INF4411 Analog Microelectronics
Mandatory lab exercise 2

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Introduction

This lab is the second in the course INF4411 Analog Microelectronics. The purpose of the lab is to model, simulate and measure MOSFETs, metal–oxide –semiconductor field-effect transistors. The measurements, simulations and modeling will be done for all regions of the MOSFET: weak inversion, strong inversion, triode and active. To model both weak inversion and strong inversion at the same time, the EKV-model by Vittoz [1] is used. In addition to this, measurements will be taken of the common-source amplifier, one of the simplest transistor level amplifiers.

Equipment

We used the following equipment on workbench 5 in the laboratory (room 5419):

- CD4007UBE Dual Complementary Pair Plus inverter[2]
- ICL7621 op-amp[3]
- GPIB I/O
- MATLAB
  - GPIB-functions (from /hom/mes/src/matlab/gpib/linux at the UiO-server)
- Hewlett Packard 34401A Digital Multimeter
- Agilent E3631 DC power-supply
- General resistors (5% tolerance)

1 The EKV model in MATLAB

In this task, we were asked to use the EKV-model to produce plots showing the characteristics of a typical $0.35\mu m$ CMOS transistor with MATLAB, using the simplifying assumption that V_S and V_B are connected to ground potential. Further specifications given in the lab task are listed below:

- Constant slope factor value, $n = 1.0$
- Channel length modulation by means of multiplicative term $(1 + \lambda v_{ds})$.
- Typical values for a $0.35\mu m$ process as initial values:
  - $V_{tn} = 0.57V$
  - $V_{tp} = 0.71V$
  - $\lambda L = 0.15\frac{\mu m}{V}$
  - $Vdd = 3.3V$
- Other assumptions:
  - $W = L = 1\mu m$
  - $\beta_N = 190\frac{\mu A}{V^2}$
  - $\beta_P = 55\frac{\mu A}{V^2}$
- $U_T = \frac{kT}{q} = 26mV$ at room temperature.

Background information

The EKV model [1] is based on the fact that the drain current of the transistor can be expressed as:

$$I_D = I_F - I_R \tag{1}$$

where I_F is the forward component of the current and independent of the drain voltage V_D, and I_R is the reverse component of the current and independent of the source voltage V_S.

The operating mode of the transistor is dependant on whether the current components are in weak, moderate or strong inversion. The requirements for the current components to be in either mode are listed below.

1. Weak Inversion
   - $I_F \ll I_s$, $V_S > V_P$[1]
   - $I_R \ll I_s$, $V_D > V_P$
2. Strong Inversion
   - $I_F \gg I_s$, $V_S < V_P$
   - $I_R \gg I_s$, $V_D < V_P$
3. Moderate Inversion
   - I_F is neither much smaller or larger than I_s. $V_S \simeq V_P$
   - I_R is neither much smaller or larger than I_s. $V_D \simeq V_P$

For the transistor to be in weak inversion mode, both I_F and I_R must be in weak inversion. For the transistor to be in strong inversion mode, both I_F and I_R must be in strong inversion.[2] Furthermore, we can distinguish triode and saturation operation modes for a given transistor which is weakly inverted or strongly inverted separately.

The way the EKV model distinguishes weak inversion from strong inversion is not very enlightening in regards to what actually happens in the transistor. To clarify this, we note that the term "inversion" refers to the inversion layer of mobile electrons that is induced under the gate electrode in the transistor, and that allows current to flow from the drain electrode to the source electrode. [4]

In strong inversion, this inversion is established by the presence of a voltage on the gate electrode which is above the threshold voltage of the transistor. Said voltage attracts mobile electrons to the region of the substrate directly under the gate that balance out the most of the charges present in the gate electrode.

In weak inversion, however, the presence of electrons in the layer under the gate, does not depend on the gate voltage but rather on the physical properties of the transistor and the substrate. A parameter of particular importance for weak inversion is the slope factor, n. [5]

[1] V_P is the pinch-off voltage

[2] Refered to as conduction mode in the EKV paper.

1.1 $i_D(v_{GS})$, NMOS, active region

For this task we set the value of V_D to V_{dd}. We do so because, for the transistor to be in the active region, the following requirement must be fulfilled [6]:

$$v_{DS} \geq V_{ov} \tag{2}$$

We assume the source is grounded, so $V_{DS} = V_D$.

The maximum value of V_{ov}, $V_{ov_{max}}$[3], is found:

$$\begin{aligned} V_{ov_{max}} &= V_{G_{max}} - V_{tn} \\ &= 3.3V - 0.57V \\ &= 2.53V \end{aligned} \tag{3}$$

Thus we consider V_{dd} to be a suitable value for V_D in order to show the operation in the active region. We note, however, that in this exercise we want to plot the active region operation for both weak and strong inversion, and the above used condition is valid for strong inversion. The condition for active region operation in weak inversion is [1]:

$$\begin{aligned} v_{DS} &\geq 4 \cdot U_t \\ v_{DS} &\geq 4 \cdot 26mV \\ v_{DS} &\geq 104mV \end{aligned} \tag{4}$$

This condition is obviously satisfied when we set V_D to V_{dd}. This means that for the entirety of both weak and strong inversion $V_D \geq V_{D,sat} = V_{ov}$.

[3] In the EKV paper, V_{Dsat} is used instead of v_{ov}

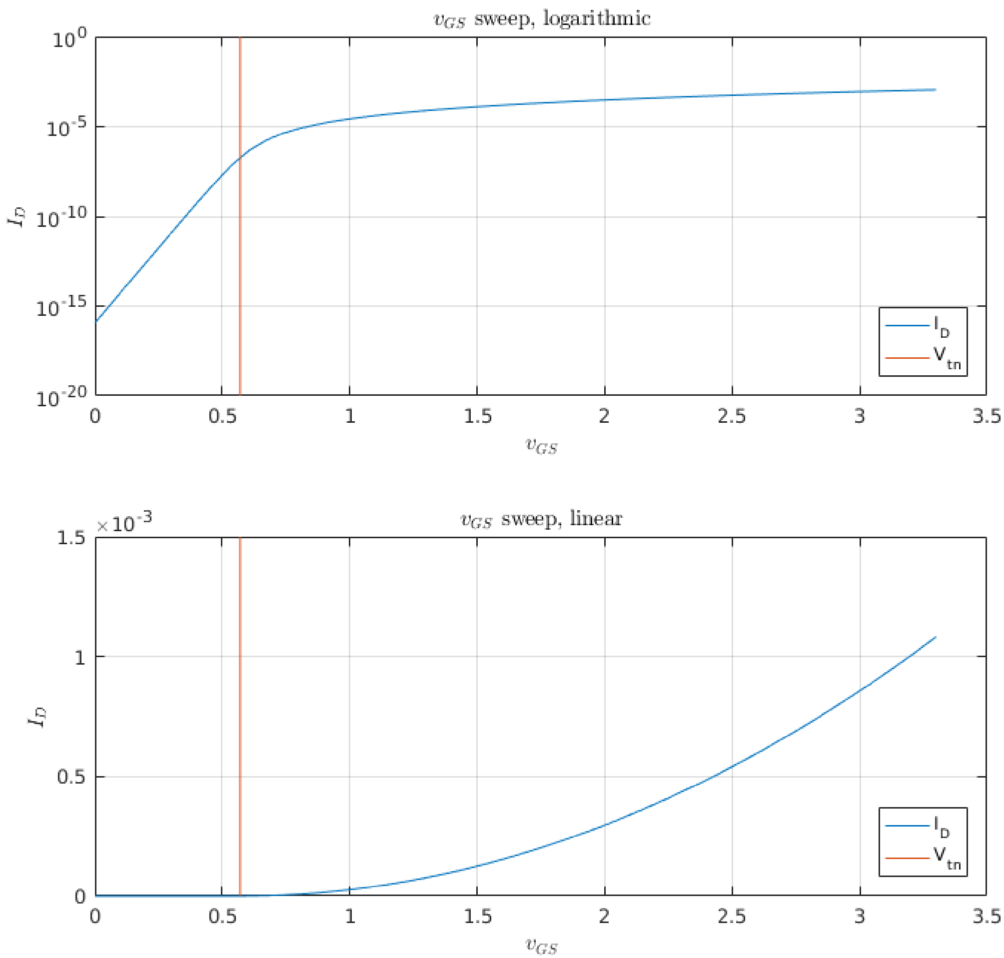


Figure 1.1: EKV-model, active region

1.2 $i_D(v_{GS})$, NMOS, triode region

For this task we set the value of V_D to $0.1V$. We want to plot triode region operation in both weak and strong inversion. The condition for triode operation in strong inversion [6], shown below, is insufficient because it does not take into account the lowest v_{GS} voltages of weak inversion.

$$v_{DS} < v_{ov} \tag{5}$$

$$\begin{aligned} V_{ov_{min}} &= V_{G_{min}} - V_{tn} \\ &= 0V - 0.57V = -570mV \end{aligned} \tag{6}$$

$$v_{DS} < -570mV \tag{7}$$

The following is the condition for triode operation in weak inversion.[5] As we can see, this condition constrains the value of v_{DS} even more.

$$\begin{aligned} v_{DS} &< 4 \cdot U_t \\ v_{DS} &< 4 \cdot 26mV \\ v_{DS} &< 104mV \end{aligned} \tag{8}$$

We base our choice of V_D on this condition, and choose V_D = 100mV. This ensures the transistor is in the triode region during the weak inversion and most of the strong inversion. However, when $v_{GS} = V_{tn}$ we are not in the triode region, but in the active, as $V_D \geq v_{GS} - V_{tn} = 0V$. This means we are in the active region until v_{GS} is V_D above the threshold.

$$v_{GS,active} = [V_{tn}, V_{tn} + V_D] = [0.57V, 0.67V]$$

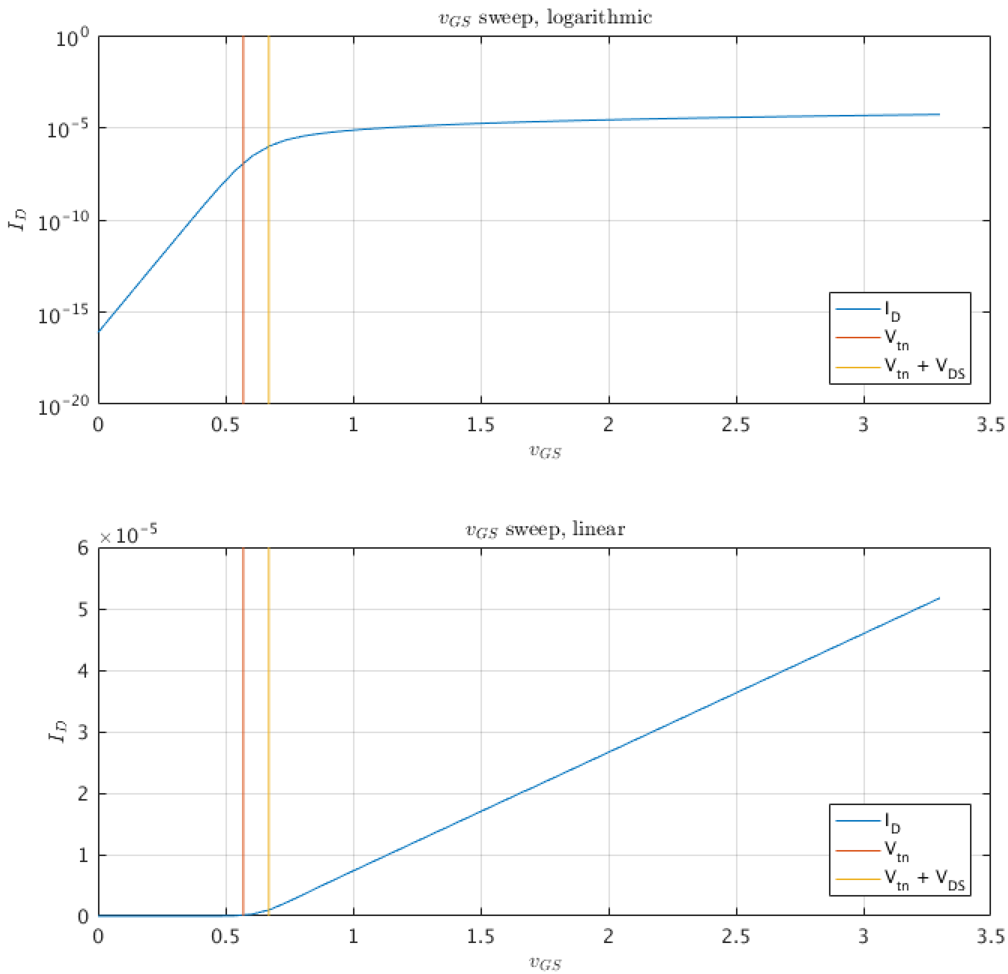


Figure 1.2: EKV-model, triode region

1.3 $i_D(v_{DS})$, NMOS, strong inversion

Whereas in the above exercises we have swept the value of v_{GS} to show the behaviour in triode and saturation modes separately, we now wish to narrow down and look at the behaviour of the transistor only in strong inversion. This approach should yield a result that is similar to the model used by Sedra and Smith [6].

