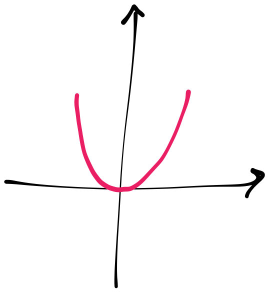
27/2/2018

368 $2^x = |x^2 - 2|$ Risolvere graficamente

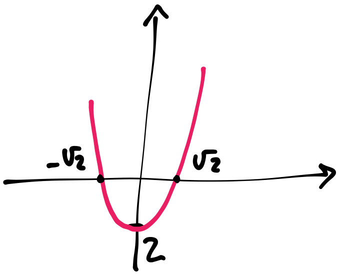
[tre sol.; $-2 < x_{1,2} < -1$, $0 < x_3 < 1$]

$$\begin{cases} y = 2^x \\ y = |x^2 - 2| \end{cases}$$

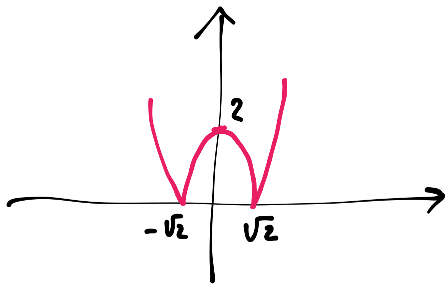
$y = x^2$

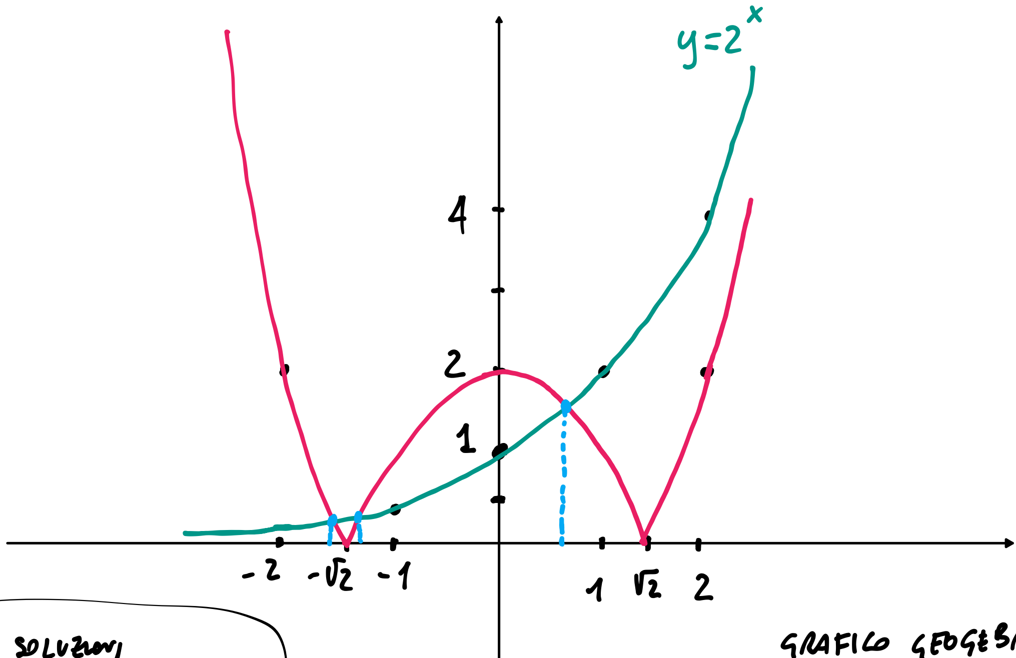


$y = x^2 - 2$



$y = |x^2 - 2|$





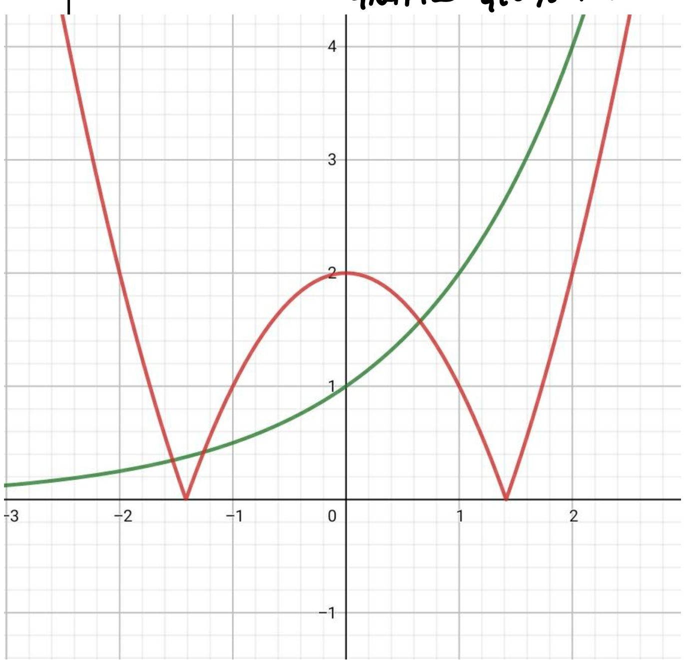
3 SOLUZIONI

$-2 < x_1 < -1$

$-2 < x_2 < -1$

$0 < x_3 < 1$

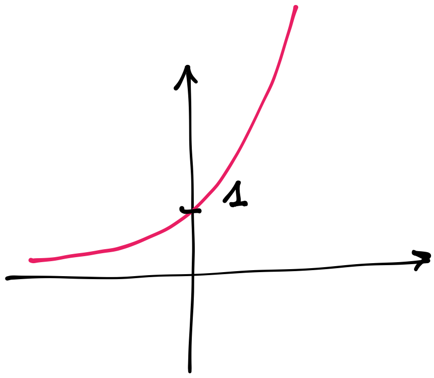
GRAFICO GEOGEBRA



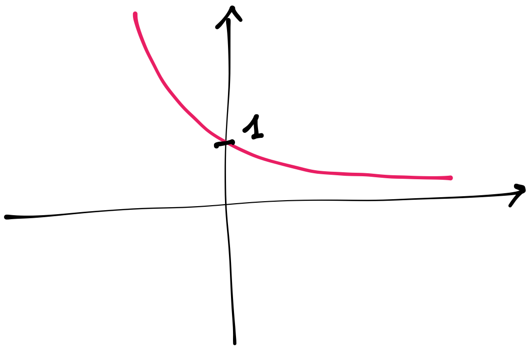
PUNTO DELLA SITUAZIONE

$y=a^x$

$a>1$



$0<a<1$



$\exp_a(x)=a^x$

$\exp_a : \underbrace{\mathbb{R}}_{\text{DOMINIO}} \to \underbrace{\mathbb{R}^+}_{\text{CODOMINIO}}$

BIETTIVA ↗ CRESCENTE SE $a>1$

BIETTIVA ↘ DECRESCENTE SE $0<a<1$

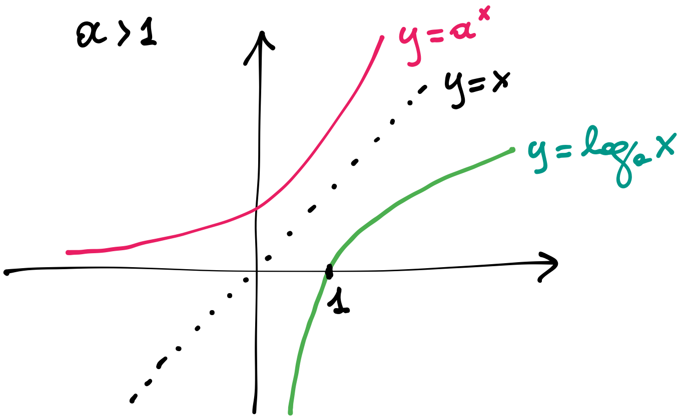
$\Downarrow$

INVERTIBILE

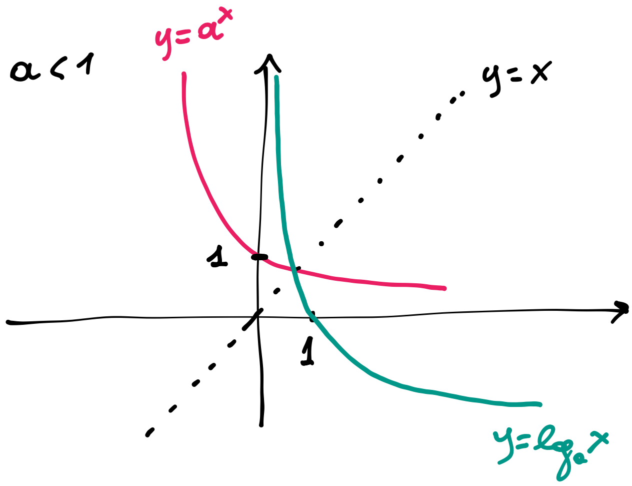
$$\boxed{\text{FUNZIONE INVERSA DI } \exp_a \text{ SI CHIAMA LOGARITMO IN BASE } a}$$

$\log_a : \underbrace{\mathbb{R}^+}_{\text{DOMINIO}} \to \underbrace{\mathbb{R}}_{\text{CODOMINIO}}$

$a>1$



$0<a<1$



ESEMPI

BASE $a = 2$

$\exp_2(3) = 2^3 = 8$

