UNIT-3

MATHS-7B

CLASS NO. 1

Inverse Laplace Transforms

Formula's :-

$L(1) = \frac{1}{p}$

$1 = L^{-1}\left[\frac{1}{p}\right]$

$\boxed{L(e^{at}) = \frac{1}{p-a}}$

1) $L^{-1}\left[\frac{1}{p}\right] = 1$

2) $L^{-1}\left[\frac{1}{p-a}\right] = e^{at}$

3) $L^{-1}\left[\frac{1}{p+a}\right] = e^{-at}$

4) $L^{-1}\left(\frac{p}{p^2+a^2}\right) = \cos at$

5) $L^{-1}\left(\frac{a}{p^2+a^2}\right) = \sin at$

6) $L^{-1}\left(\frac{a}{p^2-a^2}\right) = \sinh at$

7) $L^{-1}\left(\frac{p}{p^2-a^2}\right) = \cosh at$

8) $L^{-1}\left(\frac{n!}{p^{n+1}}\right) = t^n \quad \forall\, n = 1, 2, 3, \ldots$

9) $L^{-1}\left[\frac{p-a}{(p-a)^2+b^2}\right] = e^{at}\cdot\cos bt$

10) $L^{-1}\left[\frac{p-a}{(p-a)^2-b^2}\right] = e^{at}\cosh bt$

11) $L^{-1}\left[\frac{n!}{(p-a)^{n+1}}\right] = e^{at}\,t^n$

12) $L^{-1}\left[\frac{b}{(p-a)^2+b^2}\right] = e^{at}\sin bt$

13) $L^{-1}\left[\frac{b}{(p-a)^2-b^2}\right] = e^{at}\sinh bt$

14) $L^{-1}\left[\frac{1}{(p+a)^n}\right] = e^{-at}\,L^{-1}\left[\frac{1}{p^n}\right]$

15) $L^{-1}\,\frac{p}{(p^2+a^2)^2} = \frac{t}{2a}\sin at$

$L^{-1}\,\frac{p}{(p^2-a^2)^2} = \frac{t}{2a}\sinh at.$

→ Partial Fractions :-

→ $\dfrac{a+b}{2} = \dfrac{a}{2} + \dfrac{b}{2}$ ✓

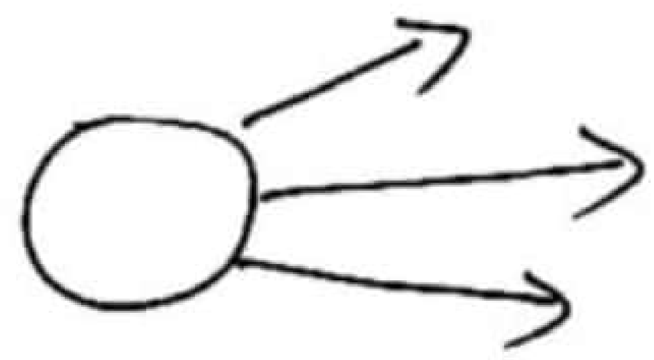
→ $\dfrac{2}{a+b} = \ ?$

① → $\dfrac{5}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$.

② → $\dfrac{x^2+2x}{(x-1)(x+1)^3} = \dfrac{A}{x-1} + \dfrac{B}{(x+1)^1} + \dfrac{C}{(x+1)^2} + \dfrac{D}{(x+1)^3}$

③ → $\dfrac{x+5}{(x+2)(x^2+4)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$

power → [only → x]



Idealogy: ① $\dfrac{1}{p^2+10p+26}$

$p^2 + 2 \times p \times 5 + 25 + 1$

$(p+5)^2 + 1$

② $(p+3)^2 = \left\{p\left[1+\dfrac{3}{p}\right]\right\}^2$

PROBLEMS :-

① Evaluate $L^{-1}\left[\frac{(s+1)}{s^2+6s+25}\right]$.

