6/3/2018

pg. 620 N 24)

$$\log_a a^x = x$$

$$\log_2 \frac{4}{\sqrt{2}} = \log_2 \left(2^2 \cdot 2^{-\frac{1}{2}}\right) = \log_2 2^{2-\frac{1}{2}} = 2 - \frac{1}{2} = \frac{3}{2}$$

$$\log_3 \left(3 \cdot \sqrt[4]{3}\right) = \log_3 \left(3 \cdot 3^{\frac{1}{4}}\right) = 1 + \frac{1}{4} = \frac{5}{4}$$

N 36)

$$\log_{\frac{4}{9}} \frac{27}{8} = x \iff \left(\frac{4}{9}\right)^x = \frac{27}{8}$$

$$\left(\frac{2}{3}\right)^{2x} = \left(\frac{3}{2}\right)^3 \qquad \left(\frac{2}{3}\right)^{2x} = \left(\frac{2}{3}\right)^{-3}$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

quindi $\log_{\frac{4}{9}} \frac{27}{8} = -\frac{3}{2}$

$$\log_{\sqrt[3]{9}} \sqrt[4]{27} = x \iff \left(\sqrt[3]{9}\right)^x = \sqrt[4]{27}$$

$$3^{\frac{2x}{3}} = 3^{\frac{3}{4}} \qquad \frac{2x}{3} = \frac{3}{4} \qquad x = \frac{9}{8}$$

quindi $\log_{\sqrt[3]{9}} \sqrt[4]{27} = \frac{9}{8}$

48)

$$\log_4 b = -2 \iff 4^{-2} = b \implies b = \frac{1}{16}$$

$$\log_{\frac{2}{3}} b = -\frac{1}{2} \iff \left(\frac{2}{3}\right)^{-\frac{1}{2}} = b \implies b = \sqrt{\frac{3}{2}}$$

56)

$$\log_a \frac{1}{81} = -4 \iff a^{-4} = \frac{1}{81}$$

$$a^4 = 81$$

$$a = \sqrt[4]{81} = 3$$

BASE $\boxed{a > 0}$

77)

$$\log_2 32 - 4\log_4 16 + \log\left[\log_2(\log 100)\right] =$$

$\boxed{\log = \log_{10}}$ → SECONDO IL LIBRO

$$= 5 - 4 \cdot 2 + \log\left[\log_2 2\right] =$$

$$= 5 - 8 + \log 1 = 5 - 8 + 0 = -3$$

PROPRIETÀ DEI LOGARITMI

$\boxed{a>0 \quad a\neq 1}$

ESPONENZIALI	LOGARITMI
$\forall x,y \quad a^x \cdot a^y = a^{x+y}$	$\forall x,y>0 \quad \log_a(x\cdot y) = \log_a x + \log_a y$
$\forall x,y \quad a^x : a^y = a^{x-y}$	$\forall x,y>0 \quad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
$\forall x,y \quad (a^x)^y = a^{xy}$	$\forall x>0 \quad \forall y \quad \log_a x^y = y\log_a x$

DIMOSTRAZIONI

1) $\forall x,y>0 \quad \log_a(x\cdot y) = \log_a x + \log_a y$

$$\Updownarrow$$

$$a^{\log_a(xy)} = a^{\log_a x + \log_a y}$$

$$\Updownarrow$$

$$xy = a^{\log_a x}\cdot a^{\log_a y}$$

$$\Updownarrow$$

$$xy = xy$$

3) $\forall x>0 \quad \forall y \quad \log_a x^y = y\log_a x$

$$a^{\log_a x^y} = a^{y\log_a x}$$

$$x^y = \left(a^{\log_a x}\right)^y$$

$$x^y = x^y$$

2) $\forall x,y>0 \quad \log_a \frac{x}{y} = \log_a x - \log_a y$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x\cdot y^{-1}) =$$

$$= \log_a x + \log_a y^{-1} =$$

PROPR. 1) PROPR. 3)

$$= \log_a x - \log_a y$$

$$\boxed{\log_a \sqrt[n]{x} = \frac{1}{n}\log_a x \qquad \forall x>0}$$ → DIRETTA CONSEGUENZA DELLA 3)

$$\log_3 \sqrt{7} = \log_3 7^{\frac{1}{2}} = \frac{1}{2}\log_3 7$$

IL CAMBIAMENTO DI BASE

$a>0 \quad a\neq 1$

$n>0 \quad n\neq 1$

$x>0$

$$\log_a x = \frac{\log_n x}{\log_n a}$$

n = NUOVA BASE

$$\log_3 2 = \frac{\log 2}{\log 3} = 0{,}630929753\ldots = \frac{\ln 2}{\ln 3}$$

DA CALCOLATRICE

$$3^{0{,}6309297} = 1{,}99999882\ldots$$

$$\boxed{\ln = \log_e}$$

$e \simeq 2{,}718\ldots$

$$\boxed{3^x = 2 \iff x = \log_3 2}$$

DIMOSTRAZIONE DEL CAMBIAMENTO DI BASE

$$\log_a x = \frac{\log_n x}{\log_n a}$$

$$\log_n a \cdot \log_a x = \log_n x$$

$$\log_a x \cdot \log_n a$$

PROPR. 3)

