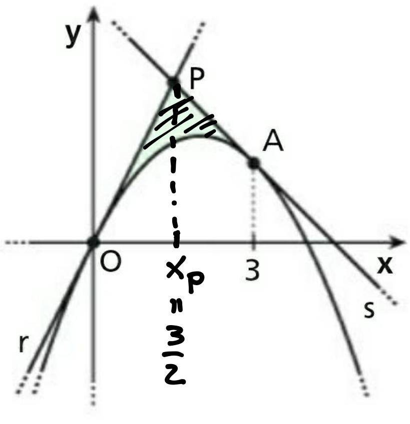
14/5/2021

330 Le rette r e s sono tangenti alla parabola di equazione

$$y = -\frac{1}{2}x^2 + 2x$$

rispettivamente in O e A. Trova l'area della zona colorata.

$\left[\frac{9}{8}\right]$



$O(0,0)$

$A(3, f(3)) = \left(3, \frac{3}{2}\right)$

$y = f(x) = -\frac{1}{2}x^2 + 2x$

$f(3) = -\frac{1}{2}9 + 6 =$

$= \frac{-9+12}{2} = \frac{3}{2}$

$f'(x) = -x + 2$

$f'(0) = 2 \quad f'(3) = -3 + 2 = -1$

TANGENTE $\quad y - f(x_0) = f'(x_0)(x - x_0)$

$r: \quad y - 0 = 2(x - 0) \qquad y = 2x \Rightarrow r(x) = 2x$

$s: \quad y - \frac{3}{2} = -(x - 3) \qquad y = -x + 3 + \frac{3}{2} \qquad y = -x + \frac{9}{2} \Rightarrow$

$\Rightarrow s(x) = -x + \frac{9}{2}$

$$\mathcal{A}_{\text{COLORATA}} = \int_0^{x_P} [r(x) - f(x)]\,dx + \int_{x_P}^{3} [s(x) - f(x)]\,dx = (*)$$

$$x_P \rightarrow \begin{cases} y = 2x \\ y = -x + \frac{9}{2} \end{cases} \qquad 2x = -x + \frac{9}{2} \qquad 3x = \frac{9}{2} \Rightarrow x_P = \frac{3}{2}$$

$$(*) = \int_0^{\frac{3}{2}} \left[\cancel{2x} + \frac{1}{2}x^2 - \cancel{2x}\right] dx + \int_{\frac{3}{2}}^{3} \left[-x + \frac{9}{2} + \frac{1}{2}x^2 - 2x\right] dx =$$

$$= \int_0^{\frac{3}{2}} \left(\frac{1}{6}x^3\right)' dx + \int_{\frac{3}{2}}^{3} \left(\frac{1}{6}x^3 - \frac{3}{2}x^2 + \frac{9}{2}x\right)' dx =$$

$$= \int_0^{\frac{3}{2}} \left(\frac{1}{6}x^3\right)' dx + \int_{\frac{3}{2}}^{3} \left(\frac{1}{6}x^3 - \frac{3}{2}x^2 + \frac{9}{2}x\right)' dx =$$

$$= \left[\frac{1}{6}x^3\right]_0^{\frac{3}{2}} + \left[\frac{1}{6}x^3 - \frac{3}{2}x^2 + \frac{9}{2}x\right]_{\frac{3}{2}}^{3} =$$

$$= \cancel{\frac{1}{6}\left(\frac{3}{2}\right)^3} + \frac{1}{6}\cdot 3^3 - \frac{3}{2}\cdot 3^2 + \frac{9}{2}\cdot 3 - \cancel{\frac{1}{6}\left(\frac{3}{2}\right)^3} + \frac{3}{2}\left(\frac{3}{2}\right)^2 - \frac{9}{2}\left(\frac{3}{2}\right) =$$

$$= \frac{27}{6} - \cancel{\frac{27}{2}} + \cancel{\frac{27}{2}} + \frac{27}{8} - \frac{27}{4} = 27\left(\frac{1}{6} + \frac{1}{8} - \frac{1}{4}\right) = 27\,\frac{4+3-6}{24} =$$

$$= \frac{\cancel{27}^{\,9}}{\cancel{24}_{\,8}} = \boxed{\frac{9}{8}}$$

519 $\displaystyle\int_2^{+\infty} \frac{1}{x\ln^3 2x}\,dx$ $\left[\dfrac{1}{2\ln^2 4}\right]$

INT. INDEFINITO $\displaystyle\int \frac{1}{x\ln^3 2x}\,dx = \int \frac{1}{\cancel{\frac{e^t}{2}}\cdot t^3}\,\cancel{\frac{e^t}{2}}\,dt =$

