ISCI 422: Assign 5 – Analytic Model Sol'n

Oct 20/09

	Reactions	Rate	ΔN	ΔZ	dn/dt	dz/dt
"Birth"	$N \xrightarrow{\beta} 2N$	βn	$+1$	0	$+\beta n$	0
"Death"	$N \xrightarrow{\delta} Z$	δn	-1	$+1$	$-\delta n$	$+\delta n$
"Competition"	$2N \xrightarrow{\alpha} N+Z$	αn^2	-1	$+1$	$-\alpha n^2$	$+\alpha n^2$
"Predation"	$N+Z \xrightarrow{\gamma} 2Z$	$\gamma n z$	-1	$+1$	$-\gamma n z$	$+\gamma n z$
New	$N+Z \xrightarrow{\epsilon} N$	$\epsilon n z$	0	-1	0	$-\epsilon n z$

New reaction could represent a ~~humans~~ human fighting back and killing ~~zombies~~ a zombie.

Rates: $\dfrac{dn}{dt} = +(\beta-\delta)n - \alpha n^2 - \gamma n z$

$\dfrac{dz}{dt} = +\delta n + \alpha n^2 + \gamma n z - \epsilon n z$

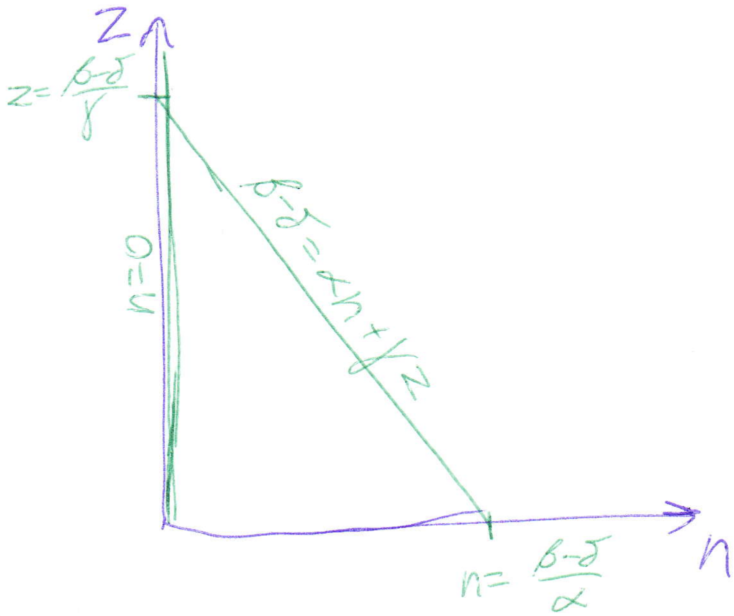
Nullclines: $\dfrac{dn}{dt} = 0$

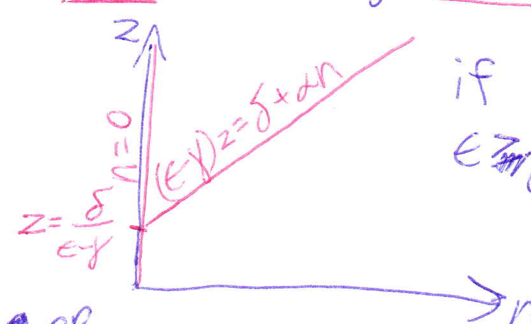
note: $\beta > \delta$

$\Rightarrow n = 0$ OR $\beta - \delta = \alpha n + \gamma z$

$\dfrac{dz}{dt} = 0$

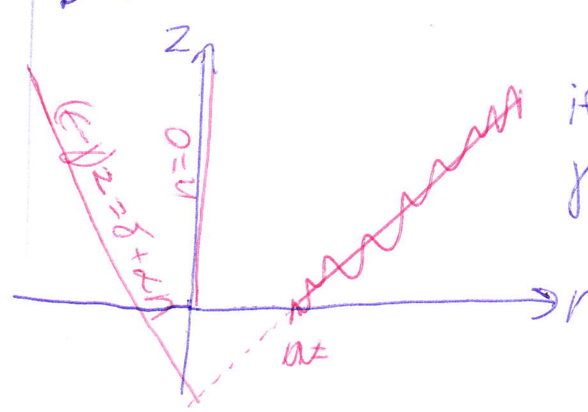
$\Rightarrow n = 0$ OR $(\epsilon - \gamma) z = \delta + \alpha n$





