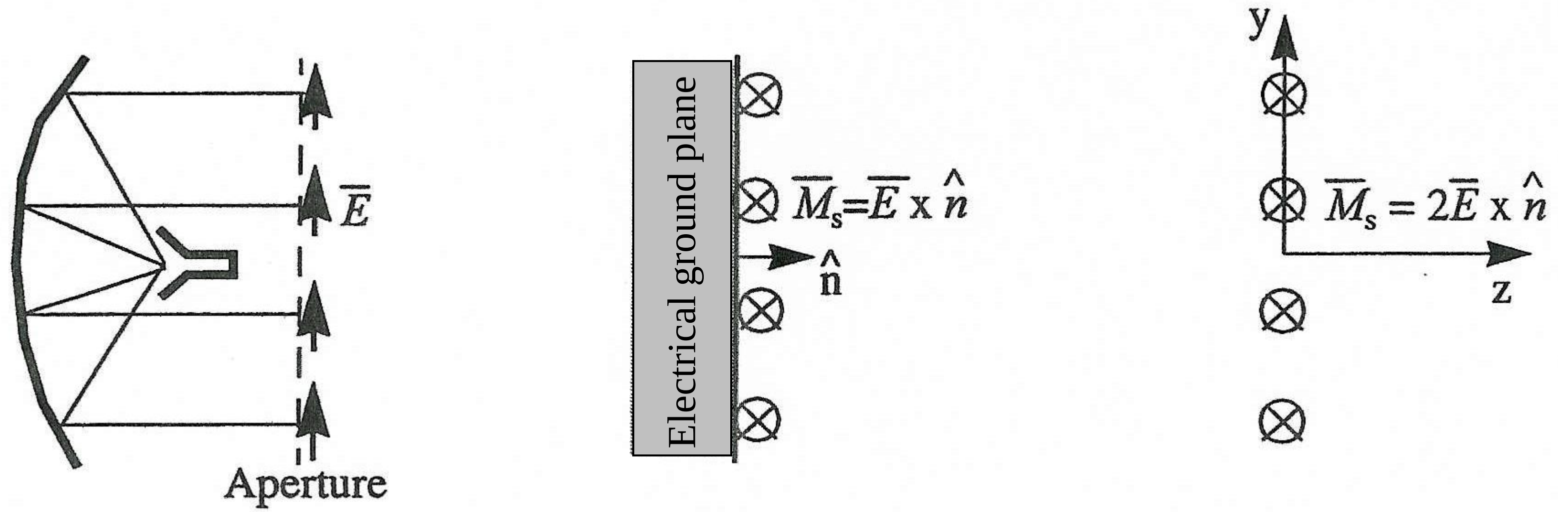
This part of reflector antennas is not in the book.

Radiation from apertures



Use ray tracing from feed antenna (e.g., horn) via main reflector to establish the field in a plane, the aperture plane. Use this field to find the far field.
The method is to use equivalent current sources on the aperture plane to easier find the far field, as illustrated in the three steps above.
The magnetic surface current becomes $\boldsymbol{M}_s = 2(\hat{x}E_x + \hat{y}E_y) \times \hat{z} = 2(-\hat{y}E_x + \hat{x}E_y)$ when the aperture is in the xy-plane.

The far field from a reflector antenna

This part of reflector antennas is not in the book.

Magnetic fields in the aperture *A* give radiation vector $\mathbf{L} = \int_{V'} \mathbf{M} e^{jk0r'\cos\psi} dv'$

$$\boldsymbol{L} = \int_A \boldsymbol{M}(\boldsymbol{r}') e^{jk_0 r'\cos\psi} dx'dy' = \int_A 2\left(-\hat{y}E_x + \hat{x}E_y\right) e^{jk_0 r'\cos\psi} dx'dy'$$

The far field components become for *z*'=0 and $r'\cos\psi = (x'\cos\phi + y'\sin\phi)\sin\theta$

