14/2/2019

175 $y = \dfrac{1 - x^3 - x^5}{x^5} = \dfrac{1}{x^5} - \dfrac{\cancel{x^3}}{\cancel{x^5}\,{}^2} - \dfrac{\cancel{x^5}}{\cancel{x^5}} =$

$= \dfrac{1}{x^5} - \dfrac{1}{x^2} - 1 = x^{-5} - x^{-2} - 1$

$y' = -5x^{-6} + 2x^{-3} = \boxed{-\dfrac{5}{x^6} + \dfrac{2}{x^3}}$

176 $y = \dfrac{4 + x^4}{x^5} + \dfrac{1}{2} = \dfrac{4}{x^5} + \dfrac{\cancel{x^4}}{x^{\cancel{5}}} + \dfrac{1}{2} =$

$= 4x^{-5} + x^{-1} + \dfrac{1}{2}$

$y' = -20x^{-6} - x^{-2} = \boxed{-\dfrac{20}{x^6} - \dfrac{1}{x^2}}$

182 $y = \dfrac{1}{4}x^8 - \dfrac{2}{\sqrt{x}} + \dfrac{1}{x^3} = \dfrac{1}{4}x^8 - 2x^{-\frac{1}{2}} + x^{-3}$

$y' = \dfrac{8}{4}x^7 - 2\left(-\dfrac{1}{2}\right)x^{-\frac{1}{2}-1} - 3x^{-4} = \dfrac{8}{4}x^7 + x^{-\frac{3}{2}} - 3x^{-4} =$

$= 2x^7 + \dfrac{1}{x^{\frac{3}{2}}} - 3x^{-4} = \boxed{2x^7 + \dfrac{1}{\sqrt{x^3}} - \dfrac{3}{x^4}}$

DERIVATA DI UN PRODOTTO

$$y = f(x) \cdot g(x) \longrightarrow y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

ESEMPIO

$$y = x^2 \sin x \qquad y' = 2x \cdot \sin x + x^2 \cdot \cos x$$

$f(x) = x^2 \quad g(x) = \sin x$

204 $y = \overbrace{(e^x + 3)}^{f(x)}\overbrace{\ln x}^{g(x)}$ $\left[y' = e^x \ln x + \frac{1}{x}(e^x + 3)\right]$

$$y' = e^x \cdot \ln x + (e^x + 3) \cdot \frac{1}{x}$$

195 $y = e^x \sin x$

$$y' = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$$

209 $y = 2xe^x + (x - 2)e^x$

$$y' = 2e^x + 2xe^x + e^x + (x-2)e^x =$$

$$= \cancel{2e^x} + 2xe^x + e^x + xe^x - \cancel{2e^x} = 3xe^x + e^x =$$

$$= e^x(3x + 1)$$

SI POTEVA ANCHE FARE COSÌ:

$$y = 2xe^x + xe^x - 2e^x = 3xe^x - 2e^x = e^x(3x - 2)$$

$$y' = e^x(3x - 2) + e^x \cdot 3 = e^x(3x - 2 + 3) = e^x(3x + 1)$$

DERIVATA DEL QUOZIENTE

$$y = \frac{f(x)}{g(x)} \qquad y' = \frac{f'(x)\cdot g(x) - f(x)\,g'(x)}{[g(x)]^2}$$

ESEMPIO

$$y = \frac{\overbrace{x^2+1}^{f(x)}}{\underbrace{x^3-1}_{g(x)}}$$

$$y' = \frac{2x(x^3-1) - (x^2+1)\cdot 3x^2}{(x^3-1)^2} =$$

$$= \frac{2x^4 - 2x - 3x^4 - 3x^2}{(x^3-1)^2} =$$

$$= \frac{-x^4 - 3x^2 - 2x}{(x^3-1)^2}$$

255 $\quad y = \dfrac{x^3 - \ln x}{x} \qquad \left[y' = \dfrac{2x^3 - 1 + \ln x}{x^2} \right]$

$$y' = \frac{\left(3x^2 - \frac{1}{x}\right)\cdot x - 1\cdot(x^3 - \ln x)}{x^2} = \frac{3x^3 - 1 - x^3 + \ln x}{x^2} =$$

$$= \frac{2x^3 - 1 + \ln x}{x^2}$$

CASO IMPORTANTE: DERIVATA DELLA TANGENTE

$$y = \tan x \qquad y = \frac{\sin x}{\cos x} \qquad y' = \frac{\cos x \cdot \cos x - (-\sin x)\cdot \sin x}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \boxed{\frac{1}{\cos^2 x}}$$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \boxed{1 + \tan^2 x}$$

