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218 $5^{2x-1} = 7^{2x-1}$ $\left[\frac{1}{2}\right]$

$$\frac{5^{2x-1}}{7^{2x-1}} = 1$$

$$\left(\frac{5}{7}\right)^{2x-1} = 1$$

$$\Downarrow$$

$$\left(\frac{5}{7}\right)^{2x-1} = \left(\frac{5}{7}\right)^{0}$$

$$2x-1=0 \qquad \boxed{x = \frac{1}{2}}$$

245 $2^{2x-1} \cdot 3^{x} = \frac{1}{2 \cdot 3^{x}}$ [0]

$$2^{2x} \cdot 2^{-1} \cdot 3^{x} = \frac{1}{2 \cdot 3^{x}}$$

$$2^{2x} \cdot \frac{1}{\cancel{2}} \cdot 3^{x} = \frac{1}{\cancel{2} \cdot 3^{x}}$$

$$3^{x} \cdot \quad 2^{2x} \cdot 3^{x} = \frac{1}{3^{x}} \quad \cdot 3^{x}$$

$$2^{2x} \cdot 3^{2x} = 1$$

$$(2 \cdot 3)^{2x} = 1 \longrightarrow 2x = 0 \qquad \boxed{x = 0}$$

186 $2^{x+2} - 4 \cdot 5^{x+2} = 25 \cdot 5^{x} - 4 \cdot 2^{x}$

$$2^x \cdot 2^2 - 4 \cdot 5^x \cdot 5^2 = 25 \cdot 5^x - 4 \cdot 2^x$$

$$4 \cdot 2^x + 4 \cdot 2^x = 25 \cdot 5^x + 100 \cdot 5^x$$

$$8 \cdot 2^x = 125 \cdot 5^x$$

$$\frac{2^x}{5^x} = \frac{125}{8} \rightarrow \left(\frac{5}{2}\right)^3$$

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^{-3}$$

$$\boxed{x = -3}$$

262 $\left\{\begin{array}{l} 4^{y^2} - 2^{4x} = 0 \\ \dfrac{625^x \cdot 25^x}{\sqrt{125}} = \left(\dfrac{1}{5}\right)^y \end{array}\right.$ $\left[\left(\frac{1}{2}; -1\right); \left(\frac{9}{50}; \frac{3}{5}\right)\right]$

$$\left\{\begin{array}{l} 2^{2y^2} = 2^{4x} \rightsquigarrow 2y^2 = 4x \\ \dfrac{5^{4x} \cdot 5^{2x}}{5^{\frac{3}{2}}} = 5^{-y} \end{array}\right. \qquad \left\{\begin{array}{l} y^2 = 2x \\ 5^{4x+2x-\frac{3}{2}} = 5^{-y} \end{array}\right.$$

$$\left\{\begin{array}{l} y^2 = 2x \\ 6x - \frac{3}{2} = -y \end{array}\right. \qquad \left\{\begin{array}{l} x = \dfrac{y^2}{2} \\ 6\left(\dfrac{y^2}{2}\right) - \dfrac{3}{2} + y = 0 \end{array}\right.$$

$$\downarrow \quad 6y^2 + 2y - 3 = 0$$

$$\frac{\Delta}{4} = 1 + 18 = 19 \qquad y = \frac{-1 \pm \sqrt{19}}{6}$$

$$\boxed{\left\{\begin{array}{l} x = \dfrac{1}{2}\left(\dfrac{-1-\sqrt{19}}{6}\right)^2 \\ y = \dfrac{-1-\sqrt{19}}{6} \end{array}\right. \vee \left\{\begin{array}{l} x = \dfrac{1}{2}\left(\dfrac{-1+\sqrt{19}}{6}\right)^2 \\ y = \dfrac{-1+\sqrt{19}}{6} \end{array}\right.}$$

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552 $2^x + 2^{x+1} + 2^{x-1} = 15$ $\left[\dfrac{\log 30 - \log 7}{\log 2}\right]$

$2^x + 2^x \cdot 2 + 2^x \cdot 2^{-1} = 15$ $\quad 2^x = t$

$t + 2t + \frac{1}{2}t = 15$

$$\frac{2t + 4t + t}{\cancel{2}} = \frac{30}{\cancel{2}}$$

$7t = 30 \qquad t = \frac{30}{7}$

$$2^x = \frac{30}{7}$$

$$x = \log_2 \frac{30}{7} =$$

$$= \frac{\log \frac{30}{7}}{\log 2} = \frac{\log 30 - \log 7}{\log 2}$$

$$\boxed{x = \frac{\log 30 - \log 7}{\log 2}}$$

594 $3^{\frac{x+1}{2}} \cdot 7^{x-1} = \dfrac{1}{49^x \cdot 9^x}$ $\left[\dfrac{2\ln 7 - \ln 3}{5\ln 3 + 6\ln 7}\right]$

