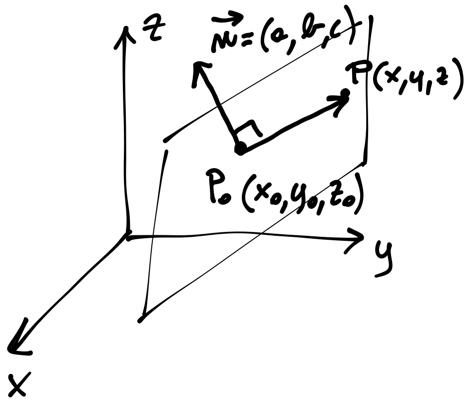
16/4/2018

RETTE E PIANI NELLO SPAZIO



$$\vec{n}\cdot\overrightarrow{P_0P}=0$$

$$(a,b,c)\cdot(x-x_0,y-y_0,z-z_0)=0$$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$ax+by+cz\underbrace{-ax_0-by_0-cz_0}_{d}=0$$

$$\boxed{ax+by+cz+d=0}$$

$(a,b,c)=\vec{n}$ direzione della normale al piano

$$ax+by+cz+d=0$$
$$a'x+b'y+c'z+d'=0$$

$\vec{n}=(a,b,c)$ $\quad\vec{n}'=(a',b',c')$

PERPENDICOLARI

$\perp$ $\quad\boxed{aa'+bb'+cc'=0}$

PARALLELI

$//$ $\quad\boxed{\dfrac{a}{a'}=\dfrac{b}{b'}=\dfrac{c}{c'}}$

ESEMPIO

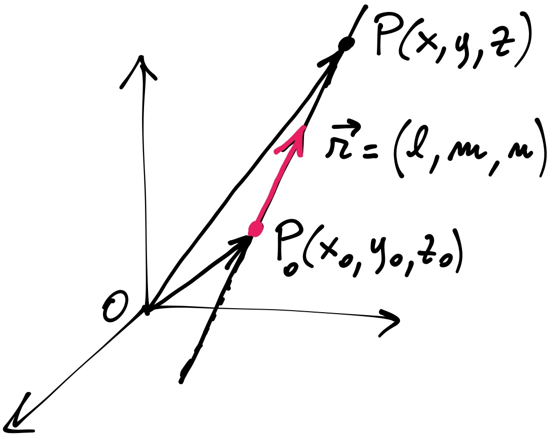
117 $A(1;0;0)$, $\quad B(0;-3;1)$, $\quad C(2;-2;0)$. $\quad[2x+y+5z-2=0]$

$$ax+by+cz+d=0$$

$A(1,0,0)\to$, $B(0,-3,1)\to$, $C(2,-2,0)\to$

$$\begin{cases}a+d=0\\-3b+c+d=0\\2a-2b+d=0\end{cases}\quad\begin{cases}a=-d\\-3b+c+d=0\\2a-2b-a=0\end{cases}\quad\begin{cases}a=-d\\c=3b-d\\a=2b\end{cases}\quad\begin{cases}a=-d\\c=-\frac{5}{2}d\\b=-\frac{d}{2}\end{cases}$$

$$d=-2\to\begin{cases}a=2\\c=5\\b=1\end{cases}\qquad\boxed{2x+y+5z-2=0}$$



$$\vec{OP} = \vec{OP_0} + \vec{P_0P} =$$

$$= (x_0, y_0, z_0) + t\vec{r}$$

$$\vec{OP} = (x, y, z)$$

$$\begin{cases} x = x_0 + tl \\ y = y_0 + tm \\ z = z_0 + tn \end{cases} \Rightarrow \begin{cases} t = \dfrac{x - x_0}{l} \\ t = \dfrac{y - y_0}{m} \\ t = \dfrac{z - z_0}{n} \end{cases}$$

$$t = \boxed{\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}}$$

2017

Dati i punti $A(-2,3,1)$, $B(3,0,-1)$, $C(2,2,-3)$, determinare l'equazione della retta r passante per A e per B e l'equazione del piano π perpendicolare ad r e passante per C.

retta per A e per B

$$\begin{cases} x = x_0 + t\,l \\ y = y_0 + t\,m \\ z = z_0 + t\,n \end{cases}$$

DIREZIONE DELLA RETTA $(l, m, n) = \vec{AB} = (x_B - x_A,\ y_B - y_A,\ z_B - z_A) =$

$$= (3+2,\ 0-3,\ -1-1) = (5, -3, -2)$$

RETTA AB $\begin{cases} x = -2 + 5t \\ y = 3 - 3t \\ z = 1 - 2t \end{cases}$ in forma parametrica

$$\downarrow$$

$$\vec{OP} = \vec{OA} + t\,\vec{AB}$$

modo alternativo

$$\frac{x - x_A}{\underbrace{x_B - x_A}_{l}} = \frac{y - y_A}{\underbrace{y_B - y_A}_{m}} = \frac{z - z_A}{\underbrace{z_B - z_A}_{n}}$$

