26/3/2021

INTEGRAZIONE PER SOSTITUZIONE

311 $\int \frac{\cos x \cdot e^{\sqrt{\sin x}}}{2\sqrt{\sin x}}\,dx =$

$t = \sqrt{\sin x}$

$t^2 = \sin x$

$x = \arcsin t^2 \qquad \frac{dx}{dt} = \frac{1}{\sqrt{1-t^4}} \cdot 2t \Rightarrow dx = \frac{2t}{\sqrt{1-t^4}}\,dt$

$\cos x = \cos(\arcsin t^2) = \sqrt{1-\sin^2(\arcsin t^2)} = \sqrt{1-t^4}$

$= \int \frac{\cancel{\sqrt{1-t^4}} \cdot e^t}{\cancel{2t}} \cdot \frac{\cancel{2t}}{\cancel{\sqrt{1-t^4}}}\,dt = \int e^t\,dt = e^t + c =$

$= e^{\sqrt{\sin x}} + c$

352 $\int \frac{6}{\sqrt{8-3x}}\,dx =$ $[-4\sqrt{8-3x}+c]$

$t = 8-3x \Rightarrow 3x = 8-t \Rightarrow x = \frac{8}{3} - \frac{t}{3}$

$\frac{dx}{dt} = -\frac{1}{3}$

$dx = -\frac{1}{3}\,dt$

$= \int \frac{\overset{2}{\cancel{6}}}{\sqrt{t}} \left(-\frac{1}{\cancel{3}}\right) dt =$

$= -2 \int t^{-\frac{1}{2}}\,dt =$

$= -2 \cdot \frac{1}{-\frac{1}{2}+1}\, t^{-\frac{1}{2}+1} + c = -2 \cdot \frac{1}{\frac{1}{2}}\, t^{\frac{1}{2}} + c =$

$= -4\sqrt{t} + c = \boxed{-4\sqrt{8-3x} + c}$

375 $\int \frac{e^x}{e^{2x}+1} dx; \quad t = e^x.$ $[\arctan e^x + c]$

$t = e^x \qquad x = \ln t$

$dx = \frac{1}{t}\, dt$

$$= \int \frac{\cancel{t}}{t^2+1} \cdot \frac{1}{\cancel{t}}\, dt =$$

$$= \int \frac{1}{t^2+1}\, dt = \arctan t + c = \boxed{\arctan e^x + c}$$

357 $\int \frac{e^x}{e^x - e^{-x}} dx =$ $\left[\frac{1}{2}\ln\left|e^{2x}-1\right| + c\right]$

$t = e^x \qquad x = \ln t$

$dx = \frac{1}{t}\, dt$

$$= \int \frac{\cancel{t}}{t - \frac{1}{t}} \cdot \frac{1}{\cancel{t}}\, dt =$$

$$= \int \frac{1}{\frac{t^2-1}{t}}\, dt = \int \frac{t}{t^2-1}\, dt = \frac{1}{2} \int \frac{2t}{t^2-1}\, dt =$$

$$= \frac{1}{2} \ln\left|t^2-1\right| + c = \frac{1}{2} \ln\left|e^{2x}-1\right| + c$$

382 $\int \frac{\sin x}{4+\cos^2 x}\,dx =$ $\left[-\frac{1}{2}\arctan\frac{\cos x}{2}+c\right]$

$$= \int \frac{\sin x}{4\left(1+\left(\frac{\cos x}{2}\right)^2\right)}\,dx =$$

$$= \int \frac{\cancel{\sqrt{1-4t^2}}}{\underset{2}{\cancel{4}}\left(1+t^2\right)}\left(-\frac{\cancel{2}}{\cancel{\sqrt{1-4t^2}}}\right)dt$$

$$= -\frac{1}{2}\int \frac{1}{1+t^2}\,dt =$$

$$= -\frac{1}{2}\arctan t + c =$$

$$= \boxed{-\frac{1}{2}\arctan\left(\frac{\cos x}{2}\right)+c}$$

$$t = \frac{\cos x}{2}$$

$$2t = \cos x$$

$$x = \arccos(2t)$$

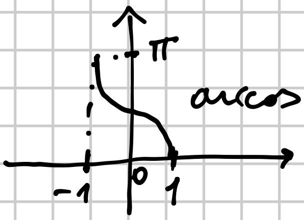
$$\frac{dx}{dt} = -\frac{1}{\sqrt{1-4t^2}}\cdot 2$$

$$dx = -\frac{2}{\sqrt{1-4t^2}}\,dt$$

$$\sin x = \sin(\arccos(2t)) =$$

$$= \sqrt{1-\cos^2(\arccos(2t))} =$$

$$= \sqrt{1-4t^2}$$



381 $\int \tan^3 x \, dx$ $=$ $\left[\frac{1}{2\cos^2 x} + \ln|\cos x| + c\right]$

1° TENTATIVO

$t = \tan x$

$x = \arctan t$

$dx = \frac{1}{1+t^2} dt$

$$= \int \frac{t^3}{1+t^2} dt = (*)$$

$$\frac{t^3}{1+t^2} = \frac{t^3+t-t}{1+t^2} = \frac{t^3+t}{1+t^2} - \frac{t}{1+t^2} =$$

$$= \frac{t\cancel{(t^2+1)}}{\cancel{1+t^2}} - \frac{t}{1+t^2}$$

$$(*) = \int t \, dt - \int \frac{t}{1+t^2} dt =$$

$$= \frac{1}{2}t^2 - \frac{1}{2}\int \frac{2t}{1+t^2} dt =$$

$$= \frac{1}{2}t^2 - \frac{1}{2}\ln(1+t^2) + c =$$

$$= \frac{1}{2}\tan^2 x - \frac{1}{2}\ln(1+\tan^2 x) + c$$

$$1+\tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

TENTATIVO OK!

384 $\int \frac{1}{\tan^3 x}\,dx =$ $\left[-\frac{1}{2\sin^2 x} - \ln|\sin x| + c\right]$

1° TENTATIVO

$t = \tan x$

$x = \arctan t$

$dx = \frac{1}{1+t^2}\,dt$

$$= \int \frac{1}{t^3} \cdot \frac{1}{1+t^2}\,dt = \int \frac{1}{t^3+t^5}\,dt$$

...... TENTATIVO FALLITO!

2° TENTATIVO

$\cot x = \frac{1}{\tan x}$

$t = \cot x$

$x = \operatorname{arccot} t$

$\frac{dx}{dt} = -\frac{1}{1+t^2}$

$dx = -\frac{1}{1+t^2}\,dt$

$$\int \frac{1}{\tan^3 x}\,dx = \int \cot^3 x\,dx =$$

$$= \int t^3 \left(-\frac{1}{1+t^2}\right) dt =$$

$$= -\int \frac{t^3}{1+t^2}\,dt = \ldots$$

e viene come prima (col segno cambiato)