19/3/2021

88 $\int \frac{4-x}{\sqrt{x}+2}dx =$ $\left[2x - \frac{2}{3}x\sqrt{x} + c\right]$

$$= -\int \frac{x-4}{\sqrt{x}+2}dx = -\int \frac{\cancel{(\sqrt{x}+2)}(\sqrt{x}-2)}{\cancel{\sqrt{x}+2}}dx =$$

$$= -\int(\sqrt{x}-2)dx = -\int\left(x^{\frac{1}{2}}-2\right)dx = -\int x^{\frac{1}{2}}dx + 2\int dx =$$

$$= -\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + 2x + c = \boxed{-\frac{2}{3}x\sqrt{x} + 2x + c}$$

87 $\int \frac{x^3-5x^2+4x}{x-1}dx =$ $\left[\frac{x^3}{3} - 2x^2 + c\right]$

$$= \int \frac{x(x^2-5x+4)}{x-1}dx = \int \frac{x\cancel{(x-1)}(x-4)}{\cancel{x-1}}dx =$$

$$= \int(x^2-4x)dx = \frac{1}{3}x^3 - \frac{4}{2}x^2 + c = \boxed{\frac{1}{3}x^3 - 2x^2 + c}$$

101 $\int(4e^x + 5\cdot 3^x)dx =$

$$= 4\int e^x\,dx + 5\int 3^x\,dx =$$

$(3^x)' = 3^x \cdot \ln 3$

basta considerare:

$3^x = e^{x\ln 3}$

$$= 4e^x + 5\cdot\frac{1}{\ln 3}\int 3^x\cdot\ln 3\,dx =$$

$$= 4e^x + \frac{5}{\ln 3}\int (3^x)'\,dx =$$

$$= \boxed{4e^x + \frac{5}{\ln 3}\cdot 3^x + c}$$

111 $\int 4^{x-1}\cdot 2^{-x+2}dx =$

$$= \int 2^{2(x-1)}\cdot 2^{-x+2}\,dx = \int 2^{2x-\cancel{2}-x+\cancel{2}}\,dx =$$

$$= \int 2^x\,dx = \frac{1}{\ln 2}\int 2^x\cdot\ln 2\,dx = \frac{1}{\ln 2}\int (2^x)'\,dx =$$

$$= \boxed{\frac{2^x}{\ln 2} + c}$$

$\int \sin x\, dx = -\cos x + c$; $\int \cos x\, dx = \sin x + c$; $\int \frac{1}{\sin^2 x} dx = -\cot x + c$; $\int \frac{1}{\cos^2 x} dx = \tan x + c$.

125 $\int \tan^2 x\, dx =$ $[\tan x - x + c]$

$(\tan x)' = 1 + \tan^2 x$

$= \int (1 + \tan^2 x - 1)\, dx =$

$= \int (1 + \tan^2 x)\, dx - \int dx = \int (\tan x)'\, dx - \int dx =$

$= \boxed{\tan x - x + c}$

126 $\int \frac{\cos 2x}{4\cos^2 x} dx =$ $\left[\frac{1}{2}x - \frac{1}{4}\tan x + c\right]$

$= \int \frac{\cos^2 x - \sin^2 x}{4\cos^2 x} dx = \frac{1}{4} \int (1 - \tan^2 x)\, dx =$

$= \frac{1}{4} \int (1 + 1 - 1 - \tan^2 x)\, dx = \frac{1}{4} \int \left(2 - (1 + \tan^2 x)\right) dx =$

$= \frac{1}{4} \int 2\, dx - \frac{1}{4} \int \underbrace{(1 + \tan^2 x)}_{(\tan x)'} dx = \frac{1}{4} \cdot 2x - \frac{1}{4} \tan x + c =$

$= \boxed{\frac{1}{2}x - \frac{1}{4}\tan x + c}$

129 $\int \frac{\cos 2x}{\sin^2 2x}dx =$ $\left[-\frac{1}{2\sin 2x}+c\right]$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$\sin 2x = 2\sin x \cos x$

$$= \int \frac{\cos^2 x - \sin^2 x}{4\sin^2 x \cos^2 x}\,dx =$$

$$= \int \frac{\cancel{\cos^2 x}}{4\sin^2 x\cancel{\cos^2 x}}\,dx - \int \frac{\cancel{\sin^2 x}}{4\cancel{\sin^2 x}\cos^2 x}\,dx =$$

$$= \frac{1}{4}\int \frac{1}{\sin^2 x}\,dx - \frac{1}{4}\int \frac{1}{\cos^2 x}\,dx =$$

$$= -\frac{1}{4}\cot x - \frac{1}{4}\tan x + c =$$

$$= -\frac{1}{4}\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right) + c = -\frac{1}{4}\,\frac{\cos^2 x + \sin^2 x}{\sin x \cos x} + c =$$

$$= \boxed{-\frac{1}{2\sin 2x} + c}$$

$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c = -\arccos x + c$; $\int \frac{1}{1+x^2}dx = \arctan x + c = -\operatorname{arccot} x + c.$

135 $\int\left(\frac{1}{x}+\frac{1}{4+4x^2}\right)dx =$ $\left[\ln|x| + \frac{1}{4}\arctan x + c\right]$

$$= \int \frac{1}{x}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx = \boxed{\ln|x| + \frac{1}{4}\arctan x + c}$$