$$3^{\frac{x+1}{2}} \cdot 7^{x-1} = 7^{-2x} \cdot 3^{-2x}$$

$$\ln\left[3^{\frac{x+1}{2}} \cdot 7^{x-1}\right] = \ln\left[7^{-2x} \cdot 3^{-2x}\right]$$

$$\ln 3^{\frac{x+1}{2}} + \ln 7^{x-1} = \ln 7^{-2x} + \ln 3^{-2x}$$

$$\frac{x+1}{2} \cdot \ln 3 + (x-1)\ln 7 = -2x\ln 7 - 2x\ln 3$$

$$(x+1)\ln 3 + 2(x-1)\ln 7 = -4x\ln 7 - 4x\ln 3$$

$$x\ln 3 + \ln 3 + (2\ln 7)\cdot x - 2\ln 7 = -4x\ln 7 - 4x\ln 3$$

$$(\ln 3)x + (2\ln 7)x + (4\ln 7)x + (4\ln 3)x = 2\ln 7 - \ln 3$$

$$\left[\ln 3 + 2\ln 7 + 4\ln 7 + 4\ln 3\right]x = 2\ln 7 - \ln 3$$

$$(5\ln 3 + 6\ln 7)\,x = 2\ln 7 - \ln 3$$

$$\boxed{x = \frac{2\ln 7 - \ln 3}{5\ln 3 + 6\ln 7}}$$

589 $3^{x+1}-2\cdot3^x+3^{x+2}=5^{x-1}$ $\left[\frac{2\log 5+\log 2}{\log 5-\log 3}\right]$

$$3^x\cdot3-2\cdot3^x+3^x\cdot3^2=5^{x-1}$$

$$3^x\left[3-2+9\right]=5^{x-1}$$

$$10\cdot3^x=5^{x-1}$$

$$\log\left(10\cdot3^x\right)=\log\left(5^{x-1}\right)$$

$$\log10+x\log3=(x-1)\log5$$

$$x\log3-x\log5=-\log5-\log10$$

$$x\left(\log3-\log5\right)=-\log5-\log10$$

$$x=\frac{\log5+\log10}{\log5-\log3}=\frac{2\log5+\log2}{\log5-\log3}$$

$$1=\log(5\cdot2)=\log5+\log2$$

$$10\cdot3^x=5^x\cdot5^{-1}$$

$$\left(\frac{3}{5}\right)^x=\frac{1}{50}\qquad x=\log_{\frac{3}{5}}\left(\frac{1}{50}\right)=$$

$$=\frac{\overbrace{\log1}^{=0}-\log50}{\log3-\log5}$$

$$=\ldots\ldots$$

326 $\log_2(x^2+1) = 1 + \frac{2}{3}\log_2 x + \log_8 x$

C.E.

$\begin{cases} x^2+1>0 & \forall x \\ x>0 \end{cases}$ $\boxed{x>0}$

$\log_2(x^2+1) = \log_2 2 + \frac{2}{3}\log_2 x + \frac{\log_2 x}{\log_2 8}$

$\log_2(x^2+1) = \log_2 2 + \frac{2}{3}\log_2 x + \frac{1}{3}\log_2 x$

$\log_2(x^2+1) = \log_2 2 + \log_2 x$

$\log_2(x^2+1) = \log_2(2x) \rightsquigarrow x^2+1 = 2x$

$x^2-2x+1=0$

$(x-1)^2=0$

$\boxed{x=1}$ OK

325 $\log_2(x^2-4)+2\log_2 x = 1+\log_2(5x^2+16)$

C.E.

$$\begin{cases} x^2-4>0 & x<-2 \vee x>2 \\ x>0 & \Rightarrow \boxed{x>2} \\ 5x^2+16>0 & \forall x \end{cases}$$

$$\log_2(x^2-4)+\log_2 x^2 = \log_2 2+\log_2(5x^2+16)$$

$$\log_2\left[x^2(x^2-4)\right] = \log_2\left[2(5x^2+16)\right]$$

$$x^4-4x^2 = 10x^2+32$$

$$x^4-14x^2-32=0$$

$$x^2 = 7\pm\sqrt{49+32} = 7\pm\sqrt{81} = 7\pm 9 = \begin{cases} -2 & \text{N.A.} \\ 16 \end{cases}$$

$$\Downarrow$$

$$x^2=16$$

$$x=\pm 4$$

$x=-4$ N.A. $\vee$ $x=4$

$$\boxed{x=4}$$

316 $2\log_2\sqrt{x-2}+\log_2 x = 3$

C.E.

$\begin{cases} x-2>0 \\ x>0 \end{cases}$ $\boxed{x>2}$

$$\log_2\left(\sqrt{x-2}\right)^2+\log_2 x = 3\cdot\log_2 2$$

$$\log_2(x-2)+\log_2 x = \log_2 2^3$$

$$\log_2\left[(x-2)\cdot x\right] = \log_2 8$$

$$(x-2)\cdot x = 8$$

$$x^2-2x-8=0$$

$$(x-4)(x+2)=0$$

$x = 4$ V

$x = -2$ N.A.

$\boxed{x=4}$