15/2/2019

PAG. 1011

22 $y = \dfrac{\sqrt{x^2+4x}}{x} = \dfrac{(x^2+4x)^{\frac{1}{2}}}{x}$

$$y' = \frac{\left[(x^2+4x)^{\frac{1}{2}}\right]' \cdot x - 1 \cdot (x^2+4x)^{\frac{1}{2}}}{x^2} =$$

$$= \frac{\left[\frac{1}{2}(x^2+4x)^{-\frac{1}{2}} \cdot (2x+4)\right] x - (x^2+4x)^{\frac{1}{2}}}{x^2} =$$

$$= \frac{\left[\frac{1}{2} \cdot \frac{1}{\sqrt{x^2+4x}} \cdot (2x+4)\right] \cdot x - \sqrt{x^2+4x}}{x^2} =$$

$$= \frac{\frac{\cancel{2}(x+2)x}{\cancel{2}\sqrt{x^2+4x}} - \sqrt{x^2+4x}}{x^2} =$$

$$= \frac{\frac{x^2+2x-(x^2+4x)}{\sqrt{x^2+4x}}}{x^2} =$$

$$= \frac{\cancel{x^2}+2x-\cancel{x^2}-4x}{x^2\sqrt{x^2+4x}} = -\frac{2\cancel{x}}{\cancel{x^2}\sqrt{x^2+4x}} =$$

$$= \boxed{-\frac{2}{x\sqrt{x^2+4x}}}$$

374 $y = \dfrac{\sqrt{1+x^2}}{2x} = \dfrac{(1+x^2)^{\frac{1}{2}}}{2x}$

$$y' = \frac{\left[(1+x^2)^{\frac{1}{2}}\right]' \cdot 2x - 2(1+x^2)^{\frac{1}{2}}}{4x^2} =$$

$$= \frac{\frac{1}{\cancel{2}}(1+x^2)^{-\frac{1}{2}} \cdot \cancel{2}x \cdot 2x - 2\sqrt{1+x^2}}{4x^2} = \frac{\frac{2x^2}{\sqrt{1+x^2}} - 2\sqrt{1+x^2}}{4x^2} =$$

$$= \frac{2x^2 - 2(1+x^2)}{4x^2\sqrt{1+x^2}} = \frac{\cancel{2x^2} - 2 - \cancel{2x^2}}{4x^2\sqrt{1+x^2}} =$$

$$= -\frac{\cancel{2}^{\,1}}{{}_{2}\cancel{4}x^2\sqrt{1+x^2}} = \boxed{-\frac{1}{2x^2\sqrt{1+x^2}}}$$

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$$y = \sqrt[3]{x^3 - x^2} = (x^3 - x^2)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}(x^3 - x^2)^{\frac{1}{3}-1} \cdot (3x^2 - 2x) =$$

$$= \frac{1}{3}(x^3 - x^2)^{-\frac{2}{3}} \cdot (3x^2 - 2x) =$$

$$= \frac{1}{3} \cdot \frac{1}{(x^3 - x^2)^{2/3}} \cdot (3x^2 - 2x) =$$

$$= \boxed{\frac{3x^2 - 2x}{3\sqrt[3]{(x^3 - x^2)^2}}}$$

Calcolare l'equazione della retta tangente al grafico della funzione

$$f(x) = \frac{x^2 + \ln x}{x}$$

nel punto $x_0 = 1$.

PUNTO $\rightarrow$ $P(x_0, f(x_0)) = (1, f(1)) = (1, 1)$

$$f(1) = \frac{1 + \ln 1}{1} = \frac{1+0}{1} = 1$$

CALCOLO LA DERIVATA

$$f'(x) = \frac{\left(2x + \frac{1}{x}\right) \cdot x - 1 \cdot (x^2 + \ln x)}{x^2} = \frac{2x^2 + 1 - x^2 - \ln x}{x^2} = \frac{x^2 + 1 - \ln x}{x^2}$$

$$f'(x_0) = f'(1) = \frac{1 + 1 - \ln 1}{1^2} = 2$$ $\leftarrow$ COEFF. ANGOLARE DELLA TANGENTE

RETTA TANGENTE

$$y - y_0 = m(x - x_0)$$

$P(1, 1)$ $\quad m = 2$

$$y - 1 = 2(x - 1)$$

$$y = 2x - 2 + 1 \qquad \boxed{y = 2x - 1}$$

FORMULA IMMEDIATA $\rightsquigarrow$ $\boxed{y - f(x_0) = f'(x_0)(x - x_0)}$