As discussed in the section on background information, for the transistor to be in strong inversion, both the forward and reverse components of the drain current must be in strong inversion. This is achieved when:

$$V_{GS} > V_t + nV_S \tag{9}$$

We are using the simplifying assumptions that $n = 1$ and $V_S = 0V$. Which in turn simplifies the condition for strong inversion operation to:

$$\begin{aligned} V_{GS} &> V_t \\ V_{GS} &> 0.57V \end{aligned} \tag{10}$$

We have chosen to use nine values for V_{GS} ranging from 1V to 3V with linear increments of 0.25 V. We do this to show that, in strong inversion, the knee-points separating the triode and saturation region have a quadratic behaviour with increasing v_{GS}. See knee-points marked as $V_{D,sat}$ in figure 1.3. The transition from the triode to saturation region happens when $V_D \geq V_{D,sat} = v_{GS} - V_{tn}$. This means the knee-points are dependent on v_{GS}. Note also the linear increase proportional to v_{DS} in the saturation region as a result of channel length modulation from the multiplicative term $(1 + \lambda v_{DS})$.

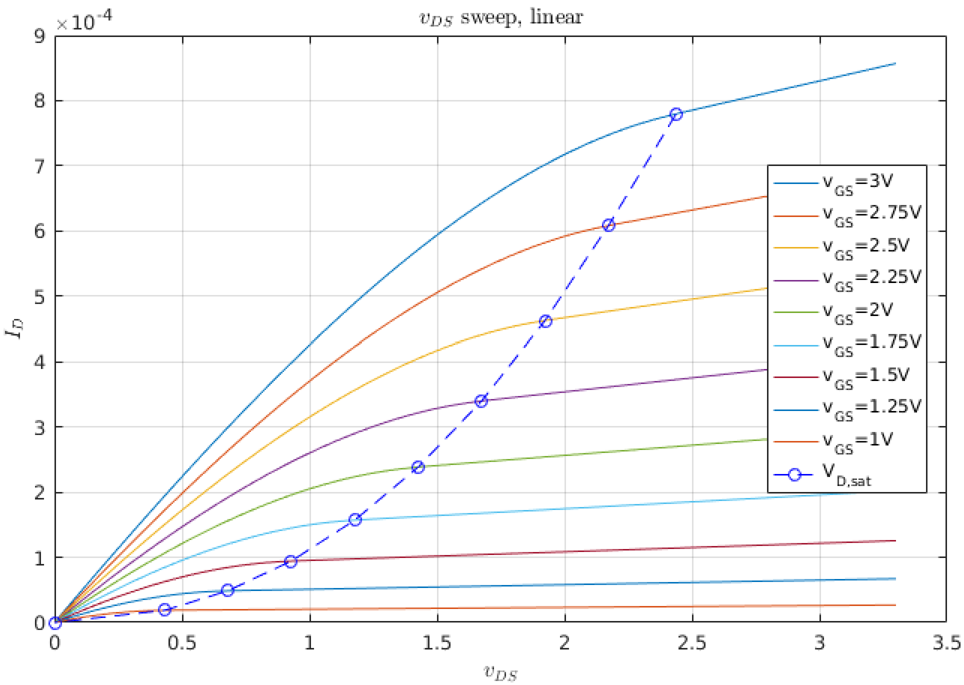


Figure 1.3: EKV-model, strong inversion

1.4 $i_D(v_{DS})$, NMOS, weak inversion

The EKV model condition for weak inversion operation is that both the forward and reverse components of the drain current are in weak inversion. This is achieved when:

$$V_{GS} < V_t + nV_S \tag{11}$$

Which with our assumptions simplifies to:

$$\begin{aligned} V_{GS} &< V_t \\ V_{GS} &< 0.57V \end{aligned} \tag{12}$$

We choose nine values of V_{GS} ranging from $0.32V$ to $0.40V$ with linear increments of 0.01V. The transition from the triode to saturation region happens when $V_D \geq V_{D,sat} = 4 \cdot U_t$. This means the knee-points are independent of v_{GS}.

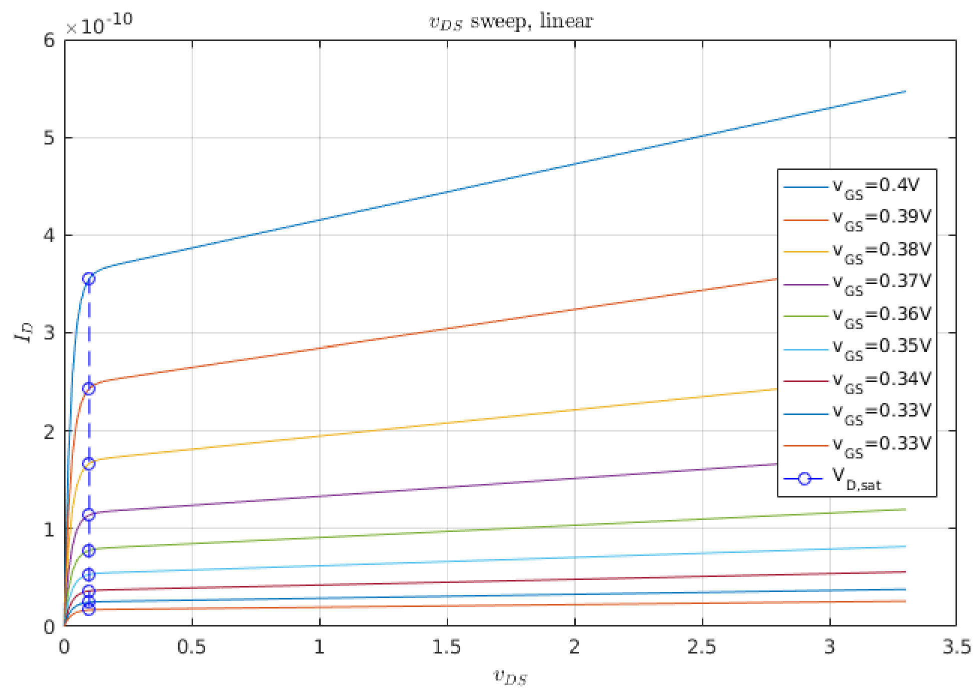


Figure 1.4: EKV-model, weak inversion

2 Cadence simulation

In this task the "nmos4" cell from the "PRIMLIB" library was used as the model for the transistor whose characteristics were simulated in cadence. The use of this particular cell/model was requested in the laboratory assignment.

The length and width parameters of the transistor had to be modified to $1.0\mu m$ to match the assignment specification.

2.1 $i_D(v_{GS})$, NMOS, active region

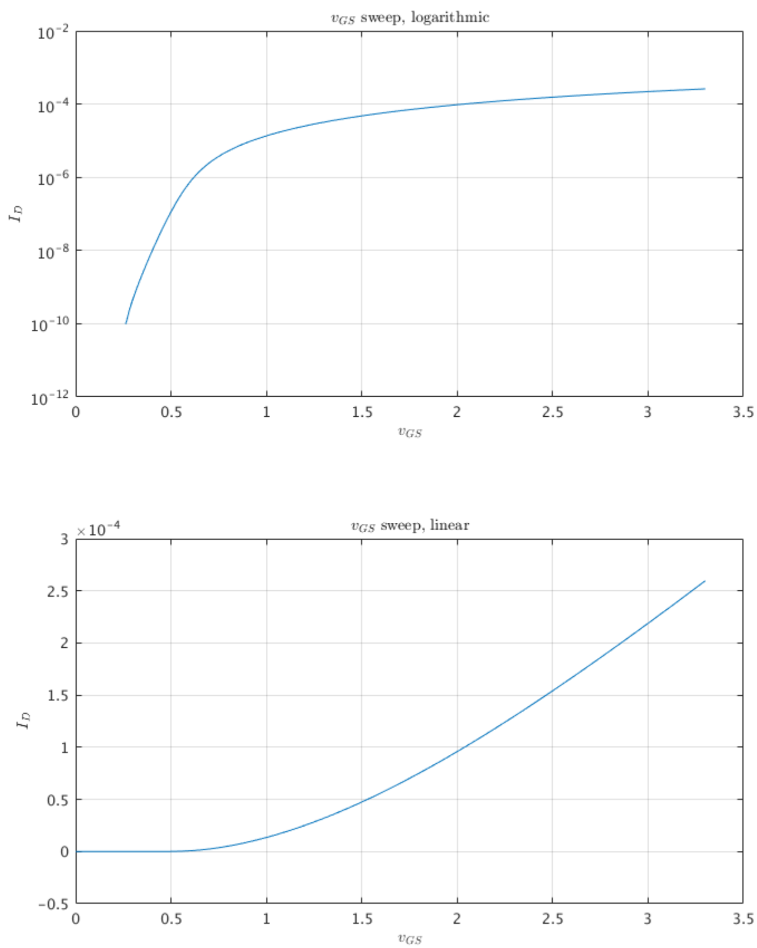
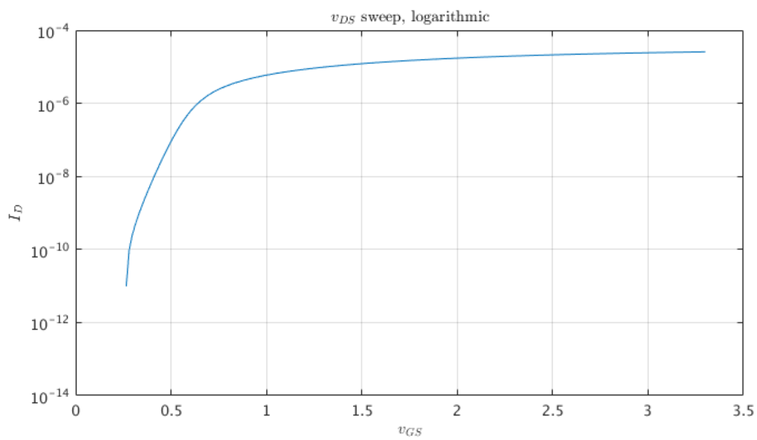


Figure 2.1: Cadence simulation of NMOS transistor in active region.

2.2 $i_D(v_{GS})$, NMOS, triode region



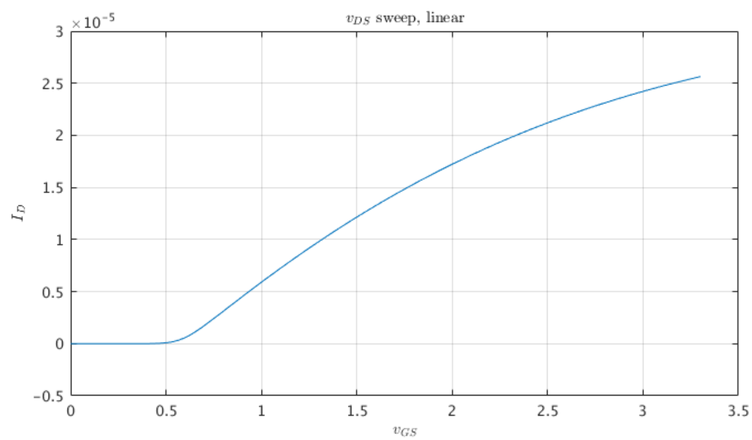


Figure 2.2: Cadence simulation of NMOS transistor in triode region.

