Analytic Side-Blotched Lizards

Mar 16/08

R = oRange males, Y = Yellow males, B = Blue males

oRange dominates Blue:	$B + R \rightarrow 2R$	assume all rate constants are unity. (Doesn't affect <u>qualitative</u> dynamics.)
Blue dominates Yellow:	$Y + B \rightarrow 2B$	
Yellow tricks oRange:	$R + Y \rightarrow 2Y$	

<u>Rates</u>:

$$\frac{dr}{dt} = +br - yr$$

$$\frac{dy}{dt} = +yr - yb$$

$$\frac{db}{dt} = +yb - br$$

<u>Fixed pts</u>:

$$\frac{dr}{dt} = 0 \Rightarrow r = 0 \quad \text{or} \quad y = b$$

$$\frac{dy}{dt} = 0 \Rightarrow y = 0 \quad \text{or} \quad b = r$$

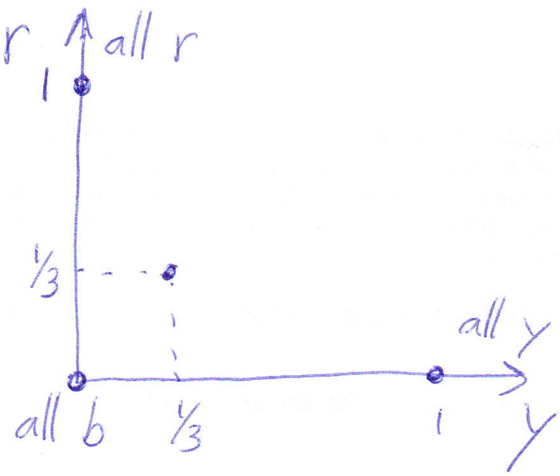
$$\frac{db}{dt} = 0 \Rightarrow b = 0 \quad \text{or} \quad r = y$$

Assume total density $= r + y + b = 1$.

Then all possible fixed pts are

r	y	b	
0	0	1	($r=0$, $y=0$, $y=r$)
0	1	0	($r=0$, $b=r$, $b=0$)
1	0	0	($y=b$, $y=0$, $b=0$)
1/3	1/3	1/3	($y=b$, $b=r$, $r=y$)

Can plot any 2 of 3 types (3rd is determined by $r+y+b=1$):

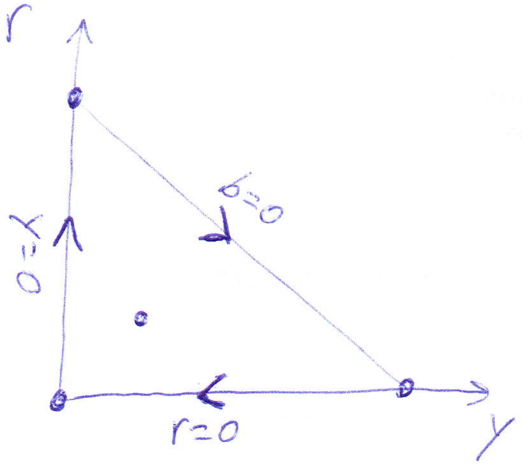


<u>Flows</u>: Flows are difficult to analyze except in special cases:

if $r=0$ then $\frac{dy}{dt} \le 0$, $\frac{db}{dt} \ge 0$

if $y=0$ then $\frac{db}{dt} \le 0$, $\frac{dr}{dt} \ge 0$

if $b=0$ then $\frac{dr}{dt} \le 0$, $\frac{dy}{dt} \ge 0$

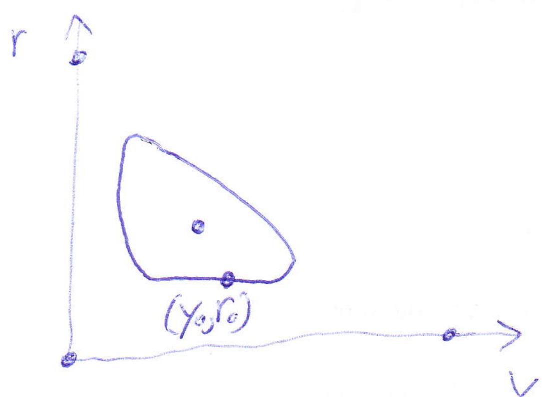


Also notice $\frac{d}{dt}(r+y+b) = 0$ and $\frac{d}{dt}(ryb) = 0$.

So trajectory for any $r=r_0, y=y_0$ constrained to

$$\underbrace{ry(1-r-y)}_{b} = r_0 y_0 \underbrace{(1-r_0-y_0)}_{b_0} = \text{constant}$$

Trajectory looks like

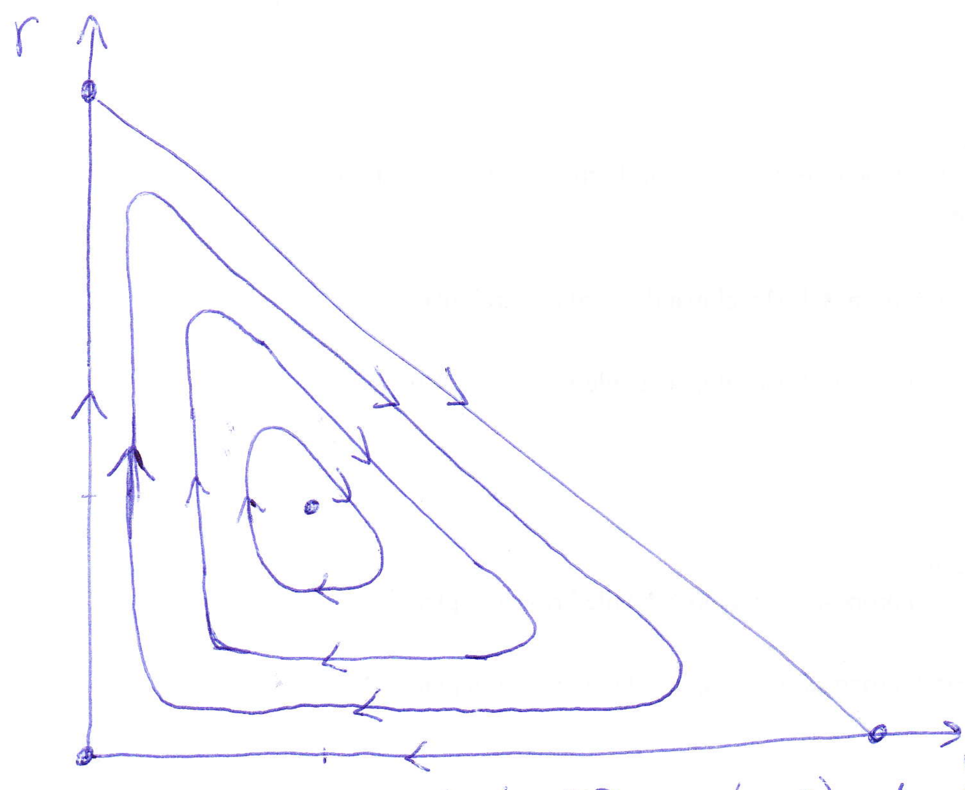


As before, when r small then flow ←

" y " ↑

" b " ↘

So phase portrait is ...



No matter where we start (if $r, y, b > 0$) densities will oscillate, periodically returning to initial conditions.