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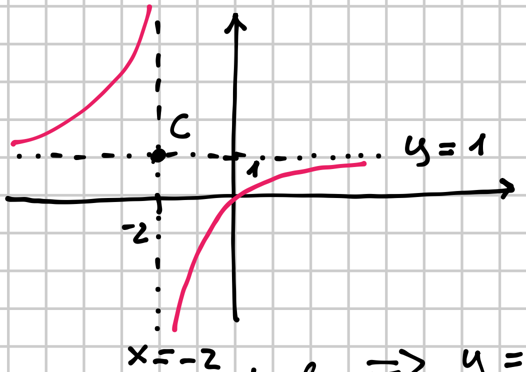
361 La funzione $y = \log_3 \frac{3a+x}{-4-x}$ passa per il punto di intersezione tra la funzione omografica di centro $C(-2; 1)$ e passante per $O(0; 0)$ e la retta di equazione $y = -1$. Determina il valore del parametro a, studia l'andamento della funzione e disegna il suo grafico. [$a = 0$]

$$y = \frac{ax+b}{x+c}$$



as. verticale $\Rightarrow y = \frac{ax+b}{x+2}$ $(c = 2)$

$$\lim_{x \to \pm\infty} \frac{ax+b}{x+c} = a = 1$$

y=1 è asintoto orizzontale

$a = 1$
$c = 2$ $\quad y = \frac{x+b}{x+2}$ $\quad$ passaggio per $O(0,0)$ $\quad 0 = \frac{0+b}{0+2} \Rightarrow b = 0$

La funzione omografica è $y = \frac{x}{x+2}$

$$\begin{cases} y = \frac{x}{x+2} \\ y = -1 \end{cases} \Rightarrow \frac{x}{x+2} = -1 \qquad x = -x-2 \qquad 2x = -2 \qquad x = -1$$

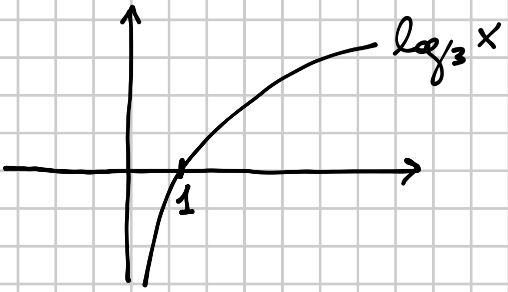
$P(-1, -1)$

$y = \log_3 \frac{3a+x}{-4-x}$ passaggio

$$-1 = \log_3 \frac{3a-1}{-3}$$

$$\begin{cases} \frac{3a-1}{-3} > 0 \\ 3^{-1} = \frac{3a-1}{-3} \end{cases} \quad \begin{cases} a < \frac{1}{3} \\ \frac{1}{\cancel{3}} = \frac{-3a+1}{\cancel{3}} \end{cases} \Rightarrow a = 0$$

