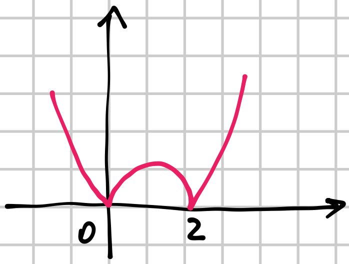
24/2/2021

Trovare max, min, flessi orizz.

178 $y = |2x^2 - 4x|$

[$x = 0$ min (p. ang.); $x = 2$ min (p. ang.); $x = 1$ max]

$D = \mathbb{R}$ $\qquad f(x) = 2\,|x(x-2)|$



$2x^2 - 4x \geq 0$

$2x(x-2) \geq 0 \qquad x \leq 0 \ \vee \ x \geq 2$

$$f(x) = \begin{cases} 2x^2 - 4x & \text{se } x \leq 0 \ \vee \ x \geq 2 \\ -2x^2 + 4x & \text{se } 0 < x < 2 \end{cases}$$

$$f'(x) = \begin{cases} 4x - 4 & \text{se } x < 0 \ \vee \ x > 2 \\ -4x + 4 & \text{se } 0 < x < 2 \end{cases}$$

$$f'_-(0) = \lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} (4x - 4) = -4$$
$$f'_+(0) = \lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (-4x + 4) = 4$$

0 è punto angoloso

$$f'_-(2) = \lim_{x \to 2^-} f'(x) = \lim_{x \to 2^-} (-4x + 4) = -4$$
$$f'_+(2) = \lim_{x \to 2^+} f'(x) = \lim_{x \to 2^+} (4x - 4) = 4$$

2 è p.to angoloso

$f'(x) = 0$

$$f'(x) = \begin{cases} 4x - 4 & \text{se} \quad x < 0 \ \vee \ x > 2 \\ -4x + 4 & \text{se} \quad 0 < x < 2 \end{cases}$$

$$\begin{cases} 4x - 4 = 0 \\ x < 0 \ \vee \ x > 2 \end{cases} \ \vee \ \begin{cases} -4x + 4 = 0 \\ 0 < x < 2 \end{cases}$$

$$\begin{cases} x = 1 \\ x < 0 \ \vee \ x > 2 \end{cases} \ \vee \ \begin{cases} x = 1 \\ 0 < x < 2 \end{cases}$$

$$\emptyset \qquad\qquad \Downarrow \quad \boxed{x = 1}$$

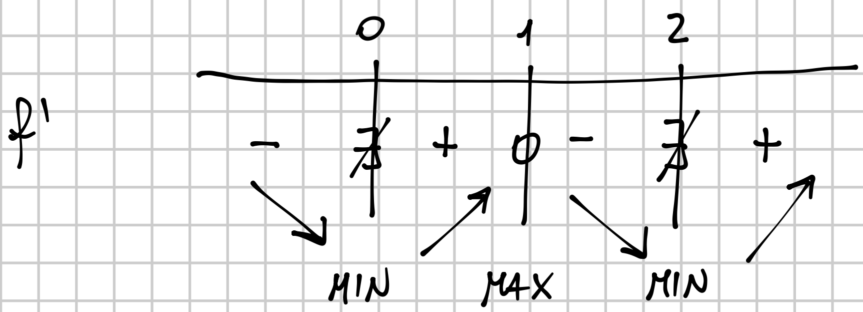
1 candidato max
min
flesso orizz.

$f'(x) > 0$

$$\begin{cases} 4x - 4 > 0 \\ x < 0 \ \vee \ x > 2 \end{cases} \ \vee \ \begin{cases} -4x + 4 > 0 \\ 0 < x < 2 \end{cases}$$

$$\begin{cases} x > 1 \\ x < 0 \ \vee \ x > 2 \end{cases} \ \vee \ \begin{cases} x < 1 \\ 0 < x < 2 \end{cases}$$

$$x > 2 \quad \vee \quad 0 < x < 1$$



0, 2 p.ti di minimo (p.ti angolosi)

