2/3/2021

226 $x^4 - 2\sqrt{2}\,x^2 + 2 = 0$ $[\pm\sqrt[4]{2}]$

$$\left(x^2 - \sqrt{2}\right)^2 = 0$$

$$\Downarrow$$

$$x^2 - \sqrt{2} = 0 \qquad x^2 = \sqrt{2} \qquad x = \pm\sqrt{\sqrt{2}} = \pm\sqrt[4]{2}$$

ALTERNATIVO: LA TRATTO COME BIQUADRATICA

$$x^2 = t \implies t^2 - 2\sqrt{2}\,t + 2 = 0 \qquad \frac{\Delta}{4} = \left(-\sqrt{2}\right)^2 - 2 = 0$$

$$t = \sqrt{2} \pm 0 = \sqrt{2}$$

$$x^2 = \sqrt{2} \qquad x = \pm\sqrt[4]{2}$$

227 $x^6 + 6x^3 - 7 = 0$ $[-\sqrt[3]{7};\ 1]$

$$x^3 = t \qquad t^2 + 6t - 7 = 0$$

$$(t+7)(t-1) = 0$$

$\nearrow$ $t = -7 \qquad x^3 = -7 \qquad x = -\sqrt[3]{7}$

$\searrow$ $t = 1 \qquad x^3 = 1 \qquad x = 1$

$$\boxed{x = -\sqrt[3]{7} \ \vee\ x = 1}$$

52 $(2x-1)^3=8$ $\left[\frac{3}{2}\right]$

$t = 2x - 1$ $\qquad t^3 = 8$

$t = \sqrt[3]{8} = 2$

$2x - 1 = 2 \qquad 2x = 3 \qquad \boxed{x = \frac{3}{2}}$

57 $(3x-1)^3+3=0$ $\left[\frac{1-\sqrt[3]{3}}{3}\right]$

$t = 3x - 1$ $\qquad t^3 + 3 = 0 \qquad t^3 = -3$

$t = -\sqrt[3]{3}$

$3x - 1 = -\sqrt[3]{3}$

$3x = 1 - \sqrt[3]{3}$

$$\boxed{x = \frac{1-\sqrt[3]{3}}{3}}$$

64 $\left(\frac{1}{x}+\frac{1}{x-1}\right)^2=\frac{9}{4}$ $\left[-1;\ 2;\ \frac{1}{3};\ \frac{2}{3}\right]$

C.E.

$x \neq 0$

$x \neq 1$

$t = \frac{1}{x} + \frac{1}{x-1}$

$$t^2 = \frac{9}{4}$$

$$t = \pm \frac{3}{2}$$

$\frac{1}{x} + \frac{1}{x-1} = -\frac{3}{2}$ $\quad \vee \quad$ $\frac{1}{x} + \frac{1}{x-1} = \frac{3}{2}$

$\frac{2(x-1)+2x}{\cancel{2x(x-1)}} = \frac{-3x(x-1)}{\cancel{2x(x-1)}}$ $\qquad$ $\frac{2(x-1)+2x}{\cancel{2x(x-1)}} = \frac{3x(x-1)}{\cancel{2x(x-1)}}$

$2x-2+2x = -3x^2+3x$ $\qquad$ $2x-2+2x = 3x^2-3x$

$3x^2+x-2=0$ $\qquad$ $3x^2-7x+2=0$

$\Delta = 1+24 = 25$ $\qquad$ $\Delta = 49-24 = 25$

$x = \frac{-1\pm 5}{6} = \begin{cases} \frac{2}{3} \\ -1 \end{cases}$ $\quad \vee \quad$ $x = \frac{7 \pm 5}{6} = \begin{cases} \frac{1}{3} \\ 2 \end{cases}$

$$\boxed{x = \frac{2}{3} \quad \vee \quad x=-1 \quad \vee \quad x = \frac{1}{3} \quad \vee \quad x = 2}$$

236 $3x^3 - 5x^2 - 8x - 2 = 0$ $\left[-\frac{1}{3}; 1 \pm \sqrt{3}\right]$

RUFFINI: $\pm 1, \pm 2$ (divisori termine noto)

$1 \mapsto 3-5-8-2 \neq 0$ NO!