250 $y = \dfrac{x + \cos x}{\sin x}$

$$y' = \frac{(1 - \sin x)\sin x - \cos x(x + \cos x)}{\sin^2 x} =$$

$$= \frac{\sin x - \sin^2 x - x\cos x - \cos^2 x}{\sin^2 x} =$$

$$= \frac{\sin x - x\cos x - 1}{\sin^2 x}$$

DERIVATA DELLA FUNZIONE COMPOSTA

$y = \sin(x^2)$ $x \longmapsto x^2 \longmapsto \sin(x^2)$

$y = f(g(x))$ $x \overset{g}{\longmapsto} x^2 \overset{f}{\longmapsto} \sin(x^2)$

$g(x) = x^2 \longrightarrow g'(x) = 2x$

$f(x) = \sin x \longrightarrow f'(x) = \cos x$

$f(x^2) = \sin x^2$

$\downarrow$

$y' = f'(g(x)) \cdot g'(x)$ $y' = \cos(x^2) \cdot 2x$

$\uparrow$

DERIVATA DELLA FUNZIONE
ESTERNA CALCOLATA IN QUELLA INTERNA
MOLTIPLICATA PER LA DERIVATA DELLA FUNZIONE INTERNA

ESEMPIO

$y = e^{-x}$ $f(x) = e^x$ $g(x) = -x$

$f(g(x)) = f(-x) = e^{-x}$

$f'(x) = e^x$

$g'(x) = -1$

$y' = f'(g(x)) \cdot g'(x) = e^{-x} \cdot (-1) = -e^{-x}$

275 $y = \ln(x^2+3)$ $\left[y' = \dfrac{2x}{x^2+3}\right]$

$y' = f'(g(x)) \cdot g'(x)$

INTERNA $g(x) = x^2+3 \rightsquigarrow g'(x) = 2x$

ESTERNA $f(x) = \ln x \rightsquigarrow f'(x) = \dfrac{1}{x}$

$$y' = \underbrace{\frac{1}{x^2+3}}_{f'(g(x))} \cdot 2x = \frac{2x}{x^2+3}$$

281 $y = \sqrt{x^3 - x^2} = (x^3 - x^2)^{\frac{1}{2}}$

$f(g(x)) = (x^3 - x^2)^{\frac{1}{2}}$

EST. $f(x) = x^{\frac{1}{2}}$ $f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

INT. $g(x) = x^3 - x^2$ $g'(x) = 3x^2 - 2x$

$$y' = f'(g(x)) \cdot g'(x) = \frac{1}{2}(x^3 - x^2)^{-\frac{1}{2}} \cdot (3x^2 - 2x) =$$

$$= \frac{1}{2\sqrt{x^3 - x^2}} \cdot (3x^2 - 2x) = \frac{3x^2 - 2x}{2\sqrt{x^3 - x^2}}$$

OSSERVAZIONE

$$y = (2x^2 - 1)^2$$

1° MODO : SVILUPPO

$$y = 4x^4 - 4x^2 + 1 \qquad \boxed{y' = 16x^3 - 8x}$$

2° MODO : FORMULA DELLA COMPOSTA $\quad f(g(x)) = (2x^2 - 1)^2$

ESTERNA : $f(x) = x^2 \rightsquigarrow f'(x) = 2x$

INTERNA : $g(x) = 2x^2 - 1 \rightsquigarrow g'(x) = 4x$

$$y' = f'(g(x)) \cdot g'(x) = 2(2x^2 - 1) \cdot 4x = \boxed{16x^3 - 8x}$$

299 $\quad y = \sqrt[3]{3x + 1} = (3x+1)^{\frac{1}{3}}$

$$y' = \frac{1}{\cancel{3}}(3x+1)^{\frac{1}{3}-1} \cdot \cancel{3} = (3x+1)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(3x+1)^2}}$$

308 $y = (2x-1)^5 + \cos^2 x \quad [y' = 10(2x-1)^4 - \sin 2x]$

$$y' = 5(2x-1)^4 \cdot 2 + 2\cos x(-\sin x) =$$

$$= 10(2x-1)^4 - \sin 2x$$

372 $y = (x^3-1)^2(x+2)$

$$y' = \left[(x^3-1)^2\right]'(x+2) + (x^3-1)^2 \cdot (x+2)' =$$

$$= \left[2(x^3-1) \cdot 3x^2\right] \cdot (x+2) + (x^3-1)^2 \cdot 1 =$$

$$= 6x^2(x^3-1) \cdot (x+2) + (x^3-1)^2 =$$

$$= (x^3-1)\left[6x^2(x+2) + x^3 - 1\right] =$$

$$= (x^3-1)(6x^3 + 12x^2 + x^3 - 1) =$$

$$= (x^3-1)(7x^3 + 12x^2 - 1)$$