2.3 $i_D(v_{DS})$, NMOS, strong inversion

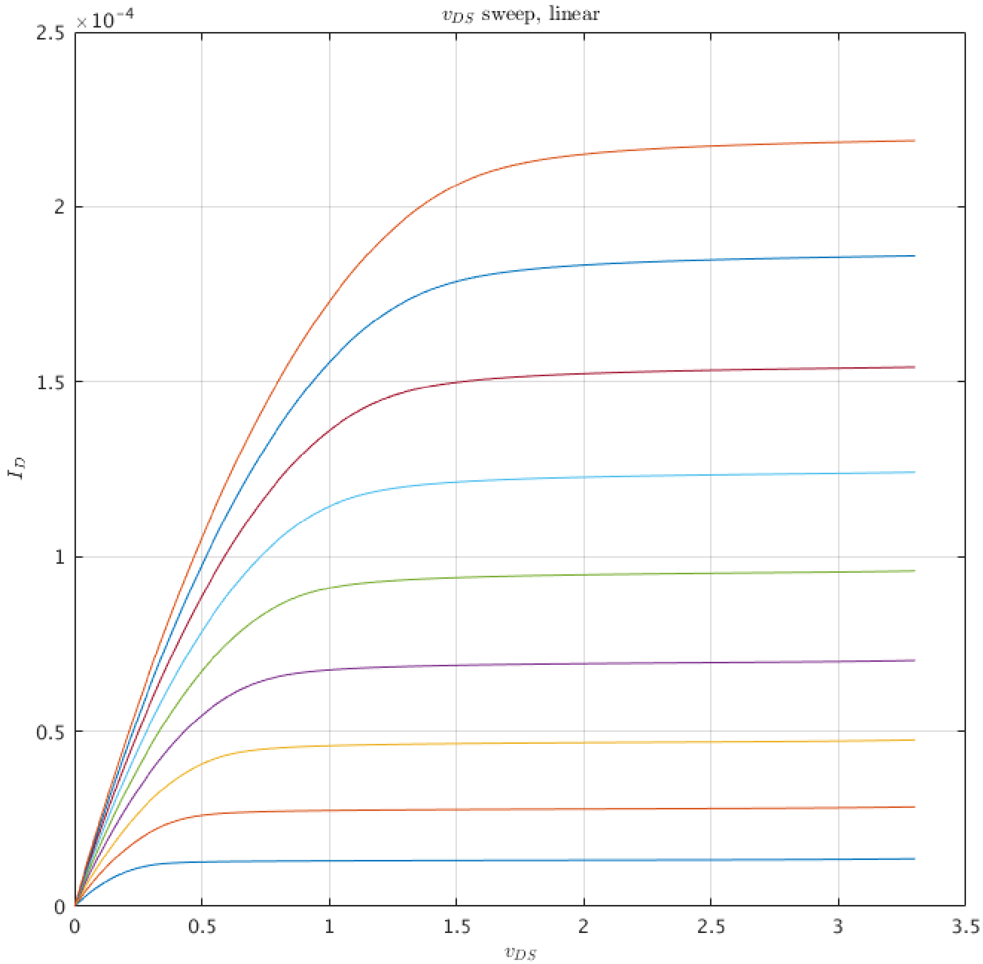


Figure 2.3: Cadence simulation of NMOS transistor in strong inversion.

2.4 $i_D(v_{DS})$, NMOS, weak inversion

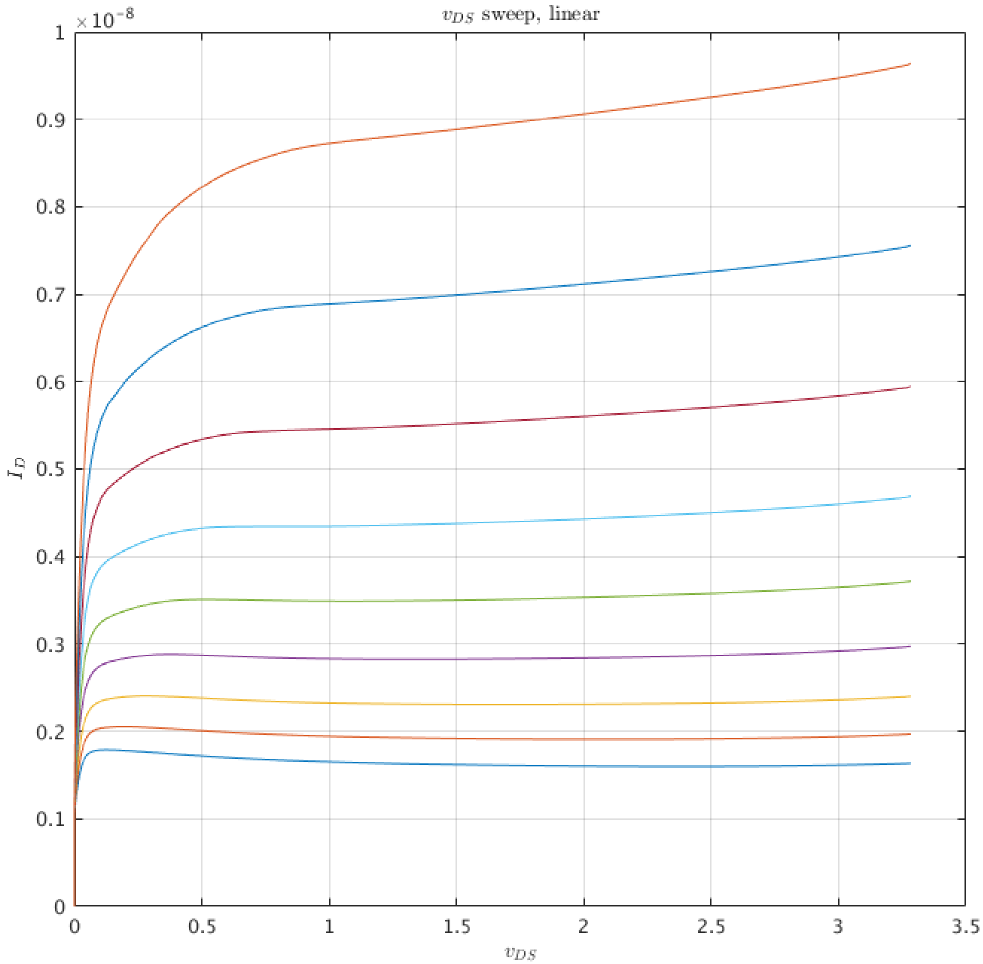


Figure 2.4: Cadence simulation of NMOS transistor in weak inversion.

3 Matching the EKV-model to simulation

In this task we try to match the EKV-model with the simulation results from Cadence. In figure 3.1, 3.2, 3.3 and 3.4 we can see the original EKV-model from task 1 plotted together with the Cadence simulation from task 2. It is easy to see that the model implemented in task 1 does not match the Cadence simulation. Tuning this model means we, by trial and error, changed the assumed $0.35\mu m$ CMOS typical values[4] from task 1: λ, k and V_t. Changing V_t moves the transition between weak and strong inversion, as well as the transition between triode and saturation in strong inversion. Changing k affects the plot in weak inversion exponentially, as well as the slope in the triode region linearly. Changing λ affects the slope in the saturation region proportional to v_{DS}.