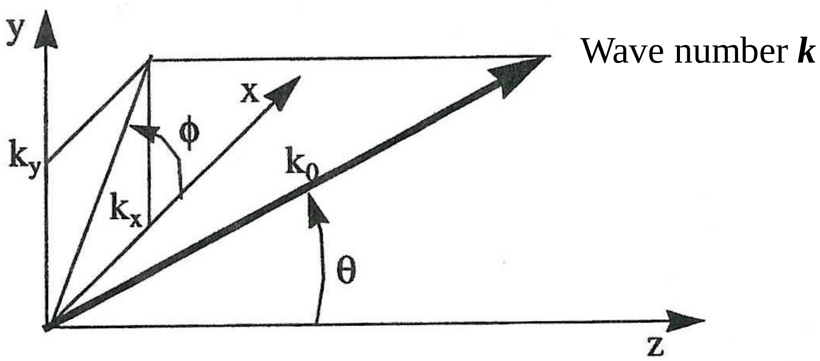
$$E_\theta = -j\frac{1}{2\lambda}\frac{e^{-jk_0 r}}{r} L_\phi = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(E_y \sin\phi + E_x \cos\phi\right) e^{jk_0(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)} dx'dy'$$

$$E_\phi = j\frac{1}{2\lambda}\frac{e^{-jk_0 r}}{r} L_\theta = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(E_y \cos\phi - E_x \sin\phi\right)\cos\theta\, e^{jk_0(x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)} dx'dy'$$

Far field expressed with the wave number *k*

This part of reflector antennas is not in the book.

Define the wave number $\boldsymbol{k}_0$ with he same length as k_0.



$$k_x = k_0 \sin\theta\cos\phi$$
$$k_y = k_0 \sin\theta\sin\phi$$

$$E_\theta(r,\theta,\phi) = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(E_x(x',y')\cos\phi + E_y(x',y')\sin\phi\right) e^{jk_x} e^{jk_y}\, dx'dy'$$

$$E_\phi(r,\theta,\phi) = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(-E_x(x',y')\cos\phi + E_y(x',y')\sin\phi\right)\cos\theta\, e^{jk_x} e^{jk_y}\, dx'dy'$$

Now the far field can be found using the aperture electrical field components

Radiation intensity and directivity

This part of reflector antennas is not in the book.

Radiated power is the power passing through the aperture, as for a plane wave

$$P_{rad} = \frac{1}{2Z_0} \int_A \left(\left| E_x(x', y') \right|^2 + \left| E_y(x', y') \right|^2 \right) dx' dy'$$

Referring to directivity for uniform illumination: The field has the same direction, amplitude and phase of the whole aperture.

Radiation intensity *U* in maximum direction is

$$U_{\max} = \frac{r^2}{2Z_0} |E|^2 = \frac{A^2 |E_0|^2}{2Z_0 \lambda^2}$$

where *E* chosen $E_x = 0$ and $E_y = E_0$ such that

$$E_\theta(r, 0, \pi/2) = E_\phi(r, 0, 0) = \frac{j}{\lambda} \frac{e^{-jk_0 r}}{r} \int_A E_0 e^{j0} e^{j0} dx' dy' = \frac{j}{\lambda} \frac{e^{-jk_0 r}}{r} E_0 A$$

