5/5/2022

157 $\log_{25}36 + \log_5 \frac{1}{6} =$ [0]

$$= \frac{\log_5 36}{\log_5 25} + \log_5 \frac{1}{6} = \frac{1}{2} \log_5 6^2 + \log_5 6^{-1} =$$

$$= \frac{2}{2} \log_5 6 - \log_5 6 = \log_5 6 - \log_5 6 = 0$$

159 $\log_4 10 + \frac{1}{2\log_{10}4} + \log_2 10 =$ $\left[\frac{7}{4}\log_2 10\right]$

$$= \frac{\log_2 10}{\log_2 4} + \frac{1}{2\,\frac{\log_2 4}{\log_2 10}} + \log_2 10 =$$

$$= \frac{\log_2 10}{2} + \frac{1}{\frac{4}{\log_2 10}} + \log_2 10 =$$

$$= \log_2 10 \left(\frac{1}{2} + \frac{1}{4} + 1\right) = \frac{7}{4} \log_2 10$$

91 $\log\sqrt{a\sqrt[3]{ab^2}} =$ $\left[\frac{2}{3}\log a + \frac{1}{3}\log b\right]$

$$= \log\left(a\sqrt[3]{ab^2}\right)^{\frac{1}{2}} = \frac{1}{2}\log\left(a\cdot\sqrt[3]{ab^2}\right) =$$

$$= \frac{1}{2}\left[\log a + \log\left(ab^2\right)^{\frac{1}{3}}\right] =$$

$$= \frac{1}{2}\log a + \frac{1}{2}\cdot\frac{1}{3}\log\left(a\cdot b^2\right) =$$

$$= \frac{1}{2}\log a + \frac{1}{6}\left[\log a + \log b^2\right] =$$

$$= \frac{1}{2}\log a + \frac{1}{6}\left[\log a + 2\log b\right] =$$

$$= \frac{1}{2}\log a + \frac{1}{6}\log a + \frac{1}{3}\log b =$$

$$= \frac{2}{3}\log a + \frac{1}{3}\log b$$

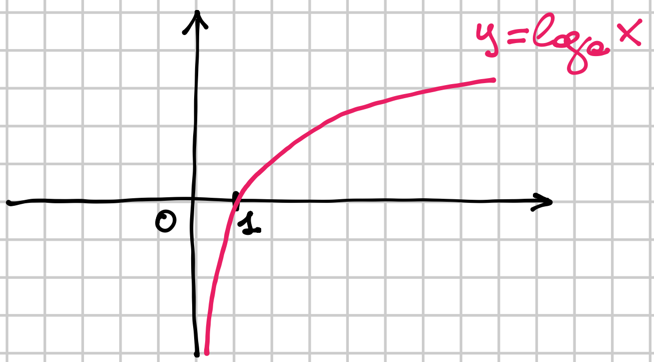
161 $\dfrac{\log_8 27 + \log_2 5}{\log_4 9 - \log_{\frac{1}{4}} 25} =$ [1]

$$= \frac{\dfrac{\log_2 27}{\log_2 8} + \log_2 5}{\dfrac{\log_2 9}{\log_2 4} - \dfrac{\log_2 25}{\log_2 \frac{1}{4}}} = \frac{\dfrac{\log_2 27}{3} + \log_2 5}{\dfrac{\log_2 9}{2} - \dfrac{\log_2 25}{-2}} =$$

$$= \frac{\dfrac{\cancel{3}\log_2 3}{\cancel{3}} + \log_2 5}{\dfrac{\cancel{2}\log_2 3}{\cancel{2}} + \dfrac{\cancel{2}\log_2 5}{\cancel{2}}} = 1$$

