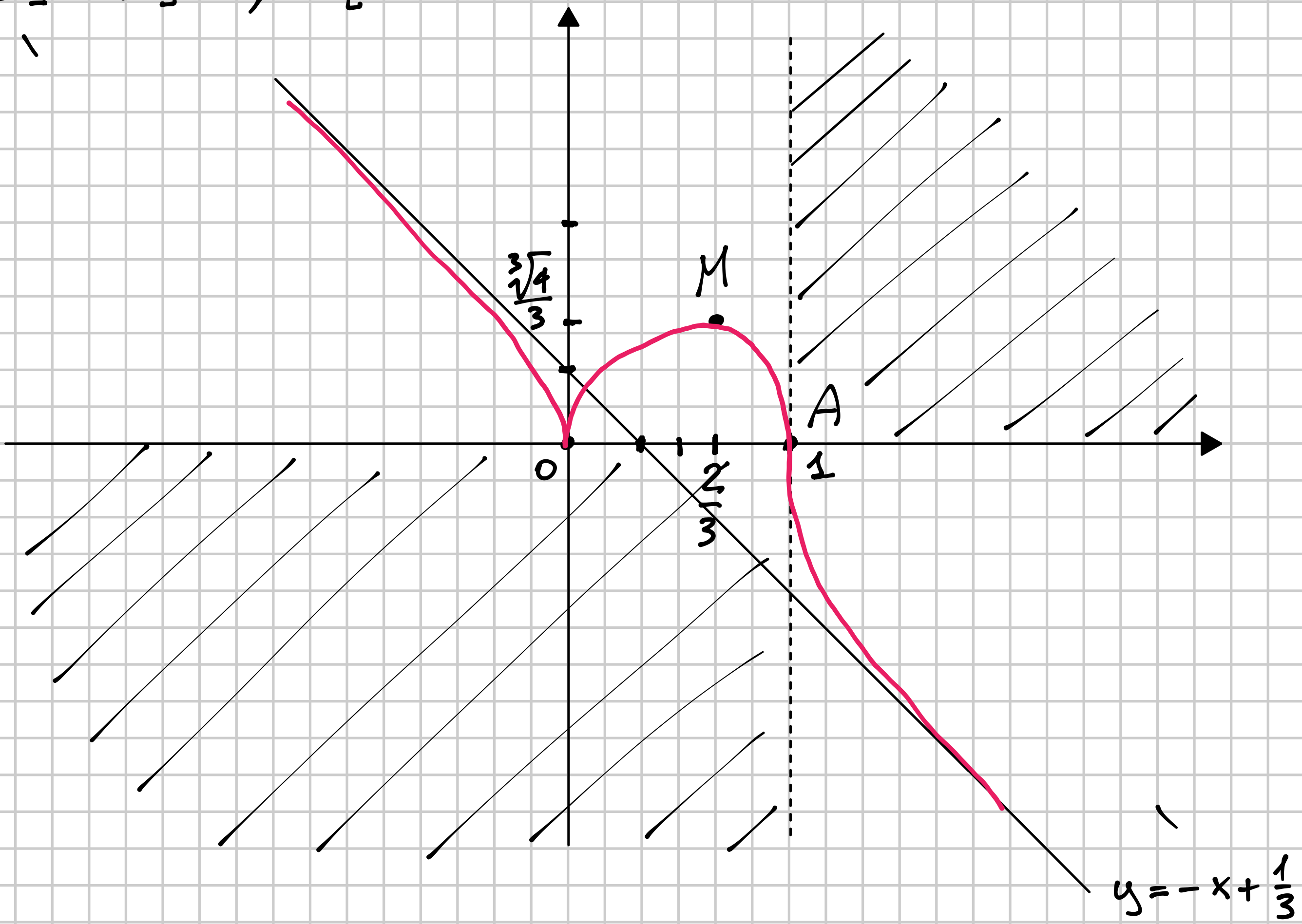
10/3/2021

127 $y=\sqrt[3]{x^2(1-x)}$

1) $D=\mathbb{R}=]-\infty,+\infty[$



2) INT. ASSI

$f(x)=0 \qquad \sqrt[3]{x^2(1-x)}=0 \Rightarrow x=0 \lor x=1$

$O(0,0) \quad A(1,0)$

3) SEGNO

$f(x)>0 \qquad x^2(1-x)>0 \Rightarrow 1-x>0 \Rightarrow x<1$

		0		1	
f	+	0	+	0	−

4) LIMITI

$\lim_{x\to-\infty} \sqrt[3]{\underbrace{x^2}_{+\infty}\underbrace{(1-x)}_{+\infty}}=+\infty \qquad \lim_{x\to+\infty} \sqrt[3]{\underbrace{x^2}_{+\infty}\underbrace{(1-x)}_{-\infty}}=-\infty$

5) DERIVATA PRIMA

$$f(x)=\sqrt[3]{x^2(1-x)}=\left[x^2-x^3\right]^{\frac{1}{3}}$$

$$f'(x)=\frac{1}{3}\left(x^2-x^3\right)^{-\frac{2}{3}}\cdot\left(2x-3x^2\right)=\frac{2x-3x^2}{3\sqrt[3]{(x^2-x^3)^2}}=$$

$$=\frac{x(2-3x)}{3\sqrt[3]{x^4(1-x)^2}}=\frac{\cancel{x}(2-3x)}{3\cancel{x}\sqrt[3]{x(1-x)^2}}=\frac{2-3x}{3\sqrt[3]{x(1-x)^2}}\qquad \begin{array}{l}x\neq 0\\ x\neq 1\end{array}$$