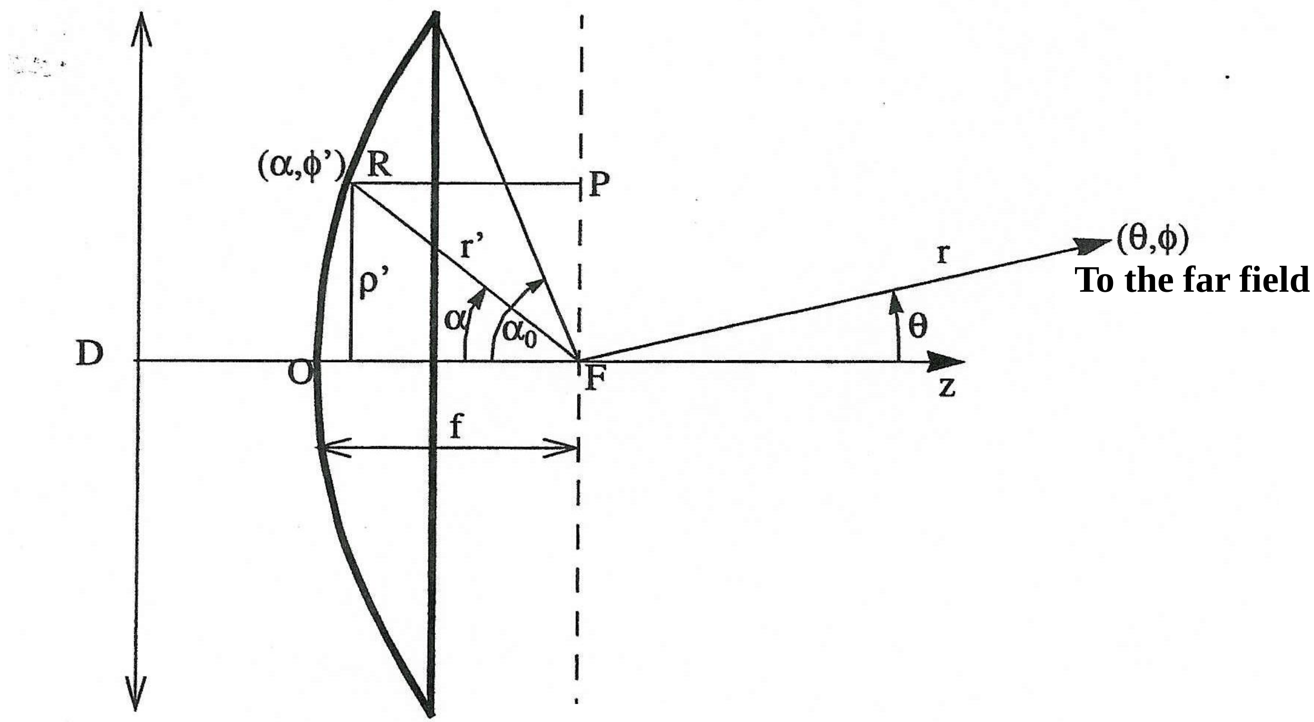
P_{rad} becomes $P_{rad} = \frac{1}{2Z_0} \int_A \left(\left| E_y \right|^2 \right) dx' dy' = \frac{|E_0|^2 A}{2Z_0}$ and directivity $D_{\max} = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi}{\lambda^2} A$

$G_{max} = D_{max}$ as there is no loss in the aperture. The effective area A_e and efficiency are:

$$A_{e,\max} = \frac{\lambda^2}{4\pi} G_{\max} = \frac{\lambda^2 4\pi}{4\pi\lambda^2} A = A \qquad \varepsilon_{ap} = \frac{A_{e,\max}}{A} = 1$$

This part of reflector antennas is not in the book.

Front fed parabolic antenna

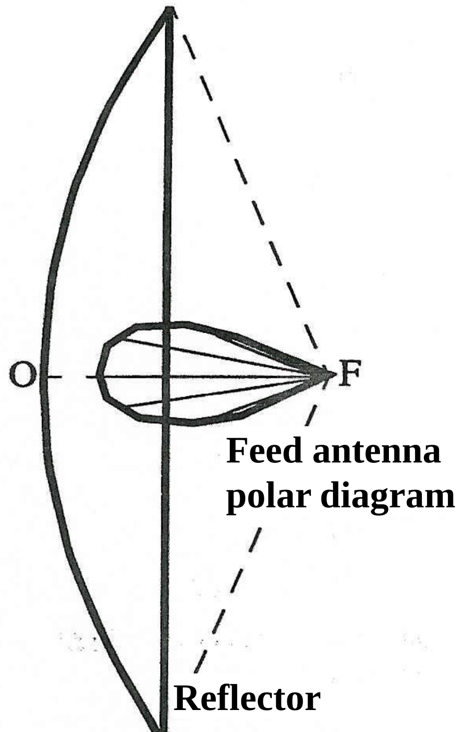


Aperture plane

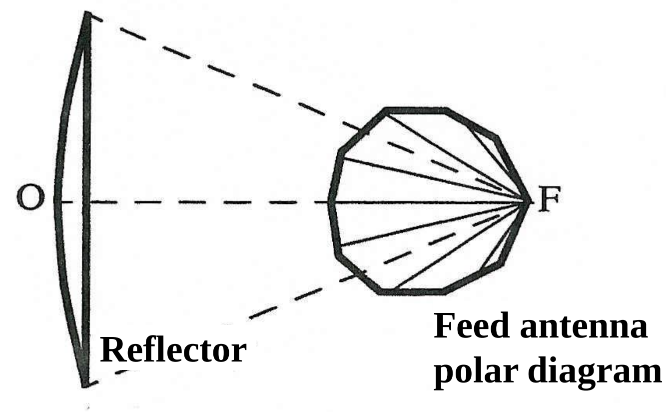
Antenna with diameter *D* and a number of angles and points defined.
The form is precisely determined by the focal distance *f* and diameter *D*.
The ratio *f*/*D* is decisive and normally between 0.3 and 1

Optimum relationship between the feed antenna and the main reflector

This part of reflector antennas is not in the book.



