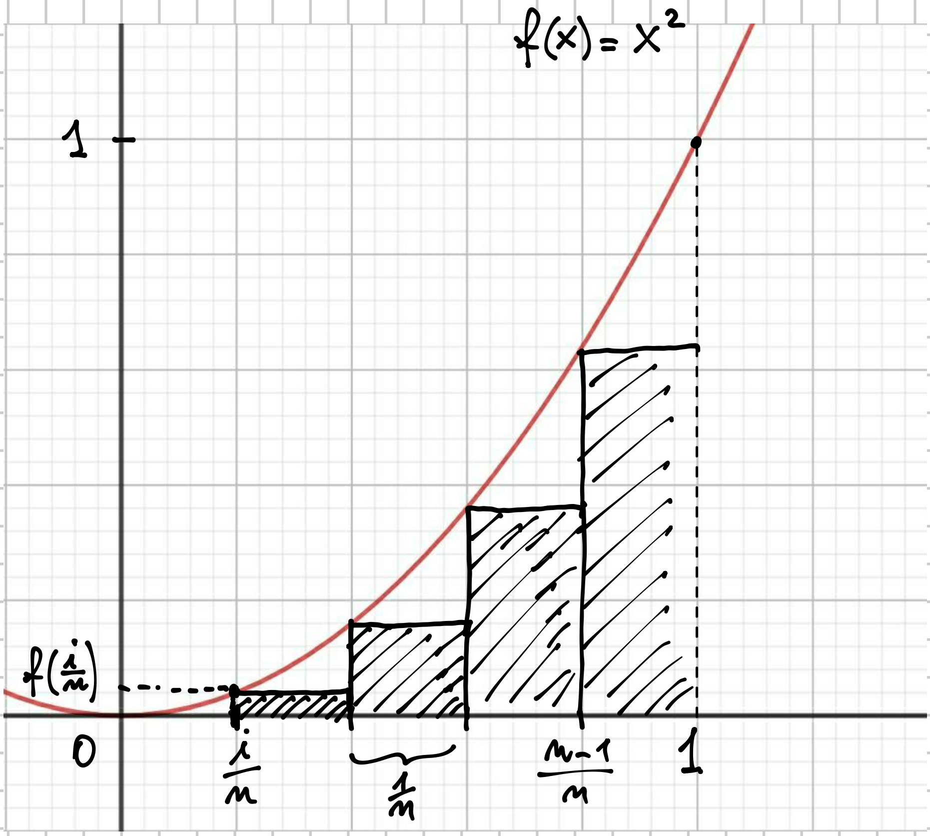
PROBLEMA: CALCOLARE L'AREA DELLA REGIONE TRA IL GRAFICO DELLA PARABOLA $y = x^2$ E L'ASSE x (TRA 0 E 1)



$$\left[0, \frac{1}{n}\right] \left[\frac{1}{n}, \frac{2}{n}\right] \left[\frac{2}{n}, \frac{3}{n}\right] \ldots \left[\frac{n-1}{n}, 1\right]$$

ESTREMI DI OGNI INTERVALLO

$$\frac{i}{n} \qquad i = 0, 1, \ldots, n-1$$

$$\downarrow$$

$$f\left(\frac{i}{n}\right)$$

Area di ciascun rettangolo $f\left(\frac{i}{n}\right) \cdot \frac{1}{n}$ $\quad i = 0, 1, \ldots, n-1$

$$\text{Area (Unione dei rettangoli)} = \sum_{i=0}^{n-1} f\left(\frac{i}{n}\right) \cdot \frac{1}{n}$$

$$\text{Area della regione} \ldots = \lim_{n \to +\infty} \sum_{i=0}^{n-1} f\left(\frac{i}{n}\right) \cdot \frac{1}{n} =$$

$$= \lim_{n \to +\infty} \sum_{i=0}^{n-1} \frac{i^2}{n^2} \cdot \frac{1}{n} = \lim_{n \to +\infty} \sum_{i=0}^{n-1} \frac{i^2}{n^3} =$$

$$= \lim_{n \to +\infty} \frac{1}{n^3} \sum_{i=0}^{n-1} i^2 =$$

SOMMA DEI PRIMI n QUADRATI

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$n=1 \qquad 1^2 = 1$

$n=2 \qquad 1^2+2^2 = 5$

$n=3 \qquad 1^2+2^2+3^2 = 14$

......

