Kathmandu University
Scientific Computing with Python
Assignmnet-I

Level: BE/B.Sc 2nd sem/2nd year   **Course: Numerical Methods (MCSC-202)**
Instructor: Dr. Samir Shrestha

Implement the following problems using Python 3 in Script file to get the results.

1. A ball is thrown vertically up in the air from a height h_0 above the ground at an initial velocity v_0. Its subsequent height h and velocity v are given by the equations

$$h = h_0 + v_0 t - \frac{1}{2}gt^2$$
$$v = v_0 - gt$$

where $g = 9.8$ is the acceleration due to gravity in m/s^2. Write a script that finds the height h and velocity v at a time t after the ball is thrown. Start the script by setting $h_0 = 1.2$ (meters) and $v_0 = 5.4$ (m/s) and have your script print out the values of height and velocity. Then use the script to find the height and velocity after 0.5 seconds. Then modify your script to find them after 2.0 seconds.

2. Write a single Python script that calculates the following expressions:

(i) $\frac{2+e^{2.8}}{\sqrt{13}-2}$ (ii) $\frac{1-(1+ln\,2)^{-3.5}}{1+\sqrt{5}}$ (iii) $sin\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)$

3. Create an array of 9 evenly spaced numbers going from 0 to 29 (inclusive) and give it the variable name r. Find the square of each element of the array (as simply as possible). Find twice the value of each element of the array in two different ways:
(i)   using addition and   (ii) using multiplication.

4. The position of a ball at time t dropped with zero initial velocity from a height h_0 is given by

$$y = h_0 - 0.5gt^2$$

where $g = 9.8\, m/s^2$. Suppose $h_0 = 10\, m$. Find the sequence of times when the ball passes each half meter assuming the ball is dropped at $t = 0$. Hint: Create a NumPy array for y that goes from 10 to 0 in increments of -0.5 using the arrange function. Solving the above equation for t, show that

$$t = \frac{\sqrt{2(h_0-y)}}{g}.$$

(i)   Using this equation and the array you created, find the sequence of times when the ball passes each half meter. Save your code as a Python script.
(ii)   Recalling that the average velocity over an interval Δt is defined as $\bar{v} = \frac{\Delta y}{\Delta t}$, find the average velocity for each time interval in the problem using NumPy arrays.

5. Plot the function $y = 3x^2$ for $-3 \leq x \leq 3$. Include enough points (say 100 points) so that the curve you plot appears smooth. Label the axes x and y.

6. Plot the functions $y = \sin x$ and $y = \cos x$ for $-2\pi \le x \le 2\pi$ on the same plot. Plot $y = \sin x$ in the color red and $y = \cos x$ in the color blue and include a legend to label the two curves. Place the legend within the plot, but such that it does not cover either of the sine or cosine traces.

7. Write a program to tell the nature of the roots and values of the roots of a quadratic equation $ax^2 + bx + c = 0, a \ne 0$.
8. Write a program to calculate the factorial of a positive integer input by the user.
   (a) Write the factorial function using a Python while loop.
   (b) Write the factorial function using a Python for loop.
   Check your programs to make sure they work for 0,1, 2, 3, 4,5
9. Write a program to calculate the sum of natural n natural numbers $1 + 2 + \cdots + n$. Calculate the sum when $n = 10$
10. Write a program to calculate the sum of squares of natural n natural numbers $1^2 + 2^2 + \cdots + n^2$. Calculate the sum when $n = 5$.
11. Write a program to find the smallest integer n such that $3^n \ge 2000$