MATHS - 6B

UNIT - 2

CLASS NO. 1

LONGS :-

① Using cylindrical Co-ordinates, find the volume of the sphere $x^2+y^2+z^2=a^2$.

SOL:

Let the equation of the sphere be $x^2+y^2+z^2=a^2$.

$\therefore$ Now, we will solve this problem using cylindrical co-ordinates

$x = r\cos\theta$, $y = r\sin\theta$

$\boxed{x^2+y^2=a^2}$ ↓ Circle ↓ $\boxed{0 \text{ to } a}$

$\therefore x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(1) = r^2$

$\therefore x^2+y^2+z^2=a^2$

$r^2+z^2=a^2$

$\therefore$ limits of z are : $\pm\sqrt{a^2-r^2}$

$\therefore \sqrt{a^2-x^2-y^2}$ & $-\sqrt{a^2-x^2-y^2}$

limits of r are : 0 to a.

limits of θ are: 0 to 2π.

$\therefore$ Req. Volume: $\iiint dx\, dy\, dz$

$$= \iiint r\, d\theta\, dr\, dz$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{a} \int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} r\, \underline{d\theta}\, dr\, dz$$

$$\boxed{\int_{-a}^{a} f(x) = 2\cdot\int_{0}^{a} f(x)}$$

$$= 2\int_{0}^{2\pi} d\theta \int_{0}^{a} r\, dr\, [z]_{0}^{\sqrt{a^2-r^2}}$$

$$= 2\int_{0}^{2\pi} d\theta \int_{0}^{a} r\, dr\, (a^2-r^2)^{1/2}$$

$$\boxed{f^n(x)\cdot f'(x) = \frac{f^{n+1}(x)}{n+1}}$$

$$= \frac{\cancel{2}}{-\cancel{2}}\int_{0}^{2\pi} d\theta \int_{0}^{a} (a^2-r^2)^{\frac{1}{2}} \cdot -2r\, dr$$

$$-\int_{0}^{2\pi} d\theta \left[\frac{(a^2-r^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_{0}^{a}$$

$$= -\int_0^{2\pi} d\theta \left[-(a^2)^{\frac{3}{2}} \cdot \frac{2}{3} \right]$$

$$= \frac{2}{3} a^3 \int_0^{2\pi} d\theta$$

$$= \frac{2}{3} a^3 \left[\theta \right]_0^{2\pi}$$

$$= \frac{2}{3} a^3 (2\pi - 0)$$

$$= \frac{4\pi a^3}{3} \quad //$$

Unit-2

MI-2

Triple Integrals

9581234096

MATHS-6B

UNIT-2

CLASS NO.2

LONGS :-

(2) Evaluate $\iiint xyz \, dx\,dy\,dz$ over the positive octant of sphere $x^2+y^2+z^2=a^2$.

SOL: Given sphere is $x^2+y^2+z^2=a^2$.

$$z = \sqrt{a^2-x^2-y^2}.$$

Projection of the sphere on the xy-plane is the circle $x^2+y^2=a^2$

$\therefore$ y varies from: 0 to $\sqrt{a^2-x^2}$.

$\therefore$ x Varies from: 0 to a.

$$\therefore \iiint xyz \, dx\,dy\,dz$$

$$= \int_0^a x\,dx \int_0^{\sqrt{a^2-x^2}} y\,dy \int_0^{\sqrt{a^2-x^2-y^2}} z\,dz$$

$$= \int_0^a x\,dx \int_0^{\sqrt{a^2-x^2}} y\,dy \left[\frac{z^2}{2}\right]_0^{\sqrt{a^2-x^2-y^2}}$$

$$= \int_0^a x\,dx \int_0^{\sqrt{a^2-x^2}} y\,dy \cdot \left[\frac{a^2-x^2-y^2}{2}\right]$$

$$= \frac{1}{2} \int_0^a x\,dx \int_0^{a^2-x^2} \left(ya^2 - x^2y - y^3\right) dy$$

$$= \frac{1}{2} \int_0^a x\,dx \left[\frac{a^2y^2}{2} - x^2\frac{y^2}{2} - \frac{y^4}{4}\right]_0^{\sqrt{a^2-x^2}}$$

$$= \frac{1}{2} \int_0^a x\,dx \left[\frac{a^2}{2}\left(\sqrt{a^2-x^2}\right)^2 - \frac{x^2}{2}\left(\sqrt{a^2-x^2}\right)^2 - \frac{\left(\sqrt{a^2-x^2}\right)^4}{4}\right]$$

