612 Studia il fascio di rette di equazione $(k + 1) x + (2 - 3k) y - 7 + 3k = 0$ e determina:

  a. le rette parallele agli assi cartesiani;

  b. la retta del fascio parallela alla retta di equazione $y = x - 3$;

  c. la retta passante per il punto $A(4; 1)$;

  d. le rette che hanno distanza dall'origine uguale a $\frac{4}{5}\sqrt{5}$.

$$[a) y = 2; x = 3; b) x - y - 1 = 0; c) x + y - 5 = 0; d) 2x - y - 4 = 0, 2x - 29y + 52 = 0]$$

a) $\parallel$ asse x $\Rightarrow$ $k + 1 = 0$ $\qquad K = -1$

$$\Downarrow$$

$$(2 + 3) y - 7 - 3 = 0$$

$$5y - 10 = 0 \Rightarrow \boxed{y = 2}$$

$\parallel$ asse y $\Rightarrow$ $2 - 3k = 0$ $\qquad K = \frac{2}{3}$

$$\Downarrow$$

$$\left(1 + \frac{2}{3}\right) x - 7 + 2 = 0$$

$$\frac{5}{3} x - 5 = 0 \Rightarrow \boxed{x = 3}$$

CENTRO DEL FASCIO

$C(3, 2)$

b) $\parallel$ $y = x - 3$ $\quad m = 1$

$$-\underbrace{\frac{k + 1}{2 - 3k}}_{\substack{\text{coeff. angolare} \\ \text{(generico) del fascio}}} = 1 \qquad -k - 1 = 2 - 3k$$

$$2K = 3 \qquad K = \frac{3}{2}$$

$$\Downarrow$$

$$\left(\frac{3}{2} + 1\right) x + \left(2 - \frac{9}{2}\right) y - 7 + \frac{9}{2} = 0$$

$$\frac{5}{2} x - \frac{5}{2} y - \frac{5}{2} = 0$$

$$\Downarrow$$

$$x - y - 1 = 0 \Rightarrow \boxed{y = x - 1}$$

c) $A(4,1)$

$$(k+1)x + (2-3k)y - 7 + 3k = 0$$

$$(k+1) \cdot 4 + (2-3k) \cdot 1 - 7 + 3k = 0$$

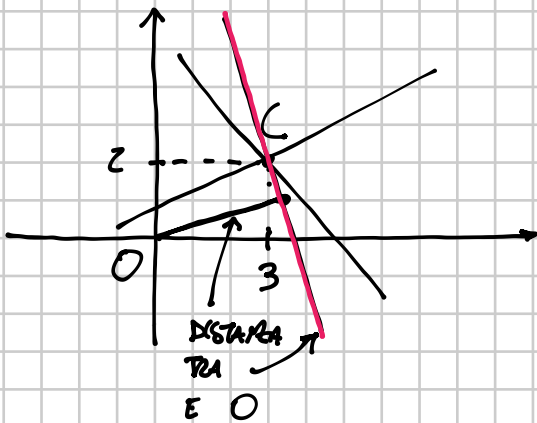$$4k + 4 + 2 - \cancel{3k} - 7 + \cancel{3k} = 0$$

$$4k - 1 = 0 \qquad k = \frac{1}{4}$$

$$\left(\frac{1}{4}+1\right)x + \left(2-\frac{3}{4}\right)y - 7 + \frac{3}{4} = 0$$

$$\frac{5}{\cancel{4}}x + \frac{5}{\cancel{4}}y - \frac{25}{\cancel{4}} = 0$$

$$x + y - 5 = 0 \qquad \boxed{y = -x + 5}$$

d) Le rette che distano $\frac{4}{5}\sqrt{5}$ da $O(\overset{x_0}{0},\overset{y_0}{0})$

DISTANZA DI r: $ax + by + c = 0$ DA $P(x_0, y_0)$ È $\dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

$$\underbrace{(k+1)}_{a}x + \underbrace{(2-3k)}_{b}y \underbrace{- 7 + 3k}_{c} = 0$$

$$\frac{\left|(k+1)\cdot 0 + (2-3k)\cdot 0 - 7 + 3k\right|}{\sqrt{(k+1)^2 + (2-3k)^2}} = \frac{4}{5}\sqrt{5}$$

$$\frac{|-7+3k|}{\sqrt{(k+1)^2 + (2-3k)^2}} = \frac{4}{5}\sqrt{5}$$

$$|-7+3k| = \frac{4}{5}\sqrt{5} \cdot \sqrt{(k+1)^2 + (2-3k)^2}$$

$\downarrow$ elevo al quadrato

$$(-7+3k)^2 = \frac{16}{5}\left[(k+1)^2 + (2-3k)^2\right]$$

DISTANZA
TRA
r E O

$$(-7+3K)^2 = \frac{16}{5}\left[(K+1)^2 + (2-3K)^2\right]$$

$$5\left(49 + 9K^2 - 42K\right) = 16\left[K^2 + 1 + 2K + 4 + 9K^2 - 12K\right]$$

$$5\left(49 + 9K^2 - 42K\right) = 16\left[10K^2 - 10K + 5\right]$$

$$\cancel{5}\left(49 + 9K^2 - 42K\right) = 16 \cdot \cancel{5} \cdot \left[2K^2 - 2K + 1\right]$$

$$49 + 9K^2 - 42K = 32K^2 - 32K + 16$$

$$23K^2 + 10K - 33 = 0 \qquad\qquad \frac{\Delta}{4} = 25 + 759 = 784 = 28^2$$

$$K = \frac{-5 \pm 28}{23} = \begin{cases} -\dfrac{33}{23} \\[2mm] \dfrac{23}{23} = 1 \end{cases}$$

$$(K+1)x + (2-3K)y - 7 + 3K = 0 \quad \text{eq. fascis}$$

$K = -\dfrac{33}{23}$

$$-\frac{10}{23}x + \left(2 + \frac{99}{23}\right)y - 7 - \frac{99}{23} = 0$$

$$-\frac{10}{\cancel{23}}x + \frac{145}{\cancel{23}}y - \frac{260}{\cancel{23}} = 0$$

$$-2x + 29y - 52 = 0 \qquad\qquad \boxed{2x - 29y + 52 = 0}$$

$K = 1$

$$\boxed{2x - y - 4 = 0}$$