = \frac{p\cos b - a\sin b}{p^2+a^2}$$

iii) $L\left[\sin t-\cos t\right]^2$

$= L\left[\sin^2 t+\cos^2 t-2\sin t\cos t\right]$

$= L\left[1-\sin 2t\right]$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$\left[\because \sin 2\theta = 2\sin\theta\cos\theta\right]$

$= L(1)-L(\sin 2t)$

$= \frac{1}{p}-\frac{2}{p^2+2^2}$ $\left[\because L(1)=\frac{1}{p}\right]$

$= \frac{1}{p}-\frac{2}{p^2+4}$ .. $\left[\because L(\sin at)=\frac{a}{p^2+a^2}\right]$

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MATHS - 7B

UNIT-1

CLASS NO. 2

Formula's :-

$$L(1) = \frac{1}{p}$$

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$$L(2p) = \cancel{\frac{1}{2p}} = 2\,L(1)$$
$$= 2\,\frac{1}{p}$$
$$= \frac{2}{p}$$

* For a function

$$L\left[f(t)\right] = \int_0^\infty e^{-pt} f(t)\, dt$$

$$L(t^n) = \frac{n!}{p^{n+1}} \quad (n \text{ is Natural})$$

$$L(t^n) = \frac{\Gamma(n+1)}{p^{n+1}}$$

$$L(e^{at}) = \frac{1}{p-a}$$

$$\boxed{\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}}$$

$$L(e^{-at}) = \frac{1}{p+a}$$

$$\boxed{\Gamma\left(-\tfrac{1}{2}\right) = 2\sqrt{\pi}}$$

$$L(\sin at) = \frac{a}{p^2+a^2}$$

$$L(\cos at) = \frac{p}{p^2+a^2}$$

① Find the laplace transforms of

(i) $\cos(at+b)$ (ii) $(\sin t - \cos t)^2$.

Sol:

(i) $L(\cos(at+b)) = L[\cos at \cos b - \sin at \sin b]$

$= L(\cos at \cos b) - L(\sin at \sin b)$

$= \cos b \, L(\cos at) - \sin b \, L(\sin at)$ $\left[\because \cos at = \frac{p}{p^2+a^2}\right]$

Any old formula ↓ Lap. form.

$= \cos b \cdot \frac{p}{p^2+a^2} - \sin b \cdot \frac{a}{p^2+a^2}$

$= \frac{p\cos b - a\sin b}{p^2+a^2}$.

(ii) $L[\sin t - \cos t]^2 = L[\sin^2 t + \cos^2 t - 2\sin t\cos t]$

$= L[1 - \sin 2t]$ $[\because \sin^2\theta + \cos^2\theta = 1]$ $[\sin 2\theta = 2\sin\theta\cos\theta]$

$= L(1) - L(\sin 2t)$

$= \frac{1}{p} - \frac{2}{p^2+4}$ //.

$L(\sin at) = \frac{a}{p^2+a^2}$

② Find the Laplace transforms of

(i) $\sin^2 at$ (ii) $(t^2+1)^2$

Sol:

(i) $L(\sin^2 at) = L\left[\frac{1-\cos 2at}{2}\right]$

$\left[\cos 2\theta = 1-2\sin^2\theta\right.$

$2\sin^2\theta = 1-\cos 2\theta$

$\left.\sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$

$= \frac{1}{2}\left[L(1-\cos 2at)\right]$

$= \frac{1}{2}\left[L(1) - L(\cos 2at)\right]$

$= \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+(2a)^2}\right]$ $\left[\because \cos at = \frac{p}{p^2+a^2}\right]$

$= \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+4a^2}\right]$ //..

(ii) $L\left[(t^2+1)^2\right] = L\left[t^4+1+2t^2\right]$

$= L(t^4) + L(1) + 2L(t^2)$ $\left[\because L[t^n] = \frac{n!}{p^{n+1}}\right]$

$= \frac{4!}{p^{4+1}} + \frac{1}{p} + 2 \cdot \frac{2!}{p^{2+1}}$

$= \frac{24}{p^5} + \frac{1}{p} + \frac{4}{p^3}$ //..

