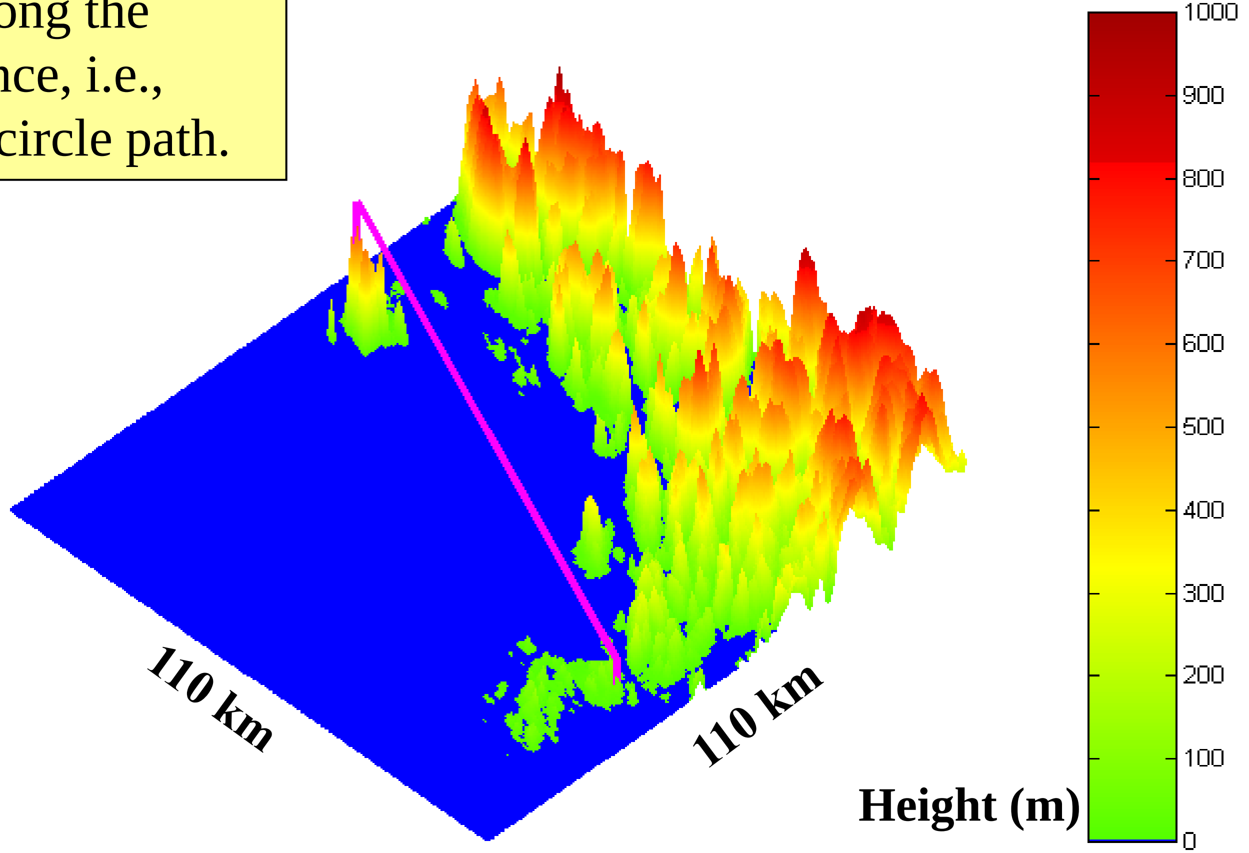
Chapter 6. Terrestrial fixed links

- Links up to several tens or hundreds of kilometres
- Stations on towers with directive antennas
- Path or terrain profile
- Tropospheric refraction, variability, ducting
- Obstruction
- Single knife-edge, multiple knife-edge
- Other objects
- Various methods

Link at the coast of Norway

Propagates along the shortest distance, i.e., along a great circle path.



Path profile

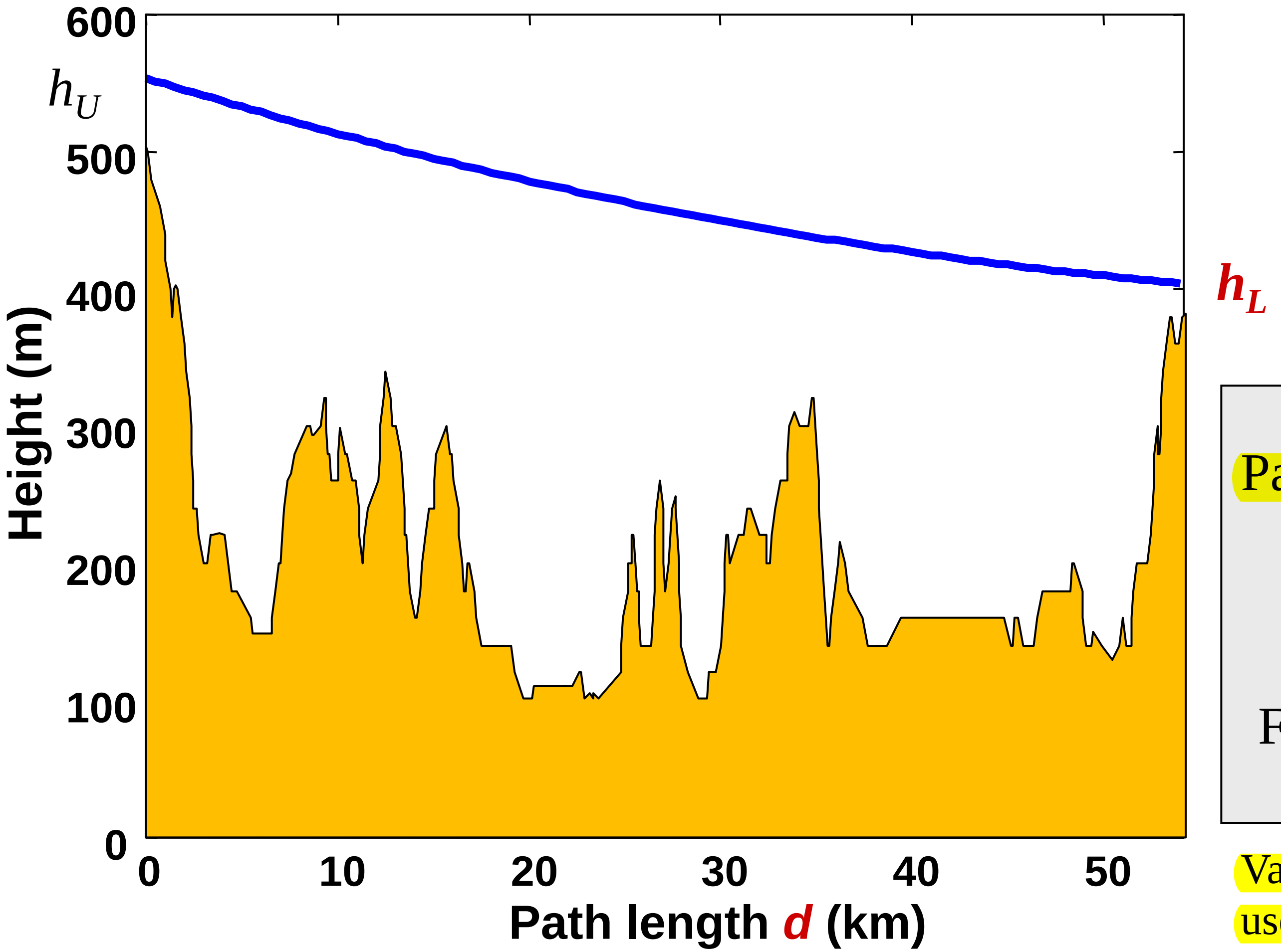
Great circle path length *r*

$$r = R\cos^{-1}\left(\cos\theta_1\cos\theta_2\cos\left(\lambda_1-\lambda_2\right)+\sin\theta_1\sin\theta_2\right)$$

where *R* is the radius of the Earth, θ_1 and θ_2 are latitudes, and λ_1 and λ_2 are longitudes.

The Earth is not perfectly spherical, but this is a good approximation and *R* = 6370 km.

Path profile over flat earth



Path inclination ε_P mrad:

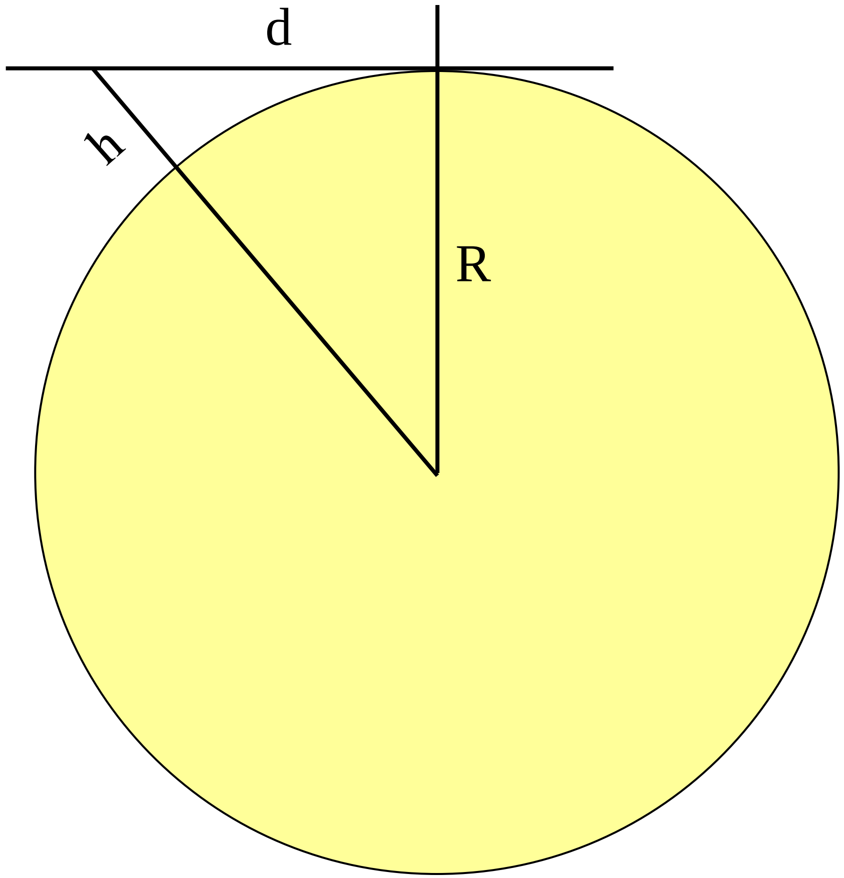
$$\varepsilon_P = \frac{(h_t - h_r)}{d}$$

Frequency f GHz

Variables marked in **red** are used for multipath prediction

Distance to the horizon

Under normal atmospheric conditions the distance to the horizon is:



$$d = \sqrt{(R+h)^2 - R^2}$$

$$d \approx 4.12\sqrt{h}$$

where d is in km and *h* in m.

R is a modified or an effective earth radius *R* = 4*r*/3, where *r* is the earth radius = 6370 km.

Note that *d* can be very much modified by atmospheric layers.

Electromagnetic waves in the troposphere

Electromagnetic waves is generally refracted and scattered in the troposphere caused by variations in the refractive index *n.*

A plane wave in a medium with constant *n* varies with position ***r*** and tine *t*

$$E(\mathbf{r},t) = E_0 e^{\{j(n\mathbf{k}_0\cdot\mathbf{r}-\omega t)\}}$$

Where the wave number $\boldsymbol{k}_0=2\pi/\lambda$ is a vector along the propagation direction and $\omega=2\pi f$ the angle frequency.

The refractive index is

$$n = \sqrt{\mu_r \varepsilon_r} \approx \sqrt{\varepsilon_r}$$

where μ_r is the relative permeability (close to 1) and ε_r the relative permittivity.

Refractive index

The refractive index n can be established for air as a mixture of gases (nitrogen, oxygen, carbon dioxide) and water vapour.

n deviates from 1 because of molecule polarisation under the exposure of an external electric field and quantum mechanical resonances.

n is very close to 1 near the surface of the earth, n = 1.0003. Because of this it is usual to work with N, the refractivity, $N=(n-1)10^6$. Given P is the pressure, T absolute temperature and e the water vapour partial pressure then

$$\underbrace{N = 77.6\, P/T}_{\text{dry}} + \underbrace{3.73\; 10^5\, e/T^2}_{\text{wet}}$$

Refractivity variability

$$N = 77.6\ P/T + 3.73\ 10^5\ e/T^2$$

The first part (dry) is dominated by non-polar gases oxygen and nitrogen, while the last part (wet) is dominated by polar water vapour molecule.

