MATHS - 6(B)

MULTIPLE INTEGRALS AND VECTOR CALCULUS

Syllabus :-

UNIT-1 : Multiple Integrals - I 10 PROB

UNIT-2 : Multiple Integrals - II 10 PROB

* UNIT-3 : Vector Differentiation 3TH 5L+5S

* UNIT-4 : Vector Integration 3TH

* UNIT 5 : Vector Integration's Applications 3TH

→ 6B & 7B → Diff

→ Live ✓ ST40

→ Download option ✓

→ Pdf's ✓ (TM)

→ My guidance ✓

→ 9581234096

UNIT-1

Formula's :- How to memorize ✓

→ $\int \text{constant}\, dx = cx$ yt ↓ Tm ✓ App ✓ Study mat →

Ex: $\int 5\, dx = 5x$

$\int 10\, dx = 10x$

$\int y\, dx = yx.$ 9581234096 →

$\int y^2\, dx = y^2x$

→ $\int x^n\, dx = \frac{x^{n+1}}{n+1}.$

Ex: $\int x^5\, dx = \frac{x^6}{6}$

$\int x^2\, dx = \frac{x^3}{3}$

$\int x^1\, dx = \frac{x^2}{2}.$

→ $\int 2x\, dx = \cancel{2}\frac{x^2}{\cancel{2}} = x^2$ //..

→ $\int xy\, dx = y \cdot \frac{x^2}{2}$ //..

$\rightarrow \iint (f+g)\, dx\, dy$

$$= \iint f\, dx\, dy + \iint g\, dx\, dy$$

$\rightarrow \iint (Kf)\, dx\, dy$

$$= K \iint f\, dx \cdot dy$$

$\rightarrow \iint_R f\, dx\, dy = \int_{R_1} f\, dx\, dy + \int_{R_2} f\, dx\, dy$

$$\boxed{R = R_1 \cup R_2}$$

$\rightarrow$ Definite Integral:

$$\int_0^1 x^2\, dx = \left[\frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{1^3}{3} - \frac{0^3}{3}\right)$$

$$= \frac{1}{3} - 0 = 1/3$$

PROBLEMS :-

① Evaluate i) $\int_0^2 \int_0^x y \, dy \, dx$

ii) $\int_0^3 \int_{-x}^{x} xy \, dx \, dy$.

SOL:

$$\int_0^2 \int_0^x y \, dy \, dx = \int_0^2 \left[\int_0^x y \, dy \right] dx$$

$$= \int_0^2 \left[\frac{y^2}{2} \right]_0^x dx$$

$$= \int_0^2 \left(\frac{x^2}{2} - \frac{0^2}{2} \right) dx$$

$$= \int_0^2 \frac{x^2}{2} \, dx \qquad \boxed{\int x^n = \frac{x^{n+1}}{n+1}}$$

$$= \frac{1}{2} \int_0^2 x^2 \, dx$$

$$= \frac{1}{2} \left[\frac{x^3}{3} \right]_0^2 = \frac{1}{2} \left[\frac{2^3}{3} - \frac{0^2}{2} \right]$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{8}\,4}{3} = 4/3 \,//..$$

iii, $\int_0^3 \int_{-x}^{x} xy \; dx\,dy$

$$= \int_0^3 \left[\int_{-x}^{x} y\,dy \right] x\,dx$$

$$= \int_0^3 \left[\frac{y^2}{2} \right]_{-x}^{x} x\,dx$$

$$= \int_0^3 \left(\frac{x^2}{2} - \frac{(-x)^2}{2} \right) x\,dx$$

$$= \int_0^3 \left(\cancel{\frac{x^2}{2}} - \cancel{\frac{x^2}{2}} \right) x\,dx = 0$$

② Evaluate :- $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy$

iii, $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$.

Sol:

$$\int_0^1 \left[\int_x^{\sqrt{x}} (x^2+y^2)\,dy \right] dx$$

$$= \int_0^1 \left[x^2 y + \frac{y^3}{3}\right]_x^{\sqrt{x}} dx$$

$\frac{1}{2}+2 = \frac{5}{2}$

$$= \int_0^1 \left[\left(x^2\sqrt{x} + \frac{(\sqrt{x})^3}{3}\right) - \left(x^2(x) + \frac{x^3}{3}\right)\right] dx$$

$$= \int_0^1 \left[x^{5/2} + \frac{x^{3/2}}{3} - \left(\frac{x^3}{1} + \frac{x^3}{3}\right)\right] dx$$

$$= \int_0^1 \left(x^{5/2} + \frac{x^{3/2}}{3} - \frac{4x^3}{3}\right) dx$$

$$= \left[\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{1}{3}\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{4}{3}\cdot\frac{x^{3+1}}{3+1}\right]_0^1$$

$$= \left[\frac{x^{7/2}}{7/2} + \frac{1}{3}\frac{x^{5/2}}{5/2} - \frac{\cancel{4}}{3}\frac{x^4}{\cancel{4}}\right]_0^1$$

$$= \left(\frac{2}{7}x^{7/2} + \frac{2}{15}x^{5/2} - \frac{1}{3}x^4\right)_0^1$$

$$= \left(\frac{2}{7} + \frac{2}{15} - \frac{1}{3}\right) - 0 = \frac{30+14-35}{105} = \frac{9}{105} = \frac{3}{35}$$

ii, $$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$$

SOL:

Rewrite the given integral as,

$$\int_0^1 \left[\int_{y=0}^{\sqrt{1+x^2}} \frac{1}{(1+x^2)+y^2}\, dy \right] dx$$

Where, $\sqrt{1+x^2} = p$

$1+x^2 = p^2$

$$\int_0^1 \left[\int_0^p \frac{1}{p^2+y^2}\, dy \right] dx$$

$$\boxed{\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a} \tan^{-1}\frac{x}{a}}$$

$$\int_0^1 \left[\frac{1}{p} \tan^{-1}\left(\frac{y}{p}\right) \right]_0^p dx$$

$$= \int_0^1 \left[\frac{1}{p} \left(\tan^{-1}\left(\cancel{\frac{p}{p}}\right) - \tan^{-1}\left(\frac{0}{p}\right) \right] dx$$

$$= \int_0^1 \left[\frac{1}{p} \tan^{-1} (1) - \tan^{-1}(0) \right] dx$$

$$= \int_0^1 \frac{1}{\sqrt{1+x^2}} \cdot \left[\tan^{-1}\left(\tan \frac{\pi}{4}\right) - 0 \right] dx$$

$$= \frac{\pi}{4} \int_0^1 \frac{1}{\sqrt{1+x^2}}\, dx.$$

$$= \frac{\pi}{4} \left(\sinh^{-1} x \right)_0^1$$

$$= \frac{\pi}{4} \sinh^{-1}(1)..$$

$\boxed{9581234096}$ ✓

↓

Short tricks 40

↓

yt ✓

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~~1600~~/-

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MATHS - 6 (B)

UNIT - 1

CLASS NO. 2

① Evaluate (i) $\int_0^4 \int_{y=0}^{x^2} e^{y/x}\, dx\, dy$

(ii) $\int_0^5 \int_0^{x^2} x(x^2+y^2)\, dx\, dy$

(iii) $\int_0^2 \int_0^x e^{x+y}\, dy\, dx$

SOL:

(i) $\int_0^4 \left[\int_0^{x^2} e^{\frac{y}{x}}\, dy\right] dx$ → Inte 'x' → dy →

$= \int_0^4 \left[\frac{e^{y/x}}{1/x}\right]_0^{x^2} dx$ $\left[\int e^{ax} = \frac{e^{ax}}{a}\right]$

$= \int_0^4 x\left[e^{\frac{x^2}{x}} - e^{0/x}\right] dx$

ILATE

↓

$= \int_0^4 (xe^x - x)\, dx$ $\left[\int uv = u\int v\,dx - \int \frac{d}{dx}u \int v\,dx\right]$

$= \left[xe^x - \int 1.e^x\, dx - \frac{x^2}{2}\right]_0^4$

$= \left[xe^x - e^x - \frac{x^2}{2}\right]_0^4 = (4e^4 - e^4 - 8) + 1$

$= 3e^4 + 1$ //

ii) $\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dx\,dy$

$$= \int_0^5 \left[\int_0^{x^2} (x^3 + xy^2)\,dy\right] dx$$

$$= \int_0^5 \left[\left(x^3.(y) + x.\frac{y^3}{3}\right)_0^{x^2}\right] dx$$

$$= \int_0^5 \left[\left(x^3(x^2) + x.\frac{(x^2)^3}{3}\right) - 0\right] dx$$

$$= \int_0^5 \left(x^5 + \frac{x^7}{3}\right) dx$$

$$\boxed{\int x^n = \frac{x^{n+1}}{n+1}}$$

$$= \left[\frac{x^6}{6} + \frac{1}{3}\left(\frac{x^8}{8}\right)\right]_0^5$$

$$\boxed{\text{Inte.} \downarrow \text{limits}}$$

$$= \left[\frac{5^6}{6} + \frac{1}{3}\left(\frac{5^8}{8}\right)\right] - 0$$

2	6, 24
3	3, 12
	1, 4

$= 24$

$$= 5^6\left[\frac{1}{6} + \frac{1}{3} \times \frac{25}{8}\right]$$

$$= 5^6\left(\frac{1}{6} + \frac{25}{24}\right) = 5^6\left(\frac{4+25}{24}\right)$$

$\times 4$ $\times 1$

$$= 5^6\,(29/24) \;//$$

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iii) $$\int_0^2 \int_0^x e^{x+y}\, dy\, dx$$

$$= \int_0^2 \left[\int_0^x e^{x+y}\, dy \right] dx$$

$$= \int_0^2 e^x \left[\int_0^x e^y\, dy \right] dx$$

$$= \int_0^2 e^x \left[e^y \right]_0^x dx$$

$\int e^y = e^y$

Share Unit-1

$$= \int_0^2 e^x \left(e^x - e^0 \right) dx$$

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$$= \int_0^2 e^x \left(e^x - 1 \right) dx$$

$$= \int_0^2 \left(e^{2x} - e^x \right) dx$$

$$= \left[\frac{e^{2x}}{2} - e^x \right]_0^2$$

$\left[\frac{e^{ax}}{a} \right]$

$$= \left(\frac{e^4}{2} - e^2 \right) - \left(\frac{1}{2} - \frac{1}{1} \right)$$

$$= \frac{e^4}{2} - e^2 - \left(\frac{1-2}{2} \right) = \frac{e^4}{2} - e^2 + \frac{1}{2}$$