$1! = 1$
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$

③ Find $L(f(t))$, where → Break function

$$f(t) = \begin{cases} 4 ; & 0 < t < 1 \\ 3 ; & t > 1 \end{cases}.$$

Sol:

We know that,

$$* \; L[f(t)] = \int_0^\infty e^{-pt} f(t)\,dt$$

$$= \int_0^1 e^{-pt}\, 4 \,dt + \int_1^\infty e^{-pt}\, 3\, dt$$

$$= 4\int_0^1 e^{-pt}\,dt + 3\int_1^\infty e^{-pt}\,dt$$

$$= 4\left[\frac{e^{-pt}}{-p}\right]_0^1 + 3\left[\frac{e^{-pt}}{-p}\right]_1^\infty$$

$$= 4\left[\frac{e^{-p(1)}}{-p} - \frac{e^{-p(0)}}{-p}\right] +$$

$$3\left[\frac{e^{-p(\infty)}}{-p} - \frac{e^{-p(1)}}{-p}\right]$$

$$= 4\left[\frac{e^{-p}}{-p} + \frac{1}{p}\right] + 3\left[0 + \frac{e^{-p}}{p}\right]$$

$$= -4\frac{e^{-p}}{p} + \frac{4}{p} + \frac{3e^{-p}}{p}$$

$$= \frac{4}{p} - \frac{e^{-p}}{p} \quad //..$$

(4) Find laplace transforms of the function

$$f(t) = \begin{cases} 2t & ; \; 0 \le t \le 5 \\ 1 & \quad t > 5 \end{cases}$$

SOL: We know that,

$$L[f(t)] = \int_0^{\infty} e^{-pt} f(t)\, dt$$

$$= \int_0^5 e^{-pt}\, 2t\, dt + \int_5^{\infty} e^{-pt}\, 1\, dt$$

$$= 2\int_0^5 \underset{\substack{\downarrow \\ v}}{e^{-pt}} \cdot \underset{\substack{\downarrow \\ u}}{t}\, dt + \int_5^{\infty} e^{-pt}\, dt$$

$$= 2\left[t\frac{e^{-pt}}{-p} - \int_0^5 1 \int \frac{e^{-pt}}{-p}\, dt\right] + \left[\frac{e^{-pt}}{-p}\right]_5^{\infty}$$

$$\left[\int uv\, dx = u\int v\, dx - \int\left\{\frac{d}{dx}u\int v\, dx\right\}\right]$$

I LATE

$$= 2\left[\frac{te^{-pt}}{-p} + \frac{1}{p}\int_0^5 e^{-pt}\, dt\right] + \left[0 + \frac{e^{-5p}}{p}\right]$$

$$= \frac{2te^{-pt}}{-p} + \frac{2}{p}\left[\frac{e^{-pt}}{-p}\right]_0^5 + \frac{e^{-5p}}{p}$$

$$= \frac{2te^{-pt}}{-p} + \frac{2}{p}\left[\frac{e^{-p(5)}}{-p} - \frac{e^{-p(0)}}{-p}\right] + \frac{e^{-5p}}{p}$$

$$= \frac{2te^{-pt}}{-p} + \frac{2}{p}\left[\frac{e^{-5p}}{-p} + \frac{1}{p}\right] + \frac{e^{-5p}}{p}$$

$$= \frac{2te^{-pt}}{-p} - \frac{2e^{-5p}}{p^2} + \frac{2}{p^2} + \frac{e^{-5p}}{p}$$

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MATHS – 7(B)

Integral Transforms with its app's

UNIT – 1

CLASS NO. 3

① State and prove first shifting theorem? Also find the Laplace transforms of $e^{-3t}(2\cos 5t - 3\sin 5t)$.

e^{at}

$P \rightarrow P-a$

Statement:

