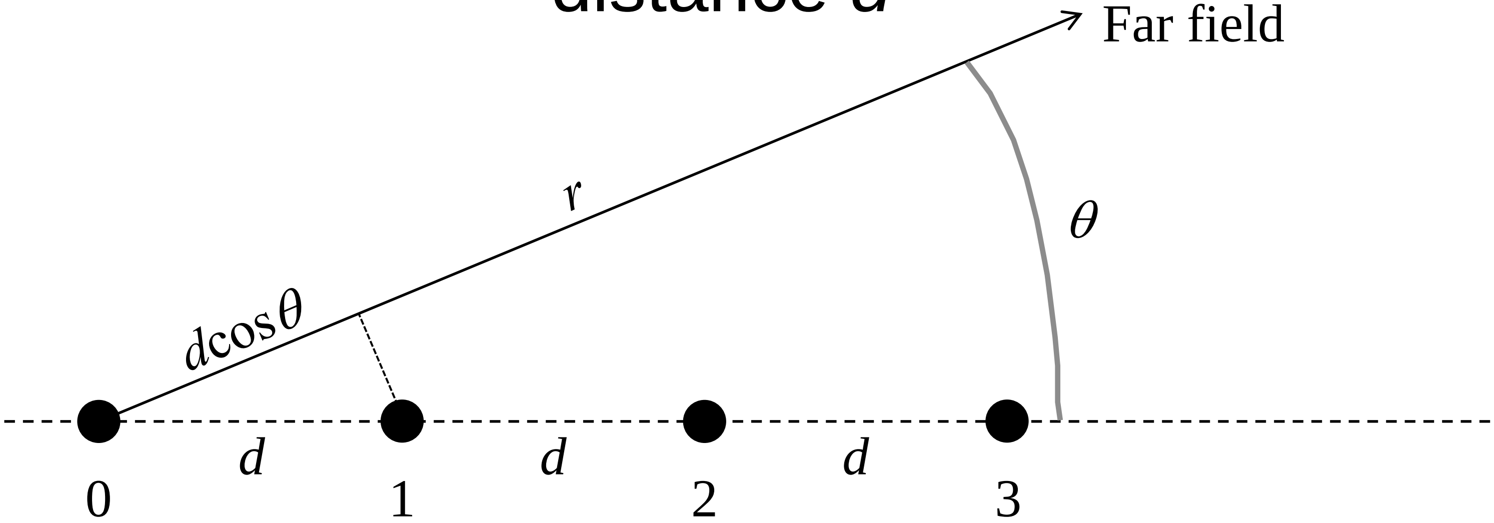
Chapter 4.5-4.10 Antenna fundamentals

- Arrays
- Reflector antennas for radio links and satellite communication
- Horn

Array of equal aligned antennas separated by distance *d*



Write the far field from the first element on the form $E_0(r,\theta,\phi) = I_0 \dfrac{e^{-jkr}}{r} e(\theta,\phi)$ where *e* is called the element factor

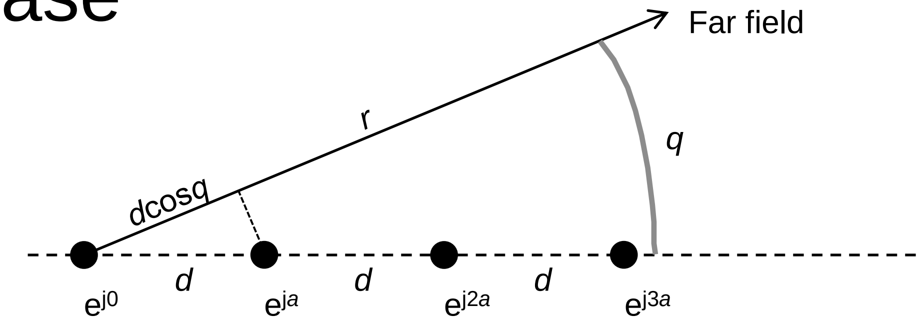
The field from element *n* is of the same form except for phase change caused by the different distance $r - nd\cos\theta$

For *N* elements the total field becomes

$$E(r,\theta,\phi) = \frac{e^{-jkr}}{r} e(\theta,\phi) \sum_{n=0}^{N-1} I_n e^{jknd\cos\theta} = \frac{e^{-jkr}}{r} e(\theta,\phi) F_a$$

where F_a is called the array factor

Assume array with equal amplitude and linear phase



Let I_n constant and equal to 1, and given constant phase shift α then

$$I_n = e^{jn\alpha}$$

The array factor becomes

$$F_a = \sum_{n=0}^{N-1} I_n e^{jknd\cos\theta} = \sum_{n=0}^{N-1} I_n e^{jn(\alpha + kd\cos\theta)}$$

Set $\psi=\alpha+kd\cos\theta$ to write $F_a = \sum_{n=0}^{N-1} I_n e^{jn\psi}$

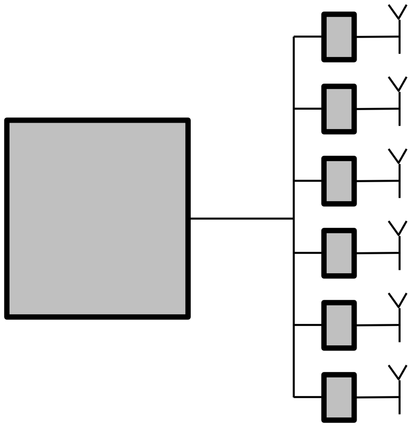
The sum of a final geometrical series gives $F_a = \dfrac{1-e^{jN\psi}}{1-e^{j\psi}}$

$$F_a = \frac{1-e^{jN\psi}}{1-e^{j\psi}} = \left(\frac{e^{-jN\psi/2}-e^{jN\psi/2}}{e^{-j\psi/2}-e^{jN\psi/2}}\right)\left(\frac{e^{jN\psi/2}}{e^{j\psi/2}}\right) = \frac{\sin(N\psi/2)}{\sin(\psi/2)} e^{j(N-1)\psi/2}$$

$$|F_a| = \left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right|$$

Array of antennas

Line array with equal distance *d* between equal elements



Array factor F_a normalised and denominator approx.

$$F_a = \frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad \text{where } \psi = \frac{2\pi}{\lambda}\cos\theta + \alpha$$

The array antenna radiation patterns is the array factor, F_a, multiplied with the antenna element pattern.

Phase shift between elements is α.

Weight applied to the i^{th} element is $a_i = e^{j(i-1)\alpha}$

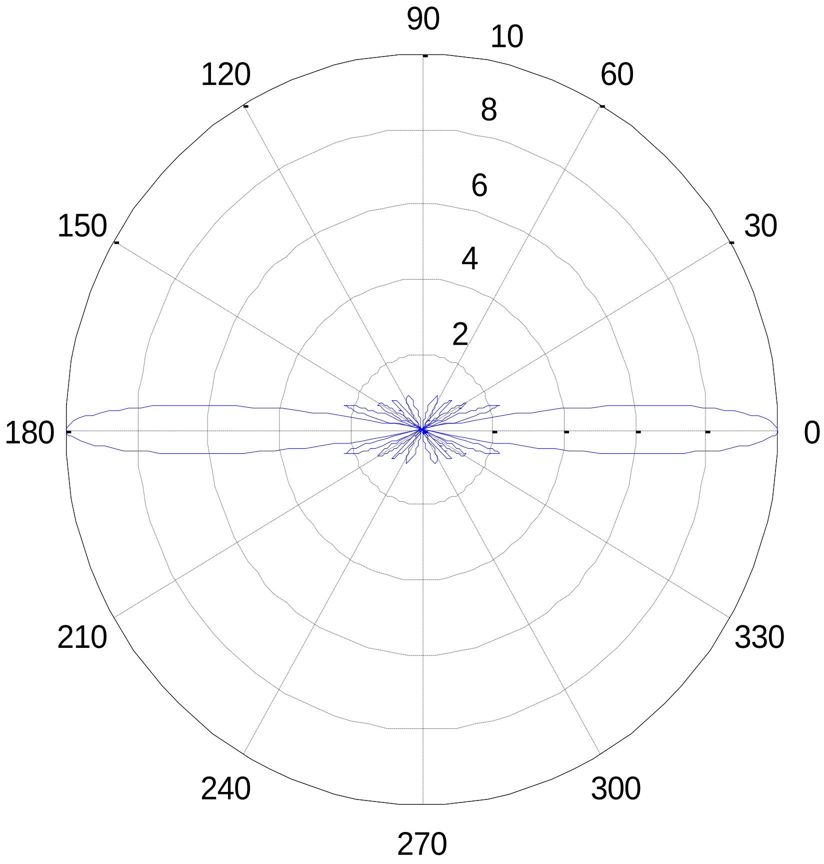
Maximum gain 20log(*n*) (dB) greater than the individual elements.

