

[fascio improprio;  $3x + 2y - 2 = 0$ ]

$$3x + 2y - 1 + K(6x + 4y + 3) = 0 \text{ eq. locus}$$

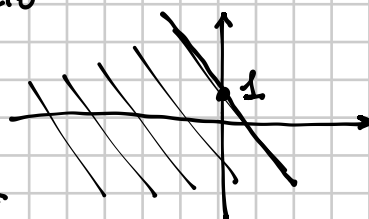
$$\begin{cases} 3x + 2y - 1 = 0 \\ 6x + 4y + 3 = 0 \end{cases} \quad \text{SIST. IMPOSSIBILE}$$

$$\times 2 \begin{cases} 6x + 4y - 2 = 0 \\ -6x + 4y + 3 = 0 \end{cases} \quad \downarrow \text{ sottraggendo}$$


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$$\quad \quad \quad // \quad // \quad -5 = 0 \quad \text{M.P.}$$

le 2 rette sono parallele  $\Rightarrow$  FASCIO IMPROPRIO



$P(0,1) \Rightarrow$  sostituisco nell'eq. del fascio e trovo  $K$  che "va bene"

$$3 \cdot 0 + 2 \cdot 1 - 1 + K(6 \cdot 0 + 4 \cdot 1 + 3) = 0$$

$$2 - 1 + K(4 + 3) = 0 \quad K = -\frac{1}{7}$$

$$3x + 2y - 1 - \frac{1}{7}(6x + 4y + 3) = 0$$

$$21x + 14y - 7 - 6x - 4y - 3 = 0$$

$$15x + 10y - 10 = 0$$

$$3x + 2y - z = 0$$

ALTERNATIVA  $\Rightarrow$  calcolo il coeff. angolare del fascio  $m = -\frac{3}{2}$ , so che l'intercetta è 1

$$y = -\frac{3}{2}x + 1$$

$$(3-k)x + (k+1)y + 4k - 8 = 0$$

STUDIARE

IL FASCIO

- ↓
- proprio/improprio
  - centro eventuale

$$3x - kx + ky + y + 4k - 8 = 0$$

$$\underbrace{3x + y - 8}_{1^a \text{ generatrice}} + k \underbrace{(-x + y + 4)}_{2^a \text{ generatrice (ESCLUSA)}} = 0$$

⇓

$$3x + y - 8 = 0$$

⇓

$$-x + y + 4 = 0$$

FASCIO PROPRIO

$$\begin{cases} 3x + y - 8 = 0 \\ -x + y + 4 = 0 \end{cases} \quad \begin{cases} y = -3x + 8 \\ -x - 3x + 8 + 4 = 0 \end{cases} \quad \begin{cases} // \\ -4x = -12 \end{cases} \quad \begin{cases} y = -1 \\ x = 3 \end{cases}$$

C(3, -1) CENTRO

Studia il fascio di rette di equazione  $(k+1)x + (2-3k)y - 7 + 3k = 0$  e determina:

- le rette parallele agli assi cartesiani;
- la retta del fascio parallela alla retta di equazione  $y = x - 3$ ;
- la retta passante per il punto  $A(4; 1)$ ;
- le rette che hanno distanza dall'origine uguale a  $\frac{4}{5}\sqrt{5}$ .

[a)  $y = 2$ ; x = 3; b)  $x - y - 1 = 0$ ; c)  $x + y - 5 = 0$ ; d)  $2x - y - 4 = 0$ ,  $2x - 29y + 52 = 0$ ]

$$Kx + x + 2y - 3Ky - 7 + 3K = 0$$

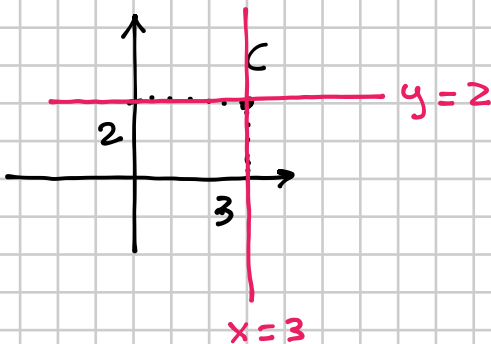
$$x + 2y - 7 + K(x - 3y + 3) = 0$$

$$1^a \text{ gen. } x + 2y - 7 = 0$$

$$2^a \text{ gen. } x - 3y + 3 = 0 \quad (\text{esclusa})$$

$$\begin{cases} x + 2y - 7 = 0 \\ x - 3y + 3 = 0 \end{cases} \quad \begin{cases} x = 7 - 2y \\ 7 - 2y - 3y + 3 = 0 \end{cases} \quad \begin{cases} x = 3 \\ y = 2 \end{cases} \quad C(3, 2)$$