SOL:

$$L^{-1}\left[\frac{s+1}{s^2+6s+25}\right]$$

$$= L^{-1}\left[\frac{s+1}{s^2+2\cdot s\cdot 3+3^2+16}\right]$$

$$= L^{-1}\left[\frac{s+1}{(s+3)^2+16}\right]$$

$$= L^{-1}\left[\frac{(s+3)-2}{(s+3)^2+16}\right]$$

$$= L^{-1}\left[\frac{s+3}{(s+3)^2+16}\right] - 2L^{-1}\left[\frac{1}{(s+3)^2+16}\right]$$

$$= L^{-1}\left[\frac{s+3}{(s+3)^2+4^2}\right] - 2L^{-1}\left[\frac{1}{(s+3)^2+4^2}\right]$$

$$= e^{-3t}\cdot \cos 4t - 2\cdot e^{-3t}\sin 4t$$

$$= e^{-3t}\left[\cos 4t - 2\sin 4t\right]$$

$\left[\because L^{-1}\left[\frac{p-a}{(p-a)^2+b^2}\right] = e^{at}\cos bt\right]$

$\left[\because L^{-1}\frac{b}{(p-a)^2+b^2} = e^{at}\sin bt\right]$

Pdf → TM ✓ (5)

videos → Rec.

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$$\frac{a-b}{2} = \frac{a}{2} - \frac{b}{2}$$

② Find the Laplace Inverse of $\frac{s+3}{s^2-4s+13}$.

SOL:

$$L^{-1}\left[\frac{s+3}{s^2-4s+13}\right]$$

$$= L^{-1}\left[\frac{s+3}{s^2-2\cdot s\cdot 2+2^2+9}\right]$$

$$= L^{-1}\left[\frac{s+3}{(s-2)^2+3^2}\right]$$

$$= L^{-1}\left[\frac{(s-2)+5}{(s-2)^2+3^2}\right]$$

$$= L^{-1}\left[\frac{s-2}{(s-2)^2+3^2}\right] + L^{-1}\left[\frac{5}{(s-2)^2+3^2}\right]$$

$$= e^{2t}\cos 3t + 5e^{2t}\sin 3t$$

$$= e^{2t}\left[\cos 3t + 5\sin 3t\right].$$

MATHS-7B

UNIT-3

CLASS NO. 2

① Find the inverse Laplace transforms of (i) $\frac{s^2}{(s-2)^3}$ (ii) $\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9}$.

SOL:

(i) $$\frac{s^2}{(s-2)^3} = \frac{A}{(s-2)^1} + \frac{B}{(s-2)^2} + \frac{C}{(s-2)^3}$$

$$\frac{s^2}{\cancel{(s-2)^3}} = \frac{A(s-2)^2 + B(s-2) + C}{\cancel{(s-2)^3}}$$

$$s^2 = A(s-2)^2 + B(s-2) + C$$

$$s^2 = As^2 + 4A - 4As + Bs - 2B + C$$

Put $s = 2$;

$$4 = A(0)^2 + B(0) + C$$

$$\boxed{C = 4}$$

Compare the coff. of s^2

$$\boxed{1 = A}$$

Compare the coff. of s

$$0 = -4A + B$$

$$0 = -4(1) + B = \boxed{B = 4}$$

$$L^{-1}\left[\frac{s^2}{(s-2)^3}\right] = L^{-1}\left[\frac{1}{s-2} + \frac{4}{(s-2)^2} + \frac{4}{(s-2)^3}\right]$$

$$= L^{-1}\left(\frac{1}{s-2}\right) + L^{-1}\left(\frac{4}{(s-2)^2}\right) + L^{-1}\left(\frac{4}{(s-2)^3}\right)$$

$$= e^{2t} + 4.e^{2t}t^1 + 4.e^{2t}t^2$$

$$= e^{2t} + 4te^{2t} + 4t^2e^{2t} \ //..$$

iii)

$$L^{-1}\left[\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9}\right]$$

$$= L^{-1}\left[\frac{6}{2s-3}\right] - L^{-1}\left[\frac{3+4s}{9s^2-16}\right] + L^{-1}\left[\frac{8-6s}{16s^2+9}\right]$$

