513 $\lim_{x \to 0} \frac{2x^2 \sin^2 x}{\ln(1+4x^4)} = \frac{0}{0}$ F.I.

$$= \lim_{x \to 0} \frac{2x^2 \cdot \sin^2 x}{\ln(1+4x^4)} \cdot \frac{x^2}{x^2} \cdot \frac{2}{2} =$$

$$= \lim_{x \to 0} \underbrace{\frac{\sin^2 x}{x^2}}_{1^2} \frac{4x^4}{2\ln(1+4x^4)} = \frac{1}{2}$$

523 $\lim_{x \to 0} \frac{1-\cos^2 2x}{x\sin x} = \frac{0}{0}$ F.I.

$$= \lim_{x \to 0} \frac{\sin^2 2x}{x \sin x} = \lim_{x \to 0} \frac{(2\sin x \cos x)^2}{x \sin x} =$$

$$= \lim_{x \to 0} \frac{4 \sin^{\cancel{2}} x \cos^2 x}{x \cancel{\sin x}} = \lim_{x \to 0} \frac{4 \sin x \cos^2 x}{x} = 4$$

ALTRO MODO:

$$\lim_{x \to 0} \frac{(1-\cos 2x)(1+\cos 2x)}{x \sin x} \cdot \frac{4x}{4x} = \lim_{x \to 0} \underbrace{\frac{1-\cos 2x}{(2x)^2}}_{\frac{1}{2}} \frac{1+\cos 2x}{\sin x} \cdot 4x =$$

$$= \frac{1}{2} \cdot \frac{1+1}{1} \cdot 4 = 4$$

544 $\lim_{x \to 0} \frac{e^x - e^{-x}}{\ln(1+x)} = \frac{0}{0}$ F. I.

$$= \lim_{x \to 0} \frac{e^x - e^{-x}}{\ln(1+x)} \cdot \frac{x}{x} = \lim_{x \to 0} \frac{x}{\ln(1+x)} \cdot \frac{e^x - 1 + 1 - e^{-x}}{x} =$$

$$= \lim_{x \to 0} \underbrace{\frac{x}{\ln(1+x)}}_{1} \cdot \left(\underbrace{\frac{e^x - 1}{x}}_{1} + \underbrace{\frac{e^{-x} - 1}{-x}}_{1} \right) = 1 \cdot (1 + 1) = 2$$

527 $\lim_{x \to 0} \frac{\cos 6x - \cos 3x}{x^2} = \frac{0}{0}$ F. I.

$$= \lim_{x \to 0} \frac{\cos 6x - 1 + 1 - \cos 3x}{x^2} =$$

$$= \lim_{x \to 0} \left(\frac{\cos 6x - 1}{x^2} + \frac{1 - \cos 3x}{x^2} \right) =$$

$$= \lim_{x \to 0} \left(-\frac{1 - \cos 6x}{x^2} \cdot \frac{36}{36} + \frac{1 - \cos 3x}{x^2} \cdot \frac{9}{9} \right) =$$

$$= \lim_{x \to 0} \left(- \underbrace{\frac{1 - \cos 6x}{(6x)^2}}_{\frac{1}{2}} \cdot 36 + \underbrace{\frac{1 - \cos 3x}{(3x)^2}}_{\frac{1}{2}} \cdot 9 \right) =$$

$$= -\frac{1}{2} \cdot 36 + \frac{1}{2} \cdot 9 = -18 + \frac{9}{2} = \frac{-36 + 9}{2} = \boxed{-\frac{27}{2}}$$

519 $\lim_{x \to 0} \frac{e^{5x} - \ln[e(x+1)]}{x} = \frac{0}{0}$ F.I.

$\ln(a \cdot b) = \ln a + \ln b$

$$= \lim_{x \to 0} \frac{e^{5x} - (\overbrace{\ln e}^{1} + \ln(x+1))}{x} =$$

$$= \lim_{x \to 0} \frac{e^{5x} - 1 - \ln(x+1)}{x} =$$

$$= \lim_{x \to 0} \left[\underbrace{\frac{e^{5x} - 1}{x} \cdot \frac{5}{5}}_{\to 1} - \underbrace{\frac{\ln(x+1)}{x}}_{\to 1} \right] = 5 - 1 = 4$$

529 $\lim_{x \to 0} (1 - \sin x)^{\frac{\cos x}{x}} = 1^{\infty}$ F.I.

