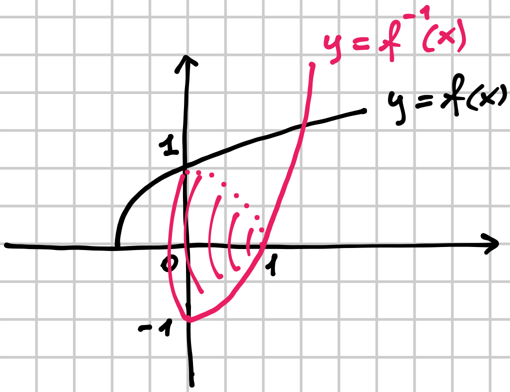
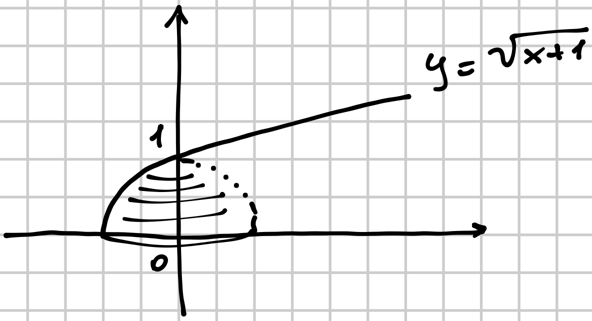
5/5/2021

404 $y=\sqrt{1+x}$, $\quad 0 \le y \le 1$. $\quad \left[\frac{8}{15}\pi\right]$

SOLIDO OTTENUTO DALLA ROTAZIONE ATTORNO ALL'ASSE y





$y=\sqrt{1+x}$

$y^2=1+x$

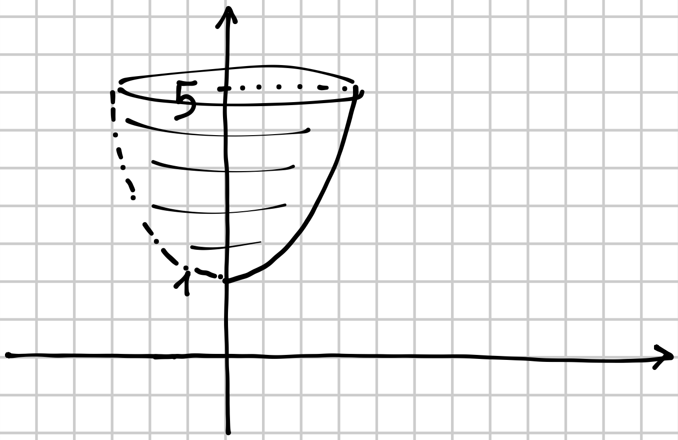
$x=y^2-1 \implies y=x^2-1$

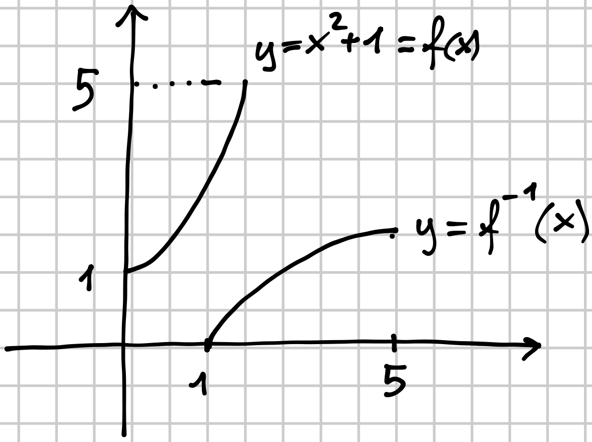
$$V=\pi\int_0^1 (x^2-1)^2\,dx =$$

$$=\pi\int_0^1 (x^4-2x^2+1)\,dx = \pi\left[\frac{1}{5}x^5-\frac{2}{3}x^3+x\right]_0^1 =$$

$$=\pi\left[\frac{1}{5}-\frac{2}{3}+1\right] = \frac{3-10+15}{15}\pi = \boxed{\frac{8}{15}\pi}$$

406 $y = x^2 + 1, \qquad 1 \leq y \leq 5.$





$$y = x^2+1 \implies x^2 = y-1 \implies x = \sqrt{y-1} \implies y = \sqrt{x-1}$$

$$V = \pi \int_1^5 (x-1)\,dx = \pi\left[\frac{1}{2}x^2 - x\right]_1^5 = \pi\left[\frac{25}{2} - 5 - \frac{1}{2} + 1\right] = \boxed{8\pi}$$

416 $y = 4x - x^2, \qquad [0; 2].$ METODO DEI GUSCI CILINDRICI

$$V = 2\pi \int_0^2 x(4x - x^2)\,dx = 2\pi \int_0^2 (4x^2 - x^3)\,dx =$$

$$= 2\pi\left[\frac{4}{3}x^3 - \frac{1}{4}x^4\right]_0^2 = 2\pi\left[\frac{32}{3} - 4\right] = \boxed{\frac{40}{3}\pi}$$

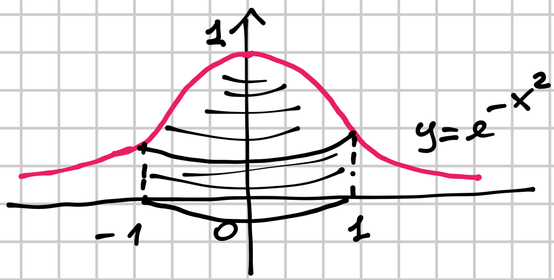
419 $y = e^{-x^2}$, $[0; 1]$. $\left[\left(1-\frac{1}{e}\right)\pi\right]$

GUSCI CILINDRICI

$$V = 2\pi \int_0^1 x e^{-x^2} dx = -\pi \int_0^1 (-2x) e^{-x^2} dx =$$

$$= -\pi \int_0^1 \left(e^{-x^2}\right)' dx = -\pi \left[e^{-x^2}\right]_0^1 =$$

$$= -\pi \left[e^{-1} - 1\right] = \boxed{\left(1-\frac{1}{e}\right)\pi}$$



443 $y = -x^2 + 6x$, $[1; 3]$; sezioni: semicerchi.

VOLUME DEL SOLIDO

$\hookrightarrow$ diametro $f(x)$

$\Rightarrow$ raggio $\frac{f(x)}{2}$

$$dV = \frac{1}{2}\left(\frac{f(x)}{2}\right)^2 \pi dx$$

$$\Downarrow$$

$$V = \pi\int_1^3 \frac{1}{2}\,\frac{f^2(x)}{4}\,dx = \frac{\pi}{8}\int_1^3 (-x^2+6x)^2\,dx =$$

$$= \frac{\pi}{8}\int_1^3 (x^4 - 12x^3 + 36x^2)\,dx = \frac{\pi}{8}\left[\frac{1}{5}x^5 - 3x^4 + 12x^3\right]_1^3 =$$

$$= \frac{\pi}{8}\left[\frac{243}{5} - 243 + 324 - \frac{1}{5} + 3 - 12\right] =$$

$$= \frac{\pi}{8}\left[\frac{242}{5} + 72\right] = \frac{\overset{301}{\cancel{602}}}{\underset{4}{\cancel{8}}\cdot 5}\pi = \boxed{\frac{301}{20}\pi}$$