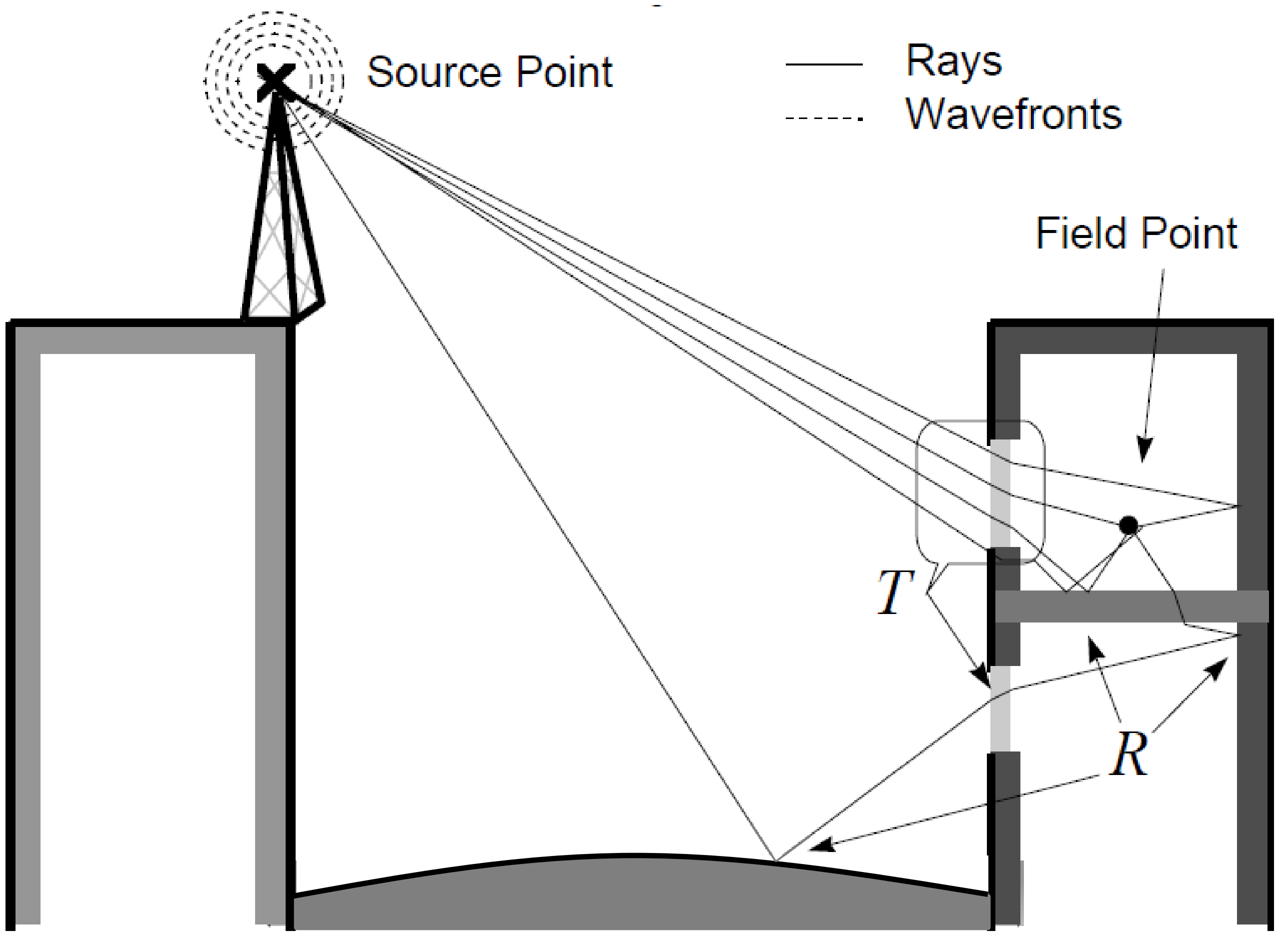
Chapter 3 Propagation mechanisms
3.4 - 3.6

- Geometrical optics
- Diffraction

Real system example



Geometrical optics (GO)

- Calculate rays between the source and the field points consistent with Snell's law
- Calculate Fresnel reflection and transmission coefficients
- Correct amplitude to account for wave front curvature from source and due to curvature of boundaries
- Sum all rays with regard to both amplitude and phase

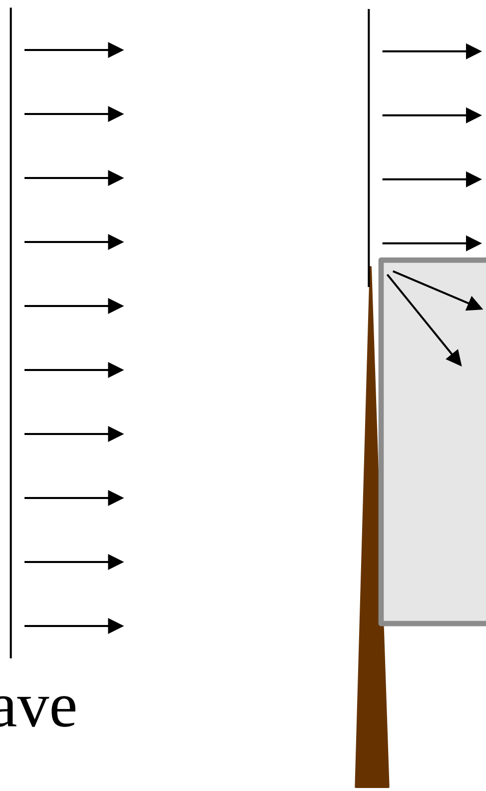
GO formulation

$$\mathbf{E} = \mathbf{E}_0 A_0 e^{-jk_0 r_0} + \sum_{m=1}^{N_r} \mathbf{R}\mathbf{E}_m A_m e^{-jk_m r_m} + \sum_{k=1}^{N_t} \mathbf{T}\mathbf{E}_k A_k e^{-jk_k r_k}$$

- N_r is number of reflected rays
- N_t is number of transmitted rays
- r is distance along the ray
- k is the wave number for the medium
- A is spreading factor
- $\mathbf{E}_m$ or $\mathbf{E}_k$ is the incident field
- Parameters with subscript 0 account for the direct ray

Diffraction

A plane wave propagating towards an absorbing obstacle



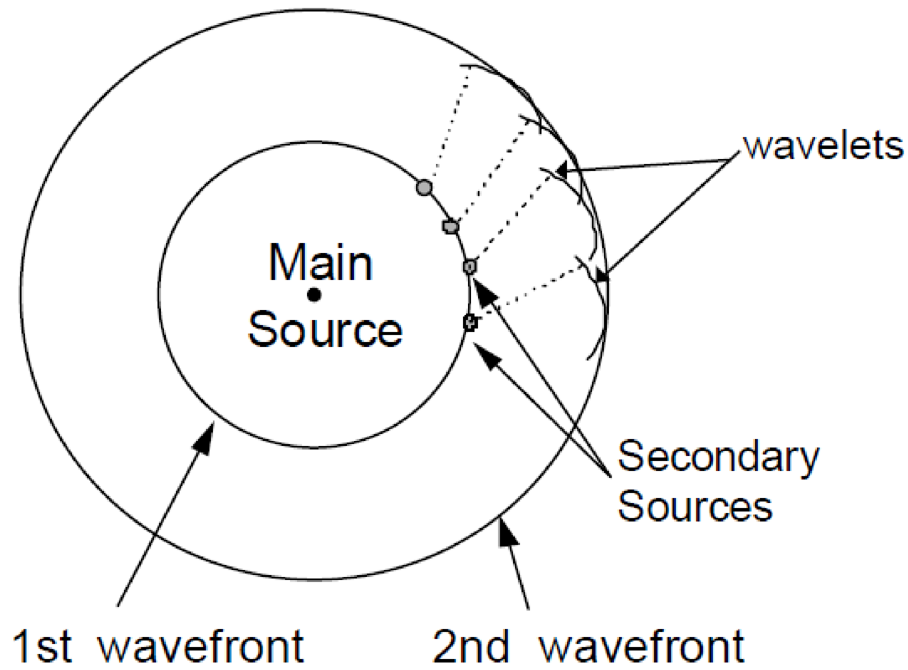
Wave

Obstacle

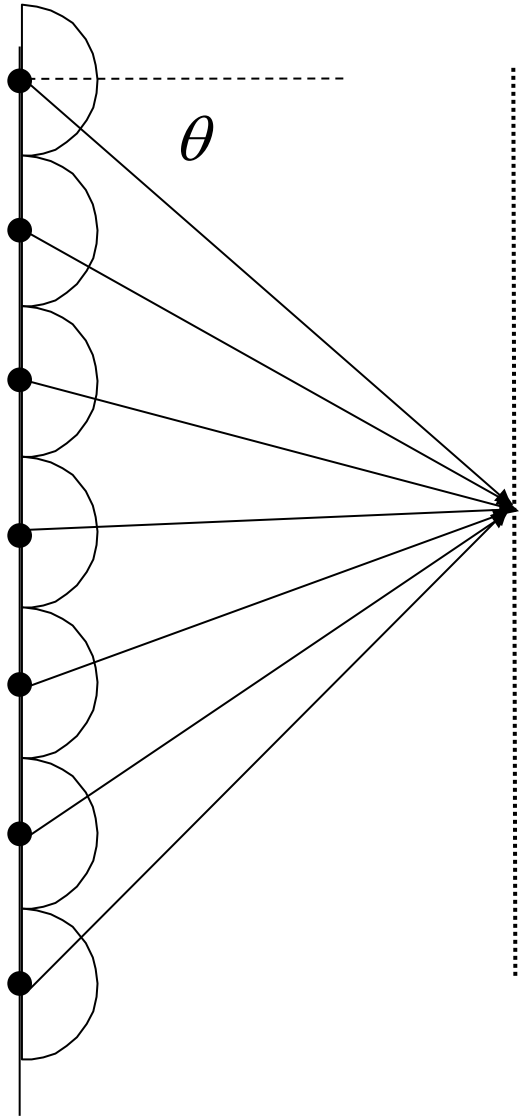
optical approximation
): straight lines
=> no field behind the obstacle

However, there is field in the shadow.
All points on the wave front radiates.
Need them all to maintain the wave.

Huygen's principle

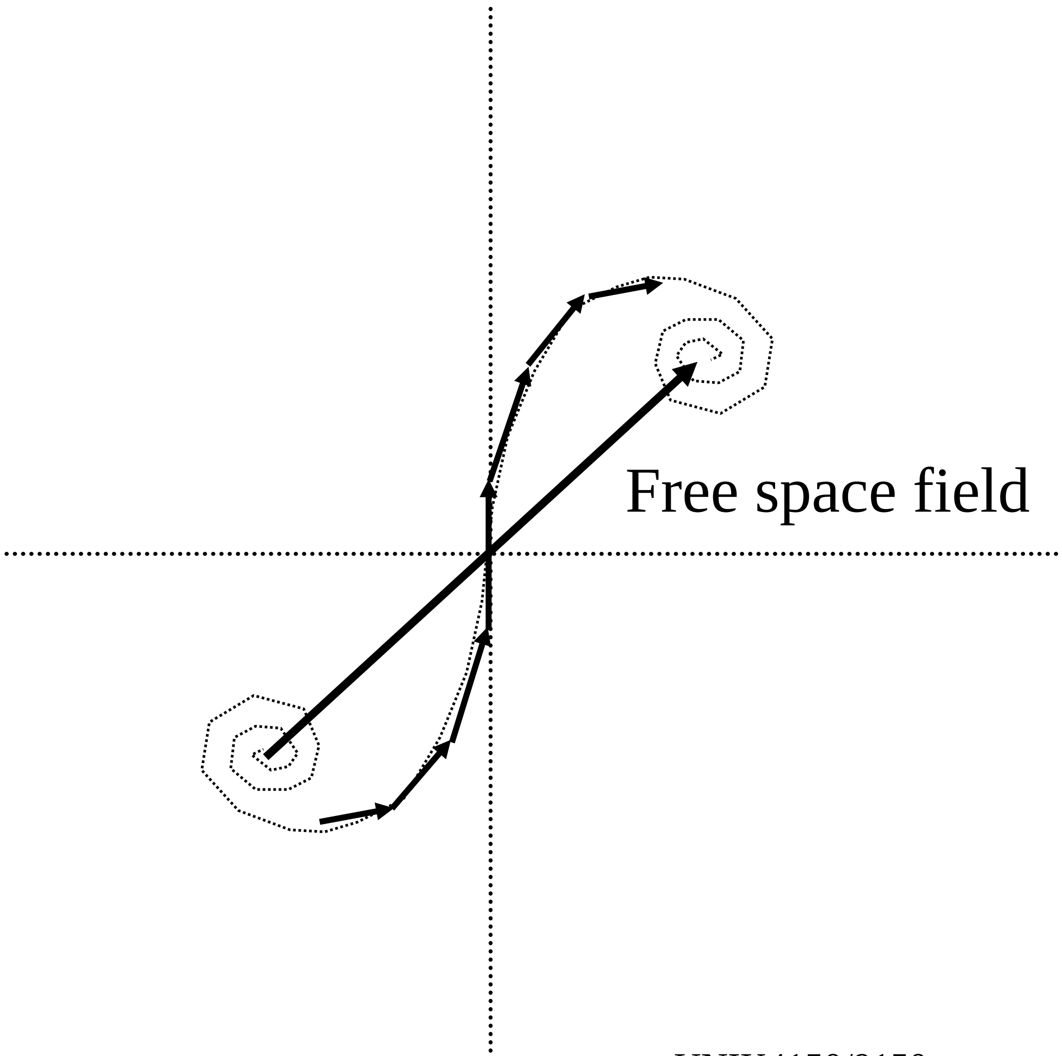


Apply Huygen’s principle



- Every wave front point create secondary waves.
- The amplitude on the secondary wave is proportional to 1+cosθ, where θ the angle with propagation direction.
- The field is the sum of all secondary waves.

