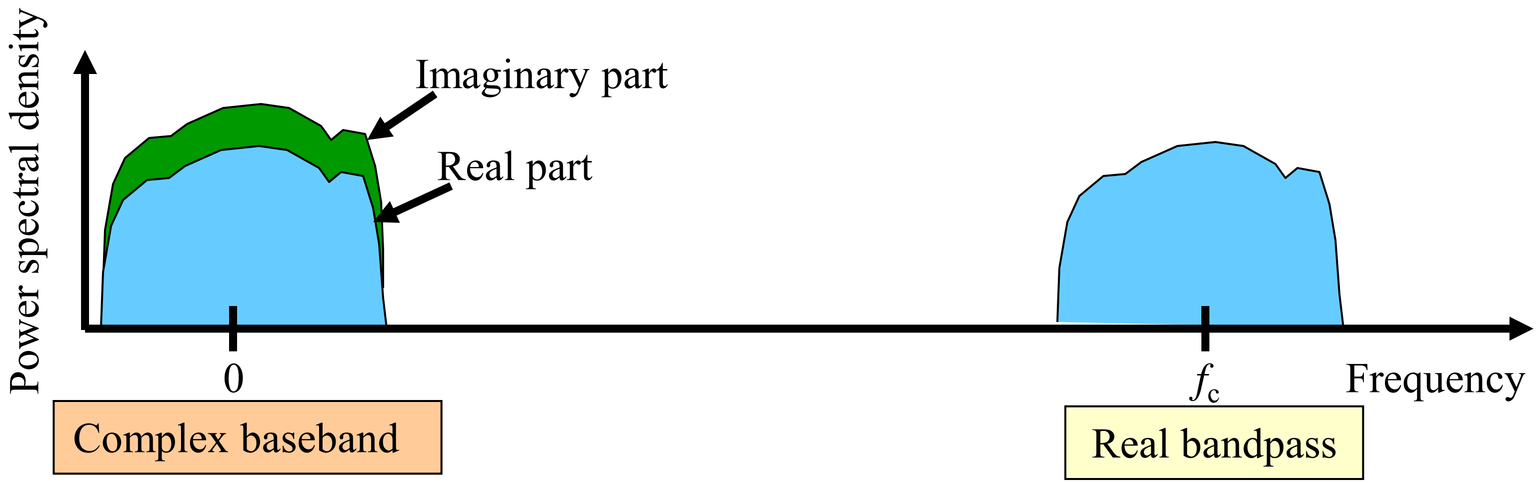
Chapter 10 Narrowband fast fading

- Channel representation
- The AWGN channel
- First order fading statistics: Rayleigh and Rician multipath fading channels
- Second order fading statistics: Doppler spread

Baseband signal representation

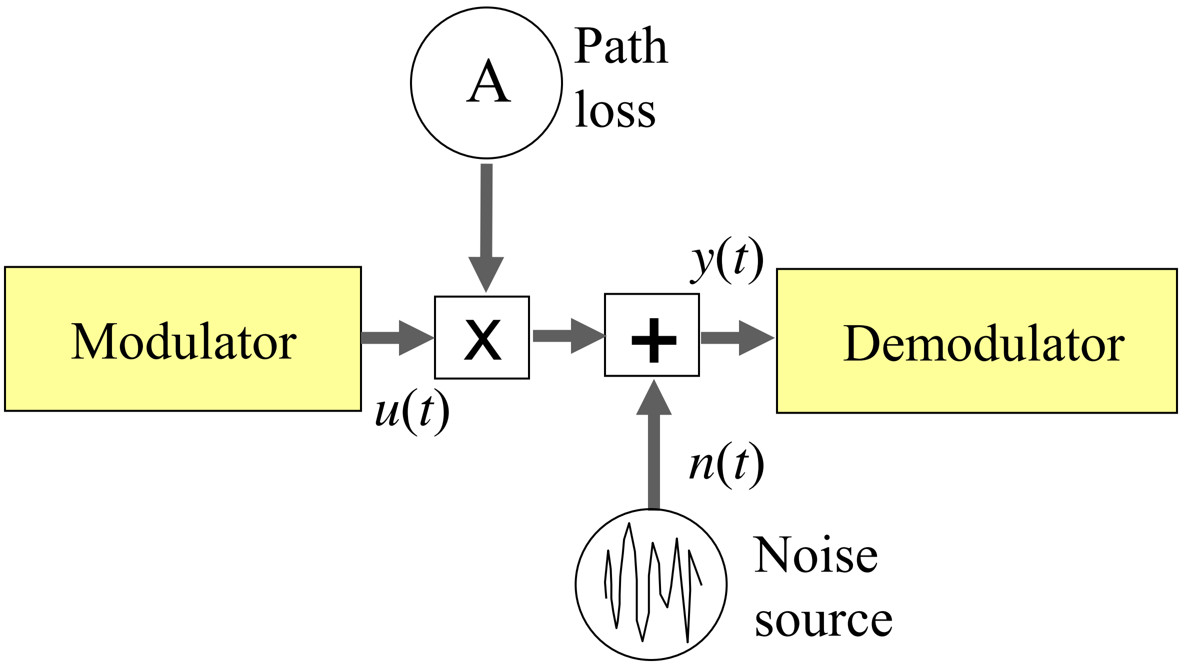
- A signal transmitted over radio has usually a narrow bandwidth compared with the centre of the radio channel
- Called bandpass signal: $s(t) = a(t)\cos[2\pi f_c t+\theta(t)]$
  where $a(t)$ is the envelope, $\theta(t)$ phase and f_c carrier frequency
- Complex baseband: $u(t) = a(t)e^{j\theta(t)}$
  such that $s(t) = \mathrm{Re}[u(t)e^{j2\pi fc(t)}]$
- Mean power $P_s = E[|u(t)|^2]/2 = E[u(t)u^*(t)]/2$



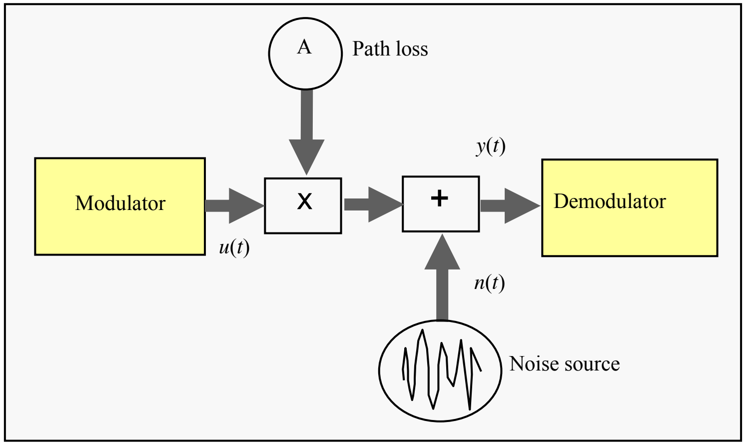
The AWGN channel

Simplest channel: Additive white Gaussian noise (AWGN)

- White: constant power spectral density
- Gaussian: the noise power follows a normal distribution



AWGN characterisation



- Received signal (voltage):
  $y(t) = A \cdot u(t) + n(t)$
  where A represents path loss and shadowing, $u(t)$ the modulated signal and $n(t)$ the noise
- Assume complex baseband representation of signals; noise is composed of real and imaginary components:
  $n(t) = x_n(t) + jy_n(t)$
  where x and y are zero mean, independent, real Gaussian processes with standard deviation σ_n
- Mean noise power:

$$P_n = \frac{E[n(t)n^*(t)]}{2} = \frac{E[(x_n(t) + jy_n(t))(x_n(t) + jy_n(t))^*]}{2} = \frac{E[x^2 + y^2]}{2} = \frac{\sigma_n^2 + \sigma_n^2}{2} = \sigma_n^2$$

Signal to noise ratio (SNR) at demodulator input

$$\frac{\text{Signal power}}{\text{Noise power}} = \gamma = \frac{E\left[A^2 u^2(t)\right]}{2P_n} = \frac{A^2 E\left[u^2(t)\right]}{2P_n} = \frac{A^2}{2P_n}$$

assuming the variance equals 1 for modulator output signal.

Alternative SNR expression for digital signals consisting of symbols each lasting a finite time T:

If each symbol has energy E_s then $E_s=(A^2/2)\cdot T=A^2T/2$.

Noise contained in bandwidth $B=1/T$ and noise power spectral density N_0, then $P_n = BN_0=N_0/T=\sigma^2$.

Usual to write $\gamma = E_s/N_0$, or $\gamma_b = \gamma/m = E_b/N_0$, where m is bits per symbol, as parameter when expressing error ratio performance.

Binary phase shift keying (BPSK) in AWGN

From literature (Proakis) the error rate of any modulation scheme in AWGN is

$$P_e = Q\left(\sqrt{\frac{A^2 d^2}{2N_0}}\right)$$

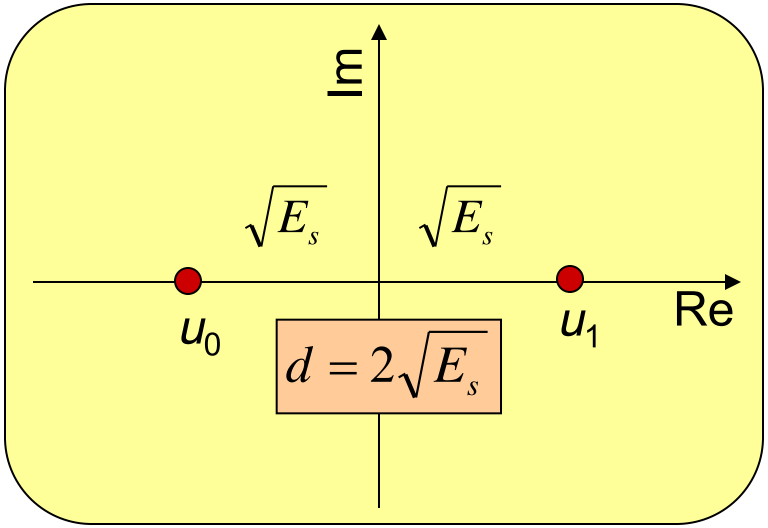
where d is he Euclidan distance between the transmitted waveform corresponding to the different bits and Q the complementary cumulative normal distribution (Ch. 9).

For BPSK the bits 1 and 0 can be represented

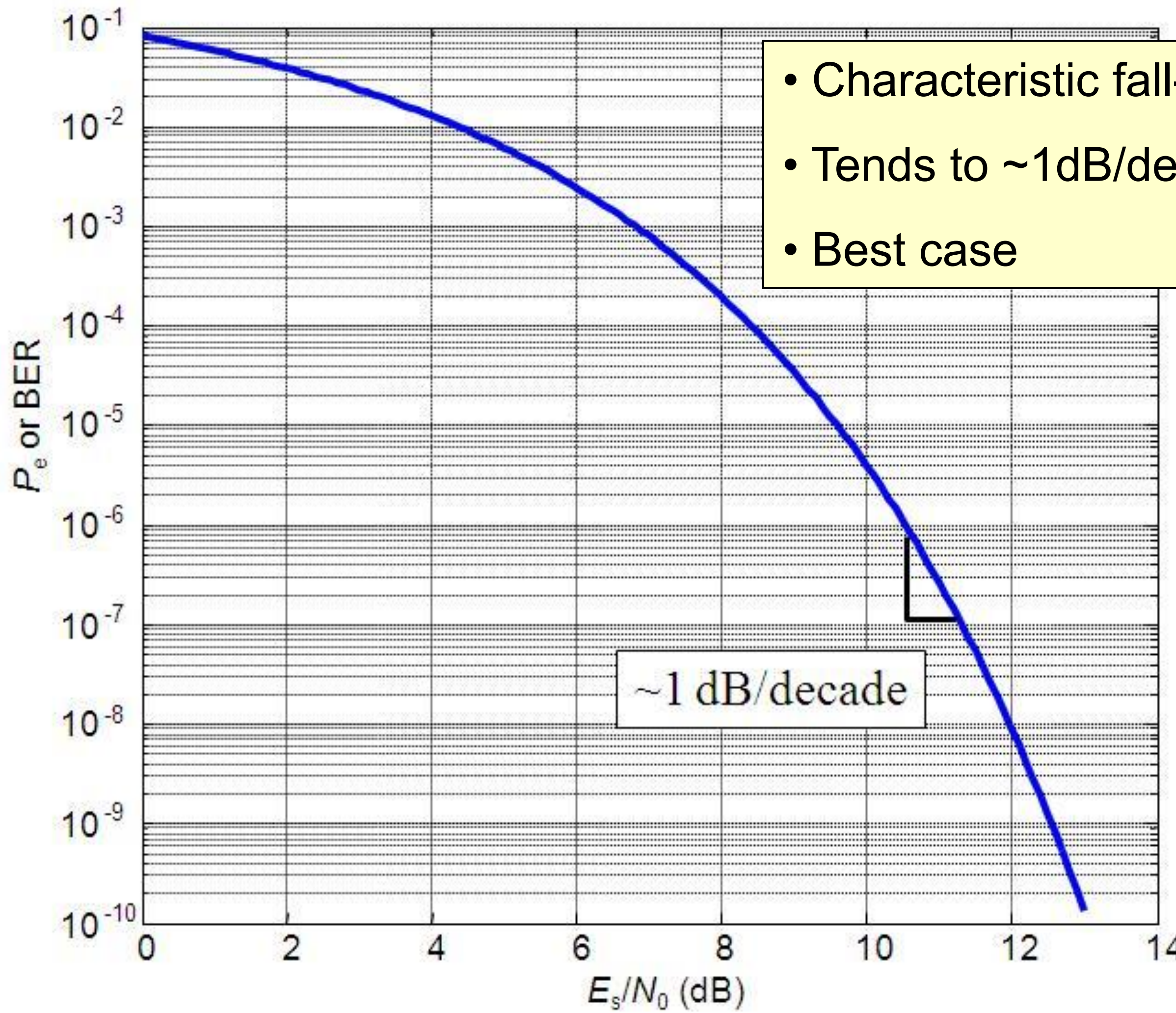
$$u_1 = \sqrt{\frac{2E_s}{T}} \qquad u_0 = -\sqrt{\frac{2E_s}{T}}$$

for symbol energy and duration, E_S and T, and A = 1. In the case of BPSK the BER is

$$P_e = Q\left(\sqrt{\frac{A^2 d^2}{2N_0}}\right) = Q\sqrt{\left(\frac{4E_s}{2N_0}\right)} = Q\left(\sqrt{2\gamma}\right)$$



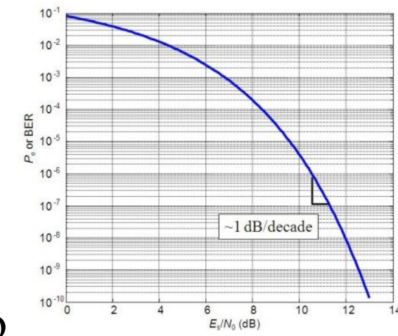
Bit error rate for BPSK in AWGN



- Characteristic fall-off
- Tends to ~1dB/decade
- Best case

Example: Necessary transmitter power for a QPSK system

Transmit R =20·10^6 bit/s at carrier frequency of f = 2.4 GHz.
Let the fade margin be M = 30 dB and need to operate satisfactory at d = 0.1 km.



Steps:

1. Determine E_b/N_0 for the desired BER
2. Convert E_b/N_0 to C/N at the receiver using the bit rate
3. Add the path loss and fade margin and calculate power

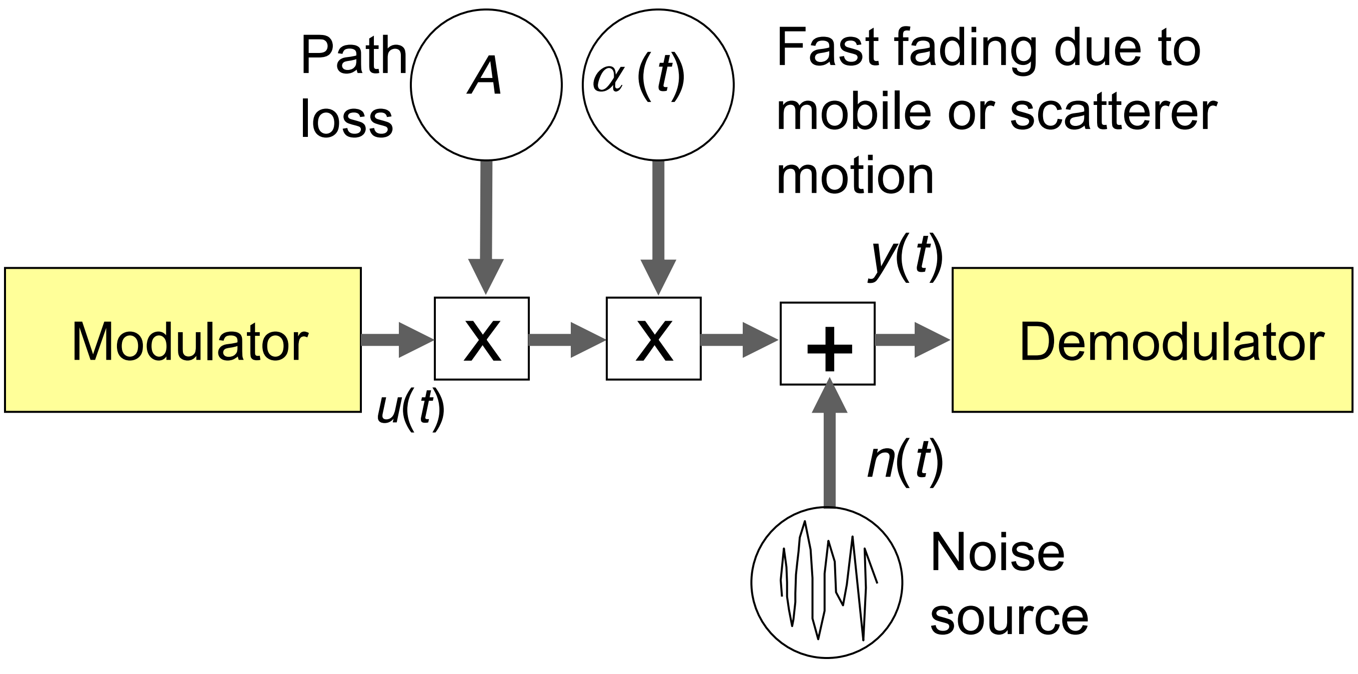
Let BER be less than 10^{-6} for satisfactory operation. Use the figure to see that E_b/N_0 must be greater than 10.5 dB, same as for BPSK. C/N is: $\frac{C}{N} = \frac{E_b}{N_0} \cdot \frac{R}{B}$
where *B=10 MHz* is the receiver bandwidth.