Array factor

10 elements

$d = \lambda/2$

$\alpha = 0$

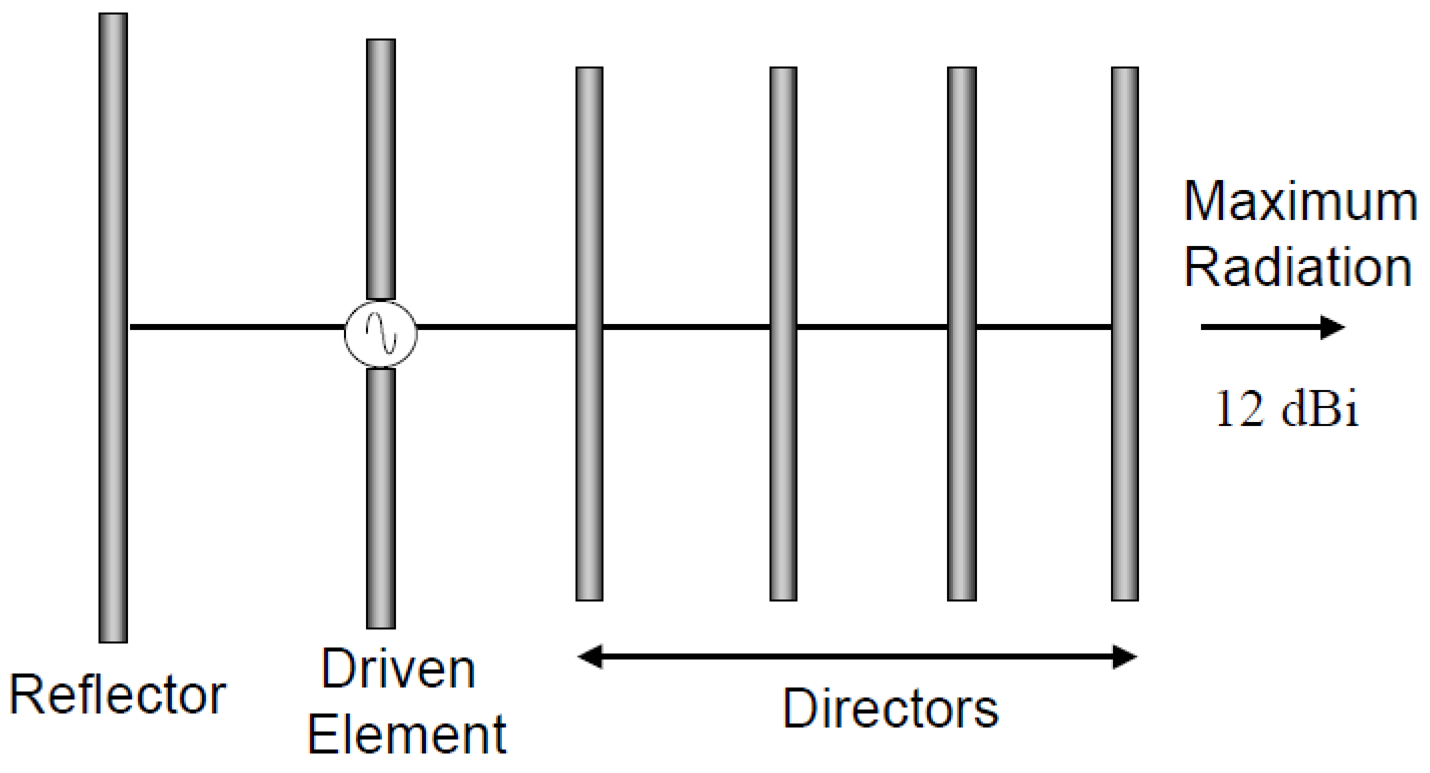


Array factor as function of angle perpendincular to the array line

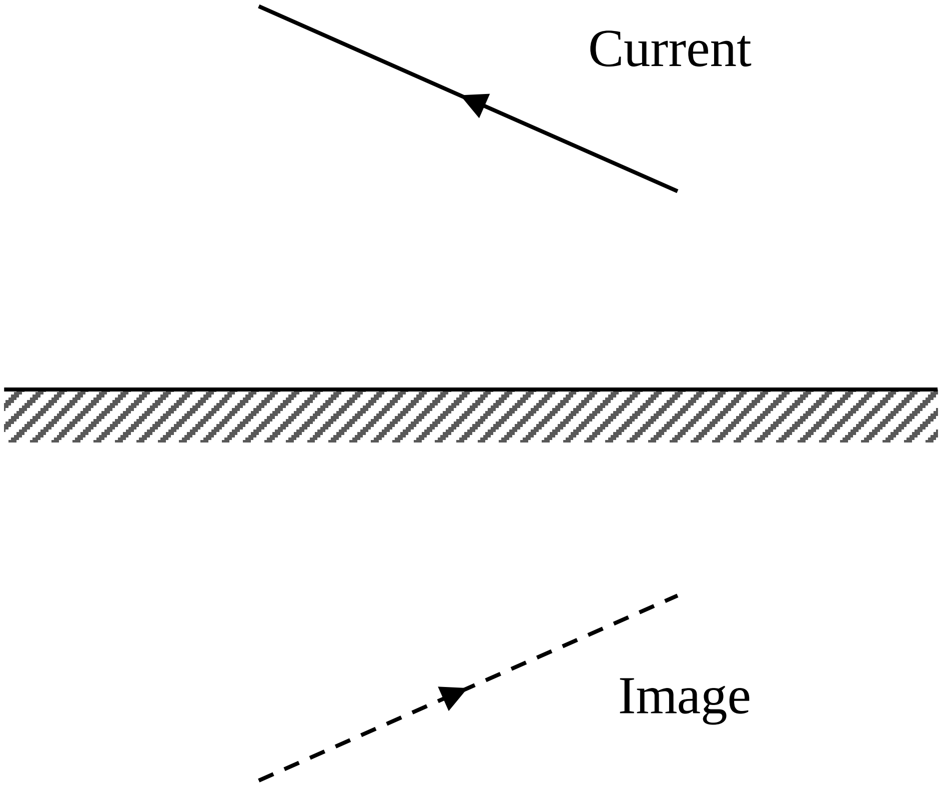
Reflector antennas

- Typical for radio links, satellite (communication and television), space communications
- Reflector antennas have large gain
- Antenna size and radius of curvatures are large compared to the wave length
- Geometrical optical methods
- Classical conical section, parabola, hyperbola, and ellipsoid

Parasitic elements: UDA-Yagi antenna

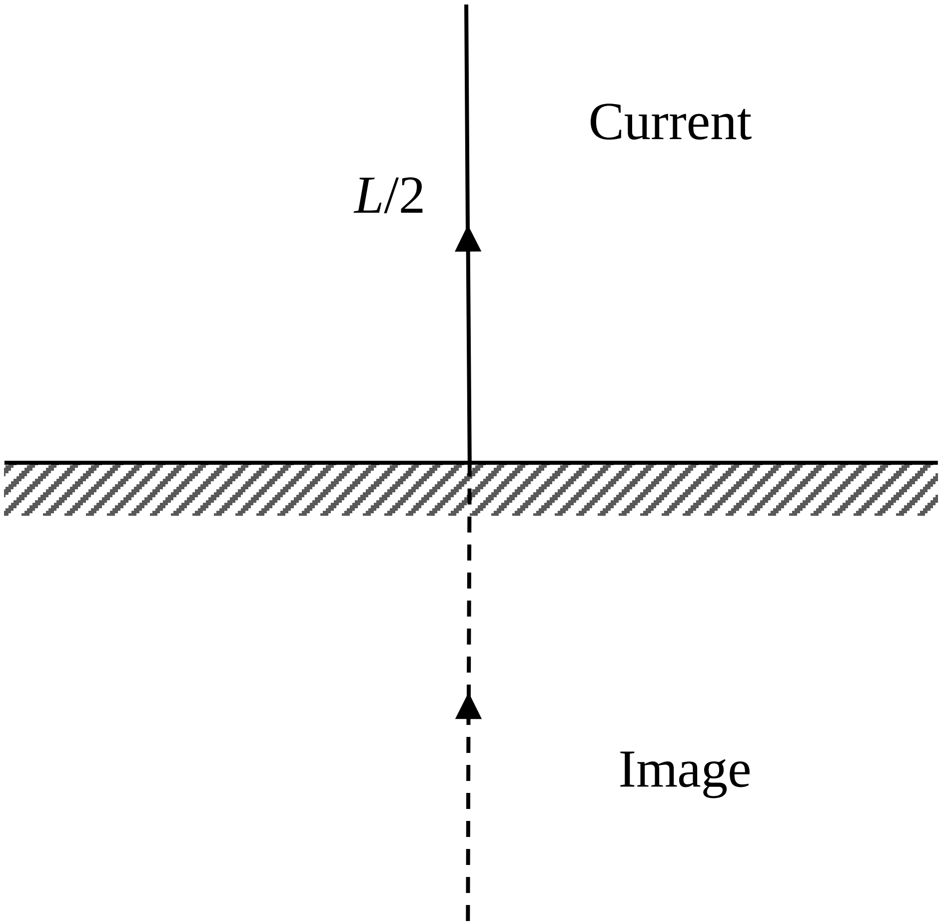


Reflector antennas



Application of image theory. Antenna current placed adjacent to a perfect conducting ground plane. The combined system has the same fields as if an image of the antenna is placed below the plane at the same distance. The image current is of equal magnitude as the current in the real antenna, but opposite direction. Consequence of Snell's reflection law.

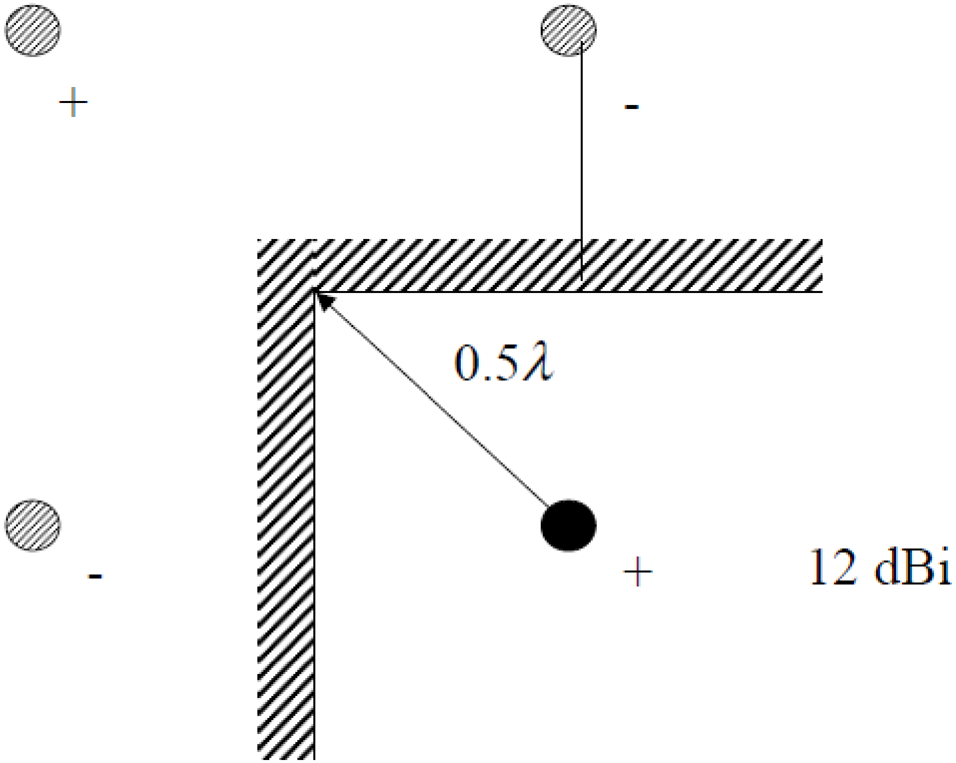
Monopole antenna



Conducting plane placed below a single $L/2$ antenna the combined system acts as a dipole of length L where the radiation only takes place above the ground plane. The quarter-wave monopole ($L/2=\lambda/4$) approximates the half-wave dipole and is useful for car mounted antennas and handsets.