if $\epsilon > \gamma$

OR

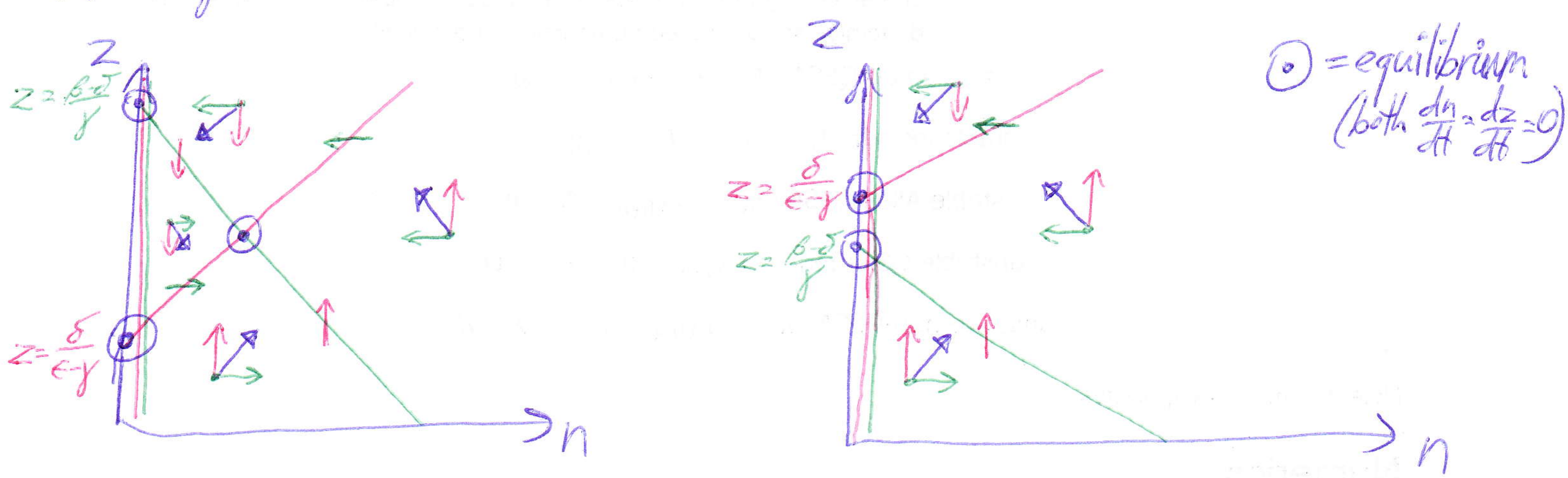


if $\gamma \geq \epsilon$

If $\gamma > \epsilon$ then there is only one z-nullcline for biologically-meaningful $n \geq 0$ & $z \geq 0$. The nullcline $(\epsilon - \gamma) z = \delta + \alpha n$ is outside this region and irrelevant. Without it we expect the dynamics to be qualitatively identical to those we found in class, meaning that $n \to 0$ ie. humans will go extinct.

To rescue humanity we need $\epsilon > \gamma$ so that in fights between humans and zombies ($N + Z \to \ldots$) humans are more likely to kill zombies than the other way around. So we'll just consider the case where $\epsilon > \gamma$.

<u>Flows:</u>
Two possibilities:

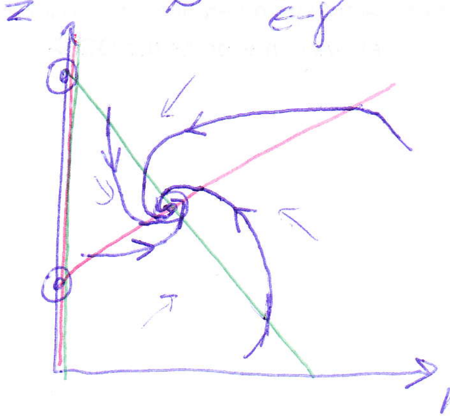


if $\frac{\beta - \delta}{\gamma} > \frac{\delta}{\epsilon - \gamma}$ $\left(\text{or } \beta > \frac{\epsilon\delta}{\epsilon - \gamma}\right)$ if $\frac{\delta}{\epsilon - \gamma} > \frac{\beta - \delta}{\gamma}$ $\left(\text{or } \beta < \frac{\epsilon\delta}{\epsilon - \gamma}\right)$

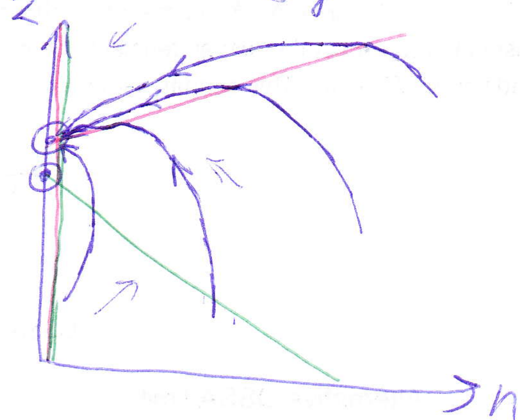
So outcome depends on parameters. If β is ~~small~~ large enough, then the nullclines cross and we have a non-trivial equilibrium. Otherwise, the only equilibria are found at $n = 0$ (see right graph).

For the left graph our nullcline & flow analysis doesn't tell us enough to be sure the coexisting equilibrium is stable. We need more advanced analysis or numerics to be sure. But it turns out it is — here are some trajectories.

if ~~$\gamma\delta < (\beta-\delta)(\epsilon-\gamma)$~~ $\beta > \frac{\epsilon\delta}{\epsilon-\gamma}$



if ~~$\delta > (\beta-\delta)(\epsilon-\gamma)$~~ $\beta < \frac{\epsilon\delta}{\epsilon-\gamma}$



So, for humans to survive they need:

(1) to be effective zombie killers, $\epsilon > \gamma$, and

~~(2) to live long enough (have a small enough "spontaneous" death rate), $\delta < \beta$~~

(2) to reproduce fast enough, $\beta > \frac{\epsilon\delta}{\epsilon-\gamma}$.

~~(or equivalently, to be~~

Actually, that reduces to one condition because the second condition can be written as $\epsilon > \gamma\frac{\beta}{\beta-\delta}$. Since $\beta > \beta-\delta$ then $\frac{\gamma\beta}{\beta-\delta} > \gamma$ so we just need humans to be very effective zombie killers: $\boxed{\epsilon > \frac{\gamma\beta}{\beta-\delta}}$.

Then the dynamics will spiral in towards the coexistence equilibrium.