

Calcola seno, coseno e tangente dei seguenti angoli, sfruttando le conoscenze sugli angoli particolari.

440 $330^\circ, 225^\circ, 495^\circ$.

442 $\frac{11}{4}\pi, \frac{17}{6}\pi, -\frac{7}{3}\pi, \frac{13}{6}\pi$.

$$\sin 330^\circ = \sin(360^\circ - 30^\circ) = \sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cos 330^\circ = \dots = \cos(-30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 330^\circ = \tan(360^\circ - 30^\circ) = \tan(-30^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

$$\sin 225^\circ = \sin(180^\circ + 45^\circ) = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

⋮

$$\sin 495^\circ = \sin(360^\circ + 135^\circ) = \sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\sin \frac{11}{4}\pi = \sin\left(2\pi + \frac{3}{4}\pi\right) = \sin \frac{3}{4}\pi = \sin\left(\pi - \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \frac{17}{6}\pi = \cos\left(2\pi + \frac{5}{6}\pi\right) = \cos \frac{5}{6}\pi = \cos\left(\pi - \frac{\pi}{6}\right) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{7}{3}\pi\right) = -\tan \frac{7}{3}\pi = -\tan\left(2\pi + \frac{\pi}{3}\right) = -\tan \frac{\pi}{3} = -\sqrt{3}$$

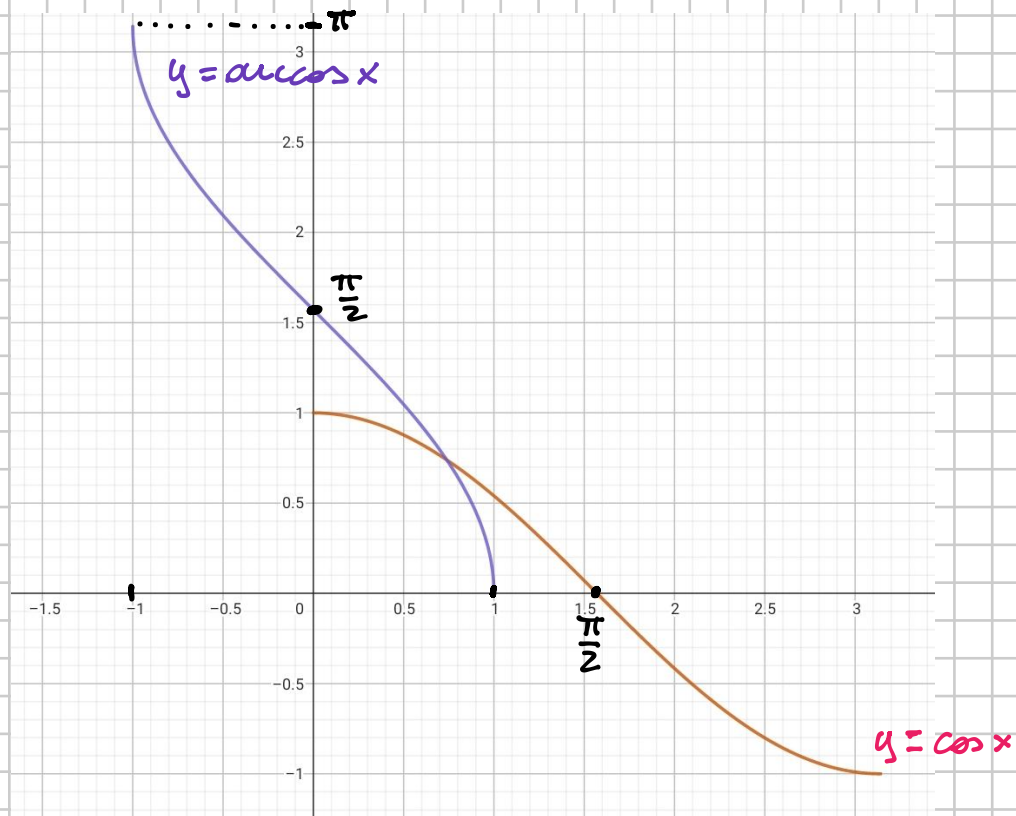
$$\sin \frac{13}{6}\pi = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos \frac{13}{6}\pi = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