EKV-model from task 1

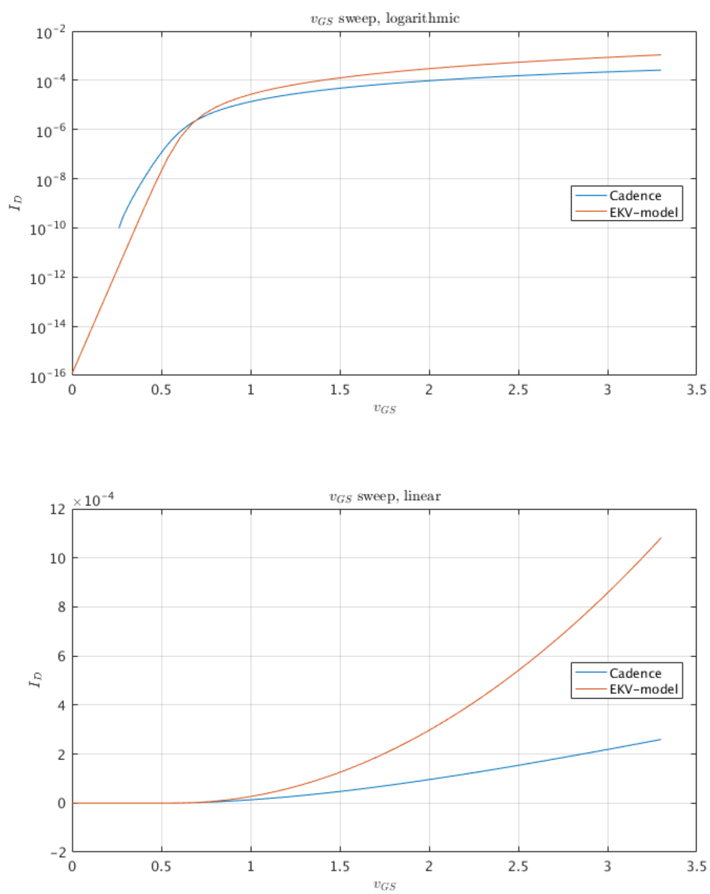


Figure 3.1: Cadence simulation and unmatched EKV-model in the active region

[4] In the EKV paper [1] k is written as β

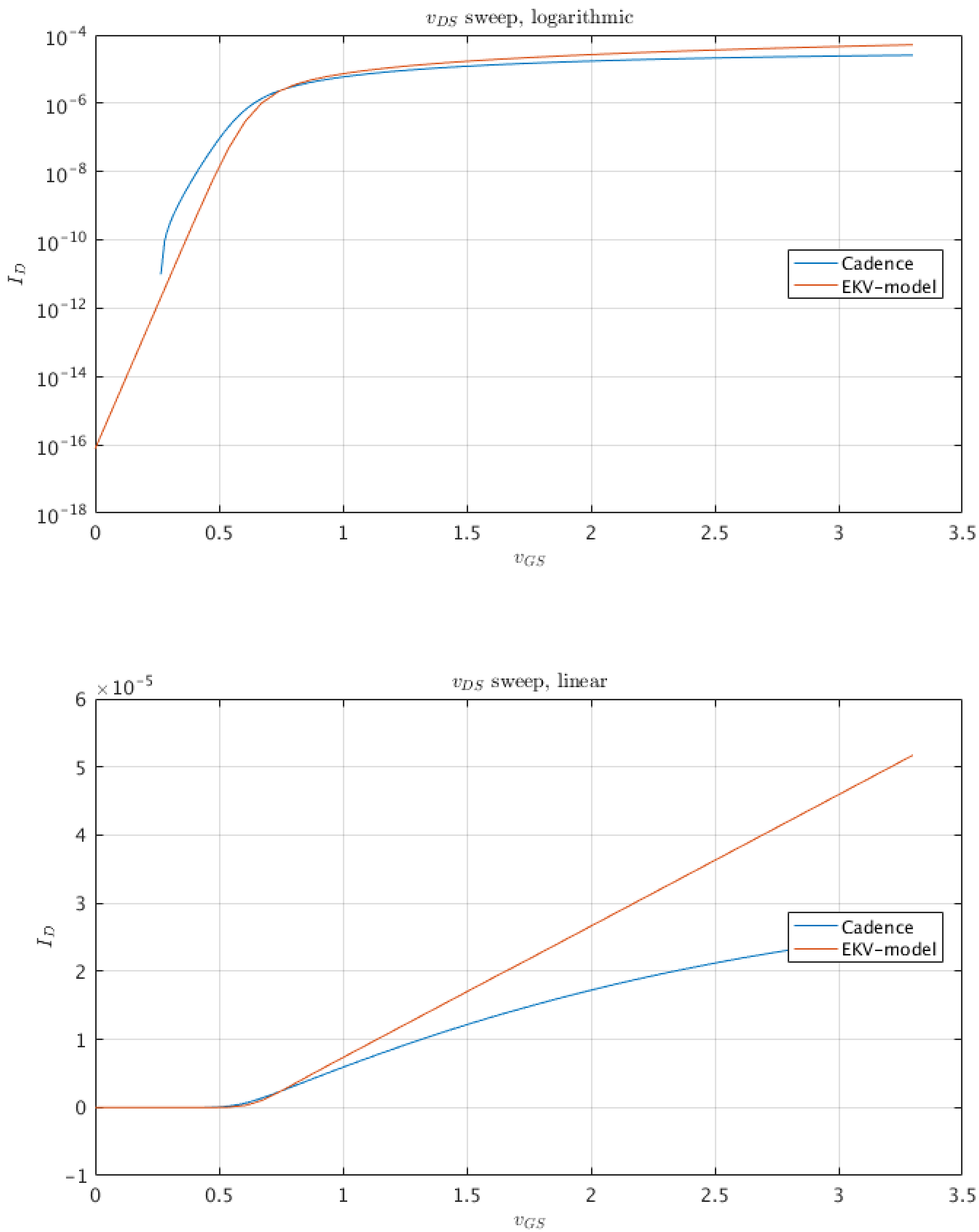


Figure 3.2: Cadence simulation and unmatched EKV-model in the triode region

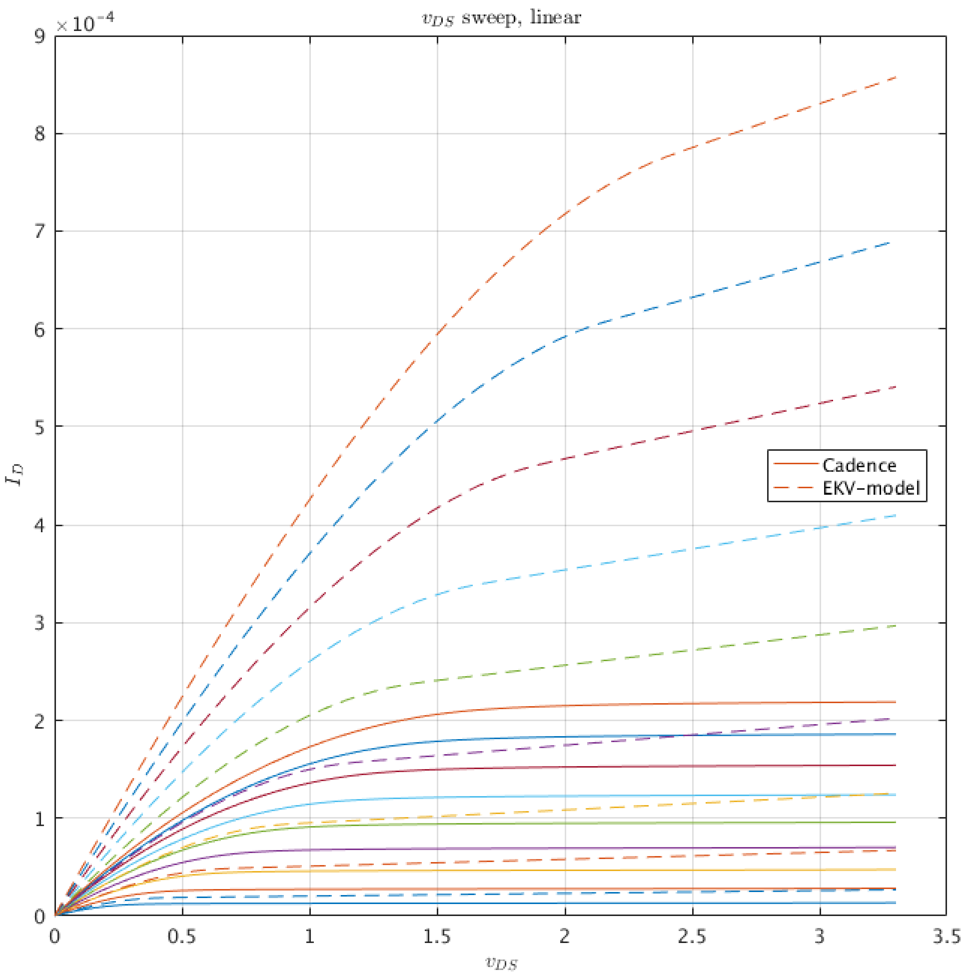


Figure 3.3: Cadence simulation and unmatched EKV-model in strong inversion

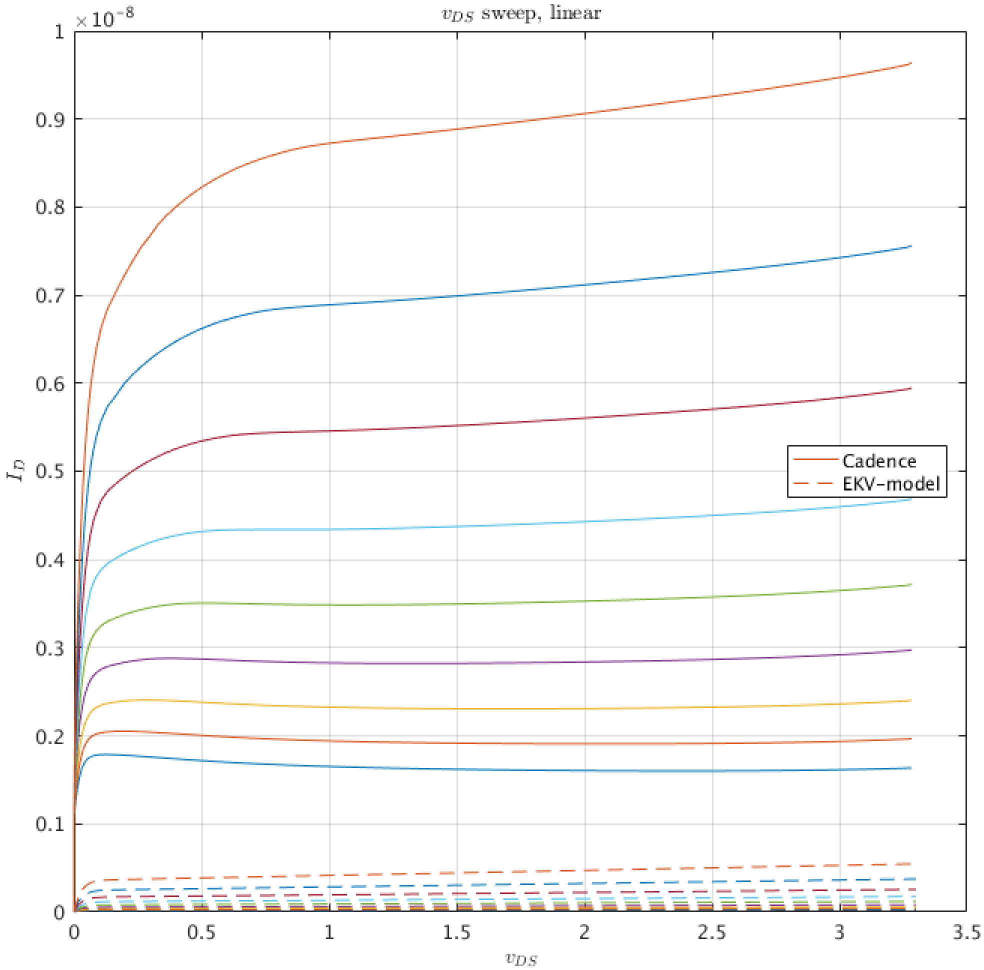


Figure 3.4: Cadence simulation and unmatched EKV-model in weak inversion

Matched EKV-model

Matching the EKV-model so that it is matched for all regions proved impossible. The plot of the active region was chosen as basis for the parameter estimation, and the parameters were changed so that the simulation and EKV-model of that region were as similar as possible. See figure 3.5. The plots of the other regions using the same parameters can be seen in figure 3.6, 3.7 and 3.8. The matched parameters were as follows:

$$V_{tn} = 0.45V$$
$$\lambda L = 0.6\frac{\mu m}{V}$$
$$k_n = 30\frac{\mu A}{V^2};$$

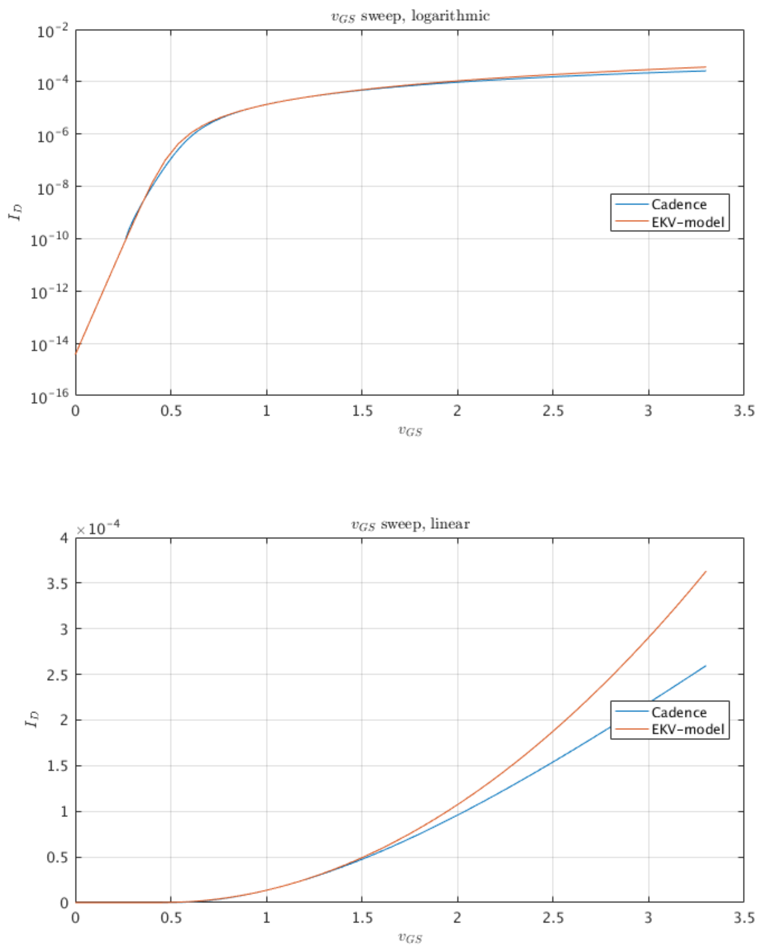
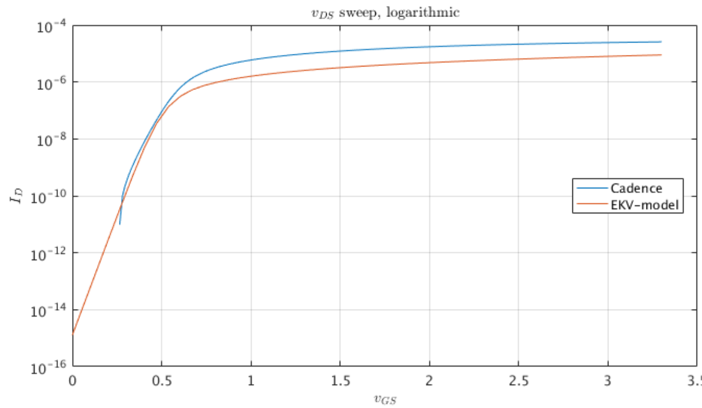


