Incorporating Aerodynamic Effects into Model Based Control for Multirotors

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Semester Project

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Motivation

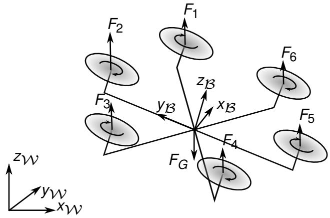
- Multirotors
- autonomous flight
  position control
- Wind

AscTec Falcon 8 at wind turbine inspection, [1]

Overview

Model

Equations of Motion

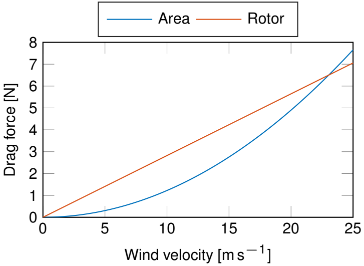


$$m \cdot \mathbf{a} = \mathbf{R}_{\mathcal{W}\mathcal{B}} \sum_{i=1}^{n} \underbrace{(\mathbf{F}_{T,i} + \mathbf{F}_{D,i})}_{=:\mathbf{F}_i} + \mathbf{F}_G$$

$$\mathbf{J} \cdot \dot{\omega} + \omega \times \mathbf{J} \cdot \omega = \sum_{i=1}^{n} (\mathbf{M}_{R,i} + \mathbf{M}_{D,i} + \mathbf{F}_i \times \mathbf{l}_i)$$

Wind Drag

- Air speed
  - $\boldsymbol{\nu} = \mathbf{v} - \mathbf{w}$
- Area drag [2]
  - $\mathbf{F}_A = -\frac{1}{2} C_A \rho \left\| \boldsymbol{\nu} \right\|_2 \boldsymbol{\nu}$
- Rotor drag
  - $\mathbf{F}_D = -\sum_{i=1}^{n} \omega_i \cdot C_D \cdot \boldsymbol{\nu}^{\perp}$

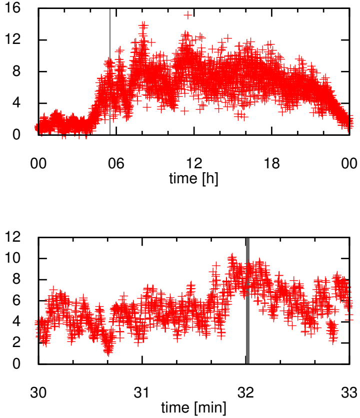


Wind Model

Stationary stochastic process

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \boldsymbol{\sigma}\varepsilon(k)$$

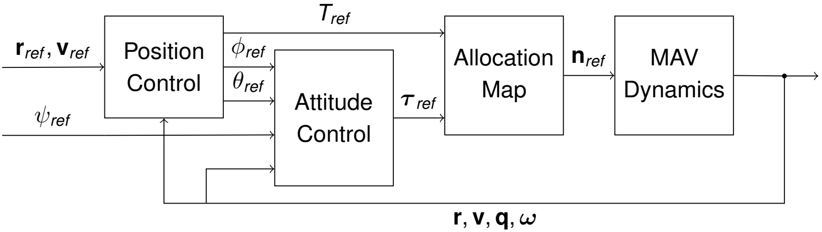
$$\varepsilon_i(k) \sim \mathcal{N}(0,1)\ \forall i \in \{x, y, z\}$$



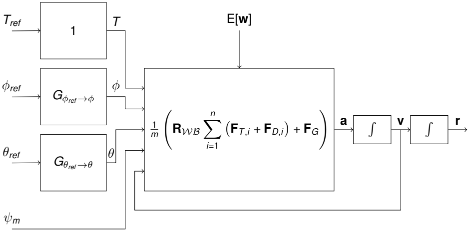
Wind speed observations in $[\mathrm{m\,s^{-1}}]$, [3]

Control Approach

Cascaded Control

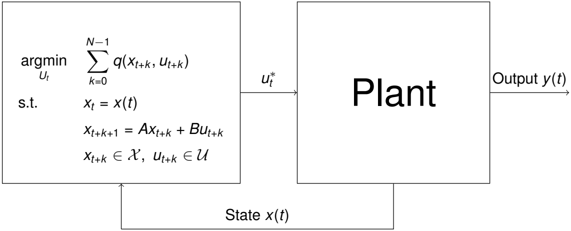


Control System



$$f\left(\mathbf{x}, \mathbf{u}, \mathbf{w}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \begin{bmatrix} \mathbf{A}_{\phi} \begin{bmatrix} \frac{1}{8}\dot{\phi} \\ \phi \end{bmatrix} + \mathbf{B}_{\phi}\,\phi_{ref} \\ \mathbf{A}_{\theta} \begin{bmatrix} \frac{1}{8}\dot{\theta} \\ \theta \end{bmatrix} + \mathbf{B}_{\theta}\,\theta_{ref} \\ \mathbf{v} \\ \frac{1}{m}\left(\mathbf{F}_{T,total} + \mathbf{F}_{D,total}\right) + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \end{bmatrix}$$