The coefficients have been experimentally fixed on the basis of many measurements.

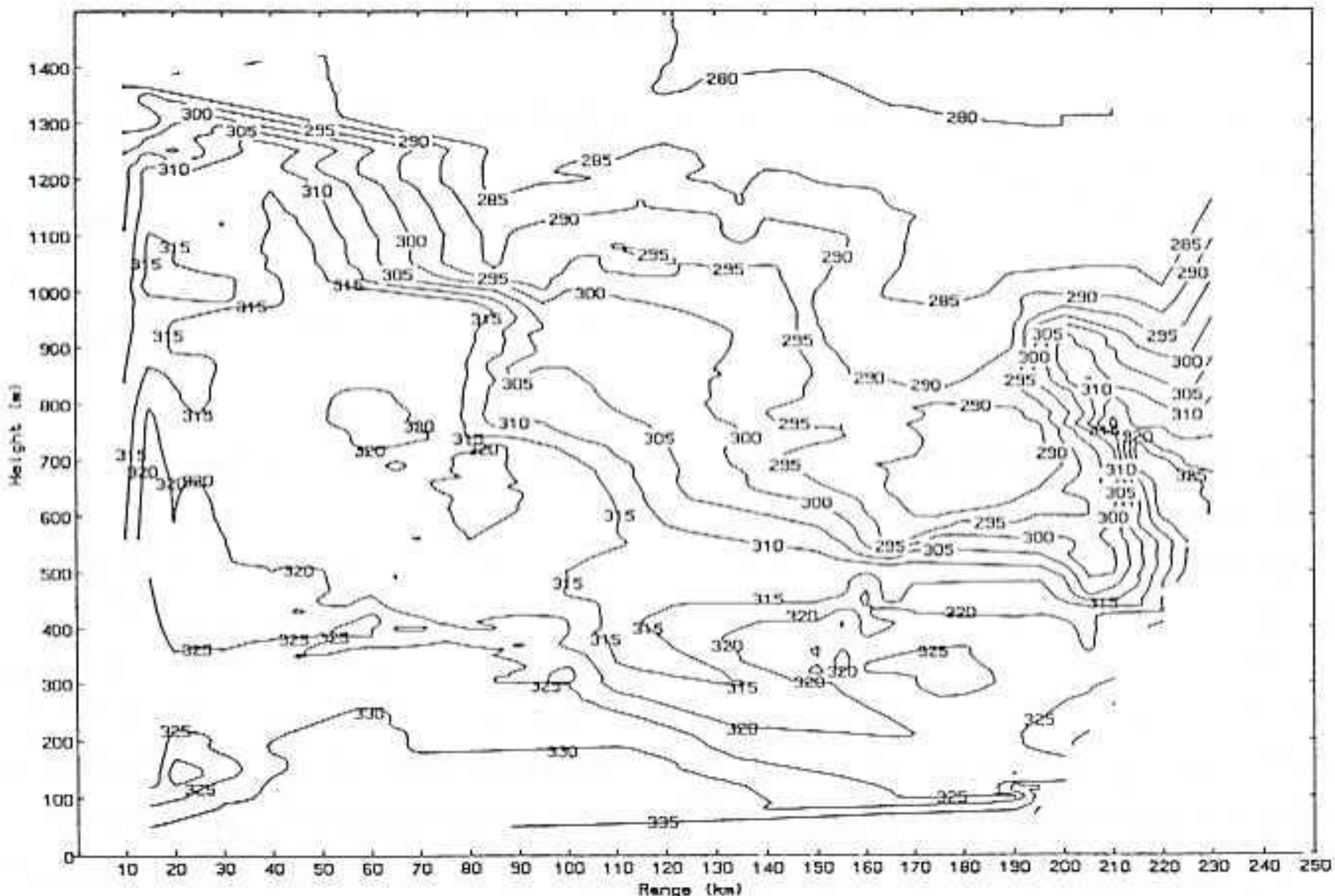
Three orders of size:

a) Large scale governed by the gravity resulting on a horizontal layered atmosphere

b) Middle scale (100 m – 100 km) resulting in local variation in time and space

c) Small scale (<100 m) resulting in turbulence

Measured refractivity example



Source: Hall, Barcley and Hewitt: Propagation of radiowaves

Refractivity variation with height

Large scale variation gives very little horizontal change compared with the vertical.

Vertically the pressure, temperature, and water vapour are reduced resulting in

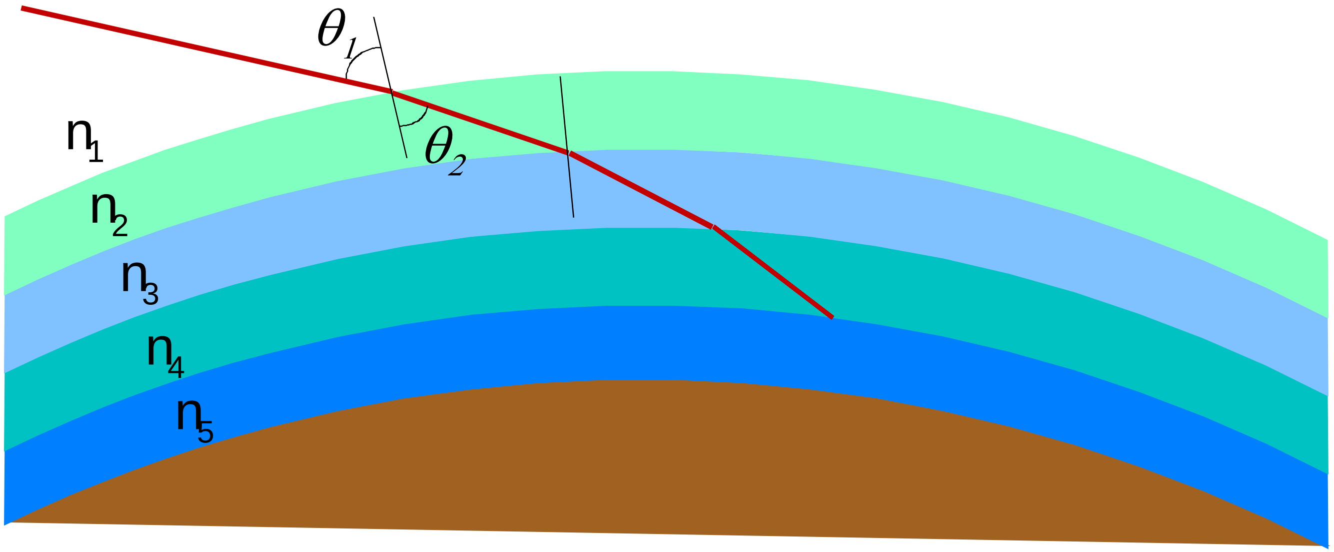
$$N = N_s e^{-z/H}$$

where N_s is N at the surface of earth, z the height and H is a constant (actually the height where N is reduced by 1/e).

Typical values are N_s = 315 and H = 7.35 km.

It is noticed that $dN/dz = -N_s e^{-z/H}/H = -40$ N-units/km at the surface of the earth of values suggested are used for H and N_s (evt. $e^x = 1+x$)

Snell's law applied for the atmosphere

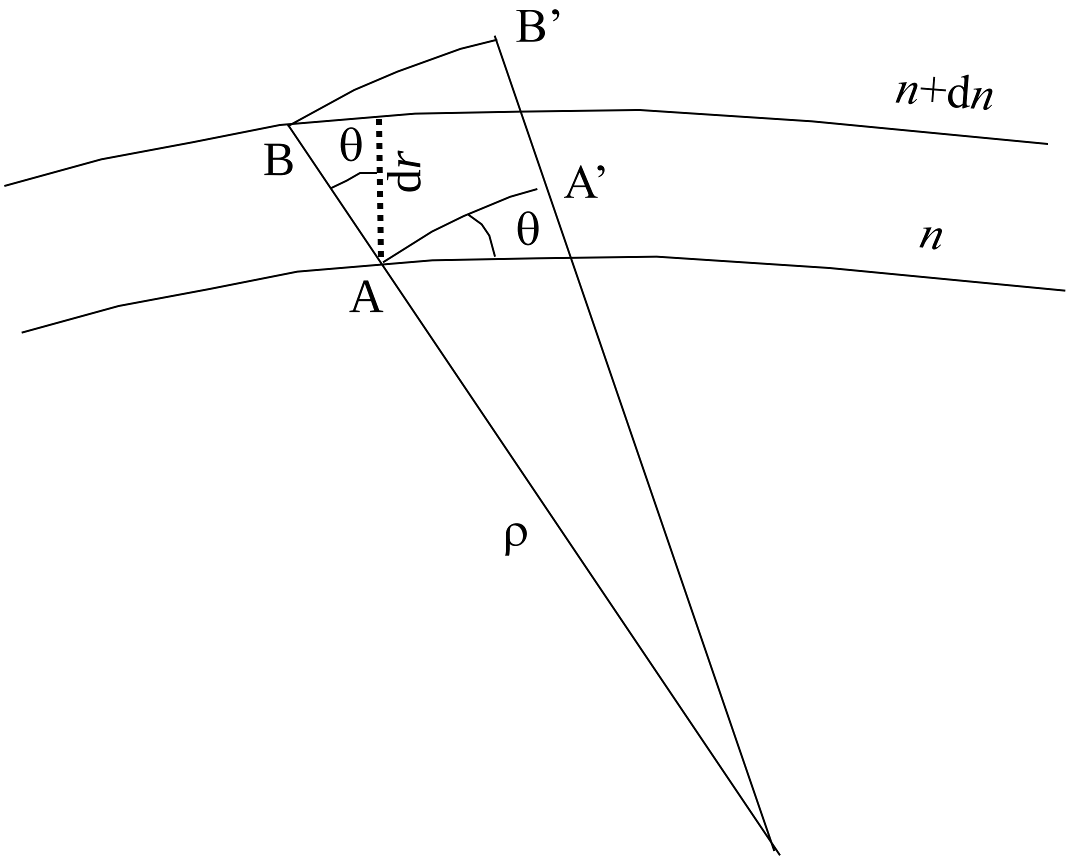


$$n_1 < n_2 < n_3 < n_4 < n_5$$
$$\theta_1 > \theta_2$$

Ray curvature basic geometry

The wave front AB propagates to A'B'.

The phase velocity along AA' is v and along BB' it is v+dv.



The angle velocity is:

$$\frac{v}{\rho} = \frac{v + dv}{\rho + d\rho}$$

$$\frac{v + dv}{v} = \frac{\rho + d\rho}{\rho}$$

$$\frac{dv}{v} = \frac{d\rho}{\rho}$$

Calculating the radius of ray curvature

The phase velocity
v=c/n

$$\frac{dv}{dn} = -\frac{c}{n^2} = -\frac{c}{n}\frac{1}{n} = \frac{-v}{n}$$

$$\Rightarrow \quad \frac{dv}{v} = -\frac{dn}{n}$$

$$\frac{d\rho}{\rho} = -\frac{dn}{n}$$

$$\frac{1}{\rho} = -\frac{dn}{n d\rho}$$

From the geometry $dr = d\rho \cos\theta$ =>

$$\frac{1}{\rho} = -\frac{1}{n}\frac{dn}{dr}\cos\theta \quad \text{or}$$

$$\rho = -\frac{1}{\frac{1}{n}\frac{dn}{dr}\cos\theta} = -\frac{n}{\cos\theta\frac{dn}{dr}}$$

Ray curvature

Ray curvature definition: 1/radius. Let the difference where effective earth radius r_e is such that the ray is *not* curved, i.e., the curvature is zero.

$$\frac{1}{r}-\frac{1}{\rho}=\frac{1}{r_e}-0$$

$$r_e=\frac{1}{\frac{1}{r}-\frac{1}{\rho}}$$

Define the constant k such that $r_e = kr$.

$$k=\frac{1}{1-\frac{k}{r}}=\frac{1}{1+\frac{r}{n}\cos\theta\frac{dn}{dr}}$$

Small θ:

$$k=\frac{1}{1+\frac{r}{n}\frac{dn}{dr}}\approx\frac{4}{3}$$

since dn/dr =-1/4r for normal atmosphere.

Radius of ray parallel with the earth

Let the ray have the curvature *C*.