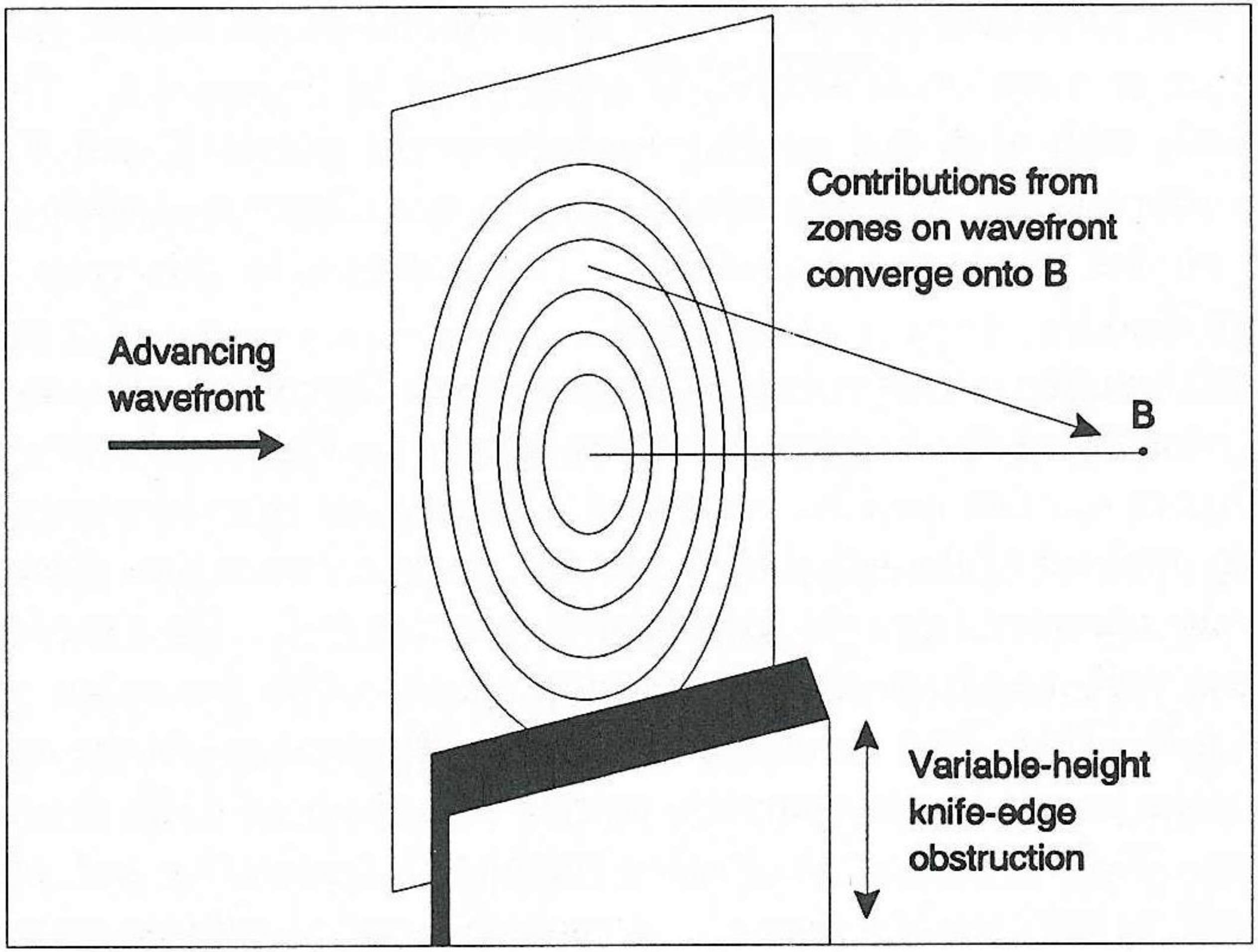
Vector addition – Cornu spiral



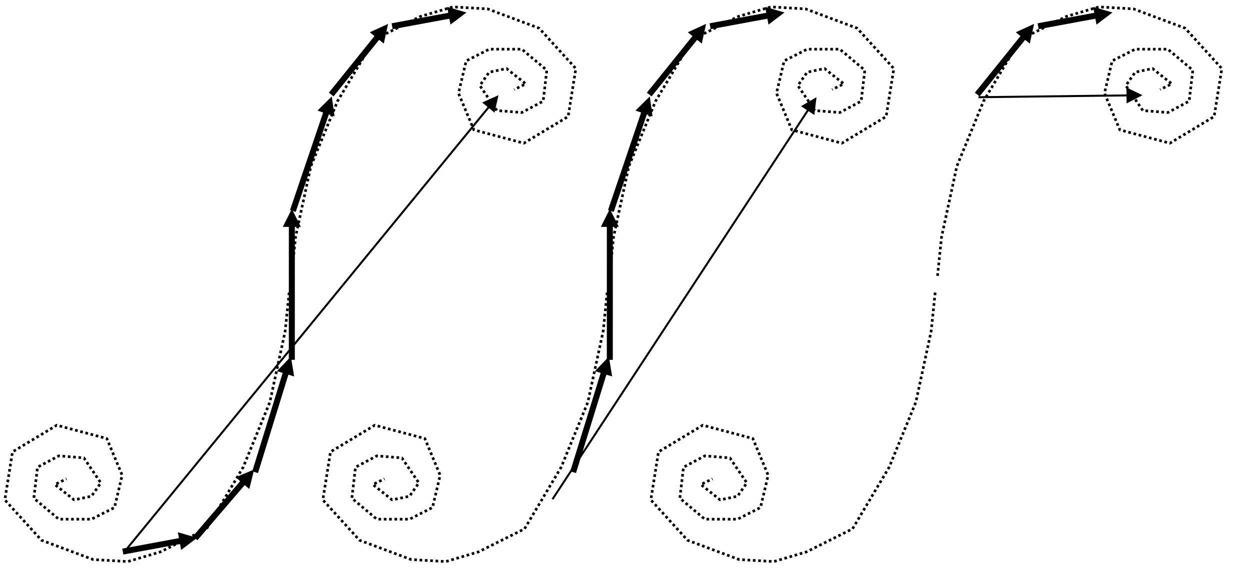
Add vectors from every secondary wave.

The length of each vector becomes shorter the longer the propagation distance.

Three dimensional picture



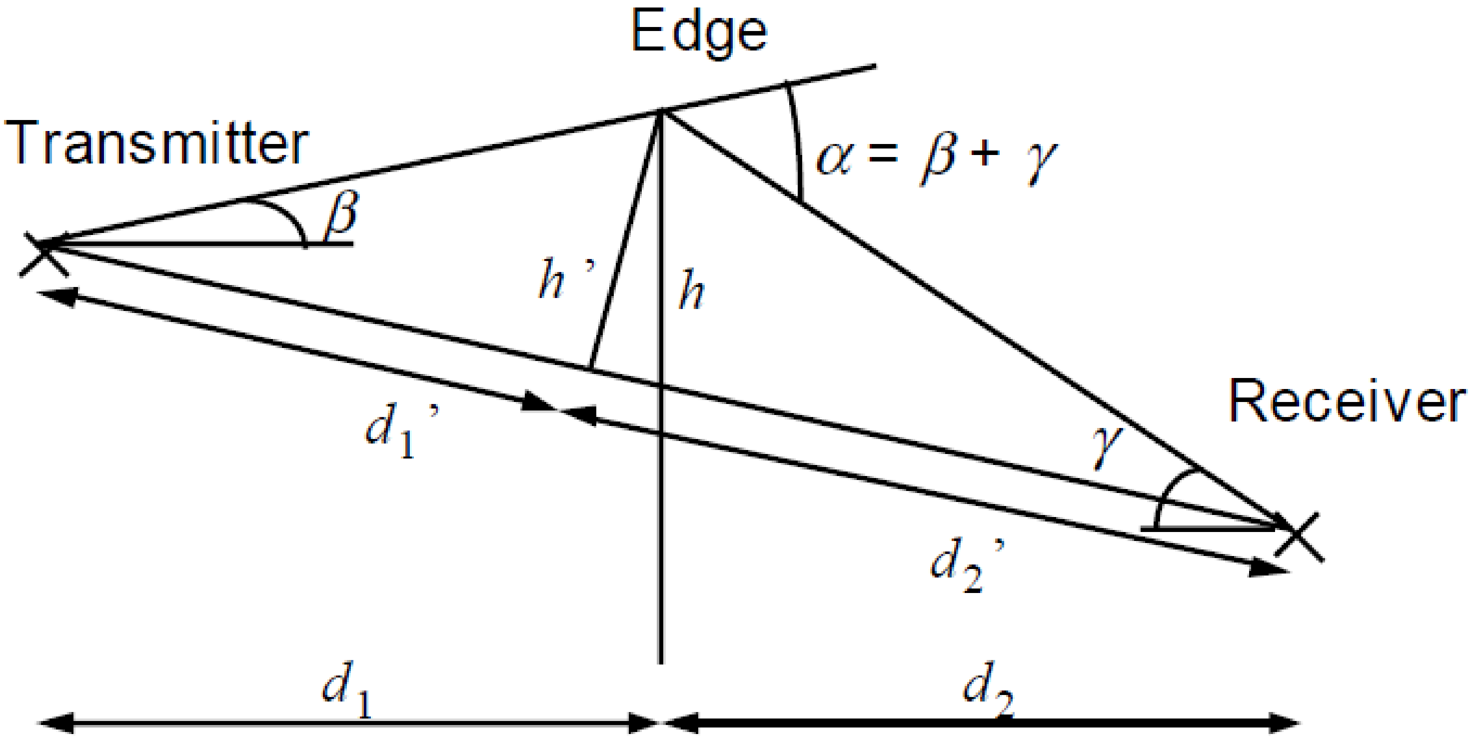
The effect of an obstacle



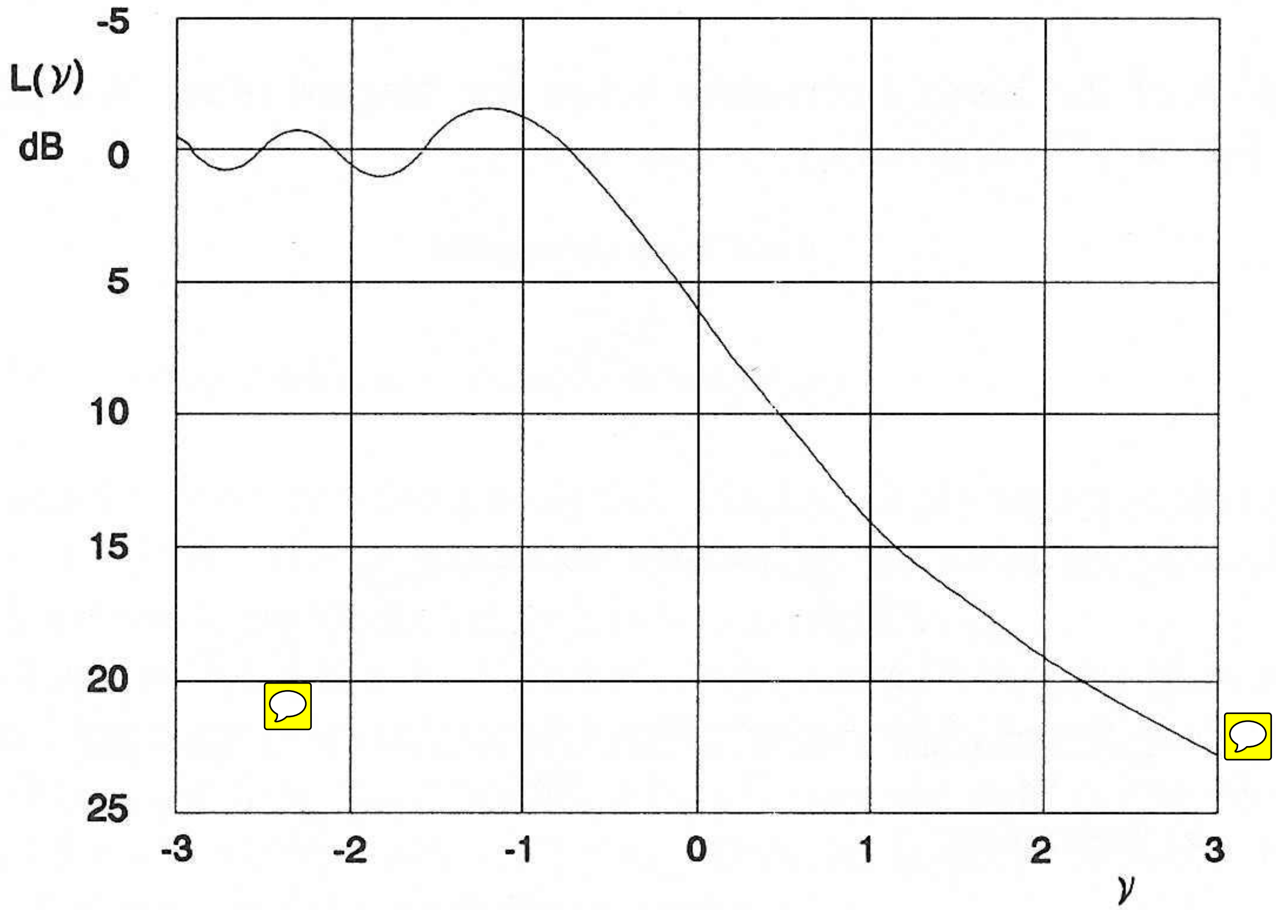
The more the absorber is coming into the wave the more vectors are removed.