② Evaluate (i) $\int_0^1 \int_0^y e^{x/y}\, dx\, dy$

(ii) $\int_0^1 \int_{\sqrt{y}}^{2-y} x^2\, dx\, dy$

Sol:

(i) $\int_0^1 \left[\int_{x=0}^{y} e^{x/y}\, dx \right] dy$

$\int_0^1 \left[\frac{e^{x/y}}{\frac{1}{y}} \right]_0^y dy$ $\qquad e^{ax} = \frac{e^{ax}}{a}$

$\int_0^1 \left(y e^{x/y} \right)_0^y dy$

$\int_0^1 \left(y e^{\cancel{y}/\cancel{y}} - y e^{0/y} \right) dy$

$\int_0^1 (ye - y)\, dy$

$\int_0^1 y(e-1)\, dy$

$(e-1) \int_0^1 y\, dy = (e-1) \left[\frac{y^2}{2} \right]_0^1$

$(e-1) \left(\frac{1^2}{2} - \frac{0^2}{2} \right) = (e-1) \times \frac{1}{2} = \frac{e-1}{2}$

(ii) $$\int_0^1 \left[\int_{\sqrt{y}}^{2-y} x^2\, dx\right] dy$$

$$= \int_0^1 \left(\frac{x^3}{3}\right)_{\sqrt{y}}^{2-y} dy$$

Formula ↓ link ✓ App ✓ Ft concept

$$= \int_0^1 \left(\frac{(2-y)^3}{3} - \frac{(\sqrt{y})^3}{3}\right) dy$$

75/75

$$= \int_0^1 \left(\frac{(2-y)^3}{3} - \frac{y^{3/2}}{3}\right) dy$$

$$= \int_0^1 \frac{(2-y)^3}{3}\, dy - \int_0^1 \frac{y^{3/2}}{3}\, dy$$

$$= \left[-\frac{1}{3}\times\frac{(2-y)^4}{4} - \frac{1}{3}\,\frac{y^{3/2+1}}{3/2+1}\right]_0^1$$

$$= \left[-\frac{1}{12} - \frac{1}{3}\left(\frac{2}{5}\right)\right] - \left[\frac{16}{-12}\right]$$

$$= \left[-\frac{1}{12} - \frac{2}{15} + \frac{4}{3}\right]$$

$$= \frac{-5-8+80}{60} = \frac{67}{60}$$

Unit-1

MATHS-6B

CLASS NO.3

① Evaluate $\int_0^4 \int_{y^2/4}^{y} \frac{y}{x^2+y^2}\, dx\, dy$.

SOL:

$$\int_0^4 \left[\int_{y^2/4}^{y} \frac{y}{x^2+y^2}\, dx \right] dy$$

$$= \int_0^4 \cancel{y} \left[\frac{1}{\cancel{y}} \tan^{-1}\left(\frac{x}{y}\right) \right]_{y^2/4}^{y} dy$$

$$= \int_0^4 \left[\tan^{-1}\left(\frac{y}{\cancel{y}}\right) - \tan^{-1}\left(\frac{y^{\cancel{2}}/4}{\cancel{y}}\right) \right] dy$$

$$= \int_0^4 \left[\tan^{-1}(1) - \tan^{-1}\left(\frac{y}{4}\right) \right] dy$$

$$= \int_0^4 \left[\cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\pi}{4}\right) - \tan^{-1}\left(\frac{y}{4}\right) \right] dy$$

$$= \frac{\pi}{4}\left[y\right]_0^4 - \int_0^4 \underset{\downarrow\, u}{\tan^{-1}\left(\frac{y}{4}\right)} \cdot \underset{\downarrow\, v}{1}\; dy$$

I LATE ⟶

Formula

- $\rightarrow \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$
- $\rightarrow \int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$
- $\rightarrow \int uv\, dx = u\int v\, dx - \int\left[\frac{d}{dx}u \int v\, dx\right] dx$
- $\rightarrow \tan\frac{\pi}{4} = 1$
- $\rightarrow \log a - \log b = \log\frac{a}{b}$
- $\rightarrow \int \frac{f'(x)}{f(x)}\, dx = \log f(x)$

$$= \frac{\pi}{4}(4-0) - \left[\tan^{-1}\left(\frac{y}{4}\right)\int 1\,dy\right]_0^4 + \int_0^4 \left[\frac{d}{dy}\tan^{-1}\frac{y}{4}\int 1\,dy\right] dy$$

$$= \frac{\pi}{\cancel{4}} \times \cancel{4} - \left[\tan^{-1}\left(\frac{y}{4}\right)\cdot y\right]_0^4 + \int_0^4 \frac{1}{1+\frac{y^2}{16}}\,\frac{1}{4}\cdot y\,dy$$

$$= \pi - \left[\tan^{-1}\left(\frac{\cancel{4}}{\cancel{4}}\right)\cdot 4 - 0\right] + \int_0^4 \frac{4\,\cancel{16}}{16+y^2} \times \frac{1}{\cancel{4}}\cdot y\,dy$$

