

5/3/2021

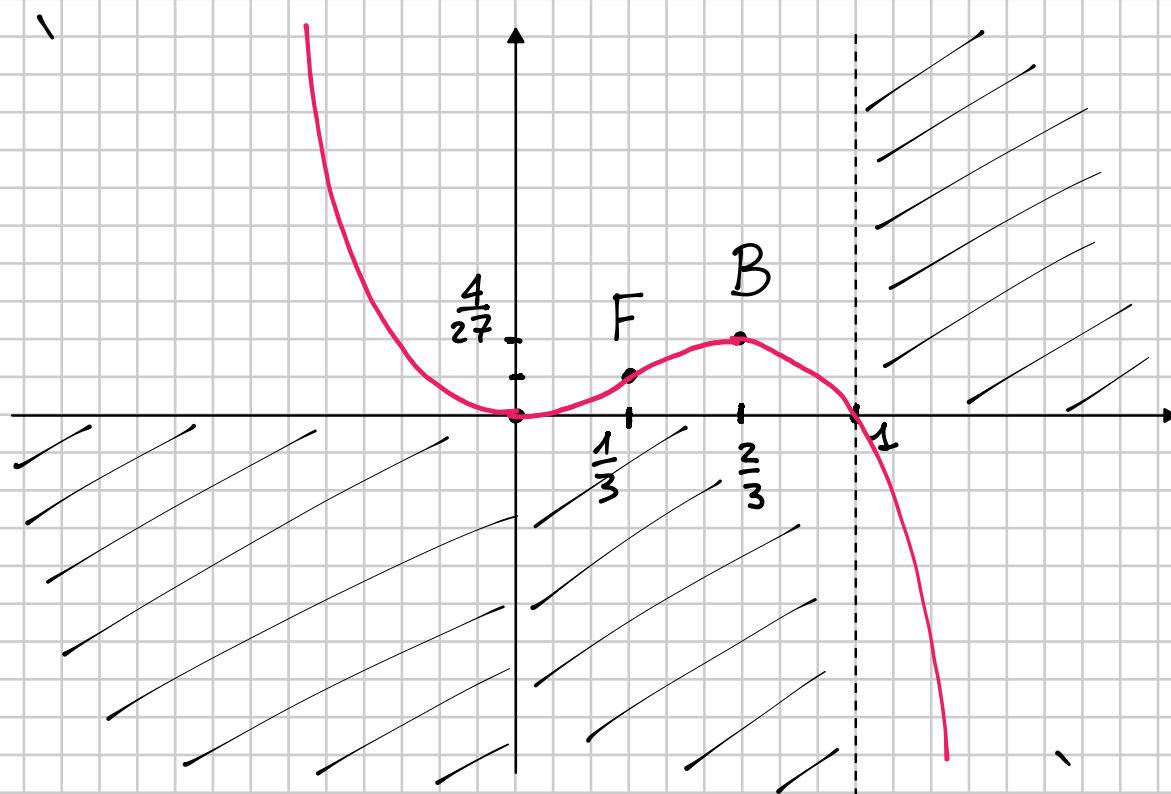
STUDIO COMPLETO DI FUNZIONE

PAG. 1858 N. 31

$$f(x) = x^2 - x^3$$

1) DOMINIO: $\mathbb{R} =]-\infty, +\infty[$

2) PARI/DISPARI: NO

3) ZERI DI f

$$f(x) = 0 \quad x^2 - x^3 = 0 \quad x^2(1-x) = 0 \quad x = 0 \quad \vee \quad x = 1$$

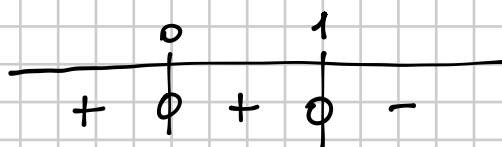
O(0,0) A(1,0)

INTERSEZ. CON ASSE X

4) SEGNO DI f

$$f(x) > 0 \quad x^2 - x^3 > 0 \quad x^2(1-x) > 0 \quad x < 1$$

f



5) LIMITI AGLI ESTREMI DEL DOMINIO

$$\lim_{x \rightarrow -\infty} (x^2 - x^3) = +\infty + \infty = +\infty$$

$$\lim_{x \rightarrow +\infty} (x^2 - x^3) =$$

$$= \lim_{x \rightarrow +\infty} \underbrace{x^3}_{+\infty} \left(\underbrace{\left(\frac{1}{x} - 1 \right)}_{\begin{matrix} \nearrow -1 \\ \downarrow 0 \end{matrix}} \right) = +\infty \cdot (-1) = -\infty$$

