7/4/2021

INTEGRAZIONE DI FUNZIONI RAZIONALI FRATTE

463 $\int \frac{x^2-1}{x^3-3x+1}\,dx =$ $\left[\ln\sqrt[3]{|x^3-3x+1|}+c\right]$

$$= \frac{1}{3}\int \frac{3(x^2-1)}{x^3-3x+1}\,dx = \frac{1}{3}\int \frac{3x^2-3}{x^3-3x+1}\,dx =$$

$$= \frac{1}{3}\ln|x^3-3x+1|+c = \ln|x^3-3x+1|^{\frac{1}{3}}+c$$

478 $\int \frac{x^2+x+1}{x-4}\,dx$

← GRADO DEL NUMERATORE ≥ GRADO DEN.

⇓

DIVISIONE DI POLINOMI

$$\begin{array}{r|l} x^2+x+1 & x-4 \\ \hline -x^2+4x & x+5 \\ \hline // \quad 5x+1 & \\ -5x+20 & \\ \hline // \quad 21 & \end{array}$$

$x+5 = Q(x)$; $21 = R(x)$

$$x^2+x+1 = (x+5)(x-4)+21$$

⇓

$$\frac{x^2+x+1}{x-4} = x+5+\frac{21}{x-4}$$

$$\int \frac{x^2+x+1}{x-4}\,dx = \int (x+5)\,dx + \int \frac{21}{x-4}\,dx =$$

$$= \boxed{\frac{1}{2}x^2+5x+21\ln|x-4|+c}$$

OSSERVAZIONE

$$\int \frac{x^2+x+1}{x-4}\,dx = \int \frac{\overbrace{x^2+x-20}^{(x-4)(x+5)}+20+1}{x-4}\,dx =$$

$x-4$ → devo in modo che sommato a -4 dà $+1$

$(x-4)(x+5)$ → x^2+x-20 → INDIVIDUO e aggiungo e tolgo

$$= \int \frac{(x-4)(x+5)+21}{x-4}\,dx = \dots$$

479 $\displaystyle\int \frac{x^2-x+3}{3-x}\,dx = -\int \frac{x^2-x+3}{x-3}\,dx =$

$$= -\int \frac{x^2-x-6+6+3}{x-3}\,dx = -\int \frac{(x-3)(x+2)+9}{x-3}\,dx =$$

$x^2-x+\dots(-6) = (x-3)(x+2)$

$$= -\int (x+2)\,dx - \int \frac{9}{x-3}\,dx = \boxed{-\frac{1}{2}x^2-2x-9\ln|x-3|+c}$$

in alternativa:

$$\begin{array}{r|l} x^2-x+3 & x-3 \\ \hline -x^2+3x & x+2 \\ \hline //\ \ 2x+3 & \\ \quad -2x+6 & \\ \hline \quad //\ \ 9 & \end{array}$$

$$\frac{x^2-x+3}{x-3} = x+2+\frac{9}{x-3}$$

486 $\int \frac{3x-9}{x^2-x-2}dx =$

NUM. → 1° grado

DEN. → 2° grado $\Delta > 0$

$ax^2+bx+c = a(x-x_1)(x-x_2)$

con $\Delta > 0$ $\quad x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$

$$= \int \frac{3x-9}{(x+1)(x-2)}dx = 3\int \frac{x-3}{(x+1)(x-2)}dx = (*)$$

$$\frac{x-3}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} = \frac{A(x-2)+B(x+1)}{(x+1)(x-2)} =$$

$$= \frac{Ax-2A+Bx+B}{(x+1)(x-2)} = \frac{(A+B)x-2A+B}{(x+1)(x-2)}$$

$$\begin{cases} A+B=1 \\ -2A+B=-3 \end{cases} \quad \begin{cases} A=1-B \\ -2(1-B)+B=-3 \end{cases} \quad \begin{cases} A=1-B \\ -2+2B+B=-3 \end{cases}$$

$$\begin{cases} // \\ 3B=-1 \end{cases} \quad \begin{cases} A=1+\frac{1}{3}=\frac{4}{3} \\ B=-\frac{1}{3} \end{cases}$$

$$(*) = 3\left[\frac{4}{3}\int \frac{1}{x+1}dx - \frac{1}{3}\int \frac{1}{x-2}dx\right] = 4\ln|x+1| - \ln|x-2| + c$$

$$= \ln(x+1)^4 - \ln|x-2| + c$$

$$= \boxed{\ln \frac{(x+1)^4}{|x-2|} + c}$$