22/3/2021

140 $$\int \frac{2x^2-9}{3x^2+3}\,dx =$$

1° MODO

$$= \int \frac{2x^2}{3x^2+3}\,dx - \int \frac{9}{3x^2+3}\,dx =$$

$$= \frac{2}{3}\int \frac{x^2}{x^2+1}\,dx - 3\int \frac{1}{x^2+1}\,dx =$$

$$= \frac{2}{3}\int \frac{x^2+1-1}{x^2+1}\,dx - 3\arctan x = \frac{2}{3}\int \frac{x^2+1}{x^2+1}\,dx - \frac{2}{3}\int \frac{1}{x^2+1}\,dx$$

$$-3\arctan x =$$

$$= \frac{2}{3}x - \frac{2}{3}\arctan x - 3\arctan x + c =$$

$$= \boxed{\frac{2}{3}x - \frac{11}{3}\arctan x + c}$$

2° MODO

$$\frac{2x^2-9}{3x^2+3} = \frac{2x^2-9}{3(x^2+1)}$$

$$\begin{array}{r|l} 2x^2-9 & x^2+1 \\ \hline -2x^2-2 & 2 \leftarrow \text{QUOZIENTE} \\ \hline // \quad -11 & \\ \uparrow \text{RESTO} & \end{array}$$

$$\frac{2x^2-9}{x^2+1} = 2 + \frac{-11}{x^2+1}$$

$$\int \frac{2x^2-9}{3x^2+3}\,dx = \frac{1}{3}\int \frac{2x^2-9}{x^2+1}\,dx = \frac{1}{3}\left[\int 2\,dx - 11\int \frac{1}{x^2+1}\,dx\right] =$$

$$= \frac{1}{3}\left(2x - 11\arctan x\right) + c$$

$$\frac{2x^2-9}{x^2+1} = \frac{2x^2+2-2-9}{x^2+1} = \frac{2(x^2+1)-11}{x^2+1} =$$

$$= 2 - \frac{11}{x^2+1}$$

grado $P(x) \geq$ grado $Q(x)$

$P(x), Q(x)$ polinomi

$$\frac{P(x)}{Q(x)} = A(x) + \frac{R(x)}{Q(x)}$$

$A(x) =$ quoziente

$R(x) =$ resto grado $R(x) <$ grado $Q(x)$

$$P(x) = Q(x) \cdot A(x) + R(x)$$

142 $\int \frac{4x^2-1}{2x^2+2}dx =$

$$= \frac{1}{2}\int \frac{4x^2-1}{x^2+1}dx = \frac{1}{2}\int \frac{4x^2+4-4-1}{x^2+1}dx =$$

$$= \frac{1}{2}\left[\int \frac{4\cancel{(x^2+1)}}{\cancel{x^2+1}}dx - 5\int \frac{1}{x^2+1}dx\right] =$$

$$= \frac{1}{2}\left[4x - 5\arctan x\right] + c = \boxed{2x - \frac{5}{2}\arctan x + c}$$

$$\begin{array}{cc|l} 4x^2 & -1 & x^2+1 \\ \hline -4x^2 & -4 & 4 \\ \hline // & -5 & \end{array} \Rightarrow \frac{4x^2-1}{x^2+1} = 4 - \frac{5}{x^2+1}$$

146 $\int \underbrace{4x}_{f'(x)}\underbrace{(2x^2+3)^6}_{[f(x)]^\alpha}dx =$

INTEGRALE DEL TIPO

$$\int [f(x)]^\alpha \cdot f'(x)\,dx =$$

$$= \frac{f(x)^{\alpha+1}}{\alpha+1} + c$$

$$= \frac{(2x^2+3)^{6+1}}{6+1} + c =$$

$$= \boxed{\frac{(2x^2+3)^7}{7} + c}$$

$$\int [f(x)]^{\alpha} f'(x)\, dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + c$$

148 $\int 15\sqrt{6-5x}\, dx =$ $\left[-2\sqrt{(6-5x)^3}+c\right]$

$$= \int 15\,(6-5x)^{\frac{1}{2}}\, dx = \int (-3)\cdot(-5)\,(6-5x)^{\frac{1}{2}}\, dx =$$

$$= -3\int(-5)\,(6-5x)^{\frac{1}{2}}\, dx = -3\,\frac{(6-5x)^{\frac{3}{2}}}{\frac{3}{2}} + c =$$

$$= -2\,(6-5x)^{\frac{3}{2}} + c = \boxed{-2\sqrt{(6-5x)^3}+c}$$

151 $\int (x^2+2x-1)^5(x+1)\,dx =$ $\left[\frac{(x^2+2x-1)^6}{12}+c\right]$

$$= \frac{1}{2}\int (x^2+2x-1)^5\,(2x+2)\, dx = \frac{1}{2}\,\frac{(x^2+2x-1)^6}{6} + c =$$

$$= \boxed{\frac{(x^2+2x-1)^6}{12} + c}$$

154 $\int \frac{x}{\sqrt{x^2+4}}\,dx =$

$$= \int x\,(x^2+4)^{-\frac{1}{2}}\,dx = \frac{1}{2}\int 2x\,(x^2+4)^{-\frac{1}{2}}\,dx =$$

$$= \frac{1}{\cancel{2}}\,\frac{(x^2+4)^{\frac{1}{2}}}{\frac{1}{\cancel{2}}} + c = \boxed{\sqrt{x^2+4} + c}$$

