Vaibhav Degree College - koilkuntla

Maths Important questions for Sem VI final year

Maths Paper - VI

Unit - I Vector Space I

1. A is non-empty set W is a sub set of vector space V(F). W is a subspace of W if and only if $a \in F$, and $\alpha, \beta \in V \Rightarrow a\alpha + \beta = W$.
2. let V(F) be a vector space. A non-empty set $W \subseteq V$. The necessary and sufficient condition for W to be a sub set of V is $a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$
3. The Intersection of two sub spaces is also a subspace of V(F).
4. The linear span of any subset of a vector space V(F) is a subspace of V(F). (or) Prove that $L(S) = V$.
5. If W_1 and W_2 are two sub spaces in V(F). Then show that $W_1 + W_2$ is a sub space in V and $W_1 + W_2 = L(W_1 \cup W_2)$.
6. Show that the system of vectors $(1,3,2), (1,-7,-8), (2,1,-1)$ is L.D.
7. The union of two subspaces W_1, W_2 of V(F) is a sub space, if and only if, one is contained in other. $W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$

Unit - II (Basis and Dimensions)

1. Show that the vectors $(1,2,1), (2,1,0), (1,-1,2)$ form a basis of R^3.
2. Find the co-ordinates of $\alpha = (4,5,6)$ w.r.to the basis set $x = (1,1,1)$ $y = (-1,1,1)$ and $z = (1,0,-1)$.
3. Let W_1 and W_2 be two subspaces of a finite dimensional vectorspa V(F). Then P.T. $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.
4. Let W be a sub space of finite dimensional vector space V(F), then P.T. $\dim(V/W) = \dim V - \dim W$

Invariance Theory

5. let V(F) be the finite dimensional vector space. Then prove that any two bases of V have the same number of elements.
6. $W_1 = \{(a,b,c,d) : b - 2c + d = 0\}$, $W_2 = \{(a,b,c,d) : a = d, b = 2c\}$ Find the basis and dimensions of (i) W_1 (ii) W_2 (iii) $W_1 \cap W_2$ Hence find $\dim(W_1 + W_2)$.

Unit - III Linear Transformations

1. $T: V_3(R) \to V_2(R)$ is defined by $T(x,y,z) = (x-y,\ x-z)$. Show that T is a Linear transformation.

2. $T: R^3 \to R^2$ defined by $T(x,y,z) = (|x|, 0)$. S.T. T is Linear transformation.

3. Describe explicitly the L.T. $T: R^2 \to R^2$ such that $T(2,3) = (4,5)$, $T(1,0) = (0,0)$

4. Find $T(x,y,z)$ where $T: R^3 \to R$ is defined by $T(1,1,1) = 3$, $T(0,1,-2) = 1$, and $T(0,0,1) = -2$

5. Let $U(F)$ and $V(F)$ be two vector spaces and $T: U \to V$ is a L.T. Then prove that The null space $N(T)$ is a subspace of $U(F)$.

6. $T: U \to V$ be a L.T. then P.T. $R(T)$ is a subspace of $V(F)$.

7. State and prove Rank- Nullity Theorem.

8. $T: R^3 \to R^2$ be a L.T. defined by $T(x,y,z) = (x+y,\ y+z)$. Find basis dimension of range and Null space of T.

9. The linear mapping $T: R^3 \to R$ is defined by $T(x,y,z) = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta,\ z)$. Show that T is non-singular.

10. $T: R^3 \to R^3$ is defined by $T(x,y,z) = (2x,\ 4x-y,\ 2x+3y-z)$. Find T^{-1}.

11. If α, β, γ are the Linearly Independent vectors of $V(F)$. Show that $\alpha+\beta$, $\beta+\gamma$, $\gamma+\alpha$ are also L.I.

12. State and prove Fundamental Theorem of Homomorphism.

13. Let $T: R^3 \to R^3$ be the Linear mapping defined by $T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$. Find the Rank, nullity and a basis for each of the range and null space of T.

14. $T: V_3(R) \to V_1(R)$ is defined by $T(a,b,c) = a^2+b^2+c^2$, can T be a L.T.?

Unit – IV :- Matrix

1. State and prove Cayley – Hamilton Theorem

2. Find the Eigen roots and the corresponding eigen vectors of the Matrix (i) $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (Model Imp)

3. Find the characteristic polynomial of the Matrix

$A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 6 \\ 3 & 1 & 4 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$

4. Verify Cayley Hamilton theorem and Hence find A^{-1} of the Matrix

$A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$

5. Find the Rank of the Matrix

$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$

Unit – V Inner product Space

1. State and prove (i) Triangle inequality (ii) Parallelogram law.
2. State and Prove : Cauchy – Schwarz's Inequality.
3. State and Prove : (i) Bessel's Inequality (ii) Parseval's Identity.
4. P.T. $S = \{(\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}), (\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}), (\frac{2}{3}, \frac{2}{3}, -\frac{1}{3})\}$ is an orthonormal set of V(F
5. Construct an orthonormal basis of R^3 using Gram- Schmidt Orthogonalization process from $\{(1,2,3), (2,0,1), (1,3,0)\}$

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Important Questions for Sem - IV

Sequences - Unit I

1. Prove that every convergent sequence is bounded.
2. A monotone sequence is convergent iff it is bounded.
3. Every Cauchy sequence is convergent.
4. Test the convergence of $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}$
5. P.T. $\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\cdots+\frac{1}{(n+n)^2}=0$.
6. P.T. $\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n^2+1}}+\frac{1}{\sqrt{2n^2+2}}+\cdots+\frac{1}{\sqrt{2n^2+n}}\right]=\frac{1}{\sqrt{2}}$
7. P.T. the sequence $S_n=\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}$ is convergent.
8. If $S_n=\sqrt{n+1}-\sqrt{n}$ then prove that $S_n=0$.
9. Prove that $\lim_{n\to\infty}\frac{\sin\left(\frac{n\pi}{3}\right)}{\sqrt{n}}=0$
10. If $S_n=2-\frac{1}{2^{n-1}}$ then P.T. $\{S_n\}$ is convergent.