Corner reflector

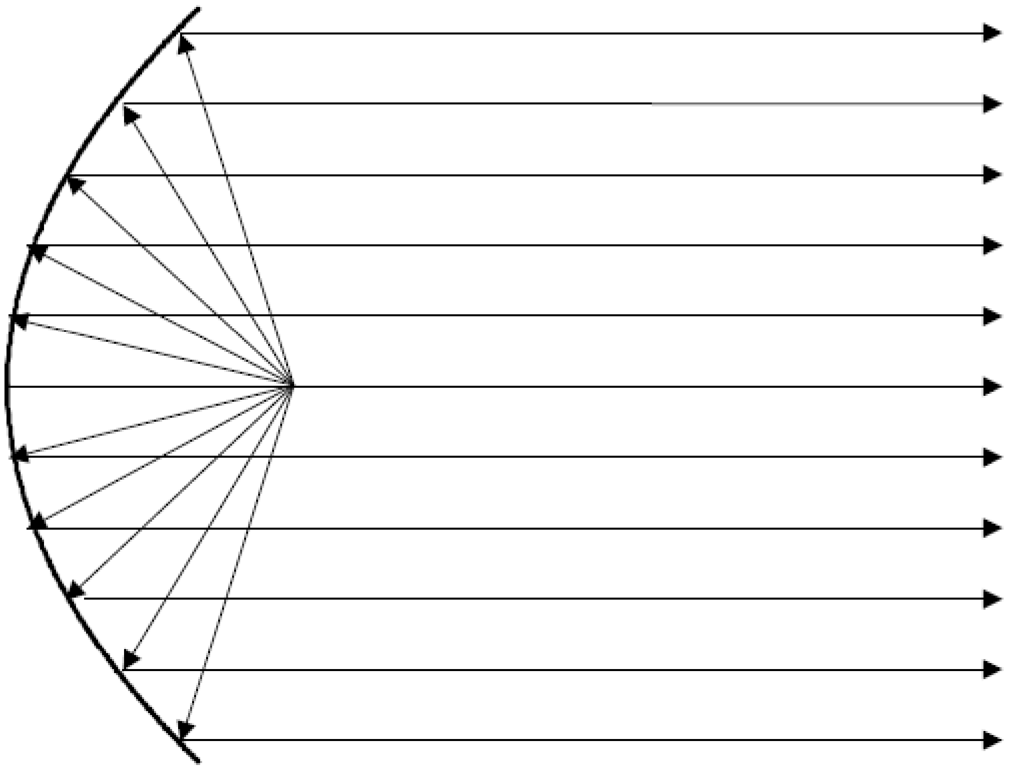
- Multiple Reflections



Monopole extended using several ground plane reflectors.

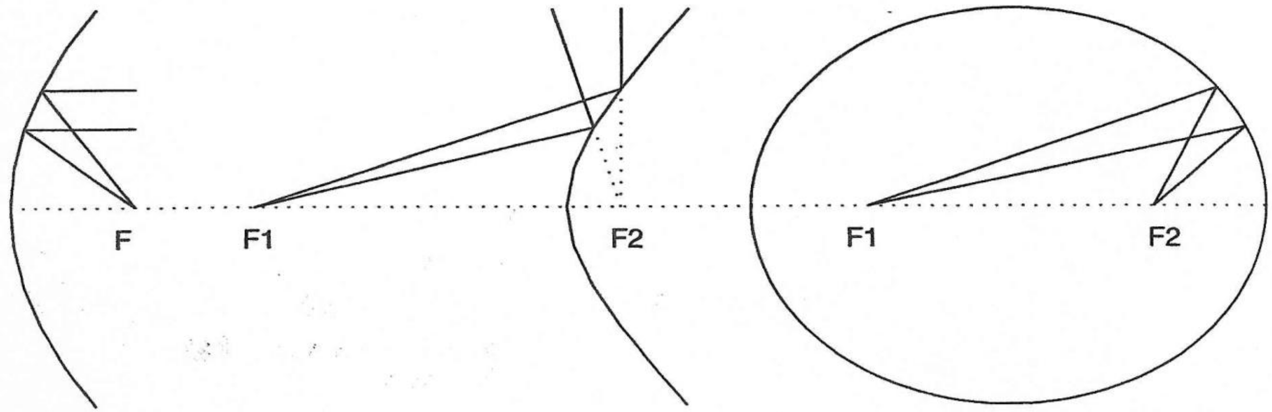
Parabolic reflector antenna

- Infinite Uniform Array



$$P = \frac{\sin \frac{\pi D \cos \phi}{\lambda}}{\frac{\pi D \cos \phi}{\lambda}}$$

Parabola, hyperbola, and ellipsoid focussing properties



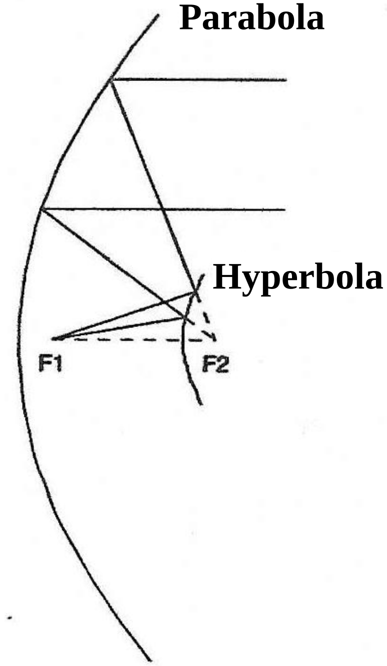
Parabola Hyperbola Ellipsoid

Parabola: F is the focus point. Spherical waves from F reflected to a plane wave along the symmetry axis, or a plane wave focused to F

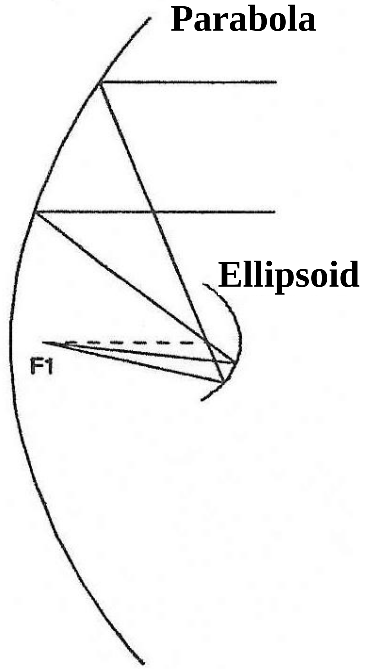
Hyperbola: Spherical wave from F1 reflected to spherical wave apparently from F2

Ellipsoid: Spherical wave from F1 reflected to new spherical waves focussed to F2

Parabola antennas using two reflectors



Cassegrain-antenna with hyperbola sub-reflector

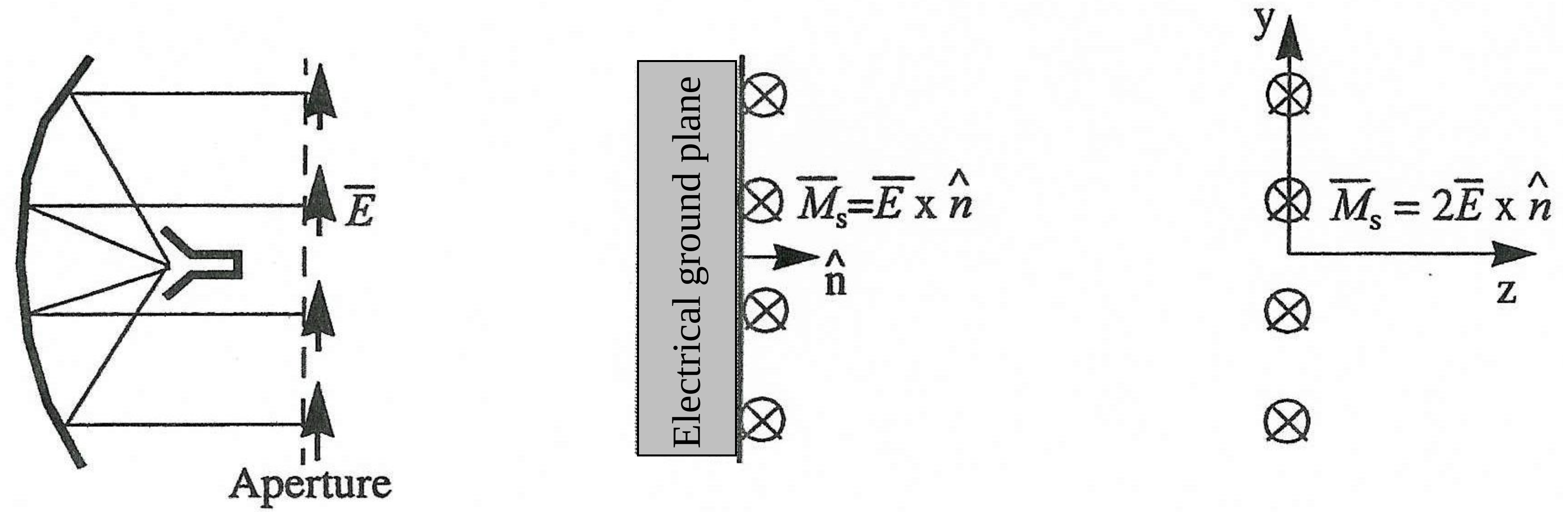


Gregory-antenna with ellipsoid as sub-reflector

Two-reflector antenna, good position of the feed and less noise. Negative side is shadowing, but can be avoided letting the sub-reflector be out of the main direction.

