317 $2^x - 1 > \sqrt{3 \cdot 2^x - 3}$ $[x > 2]$

$2^x = t$

$t - 1 > \sqrt{3t - 3}$

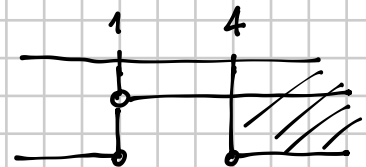$\sqrt{3t - 3} < t - 1$

$$\begin{cases} 3t - 3 \geq 0 \\ t - 1 > 0 \\ 3t - 3 < t^2 - 2t + 1 \end{cases}$$

$$\begin{cases} t \geq 1 \\ t > 1 \end{cases} \Rightarrow t > 1 \qquad \begin{cases} t > 1 \\ (t-4)(t-1) > 0 \end{cases} \qquad \begin{cases} t > 1 \\ t < 1 \ \lor \ t > 4 \end{cases}$$

$t^2 - 5t + 4 > 0$

$\sqrt{f(x)} < g(x)$

$$\begin{cases} f(x) \geq 0 \\ g(x) > 0 \\ f(x) < g^2(x) \end{cases}$$

$t > 4$

$2^x > 4$

$2^x > 2^2$

$\boxed{x > 2}$

328 $\left| \dfrac{4^{-x}}{2^{x+2} : 2^6} \right| < 1$

$\left| \dfrac{2^{-2x}}{2^{x+2-6}} \right| < 1$

$\left| 2^{-2x-(x-4)} \right| < 1$

$\left| 2^{-2x-x+4} \right| < 1$

$\left| 2^{-3x+4} \right| < 1$

$-1 < 2^{-3x+4} < 1 \quad \Rightarrow \quad \begin{cases} -1 < 2^{-3x+4} & \Leftarrow \text{ verificata sempre} \\ & \text{perchè l'esponenziale} \\ & \text{è positivo } \forall x \\ 2^{-3x+4} < 1 \end{cases}$

$\Downarrow$

$2^{-3x+4} < 2^0$

$-3x + 4 < 0$

$\boxed{x > \dfrac{4}{3}}$

326 $\overset{\overbrace{4^x \cdot 4}}{4^x(4^{x+1} - 33)} > -8$ $\left[x < -1 \lor x > \dfrac{3}{2} \right]$

$t = 4^x$

$t(4t - 33) + 8 > 0$

$4t^2 - 33t + 8 > 0$ $\Delta = 1089 - 128 = 961 = 31^2$

$t = \dfrac{33 \pm 31}{8} = \begin{cases} \dfrac{2}{8} = \dfrac{1}{4} \\ \dfrac{64}{8} = 8 \end{cases}$ $t < \dfrac{1}{4} \quad \lor \quad t > 8$

$4^x < 4^{-1} \quad \lor \quad 4^x > 8$

$x < -1 \quad \lor \quad 2^{2x} > 2^3$

$2x > 3$

$x > \dfrac{3}{2}$

$\boxed{x < -1 \quad \lor \quad x > \dfrac{3}{2}}$

$|f(x)| \geq g(x) \iff f(x) \leq -g(x) \quad \lor \quad f(x) \geq g(x)$

① $16^x - 4 \leq -4 - 2 \cdot 4^x$ $\qquad \lor \qquad$ ② $16^x - 4 \geq 4 + 2 \cdot 4^x$

① $16^x \leq \underbrace{-2 \cdot 4^x}_{<0}$ $\qquad \lor \qquad$ ② $4^{2x} - 2 \cdot 4^x - 8 \geq 0$

IMPOSSIBILE $\quad \emptyset$

$\qquad\qquad\qquad\qquad\qquad\qquad 4^x = t \qquad t^2 - 2t - 8 \geq 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (t-4)(t+2) \geq 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad t = 4 \quad t = -2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad t \leq -2 \quad \lor \quad t \geq 4$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 4^x \leq -2 \quad \lor \quad 4^x \geq 4$