6) STUDIO f'

$$f(x) = x^2 - x^3$$

$$f'(x) = 2x - 3x^2$$

ZERI DI f'

$$2x - 3x^2 = 0$$

$$x = 0 \vee x = \frac{2}{3}$$

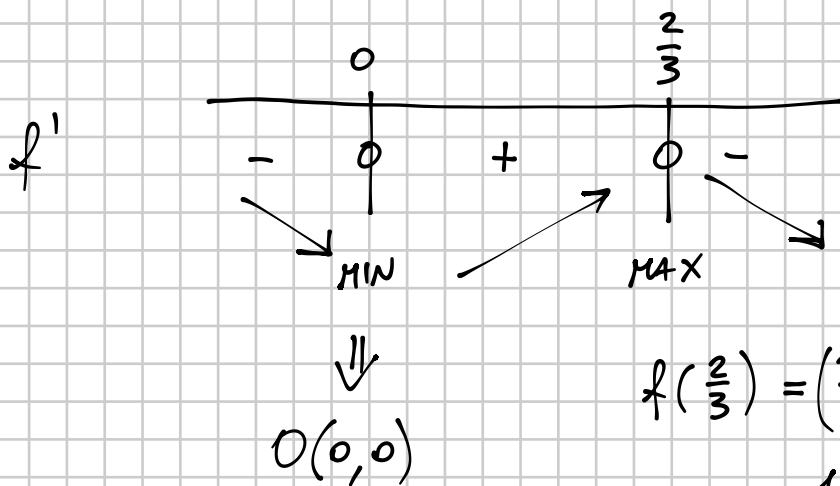
$$x(2 - 3x) = 0$$

SEGNO DI f'

$$2x - 3x^2 > 0$$

$$3x^2 - 2x < 0$$

$$0 < x < \frac{2}{3}$$



$$f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3 = \frac{4}{9} - \frac{8}{27} = \frac{12-8}{27} = \frac{4}{27} \approx 0,15$$

$$B\left(\frac{2}{3}, \frac{4}{27}\right)$$

7) STUDIO DI f''

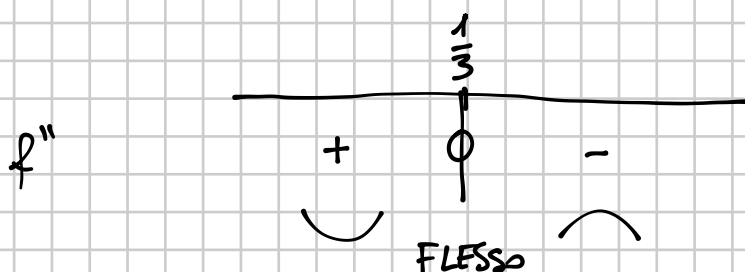
$$f'(x) = 2x - 3x^2 \quad f''(x) = 2 - 6x$$

ZERI DI f''

$$f''(x) = 0 \quad 2 - 6x = 0 \Rightarrow x = \frac{1}{3}$$

SEGNO DI f''

$$f''(x) > 0 \quad 2 - 6x > 0 \Rightarrow x < \frac{1}{3}$$



$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^3 = \frac{1}{9} - \frac{1}{27} = \frac{3-1}{27} = \frac{2}{27}$$

$$F\left(\frac{1}{3}, \frac{2}{27}\right)$$

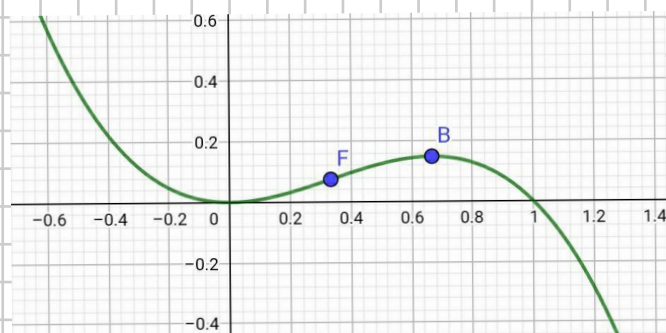
8) EVENTUALI ASINTOTI OBLIQUI

$$m = \lim_{x \rightarrow \infty} \frac{f(x)}{x}$$

$$q = \lim_{x \rightarrow \infty} [f(x) - mx]$$

$$m = \lim_{x \rightarrow \infty} \frac{x^2 - x^3}{x} = \lim_{x \rightarrow \infty} (x - x^2) = +\infty$$

