



Incorporating Aerodynamic Effects into Model Based Control for Multirotors

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Semester Project

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Motivation

- Trajectory following
- Model predictive control
- Wind compensation



AscTec Falcon 8 at wind turbine inspection [1].

Overview

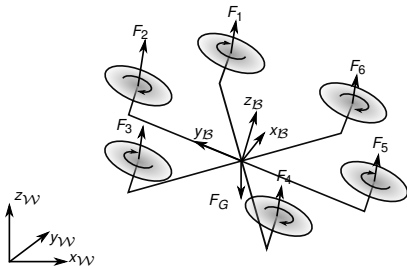
Modelling

Control

Simulation

Conclusion and Outlook

Equations of Motion

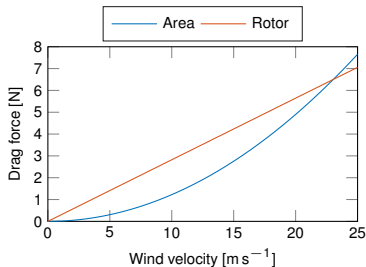


$$m \cdot \mathbf{a} = \mathbf{R}_{WB} \sum_{i=1}^n \underbrace{(\mathbf{F}_{T,i} + \mathbf{F}_{D,i})}_{=:\mathbf{F}_i} + \mathbf{F}_G$$

$$\mathbf{J} \cdot \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J} \cdot \boldsymbol{\omega} = \sum_{i=1}^n (\mathbf{M}_{R,i} + \mathbf{M}_{D,i} + \mathbf{F}_i \times \mathbf{l}_i)$$

Wind Drag

- Area drag [2]
 - $\mathbf{F}_A = -\frac{1}{2} C_A \rho \|\boldsymbol{\nu}\|_2 \boldsymbol{\nu}$
- Rotor drag
 - $\mathbf{F}_D = -\sum_{i=1}^n \omega_i \cdot C_D \cdot \boldsymbol{\nu}^\perp$
- Air speed
 - $\boldsymbol{\nu} = \mathbf{v} - \mathbf{w}$

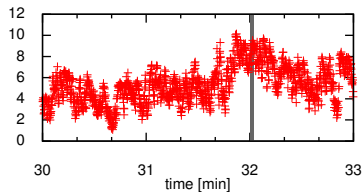
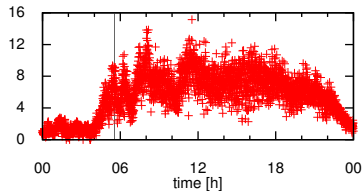


Wind Model

Stationary stochastic process

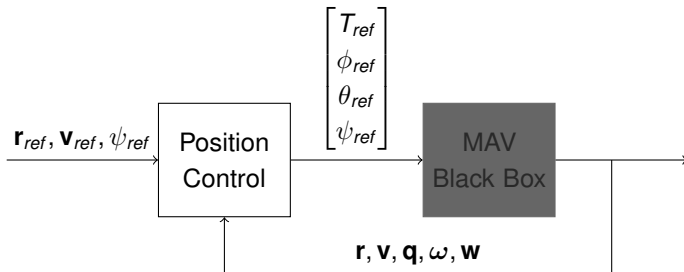
$$\mathbf{w}(k+1) = \mathbf{w}(k) + \sigma \varepsilon(k)$$

$$\varepsilon_i(k) \sim \mathcal{N}(0, 1) \quad \forall i \in \{x, y, z\}$$



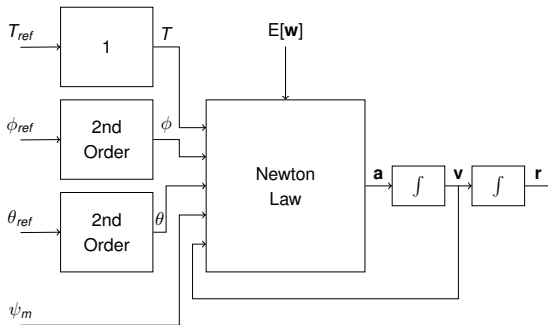
Wind speed observations in $[\text{m s}^{-1}]$ [3].

System Overview



- Cascaded control
- Given attitude control
- Given state and wind estimation
- SysId attitude and thrust dynamics

Black Box Dynamics Model



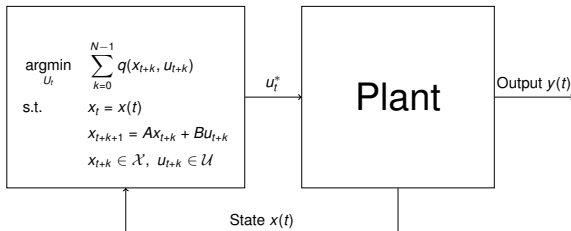
$$\mathbf{x} = [\phi, \theta, \dot{\phi}, \dot{\theta}, x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$$

$$\mathbf{u} = [T, \phi_{ref}, \theta_{ref}]^T$$

$$\mathbf{w} = [w_x, w_y, w_z]^T$$

$$\mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \frac{d}{dt} \mathbf{x}$$

Receding Horizon Control



At each sample time:

- Measure/estimate current state $x(t)$
- Find the optimal input sequence for the entire planning window N :

$$U_t^* = \{u_t^*, u_{t+1}^*, \dots, u_{t+N-1}^*\}$$
- Implement the first control action u_t^*