$$\log_n a^{\log_a x} = \log_n x$$

$$\log_n x = \log_n x$$

pg. 623 N 88)

$$\log_5 (3ab^2) = \log_5 3 + \log_5 a + \log_5 b^2 =$$

$$= \log_5 3 + \log_5 a + 2 \log_5 b$$

94)
$$\log \frac{a^3(a^2+1)}{b^2} = \log [a^3(a^2+1)] - \log b^2 =$$

$$= \log a^3 + \log (a^2+1) - 2 \log b =$$

$$= 3 \log a + \log (a^2+1) - 2 \log b$$

97)
$$\log \sqrt{a \sqrt[3]{ab^2}} = \frac{1}{2} \log [a \sqrt[3]{ab^2}] =$$

$$= \frac{1}{2} \left[\log a + \log \sqrt[3]{ab^2} \right] = \frac{1}{2} \left[\log a + \frac{1}{3} \log (ab^2) \right] =$$

$$= \frac{1}{2} \log a + \frac{1}{6} \left(\log a + \log b^2 \right) =$$

$$= \frac{1}{2} \log a + \frac{1}{6} \log a + \frac{1}{\cancel{6}_{3}} \cdot \cancel{2}^{1} \log b =$$

$$= \frac{3+1}{6} \log a + \frac{1}{3} \log b = \frac{4}{6} \log a + \frac{1}{3} \log b =$$

$$= \frac{2}{3} \log a + \frac{1}{3} \log b$$

pag. 624 N 116)

portare tutto a 1 solo logaritmo

$$\frac{1}{2}\log_3 x + 2\log_3(x+1) - \log_3 7 =$$

$$= \log_3 x^{\frac{1}{2}} + \log_3(x+1)^2 - \log_3 7 =$$

$$= \log_3 \frac{\sqrt{x}\,(x+1)^2}{7}$$

120)

$$\log_2(x+1) + 5\log_2(x-1) - 4\log_2(x^2-1) =$$

$$= \log_2(x+1) + \log_2(x-1)^5 - \log_2(x^2-1)^4 =$$

$$= \log_2 \frac{(x+1)(x-1)^5}{(x^2-1)^4} = \log_2 \frac{\cancel{(x+1)}(x-1)^{\cancel{5}}}{(x+1)^{\cancel{4}\,3}\cancel{(x-1)^4}} =$$

$$= \log_2 \frac{x-1}{(x+1)^3}$$

CAMBIAMENTO DI BASE

140)

$$\log_4 7 \cdot \underbrace{\log_7 16}_{\text{APPLICO IL CAMBIAMENTO IN BASE 4}} = \cancel{\log_4 7} \cdot \frac{\log_4 16}{\cancel{\log_4 7}} = \log_4 16 = 2$$

150)

$$\frac{\log_3 12 - \log_9 4}{\log_{\frac{1}{3}} 6} = \frac{\log_3 12 - \dfrac{\log_3 4}{\log_3 9}}{\dfrac{\log_3 6}{\underbrace{\log_3 \frac{1}{3}}_{-1}}} =$$

$$= \frac{\log_3 \overbrace{12}^{3\cdot 4} - \dfrac{\log_3 4}{2}}{-\log_3 6} = -\frac{\log_3 3 + \log_3 \overbrace{4}^{2^2} - \frac{1}{2}\log_3 4}{\log_3 6} =$$

$$= -\frac{\log_3 3 + 2\log_3 2 - \frac{1}{\cancel{2}}\cdot \cancel{2}\log_3 2}{\log_3 6} = -\frac{\log_3 3 + \log_3 2}{\log_3 6} =$$

$$= -\frac{\log_3 (3\cdot 2)}{\log_3 6} = -1$$

151)

$$\frac{\log_8 27 + \log_2 5}{\log_4 9 - \log_{\frac{1}{4}} 25} = \frac{\frac{\log_2 27}{\log_2 8} + \log_2 5}{\frac{\log_2 9}{\log_2 4} - \frac{\log_2 25}{\log_2 \frac{1}{4}}} =$$

$$= \frac{\frac{1}{3}\log_2 27 + \log_2 5}{\frac{1}{2}\log_2 9 - \frac{\log_2 25}{-2}} = \frac{\log_2 \sqrt[3]{27} + \log_2 5}{\log_2 \sqrt{9} + \log_2 \sqrt{25}} =$$

$$= \frac{\log_2 3 + \log_2 5}{\log_2 3 + \log_2 5} = 1$$

$$\frac{\dfrac{\log_2 27}{\log_2 8} + \log_2 5}{\dfrac{\log_2 9}{\log_2 4} - \dfrac{\log_2 25}{\log_2 \frac{1}{4}}} = \frac{\dfrac{\cancel{3}\log_2 3}{\cancel{3}\log_2 2} + \log_2 5}{\dfrac{\cancel{2}\log_2 3}{\cancel{2}\log_2 2} - \dfrac{\cancel{2}\log_2 5}{-\cancel{2}\log_2 2}} =$$

$$= \frac{\log_2 3 + \log_2 5}{\log_2 3 + \log_2 5} = 1$$