(a)



(b)

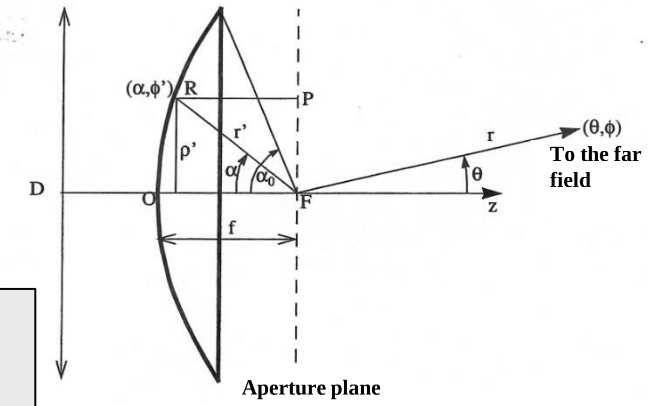
b) Lot of spill over.

a) Poorly deployed main reflector

Geometrical relations for a front fed parabolic antenna

This part of reflector antennas is not in the book.

Have to develop some geometrical relations for later use. Important to express the reflectors distance ρ' from the axis and distance r' from the focal point using the angle α.



All rays from F via R to P have equal lengths. This gives

$$\mathrm{FR}+\mathrm{RP}=\mathrm{FO}+\mathrm{OF}$$

$$r'+r'\cos\alpha = f+f$$

$$r'=\frac{2f}{1+\cos(\alpha)}=\frac{f}{\cos^2\left(\frac{\alpha}{2}\right)}$$

Use that

$$1+\cos\alpha = 2\cos^2\left(\frac{\alpha}{2}\right)$$

Then, from the figure:

$$\rho'=r'\sin(\alpha)$$

Insert for r' and use $\sin\alpha = 2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)$

Then we get $\rho'=2f\tan\left(\frac{\alpha}{2}\right)$

The outermost ray, $\rho'=D/2$: $\frac{D}{2}=2f\tan\left(\frac{\alpha_0}{2}\right)$ results in $\frac{f}{D}=\frac{1}{4\tan\left(\frac{\alpha_0}{2}\right)}$

Unique relation between f/D and α_0.

This part of reflector antennas is not in the book.

Optimum antenna gain

Use a simplified radiation pattern from the feed antenna. Calculate first the total radiated power and then the effective power from the reflector. Have now introduced spill over efficiency besides the aperture efficiency and the gain of the reflector antenna can be written

$$G = \frac{4\pi}{\lambda^2} A\varepsilon_s \varepsilon_t$$

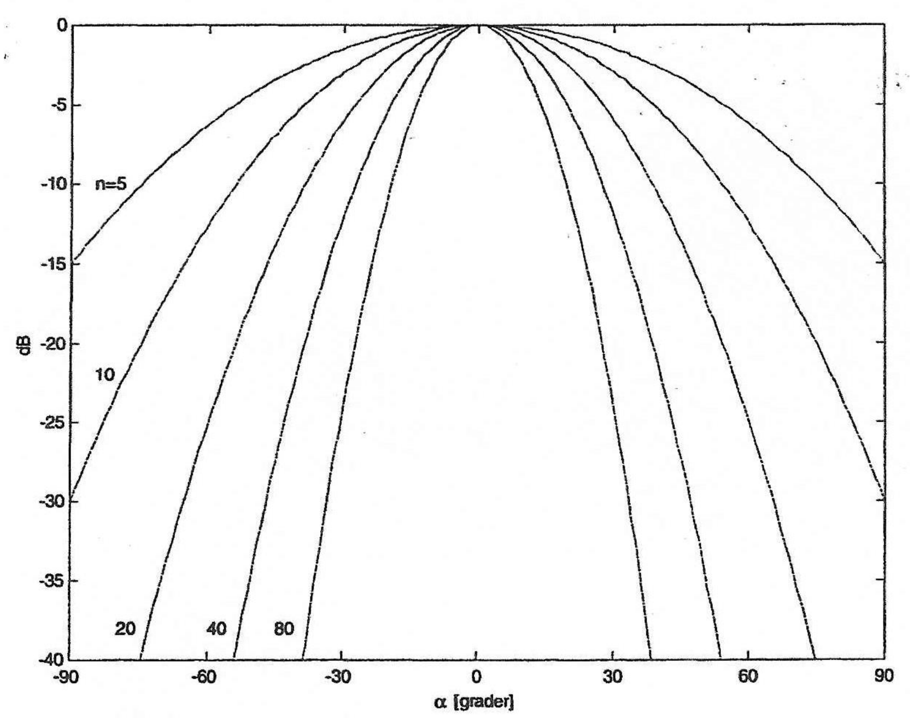
where $A=\pi(D/2)^2$ aperture area. The goal is to maximise the product $\varepsilon_s\varepsilon_t$.

