208 $4\cos x + \tan^2 \dfrac{x}{2} = 3$ $\qquad \left[\pm\dfrac{\pi}{4} + k\pi\right]$

$\tan \dfrac{x}{2} = \pm\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$4\cos x + \dfrac{1-\cos x}{1+\cos x} = 3$

$\dfrac{4\cos x(1+\cos x)+1-\cos x}{1+\cos x} = \dfrac{3(1+\cos x)}{1+\cos x}$

$4\cos x + 4\cos^2 x + 1 - \cos x = 3 + 3\cos x$
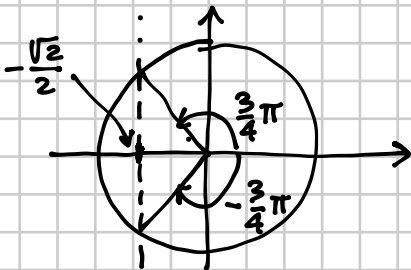
$4\cos^2 x = 3 - 1$

$\cos^2 x = \dfrac{1}{2}$

$\cos x = \pm\dfrac{1}{\sqrt{2}}$

$\cos x = \pm\dfrac{\sqrt{2}}{2}$
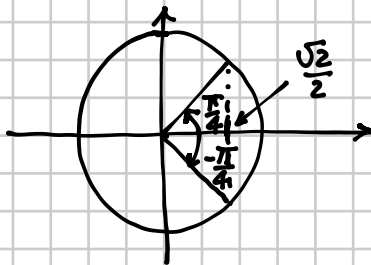
$\cos x = -\dfrac{\sqrt{2}}{2}$ $\qquad\qquad$ V $\qquad\qquad$ $\cos x = \dfrac{\sqrt{2}}{2}$

$x = \pm\dfrac{3}{4}\pi + 2k\pi$ $\qquad$ V $\qquad$ $x = \pm\dfrac{\pi}{4} + 2k\pi$

$\boxed{x = \dfrac{\pi}{4} + k\dfrac{\pi}{2}}$

227 $\tan\left(x + \dfrac{\pi}{4}\right) - (1 + \tan x) = 0$ $\left[k\pi; \dfrac{3}{4}\pi + k\pi\right]$

$$\frac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \cdot \tan\frac{\pi}{4}} - 1 - \tan x = 0$$

C.E.

1) $x \neq \dfrac{\pi}{2} + K\pi$

2) $x + \dfrac{\pi}{4} \neq \dfrac{\pi}{2} + K\pi$

$$\frac{\tan x + 1}{1 - \tan x} - 1 - \tan x = 0$$

$x \neq \dfrac{\pi}{4} + K\pi$

DA ESCLUDERE

METODO 1

$$\frac{\tan x + 1}{1 - \tan x} = 1 + \tan x$$

→ lo posso fare se prima "metto da parte" la soluzione $1 + \tan x = 0$

$$\frac{1}{1 - \tan x} = 1$$

$1 + \tan x = 0 \implies \tan x = -1$

$1 = 1 - \tan x$

$x = -\dfrac{\pi}{4} + K\pi$

$\tan x = 0$

$x = K\pi$

$$\boxed{x = -\dfrac{\pi}{4} + K\pi \quad \vee \quad x = K\pi}$$

METODO 2

$$\frac{\tan x + 1 - (1 + \tan x)(1 - \tan x)}{1 - \tan x} = 0$$

$\tan x + 1 - 1 + \tan^2 x = 0 \implies \tan^2 x + \tan x = 0$

$\tan x (\tan x + 1) = 0$

$\implies$
- $\tan x = 0 \qquad x = K\pi$
- $\tan x + 1 = 0 \qquad x = -\dfrac{\pi}{4} + K\pi$
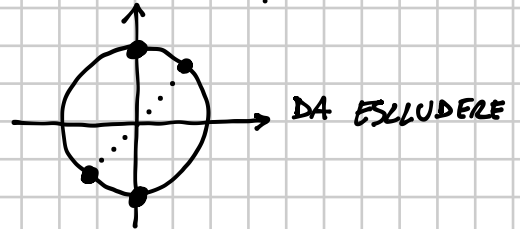
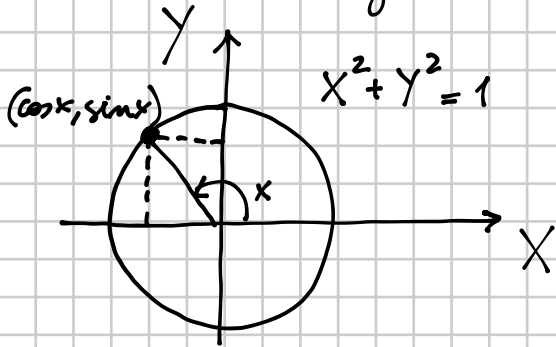# EQUAZIONI   LINEARI

$$a\cos x + b\sin x = c$$

245 $\cos x - \sin x = 1$ $\left[-\dfrac{\pi}{2} + 2k\pi; \; 2k\pi\right]$

a, b non entrambi nulli (in realtà entrambi $\neq 0$)