a) // asse x  $\Rightarrow$  coeff. di x deve essere 0  $\Rightarrow k+1=0 \quad k=-1$



$$(k+1)x + (2-3k)y - 7 + 3k = 0$$

$$5y - 10 = 0 \Rightarrow \boxed{y=2}$$

// asse y  $\Rightarrow$  coeff. di y deve essere 0  $\Rightarrow 2-3k=0 \quad k=\frac{2}{3}$

$\downarrow$   
ret. nel fascio

$$\left(\frac{2}{3}+1\right)x - 7 + 2 = 0$$

$$\frac{5}{3}x - 5 = 0 \Rightarrow \boxed{x=3}$$

Usando il centro  $C(3, 2)$  trovo subito

le 2 rette parallele agli assi  $x=3$   
 $y=2$

$$b) (k+1)x + (2-3k)y - 7 + 3k = 0$$

$$y = x - 3$$

$$\Downarrow$$

$$m = -\frac{k+1}{2-3k}$$

$$\Downarrow$$

$$m' = 1$$

$$m = m'$$

$$-\frac{k+1}{2-3k} = 1$$

$$-k-1 = 2-3k$$

$$2k = 3 \Rightarrow k = \frac{3}{2}$$

$$\left(\frac{3}{2} + 1\right)x + \left(2 - 3 \cdot \frac{3}{2}\right)y - 7 + 3 \cdot \frac{3}{2} = 0$$

$$\frac{5}{2}x - \frac{5}{2}y - \frac{5}{2} = 0 \Rightarrow \boxed{x - y - 1 = 0}$$

c) retta per C(3,2) e P(4,1)

$$\frac{y-2}{1-2} = \frac{x-3}{4-3}$$

$$-(y-2) = x-3$$

$$-y+2 = x-3$$

$$\boxed{x + y - 5 = 0}$$

d) la retta con distanza da O uguale a  $\frac{4}{5}\sqrt{5}$

$$(k+1)x + (2-3k)y - 7 + 3k = 0$$

$$\frac{|(k+1) \cdot 0 + (2-3k) \cdot 0 - 7 + 3k|}{\sqrt{(k+1)^2 + (2-3k)^2}} = \frac{4}{5}\sqrt{5}$$

$$|3k-7| = \frac{4}{5}\sqrt{5} \sqrt{(k+1)^2 + (2-3k)^2}$$

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$$(3k-7)^2 = \frac{16}{25} \cdot 5 \left[ (k+1)^2 + (2-3k)^2 \right]$$

$$5(9k^2 + 49 - 42k) = 16(k^2 + 1 + 2k + 4 + 9k^2 - 12k)$$

$$45k^2 + 245 - 210k = 16(10k^2 - 10k + 5)$$

$$45k^2 + 245 - 210k = 160k^2 - 160k + 80$$

$$115k^2 + 50k - 165 = 0$$

$$23k^2 + 10k - 33 = 0$$

$$\frac{\Delta}{4} = 25 + 33 \cdot 23 = 784 = 28^2$$

$$k = \frac{-5 \pm 28}{23} = \begin{cases} -\frac{33}{23} \\ 1 \end{cases}$$

$$(k+1)x + (2-3k)y - 7 + 3k = 0 \quad \text{FASCIO}$$

$$k=1 \Rightarrow (1+1)x + (2-3 \cdot 1)y - 7 + 3 \cdot 1 = 0$$

$$2x - y - 4 = 0$$

$$k = -\frac{33}{23} \Rightarrow \left(-\frac{33}{23} + 1\right)x + \left(2 - 3 \cdot \left(-\frac{33}{23}\right)\right)y - 7 + 3 \cdot \left(-\frac{33}{23}\right) = 0$$

$$-\frac{10}{23}x + \left(2 + \frac{99}{23}\right)y - 7 - \frac{99}{23} = 0$$

$$-\frac{10}{23}x + \frac{145}{23}y - \frac{260}{23} = 0$$

$$10x - 145y + 260 = 0$$

$$2x - 29y + 52 = 0$$