Fresnel diffraction parameter

$$v = h'\sqrt{\frac{2(d_1'+d_2')}{\lambda d_1' d_2'}} = \alpha\sqrt{\frac{2d_1'd_2'}{\lambda(d_1'+d_2')}}$$



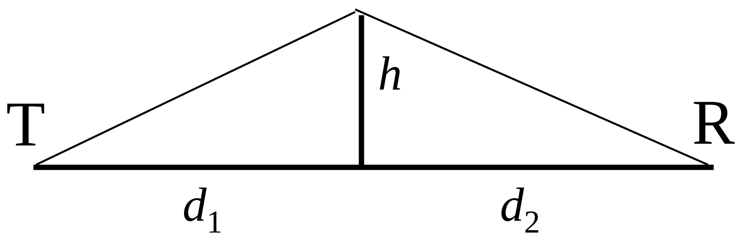
Knife edge diffraction



Knife edge diffraction of sea waves

Absorbing knife edge

Integrating over the visible part of the wave front. The transmitted signal is obstructed between T and R. The received diffracted electrical field amplitude E_d is given by the following expression , the complex Fresnel integral (from theory):



$$\frac{E_d}{E_0} = \frac{1+j}{2}\int_{\nu}^{\infty} e^{-\frac{j\pi t^2}{2}}\,dt$$

where E_0 is the field without the obstacle and ν the diffraction parameter given by the wave length and geometry.

Knife edge diffraction with cosine and sine Fresnel integrals

The complex Fresnel integral can be written

$$\int_{\nu}^{\infty} e^{-\frac{j\pi t^2}{2}} dt = \int_{\nu}^{\infty} \cos\left(\frac{\pi t^2}{2}\right) dt - j\int_{\nu}^{\infty} \sin\left(\frac{\pi t^2}{2}\right) dt$$

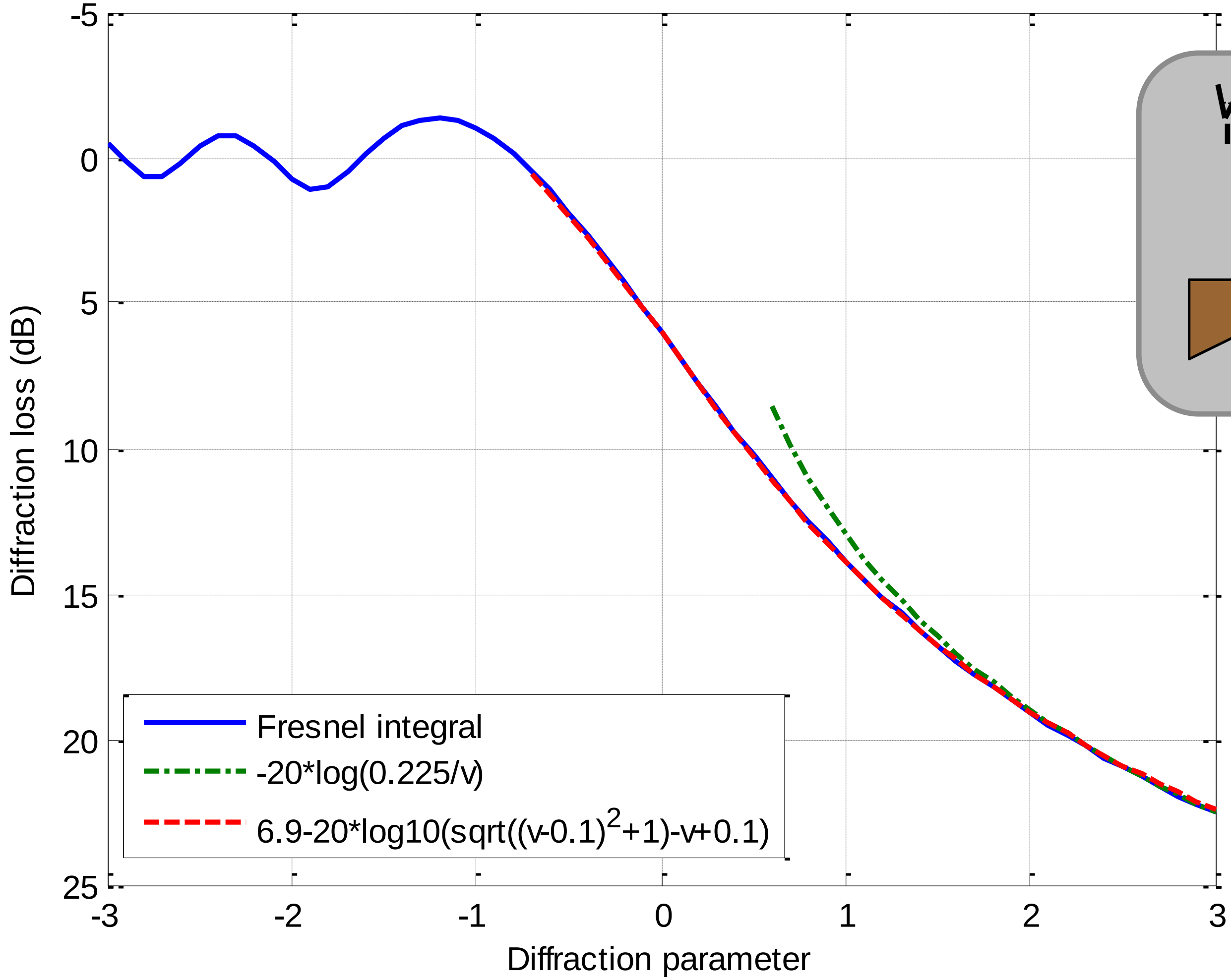
and $$\int_{\nu}^{\infty} \cos\left(\frac{\pi t^2}{2}\right) dt = \frac{1}{2} - \int_{0}^{\nu} \cos\left(\frac{\pi t^2}{2}\right) dt = \frac{1}{2} - C(\nu)$$

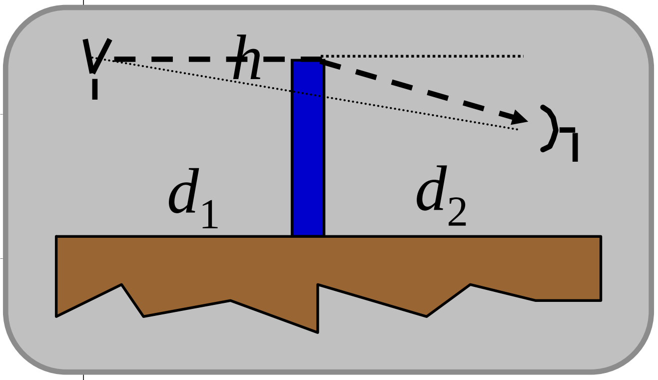
where *C* denotes the cosine Fresnel integral

Similarly $$\int_{\nu}^{\infty} \sin\left(\frac{\pi t^2}{2}\right) dt = \frac{1}{2} - \int_{0}^{\nu} \sin\left(\frac{\pi t^2}{2}\right) dt = \frac{1}{2} - S(\nu)$$

Then $$\frac{E_d}{E_0} = \frac{1+j}{2}\left[\left(\frac{1}{2} - C(\nu)\right) - j\left(\frac{1}{2} - S(\nu)\right)\right]$$

Knife-edge diffraction





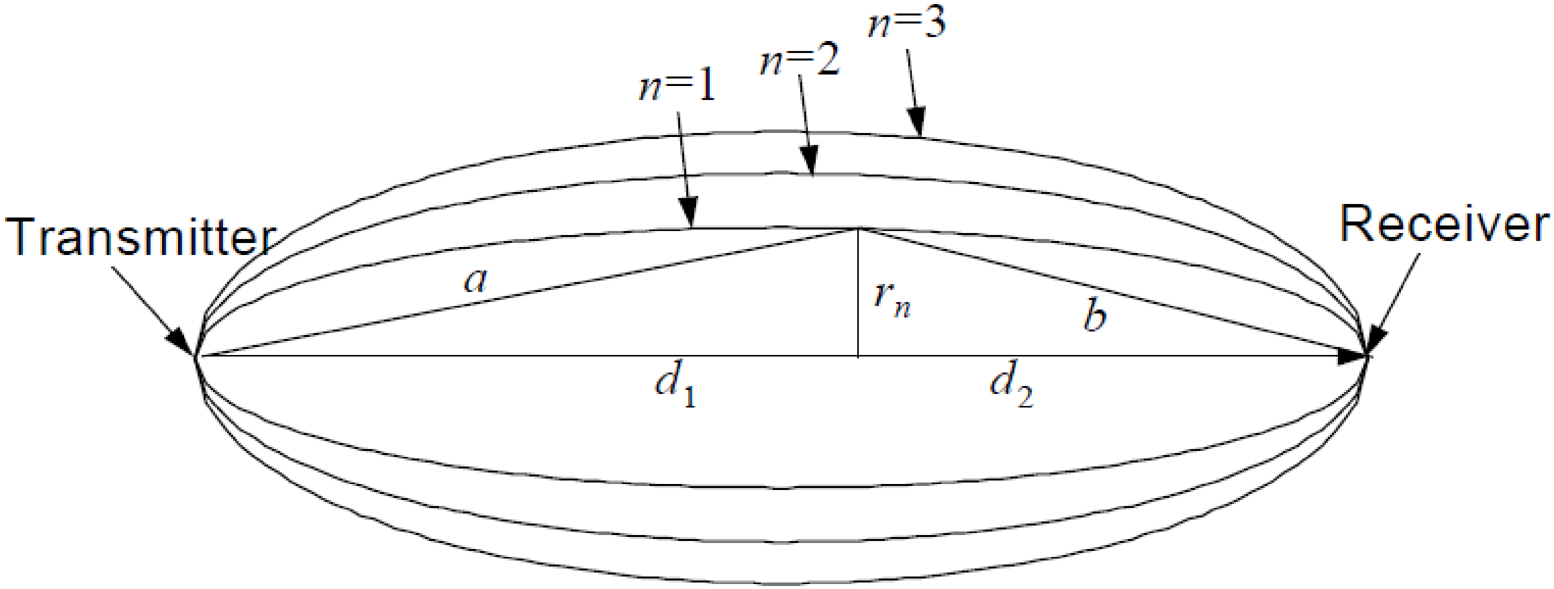
$$\upsilon \approx h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}}$$

