Problem FYS7n4310 400-7(4.11 in 3rd Ed) suggested solution

```
> restart;
```

Dry oxidation Si at 1000 °C but with oxygen ions
It is a reasonable intepretation of the problem text to write it like

B(O2-)=2*B(O2) and (B/A(O2-) = 10 (B/A(O2))

where B is the parabolic growth rate which is proportional to D (the diffusivity) and B/A is the linear growth rate
limiting the growth when the growth is reaction limited.

Let us call A(O2-) for A and A(O2) for An - for A-neutral. And similarly for B. We then have.

```
> eqa:=B =2*Bn;eqb:=B/A=10*Bn/An;solve({eqa,eqb},{A,B});
```

$$eqa := B = 2\,Bn$$

$$eqb := \frac{B}{A} = \frac{10\,Bn}{An}$$

$$\left\{A = \frac{1}{5}\,An,\ B = 2\,Bn\right\}$$

We have no clues as to what should be done with the value of tau, taking care of the initial fast growth.
So we let the value of tau be as in table 4.1

We want a thickness of 100 nm
The question is What time is needed? x is the thickness of the oxide (t_o in the book)

```
> eq411:=x^2+A*x=B*(t+tau);
```

$$eq411 := A\,x + x^2 = B\,(t + \tau)$$

We set in for An and Bn from table 4.1 From table 4.1
and multiply by the values obove (1/5 and 2) to get A and B

```
> consts:={A=1/5*0.165e-6*m,B=2*0.0117*(1e-6)^2*m^2/(60*60*s), tau=
  0.37*60*60*s, x=100e-9*m};
```

$$consts := \left\{A = 3.300000000\ 10^{-8}\ m,\ B = \frac{6.500000000\ 10^{-18}\ m^2}{s},\ \tau = 1332.00\ s,\ x = 1.00\ 10^{-7}\ m\right\}$$

```
> eq411n:=subs(consts,eq411);
```

$$eq411n := 1.330000000\ 10^{-14}\ m^2 = \frac{6.500000000\ 10^{-18}\ m^2\ (t + 1332.00\ s)}{s}$$

```
> solve(eq411n,t);%/60/s*min;
```

$$714.1538462\ s$$

$$11.90256410\ \mathrm{min}$$

Answer: We must oxidize for 12 minutes

Question: Are we in the linear regime or the parabolic regime?

We could try to see what gives the best agreement 4.13 or 4.14

```
> eq413:=x=B/A*(t+tau);
```

$$eq413 := x = \frac{B\,(t+\tau)}{A}$$

```
> eq413n:=subs(consts,eq413);
```

$$eq413n := 1.00\;10^{-7}\;m = \frac{1.969696970\;10^{-10}\;m\;(t+1332.00\;s)}{s}$$

```
> solve(eq413n,t);
```

$$-824.3076924\;s$$

```
> eq414:=x^2=B*(t+tau);
```

$$eq414 := x^2 = B\,(t+\tau)$$

```
> eq414n:=subs(consts,eq414);
```

$$eq414n := 1.0000\;10^{-14}\;m^2 = \frac{6.500000000\;10^{-18}\;m^2\;(t+1332.00\;s)}{s}$$

```
> solve(eq414n,t);
```

