Calcola i seguenti limiti

229 $\displaystyle\lim_{x \to 0} \frac{\int_0^x \cos t \, dt}{4x} = \frac{\int_0^0 \cos t \, dt}{4 \cdot 0} = \frac{0}{0}$ F.I.

$$\overset{H}{=} \lim_{x \to 0} \frac{\left(\int_0^x \cos t \, dt\right)'}{(4x)'} \xrightarrow{1^\circ \text{ TH. FOND. CALCOLO}} = \lim_{x \to 0} \frac{\cos x}{4} = \frac{1}{4}$$

232 $\displaystyle\lim_{x \to 0} \frac{\int_0^{x^3} e^{t^2} dt}{6x^3} = \frac{0}{0}$ F.I. $\left[\frac{1}{6}\right]$

$$\overset{H}{=} \lim_{x \to 0} \frac{\left(\int_0^{x^3} e^{t^2} dt\right)'}{18x^2} = \lim_{x \to 0} \frac{\cancel{3x^2} e^{x^6}}{{}_6\cancel{18x^2}} = \frac{1}{6}$$

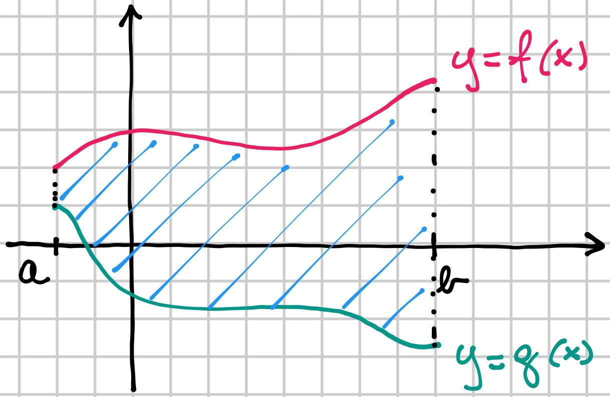
$$F(x) = \int_0^x e^{t^2} dt \qquad g(x) = x^3 \qquad F'(x) = e^{x^2} \qquad g'(x) = 3x^2$$

$$F(g(x)) = \int_0^{x^3} e^{t^2} dt \qquad [F(g(x))]' = F'(g(x)) \cdot g'(x) = e^{(x^3)^2} \cdot 3x^2 =$$

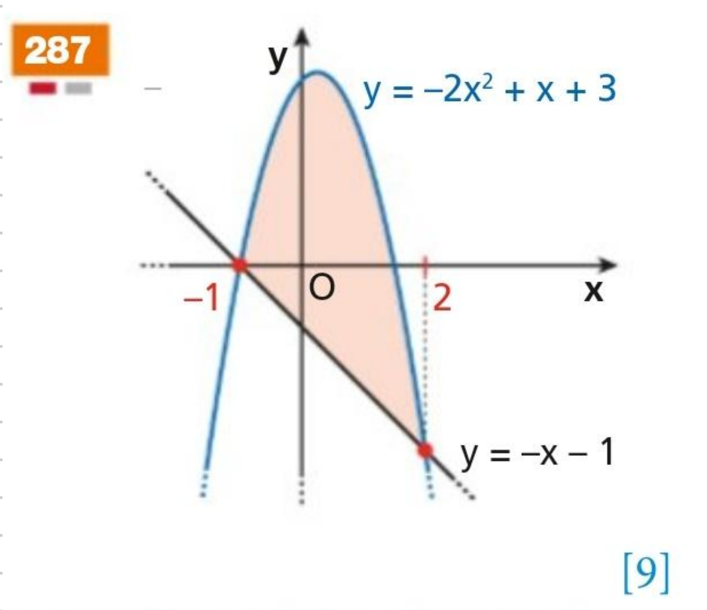
$$= 3x^2 e^{x^6}$$

AREA COMPRESA FRA DUE CURVE

$f, g : [a, b] \rightarrow \mathbb{R}$ CONTINUE $\qquad f(x) \geq g(x) \quad \forall x \in [a, b]$



$$\text{Area tra le 2 curve} = \int_a^b [f(x) - g(x)]\, dx$$



$$\text{Area} = \int_{-1}^{2} \left(-2x^2 + x + 3 - (-x-1)\right) dx =$$

$$= \int_{-1}^{2} \left(-2x^2 + x + 3 + x + 1\right) dx =$$

$$= \int_{-1}^{2} \left(-2x^2 + 2x + 4\right) dx =$$

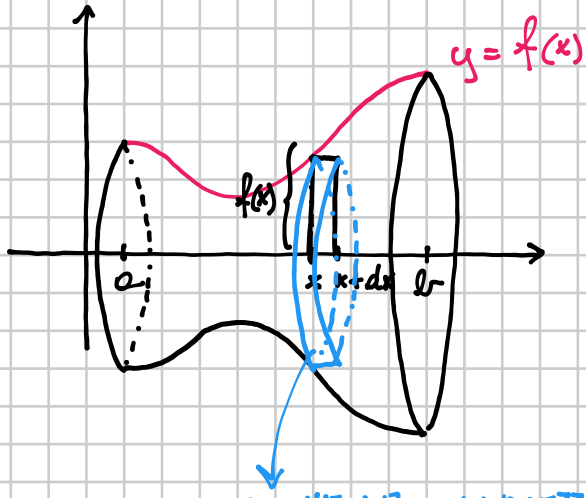
$$= 2\int_{-1}^{2} \left(-x^2 + x + 2\right) dx = 2\left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right]_{-1}^{2} =$$