C/N = 10.5 dB + 10log(20·10^6/10·10^6) = 10.5 dB + 3dB = 13.5 dB. (Note dBs)
The noise $N = kTB+F$, where k = 1.380650·10^{-23} J/K is Boltzmann's constant, T =290 K is the absolute temperature in Kelvin, and F = 6 dB the receiver noise figure.
$N = kTB+F$ = 10log(1.380650·10^{-23} · 290 ·10·10^6) +6 = -134 + 6 = -128 dBW=-98 dBm
Carrier C = 13.5 + N = -84.5 dBm
Path loss L = 92.4 + 20log$d$ + 20log$f$ = 80 dB
Transmit power, antennas G = 6 dB: $P = C-G+M+L$ =-84.5-6+80+30=19.5 dBm=89 mW

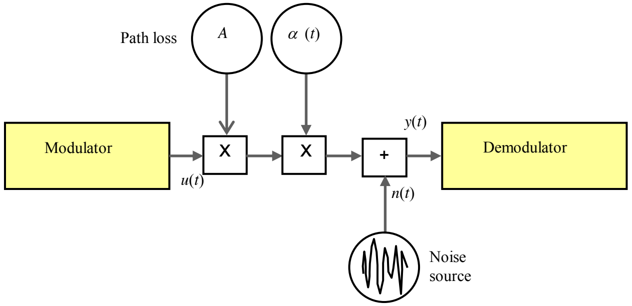
Narrowband fading channel

Simple AWGN normally not good enough

- Need to find mean SNR
- Need to describe how the instantaneous SNR various around mean



Characteristics of narrowband fading channel



Received signal: $y(t) = A \cdot \alpha(t) \cdot u(t) + n(t)$

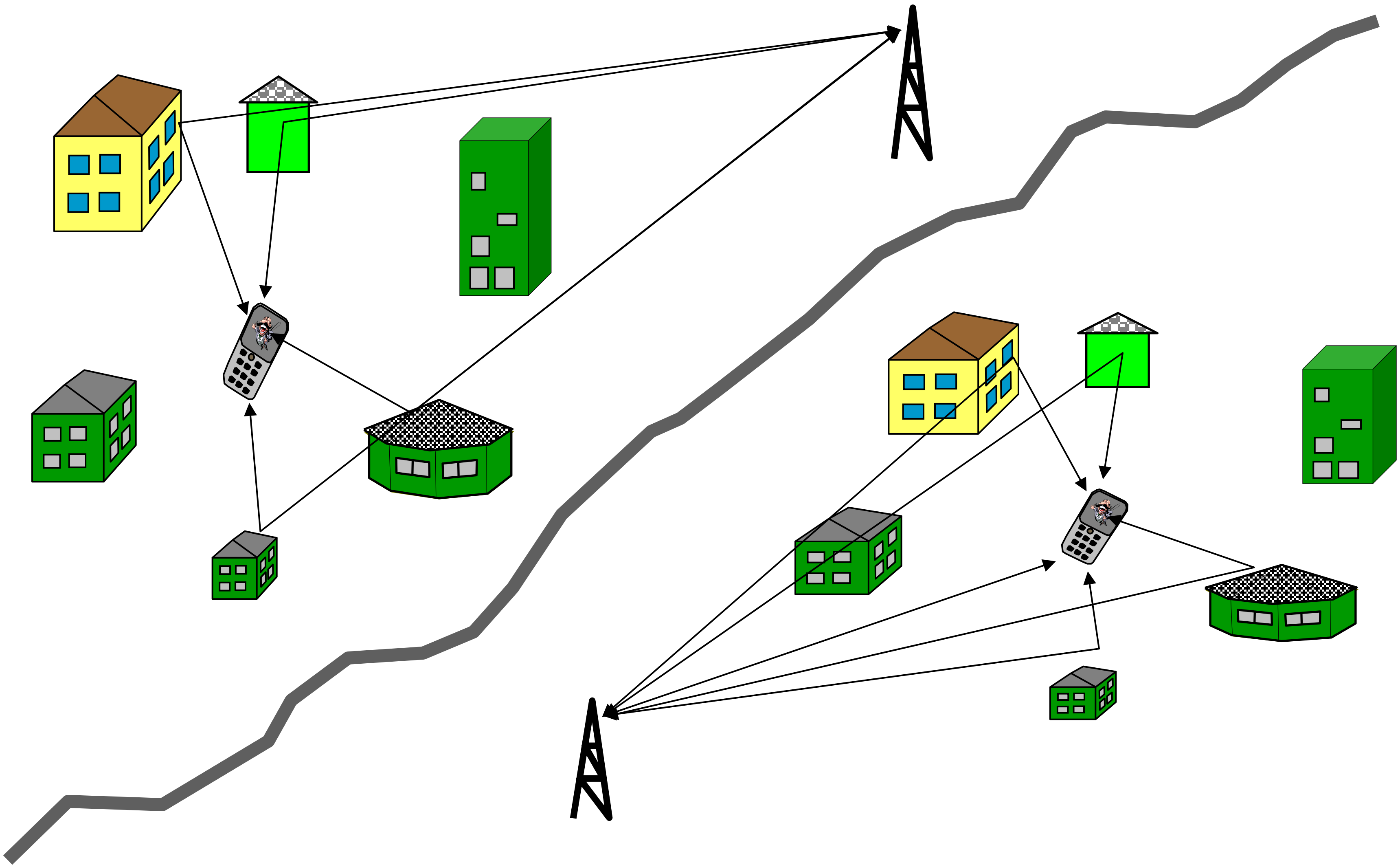
Instantaneous SNR: $\gamma = \frac{\text{Signal power}}{\text{Noise power}} = \frac{A^2 |\alpha(t)|^2 E\left[|u(t)|^2\right]}{2P_n} = \frac{A^2 |\alpha(t)|^2}{2P_n}$

assuming variance of modulator output being 1.

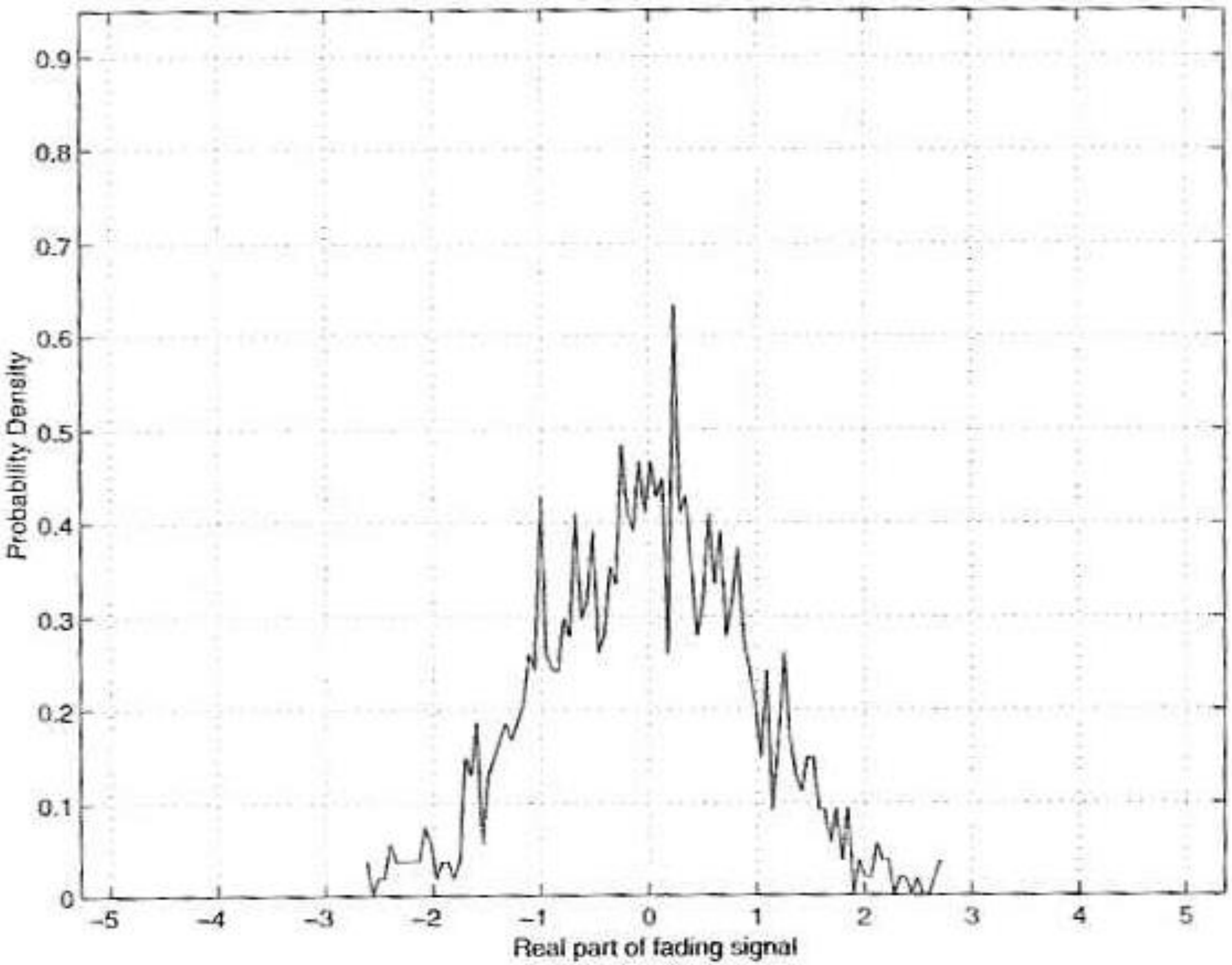
Mean SNR, taking fading to have unit variance change in mean into path loss :

$$\Gamma = E[\gamma(t)] = \frac{A^2}{2P_n}$$

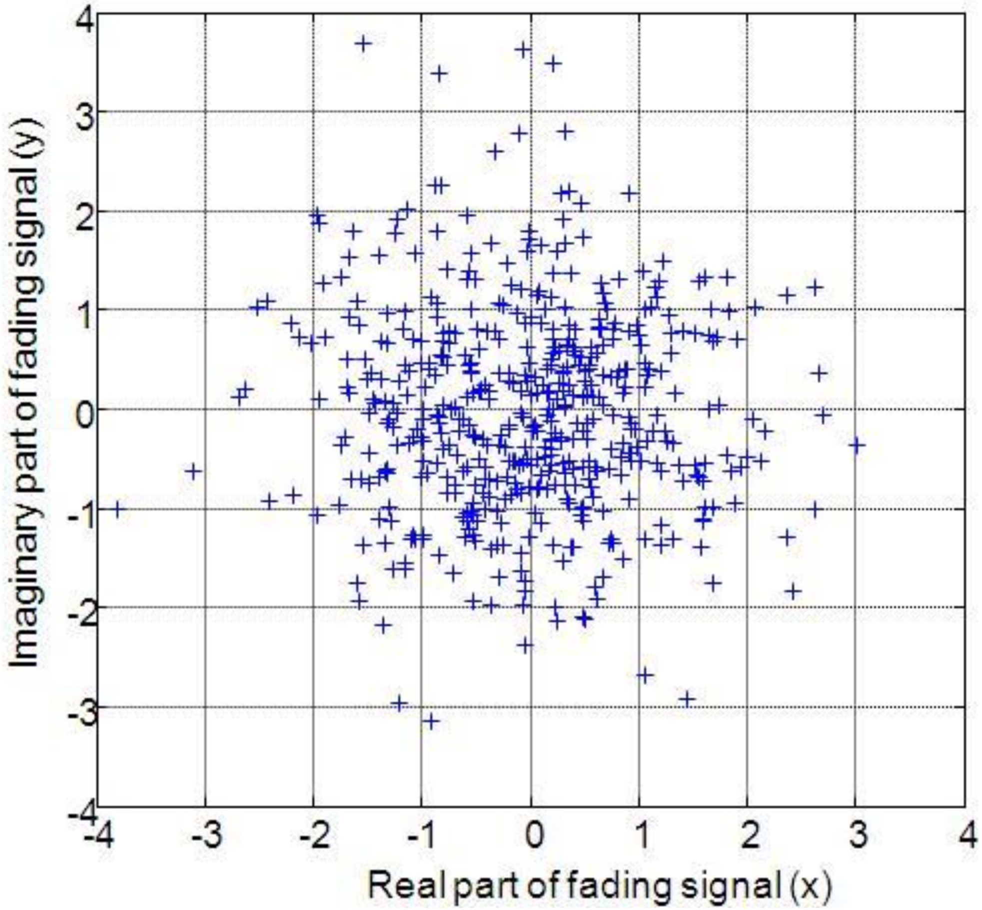
NLOS and LOS



pdf for NLOS fading

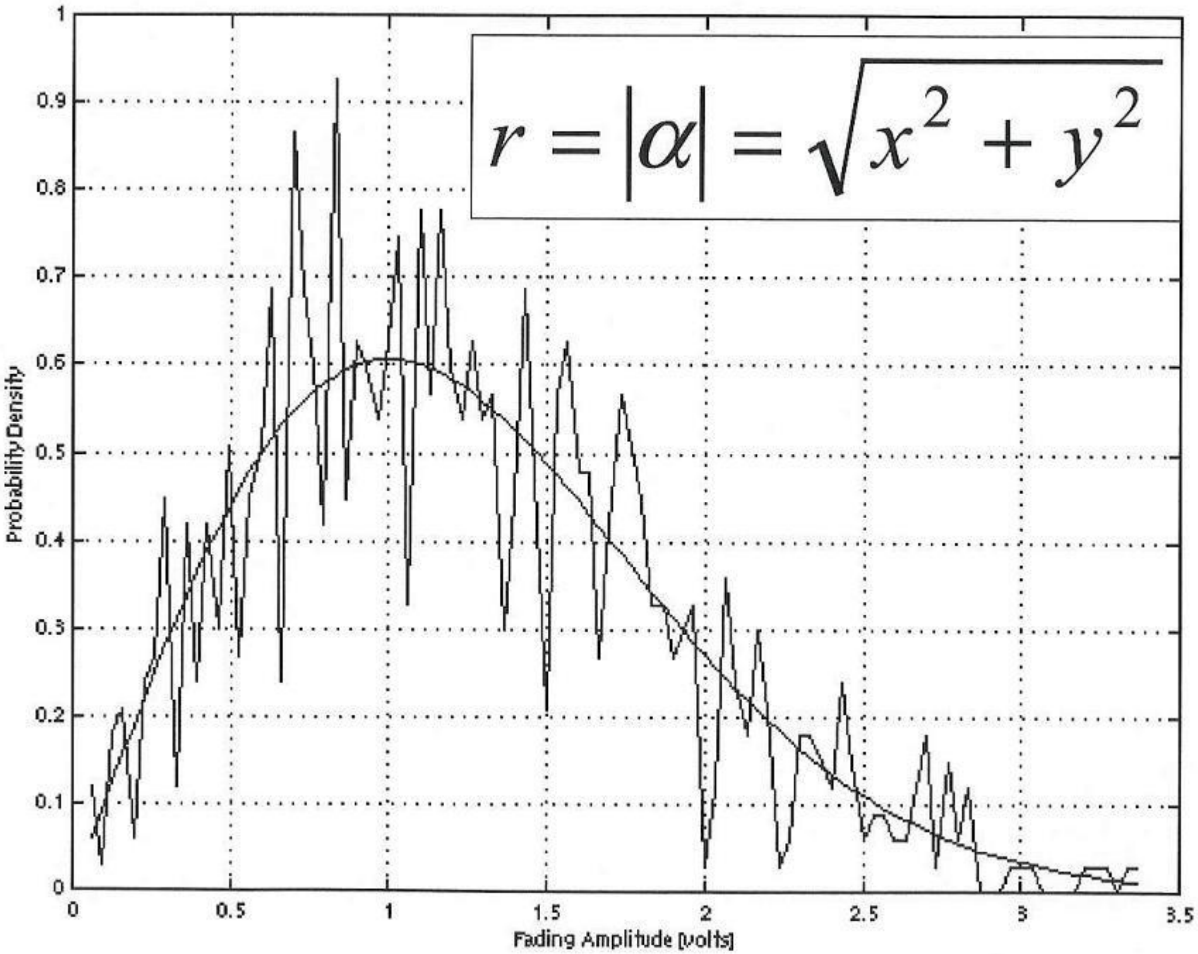


NLOS case



Imaginary part of fading signal (y)
Real part of fading signal (x)
4
3
2
1
0
-1
-2
-3
-4
-4 -3 -2 -1 0 1 2 3 4

Rayleigh distribution



Properties

$$\text{Mean}(r) = \sigma\sqrt{\frac{\pi}{2}}$$

$$\text{Median}(r) = \sigma\sqrt{\ln(4)}$$

$$\text{Mode}(r) = \sigma$$

$$\text{Variance}(r) = \frac{4-\pi}{2}\sigma^2$$

where σ is the standard deviation for x and for y.