$t = \ln 2x$

$2x = e^t$

$x = \dfrac{e^t}{2}$ $\quad dx = \dfrac{e^t}{2}\,dt$

$$= \int t^{-3}\,dt = \frac{1}{-3+1}t^{-3+1} + c$$

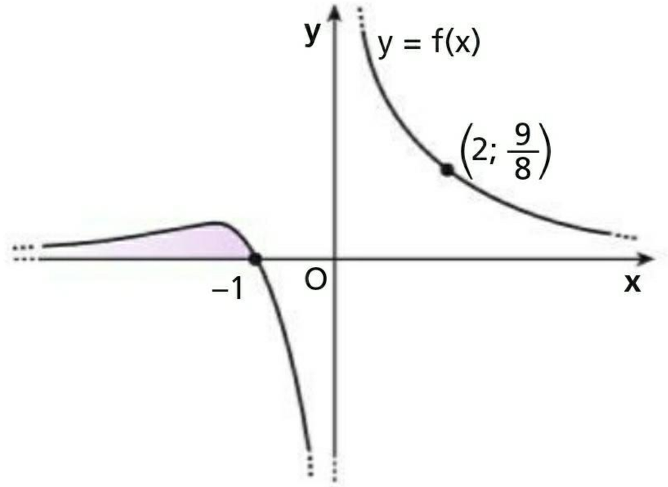
$$= -\frac{1}{2}t^{-2} + c = -\frac{1}{2t^2} + c =$$

$$= -\frac{1}{2\ln^2 2x} + c$$

$$\int_2^{+\infty} \frac{1}{x \ln^3 2x}\, dx = \lim_{x \to +\infty} \int_2^{x} \frac{1}{t \ln^3 2t}\, dt =$$

$$= \left[-\frac{1}{2\ln^2 2x} \right]_2^{+\infty} = \underbrace{-\frac{1}{2\ln^2(+\infty)}}_{\text{è un limite!} \ \downarrow \ 0} + \frac{1}{2\ln^2 4} = \boxed{\frac{1}{2\ln^2 4}}$$

532 **LEGGI IL GRAFICO** Nel grafico è rappresentata la funzione $f(x) = \dfrac{ax+b}{x^3}$.



a. Trova a e b.

b. Calcola la misura dell'area colorata.

$\left[\text{a) } a = b = 3; \text{ b) } \dfrac{3}{2}\right]$

$A(-1,0) \quad B\left(2, \frac{9}{8}\right)$

$$\begin{cases} 0 = a - b \\ \dfrac{9}{\cancel{8}} = \dfrac{2a+b}{\cancel{8}} \end{cases} \qquad \begin{cases} a = b \\ 3a = 9 \Rightarrow a = 3 \end{cases}$$

$a = b = 3$

$$f(x) = \frac{3x+3}{x^3}$$

$$\mathcal{A} = \int_{-\infty}^{-1} \frac{3x+3}{x^3}\,dx = 3\int_{-\infty}^{-1} \frac{x+1}{x^3}\,dx = 3\int_{-\infty}^{-1} \left[\frac{1}{x^2} + \frac{1}{x^3}\right] dx =$$

$$= 3\left[-\frac{1}{x} + \frac{1}{1-3}x^{1-3}\right]_{-\infty}^{-1} = 3\left[-\frac{1}{x} - \frac{1}{2x^2}\right]_{-\infty}^{-1} =$$

$$= 3\left[1 - \frac{1}{2} + \underbrace{\frac{1}{-\infty}}_{0} + \underbrace{\frac{1}{2(-\infty)^2}}_{0}\right] = 3\left(1 - \frac{1}{2}\right) = \frac{3}{2}$$

SONO LIMITI!

534 Rappresenta graficamente la funzione $y = xe^{-x^2}$ e calcola l'area della regione illimitata contenuta nel terzo quadrante e delimitata dall'asse x e dal grafico della funzione. $\left[\frac{1}{2}\right]$

$y = xe^{-x^2}$ $\quad D =]-\infty, +\infty[= \mathbb{R}$

DISPARI $\quad f(-x) = -xe^{-(-x)^2} = -xe^{-x^2} = -f(x) \quad \forall x \in \mathbb{R}$

SEGNO $\quad f(x) > 0$ per $x > 0 \qquad f(x) < 0$ per $x < 0$

INT. ASSI $\quad O(0,0)$

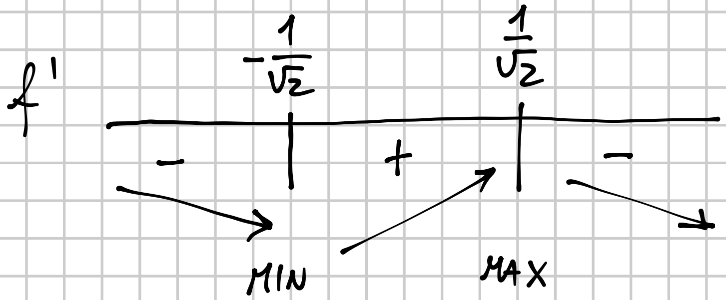
$$\lim_{x\to-\infty} xe^{-x^2} = \lim_{x\to-\infty} \frac{x}{e^{x^2}} = \overset{F.I.}{\frac{\infty}{\infty}} \overset{H}{=} \lim_{x\to-\infty} \frac{1}{e^{x^2}\cdot 2x} = \frac{1}{-\infty} = 0^-$$