$[f(x)]^{g(x)} = e^{g(x) \ln f(x)}$

$$= \lim_{x \to 0} e^{\frac{\cos x}{x} \ln(1 - \sin x)} = \ldots = e^{-1}$$

A PARTE

$$\lim_{x \to 0} \frac{\cos x}{x} \ln(1 - \sin x) = \lim_{x \to 0} \frac{\cos x}{x} \ln(1 - \sin x) \cdot \frac{-\sin x}{-\sin x} =$$

$$= \lim_{x \to 0} \underbrace{\frac{\ln(1 - \sin x)}{-\sin x}}_{\to 1} \cdot \underbrace{\frac{-\sin x}{x}}_{\to -1} \cdot \underbrace{\cos x}_{\to 1} = -1$$

548 $\lim_{x \to 0^+} e^{-\frac{1}{x}} (e + 2x)^{\frac{1}{x}} = 0 \cdot \infty$ F.I.

$$= \lim_{x \to 0^+} \left[e^{-1} (e + 2x) \right]^{\frac{1}{x}} = \lim_{x \to 0^+} \left(1 + \frac{2}{e} x \right)^{\frac{1}{x}} =$$

$$= \lim_{t \to +\infty} \left(1 + \frac{1}{t} \right)^{\frac{2}{e} t} = \lim_{t \to +\infty} \left[\left(1 + \frac{1}{t} \right)^{t} \right]^{\frac{2}{e}} =$$

$$= e^{\frac{2}{e}}$$

$\frac{1}{x} = \frac{2}{e} t$

$\Uparrow$

$\frac{2x}{e} = \frac{1}{t}$

$t = \frac{e}{2x}$

$x \to 0^+ \Rightarrow t \to +\infty$

DEFINIZIONE GENERALE DI LIMITE

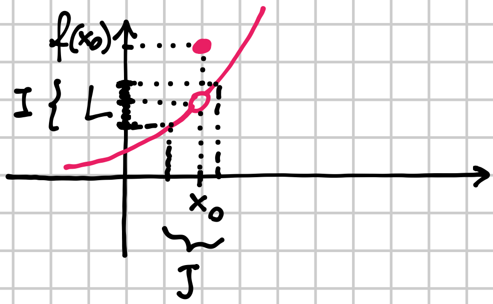
Siano $x_0, L \in \overline{\mathbb{R}} = [-\infty, +\infty]$ $\qquad f: A \to \mathbb{R} \qquad A \subseteq \mathbb{R}$

x_0 di accumulazione per A

$$\lim_{x \to x_0} f(x) = L$$

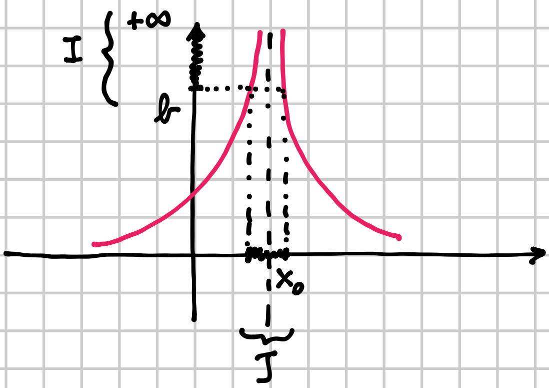
se e solo se

per ogni intorno I di L esiste un intorno J di x_0 tale che per ogni $x \in A \cap J \setminus \{x_0\}$ si abbia $f(x) \in I$



caso L finito

$I = (b, +\infty)$



caso L infinito

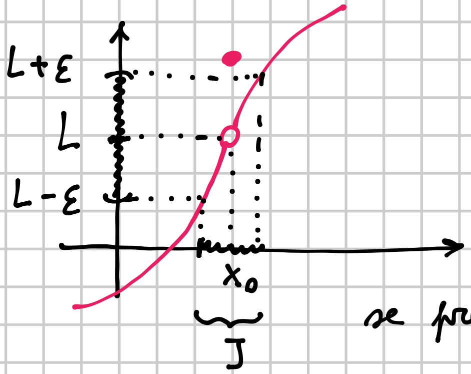
$$\lim_{x \to x_0} f(x) = +\infty$$

Questa definizione di limite comprende tutti i casi possibili (x_0, L possono essere finiti o infiniti)