Figure 3.5: Cadence simulation and matched EKV-model in the active region



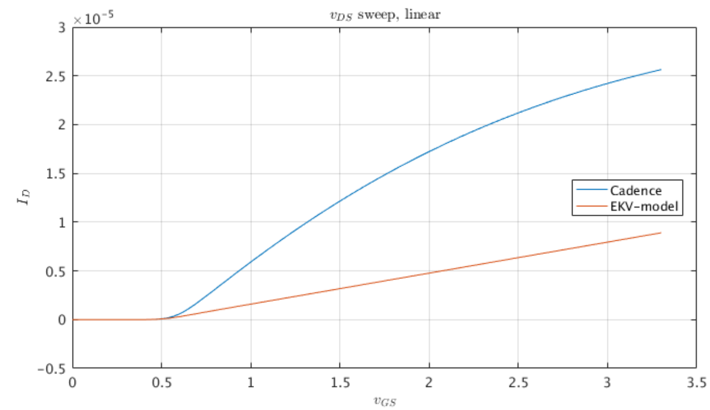


Figure 3.6: Cadence simulation and matched EKV-model in the triode region

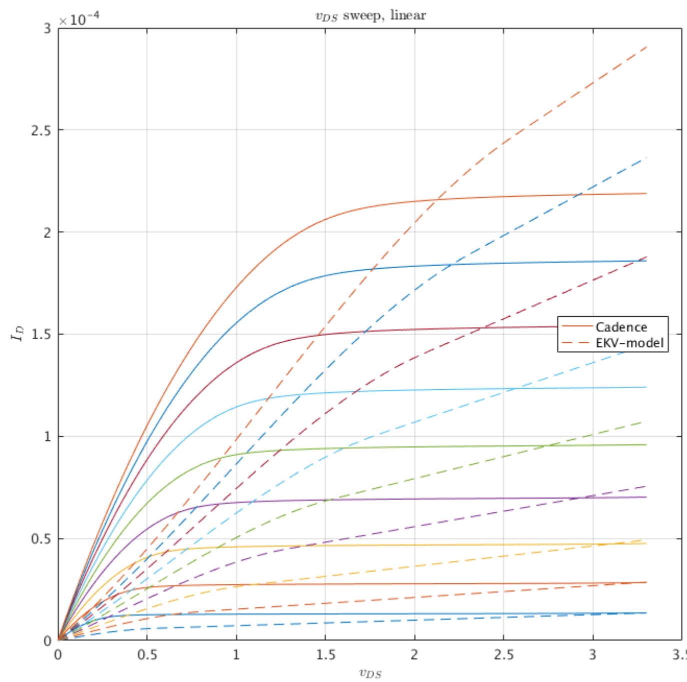


Figure 3.7: Cadence simulation and matched EKV-model in strong inversion

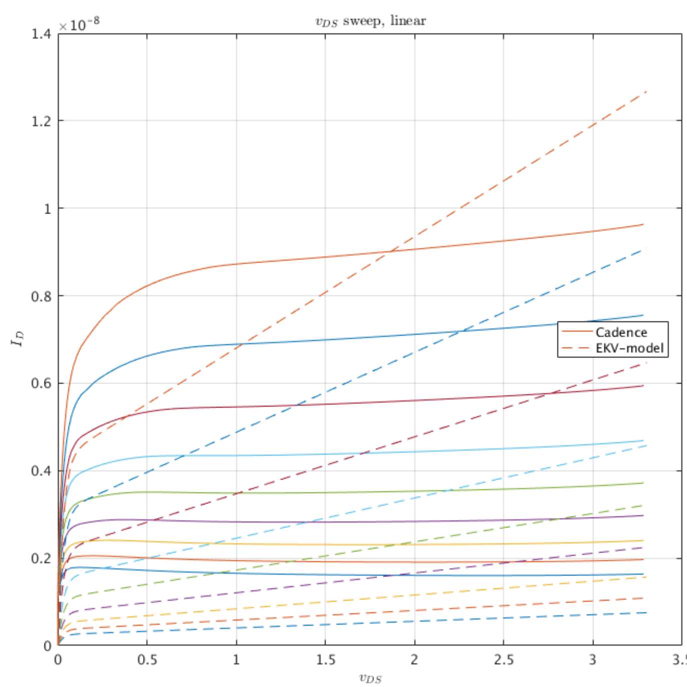


Figure 3.8: Cadence simulation and matched EKV-model in weak inversion

To illustrate the effect of matching a single plot, see figure 3.9 and 3.10. In this, the plot for strong inversion was used to match the parameters, by trying to fit the lowest v_{GS} value. As one can see, using these matched parameters for weak inversion produced a plot that does not look matched. For this reason, the parameters were matched using the active region instead.

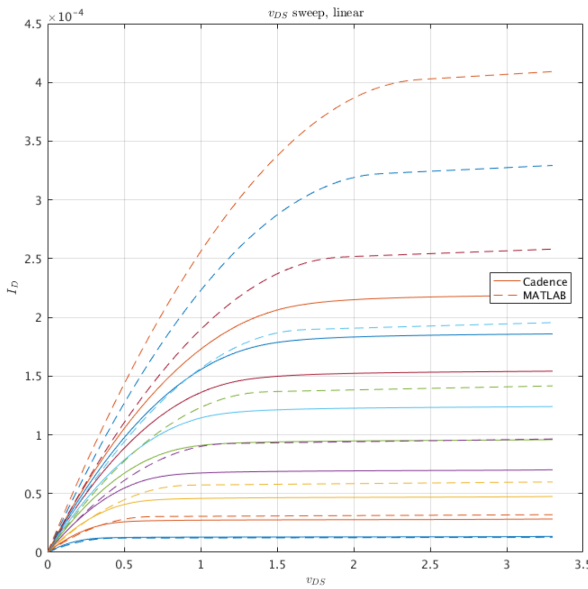


Figure 3.9: Example of matched EKV-model in strong inversion

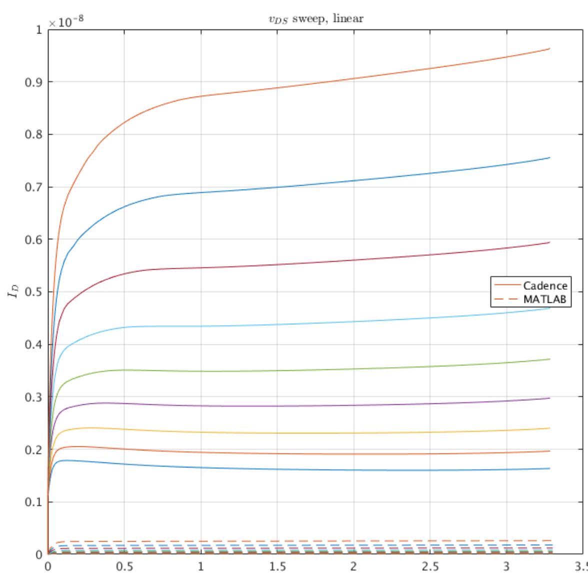


Figure 3.10: Example of matched EKV-model in weak inversion

4 Measurements of TI CD4007UB

Background information

The IC available from the lab differs from the one mentioned in the task text. The one available is the Texas Instruments CD4007UBE. From the datasheet we find that the supply voltage, V_{dd} should be between $-0.5V$ and $20V$. The pinout is shown in figure 4.1

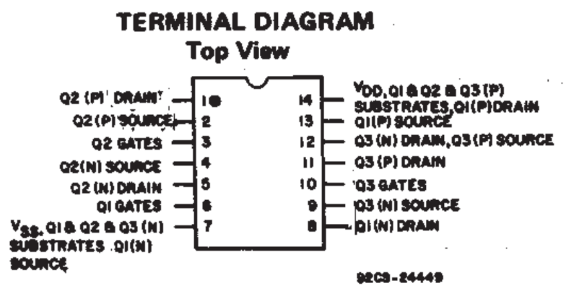


Figure 4.1: CD4007UBE Pinout

The measuring setup is shown in figure 4.2. We have made a drawing of the setup in order to make clear how the probes from the voltage supply and the ammeter were connected to obtain the desired results, as this was impossible to show clearly by photographing the bench.

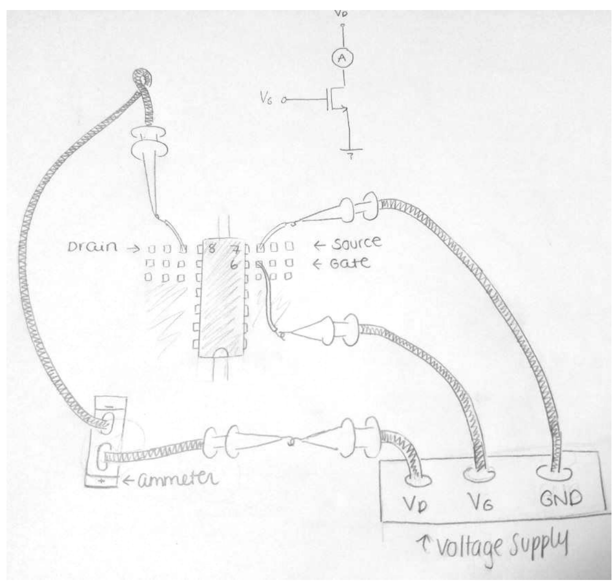


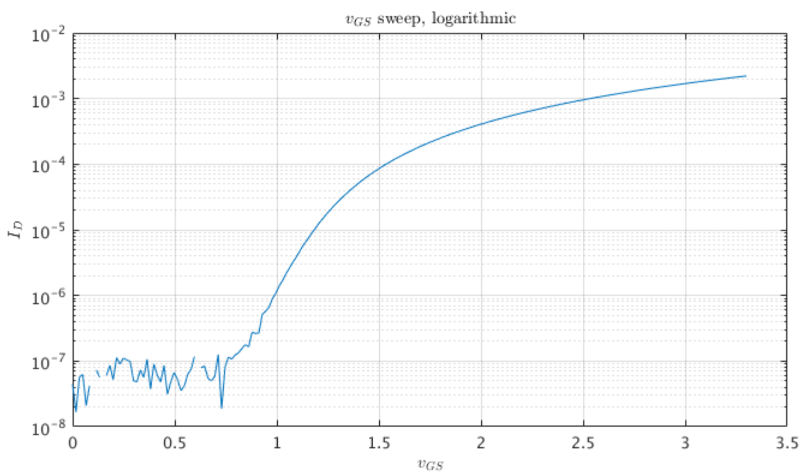
Figure 4.2: Measuring setup

4.1 $i_D(v_{GS})$, NMOS, active region

To measure the active region we use the same parameters as in the EKV-modelling from the first task. Therefore we set the sweep settings to what is shown in listing 1. This will sweep v_{GS} linearly from $0V$ to v_{DS}.

Listing 1: Settings for active region sweep

```
vdd = 3.3;
v_gs = linspace(0,3.3,200);
```



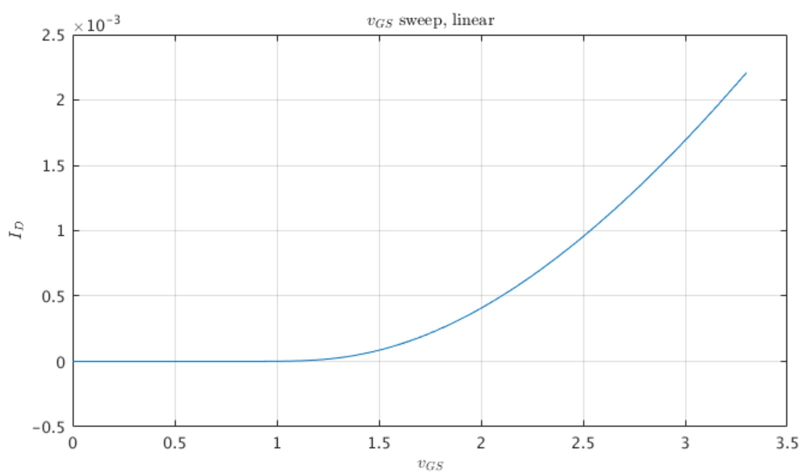


Figure 4.3: Measuring the active region

4.2 $i_D(v_{GS})$, NMOS, triode region

To measure the triode region we use the same parameters as in the EKV-modelling from the first task. Therefore we set the sweep settings to what is shown in listing 2.This will sweep v_{gs} linearly from $0V$ to $3.3V$ and set $v_{ds} = 0.1V$.

