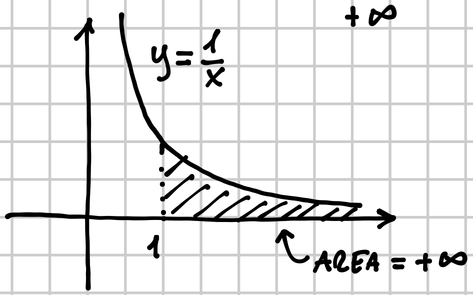
INTEGRALI IMPROPRI

1) $$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{t\to+\infty} \int_1^{t} \frac{1}{x}\,dx = \lim_{t\to+\infty} \left[\ln x\right]_1^t =$$

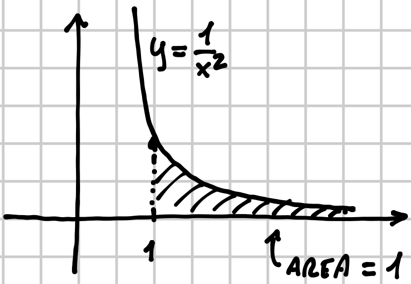
$$= \lim_{t\to+\infty} \left[\underbrace{\ln t}_{\downarrow\, +\infty} - \underbrace{\ln 1}_{\downarrow\, 0}\right] = +\infty$$

Si dice che l'integrale DIVERGE a $+\infty$



2) $$\int_1^{+\infty} \frac{1}{x^2}\,dx = \lim_{t\to+\infty} \int_1^{t} \frac{1}{x^2}\,dx = \lim_{t\to+\infty} \left[-\frac{1}{x}\right]_1^t = \lim_{t\to+\infty} \left[\underbrace{-\frac{1}{t}}_{\downarrow\, 0} + 1\right] = 1$$

L'integrale CONVERGE



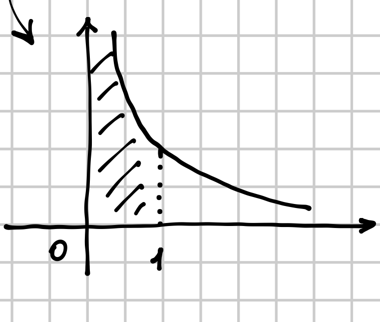
3) $$\int_0^{+\infty} \frac{1}{x}\,dx = \int_0^{1} \frac{1}{x}\,dx + \underbrace{\int_1^{+\infty} \frac{1}{x}\,dx}_{=+\infty} = +\infty$$

dunque posso dire che l'integrale diverge

in realtà anche $\int_0^1 \frac{1}{x}\,dx$ diverge a $+\infty$, infatti

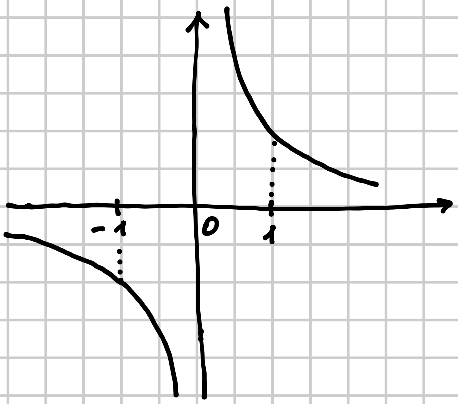
$$\int_0^1 \frac{1}{x}\,dx = \lim_{t\to0^+} \int_t^{1} \frac{1}{x}\,dx = \lim_{t\to0^+} \left[\ln x\right]_t^1 = \lim_{t\to0^+} \left[\overbrace{\ln 1}^{0} - \overbrace{\ln t}^{-\infty}\right] =$$

$$= +\infty$$



4) $\int_{-1}^{1} \frac{1}{x}\, dx = \int_{-1}^{0} \frac{1}{x}\, dx + \int_{0}^{1} \frac{1}{x}\, dx = -\infty + \infty$

L'integrale NON CONVERGE, ovvero la funzione $\frac{1}{x}$ non è integrabile fra -1 e 1