$-1 \mapsto -3-5+8-2 \neq 0$ NO!

$2 \mapsto 3 \cdot 8 - 5 \cdot 4 - 8 \cdot 2 - 2 = 24 - 20 - 16 - 2 \neq 0$ NO!

$-2 \mapsto 3(-8) - 5 \cdot 4 - 8(-2) - 2 = -24 - 20 + 16 - 2 \neq 0$ NO!

coefficiente di grado massimo: 3 divisori $\cancel{\pm 1}$ ± 3

NUOVI TENTATIVI: rapporti fra i divisori del termine noto e i divisori del coeff. di grado max

$\pm \frac{1}{3}, \pm \frac{2}{3}$

$\frac{1}{3} \mapsto 3\left(\frac{1}{3}\right)^3 - 5\left(\frac{1}{3}\right)^2 - 8 \cdot \frac{1}{3} - 2 = \frac{3}{27} - \frac{5}{9} - \frac{8}{3} - 2$

$= \frac{3-15-72-54}{27} \neq 0$ NO!

$-\frac{1}{3} \mapsto 3\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 - 8 \cdot \left(-\frac{1}{3}\right) - 2 = -\frac{1}{9} - \frac{5}{9} + \frac{8}{3} - 2 =$

$= \frac{-1-5+24-18}{9} = 0$ OK!!!

$-\frac{1}{3}$ è una soluzione!!!!

$$3x^3-5x^2-8x-2=0$$

$$\begin{array}{c|ccc|c}
 & 3 & -5 & -8 & -2 \\
-\frac{1}{3} & & -1 & 2 & 2 \\
\hline
 & 3 & -6 & -6 & /\!/
\end{array}$$

$$\left(x+\frac{1}{3}\right)\left(3x^2-6x-6\right)=0$$

$\downarrow$ LEGGE DI ANNULLAMENTO DEL PRODOTTO

$$3x^2-6x-6=0$$

$$x^2-2x-2=0 \qquad \frac{\Delta}{4}=1+2=3$$

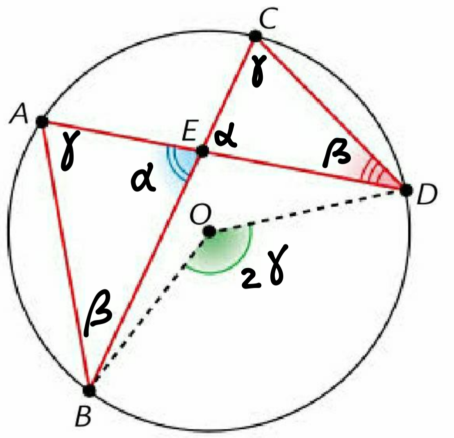
$$x=1\pm\sqrt{3}$$

Soluzioni: $\boxed{x=-\frac{1}{3} \quad \vee \quad x=1\pm\sqrt{3}}$

135 In riferimento agli angoli rappresentati nella figura è noto che:

- l'ampiezza dell'angolo $A\hat{E}B$ supera di 40° quella di $A\hat{D}C$;
- l'ampiezza di $B\hat{O}D$ è il quadruplo di quella di $A\hat{D}C$.

Qual è l'ampiezza di $A\hat{D}C$?