Listing 2: Settings for triode region sweep

```
vdd = 0.1;
v_gs = linspace(0,3.3,200);
```

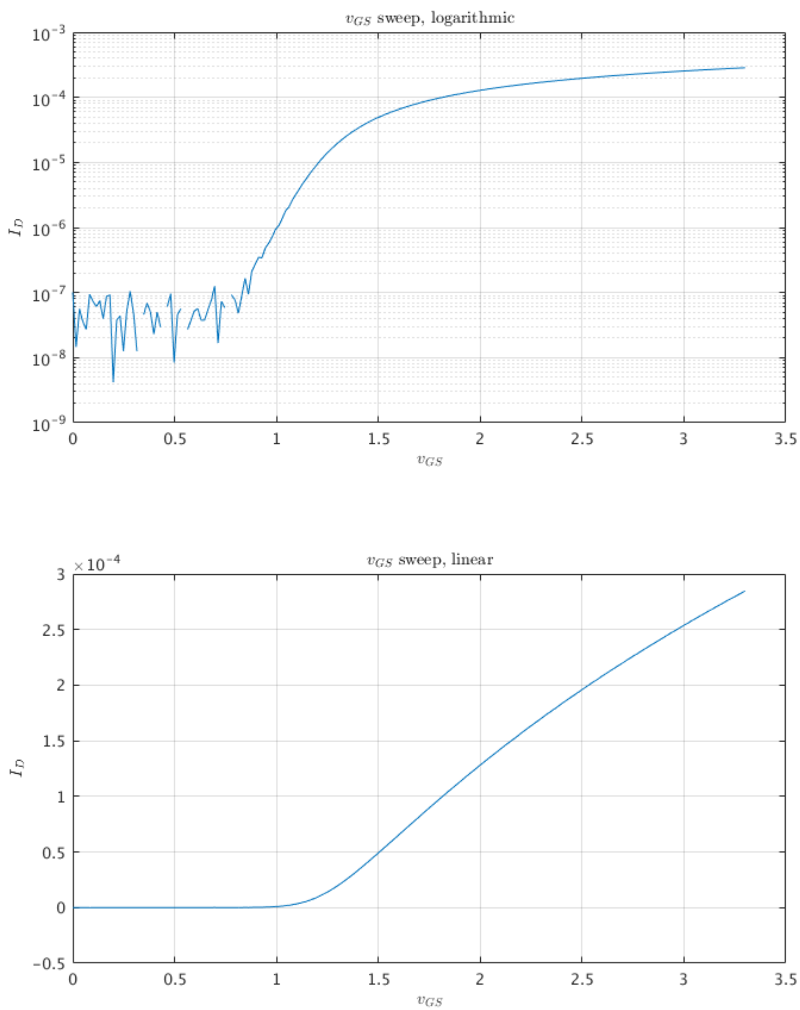


Figure 4.4: Measuring the triode region

4.3 $i_D(v_{DS})$, NMOS, strong inversion

To measure the transistor in strong inversion we use the same parameters as in the EKV-modelling from the first task. Therefore we set the sweep settings to what is shown in listing 3. This will sweep v_{ds} linearly from $0V$ to $3.3V$ and increment v_{gs} in $0.25V$ steps from $1V$ to $3V$.

Listing 3: Settings strong inversion sweep

```
v_gs = (1:+0.25:3);
v_ds = linspace(0,3.3,200);
```

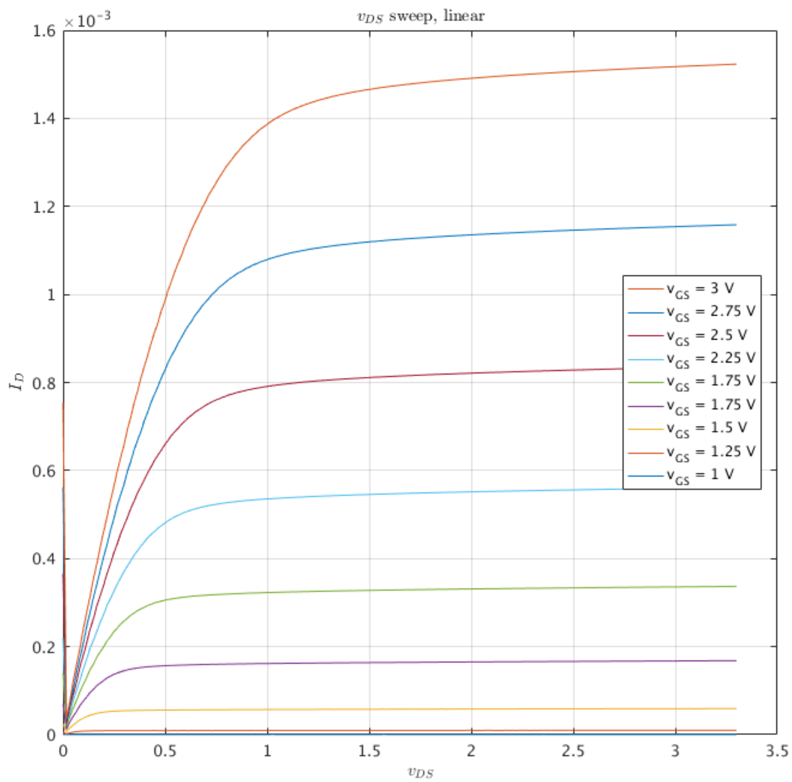


Figure 4.5: Measuring the transistor in strong inversion

4.4 $i_D(v_{DS})$, NMOS, weak inversion

Measuring the characteristic of the transistor in weak inversion is very hard. The expected drain currents are in the order of $10^{-10}A$, which is far below what our instrument is capable of measuring. The maximum resolution of the ammeter is 10nA on a 10mA range, according to the user manual.

We attempted to bypass this shortcoming using two different measuring circuits, which are shown below. To our dismay neither of the approaches yielded any useful results.

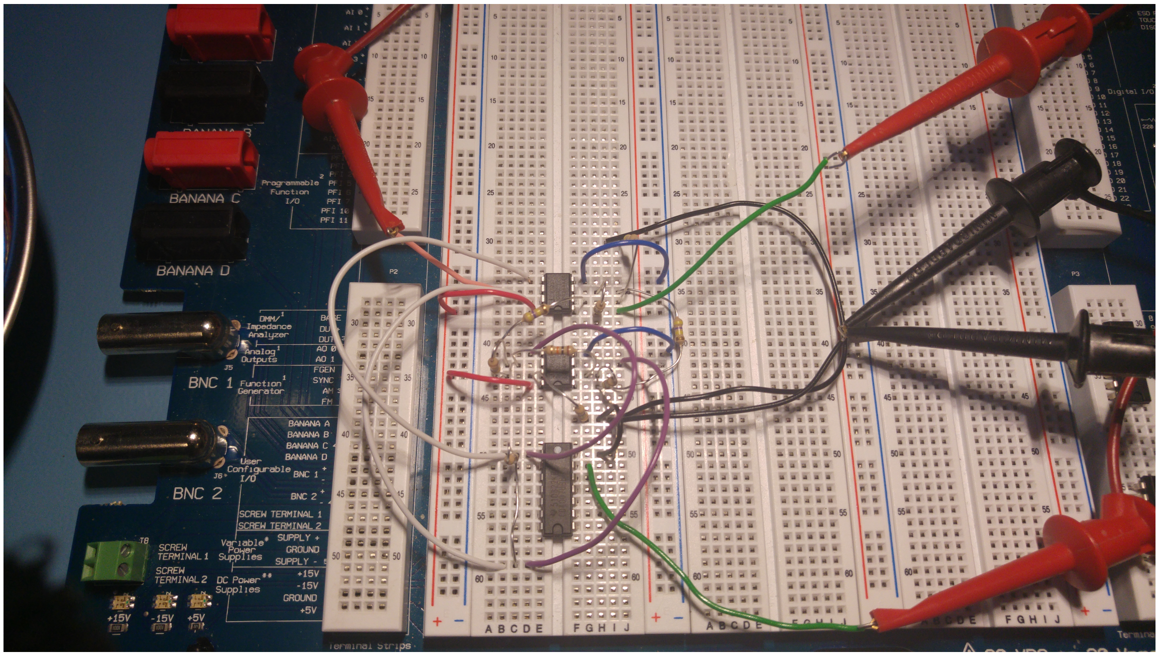


Figure 4.6: First setup for measuring I_d in weak inversion.

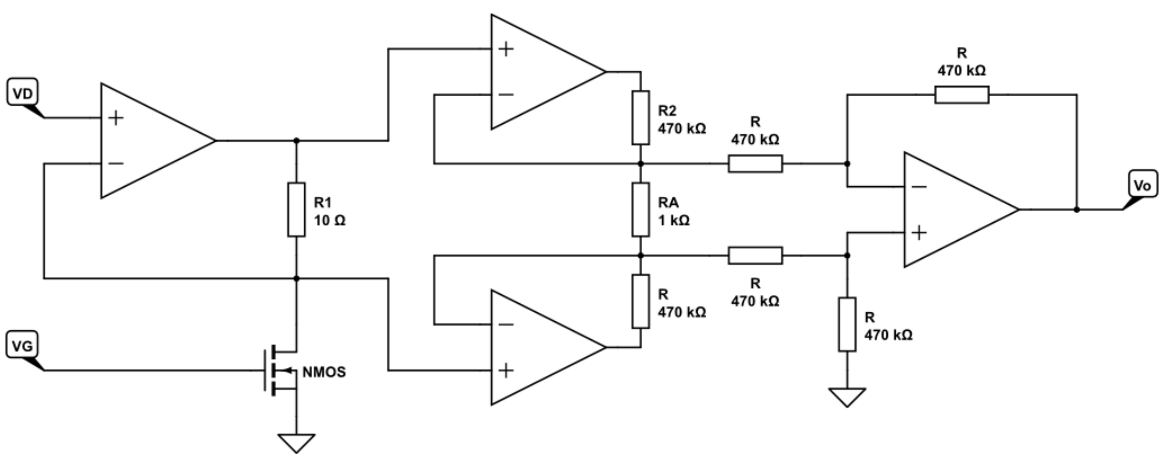


Figure 4.7: Setup with instrumentation amplifier.

The purpose of the first operational amplifier is to hold the drain voltage constant in such a manner that the voltage drop across the resistor does not affect the characteristic we are attempting to measure.

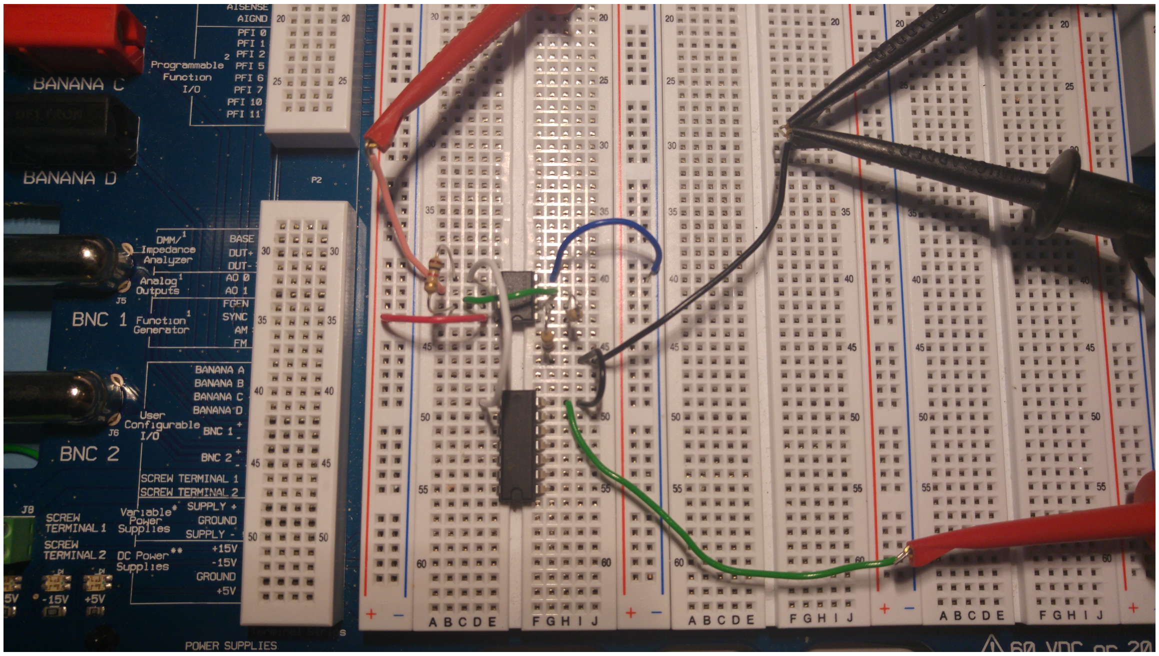


Figure 4.8: Second setup for measuring I_d in weak inversion.