NON CI
SONO ASINTOTI
OBLIQUI

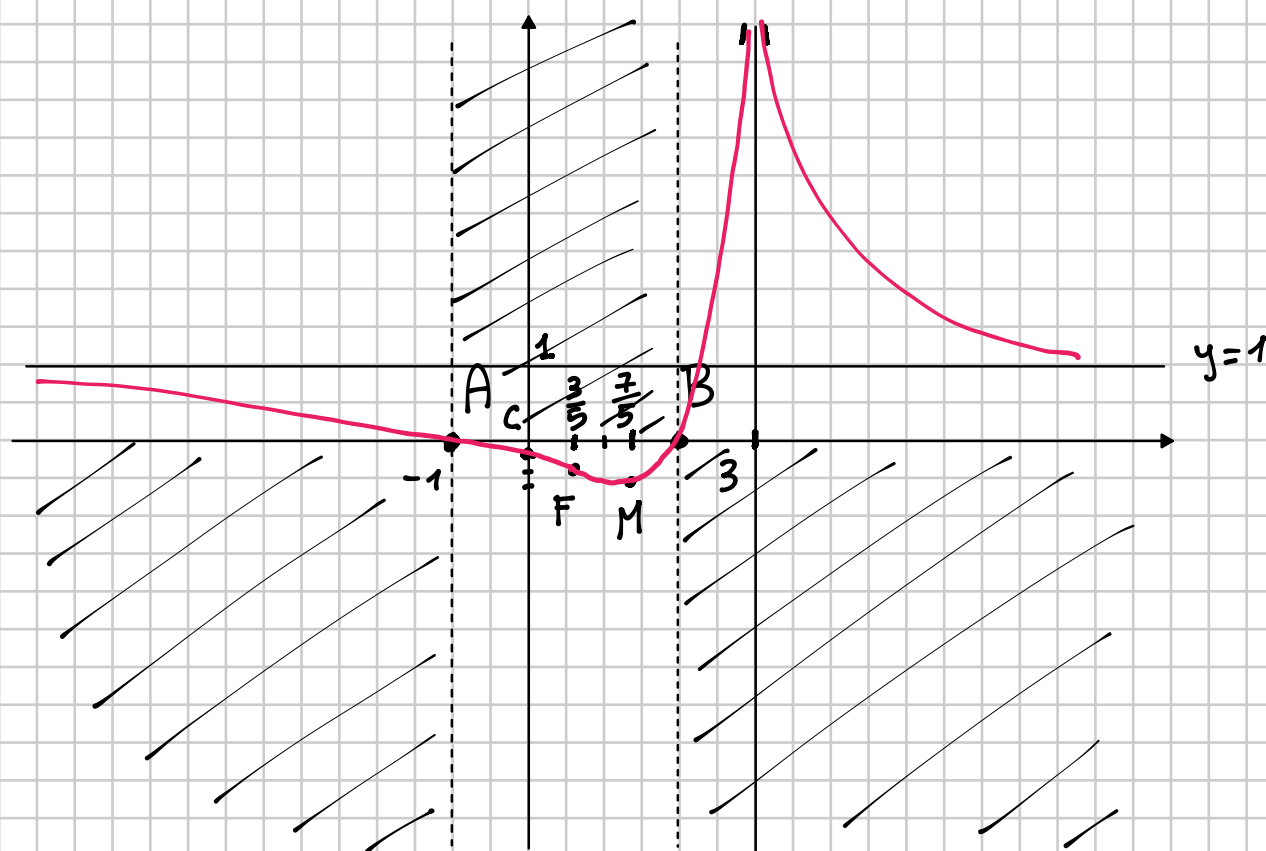


$$f(x) = \frac{x^2 - x - 2}{x^2 - 6x + 9}$$

1) DOMINIO: $x^2 - 6x + 9 \neq 0$ $(x-3)^2 \neq 0$ $x \neq 3$

$$D =]-\infty, 3[\cup]3, +\infty[$$

2) PARI/DISPARI: non è né pari né dispari perché il dominio non è simmetrico risp. a 0



3) INTERSEZIONI CON GLI ASSI

• ZERI DI f (INT. CON ASSE X)

$$\begin{cases} y = \frac{x^2 - x - 2}{(x-3)^2} \\ y = 0 \end{cases} \Rightarrow \begin{cases} x^2 - x - 2 = 0 \\ (x-2)(x+1) = 0 \\ x = 2 \vee x = -1 \end{cases}$$

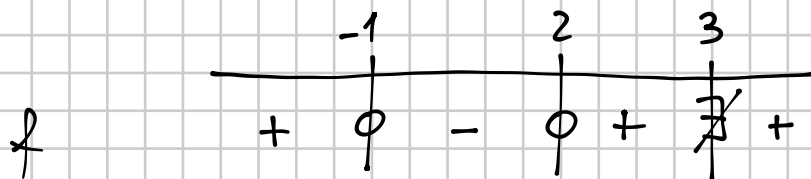
$$A(-1, 0) \quad B(2, 0)$$

• INT. CON ASSE Y

$$\begin{cases} y = \frac{x^2 - x - 2}{(x-3)^2} \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = -\frac{2}{9} \\ x = 0 \end{cases} \quad C\left(0, -\frac{2}{9}\right)$$

