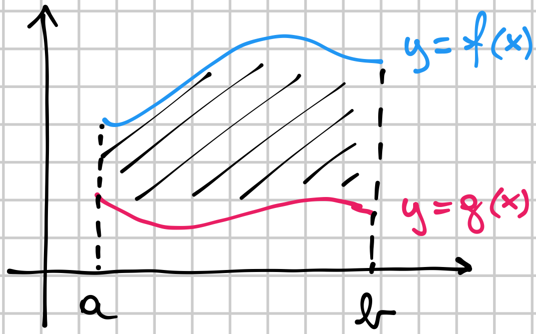
30/4/2021

AREA DELLA REGIONE TRA DUE CURVE



$f, g : [a, b] \to \mathbb{R}$ continue

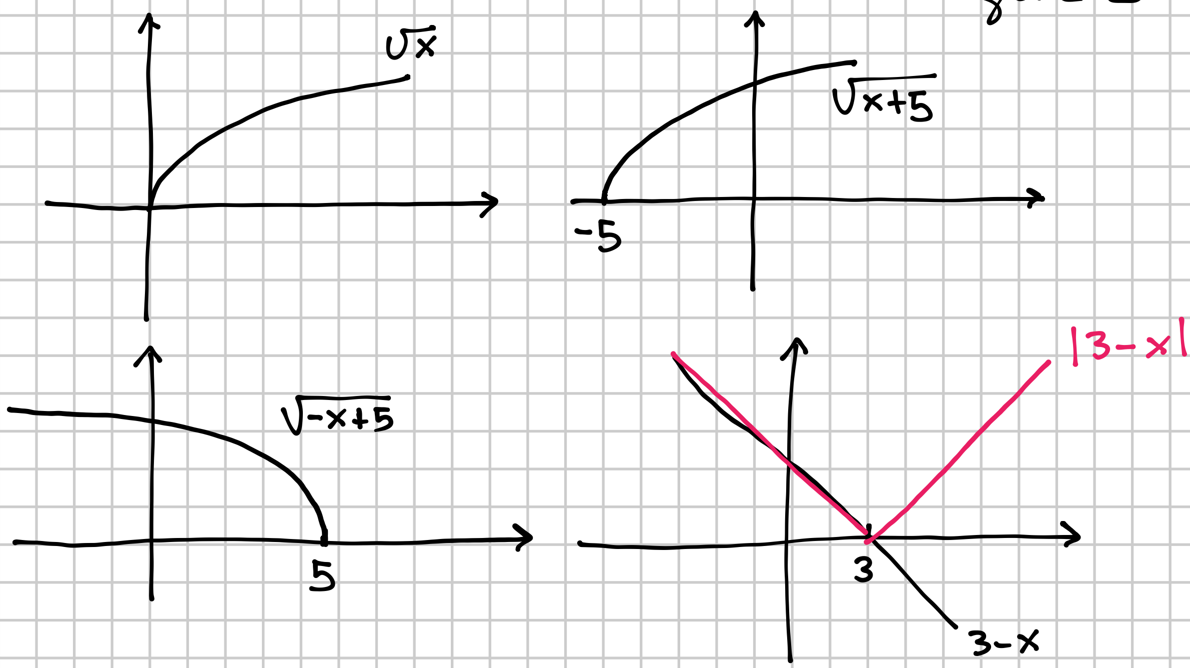
$f(x) \geq g(x) \quad \forall x \in [a, b]$

$$\mathcal{A} = \int_a^b [f(x) - g(x)]\,dx$$

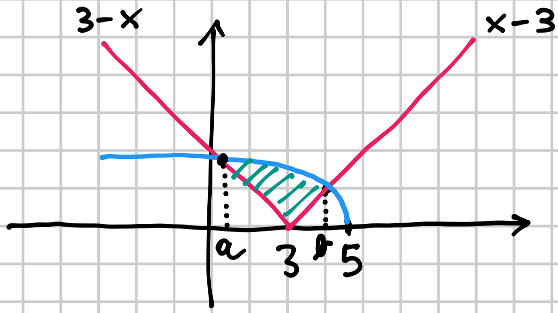
AREA FRA f E g

295 $y = \sqrt{5-x}$; $y = |3-x|$. $\left[\frac{13}{6}\right]$

Calcolare l'area della regione compresa



$$\begin{cases} y = \sqrt{5-x} \\ y = |3-x| \end{cases}$$



$5 - x \geq 0$

$x \leq 5$ $\qquad \sqrt{5-x} = |3-x| \qquad 5 - x = 9 + x^2 - 6x \qquad x^2 - 5x + 4 = 0$

$$x = \frac{5 \pm 3}{2} = \begin{cases} 1 \\ 4 \end{cases}$$

$$\mathcal{A} = \int_1^4 \left[\sqrt{5-x} - |3-x|\right] dx = \qquad |3-x| = \begin{cases} 3-x & \text{se } x \le 3 \\ x-3 & \text{se } x > 3 \end{cases}$$

$$= \int_1^3 \left[\sqrt{5-x} - (3-x)\right] dx + \int_3^4 \left[\sqrt{5-x} + (3-x)\right] dx =$$

$$= \int_1^3 \left[\sqrt{5-x} - 3 + x\right] dx + \int_3^4 \left[\sqrt{5-x} + 3 - x\right] dx =$$

$$= \left[-\frac{1}{1+\frac{1}{2}}(5-x)^{1+\frac{1}{2}} - 3x + \frac{1}{2}x^2\right]_1^3 + \left[-\frac{1}{1+\frac{1}{2}}(5-x)^{1+\frac{1}{2}} + 3x - \frac{1}{2}x^2\right]_3^4 =$$

$$= \left[-\frac{2}{3}(5-x)^{\frac{3}{2}} - 3x + \frac{1}{2}x^2\right]_1^3 + \left[-\frac{2}{3}(5-x)^{\frac{3}{2}} + 3x - \frac{1}{2}x^2\right]_3^4 =$$

$$= -\frac{2}{3}2^{\frac{3}{2}} - 9 + \frac{9}{2} - \left(-\frac{2}{3}\cdot 4^{\frac{3}{2}} - 3 + \frac{1}{2}\right) - \frac{2}{3} + 12 - 8$$

$$- \left(-\frac{2}{3}2^{\frac{2}{3}} + 9 - \frac{9}{2}\right) = -9 + \frac{2}{3}\cdot\sqrt{4^3} + 3 - \frac{1}{2} - \frac{2}{3} + 12 - 8 =$$

$$= \frac{16}{3} - 2 - \frac{1}{2} - \frac{2}{3} = \frac{14}{3} - 2 - \frac{1}{2} = \frac{28 - 12 - 3}{6} = \boxed{\frac{13}{6}}$$