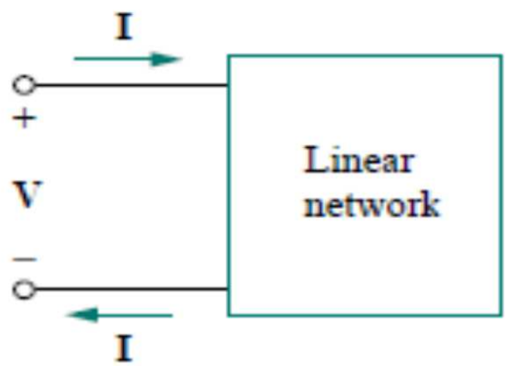
TWO-PORT CIRCUITS

INTODUCTION

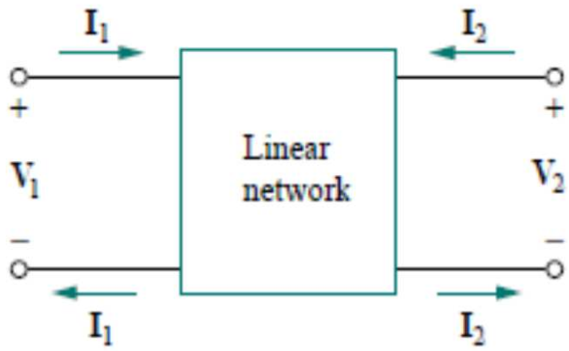
- A pair of terminals through which a current may enter or leave a network is known as a *port*.
- Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks.
- Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in the following figure 1(a):



(a)

INTRODUCTION

- The four-terminal or *two-port circuits* are involving op amps, transistors, and transformers, as shown in the following figure 1(b):



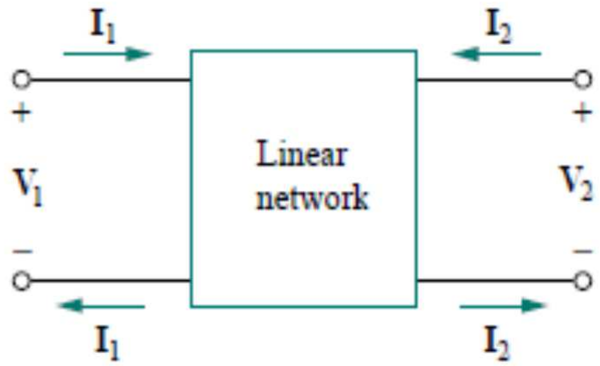
(b)

- *A port is an access to the network and consists of a pair of terminals.*
- The current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

INTRODUCTION

- *A two-port network is an electrical network with two separate ports for input and output.*
- A two-port network has *two terminal pairs* acting as access points.
- Such networks are useful in communications, control systems, power systems, and electronics. *For example*, they are used in electronics to model transistors and to apply the analysis of transistor circuits.
- The current entering one terminal of a pair leaves the other terminal in the pair.

To characterize a two-port network requires that we relate the terminal quantities V1, V2, I1, and I2 in Fig. 1(b).



(b)

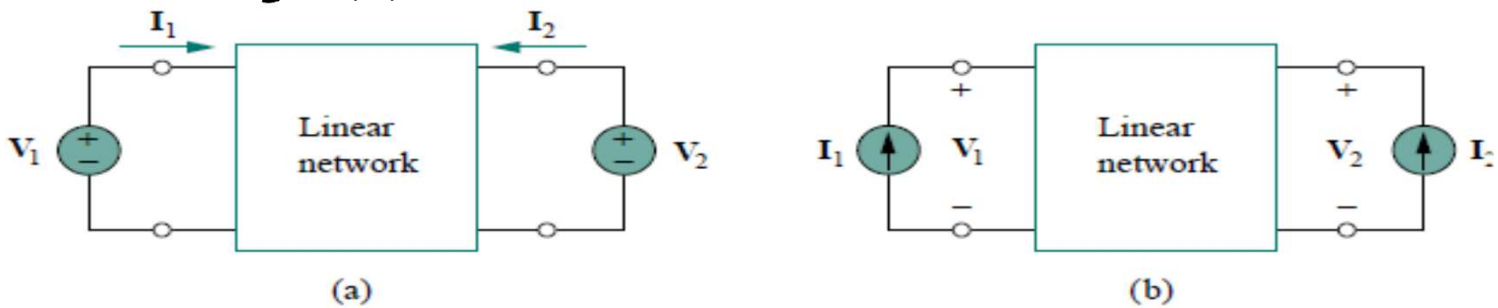
- The various terms that relate these voltages and currents are called *parameters.*

- Our goal is to derive six sets of these parameters.

- We will show the relationship between these parameters and how two-port networks can be connected in series, parallel, or cascade.

IMPEDANCE PARAMETERS

- *Impedance and admittance parameters are commonly used in the synthesis of filters.*
- A two-port network may be voltage-driven as in Fig. 2(a) or current-driven as in Fig. 2(b).



- The terminal voltages can be related to the terminal currents as:

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned}}$$

or in matrix form as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

the **z** terms are called the *z parameters*, and have units of ohms.

- The values of the parameters can be evaluated by setting $\mathbf{I}_1 = 0$
- or $\mathbf{I}_2 = 0$ .Thus,

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

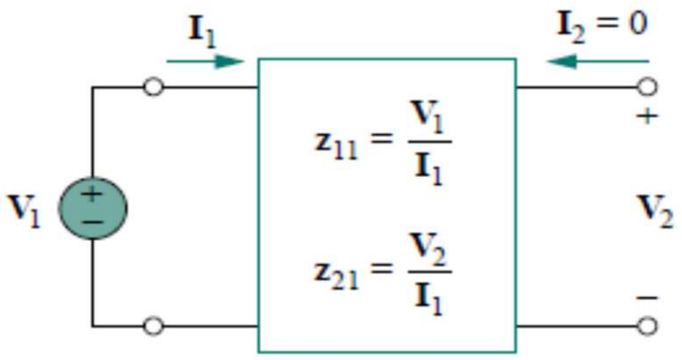
$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

- Since the *Z* parameters are obtained by open-circuiting the input or output
- port, they are also called the *open-circuit impedance parameters*.

