PROPRIETÀ DEI LOGARITMI

$$\log_a(xy) = \log_a x + \log_a y \qquad \forall x, y > 0,$$

$$\log_a(x/y) = \log_a x - \log_a y \qquad \forall x, y > 0,$$

$$\log_a x^y = y \log_a x \qquad \forall x > 0 \quad \forall y,$$

$$\log_a 1 = 0,$$

$$\log_x y = \frac{\log_a y}{\log_a x} \qquad \forall x, y > 0 \quad x \neq 1.$$

PROPRIETÀ DEGLI ESPONENZIALI

$$a^{x+y} = a^x a^y \qquad \forall x, y,$$

$$a^{x-y} = a^x / a^y \qquad \forall x, y,$$

$$(a^x)^y = a^{xy} \qquad \forall x, y,$$

$$a^0 = 1,$$

DIMOSTRAZIONI

1) $\log_a(x \cdot y) = \log_a x + \log_a y \qquad \forall x, y > 0$

$$a^{\log_a xy} = a^{\log_a x + \log_a y}$$

$$xy = \underbrace{a^{\log_a x}}_{x} \cdot \underbrace{a^{\log_a y}}_{y}$$

$$xy = xy$$

3) $\log_a x^y = y \log_a x \qquad \forall x > 0 \quad \forall y$

$$x = a^{\log_a x}$$

$$x^y = \left[a^{\log_a x}\right]^y$$

$$x^y = a^{y \cdot \log_a x} \qquad\qquad x^y = a^{\log_a x^y}$$
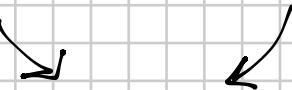
$$a^{y \log_a x} = a^{\log_a x^y} \qquad \Rightarrow \quad y \log_a x = \log_a x^y$$

2) $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\qquad \forall x, y > 0$

$\log_a \frac{x}{y} = \log_a (x \cdot y^{-1}) \underset{\substack{\uparrow \\ \text{per la 1)}}{=} \log_a x + \log_a y^{-1} \underset{\substack{\uparrow \\ \text{per la 2)}}{=} \log_a x - \log_a y$

4) $\log_x y = \frac{\log_a y}{\log_a x}$ $\qquad \forall x, y > 0 \qquad x \neq 1$

$\log_a x \cdot \log_x y = \log_a y$

$\log_x y \cdot \log_a x = \log_a y$

$\log_a x^{\underbrace{\log_x y}_{y}} = \log_a y$

$\log_a y = \log_a y$

Applicare le proprietà per scrivere un solo logaritmo

107 $\frac{1}{2}\log_3 x + 2\log_3(x+1) - \log_3 7 =$ $\qquad\left[\log_3\dfrac{\sqrt{x}\,(x+1)^2}{7}\right]$

$$= \log_3 x^{\frac{1}{2}} + \log_3(x+1)^2 - \log_3 7 =$$

$$= \log_3\left[\sqrt{x}\,(x+1)^2\right] - \log_3 7 =$$

$$= \log_3 \frac{\sqrt{x}\,(x+1)^2}{7}$$

101 $\frac{1}{3}\left[\log_3 35 - (\log_3 7 - 2\log_3 5)\right] =$ $\qquad[\log_3 5]$

$$= \frac{1}{3}\left[\log_3 35 - \log_3 7 + \log_3 5^2\right] =$$

$$= \frac{1}{3}\left[\log_3 35 + \log_3 7^{-1} + \log_3 25\right] =$$

$$= \frac{1}{3}\log_3\left(\frac{\overset{5}{\cancel{35}}\cdot 25}{\cancel{7}}\right) = \frac{1}{3}\log_3 5^3 = \log_3\left(5^3\right)^{\frac{1}{3}} = \log_3 5 =$$

$$= \frac{\log 5}{\log 3} \simeq 1,4649\ldots$$