18/3/2021

318 $2x^4 - x^3 + 8x^2 - 4x < 0$ $\left[0 < x < \frac{1}{2}\right]$

$$x^3(2x-1) + 4x(2x-1) < 0$$

$$(2x-1)(x^3+4x) < 0$$

$$\underset{1}{x}\,\underset{2}{(2x-1)}\,\underset{3}{(x^2+4)} < 0$$

1. $x > 0$

2. $2x - 1 > 0 \qquad x > \frac{1}{2}$

3. $x^2 + 4 > 0 \qquad \forall x \in \mathbb{R}$

$\Delta = 0^2 - 4 \cdot 1 \cdot 4 = -16 < 0$

		0		$\frac{1}{2}$	
1	$-$	0	$+$		$+$
2	$-$		$-$	0	$+$
3	$+$		$+$		$+$
	$+$	0	$-$	0	$+$

$$\boxed{0 < x < \frac{1}{2}}$$

333 $x^6 - 7x^3 - 8 < 0$

1° MODO

$$(x^3+1)(x^3-8) < 0$$

$$\underbrace{(x+1)}_{①}\underbrace{(x^2-x+1)}_{② \ \Delta<0}\underbrace{(x-2)}_{③}\underbrace{(x^2+2x+4)}_{④ \ \Delta<0} < 0$$

① $x+1>0 \quad x>-1$

② $x^2-x+1>0 \quad \forall x \in \mathbb{R}$

$\Delta = 1-4<0$

③ $x-2>0 \quad x>2$

④ $x^2+2x+4>0 \quad \forall x\in\mathbb{R}$

$\Delta = 4-16<0$

		-1		2	
①	$-$	0	$+$		$+$
②	$+$		$+$		$+$
③	$-$		$-$	0	$+$
④	$+$		$+$		$+$
	$+$	0	$-$	0	$+$

$$\boxed{-1<x<2}$$

2° MODO

$x^6-7x^3-8<0$

$t=x^3 \qquad t^2-7t-8<0 \qquad \Delta = 49+32=81$

$$t_{1,2} = \frac{7\pm 9}{2} = \begin{cases} -1 \\ 8 \end{cases}$$

$-1<t<8 \Rightarrow -1<x^3<8$

$\Downarrow$

applico la radice CUBICA!!

$$\boxed{-1<x<2}$$

332 $x^4 + 5x^2 - 14 \geq 0$ $\quad [x \leq -\sqrt{2} \vee x \geq \sqrt{2}]$

1° MODO

$$(x^2+7)(x^2-2) \geq 0$$

① ②

① $x^2+7>0 \quad \forall x \in \mathbb{R}$

② $x^2-2>0 \quad x<-\sqrt{2} \vee x>\sqrt{2}$

$x_{1,2} = \pm\sqrt{2}$

		$-\sqrt{2}$		$\sqrt{2}$	
①	+		+		+
②	+	0	−	0	+
	+	0	−	0	+

$$\boxed{x \leq -\sqrt{2} \quad \vee \quad x \geq \sqrt{2}}$$

2° MODO (più "delicato")

$x^4+5x^2-14 \geq 0 \qquad x^2 = t$

$t^2+5t-14 \geq 0$

$(t+7)(t-2) \geq 0$

$t_1 = -7 \quad t_2 = 2 \qquad t \leq -7 \vee t \geq 2$

$\underbrace{x^2 \leq -7}_{\text{IMPOSSIBILE}} \vee x^2 \geq 2$

$\Downarrow$

$x^2 \geq 2$

$x^2-2 \geq 0 \qquad x_{1,2} = \pm\sqrt{2}$

$$\boxed{x \leq -\sqrt{2} \vee x \geq \sqrt{2}}$$

336 $3x^4 - 2x^2 + 5 < 0$

$x^2 = t$

$3t^2 - 2t + 5 < 0$ IMPOSSIBILE

$\Delta = 4 - 60 = -56 < 0$

É difficile scomporre $3x^4 - 2x^2 + 5$, però esiste la sua scomposizione nel prodotto di due polinomi di 2° grado, entrambi con $\Delta < 0$.