Unit - II (Series)

1. State and prove Cauchy's n^{th} root test. and Hence Test for the convergence of $\sum n\cdot e^{-n^2}$
2. State and prove D'Alembert's ratio test.
3. State and prove comparision test on limits.
4. Test for convergence of the following.

(i) $\sum_{n=1}^{\infty}\sqrt{n^4+1}-n^2$ (ii) $\sum_{n=1}^{\infty}\sqrt{n^3+1}-\sqrt{n^3}$ (iii) $\sum_{n=1}^{\infty}\frac{2n-1}{n(n+1)(n+2)}$

(iv) $\sum_{n=1}^{\infty}\sqrt{n^2+1}-n$ (v) $\sum_{n=1}^{\infty}\frac{(n+1)!}{3^n}$

5. P.T. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{1}{3n}\right]=3/8$

Unit-III Limits & Continuity

1. f is continuous on $[a,b]$ then S.T. f is uniformly continuous on $[a,b]$
2. Discuss any three kinds of discontinuity with suitable examples.
3. State and prove Intermediate value Theorem.
4. If f is continuous on $[a,b]$ and $f(a), f(b)$ have opposite signs then there exist $c \in (a,b)$ such that $f(c)=0$.
5. If $f:[a,b] \to R$ is continuous on $[a,b]$ then f is bounded on $[a,b]$
6. Examine the continuity of the function $f(x) = |x| + |x-1|$ at $x = 0,1$
7. Determine the values of a,b,c so that the function f is defined by

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & ; \text{ if } x<0 \\ c & ; \text{ if } x=0 \\ \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & ; \text{ if } x>0. \end{cases}$$

8. Check the continuity of the following functions.

(i) $f(x) = x\sin(1/x)$ (ii) $f(x) = \dfrac{e^{1/x}}{1+e^{1/x}}$ (iii) $f(x) = \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ and $f(0)=1$ at $x=0$

Unit-IV Derivatives

1. P.T. derivable function is continuous. and S.T. $|x-1|$ is not derivable at $x=1$
2. If $f(x) = x\left(\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\right)$ if $x \neq 0$ and $f(0)=0$ S.T. f is not derivable at '0'.
3. S.T. $f(x) = |x-1| + |x-2|$ is continuous but not derivable at $x = 1,2$.
4. State and prove the following theorems

(i) Rolle's Theorem (ii) Lagrange's Theorem (iii) Cauchy's (iv) Taylor's Theorem.

5. (i) Discuss the applicability of Lagrange's Theorem for $f(x) = x(x-1)(x-2)$ on $[0, 1/2]$ (ii) Find 'c' of cauchy's Theorem $f(x) = \frac{1}{x^2}$ and $g(x) = 1/x$ on $[a,b]$
6. Discuss the applicability of Rolle's Theorem $f(x) = x^3 - 6x^2 + 11x - 6$ on $[1,3]$
7. Show that $\dfrac{v-u}{1+v^2} < \tan^{-1}v - \tan^{-1}u < \dfrac{v-u}{1+u^2}$ for $0<u<v$ Hence deduce that

$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

d 'c' of Cauchy's Theorem for $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ on $[a,b]$

· Using Taylor's Theorem, P.T. $x - \frac{x^3}{3!} \leq \sin x \leq x - \frac{x^3}{3!} + \frac{x^5}{5!}$ for $x \geq 0$.

10. P.T. $f(x) = x^2 \sin(\frac{1}{x})$, $x \neq 0$ and $f(0) = 0$ is derivable at origin.

Unit - II Riemann Integration

1. State and prove the fundamental Theorem of Integral calculus.
2. State and prove necessary and sufficient condition on Riemann Integral.
3. P.T. $f(x) = 3x+1$ is Integrable on $[1,2]$ and $\int_1^2 (3x+1)\,dx = 11/2$.
4. P.T. $f(x) = x^2$ is Integrable on $[0,a]$ and $\int_0^a x^2\,dx = a^3/3$.
5. If $f: [a,b] \to R$ is a bounded function, then P.T. $\underline{\int_a^b} f(x)\,dx \leq \overline{\int_a^b} f(x)\,dx$.
6. If $f \in [a,b]$ then P.T. $|f| \in R[a,b]$.
7. If $f: [a,b] \to R$ is monotonic on $[a,b]$ then P.T. f is Riemann Integral.
8. If $f(x) = x$ on $[0,1]$ and $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ find $U(P,f)$ and $L(P,f)$.
9. Evaluate: $\int_0^{\pi/4} (\sec^4 x - \tan^4 x)\,dx$.
10. Show that (i) $\int_a^b \cos x\,dx = \sin b - \sin a$ (ii) $\int_0^1 x^4\,dx = 1/5$ (iii) $\int_0^3 [x]\,dx = 3$.

By
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