$$= \pi - \left[4\,\cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\pi}{4}\right)\right] + 2\int_0^4 \frac{2y}{y^2+16}\,dy$$

$$= \pi - \left[\cancel{4}\cdot\frac{\pi}{\cancel{4}}\right] + 2\left\{\log(y^2+16)\right\}_0^4$$

$$\left[\because \int \frac{f'(x)}{f(x)}\,dx = \log|f(x)|\right]$$

$$= \cancel{\pi} - \cancel{\pi} + 2\left[\log 32 - \log 16\right]$$

$$= 2\log \frac{\cancel{32}^{\,2}}{\cancel{16}}$$

$$= 2\log 2$$

② Evaluate $\int_0^2 \int_0^3 xy \, dx \, dy$

SOL :-

Given that,

$$\int_0^2 \int_0^3 x y \, dx \, dy$$

$$= \int_0^2 x \, dx \int_0^3 y \, dy$$

$$= \left[\frac{x^2}{2}\right]_0^2 \left[\frac{y^2}{2}\right]_0^3 \quad \because x^n = \frac{x^{n+1}}{n+1}$$

$$= \left[\frac{2^2}{2} - 0\right] \cdot \left[\frac{3^2}{2} - 0\right]$$

$$= (2-0) \cdot \left(\frac{9}{2} - 0\right)$$

$$= \cancel{2} \cdot \frac{9}{\cancel{2}} = 9 //..$$

③ Evaluate i) $\iint_R \frac{dx}{(1+x^2)(1+y^2)}$ Over $[0,1; 0,1]$

ii) $\int_0^a \int_0^b (x^2+y^2) \, dx \, dy$

iii) $\int_0^3 \int_0^1 (x^2+3y^2) \, dx \, dy$

(i) $$\int_0^1\int_0^1 \frac{1}{(1+x^2)(1+y^2)}\,dx\,dy$$

$$= \int_0^1 \frac{1}{1+x^2}\,dx \int_0^1 \frac{1}{1+y^2}\,dy$$

$$= \left[\tan^{-1}x\right]_0^1 \cdot \left[\tan^{-1}y\right]_0^1$$

$$= \left[\tan^{-1}(1)-\tan^{-1}(0)\right]\left[\tan^{-1}(1)-\tan^{-1}(0)\right]$$

$$= \cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\pi}{4}\right)\cdot\cancel{\tan^{-1}}\left(\cancel{\tan}\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}\cdot\frac{\pi}{4} = \frac{\pi^2}{16}$$ //..

(ii) $$\int_0^a\int_0^b (x^2+y^2)\,dx\,dy$$

$$\int_0^a\left[\int_0^b (x^2+y^2)\,dy\right]dx$$

$$\int_0^a\left[x^2\cdot y+\frac{y^3}{3}\right]_0^b dx$$

$$\int_0^a\left(x^2 b+\frac{b^3}{3}\right)dx = \left[b\frac{x^3}{3}+\frac{b^3}{3}x\right]_0^a$$

$$b\,\frac{a^3}{3} + \frac{b^3}{3}\,a = \frac{ab}{3}\left(a^2+b^2\right) \;//..$$

iii) $\int_0^3 \int_0^1 (x^2+3y^2)\,dx\,dy$

$$= \int_0^3 \left[\int_0^1 (x^2+3y^2)\,dy\right] dx$$

$$= \int_0^3 \left[x^2 \cdot y + \cancel{3}\,\frac{y^3}{\cancel{3}}\right]_0^1 dx$$

$$= \int_0^3 \left(x^2(1) + 1^3\right) dx$$

$$= \int_0^3 \left(x^2+1\right) dx$$

$$= \left[\frac{x^3}{3} + x\right]_0^3$$

$$= \left(\frac{3^3}{3}+3\right) - \left(\frac{0^3}{3}+0\right)$$

$$= 9+3 = \underline{\underline{12}}\,..$$

MATHS-6B

UNIT-1

CLASS-4

Region of Integration:

(*) If the region of integration is a rectangle between the lines $x=a, y=c$

$x=b, y=d$ then

$$\int_a^b \int_c^d f(x,y)\,dx\,dy = \int_c^d \int_a^b f(x,y)\,dx\,dy.$$

Steps
→ Lower limit & Upper limit, We have to find.

→ Old process $\left\langle \begin{array}{l} y \to dx \\ x \to dy \end{array} \right.$

(1) Evaluate $\iint_R xy\,dx\,dy$, R is the Region bounded by $y=x, y^2=4x$.

