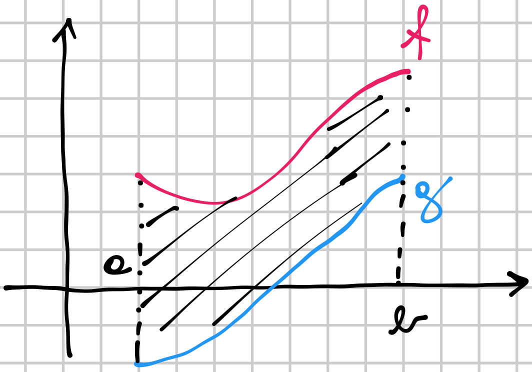
$f, g : [a, b] \to \mathbb{R}$ $\quad f \geq g$

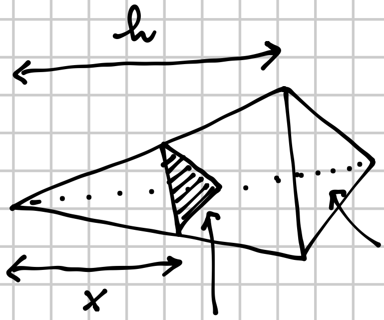



AREA DELLA REGIONE COMPRESA FRA I 2 GRAFICI

$$\downarrow$$

$$\int_a^b [f(x) - g(x)]\, dx$$

VOLUME DELLA PIRAMIDE DI BASE $B$ E ALTEZZA $h$




$B$ = AREA DI BASE

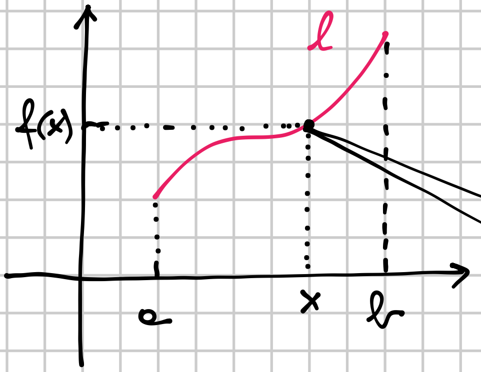
$S(x)$ = AREA DELLA SEZIONE CHE DISTA $x$ DAL VERTICE

$$S(x) : B = x^2 : h^2 \implies S(x) = \frac{B \cdot x^2}{h^2}$$

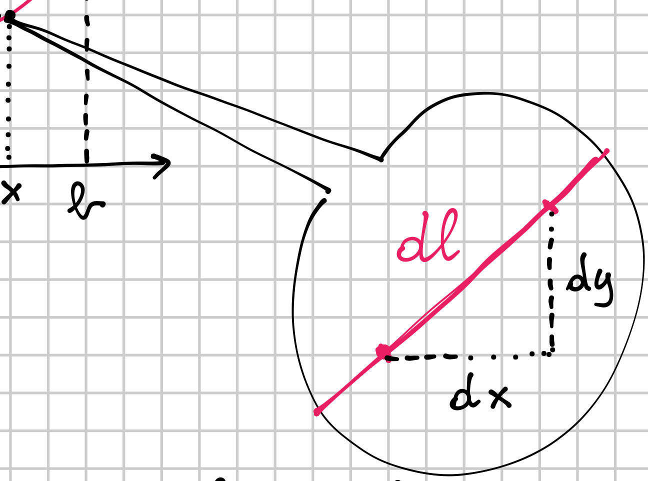
$$V_{\text{PIRAMIDE}} = \int_0^h S(x)\, dx = \int_0^h \frac{B \cdot x^2}{h^2}\, dx = \frac{B}{h^2} \int_0^h \left(\frac{1}{3} x^3\right)' dx =$$

$$= \frac{B}{h^2} \left[\frac{1}{3} x^3\right]_0^h = \frac{B}{\cancel{h^2}} \cdot \frac{1}{3} h^{\cancel{3}} = \boxed{\frac{1}{3} B h}$$

# LUNGHEZZA DI UNA CURVA $y = f(x)$




$f : [a, b] \to \mathbb{R}$ continua



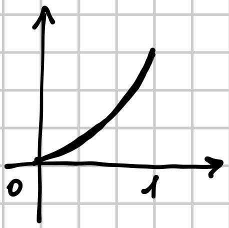

$$dl = \sqrt{dx^2 + dy^2}$$

LUNGHEZZA $\quad l = \int_0^l dl = \int_a^b \sqrt{dx^2 + dy^2} = \int_a^b dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2} =$

$$= \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

## ESEMPIO

Calcolare la lunghezza della parabola $y = \frac{1}{2}x^2$ tra 0 e 1




$f : [0, 1] \to \mathbb{R} \quad f(x) = \frac{1}{2}x^2 \quad f'(x) = x$

$$l = \int_0^1 \sqrt{1 + x^2}\, dx = \ldots$$

calcolo l'integrale indefinito:

$$\int \sqrt{1+x^2}\, dx = \int (x)' \sqrt{1+x^2}\, dx = x\sqrt{1+x^2} - \int x \cdot \frac{1}{\cancel{2}\sqrt{1+x^2}} \cdot \cancel{2}x\, dx$$

$$= x\sqrt{1+x^2} - \int \frac{x^2}{\sqrt{1+x^2}}\, dx = x\sqrt{1+x^2} - \int \frac{x^2+1-1}{\sqrt{1+x^2}}\, dx =$$

$$= x\sqrt{1+x^2} - \int \frac{x^2+1}{\sqrt{1+x^2}}\,dx + \int \frac{1}{\sqrt{1+x^2}}\,dx =$$

$$= x\sqrt{1+x^2} - \int \sqrt{1+x^2}\,dx + \ln\left(x+\sqrt{1+x^2}\right)$$

$$\int \sqrt{1+x^2}\,dx = x\sqrt{1+x^2} - \int \sqrt{1+x^2}\,dx + \ln\left(x+\sqrt{1+x^2}\right)$$

$$2\int \sqrt{1+x^2}\,dx = x\sqrt{1+x^2} + \ln\left(x+\sqrt{1+x^2}\right) + c$$

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right) + c$$

$$\ell = \int_0^1 \sqrt{1+x^2}\,dx = \left[\frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right)\right]_0^1 =$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2}\ln\left(1+\sqrt{2}\right)$$

DA VEDERE: $\int \frac{1}{\sqrt{1+x^2}}\,dx = \ln\left(x+\sqrt{1+x^2}\right) + c$

Moltiplicando e dividendo per $\sqrt{1+x^2}+x$ si ha:

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}+x}\,dx =$$

$$= \int \left(1+\frac{x}{\sqrt{1+x^2}}\right) \cdot \frac{1}{\sqrt{1+x^2}+x}\,dx = \int \frac{1+\frac{2x}{2\sqrt{1+x^2}}}{\sqrt{1+x^2}+x}\,dx =$$

$$= \ln\left(\sqrt{1+x^2}+x\right) + c$$