METODO GRAFICO

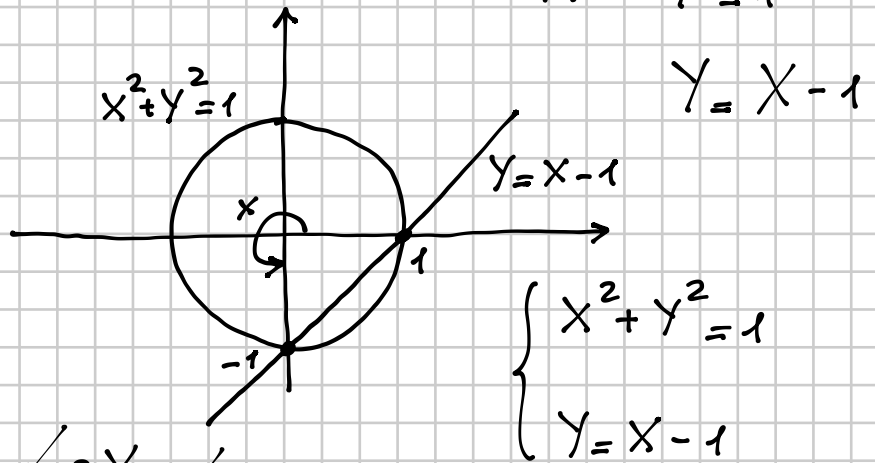$(\cos x, \sin x)$ è un punto della circonferenza goniometrica $X^2 + Y^2 = 1$

$X^2 + Y^2 = 1$

(cosx, sinx)

$Y = \sin x$
$X = \cos x$

$\cos x - \sin x = 1$

$X - Y = 1$

$Y = X - 1$

$X^2 + Y^2 = 1$

$Y = X - 1$

$\begin{cases} X^2 + Y^2 = 1 \\ Y = X - 1 \end{cases}$

$\begin{cases} X^2 + (X-1)^2 = 1 \\ Y = X - 1 \end{cases}$ $\begin{cases} X^2 + X^2 + 1 - 2X = 1 \\ Y = X - 1 \end{cases}$

$\begin{cases} 2X^2 - 2X = 0 \\ Y = X - 1 \end{cases}$ $\begin{cases} 2X(X-1) = 0 \\ Y = X - 1 \end{cases}$

$\begin{cases} X = 0 \\ Y = -1 \end{cases}$ $\begin{cases} \cos x = 0 \\ \sin x = -1 \end{cases}$ $x = \dfrac{3}{2}\pi + 2k\pi$

$\begin{cases} X = 1 \\ Y = 0 \end{cases}$ $\begin{cases} \cos x = 1 \\ \sin x = 0 \end{cases}$ $x = 2k\pi$

$$\boxed{x = 2k\pi \quad \lor \quad x = \dfrac{3}{2}\pi + 2k\pi}$$

251 $\sqrt{3}\,\sin x + \cos x = \sqrt{3}$

$$\begin{cases} \cos x = X \\ \sin x = Y \end{cases}$$

$$\begin{cases} X^2 + Y^2 = 1 \\ \sqrt{3}\,Y + X = \sqrt{3} \end{cases} \qquad \begin{cases} (\sqrt{3} - \sqrt{3}\,Y)^2 + Y^2 = 1 \\ X = \sqrt{3} - \sqrt{3}\,Y \end{cases} \qquad \begin{cases} 3 + 3Y^2 - 6Y + Y^2 = 1 \\ X = \sqrt{3} - \sqrt{3}\,Y \end{cases}$$

$$\begin{cases} 4Y^2 - 6Y + 2 = 0 \\ X = \ldots \end{cases} \qquad \begin{cases} 2Y^2 - 3Y + 1 = 0 \\ X = \ldots \end{cases} \qquad Y = \frac{3 \pm \sqrt{9-8}}{4}$$

$$\begin{cases} X = \sqrt{3} - \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{2} \\ Y = \dfrac{1}{2} \end{cases} \qquad \begin{cases} X = \dfrac{\sqrt{3}}{2} \\ Y = \dfrac{1}{2} \end{cases} \qquad \begin{cases} \cos x = \dfrac{\sqrt{3}}{2} \\ \sin x = \dfrac{1}{2} \end{cases} \qquad x = \dfrac{\pi}{6} + 2K\pi$$

$\vee$

$$\begin{cases} X = \sqrt{3} - \sqrt{3} = 0 \\ Y = 1 \end{cases} \qquad \begin{cases} \cos x = 0 \\ \sin x = 1 \end{cases} \qquad x = \dfrac{\pi}{2} + 2K\pi$$

$$\boxed{x = \dfrac{\pi}{2} + 2K\pi \quad \vee \quad x = \dfrac{\pi}{6} + 2K\pi}$$

$\sqrt{3}\,Y + X = \sqrt{3}$