Receding Horizon Control

Advantages:

- Input and state constraints
- Optimal solution for horizon
- Parameter tuning

Disadvantages:

- Stability, robustness guarantees
- OCP computationally expensive

Solution:

- Code generation for optimization
 - CVXGEN
 - ACADO

Linear and nonlinear MPC

- deterministic wind feed-forward
- offset-free constant reference tracking
- trajectory tracking

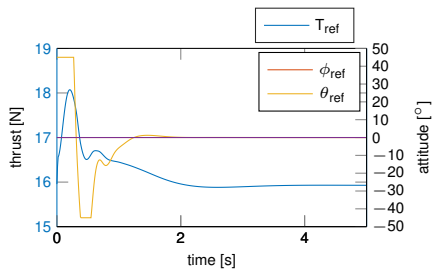
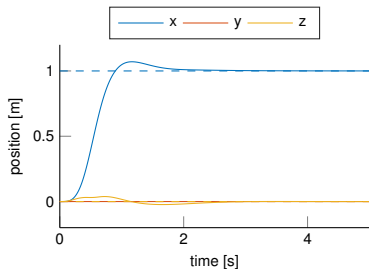
Difference:

- MPC linearization about hovering
- NMPC nonlinear dynamics

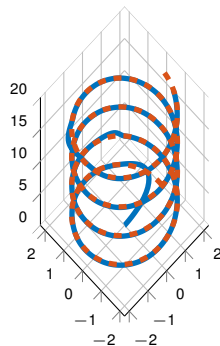
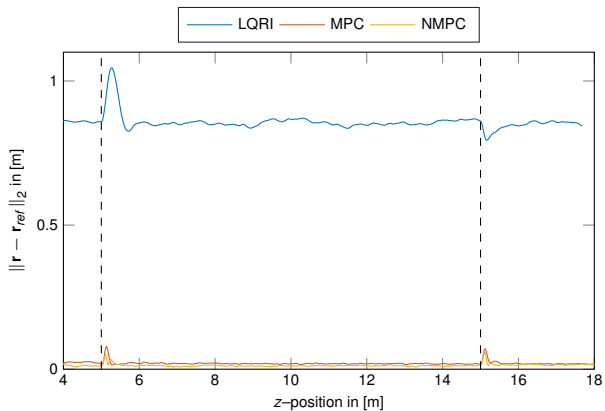
Nonlinear MPC OCP

$$\begin{aligned}
 & \min_{x(\cdot), u(\cdot)} && \int_t^{t+NT_s} \left(\|x(\tau) - x_{ref}(\tau)\|_Q^2 + \|u(\tau) - u_{ref}(\tau)\|_R^2 \right) d\tau \\
 & && + \|x(t + NT_s) - x_{ref}(t + NT_s)\|_P^2 \\
 \text{s.t.} & && \dot{x}(\tau) = f(x(\tau), u(\tau), w(\tau), \psi(\tau)) \\
 & && \dot{w}(\tau) = 0 \\
 & && \dot{\psi}(\tau) = 0 \\
 & && \begin{bmatrix} T_{min} \\ -45^\circ \\ -45^\circ \end{bmatrix} \leq u(\tau) \leq \begin{bmatrix} T_{max} \\ 45^\circ \\ 45^\circ \end{bmatrix}, \quad \forall \tau \in [t, t + NT_s] \\
 & && x(t) = \hat{x}(t), \quad w(t) = \hat{w}(t), \quad \psi(t) = \hat{\psi}(t) \\
 & && N = 100
 \end{aligned}$$

Step Response: Nonlinear MPC

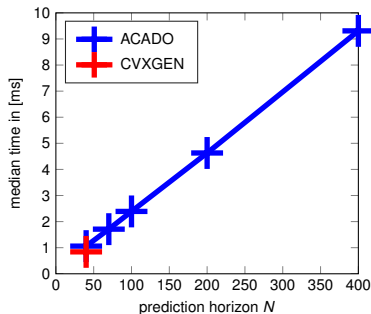


Wind Experiment



Optimization Times

- Real-time capability
- ACADO optimization time grows linearly in N
- CVXGEN bounded in problem size



index	CVXGEN	ACADO
mean	0.84	1.09
std	0.14	0.13
median	0.83	1.08
max	1.15	1.52
states	10	13
inputs	3	3
horizon	40	40

Conclusion and Outlook

- Two trajectory following controller
- Wind gust rejection for measured winds
- Real-time linear and nonlinear MPC
- MPC just as easy as LQRI but superior

Outlook:

- Validate results in real experiments
- Improve controller
 - e.g. quaternion NMPC, disturbance estimation, stability analysis, model...

Thank you! Questions?

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References

- [1] Ascending Technologies (AscTec), “Industrial Civil Infrastructure Inspection,” <http://www.ascotec.de/en/industrial-civil-infrastructure-inspection/>, 2015.
- [2] F. Schiano, J. Alonso-Mora, K. Rudin, P. Beardsley, R. Siegwart, and B. Siciliano, “Towards Estimation and Correction of Wind Effects on a Quadrotor UAV,” *IMAV 2014: International Micro Air Vehicle Conference and Competition 2014, Delft, The Netherlands, August 12-15, 2014*, Aug. 2014.
- [3] Google, “Wind Observation,” http://www.google.org/pdfs/google_heliostat_wind_data_collection.pdf, 2015.