Let the feed antenna radiate a field proportional with $\cos^n(\alpha/2)$ where n is positive, but not necessarily an integer.
Simplify (polarisation not considered) such that the feed antenna radiates a scalar field E with the radiation intensity U

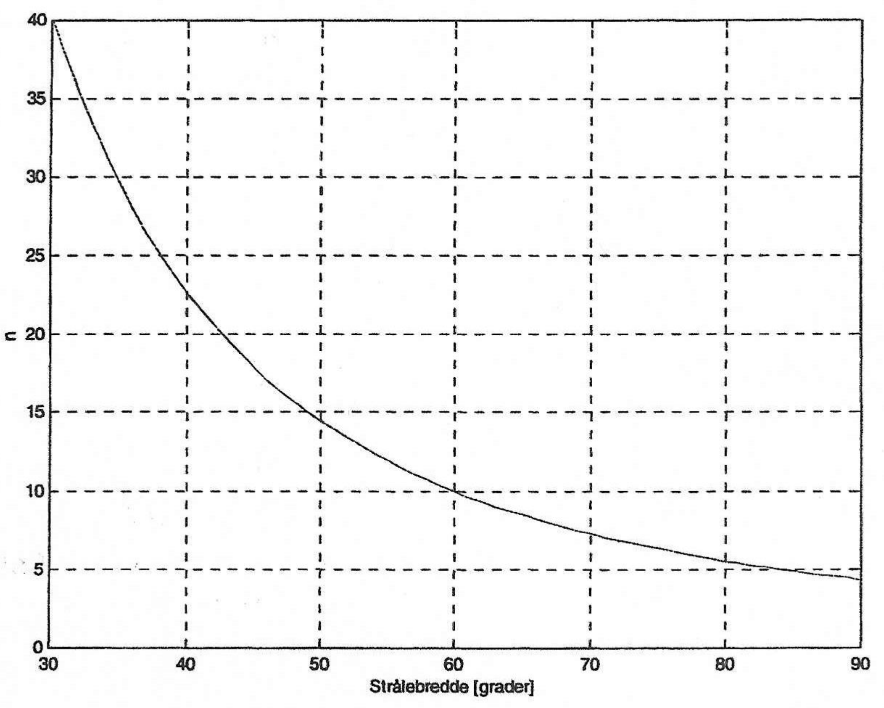
$$E(r',\alpha,\phi') = F(\alpha)\frac{e^{-jk_0 r'}}{r'} = \cos^n\left(\frac{\alpha}{2}\right)\frac{e^{-jk_0 r'}}{r'} \quad U(\alpha,\phi') = \frac{1}{2Z_0}\left|U(\alpha)\right|^2 = \frac{1}{2Z_0}\cos^{2n}\left(\frac{\alpha}{2}\right)$$

Feed antenna radiation pattern and beam width

This part of reflector antennas is not in the book.



Radiation pattern



n as function of beam width

Total radiated power and effective power from the reflector

This part of reflector antennas is not in the book.