$$= 2\left[-\frac{1}{3}\cdot 8 + \frac{1}{2}\cdot 4 + 4 + \frac{1}{3}(-1) - \frac{1}{2} + 2\right] = 2\left[-\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2\right] =$$

$$= 2\left[5 - \frac{1}{2}\right] = 2\,\frac{10-1}{2} = \boxed{9}$$

VOLUME DI UN SOLIDO DI ROTAZIONE

$f : [a, b] \to \mathbb{R}$



↘ ROTAZIONE ATTORNO ALL'ASSE x DI 360°

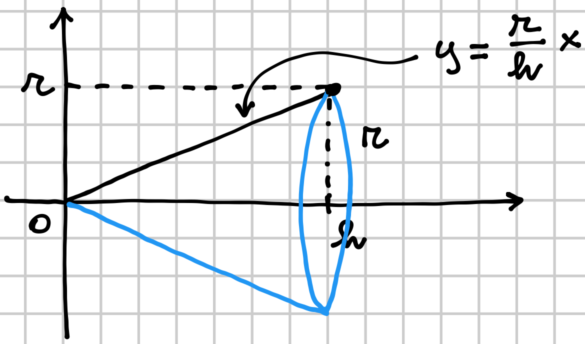
VOLUME DEL CILINDRETTO INFINITESIMO

$$dV = \underbrace{f^2(x) \cdot \pi}_{\text{AREA DI BASE}} \cdot \underbrace{dx}_{\text{ALTEZZA}}$$

VOLUME SOLIDO $\quad V = \int_a^b dV = \int_a^b f^2(x) \pi \, dx$

$$\boxed{V = \pi \int_a^b f^2(x) \, dx}$$

ESEMPIO - VOLUME DEL CONO



$$\text{VOLUME CONO} = \pi \int_0^h \left(\frac{r}{h} x\right)^2 dx =$$

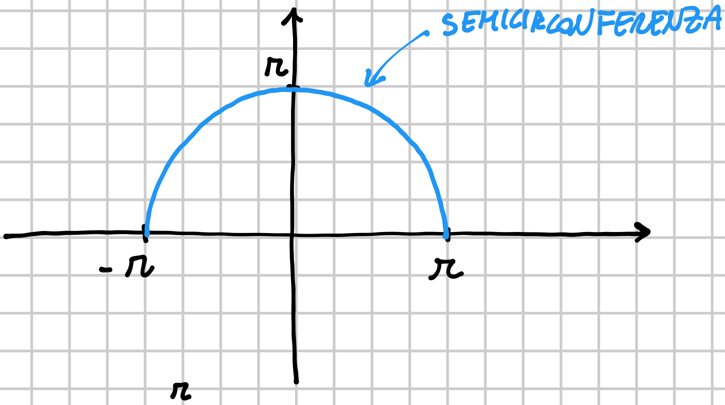
$$= \pi \int_0^h \frac{r^2}{h^2} x^2 \, dx =$$

$$= \pi \frac{r^2}{h^2} \int_0^h x^2 \, dx =$$

$$= \pi \frac{r^2}{h^2} \left[\frac{1}{3} x^3 \right]_0^h =$$

$$= \pi \frac{r^2}{\cancel{h^2}} \frac{1}{3} h^{\cancel{3}} = \frac{1}{3} \pi r^2 h$$

ESEMPIO - VOLUME SFERA



$x^2+y^2=r^2$ CIRC. CENTRO O RAGGIO r

$y^2=r^2-x^2$

$y=\sqrt{r^2-x^2}$ SEMICIRC. SUPERIORE

$$\text{VOLUME SFERA} = \pi\int_{-r}^{r}(r^2-x^2)\,dx = \pi\left[r^2x-\frac{1}{3}x^3\right]_{-r}^{r} =$$

$$= \pi\left[r^3-\frac{1}{3}r^3-\left(-r^3+\frac{1}{3}r^3\right)\right] =$$

$$= \pi\left[r^3-\frac{1}{3}r^3+r^3-\frac{1}{3}r^3\right] = \pi\,\frac{3-1+3-1}{3}\,r^3 =$$

$$= \frac{4}{3}\pi r^3$$