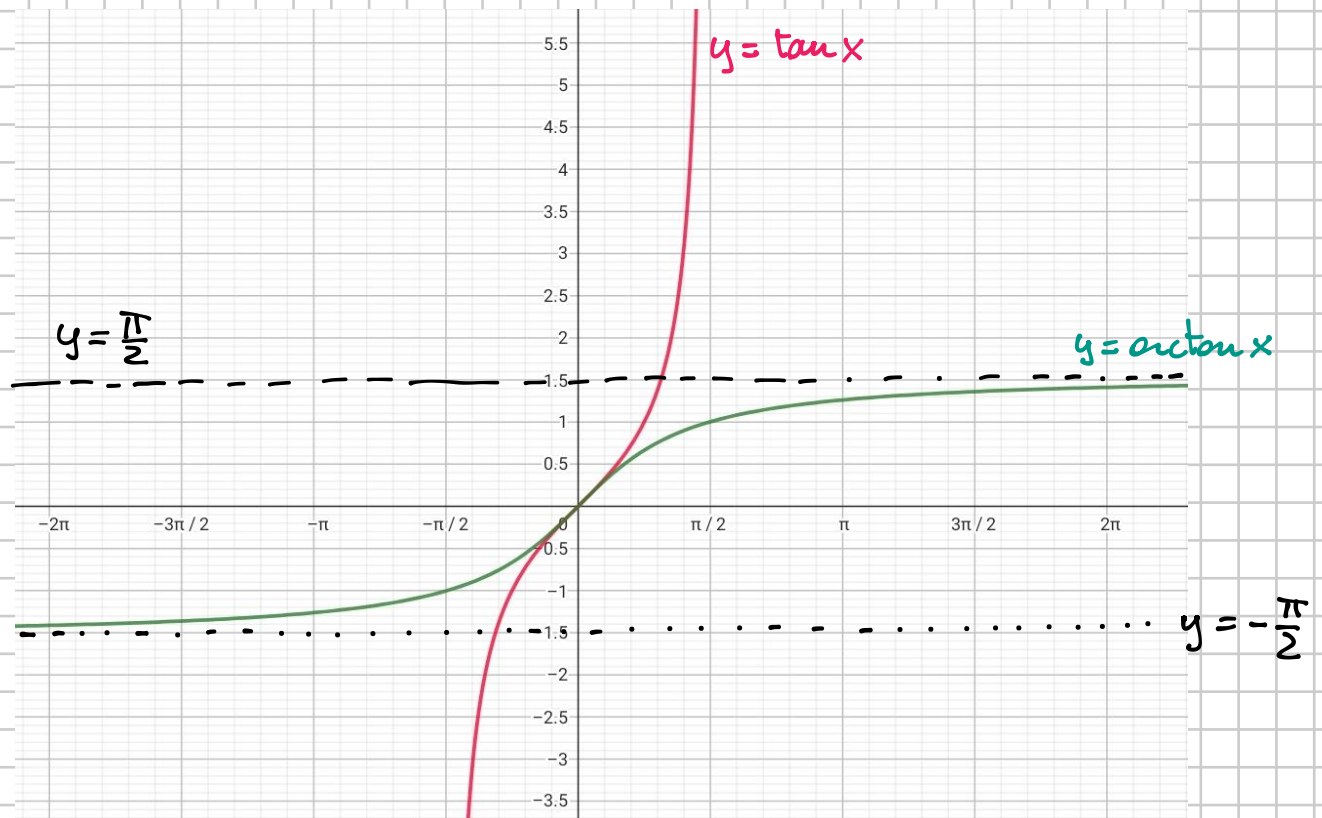
$$\tan \frac{13}{6}\pi = \tan\left(2\pi + \frac{\pi}{6}\right) = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

ARCO COSENO = inversa della RESTRIZIONE della funzione coseno a $[0, \pi]$

$$\arccos : [-1, 1] \rightarrow [0, \pi]$$

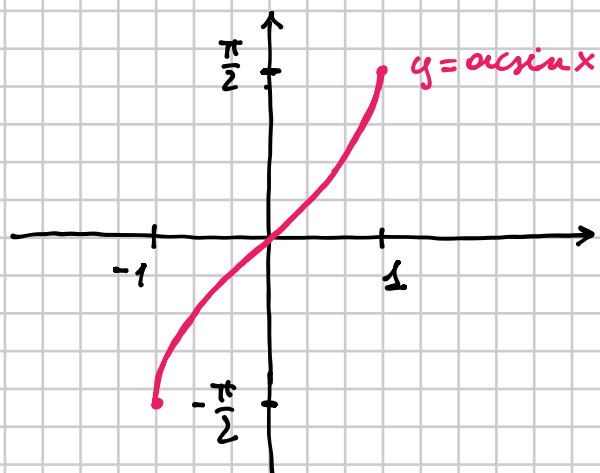


ARCO TANGENTE = inversa della restrizione della funzione tangente
a $(-\frac{\pi}{2}, \frac{\pi}{2})$ $\arctan : \mathbb{R} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$