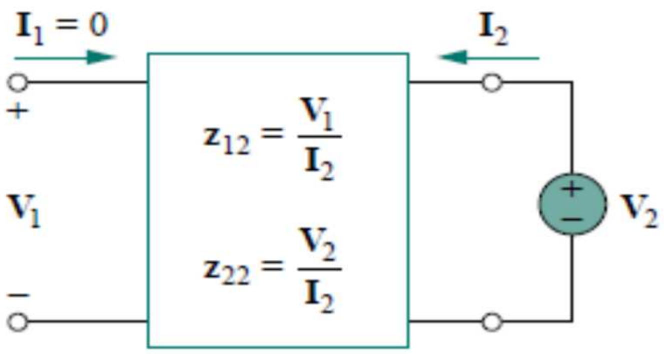
- $\mathbf{z}_{11}$ = Open-circuit input impedance
- $\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2
- $\mathbf{z}_{21}$ = Open-circuit transfer impedance from port 2 to port 1
- $\mathbf{z}_{22}$ = Open-circuit output impedance

We obtain Z_{11} and Z_{21} by connecting a voltage V_1 (or a current source I_1) to port 1 with port 2 open-circuited as in Fig.3(a) and finding I_1 and V_2

- Similarly, we obtain z_{12} and z_{22} by connecting a voltage V_2 (or a current source I_2) to port 2 with port 1 open-circuited as in Fig. 3(b) and finding I_2 and V_1



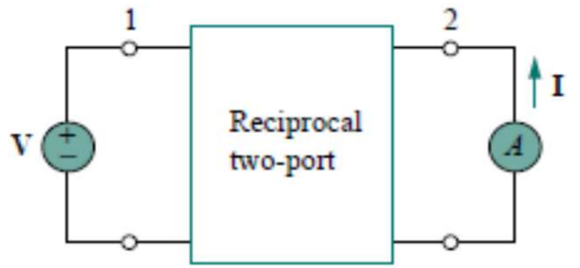
(a)



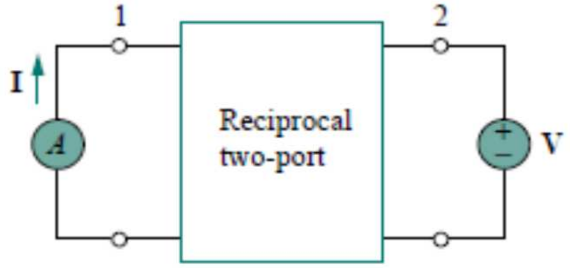
(b)

Sometimes $\mathbf{Z}_{11}$ and $\mathbf{Z}_{22}$ are called *driving-point impedances*, while $\mathbf{Z}_{21}$ and $\mathbf{Z}_{12}$ are called *transfer impedances*.

- When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be *symmetrical*.
- When the two-port network has no dependent sources, the transfer impedances are equal ($\mathbf{Z}_{12} = \mathbf{Z}_{21}$), and the two-port is said to be *reciprocal*.
- A two-port is *reciprocal* if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.
- The reciprocal network yields $\mathbf{V} = \mathbf{z}_{12}\mathbf{I}$ when connected as in Fig. 4(a), but yields $\mathbf{V} = \mathbf{z}_{21}\mathbf{I}$ when connected as in Fig. 4(b).



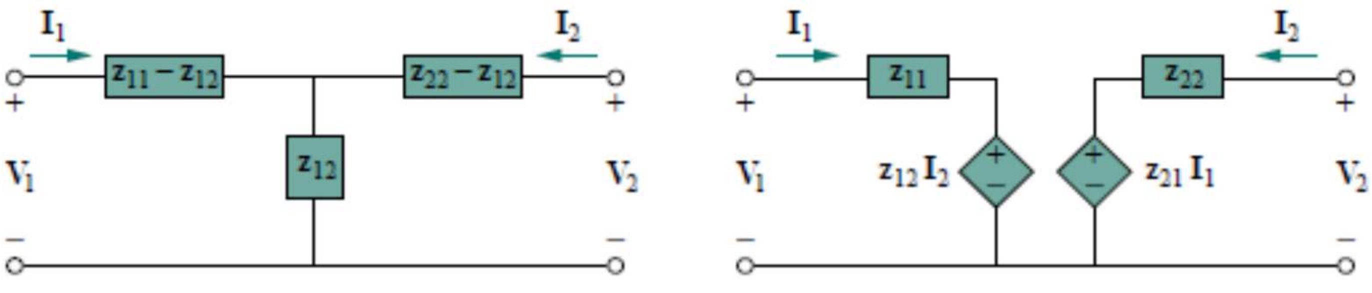
(a)



(b)

For a reciprocal network, the **T-equivalent** circuit in Fig.5(a) can be used.

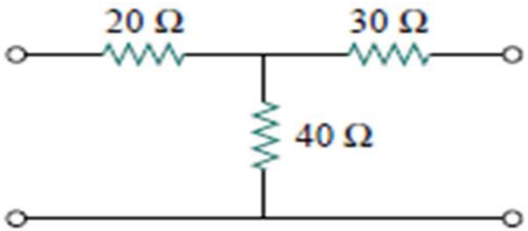
- If the network is not reciprocal, a more general equivalent network is shown in Fig. 5(b)

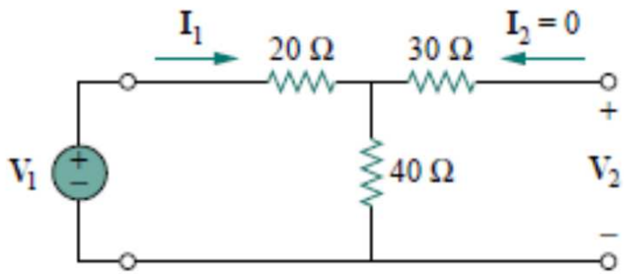


(a) (b)

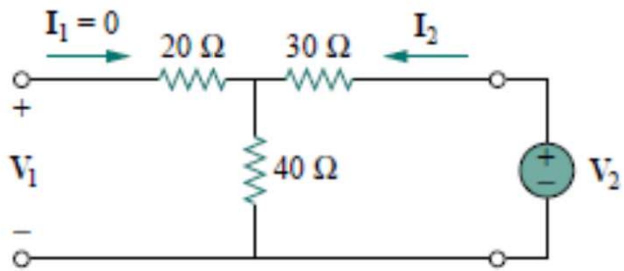
Example. 1

- Determine the *z* parameters for the circuit in the following figure:





(a)



(b)

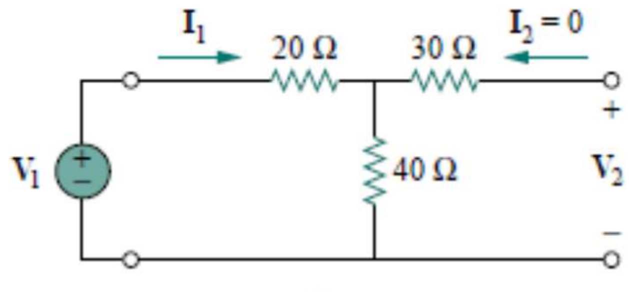
Solution:

To determine $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, we apply a voltage source $\mathbf{V}_1$ to the input port and leave the output port open as in Fig. (a). Then,

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20+40)\mathbf{I}_1}{\mathbf{I}_1} = 60\ \Omega$$

that is, $\mathbf{z}_{11}$ is the input impedance at port 1.

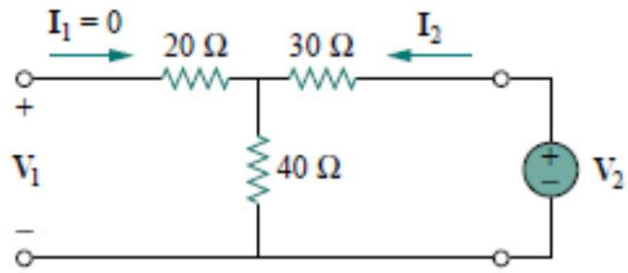
$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\ \Omega$$



(a)

To find $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, we apply a voltage source $\mathbf{V}_2$ to the output port and leave the input port open as in Fig (b). Then,

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\ \Omega, \qquad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(30+40)\mathbf{I}_2}{\mathbf{I}_2} = 70\ \Omega$$



(b)

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

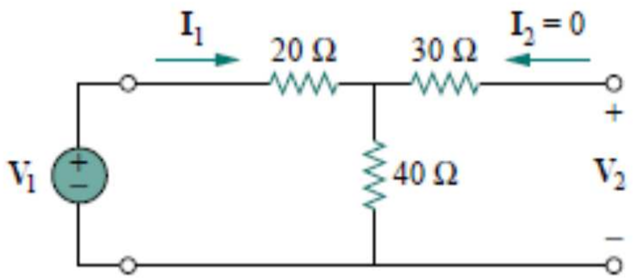
Method 2

Alternatively, since there is no dependent source in the given circuit, $z_{12} = z_{21}$ and we can use Fig. 5(a). Comparing with Fig. 5(a), we get

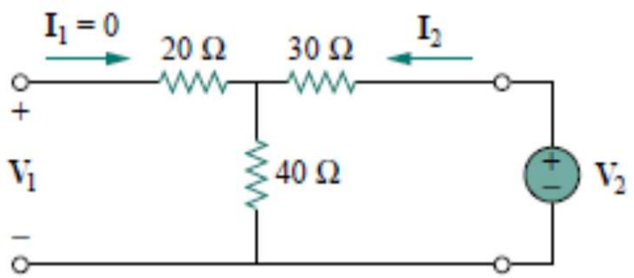
$$z_{12} = 40\ \Omega = z_{21}$$

$$z_{11} - z_{12} = 20 \quad \Longrightarrow \quad z_{11} = 20 + z_{12} = 60\ \Omega$$

$$z_{22} - z_{12} = 30 \quad \Longrightarrow \quad z_{22} = 30 + z_{12} = 70\ \Omega$$



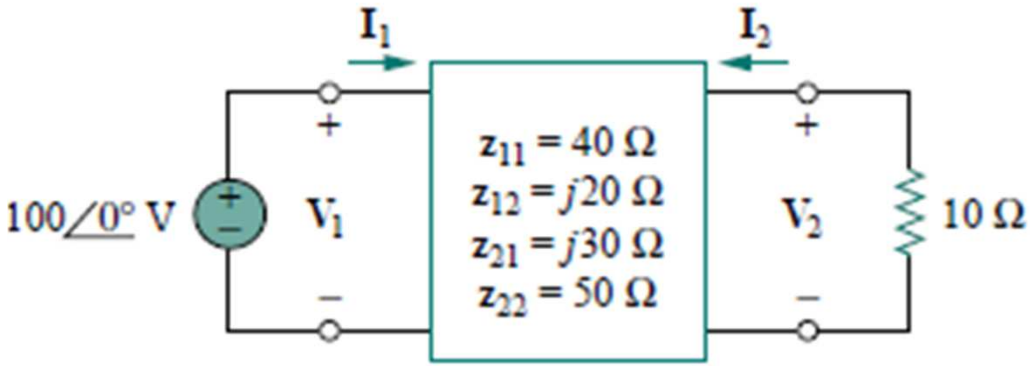
(a)



(b)

Example 2

- Find **I**$_1$ and **I**$_2$ in the circuit of the following figure:

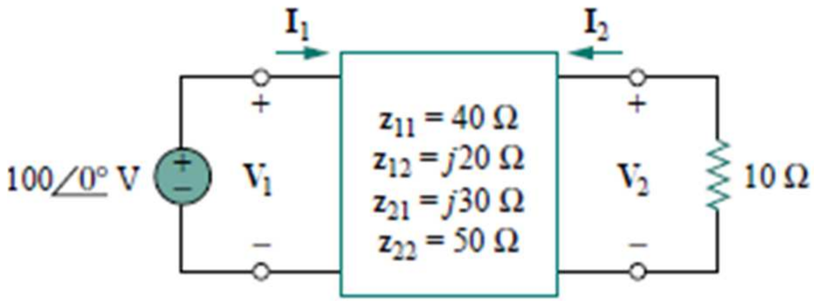


Example 2 Find I1 and I2 in the circuit in the following figure.