When the difference between two curvatures (C, and 1/r), is zero: $C - 1/r = 0$ it means that the ray radius is the same (equal to r).

If *r* is the earth radius the ray will follow the surface of the earth. It was derived that $C = 1/\rho = -dn/dz = -10^{-6} dN/dz$. Therefore this situation will happen when $dN/dz = -10^{6}/r = -157$ N-units/km (r=6370 km)

Curved trajectory

Remember that a point on the wave front follows the curved path with radius ρ or curvature 1/ρ :

$$\frac{1}{\rho} = -\frac{1}{n}\frac{dn}{dr}\cos\theta \quad \Rightarrow \quad \rho = -\frac{1}{\frac{1}{n}\frac{dn}{dr}\cos\theta} = -\frac{n}{\cos\theta\frac{dn}{dr}}$$

With normal values for *n, dn/dz* = -39 (N-units/km) , then $\rho \approx 25600 \quad \text{km}$

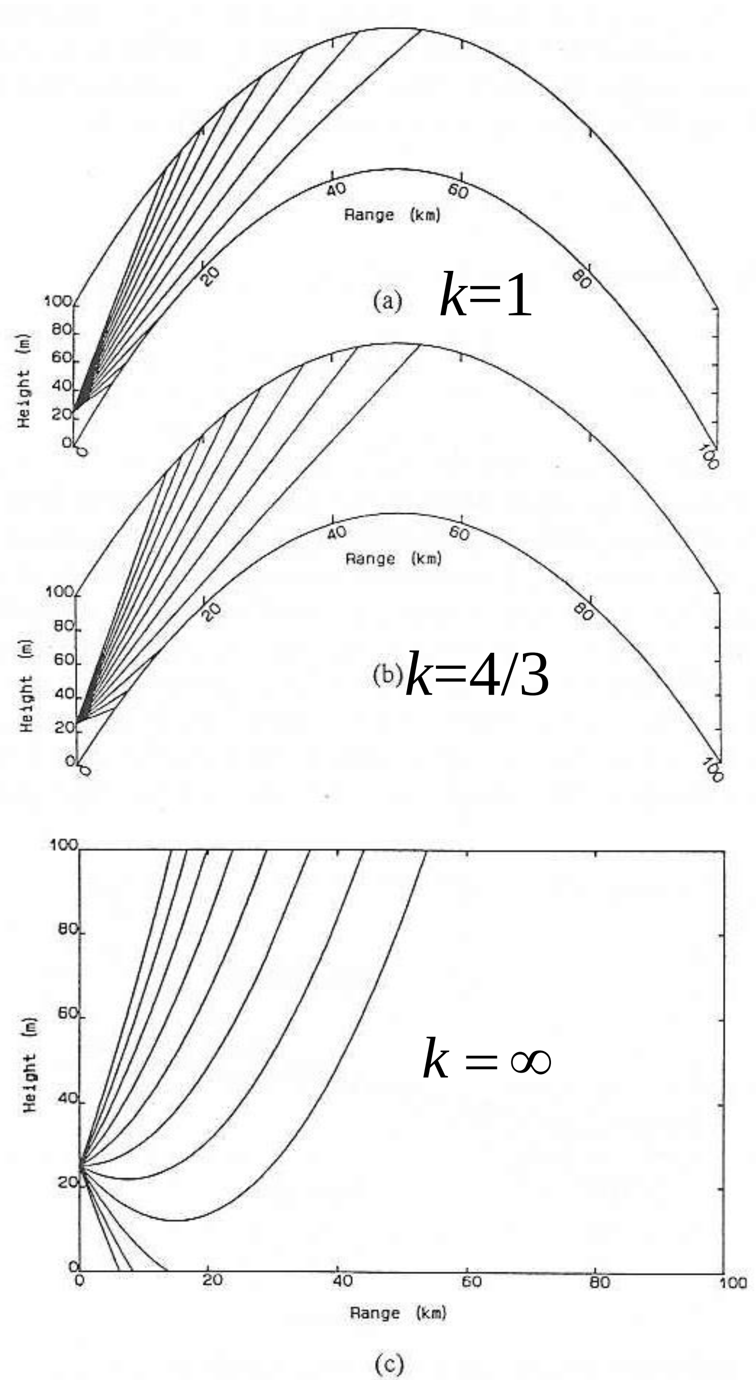
Normally three alternatives are used for the "effective" earth radius:

a) Normal curvature and ρ as above, gives $k = 1$

b) "Effective" earth radius such that $\rho = \infty$ (curvature = 0), gives $k = 4/3$

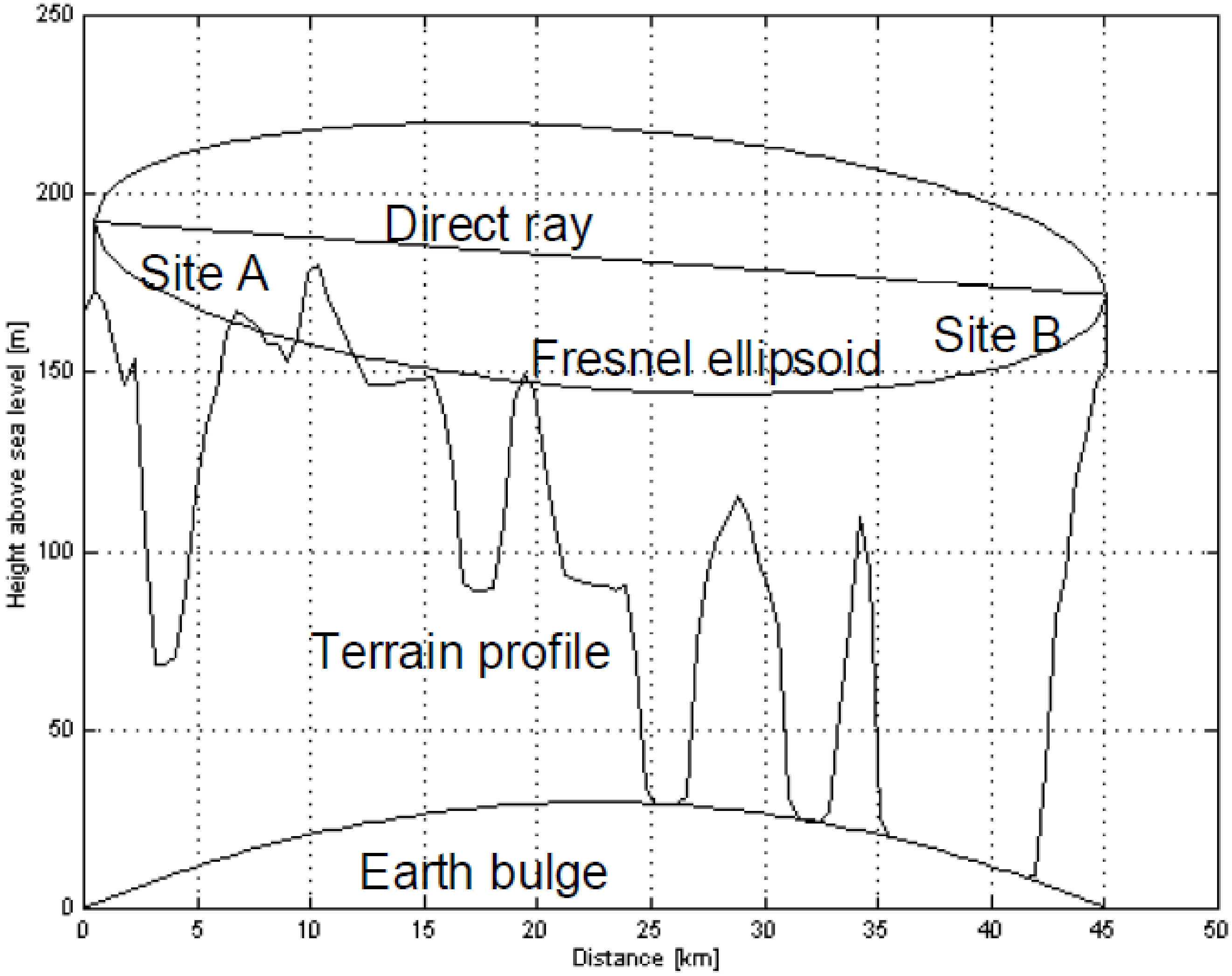
c) "Flat" earth such that $k = \infty$

k-factor representations for rays propagating in a normal atmosphere

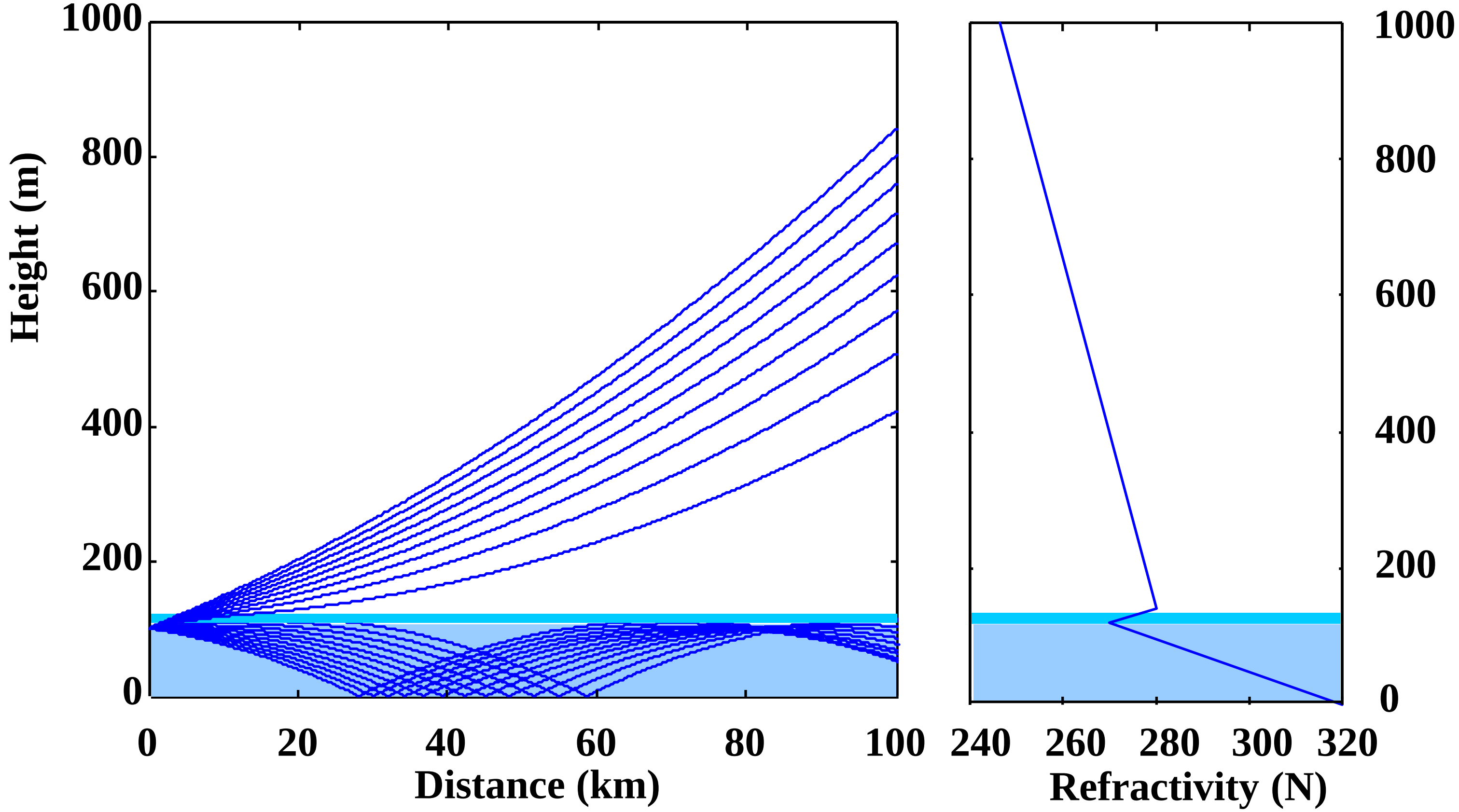


Source: Hall, Barcley and Hewitt: Propagation of radiowaves

Path profile and clearance



Ducting


1000
800
600
400
200
0
Height (m)
0
20
40
60
80
100
Distance (km)
1000
800
600
400
200
0
240
260
280
300
320
Refractivity (N)

