16/4/2021

508 $\int \frac{x^3+4x+4}{x^2+4}\,dx = (*)$ $\left[\frac{x^2}{2} + 2\arctan\frac{x}{2} + c\right]$

$$\begin{array}{r|l} x^3+4x+4 & x^2+4 \\ \hline -x^3-4x & x \\ \hline // \quad // \quad 4 & \end{array}$$

$$\frac{x^3+4x+4}{x^2+4} = x + \frac{4}{x^2+4}$$

$$(*) = \int x\,dx + 4\int \frac{1}{x^2+4}\,dx = \frac{1}{2}x^2 + \cancel{4}\int \frac{1}{\cancel{4}\left(1+\left(\frac{x}{2}\right)^2\right)}\,dx =$$

$$= \frac{1}{2}x^2 + \int \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx = \frac{1}{2}x^2 + 2\int \frac{1}{2}\cdot\frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx =$$

$$= \frac{1}{2}x^2 + 2\int \left(\arctan\frac{x}{2}\right)'dx = \boxed{\frac{1}{2}x^2 + 2\arctan\frac{x}{2} + c}$$

586 $\int \sqrt{1-9x^2}\,dx \underset{(*)}{=} \left[\frac{1}{6}\arcsin 3x + \frac{x\sqrt{1-9x^2}}{2} + c\right]$

$$9x^2 = \sin^2 t \Rightarrow \sin t = 3x \Rightarrow x = \frac{\sin t}{3}$$

$$t = \arcsin 3x \qquad \Downarrow$$

$$dx = \frac{1}{3}\cos t\, dt$$

$$(*) = \int \overbrace{\sqrt{1-\sin^2 t}}^{\cos t} \cdot \frac{1}{3}\cos t\, dt = \frac{1}{3}\int \cos^2 t\, dt =$$

$$\cos 2t = \cos^2 t - \sin^2 t =$$
$$= 2\cos^2 t - 1$$
$$\Downarrow$$
$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$= \frac{1}{3}\int \frac{1+\cos 2t}{2}\, dt =$$

$$= \frac{1}{6}\int [1+\cos 2t]\, dt =$$

$$= \frac{1}{6}\int dt + \frac{1}{6}\int \cos 2t\, dt =$$

$$= \frac{1}{6}t + \frac{1}{6}\cdot\frac{1}{2}\int \underbrace{2\cos 2t}_{(\sin 2t)'}\, dt =$$

$t = \arcsin 3x$

$$= \frac{1}{6}t + \frac{1}{12}\sin 2t + c = \frac{1}{6}\arcsin 3x + \frac{1}{12}\sin(2\cdot\arcsin 3x) + c =$$

$$= \frac{1}{6}\arcsin 3x + \frac{1}{12}\cdot 2\cdot \sin(\arcsin 3x)\cdot\cos(\arcsin 3x) + c =$$

$$= \frac{1}{6}\arcsin 3x + \frac{1}{12}\cdot 2\cdot 3x\cdot\sqrt{1-\sin^2(\arcsin 3x)} + c =$$

$$= \boxed{\frac{1}{6}\arcsin 3x + \frac{1}{2}x\sqrt{1-9x^2} + c}$$