$$206.4615385\;s$$

Well, none fits real well

```
> f1:=solve(eq411,x)[1];
```

$$f1 := -\frac{1}{2}A + \frac{1}{2}\sqrt{A^2 + 4\,B\,t + 4\,B\,\tau}$$

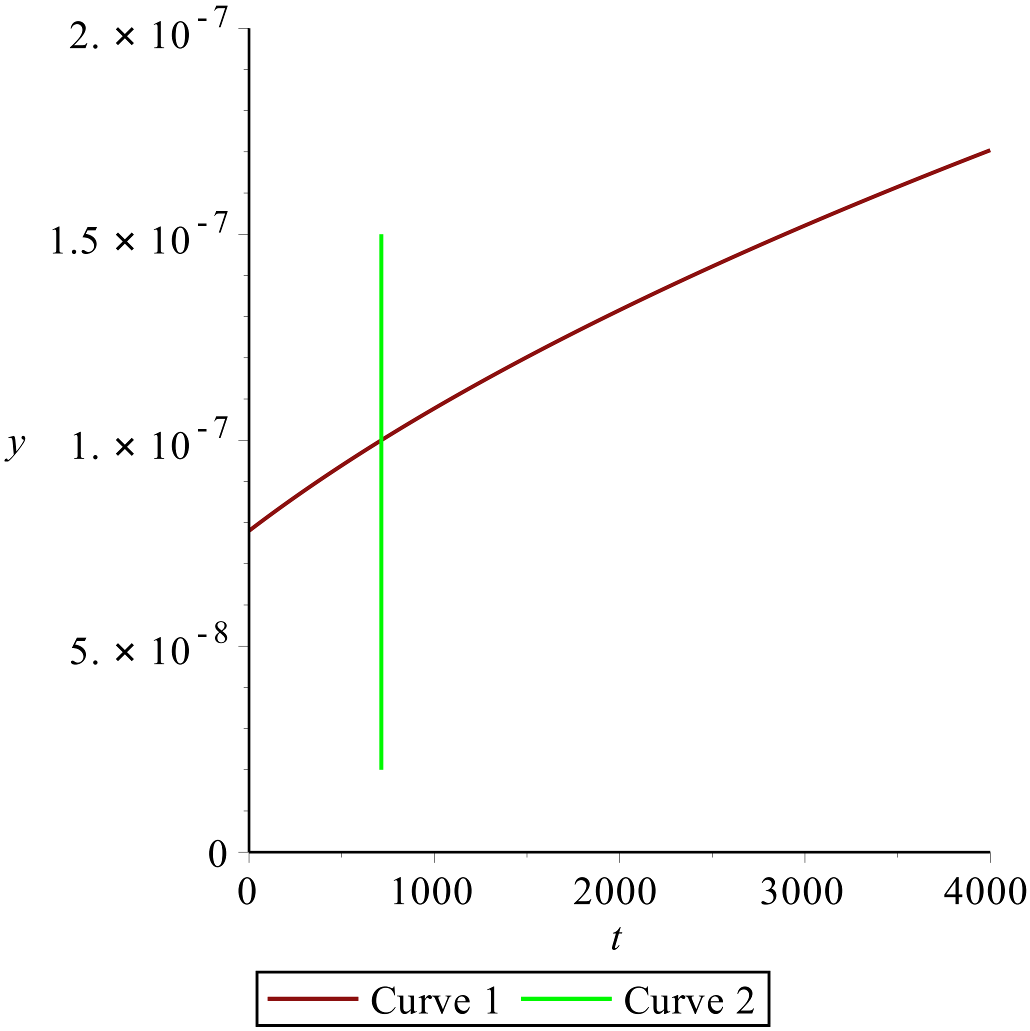
```
> f2:=subs(consts,f1);
```

$$f2 := -1.650000000\;10^{-8}\;m + \frac{1}{2}\sqrt{3.572100000\;10^{-14}\;m^2 + \frac{2.600000000\;10^{-17}\;m^2\;t}{s}}$$

```
> fx := t->-.1650000000e-7+1/2*sqrt(.3572100000e-13+.2600000000e-16*
  t);
```

$$fx := t \to -1.650000000\;10^{-8} + \frac{1}{2}\sqrt{3.572100000\;10^{-14} + 2.600000000\;10^{-17}\;t}$$

```
> with(plots):
> plot1:=plot(fx(t),t=0..4000,y=0..2e-7):
  plot2:=plot([[714,2e-8],[714,1.5e-7]],color=green):
> display({plot1,plot2});
```



The figure above show how the thickness varies with time. The large offset at t=0 is to account for fast initial growth. The green line is the time for 1000 Aangstroms.

