4/5/2018

412 $\int\sqrt{9-x^2}\,dx$ $\quad\left[\frac{9}{2}\arcsin\frac{x}{3}+\frac{x}{2}\sqrt{9-x^2}+c\right]$

DOMINIO DI $\sqrt{9-x^2}$

È $[-3,3]$ cioè $-3\le x\le 3$

SOSTITUZIONE

$x=3\sin t \; : \left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to[-3,3]$

È BIETTIVA E INVERTIBILE

$$\int\sqrt{9-x^2}\,dx=\int\sqrt{9-9\sin^2 t}\cdot 3\cos t\,dt=$$

$dx=3\cos t\,dt$

$$=3\int\sqrt{9(1-\sin^2 t)}\cos t\,dt$$

$$=9\int\sqrt{1-\sin^2 t}\cos t\,dt=$$

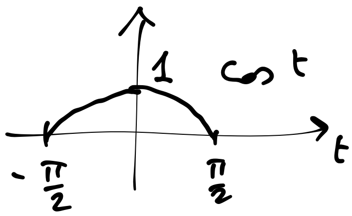
$\sqrt{1-\sin^2 t}=\cos t$

perché $t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

e dunque $\cos t\geq 0$

$$=9\int\cos t\cdot\cos t\,dt=$$

$$=9\int\cos^2 t\,dt=9\int\left[\frac{1}{2}+\frac{1}{2}\cos 2t\right]dt=$$



$\cos 2\alpha=2\cos^2\alpha-1$

$2\cos^2\alpha=1+\cos 2\alpha$

$\cos^2\alpha=\frac{1}{2}+\frac{1}{2}\cos 2\alpha$

$$=\frac{9}{2}\int dt+\frac{9}{2}\int\cos 2t\,dt=$$

$$=\frac{9}{2}t+\frac{9}{2}\cdot\frac{1}{2}\int 2\cos 2t\,dt=$$

$$=\frac{9}{2}t+\frac{9}{4}\sin 2t+c=$$

$x=3\sin t$

$\sin t=\frac{x}{3}$

$t=\arcsin\left(\frac{x}{3}\right)$

$$=\frac{9}{2}\arcsin\frac{x}{3}+\frac{9}{\cancel{4}_{2}}\cancel{2}\sin t\cos t+c=$$

$$=\frac{9}{2}\arcsin\frac{x}{3}+\frac{\cancel{9}^{3}}{2}\cdot\frac{x}{\cancel{3}}\cdot\cos\left(\arcsin\frac{x}{3}\right)+c=$$

$$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{\cancel{9}^{3}}{2}\cdot\frac{x}{\cancel{3}}\cdot\cos\left(\arcsin\frac{x}{3}\right) + c =$$

$$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{3}{2}x\sqrt{1-\sin^2\left(\arcsin\frac{x}{3}\right)} + c =$$

$$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{3}{2}x\sqrt{1-\left(\frac{x}{3}\right)^2} + c =$$

$$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{3}{2}x\sqrt{\frac{9-x^2}{9}} + c =$$

$$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{x}{2}\sqrt{9-x^2} + c$$

OSSERVAZIONE

$$\int\cos^2 x\,dx = \int\underbrace{\cos x}_{(\sin x)'}\cdot\cos x\,dx \overset{\text{PER PARTI}}{=} \sin x\cos x - \int\sin x\cdot(\cos x)'dx$$

$$= \sin x\cos x + \int\sin^2 x\,dx = \sin x\cos x + \int(1-\cos^2 x)\,dx$$

$$= \sin x\cos x + x - \int\cos^2 x\,dx$$

PORTO AL 1° MEMBRO

$$2\int\cos^2 x\,dx = \sin x\cos x + x + c$$

$$\int\cos^2 x\,dx = \frac{1}{2}\sin x\cos x + \frac{x}{2} + c$$

415 $\int \sqrt{36 - 4x^2}\, dx$ $\left[9 \arcsin \frac{x}{3} + x\sqrt{9 - x^2} + c\right]$

VA RICONDOTTO A UN INTEGRALE DEL TIPO PRECEDENTE

$$\int \sqrt{4(9 - x^2)}\, dx = 2 \int \sqrt{9 - x^2} = \dots\dots$$

523 $\int \frac{1}{x^2 + 4x + 5}\, dx$ $[\arctan(x + 2) + c]$

$$\frac{1}{x^2 + 4x + 5} = \frac{1}{\underbrace{x^2 + 4x + 4}_{(x+2)^2} - 4 + 5} = \frac{1}{(x+2)^2 + 1}$$

$\Delta = 16 - 20 = -4 < 0$

$$\int \frac{1}{x^2 + 4x + 5}\, dx = \int \frac{1}{(x+2)^2 + 1}\, dx = \arctan(x+2) + c$$

oppure $x + 2 = t$

$x = t - 2$

$dx = dt$

$$\int \frac{1}{t^2 + 1}\, dt =$$

$$= \arctan(t) + c =$$

$$= \arctan(x + 2) + c$$