$$= \cancel{\frac{6}{2}}\, L^{-1}\left[\frac{1}{s-\frac{3}{2}}\right] - \frac{1}{9}L^{-1}\left[\frac{3+4s}{s^2-\frac{16}{9}}\right] + \frac{1}{16}L^{-1}\left[\frac{8-6s}{s^2+\frac{9}{16}}\right]$$

$$= 3\,L^{-1}\left[\frac{1}{s-\frac{3}{2}}\right] - \frac{1}{9}L^{-1}\left[\frac{3}{s^2-\frac{16}{9}}\right] - \frac{1}{9}L^{-1}\left[\frac{4s}{s^2-\frac{16}{9}}\right]$$

$$+ \frac{1}{16}L^{-1}\left[\frac{8}{s^2+\frac{9}{16}}\right] + \frac{1}{16}L^{-1}\left[\frac{-6s}{s^2+\frac{9}{16}}\right]$$

$$= 3L^{-1}\left[\frac{1}{s-\frac{3}{2}}\right] - \frac{1}{3}L^{-1}\left[\frac{1}{s^2-\left(\frac{4}{3}\right)^2}\right] -$$

$$-\frac{4}{9}L^{-1}\left[\frac{s}{s^2-\left(\frac{4}{3}\right)^2}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{s^2+\left(\frac{3}{4}\right)^2}\right] - \frac{3}{8}L^{-1}\left[\frac{s}{s^2+\left(\frac{3}{4}\right)^2}\right]$$

$$= 3e^{\frac{3}{2}t} - \frac{1}{\cancel{3}}\cdot\frac{1}{4/\cancel{3}}\sinh\frac{4}{3}t - \frac{\cancel{4}}{3\cancel{9}}\frac{1}{\cancel{4}/\cancel{3}}\cosh\frac{4}{3}t$$

$$+\frac{1}{\cancel{2}}\cdot\frac{1}{3/\cancel{4}2}\sin\frac{3}{4}t - \frac{\cancel{3}}{2\cancel{8}}\frac{1}{\cancel{3/4}}\cos\frac{3}{4}t$$

$$= 3e^{\frac{3}{2}t} - \frac{1}{4}\sinh\frac{4}{3}t - \frac{1}{3}\cosh\frac{4}{3}t +$$

$$\frac{2}{3}\sin\frac{3}{4}t - \frac{1}{2}\cos\frac{3}{4}t$$

$$\boxed{L^{-1}\left(\frac{1}{p-a}\right) = e^{at}}$$

$$\boxed{L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a}\sinh at}$$

$$\boxed{L^{-1}\left(\frac{s}{s^2-a^2}\right) = \frac{1}{a}\cosh at}$$

$$\boxed{L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\sin at}$$

$$\boxed{L^{-1}\left(\frac{s}{s^2+a^2}\right) = \frac{1}{a}\cos at}$$

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② Find: i) $L^{-1}\left[\frac{3(s^2-2)^2}{2s^5}\right]$

ii) $L^{-1}\left[\frac{2s-5}{4s^2+25}+\frac{4s-18}{9-s^2}\right]$.

Sol:

i) $L^{-1}\left[\frac{3(s^2-2)^2}{2s^5}\right] = \frac{3}{2}L^{-1}\left[\frac{s^4-4s^2+4}{s^5}\right]$

$= \frac{3}{2}L^{-1}\left[\frac{1}{s}-\frac{4}{s^3}+\frac{4}{s^5}\right]$

$= \frac{3}{2}\left[L^{-1}\left(\frac{1}{s}\right)-4L^{-1}\left[\frac{1}{s^3}\right]+4L^{-1}\left[\frac{1}{s^5}\right]\right]$

$= \frac{3}{2}\left[1-4\frac{t^{3-1}}{3-1}+4.\frac{t^{5-1}}{5-1}\right]$

$= \frac{3}{2}\left[1-\cancel{4}^{2}.\frac{t^2}{\cancel{2}}+\cancel{4}\frac{t^4}{\cancel{4}}\right]$

$\boxed{L^{-1}\left(\frac{1}{p}\right)=1}$

$= \frac{3}{2}\left[1-2t^2+t^4\right]$ //..