Signal to noise ratio for a Rayleigh channel

Instantaneous SNR: $$\gamma = \frac{\text{Signal power}}{\text{Noise power}} = \frac{A^2 r^2 / 2}{P_n} = \frac{A^2 r^2}{2P_n}$$

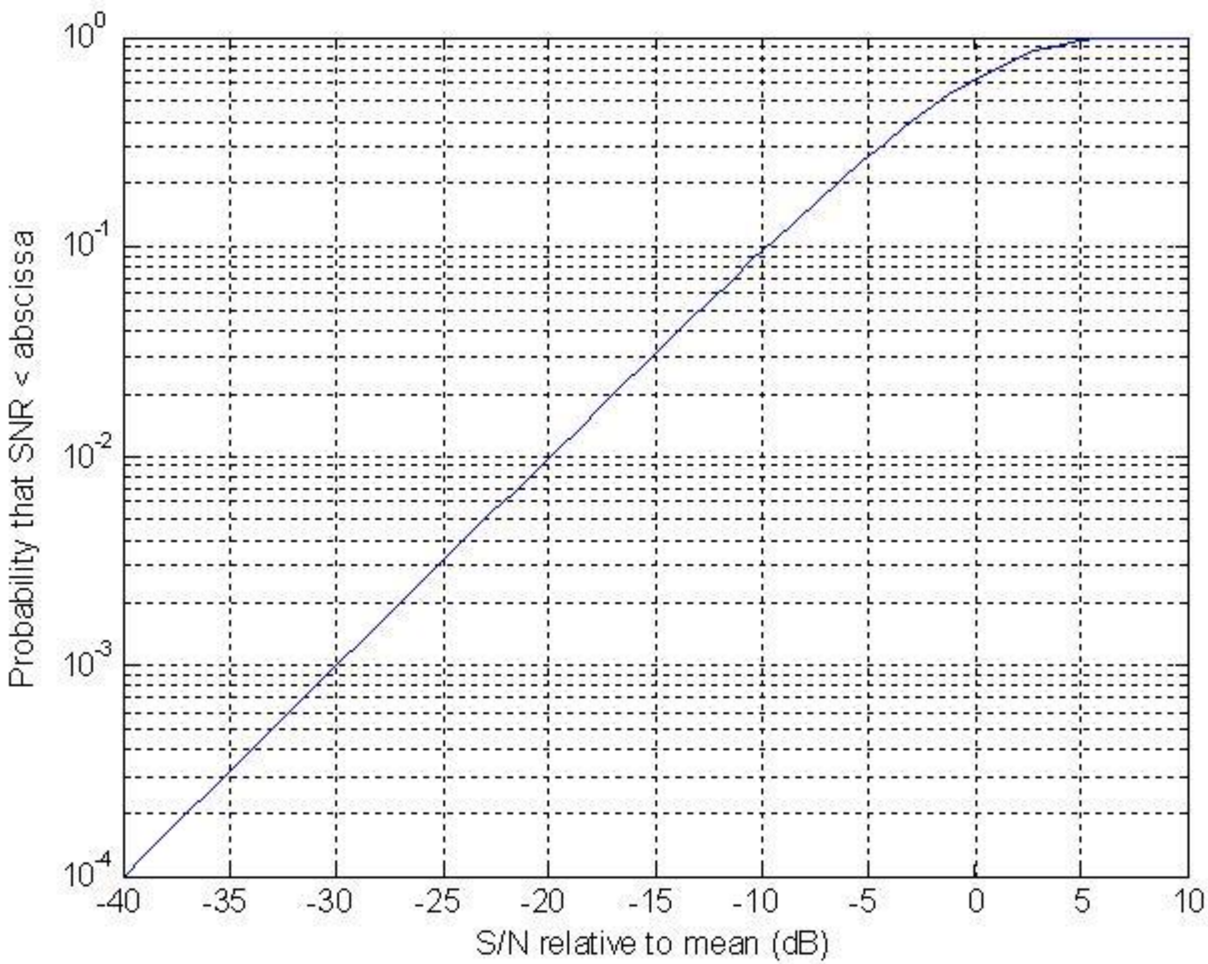
Mean SNR: $$\Gamma = \frac{A^2 E\left[r^2\right]}{2P_n} = \frac{2A^2\sigma^2}{2P_n} = \frac{A^2\sigma^2}{P_n}$$

Using the identify $$p_\gamma(\gamma) = P_R(r)\frac{dr}{d\gamma}$$

then PDF: $$p_\gamma(\gamma) = \frac{1}{\Gamma} e^{-\gamma/\Gamma} \text{ for } \quad \gamma > 0$$

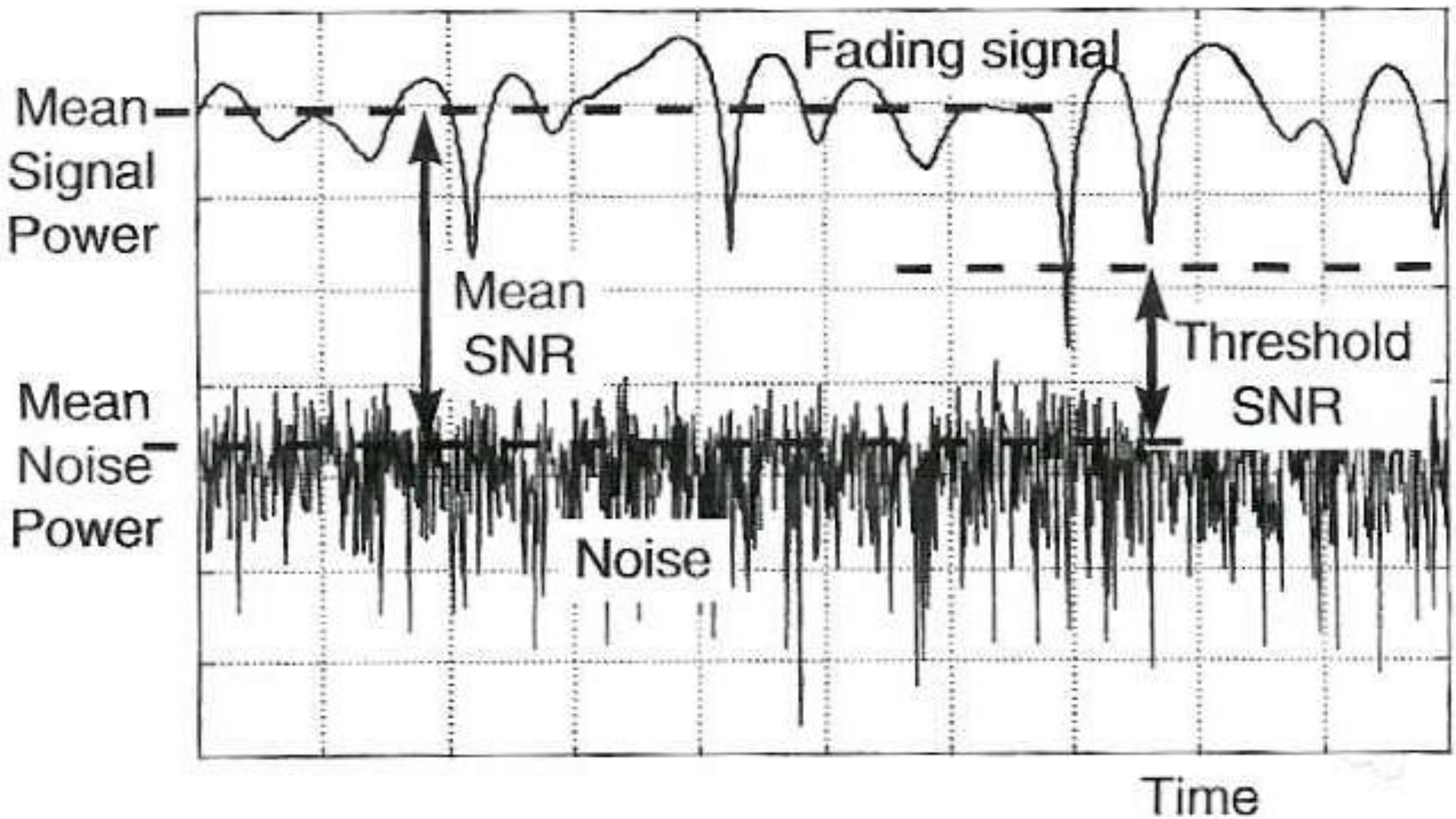
and CDF: $$P_R(\gamma < \gamma_s) = 1 - e^{-\gamma/\Gamma}$$

NLOS case



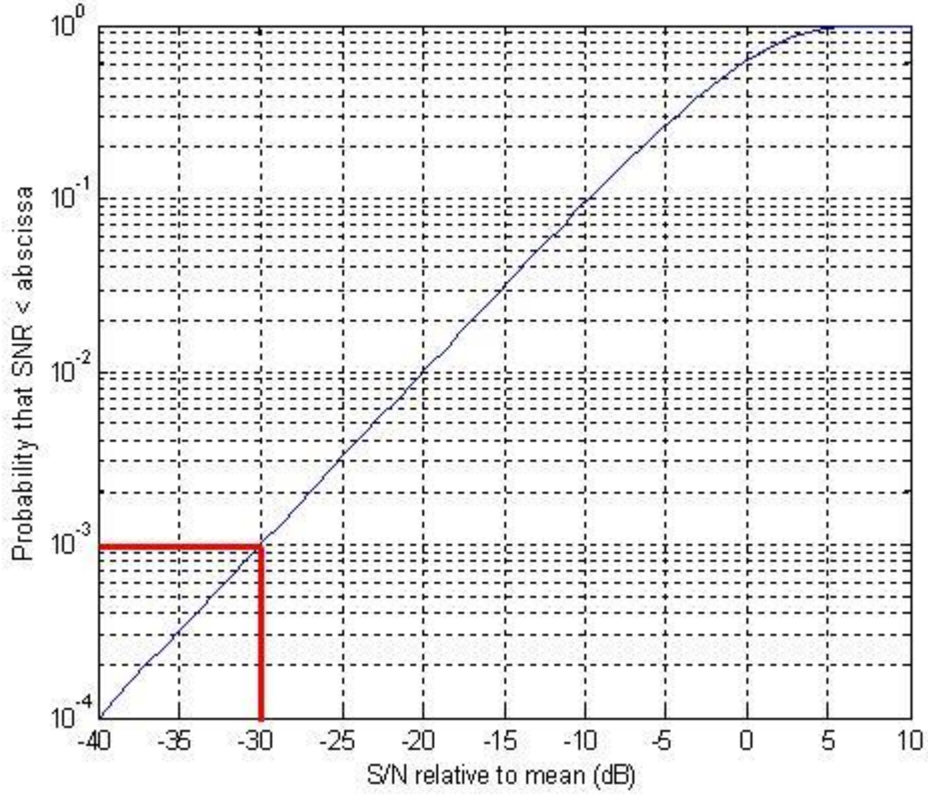
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Probability that SNR < abscissa
-40
-35
-30
-25
-20
-15
-10
-5
0
5
10
S/N relative to mean (dB)

Variation of instantaneous SNR



Example Rayleigh channel

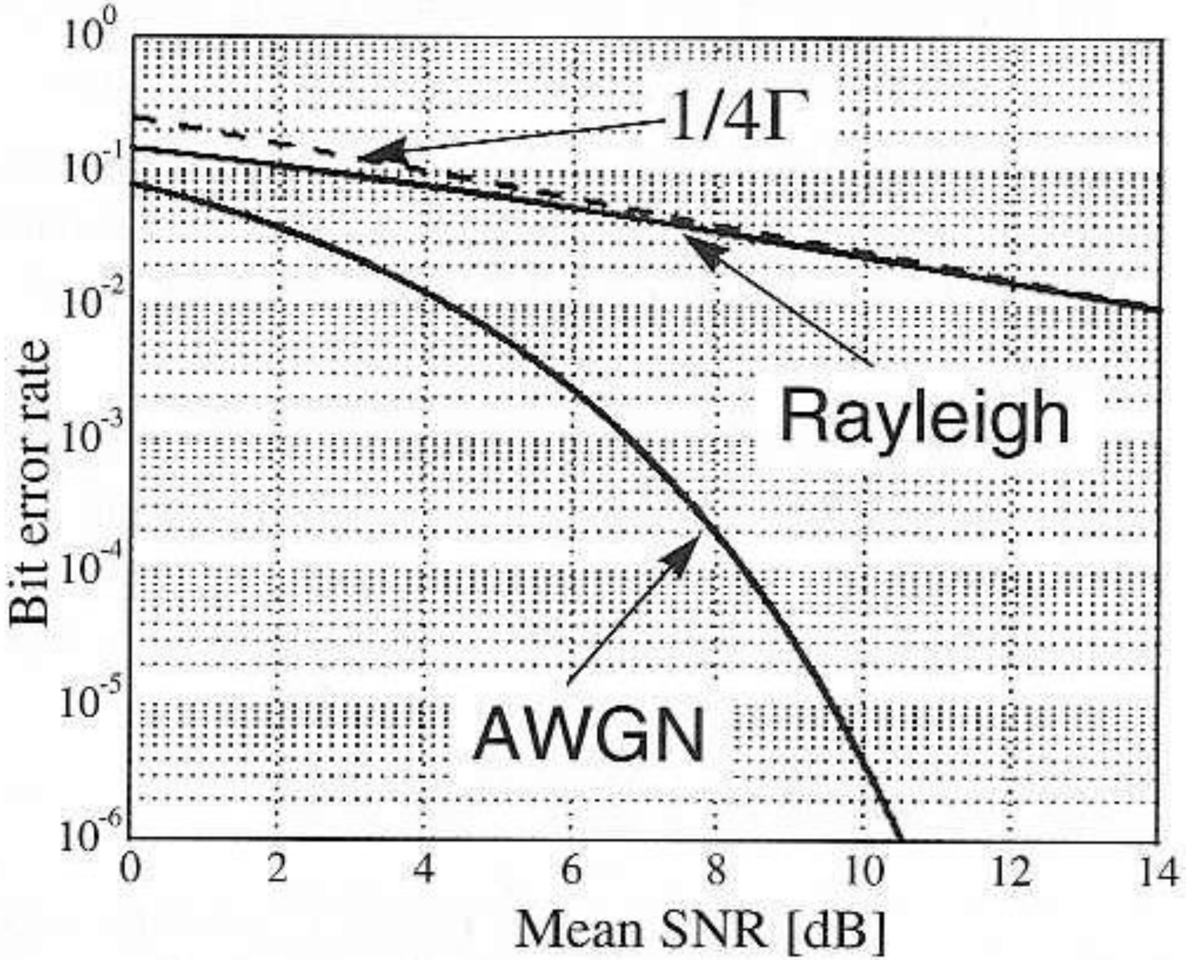
Acceptable bit error ratio (BER) if instantaneous SNR > 9 dB. What is the mean SNR required in a Rayleigh channel for acceptable BER 99.9 % of the time?



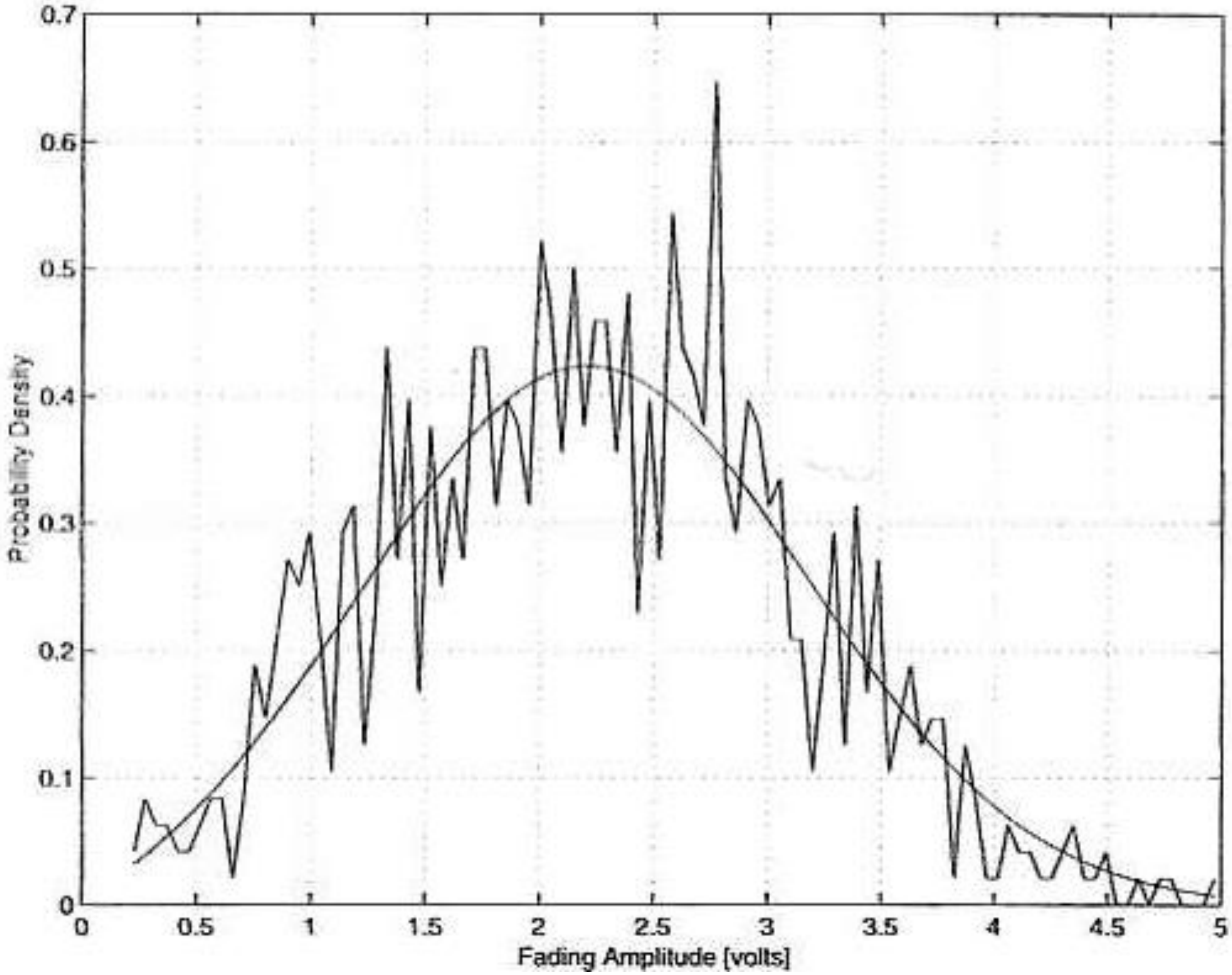
Solution

99.9 % success means 0.1% failure, i.e., probability 0.001. Using the figure (or Equation 10.28) the SNR relative to the mean at this probability is -30 dB, therefore the average SNR has to be 39 dB.