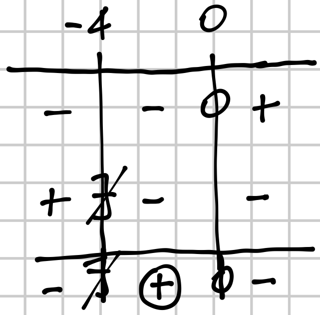
$$f(x) = \log_{1/3} \frac{x}{-4-x}$$



$$D: \frac{x}{-4-x} > 0$$

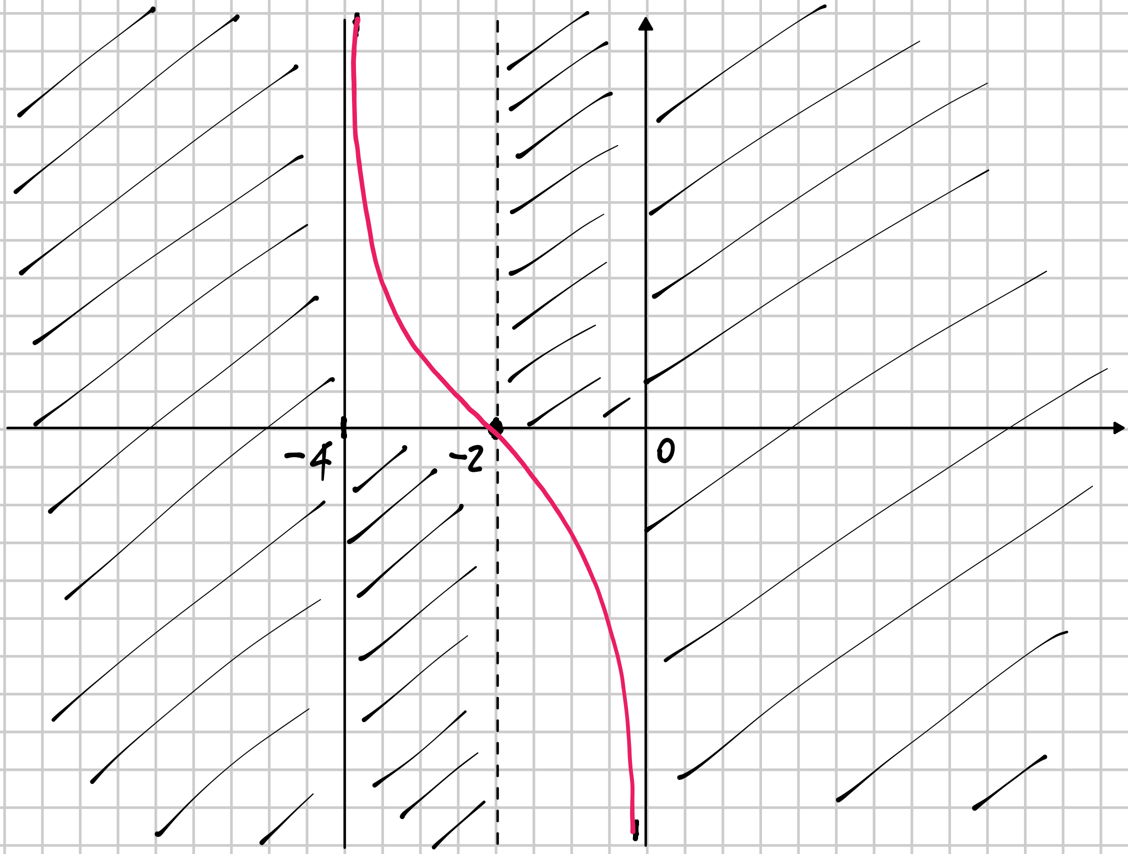
N] $x > 0$

D] $-4-x > 0 \Rightarrow x < -4$



$-4 < x < 0$

$$D =]-4, 0[$$



INT. ASSE X

$$\log_3 \frac{x}{-4-x} = 0 \Rightarrow \frac{x}{-4-x} = 1 \Rightarrow x = -4-x \Rightarrow x = -2$$

$A(-2, 0)$

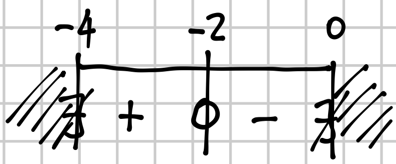
SEGNO

$$\log_{1/3} \frac{x}{-4-x} > 0 \Rightarrow \frac{x}{-4-x} > 1 \Rightarrow x < -4-x \qquad 2x < -4$$

$\Downarrow$ $x < -2$

$x \in]-4, 0[$

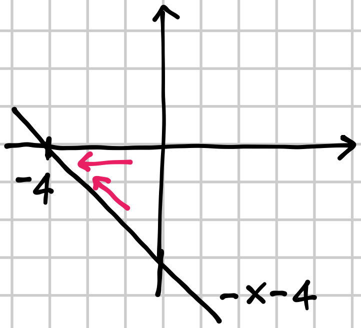
$-4-x < 0 \quad \forall x \in]-4, 0[$



LIMITI

$D =]-4, 0[$

$$\lim_{x \to -4^+} \log_3 \frac{x}{-4-x} = \log_3 \frac{-4}{0^-} = \log_3 (+\infty) = +\infty$$



$$\lim_{x \to 0^-} \log_3 \frac{x}{-4-x} = \log_3 \frac{0^-}{-4} = \log_3 0^+ = -\infty$$

DERIVATA PRIMA

$\boxed{x \in]-4, 0[}$

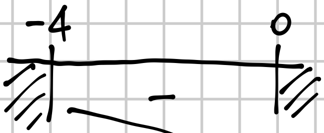
$$f(x) = \log_3 \frac{x}{-4-x}$$

$$y = \log_a x$$

$$y' = \frac{1}{x \ln a}$$

$$f'(x) = \frac{1}{\ln 3} \cdot \frac{\cancel{-4-x}}{x} \cdot \frac{-4-x-(-1)\cdot x}{(-4-x)^{\cancel{2}}} =$$

$$= \frac{1}{\ln 3} \cdot \frac{-4}{x(-4-x)} = \frac{1}{\ln 3} \frac{4}{\underbrace{x}_{<0}\underbrace{(x+4)}_{>0 \text{ in }]-4,0[}} < 0 \quad \forall x \in]-4, 0[$$



f è strettamente decrescente

DERIVATA SECONDA

$$f'(x) = \frac{1}{\ln 3} \cdot \frac{4}{\underset{x^2+4x}{x(x+4)}} = \frac{4}{\ln 3} \left(x^2+4x\right)^{-1}$$

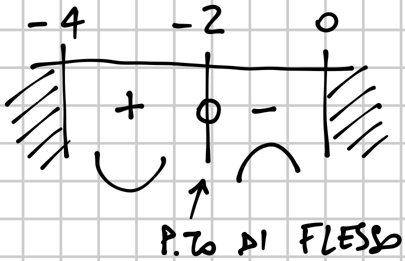
$$f''(x) = -\frac{4}{\ln 3}\left(x^2+4x\right)^{-2} \cdot (2x+4) = -\frac{4}{\ln 3} \frac{2x+4}{\left(x^2+4x\right)^2}$$

ZERI DI f'' $\qquad f''(x)=0 \qquad 2x+4=0 \implies x=-2$ CANDIDATO FLESSO

$x \in \,]-4,0[$

SEGNO DI f'' $\qquad f''(x)>0 \implies -(2x+4)>0$

$2x+4<0 \qquad x<-2$



$F(-2,0)$

$y = \log_3 \frac{x}{-4-x}$