Propagation in ducting conditions

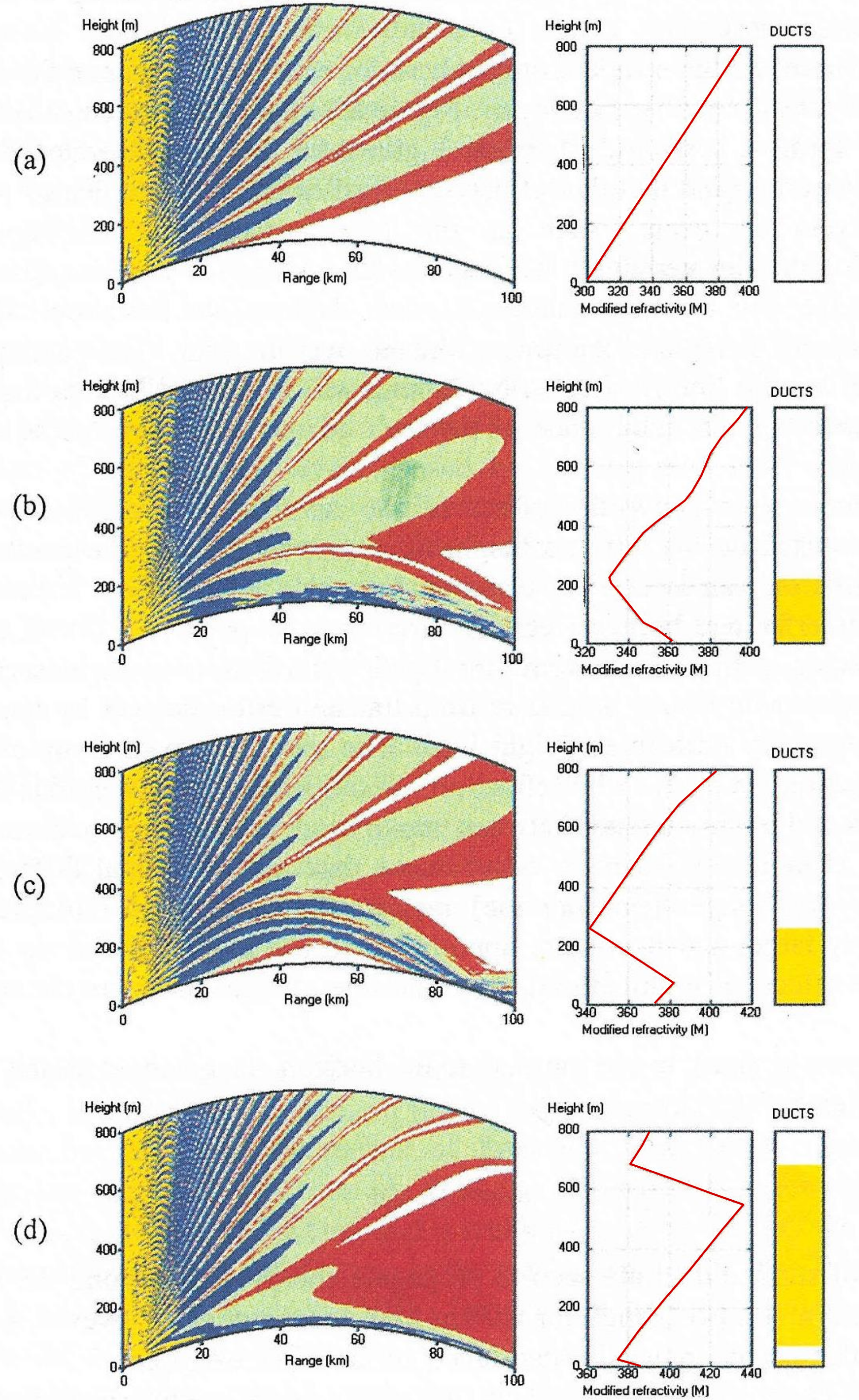
Given an atmospheric duct the following may happen. Interpret full wave calculations at 3 GHz 20 m above ground:

a) Normal

b) Surface duct

c) Surface-elevated duct

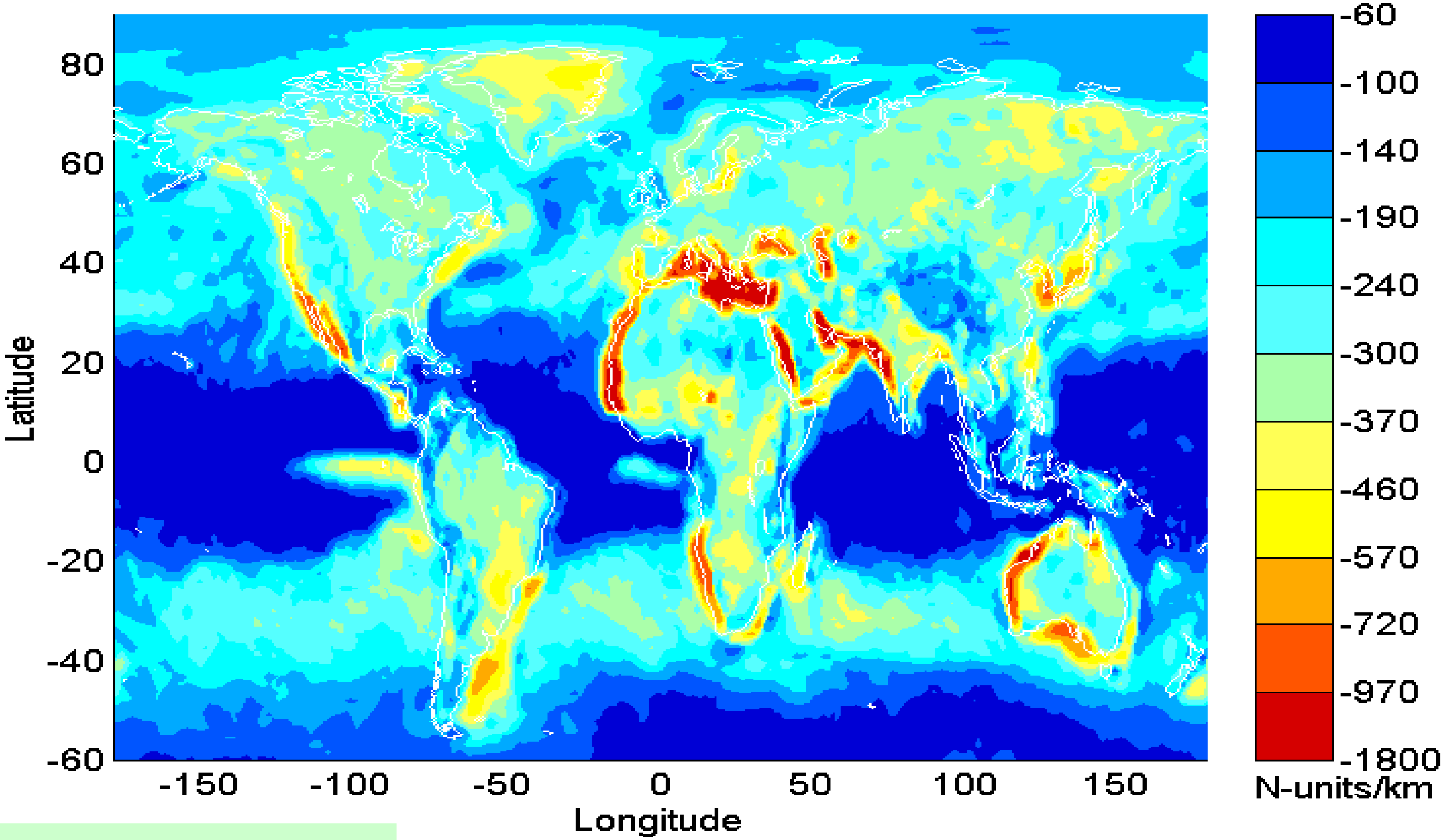
d) Elevated ducts



Modified refractivity M
$M = N + 157z$

Source: Hall, Barcley and Hewitt: Propagation of radiowaves

Average year *dN/dz* not exceeded at 1 % of the time, dN_1

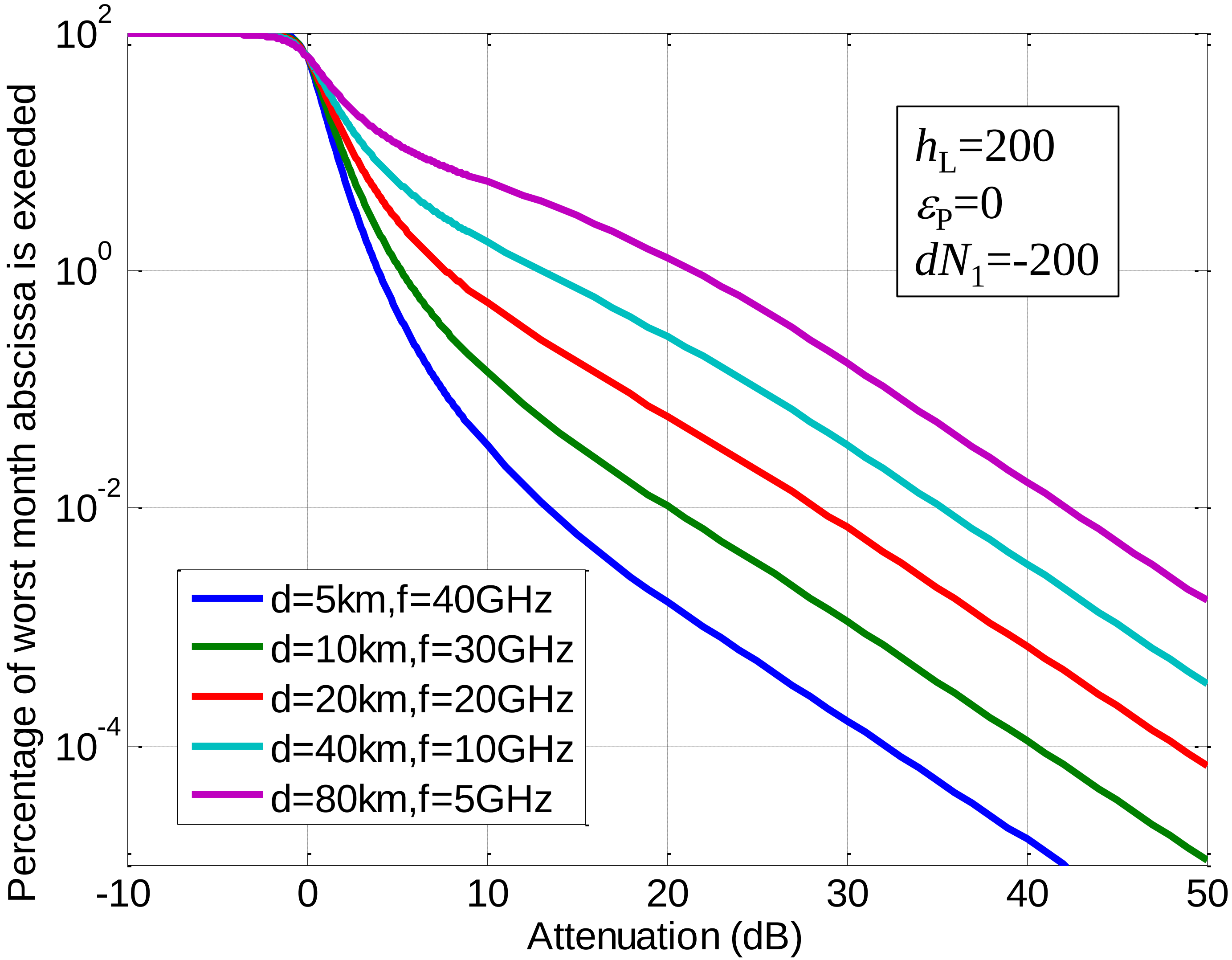


$dN_1 = (N_2 - N_1)/(h_2 - h_1)$

h_1 = ground, h_2 = ~65 m

Variables marked in **red** are used for multipath prediction

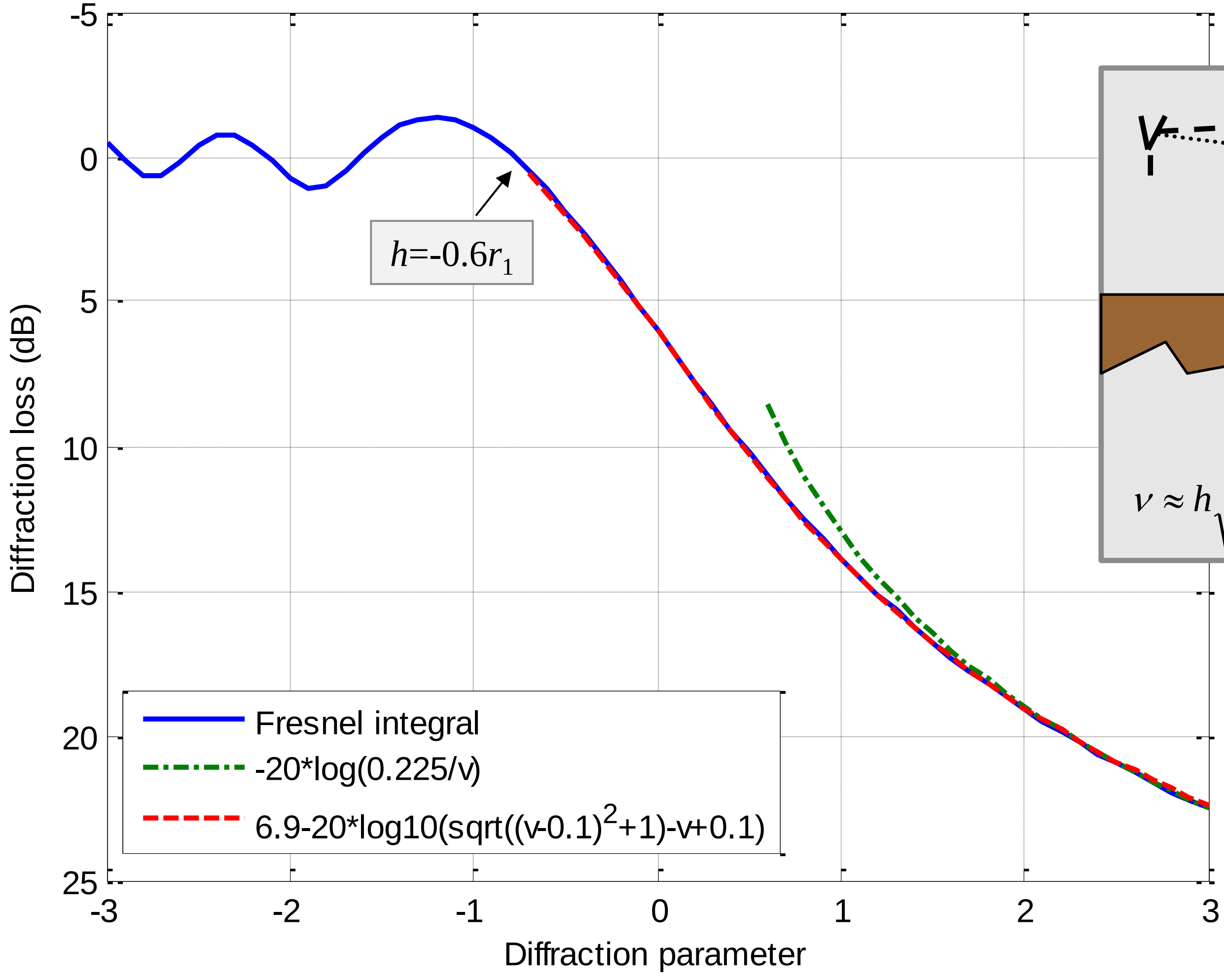
Multipath fading and enhancement prediction

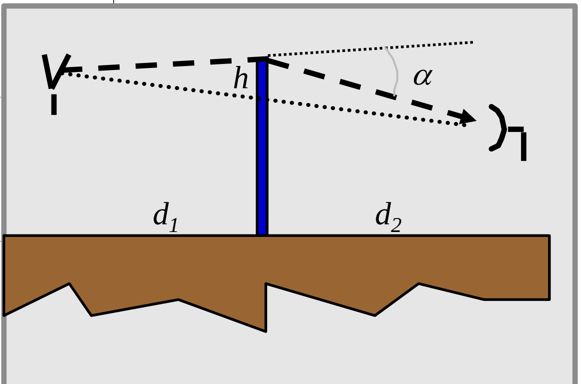


Obstruction

- Multiple edges
- Some methods and examples

Obstruction loss

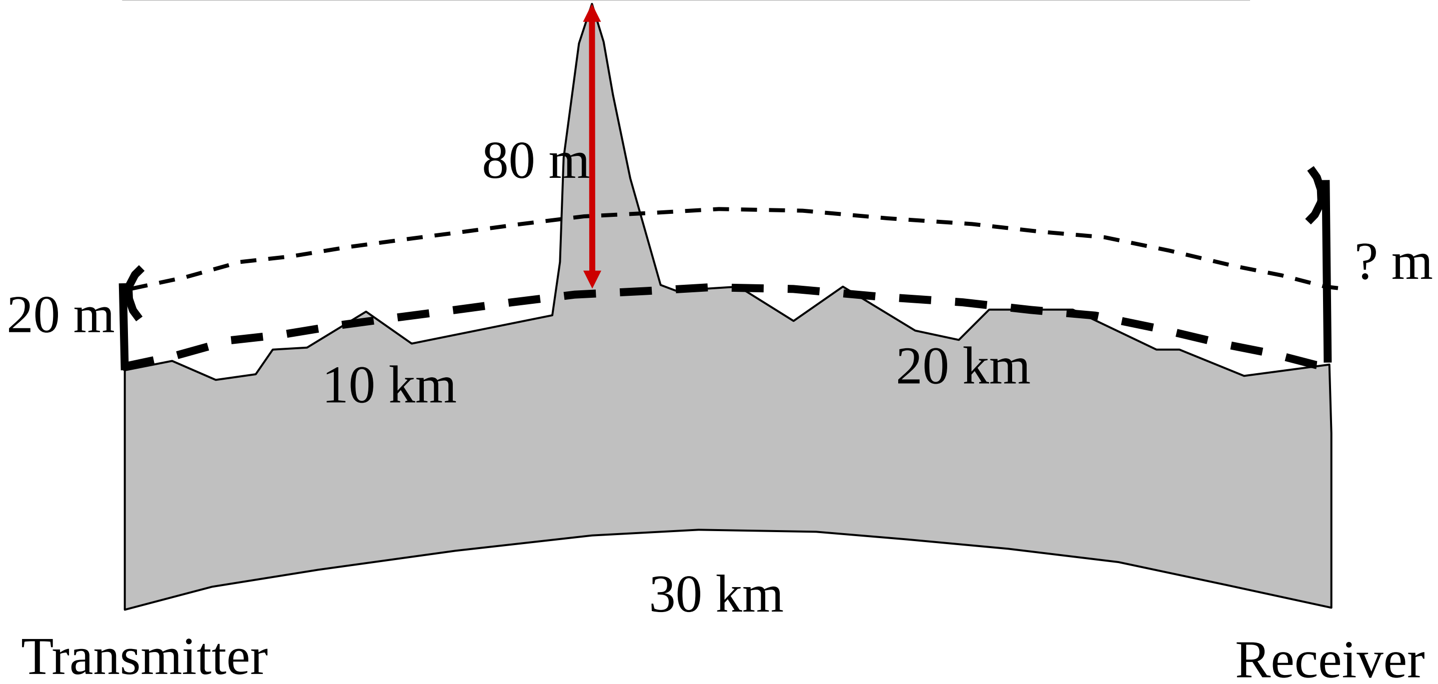




$$\nu \approx h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2d_1 d_2}{\lambda(d_1+d_2)}}$$

Diffraction example

A 10 GHz link has acceptable path loss of 169 dB.



a) Total loss with receiver at 20 m?
b) Receiver height for just acceptable loss?

Example solution

a) Total loss

Free space loss:

$$L_{F(dB)} = 32.4 + 20\log R_{km} + 20\log f_{MHz}$$
$$= 32.4 + 20\log 30 + 20\log 10000$$
$$\approx 142\ dB$$

Diffraction loss:

$$\nu = \mathrm{h}\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}}$$
$$= 60\sqrt{\frac{2\cdot 30\cdot 10^3}{3\cdot 10^{-2}\cdot 10\cdot 10^3\cdot 20\cdot 10^3}} = 6$$
$$L_{ke} \approx -20\cdot\log\frac{1}{\pi\nu\sqrt{2}} \approx -20\cdot\log\frac{0.225}{\nu}$$
$$= 28.5\ dB$$

Total loss = 142 + 28.5 = 170.5 dB

b) Receiver height

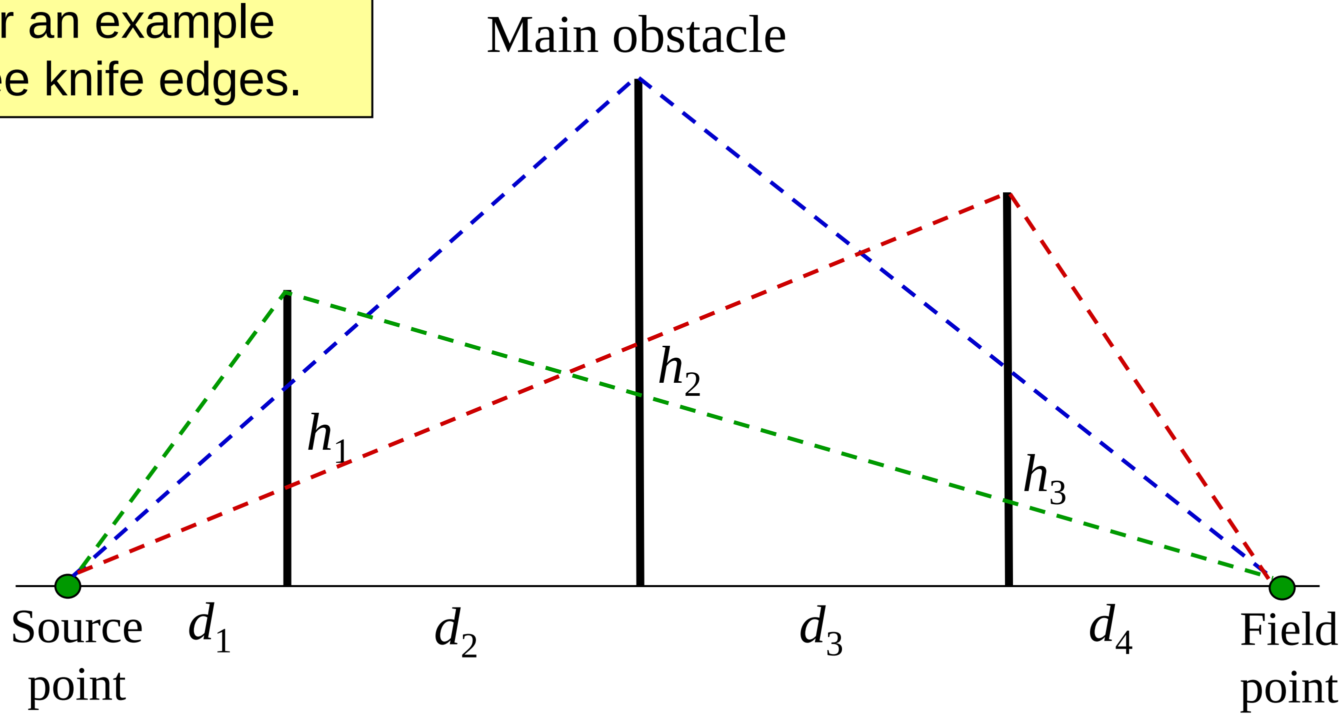
Knife edge loss not greater than 169 - 142 = 27 dB

$$L_{ke} \approx -20\cdot\log\frac{0.225}{\nu} = 27dB$$
$$\nu = \frac{0.225}{10^{-27/20}} = 5$$
$$h = \nu\sqrt{\frac{\lambda d_1 d_2}{2(d_1 + d_2)}} = 50\ m$$

Using a flat earth and no curvature gives height of antenna equal 50 m also.

Multiple knife-edge diffraction: Deygout method

Consider an example with three knife edges.



