

8/3/2021

**317**  $3x^3 - 7x^2 - 3x + 2 = 0$

$$\left[ -\frac{2}{3}; \frac{3 \pm \sqrt{5}}{2} \right]$$

$\pm 1 \pm 2$

Scomporre con Ruffini

$$1 \mapsto 3 - 7 - 3 + 2 \neq 0$$

$$2 \mapsto 3 \cdot 8 - 7 \cdot 4 - 6 + 2 =$$

$$= 24 - 28 - 6 + 2 \neq 0$$

$$-1 \mapsto -3 - 7 + 3 + 2 \neq 0$$

$$-2 \mapsto -24 - 28 + 6 + 2 \neq 0$$

Divisori del coeff. di grado max:  $\pm 3$

NUOVI TENTATIVI

$$\pm \frac{1}{3}$$

$$\pm \frac{2}{3}$$

(rapporti fra divisori del termine noto e divisori del coefficiente di grado massimo)

$$\frac{1}{3} \mapsto 3 \left(\frac{1}{3}\right)^3 - 7 \left(\frac{1}{3}\right)^2 - 3 \cdot \frac{1}{3} + 2 =$$

$$= \frac{1}{9} - \frac{7}{9} - 1 + 2 = -\frac{2}{3} + 1 = \frac{1}{3} \neq 0$$

.....

$$-\frac{2}{3} \mapsto 3 \left(-\frac{2}{3}\right)^3 - 7 \left(-\frac{2}{3}\right)^2 - 3 \left(-\frac{2}{3}\right) + 2 = -\frac{8}{9} - \frac{28}{9} + 2 + 2 =$$

$$= -\frac{36}{9} + 4 = 0 \quad \text{ok!!!}$$

$$\begin{array}{c|ccc|c} -\frac{2}{3} & 3 & -7 & -3 & 2 \\ & & -2 & 6 & -2 \\ \hline & 3 & -9 & 3 & // \end{array}$$

$-\frac{2}{3}$  è soluzione

$$(3x^2 - 9x + 3) \left(x + \frac{2}{3}\right) = 0$$

$$3x^2 - 9x + 3 = 0$$

$$x^2 - 3x + 1 = 0$$

$$\Delta = 9 - 4 = 5$$

$$x = \frac{3 \pm \sqrt{5}}{2} \vee x = -\frac{2}{3}$$

**316**  $(x^6 + 4x^4 + x^2)(x^7 - 4x^4 + 3x) = 0$

$[0; 1; \sqrt[3]{3}]$

$$x^6 + 4x^4 + x^2 = 0$$

✓

$$x^7 - 4x^4 + 3x = 0$$

$$x^2(x^4 + 4x^2 + 1) = 0$$

$$x(x^6 - 4x^3 + 3) = 0$$

$$x^2 = 0$$

✓

$$x^4 + 4x^2 + 1 = 0$$

✓

$$x = 0$$

✓

$$x^6 - 4x^3 + 3 = 0$$

①

②

①  $x^4 + 4x^2 + 1 = 0$

$$x^2 = t$$

$$t^2 + 4t + 1 = 0$$

$$\frac{\Delta}{4} = 4 - 1 = 3$$

$$t = -2 \pm \sqrt{3}$$

$$\nearrow -2 - \sqrt{3} < 0 \text{ N.A.}$$

$$\searrow -2 + \sqrt{3} < 0 \text{ N.A.}$$

IMPOSSIBLE IN  $\mathbb{R}$

②  $x^6 - 4x^3 + 3 = 0$

$$x^3 = t$$

$$t^2 - 4t + 3 = 0$$

$$(t - 3)(t - 1) = 0$$

$$t = 3 \quad \vee \quad t = 1$$

$$x^3 = 3$$

$$x^3 = 1$$

$$x = \sqrt[3]{3}$$

$$\vee \quad x = 1$$

$$\boxed{x = 0 \quad \vee \quad x = 1 \quad \vee \quad x = \sqrt[3]{3}}$$

0 è soluzione di molteplicità 3

**357** Data la frazione algebrica  $\frac{8x^6 - 2x^3 - 1}{2x^4 - x^2 - 1}$ , determina:

- a. i valori di  $x$  per cui non è definita;
- b. i valori di  $x$  per cui si annulla;
- c. i valori di  $x$  per cui vale 1.