FUNZIONE LOGARITMICA

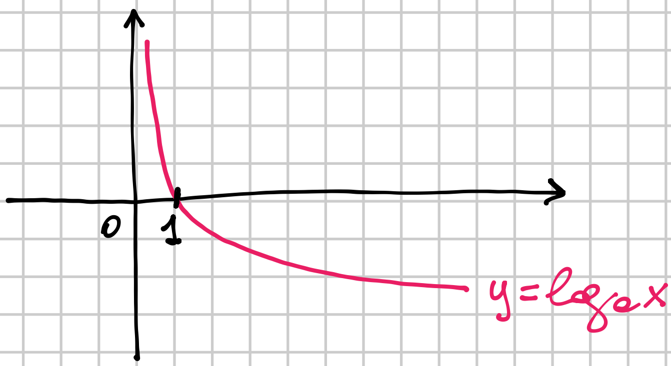
$a > 1$



$0 < x < 1 \Rightarrow \log_a x < 0$

$x > 1 \Rightarrow \log_a x > 0$

$0 < a < 1$



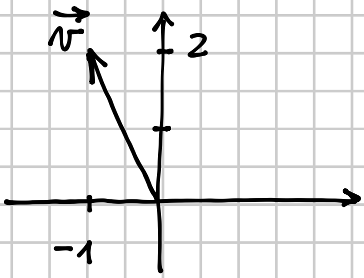
$0 < x < 1 \Rightarrow \log_a x > 0$

$x > 1 \Rightarrow \log_a x < 0$

In entrambi i casi $\log_a 1 = 0$

184 $y = \ln x$, traslazione di vettore $\vec{v}(-1; 2)$. $[y = \ln(x+1) + 2]$

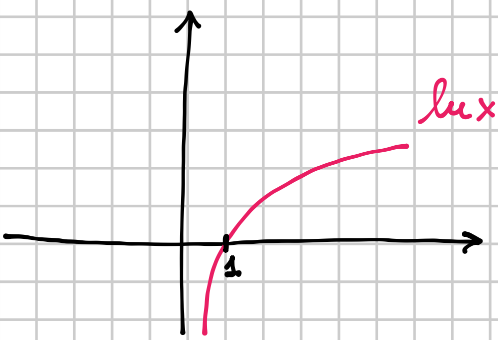
VETTORE $\vec{v}\,(-1, 2)$

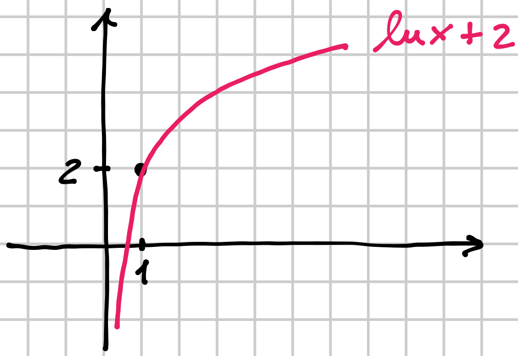


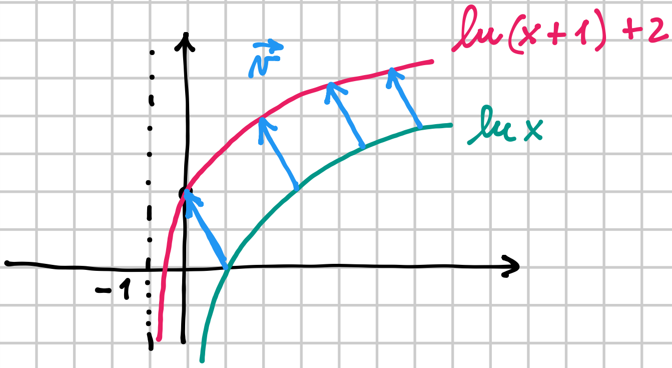
$y = \ln x \Rightarrow f(x) = \ln x$

TRASLO IN SU DI 2 $\quad \ln x + 2$

TRASLO A SINISTRA DI 1 $\quad \ln(x+1) + 2$







Determina il dominio delle seguenti funzioni.

220 $y = \ln \dfrac{x^2 - 1}{x^2 + 4}$ $\quad [x < -1 \vee x > 1]$

REGOLA IMPORTANTE: l'argomento di un logaritmo deve essere sempre > 0

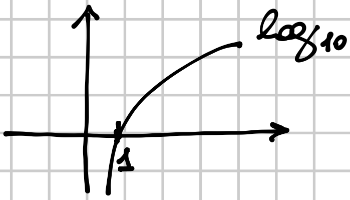
$$\frac{x^2-1}{x^2+4} > 0 \Rightarrow x^2 - 1 > 0 \Rightarrow x < -1 \quad \vee \quad x > 1$$

$$D = (-\infty, -1) \cup (1, +\infty)$$

232 $y=\sqrt{\log\frac{x}{x-3}}$ $[x>3]$ Calcolare il dominio

$$\begin{cases} \dfrac{x}{x-3}>0 \\ \log\dfrac{x}{x-3}\geq 0 \Rightarrow \dfrac{x}{x-3}\geq 1 \end{cases}$$

BASTA QUESTA PERCHÉ $\frac{x}{x-3}>0$ È AUTOMATICA



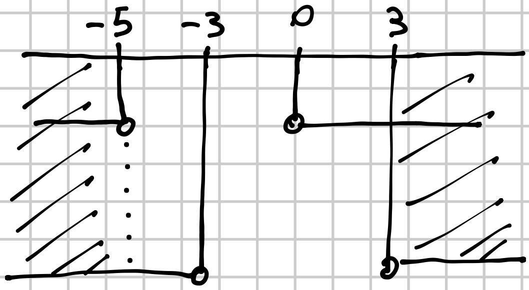
il logaritmo in base 10 di qualcosa è ≥ 0 quando l'argomento è ≥ 1

$$\frac{x}{x-3}\geq 1 \qquad \frac{x}{x-3}-1\geq 0 \qquad \frac{\cancel{x}-\cancel{x}+3}{x-3}\geq 0 \qquad \frac{3}{x-3}\geq 0 \Rightarrow x>3$$