4) SEGNO DI f

$$\frac{x^2 - x - 2}{(x-3)^2} > 0 \quad \frac{(x-2)(x+1)}{(x-3)^2} > 0 \quad \begin{cases} x < -1 \vee x > 2 \\ x \neq 3 \end{cases}$$

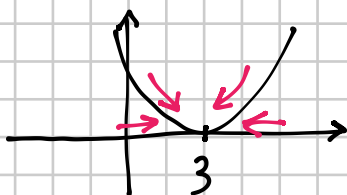


5) LIMITI AGLI ESTREMI DEL DOMINIO $D =]-\infty, 3[\cup]3, +\infty[$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2 - x - 2}{x^2 - 6x + 9} = 1$$

$y = 1$ ASINTOTO ORIZZONTALE
per $x \rightarrow \pm\infty$

$$\lim_{x \rightarrow 3^\pm} \frac{x^2 - x - 2}{(x-3)^2} = \frac{9 - 3 - 2}{0^+} = \frac{4}{0^+} = +\infty \quad x = 3 \text{ ASINTOTO VERTICALE}$$



6) STUDIO DERIVATA PRIMA

$$f'(x) = \frac{(2x-1)(x-3)^2 - 2(x-3)(x^2-x-2)}{(x-3)^4} =$$

$$= \frac{(x-3)(\cancel{2x^2} - 6x - x + 3 - \cancel{2x^2} + 2x + 4)}{(x-3)^4} = \frac{\cancel{(x-3)}(7-5x)}{(x-3)^4 \cancel{3}} =$$

$$= \frac{7-5x}{(x-3)^3}$$

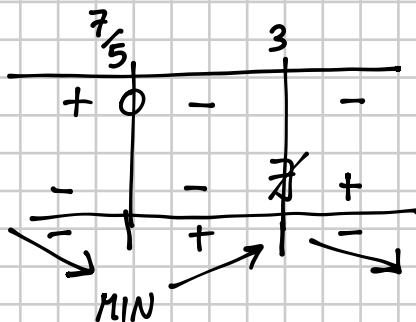
ZERI $\Rightarrow x = \frac{7}{5}$

SEGNO $\Rightarrow f'(x) > 0$

$$\frac{7-5x}{(x-3)^3} > 0$$

$$N) 7-5x > 0 \quad x < \frac{7}{5}$$

$$D) (x-3)^3 > 0 \quad x > 3$$



$$f\left(\frac{7}{5}\right) = \frac{\left(\frac{7}{5}\right)^2 - \frac{7}{5} - 2}{\left(\frac{7}{5} - 3\right)^2} = \frac{\frac{49}{25} - \frac{7}{5} - 2}{\left(\frac{7-15}{5}\right)^2} = \frac{\frac{49-35-50}{25}}{\frac{64}{25}} =$$

$$= -\frac{36}{64} = -\frac{9}{16}$$

$$M\left(\frac{7}{5}, -\frac{9}{16}\right)$$

7) STUDIO DERIVATA SECONDA

$$f'(x) = \frac{7-5x}{(x-3)^3}$$

$$f''(x) = \frac{-5(x-3)^3 - 3(x-3)^2(7-5x)}{(x-3)^6} =$$

$$= \frac{(x-3)^2(-5x+15-21+15x)}{(x-3)^6} =$$

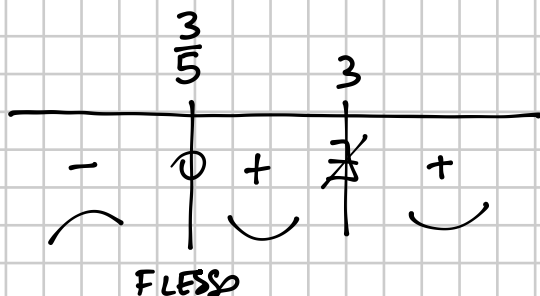
$$= \frac{10x-6}{(x-3)^4}$$

ZERI DI f''

$$10x-6=0 \Rightarrow x = \frac{3}{5}$$

SEGNO DI f''

$$\frac{10x-6}{(x-3)^4} > 0 \Rightarrow \begin{cases} x > \frac{3}{5} \\ x \neq 3 \end{cases}$$



$$f\left(\frac{3}{5}\right) = \frac{\left(\frac{3}{5}\right)^2 - \frac{3}{5} - 2}{\left(\frac{3}{5} - 3\right)^2} = \dots = -\frac{7}{18}$$

$$F\left(\frac{3}{5}, -\frac{7}{18}\right)$$