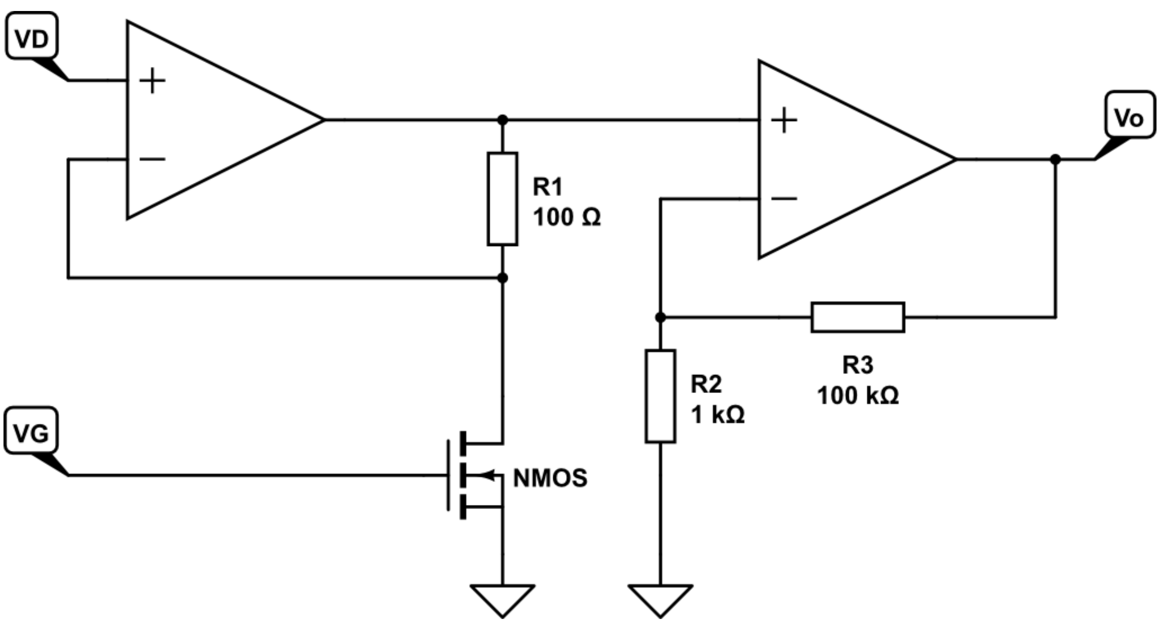


Figure 4.9: Setup with single non-inverting amplifier.

In the second setup we attempted using a larger resistor to measure over, seeing as the drain voltage would be held constant due to the

5 Matching the EKV-model to measurements

For the same reasons as mentioned in task 3, the parameters were matched using the plot for the active region. The matched plot can be seen in figure 5.1. The parameters were estimated to be as follows:

$$
\begin{aligned}
V_{tn} &= 1V \\
\lambda L &= 0.7\frac{\mu m}{V} \\
k_n &= 250\frac{\mu A}{V^2};
\end{aligned}
$$

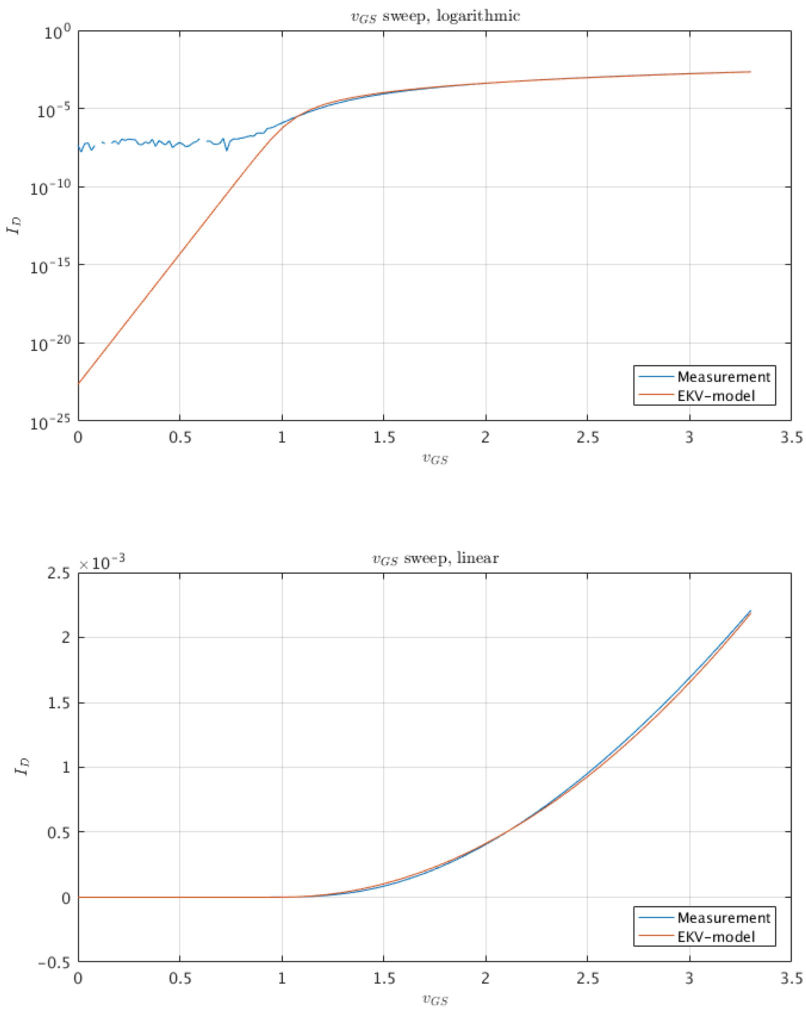


Figure 5.1: Measurements and matched EKV-model in the active region

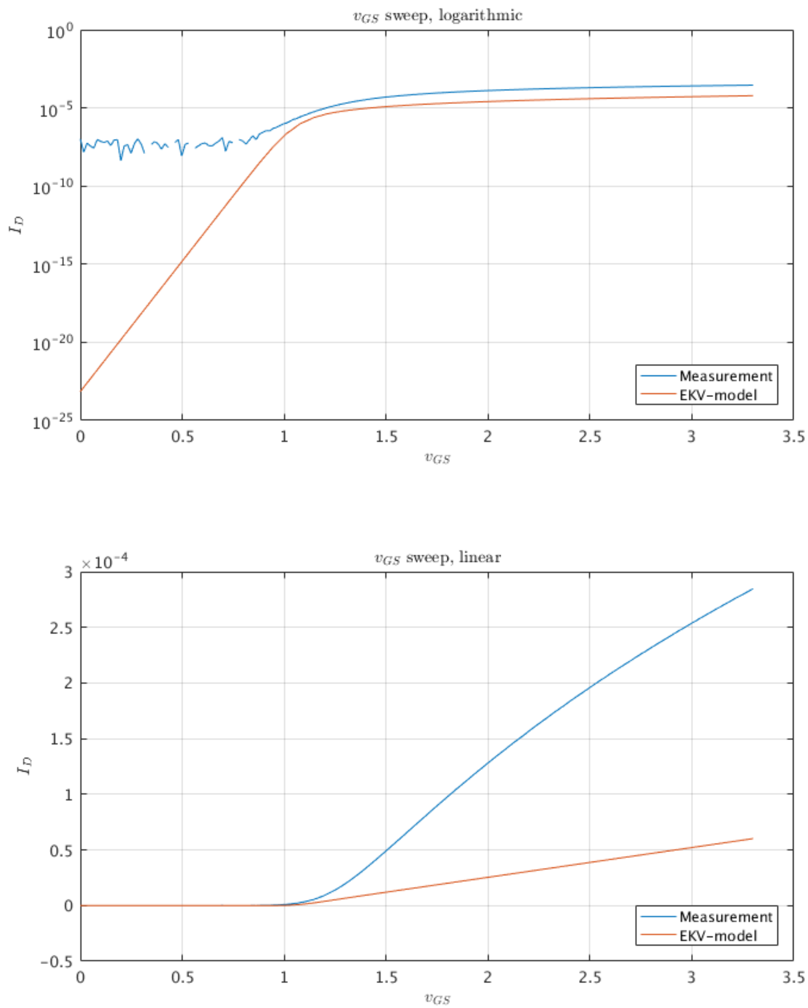


Figure 5.2: Measurements and matched EKV-model in the triode region

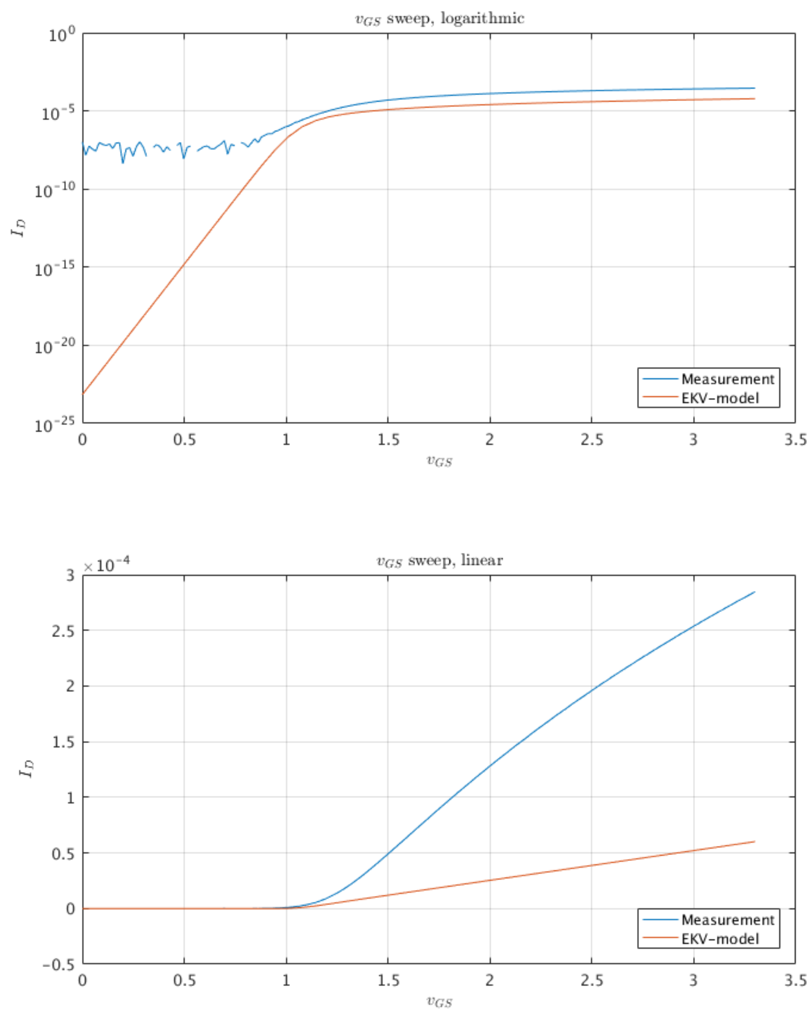


Figure 5.3: Measurements and matched EKV-model in strong inversion

6 Common Source Amplifier

The circuit used in these measurements is shown in figure 6.1. For this circuit we used resistors with nominal values of $2.2k\Omega$, $220k\Omega$ and $2M\Omega$ (2x $1M\Omega$ in series), and for simplicity we used the same supply voltage as for the previous tasks of $3.3V$. In figure 6.2 we compare the voltage outputs of the amplifier with the three different resistors. With a fixed resistor, the only variable able to change the biasing point is v_{GS}. We would like to bias it at a point where the response is as linear as possible.

