

Trasforma l'espressione in funzione soltanto di $\tan \alpha$, sapendo che $\frac{\pi}{2} < \alpha < \pi$.

$$\frac{\sin^2 \alpha + \cot \alpha - 1}{2 \cot^2 \alpha + \cos^2 \alpha}$$

$$\left[\frac{\tan \alpha (\tan^2 \alpha - \tan \alpha + 1)}{3 \tan^2 \alpha + 2} \right]$$

$$\tan^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} \Rightarrow \tan^2 \alpha = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha} - 1$$

$$\Rightarrow \tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \Rightarrow \boxed{\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha}}$$

$$\tan^2 \alpha = \frac{\sin^2 \alpha}{1 - \sin^2 \alpha} \Rightarrow \frac{1}{\tan^2 \alpha} = \frac{1 - \sin^2 \alpha}{\sin^2 \alpha} = \frac{1}{\sin^2 \alpha} - 1$$

$$\frac{1}{\sin^2 \alpha} = \frac{1}{\tan^2 \alpha} + 1 \Rightarrow \frac{1}{\sin^2 \alpha} = \frac{1 + \tan^2 \alpha}{\tan^2 \alpha}$$

$$\Rightarrow \boxed{\sin^2 \alpha = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}}$$

$$\frac{\sin^2 \alpha + \cot \alpha - 1}{2 \cot^2 \alpha + \cos^2 \alpha} = \frac{\sin^2 \alpha + \frac{1}{\tan \alpha} - 1}{2 \frac{1}{\tan^2 \alpha} + \cos^2 \alpha} = \frac{\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} + \frac{1}{\tan \alpha} - 1}{\frac{2}{\tan^2 \alpha} + \frac{1}{1 + \tan^2 \alpha}} =$$

$$= \frac{\frac{\tan^3 \alpha + 1 + \tan^2 \alpha - (1 + \tan^2 \alpha) \tan \alpha}{(1 + \tan^2 \alpha) \tan \alpha}}{\frac{2(1 + \tan^2 \alpha) + \tan^2 \alpha}{(1 + \tan^2 \alpha) \tan^2 \alpha}} = \frac{\cancel{\tan^3 \alpha} + 1 + \tan^2 \alpha - \tan \alpha - \cancel{\tan^3 \alpha}}{\frac{2 + 2 \tan^2 \alpha + \tan^2 \alpha}{\tan \alpha}} =$$

$$= \frac{(1 + \tan^2 \alpha - \tan \alpha) \tan \alpha}{2 + 3 \tan^2 \alpha}$$

293

$$\frac{\cot \alpha}{\sin \alpha} + \csc \alpha - \frac{\csc \alpha}{\tan \alpha} =$$

$$\left[\frac{1}{\sin \alpha} \right]$$

$$= \frac{\cancel{\cos \alpha}}{\cancel{\sin^2 \alpha}} + \frac{1}{\sin \alpha} - \frac{1}{\cancel{\sin \alpha}} \cdot \frac{\cancel{\cos \alpha}}{\cancel{\sin \alpha}} = \frac{1}{\sin \alpha}$$

414

$$\sin(\alpha - 2\pi) \cot(-\alpha) + \sin\left(\alpha + \frac{3}{2}\pi\right) \cos(\alpha + 5\pi) + \sin^2(6\pi - \alpha) =$$

$$[1 - \cos \alpha]$$

$$= \sin \alpha \cdot (-\cot \alpha) + \sin\left(\alpha + 2\pi - \frac{\pi}{2}\right) \cos(\alpha + \pi + 4\pi) + \sin^2(-\alpha) =$$

$$= \cancel{\sin \alpha} \cdot \left(-\frac{\cancel{\cos \alpha}}{\cancel{\sin \alpha}}\right) + \sin\left(\alpha - \frac{\pi}{2}\right) \cdot \cos(\alpha + \pi) + [-\sin \alpha]^2 =$$

$$= -\cos \alpha + \sin\left(-\left(\frac{\pi}{2} - \alpha\right)\right) \cdot (-\cos \alpha) + \sin^2 \alpha =$$

$$= -\cos \alpha - \sin\left(\frac{\pi}{2} - \alpha\right) \cdot (-\cos \alpha) + \sin^2 \alpha =$$

$$= -\cos \alpha - \cos \alpha \cdot (-\cos \alpha) + \sin^2 \alpha =$$

$$= -\cos \alpha + \underbrace{\cos^2 \alpha + \sin^2 \alpha}_1 = \boxed{1 - \cos \alpha}$$