The total power from the feed antenna is

$$P_{rad}=\int_0^{2\pi}\int_0^{\pi}U(\alpha,\phi')\sin\alpha\, d\alpha\, d\phi'=\frac{2\pi}{2Z_0}\int_0^{2\pi}\cos^{2n}\left(\frac{\alpha}{2}\right)2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)d\alpha$$

Solve by substituting $u=\cos(\alpha/2)$, i.e., $du=-\sin(\alpha/2)d\alpha/2$

$$P_{rad}=-\frac{2\pi}{2Z_0}\int_1^0 u^{2n}2u2du=-\frac{4\pi}{Z_0}\int_1^0 u^{2n+1}du=\frac{4\pi}{Z_0}\frac{1}{2n+2}\left(u_{2n+2}\right)\Big|_0^1=\frac{2\pi}{\eta_0(n+1)}$$

Note that only rays within α_0 reach the reflector

$$P_{rad}=\int_0^{2\pi}\int_0^{\alpha_0}F(\alpha,\phi')\sin\alpha\, d\alpha\, d\phi'=\frac{2\pi}{Z_0(n+1)}\left(1-\cos^{2(n+1)}\left(\frac{\alpha_0}{2}\right)\right)$$

Spill over efficiency therefore becomes

$$\varepsilon_s=\frac{P_{refl}}{P_{rad}}=\left(1-\cos^{2(n+1)}\left(\frac{\alpha_0}{2}\right)\right)$$

This part of reflector antennas is not in the book.

The reflector's directivity (1)

Establish the reflector's directivity by determining the far field at *q*=0, given by the aperture field found earlier and the fact that θ=0 for k_x=k_y=0. Neglects polarisation effects. The scalar far field therefore becomes

$$E_{far} = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A E_{ap}(x',y')dx'dy'$$

where E_{ap} is the aperture field. Perform the integration in cylindrical coordinates since the feed antenna radiation pattern is circular symmetric and the aperture is circular

$$E_{far} = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A E_{ap}(\rho')\rho' d\rho' d\phi' = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}2\pi\int_0^{D/2} E_{ap}(\rho')\rho' d\rho'$$

Convert to an integral over α using the geometrical relations (form earlier)

$$d\rho' = 2f\frac{1}{\cos^2\left(\frac{\alpha}{2}\right)}\frac{1}{2}d\alpha = \frac{f}{\cos^2\left(\frac{\alpha}{2}\right)}d\alpha \quad \Rightarrow \quad \rho' d\rho' = 2f\tan\left(\frac{\alpha}{2}\right)\frac{f}{\cos^2\left(\frac{\alpha}{2}\right)}d\alpha$$

The reflector's directivity (2)

This part of reflector antennas is not in the book.

The aperture field's is given by the propagation to the point R, i.e.:

$$E(r',\alpha,\phi') = \cos^n\left(\frac{\alpha}{2}\right)\frac{1}{r'} = \cos^n\left(\frac{\alpha}{2}\right)\frac{1}{\dfrac{f}{\cos^2\left(\dfrac{\alpha}{2}\right)}} = \frac{\cos^{n+2}\left(\dfrac{\alpha}{2}\right)}{f}$$

where the phase $e^{-jk_0 r'}$ has been suppressed since the parabolic reflector exactly gives a field in phase. Use this expression in E_{far}

$$E_{far} = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}2\pi\int_0^{D/2} E_{ap}(\rho')\rho' d\rho' = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}2\pi\int_0^{\alpha_0}\frac{\cos^{n+2}\left(\dfrac{\alpha}{2}\right)}{f}2f\tan\left(\frac{\alpha}{2}\right)\frac{f}{\cos^2\left(\dfrac{\alpha}{2}\right)}d\alpha$$

$$\left|E_{far}\right| = \frac{4\pi f}{\lambda r}\int_0^{\alpha_0}\cos^{n-1}\left(\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right)d\alpha = -\frac{4\pi f}{\lambda r}\int_1^{\cos(\alpha_0/2)} u^{n-1}2du = \frac{4\pi f}{\lambda r}\left.\frac{u^n}{n}\right|_{\cos(\alpha_0/2)}^{1}$$

$$= \frac{8\pi f}{\lambda r}\frac{1}{n}\left(1-\cos^n\left(\frac{\alpha_0}{2}\right)\right)$$

This part of reflector antennas is not in the book.

The reflector's directivity (3)

The radiation intensity *U* becomes

$$U_{far} = \frac{1}{2Z_0} r^2 \left| E_{far} \right|^2 = \frac{32\pi^2 f^2}{Z_0 \lambda^2} \frac{1}{n^2} \left(1 - \cos^n \left(\frac{\alpha_0}{2} \right) \right)^2$$

hence the directivity

$$D_{refl} = \frac{4\pi U_{far}}{P_{refl}} = \frac{4\pi \dfrac{32\pi^2 f^2}{Z_0 \lambda^2} \dfrac{1}{n^2} \left(1 - \cos^n \left(\dfrac{\alpha_0}{2} \right) \right)^2}{\dfrac{2\pi}{Z_0 (n+1)} \left(1 - \cos^{2(n+1)} \left(\dfrac{\alpha_0}{2} \right) \right)}$$

$$= \frac{64\pi^2 f^2 (n+1) \left(1 - \cos^n \left(\dfrac{\alpha_0}{2} \right) \right)^2}{\lambda^2 n^2 \left(1 - \cos^{2(n+1)} \left(\dfrac{\alpha_0}{2} \right) \right)}$$

Spill over, aperture, and total efficiency

This part of reflector antennas is not in the book.