Caso x_0, L finiti $\qquad f: A \to \mathbb{R} \quad x_0$ di acc. per A

$$\lim_{x \to x_0} f(x) = L$$

$\forall \varepsilon > 0 \quad \exists J$ intorno di $x_0 \quad : \quad \forall x \in A \cap J \setminus \{x_0\} \quad |f(x) - L| < \varepsilon$



intorno I di L

se prendo x in $J \cap A \setminus \{x_0\}$, ho che $f(x) \in (L - \varepsilon, L + \varepsilon)$

$$\Updownarrow$$

$$L - \varepsilon < f(x) < L + \varepsilon$$

$$\Updownarrow$$

$$-\varepsilon < f(x) - L < \varepsilon$$

$$\Updownarrow$$

$$|f(x) - L| < \varepsilon$$

107 $\lim_{x \to 1} (2 - 3x) = -1$

Usando la definizione, verificare questo limite

$\forall \varepsilon > 0 \quad \exists J$ intorno di $\underset{x_0}{\overset{}{1}}$ $: \forall x \in \underbrace{J \setminus \{1\}}_{A=\mathbb{R}} \quad |f(x) - \underbrace{(-1)}_{L = \text{limite}}| < \varepsilon$

Dato $\varepsilon > 0$ devo trovare questo intorno J tale per cui $|f(x) + 1| < \varepsilon$ per ogni x in $J \setminus \{1\}$.

ε è un numero dato "piccolo"

parto da $|f(x) + 1| < \varepsilon$

$$|2 - 3x + 1| < \varepsilon \qquad |3 - 3x| < \varepsilon$$

$$-\varepsilon < 3 - 3x < \varepsilon$$

$$-3 - \varepsilon < -3x < -3 + \varepsilon$$

$$\frac{-3 - \varepsilon}{-3} > x > \frac{-3 + \varepsilon}{-3}$$

$$1 + \frac{\varepsilon}{3} > x > 1 - \frac{\varepsilon}{3}$$

$$1 - \frac{\varepsilon}{3} < x < 1 + \frac{\varepsilon}{3} \quad \Leftarrow$$ ho trovato l'intorno J di 1

$$J = \left(1 - \frac{\varepsilon}{3}, 1 + \frac{\varepsilon}{3}\right)$$

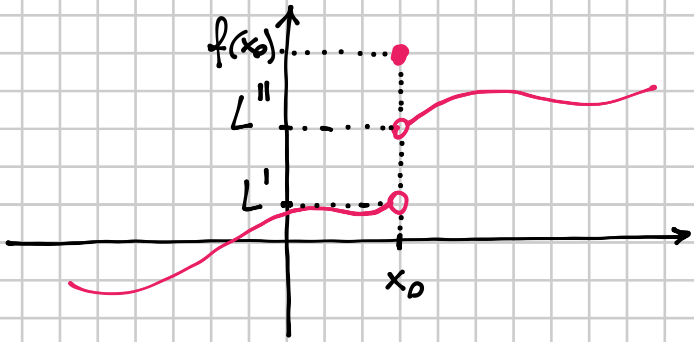
Se prendo un qualsiasi x in J (tranne $x_0 = 1$) ho che $|f(x) - (-1)| < \varepsilon$

LIMITE DESTRO E SINISTRO

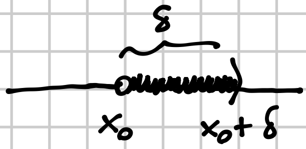
IDEA

$$\lim_{x \to x_0^-} f(x) = L'$$

$$\lim_{x \to x_0^+} f(x) = L''$$

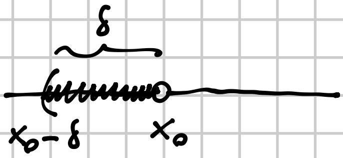


Dato $x_0 \in \mathbb{R}$, un INTORNO DESTRO di x_0 è un intervallo $(x_0, x_0+\delta)$ con $\delta > 0$



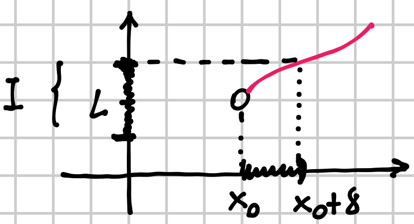
Analogamente, un INTORNO SINISTRO di x_0 è un intervallo $(x_0-\delta, x_0)$ con $\delta > 0$

x_0 è già escluso!