[35°]

$$\begin{cases} \alpha = 40° + \beta \\ \cancel{2}\gamma = \overset{2}{\cancel{4}}\beta \\ \alpha + \beta + \gamma = 180° \end{cases}$$

$$\begin{cases} \alpha = 40° + \beta \\ \gamma = 2\beta \\ \alpha + \beta + 2\beta = 180° \end{cases}$$

$$\begin{cases} \alpha = 40° + \beta \\ // \\ 40° + \beta + \beta + 2\beta = 180° \end{cases} \Rightarrow 4\beta = 140° \Rightarrow \beta = \frac{140°}{4} = 35°$$

$$\Downarrow$$

$$A\hat{D}C = 35°$$

$$\begin{cases} \alpha = 40° + 35° = 75° \\ \beta = 35° \\ \gamma = 2 \cdot 35° = 70° \end{cases}$$

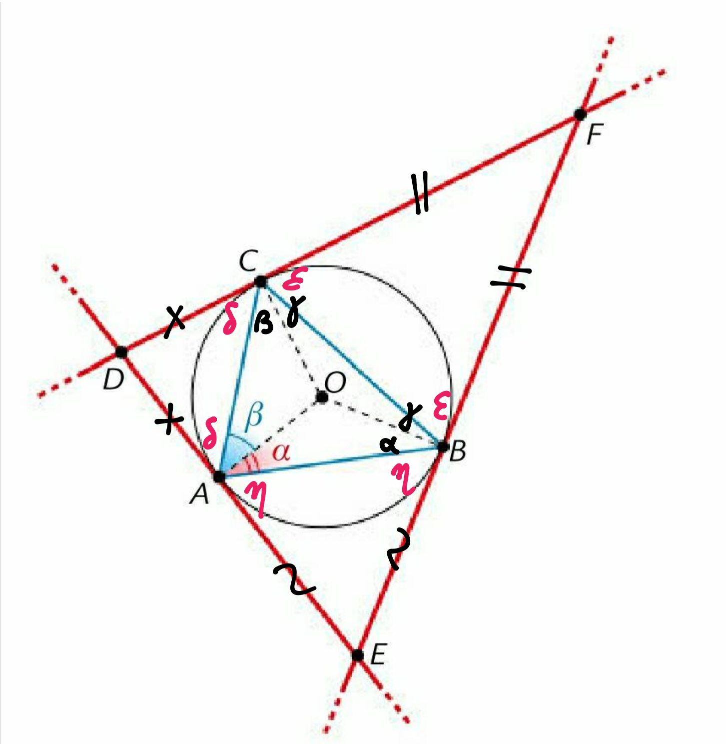
136 Fai riferimento alla figura, in cui le rette rappresentate sono le tangenti alla circonferenza nei punti A, B, C.

a. Supposto che $\alpha = 30°$ e $\beta = 40°$, determina le ampiezze degli angoli del triangolo ABC e le ampiezze degli angoli del triangolo DEF.

b. Supposto ora che α e β siano due variabili esprimi in funzione di α e β le ampiezze degli angoli del triangolo DEF.

c. Determina α e β, in modo che α sia il doppio di β e $D\hat{F}E = 30°$.

[**a.** $\widehat{A} = 70°$, $\widehat{B} = 50°$, $\widehat{C} = 60°$, $\widehat{D} = 80°$, $\widehat{E} = 60°$, $\widehat{F} = 40°$;
b. $\widehat{D} = 2\beta$, $\widehat{E} = 2\alpha$, $\widehat{F} = 180° - 2\alpha - 2\beta$; **c.** $\alpha = 50°$, $\beta = 25°$]



a)

$A\hat{O}B = 180° - 2\alpha = 120°$

$C\hat{O}A = 180° - 2\beta = 100°$

$C\hat{O}B = 360° - (A\hat{O}B + C\hat{O}A) =$

$= 360° - 220° = 140°$

$\gamma = \dfrac{180° - 140°}{2} = 20°$

$\hat{A} = \alpha + \beta = 70°$

$\hat{B} = \alpha + \gamma = 30° + 20° = 50°$

$\hat{C} = \beta + \gamma = 40° + 20° = 60°$

α = ALPHA $\quad$ δ = DELTA

β = BETA $\quad$ ε = EPSILON

γ = GAMMA $\quad$ η = ETA

$$\begin{cases} \delta + \eta + 70° = 180° \\ \eta + \varepsilon + 50° = 180° \\ \delta + \varepsilon + 60° = 180° \end{cases} \quad \begin{cases} \delta + \eta = 110° \\ \eta + \varepsilon = 130° \\ \delta + \varepsilon = 120° \end{cases} \quad \begin{cases} \eta = 110° - \delta \\ 110° - \delta + \varepsilon = 130° \\ \varepsilon = 120° - \delta \end{cases}$$