$3 \xmapsto{\exp_2} 8$

$\vdots$

$5 \xmapsto{\exp_2} 32$

$\log_2(8) = 3$

$8 \xmapsto{\log_2} 3$

$\vdots$

$32 \xmapsto{\log_2} 5$

$$\log_2(2^3) = 3$$
$$\log_2(2^5) = 5$$

$\log_3 81 = 4$ quindi $\log_3 81$ è l'esponente da dare a 3 per ottenere 81

DEFINIZIONE

Dati due numeri reali positivi a e b, con $a \neq 1$, chiamiamo **logaritmo in base *a* di *b*** l'esponente x da assegnare alla base a per ottenere il numero b.

$\log_a b = x \leftrightarrow a^x = b$

$a > 0,\ a \neq 1,\ b > 0$

PRIME OSSERVAZIONI

$\log_a 1 = 0$

$\log_a a = 1$

$a^{\log_a b} = b$

$\log_a a^b = b$

$$\log_2 16 = 4 \quad \text{perché} \quad 2^4 = 16$$

$$\log_3\left(\frac{1}{9}\sqrt{3}\right) = \log_3\left(3^{-2}\cdot 3^{\frac{1}{2}}\right) = \log_3 3^{-2+\frac{1}{2}} = -2+\frac{1}{2} = -\frac{3}{2}$$

$$\log_{25}\frac{5}{\sqrt[3]{5}} = \log_{25}\left(5^{1-\frac{1}{3}}\right) = \log_{25} 5^{\frac{2}{3}} =$$

$$= \log_{25}\left(5^{2}\right)^{\frac{1}{3}} = \frac{1}{3}$$

In pratica si tratta di risolvere un'eq. esponenziale:

$$\log_{25}\frac{5}{\sqrt[3]{5}} = x \rightsquigarrow 25^x = \frac{5}{\sqrt[3]{5}}$$

$$\Downarrow$$

$$5^{2x} = 5^{1-\frac{1}{3}} \qquad 2x = \frac{2}{3} \qquad x = \frac{1}{3}$$