$\boxed{L^{-1}\left(\frac{1}{p^n}\right)=\frac{t^{n-1}}{n-1}}$

iii) $L^{-1}\left[\frac{2s-5}{4s^2+25}+\frac{4s-18}{9-s^2}\right]$

SOL:

$L^{-1}\left[\frac{2s}{4s^2+25}\right]-L^{-1}\left[\frac{5}{4s^2+25}\right]+$

$L^{-1}\left[\frac{4s}{9-s^2}\right]-L^{-1}\left[\frac{18}{9-s^2}\right]$

$=\frac{\cancel{2}}{\cancel{4}_2}L^{-1}\left[\frac{s}{s^2+\frac{25}{4}}\right]-\frac{5}{4}L^{-1}\left[\frac{1}{s^2+\frac{25}{4}}\right]+$

$\frac{4}{-1}L^{-1}\left[\frac{s}{s^2-9}\right]+18L^{-1}\left[\frac{1}{s^2-9}\right]$

$=\frac{1}{2}L^{-1}\left[\frac{s}{s^2+\left(\frac{5}{2}\right)^2}\right]-\frac{5}{4}L^{-1}\left[\frac{1}{s^2+\left(\frac{5}{2}\right)^2}\right]$

$-4L^{-1}\left[\frac{s}{s^2-3^2}\right]+18L^{-1}\left[\frac{1}{s^2-3^2}\right]$

$\frac{1}{\cancel{2}}\frac{1}{5/\cancel{2}}\cos\frac{5}{2}t-\frac{\cancel{5}}{{}_2\cancel{4}}\frac{1}{\cancel{5}/\cancel{2}}\sin\frac{5}{2}t-$

$4\frac{1}{3}\cosh 3t+\overset{6}{\cancel{18}}\frac{1}{\cancel{3}}\sinh 3t$

$=\frac{1}{5}\cos\frac{5}{2}t-\frac{1}{2}\sin\frac{5}{2}t-\frac{4}{3}\cosh 3t+6\sinh 3t$

MATHS-7B

UNIT-3

CLASS-NO.3

① Define Inverse Laplace Transform?

SOL :-

Let $f(t)$ be a function of t for all positive values of t.

Then the Laplace transforms of $f(t)$ denoted by $L[f(t)]$ is defined as

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt,$$

where, 's' is a parameter which may be real or complex.

$L[f(t)]$ being clearly a function of 's'

$\therefore$ It can be written as $\bar{f}(s)$.

$$\therefore \bar{f}(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

which can also be written as,

$$f(t) = L^{-1}[\bar{f}(s)].$$

Then $f(t)$ is called Inverse Laplace transforms of $\bar{f}(s)$.

And the symbol L which transforms $f(t)$ into $\bar{f}(s)$ is called Laplace transform operator.

② State and prove Linear property?

Statement:-

Let $f_1(P)$ and $f_2(P)$ be the two Laplace transforms of functions $F_1(t)$ and $F_2(t)$ respectively and c_1, c_2 be two constand

then,

$$L^{-1}\{c_1 f_1(P) + c_2 f_2(P)\} = c_1 L^{-1}[f_1(P)] + c_2 L^{-1}[f_2(P)]$$

$$= c_1 F_1(t) + c_2 F_2(t).$$

Proof:

$$L\left\{c_1 F_1(t) + c_2 F_2(t)\right\} = c_1 L\left\{F_1(t)\right\} + c_2 L\left\{F_2(t)\right\}.$$

$$= c_1 f_1(P) + c_2 f_2(P)$$

$$\therefore \bar{L}^1\left\{c_1 f_1(P) + c_2 f_2(P)\right\} = \bar{L}^1(c_1 f_1(P)) + \bar{L}^1[c_2 f_2(P)]$$

$$= c_1 F_1(t) + c_2 F_2(t)$$

$$= c_1 \bar{L}^1\left\{f_1(P)\right\} + c_2 \bar{L}^1\left\{f_2(P)\right\}$$

③ State and prove First shifting Theorem On Inverse Laplace transform.