SCOMPOSIZIONE

$$3x^4 - 2x^2 + 5 = 3x^4 + 5 - 2x^2 =$$

$$= 3x^4 + 5 + 2\sqrt{15}\,x^2 - 2\sqrt{15}\,x^2 - 2x^2 =$$

$$= \left(\sqrt{3}x^2 + \sqrt{5}\right)^2 - 2\sqrt{15}\,x^2 - 2x^2 =$$

$$= \left(\sqrt{3}x^2 + \sqrt{5}\right)^2 - \left(2\sqrt{15} + 2\right)x^2 =$$

$$= \left(\sqrt{3}x^2 + \sqrt{5} - \sqrt{2\sqrt{15} + 2}\;x\right)\left(\sqrt{3}x^2 + \sqrt{5} + \sqrt{2\sqrt{15} + 2}\;x\right)$$

385 $$\frac{x^4-4x^2+3}{x(9-x^2)}>0$$

$$\frac{(x^2-3)(x^2-1)}{-x(x^2-9)}>0$$

elimino e inverto

$$\frac{\overset{N_1}{(x^2-3)}\overset{N_2}{(x^2-1)}}{\underset{D_1}{x}\underset{D_2}{(x^2-9)}}<0$$

$N_1>0 \quad x^2-3>0 \quad x<-\sqrt{3} \lor x>\sqrt{3}$

$N_2>0 \quad x^2-1>0 \quad x<-1 \lor x>1$

$D_1>0 \quad x>0$

$D_2>0 \quad x^2-9>0 \quad x<-3 \lor x>3$

		-3		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$		3	
N_1	+		+	0	−		−		−		−	0	+		+
N_2	+		+		+	0	−		−	0	+		+		+
D_1	−		−		−		−	∄	+		+		+		+
D_2	+	∄	−		−		−		−		−		−	∄	+
	−	∄	+	0	−	0	+	∄	−	0	+	0	−	∄	+

$$\boxed{x<-3 \lor -\sqrt{3}<x<-1 \lor 0<x<1 \lor \sqrt{3}<x<3}$$

386 $\dfrac{(x+3)^3}{(x^2-x)(4-x^2)} \geq 0$

$$\frac{(x+3)^3}{-\,x(x-1)(x^2-4)} \geq 0$$

$$\Downarrow$$

$$\frac{\overset{\boxed{N_1}}{(x+3)^3}}{\underset{\boxed{D_1}}{x}\,\underset{\boxed{D_2}}{(x-1)}\,\underset{\boxed{D_3}}{(x^2-4)}} \leq 0$$

$N_1 > 0 \qquad (x+3)^3 > 0 \Rightarrow \qquad x+3>0 \qquad x>-3$

$D_1 > 0 \qquad x > 0$

$D_2 > 0 \qquad x-1>0 \qquad x>1$

$D_3 > 0 \qquad x^2-4>0 \qquad x<-2 \ \vee \ x>2$

		-3		-2		0		1		2	
N_1	$-$	0	$+$		$+$		$+$		$+$		$+$
D_1	$-$		$-$		$-$	$\nexists$	$+$		$+$		$+$
D_2	$-$		$-$		$-$		$-$	$\nexists$	$+$		$+$
D_3	$+$		$+$	$\nexists$	$-$		$-$		$-$	$\nexists$	$+$
	$-$	0 =	$+$	$\nexists$	$-$	$\nexists$	$+$	$\nexists$	$-$	$\nexists$	$+$

$$\boxed{x \leq -3 \ \vee \ -2<x<0 \ \vee \ 1<x<2}$$

387 $$\frac{\overbrace{(x^2-1)^2}^{\boxed{N_1}}\overbrace{(x^2-4x)^3}^{\boxed{N_2}}}{\underbrace{2x^2+1}_{\boxed{D_1}}} \geq 0 \qquad [x \leq 0 \vee x = 1 \vee x \geq 4]$$

$\boxed{N_1}$ $(x^2-1)^2 > 0 \quad \forall x \neq \pm 1$

$(x^2-1)^2$ è sempre positivo tranne che per gli x che annullano x^2-1, cioè ± 1

$\boxed{N_2}$ $(x^2-4x)^3 > 0 \implies x^2-4x > 0 \qquad x < 0 \vee x > 4$

$x(x-4) > 0$

$x_1 = 0 \quad x_2 = 4$

$\boxed{D_1}$ $2x^2+1 > 0 \quad \forall x$

$\Delta < 0$

		-1		0		1		4	
N_1	$+$	0	$+$		$+$	0	$+$		$+$
N_2	$+$		$+$	0	$-$		$-$	0	$+$
D_1	$+$		$+$		$+$		$+$		$+$
	$+$	0	$+$	0	$-$	0	$-$	0	$+$

$$\boxed{x \leq 0 \quad \vee \quad x = 1 \quad \vee \quad x \geq 4}$$