

18/3/2021

318 $2x^4 - x^3 + 8x^2 - 4x < 0$

$$\left[0 < x < \frac{1}{2} \right]$$

$$x^3(2x-1) + 4x(2x-1) < 0$$

$$(2x-1)(x^3+4x) < 0$$

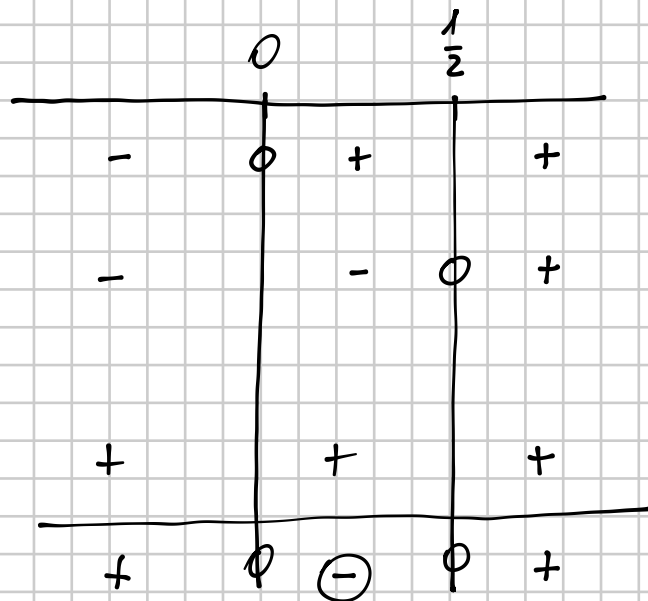
$$\underset{\textcircled{1}}{x} \underset{\textcircled{2}}{(2x-1)} \underset{\textcircled{3}}{(x^2+4)} < 0$$

$\textcircled{1} \quad x > 0$

$\textcircled{2} \quad 2x-1 > 0 \quad x > \frac{1}{2}$

$\textcircled{3} \quad x^2+4 > 0 \quad \forall x \in \mathbb{R}$

$$\Delta = 0^2 - 4 \cdot 1 \cdot 4 = -16 < 0$$



$$\boxed{0 < x < \frac{1}{2}}$$

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$$x^6 - 7x^3 - 8 < 0$$

1° Modo

$$(x^3 + 1)(x^3 - 8) < 0$$

$$(x+1)(x^2-x+1)(x-2)(x^2+2x+4) < 0$$

①

②

$$\Delta < 0$$

③

④

$$\Delta < 0$$

$$\textcircled{1} \quad x+1 > 0 \quad x > -1$$

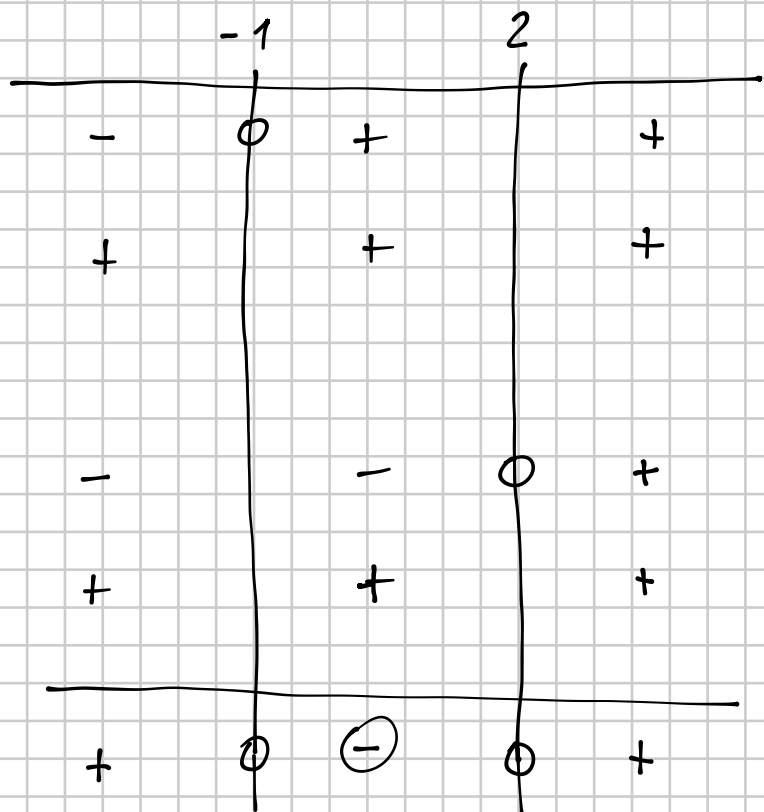
$$\textcircled{2} \quad x^2 - x + 1 > 0 \quad \forall x \in \mathbb{R}$$

$$\Delta = 1 - 4 < 0$$

$$\textcircled{3} \quad x - 2 > 0 \quad x > 2$$

$$\textcircled{4} \quad x^2 + 2x + 4 > 0 \quad \forall x \in \mathbb{R}$$

$$\Delta = 4 - 16 < 0$$



$$-1 < x < 2$$

2° Modo

$$x^6 - 7x^3 - 8 < 0$$

$$t = x^3$$

$$t^2 - 7t - 8 < 0$$

$$\Delta = 49 + 32 = 81$$

$$t_{1,2} = \frac{7 \pm 9}{2} = \begin{cases} -1 \\ 8 \end{cases}$$

$$-1 < t < 8 \Rightarrow -1 < x^3 < 8$$

applies to
radice cubica!!