To analyze better we
will plot in log log plot after subtracting the 0 time thickness.
In a log log plot we will get a straight line for the linear asymptote as well as the parabolic one, the difference is that their slopes are different

We introduce a function fxmark that is exactly as x exept it is offset in time such that the value (thickness) is zero at zero time

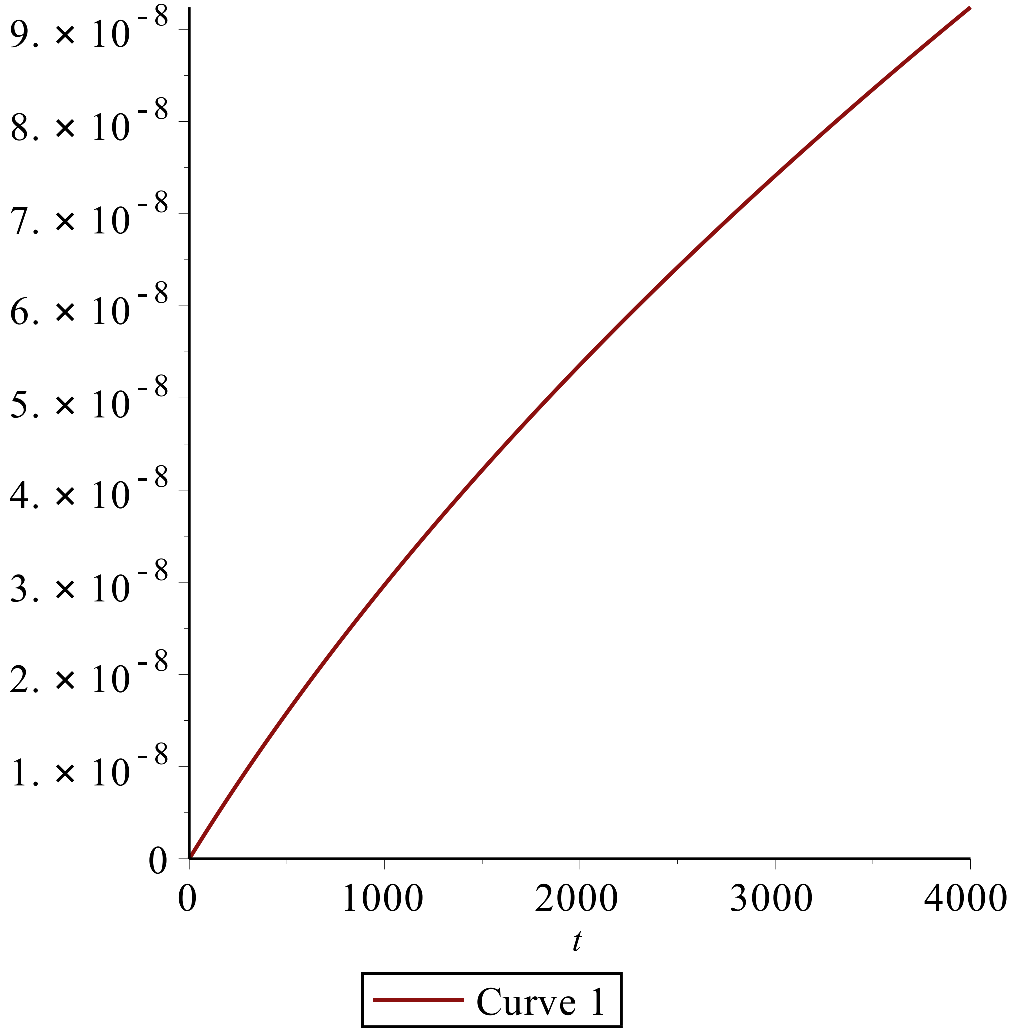
```
> fx(0);
```

$$7.800000000\ 10^{-8}$$

```
> fxmark:=t->-.1650000000e-7+1/2*sqrt
  (.3572100000e-13+.2600000000e-16*t)-.7800000000e-7;
```

$$fxmark := t \rightarrow -1.650000000\ 10^{-8} + \frac{1}{2}\sqrt{3.572100000\ 10^{-14} + 2.600000000\ 10^{-17}\ t} - 7.800000000\ 10^{-8}$$

```
> plot(fxmark(t),t=0..4000);
```