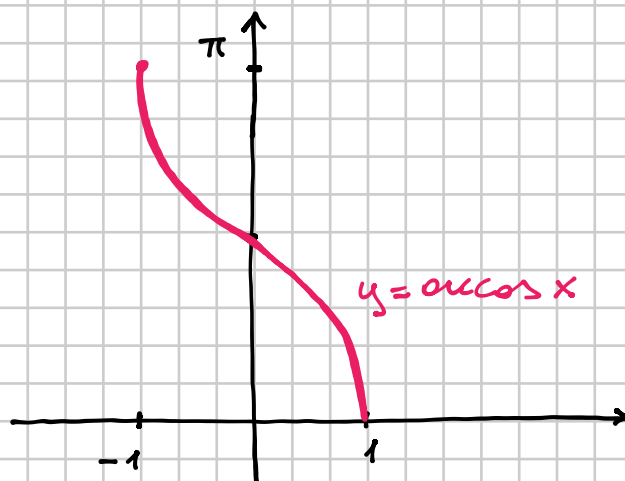
$$y = \arcsin x$$

$$\arcsin: [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



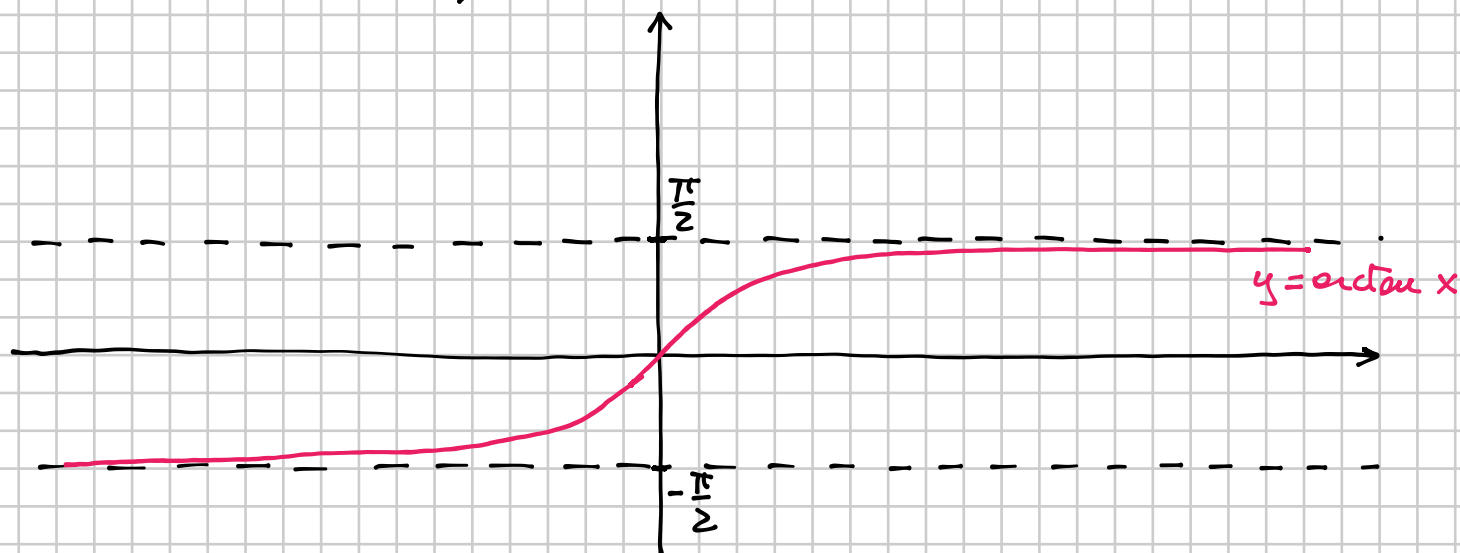
$$y = \arccos x$$

$$\arccos: [-1, 1] \rightarrow [0, \pi]$$



$$y = \arctan x$$

$$\arctan: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$



$$\arccos\left[\sin\left(-\frac{\pi}{6}\right)\right] = \text{ },$$

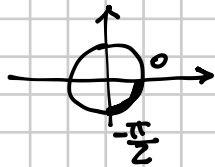
$$\sin\left[\arctan\left(-\frac{4}{3}\right)\right] = \text{ },$$

$$\arccos\left[\sin\left(-\frac{\pi}{6}\right)\right] = \arccos\left[-\frac{1}{2}\right] = \frac{2}{3}\pi$$

qual è l'angolo
tra 0 e π il
cui coseno è $-\frac{1}{2}$?

