26/4/2021

122 $\displaystyle\int_1^{e^2} \frac{\ln x}{\sqrt{x}}\,dx =$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$x = 1 \Rightarrow t = 1$

$x = e^2 \Rightarrow t = e$

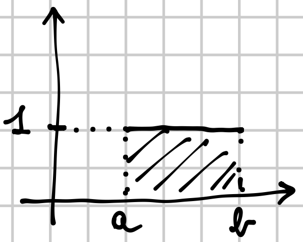
$$= \int_1^e \frac{\ln t^2}{\cancel{t}}\, 2\cancel{t}\,dt =$$

$\ln t^2 = 2\ln t$

$$= 2\int_1^e \ln t^2\,dt = 4\int_1^e \ln t\,dt =$$

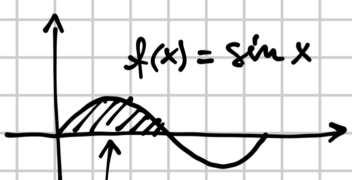
$$= 4\int_1^e \underbrace{1}_{(t)'}\cdot \ln t\,dt = 4\left[\left[t\ln t\right]_1^e - \int_1^e t\cdot \overbrace{\frac{1}{t}}^{(\ln t)'}\,dt\right] =$$

$$= 4\left[e\ln e - \int_1^e dt\right] = 4\left[e-(e-1)\right] = 4\left[\cancel{e}-\cancel{e}+1\right] = 4$$



$$\int_a^b dt = b-a$$

OSSERVAZIONE



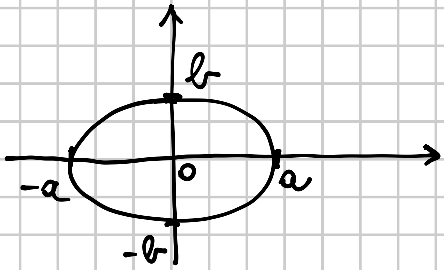
quanto vale questa area?

$$A = \int_0^{\pi} \sin x\,dx = \int_0^{\pi} (-\cos x)'\,dx =$$

$$= \left[-\cos x\right]_0^{\pi} = -\cos\pi - (-\cos 0) =$$

$$= 1 + 1 = 2$$

AREA DELL'ELLISSE



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2} \qquad y^2=b^2\left(1-\frac{x^2}{a^2}\right)$$

$$y=\pm b\sqrt{1-\frac{x^2}{a^2}}$$

PARTE SOPRA $\quad y=b\sqrt{1-\frac{x^2}{a^2}}$

$$A_{ELLISSE}=4\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx=4b\int_0^a\sqrt{1-\frac{x^2}{a^2}}\,dx=$$

$x=a\sin t$

$dx=a\cos t\,dt$

$x=0 \Rightarrow t=0$

$x=a \Rightarrow \sin t=1 \Rightarrow t=\frac{\pi}{2}$

$$=4b\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2 t}\cdot a\cos t\,dt=$$

$$=4ab\int_0^{\frac{\pi}{2}}\cos^2 t\,dt=$$

$$=\overset{2}{\cancel{4}}ab\int_0^{\frac{\pi}{2}}\frac{1+\cos 2t}{\cancel{2}}\,dt=$$

$\cos 2t=2\cos^2 t-1$

$\Downarrow$

$\cos^2 t=\frac{1+\cos 2t}{2}$

$$=2ab\left[\int_0^{\frac{\pi}{2}}dt+\int_0^{\frac{\pi}{2}}\cos 2t\,dt\right]=$$

$$=2ab\left[\frac{\pi}{2}+\frac{1}{2}\int_0^{\frac{\pi}{2}}2\cos 2t\,dt\right]=$$

$$=2ab\left[\frac{\pi}{2}+\frac{1}{2}\left[\sin 2t\right]_0^{\frac{\pi}{2}}\right]=2ab\left[\frac{\pi}{2}+\frac{1}{2}(0-0)\right]=\cancel{2}ab\frac{\pi}{\cancel{2}}=\boxed{ab\pi}$$