1 p.to di massimo

391 $y = e^{\frac{x-1}{x+2}}$

Trovare max, min, flessi

$D =]-\infty, -2[\cup]-2, +\infty[$

$$f'(x) = e^{\frac{x-1}{x+2}} \cdot \left(\frac{x-1}{x+2}\right)' = e^{\frac{x-1}{x+2}} \cdot \frac{\cancel{x}+2-\cancel{x}+1}{(x+2)^2} =$$

$$= e^{\frac{x-1}{x+2}} \cdot \frac{3}{(x+2)^2} > 0 \quad \forall x \in D$$

$\Downarrow$

quindi f è strett. crescente in $]-\infty, -2[$ e in $]-2, +\infty[$

$$f''(x) = \left(e^{\frac{x-1}{x+2}}\right)' \cdot \frac{3}{(x+2)^2} + e^{\frac{x-1}{x+2}} \cdot \left(\frac{3}{(x+2)^2}\right)' =$$

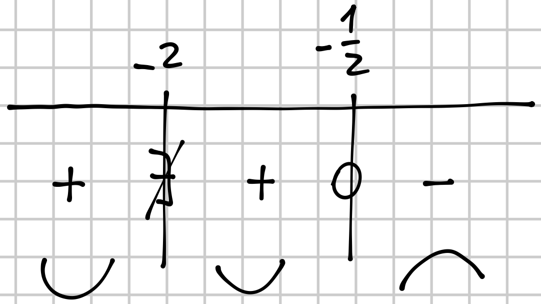
$$= e^{\frac{x-1}{x+2}} \cdot \frac{3}{(x+2)^2} \cdot \frac{3}{(x+2)^2} + e^{\frac{x-1}{x+2}} \cdot \frac{-6}{(x+2)^3} =$$

$$= e^{\frac{x-1}{x+2}} \cdot \frac{3}{(x+2)^3}\left[\frac{3}{x+2} - 2\right] =$$

$$= \frac{3e^{\frac{x-1}{x+2}}}{(x+2)^2} \cdot \frac{1}{x+2}\left[\frac{3-2x-4}{x+2}\right] = \frac{3e^{\frac{x-1}{x+2}}}{(x+2)^2} \cdot \frac{-1-2x}{(x+2)^2}$$

$f''(x) = 0 \Rightarrow -1-2x = 0 \Rightarrow x = -\frac{1}{2}$ candidato flesso

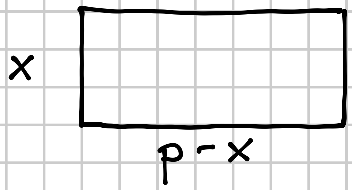
$$f''(x) > 0 \implies \begin{cases} -1-2x > 0 \\ x \neq -2 \end{cases} \qquad \begin{cases} x < -\frac{1}{2} \\ x \neq -2 \end{cases}$$



P.TO DI FLESSO

$-\frac{1}{2}$ è punto di flesso (obliquo)

PROBLEMA: Fra tutti i rettangoli di perimetro $2p$ dato, trovare quello di area massima.



$0 < x < p$

funzione AREA $\quad \mathcal{A}(x) = x(p-x) \qquad D =]0, p[$

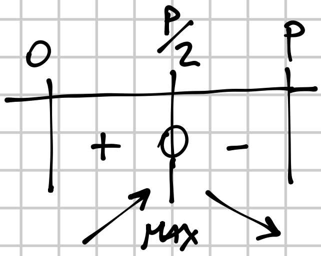
Studio la funzione $\mathcal{A}$ per vedere in corrispondenza di quale x c'è il max

$$\mathcal{A}(x) = px - x^2$$

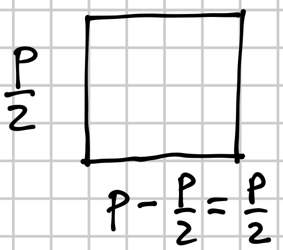
$$\mathcal{A}'(x) = p - 2x$$

$\mathcal{A}'(x) = 0 \qquad p - 2x = 0 \qquad x = \frac{p}{2}$ CANDIDATO MAX

$\mathcal{A}'(x) > 0 \qquad p - 2x > 0 \qquad x < \frac{p}{2}$



L'area max si ha quando $x = \frac{p}{2}$



$\Rightarrow$ il rettangolo di area massima è il quadrato