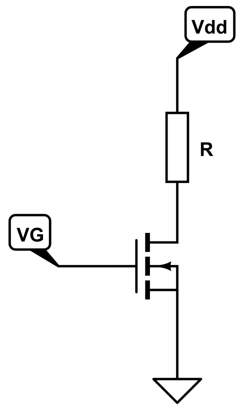


Figure 6.1: Common Source Amplifier Circuit

From the plots in figure 6.2 we can see that for $R = 2.2k\Omega$ a suitable Q-point can be found at $v_{out} = v_{ds} \simeq 2V$. We can find similar values for the other resistances as well. Using the measured values we find the Q-points shown in table 6.1. We found the gain by calculating the incline of the linear region. To calculate the current we simply use ohms law as shown in equation 13 for each measurement required.

A striking feature of the plot is that the height of the flat region differs with the magnitude of the drain resistance. According to Sedra & Smith, the height should correspond to Vdd. However, Sedra & Smith work with a strong inversion model that does not take into account the weak inversion currents. These small currents cause a voltage to drop across the drain resistor, and this voltage will naturally be larger with increasing resistance according to Ohms law.

These A-, B- and C-points in figure 6.2 are found by using formulas from Sedra & Smith [6]. The A-point marks the transition cutoff-saturation, the B-point marks the transition saturation-triode, while the C-point marks the end of triode. The formula calculating the B-point does not take channel length modulation into account, nor the possibility of weak inversion, so it is quite simple.

$$
\begin{aligned}
v_{GS,A} &= V_t \\
v_{GS,B} &= v_{DS,B} - V_t \\
v_{GS,C} &= V_{dd}
\end{aligned}
$$

$$I_D = \frac{V_{DD} - V_{out}}{R} \tag{13}$$

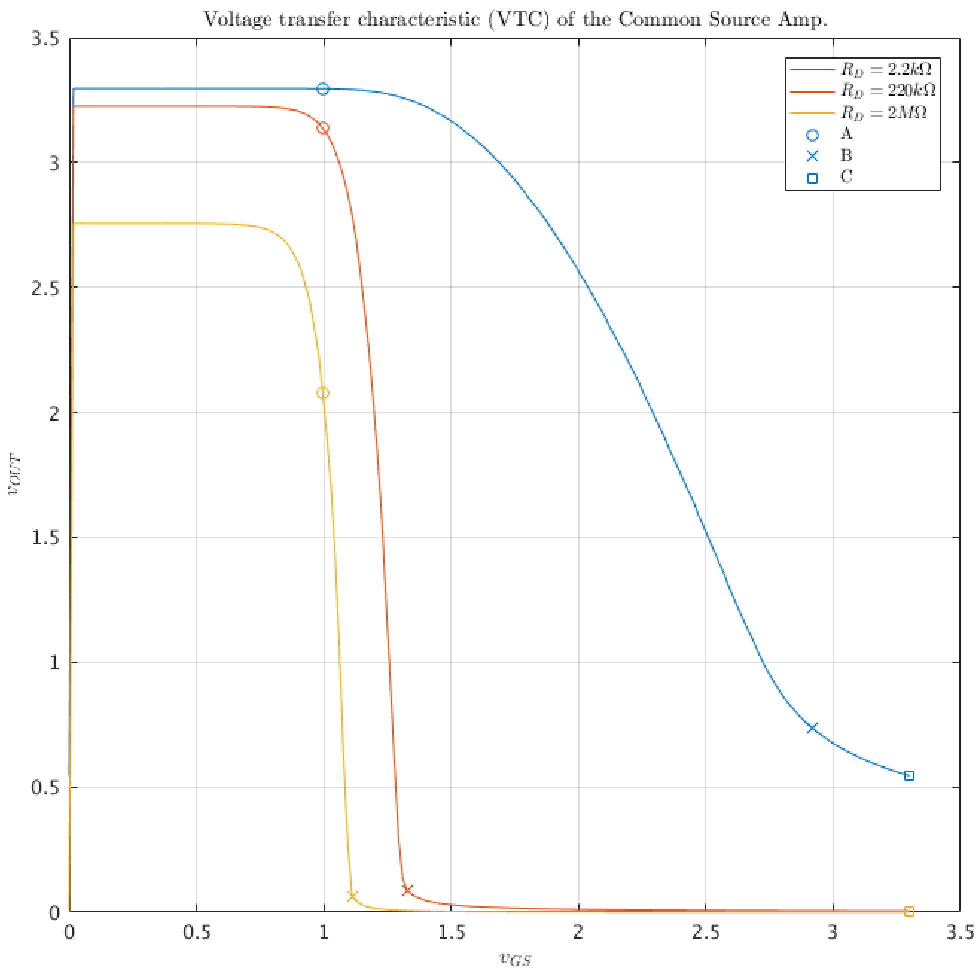


Figure 6.2: Voltage Transfer Characteristic of Common Source Amplifier

Table 6.1: Q-points of Common Source Amplifier

Resistance	$V_{GS}[V]$	$V_{out}[V]$	$I_D[\mu A]$	Gain
$2.2k\Omega$	2.2884	2.0071	587.66	-1.930
$220k\Omega$	1.2271	1.5836	7.8019	-14.00
$2M\Omega$	1.0613	1.0597	1.1201	-19.95

If we bias the amplifier at the points in table 6.1 none of them should be in weak inversion. However for the $220k\Omega$ and $2M\Omega$ circuits V_G is not far above the threshold voltage, which has been measured in previous tasks to be $V_t \simeq 1V$ and can easily be pushed out of the linear region by a sufficiently large input signal. In addition to this for the $2.2k\Omega$ resistor when $V_{GS} = 2.5V$ we get that $V_{DS} = V_{OV}$. This means that beyond this point the transistor is no longer in the saturation region, but in the triode region. We suspect that increasing the supply voltage would increase our linear region and raise point B out of the triode region.

In figure 6.3 we can see a graphical analysis of the bias point for the $2.2k\Omega$ resistor. Doing this type of analysis for the two other resistors was not possible as the currents are too small (μA range). The straight line is $I_D(V_{GS})$ overlaid on the calculated values of $I_D(V_{DS})$ from the EKV-model. The circuit will have the highest bandwidth when using the $2.2k\Omega$, this is because this circuit has the lowest gain. If we had a GBW for the transistor we could estimate the total bandwidth of the circuit. Consequently the circuit will have the lowest bandwidth when using the $2M\Omega$ resistor.

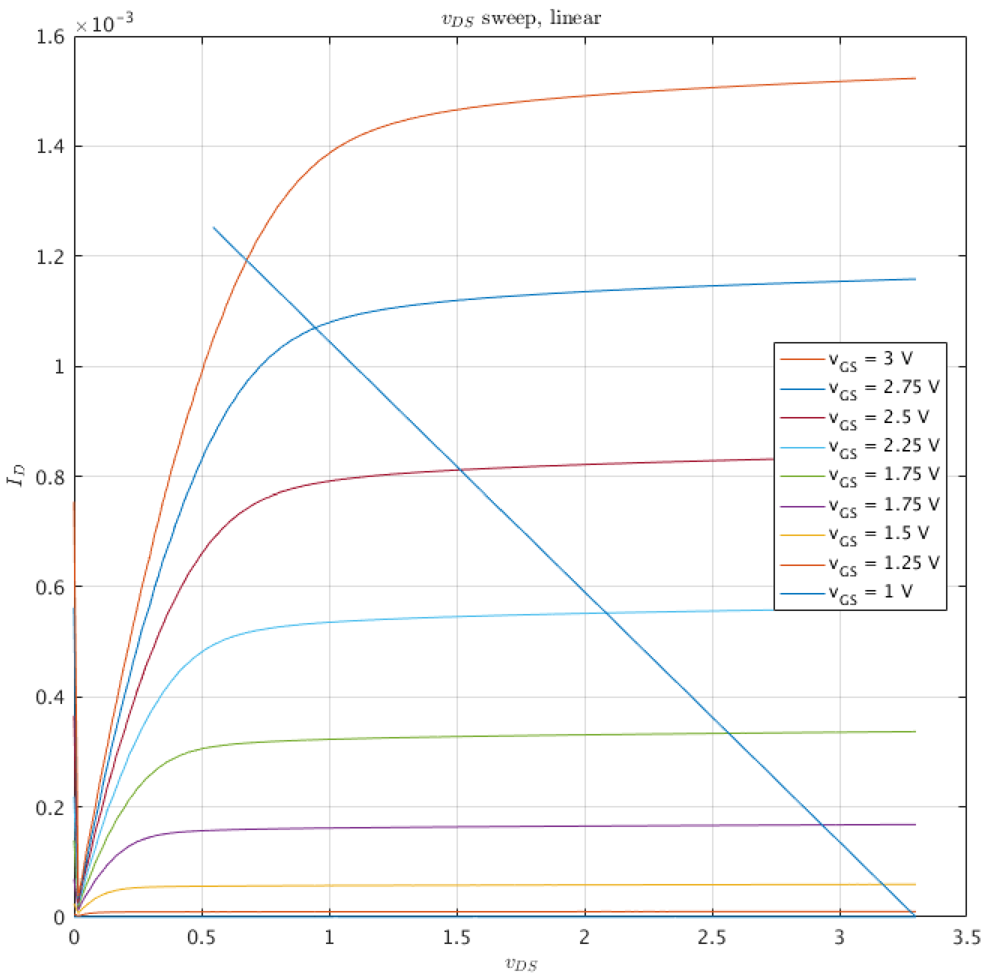


Figure 6.3: Graphical method to determine VTC of a simple amplifier

Code listings

In this section some essential MATLAB code from the project is listed.

Listing 4: EKV-model example

```
for VG=1:0.25:3
for VD=0:step:Vdd %nested for-loop to avoid matrix multplicative error
I_F = IS*((log(1+exp((VG-VT-n*VS)/(2*n*UT)))).^2)*(1+lambda*VD); % mulitplicative term (1+
    lambda*VD) for clm
I_R = IS*((log(1+exp((VG-VT-n*VD)/(2*n*UT)))).^2)*(1+lambda*VD);
I_D(j) = I_F-I_R;
V_D(j) = VD;
j=j+1;
if j==201
    j=1;
end
end
plot(V_D,I_D)
hold on

%finding overdrive voltage points on the curves:
VP(i)= (VG-VT)/n; %from EKV paper, n=1 => same as overdrive voltage
[dist index] = min(abs(VP(i)-V_D)); %finding index of closest value to pinchoff in VD-
    vector
distance(i)=dist; %to check distances
VP_vect(i,1) = V_D(index);
VP_vect(i,2) = I_D(index);
i = i+1;
end
```

Listing 5: GPIB Measurement example

```
%% Task 5.1
% Plotting ID as a funciton of VGS in the saturation region
% Sweep V_GS 0:3.3, Vdd = 3.3V
vdd = 3.3;
HPE3631_SetVolt(2,vdd);
HPE3631_Operate(psu_adr);

v_gs = linspace(0,3.3,200);
I_D1 = zeros(length(v_gs),1);

display('Starting Sweep 5.1')
for i = 1:length(v_gs)
    str = sprintf('Step %d of %d',i,length(v_gs));
    display(str)
    HPE3631_SetVolt(1,v_gs(i)); %Set v_gs
    pause(0.5)
    HPE3631_Operate(psu_adr);
    pause(0.5)
    I_D1(i) = HP34401_ReadQuick();
    pause(1)
end

display('Task 5.1 Sweep Complete')
```

References

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[2] *CMOS Dual Complementary Pair Plus Inverter*. CD4007UB. Texas Instruments. 2003. http://www.ti.com/lit/ds/symlink/cd4007ub.pdf (visited on 10/22/2016).

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[4] Reid R. Harrison. *Introduction to MOSFET Operation*. http://www.ece.utah.edu/~harrison/ece5720/Intro.pdf (visited on 10/25/2016).

[5] Reid R. Harrison. *MOSFET Operation in Weak and Moderate Inversion*. http://www.ece.utah.edu/~harrison/ece5720/Subthreshold.pdf (visited on 10/25/2016).

[6] Adel S. Sedra and Kenneth C. Smith. *Microelectronic Circuits*. Oxford University Press, 2011. ISBN: 9780199738519.