A criterion Proposed for Inductive Coupling and Magnetic Resonance Coupling in Wireless Power Transfer System

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***Abstract* — A criterion of inductive coupling and magnetic resonance coupling in wireless power transfer system is proposed. There are two methods that are widely used in wireless power transfer system; Inductive coupling and Magnetic resonance coupling. However, these two terms have been ambiguously used without clear definitions. In this paper, we present an analytic formula which is able to define two couplings separately using Coupled-mode theory.**

Index Terms — Coupled-mode theory, Inductive coupling, Magnetic resonance coupling, Critical coupling coefficient.

I. Introduction

In recent years, there has been increasing interest in research and development of wireless power transfer system (WPTS) using many theoretical analysis and experimental demonstration [1].

The inductive coupling (IC) are one of the coupling method for WPTS. Instead of relying on propagating electromagnetic waves, IC techniques operate at distance less than a wavelength of the transmitted signal [2]. The mutual coupling of IC system is generally weak, so it is comprised of two magnetically coupled electrical systems driven by a high frequency switching power supply [3].

The magnetic resonance coupling (MRC) in WPTS can transfer power more efficiently at longer distances than inductively coupled WPTS. An impedance matching generates the resonant at each coil, and at the resonant peaks, the WPTS get the maximum power transfer efficiency. In the case of tight magnetic coupling, the resonant peak of the input impedance is divided into several peaks due to increased mutual inductance between coupled coils [4].

These two coupling methods are important technology in WPTS. However, when we define the type of coupling, these two terms are used indiscriminately. To analyze WPT system accurately, it is important to clearly define which type of coupling is used.

In this paper, we extend prior theoretical analysis of WPTS to define the two terms; IC and MRC using coupled mode theory. The key term, critical coupling coefficient is proposed. We took consideration into two sources of WPTS; power and voltage source. The Coupled-mode theory is analyzed and compared with transient circuit analysis.

II. The Coupling of Two Resonator Modes

A. The Analysis of Coupled-mode theory

The structure of proposed WPTS is illustrated in Fig. 1. The transmitting and receiving antenna consist of multi-turn spiral coil. Each coil acts as a high-Q LCR tank resonator. Two coils share a mutual inductance which is a function of the geometry of the coils and the distance between them. These two coupled oscillators transfer energy generated from source of one resonator to Load of the other resonator.

The simple formalism for the time evolution of a resonant mode is particularly useful for the description of coupling between two resonant mode. Consider the equation of motion of the amplitudes a_1 and a_2 of the modes of two coupled lossy resonator at natural frequencies ω_1 and ω_2:

$$\begin{aligned} \frac{da_1}{dt} &= \left(j\omega_1 - \frac{1}{\tau_1} - \frac{1}{\tau_{ext_1}} \right) a_1 + \kappa_{12} a_2 \\ \frac{da_2}{dt} &= \left(j\omega_2 - \frac{1}{\tau_2} - \frac{1}{\tau_{ext_2}} \right) a_2 + \kappa_{21} a_1 \end{aligned} \tag{1}$$

where $1/\tau$ is the decay rate due to not only internal loss but also escapes into the waveguide or outside waveguide. These are proportional to Q factors. κ_{12}, κ_{21} are the coupling coefficients. a(t) has the dependence $\exp(j\omega_0 t)$ and is so normalized that

$$|a|^2 = \frac{C}{2}|V|^2 = W \tag{2}$$

where W is the energy in the circuit [6]. Using the general solution of Eq. (1) and the field-amplitude of reach resonator can be obtained

$$a_1(t) = \left[a_1(0)\cos\Omega t + \left(\frac{\kappa_{12} a_2(0) + a_1(0)\left(\sqrt{\Omega^2 - \kappa_{12}^2}\right)}{\Omega} \right) \sin\Omega t \right] \cdot e^{\frac{j(\omega_1+\omega_2) - \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_{ext_1}} + \frac{1}{\tau_{ext_2}} \right)}{2} t}$$

$$a_2(t)=\left[a_2(0)\cos\Omega t+\left(\frac{\kappa_{21}a_1(0)-a_2(0)\left(\sqrt{\Omega^2-\kappa_{12}^2}\right)}{\Omega}\right)\sin\Omega t\right]\cdot e^{\frac{j(\omega_1+\omega_2)-\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_{ext_1}}+\frac{1}{\tau_{ext_2}}\right)}{2}t}$$

$$\text{where,}\qquad \Omega=\sqrt{\left(\frac{j(\omega_1-\omega_2)-\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}+\frac{1}{\tau_{ext_1}}-\frac{1}{\tau_{ext_2}}\right)}{2}\right)^2+\kappa_{12}^2} \tag{3}$$

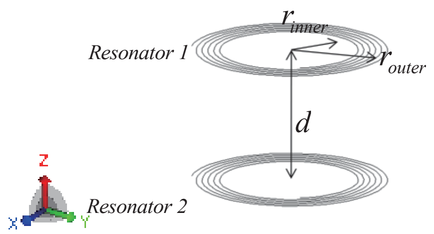


Fig. 1. Structure of the proposed WPTS all in the unit of m.