166 $\int \frac{\sin x - \sin^2 x}{\cos^4 x}\,dx =$ $\left[\frac{1}{3\cos^3 x} - \frac{\tan^3 x}{3} + c\right]$

$$= \int \frac{\sin x}{\cos^4 x}\,dx - \int \frac{\sin^2 x}{\cos^4 x}\,dx =$$

$$= -\int (-\sin x)\,(\cos x)^{-4}\,dx - \int \overbrace{\frac{\sin^2 x}{\cos^2 x}}^{\tan^2 x} \cdot \frac{1}{\cos^2 x}\,dx =$$

$$= -\frac{(\cos x)^{-3}}{-3} - \frac{(\tan x)^3}{3} + C =$$

$$= \boxed{\frac{1}{3\cos^3 x} - \frac{\tan^3 x}{3} + c}$$

168 $\int \frac{\arctan x + 3}{1+x^2}\,dx =$

$$= \int \frac{\arctan x}{1+x^2}\,dx + 3\int \frac{1}{1+x^2}\,dx =$$

$$= \boxed{\frac{1}{2}\arctan^2 x + 3\arctan x + c}$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

175 $\int \frac{x^2}{x^3+2}\,dx =$ $\left[\frac{1}{3}\ln|x^3+2| + c\right]$

$$= \frac{1}{3}\int \frac{3x^2}{x^3+2}\,dx = \boxed{\frac{1}{3}\ln|x^3+2| + c}$$

179 $\int 3\tan x\,dx =$ $\left[-3\ln|\cos x| + c\right]$

$$= 3\int \frac{\sin x}{\cos x}\,dx = -3\int \frac{-\sin x}{\cos x}\,dx =$$

$$= \boxed{-3\ln|\cos x| + c}$$

In generale si ha

$$\int g'(f(x)) \cdot f'(x)\, dx = g(f(x)) + c$$

199 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx =$ $[2e^{\sqrt{x}} + c]$

$$= 2\int e^{\sqrt{x}} \cdot \underbrace{\frac{1}{2\sqrt{x}}}_{(\sqrt{x})'}\, dx = \boxed{2e^{\sqrt{x}} + c}$$

201 $\int e^x 5^{2e^x}\, dx =$

$$(a^x)' = a^x \cdot \ln a$$

$$(5^x)' = 5^x \cdot \ln 5$$

$$= \frac{1}{2\ln 5} \int \underbrace{(2e^x)}_{f'(x)} \underbrace{5^{2e^x} \cdot \ln 5}_{g'(f(x))}\, dx =$$

$g(x) = 5^x$

$f(x) = 2e^x$

$$= \boxed{\frac{5^{2e^x}}{2\ln 5} + c}$$

202 $\int 2^{x^3-x^2}(6x^2-4x)\,dx =$

$$= 2\int 2^{x^3-x^2}(3x^2-2x)\,dx =$$

$$= \frac{2}{\ln 2}\int 2^{x^3-x^2}\cdot \ln 2\,(3x^2-2x)\,dx =$$

$$= \frac{2\cdot 2^{x^3-x^2}}{\ln 2} + c = \boxed{\frac{2^{x^3-x^2+1}}{\ln 2} + c}$$

218 $\int \frac{x}{\cos^2 4x^2}\,dx = \frac{1}{8}\int \frac{8x}{\cos^2 4x^2}\,dx =$

$$= \boxed{\frac{1}{8}\tan 4x^2 + c}$$

299 $\int \frac{1}{25+4x^2}\,dx =$ $\left[\frac{1}{10}\arctan\frac{2x}{5}+c\right]$

$$= \int \frac{1}{25\left(1+\frac{4}{25}x^2\right)}\,dx = \frac{1}{25}\int \frac{1}{1+\left(\frac{2}{5}x\right)^2}\,dx =$$

$$= \frac{1}{\cancel{25}_{5}}\cdot\frac{\cancel{5}^{1}}{2}\int \frac{\frac{2}{5}}{1+\left(\frac{2}{5}x\right)^2}\,dx = \frac{1}{10}\int \left[\arctan\frac{2}{5}x\right]'\,dx =$$

$$= \boxed{\frac{1}{10}\arctan\frac{2}{5}x + c}$$

275 $\int \frac{9x-3}{x^2+1}\,dx = \int \frac{9x}{x^2+1}\,dx - 3\int \frac{1}{x^2+1}\,dx =$

$$= \frac{9}{2}\int \frac{2x}{x^2+1}\,dx - 3\arctan x + c =$$

$$= \frac{9}{2}\ln|x^2+1| - 3\arctan x + c =$$

$$= \boxed{\frac{9}{2}\ln(x^2+1) - 3\arctan x + c}$$