Fresnel ellipsoids

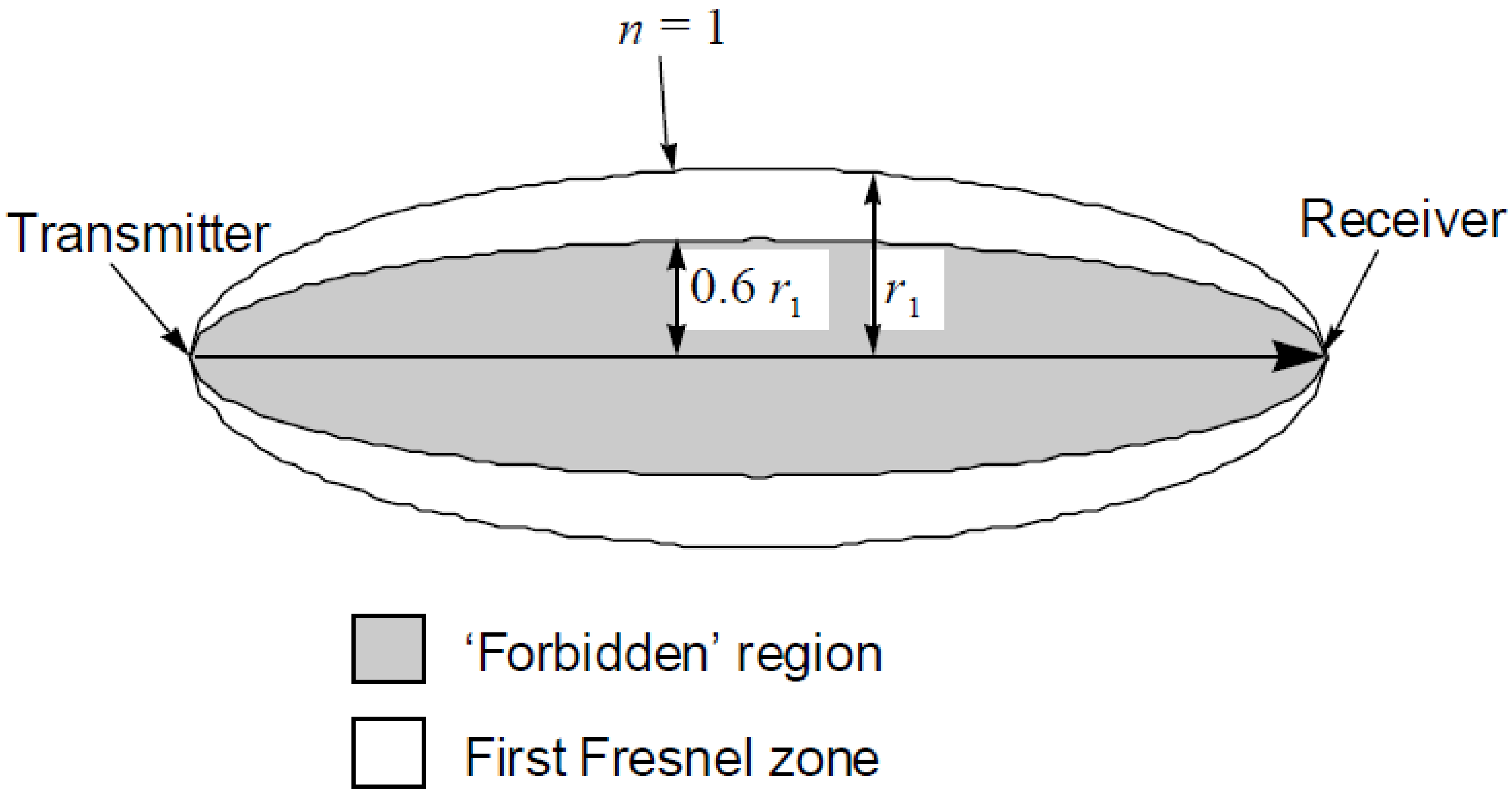
$$a + b = d_1 + d_2 + \frac{n\lambda}{2}$$

$$r_n \approx \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$

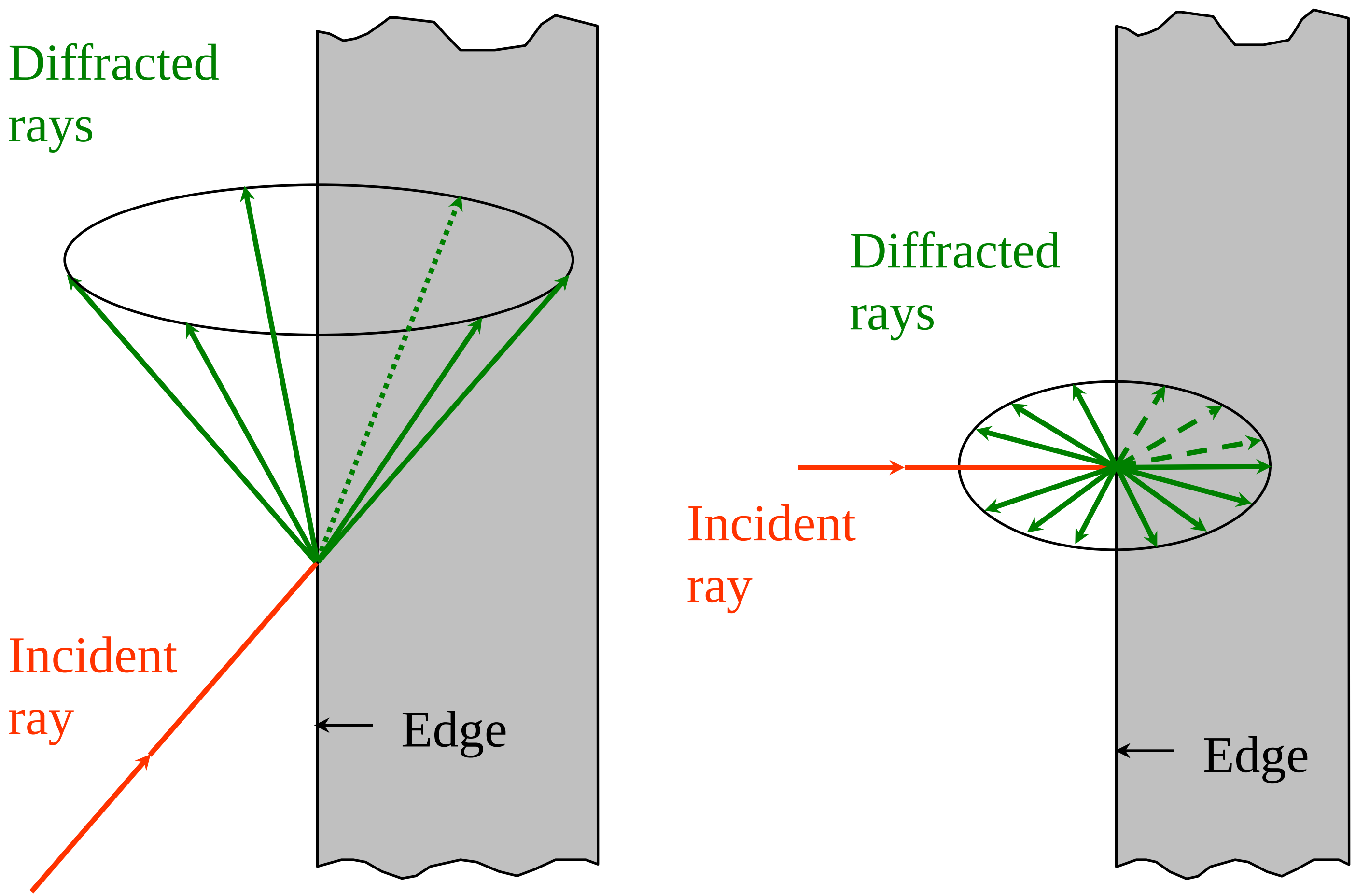
$$v \approx h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \frac{h}{r_n}\sqrt{2n}$$



Path clearance for full signal strength



Geometrical theory of diffraction (GTD)



$$\mathbf{E}_d = \mathbf{D}\mathbf{E}_i A_d$$

GTD field

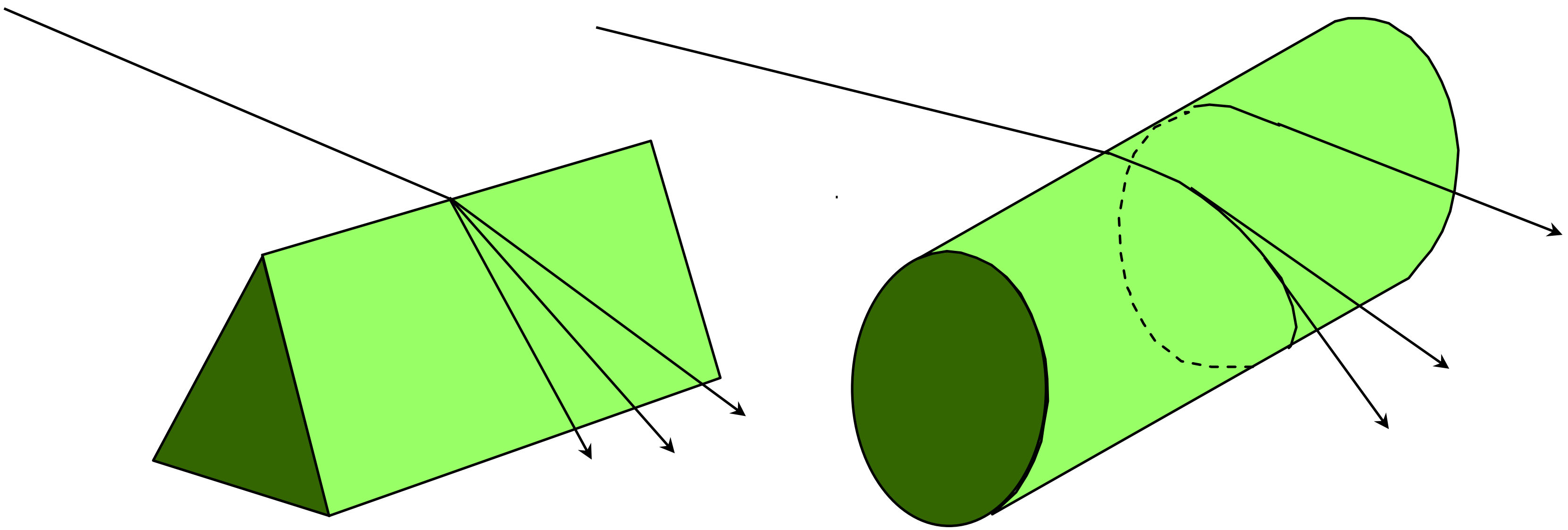
$$\mathbf{E}_i = \begin{bmatrix} E_{i\|} \\ E_{i\perp} \end{bmatrix}$$

$$\mathbf{E}_d = \begin{bmatrix} E_{d\|} \\ E_{d\perp} \end{bmatrix}$$

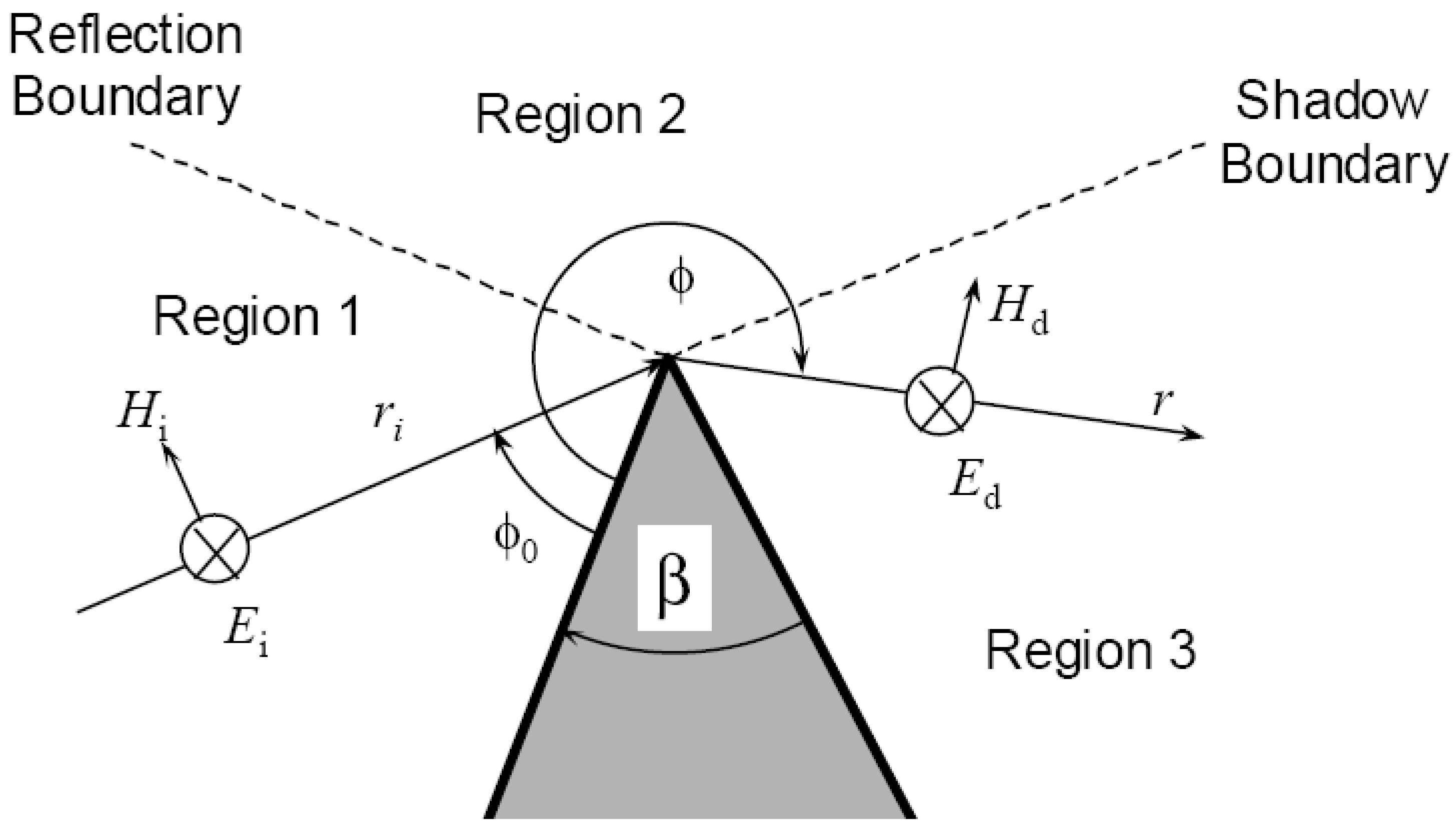
$$\Rightarrow \quad \mathbf{D} = \begin{bmatrix} D_{\|} & 0 \\ 0 & D_{\perp} \end{bmatrix} \quad \Rightarrow \quad \boxed{\mathbf{E}_d = \mathbf{D}\mathbf{E}_i A}$$