Calculate diffraction parameter as if each obstacle were present alone using

$$\nu(d_a, d_b, h) = h\sqrt{\frac{2(d_a + d_b)}{\lambda d_a d_b}}$$

$$\nu_1 = \nu(d_1, d_2 + d_3 + d_4, h_1)$$

$$\nu_2 = \nu(d_1 + d_2, d_3 + d_4, h_2)$$

$$\nu_3 = \nu(d_1 + d_2 + d_3, d_4, h_3)$$

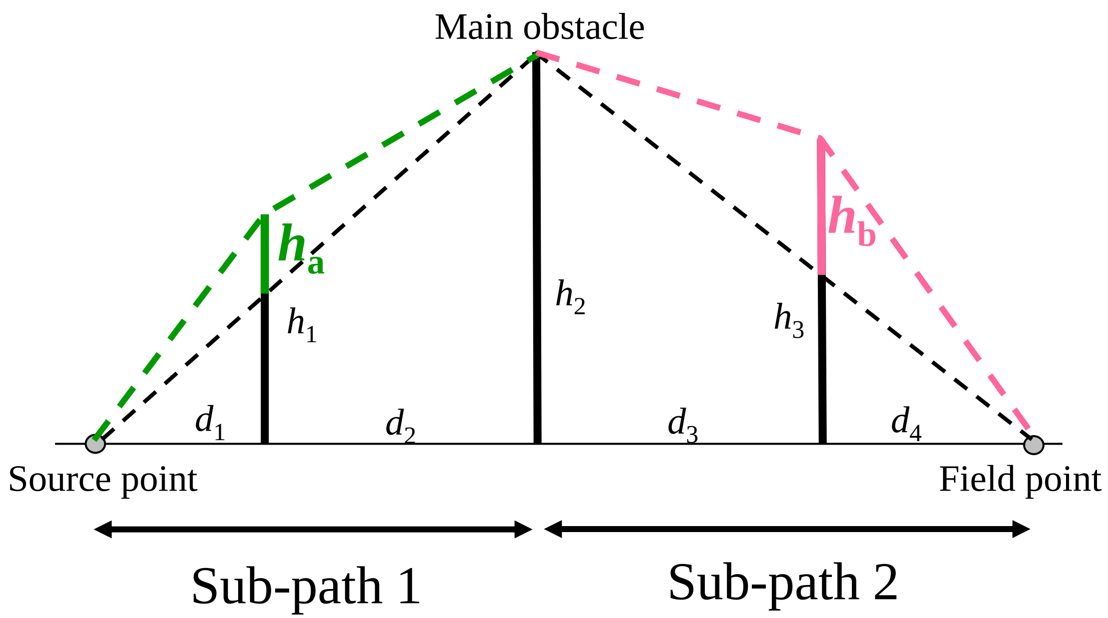
Deygout method sub-paths

The edge with largest ν is called the *Main obstacle.*

The main edge is used to split the path into two sub-paths with diffraction parameters given by

$$\nu_1^{'} = \nu(d_1, d_2, h_a)$$

$$\nu_3^{'} = \nu(d_3, d_4, h_b)$$

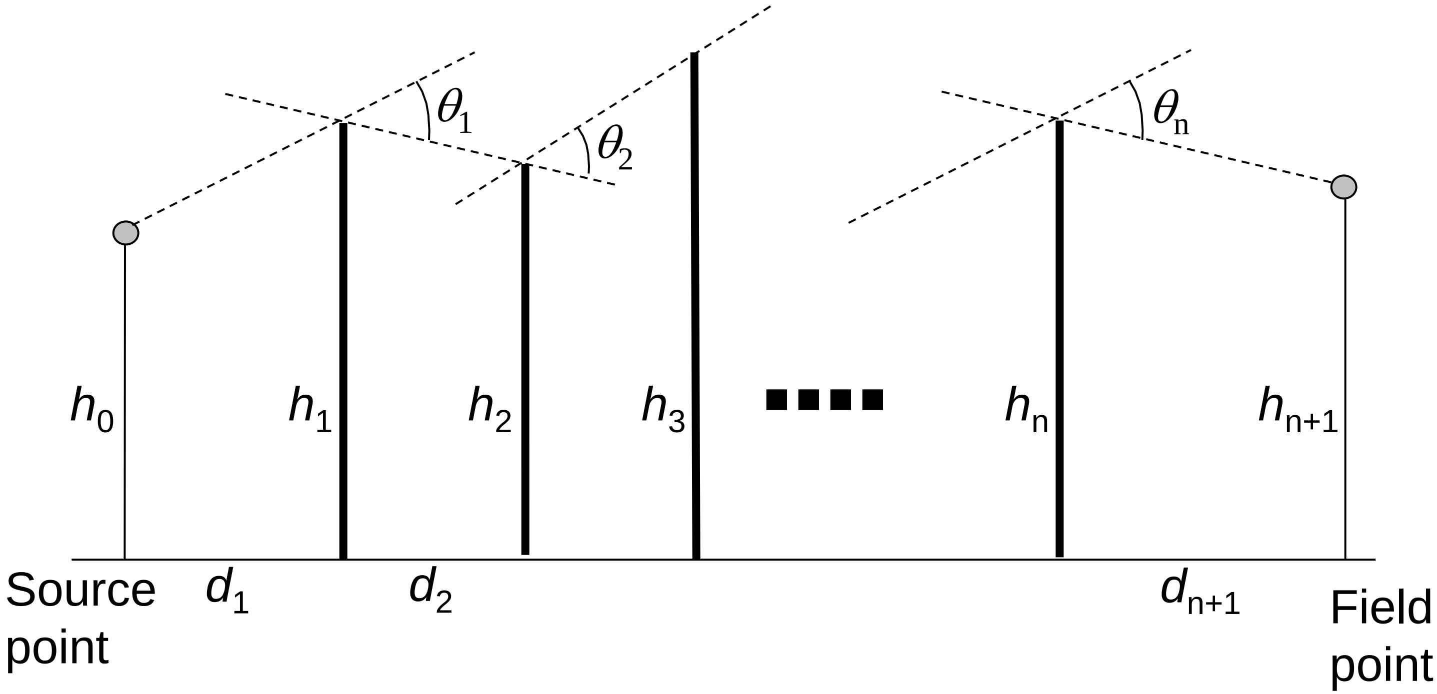


The total loss is now calculated combining the single Main obstacle and the two sub-paths.

$$L_{ex} = L_{ke}\left(\nu_1^{'}\right) + L_{ke}\left(\nu_2\right) + L_{ke}\left(\nu_3^{'}\right)$$

With several obstacles the method is used on sub-paths. However, usual to find the main and the two dominating on each side.

Multiple edge diffraction diffraction integral



Brief presentation and discussion of a more accurate method by Furutsu and transformed by Vogler into multiple integral representation. Numerically unstable for large number of obstacles.

Multiple-edge diffraction integral (Vogler)

Vogler expresses the excess diffraction loss (as a ratio between the received field strengths with and without the edges present) due to n knife-edges as

$$A_n = \sqrt{L_{ex}} = C_n \pi^{-n/2} e^{\sigma_n} I_n \tag{6.25}$$

where

$$I_n = \int_{x_n=\beta_n}^{\infty} \dots \int_{x_1=\beta_1}^{\infty} \exp\left(2f - \sum_{m=1}^{n} x_m^2\right) dx_1 \dots dx_n \tag{6.26}$$

with

$$f = \sum_{m=1}^{n-1} \alpha_m (x_m - \beta_m)(x_{m+1} - \beta_{m+1}) \quad \text{for } n \geq 2 \tag{6.27}$$

where

$$\alpha_m = \sqrt{\frac{d_m d_{m+2}}{(d_m + d_{m+1})(d_{m+1} + d_{m+2})}} \tag{6.28}$$

$$\beta_m = \theta_m \sqrt{\frac{jk d_m d_{m+1}}{2(d_m + d_{m+1})}} \tag{6.29}$$

$$\sigma_n = \beta_1^2 + \dots + \beta_n^2 \tag{6.30}$$

$$C_n = \sqrt{\frac{d_2 d_3 \times \dots \times d_n d_T}{(d_1 + d_2)(d_2 + d_3) \times \dots \times (d_n + d_{n+1})}} \tag{6.31}$$

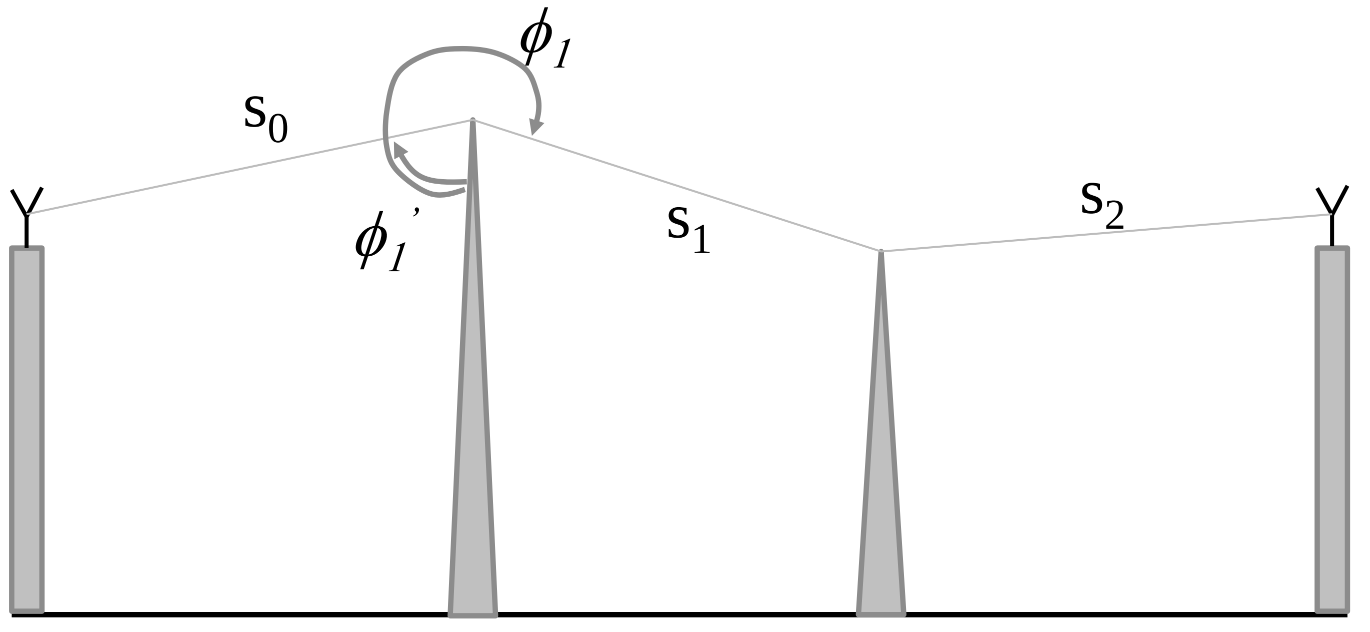
$$d_T = d_1 + \dots + d_{n+1} \tag{6.32}$$

and the geometrical parameters are defined in Figure 6.18 (note that θ_1 and θ_3 are positive in this diagram, θ_2 is negative).

Slope-UTD multiple-edge diffraction model (Andersen)

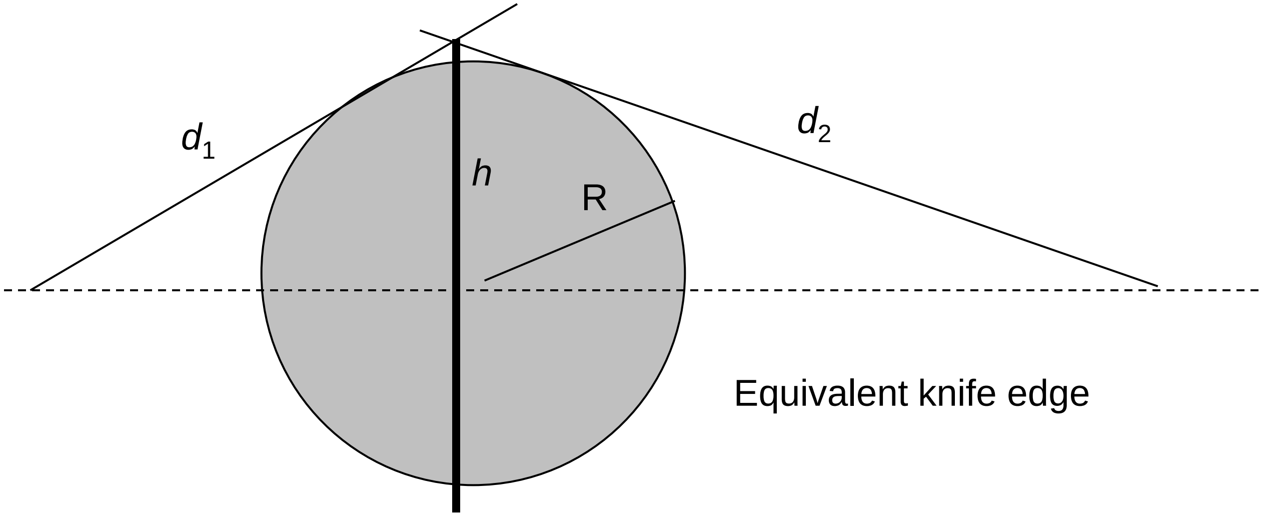
- From Chapter 3 the uniform theory of diffraction (UTD) allows the geometrical theory of diffraction (GTD) to be used in the Fresnel zones
- The UTD diffraction coefficients can be cascaded in the multiple edge case, but the result is inaccurate since diffracted field illuminating the next obstacle may not be a plane wave
- A “slope” diffraction coefficient is added to account for spatial rate of change of the diffracted fields

Geometry slope UTD model



Fast compared to the Vogler approach.
Equations and example see text book.