Receding Horizon Control



At each sample time:

- Measure/estimate current state $x(t)$
- Find the optimal input sequence for the entire planning window N:
  $U_t^* = \{u_t^*, u_{t+1}^*, \ldots, u_{t+N-1}^*\}$
- Implement the first control action u_t^*

Receding Horizon Control

Advantages:

- Input and output constraints
- Predictive
- Intuitive

Disadvantages:

- No robustness guarantees
- OCP computationally expensive

Control Realization

Linear MPC

- Linearization: About hovering
- Offset-free: System augmented with constant input disturbance
- Constant Disturbances: Estimated with Luenberger observer
- Reference tracking: Tracking all N future reference positions and velocities (feed-forward)
- Input constraints: Limited thrust, roll and pitch
- Terminal penalty P: Solution of algebraic Riccati equation
- CVXGEN: Generate hard-coded convex optimization solver

Linear MPC OCP

$$
\begin{aligned}
\min_{u(\cdot),x(\cdot)} \quad & ||x_N - \bar{x}_t||_P^2 + \sum_{k=0}^{N-1} ||x_k - \bar{x}_t||_Q^2 + ||u_k - \bar{u}_t||_R^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k + B_w w_k + B_d d_k, && k = 0, \dots, N-1 \\
& w_{k+1} = w_k && k = 0, \dots, N-1 \\
& d_{k+1} = d_k, && k = 0, \dots, N-1 \\
& \begin{bmatrix} T_{min} \\ -30^\circ \\ -30^\circ \end{bmatrix} \leq u_k \leq \begin{bmatrix} T_{max} \\ 30^\circ \\ 30^\circ \end{bmatrix} && k = 0, \dots, N-1 \\
& x_0 = \hat{x}(t), \; d_0 = \hat{d}(t), \; w_0 = \hat{w}(t) \\
& N = 40
\end{aligned}
$$

Linear MPC Algorithm

At every sample time:

- Update state $\hat{x}(t)$ and disturbance $\hat{d}(t)$ estimation
- Calculate $N + 1$ desired steady-state inputs $\bar{u}_t$ and states $\bar{x}_t$
- Solve finite-horizon optimal control problem (CVXGEN)
- Apply first optimal control action to system

Nonlinear MPC

No stability guarantees, no optimality guarantees

- Offset-free: System augmented with integrator
- Reference tracking: Tracking all N future reference positions and velocities (feed-forward)
- Input constraints: Limited thrust, roll and pitch
- Terminal penalty P: Solution of algebraic Riccati equation of linearized system
- ACADO: Generate hard-coded nonlinear optimization solver
- qpDUNES: Interior point method/active set for solving sparse OCP

Nonlinear MPC OCP

$$
\begin{aligned}
\min_{x(\cdot),u(\cdot)} \quad & \int_{t}^{t+NT_s} \Big(||x(\tau) - x_{ref}(\tau)||_Q^2 + ||u(\tau) - u_{ref}(\tau)||_R^2 \, \mathrm{d}\tau \Big) \\
& + ||x(t+NT_s) - x_{ref}(t+NT_s)||_P^2 \\
s.t. \quad & \dot{x}(\tau) = f(x(\tau), u(\tau), w(\tau), \psi(\tau)) \\
& \dot{w}(\tau) = 0 \\
& \dot{\psi}(\tau) = 0 \\
& \begin{bmatrix} T_{min} \\ -45^\circ \\ -45^\circ \end{bmatrix} \leq u(\tau) \leq \begin{bmatrix} T_{max} \\ 45^\circ \\ 45^\circ \end{bmatrix}, \quad \forall \tau \in [t, t+NT_s] \\
& x(t) = \hat{x}(t), \;\; w(t) = \hat{w}(t), \;\; \psi(t) = \hat{\psi}(t) \\
& N = 100
\end{aligned}
$$

