

1/3/2021

Three max, min, flessi

395

$$y = \frac{x^2 - 4x + 5}{1 - |x - 1|}$$

$$D: 1 - |x - 1| \neq 0 \quad |x - 1| \neq 1 \quad x - 1 \neq \pm 1$$

$$x \neq 0 \wedge x \neq 2$$

$$D =]-\infty, 0[\cup]0, 2[\cup]2, +\infty[$$

$$f(x) = \frac{x^2 - 4x + 5}{1 - |x - 1|}$$

$$f(x) = \begin{cases} \frac{x^2 - 4x + 5}{1 - (x - 1)} & \text{se } x - 1 \geq 0 \wedge x \neq 2 \\ \frac{x^2 - 4x + 5}{1 + (x - 1)} & \text{se } x - 1 < 0 \wedge x \neq 0 \end{cases}$$

$$f(x) = \begin{cases} \frac{x^2 - 4x + 5}{2 - x} & \text{se } x \geq 1 \wedge x \neq 2 \\ \frac{x^2 - 4x + 5}{x} & \text{se } x < 1 \wedge x \neq 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{(2x - 4)(2 - x) + (x^2 - 4x + 5)}{(2 - x)^2} = \frac{-x^2 + 4x - 3}{(2 - x)^2} \\ \frac{(2x - 4)x - x^2 + 4x - 5}{x^2} = \frac{2x^2 - 4x - x^2 + 4x - 5}{x^2} = \frac{x^2 - 5}{x^2} \end{cases}$$

$$f'(x) = \begin{cases} \frac{-x^2+4x-3}{(2-x)^2} & \text{se } x > 1 \wedge x \neq 2 \\ 1 - \frac{5}{x^2} & \text{se } x < 1 \wedge x \neq 0 \end{cases}$$

$$f'_+(1) = \lim_{x \rightarrow 1^+} \frac{-x^2+4x-3}{(2-x)^2} = -1+4-3=0$$

$$f'_-(1) = \lim_{x \rightarrow 1^-} \left(1 - \frac{5}{x^2}\right) = 1-5=-4$$

1 è un punto angoloso

ZERI DI f'

$$f'(x) = 0$$

$$\begin{cases} \frac{-x^2+4x-3}{(2-x)^2} = 0 \Rightarrow -x^2+4x-3=0 \\ x > 1 \wedge x \neq 2 \end{cases}$$

$$x^2-4x+3=0$$

$$(x-3)(x-1)=0 \begin{matrix} \nearrow x=3 \\ \searrow x=1 \text{ N.A.} \end{matrix}$$

$$\begin{cases} 1 - \frac{5}{x^2} = 0 \Rightarrow x^2 = 5 \\ x < 1 \wedge x \neq 0 \end{cases}$$

$$x = \pm\sqrt{5}$$

$$\Downarrow \\ x = -\sqrt{5}$$

$$(x = +\sqrt{5} \text{ N.A.})$$

SEGNO DI f'

$$f'(x) > 0$$

$$\begin{cases} \frac{-x^2+4x-3}{(2-x)^2} > 0 \Rightarrow x^2-4x+3 < 0 \\ x > 1 \wedge x \neq 2 \end{cases}$$