$f: A \to \mathbb{R}$ $\quad$ x_0 di accumulazione per A

$\lim_{x \to x_0^+} f(x) = L \iff$ Per ogni I intorno di L esiste J INTORNO DESTRO di x_0 tale che $\forall x \in J \cap A$ si abbia $f(x) \in I$

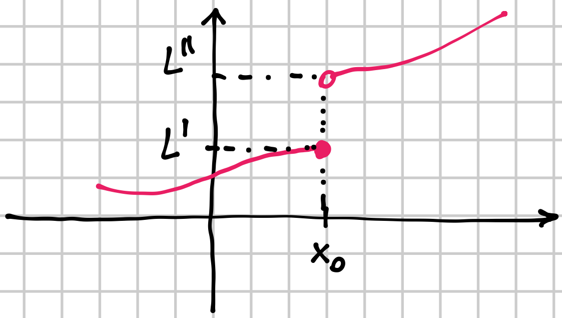


MI LIMITO A PRENDERE x NELL'INTORNO DESTRO

$\lim_{x \to x_0^-} f(x) = L \iff$ Per ogni I intorno di L esiste J INTORNO SINISTRO di x_0 tale che $\forall x \in J \cap A$ si abbia $f(x) \in J$

TEOREMA

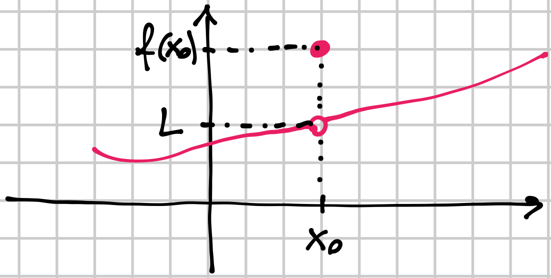
$$\lim_{x \to x_0} f(x) = L \quad \Longleftrightarrow \quad \begin{cases} \lim_{x \to x_0^+} f(x) = L \\ \text{e} \\ \lim_{x \to x_0^-} f(x) = L \end{cases}$$



$$\lim_{x \to x_0^+} f(x) = L''$$

$$\lim_{x \to x_0^-} f(x) = L'$$

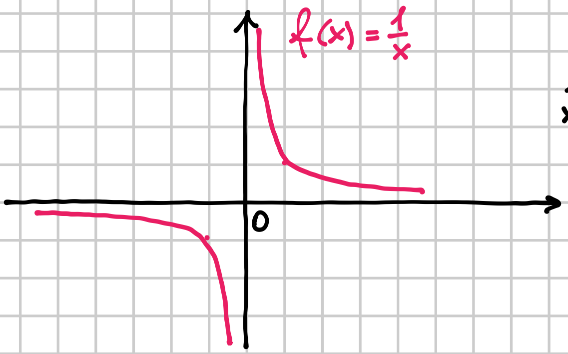
$\lim_{x \to x_0} f(x)$ NON ESISTE



$$\lim_{x \to x_0^-} f(x) = L = \lim_{x \to x_0^+} f(x)$$

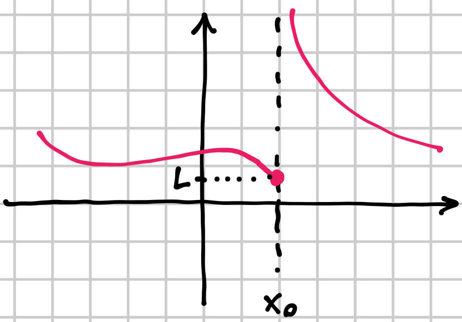
$$\Updownarrow$$

$$\lim_{x \to x_0} f(x) = L$$



$$\lim_{x \to 0^+} \frac{1}{x} = +\infty \qquad \lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$\lim_{x \to 0} \frac{1}{x}$ NON ESISTE oppure $\lim_{x \to 0} \frac{1}{x} = \infty$



$$\lim_{x \to x_0^+} f(x) = +\infty \qquad \lim_{x \to x_0^-} f(x) = L$$

$\lim_{x \to x_0} f(x)$ NON ESISTE