ISCI 422 HW #5: Analytic

Feb 22/06

Problem: Solve the following system of processes for the nullclines and flows:

(H = "host", P = "predator")

$$H \xrightarrow{\beta} 2H$$
$$2H \xrightarrow{c} H$$
$$H + P \xrightarrow{\alpha} 2P$$
$$P \xrightarrow{\delta} \emptyset$$

(assume all rate constants > 0)

Solution:

Reaction	Rate	H change	P change	$\frac{dh}{dt}$	$\frac{dp}{dt}$
$H \xrightarrow{\beta} 2H$	βh	+1	0	$+\beta h$	0
$2H \xrightarrow{c} H$	ch^2	−1	0	$-ch^2$	0
$H + P \xrightarrow{\alpha} 2P$	αhp	−1	+1	$-\alpha hp$	$+\alpha hp$
$P \xrightarrow{\delta} \emptyset$	δp	0	−1	0	$-\delta p$

Net rate eqns:

$$\frac{dh}{dt} = \beta h - ch^2 - \alpha hp = h(\beta - ch - \alpha p)$$

$$\frac{dp}{dt} = \alpha hp - \delta p = p(\alpha h - \delta)$$

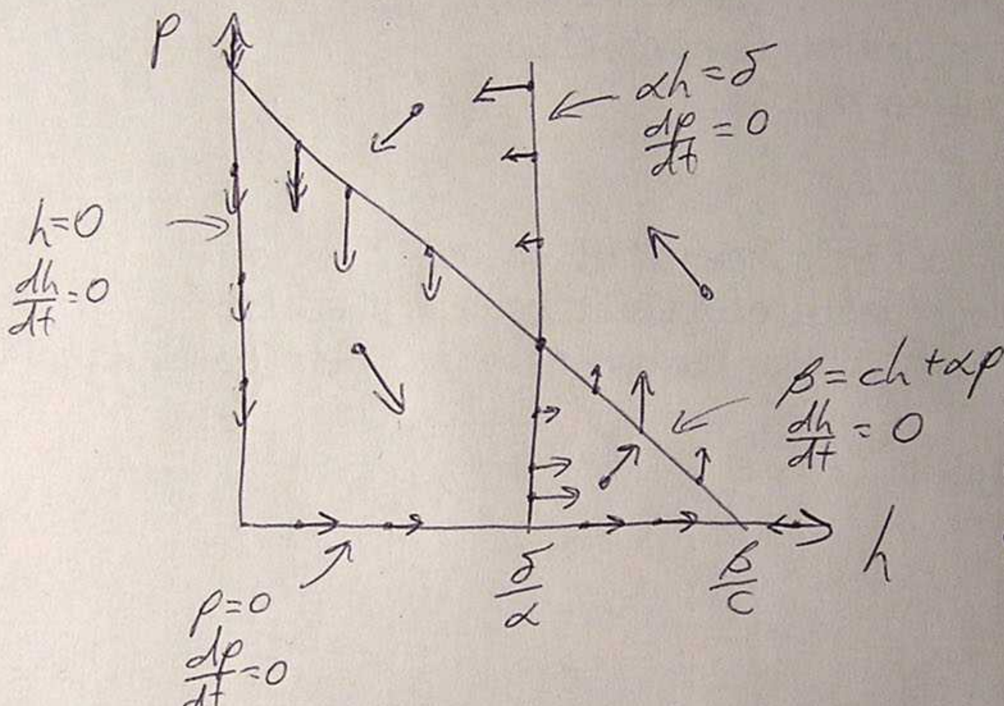
Nullclines:

$$\frac{dh}{dt} = 0 \Rightarrow h = 0 \quad \text{or} \quad \beta = ch + \alpha p$$

$$\frac{dp}{dt} = 0 \Rightarrow p = 0 \quad \text{or} \quad \alpha h = \delta$$

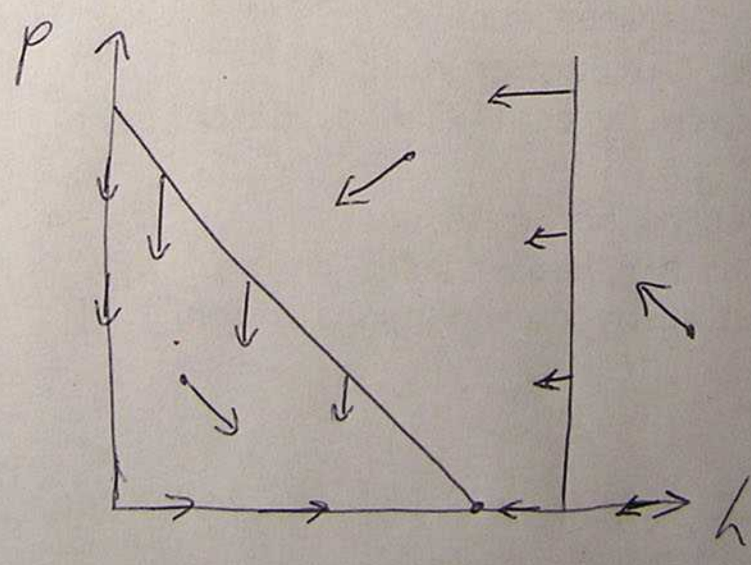
Flows: 2 possibilities — nullcline $\beta = ch + \alpha p$ may or may not intersect $\alpha h = \delta$, depending on α, β, c & δ.

Case 1: $\frac{\beta}{c} > \frac{\delta}{\alpha}$ (intersection)



So it appears host & predator populations will spiral around coexisting equilibrium. Need more detailed analysis to determine if spiral grows or shrinks.

Case 2: $\frac{\beta}{c} \leq \frac{\delta}{\alpha}$ (no intersection)



In this case predators may have an early boom but ultimately they die out leaving only hosts.