29/3/2021

INTEGRAZIONE PER PARTI

$$[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\Downarrow$$

$$f'(x)\cdot g(x) = [f(x)\cdot g(x)]' - f(x)\cdot g'(x)$$

$$\int f'(x)\cdot g(x)\,dx = \int [f(x)\cdot g(x)]'dx - \int f(x)\cdot g'(x)\,dx$$

$$\boxed{\int f'(x)\,g(x)\,dx = f(x)\cdot g(x) - \int f(x)\cdot g'(x)\,dx}$$

FORMULA DI INTEGRAZIONE PER PARTI

ESEMPIO

$$\int e^x\cdot x\,dx = \int (e^x)'x\,dx = e^x\cdot x - \int e^x\cdot \underset{(x)'}{1}\,dx =$$

$f(x) = e^x$

$g(x) = x$

$$= xe^x - \int e^x\,dx = \boxed{xe^x - e^x + c}$$

ATTENZIONE

$$\int x\cdot e^x\,dx = \int \left(\frac{1}{2}x^2\right)' e^x\,dx = \frac{1}{2}x^2\cdot e^x - \int \frac{1}{2}x^2\cdot e^x\,dx =$$

.....

e ho peggiorato la situazione!!

Bisogna applicare la formula in modo opportuno!!

407 $\int 2x\ln x\, dx =$ $\left[x^2\left(\ln x - \frac{1}{2}\right) + c\right]$

$$= \int (x^2)' \cdot \ln x\, dx = x^2 \ln x - \int x^2 \cdot \underbrace{\frac{1}{x}}_{(\ln x)'} dx =$$

$$= x^2 \ln x - \int x\, dx = x^2 \ln x - \frac{1}{2} x^2 + c = x^2 \left(\ln x - \frac{1}{2}\right) + c$$

ALTRO ESEMPIO

$$\int \ln x\, dx = \int 1 \cdot \ln x\, dx = \int (x)' \ln x\, dx =$$

$$= x \ln x - \int x \cdot \frac{1}{x}\, dx = x \ln x - x + c$$

417 $\int (x+2)\sin x\, dx =$ $[-(x+2)\cos x + \sin x + c]$

$$= \int (x+2)(-\cos x)'\, dx = (x+2)(-\cos x) - \int (x+2)'(-\cos x)\, dx$$

$$= -(x+2)\cos x + \int \cos x\, dx = -(x+2)\cos x + \sin x + c$$

$$I = \int \sin^2 x \, dx = \int \sin x \cdot \sin x \, dx = \int \sin x \cdot (-\cos x)' \, dx =$$

$$= \sin x \cdot (-\cos x) - \int (\sin x)' \cdot (-\cos x) \, dx =$$

$$= -\sin x \cos x + \int \cos^2 x \, dx = -\sin x \cos x + \int (1 - \sin^2 x) \, dx =$$

$$= -\sin x \cos x + \int dx - \int \sin^2 x \, dx =$$

$$= -\sin x \cos x + x - \underbrace{\int \sin^2 x \, dx}_{I}$$

$$I = -\sin x \cos x + x - I$$

$$2I = -\sin x \cos x + x + c$$

$$\Rightarrow \quad I = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + c$$

$$\int \sin^2 x \, dx = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + c$$

436 $\int \frac{\ln x^2}{x^2}\,dx =$ $\left[-\frac{1}{x}(2+\ln x^2)+c\right]$

$\ln x^2 = 2\ln|x|$

$$= 2\int \frac{1}{x^2}\cdot \ln|x|\,dx = 2\int \left(\frac{x^{-2+1}}{-2+1}\right)'\cdot \ln|x|\,dx =$$

$$= 2\left[-\frac{1}{x}\cdot \ln|x| - \int\left(-\frac{1}{x}\right)\cdot \underbrace{\frac{1}{x}}_{(\ln|x|)'}\,dx\right] =$$

$$= 2\left[-\frac{\ln|x|}{x} + \int x^{-2}\,dx\right] = 2\left[-\frac{\ln|x|}{x} + \frac{x^{-2+1}}{-2+1} + c\right] =$$

$$= -\frac{2\ln|x|}{x} - \frac{2}{x} + c = \boxed{-\frac{\ln x^2}{x} - \frac{2}{x} + c}$$

438 $\int 8x\sin x\cos x\,dx =$ $[-2x\cos 2x + \sin 2x + c]$

$\sin 2x = 2\sin x\cos x$

$$= \int 4x\cdot \sin 2x\,dx =$$

$$= \int 2x\cdot \underbrace{2\sin 2x}_{(-\cos 2x)'}\,dx = \int (2x)\cdot(-\cos 2x)'\,dx =$$

$$= 2x\cdot(-\cos 2x) - \int \overset{(2x)'}{2}\cdot(-\cos 2x)\,dx = -2x\cos 2x + \int \underbrace{2\cos 2x}_{(\sin 2x)'}\,dx$$

$$= \boxed{-2x\cos 2x + \sin 2x + c}$$