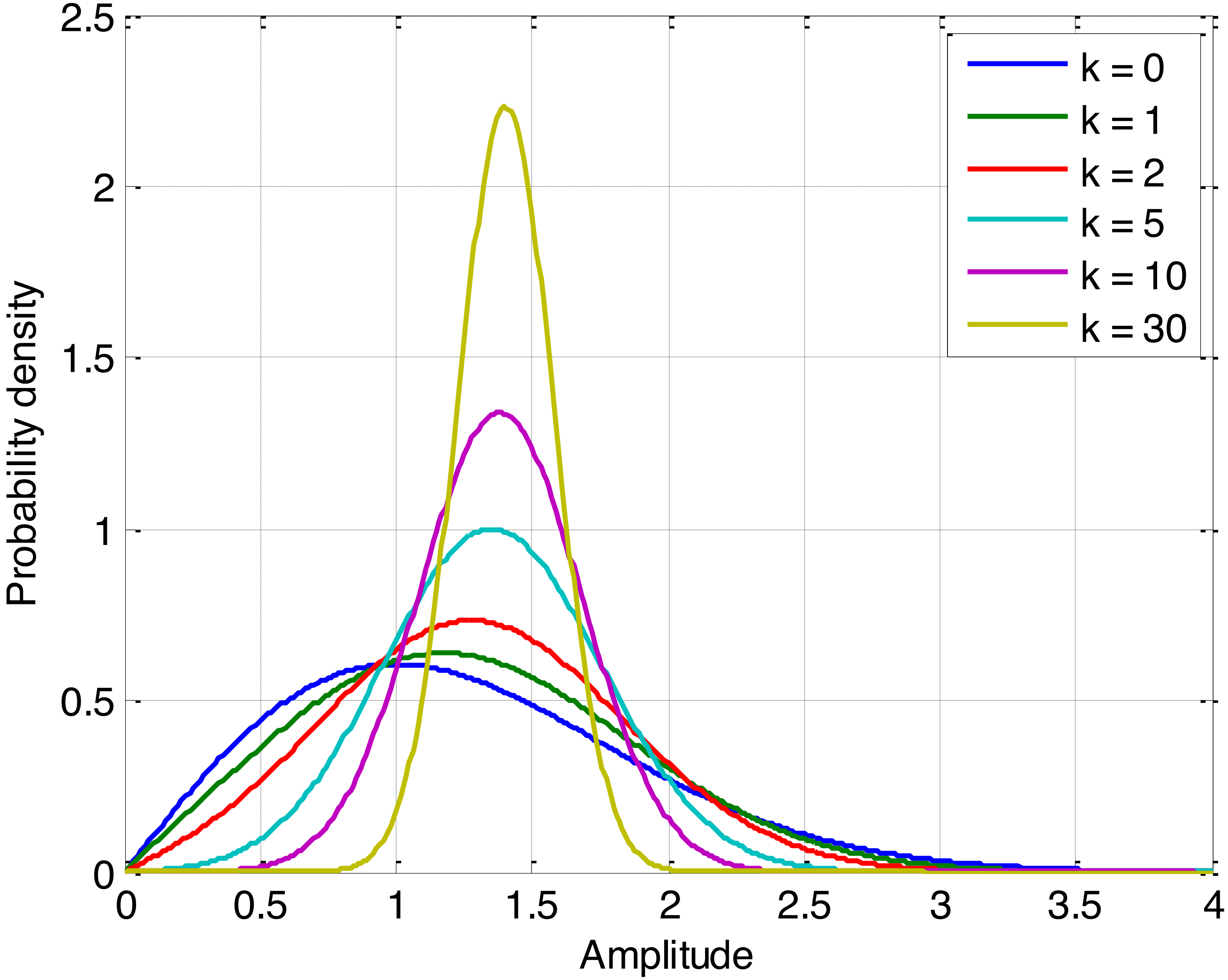
BPSK performance in a Rayleigh channel



Rice distribution

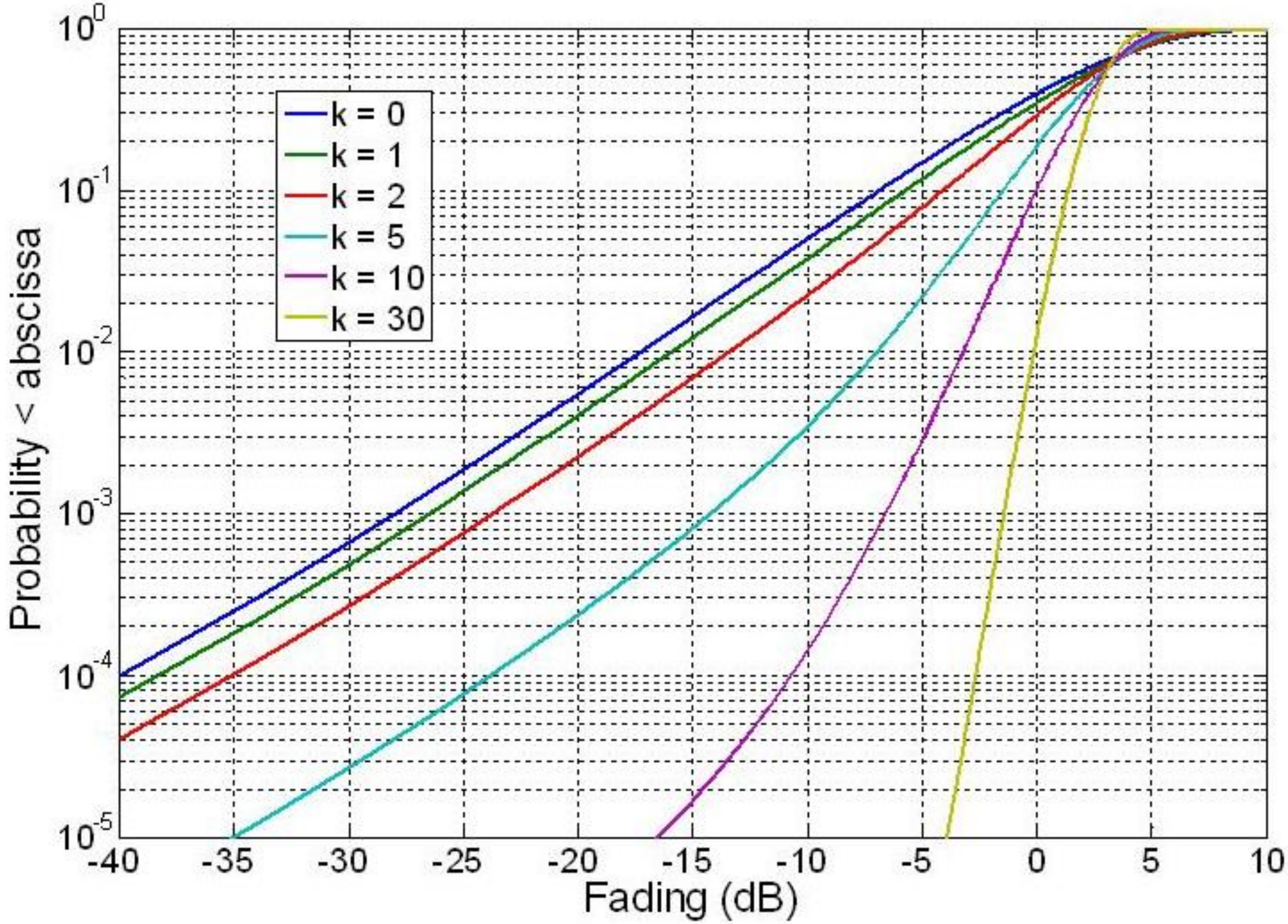


Rice pdf

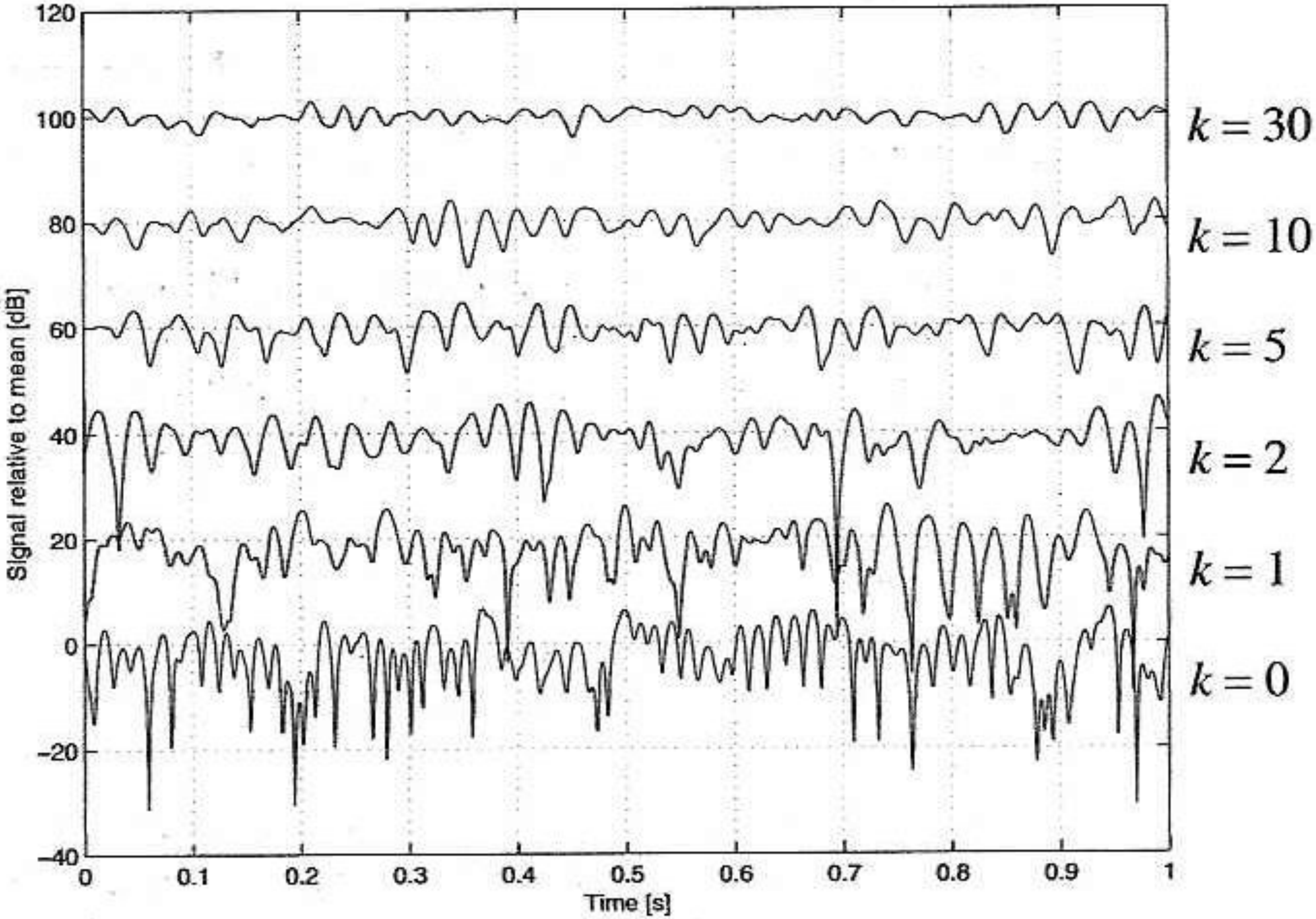


Probability density
Amplitude
k = 0
k = 1
k = 2
k = 5
k = 10
k = 30
0
0.5
1
1.5
2
2.5
3
3.5
4

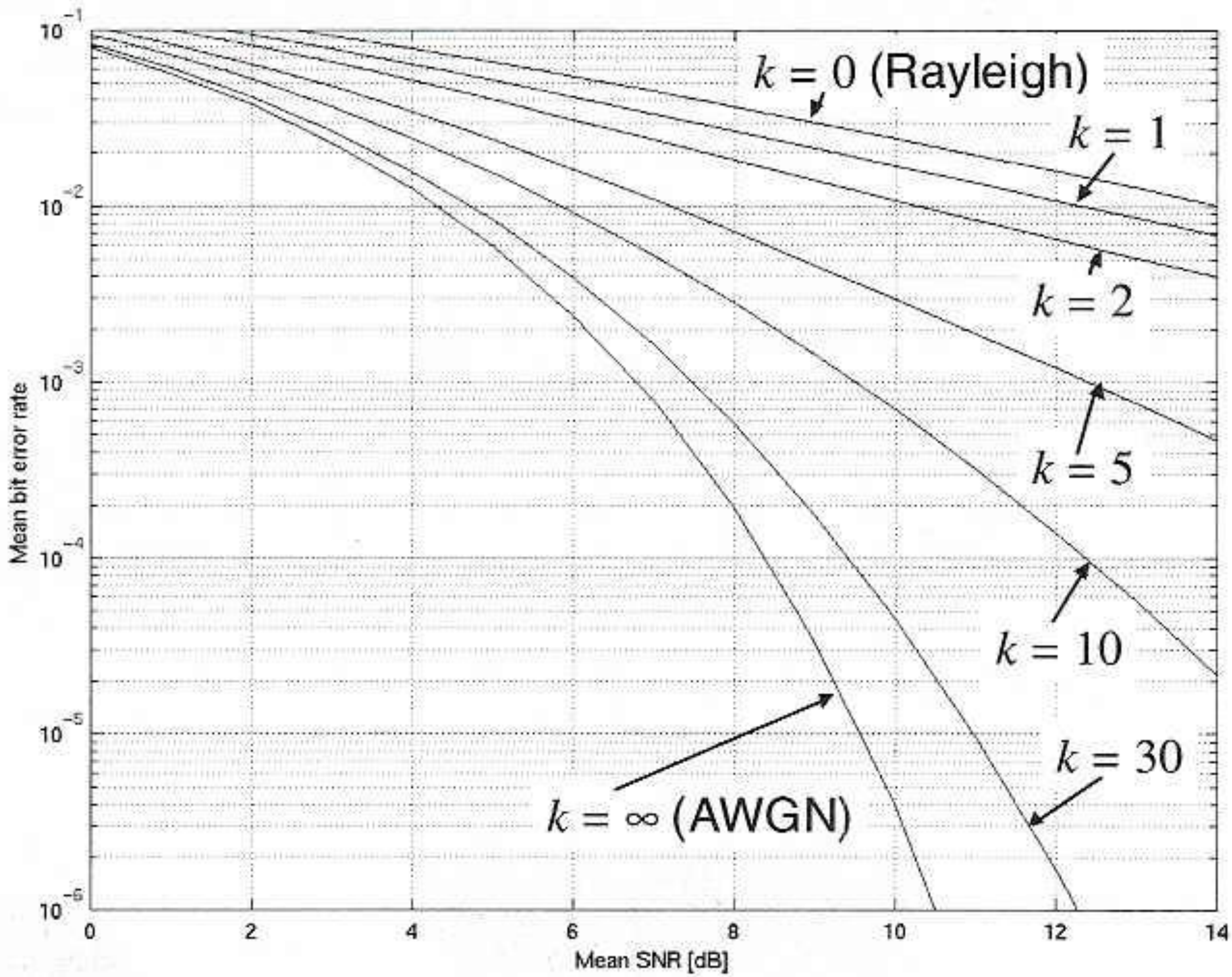
Rice distributions for various *k*



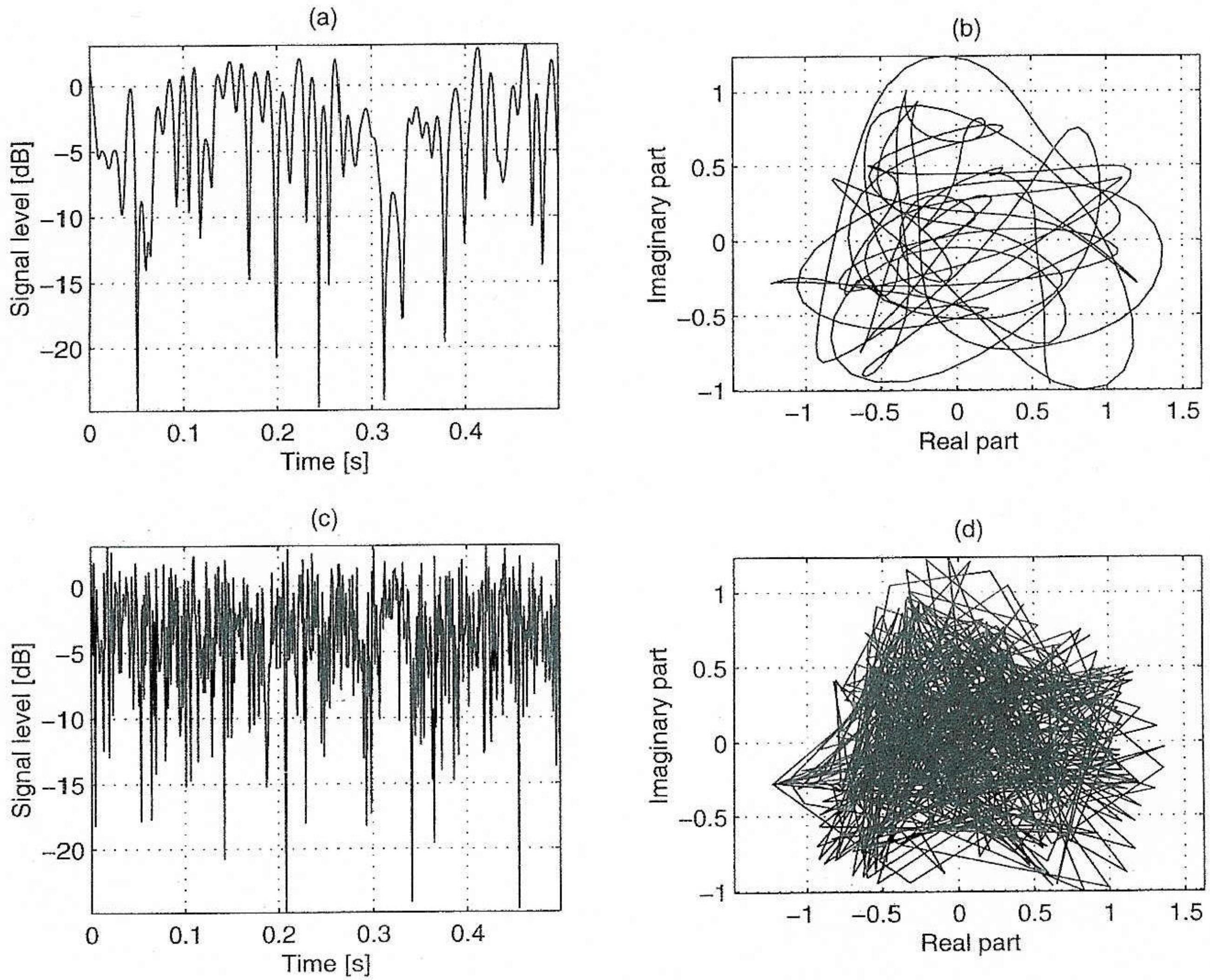
Rice fading signals



Performance for Rice channels

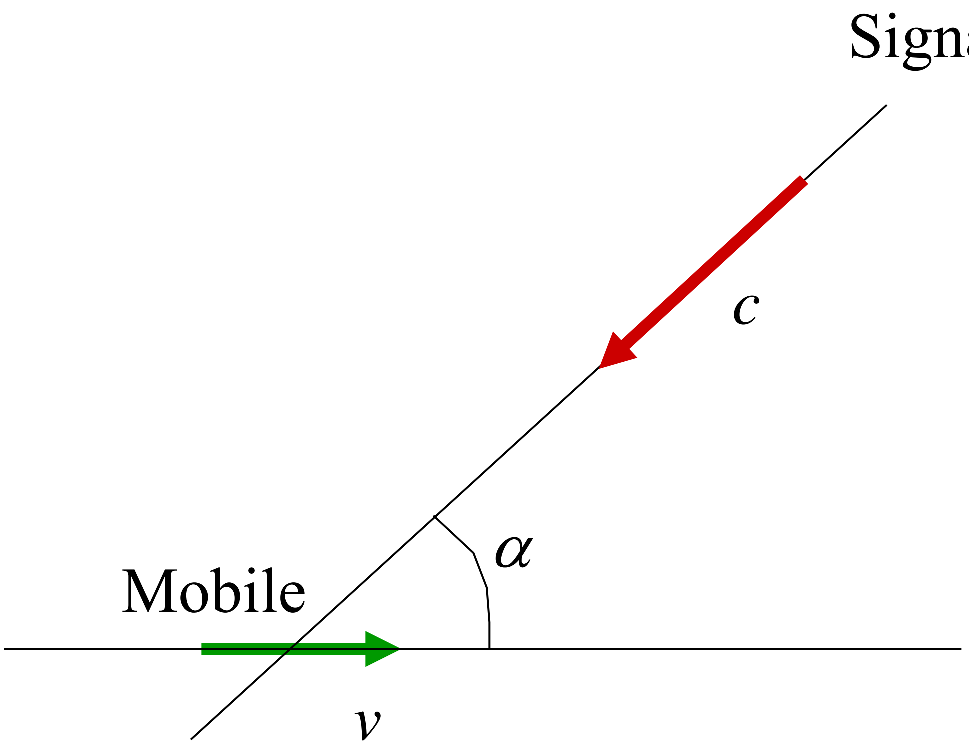


Effects of second order statistics



Doppler shift

If the mobile or the reflector moves the frequency may change.