Radiation from apertures

This part of reflector antennas is not in the book.



Use ray tracing from feed antenna (e.g., horn) via main reflector to establish the field in a plane, the aperture plane. Use this field to find the far field.

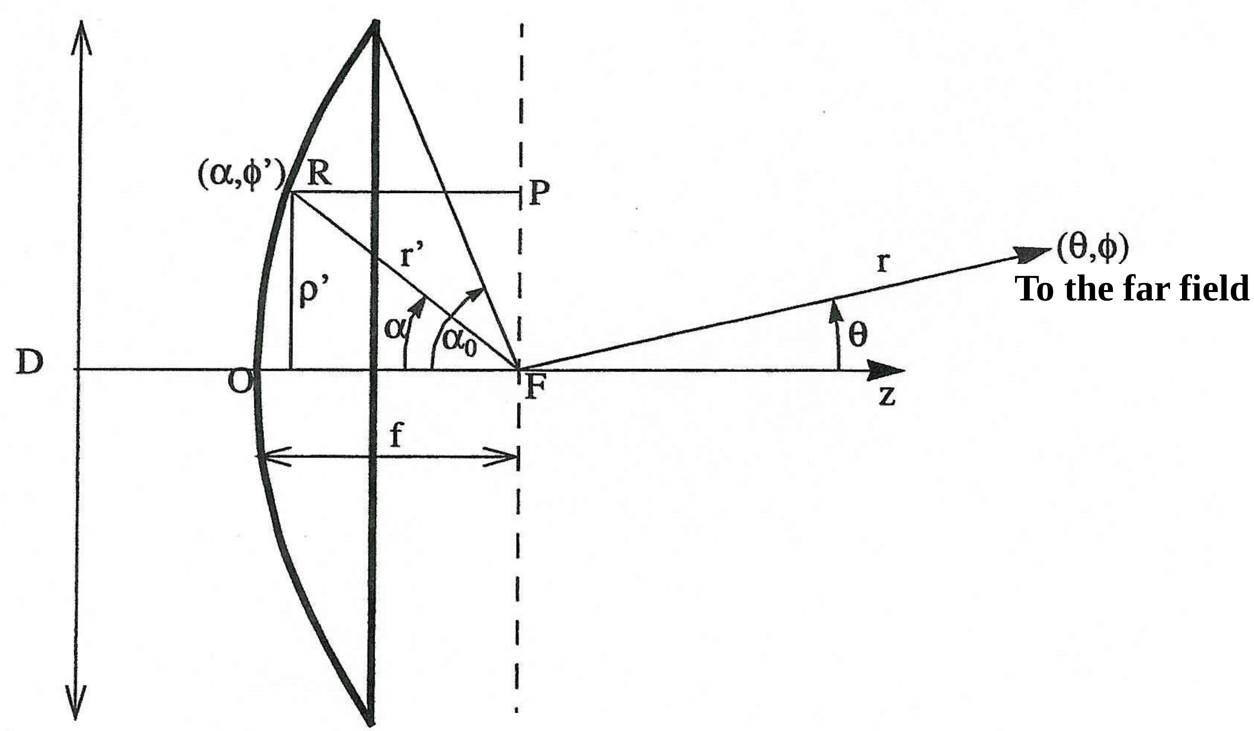
The method is to use equivalent current sources on the aperture plane to easier find the far field, as illustrated in the three steps above.

The magnetic surface current becomes when the aperture is in the xy-plane.

$$\boldsymbol{M}_s = 2\left(\hat{x}E_x + \hat{y}E_y\right) \times \hat{z} = 2\left(-\hat{y}E_x + \hat{x}E_y\right)$$

This part of reflector antennas is not in the book.

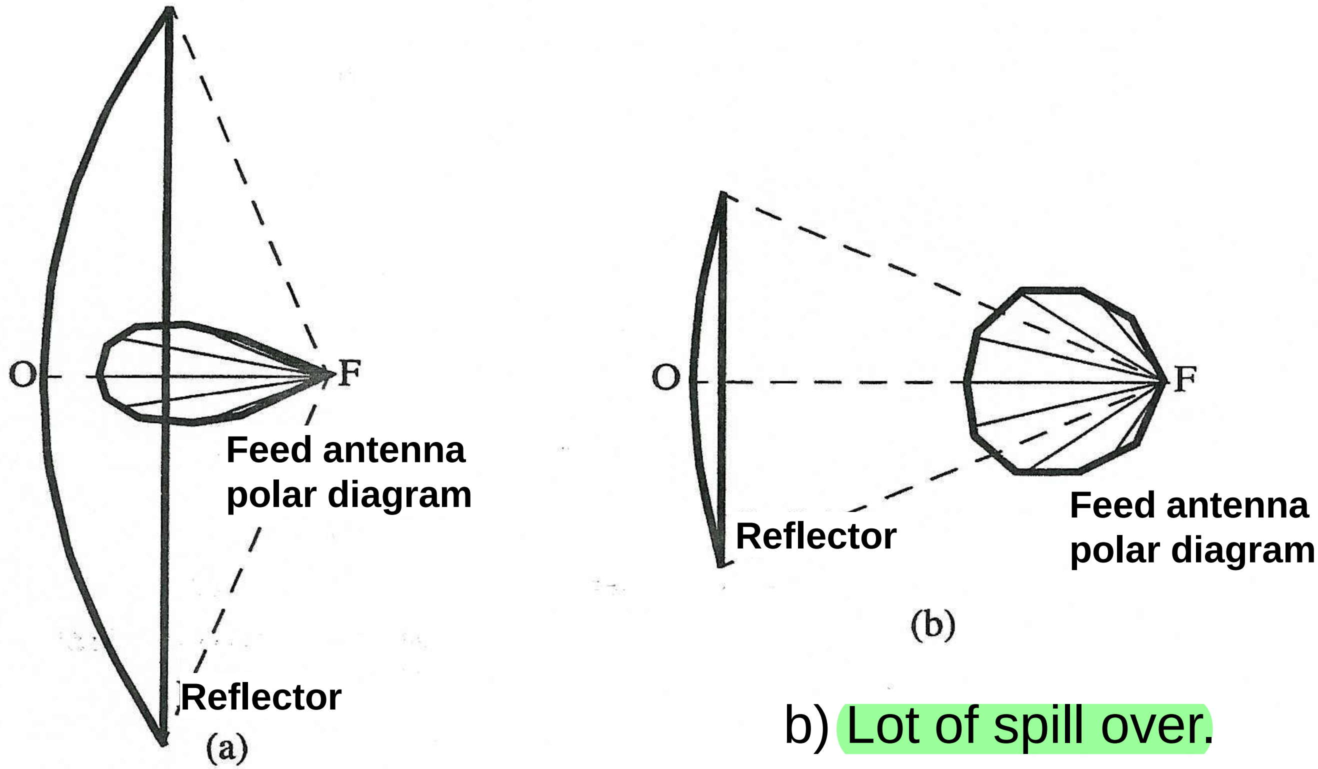
Front fed parabolic antenna



Antenna with diameter *D* and a number of angles and points defined. The form is precisely determined by the focal distance *f* and diameter *D*.
The ratio *f*/*D* is decisive and normally between 0.3 and 1

Optimum relationship between the feed antenna and the main reflector

This part of reflector antennas is not in the book.



b) Lot of spill over.

a) Poorly deployed main reflector

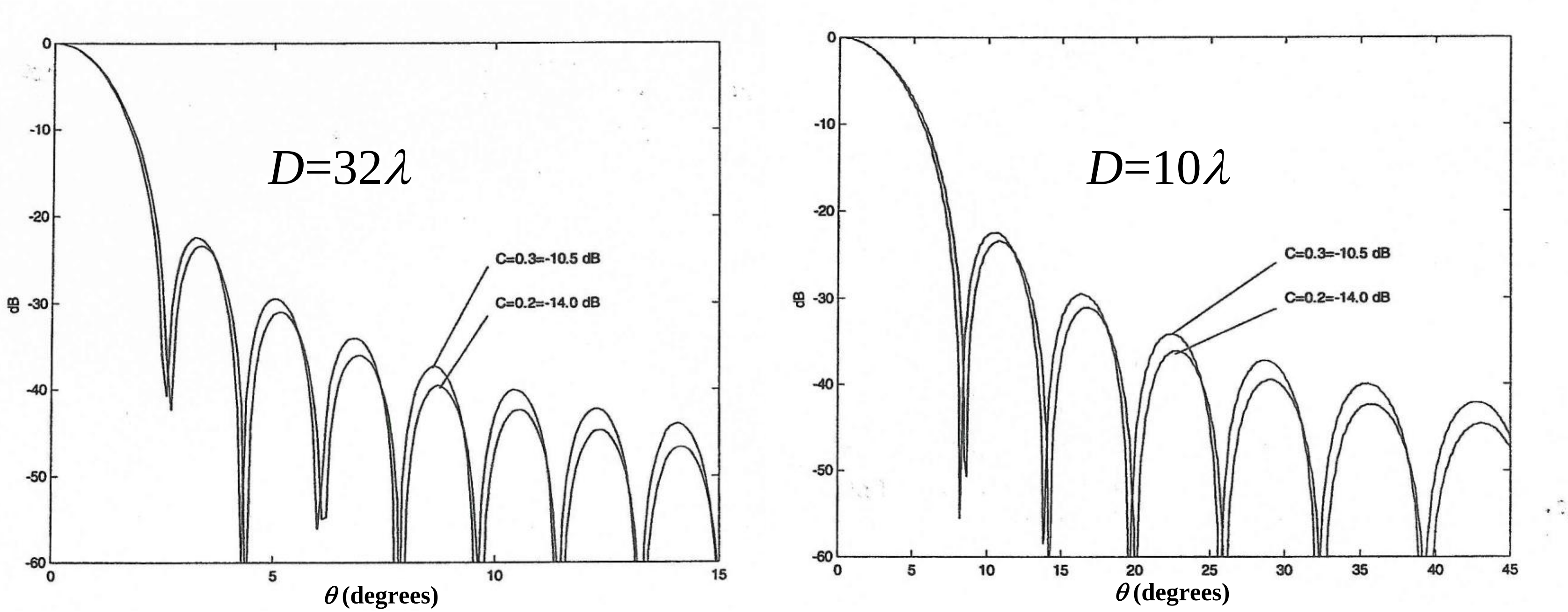
Radiation pattern

This part of reflector antennas is not in the book.