$$\sin\left[\arctan\left(-\frac{4}{3}\right)\right] = (*)$$

$\arctan(-\frac{4}{3})$ è negativo, e dà quindi un
angolo compreso fra $-\frac{\pi}{2}$ e 0. Il seno di
un tale angolo è negativo



$$(*) = \frac{\tan(\arctan(-\frac{4}{3}))}{\sqrt{1 + \tan^2(\arctan(-\frac{4}{3}))}} =$$

$$= \frac{-\frac{4}{3}}{\sqrt{1 + (-\frac{4}{3})^2}} = \frac{-\frac{4}{3}}{\sqrt{1 + \frac{16}{9}}} =$$

$$= \frac{-\frac{4}{3}}{\sqrt{\frac{25}{9}}} = \frac{-\frac{4}{3}}{\frac{5}{3}} = -\frac{4}{5}$$

A PARTE

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\tan^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha}$$

$$\tan^2 \alpha = \frac{\sin^2 \alpha}{1 - \sin^2 \alpha}$$

$$\tan^2 \alpha - \tan^2 \alpha \sin^2 \alpha = \sin^2 \alpha$$

$$\sin^2 \alpha + \tan^2 \alpha \sin^2 \alpha = \tan^2 \alpha$$

$$\sin^2 \alpha (1 + \tan^2 \alpha) = \tan^2 \alpha$$

$$\sin^2 \alpha = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$\sin \alpha = \pm \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$$

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$$y = 4\cos\left(2x + \frac{\pi}{4}\right)$$

DISEGNARE IL GRAFICO DI QUESTA FUNZIONE

1]

$$y = \cos x$$

2]

$$y = \cos 2x$$

3]

$$y = \cos 2\left(x + \frac{\pi}{8}\right)$$

4]

$$y = 4\cos\left(2x + \frac{\pi}{4}\right)$$

ALTERNATIVA

1]

$$y = \cos x$$

2]

$$y = \cos\left(x + \frac{\pi}{4}\right)$$

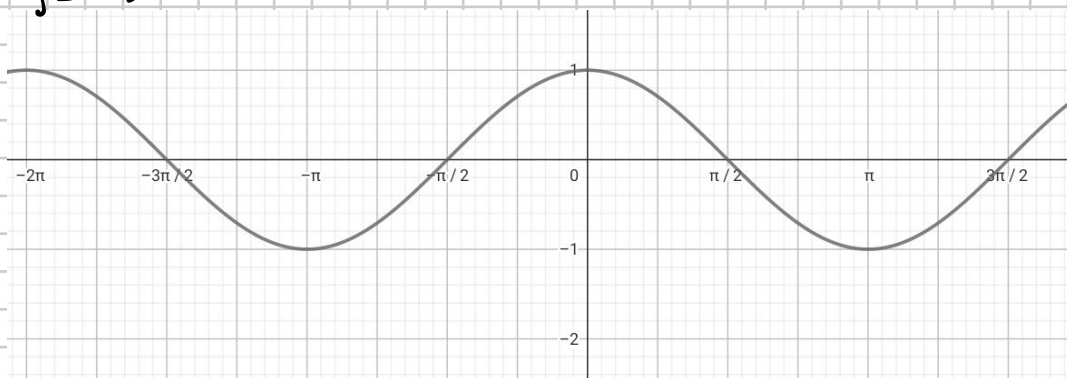
3]

$$y = \cos\left(2x + \frac{\pi}{4}\right)$$

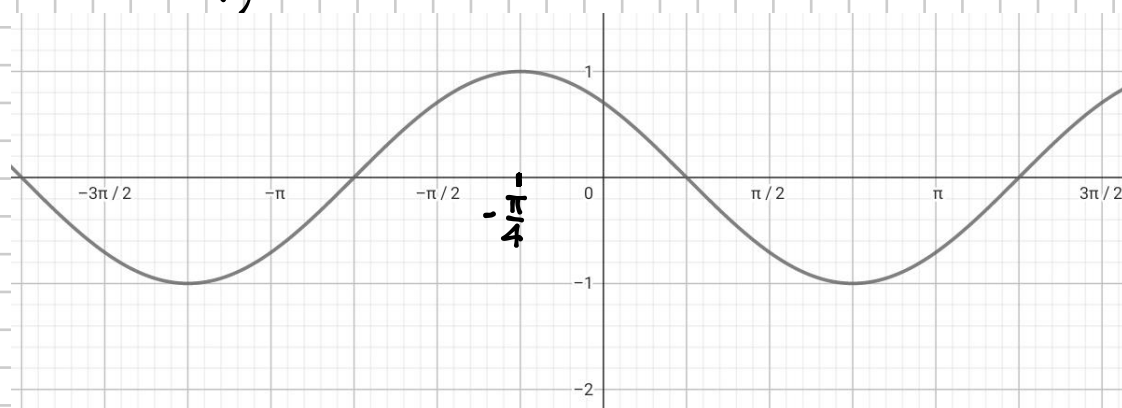
4]