$$= \frac{1}{2} \int_0^a x\,dx \left[\frac{a^2}{2}(a^2-x^2) - \frac{x^2}{2}(a^2-x^2) - \frac{(a^2-x^2)^2}{4}\right]$$

$$= \frac{1}{2} \int_0^a x\,dx \left[(a^2-x^2)\left[\frac{a^2}{2} - \frac{x^2}{2} - \frac{a^2-x^2}{4}\right]\right]$$

$$= \frac{1}{2} \int_0^a x\,dx \left[(a^2-x^2)\left[\frac{(a^2-x^2)}{2} - \frac{(a^2-x^2)}{4}\right]\right]$$

$$= \frac{1}{2} \int_0^a x\,dx \left[(a^2-x^2)\left[\frac{2a^2-2x^2-a^2+x^2}{4}\right]\right]$$

$$= \frac{1}{2}\int_0^a x\,dx \left[(a^2-x^2)\left(\frac{a^2-x^2}{4}\right)\right]$$

$$= \frac{1}{8}\int_0^a x\,dx\,(a^2-x^2)^2.$$

$$= \frac{1}{8}\int_0^a (a^4+x^4-2a^2x^2)\;x\,dx$$

$$= \frac{1}{8}\int_0^a \left[a^4x+x^5-2a^2x^3\right]dx$$

$$= \frac{1}{8}\left[a^4\cdot\frac{x^2}{2}+\frac{x^6}{6}-2a^2\cdot\frac{x^4}{4}\right]_0^a$$

$$= \frac{1}{8}\left[\frac{a^6}{2}+\frac{a^6}{6}-\frac{2a^6}{4}\right]$$

$$= \frac{1}{8}\left[\frac{\cancel{6a^6}+2a^6-\cancel{6a^6}}{12}\right]$$

$$= \frac{1}{8}\left[\frac{\cancel{2}a^6}{\cancel{12}_{6}}\right] = \frac{a^6}{48}$$

$\boxed{9581234096}$

MATHS-6B

UNIT-2

CLASS NO.3

LONGS :-

① Find the volume bounded by the ellipse parabolids $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

Sol: Given Two Surfaces :

$$x^2 + 3y^2 = z = 8 - x^2 - y^2$$

$$\Rightarrow x^2 + 3y^2 = 8 - x^2 - y^2$$

$$\Rightarrow 2x^2 + 4y^2 = 8$$

$$\Rightarrow x^2 + 2y^2 = 4.$$

z : from x^2+3y^2 to $8-x^2-y^2$

y : $2y^2 = 4 - x^2$

$$y^2 = \frac{4-x^2}{2}$$

$$y = \pm\sqrt{\frac{4-x^2}{2}}$$

x : -2 to 2.

$\boxed{9581234096}$

$\therefore$ Required volume:

$$\int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}}\int_{x^2+3y^2}^{8-x^2-y^2} dx\, dy\, dz$$

$$= \int_{-2}^{2} dx \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dy \left[z\right]_{x^2+3y^2}^{8-x^2-y^2}$$

$$= \int_{-2}^{2} dx \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dy \left[8-x^2-y^2-x^2-3y^2\right]$$

$$= \int_{-2}^{2} dx \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dy \left(8-2x^2-4y^2\right)$$

$$= 2\int_{-2}^{2} dx \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dy \left(4-x^2-2y^2\right)$$

$$\boxed{\int_{-a}^{+a} f(x) = 2\int_{0}^{a} f(x)}$$

$$= 2\cdot 2 \int_{-2}^{2} dx \int_{0}^{\sqrt{\frac{4-x^2}{2}}} dy \left(4 - x^2 - 2y^2\right)$$

$$= 4 \int_{-2}^{2} dx \left[4y - x^2 \cdot y - 2\frac{y^3}{3} \right]_{0}^{\sqrt{\frac{4-x^2}{2}}}$$

$$= 4 \int_{-2}^{2} dx \left[(4-x^2) y - \frac{2y^3}{3} \right]_{0}^{\sqrt{\frac{4-x^2}{2}}}$$

$$= 4 \int_{-2}^{2} dx \left[(4-x^2)^1 \frac{(4-x^2)^{\frac{1}{2}}}{\sqrt{2}} - \frac{2}{3}\left[\sqrt{\frac{4-x^2}{2}}\right]^3 \right]$$

$$= 4 \int_{-2}^{2} dx \left[\frac{1}{\sqrt{2}} (4-x^2)^{\frac{3}{2}} - \frac{2}{3\cdot 2^{3/2}} (4-x^2)^{3/2} \right]$$