Solution:

we can use Eq. (1) directly. Substituting the given z parameters into Eq. (1),

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$
$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2$$



since we are looking for $\mathbf{I}_1$ and $\mathbf{I}_2$, we substitute

$$\mathbf{V}_1 = 100\angle 0^\circ, \qquad \mathbf{V}_2 = -10\mathbf{I}_2$$

into the above Eqs., which become

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$
$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \qquad \Longrightarrow \qquad \mathbf{I}_1 = j2\mathbf{I}_2$$

Substituting we get

$$100 = j80\mathbf{I}_2 + j20\mathbf{I}_2 \qquad \Longrightarrow \qquad \mathbf{I}_2 = \frac{100}{j100} = -j$$

$\mathbf{I}_1 = j2(-j) = 2$. Thus,

$$\mathbf{I}_1 = 2\angle 0^\circ \text{ A}, \qquad \mathbf{I}_2 = 1\angle -90^\circ \text{ A}$$

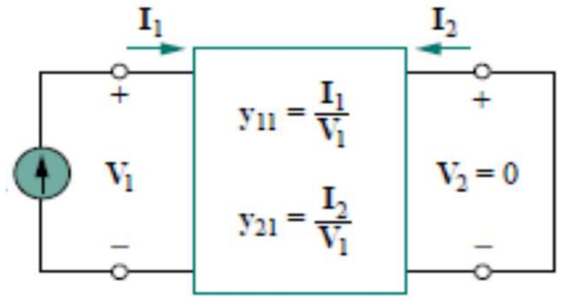
ADMITTANCE PARAMETERS

- Impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need is met by the second set of parameters,
- In either Fig. 6(a) or (b), the terminal currents can be expressed in terms of the terminal voltages:
- The **y** terms are known as the *admittance parameters*
- (or, simply, *y parameters*)
- and have units of siemens.

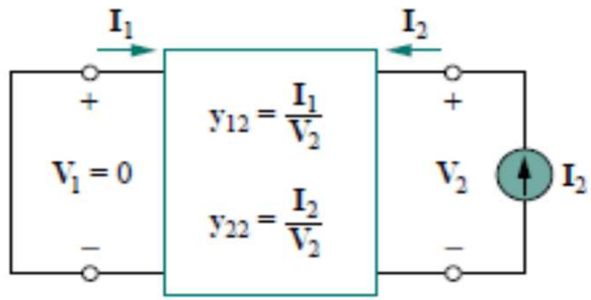
$$\boxed{\begin{aligned} \mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{aligned}}$$

or in matrix form as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$



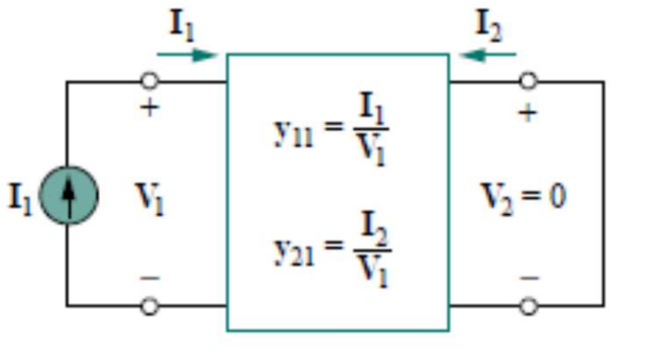
(a)



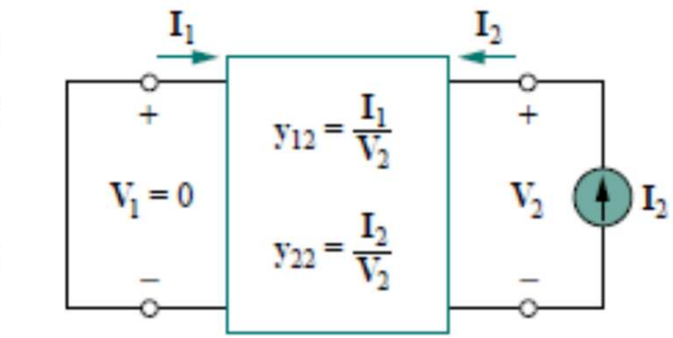
(b)

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



(a)



(b)

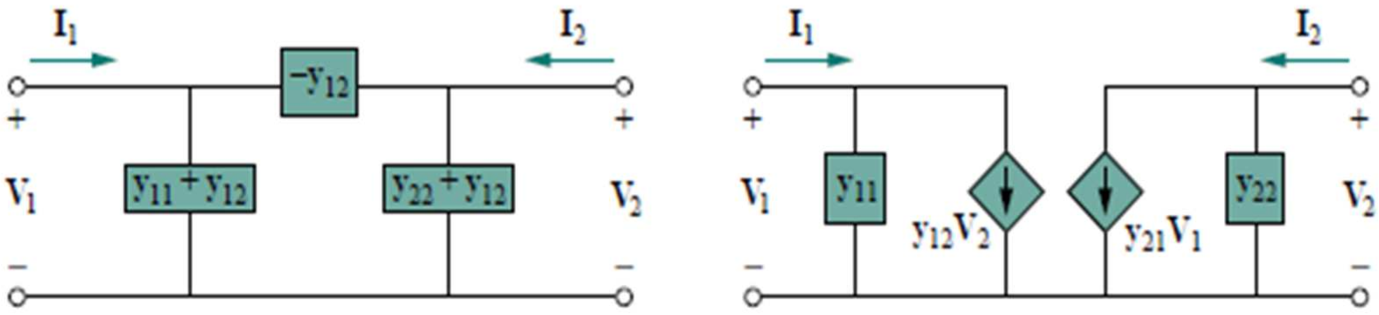
- y_{11} = Short-circuit input admittance
- y_{12} = Short-circuit transfer admitta
  from port 2 to port 1
- y_{21} = Short-circuit transfer admitta
  from port 1 to port 2
- y_{22} = Short-circuit output admittanc

- We obtain $\mathbf{y_{11}}$ and $\mathbf{y_{21}}$ by connecting a current $\mathbf{I_1}$ to port 1 and short-circuiting port 2 as in Fig. 6(a), finding $\mathbf{V_1}$ and $\mathbf{I_2}$

When a two-port network has no dependent sources, the transfer admittances are equal ($\mathbf{y}_{12} = \mathbf{y}_{21}$).

- A reciprocal network ($\mathbf{y}_{12} = \mathbf{y}_{21}$) can be modeled by the -equivalent circuit in Fig. 7(a).

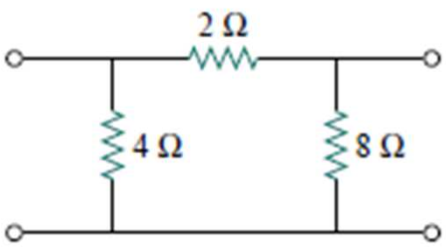
- If the network is not reciprocal, a more general equivalent network is shown in Fig. 7(b).