- Direct analogy with geometrical optics

Basic shapes

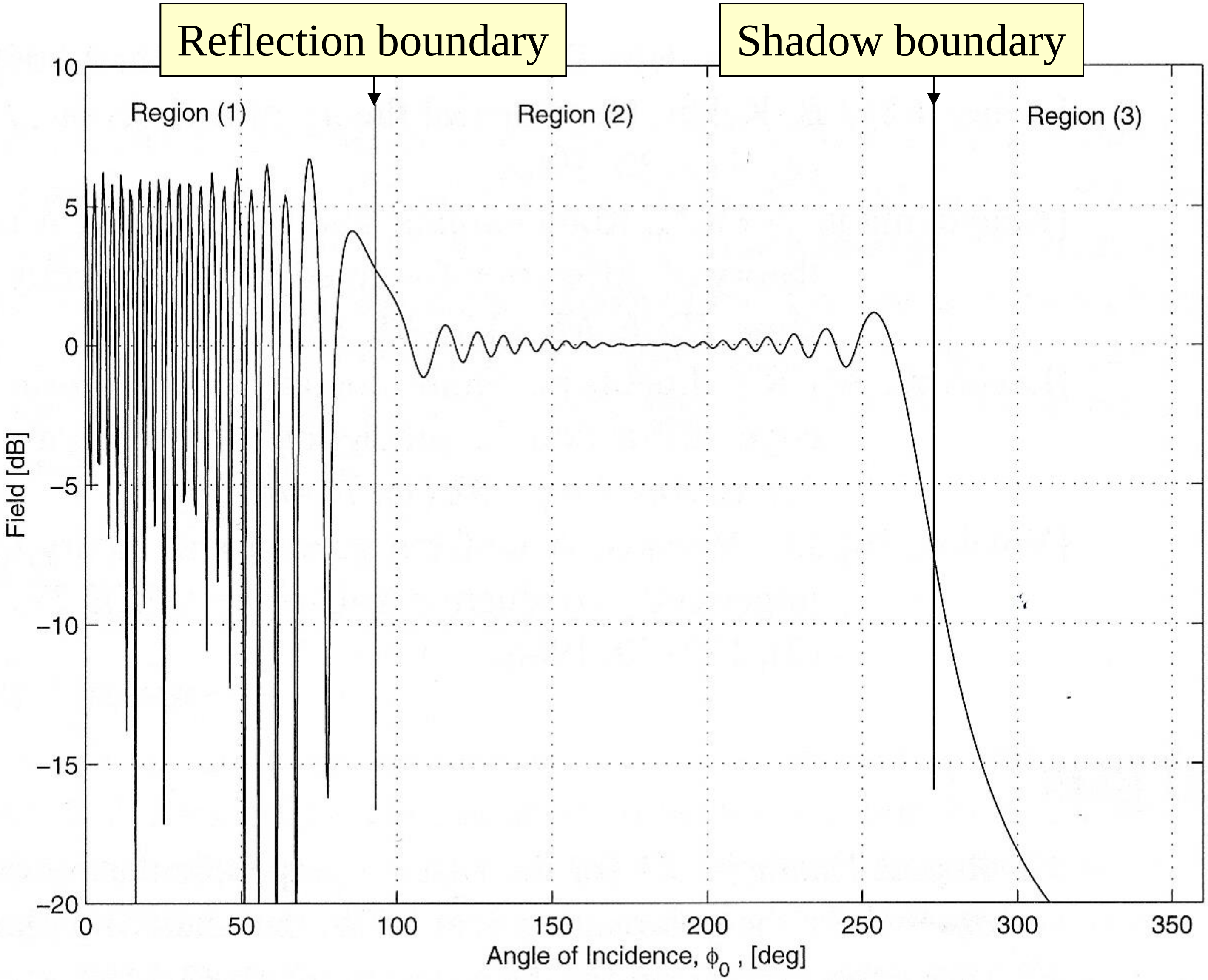


Geometry for wedge diffraction



$$\boxed{\mathbf{E}_t = \mathbf{E}_i \left(A_0 + U_r \mathbf{R} A_d + \mathbf{D} A_d \right)}$$

UTD solution around a conducting half-plane



Conclusion

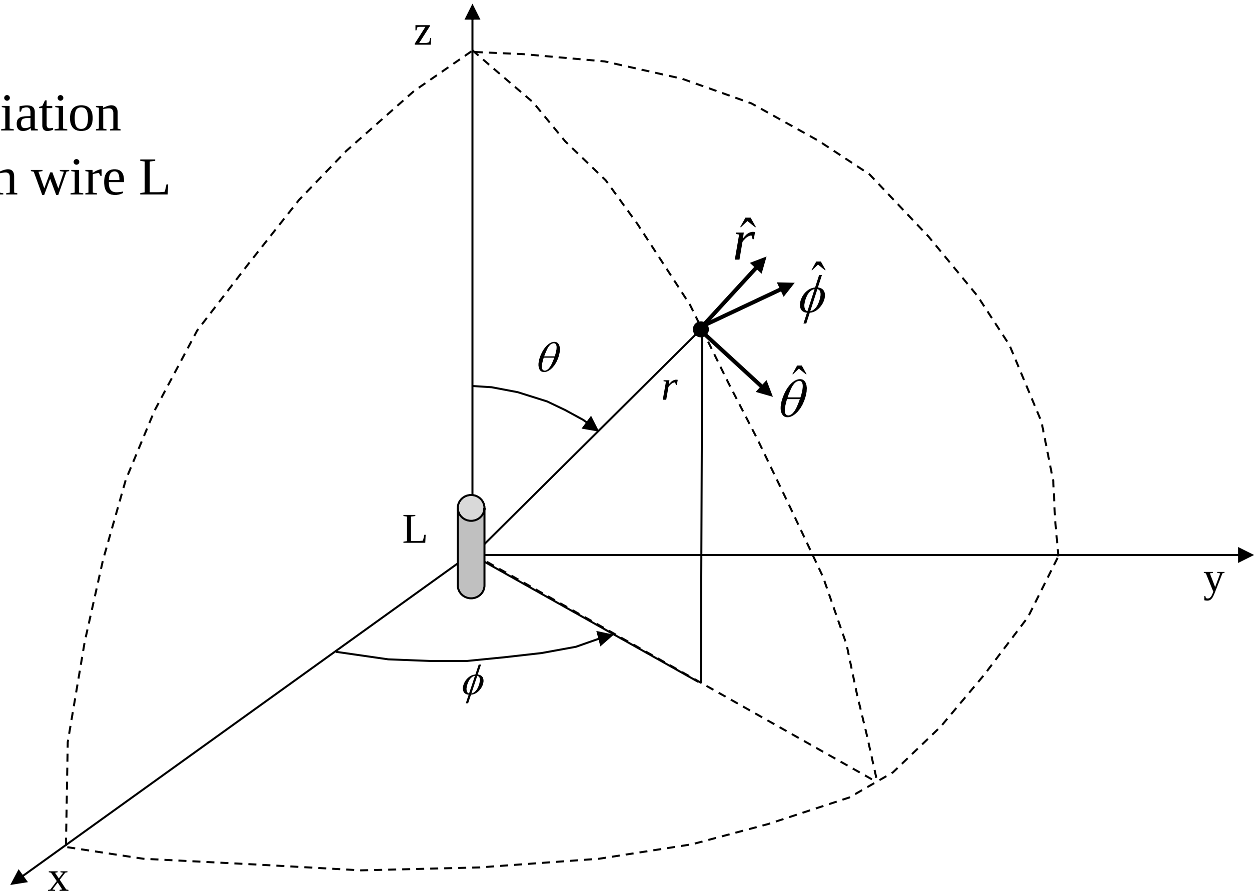
- Diffraction simplified illustration for the knife edge case
- Better for data program using Fresnel integral method or geometrical diffraction theory

Chapter 4 Antenna fundamentals additional

- Fundamental theory
- Small antennas for mobile communication

Spherical coordinate system

Radiation
from wire L



Radiation from an infinitesimal dipole *L*

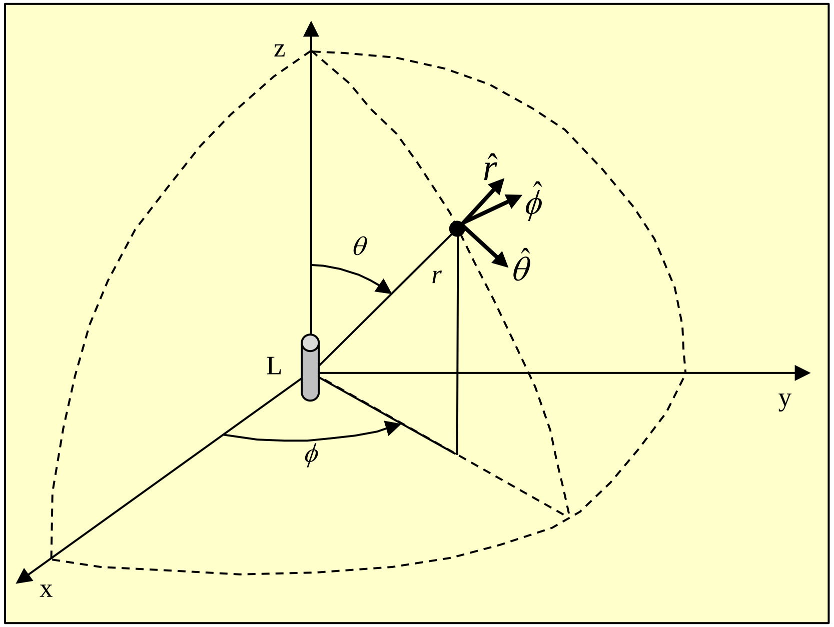
Electric field

$$\mathbf{E} = \frac{jZ_0 IL}{2\pi k_0}\cos\theta\left(\frac{jk_0}{r^2}+\frac{1}{r^3}\right)e^{-jk_0 r}\mathbf{a}_r - \frac{jZ_0 IL}{4\pi k_0}\sin\theta\left(-\frac{k_0^2}{r}+\frac{jk_0}{r^2}+\frac{1}{r^3}\right)e^{-jk_0 r}\mathbf{a}_\theta$$

$$= E_r\mathbf{a}_r + E_\theta\mathbf{a}_\theta$$

Magnetic field

$$\mathbf{H} = j\frac{k_0 IL\sin\theta}{4\pi r}\left(1+\frac{1}{jk_0 r}\right)e^{-jk_0 r}\mathbf{a}_\phi$$



Note that the term e^{jwt} is dropped for simplicity

Far-field equations

Can neglect terms of r^2 or higher

$$E_\theta = jZ_0 \frac{k_0 IL e^{-jk_0 r}}{4\pi r} \sin\theta$$

$$H_\phi = j \frac{k_0 IL e^{-jk_0 r}}{4\pi r} \sin\theta$$

$$E_r = 0$$
$$E_\phi = 0$$
$$H_r = 0$$
$$H_\theta = 0$$

- The radiated field has transverse components
- Ratio $E_\theta / H_\phi = Z_0$: fields in phase and the wave impedance is $120\pi\ \Omega$
- The field is inversely proportional to r
- The fields re zero at $\theta = 0$ and π, but maximum at $\pi/2$; the x-y plane

Develop from Maxwell equations

Theoretical derivation of the radiation from a wire, not in the book.

Given only the current source ***J*** causing the radiation, then Maxwell's equations:

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon_0 \mathbf{E} + \mathbf{J}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mu_0 \mathbf{H} = 0$$

Since $\nabla \cdot \frac{1}{\mu_0} \boldsymbol{H} = 0$, ***H*** can be expressed $\boldsymbol{H} = \frac{1}{\mu_0}(\nabla \times \boldsymbol{A})$, because $\nabla \cdot \nabla \times \mathbf{A} = 0$, where ***A*** is called the magnetic vector potential.