Aperture distribution: second order polygon: $E_y = C + (1-C)(1-(2\rho'/D)^2)$.

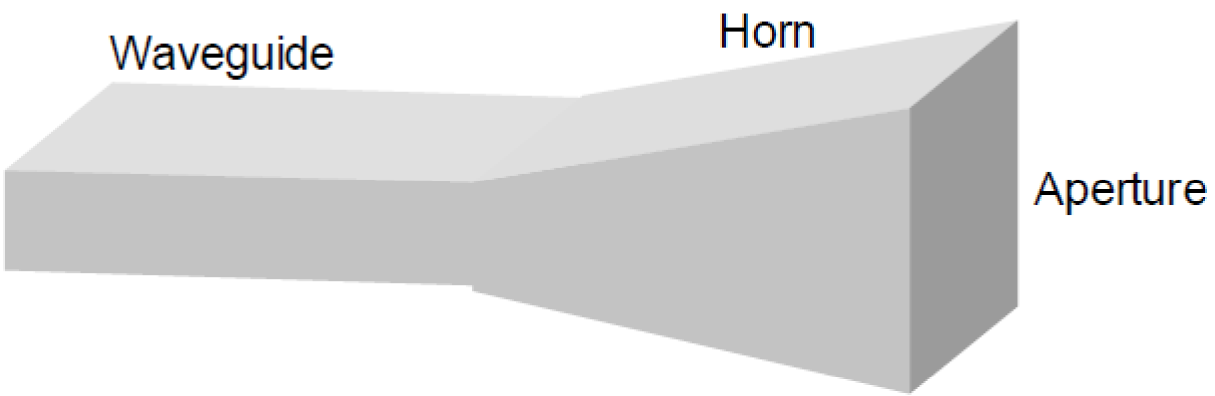
$$E\left(r,\theta,\frac{\pi}{2}\right) = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(C + (1-C)\left(1-\left(\frac{2\rho'}{D}\right)^2\right)\right) e^{jk_0 y'\sin\theta} dx'\,dy'$$
$$= \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(C + (1-C)\left(1-\frac{4(x'^2+y'^2)}{D^2}\right)\right) e^{jk_0 y'\sin\theta} dx'\,dy'$$

Solved numerically for different C and D.



Rule of thumb: BW=66°/(D/λ)

