28/1/2019

439 Risolvi in $\mathbb{C}$ l'equazione $(z-1)^3+8=0$. Detta z_0 la radice che ha il coefficiente della parte immaginaria positivo, calcola $(z_0-1)^6$. $[2+i\sqrt{3}, -1, 2-i\sqrt{3}; \cancel{+}\, 64]$

$$(z-1)^3+8=0$$

$$(z-1)^3=-8 \qquad z-1=w$$

$$w^3=-8 \qquad \rho=8 \quad \vartheta=\pi$$

$$w^3=8e^{i\pi}$$

Calcoliamo le radici cubiche di $8e^{i\pi}$

$$w_0=\sqrt[3]{8}\,e^{i\frac{\pi}{3}} = 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)=1+\sqrt{3}\,i$$

$$w_1=\sqrt[3]{8}\,e^{i\frac{\pi+2\pi}{3}} = 2\left(\cos\pi+i\sin\pi\right)=-2$$

$$w_2=\sqrt[3]{8}\,e^{i\frac{\pi+4\pi}{3}} = 2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)=1-\sqrt{3}\,i$$

$$\boxed{w_k=\sqrt[n]{\rho}\,e^{i\frac{\vartheta+2k\pi}{n}} \qquad k=0,1,\dots,n-1}$$

$$z-1=1+\sqrt{3}\,i \implies z=2+\sqrt{3}\,i$$

$$z-1=-2 \implies z=-1$$

$$z-1=1-\sqrt{3}\,i \implies z=2-\sqrt{3}\,i$$

RISOLUZIONE VELOCE (BY BEPPE)

$$(z_0-1)^6=\Big[\underbrace{(z_0-1)^3}_{-8}\Big]^2=$$

per l'es. prec.

$$=(-8)^2=64$$

z_0: $\rho=2$ $\vartheta=\frac{\pi}{3}$

$$z_0=2+\sqrt{3}\,i = \qquad (z_0-1)^6=(1+\sqrt{3}\,i)^6=$$

$$=\left[2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right]^6=\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^6=$$

$$=2^6\left(\cos\frac{6\pi}{3}+i\sin\frac{6\pi}{3}\right)=64\,(1+i\cdot 0)=\boxed{64}$$

$$\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^2=\cancel{\frac{1}{2}}-\cancel{\frac{1}{2}}+\cancel{2}\frac{\cancel{\sqrt{2}}}{\cancel{2}}\frac{\cancel{\sqrt{2}}}{\cancel{2}}i=i$$

$$\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)^2=\frac{1}{2}-\frac{1}{2}+2\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}i=i$$

$-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ e $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ sono le 2 radici quadrate di i

453 $e^{i\pi+\frac{\pi}{3}i}$ $\left[-\frac{1}{2}(1+i\sqrt{3})\right]$

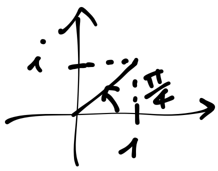
$\downarrow$

trasformare in forma algebrica

$$e^{i\pi+\frac{\pi}{3}i}=e^{\frac{4}{3}\pi i}=\cos\frac{4}{3}\pi+i\sin\frac{4}{3}\pi=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$$

471 $(1+i)^2$ $\left[2e^{i\frac{\pi}{2}}\right]$

$$1+i=\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)=\sqrt{2}e^{i\frac{\pi}{4}}$$



$$(1+i)^2=\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^2=2e^{i\frac{\pi}{2}}$$

40 $x^5 + 4x^3 + x^2 + 4 = 0$ Risolvere in $\mathbb{C}$

$$x^3(x^2+4)+(x^2+4)=0$$

$$(x^2+4)(x^3+1)=0$$

$$x^2+4=0 \implies x^2=-4 \qquad x=\pm 2i$$

$$x^3+1=0 \implies x^3=-1=1(\cos\pi+i\sin\pi)$$

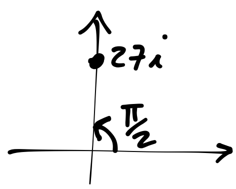
$$x_0=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$x_1=\frac{1}{2}-\frac{\sqrt{3}}{2}i \quad \text{(coniugato di } x_0\text{)}$$

$$x_2=-1$$

$$\boxed{x=\pm 2i \quad \vee \quad x=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i \quad \vee \quad x=-1}$$

41 $(x^2 + 4)(x^3 - 27i) = 0$



$$x^2+4=0 \implies \boxed{x=\pm 2i}$$

$$x^3-27i=0 \implies x^3=27i$$ trovare le 3 radici cubiche di $27i$

$$x^3=27e^{i\frac{\pi}{2}} \qquad \rho=27 \quad \vartheta=\frac{\pi}{2}$$

$$x_0=\sqrt[3]{27}\,e^{i\frac{\pi}{6}}=3\left(\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)=\boxed{\frac{3\sqrt{3}}{2}+\frac{3}{2}i}$$

$$x_1=\sqrt[3]{27}\,e^{i\frac{\frac{\pi}{2}+2\pi}{3}}=3e^{i\frac{5}{6}\pi}=3\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\boxed{-\frac{3\sqrt{3}}{2}+\frac{3}{2}i}$$

$$x_2=3e^{i\frac{\frac{\pi}{2}+4\pi}{3}}=3e^{i\frac{3}{2}\pi}=3(0-1i)=\boxed{-3i}$$