Replace ***H*** in the first equation above: $\nabla \times (\boldsymbol{E} + j\omega \boldsymbol{A}) = 0$

Since the curl is 0 the expression in the bracket can be expressed as the gradient of a scalar; *Φ*, called the electric scalar potential

$$\mathbf{E} + j\omega\mathbf{A} = -\nabla\Phi$$

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi$$

Vector potential $\boldsymbol{A}$ and scalar potential Φ

Theoretical derivation of the radiation from a wire, not in the book.

Use expressions for $\boldsymbol{E}$ and $\boldsymbol{H}$, $\boxed{\boldsymbol{H} = \frac{1}{\mu_0}(\nabla \times \boldsymbol{A}) \quad \mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi}$, to get:

$$\nabla \times \nabla \times \mathbf{A} = \mu_0(\nabla \times \mathbf{H}) = j\omega\mu_0\varepsilon_0\mathbf{E} + \mu_0\mathbf{J} = \omega^2\mu_0\varepsilon_0\mathbf{A} - j\omega\mu_0\varepsilon_0\nabla\Phi + \mu_0\mathbf{J}$$

$$-j\omega\nabla \cdot \mathbf{A} - \nabla^2\Phi = \frac{\rho}{\varepsilon_0}$$

Use $k_0^2 = \omega^2\varepsilon_0\mu_0$ and $\nabla \times \nabla \times \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A}$ to get:

$$\nabla^2\boldsymbol{A} + k_0^2\boldsymbol{A} - \nabla(\nabla \cdot \boldsymbol{A} + j\omega\mu_0\varepsilon_0\Phi) = -\mu_0\boldsymbol{J}$$

$$\nabla^2\Phi + j\omega\nabla \cdot \boldsymbol{A} = -\frac{\rho}{\varepsilon_0}$$

Decouple the equations using the Lorentz-condition: $\nabla \cdot \boldsymbol{A} = -j\omega\mu_0\varepsilon_0\Phi$

Use expression for div$\boldsymbol{A}$ and k_0^2

$$\nabla^2\boldsymbol{A} + k_0^2\boldsymbol{A} = -\mu_0\boldsymbol{J}$$

$$\nabla^2\Phi + k_0^2\Phi = -\frac{\rho}{\varepsilon_0}$$

Theoretical derivation of the radiation from a wire, not in the book.

Solution for the electric field

Can now find the electric and magnetic fields by solving for scalar potential Φ and the vector potential ***A***. However, also possible to find the electric field in terms of the vector potential directly.

Lorentz condition: $\nabla \cdot \boldsymbol{A} = -j\omega\mu_0\varepsilon_0\Phi$

Previously found: $\mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi$

Then: $\mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi = -j\omega\mathbf{A} + \dfrac{\nabla\nabla \cdot \mathrm{A}}{j\omega\mu_0\varepsilon_0}$

Apply the solution for a current along the z-axis for $\nabla^2\mathbf{A} + k_0^2\mathbf{A} = -\mu_0\mathbf{J}$

$\mathbf{J} = J_z\mathbf{a}_z$ and $\mathbf{A} = A_z\mathbf{a}_z$ then

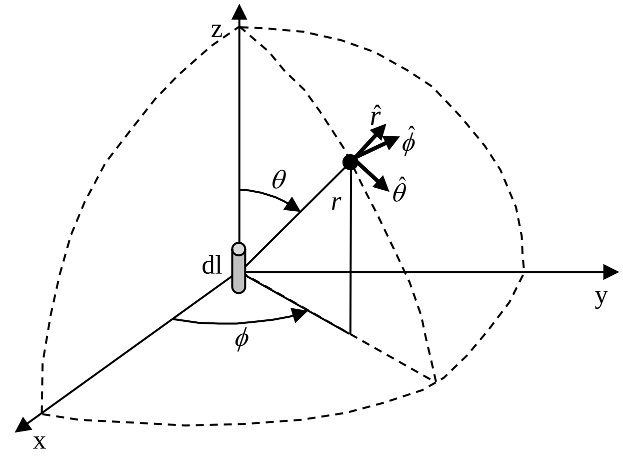
$$\left(\nabla^2 + k_0^2\right)A_z = -\mu_0 J_z$$

Theoretical derivation of the radiation from a wire, not in the book.

Radiation from a short wire

The antenna is a thin wire located at the origin

$$\left(\nabla^2 + k_0^2\right)A_z = -\mu_0 J_z$$



where the current and the vector potential has only a z-component, $J_z=I/\mathrm{d}S$, dS the cross-sectional area of the wire of length dl. The volume is dV=dSdl of infinitesimal size such that the current can be considered located at a point. There is then spherical symmetry and the field will only be a function of r.

For $r \neq 0$ A_z satisfies $$\frac{1}{r^2}\frac{\partial}{\partial r}r2\frac{\partial A_z}{\partial r} + k_0^2 A_z = 0$$

where ∇^2 is in spherical coordinates, not include derivates on θ and ϕ.

Theoretical derivation of the radiation from a wire, not in the book.

Harmonic motion equation and solution

Substitute $A_z = \psi/r$ then $dA_z/dr = r^{-1}d\psi/dr - r^{-2}\psi$ to get $\dfrac{d^2\psi}{dr^2} + k_0^2\psi = 0$

which is the harmonic-motion equation with solutions $C_1 e^{-jk_0 r}$ and $C_2 e^{jk_0 r}$

Choose the first and restore time-dependency $\psi(r,t) = C_1 e^{-jk_0 r + j\omega t}$

Since $k_0 = \omega/c$ where speed of light $c = (\mu_0\varepsilon_0)^{-1/2}$ then $\psi(r,t) = C_1 e^{j\omega(t - r/c)}$

This is a solution for a wave propagating away from the source, since the phase is retarded by $k_0 r$ corresponding to the time delay of r/c. The other solution with C_2 corresponds to an inward propagating spherical wave not part of the solution for radiation from a current element a $r = 0$.

The solution for A_z is now of the form $A_z = C_1 \dfrac{e^{-jk_0 r}}{r}$

Constant C_1 and the source strength to find the vector potential final solution

Theoretical derivation of the radiation from a wire, not in the book.

Integrate both sides of the equation $\left(\nabla^2 + k_0^2\right)A_z = -\mu_0 J_z$ over a small spherical volume V with radius r_0

Note $\nabla^2 A_z = \nabla \bullet \nabla A_z$ such that

$$\int_V \nabla^2 A_z dV = \int_V \nabla \cdot \nabla A_z dV = \oint_S \nabla A_z \cdot \mathbf{a}_r r_0^2 \sin\theta d\theta d\phi = -k_0^2 \int_V A_z dV - \mu_0 \int_V J_z dV$$

Both $dV = r^2 \sin\theta d\theta dr$ and A_z varies with $1/r$ such that for $r_0 \to 0$ the volume integral of A_z vanishes. The volume integral of J_z gives $J_z s dS dl = I dl$.

In addition $$\nabla A_z \cdot \mathbf{a}_r = \frac{\partial A_z}{\partial r} = -(1 + jk_0 r) C_1 \frac{e^{-jk0r}}{r}$$

such that $$\lim_{r_0 \to 0} \int_0^{2\pi} \int_0^{\pi} -(1 + jk_0 r_0) C_1 e^{-jk_0 r_0} \sin\theta d\theta \phi = -4\pi C_1 = -\mu_0 I dl$$

Vector potential final solution $$\mathbf{A} = \mu_0 I dl \frac{e^{-jk_0 r}}{4\pi r} \mathbf{a}_z$$

Electric and magnetic field from an infinitesimal dipole

Theoretical derivation of the radiation from a wire, not in the book.

$$\mathbf{A} = \mu_0 Idl \frac{e^{-jk_0 r}}{4\pi r}\mathbf{a}_z$$

An outward propagating spherical wave with amplitude decreasing with distance and phase velocity of c.

Electromagnetic fields in spherical coordinates noting $\mathbf{a}_z = \mathbf{a}_r\cos\theta - \mathbf{a}_\theta\sin\theta$ and

$$\mathbf{A} = \frac{\mu_0 Idl}{4\pi r}e^{-jk_0 r}\left(\mathbf{a}_r\cos\theta - \mathbf{a}_\theta\sin\theta\right)$$

Magnetic field

$$\mathbf{H} = \frac{1}{\mu_0}\left(\nabla\times\mathbf{A}\right) = \frac{Idl\sin\theta}{4\pi}\left(\frac{jk_0}{r} + \frac{1}{r^2}\right)e^{-jk_0 r}\mathbf{a}_\phi$$

Electric field

$$\begin{aligned}\mathbf{E} &= -j\omega\mathbf{A} + \frac{\nabla\nabla\cdot\mathrm{A}}{j\omega\mu_0\varepsilon_0}\\ &= \frac{jZ_0 Idl}{2\pi k_0}\cos\theta\left(\frac{jk_0}{r^2} + \frac{1}{r^3}\right)e^{-jk_0 r}\mathbf{a}_r - \frac{jZ_0 Idl}{4\pi k_0}\sin\theta\left(-\frac{k_0^2}{r} + \frac{jk_0}{r^2} + \frac{1}{r^3}\right)e^{-jk_0 r}\mathbf{a}_\theta\\ &= E_r\mathbf{a}_r + E_\theta\mathbf{a}_\theta\end{aligned}$$

Far-field equations

Can neglect terms of r^2 or higher

$$E_\theta = jZ_0 \frac{k_0 Idl e^{-jk_0 r}}{4\pi r} \sin\theta$$

$$H_\phi = j \frac{k_0 Idl e^{-jk_0 r}}{4\pi r} \sin\theta$$

$$E_r = 0$$

$$E_\phi = 0$$

$$H_r = 0$$

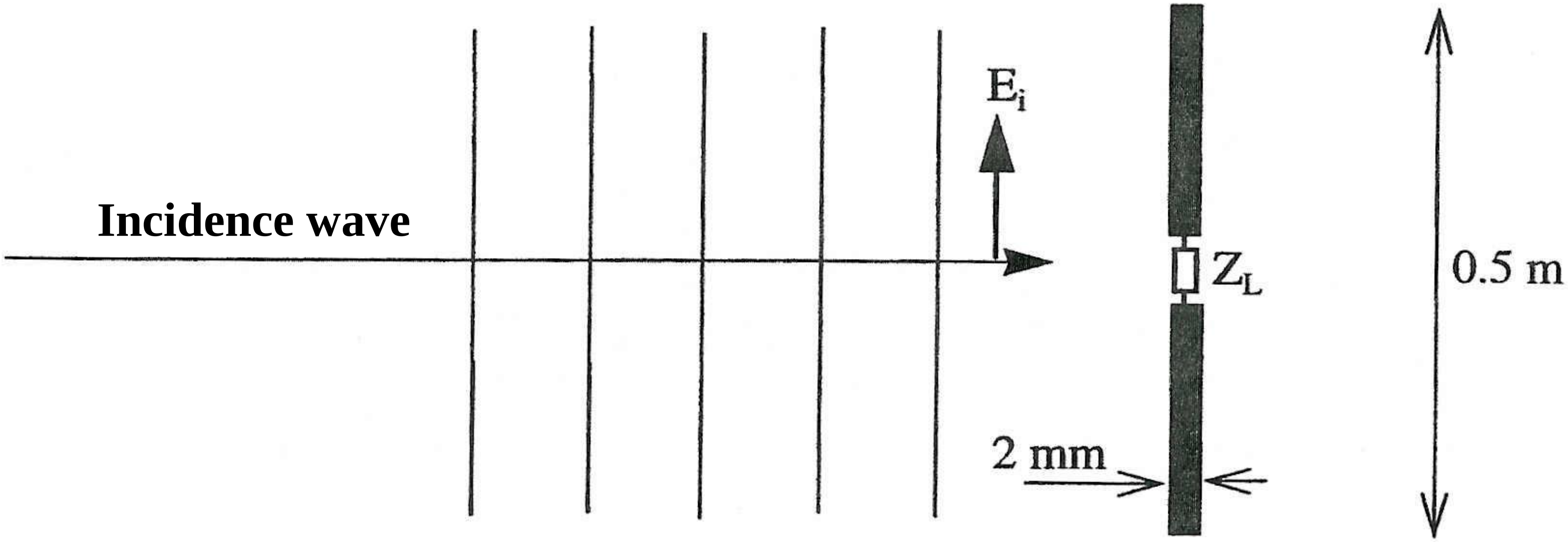
$$H_\theta = 0$$

- The radiated field has transverse components
- Ration $E_\theta/H_\phi = Z_0$: fields in phase and the wave impedance is $120\pi\ \Omega$
- The field is inversely proportional to r
- The fields re zero at $\theta = 0$ and π, but maximum at $\pi/2$; the x-y plane

Small antennas for mobile communication

- Small mobile phones require small antennas
- Radiated field strength is proportional with the integral of the current: larger current gives stronger field and the smaller the antenna is the larger the current must be to achieve the same field strength
- Large current is achieved if the antenna is in resonance with the frequency used, but with reduced bandwidth as a result also
- The focus is on the widely used dipole antenna
- The dipole can be realised as a straight electrical conducting wire with a feed gap in the middle, and also pipe or metal film on a dielectric card