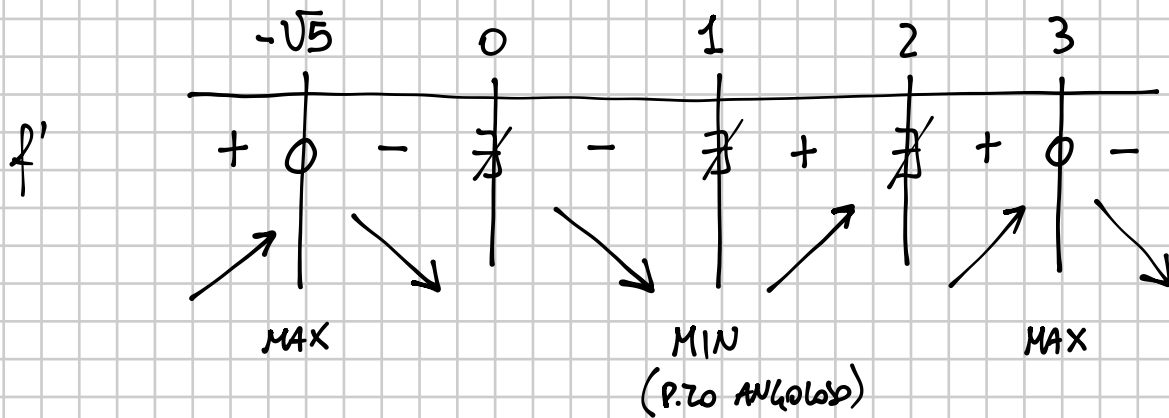
$$1 < x < 3 \wedge x \neq 2$$

$$\begin{cases} 1 - \frac{5}{x^2} > 0 \Rightarrow \frac{5}{x^2} < 1 & 5 < x^2 \\ & x^2 > 5 \end{cases}$$

$$x < 1 \wedge x \neq 0$$

$$\begin{cases} x < -\sqrt{5} \vee x > \sqrt{5} \\ x < 1 \wedge x \neq 0 \end{cases}$$

$$\Downarrow \\ x < -\sqrt{5}$$



$$f'(x) = \begin{cases} \frac{-x^2 + 4x - 3}{(2-x)^2} & \text{se } x > 1 \wedge x \neq 2 \\ 1 - \frac{5}{x^2} & \text{se } x < 1 \wedge x \neq 0 \end{cases}$$

$$f''(x) = \begin{cases} \frac{(-2x+4)(2-x)^2 + 2(2-x)(-x^2+4x-3)}{(2-x)^4} = (*) \\ \frac{10}{x^3} \end{cases}$$

$$(*) = \frac{(-2x+4)(4+x^2-4x) + (4-2x)(-x^2+4x-3)}{(2-x)^4} =$$

$$= \frac{-8x - 2x^3 + 8x^2 + 16 + 4x^2 - 16x - 4x^2 + 16x - 12 + 2x^3 - 8x^2 + 6x}{(2-x)^4} =$$

$$= \frac{4-2x}{(2-x)^4}$$

$$f''(x) = \begin{cases} \frac{4-2x}{(2-x)^4} & x > 1 \wedge x \neq 2 \\ \frac{10}{x^3} & x < 1 \wedge x \neq 0 \end{cases}$$

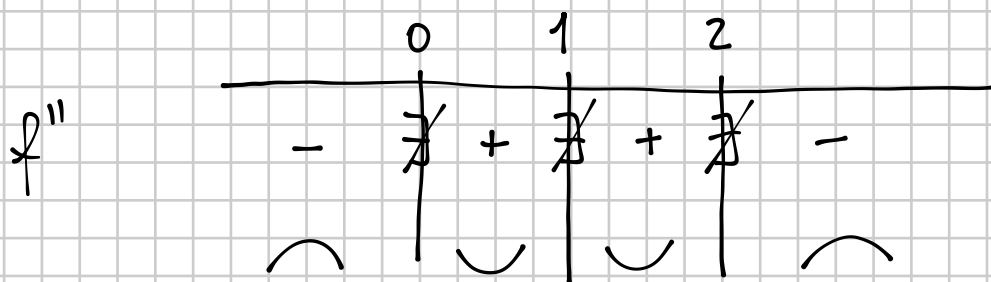
Non ci sono zeri di f''

$$f''(x) = \begin{cases} \frac{4-2x}{(2-x)^4} & x > 1 \wedge x \neq 2 \\ \frac{10}{x^3} & x < 1 \wedge x \neq 0 \end{cases}$$

SEGNO DI f''

$$\begin{cases} \frac{4-2x}{(2-x)^4} > 0 \Rightarrow 4-2x > 0 & x < 2 \\ x > 1 \wedge x \neq 2 \end{cases} \Rightarrow 1 < x < 2$$

$$\begin{cases} \frac{10}{x^3} > 0 \\ x < 1 \wedge x \neq 0 \end{cases} \begin{cases} x > 0 \\ x < 1 \wedge x \neq 0 \end{cases} \Rightarrow 0 < x < 1$$



f ha la concavità verso il basso in $]-\infty, 0[$ e in $]2, +\infty[$

f ha la concavità verso l'alto in $]0, 2[$ (1 punto angoloso)

Non ci sono punti di flesso

