

23/3/2021

INTEGRAZIONE PER PARTI

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

\Downarrow

$$f'(x) \cdot g(x) = [f(x) \cdot g(x)]' - f(x) \cdot g'(x)$$

$$\int f'(x) \cdot g(x) dx = \int [f(x) \cdot g(x)]' dx - \int f(x) \cdot g'(x) dx$$

$$\boxed{\int f'(x) g(x) dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) dx}$$

ESEMPIO

FORMULA DI INTEGRAZIONE PER PARTI

$$\int e^x \cdot x dx = \int (e^x)' x dx = e^x \cdot x - \int e^x \cdot 1 dx =$$

$$f(x) = e^x$$

$$g(x) = x$$

\uparrow
(x)'

$$= x e^x - \int e^x dx = \boxed{x e^x - e^x + c}$$

ATTENZIONE

$$\int x \cdot e^x dx = \int \left(\frac{1}{2} x^2\right)' e^x dx = \frac{1}{2} x^2 \cdot e^x - \int \frac{1}{2} x^2 \cdot e^x dx =$$

.....

e ho peggiorato la situazione!!

Bisogna applicare la formula in modo opportuno!!

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$$\int 2x \ln x \, dx = \left[x^2 \left(\ln x - \frac{1}{2} \right) + c \right]$$

$$= \int (x^2)' \cdot \ln x \, dx = x^2 \ln x - \int \underbrace{x^2 \cdot \frac{1}{x}}_{(\ln x)'} \, dx =$$

$$= x^2 \ln x - \int x \, dx = x^2 \ln x - \frac{1}{2} x^2 + c = x^2 \left(\ln x - \frac{1}{2} \right) + c$$

ALTRO ESEMPIO

$$\int \ln x \, dx = \int 1 \cdot \ln x \, dx = \int (x)' \ln x \, dx =$$

$$= x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + c$$

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$$\int (x+2) \sin x \, dx = \left[-(x+2) \cos x + \sin x + c \right]$$

$$= \int (x+2) (-\cos x)' \, dx = (x+2) (-\cos x) - \int (x+2)' (-\cos x) \, dx$$

$$= -(x+2) \cos x + \int \cos x \, dx = -(x+2) \cos x + \sin x + c$$

$$I = \int \sin^2 x \, dx = \int \sin x \cdot \sin x \, dx = \int \sin x \cdot (-\cos x)' \, dx =$$

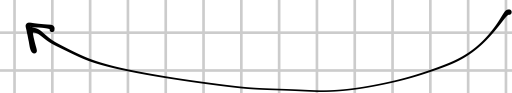
$$= \sin x \cdot (-\cos x) - \int (\sin x)' \cdot (-\cos x) \, dx =$$

$$= -\sin x \cos x + \int \cos^2 x \, dx = -\sin x \cos x + \int (1 - \sin^2 x) \, dx =$$

$$= -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx =$$

$$= -\sin x \cos x + x - \underbrace{\int \sin^2 x \, dx}_I$$

$$I = -\sin x \cos x + x - I$$



$$2I = -\sin x \cos x + x + C$$

$$\Rightarrow I = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + C$$

$$\int \sin^2 x \, dx = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + C$$

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$$\int \frac{\ln x^2}{x^2} dx =$$

$$\left[-\frac{1}{x}(2 + \ln x^2) + c \right]$$

$$\ln x^2 = 2 \ln |x|$$

$$= 2 \int \frac{1}{x^2} \cdot \ln |x| dx = 2 \int \left(\frac{x^{-2+1}}{-2+1} \right)' \cdot \ln |x| dx =$$

$$= 2 \left[-\frac{1}{x} \cdot \ln |x| - \int \underbrace{\left(-\frac{1}{x}\right)}_{(\ln |x|)'} \cdot \frac{1}{x} dx \right] =$$

$$= 2 \left[-\frac{\ln |x|}{x} + \int x^{-2} dx \right] = 2 \left[-\frac{\ln |x|}{x} + \frac{x^{-2+1}}{-2+1} + c \right] =$$

$$= -\frac{2 \ln |x|}{x} - \frac{2}{x} + c = \boxed{-\frac{\ln x^2}{x} - \frac{2}{x} + c}$$

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$$\int 8x \sin x \cos x dx =$$

$$[-2x \cos 2x + \sin 2x + c]$$

$$\sin 2x = 2 \sin x \cos x$$

$$= \int 4x \cdot \sin 2x dx =$$

$$= \int 2x \cdot \underbrace{2 \sin 2x}_{(-\cos 2x)'} dx = \int (2x) \cdot (-\cos 2x)' dx =$$

$$= 2x \cdot (-\cos 2x) - \int \overset{(2x)'}{2} \cdot (-\cos 2x) dx = -2x \cos 2x + \int \underbrace{2 \cos 2x}_{(\sin 2x)'} dx =$$

$$= \boxed{-2x \cos 2x + \sin 2x + c}$$