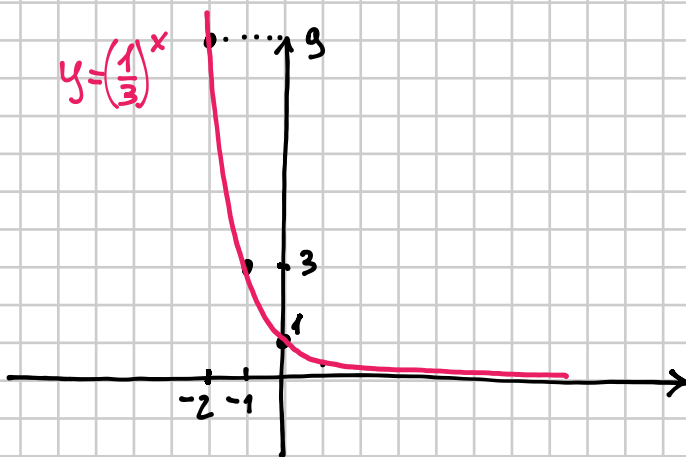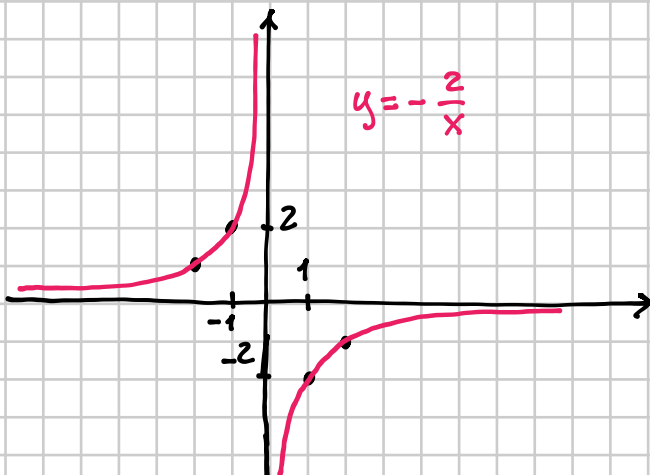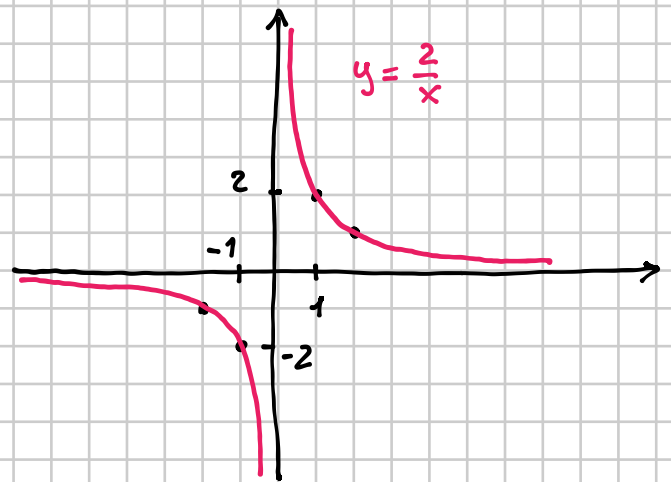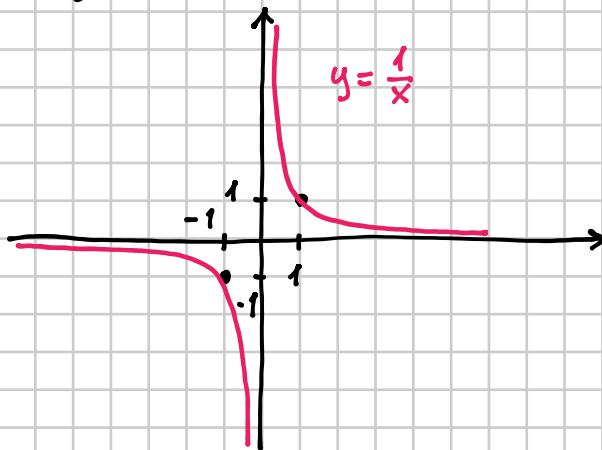$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{IMPOSS.} \qquad \boxed{x \geq 1}$

368 $\left(\dfrac{1}{3}\right)^x - 1 = -\dfrac{2}{x}$

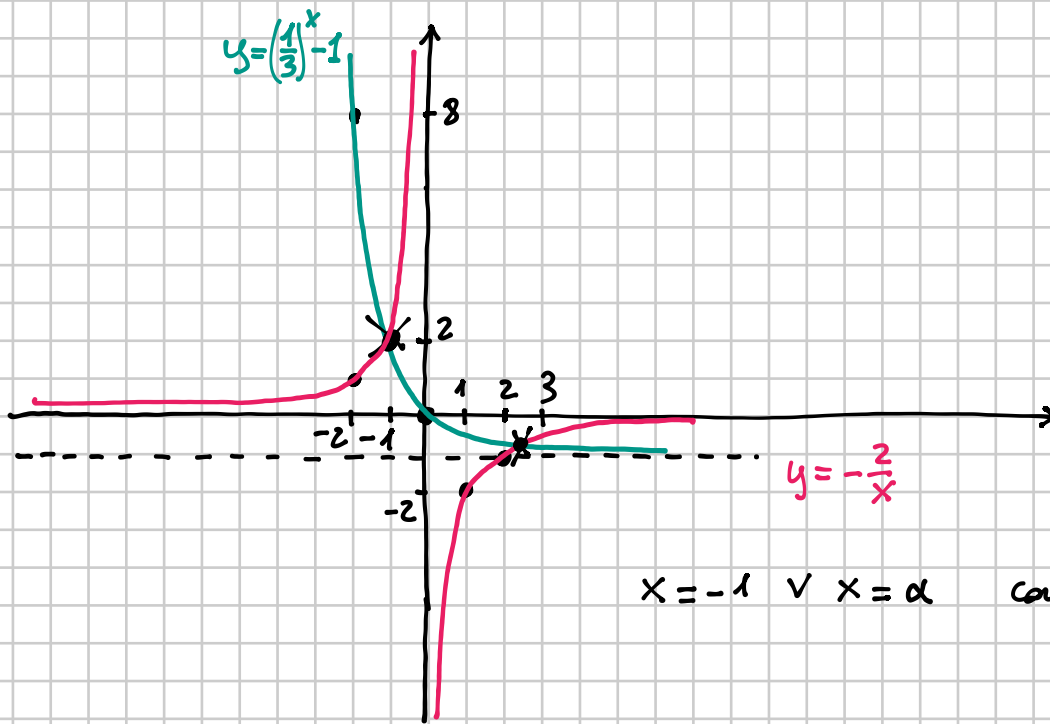$$\begin{cases} y = \left(\dfrac{1}{3}\right)^x - 1 \\[3mm] y = -\dfrac{2}{x} \end{cases}$$

Disegniamo la curva $\quad y = \left(\dfrac{1}{3}\right)^x - 1$

$y = \left(\dfrac{1}{3}\right)^x$

$y = \left(\dfrac{1}{3}\right)^x - 1$

Disegniamo la curva $y = -\dfrac{2}{x}$

$y = \dfrac{1}{x}$

$y = \dfrac{2}{x}$

$y = -\dfrac{2}{x}$

Sovrapponiamo i due grafici

$y = \left(\frac{1}{3}\right)^x - 1$

$y = -\frac{2}{x}$

$X = -1 \lor X = \alpha$ con $2 < \alpha < 3$