$\lim_{x\to+\infty} xe^{-x^2} = 0^+$ perché dispari

$$f'(x) = e^{-x^2} + x(-2x)e^{-x^2} = e^{-x^2}\left[1 - 2x^2\right]$$

ZERI DI $f'(x) \Rightarrow 1 - 2x^2 = 0 \qquad x = \pm\frac{1}{\sqrt{2}}$

SEGNO DI $f'(x) \Rightarrow 1 - 2x^2 > 0 \qquad x^2 < \frac{1}{2} \qquad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$



Per il grafico devo calcolare

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}e^{-\frac{1}{2}} = \frac{1}{\sqrt{2e}}$$

$$M_1\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2e}}\right) \quad M_2\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2e}}\right)$$

$$f'(x) = e^{-x^2}(1-2x^2)$$

$$f''(x) = -2xe^{-x^2}(1-2x^2) + e^{-x^2}(-4x) =$$

$$= e^{-x^2}\left[-2x+4x^3-4x\right] = e^{-x^2}\left[4x^3-6x\right]$$

ZERI DI f''

$$4x^3-6x=0$$

$$x(4x^2-6)=0 \quad \begin{cases} x=0 \\ 4x^2-6=0 \quad x^2=\frac{3}{2} \quad x=\pm\sqrt{\frac{3}{2}} \end{cases}$$

SEGNO DI f'' $\quad \underset{\boxed{1}}{x}\,\underset{\boxed{2}}{(4x^2-6)}>0$

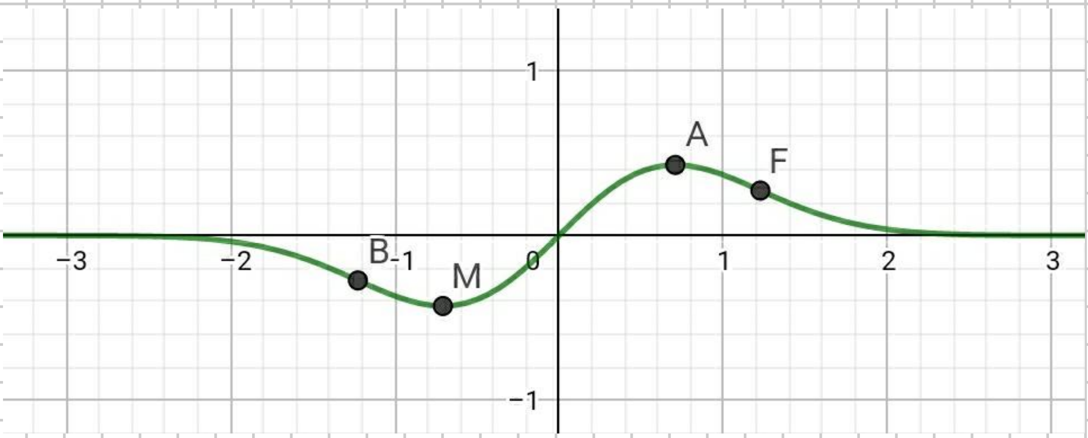
$x>0$

$4x^2-6>0 \quad x<-\sqrt{\frac{3}{2}} \lor x>\sqrt{\frac{3}{2}}$

		$-\sqrt{\frac{3}{2}}$		0		$\sqrt{\frac{3}{2}}$	
1	$-$		$-$	0	$+$		$+$
2	$+$	0	$-$		$-$	0	$+$
	$-$	0	$+$	0	$-$	0	$+$
	$\cap$	FLESSO	$\cup$	FLESSO	$\cap$	FLESSO	$\cup$

FLESSI $\quad O(0,0)$

$$F_1\left(-\sqrt{\tfrac{3}{2}}, f\left(-\sqrt{\tfrac{3}{2}}\right)\right) \quad F_2\left(\sqrt{\tfrac{3}{2}}, f\left(\sqrt{\tfrac{3}{2}}\right)\right)$$



$$\mathcal{A} = \left|\int_{-\infty}^{0} f(x)\,dx\right| \underset{\text{dato che } f \text{ è dispari}}{=} \int_{0}^{+\infty} f(x)\,dx = \int_{0}^{+\infty} x e^{-x^2}\,dx =$$

$$= \int_0^{+\infty} x e^{-x^2} dx = -\frac{1}{2} \int_0^{+\infty} (-2x) e^{-x^2} dx =$$

$$= -\frac{1}{2} \int_0^{+\infty} \left(e^{-x^2}\right)' dx = -\frac{1}{2} \left[e^{-x^2} \right]_0^{+\infty} = -\frac{1}{2} \left[\underbrace{e^{-\infty}}_{\to 0} - e^0 \right] = \boxed{\frac{1}{2}}$$