$$y = 4\cos\left(2x + \frac{\pi}{4}\right)$$

$$y = \cos x$$

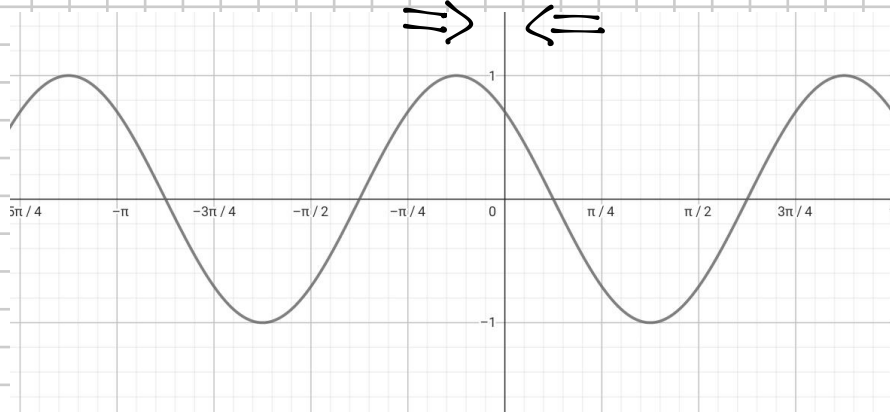


$$y = \cos\left(x + \frac{\pi}{4}\right) : \text{TRASLAZIONE DI } \frac{\pi}{4} \text{ VERSO SINISTRA}$$



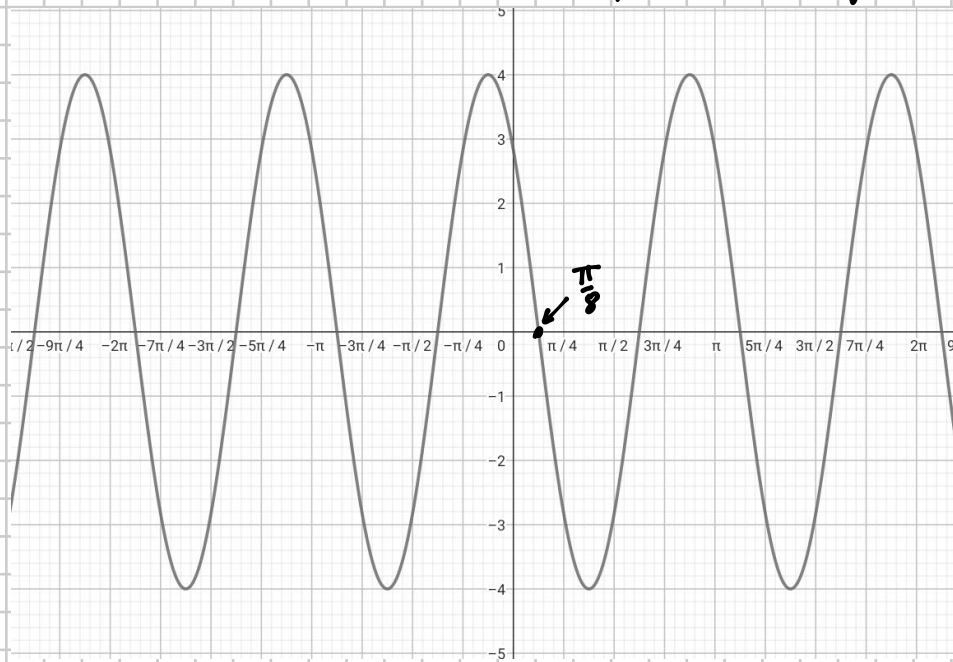
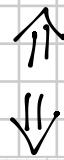
$$y = \cos\left(2x + \frac{\pi}{4}\right) : \text{"COMPILMO" DI UN FATTORE 2}$$

$\Rightarrow \Leftarrow$



$$y = 4 \cos \left(2x + \frac{\pi}{4} \right)$$

DILATAZIONE VERTICALE
DI FATTORE 4



IL PERIODO È π

In generale, dato che $\sin x$ e $\cos x$ hanno periodo 2π , le funzioni $\sin(ax + b)$ e $\cos(ax + b)$ hanno periodo $\frac{2\pi}{a}$