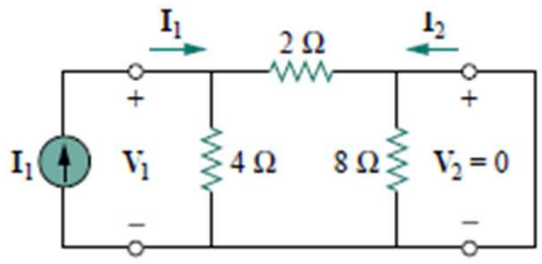


(a) (b)

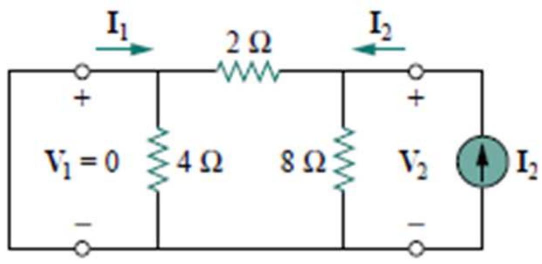
Example .3

- Obtain the *y* parameters for the network shown in the following figure:





(a)



(b)

Solution:

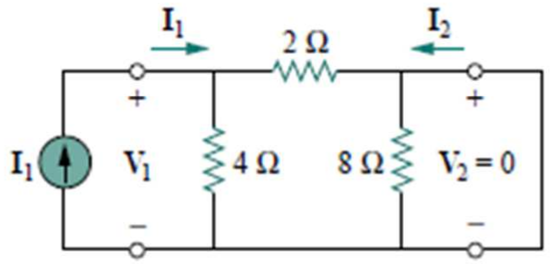
- method1

To find y_{11} and y_{21}, short-circuit the output port and connect a current source I_1 to the input port as in Fig.(a) Since the 8-Ω resistor is short-circuited, the 2-Ω resistor is in parallel with the 4-Ω resistor. Hence,

$$V_1 = I_1(4 \parallel 2) = \frac{4}{3}I_1, \qquad y_{11} = \frac{I_1}{V_1} = \frac{I_1}{\frac{4}{3}I_1} = 0.75 \text{ S}$$

By current division,

$$-I_2 = \frac{4}{4+2}I_1 = \frac{2}{3}I_1, \qquad y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}I_1}{\frac{4}{3}I_1} = -0.5 \text{ S}$$



(a)

Con.

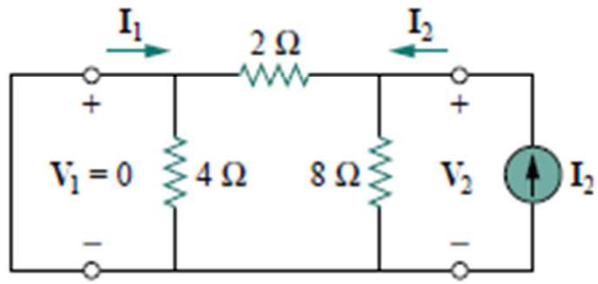
- Method 1

To get y_{12} and y_{22}, short-circuit the input port and connect a current source $\mathbf{I}_2$ to the output port as in Fig. (b). The 4-Ω resistor is short-circuited so that the 2-Ω and 8-Ω resistors are in parallel.

$$\mathbf{V}_2 = \mathbf{I}_2(8 \parallel 2) = \frac{8}{5}\mathbf{I}_2, \qquad y_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = \frac{5}{8} = 0.625 \text{ S}$$

By current division,

$$-\mathbf{I}_1 = \frac{8}{8+2}\mathbf{I}_2 = \frac{4}{5}\mathbf{I}_2, \qquad y_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\frac{4}{5}\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = -0.5 \text{ S}$$



(b)

Method 2

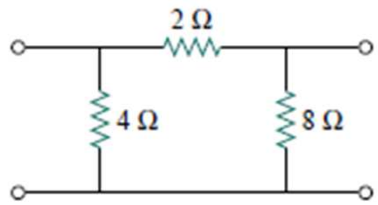
- Alternatively, comparing the original figure with Fig. (a),

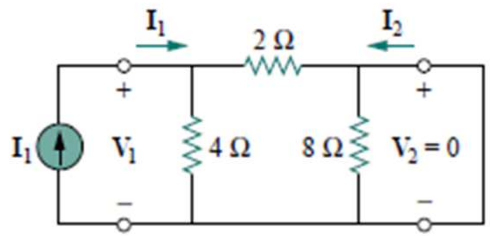
$$y_{12} = -\frac{1}{2}\,\text{S} = y_{21}$$

$$y_{11} + y_{12} = \frac{1}{4} \quad \Longrightarrow \quad y_{11} = \frac{1}{4} - y_{12} = 0.75\ \text{S}$$

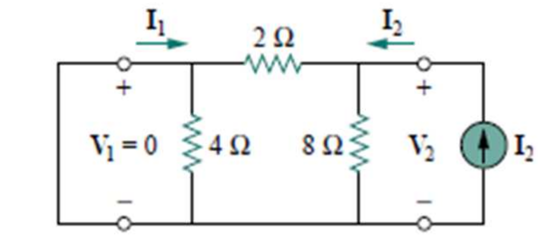
$$y_{22} + y_{12} = \frac{1}{8} \quad \Longrightarrow \quad y_{22} = \frac{1}{8} - y_{12} = 0.625\ \text{S}$$

- As obtained previously





(a)



(b)

HYBRID PARAMETERS

- This third set of parameters is based on making $\mathbf{V_1}$ and $\mathbf{I_2}$ the dependent variables.

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{aligned}}$$

or in matrix form,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

- The **h** terms are known as the *hybrid parameters* (or, *h parameters*)
- The ideal transformer can be described by the hybrid parameters.