$$= 4 \int_{-2}^{2} dx \left[\frac{1}{\sqrt{2}} (4-x^2)^{3/2} - \frac{\cancel{2}}{3\cdot \cancel{2}\sqrt{2}} (4-x^2)^{3/2} \right]$$

$$= \frac{4}{\sqrt{2}} \int_{-2}^{2} dx \left[(4-x^2)^{\frac{3}{2}} - \frac{1}{3} (4-x^2)^{3/2} \right]$$

$$= \frac{4}{\sqrt{2}} \int_{-2}^{2} dx \left[(4-x^2)^{\frac{3}{2}} \left(1 - \frac{1}{3}\right) \right]$$

$$= \frac{4}{\sqrt{2}} \int_{-2}^{2} dx \left[(4-x^2)^{3/2} \times \frac{2}{3} \right]$$

$$= \frac{8}{3\sqrt{2}} \int_{-2}^{2} dx \; (4-x^2)^{3/2} .$$

$$= \frac{8}{3\sqrt{2}} \times 2 \int_{0}^{2} (4-x^2)^{\frac{3}{2}} \, dx .$$

$$= \frac{16}{3\sqrt{2}} \int_{0}^{2} (4-x^2)^{3/2} \, dx$$

Put $x = 2\sin\theta$

$$1 = 2\cos\theta \cdot \frac{d\theta}{dx}$$

$$dx = 2\cos\theta \, d\theta .$$

$$= \frac{16}{3\sqrt{2}} \int_{0}^{\pi/2} (4 - 4\sin^2\theta)^{\frac{3}{2}} . 2\cos\theta \, d\theta$$

$$= \frac{16 \times 2}{3\sqrt{2}} \int_{0}^{\pi/2} \left[4(1-\sin^2\theta)\right]^{3/2} . \cos\theta \, d\theta$$

$$\frac{32}{3\sqrt{2}} \int_0^{\pi/2} \left[2^{\cancel{2}}\right]^{\frac{3}{\cancel{2}}} \cdot \left[\cos^{\cancel{2}}\theta\right]^{\frac{3}{\cancel{2}}} \cos\theta \, d\theta$$

$$\frac{32}{3\sqrt{2}} \int_0^{\pi/2} 8 \cdot \cos^3\theta \cdot \cos\theta \, d\theta$$

$$= \frac{32 \times 8}{3\sqrt{2}} \int_0^{\pi/2} \cos^4\theta \, d\theta.$$

$$= \frac{256}{3\sqrt{2}} \int_0^{\pi/2} \cos^4\theta \, d\theta$$

$$\boxed{\int_0^{\pi/2} \cos^n\theta = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{\pi}{2}}$$

$$= \frac{256}{3\sqrt{2}} \cdot \frac{4-1}{4} \cdot \frac{4-3}{2} \cdot \frac{\pi}{2}$$

$$= \frac{\overset{16}{\cancel{32}}\ \overset{\cancel{64}}{\cancel{256}}}{\cancel{3}\sqrt{2}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{1}{\cancel{2}} \times \frac{\pi}{\cancel{2}}$$

$$= \frac{16\pi}{\sqrt{2}} = \frac{8 \times 2 \times \pi}{\sqrt{2}}$$

$$= \frac{8 \times \sqrt{2} \times \cancel{\sqrt{2}} \times \pi}{\cancel{\sqrt{2}}} = \underline{\underline{8\sqrt{2}\,\pi}}.$$

Scanned with CamScanner

MATHS - 6B

UNIT-2

SHORTS

① Evaluate $\int_0^1 \int_0^1 \int_0^1 dx\, dy\, dz$

SOL:

$$\int_0^1 \int_0^1 \int_0^1 dx\, dy\, dz = \int_0^1 dx \int_0^1 dy \,[z]_0^1$$

$$= \int_0^1 dx \int_0^1 dy\,(1-0)$$

$$= \int_0^1 dx \int_0^1 dy = \int_0^1 dx\,[y]_0^1$$

$$= \int_0^1 dx\,(1-0)$$

$$= \int_0^1 dx$$

$$= (x]_0^1$$

$$= 1-0 = 1$$

② Evaluate $\int_0^1 dx \int_0^2 dy \int_1^2 x^2yz\,dz$

SOL:

$$\int_0^1 dx \int_0^2 dy \left[x^2y\frac{z^2}{2}\right]_1^2$$

$$= \int_0^1 dx \int_0^2 dy \left[x^2y\frac{2^2}{2} - x^2y\cdot\frac{1^2}{2}\right]$$

