9/4/2021

488 $\displaystyle\int \frac{x^4+x^3+6}{x^2+x}\,dx = \int \left[x^2 + \frac{6}{x^2+x} \right] dx =$

$$\begin{array}{rrr|l} x^4 & +x^3 & +6 & x^2+x \\ \cline{4-4} -x^4 & -x^3 & & x^2 \\ \cline{1-3} // & // & 6 & \end{array}$$

$$= \int x^2\,dx + 6\int \frac{1}{x^2+x}\,dx =$$

$$= \frac{1}{3}x^3 + 6\int \frac{1}{x^2+x}\,dx = (*)$$

$$\frac{1}{x^2+x} = \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)} = \frac{Ax+A+Bx}{x(x+1)} =$$

$$= \frac{(A+B)x+A}{x(x+1)} \Rightarrow \begin{cases} A+B=0 \\ A=1 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$(*) = \frac{1}{3}x^3 + 6\left[\int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx\right] = \frac{1}{3}x^3 + 6\int\frac{1}{x}\,dx - 6\int\frac{1}{x+1}\,dx =$$

$$= \frac{1}{3}x^3 + 6\ln|x| - 6\ln|x+1| + c =$$

$$= \frac{1}{3}x^3 + 6\left(\ln|x| - \ln|x+1|\right) + c =$$

$$= \boxed{\frac{1}{3}x^3 + 6\ln\left|\frac{x}{x+1}\right| + c}$$

CASO $\Delta = 0$

IMMEDIATO

491 $$\int \frac{1}{4x^2+12x+9}\,dx = \int \frac{1}{(2x+3)^2}\,dx =$$

$$= \int (2x+3)^{-2}\,dx = \frac{1}{2}\int 2\,(2x+3)^{-2}\,dx =$$

$$= \frac{1}{2}\cdot\frac{1}{-2+1}(2x+3)^{-2+1} + c = -\frac{1}{2}(2x+3)^{-1} + c =$$

$$= -\frac{1}{2(2x+3)} + c$$

494 $\displaystyle\int \frac{2x-1}{x^2+2x+1}\,dx = \int \frac{2x-1}{(x+1)^2}\,dx = (*)$

$$\frac{2x-1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2} = \frac{Ax+A+B}{(x+1)^2}$$

$$\begin{cases} A = 2 \\ A+B = -1 \end{cases} \qquad \begin{cases} A = 2 \\ B = -3 \end{cases}$$

$$(*) = \int \left[\frac{2}{x+1} - \frac{3}{(x+1)^2}\right] dx = 2\int \frac{1}{x+1}\,dx - 3\int \frac{1}{(x+1)^2}\,dx =$$

$$= 2\ln|x+1| - 3\left(-\frac{1}{x+1}\right) + c = \boxed{2\ln|x+1| + \frac{3}{x+1} + c}$$

ALTERNATIVA

$$\dots = \int \frac{2x-1}{(x+1)^2}\,dx = \int \frac{2(t-1)-1}{t^2}\,dt = \int \frac{2t-3}{t^2}\,dt =$$

$t = x+1 \Rightarrow x = t-1$

$dx = dt$

$$= \int \frac{2}{t}\,dt - \int \frac{3}{t^2}\,dt =$$

$$= 2\ln|t| + \frac{3}{t} + c =$$

$$= 2\ln|x+1| + \frac{3}{x+1} + c$$

495 $\int \frac{4x+1}{4x^2+4x+1}dx = \int \frac{4x+1}{(2x+1)^2}dx =$

$2x+1 = t \Rightarrow \quad x = \frac{1}{2}(t-1)$

$dx = \frac{1}{2}dt$

$$= \int \frac{2(t-1)+1}{t^2}\,\frac{1}{2}dt = \frac{1}{2}\int \frac{2t-1}{t^2}dt =$$

$$= \frac{1}{2}\int \frac{2}{t}dt - \frac{1}{2}\int \frac{1}{t^2}dt = \ln|t| + \frac{1}{2}\cdot\frac{1}{t} + c =$$

$$= \boxed{\ln|2x+1| + \frac{1}{2(2x+1)} + c}$$