$$f'_+(0)=\lim_{x\to 0^+}\frac{2-3x}{3\underbrace{\sqrt[3]{x(1-x)^2}}_{0^+}}=\frac{2}{0^+}=+\infty\qquad f'_-(0)=\lim_{x\to 0^-}\frac{2-3x}{3\underbrace{\sqrt[3]{x(1-x)^2}}_{0^-}}=\frac{2}{0^-}=-\infty$$

0 è una CUSPIDE

$$f'_+(1)=\lim_{x\to 1^+}\frac{2-3x}{3\sqrt[3]{x(1-x)^2}}=\frac{-1}{0^+}=-\infty\qquad f'_-(1)=\lim_{x\to 1^-}\frac{2-3x}{3\sqrt[3]{x(1-x)^2}}=\frac{-1}{0^+}=-\infty$$

1 è un FLESSO A TANGENTE VERTICALE

$$f'(x) = \frac{2-3x}{3\sqrt[3]{x(1-x)^2}} \qquad \begin{matrix} x \neq 0 \\ x \neq 1 \end{matrix}$$

ZERI

$$2 - 3x = 0 \qquad x = \frac{2}{3} \quad \text{candidato max, min, flesso oriz.}$$

SEGNO DI f'

$$\frac{2-3x}{3\sqrt[3]{x(1-x)^2}} > 0$$

1] $2 - 3x > 0 \implies x < \frac{2}{3}$

2] $x > 0 \implies x > 0$

		0		$\frac{2}{3}$		1	
1]	$+$		$+$	0	$-$		$-$
2]	$-$	$\nexists$	$+$		$+$	$\nexists$	$+$
f'	$-$	$\nexists$	$+$	0	$-$	$\nexists$	$-$
	$\searrow$	CUSPIDE MIN	$\nearrow$	MAX	$\searrow$	FLESSO	$\searrow$