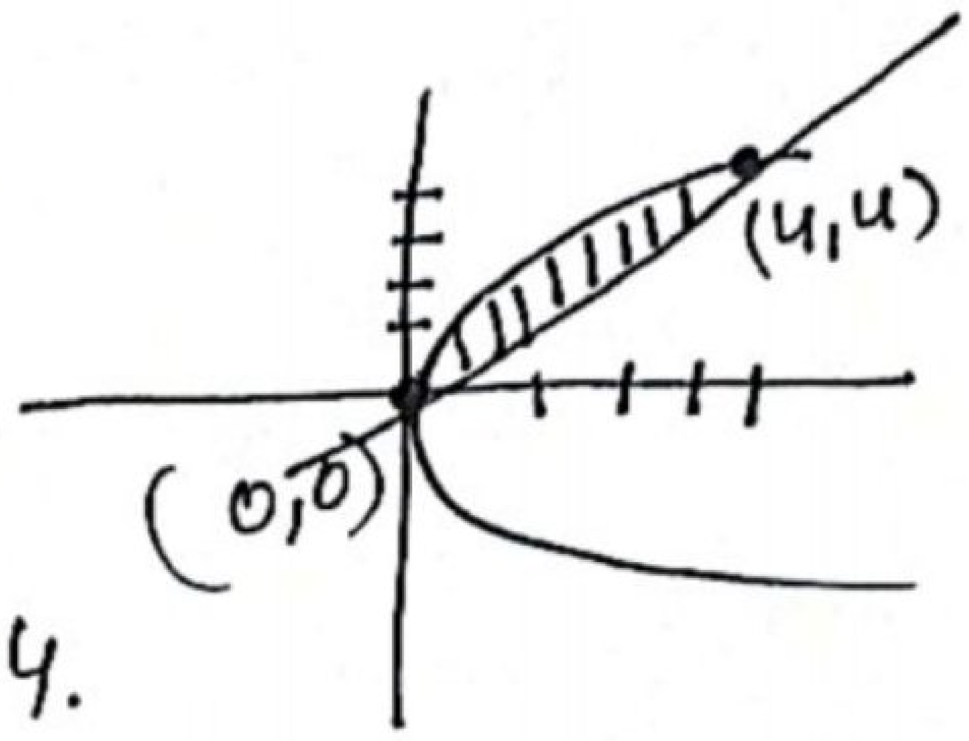
Sol: Point of intersections: $x^2=4x$

$$x^2-4x=0$$
$$x(x-4)=0$$
$$x=0; x-4=0$$
$$x=0,4.$$

$\therefore$ If $x=0$; $\boxed{y=0}$

If $x=4$; $y^2=16 \Rightarrow \boxed{y=4}$

$\therefore$ point of intersections are $(0,0), (4,4)$.



$\therefore$ From the figure,

x varies from : $x=0$ to $x=4$.

y varies from : $y=x$ to $y=2\sqrt{x}$

Now,

$$\iint_R xy\,dx\,dy$$

$$= \int_{x=0}^{4}\int_{x}^{2\sqrt{x}} xy \;dx\,dy$$

$$= \int_0^4 \left[\int_x^{2\sqrt{x}} xy\;dy\right] dx$$

$$= \int_0^4 x\left[\frac{y^2}{2}\right]_x^{2\sqrt{x}} dx$$

$$= \int_0^4 x\left[\frac{(2\sqrt{x})^2}{2} - \frac{x^2}{2}\right] dx$$

$$= \int_0^4 x\left(\frac{\cancel{4}^{2}x}{\cancel{2}} - \frac{x^2}{2}\right) dx$$

$$= \int_0^4 \left(2x^2 - \frac{x^3}{2}\right) dx$$

$$= \left[2\frac{x^3}{3} - \frac{x^4}{2\times 4}\right]_0^4$$

4^4

$4^2 \times 4^2$

16×16

256

$$= 2\cdot\frac{64}{3} - \frac{\cancel{256}\,\cancel{64}\,32}{\cancel{8}\,2}$$

$$= \frac{128}{3} - \frac{\cancel{64}\,32}{\cancel{6}}$$

$$\cancel{= \frac{256-64}{6}}$$

$$\cancel{= \frac{\cancel{192}\,\cancel{96}\,32}{\cancel{6}\,3}}$$

$$\cancel{= 32}$$

$$= \frac{128}{3} - 32$$

$$= \frac{128-96}{3}$$

$$= \frac{32}{3} \;//..$$

② Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y=x^2$, $y=x$.