Maximum $D_{\max}$ for uniform illumination:

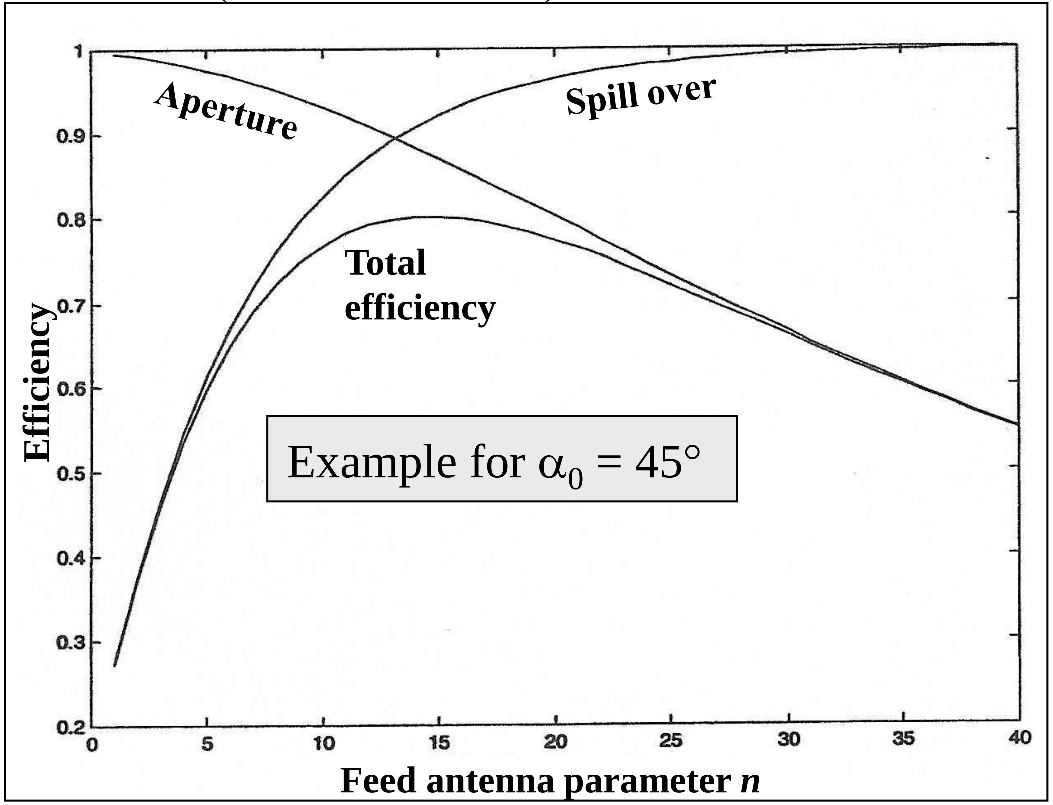
$$D_{\max} = \frac{4\pi}{\lambda^2} A = \frac{4\pi}{\lambda^2}\pi\left(\frac{D}{2}\right)^2 = \frac{\pi^2 D^2}{\lambda^2} = \frac{16\pi^2 f^2 \tan\left(\frac{\alpha_0}{2}\right)}{\lambda^2}$$

Aperture efficiency:

$$\varepsilon_t = \frac{D_{refl}}{D_{\max}} = \frac{4(n+1)\left(1-\cos^n\left(\frac{\alpha_0}{2}\right)\right)^2}{n^2\left(1-\cos^{2(n+1)}\left(\frac{\alpha_0}{2}\right)\right)\tan\left(\frac{\alpha_0}{2}\right)}$$

Total efficiency:

$$\varepsilon_s\varepsilon_t = \frac{4(n+1)\left(1-\cos^n\left(\frac{\alpha_0}{2}\right)\right)^2}{n^2\tan\left(\frac{\alpha_0}{2}\right)}$$