Statement: If $\bar{L}^1[\bar{f}(s)] = F(t)$

Then, $\bar{L}^1[\bar{f}(s-a)] = e^{at} f(t)$.

Proof:

$$\text{Let } \bar{f}(s) = \int_0^\infty e^{-st} \bar{f}(s)\, dt$$

$$\bar{f}(s) = \int_0^\infty e^{-st} F(t)\, dt$$

$$\bar{f}(s-a) = \int_0^\infty e^{-(s-a)t} F(t)\, dt$$

$$= \int_0^\infty e^{-st+at} F(t)\, dt$$

$$= \int_0^\infty e^{-st} \cdot e^{at} F(t)\, dt$$

$$= e^{at} \int_0^\infty e^{-st} F(t)\, dt$$

$$\bar{f}(s-a) = L\left[e^{at} \cdot F(t)\right]$$

$$\therefore L^{-1}\left[\bar{f}(s-a)\right] = e^{at} F(t)$$

④ State and prove Change of Scale Property on Inverse Laplace transforms.

statement :-

If $\bar{f}(s) = L[F(t)]$ denotes Laplace transforms of the function $F(t)$,

Then,

$$L^{-1}\left[\bar{f}(as)\right] = \frac{1}{a} F\left[\frac{t}{a}\right], \ a>0.$$

PROOF :-

Since, $$\bar{f}(s) = \int_0^\infty e^{-st} F(t)\, dt$$

$$\bar{f}(as) = \int_0^\infty e^{-ast} F(t)\, dt$$

put, $at = x \Rightarrow t = \frac{x}{a}$

$$a\,dt = dx$$

$$dt = \frac{dx}{a}$$

$$= \int_0^\infty e^{-sx} F\left(\frac{x}{a}\right) \frac{dx}{a}$$

$$= \frac{1}{a}\int_0^\infty e^{-sx} F\left(\frac{x}{a}\right) dx$$

$$= \frac{1}{a}\int_0^\infty e^{-st} F\left(\frac{t}{a}\right) dt$$

$$\bar{f}(as) = \frac{1}{a} L\left[F\left(\frac{t}{a}\right)\right]$$

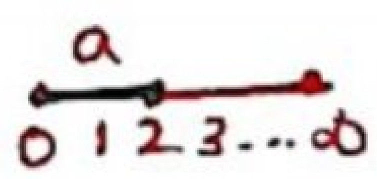
$$\therefore \quad L^{-1}\left[\bar{f}(as)\right] = \frac{1}{a} F\left(\frac{t}{a}\right).$$

(5) State and prove 2nd shifting theorem on Inverse Laplace Transform.

Statement:

If $L^{-1}\left[\bar{f}(s)\right] = F(t)$ then,

$$L^{-1}\left[e^{-as}.\bar{f}(s)\right] = G(t), \text{ where}$$



$$G(t) = \begin{cases} F(t-a) \; ; & t > a \\ 0 \; ; & t < a \end{cases}$$

PROOF :-

$$L\left[G(t)\right] = \int_0^\infty e^{-st} G(t)\, dt$$

$$= \int_0^a e^{-st} G(t)\, dt + \int_a^\infty e^{-st} G(t)\, dt$$

$$= \int_0^a e^{-st}.0\, dt + \int_0^\infty e^{-st} F(t-a)\, dt$$

$$= 0 + \int_a^\infty e^{-st} F(t-a)\, dt$$

Putting, $\boxed{t-a = u} \Rightarrow t = u+a$

$$1-0 = \frac{du}{dt}$$

$$1 = \frac{du}{dt} \Rightarrow \boxed{dt = du}$$

$$= \int_a^\infty e^{-st} F(u)\, du$$

$$= \int_a^\infty e^{-s(u+a)} F(u)\, du$$

$$= \int_a^\infty e^{-su-as} F(u)\, du$$

$$= e^{-as} \int_0^\infty e^{-su} F(u)\, du$$

$$= e^{-as} L[f(u)]$$

$$L[G(t)] = e^{-as} \bar{f}(s)$$

$$G(t) = L^{-1}\{e^{-as} \bar{f}(s)\}$$

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