SEMESTER-3

MATHEMATICS

TOP MOST IMP LONGS

* UNIT-1 [GROUPS] :-

(1) Prove that the set of all +ve rational numbers forms an abelian group with respect to the binary operation "o" defined as $a \circ b = \frac{ab}{3}\ \forall\, a,b \in Q^+$.

(2) Prove that the set of integers Z forms an abelian group w.r.t. the operation $*$ defined by $a*b = a+b+2\ \forall\, a,b \in Z$.

(3) P.T. the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\ \forall\, \alpha \in \mathbb{R}$ forms a group w.r.t. matrix multiplication if $\cos\theta = \cos\phi \Rightarrow \theta = \phi$.

(4) In a group G, for $a,b,x,y \in G$ then prove that the equations $ax=b$; $ya=b$ have unique solution.

(5) P.T. n^{th} roots of unity forms an abelian group.

UNIT - 2

MATHS - SEM 3

LONGS

* THEOREM - 1:

If H_1 and H_2 are two subgroups of group G then $H_1 \cup H_2$ is a subgroup iff $H_1 \subseteq H_2$ (or) $H_2 \subseteq H_1$

THEOREM - 2:

A non Empty complex H of a group G is subgroup if and only if

(i) $a \in H, b \in H \Rightarrow ab \in H$ (ii) $a \in H, a^{-1} \in H$.

THEOREM - II:

If H and K are two subgroups of a group G then HK is a subgroup iff $HK = KH$.

THEOREM - 4: (10M) & (5M)

Any two left cosets (right) of a subgroup of a group are disjoint or identical.

THEOREM - 5:

state and prove Lagrange's theorem on groups

[or] (Pdt unit II · long Q. No. 5)

statement: The order of a subgroup of a finite group divides the order of a group.

UNIT - 3

MATHS - SEM - 3

Theorem - ①

① A subgroup H of a group G is Normal iff each left coset of H in G is a right coset of H in G.

Theorem - ②

A subgroup H of group G is Normal iff the product of two right coset of H in G is a right coset of H in G.

Theorem - ③

If f is a homomorphism from a group G into 'G then prove that kernal of homomorphism is a normal subgroup.

Theorem - ⑦ *** (10M)

state and prove fundamental theorem of Homomorphism of Groups.

statement: Every Homomorphic Image of a group G is Isomorphic to its quotient group.

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UNIT- 4
MATHEMATICS
SEM - 3
LONGS

① Statement : Every finite group G is Isomorphic to its permutation group.

Theorem - ②

Prove that a group of prime order is cyclic (or) if P is a prime number then group of order 'P' is cyclic.

Theorem - ③

Prove that Every subgroup of cyclic group is cyclic.

UNIT - 5

MATHS - SEM - 3

LONGES

Theorem - 1:

If R is a Boolean ring then

(i) $a+a=0 \; \forall \; a \in R$ (ii) $a+b=0 \Rightarrow a=b$

(iii) R is Commutative under multiplication

(or)

Every boolean ring is abelian.

Theorem - 3:

Every field is an integral domain.

Theorem - 4:

Every finite integral domain is a field.

Theorem - 5:

The characteristics of an Integral domain Either prime (or) Zero.

1) Prove that $Q[\sqrt{2}] = \{a+b\sqrt{2} \,/\, a,b \in Q\}$ is a field with respect to ordinary and multiplication of Numbers.