Dipole sketched in an incident electric field

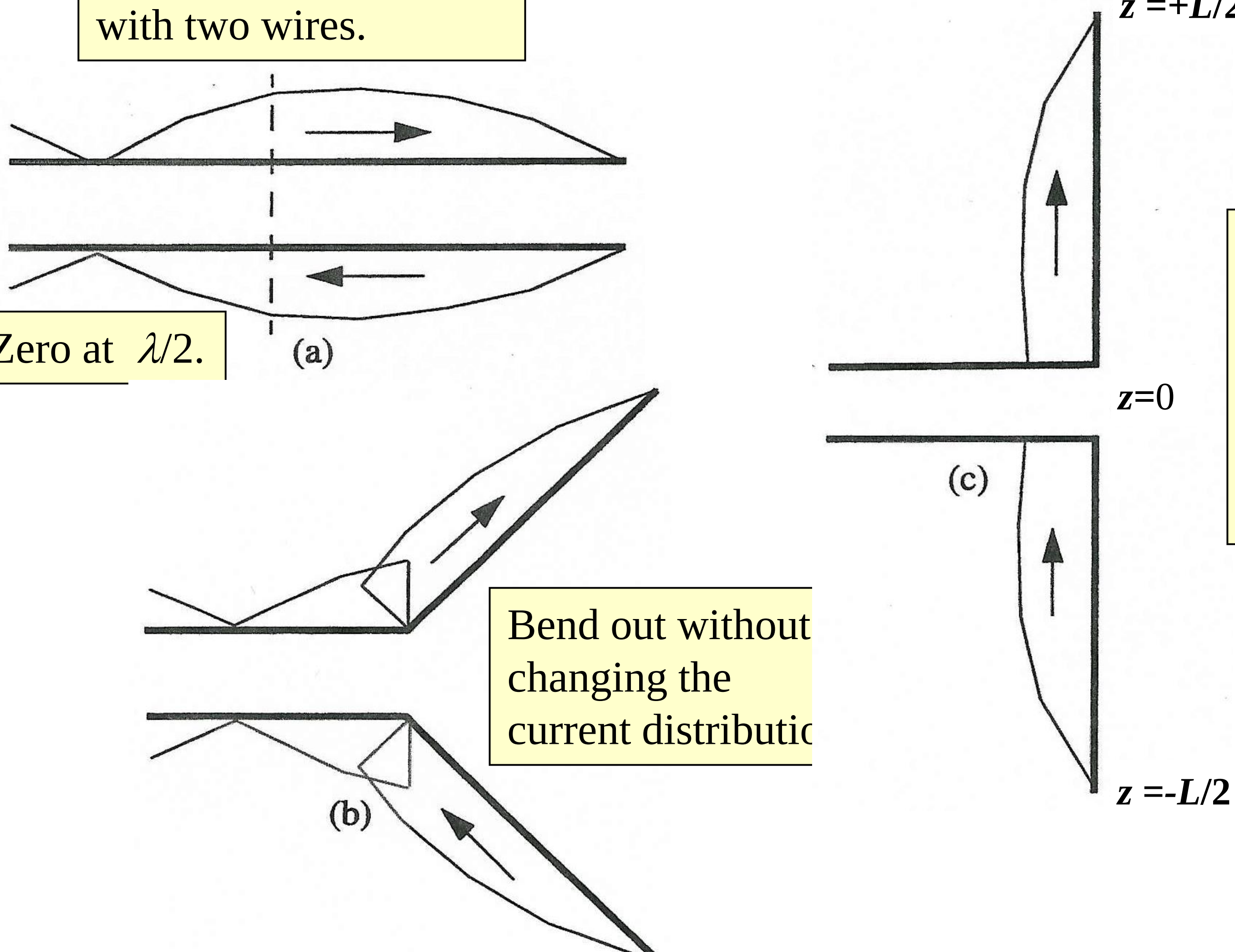


A parallel incident field felt of 1V/m will induce a current in the impedance or load Z_L. Boundary requirement is zero tangential field along the dipole arms. This will be reduced to a current I_{in} such that the voltage over Z_L becomes $V_{in}=-I_{in} Z_L$.

Simplified model for current distribution on a dipole

Open transmission line with two wires.

Zero at $\lambda/2$.



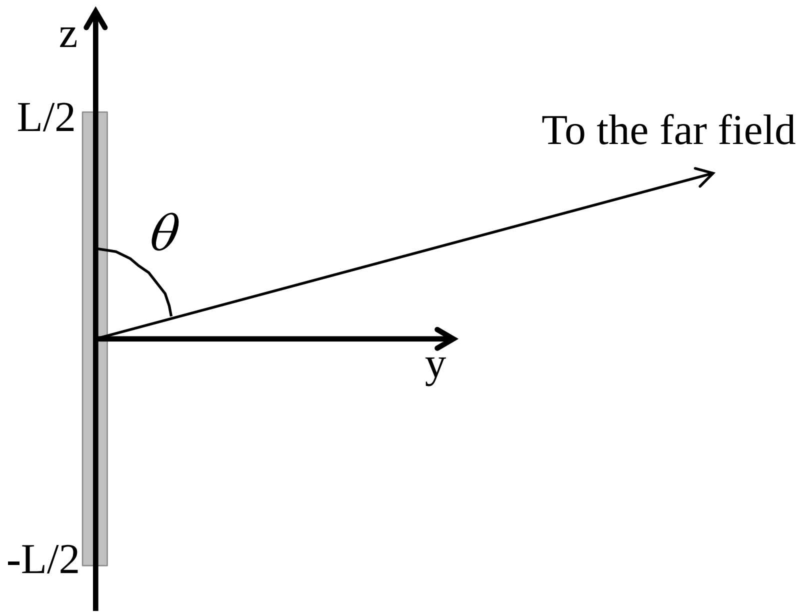
Bend out without changing the current distributio

The current now flows in the same direction on the dipole arms and it becomes an effective antenna.

Coordinate system for the dipole far field calculation

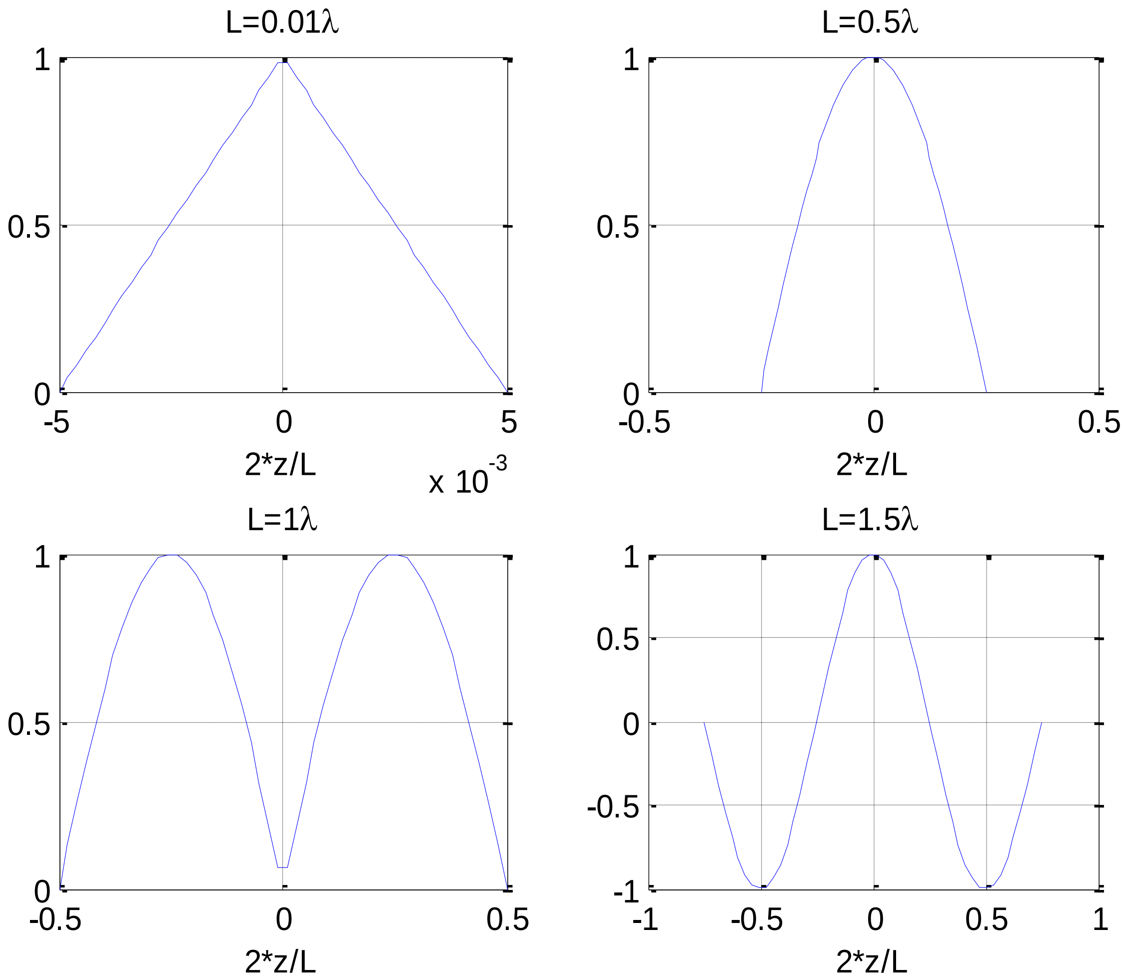
Current distribution:

$$I(z) = I(0)\sin\left(k\left(\frac{L}{2} - |z|\right)\right)$$



Use a coordinate system as shown and integrate along the dipole (a line source), i.e., along the z-axis.

Current distributions



Total E-field for dipole

$$E_\theta = jZ_0 \frac{k_0 I L e^{-jk_0 r}}{4\pi r} \sin\theta$$

$$H_\phi = j \frac{k_0 I L e^{-jk_0 r}}{4\pi r} \sin\theta$$

$E_r = 0$

$E_\phi = 0$

$H_r = 0$

$H_\theta = 0$

Small Hertzian dipole:

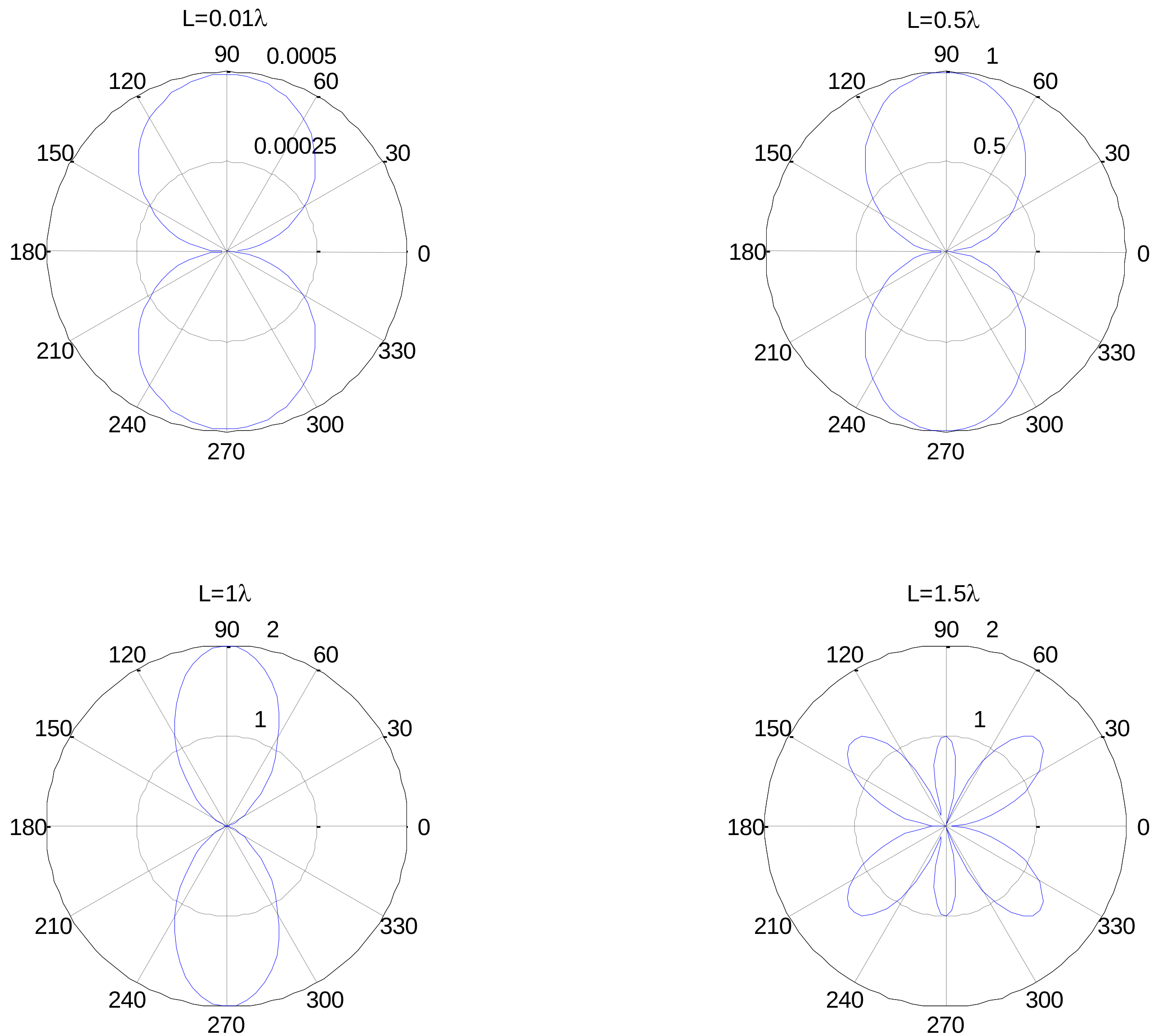
$$dE_\theta = jZ_0 \frac{kI(z) e^{-jk_0 r}}{4\pi r} e^{jkz\cos\theta} \sin\theta dL$$