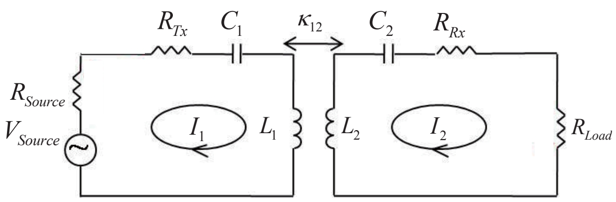


Fig. 2. Equivalent circuit model of WPTS structure

B. The Analysis of Transient circuit theory

Consider KVL in the equivalent short-circuit of the two identical coupled resonators [5].

$$\begin{aligned}&\frac{1}{C}\int_0^t I_1(t)dt+(R_1+R_s)I_1(t)+L_1\frac{dI_1(t)}{dt}+M\frac{dI_2(t)}{dt}=0\\&\frac{1}{C}\int_0^t I_2(t)dt+(R_2+R_L)I_2(t)+L_2\frac{dI_2(t)}{dt}+M\frac{dI_1(t)}{dt}=0\end{aligned} \tag{4}$$

Characteristic equation in S-domain can be obtained using Laplace transform.

$$\left(s^2+\left(\frac{\omega}{Q}+\frac{\omega}{Q_{ext}}\right)s+\omega^2\right)^2=k_{12}^2s^4 \tag{5}$$

The current of each resonator can be obtained using basis of Eq. (5) where Q_{ext} is the external Q of the resonator. The total energy in the resonator is the sum of electric energy in inductor and magnetic energy in capacitor.

$$\begin{aligned}E_1(t)&=\frac{1}{2}L_1|i_1(t)|^2+\frac{1}{2}C_1|v_1(t)|^2\\E_2(t)&=\frac{1}{2}L_2|i_2(t)|^2+\frac{1}{2}C_2|v_2(t)|^2\end{aligned} \tag{7}$$

C. Comparison of Coupled-mode and Transient circuit theory

Fig. 3,4, shows the energy of each resonator as function of time analyzed by Coupled-mode theory and transient circuit theory ($R_{source}=R_{Load}=0\ \Omega$).

As shown in Fig. 3, in the case of weak-coupling and high Q, the energy flows of each theory match well. However, as in Fig. 4, in the case of strong-coupling and low-Q, the energy flow of transient circuit theory has shorter period in that of Coupled-mode theory. And also, transient circuit theory shows more accurate energy flows. This is because the roots of characteristic equations of coupled second-order differential equations of Eq. (1), (4) are different. The four roots of characteristic equations in S-domain analyzed by transient circuit theory are

$$s_{1,2}=\frac{-\omega_0\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)}{2(1+k_{12})}\pm j\frac{\omega_0\sqrt{4(1+k_{12})-\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)^2}}{2(1+k_{12})},\ s_{3,4}=\frac{-\omega_0\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)}{2(1-k_{12})}\pm j\frac{\omega_0\sqrt{4(1-k_{12})-\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)^2}}{2(1-k_{12})} \tag{8}$$

and those analyzed by Coupled-mode theory are as follows [6]. Parameters are from [5].

$$s=\frac{-\omega_0\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)}{2}\pm j\frac{\omega_0 k_{12}}{2} \tag{9}$$

The roots match well in the case of High Q, Q_{ext} and k_{12} is nearly equal to zero. Even though the energy flows as function of time are more accurate in the case of transient circuit theory analysis, the equations are too complicated and can be applied when two resonators are exactly identical. So in this paper, Coupled-mode theory analysis is chosen for further research.