MAX $\frac{2}{3} \implies f\left(\frac{2}{3}\right) = \sqrt[3]{\left(\frac{2}{3}\right)^2\left(1-\frac{2}{3}\right)} = \sqrt[3]{\frac{4}{9}\cdot\frac{1}{3}} =$

$$= \sqrt[3]{\frac{4}{27}} = \frac{\sqrt[3]{4}}{3} \simeq 0{,}53$$

$$M\left(\frac{2}{3}, \frac{\sqrt[3]{4}}{3}\right)$$

6) DERIVATA SECONDA

$$f'(x) = \frac{2-3x}{3\sqrt[3]{x(1-x)^2}} = \qquad \begin{matrix} x \neq 0 \\ x \neq 1 \end{matrix}$$

$$= \frac{1}{3}(2-3x)\left[x(1-x)^2\right]^{-\frac{1}{3}} = \frac{1}{3}(2-3x)\left(x^3-2x^2+x\right)^{-\frac{1}{3}}$$

$$f''(x) = \frac{1}{3}\left[-3\left(x^3-2x^2+x\right)^{-\frac{1}{3}} + (2-3x)\left(-\frac{1}{3}\right)\left(x^3-2x^2+x\right)^{-\frac{4}{3}}\left(3x^2-4x+1\right)\right] =$$

$$\begin{matrix} x \neq 0 \\ x \neq 1 \end{matrix} \qquad = \frac{1}{3}\left[-\frac{3}{\sqrt[3]{x^3-2x^2+x}} - \frac{1}{3}\,\frac{(2-3x)(3x^2-4x+1)}{(x^3-2x^2+x)\sqrt[3]{x^3-2x^2+x}}\right] =$$

$$= \frac{1}{3}\left[\frac{-9(x^3-2x^2+x) - (6x^2-8x+2-9x^3+12x^2-3x)}{3(x^3-2x^2+x)\sqrt[3]{x^3-2x^2+x}}\right] =$$

$$= \frac{1}{3}\left[\frac{-\cancel{9x^3}+\cancel{18x^2}-9x-\cancel{6x^2}+8x-2+\cancel{9x^3}-\cancel{12x^2}+3x}{3(x^3-2x^2+x)\sqrt[3]{x^3-2x^2+x}}\right] =$$

$$= \frac{1}{3}\,\frac{2x-2}{3(\ldots)\sqrt[3]{\ldots}} = \frac{2}{9}\,\frac{x-1}{\underbrace{(\ldots)\sqrt[3]{\ldots}}_{>0}} \qquad \begin{matrix} x \neq 0 \\ x \neq 1 \end{matrix}$$

NON CI SONO ZERI DI f'' perché $x \neq 1$

SEGNO DI f''

		0		1	
f''	$-$	$\nexists$	$-$	$\nexists$	$+$
	$\cap$		$\cap$		$\cup$

$$f''(x) > 0 \;\Downarrow\; x > 1$$

7) RICERCA ASINTOTI

$$m = \lim_{x\to\pm\infty} \frac{\sqrt[3]{x^2(1-x)}}{x} = \lim_{x\to\pm\infty} \frac{\sqrt[3]{x^2-x^3}}{x} = \lim_{x\to\pm\infty} \frac{\sqrt[3]{x^3\left(\frac{1}{x}-1\right)}}{x} =$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{x}\sqrt[3]{\frac{1}{x}-1}}{\cancel{x}} = \sqrt[3]{-1} = -1$$

$$q = \lim_{x\to\pm\infty} \left(\sqrt[3]{x^2(1-x)} + x\right) = \lim_{x\to\pm\infty} \left(\sqrt[3]{x^2-x^3} + x\right) =$$

$$= \lim_{x\to\pm\infty} \left(\sqrt[3]{x^3\left(\frac{1}{x}-1\right)} + x\right) =$$

$$= \lim_{x\to\pm\infty} \left(\sqrt[3]{-x^3\left(1-\frac{1}{x}\right)} + x\right) =$$