where $e^{jkz\cos\theta}$ is the extra path length for element dL at position z

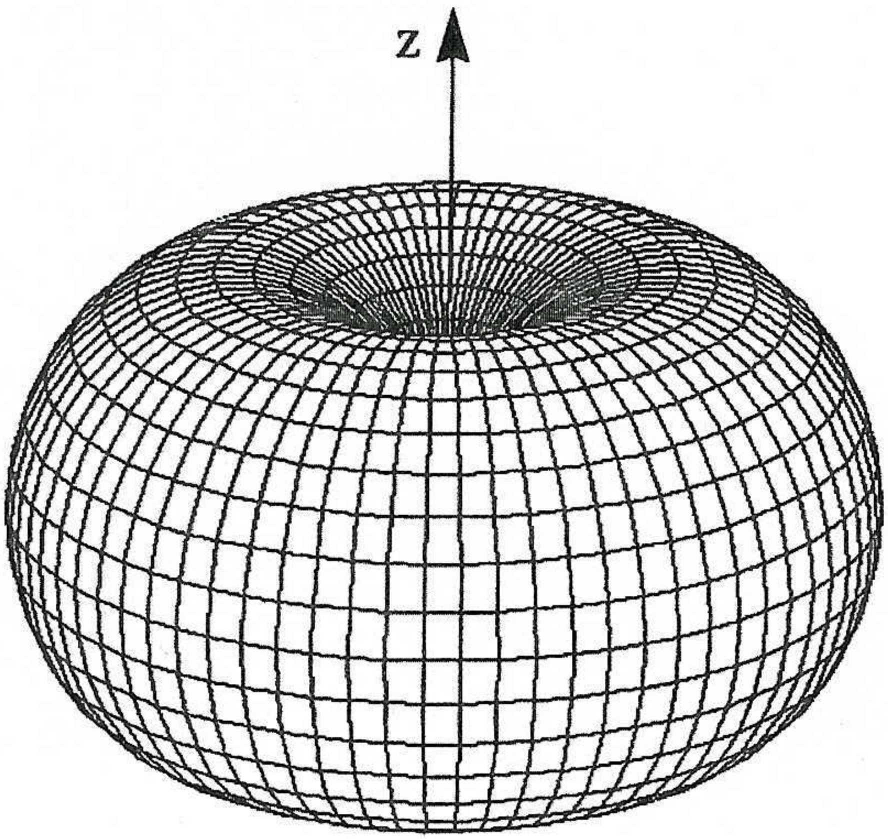
$$E_\theta = \int_{-L/2}^{L/2} dE_\theta = jZ_0 \frac{k e^{-jk_0 r}}{4\pi r} \sin\theta \int_{-L/2}^{L/2} I(z) e^{jkz\cos\theta} dz$$

$$E_\theta = \frac{jZ_0 I(0) k e^{-jk_0 r}}{4\pi r} \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta} \right]$$

Dipole radiation pattern



Radiation pattern for a half wave length dipole

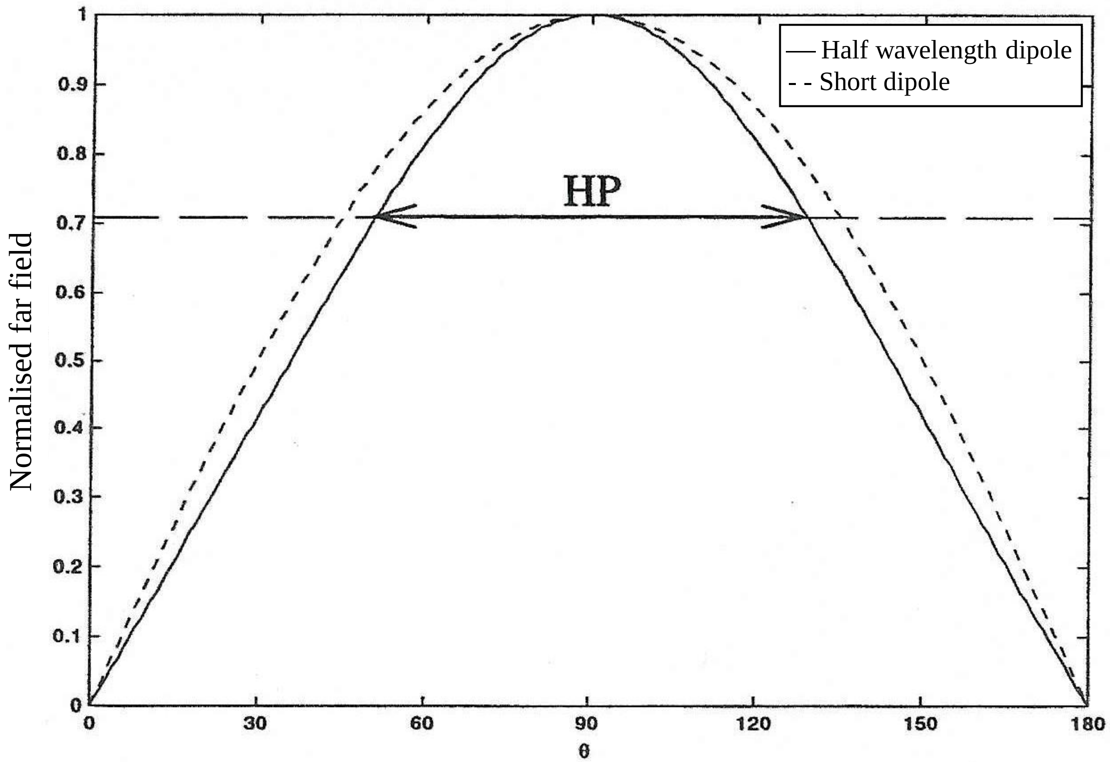


Doughnut formed with zero along the dipole axis and maximum at the perpendicular direction.

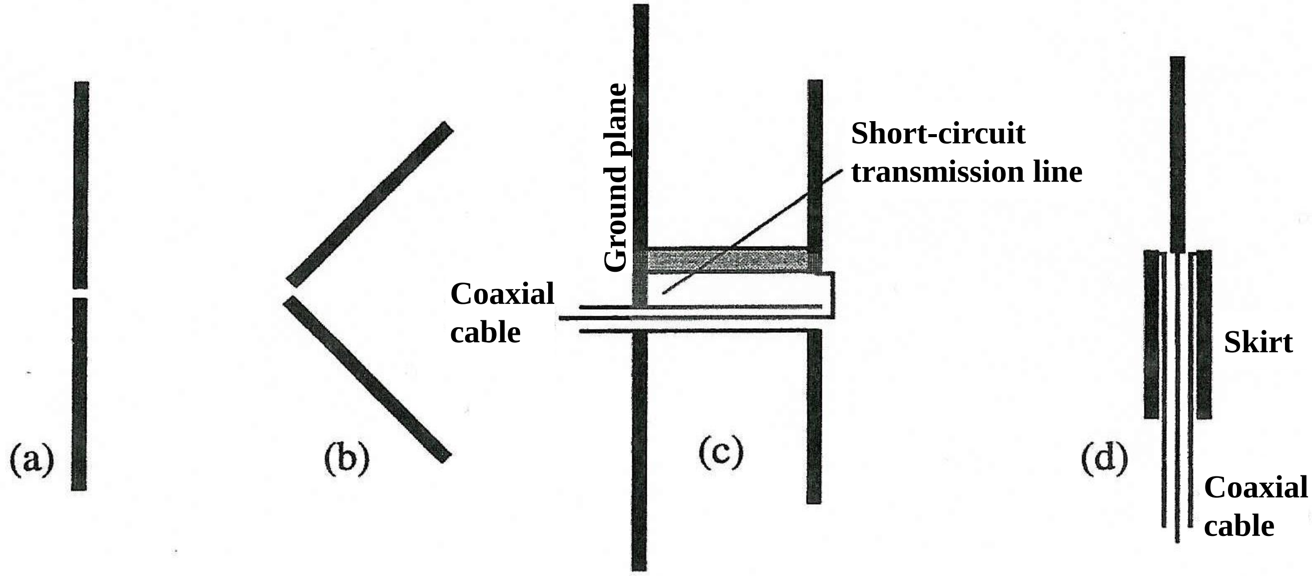
Maximum directivity D_{max}=1.64 or 2.15 dB.

Short dipole

The current distribution will become triangular on a very short dipole. However, the radiation will become very close for that derived from the half-wavelength dipole, but where the half-wavelength has a somewhat higher maximum directivity

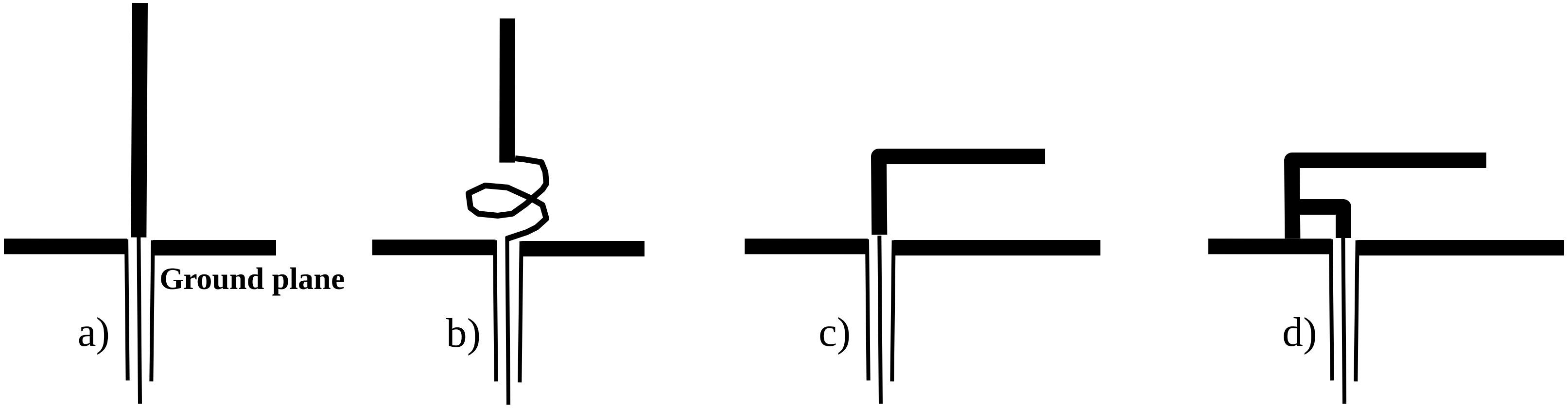


Various dipole antennas



a) Ideal dipole antenna
b) V-form to give stronger radiation in the direction the opening of the V and less backwards
c) Practical realisation taking mechanical fastening and feed with coaxial cable through the ground plane. On arm of the dipole is connected to the cable case, the other inner conductor and a pipe connected to the ground plane. The pipe and the cable case become a short-circuit transmission line in parallel to the antenna impedance. The result is a well matched dipole antenna that is shorter than a half wave length
d) Feed with coaxial cable from one end where one dipole arm is the extension of the inner conductor and the other a cylindrical skirt surrounding the cable and connected with the case of the cable

Monopole with some alternative designs



a) Idealist monopole whip antenna. Removing one branch form the dipole antenna and replacing with a ground plane it becomes a monopole antenna half the length that of the dipole antenna. It is easy to feed
b) Traditional mobile phone antenna. A monopole shorter than $\lambda/4$ gets unwanted capacitive input impedance. This is compensated by using a coil near the feed point
c) Inverted L-antenna. Close to the ground plane. Problem with small resonance
d) Inverted F-antenna. Increased input impedance and resonance

Small mobile communication antennas: conclusion

- Small antennas are needed in small equipment, such as mobile hand sets
- Small antennas work best when they are in resonance with the field
- Dipole antenna radiation pattern is symmetric in the plane of which the antenna itself is a normal
- The maximum gain is perpendicular to the dipole antenna