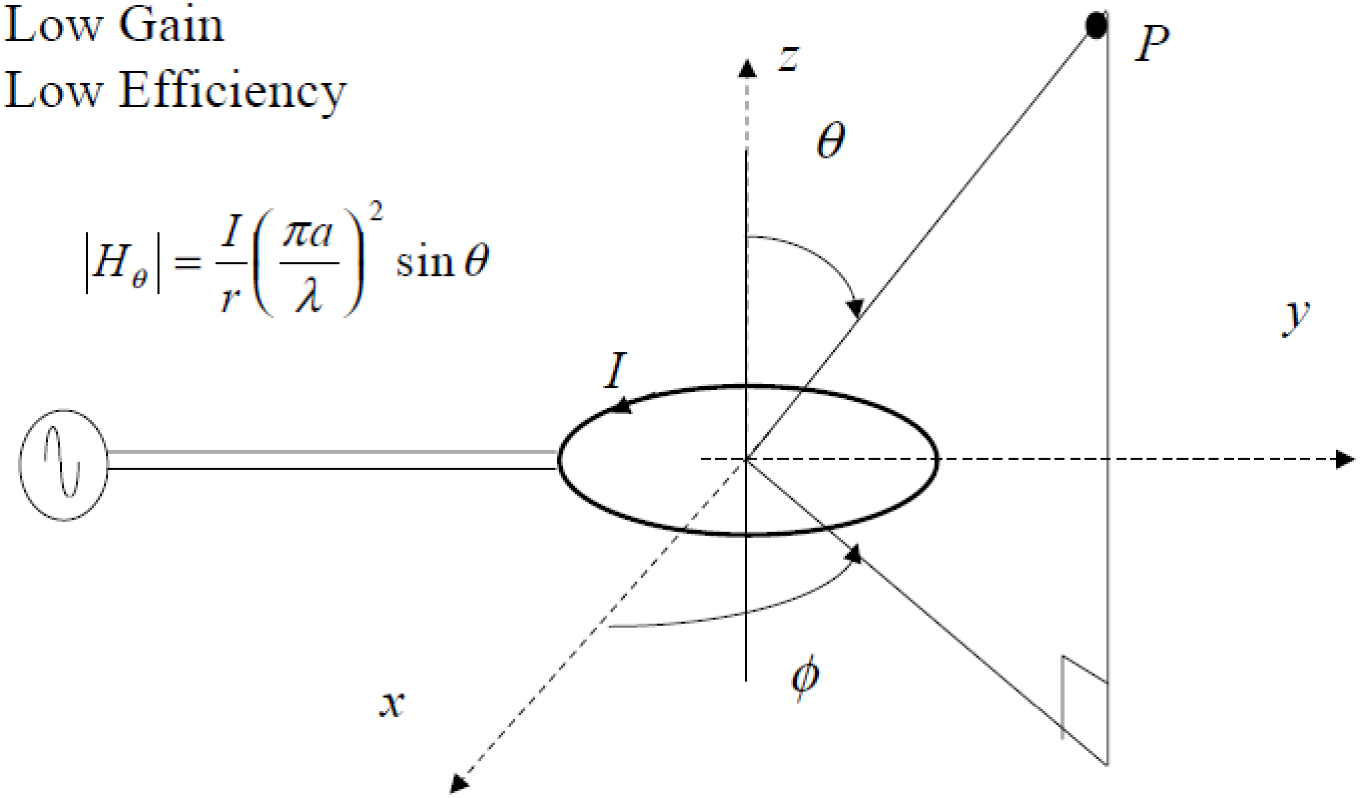
Horn antenna



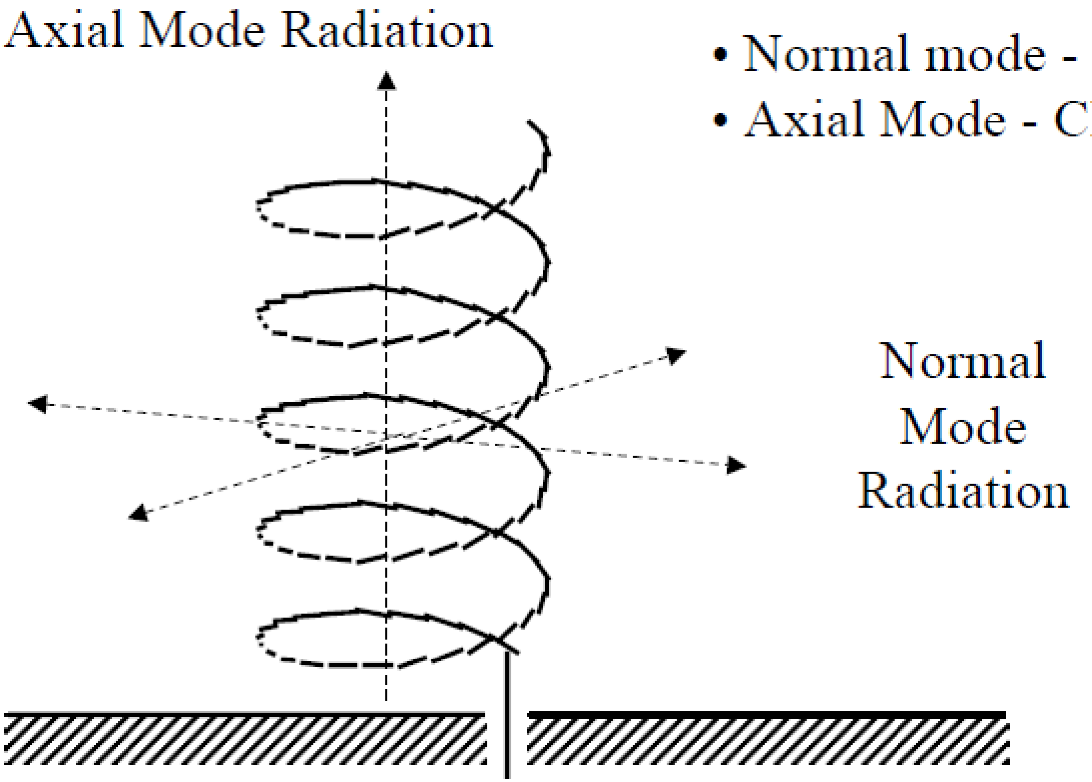
Loop antenna

- Low Gain
- Low Efficiency

$$|H_\theta| = \frac{I}{r}\left(\frac{\pi a}{\lambda}\right)^2 \sin\theta$$



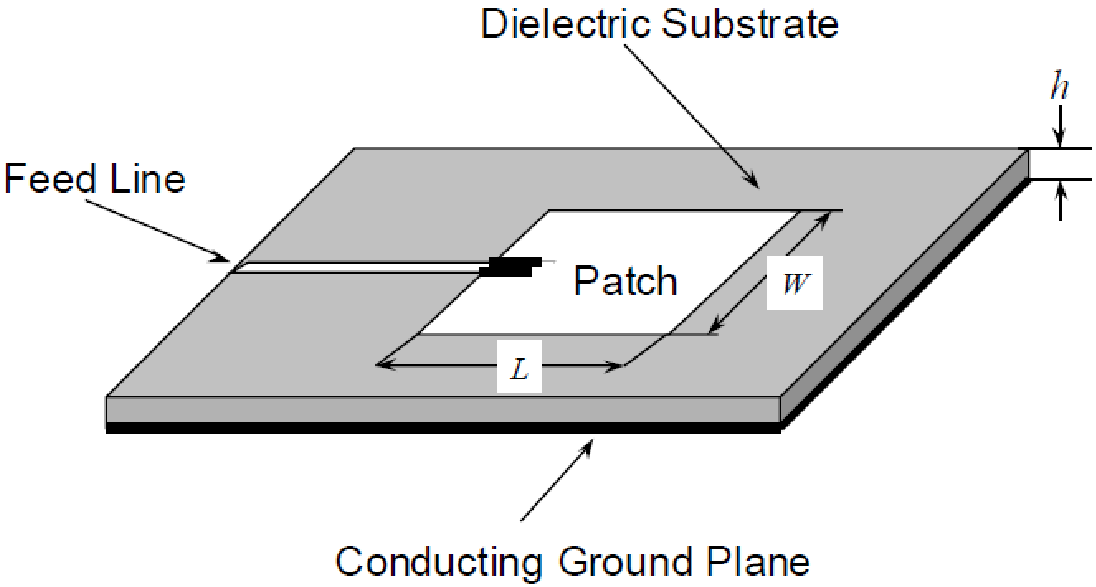
Helical antenna



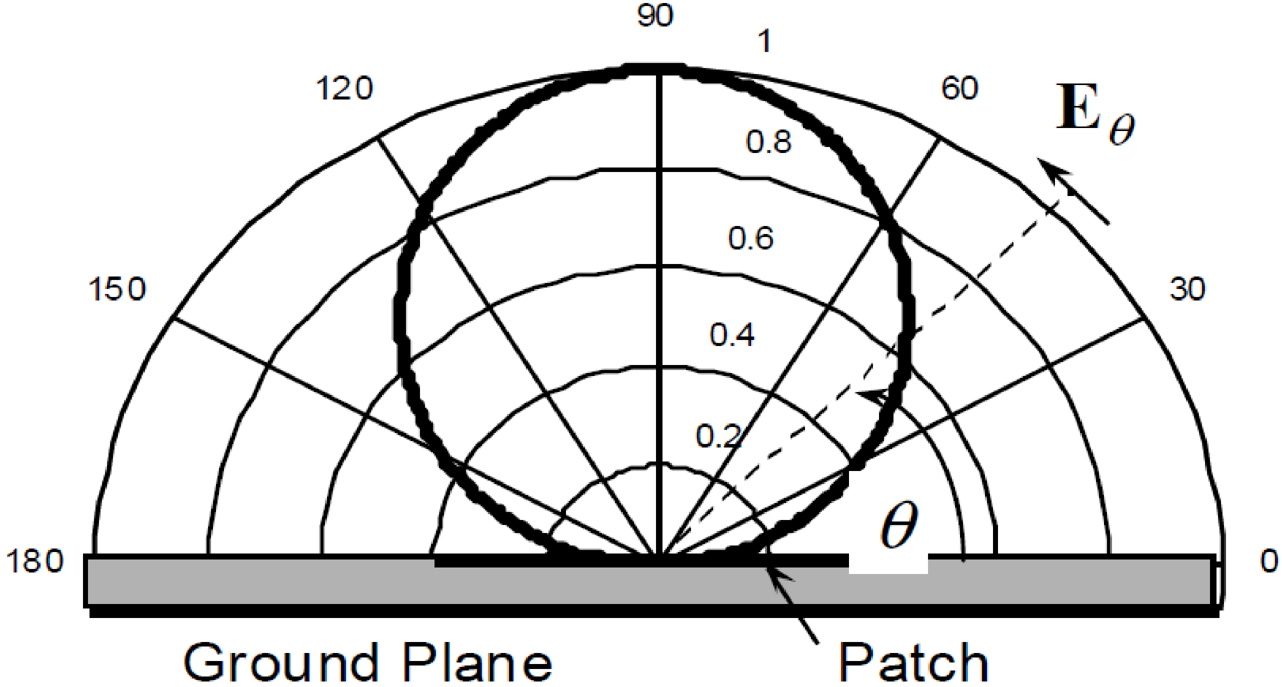
- Normal mode - Array of Loops
- Axial Mode - CP Yagi

Patch antenna

The length *L* is typically up to half the free space wave length. Incident wave in the feed line leads to fields in the dielectric immediately beneath the patch. Electrical field approximately perpendicular to the surface and the magnetic field parallel.



Patch antenna radiation pattern



The fields around the edges of the patch create radiation with contributions as if they were constituting a four-element array. The resultant radiation can be varied by changing *L* and *W.*

Antenna conclusions

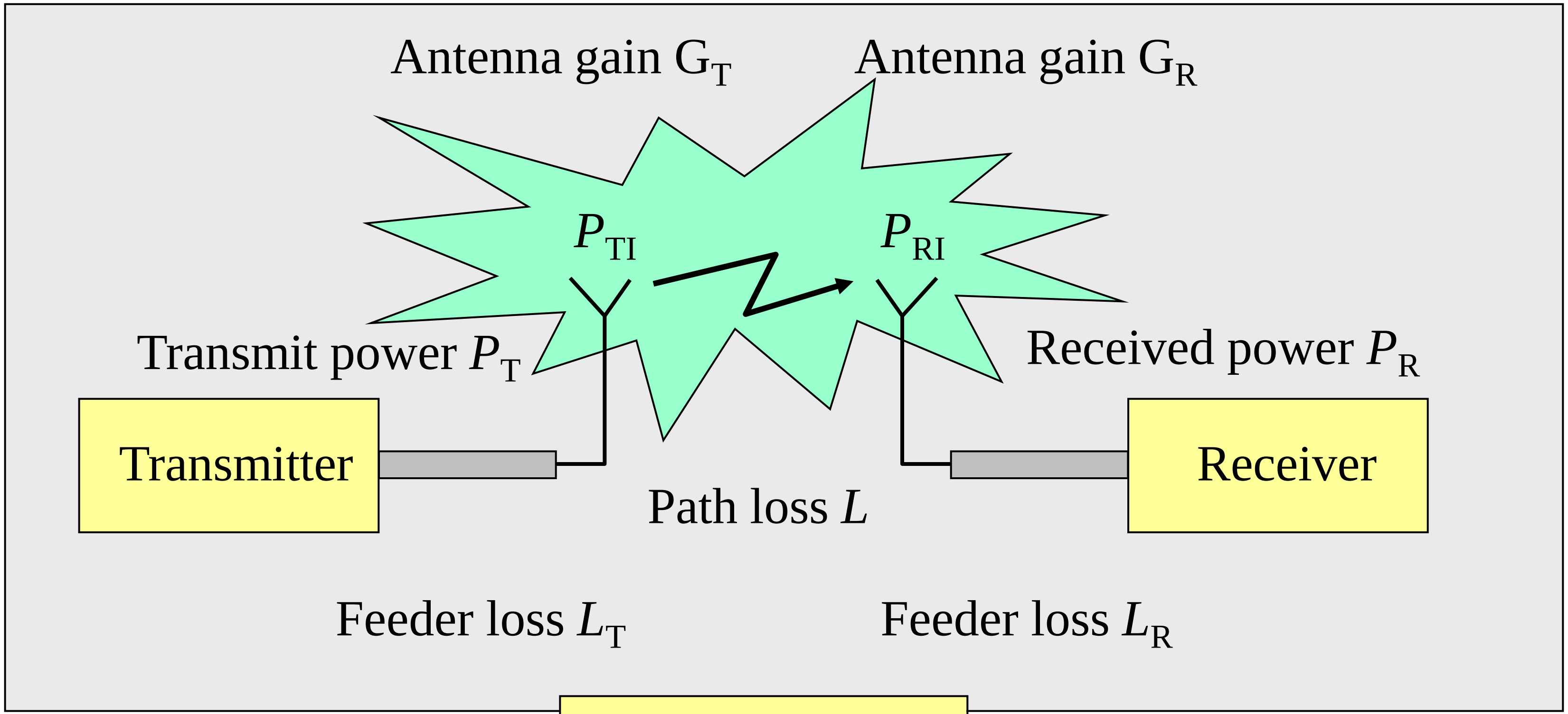
- Linear array
- Reflector antennas introduced and their deployment areas
- Type of reflectors
- Relation to the wavelength
- Antenna efficiency and radiation pattern
- Example antennas

Chapter 5: Basic propagation models

- Idealised approach
- Basic propagation
- Range of a radio system
- Link budget
- Noise factor
- Free space loss
- Plane earth reflection

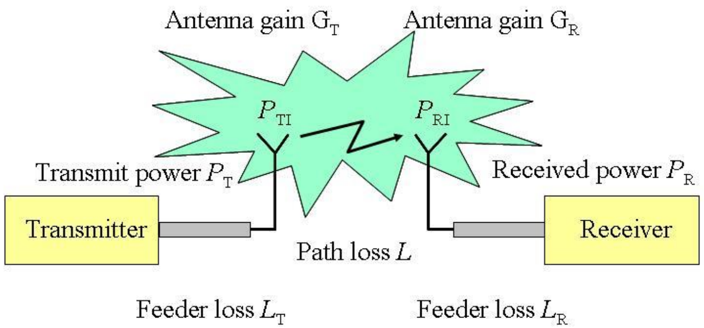
Wireless link

Model of a simple wireless link:



$$P_R = \frac{P_T G_T G_R}{L_T L L_R}$$

EIRP and path loss *L*



$$EIRP = \frac{P_T G_T}{L_T} = P_{TI} \quad \text{similarly} \quad P_{RI} = \frac{P_R L_R}{G_R}$$

P$_{TI}$: Effective isotropic radiated power

P$_{RI}$: Effective isotropic receive power

The path loss *L* now be defined independent of system parameters:

$$L = \frac{P_{TI}}{P_{RI}} = \frac{P_T G_T G_R}{P_R L_T L_R}$$

Effective isotropic radiated power (EIRP) is the equivalent power of a transmitted signal in terms of an isotropic (omnidirectional) radiator. Normally the effective isotropic radiated power equals the product of the transmitter power and the antenna gain (reduced by any coupling losses between the transmitter and antenna.)

EIRP is also often seen defined as “equivalent isotropic radiated power”.

Maximum path loss allowed

The task is to accurately predict path loss *L* such that the range of the system can be determined. Maximum range is found where the received power is above the receiver sensitivity.

Path loss often expressed in dB: $L_{dB} = 10\log\left(\frac{P_{TI}}{P_{RI}}\right)$

The transmit power and gain adds together and all losses reduces resulting in a received power to be compared with the threshold of the radio receiver. Usually the requirement for received power to be above the threshold with a certain margin depending on link quality requirements.