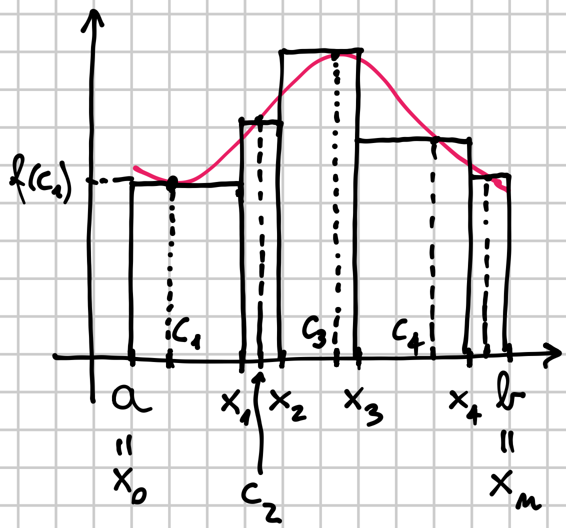
$$= \lim_{n\to+\infty} \frac{1}{n^3} \sum_{i=0}^{n-1} i^2 = \lim_{n\to+\infty} \frac{1}{n^3} \cdot \frac{(n-1)n(2n-1)}{6} =$$

$$= \lim_{n\to+\infty} \frac{2n^3 + \ldots}{6n^3} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

altri termini di grado < 3

$$\sum_{i=0}^{n-1} f\left(\frac{i}{n}\right) \cdot \Delta x \rightsquigarrow \int_0^1 f(x)\,dx$$

$f:[a,b]\to\mathbb{R}$ CONTINUA



$$\sum_a^b f(x)\Delta x = \sum_{i=1}^{n} f(c_i)\Delta x_i$$ SOMMA DI RIEMANN

$$\Delta x_i = x_i - x_{i-1} \quad i=1,\dots,n$$

INTEGRALE DEFINITO DA a A b DI $f(x)$ IN dx

$$\int_a^b f(x)\,dx = \lim_{\Delta x_{MAX}\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

Se $\Delta x_{MAX}\to 0$, tutte le somme di Riemann ottenute per ogni scelta delle suddivisioni $\{x_i\}$ dell'intervallo $[a,b]$ e dei punti $\{c_i\}$ tendono a uno stesso valore, che indico con $\int_a^b f(x)\,dx$

COME CALCOLARE GLI INTEGRALI

FORMULA FONDAMENTALE DEL CALCOLO

$$\int_a^b \underbrace{f'(x)}_{\downarrow} \, dx = f(b) - f(a)$$

CONTINUA IN $[a,b]$

$$f(b) - f(a) = \left[f(x) \right]_a^b$$

$$\int_0^1 x^2 dx = \int_0^1 \left[\frac{1}{3}x^3\right]' dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}1^3 - \frac{1}{3}\cdot 0^3 = \frac{1}{3}$$

$$\int_1^2 x^3 dx = \int_1^2 \left[\frac{1}{4}x^4\right]' dx = \left[\frac{1}{4}x^4\right]_1^2 = \frac{1}{4}\cdot 2^4 - \frac{1}{4}\cdot 1^4 = 4 - \frac{1}{4} = \frac{15}{4}$$

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N 91

$$\int_{-1}^2 x e^x dx = (*)$$

$$\int x e^x dx = \int x (e^x)' dx = x e^x - \int e^x dx = \overbrace{x e^x - e^x}^{e^x(x-1)} + c$$

$$(*) = \int_{-1}^2 \left[x e^x - e^x\right]' dx = \left[x e^x - e^x\right]_{-1}^2 = 2e^2 - e^2 - \left(-e^{-1} - e^{-1}\right) =$$

$$= e^2 + 2e^{-1} = e^2 + \frac{2}{e}$$

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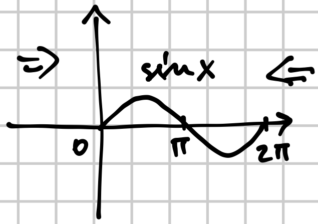
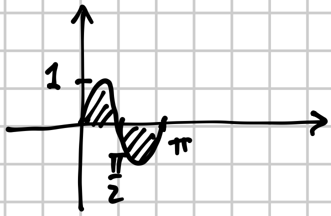
N. 84

$$\int_0^\pi \sin 2x\,dx = \int_0^\pi \left(-\frac{1}{2}\cos 2x\right)'dx = \left[-\frac{1}{2}\cos 2x\right]_0^\pi =$$





$$= -\frac{1}{2}\cos 2\pi - \left(-\frac{1}{2}\cos 0\right) =$$

$$= -\frac{1}{2} + \frac{1}{2} = 0$$

$$\int \sin 2x\,dx = \int \left(-\frac{1}{2}\cos 2x\right)'dx = -\frac{1}{2}\cos 2x + c$$