Ricordare che

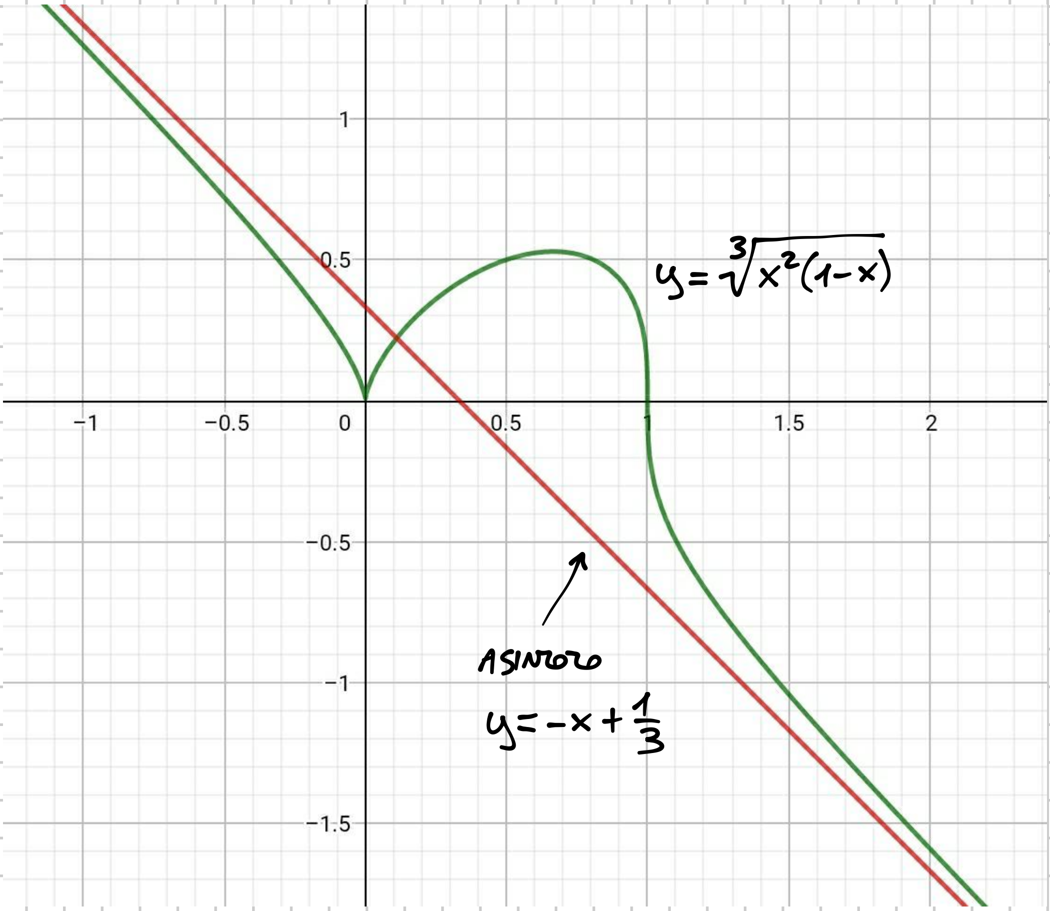
$$\boxed{\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha}$$

$$= \lim_{x\to\pm\infty} \left(-x\sqrt[3]{1-\frac{1}{x}} + x\right) = \lim_{x\to\pm\infty} -x\left[\left(1-\frac{1}{x}\right)^{\frac{1}{3}} - 1\right] =$$

$$= \lim_{x\to\pm\infty} \frac{\left(1-\frac{1}{x}\right)^{\frac{1}{3}} - 1}{-\frac{1}{x}} = \lim_{t\to 0} \frac{(1+t)^{\frac{1}{3}} - 1}{t} = \frac{1}{3}$$

$$-\frac{1}{x} = t$$

La retta $y = -x + \frac{1}{3}$ è ASINTOTO OBLIQUO per $x \to \pm\infty$


$y=\sqrt[3]{x^2(1-x)}$
ASINTOTO
$y=-x+\frac{1}{3}$