SOL: Point of intersections: $x = x^2$

$x^2 - x = 0$

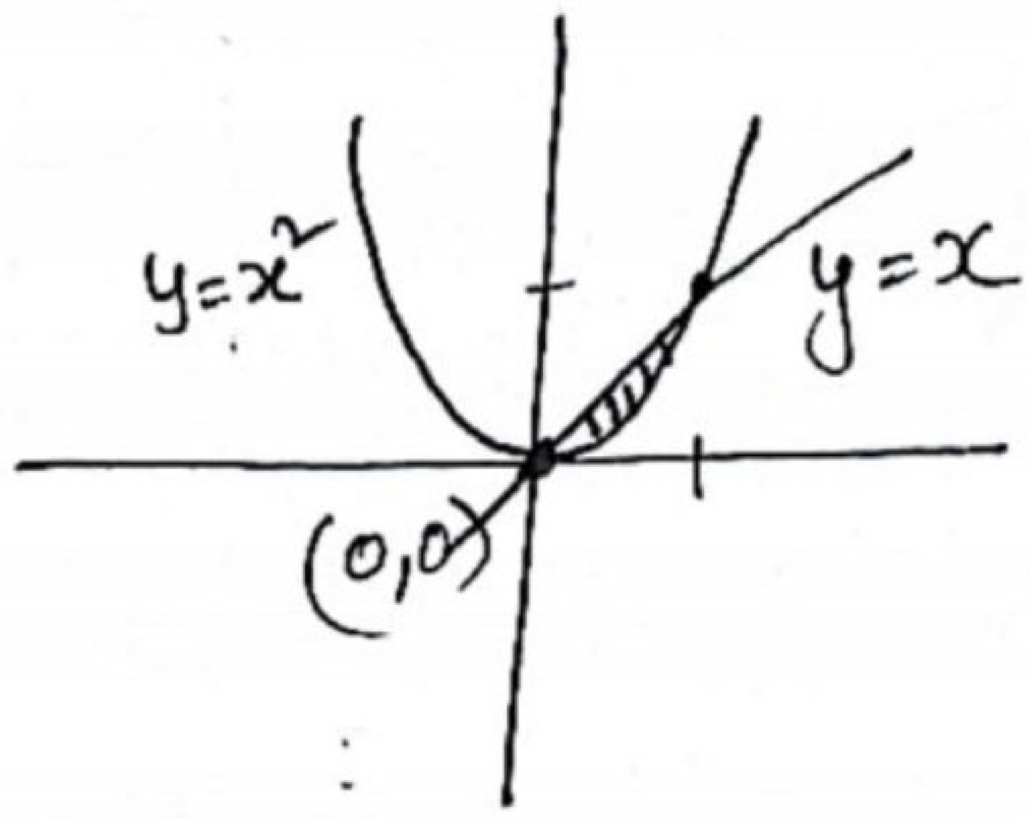
$x(x-1) = 0$

$x=0;\ x=1$

$\therefore$ If $x=0 \Rightarrow y=0$

If $x=1 \Rightarrow y=1$

$\therefore$ Point of intersections are : $(0,0)$, $(1,1)$.



x Varies from $x=0$ to $x=1$.

y Varies from $y=x^2$ to $y=x$.

Now,

$$\int_0^1 \int_{x^2}^{x} xy(x+y)\,dx\,dy$$

$$\int_0^1 \left[\int_{x^2}^{x} (x^2y + xy^2)\,dy\right] dx$$

$$\int_0^1 \left[x^2 \cdot \frac{y^2}{2} + x \cdot \frac{y^3}{3}\right]_{x^2}^{x} dx.$$

$$\int_0^1 \left(x^2 \cdot \frac{x^2}{2} + x \cdot \frac{x^3}{3}\right) - \left(x^2 \frac{(x^2)^2}{2} + x\frac{(x^2)^3}{3}\right) dx$$

$$= \int_0^1 \left(\frac{x^4}{2}+\frac{x^4}{3}-\frac{x^6}{2}-\frac{x^7}{3}\right)dx$$

$$= \left[\frac{x^5}{10}+\frac{x^5}{15}-\frac{x^7}{14}-\frac{x^8}{24}\right]_0^1$$

$$= \frac{1}{10}+\frac{1}{15}-\frac{1}{14}-\frac{1}{24}$$

$$= \frac{84+56-60-35}{840}$$

2	10, 15, 14, 24
5	5, 15, 7, 12
3	1, 3, 7, 12
2	1, 1, 7, 4
	1, 1, 7, 2

$= 120 \times 7$

$= \boxed{840}$

$$= \frac{140-95}{840}$$

$$= \frac{\cancel{45}\ 9}{\cancel{840}\ 168}$$

$$= \frac{9}{168} = \frac{3}{56}$$

$14 \mid 840 \mid 60$

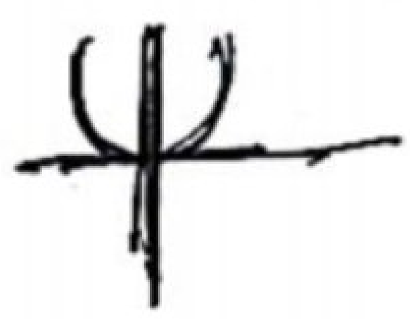
★ $\boxed{\text{RIGHTSIDE}}$

$\boxed{y^2 = 4ax}$

$y^2 = -4ax$

$x^2 = 4ay$

$\boxed{x^2 = y}$



$x^2 = -4ay$

MATHS - 6B

UNIT - 1

CLASS NO.5

$x, y \to$ Cartesian
$r, \theta \to$ Polar

Polar Co-ordinates :-

① Evaluate $\int_0^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} \, d\theta \, dr.$

Sol :-

Cartesian $\to$ Polar

$x = r\cos\theta$
$y = r\sin\theta$
$dx\,dy = r\,dr\,d\theta$

$$\int_0^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} \, d\theta \, dr$$

$$\int_0^{\pi/4} \left[\int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} \, dr \right] d\theta$$

$$\int_0^{\pi/4} \frac{1}{2} \left[\frac{-1}{(1+r^2)} \right]_0^{\sqrt{\cos 2\theta}} d\theta$$

$\left[\int \frac{1}{x^2} dx = \frac{-1}{x} \right]$

$$\frac{1}{2} \int_0^{\pi/4} \left[-\frac{1}{1+(\sqrt{\cos 2\theta})^{\cancel{2}}} + 1 \right] d\theta$$

$$\frac{1}{2} \int_0^{\pi/4} \left[-\frac{1}{1+\cos 2\theta} + 1 \right] d\theta$$

