PROPRIETÀ DEI LOGARITMI

PROPR. ESPONENZIALI

$$a^x \cdot a^y = a^{x+y}$$

$$a^x : a^y = a^{x-y}$$

$$(a^x)^y = a^{x \cdot y}$$

PROPR. LOGARITMI

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^y = y \log_a x$$

CASI PARTICOLARI

$$a^0 = 1$$

$$\log_a (1) = 0$$

DIMOSTRAZIONI

$$\boxed{\log_a (x \cdot y) = \log_a x + \log_a y}$$

$$a^{\log_a (x \cdot y)} = a^{\log_a x + \log_a y}$$

$$x \cdot y = a^{\log_a x} \cdot a^{\log_a y}$$

$$x \cdot y = x \cdot y$$

$$\boxed{\log_a x^y = y \log_a x}$$

$$a^{\log_a x^y} = a^{y \log_a x}$$

$$x^y = \left(a^{\log_a x}\right)^y$$

$$x^y = x^y$$

Se voglio dimostrare che

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a\left(x \cdot y^{-1}\right) = \log_a x + \log_a y^{-1} =$$

$$= \log_a x - \log_a y$$

$$\log_a \sqrt[n]{x} = \log_a x^{\frac{1}{n}} = \frac{1}{n} \log_a x$$

ESERCIZIO

pg. 443 N 244 → Sviluppa i seguenti logaritmi

$$\log \frac{5a}{b^4} \sqrt[7]{b} = \log \frac{5a}{b^4} + \log \sqrt[7]{b} = \log 5a - \log b^4 + \frac{1}{7} \log b =$$

$$= \log 5 + \log a - 4 \log b + \frac{1}{7} \log b = \boxed{\log 5 + \log a - \frac{27}{7} \log b}$$

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$$\log \sqrt{a \cdot \sqrt[3]{ab^2}} = \log \sqrt{\sqrt[3]{a^3 \cdot ab^2}} =$$

$$= \log \sqrt[6]{a^4 b^2} = \frac{1}{6} \log (a^4 b^2) =$$

$$\boxed{\sqrt{\sqrt[3]{x}} = \left(x^{\frac{1}{3}}\right)^{\frac{1}{2}} = x^{\frac{1}{3} \cdot \frac{1}{2}} = x^{\frac{1}{6}}}$$

$$= \frac{1}{6}\left[\log a^4 + \log b^2\right] = \frac{1}{6}\left[4 \log a + 2 \log b\right] =$$

$$= \frac{4}{6} \log a + \frac{2}{6} \log b = \frac{2}{3} \log a + \frac{1}{3} \log b$$

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$$\log_2(3x-1)+3\log_2(x-1)-4\log_2(x-2)=$$

$$=\log_2(3x-1)+\log_2(x-1)^3-\log_2(x-2)^4=$$

$$=\log_2\frac{(3x-1)(x-1)^3}{(x-2)^4}$$

CAMBIAMENTO DI BASE

$$\log_a x=\frac{\log_m x}{\log_m a}$$

ESEMPIO $\log_2 3=\dfrac{\log 3}{\log 2}\simeq 1{,}58496\ldots$

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