Radiation pattern

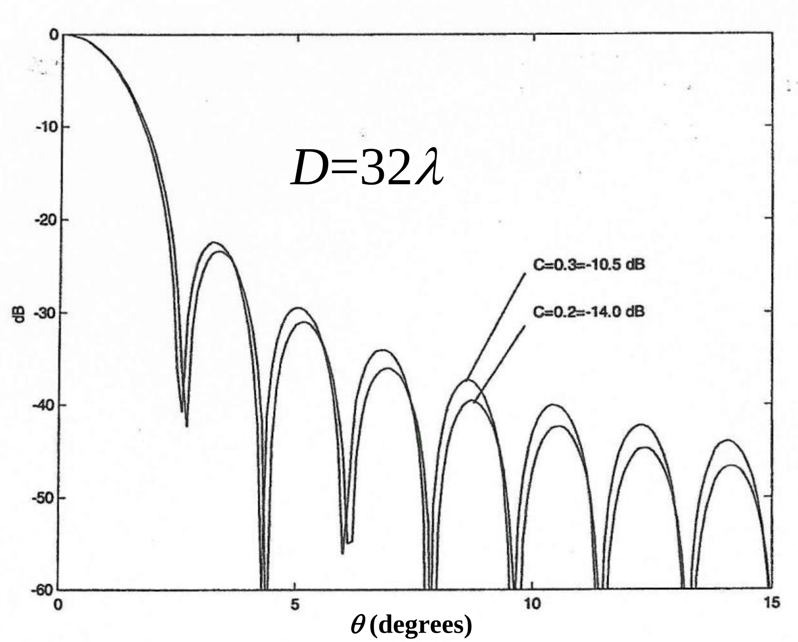
This part of reflector antennas is not in the book.

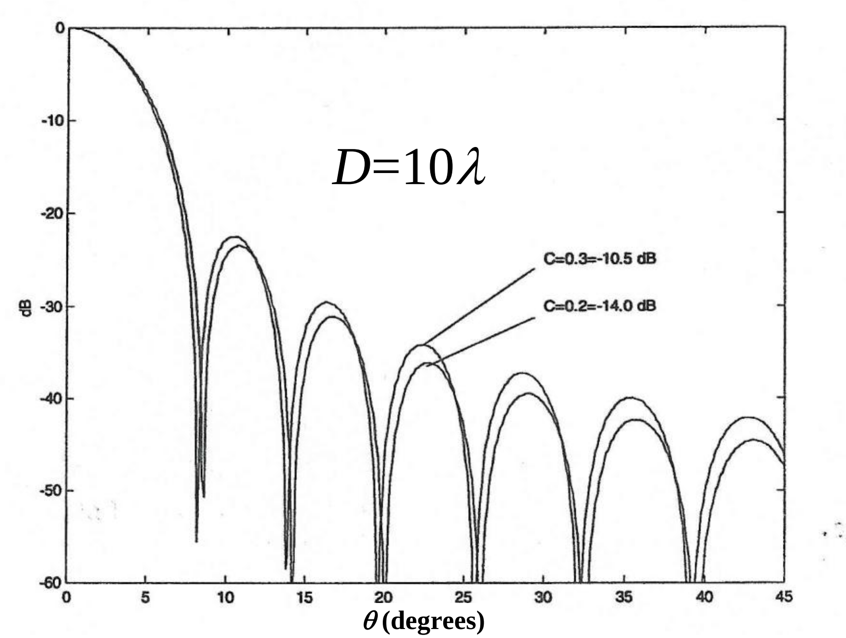
Aperture distribution: second order polygon: $E_y = C + (1-C)(1-(2r'/D)^2)$.

$$E\left(r,\theta,\frac{\pi}{2}\right) = \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(C + (1-C)\left(1-\left(\frac{2\rho'}{D}\right)^2\right)\right) e^{jk_0 y'\sin\theta} dx'\,dy'$$

$$= \frac{j}{\lambda}\frac{e^{-jk_0 r}}{r}\int_A \left(C + (1-C)\left(1-\frac{4(x'^2+y'^2)}{D^2}\right)\right) e^{jk_0 y'\sin\theta} dx'\,dy'$$

Solved numerically for different *C* and *D*.





Rule of thumb: BW=66°/(D/λ)