The mobile velocity towards the source is

$$v \cdot \cos\alpha.$$

In the time *t* the mobile has received

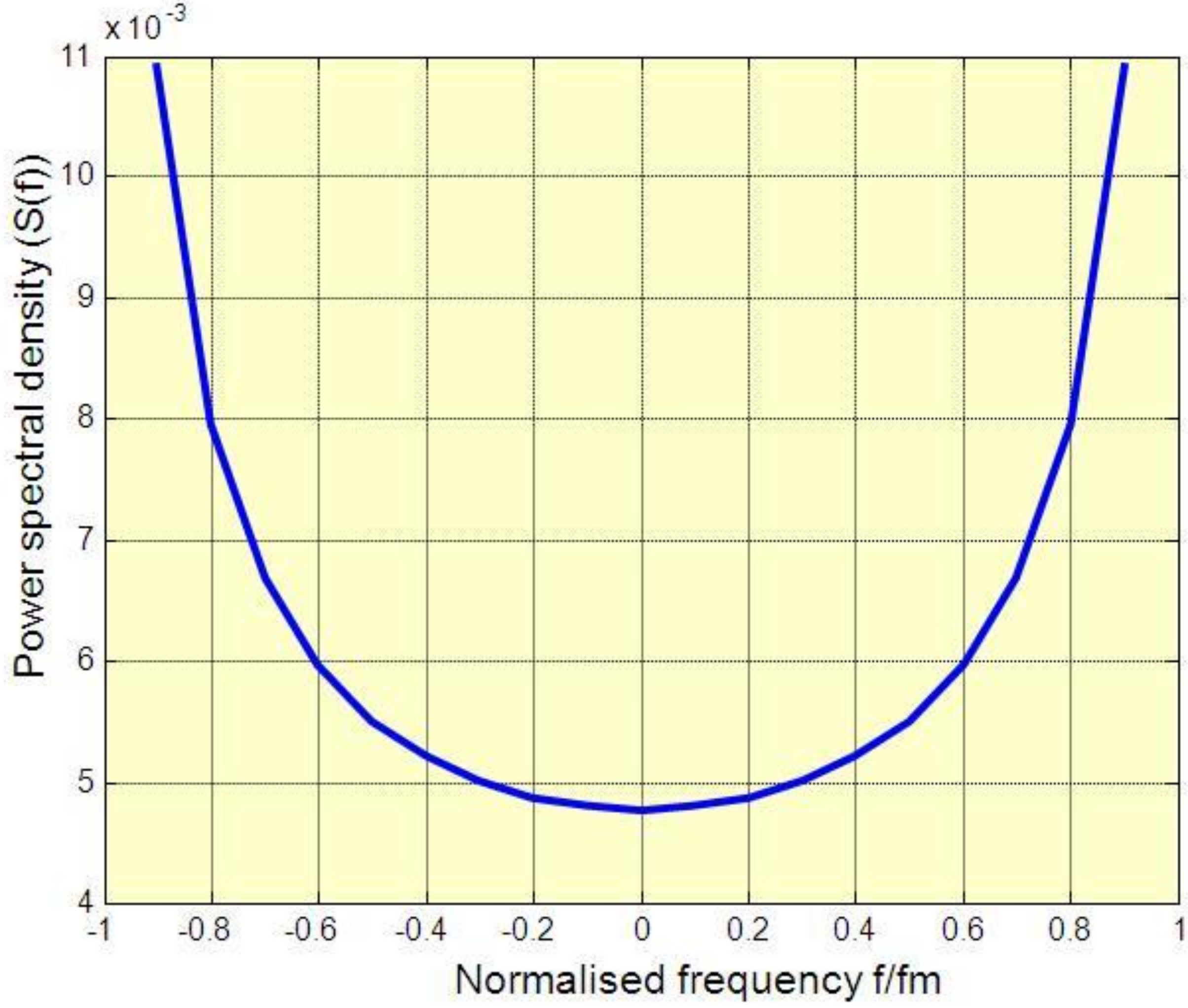
$$f \cdot t + v\cos\alpha \cdot t/\lambda$$

wave lengths.