Diffraction over finite size objects



Finite radius cylinder diffraction. First replace the cylinder with an equivalent knife edge, see figure, giving the loss

$$L_1 = L_{ke}\left(\nu\left(d_1, d_2, h\right)\right)$$

The extra loss due to the cylinder is (in dB)

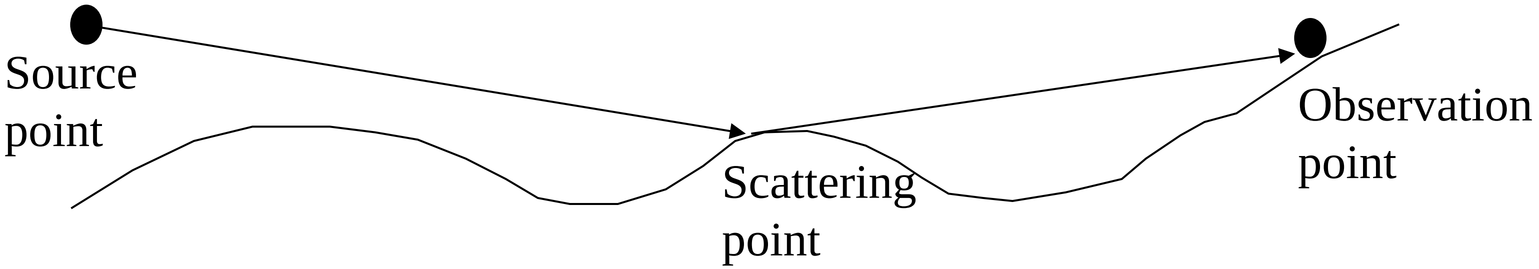
$$L_C\left(d_1, d_2, h, R\right) = \left(8.2 + 12n\right) m^{0.73+0.27\left[1-\exp\left(-1.43n\right)\right]}$$

$$m = \frac{R\left(d_1 + d_2\right)/d_1 d_2}{\left(\pi R/\lambda\right)^{1/3}}, \quad n = \frac{h}{R}\left(\frac{\pi R}{\lambda}\right)^{2/3}$$

Overall excess loss $L_{ex} = 10\log L_{ke}\left(\nu\left(d_1, d_2, h\right)\right) + L_C\left(d_1, d_2, h, R\right)$

Integral equation model

- Diffraction methods described so far based on surface modelled as canonical objects, e.g., edges, cylinders, and spheres
- Integral method using the direct signal and other scattered signals for irregular terrain



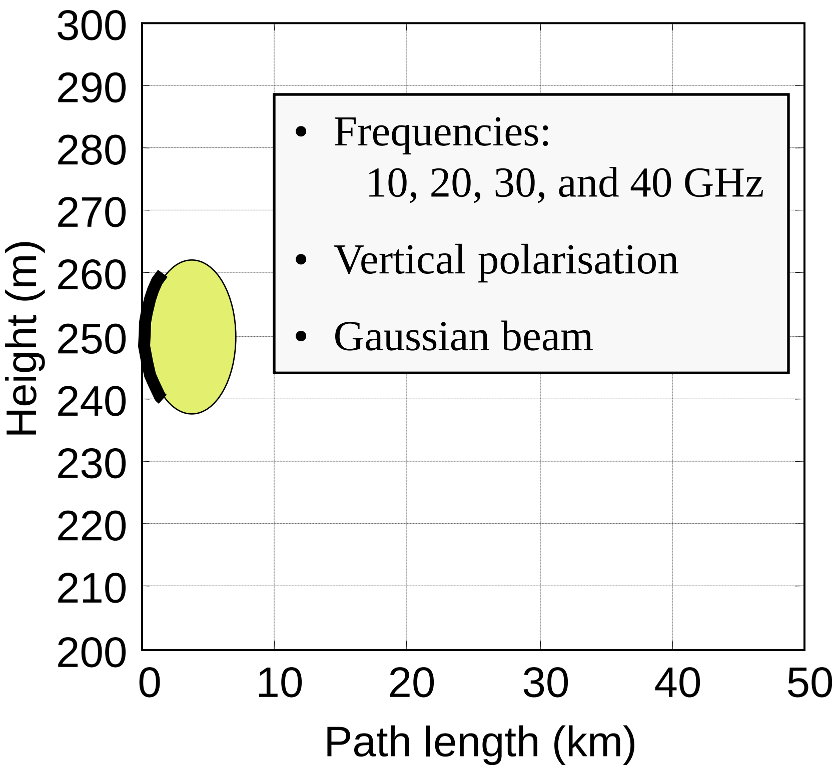
- Neglect “back scatter” and “side scatter”
- Incident field induces currents at the scatter point, that in turn radiates
- Sum up all scattering points along the path, i.e., integrate along the surface

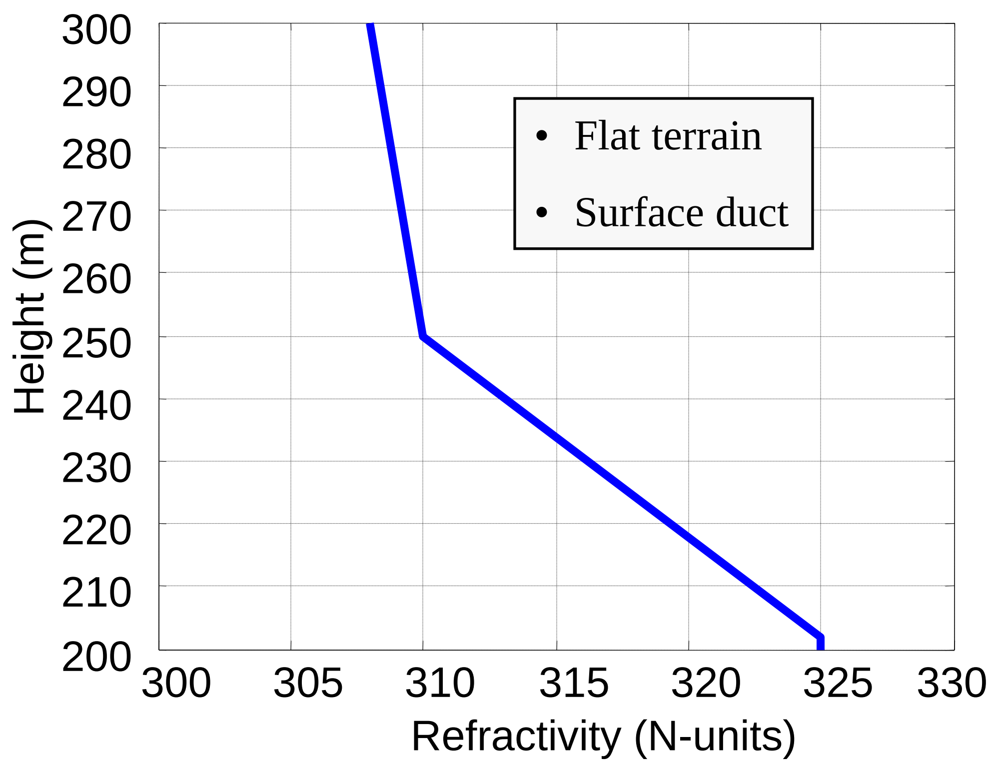
Parabolic equation (PE)

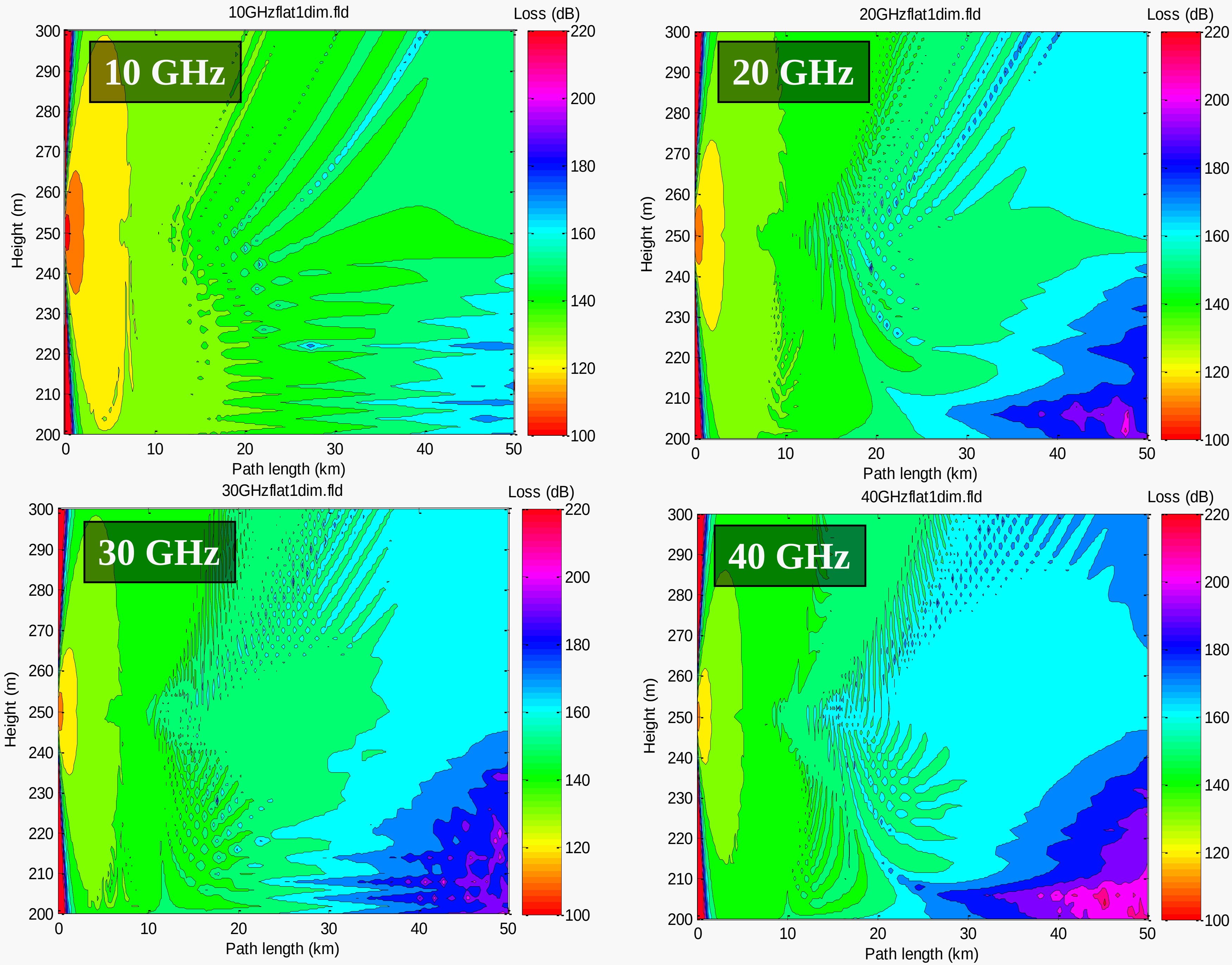
- Parabolic equation method is a full wave approach that can be efficient for numerical calculations
- Derived wave equation (from Maxwell's equations) and use the paraxial approximation, i.e., the medium change perpendicular to the propagation direction is much larger than in the propagation direction
- Can deal with any terrain and atmospheric refractive inedx, but limited narrow angles along the propagation direction
- Efficiently solved using Fourier transforms, but less flexible for various boundary conditions
- Finite difference implementation can deal with more complex boundary conditions

Check this for software: http://www.public.navy.mil/spawar/Pacific/55480/Pages/SoftwarePrograms.aspx

