7/2/2019

Calcolare la derivata di

1) $y = \frac{3}{5}x^4 - 5x^2 + 8x$ $\qquad y' = \ldots$

2) $y = -4x^{10} + 7x^5 - \frac{3}{13}x^2$

3) Calcolare la derivata in $x_0 = -6$ della funzione

$$f(x) = 2x^{14} - 30x^{11} + 2x$$

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1) $y' = \frac{12}{5}x^3 - 10x + 8$

2) $y' = -40x^9 + 35x^4 - \frac{6}{13}x$

3) $f'(x) = 2 \cdot 14x^{13} - 30 \cdot 11x^{10} + 2 =$

$= 28x^{13} - 330x^{10} + 2$

$f'(-6) = 28(-6)^{13} - 330(-6)^{10} + 2 =$

$\simeq -3{,}85653 \times 10^{11}$

DERIVATE FONDAMENTALI

1) $y = x^n \qquad y' = n x^{n-1} \qquad (n \geq 1,\ n \in \mathbb{N})$

2) $y = e^x \qquad y' = e^x$

$$e = \lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n$$

COSTANTE DI NEPERO $\simeq 2{,}718$

3) $y = \cos x \qquad y' = -\sin x$

4) $y = \sin x \qquad y' = \cos x$

ALCUNE DIMOSTRAZIONI

1) $$\frac{\Delta y}{\Delta x} = \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

$$(x+\Delta x)^n = \overbrace{(x+\Delta x)\cdot(x+\Delta x)\cdot(x+\Delta x)\cdot \ldots \cdot(x+\Delta x)}^{n \text{ FATTORI}} =$$

$= x^n + n \cdot \Delta x \cdot x^{n-1}$ + ALTRI TERMINI CHE CONTENGONO (SONO MOLTIPLICATI PER) POTENZE DI Δx CON ESPONENTE ≥ 2

$$\Delta y = (x + \Delta x)^n - x^n = \cancel{x^n} + n x^{n-1} \cdot \Delta x + \text{ALTRI}\ldots \cancel{- x^n}$$

$$\frac{\Delta y}{\Delta x} = \frac{n x^{n-1} \cdot \Delta x + \text{ALTRI}\ldots}{\Delta x} = n x^{n-1} + \frac{\text{ALTRI}\ldots}{\Delta x}$$

→ TERMINI MOLTIPLICATI PER POTENZE DI Δx

→ 0 TENDE A 0 PER $\Delta x \to 0$

$$\boxed{f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = n x^{n-1}}$$

3) $f(x)=\cos x$

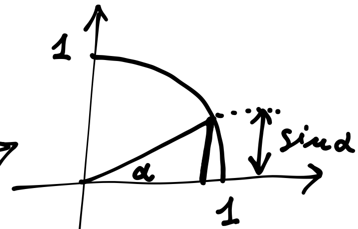
$$\frac{\Delta y}{\Delta x} = \frac{\cos(x+\Delta x)-\cos(x)}{\Delta x}$$

$$\cos(x+\Delta x) = \cos(x)\cdot\cos(\Delta x) - \sin(x)\cdot\sin(\Delta x)$$

$$\frac{\Delta y}{\Delta x} = \frac{\overbrace{\cos x\cos\Delta x - \sin x\cdot\sin\Delta x}^{\cos(x+\Delta x)} - \cos x}{\Delta x} =$$

$$= \cos x\,\underbrace{\frac{(\cos\Delta x - 1)}{\Delta x}}_{\downarrow\ 0} - \sin x\cdot\underbrace{\frac{\sin\Delta x}{\Delta x}}_{\downarrow\ 1}$$

per $\Delta x \to 0$



α = LUNGHEZZA DELL'ARCO

PER PICCOLI ANGOLI α (IN RADIANTI)

$$\sin\alpha \simeq \alpha$$

$$\Downarrow$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\frac{\cos\Delta x - 1}{\Delta x}\cdot\frac{\cos\Delta x + 1}{\cos\Delta x + 1} = \frac{\cos^2\Delta x - 1}{\Delta x(\cos\Delta x + 1)} =$$

$$= \frac{-\sin^2\Delta x}{\Delta x(\cos\Delta x + 1)} = -\overbrace{\frac{\sin\Delta x}{\Delta x}}^{\to 1}\cdot\frac{\overbrace{\sin\Delta x}^{\to 0}}{\underbrace{\underbrace{\cos\Delta x}_{\to 1} + 1}_{\to 2}} \xrightarrow[\Delta x\to 0]{} -1\cdot\frac{0}{2} = 0$$