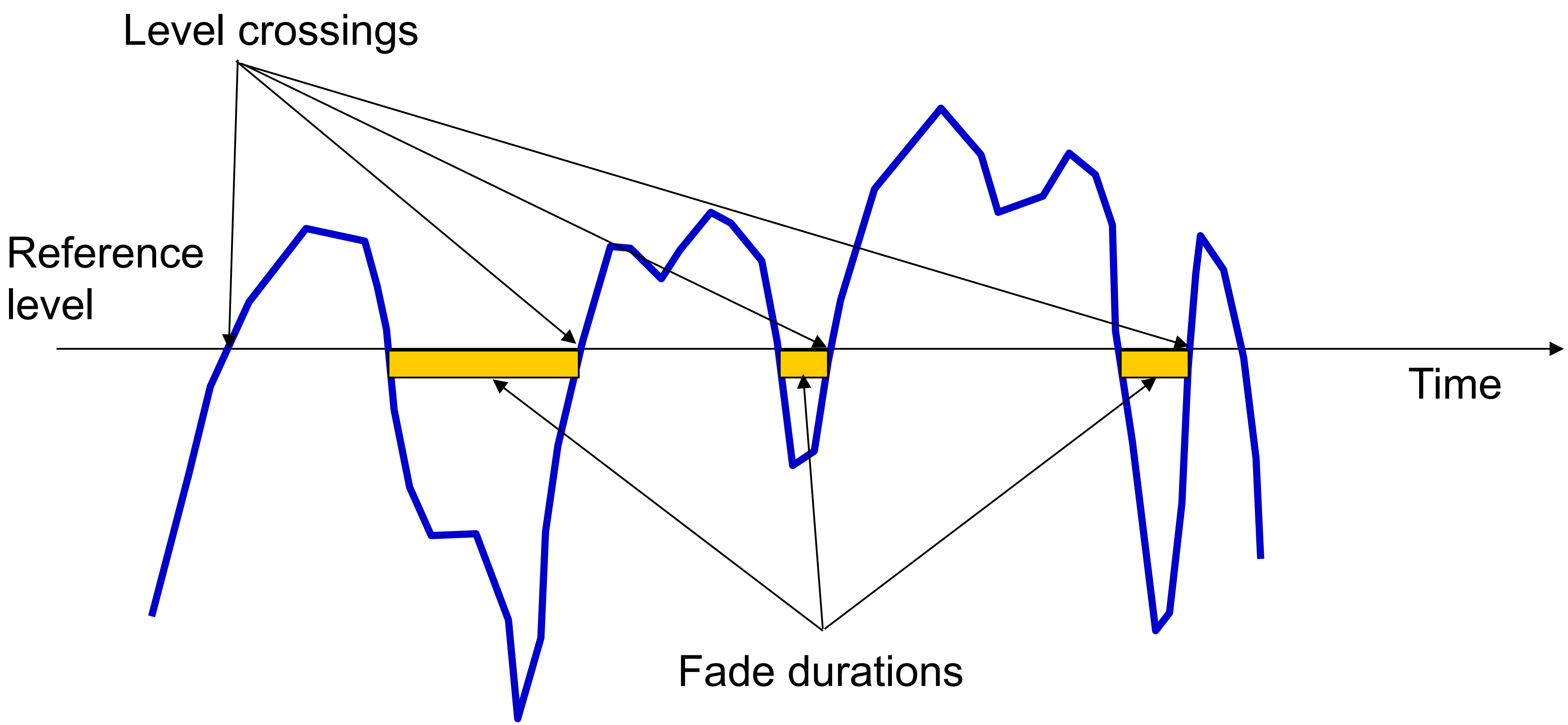
$$f_D = v \cdot \cos\alpha/\lambda$$

is called the Doppler shift

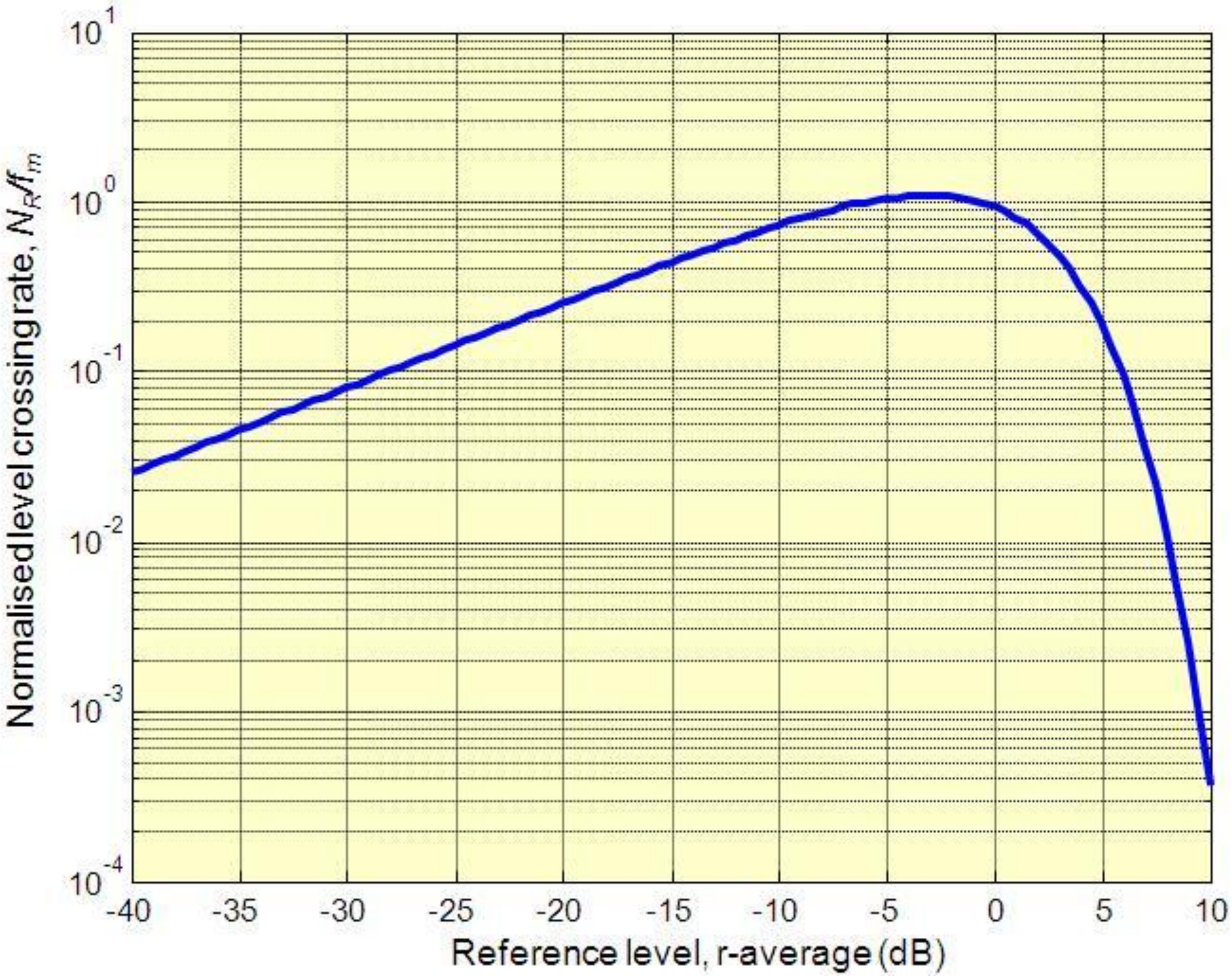
Doppler spectrum



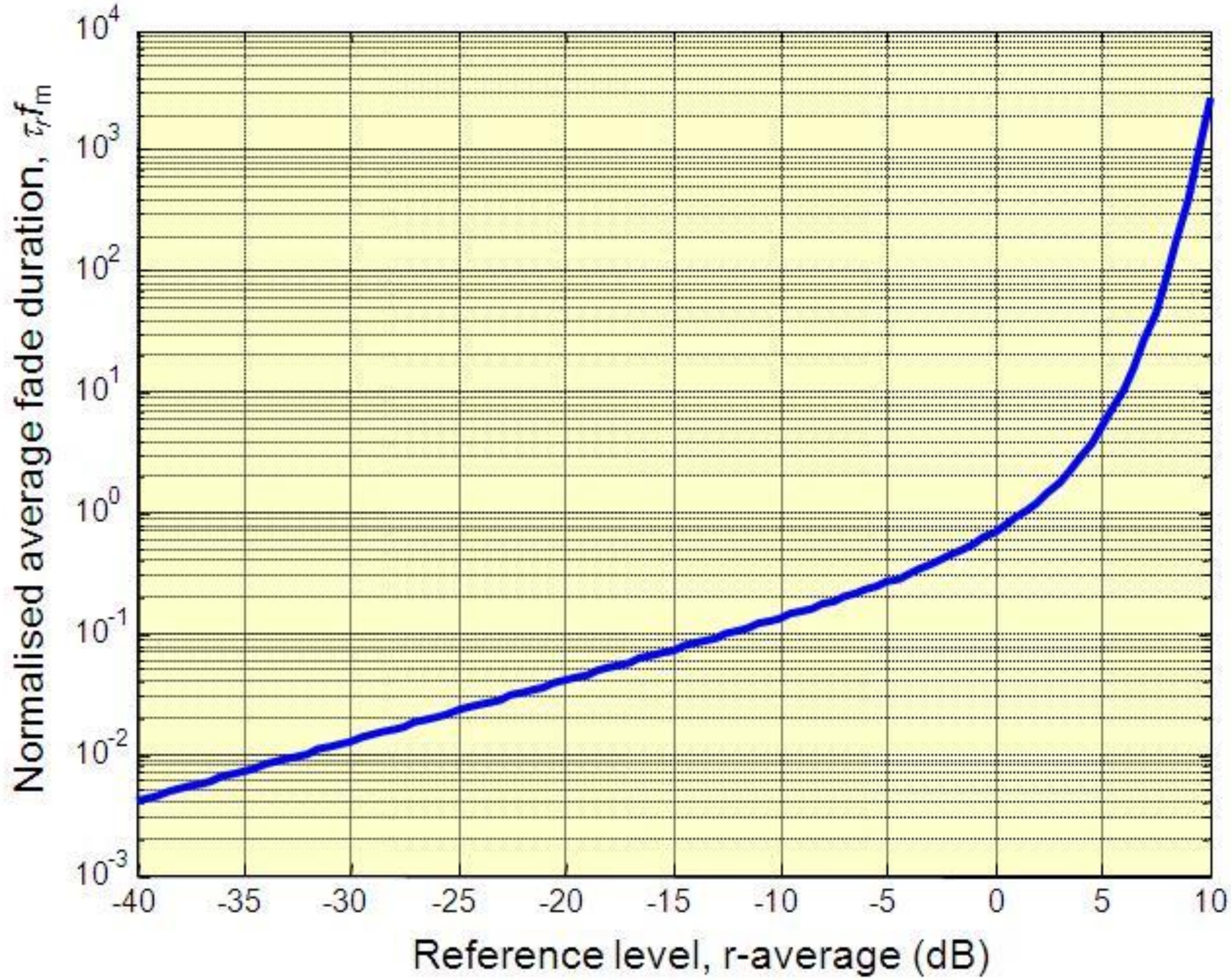
Level crossings and fade durations



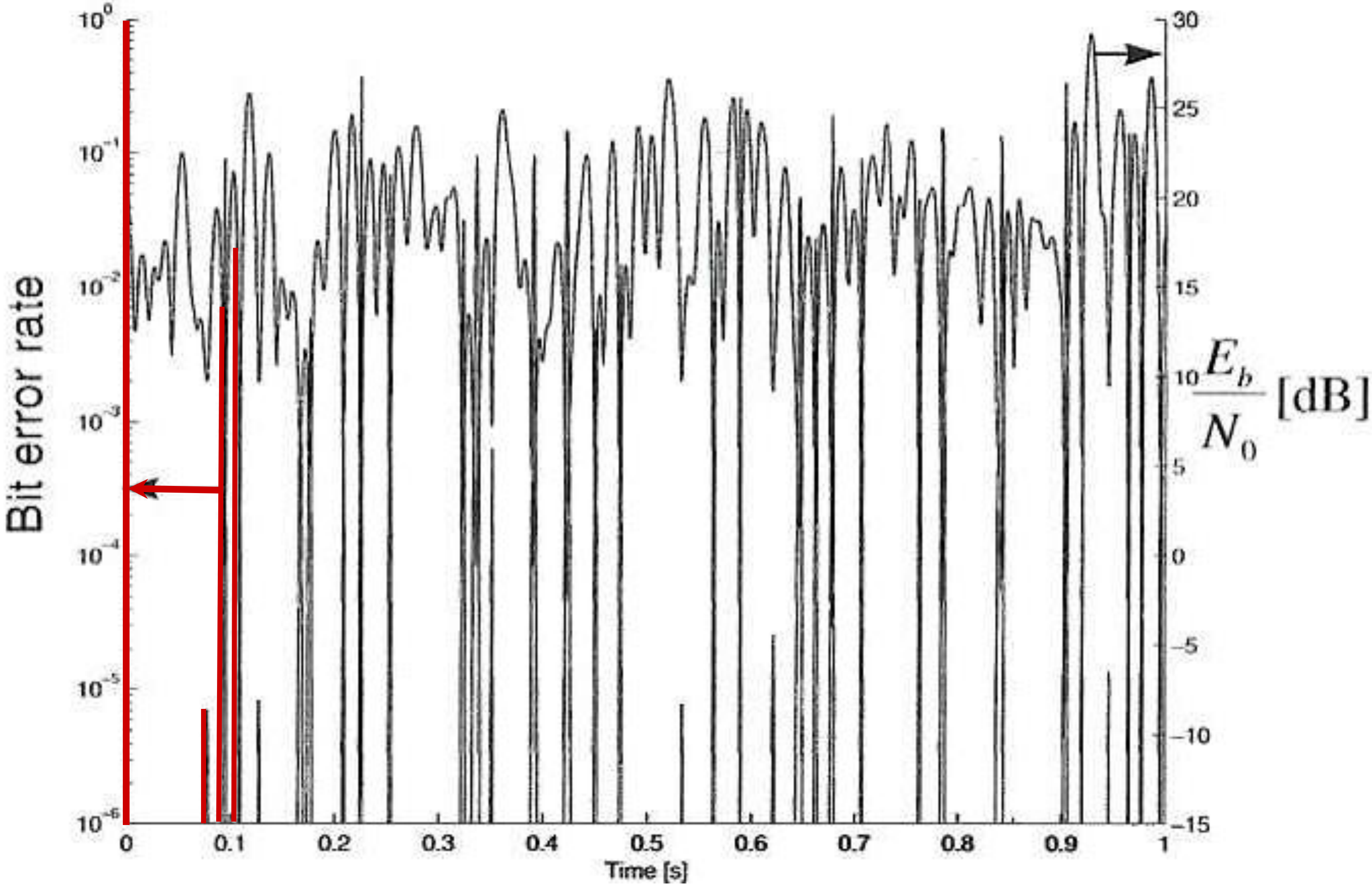
Normalised level crossings



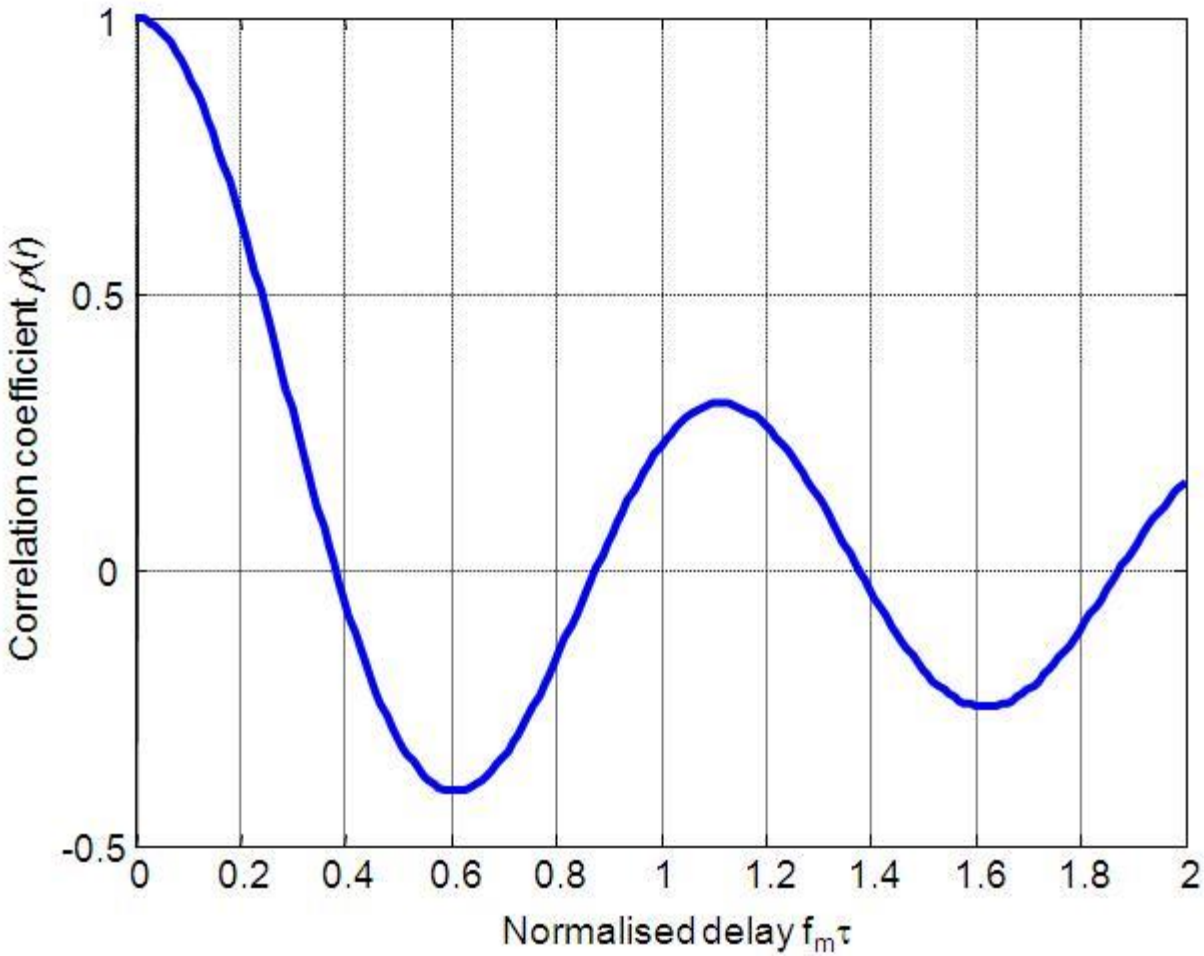
Normalised fade duration



BER for a Rayleigh channel



Autocorrelation function classical spectrum



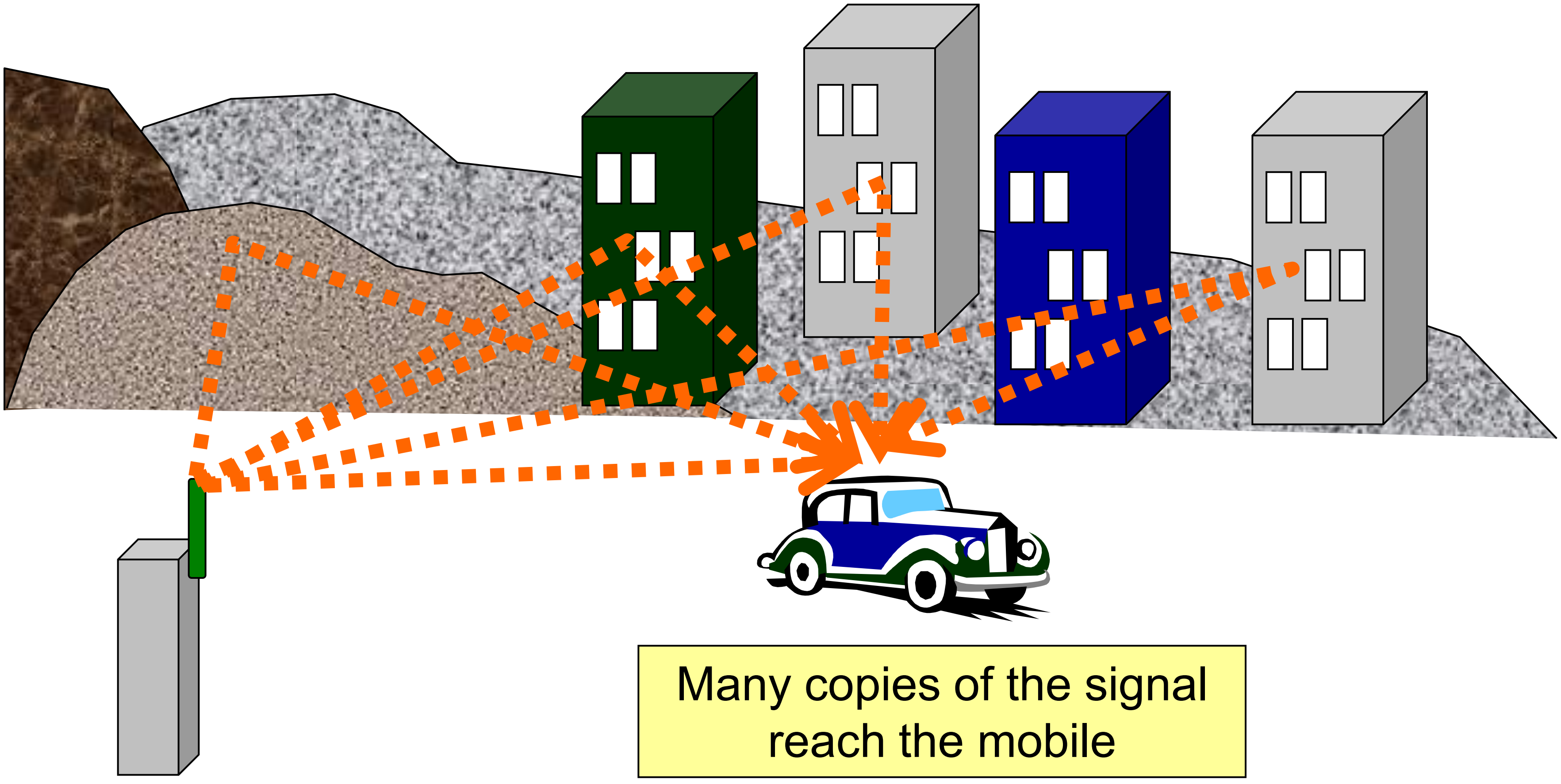
Conclusions

- AWGN channel most basic, least destructive
- Rayleigh channel worst case fading
- Rice channel intermediate
- Second order statistics crucial in real systems

Chapter 11 Wideband fast fading

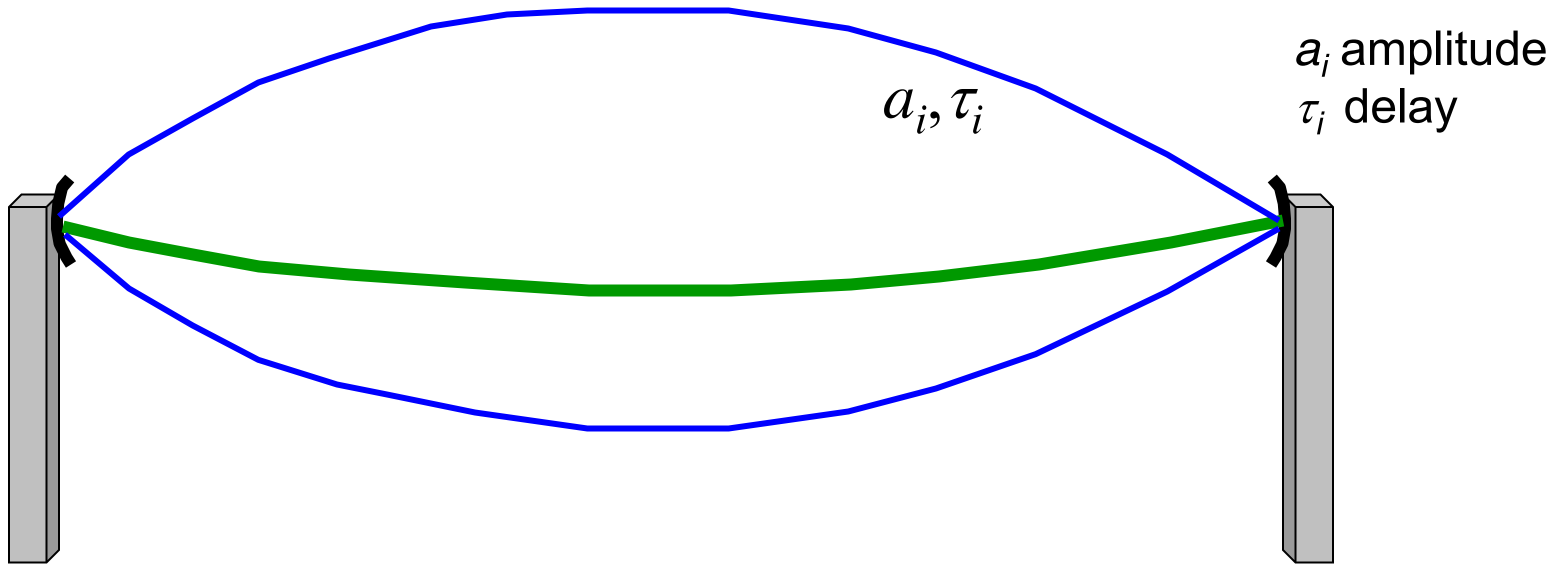
- Multipath phenomena
- Channel model
- Wideband channel parameters and characterisation
- Mitigation techniques

Example multiple path propagation



Example atmospheric and ground reflected multipath

The signal finds several routes from the transmitter to the receiver.



$$E = \sum_i a_i e^{jkd_i} = \sum_i a_i e^{jk(d+\Delta d_i)} = e^{jkd} \sum_i a_i e^{jk\Delta d_i} = e^{jkd} \sum_i a_i e^{j\omega\tau_i}$$

$$\text{remembering } k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c} \quad \text{and} \quad \frac{\omega}{c}\Delta d_i = \omega\tau_i$$

Broadband propagation

- Broadband and narrowband are not precise terms:
  something considered broad for one system may well be narrow in another
- Some think that a few 100 kbit/s is broadband, others insists on several Mbit/s
- With respect to radiowave propagation broadband is used if multipath can create frequency selective fading within the frequency band for the radio channel
- Even this definition is probably not exact but cover the topics in this lecture

Many signals

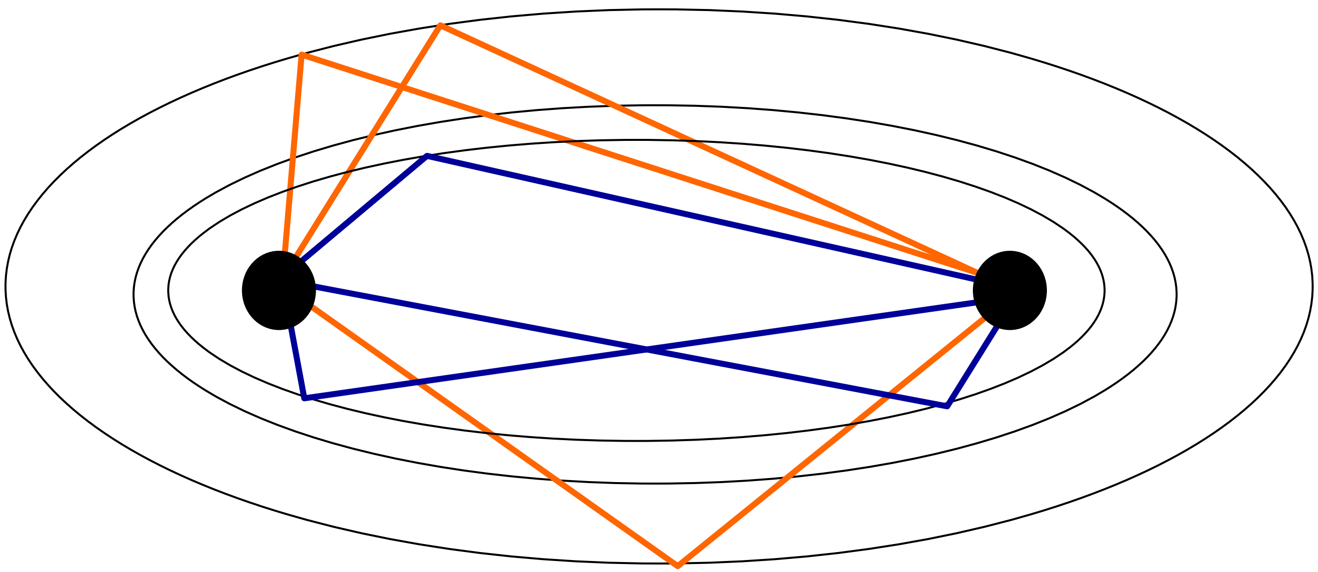
- Many signals reach the mobile, each of their own
  - amplitude
  - phase
  - time delay
  - angle of arrival
- The phase is given

$$\theta = \theta_0 + 2\pi d/\lambda$$

where θ_0 is the initial phase, d the propagation distance and λ the wave length

Many signals usual for mobile communication

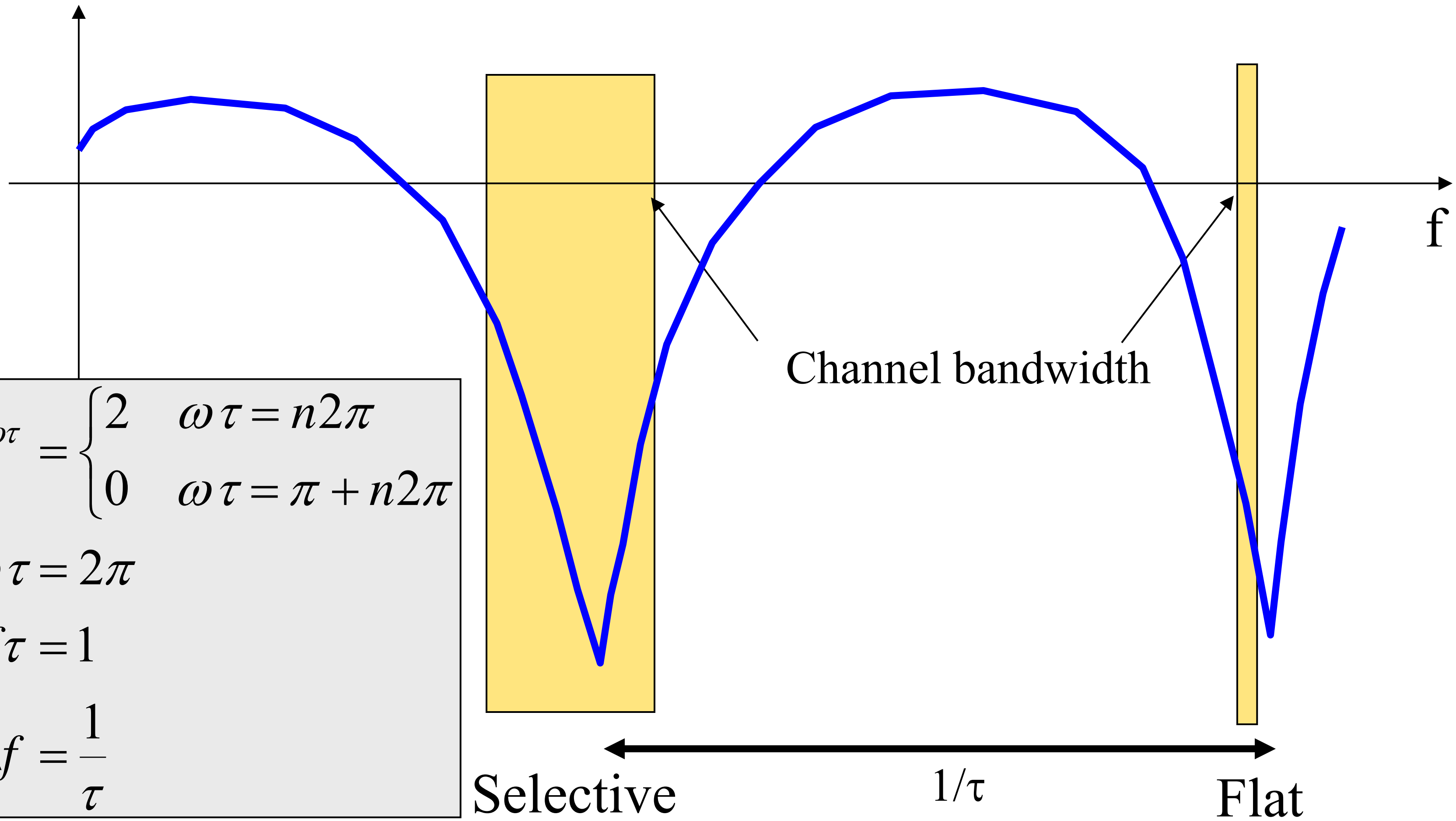
Reflection from many entirely different places may have exactly the same delay. This is often the case for mobile communication.