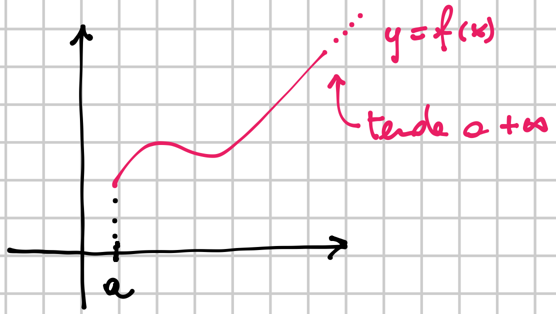


505 $\int_{2}^{+\infty} 2xe^{x^2} dx$ [diverge a $+\infty$]

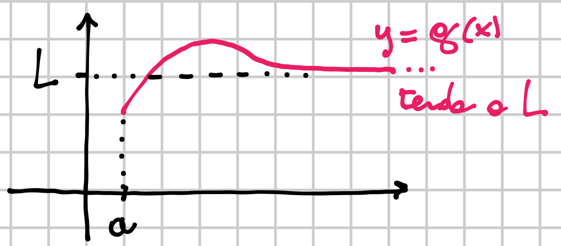
$$\int_{2}^{+\infty} 2xe^{x^2} dx = \lim_{t \to +\infty} \int_{2}^{t} 2xe^{x^2} dx = \lim_{t \to +\infty} \left[e^{x^2} \right]_2^t =$$

$$= \lim_{t \to +\infty} \left[\underbrace{e^{t^2}}_{\to +\infty} - e^4 \right] = +\infty$$

Se una funzione tende a $+\infty$ oppure a $L > 0$ per $x \to +\infty$, l'integrale di tale funzione su $[a, +\infty)$ non può convergere



$$\int_{a}^{+\infty} f(x)\, dx = +\infty$$



$$\int_{a}^{+\infty} g(x)\, dx = +\infty$$

CONDIZIONE NECESSARIA (ma non sufficiente) affinché l'integrale $\int_{a}^{+\infty} f(x)\, dx$ converga è che $\lim_{x \to +\infty} f(x) = 0$

520 $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^2-3x}\,dx$ $\left[\frac{2}{3}\ln 2\right]$

$$\int_{-\infty}^{-1} \frac{1}{x^2-3x}\,dx = \lim_{t\to-\infty} \int_{t}^{-1} \frac{1}{x^2-3x}\,dx = (**)$$

$$\int \frac{1}{x^2-3x}\,dx = \int \left[-\frac{1}{3x} + \frac{1}{3(x-3)}\right] dx = -\frac{1}{3}\int \frac{1}{x}\,dx + \frac{1}{3}\int \frac{1}{x-3}\,dx = (*)$$

$$\frac{1}{x^2-3x} = \frac{1}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3} = \frac{A(x-3)+Bx}{x(x-3)} = \frac{Ax-3A+Bx}{x(x-3)} =$$

$$= \frac{(A+B)x-3A}{x(x-3)} \qquad \begin{cases} -3A = 1 \\ A+B=0 \end{cases} \quad \begin{cases} A = -\frac{1}{3} \\ B = \frac{1}{3} \end{cases}$$

$$(*) = -\frac{1}{3}\ln|x| + \frac{1}{3}\ln|x-3| + c$$

$$(**) = \lim_{t\to-\infty} \left[-\frac{1}{3}\ln|x| + \frac{1}{3}\ln|x-3|\right]_t^{-1} =$$

$$= \lim_{t\to-\infty} \left[-\frac{1}{3}\ln|-1| + \frac{1}{3}\ln|-4| + \frac{1}{3}\ln|t| - \frac{1}{3}\ln|t-3|\right] =$$

$$= \frac{1}{3}\lim_{t\to-\infty} \left[\overbrace{\ln(1)}^{0} + \ln 4 + \ln|t| - \ln|t-3|\right] =$$

$$= \frac{1}{3}\lim_{t\to-\infty} \left[\ln 4 + \underbrace{\ln\left|\frac{t}{t-3}\right|}_{\ln 1 = 0}\right] = \frac{1}{3}\ln 4 = \frac{1}{3}\ln 2^2 = \boxed{\frac{2}{3}\ln 2}$$