$$\begin{cases} \eta = 110^\circ - \delta \\ 110^\circ - \delta + \varepsilon = 130^\circ \\ \varepsilon = 120^\circ - \delta \end{cases} \qquad \begin{cases} // \\ 110^\circ - \delta + 120^\circ - \delta = 130^\circ \\ // \end{cases}$$

$$-2\delta = -100^\circ \Rightarrow \begin{cases} \delta = 50^\circ \\ \varepsilon = 70^\circ \\ \eta = 60^\circ \end{cases} \qquad \begin{aligned} \hat{D} &= 180^\circ - 2\delta = 80^\circ \\ \hat{E} &= 180^\circ - 2\eta = 60^\circ \\ \hat{F} &= 180^\circ - 2\varepsilon = 40^\circ \end{aligned}$$

b) $$\begin{cases} \delta = 180^\circ - (\alpha + \beta) - \eta \\ \eta = 180^\circ - (\alpha + \gamma) - \varepsilon \\ \varepsilon = 180^\circ - (\gamma + \beta) - \delta \\ \gamma = \dfrac{180^\circ - 2(\alpha + \beta)}{2} = 90^\circ - (\alpha + \beta) \end{cases} \qquad \begin{cases} \delta = 180^\circ - (\alpha + \beta) - \eta \\ \eta = 180^\circ - (\cancel{\alpha} + 90^\circ \cancel{- \alpha} - \beta) - \varepsilon \\ \varepsilon = 180^\circ - (90^\circ - \alpha \cancel{- \beta} \cancel{+ \beta}) - \delta \end{cases}$$

$$\begin{cases} \delta = 180^\circ - (\alpha + \beta) - \eta \\ \eta = 90^\circ + \beta - \varepsilon \\ \varepsilon = 90^\circ + \alpha - \delta \end{cases} \Rightarrow \begin{aligned} \delta &= 180^\circ - \alpha - \beta - (90^\circ + \beta - 90^\circ - \alpha + \delta) \\ &= 180^\circ \cancel{- \alpha} - \beta \cancel{- 90^\circ} - \beta \cancel{+ 90^\circ} \cancel{+ \alpha} - \delta \end{aligned}$$

$$\Downarrow$$

$$2\delta = 180^\circ - 2\beta \qquad \delta = 90^\circ - \beta$$

$$\begin{cases} \delta = 90^\circ - \beta \\ \varepsilon = \cancel{90^\circ} + \alpha \cancel{- 90^\circ} + \beta = \alpha + \beta \\ \eta = 90^\circ \cancel{+ \beta} - \alpha \cancel{- \beta} = 90^\circ - \alpha \end{cases} \qquad \begin{aligned} \hat{D} &= 180^\circ - 2\delta = \cancel{180^\circ} - \cancel{180^\circ} + 2\beta = 2\beta \\ \hat{E} &= 180^\circ - 2\eta = 2\alpha \\ \hat{F} &= 180^\circ - 2\varepsilon = 180^\circ - 2(\alpha + \beta) \end{aligned}$$

c) $\alpha = 2\beta$

$\hat{F} = 180° - 2\alpha - 2\beta = 30°$

$$\begin{cases} \alpha = 2\beta \\ 2\alpha + 2\beta = 150° \end{cases}$$

$$\begin{cases} \alpha = 2\beta \\ 4\beta + 2\beta = 150° \end{cases} \quad \begin{cases} \alpha = 2\beta \\ \beta = \dfrac{150°}{6} = 25° \end{cases} \quad \begin{cases} \alpha = 50° \\ \beta = 25° \end{cases}$$