Ellipsoids

Channel bandwidth essential

Frequency selective only meaningful related to bandwidth occupied



$$1+e^{j\omega\tau}=\begin{cases}2 & \omega\tau=n2\pi\\ 0 & \omega\tau=\pi+n2\pi\end{cases}$$

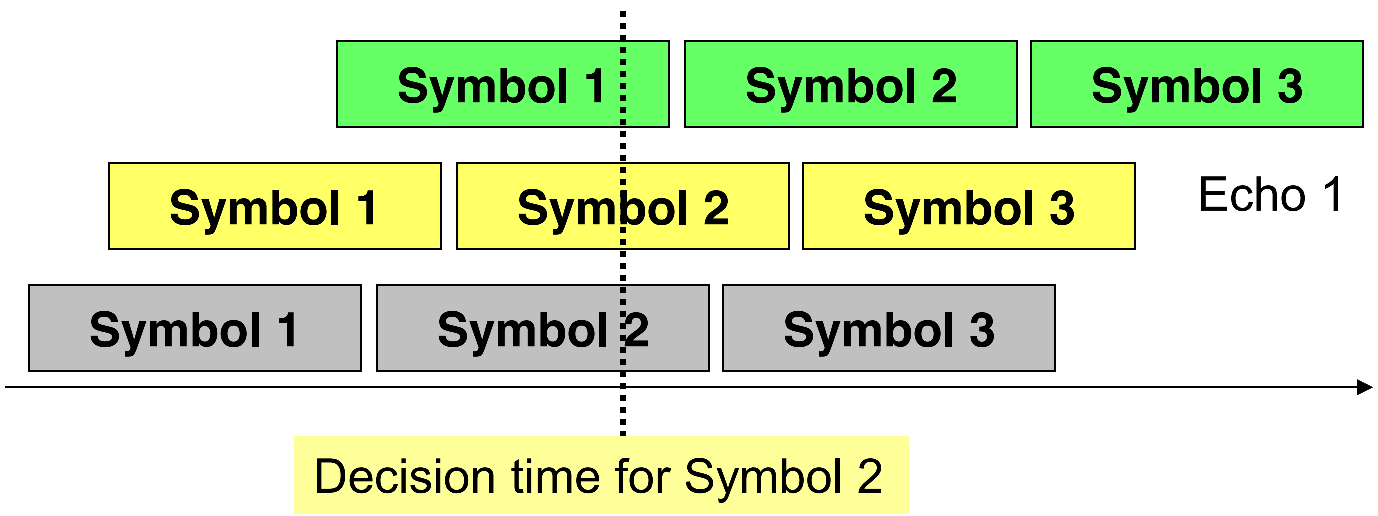
$$\Delta\omega\tau=2\pi$$

$$\Delta f\tau=1$$

$$\Delta f=\frac{1}{\tau}$$

The multipath effect

The problem that can happen is that symbols interferes, called **inter-symbol interference**

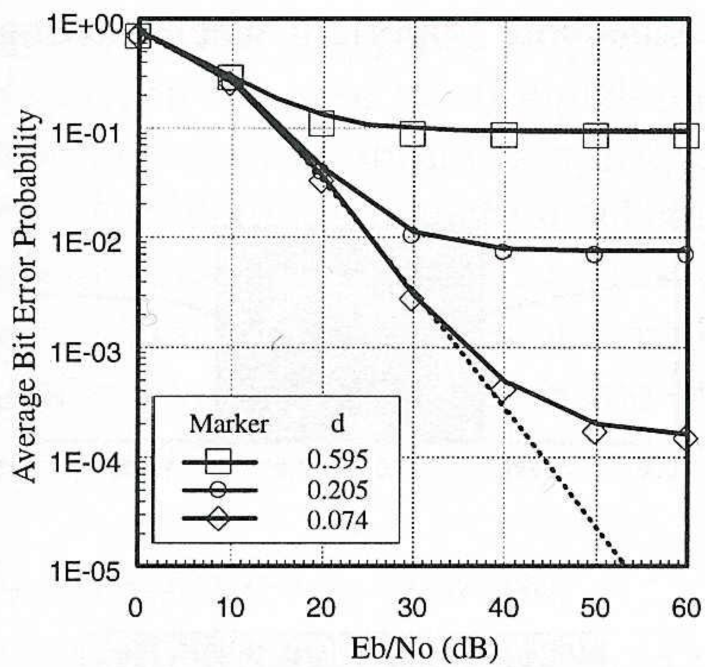


When several symbols overlap in time it may result in decision error

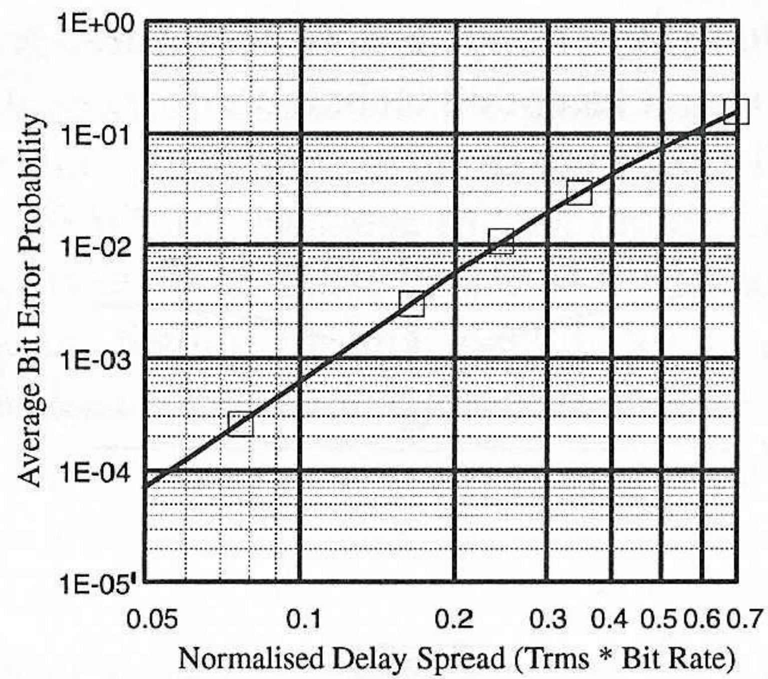
Quality (or bit error ratio)

Power delay profile connected to bit error ration. Use normalised delay spread defined : $d = \tau_{RMS} \cdot$bitrate

Typical BER

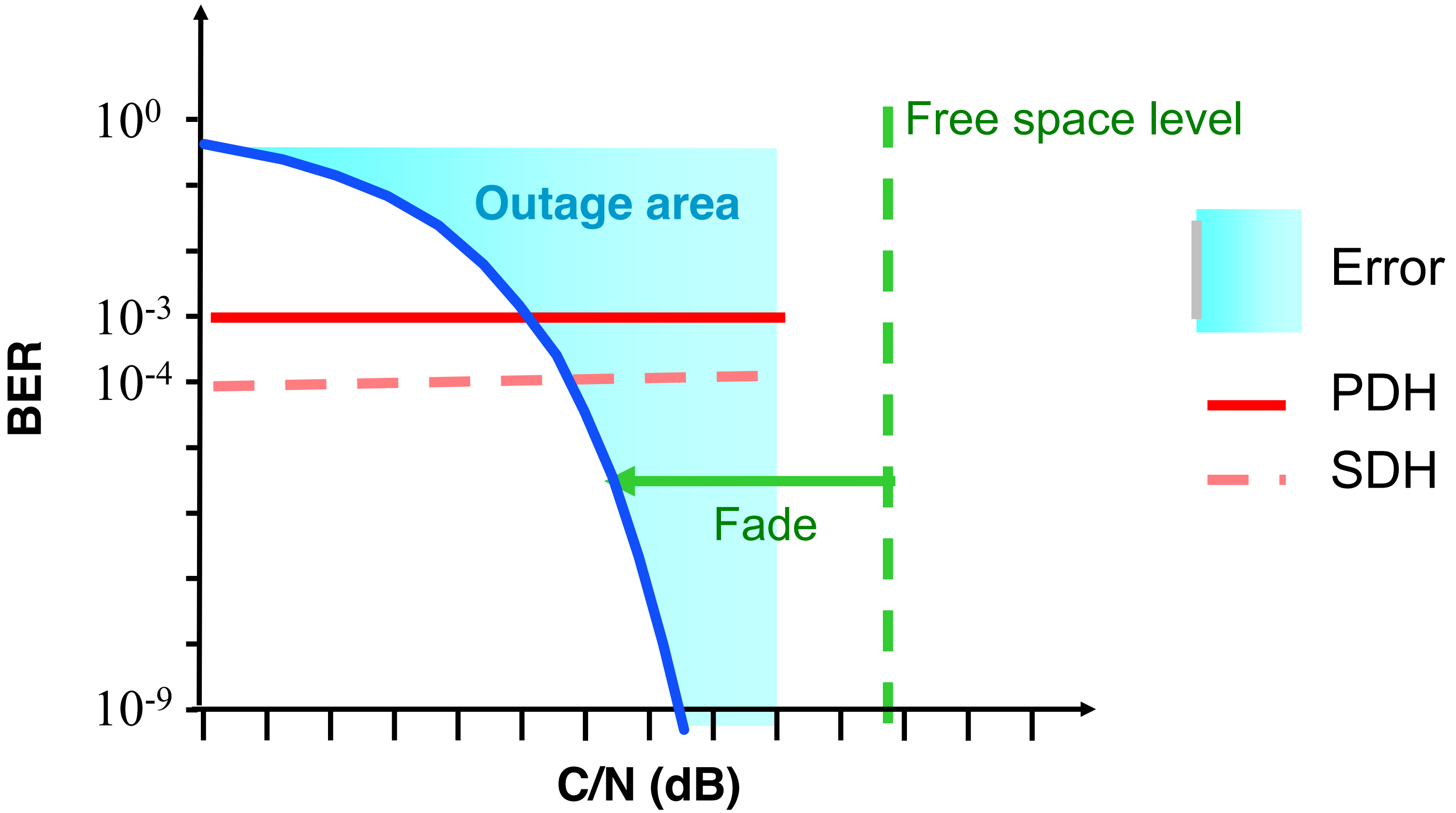


Non-reducible BER
(minimum BER)

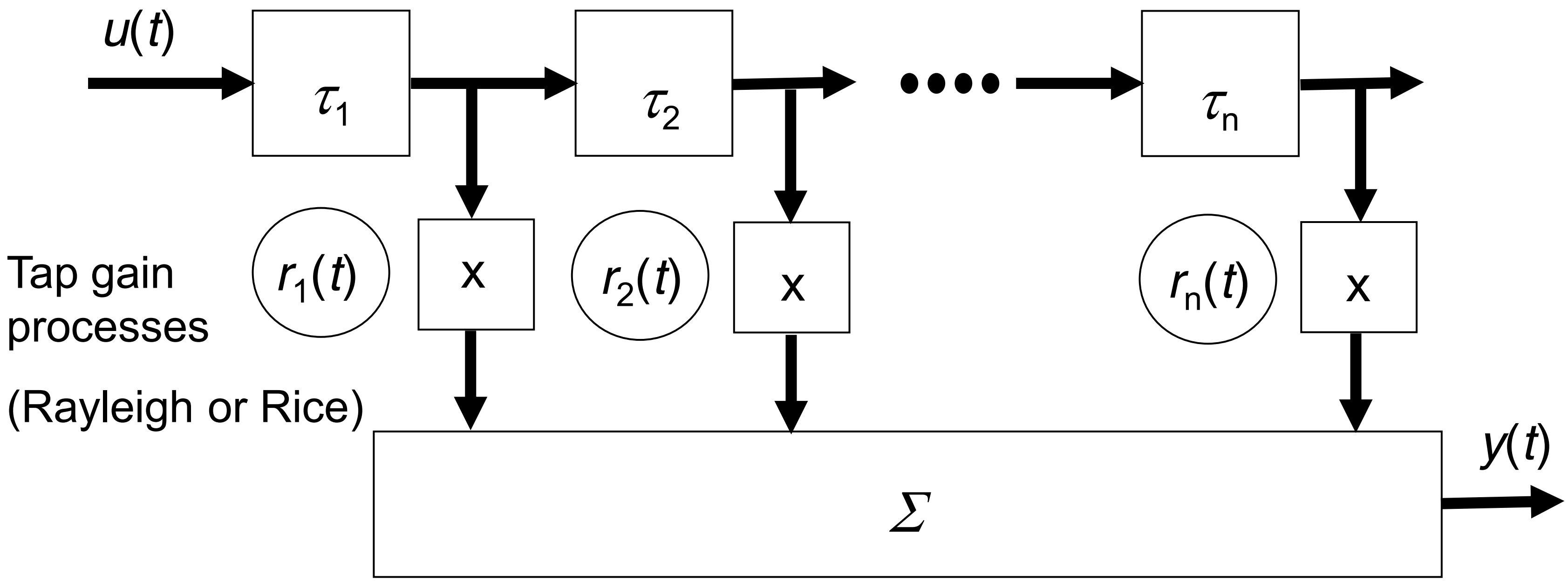


Hall, Barcleay Hewitt:
propagation of radiowaves

High capacity LOS link outage



Wideband channel model

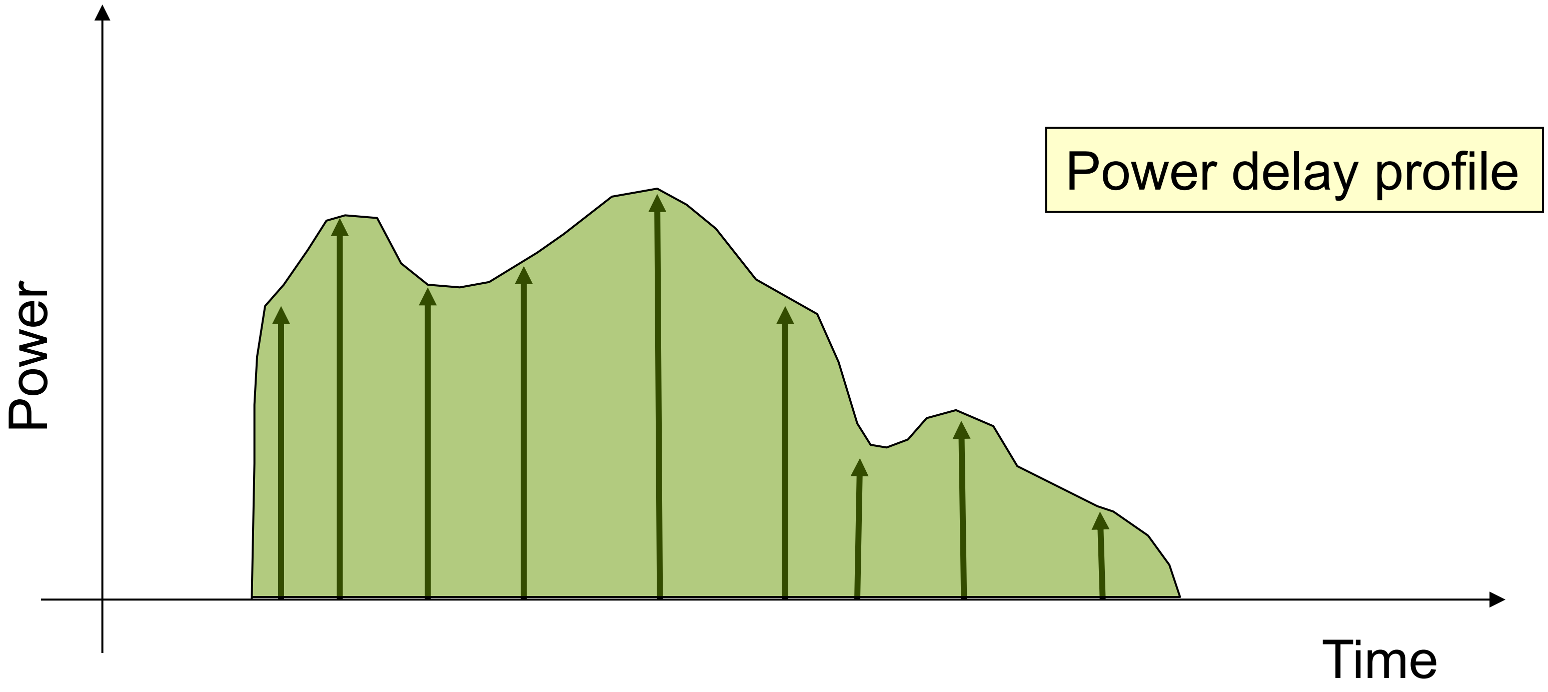


- Tapped delay line
- Linear time-variant transversal filter

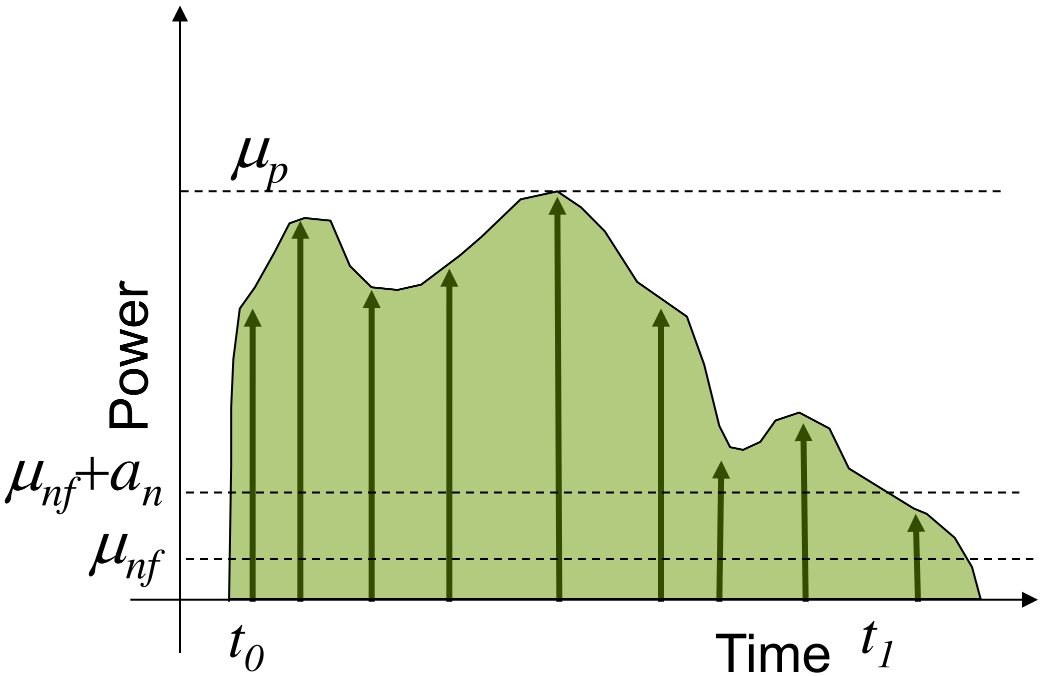
Transfer function

$$y(t) = u(t) \otimes h(t) = \int_{-\infty}^{\infty} h(t,\tau) u(t-\tau) d\tau$$

Receives multiple replica of a transmitted short pulse



Definition of delay spread



Delay spread

μ_{nf} noise floor

μ_p maximum

a_n lowest accepted level

t_0 first accepted

t_1 last accepted

t_p time for maximum

$t_m = t_1 - t_0$

Mean delay and RMS delay spread

Given a sampled profile the mean delay τ_0 or RMS delay spread are τ_{RMS}.

$$\tau_0 = \frac{\sum_{i=1}^{n} \tau_i p_i}{\sum_{i=1}^{n} p_i} \qquad \tau_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} (\tau_i - \tau_0)^2 p_i}{\sum_{i=1}^{n} p_i}}$$

Total power is $\sum_{i=1}^{n} p_i$

Parameters often used to characterise mobile channels. There is a some variation in the "delay spread" notation in use.

