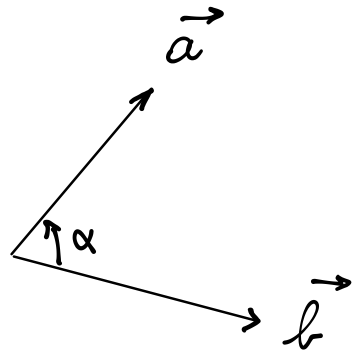
IL PRODOTTO SCALARE DI 2 VETTORI



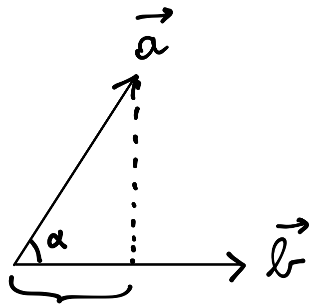
PRODOTTO SCALARE DI $\vec{a}$ E $\vec{b}$

$$\boxed{\vec{a} \cdot \vec{b} = ab\cos\alpha}$$

PER DEFINIZIONE

α = angolo (più piccolo) tra i 2 vettori

$$\underbrace{\vec{a} \cdot \vec{b}}_{\text{SI PRONUNCIA "}\vec{a}\text{ SCALAR }\vec{b}\text{"}} = ab\cos\alpha = \underbrace{(a\cos\alpha)}_{a_b}\, b$$

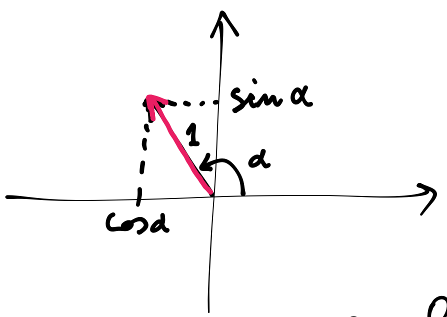


$a_b = a\cos\alpha$

PROPRIETÀ DEL PRODOTTO SCALARE

- COMMUTATIVITÀ $\quad \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- SE $\underbrace{\vec{a} = \vec{0}}$ OPPURE $\vec{b} = \vec{0}$, ALLORA $\vec{a} \cdot \vec{b} = 0$

  VETTORE NULLO
  - MODULO 0
  - DIREZIONE E VERSO INDETERMINATI
- SE $\vec{a} \perp \vec{b}$ (PERPENDICOLARI), ALLORA $\vec{a} \cdot \vec{b} = 0 \quad (\cos 90° = 0)$



$\cos 0° = 1$

$\cos 90° = 0$

$\cos 180° = -1$

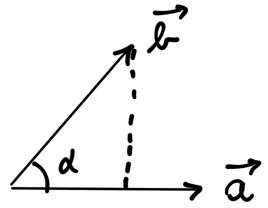
$-1 \leq \cos\alpha \leq 1$

se $0° < \alpha < 90°$, allora $\cos\alpha > 0$

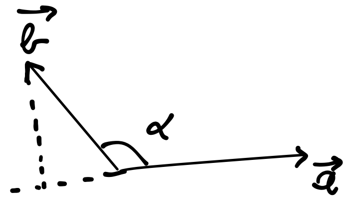
se $90° < \alpha < 180°$, allora $\cos\alpha < 0$

- SE α ACUTO, ALLORA $\vec{a}\cdot\vec{b} > 0$

  SE α OTTUSO, ALLORA $\vec{a}\cdot\vec{b} < 0$



$\cos\alpha > 0$



$\cos\alpha < 0$

- PROPRIETÀ DISTRIBUTIVA

$$(\vec{a}+\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$$

- PROPRIETÀ IMPORTANTISSIMA = SE I VETTORI SONO DATI CON LE COMPONENTI CARTESIANE

$$\vec{a} = (a_x, a_y) \qquad \vec{b} = (b_x, b_y)$$

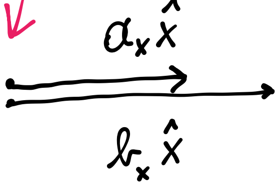
$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$$

DIMOSTRAZIONE

$$\vec{a} = (a_x, a_y) = a_x\hat{x} + a_y\hat{y} \qquad \vec{b} = (b_x, b_y) = b_x\hat{x} + b_y\hat{y}$$

$$\vec{a}\cdot\vec{b} = (a_x\hat{x} + a_y\hat{y})\cdot(b_x\hat{x} + b_y\hat{y}) =$$

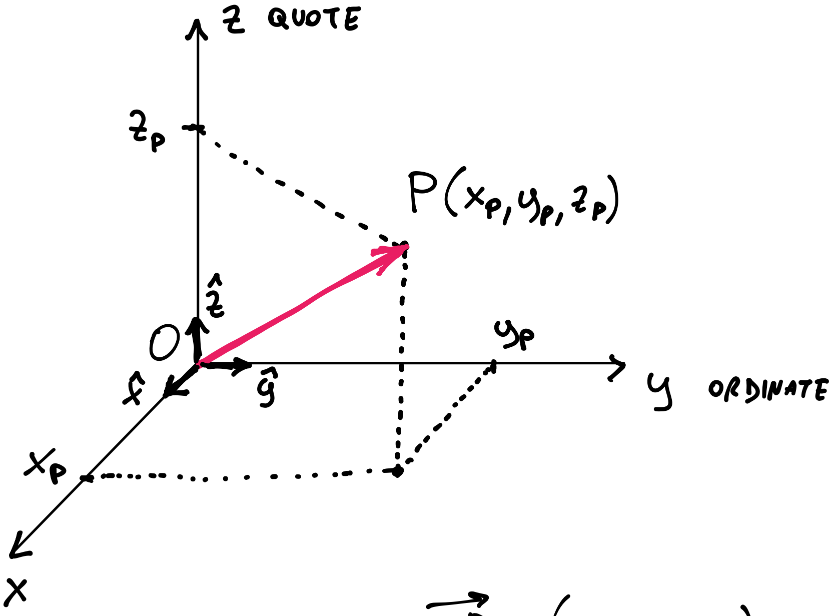
$$= a_x\hat{x}\cdot b_x\hat{x} + \underbrace{\cancel{a_x\hat{x}\cdot b_y\hat{y}}}_{=0 \text{ perché } a_x\hat{x} \perp b_y\hat{y}} + \underbrace{\cancel{a_y\hat{y}\cdot b_x\hat{x}}}_{=0 \text{ IDEM}} + a_y\hat{y}\cdot b_y\hat{y} =$$



$$= a_x b_x + a_y b_y$$

$$a_x\hat{x}\cdot b_x\hat{x} = a_x b_x$$

In generale si lavora nello spazio



$$\overrightarrow{OP} = (x_P, y_P, z_P) =$$

$$= x_P\,\hat{x} + y_P\,\hat{y} + z\,\hat{z}_P$$

$$\left.\begin{aligned}\hat{x} &= (1,0,0)\\ \hat{y} &= (0,1,0)\\ \hat{z} &= (0,0,1)\end{aligned}\right\}\ \text{VERSORI DEGLI ASSI CARTESIANI}$$