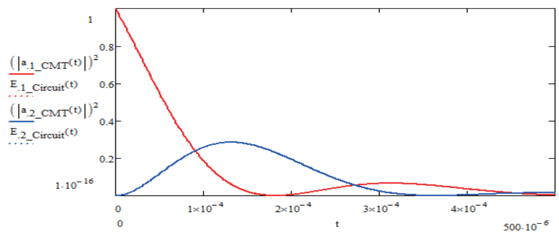
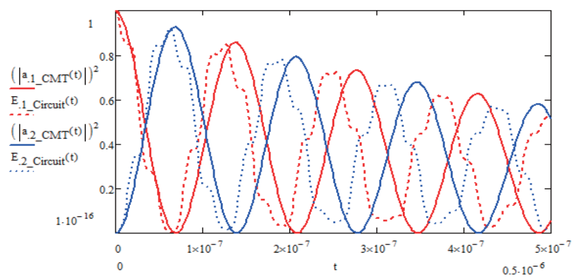


Fig. 3. Normalized energy flows analyzed by Coupled-mode theory and transient circuit theory in each resonator as function of time in



the case of Weak-coupling and High-Q

Fig. 4. Normalized energy flows analyzed by Coupled-mode theory and transient circuit theory in each resonator as function of time in the case of strong-coupling and Low-Q

III. Derivations of Critical Coupling Coefficient

The term $k_{critical}$ (i.e. critical coupling coefficient) is introduced to discriminate two couplings. Two resonators transmit and receive energy from one resonator to the other continuously. In this paper, a criterion is introduced by how much of the energy is received back after the first resonator once transmits energy. If the received energy of the first resonator after one period (1T) is greater than e^{-3} times of the initially transmitted energy (i.e., $|a_1(1\mathrm{T})|^2 > |a_1(0)|^2 \cdot e^{-3}$), the WPTS system is considered to be using magnetic resonance coupling. The reason of e^{-3} factor is based on the critical coupling between two resonators. However, If the received energy after one period is less than e^{-3} times of the first value (i.e., $|a_1(1\mathrm{T})|^2 < |a_1(0)|^2 \cdot e^{-3}$), the WPTS system is under the condition of inductive coupling. The $k_{critical}$ can be obtained when the received energy after one period is as same as e^{-3} times of the initial energy (i.e., $|a_1(1\mathrm{T})|^2 = |a_1(0)|^2 \cdot e^{-3}$). Using Eq. (3), value of one period can be derived.

$$k_{critical} = \frac{\omega_0\left(\frac{1}{Q_1}+\frac{1}{Q_2}+\frac{1}{Q_{ext_1}}+\frac{1}{Q_{ext_2}}\right)\cdot\pi}{3} \tag{10}$$

$$1\mathrm{T}(1\ \text{period}) = \frac{\pi}{\sqrt{\left(\frac{j(\omega_1-\omega_2)-\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}+\frac{1}{\tau_{ext_1}}-\frac{1}{\tau_{ext_2}}\right)}{2}\right)^2+\kappa_{12}^2}} \tag{11}$$

where $Q_{1,2}$ and Q_{ext} represents unloaded and loaded quality factor of each resonator. In other words, if the coupling coefficient k_{12} of the system is less than $k_{critical}$, the system is considered using inductive coupling. Otherwise, the system is regarded as using magnetic resonance coupling.

TABLE I
Energy of First Resonator Using Power Source

k_{12}	Energy: $\|a_1(1\mathrm{T})\|^2$	Energy: $\|a_1(0)\|^2 \cdot e^{-3}$	Type of coupling
0.05	$4.5\cdot 10^{-4}$	0.05	Inductive
0.128	0.049		Critical
0.5	0.46		Magnetic resonance

TABLE II
Energy of First Resonator Using Voltage Source

k_{12}	Energy: $\|a_1(1\mathrm{T})\|^2$	Energy: $\|a_1(0)\|^2 \cdot e^{-3}$	Type of coupling
0.05	0.013	0.05	Inductive
0.0645	0.05		Critical
0.5	0.67		Magnetic resonance

IV. Simulation Results and Discussion

To validate the theory, the size of resonators in Fig. 1 are designed to r_{inner}=0.2 m, r_{outer}=0.3m. The resonant frequency is 10.03 MHz. In the case of power source, the source and load resistance of the system is 50 Ω and in the voltage system, only the load resistance is considered. When the power source is used, the energies of the first resonator of each period satisfy the condition proposed in previous section as shown in Table I. In the case of voltage source, $k_{critical}$ using Eq. (10) is 0.068. The value of energy after one period and e^{-3} of initial transmitted energy are equal when the coupling coefficient is 0.0645. The $k_{critical}$ using Eq. (10) and simulated result have 5.15% error. This is because loaded Q becomes higher as source impedance is ignored. The results in Table II also satisfy the condition proposed in previous section.

V. Conclusion

The Coupled-mode theory and the transient circuit theory are analyzed. Using the Coupled-mode theory, the criterion of inductive coupling and magnetic resonance coupling in WPTS using power and voltage sources is proposed. Using the proposed definition of critical coupling coefficient, these two terms can be clarified analytically.

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