Now we construct the asymptotes yL (linear) and yP (parabolic)

```
> for j from 1 to 100 do
  lt:=j*4000/100;x[j]:=log(lt);lft:=fxmark(lt);y[j]:=log(lft);
  yP[j]:=log(sqrt(.6500000000e-17*lt));
  yL[j]:=log(.1969696970e-9*lt);
  od:
> plot11:=plot([seq([x[j],y[j]],j=1..100)]):
> plot21:=plot([[log(714),log(1e-9)],[log(714),log(8e-8)]], color=
  violet):

> plot31:=plot([seq([x[j],yP[j]],j=1..100)],color=green):
> plot41:=plot([seq([x[j],yL[j]],j=1..100)],color=blue):
> with(plots):display({plot11,plot21,plot31,plot41});
```

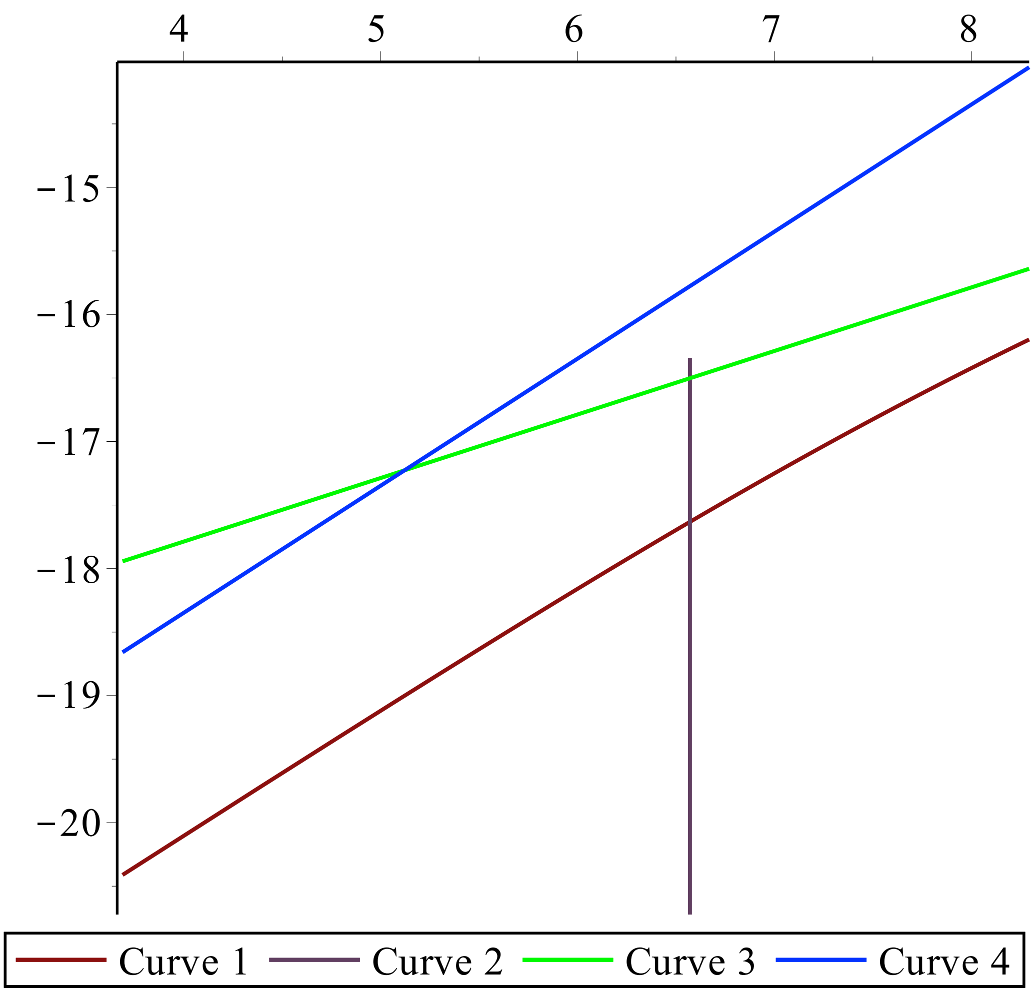


Figure above shows
red curve oxide thickness vs. time ,
logarithmic scale,
value for red curve is actually : log (x - x(0)) where x is oxide thickness
blue curve linear growth rate
green curve parabolic growth rate
violet vertical - approx time for 1000 aangstrom

The Fig shows clearly that it is reasonable to say that
we are in the linear growth regime