$$26 \cdot 2^x = 4 \cdot 5^x + 2^x$$

$$26 \cdot 2^x - 2^x = 4 \cdot 5^x$$

$$2^x(26 - 1)$$

$$26a - a$$
$$\underbrace{}_{25a}$$

$$\downarrow$$

$$25 \cdot 2^x = 4 \cdot 5^x$$

$$\frac{25 \cdot 2^x}{25 \cdot 5^x} = \frac{4 \cdot 5^x}{25 \cdot 5^x}$$

$$\frac{2^x}{5^x} = \frac{4}{25}$$

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^2$$

$$x = 2$$

62)

$$3^x - 9 \cdot \frac{\sqrt{3}}{\sqrt[5]{9}} = 0$$

$$3^x - 3^2 \cdot \frac{3^{\frac{1}{2}}}{3^{\frac{2}{5}}} = 0$$

$$3^x - 3^{2 + \frac{1}{2} - \frac{2}{5}} = 0$$

$$3^x = 3^{\frac{20 + 5 - 4}{10}}$$

$$\boxed{x = \frac{21}{10}}$$

64)

$$2^x + 2^{x+1} = -2^{x-1} + 7 \qquad\qquad t = 2^x$$

$$2^x + 2^x \cdot 2^1 = -2^x \cdot 2^{-1} + 7$$

$$t + 2t = -\frac{1}{2}t + 7$$

$$2t + 4t = -t + 14 \qquad\qquad 7t = 14 \qquad t = 2$$

$$2^x = 2$$

$$\boxed{x = 1}$$

55]

$$3^{x+2} = 2^{2x+4}$$

$$3^x \cdot 3^2 = 2^{2x} \cdot 2^4$$

$$\frac{3^x}{2^{2x}} = \frac{2^4}{3^2} \longleftarrow \quad 2^4 = (2^2)^2$$
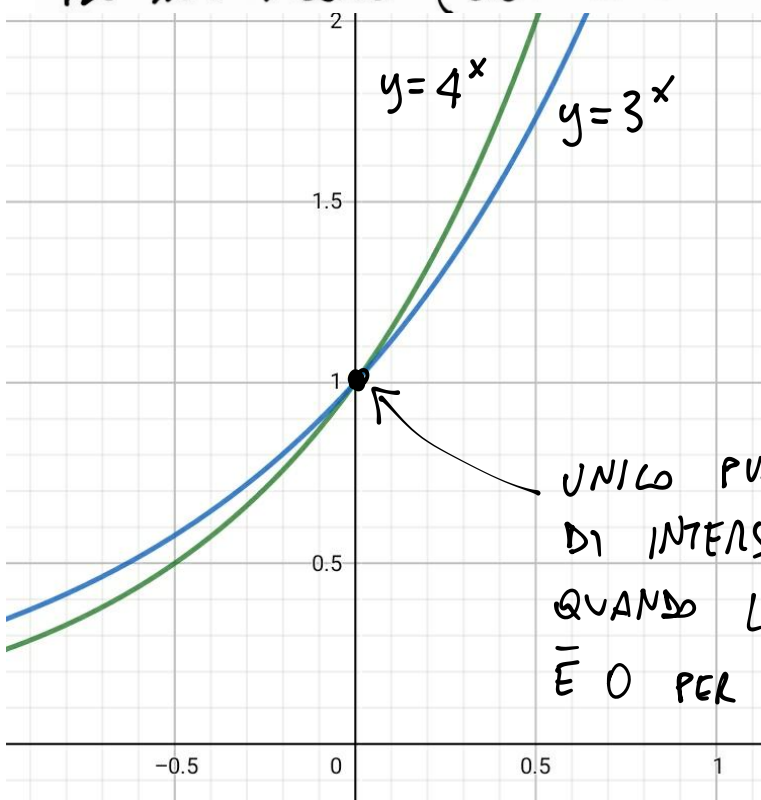
$$\frac{3^x}{4^x} = \frac{4^2}{3^2}$$

$$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^{-2} \Rightarrow \boxed{x = -2}$$

MODO ALTERNATIVO

$$3^{x+2} = 2^{2x+4} \Rightarrow 3^{x+2} = 2^{2(x+2)}$$

$$\Rightarrow 3^{x+2} = 4^{x+2}$$

I grafici delle funzioni $y = 3^x$ e $y = 4^x$
si intersecano (cioè sono "uguali") solo per $x = 0$

$$x + 2 = 0$$

$$\Downarrow$$

$$\boxed{x = -2}$$

$y = 4^x$ $y = 3^x$

UNICO PUNTO
DI INTERSEZIONE,
QUANDO L'ESPONENTE
È 0 PER ENTRAMBI

84)

$$10^x + 10^{2-x} = 101$$

$$10^x + 10^2 \cdot 10^{-x} = 101$$

$$10^x + \frac{10^2}{10^x} = 101 \qquad\qquad 10^x = t$$

$$t + \frac{100}{t} = 101$$

$$\frac{t^2 + 100}{\cancel{t}} = \frac{101t}{\cancel{t}}$$

$$t^2 - 101t + 100 = 0 \qquad t = 100 \Rightarrow 10^x = 10^2$$

$$(t-100)(t-1) = 0 \qquad\qquad\qquad \hookrightarrow \boxed{x = 2}$$

$$t = 1 \Rightarrow 10^x = 1$$

$$\hookrightarrow \boxed{x = 0}$$

$$\boxed{x = 2 \ \lor \ x = 0}$$

85)

$$2^{x+3} + 4^{x+1} = 320 \qquad\qquad 4^x = (2^x)^2$$

$$2^x \cdot 2^3 + 4^x \cdot 4 = 320 \qquad\qquad 2^x = t$$

$$2^x \cdot 2^3 + (2^x)^2 \cdot 4 = 320$$

$$8t + 4t^2 = 320 \qquad t = \frac{-1 \pm \sqrt{1+80}}{} = -1 \pm 9 = \nearrow +8 \ (*)$$

$$2t + t^2 = 80 \qquad\qquad\qquad\qquad\qquad \searrow -10 \ N.A.$$

$$t^2 + 2t - 80 = 0 \qquad\qquad\qquad\qquad\qquad 2^x = -10 \ \text{IMPOSS.}$$

$$(*) \quad 2^x = 8 \qquad 2^x = 2^3$$

$$\hookrightarrow \boxed{x = 3}$$