$\left[\cos 2\theta = 2\cos^2\theta - 1 \right]$
$\left[\frac{1}{\cos\theta} = \sec\theta \right]$

$$\frac{1}{2} \int_0^{\pi/4} \left[-\frac{1}{\cancel{1}+2\cos^2\theta \cancel{-1}} + 1 \right] d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} \left(1 - \frac{1}{2}\sec^2\theta\right) d\theta.$$

$$= \frac{1}{2}\left[\theta - \frac{1}{2}\tan\theta\right]_0^{\pi/4}$$

$\boxed{x \to \theta}$

$\left[\int 1\, d\theta = \theta\right]$

$\left[\int \sec^2 x = \tan x\right]$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\tan\frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}(1)\right]$$

$$= \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8} \;//.$$

(2) Evaluate $\int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a} r^2\, d\theta\, dr.$

Sol:

$$\int_0^{\pi/2}\left[\int_{a(1-\cos\theta)}^{a} r^2\, dr\right] d\theta$$

$$\int_0^{\pi/2}\left[\frac{r^3}{3}\right]_{a(1-\cos\theta)}^{a} d\theta$$

$\left[\int x^n = \frac{x^{n+1}}{n+1}\right]$

$$\frac{1}{3}\int_0^{\pi/2}\left[a^3 - a^3(1-\cos\theta)^3\right] d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi/2} 1-(1-\cos\theta)^3 \, d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi/2} 1-\left[1-\cos^3\theta-3\cos\theta+3\cos^2\theta\right] d\theta$$

$$\left[(a-b)^3 = a^3-b^3-3a^2b+3ab^2\right]$$

$$= \frac{a^3}{3}\int_0^{\pi/2} \left(\cancel{1}-\cancel{1}+\cos^3\theta+3\cos\theta-3\cos^2\theta\right) d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi/2} \left(3\cos\theta-3\cos^2\theta+\cos^3\theta\right) d\theta$$

$$= \frac{a^3}{3}\left[3\left[\sin\theta\right]_0^{\pi/2} - 3\int_0^{\pi/2}\cos^2\theta \, d\theta + \int_0^{\pi/2}\cos^3\theta \, d\theta\right]$$

$$n = \text{Even} = \int_0^{\pi/2}\cos^n\theta \, d\theta = \frac{n-1}{2}\cdot\cancel{\frac{n-3}{4}}\cdot\frac{\pi}{2}$$

$$n = \text{odd} = \int_0^{\pi/2}\cos^n\theta \, d\theta = \frac{n-1}{n}\cdots 1$$

$$= \frac{a^3}{3}\left[3\left[\sin\frac{\pi}{2}-\sin 0\right] - 3\ \frac{1}{2}\cdot\frac{\pi}{2} + \frac{2}{3}\cdot 1\right]$$

$$\frac{a^3}{3}\left[\frac{3}{1}-\frac{3\pi}{4}+\frac{2}{3}\right] = \frac{a^3}{3}\left[\frac{36-9\pi+8}{12}\right]$$

$$= \frac{a^3}{36}\left(44-9\pi\right)$$

③ Evaluate $\iint_R r^3\, dr\, d\theta$ where R is the region between two circle $r = 2\sin\theta$, $r = 4\sin\theta$.

Sol:

θ varies from 0 to π.

r varies from $2\sin\theta$ to $4\sin\theta$.

$$\therefore \iint_R r^3\, dr\, d\theta = \int_0^{\pi} \int_{2\sin\theta}^{4\sin\theta} r^3\, dr\, d\theta.$$

$$= \int_0^{\pi} \left[\int_{2\sin\theta}^{4\sin\theta} r^3\, dr \right] d\theta$$

$$= \int_0^{\pi} \left[\frac{r^4}{4} \right]_{2\sin\theta}^{4\sin\theta} d\theta \quad \ldots$$

$$= \frac{1}{4} \int_0^{\pi} (4\sin\theta)^4 - (2\sin\theta)^4\, d\theta$$

$$= \frac{1}{4} \int_0^{\pi} 256\sin^4\theta - 16\sin^4\theta\, d\theta$$

$$= \frac{1}{\cancel{4}} \int_0^{\pi} \overset{60}{\cancel{240}} \sin^4\theta\, d\theta$$

$$= 60 \int_0^{\pi} \sin^4\theta\, d\theta$$

$$= 60 \times 2 \int_0^{\pi/2} \sin^4\theta \, d\theta$$

$$\int_0^{\pi} f(x)\,dx = 2 \cdot \int_0^{\pi/2} f(x)\,dx$$

$$= 120 \int_0^{\pi/2} \sin^4\theta \, d\theta$$

$$\int_0^{\pi/2} \sin^n\theta \, d\theta = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{\pi}{2}.$$

$$= 120 \times \frac{4-1}{4} \times \frac{4-3}{2} \times \frac{\pi}{2}$$

$$= \overset{15}{\cancel{\overset{30}{\cancel{120}}}} \times \frac{3}{\cancel{4}} \times \frac{1}{\cancel{2}} \times \frac{\pi}{2} = \frac{45\pi}{2} \quad //.$$

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UNIT-1

MATHS-6B

CLASS-6

① Change the order of Integration and evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dx\,dy$.