$$-1 < x < 2$$

332 $x^4 + 5x^2 - 14 \geq 0$

$[x \leq -\sqrt{2} \vee x \geq \sqrt{2}]$

1º modo

$$(x^2 + 7)(x^2 - 2) \geq 0$$

(1) (2)

(1) $x^2 + 7 > 0 \quad \forall x \in \mathbb{R}$

(2) $x^2 - 2 > 0 \quad x < -\sqrt{2} \vee x > \sqrt{2}$

$x_{1,2} = \pm\sqrt{2}$

	$-\sqrt{2}$		$\sqrt{2}$	
+		+		+
+	0	-	0	+
(+)	0	-	0	(+)

$x \leq -\sqrt{2} \vee x \geq \sqrt{2}$

2º modo (pirm "delicats")

$$x^4 + 5x^2 - 14 \geq 0 \quad x^2 = t$$

$$t^2 + 5t - 14 \geq 0$$

$$(t + 7)(t - 2) \geq 0$$

$t_1 = -7 \quad t_2 = 2$

$t \leq -7 \vee t \geq 2$

$x^2 \leq -7 \vee x^2 \geq 2$

IMPOSSIBLE

\Downarrow

$x^2 \geq 2$

$x^2 - 2 \geq 0 \quad x_{1,2} = \pm\sqrt{2}$

$x \leq -\sqrt{2} \vee x \geq \sqrt{2}$

336 $3x^4 - 2x^2 + 5 < 0$

$$x^2 = t$$

$$3t^2 - 2t + 5 < 0 \quad \text{IMPOSSIBILE}$$

$$\Delta = 4 - 60 = -56 < 0$$

È difficile scomporre $3x^4 - 2x^2 + 5$, però esiste la sua scomposizione nel prodotto di due polinomi di 2° grado, entrambi con $\Delta < 0$.

SCOMPOSIZIONE

$$3x^4 - 2x^2 + 5 = 3x^4 + 5 - 2x^2 =$$

$$= 3x^4 + 5 + 2\sqrt{15}x^2 - 2\sqrt{15}x^2 - 2x^2 =$$

$$= (\sqrt{3}x^2 + \sqrt{5})^2 - 2\sqrt{15}x^2 - 2x^2 =$$

$$= (\sqrt{3}x^2 + \sqrt{5})^2 - (2\sqrt{15} + 2)x^2 =$$

$$= (\sqrt{3}x^2 + \sqrt{5} - \sqrt{2\sqrt{15} + 2}x)(\sqrt{3}x^2 + \sqrt{5} + \sqrt{2\sqrt{15} + 2}x)$$

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$$\frac{x^4 - 4x^2 + 3}{x(9 - x^2)} > 0$$

$$\frac{(x^2 - 3)(x^2 - 1)}{-x(x^2 - 9)} > 0$$

eliminate and invert

$$\frac{\begin{matrix} N_1 & N_2 \\ (x^2 - 3) & (x^2 - 1) \end{matrix}}{\begin{matrix} D_1 & D_2 \\ x & (x^2 - 9) \end{matrix}} < 0$$

$$N_1 > 0 \quad x^2 - 3 > 0 \quad x < -\sqrt{3} \vee x > \sqrt{3}$$

$$N_2 > 0 \quad x^2 - 1 > 0 \quad x < -1 \vee x > 1$$

$$D_1 > 0 \quad x > 0$$

$$D_2 > 0 \quad x^2 - 9 > 0 \quad x < -3 \vee x > 3$$

	-3	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	3
N_1	+	+	-	-	-	-	+
N_2	+	+	+	0	-	0	+
D_1	-	-	-	-	+	+	+
D_2	+	-	-	-	-	-	+
Sign	-	+	-	+	-	+	-

$$x < -3 \vee -\sqrt{3} < x < -1 \vee 0 < x < 1 \vee \sqrt{3} < x < 3$$

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$$\frac{(x+3)^3}{(x^2-x)(4-x^2)} \geq 0$$

$$\frac{(x+3)^3}{-x(x-1)(x^2-4)} \geq 0$$

$$\Downarrow$$

$$\frac{(x+3)^3}{x(x-1)(x^2-4)} \leq 0$$

$\boxed{D_1}$ $\boxed{D_2}$ $\boxed{D_3}$

$$N_1 > 0 \quad (x+3)^3 > 0 \Rightarrow x+3 > 0 \quad x > -3$$

$$D_1 > 0 \quad x > 0$$

$$D_2 > 0 \quad x-1 > 0 \quad x > 1$$

$$D_3 > 0 \quad x^2-4 > 0 \quad x < -2 \vee x > 2$$

	-3	-2	0	1	2
N_1	-	0	+	+	+
D_1	-	-	-	+	+
D_2	-	-	-	-	+
D_3	+	+	-	-	-
	-	0	+	-	+
	-	+	-	+	-

$$x \leq -3 \vee -2 < x < 0 \vee 1 < x < 2$$