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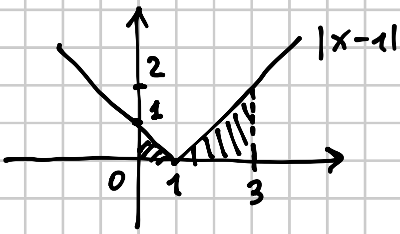
$$\int_0^3 |x-1|\,dx =$$



$$\frac{1}{2} + 2 = \frac{5}{2}$$

$$x-1 \geq 0 \Rightarrow x \geq 1$$

$$|x-1| = \begin{cases} x-1 & x \geq 1 \\ 1-x & x < 1 \end{cases}$$

$$= \int_0^1 (1-x)\,dx + \int_1^3 (x-1)\,dx =$$

$$= \int_0^1 \left[x - \frac{1}{2}x^2\right]'dx + \int_1^3 \left[\frac{1}{2}x^2 - x\right]'dx =$$

$$= \left[x - \frac{1}{2}x^2\right]_0^1 + \left[\frac{1}{2}x^2 - x\right]_1^3 = \cancel{1} - \cancel{\frac{1}{2}} + \frac{1}{2}\cdot 9 - 3 - \cancel{\frac{1}{2}} + 1 = \frac{9}{2} - 2 =$$

$$= \frac{5}{2}$$

23/4/2021

50 $\int_1^2 \left(x^2 + \frac{1}{x^2}\right) dx =$

$$= \int_1^2 \left[\frac{1}{3}x^3 - \frac{1}{x}\right]' dx = \left[\frac{1}{3}x^3 - \frac{1}{x}\right]_1^2 =$$

$$= \frac{1}{3}2^3 - \frac{1}{2} - \left(\frac{1}{3}1^3 - \frac{1}{1}\right) = \frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1 =$$

$$= \frac{7}{3} + \frac{1}{2} = \frac{14+3}{6} = \boxed{\frac{17}{6}}$$

120 $\int_0^2 \frac{x+2}{e^{x-3}} dx = (*)$

a parte mi calcolo l'integrale indefinito:

$$\int \frac{x+2}{e^{x-3}} dx = \int (x+2) \cdot e^{3-x} dx =$$

$$= \int (x+2)\left(-e^{3-x}\right)' dx = (x+2)\left(-e^{3-x}\right) - \int (x+2)'\left(-e^{3-x}\right) dx$$

$$= -(x+2)e^{3-x} + \int e^{3-x} dx = -(x+2)e^{3-x} - e^{3-x} + c$$

$$= -e^{3-x}(x+3) + c$$

$$(*) = \int_0^2 \left[-e^{3-x}(x+3)\right]' dx = \left[-e^{3-x}(x+3)\right]_0^2 = \boxed{-5e + 3e^3}$$

$$\int_a^b f'(x)\,g(x)\,dx = \left[f(x)\,g(x)\right]_a^b - \int_a^b f(x)\,g'(x)\,dx$$

$$\int_0^2 (x+2)e^{3-x}\,dx = \left[(x+2)\left(-e^{3-x}\right)\right]_0^2 - \int_0^2 \left(-e^{3-x}\right)dx =$$

$$= 4(-e) + 2e^3 + \int_0^2 e^{3-x}\,dx =$$

$$= -4e + 2e^3 + \left[-e^{3-x}\right]_0^2 = -4e + 2e^3 - e + e^3 =$$

$$= \boxed{-5e + 3e^3}$$

161 $\displaystyle\int_0^{\pi^2} \sin\sqrt{x}\,dx =$

$t = \sqrt{x}$

$t^2 = x$

$dx = 2t\,dt$

$$= \int_0^{\pi} \sin t \cdot 2t\,dt =$$

$x = \pi^2 \implies t = \sqrt{\pi^2} = \pi$

$x = 0 \implies t = \sqrt{0} = 0$

$$= 2\int_0^{\pi} t \cdot \sin t\,dt =$$

$$= 2\left(\left[t\cdot(-\cos t)\right]_0^{\pi} - \int_0^{\pi}(-\cos t)\,dt\right) =$$

$$= 2\left(\pi + \int_0^{\pi}\cos t\,dt\right) = 2\left(\pi + \left[\sin t\right]_0^{\pi}\right) =$$

$$= 2\left(\pi + \sin\pi - \sin 0\right) = 2(\pi + 0 - 0) = \boxed{2\pi}$$