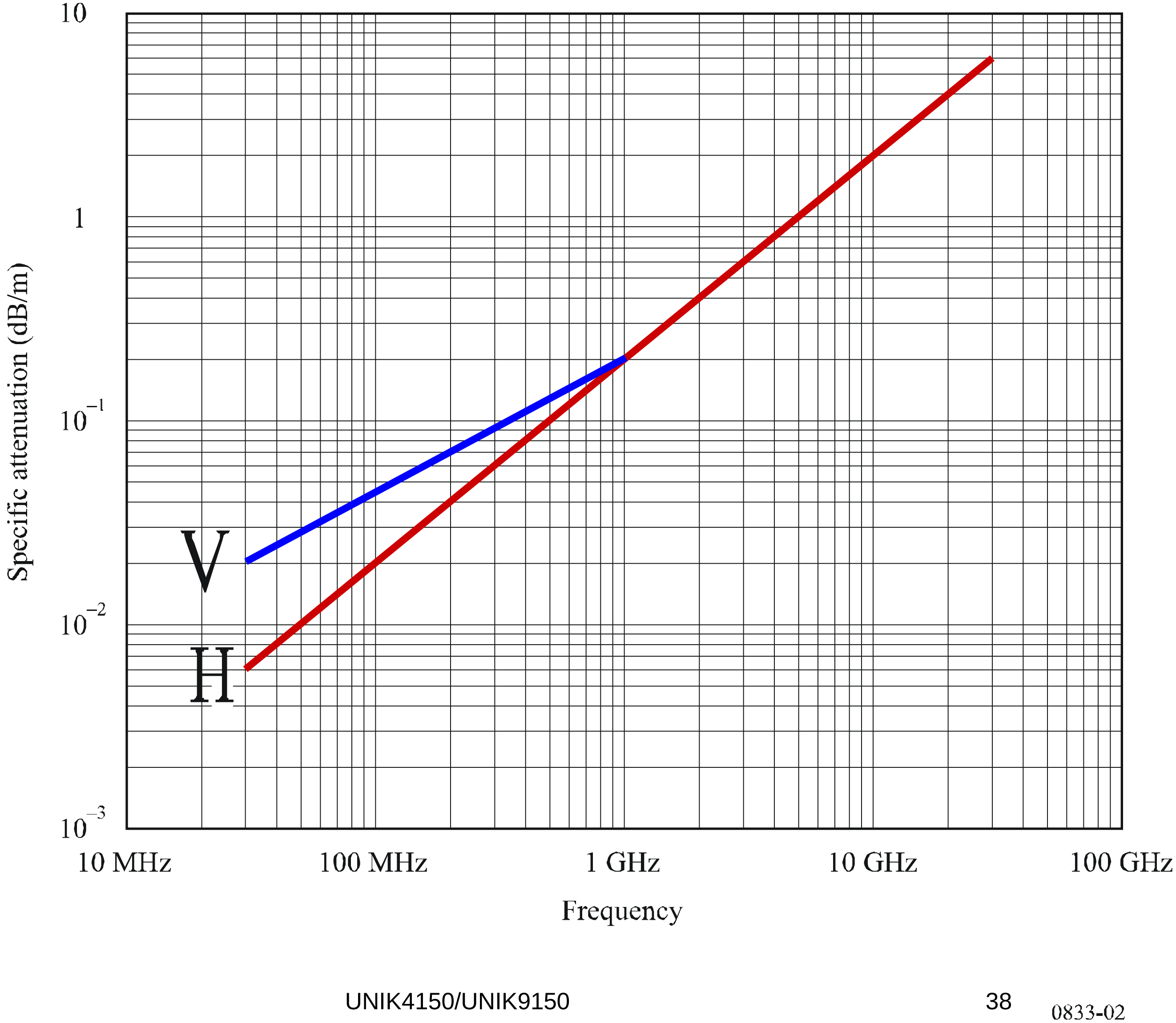
Full wave calculation over flat terrain and ducting condition






10GHzflat1dim.fld
10 GHz
Loss (dB)
Height (m)
Path length (km)
20GHzflat1dim.fld
20 GHz
30GHzflat1dim.fld
30 GHz
40GHzflat1dim.fld
40 GHz

Typical vegetation loss in woodland (ITU-R P.833-5)



Three modes of propagation

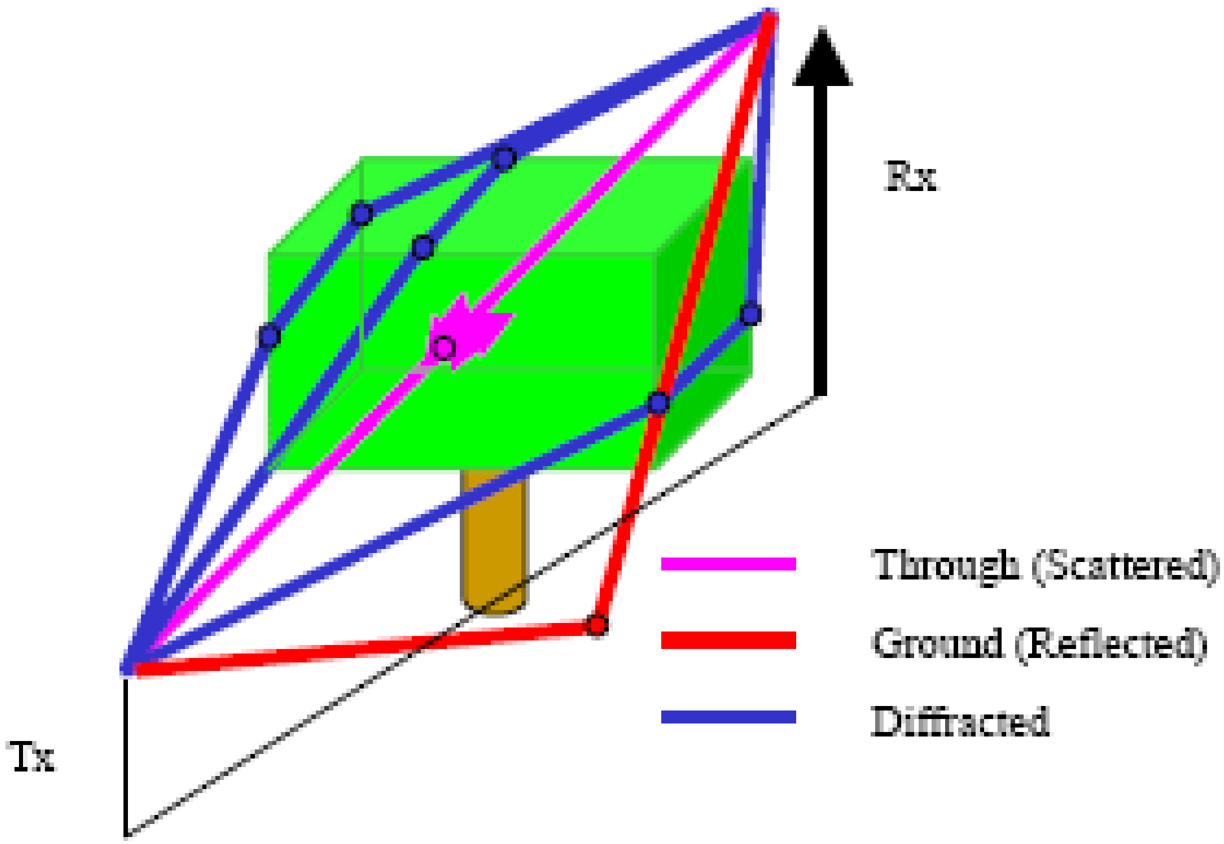
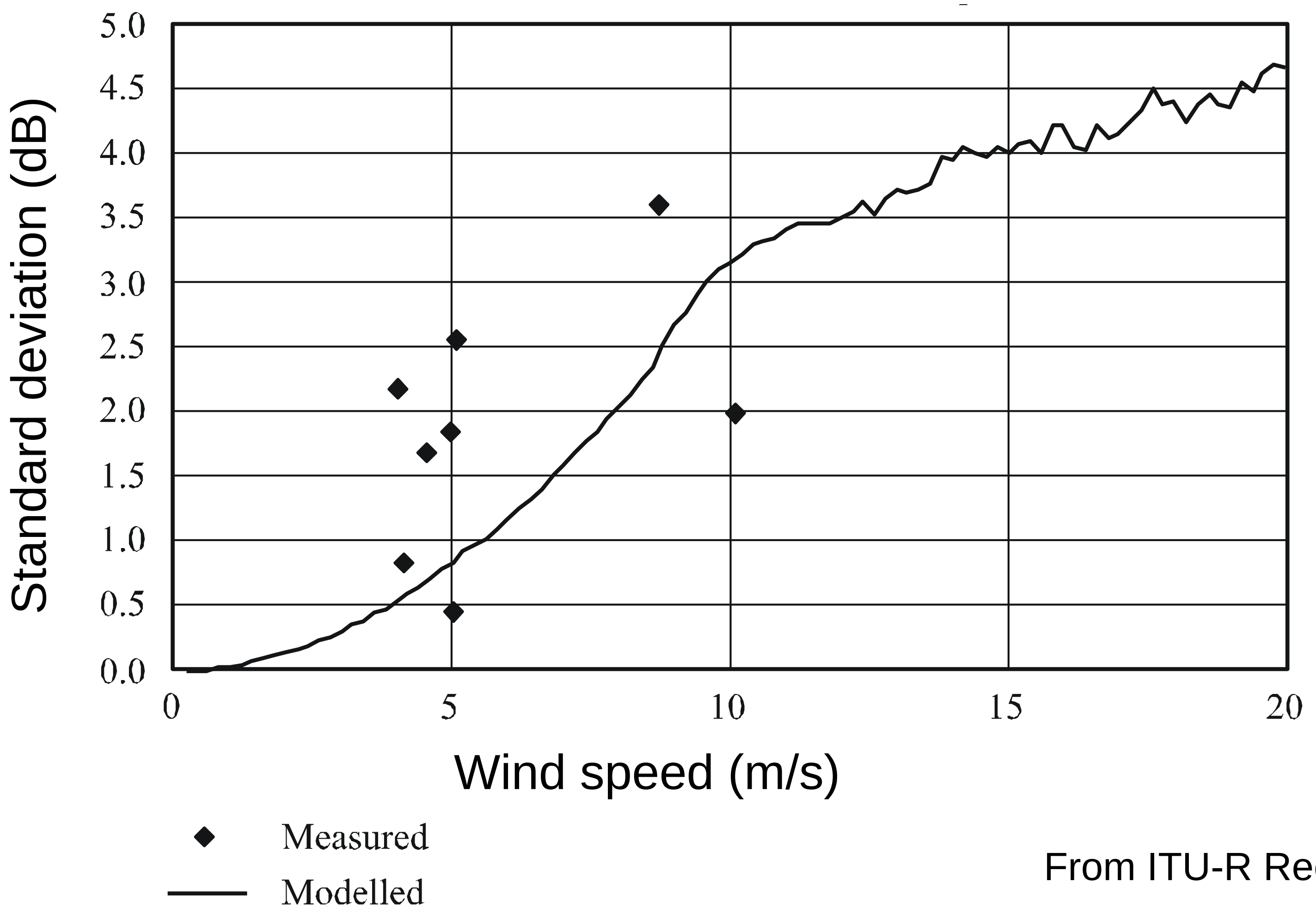


Figure from R. Ricther et al. RET input parameter estimation for a generic model of propagation through vegetation using excess attenuation and phase function measurements. ICAP 2003

Measurement at 40 GHz under periods with wind



From ITU-R Rec. P.833

Conclusions - Steps in Predicting Path Loss

- Locate the positions and heights of the antennas.
- Construct the great circle path between the antennas.
- Derive the terrain path profile; this can be done using conventional maps, or digital terrain maps.
- Uplift the terrain profile by representative heights for any known buildings along the path.
- Select a value for the effective Earth radius factor appropriate to the percentage of time being designed for; modify the path profile by this value.
- Calculate the free space loss for the path.
- If any obstructions exist within 0.6 ´ the first Fresnel zone, calculate diffraction over these obstructions and add the resulting excess loss to the link budget.
- Compute the path length which passes through trees and add the corresponding extra loss.
- For systems which require very high availability, the time variability of the signal due to multipath propagation under ducting conditions and reflections must also be accounted for.

Chapter 6. Summary

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- Tropospheric refraction, variability, ducting
- Obstruction