Ans:-

Actual limits:

$x = 0$ to $4a$

$y = \frac{x^2}{4a}$ to $2\sqrt{ax}$

$\therefore$ If $x=0 \Rightarrow y = \frac{0^2}{4a} = 0$

$y = 2\sqrt{a\times 0} = 0$

If $x = 4a \Rightarrow y = \frac{(4a)^2}{4a} = \frac{4a\times\cancel{4a}}{\cancel{4a}} = 4a$

$y = 2\sqrt{a\times 4a} = 4\sqrt{a^2} = 4a$.

Also, $y = \frac{x^2}{4a} \Rightarrow x^2 = 4ay$

$x = \sqrt{4ay}$

$\boxed{x = 2\sqrt{ay}}$ (Max)

$y = 2\sqrt{ax} \Rightarrow y^2 = 4ax$

$\boxed{x = \frac{y^2}{4a}}$ (Min)

$$\int_0^{4a}\int_{y^2/4a}^{2\sqrt{ay}} dy\,dx = \int_0^{4a}\left[\int_{y^2/4a}^{2\sqrt{ay}} dx\right] dy$$

$$= \int_0^{4a} \left[x\right]_{y^2/4a}^{2\sqrt{ay}} dy$$

$$= \int_0^{4a} \left(2\sqrt{ay} - \frac{y^2}{4a}\right) dy$$

$$\boxed{\int y^{1/2} \rightarrow \frac{y^{3/2}}{3/2}}$$

$$\boxed{y^n \rightarrow \frac{y^{n+1}}{n+1}}$$

$$= 2\sqrt{a}\left[\frac{y^{3/2}}{3/2}\right]_0^{4a} - \frac{1}{4a}\left[\frac{y^3}{3}\right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3}\left[(4a)^{3/2}\right] - \frac{1}{4a}\left(\frac{(4a)^3}{3}\right)$$

$$= \frac{4\sqrt{a}}{3}\left(2^{\cancel{2}\times\frac{3}{2}} a^{3/2}\right) - \frac{1}{\cancel{4a}}\left(\frac{\overset{16}{\cancel{64}} a^{\overset{2}{\cancel{3}}}}{3}\right)$$

$$= \frac{32a^{1/2}a^{3/2}}{3} - \frac{16}{3}a^2$$

$$= \frac{32}{3}a^{\frac{1}{2}+\frac{3}{2}} - \frac{16}{3}a^2$$

$$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{32a^2-16a^2}{3}$$

$$= \frac{16a^2}{3}$$

② Change into the polar Co-ordinates and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\,dx\,dy$.

SOL:

Here, $f(x,y) = e^{-(x^2+y^2)}$.

And the region bounded by $y=0, y=\sqrt{a^2-x^2}$ & $x=0, x=a$.

Sub. $x = r\cos\theta, y = r\sin\theta, dx\,dy = r\,dr\,d\theta$

$$\therefore f(x,y) = e^{-(r^2\cos^2\theta + r^2\sin^2\theta)}$$

$$= e^{-r^2[\cos^2\theta + \sin^2\theta]}$$

$$= e^{-r^2(1)} = e^{-r^2}.$$

The Curve $y = \sqrt{a^2-x^2}$

$$y^2 = a^2-x^2$$

$$x^2+y^2 = a^2$$

$$(r\cos\theta)^2 + (r\sin\theta)^2 = a^2$$

$$r^2 = a^2 \Rightarrow \boxed{r=a}$$

Also, θ = varies from $0°$ to $90°$.

Thus,
$$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\, dx\, dy$$

$$= \int_0^{\pi/2} \int_0^a e^{-r^2}\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \left[\int_0^a e^{-r^2}\, r\, dr\, d\theta \right]$$

Let $r^2 = t$

$2r\,dr = dt$

$r\,dr = \frac{dt}{2}$

$$= \int_0^{\pi/2} \left[\int_0^a e^{-t} \cdot \frac{dt}{2} \right] d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \left[\int_0^a e^{-t}\, dt \right] d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \left[\frac{e^{-t}}{-1} \right]_0^a d\theta$$

$$= -\frac{1}{2} \int_0^{\pi/2} \left[e^{-r^2} \right]_0^a d\theta.$$

$$\cancel{-\frac{1}{2}\int_0^{\pi/2}\left[e^{-a^2}-e^{-0}\right]_0^a d\theta}.$$

$$= -\frac{1}{2}\int_0^{\pi/2}\left(e^{-a^2}-e^{0}\right) d\theta$$

$$= -\frac{1}{2}\int_0^{\pi/2}\left(e^{-a^2}-1\right) d\theta$$

$$= -\frac{1}{2}\left(e^{-a^2}-1\right)\int_0^{\pi/2} d\theta$$

$$= -\frac{1}{2}\left(e^{-a^2}-1\right)\left[\theta\right]_0^{\pi/2}$$

$$= -\frac{1}{2}\left(e^{-a^2}-1\right)\times\frac{\pi}{2}$$

$$= \frac{\pi}{4}\left(1-e^{-a^2}\right)$$