Example 5.1

Quantity	Value in original units	Value in consistent units
P_T	10 W	10 dBW
G_T	12 dBd	14.15 dBi
G_R	0 dBd	2.15 dBi
P_R Receiver sensitivity	-104 dBm	-134 dBW
L_T	10 dB	10 dB
L_R	2 dB	2 dB

a) EIRP = $P_{TI}=P_T+G_T-L_T=10+14.15-10=14.15$ dBW=26 W

b) Maximum acceptable path loss L:
$L= P_T+ G_T+G_R-P_R-L_T -L_R=10+14.15+2.15-(-134)-10-2=148.3$ dB

Decibel

- bel is a logarithmic unit of power ratios, one bel corresponds to a 10 times increase in power with respect to a reference power

$$\text{Power ratio in bel} = \log\left(\frac{P}{P_{ref}}\right)$$

- bel is too large, and decibel is used for convenience, 1/10 of bel

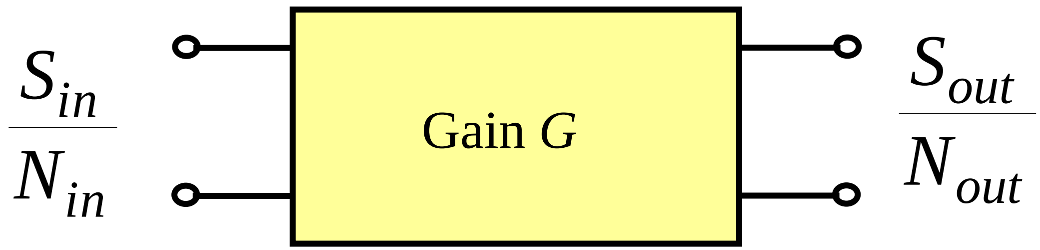
$$\text{Power ratio in decibel} = 10\log\left(\frac{P}{P_{ref}}\right)$$

- the ratio of voltage or field strength provided they appear across the same impedance

$$\text{Voltage ratio in decibel} = 20\log\left(\frac{V}{V_{ref}}\right)$$

Noise modelling

The signal to noise ratio (S/N) will determine the system performance. Most of the noise comes from the receiver itself. Model the system as a two-port network with gain *G* and noise factor *F* describing the noise of the network itself:



The noise factor *F* is defined: $F = \dfrac{S_{in}/N_{in}}{S_{out}/N_{out}} = \dfrac{\dfrac{S_{in}}{kTB}}{\dfrac{GS_{in}}{N_{out}}} = \dfrac{\dfrac{N_{out}}{G}}{kTB}$

Where *k* is Boltzmann’s constant and *B* the bandwidth.
The noise factor *F* sometimes given in dB $F_{dB}=10\log F$ and also sometimes called noise figure.

Noise temperature

The network can alternatively be described by an
equivalent noise temperature T_e:

$$N_{out} = kTB + N_{network} = GkTB + GkT_eB = Gk(T + T_e)B$$

$$F = \frac{\frac{N_{out}}{G}}{kTB} = \frac{k(T + T_e)B}{kTB} = 1 + \frac{T_e}{T} \quad \text{or} \quad T_e = T(F - 1)$$

The equivalent noise temperature T_e will on input create the same noise at the output as if the network were noiseless. And the noise created by the network $N_{network}=GkT_eB=(F-1)GkTB$.

Example

A receiver has a noise bandwidth of 200 kHz and requires at input *S*/*N* of at least 10 dB when the signal power is -104 dBm.

a) What is the maximum permitted noise figure?

In dB the SNR (signal to noise ratio) is $\mathrm{SNR}=P_s-N$ where P_s (dBW) is the input power and N (dBW) the noise power referred to the input.

$N=10\log(FkTB)$ and $\mathrm{SNR}=P_s-N=P_s-F_{dB}-10\log(kTB)$

$F_{dB}=P_s-\mathrm{SNR}-10\log(\mathrm{kTB})=(-104-30)-10-10\log(1.38\cdot10^{-23}\cdot300\cdot200\cdot10^{3})$

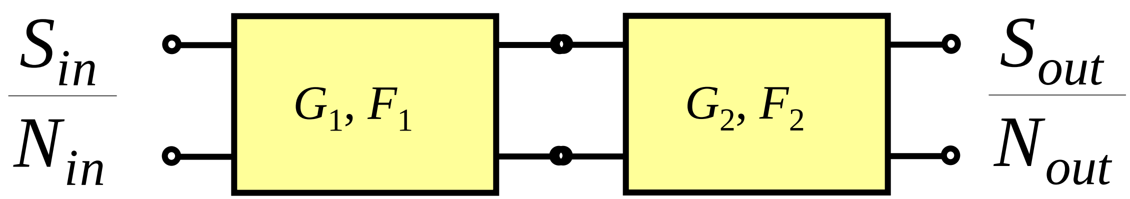
$= 7.0$ dB

b) What is the equivalent temperature T_e

$T_e=T(F-1)=290(10^{7.0/10}-1)=1163$ K

Cascaded network

Consider the case with two cascaded networks where F denotes the noise factor for the whole network.



$N_{out} = FG_1G_2kTB$ as before, but for the cascaded case.
But N_{out} can be written $N_{out} = N_1G_2 + N_2$ where N_i is the noise generated by the network component itself.
The overall F becomes
$FG_1G_2kTB = F_1G_1kTBG_2 + (F_2 - 1)G_2kTB$

$$F = \frac{F_1G_1kTBG_2}{G_1G_2kTB} + \frac{(F_2 - 1)G_2kTB}{G_1G_2kTB} = F_1 + \frac{F_2 - 1}{G_1}$$

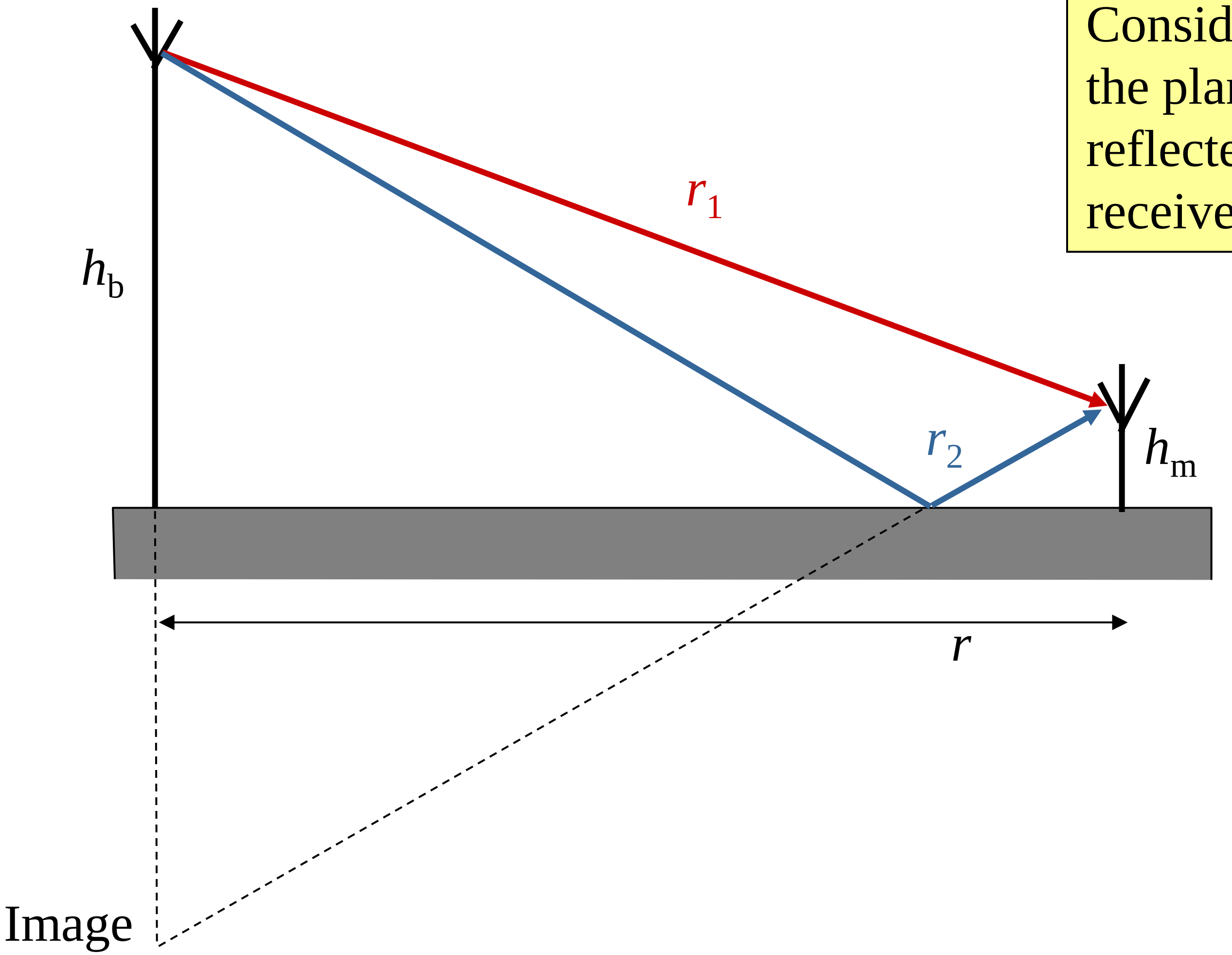
Free space loss

Friis transmission formula: $$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi r} \right)^2$$

The free space loss $$L_F = \frac{P_t G_r G_t}{P_r} = \left(\frac{4\pi r}{\lambda} \right)^2 = \left(\frac{4\pi r f}{c} \right)^2$$

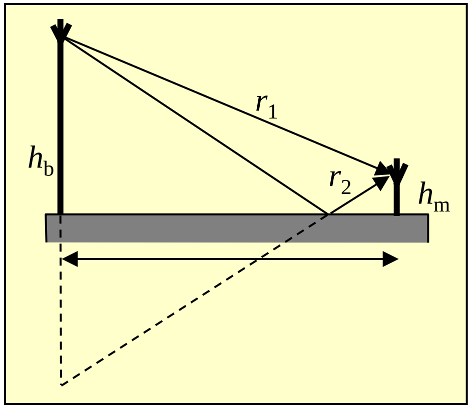
$$L_{F(dB)} = 32.4 + 20\log r_{km} + 20\log f_{MHz}$$

Plane earth reflection



Consider the direct and the plane earth reflected signals are received at the mobile.

