METODO DI SOSTITUZIONE PER INTEGRALI DEFINITI

165 $\int_{-1}^{1} \frac{x+1}{\sqrt{x+2}}\, dx =$ $\left[\frac{4}{3}\right]$

$t = x+2 \qquad x = t-2 \qquad dx = dt \qquad x=-1 \Rightarrow t=1$

$x = 1 \Rightarrow t = 3$

$$= \int_1^3 \frac{t-1}{\sqrt{t}}\, dt = \int_1^3 t^{\frac{1}{2}} dt - \int_1^3 t^{-\frac{1}{2}} dt =$$

$$= \left[\frac{2}{3} t^{\frac{3}{2}}\right]_1^3 - \left[2t^{\frac{1}{2}}\right]_1^3 = \frac{2}{3}\left(3^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) - 2\left(3^{\frac{1}{2}} - 1^{\frac{1}{2}}\right) =$$

$$= \frac{2}{3}\left(3\sqrt{3} - 1\right) - 2\left(\sqrt{3} - 1\right) = \cancel{2\sqrt{3}} - \frac{2}{3} - \cancel{2\sqrt{3}} + 2 = \boxed{\frac{4}{3}}$$

170 $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1+\cos x}\, dx =$ $[2 - \sqrt{4 - 2\sqrt{2}}\,]$

$1 + \cos x = t \qquad \cos x = t - 1 \qquad x = \arccos(t-1)$

$x = \frac{\pi}{2} \longrightarrow t = 1$

$x = \frac{\pi}{4} \longrightarrow t = 1 + \cos\frac{\pi}{4} = 1 + \frac{\sqrt{2}}{2}$

$$\frac{dx}{dt} = -\frac{1}{\sqrt{1-(t-1)^2}}$$

$$dx = -\frac{1}{\sqrt{1-(t-1)^2}}\, dt$$

$$= \int_{1+\frac{\sqrt{2}}{2}}^{1} \sqrt{t} \cdot \left(-\frac{1}{\sqrt{\cancel{1} - t^2 + 2t \cancel{-1}}}\right) dt =$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$= -\int_{1+\frac{\sqrt{2}}{2}}^{1} \frac{\sqrt{t}}{\sqrt{2t-t^2}}\, dt = \int_1^{1+\frac{\sqrt{2}}{2}} \sqrt{\frac{t}{2t-t^2}}\, dt =$$

$$= \int_1^{1+\frac{\sqrt{2}}{2}} \sqrt{\frac{t}{2t - t^2}}\, dt = \int_1^{1+\frac{\sqrt{2}}{2}} \sqrt{\frac{\cancel{t}}{\cancel{t}(2-t)}}\, dt =$$

$$= \int_1^{1+\frac{\sqrt{2}}{2}} \frac{1}{\sqrt{2-t}}\, dt = 2\int_1^{1+\frac{\sqrt{2}}{2}} \frac{1}{2\sqrt{2-t}}\, dt = -2\int_1^{1+\frac{\sqrt{2}}{2}} \frac{-1}{2\sqrt{2-t}}\, dt =$$

$$= -2\int_1^{1+\frac{\sqrt{2}}{2}} \left(\sqrt{2-t}\right)' dt = -2\left[\sqrt{2-t}\right]_1^{1+\frac{\sqrt{2}}{2}} =$$

$$= -2\left[\sqrt{2-1-\frac{\sqrt{2}}{2}} - \sqrt{2-1}\right] = -2\left[\sqrt{\frac{2-\sqrt{2}}{2}} - 1\right] =$$

$$= 2 - 2\sqrt{\frac{2-\sqrt{2}}{2}} = 2 - \sqrt{\frac{\cancel{4}^{2}(2-\sqrt{2})}{\cancel{2}}} = \boxed{2 - \sqrt{4 - 2\sqrt{2}}}$$

INTEGRAZIONE PER PARTI

123 $\int_0^1 x^2 e^x \, dx$ $[e-2]$

INTEGRALI INDEFINITI $\int f'(x) g(x) \, dx = f(x) g(x) - \int f(x) g'(x) \, dx$

INTEGRALI DEFINITI $\int_a^b f'(x) g(x) \, dx = f(x) g(x) \Big|_a^b - \int_a^b f(x) g'(x) \, dx$

$$\int_0^1 x^2 e^x \, dx = \int_0^1 x^2 \cdot (e^x)' \, dx = x^2 e^x \Big|_0^1 - \int_0^1 2x \cdot e^x \, dx =$$

$$= \left(1^2 \cdot e^1 - 0^2 \cdot e^0\right) - 2 \int_0^1 x e^x \, dx =$$

applico ancora la formula per parti

$$= e - 2\left[x e^x \Big|_0^1 - \int_0^1 e^x \, dx \right] =$$

$$= e - 2\left[e - e^x \Big|_0^1 \right] = e - 2\left[e - \left(e^1 - e^0\right)\right] =$$

$$= e - 2\left[\cancel{e} - \cancel{e} + 1 \right] = \boxed{e - 2}$$

CALCOLARE LA DERIVATA:

219 $G(x) = \int_1^{2x^2} \sqrt{4 + t^3} \, dt$ $[G'(x) = 8x\sqrt{1 + 2x^6}]$

BISOGNA CONSIDERARE LA COMPOSIZIONE DI FUNZIONI!!

$F(x) = \int_1^x \sqrt{4+t^3} \, dt$ $\quad f(x) = 2x^2$ $\quad G(x) = F(f(x))$

$$G'(x) = [F(f(x))]' = F'(f(x)) \cdot f'(x) = \sqrt{4 + (2x^2)^3} \cdot 4x =$$

$$= \sqrt{4 + 8x^6} \cdot 4x = \boxed{8x\sqrt{1 + 2x^6}}$$