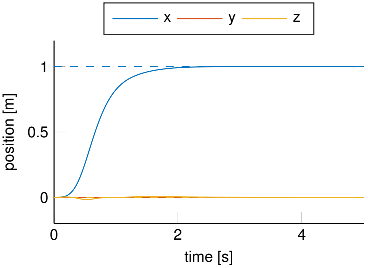
Nonlinear MPC Algorithm

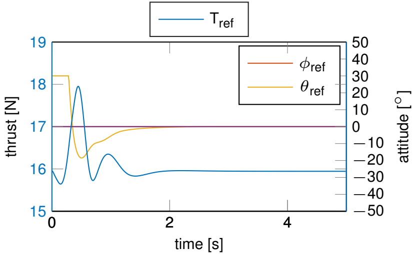
At every sample time:

- Set current state, wind and heading
- Set $N + 1$ references
- Set previous optimal solution as warm start
- Solve finite-horizon optimal control problem (ACADO)
- Update integrator
- Apply first optimal control action to system

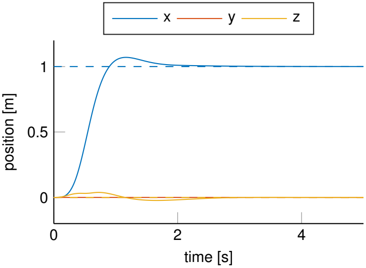
Simulation Results

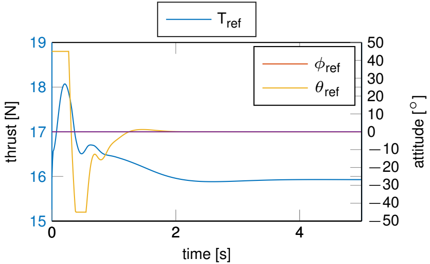
Step Response: Linear MPC



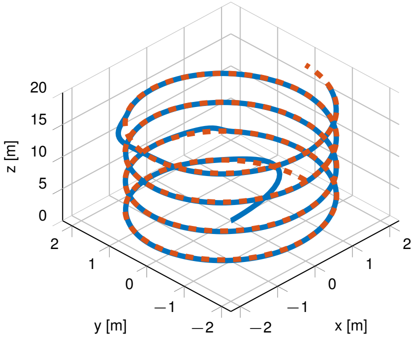


Step Response: Nonlinear MPC

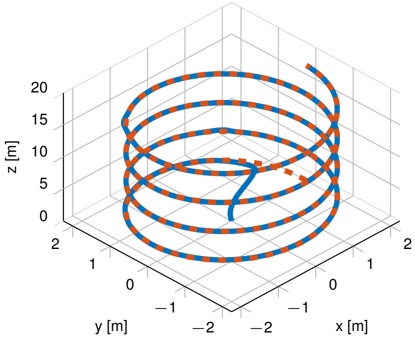




Wind Experiment: Linear MPC

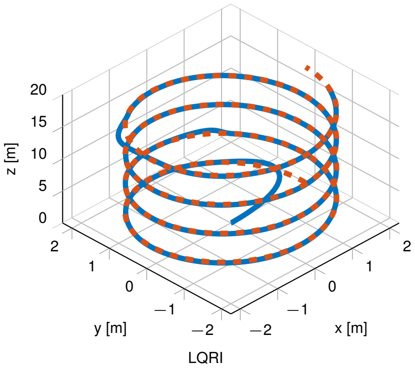


LQRI



Linear MPC

Wind Experiment: Nonlinear MPC



Computation Times

	index	LEE	LQRI	MPC	NMPC
total	mean	0.28	1.72	4.25	4.33
	median	0.28	1.67	4.19	4.22
	min	0.26	1.62	4.06	4.10
	max	0.40	2.17	5.05	6.67
online OCP	mean	–	–	0.85	2.46
	median	–	–	0.84	2.39
	min	–	–	0.80	2.31
	max	–	–	1.09	4.83
dimensions	states	–	13	10	13
	inputs	–	3	3	3
	prediction horizon	–	∞	40	100

Conclusion

Results

- Two flight position controller
- Wind gust rejection for measured winds
- High frequency MPC and NMPC is feasible

Outlook:

- Validate results in real implementation
- Improve controller

Thank you! Questions?

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References

[1] Ascending Technologies (AscTec), "Industrial Civil Infrastructure Inspection," http://www.asctec.de/en/industrial-civil-infrastructure-inspection/, 2015.

[2] F. Schiano, J. Alonso-Mora, K. Rudin, P. Beardsley, R. Siegwart, and B. Siciliano, "Towards Estimation and Correction of Wind Effects on a Quadrotor UAV," *IMAV 2014: International Micro Air Vehicle Conference and Competition 2014, Delft, The Netherlands, August 12-15, 2014*, Aug. 2014.

[3] Google, "Wind Observation," http://www.google.org/pdfs/google_heliostat_wind_data_collection.pdf, 2015.