If $L[f(t)] = F(P)$ then

$$L[e^{at} \cdot f(t)] = P(P-a).$$

PROOF: We know that,

$$L[f(t)] = \int_0^\infty e^{-Pt} f(t)\,dt = F(P).$$

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$$\therefore \text{LHS} = L\left[e^{at} f(t)\right]$$

$$= \int_0^\infty e^{-Pt}\, e^{at} f(t)\,dt$$

$$= \int_0^\infty e^{-Pt+at} f(t)\,dt$$

$$= \int_0^\infty e^{-(p-a)t} f(t)\, dt$$

$$= F(p-a) = RHS.$$

i, $L\left\{e^{-3t}\left(2\cos 5t - 3\sin 5t\right)\right\}$

$$= L\left\{e^{-3t} \cdot 2\cos 5t - e^{-3t} \cdot 3\sin 5t\right\}$$

$$= 2L\left(e^{-3t} \cdot \cos 5t\right) - 3L\left(e^{-3t} \cdot \sin 5t\right) \quad \left[\cos 5t = \frac{p}{p^2+25}\right]$$

$$= 2 \cdot \frac{p-(-3)}{[p-(-3)]^2+5^2} - 3 \cdot \frac{5}{[p-(-3)]^2+5^2} \quad \left[\sin 5t = \frac{5}{p^2+25}\right]$$

[∵ By 1st shifting theorem]

$$= 2\frac{p+3}{(p+3)^2+25} - \frac{15}{(p+3)^2+25}$$

$$= \frac{2p+6-15}{p^2+9+6p+25}$$

$$= \frac{2p-9}{p^2+6p+34} \; //..$$

② Evaluate (i) $L\left(e^{3t}\sin^2 t\right)$

(ii) $L\left(e^{-2t}\sin 4t\right)$.

Sol:

(i) $L\left(e^{3t}\cdot\sin^2 t\right)$

$$\therefore\ L\left(\sin^2 t\right) = L\left[\frac{1-\cos 2t}{2}\right]$$

$$= \frac{1}{2}L\left(1-\cos 2t\right)$$

$$= \frac{1}{2}\left[L(1) - L(\cos 2t)\right]$$

$$= \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+2^2}\right] \quad \left[\because L(1)=\frac{1}{p}\right]$$

$$\left[L(\cos at) = \frac{p}{p^2+a^2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{p} - \frac{p}{p^2+4}\right]$$

$\therefore$ By Using first shifting theorem,

$$L\left(e^{3t}\sin^2 t\right) = \frac{1}{2}\left[\frac{1}{p-3} - \frac{p-3}{(p-3)^2+4}\right]$$

$$= \frac{1}{2}\left[\frac{1}{p-3} - \frac{p-3}{p^2+13-6p}\right]$$

$\because L\left(e^{-2t}\sin 4t\right)$

$$\therefore L(\sin 4t) = \frac{4}{p^2+4^2} \quad \left[\because L(\sin at) = \frac{a}{p^2+a^2}\right]$$

$$= \frac{4}{p^2+16}$$

$\therefore$ By Using first Shifting theorem,

$$L\left(e^{-2t}\sin 4t\right) = \frac{4}{[p-(-2)]^2+16}$$

$$= \frac{4}{(p+2)^2+16}$$

$$= \frac{4}{p^2+2^2+2\times p\times 2+16}$$

$$= \frac{4}{p^2+4p+4+16}$$

$$= \frac{4}{p^2+4p+20}$$

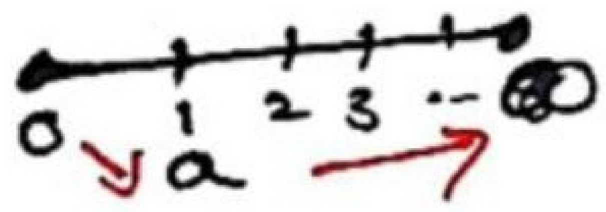
(3) State and prove Second Shifting theorem? Also find Laplace transforms of $G(t) = \begin{cases} \sin\left(t-\frac{\pi}{3}\right) & ; t > \frac{\pi}{3} \\ 0 & ; t < \frac{\pi}{3}. \end{cases}$

Statement:

If $L[F(t)] = f(p)$ then

$$G(t) = \begin{cases} F(t-a) & ; t > a \\ 0 & ; t < a \end{cases}$$ (Break function)

PROOF:



Given that,

$$L[G(t)] = \int_0^\infty e^{-pt}\, G(t)\, dt$$

$$= \int_0^a e^{-pt}\, G(t)\, dt + \int_a^\infty e^{-pt}\, G(t)\, dt$$

$$= 0 + \int_a^\infty e^{-pt}\, G(t)\, dt$$

$$= \int_a^\infty e^{-pt}\, F(t-a)\, dt$$

Let $t-a = u$.

$$t = u + a$$

$$1 = \frac{du}{dt} + 0$$

$$\boxed{dt = du}$$

Lower limit:

If $t = a \Rightarrow u = a - a = 0$

$\therefore \boxed{u=0}$

Upper limit :-

If $t = \infty \Rightarrow \boxed{u = \infty}$

$$\therefore \int_0^\infty e^{-p(u+a)} \cdot f(u)\, du$$

$$= \int_0^\infty e^{-pu} \cdot e^{-pa} f(u)\, du$$

$$= e^{-pa} \int_0^\infty e^{-pu} f(u)\, du$$

$$= e^{-pa} \bar{f}(p)$$

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(i) $$G(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right) & ; t > \frac{\pi}{3} \\ 0 & ; t < \frac{\pi}{3} \end{cases}$$

Here, $a = \frac{\pi}{3}$

$f(t) = \sin t$

$$L[f(t)] = L(\sin t)$$
$$= \frac{1}{p^2+1^2} = \frac{1}{p^2+1} = f(p)$$

$\therefore$ By second shifting theorem,

$$L[G(t)] = e^{-ap} F(p)$$
$$= e^{-ap} \frac{1}{p^2+1}$$

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③ State and Prove Change of Scale Property ? Applying change of Scale Property if $L[f(t)] = \frac{p^2-p+1}{(2p+1)^2(p-1)}$ then

S.T. $L[f(2t)] = \frac{p^2-2p+4}{4(p+1)^2(p-2)}$.

Statement:

If $L[f(t)] = \int_0^\infty e^{-pt} f(t)\, dt = F(p)$

then $L[f(at)] = \frac{1}{a} \cdot F\left(\frac{p}{a}\right)$

PROOF: Given that,

$$L[f(t)] = F(p) = \int_0^\infty e^{-pt} f(t)\, dt$$

$$L[f(at)] = \int_0^\infty e^{-pt} f(at)\, dt$$

$$\therefore \quad at = u \Rightarrow t = \frac{u}{a}$$

$$1 = \frac{1}{a}\frac{du}{dt}$$

$$dt = \frac{1}{a} du$$

$$= \int_0^{\infty} e^{-p\left(\frac{u}{a}\right)} f(u) \frac{du}{a}$$

$$= \frac{1}{a} \int_0^{\infty} e^{-\frac{p}{a}\cdot u} f(u)\, du$$

$$= \frac{1}{a} F\left(\frac{p}{a}\right) = \underline{\underline{RHS}} ..$$

Problem: —

$$L\left[f(t)\right] = \frac{p^2 - p + 1}{(2p+1)^2 (p-1)}$$

$$L\left[f(2t)\right] = \frac{1}{2} \frac{\cancel{4}\cdot\left(\frac{p}{2}\right)^2 - \cancel{2}\left(\frac{p}{2}\right) + 1}{\left(\cancel{2}\cdot\frac{p}{\cancel{2}} + 1\right)^2 \left(\frac{p}{2} - 1\right)}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\frac{p^2}{4} - \frac{p}{2} + 1}{(p+1)^2 \left(\frac{p-2}{\cancel{2}}\right)}$$

$$= \frac{\frac{p^2 - 2p + 4}{4}}{(p+1)^2 (p-2)}$$

$$= \frac{1}{4}\left[\frac{p^2 - 2p + 4}{(p+1)^2 (p-2)}\right] // ..$$