Reflected signal path length difference



The sum of the signals depends on the amplitudes and phase. Phase is derived from the path lengths involved. From the geometry it is seen that r_1 and r_2, are:

$$r_1 = \sqrt{(h_b - h_m)^2 + r^2} \qquad r_2 = \sqrt{(h_b + h_m)^2 + r^2}$$

The difference is directly related to the phase difference

$$r_2 - r_1 = r\left[\sqrt{\left(\frac{h_b + h_m}{r}\right)^2 + 1} - \sqrt{\left(\frac{h_b - h_m}{r}\right)^2 + 1}\right] \approx \frac{2h_m h_b}{r}$$

since $(1 + x)^n \approx 1 + nx$ for small x

Path loss

Let amplitudes be A, then $A_{total} = A_{direct} + A_{reflected} = A_{direct}\left|1 + Re^{jk\frac{2h_m h_b}{r}}\right|$

where k is the free space wave number and R the reflection coefficient. Since the power is proportional to the amplitudes squared

$$\frac{P_r}{P_{direct}} = \left(\frac{A_{total}}{A_{direct}}\right)^2 = \left|1 + Re^{jk\frac{2h_m h_b}{r}}\right|^2$$

where P_r is the received power.

The direct path itself undergoes free space loss where P_r is the received power. Therefore

$$P_{direct} = P_t\left(\frac{\lambda}{4\pi r}\right)^2 \quad \text{and} \quad \frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi r}\right)^2\left|1 + Re^{jk\frac{2h_m h_b}{r}}\right|^2$$

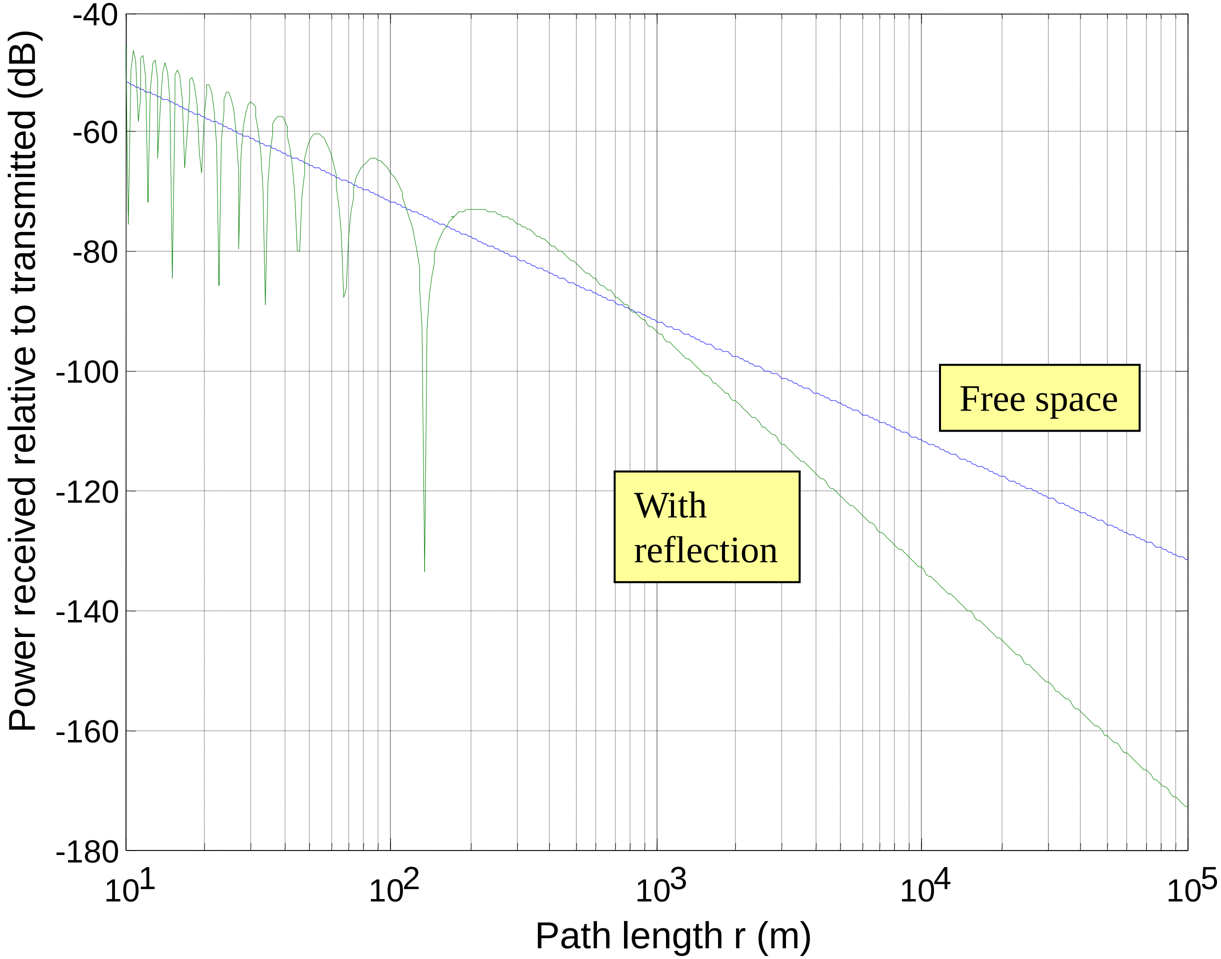
where P_T is the transmitted power.

The magnitude of R is close to 1 i.e., R=-1, and due to small angles

$$\frac{P_r}{P_t} = 2\left(\frac{\lambda}{4\pi r}\right)^2\left[1 - \cos\left(k\frac{2h_m h_b}{r}\right)\right] \approx \left(\frac{\lambda}{4\pi r}k\frac{2h_m h_b}{r}\right)^2 \approx \frac{h_m^2 h_b^2}{r^4}$$

the asymptotic solution for small angles, i.e., $\cos\theta = 1 - \frac{\theta^2}{2}$

Received power relative to transmitted



Example:
f = 900 MHz
h_m = 30 m
h_b =1.5 m

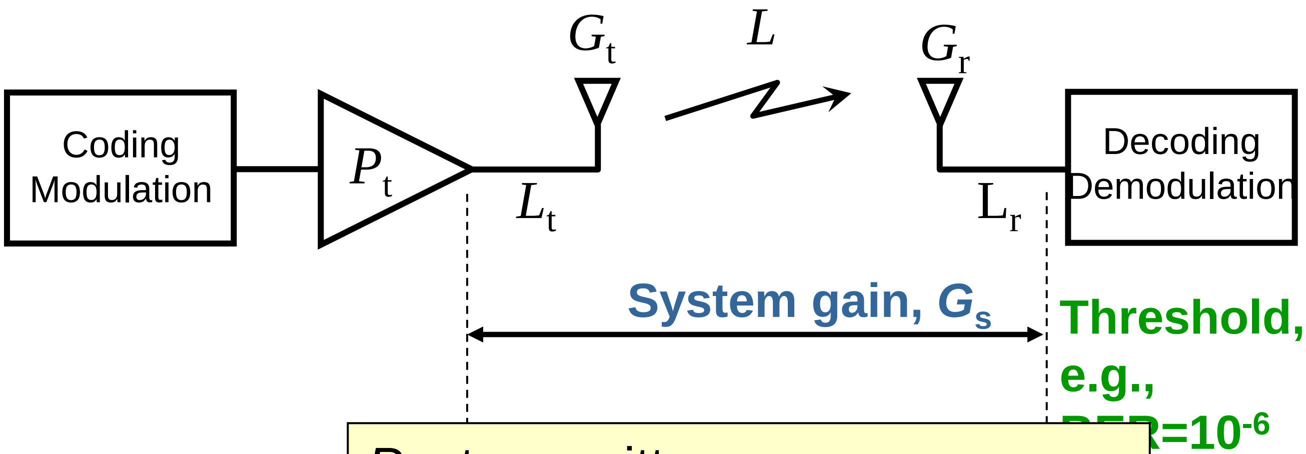
Link budget

- A link budget is a calculation of signal powers, noise powers, and/or signal-to-noise ratios
- Normally fairly simple calculations, yet commonplace and very useful
- Maximum acceptable propagation loss (dB) equals predicted loss (dB) plus fade margin (dB)
- The fade margin is directly connected to the performance of the system, but will constrain the system range

Link budget example: terrestrial mobile downlink

	Quantity	Value	Unit	Value	Unit
a	Base station transmit power	10	W	10	dBW
b	Base station feeder loss			10	dB
c	Base station antenna gain	6	dBd	8.2	dBi
d	Effective isotropic transmit power (a – b + c)			8.2	dBW
e	Maximum acceptable propagation loss			L	dB
f	Mobile antenna gain	-1	dBd	1.2	dBi
g	Body and matching loss			6	dB
h	Signal power at receiver terminals (d – e + f – g)			3.4-L	dBW
i	Receiver noise bandwidth	200	kHz	53	dBHz
j	Receiver noise figure			7	dB
k	10log(kT) with k=1.38·10^{-23} $WHz^{-1}K^{-1}$ and T=290K			-204	$dBWHz^{-1}$
l	Receiver noise power referred to input (i + j + k)			-144	dBW
m	Required signal-to-noise ratio			9	dB
n	Required input signal power (l+m)			-135	dBW
o	Maximum acceptable propagation loss by solving h=n			138.4	dB
p	Fade margin			15	dB
q	Predicted cell radius (plane earth h_m=1.5 m, h_b=15 m)			5.8	km

The concept of System Gain



P_t - transmitter power
L_t - feeder loss
G_t - transmitter antenna gain
G_r - receiver antenna gain
L_r - feeder loss
L - path loss
M - margin

$$G_s = M + L + L_t + L_r - G_t - G_r$$

Chapter 5 Summary

- Reference system illustration
- Modelling noise, noise figure, and noise temperature
- Plane-earth two-way path loss model
- What is a link budget and what is it used for