$$\frac{x - x_A}{l} = \frac{y - y_A}{m} = \frac{z - z_A}{n} = t$$

$$\begin{cases} x = x_A + l t \\ y = y_A + m t \\ z = z_A + n t \end{cases}$$

$$\begin{cases} x = t \\ 3t + 5y - 9 = 0 \\ 2y - 3z - 3 = 0 \end{cases}$$

$$\downarrow$$

$$\frac{x+2}{5} = \frac{y-3}{-3} = \frac{z-1}{-2}$$

$$\begin{cases} \dfrac{x+2}{5} = \dfrac{y-3}{-3} \\ \dfrac{y-3}{-3} = \dfrac{z-1}{-2} \end{cases} \qquad \begin{cases} -3(x+2) = 5(y-3) \\ 2(y-3) = 3(z-1) \end{cases} \qquad \begin{cases} 3x + 5y - 9 = 0 \\ 2y - 3z - 3 = 0 \end{cases}$$

$$\begin{cases} x = t \\ 3t + 5y - 9 = 0 \\ 2y - 3z - 3 = 0 \end{cases} \qquad \begin{cases} x = t \\ y = \dfrac{9-3t}{5} \\ 2\,\dfrac{9-3t}{5} - 3z - 3 = 0 \end{cases} \qquad \begin{cases} x = t \\ y = \dfrac{9}{5} - \dfrac{3}{5}t \\ \dfrac{18 - 6t - 15z - 15}{\cancel{5}} = 0 \end{cases}$$

$$\begin{cases} / \\ 15z = 3 - 6t \end{cases} \qquad \begin{cases} x = t \\ y = \dfrac{9}{5} - \dfrac{3}{5}t \\ z = \dfrac{1}{5} - \dfrac{2}{5}t \end{cases} \longrightarrow$$

retta passante per $D\left(0, \frac{9}{5}, \frac{1}{5}\right)$ di direzione $\vec{r} = \left(1, -\frac{3}{5}, -\frac{2}{5}\right)$

$$ax + by + cz + d = 0$$

$(a, b, c) \rightarrow$ direzione della normale al piano

$(5, -3, -2) \longrightarrow 5x - 3y - 2z + d = 0$ d da trovare

IMPONGO IL PASSAGGIO PER $C(2, 2, -3)$

$$5 \cdot 2 - 3 \cdot 2 - 2 \cdot (-3) + d = 0$$

$$10 - \cancel{6} + \cancel{6} + d = 0 \Rightarrow d = -10$$

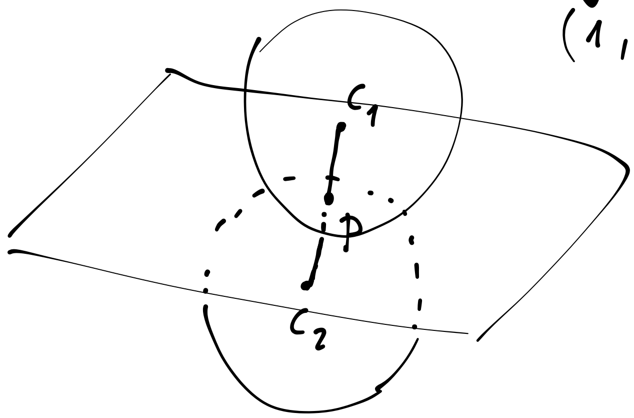
$$\boxed{5x - 3y - 2z - 10 = 0}$$

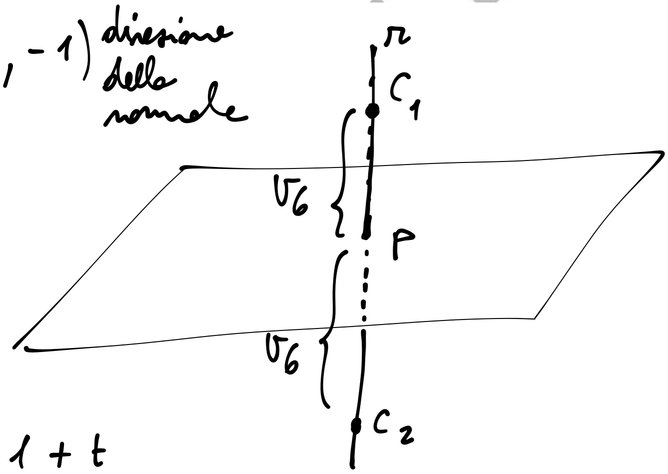
7. Determinare le coordinate dei centri delle sfere di raggio $\sqrt{6}$ tangenti al piano π di equazione:

$$x + 2y - z + 1 = 0$$

nel suo punto P di coordinate $(1, 0, 2)$.

$(1, 2, -1)$ direzione della normale





retta r
$$\begin{cases} x = 1 + t \cdot 1 \\ y = 0 + t \cdot 2 \\ z = 2 + t \cdot (-1) \end{cases} \qquad \begin{cases} x = 1 + t \\ y = 2t \\ z = 2 - t \end{cases}$$

$$\sqrt{6} = \overline{C_1P} = \sqrt{(x_{C_1} - x_P)^2 + (y_{C_1} - y_P)^2 + (z_{C_1} - z_P)^2}$$

$$(x-1)^2 + (y-0)^2 + (z-2)^2 = 6$$

$$(1+t-1)^2 + (2t)^2 + (2-t-2)^2 = 6$$

$$t^2 + 4t^2 + t^2 = 6 \qquad t^2 = 1 \Rightarrow t = \pm 1$$

$$t = 1 \rightarrow \begin{cases} x = 2 \\ y = 2 \\ z = 1 \end{cases} \qquad t = -1 \rightarrow \begin{cases} x = 0 \\ y = -2 \\ z = 3 \end{cases}$$

$C_1(2, 2, 1)$ $\qquad$ $C_2(0, -2, 3)$