9/1/2019

Da dimostrare: $|z_1 \cdot z_2| = |z_1| \cdot |z_2| \quad \forall z_1, z_2 \in \mathbb{C}$

$z_1 = a + ib$

$z_2 = c + id$

$$z_1 \cdot z_2 = (a+ib)(c+id) = $$
$$= (ac - bd) + i(ad + bc)$$

$$|z_1 \cdot z_2| = |(ac-bd) + i(ad+bc)| = \sqrt{(ac-bd)^2 + (ad+bc)^2} =$$

$$= \sqrt{a^2c^2 + b^2d^2 - \cancel{2abcd} + a^2d^2 + b^2c^2 + \cancel{2abcd}} =$$

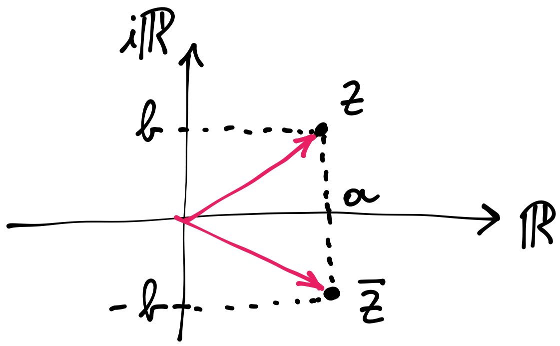
$$= \sqrt{a^2(c^2+d^2) + b^2(d^2+c^2)} =$$

$$= \sqrt{(a^2+b^2)(c^2+d^2)} = \sqrt{a^2+b^2} \cdot \sqrt{c^2+d^2} = |z_1| \cdot |z_2|$$

DEFINIZIONE

Dato un numero complesso $z = a + ib$, il suo **CONIUGATO** è il numero complesso $\bar{z} = a - ib$.

Inoltre il numero $-z = -a - ib$ si chiama l'opposto di z



$$|\bar{z}| = |z|$$

$$z \cdot \bar{z} = (a+ib)(a-ib) =$$
$$= a^2 - (ib)^2 = a^2 + b^2$$
$$= |z|^2$$

$$\bar{\bar{z}} = z$$

DIVISIONE DI NUMERI COMPLESSI

$$\frac{3-2i}{5+3i} = \frac{3-2i}{5+3i} \cdot \frac{5-3i}{5-3i} = \frac{15-9i-10i-6}{25+9} =$$

$$= \frac{9-19i}{34} = \frac{9}{34} - \frac{19}{34}i$$

RECIPROCO

$$\frac{1}{3-2i} = \frac{1}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{3+2i}{9+4} = \frac{3}{13} + \frac{2}{13}i$$

183 $\frac{4i}{1-2i} + \frac{1-i}{1+2i} + \frac{12}{5} =$ $\left[\frac{3+i}{5}\right]$

$$= \frac{20i(1+2i) + 5(1-2i)(1-i) + 12(1-2i)(1+2i)}{5(1-2i)(1+2i)} =$$

$z \cdot \bar{z} = |z|^2$

$$= \frac{20i - 40 + 5(1-i-2i-2) + 12(1^2+2^2)}{5(1^2+2^2)} =$$

$$= \frac{20i - 40 - 5 - 15i + 60}{25} = \frac{15+5i}{25} = \frac{15}{25} + \frac{5}{25}i =$$

$$= \boxed{\frac{3}{5} + \frac{1}{5}i}$$

189 $\dfrac{1}{2-i}+\dfrac{1-i}{i(1+i)}=$ $\left[\dfrac{i-3}{5}\right]$

$$=\frac{1}{2-i}\cdot\frac{2+i}{2+i}+\frac{1-i}{i-1}=\frac{2+i}{4+1}+\frac{\cancel{1-i}}{-(\cancel{1-i})}=$$

$$=\frac{2+i}{5}-1=\frac{2+i-5}{5}=\frac{i-3}{5}=-\frac{3}{5}+\frac{1}{5}i$$

191 $\dfrac{(2i)^2-(1+i)^2}{i(2+3i)}-i(2-i)=$ $\left[\dfrac{-5-12i}{13}\right]$

$$=\frac{4i^2-(\cancel{1}+\cancel{i^2}+2i)}{2i+3i^2}-2i+i^2=$$

$$=\frac{-4-2i}{2i-3}-2i-1=\frac{-4-2i}{-3+2i}\cdot\frac{-3-2i}{-3-2i}-2i-1=$$

$$=\frac{12+8i+6i-4}{9+4}-2i-1=\frac{8+14i}{13}-2i-1=$$

$$=\frac{8+14i-26i-13}{13}=\frac{-5-12i}{13}=\boxed{-\frac{5}{13}-\frac{12}{13}i}$$

195

$$\frac{18i^{18}+7i^{6}}{(2i^{52}+i^{53})^{2}} : \frac{4i^{36}-2i^{20}}{(2i^{8}+i^{7}-i^{20})^{2}} = (*)$$

$i^2=-1$

$i^3=-i$

$i^4=1$

$i^5=i$

$\vdots$

$i^{18}=i^2=-1$ $\quad$ $i^{53}=i^1=i$

$i^6=i^2=-1$ $\quad$ $i^8=1$

$i^{36}=i^0=1$ $\quad$ $i^7=i^3=-i$

$i^{20}=i^0=1$

$i^{52}=i^0=1$

$$i^{18}=\underbrace{i^4}_{1}\cdot\underbrace{i^4}_{1}\cdot\underbrace{i^4}_{1}\cdot\underbrace{i^4}_{1}\cdot i^2=i^{2}$$

RESTO DELLA DIVISIONE FRA 18 E 4

Si identificano gli esponenti che hanno lo stesso resto nella divisione per 4

$$(*)=\frac{-18-7}{(2+i)^2} : \frac{4-2}{(2-i-1)^2}=$$

$$=\frac{-25}{4-1+4i} : \frac{2}{(1-i)^2} = -\frac{25}{3+4i}\cdot\frac{\cancel{1}-\cancel{1}-2i}{2}=$$

$$=-\frac{25}{3+4i}\cdot\frac{-\cancel{2}i}{\cancel{2}} = \frac{25i}{3+4i}\cdot\frac{3-4i}{3-4i}=\frac{75i+100}{9+16}=$$

$$=\frac{100}{25}+\frac{75}{25}i=\boxed{4+3i}$$

211 $z^2 + |z|^2 - 18 = 0$

$z = a + ib$

Da trovare a e b

$|z|^2 = a^2 + b^2$

$(a+ib)^2 + (a^2+b^2) - 18 = 0$

$a^2 - \cancel{b^2} + 2abi + a^2 + \cancel{b^2} - 18 = 0$

$(2a^2 - 18) + 2abi = 0 \Rightarrow \begin{cases} 2a^2 - 18 = 0 \\ 2ab = 0 \end{cases}$

$z = 0$ se e solo se

$\begin{cases} \mathrm{Re}\, z = 0 \\ \mathrm{Im}\, z = 0 \end{cases}$

$\begin{cases} a^2 = 9 \\ \end{cases}$ $\begin{cases} a = -3 \\ b = 0 \end{cases}$ $\begin{cases} a = 3 \\ b = 0 \end{cases}$

$z = -3 + 0 \cdot i = -3 \quad \vee \quad z = 3 + 0 \cdot i = 3$

$\boxed{z = -3 \quad \vee \quad z = 3}$