$D=(3,+\infty)$

243 $y=\log\frac{x}{x+5}+\log(x^2-9)$ $[x<-5 \vee x>3]$

$$\begin{cases} \dfrac{x}{x+5}>0 \\ x^2-9>0 \end{cases} \qquad \begin{cases} x<-5 \vee x>0 \\ x<-3 \vee x>3 \end{cases}$$



$x<-5 \vee x>3$

$D=(-\infty,-5)\cup(3,+\infty)$

OSSERVIAMO che NON si sarebbe potuto applicare la proprietà dei logaritmi:

$$y=\log\frac{x(x^2-9)}{x+5}$$

e poi calcolare il dominio della funzione scritta. Infatti:

$$\frac{x(x^2-9)}{x+5}>0 \qquad \frac{x(x-3)(x+3)}{x+5}>0$$

		-5		-3		0		3	
$x>0$	$-$		$-$		$-$	0	$+$		$+$
$x-3>0 \quad x>3$	$-$		$-$		$-$		$-$	0	$+$
$x+3>0 \quad x>-3$	$-$		$-$	0	$+$		$+$		$+$
$x+5>0 \quad x>-5$	$-$	$\nexists$	$+$		$+$		$+$		$+$
	$+$	$\nexists$	$-$	0	$+$	0	$-$	0	$+$

$$x<-5 \ \vee \ -3<x<0 \ \vee \ x>3$$

<u>NON</u> è lo stesso dominio della funzione iniziale!

$$y=\log\frac{x}{x+5}+\log(x^2-9) \quad e \quad y=\log\frac{x(x^2-9)}{x+5}$$

sono <u>FUNZIONI DIVERSE!!</u>

EQUAZIONE LOGARITMICA

310 $\log_5(x^2+1) = \log_5 2 + \log_5(x^2-4)$ $[3; -3]$

C.E. $\begin{cases} x^2+1>0 \\ x^2-4>0 \end{cases}$ $\begin{cases} x \in \mathbb{R} \\ x<-2 \vee x>2 \end{cases}$ $\Rightarrow$ $x<-2 \vee x>2$

$$\log_5(x^2+1) = \log_5[2\cdot(x^2-4)]$$

$$x^2+1 = 2(x^2-4)$$

$$x^2+1 = 2x^2-8$$

$x^2 = 9$ $\boxed{x = \pm 3}$ compatibili con C.E.

331 $\log_2(x^2-4) + 2\log_2 x = 1 + \log_2(5x^2+16)$

C.E. $\begin{cases} x^2-4>0 \\ x>0 \\ 5x^2+16>0 \end{cases}$ $\begin{cases} x<-2 \vee x>2 \\ x>0 \\ x \in \mathbb{R} \end{cases}$ $\Rightarrow$ $x>2$

$$\log_2(x^2-4) + \log_2 x^2 = \log_2 2 + \log_2(5x^2+16)$$

$$\log_2[(x^2-4)\cdot x^2] = \log_2[2\cdot(5x^2+16]$$

$$(x^2-4)\cdot x^2 = 2(5x^2+16)$$

$x^4-4x^2 = 10x^2+32$ $x^4-14x^2-32=0$

$x^2 = 7 \pm 9 = \begin{cases} 16 \\ -2 \text{ N.ACC.} \end{cases}$ $\Rightarrow x^2=16 \Rightarrow x=\pm 4$ $\boxed{x=4}$

$\frac{\Delta}{4} = 49+32 = 81$

-4 NON. ACC. per C.E.

314 $\log_{\frac{1}{2}}(x^2-4x)+\log_2 2x-1=0$ [5]

C.E. $\begin{cases} x^2-4x>0 \\ 2x>0 \end{cases}$ $\begin{cases} x<0 \vee x>4 \\ x>0 \end{cases}$ $\Rightarrow x>4$

$$\frac{\log_2(x^2-4x)}{\log_2 \frac{1}{2}}+\log_2 2x-\log_2 2=0$$

$$-\log_2(x^2-4x)+\log_2 2x-\log_2 2=0$$

$$\log_2 \frac{\cancel{2}x}{\cancel{2}(x^2-4x)}=0 \leftarrow \log_2 1$$

$$\frac{x}{x^2-4x}=1$$

$$x=x^2-4x$$

$$x^2-5x=0$$

$$x=0 \vee x=5$$

N.ACC. per C.E.

$$\boxed{x=5}$$