$$= \int_0^1 dx \int_0^2 dy \left[2x^2y - \frac{x^2y}{2}\right]$$

$$= \int_0^1 dx \int_0^2 dy \ \frac{3x^2y}{2}.$$

$$= \frac{3}{2}\int_0^1 dx \int_0^2 x^2y\,dy$$

$$= \frac{3}{2}\int_0^1 dx \left[x^2\cdot\frac{y^2}{2}\right]_0^2$$

$$= \frac{3}{2}\int_0^1 dx \left(x^2\cdot\frac{4}{\cancel{2}} - 0\right) = \frac{3}{\cancel{2}}\int_0^1 \cancel{2}x^2\,dx$$

$$= 3\int_0^1 x^2\,dx = \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_0^1 = 1^3 - 0^3 = 1$$

③ Evaluate the triple integral

$$\int_0^1 \int_y^1 \int_0^{1-x} x \; dz\, dx\, dy.$$

SOL: Given,

$$\int_0^1 \int_y^1 \int_0^{1-x} x \, dz\, dx\, dy.$$

$$\int_0^1 x\, dx \int_y^1 dy \, [z]_0^{1-x}$$

$$\int_0^1 x\, dx \int_y^1 (1-x-0)$$

$$\int_0^1 \int_y^1 x - x^2 \; dx \, . dy$$

$$\int_0^1 \cancel{dx} \left[\int_y^1 (x-x^2)\, dx \right] dy$$

$$\int_0^1 \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_y^1 dy$$

$$= \int_0^1 \left[\left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{y^2}{2} - \frac{y^3}{3}\right) \right] d\cancel{x}y$$

$$= \int_0^1 \left[\left(\frac{3-2}{6}\right) - \left(\frac{3y^2 - 2y^3}{6}\right) \right] dy$$

$$= \int_0^1 \left[\frac{1}{6} - \left(\frac{3y^2 - 2y^3}{6}\right) \right] d\cancel{x}y$$

$$\cancel{= \left[\frac{1}{3}x - \left(\frac{3y^2 - 2y^3}{6}\right) \cdot x \right]_0^1}$$

$$\cancel{= \frac{1}{3}(1) - \frac{3y^2 - 2y^3}{6} \cdot 1}$$

$$\cancel{= \frac{1}{3} - \frac{3y^2 - 2y^3}{6} = \frac{2 - 3y^2 + 2y^3}{6}}$$

$$= \left[\frac{1}{6}y - \frac{3}{6}\frac{y^3}{3} + \frac{2}{6} \cdot \frac{y^4}{4} \right]_0^1$$

$$= \frac{1}{6}(1) - \frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{4} \qquad \frac{1}{12}$$

$$= \frac{1}{6} - \frac{1}{6} + \frac{1}{12} = \frac{\cancel{2} - \cancel{2} + 1}{12} = \frac{1}{12}$$

(4) Evaluate $\int_0^1 \int_0^1 \int_0^y xyz \, dx \, dy \, dz$.

SOL:

$$\int_0^1 \int_0^1 \int_0^y xyz \, dx \, dy \, dz$$

$$\int_0^1 x\,dx \int_0^1 y \, dz \left[\frac{z^2}{2}\right]_0^y$$

$$= \int_0^1 x \, dx \int_0^1 y \, dy \left(\frac{y^2}{2} - 0\right)$$

$$= \int_0^1 x \, dx \int_0^1 y \, dy \, \frac{y^2}{2}$$

$$= \frac{1}{2} \int_0^1 x \, dx \int_0^1 y^3 \, dy$$

$$= \frac{1}{2} \int_0^1 x \, dx \left[\frac{y^4}{4}\right]_0^1$$

$$= \frac{1}{8} \int_0^1 x \, dx (1-0)$$

$$= \frac{1}{8} \int_0^1 x \, dx$$

$$= \frac{1}{8} \left[\frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{8 \times 2} \left(1^2 - 0^2 \right)$$

$$= \frac{1}{16} \quad // ..$$

Competition

2m

PDF

Study

MATHS - 6B

↓

MI & Ap. VC

↓

Unit 1 → 1, 2, 3, 4, 5, 6

One Link — Teachmint — ST4U

Login → ST4U → 199/-.

6B — Download ↓ — 9581234096

Unit - 2 → One link

OLD — Unit 3, 4, 5 → One line

Open [illegible] → name [illegible]