a)  $2x^4 - x^2 - 1 = 0$

$$x^2 = t$$

$$2t^2 - t - 1 = 0$$

$$\Delta = 1 + 8 = 9$$

$$t = \frac{1 \pm 3}{4} = \begin{cases} -\frac{1}{2} & \text{N.A.} \\ 1 \end{cases}$$

$$x^2 = 1$$

$$x = \pm 1$$

b)  $8x^6 - 2x^3 - 1 = 0$

$$x \neq \pm 1$$

$$x^3 = t$$

$$8t^2 - 2t - 1 = 0$$

$$\frac{\Delta}{4} = 1 + 8 = 9$$

$$t = \frac{1 \pm 3}{8} = \begin{cases} -\frac{1}{4} \\ \frac{1}{2} \end{cases}$$

$$x^3 = -\frac{1}{4}$$

$$x = -\sqrt[3]{\frac{1}{4}} = -\frac{1}{\sqrt[3]{4}} = -\frac{1}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = -\frac{\sqrt[3]{2}}{\sqrt[3]{2^3}} = -\frac{\sqrt[3]{2}}{2}$$

$$x^3 = \frac{1}{2}$$

$$x = \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{\sqrt[3]{4}}{2}$$

$$x = -\frac{\sqrt[3]{2}}{2} \vee x = \frac{\sqrt[3]{4}}{2}$$

$$c) \quad \frac{8x^6 - 2x^3 - 1}{2x^4 - x^2 - 1} = 1$$

C.E.

$$x \neq \pm 1$$

$$\cancel{8x^6 - 2x^3 - 1} = \cancel{2x^4 - x^2 - 1}$$

$$8x^6 - 2x^4 - 2x^3 + x^2 = 0$$

$$x^2 (8x^4 - 2x^2 - 2x + 1) = 0$$

Proffini:  $\pm 1 \quad \pm \frac{1}{2} \quad \pm \frac{1}{4} \quad \pm \frac{1}{8}$

$$-\frac{1}{2} \mapsto 8 \cdot \left(-\frac{1}{2}\right)^4 - 2 \left(-\frac{1}{2}\right)^2 - 2 \left(-\frac{1}{2}\right) + 1 = \frac{1}{2} - \frac{1}{2} + 1 + 1 \neq 0$$

$$\frac{1}{2} \mapsto 8 \cdot \left(\frac{1}{2}\right)^4 - 2 \left(\frac{1}{2}\right)^2 - 2 \cdot \frac{1}{2} + 1 = \frac{1}{2} - \frac{1}{2} - 1 + 1 = 0 \quad \text{OK!}$$

8	0	-2	-2	1
$\frac{1}{2}$	4	2	0	-1
8	4	0	-2	//

$$x^2 (8x^3 + 4x^2 - 2) \left(x - \frac{1}{2}\right) = 0$$

$$8x^3 + 4x^2 - 2 = 0$$

$$\frac{1}{2} \pm \frac{1}{4}$$

$$4x^3 + 2x^2 - 1 = 0$$

$$\frac{1}{2} \mapsto 4 \left(\frac{1}{2}\right)^3 + 2 \left(\frac{1}{2}\right)^2 - 1$$

$$4 \cdot \frac{1}{8} + 2 \cdot \frac{1}{4} - 1 =$$

$$= \frac{1}{2} + \frac{1}{2} - 1 = 0 \quad \text{OK}$$

8	4	0	-2
$\frac{1}{2}$	4	4	2
8	8	4	//

$$x^2 (8x^2 + 8x + 4) \left(x - \frac{1}{2}\right)^2 = 0$$

$$x^2 (8x^2 + 8x + 4) \left(x - \frac{1}{2}\right)^2 = 0$$

$$4x^2 (2x^2 + 2x + 1) \left(x - \frac{1}{2}\right)^2 = 0$$

$$\Delta = 4 - 8 = -4 < 0$$

NON SCOMPONIBILE

Le soluzioni sono  $0$  e  $\frac{1}{2}$  (entrambe di molteplicità 2)

$$x=0 \vee x=\frac{1}{2}$$