The values of the parameters are determined as

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

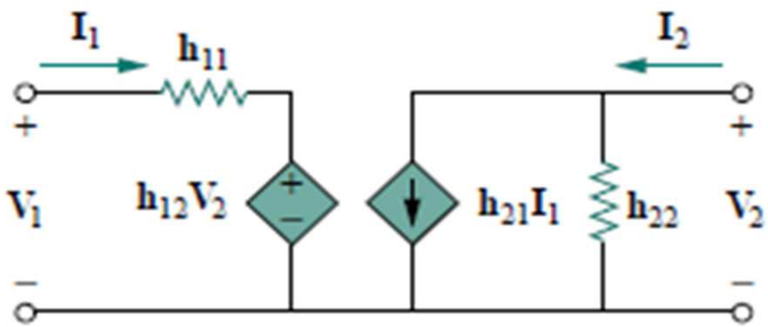
- **h**$_{11}$ = Short-circuit input impedance
- **h**$_{12}$ = Open-circuit reverse voltage gain
- **h**$_{21}$ = Short-circuit forward current gain
- **h**$_{22}$ = Open-circuit output admittance

- This is why they are called the hybrid parameters.

h parameters

- The procedure for calculating the *h* parameters is similar to that used for the *z* or *y* parameters.
- For reciprocal networks, $h_{12} = -h_{21}$. This can be proved in the same way
- as we proved that $z_{12} = z_{21}$.

- The following figure shows the hybrid model of a two-port network:



A set of parameters closely related to the *h* parameters are the *g parameters* or *inverse hybrid parameters*

- These are used to describe the terminal currents and voltages as:

$$\boxed{\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \\ V_2 &= g_{21}V_1 + g_{22}I_2 \end{aligned}}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [g] \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

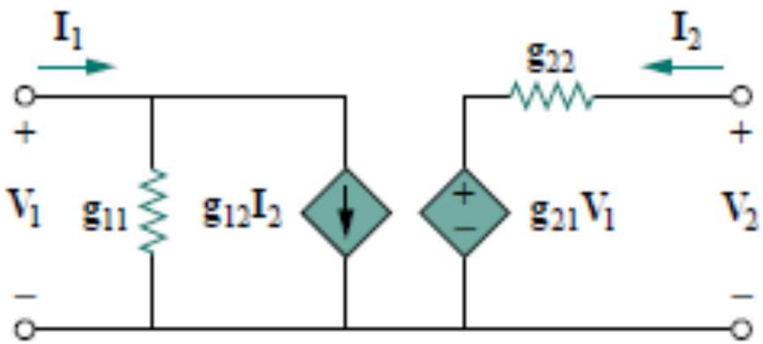
- The values of the *g* parameters are determined as:

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}, \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}, \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

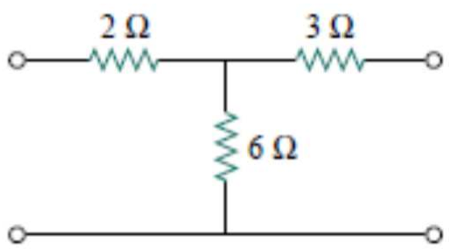
- g_{11} = Open-circuit input admittance
- g_{12} = Short-circuit reverse current gain
- g_{21} = Open-circuit forward voltage gain
- g_{22} = Short-circuit output impedance

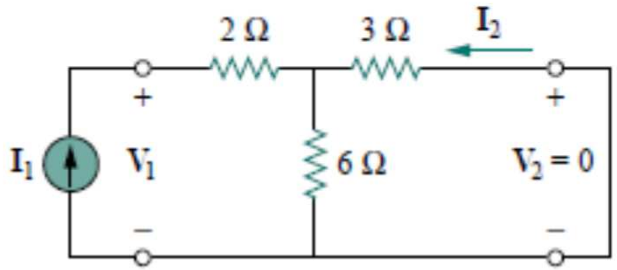
- The following figure shows the inverse hybrid model of a two-port network



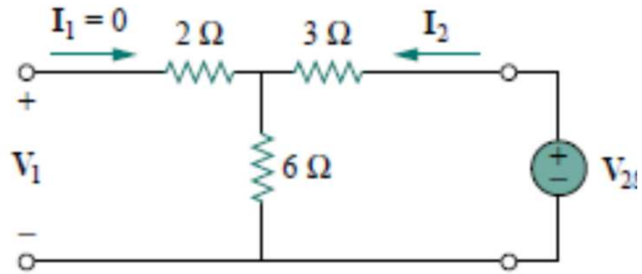
Example 5

- Find the hybrid parameters for the two-port network of the following figure:



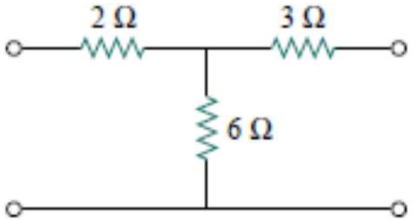


(a)



(b)

Example 5 Find the hybrid parameters for the two-port network of the following figure:



To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as shown in Fig. (a) . From Fig. (a),

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \parallel 6) = 4\mathbf{I}_1$$

Hence,

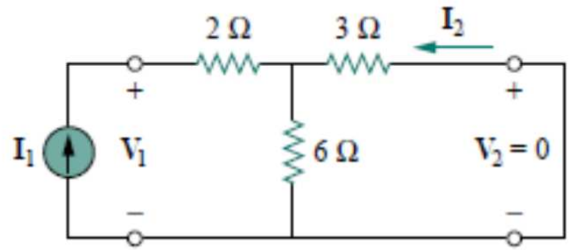
$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$$

Also, from Fig. (a) we obtain, by current division,

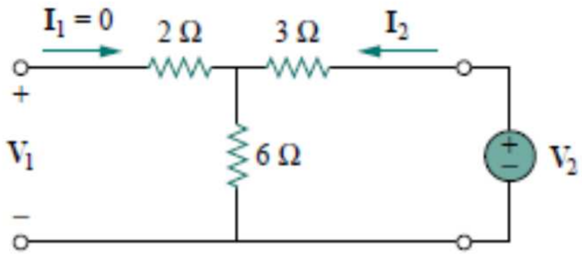
$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{2}{3}$$



(a)

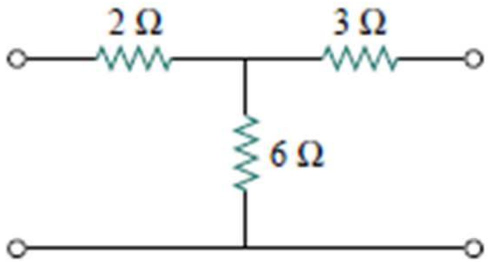


(b)

Example 5 con.