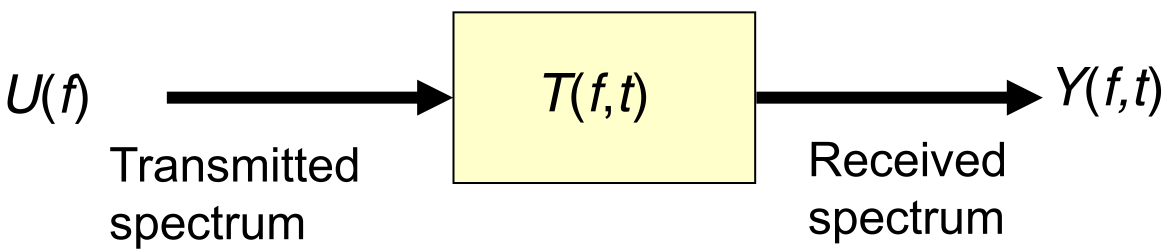
Frequency domain

Equally valid to study the phenomena in the frequency domain.

The time-variant transfer function *T* is

$$T(f,t) = F\left[h(t,\tau)\right] = \int_{-\infty}^{\infty} h(t,\tau)\, e^{-j2\pi f\tau}\, d\tau$$

where ***F*** is the Fourier transform (of the input delay spread with respect to τ)



Coherence bandwidth

In practise T is not known in advance and is specified in terms of correlation ρ between frequency components of the output spectrum:

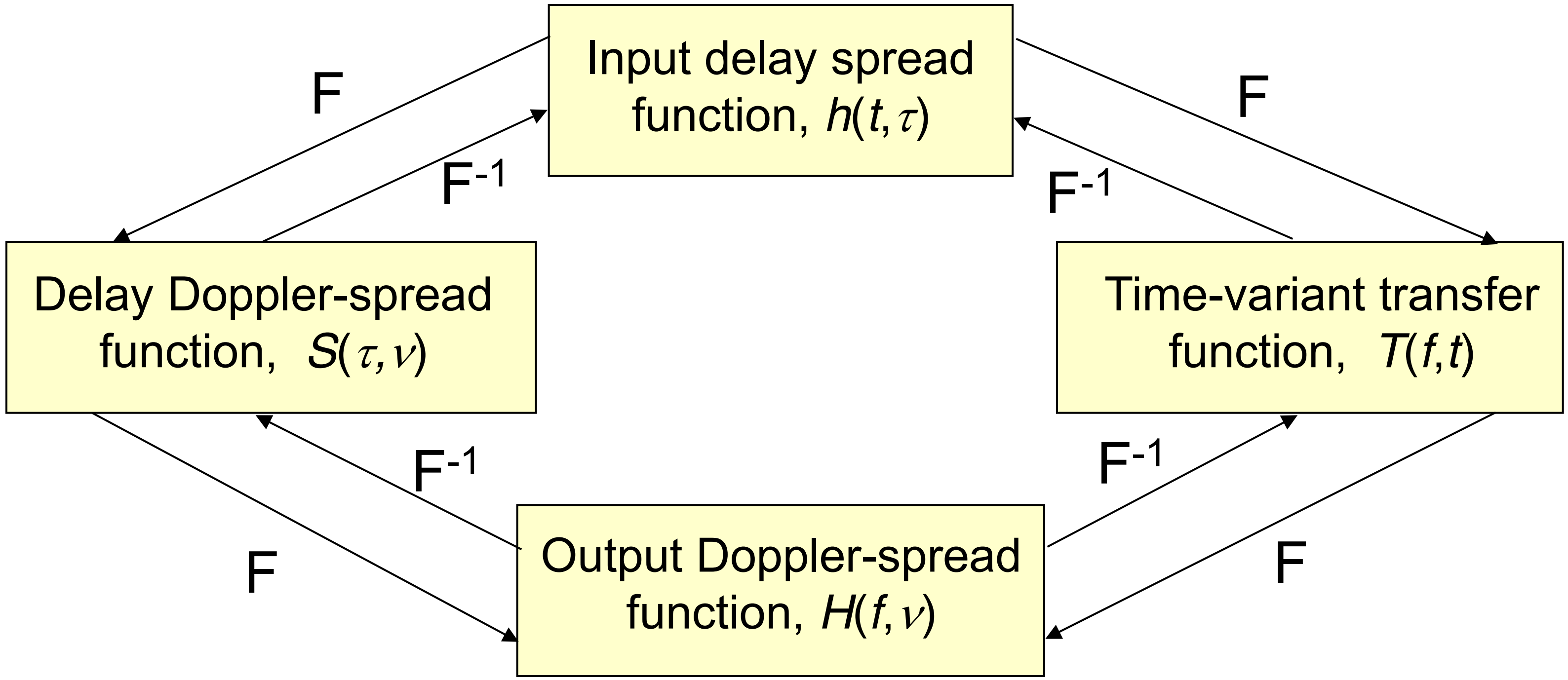
$$\rho(\Delta f, \Delta t) = \frac{E\left[T(f,t)T^*(f+\Delta f, t+\Delta t)\right]}{\sqrt{E\left[\left|T(f,t)\right|^2\right]E\left[\left|T(f+\Delta f, t+\Delta t)\right|^2\right]}}$$

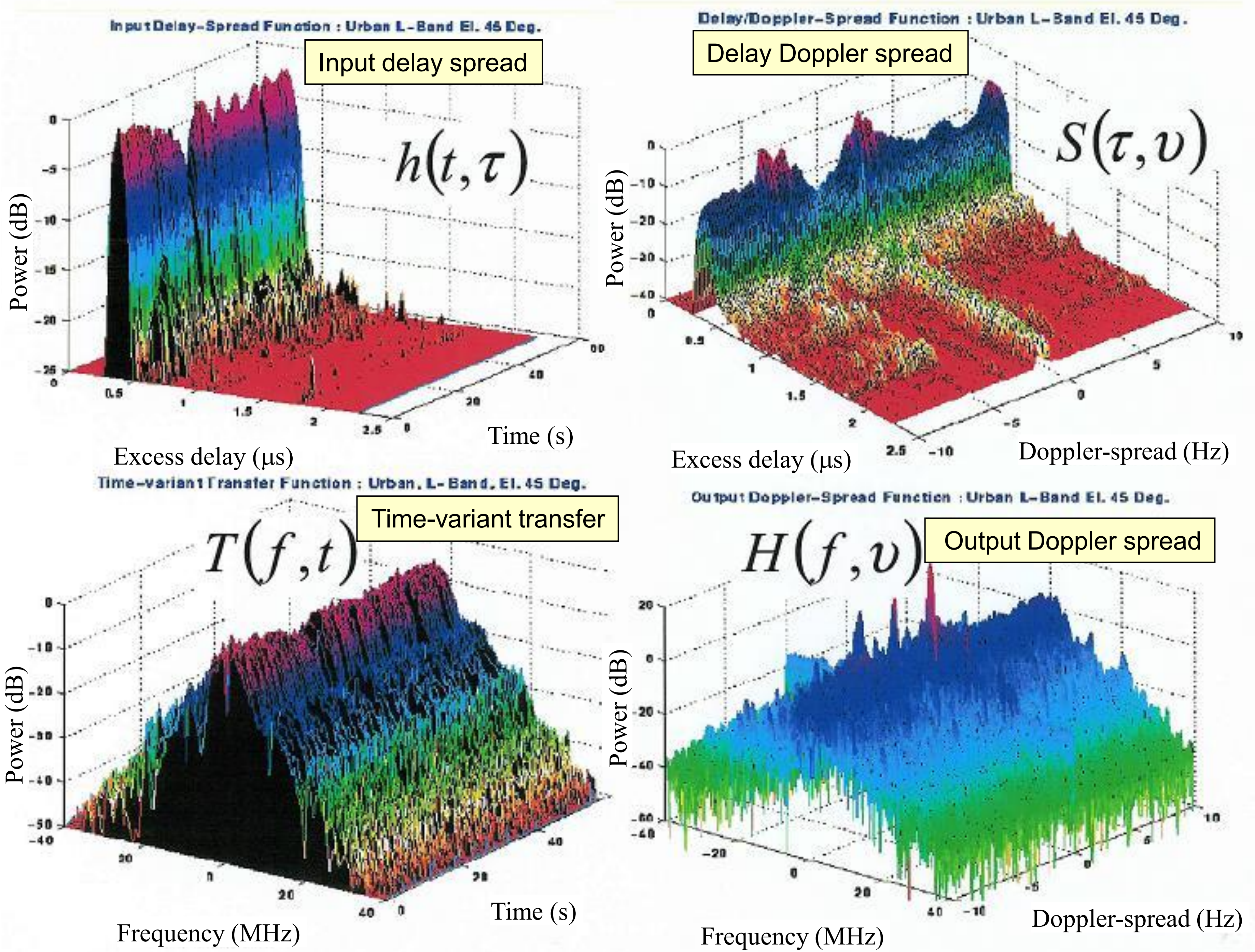
If ρ is evaluated for Δt = 0 then the coherence bandwidth, B_c, i.e. the frequency separation Δf is resulting in ρ = 0.5. If the signal bandwidth is large compared to B_c then the channel is wideband. The coherence bandwidth is proportional to the inverse of RMS delay spread:

$$B_c \propto \frac{1}{\tau_{RMS}}$$

The Bello functions

Bello-functions, defined below, useful for characterisation of the wideband channel.





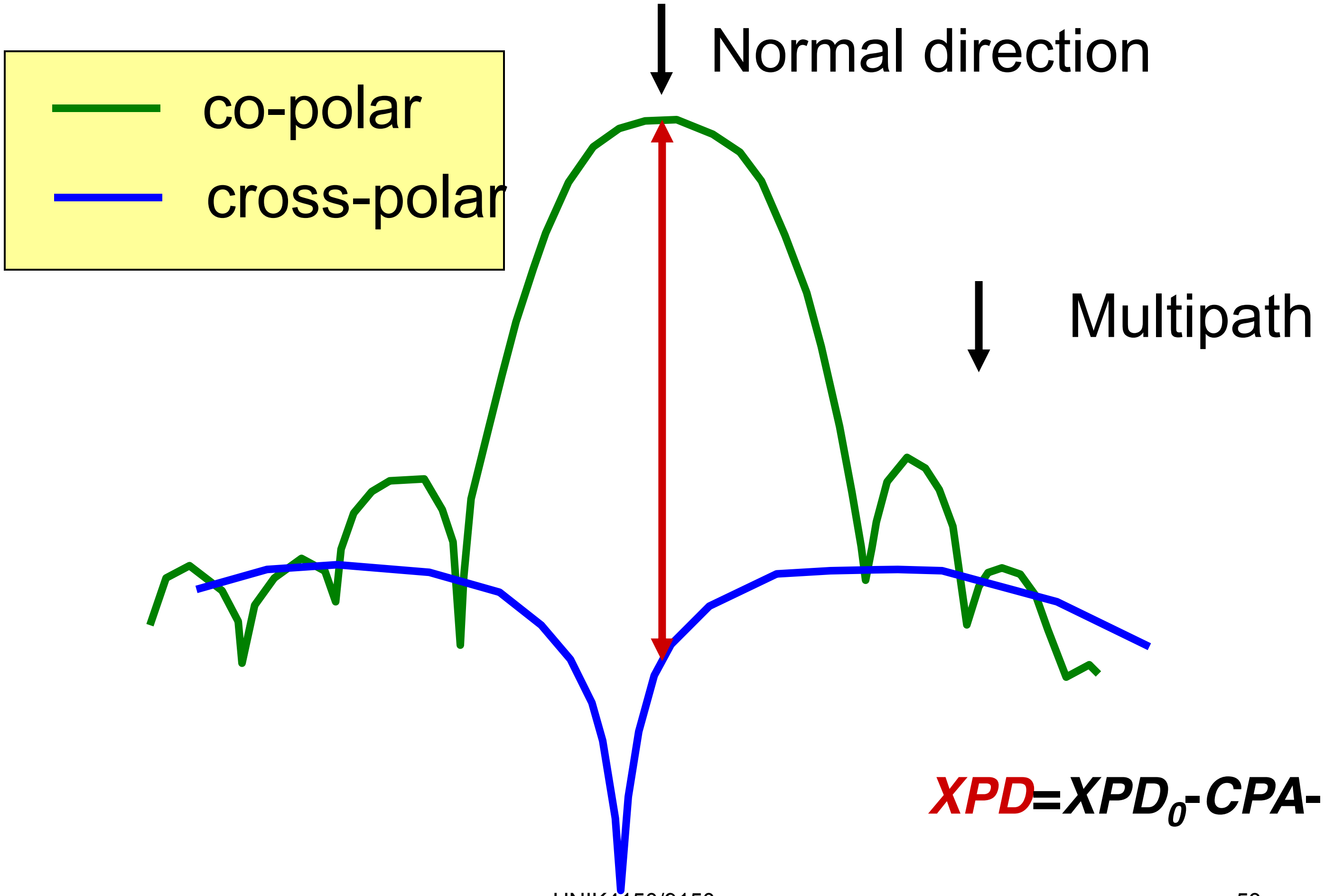
Input Delay-Spread Function : Urban L-Band El. 45 Deg.
Input delay spread
$h(t,\tau)$
Power (dB)
Excess delay (μs)
Time (s)
Delay/Doppler-Spread Function : Urban L-Band El. 45 Deg.
Delay Doppler spread
$S(\tau,\upsilon)$
Power (dB)
Excess delay (μs)
Doppler-spread (Hz)
Time-variant Transfer Function : Urban, L-Band, El. 45 Deg.
Time-variant transfer
$T(f,t)$
Power (dB)
Frequency (MHz)
Time (s)
Output Doppler-Spread Function : Urban L-Band El. 45 Deg.
Output Doppler spread
$H(f,\upsilon)$
Power (dB)
Frequency (MHz)
Doppler-spread (Hz)

Mitigating wideband channel impairments

- Directional antennas
  - Reduces far-off echoes
- Small cells
  - Limiting delay spread
- Diversity
  - Combines two or more signals, e.g., enable escaping from the deepest fades (Chapter 15)
- Equalisers
  - Adaptive filter to transfer wideband channel into a narrow one (Chapter 16)
- Data rate reduction
  - Use OFDM and transmit low rate data on each carrier

High capacity LOS links cross polar degradation

Main cause the antenna diagram combined with multipath



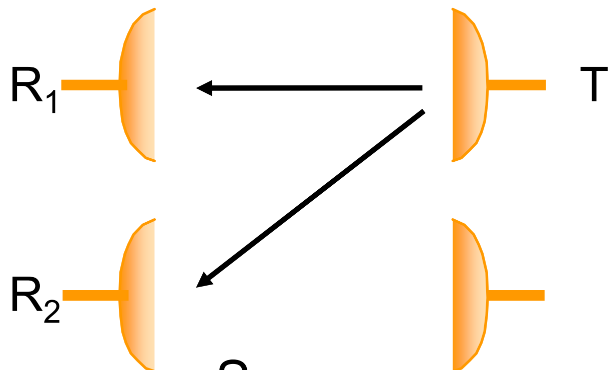
$$\color{red}{XPD}\color{black}{=XPD_0-CPA-Q(A)}$$

High capacity LOS links improved quality

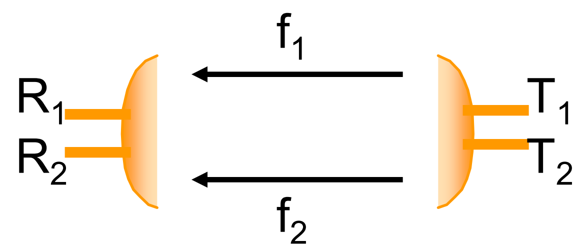
LOS links uses equalizers

- Without diversity
  i) Large path angle
  ii) Minimise ground reflection
  iii) Less clearance reduces multipath (but increases sub-refractive loss)
- With diversity
  i) Space
  ii) Angle
  iii) Frequency
  iv) Routing

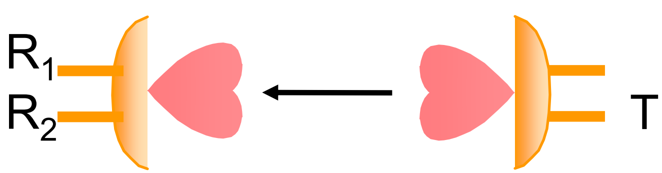
High capacity LOS links diversity reception



Space



Frequency



Angle

Also combinations of space, frequency, and route

Summary

- Wideband channel more complex than narrowband
- Can be characterised in time or frequency domains
- Environment-dependent, only partially under control of system designer
- Not necessarily undesirable if equalisation and similar techniques used
- Mitigation techniques suggested against wideband impairments
- Both mobile and high capacity fixed links covered