To obtain h_{12} and h_{22}, we open-circuit the input port and connect a voltage source V_2 to the output port as in Fig. (b). By voltage division,

$$V_1 = \frac{6}{6+3}V_2 = \frac{2}{3}V_2$$



Hence,

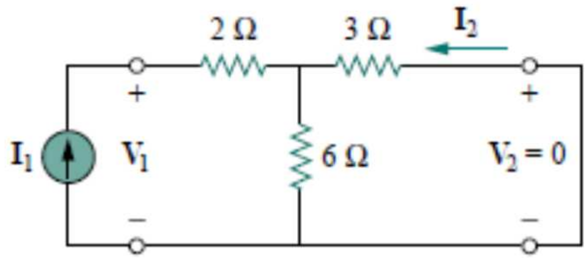
$$h_{12} = \frac{V_1}{V_2} = \frac{2}{3}$$

Also,

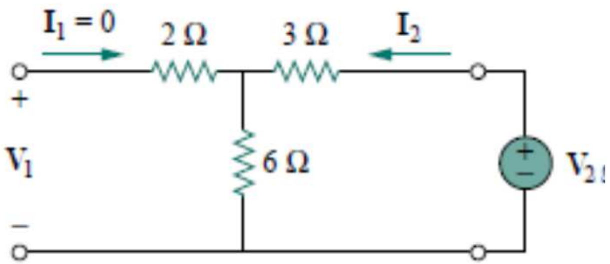
$$V_2 = (3+6)I_2 = 9I_2$$

Thus,

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{9}\ S$$



(a)



(b)

TRANSMISSION PARAMETERS

- The impedance and admittance parameters are grouped into the *immittance parameters*
- The term immittance denotes a quantity that is either an impedance or an admittance .
- The *a* parameters describe the voltage
- and current at one end of the two-port
- network in term of the voltage and current
- at the other end ,therefore they called the
- *transmission parameters*
- a_{11} = Open-circuit voltage ratio
- a_{12} =Negative short-circuit transfer impedance
- a_{22} =Open circuit transfer admittance
- a_{21} =Negative short-circuit current ratio

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \Omega$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \text{S} \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

b parameters

- The parameters *b* *are* called
- the *inverse* transmission parameters

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} \qquad b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} \Omega$$

$$b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} \text{S} \qquad b_{22} = -\frac{I_2}{I_1}\bigg|_{V_1=0}$$

- b_{11} = Open-circuit voltage gain
- b_{12} =Negative short-circuit transfer impedance
- b_{22} =Open circuit transfer admittance
- b_{21} =Negative short-circuit current gain

RELATIONSHIPS BETWEEN PARAMETERS

- Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated.
- If two sets of parameters exist, we can relate one set to the other set.
- Given the *z* parameters, let us obtain the *y* parameters.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

or

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Also, we know that :

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Comparing Eqs we see that

$$[y] = [z]^{-1}$$

The adjoint of the [z] matrix is

$$\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

and its determinant is

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

Substituting these into Eq. $[y] = [z]^{-1}$ ، we get

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{\Delta_z}$$

Equating terms yields

$$y_{11} = \frac{z_{22}}{\Delta_z}, \qquad y_{12} = -\frac{z_{12}}{\Delta_z}, \qquad y_{21} = -\frac{z_{21}}{\Delta_z}, \qquad y_{22} = \frac{z_{11}}{\Delta_z}$$

As a second example, let us determine the h parameters from the z parameters. we know that :

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

Making $\mathbf{I}_2$ the subject of second Eq.,

$$\mathbf{I}_2 = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{z}_{22}}\mathbf{V}_2$$

Substituting this into first Eq. :

$$\mathbf{V}_1 = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}\mathbf{V}_2$$

Putting Eqs in matrix form,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ -\dfrac{\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

- For ***h*** parameters,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Comparing this with the last Eq., we obtain

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{21} = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}}$$

- It can also be shown that

$$[\mathbf{g}] = [\mathbf{h}]^{-1}$$

- Table 18.1 provides the conversion formulas for the six sets of two-port parameters. Given one set of parameters.

TABLE 18.1 Conversion of two-port parameters.

	z		y		h		g		T		t	
z	$\mathbf{z}_{11}$	$\mathbf{z}_{12}$	$\frac{\mathbf{y}_{22}}{\Delta_y}$	$-\frac{\mathbf{y}_{12}}{\Delta_y}$	$\frac{\Delta_h}{\mathbf{h}_{22}}$	$\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{g}_{11}}$	$-\frac{\mathbf{g}_{12}}{\mathbf{g}_{11}}$	$\frac{\mathbf{A}}{\mathbf{C}}$	$\frac{\Delta_T}{\mathbf{C}}$	$\frac{\mathbf{d}}{\mathbf{c}}$	$\frac{1}{\mathbf{c}}$
	$\mathbf{z}_{21}$	$\mathbf{z}_{22}$	$-\frac{\mathbf{y}_{21}}{\Delta_y}$	$\frac{\mathbf{y}_{11}}{\Delta_y}$	$-\frac{\mathbf{h}_{21}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{h}_{22}}$	$\frac{\mathbf{g}_{21}}{\mathbf{g}_{11}}$	$\frac{\Delta_g}{\mathbf{g}_{11}}$	$\frac{1}{\mathbf{C}}$	$\frac{\mathbf{D}}{\mathbf{C}}$	$\frac{\Delta_t}{\mathbf{c}}$	$\frac{\mathbf{a}}{\mathbf{c}}$
y	$\frac{\mathbf{z}_{22}}{\Delta_z}$	$-\frac{\mathbf{z}_{12}}{\Delta_z}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\frac{1}{\mathbf{h}_{11}}$	$-\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}}$	$\frac{\Delta_g}{\mathbf{g}_{22}}$	$\frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}$	$\frac{\mathbf{D}}{\mathbf{B}}$	$-\frac{\Delta_T}{\mathbf{B}}$	$\frac{\mathbf{a}}{\mathbf{b}}$	$-\frac{1}{\mathbf{b}}$
	$-\frac{\mathbf{z}_{21}}{\Delta_z}$	$\frac{\mathbf{z}_{11}}{\Delta_z}$	$\mathbf{y}_{21}$	$\mathbf{y}_{22}$	$\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$	$\frac{\Delta_h}{\mathbf{h}_{11}}$	$-\frac{\mathbf{g}_{21}}{\mathbf{g}_{22}}$	$\frac{1}{\mathbf{g}_{22}}$	$-\frac{1}{\mathbf{B}}$	$\frac{\mathbf{A}}{\mathbf{B}}$	$-\frac{\Delta_t}{\mathbf{b}}$	$\frac{\mathbf{d}}{\mathbf{b}}$
h	$\frac{\Delta_z}{\mathbf{z}_{22}}$	$\frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{y}_{11}}$	$-\frac{\mathbf{y}_{12}}{\mathbf{y}_{11}}$	$\mathbf{h}_{11}$	$\mathbf{h}_{12}$	$\frac{\mathbf{g}_{22}}{\Delta_g}$	$-\frac{\mathbf{g}_{12}}{\Delta_g}$	$\frac{\mathbf{B}}{\mathbf{D}}$	$\frac{\Delta_T}{\mathbf{D}}$	$\frac{\mathbf{b}}{\mathbf{a}}$	$\frac{1}{\mathbf{a}}$
	$-\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{z}_{22}}$	$\frac{\mathbf{y}_{21}}{\mathbf{y}_{11}}$	$\frac{\Delta_y}{\mathbf{y}_{11}}$	$\mathbf{h}_{21}$	$\mathbf{h}_{22}$	$-\frac{\mathbf{g}_{21}}{\Delta_g}$	$\frac{\mathbf{g}_{11}}{\Delta_g}$	$-\frac{1}{\mathbf{D}}$	$\frac{\mathbf{C}}{\mathbf{D}}$	$\frac{\Delta_t}{\mathbf{a}}$	$\frac{\mathbf{c}}{\mathbf{a}}$
g	$\frac{1}{\mathbf{z}_{11}}$	$-\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$	$\frac{\Delta_y}{\mathbf{y}_{22}}$	$\frac{\mathbf{y}_{12}}{\mathbf{y}_{22}}$	$\frac{\mathbf{h}_{22}}{\Delta_h}$	$-\frac{\mathbf{h}_{12}}{\Delta_h}$	$\mathbf{g}_{11}$	$\mathbf{g}_{12}$	$\frac{\mathbf{C}}{\mathbf{A}}$	$-\frac{\Delta_T}{\mathbf{A}}$	$\frac{\mathbf{c}}{\mathbf{d}}$	$-\frac{1}{\mathbf{d}}$
	$\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}$	$\frac{\Delta_z}{\mathbf{z}_{11}}$	$-\frac{\mathbf{y}_{21}}{\mathbf{y}_{22}}$	$\frac{1}{\mathbf{y}_{22}}$	$-\frac{\mathbf{h}_{21}}{\Delta_h}$	$\frac{\mathbf{h}_{11}}{\Delta_h}$	$\mathbf{g}_{21}$	$\mathbf{g}_{22}$	$\frac{1}{\mathbf{A}}$	$\frac{\mathbf{B}}{\mathbf{A}}$	$\frac{\Delta_t}{\mathbf{d}}$	$-\frac{\mathbf{b}}{\mathbf{d}}$
T	$\frac{\mathbf{z}_{11}}{\mathbf{z}_{21}}$	$\frac{\Delta_z}{\mathbf{z}_{21}}$	$-\frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}$	$-\frac{1}{\mathbf{y}_{21}}$	$-\frac{\Delta_h}{\mathbf{h}_{21}}$	$-\frac{\mathbf{h}_{11}}{\mathbf{h}_{21}}$	$\frac{1}{\mathbf{g}_{21}}$	$\frac{\mathbf{g}_{22}}{\mathbf{g}_{21}}$	$\mathbf{A}$	$\mathbf{B}$	$\frac{\mathbf{d}}{\Delta_t}$	$\frac{\mathbf{b}}{\Delta_t}$
	$\frac{1}{\mathbf{z}_{21}}$	$\frac{\mathbf{z}_{22}}{\mathbf{z}_{21}}$	$-\frac{\Delta_y}{\mathbf{y}_{21}}$	$-\frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}$	$-\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}$	$-\frac{1}{\mathbf{h}_{21}}$	$\frac{\mathbf{g}_{11}}{\mathbf{g}_{21}}$	$\frac{\Delta_g}{\mathbf{g}_{21}}$	$\mathbf{C}$	$\mathbf{D}$	$\frac{\mathbf{c}}{\Delta_t}$	$\frac{\mathbf{a}}{\Delta_t}$
t	$\frac{\mathbf{z}_{22}}{\mathbf{z}_{12}}$	$\frac{\Delta_z}{\mathbf{z}_{12}}$	$-\frac{\mathbf{y}_{11}}{\mathbf{y}_{12}}$	$-\frac{1}{\mathbf{y}_{12}}$	$\frac{1}{\mathbf{h}_{12}}$	$\frac{\mathbf{h}_{11}}{\mathbf{h}_{12}}$	$-\frac{\Delta_g}{\mathbf{g}_{12}}$	$-\frac{\mathbf{g}_{22}}{\mathbf{g}_{12}}$	$\frac{\mathbf{D}}{\Delta_T}$	$\frac{\mathbf{B}}{\Delta_T}$	$\mathbf{a}$	$\mathbf{b}$
	$\frac{1}{\mathbf{z}_{12}}$	$\frac{\mathbf{z}_{11}}{\mathbf{z}_{12}}$	$-\frac{\Delta_y}{\mathbf{y}_{12}}$	$-\frac{\mathbf{y}_{22}}{\mathbf{y}_{12}}$	$\frac{\mathbf{h}_{22}}{\mathbf{h}_{12}}$	$\frac{\Delta_h}{\mathbf{h}_{12}}$	$-\frac{\mathbf{g}_{11}}{\mathbf{g}_{12}}$	$-\frac{1}{\mathbf{g}_{12}}$	$\frac{\mathbf{C}}{\Delta_T}$	$\frac{\mathbf{A}}{\Delta_T}$	$\mathbf{c}$	$\mathbf{d}$

$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}$, $\Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21}$, $\Delta_T = \mathbf{AD} - \mathbf{BC}$

$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$, $\Delta_g = \mathbf{g}_{11}\mathbf{g}_{22} - \mathbf{g}_{12}\mathbf{g}_{21}$, $\Delta_t = \mathbf{ad} - \mathbf{bc}$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}, \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}, \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}, \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$