1/3/2021

Trovare max, min, flessi

395 $y = \dfrac{x^2-4x+5}{1-|x-1|}$

D: $1-|x-1| \neq 0$ $\quad |x-1| \neq 1 \quad x-1 \neq \pm 1$

$x \neq 0 \wedge x \neq 2$

$D =]-\infty, 0[\cup]0, 2[\cup]2, +\infty[$

$$f(x) = \frac{x^2-4x+5}{1-|x-1|}$$

$$f(x) = \begin{cases} \dfrac{x^2-4x+5}{1-(x-1)} & \text{se } x-1 \geq 0 \wedge x \neq 2 \\ \dfrac{x^2-4x+5}{1+(x-1)} & \text{se } x-1 < 0 \wedge x \neq 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x^2-4x+5}{2-x} & \text{se } x \geq 1 \wedge x \neq 2 \\ \dfrac{x^2-4x+5}{x} & \text{se } x < 1 \wedge x \neq 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{(2x-4)(2-x)+(x^2-4x+5)}{(2-x)^2} = \dfrac{\overbrace{4x-2x^2-8+\cancel{4x}+x^2-\cancel{4x}+5}^{-x^2+4x-3}}{(2-x)^2} \\ \dfrac{(2x-4)x-x^2+4x-5}{x^2} = \dfrac{2x^2-\cancel{4x}-x^2+\cancel{4x}-5}{x^2} = \dfrac{x^2-5}{x^2} \end{cases}$$

$$f'(x)=\begin{cases}\dfrac{-x^2+4x-3}{(2-x)^2} & \text{se } x>1 \wedge x\neq 2\\[2ex] 1-\dfrac{5}{x^2} & \text{se } x<1 \wedge x\neq 0\end{cases}$$

$$f'_+(1)=\lim_{x\to 1^+}\frac{-x^2+4x-3}{(2-x)^2}=-1+4-3=0$$

1 è un punto angoloso

$$f'_-(1)=\lim_{x\to 1^-}\left(1-\frac{5}{x^2}\right)=1-5=-4$$

ZERI DI f'

$f'(x)=0$

$$\begin{cases}\dfrac{-x^2+4x-3}{(2-x)^2}=0\\ x>1 \wedge x\neq 2\end{cases}\Rightarrow -x^2+4x-3=0$$

$x^2-4x+3=0$

$(x-3)(x-1)=0$ $\nearrow x=3$ $\searrow x=1$ N.A.

$$\begin{cases}1-\dfrac{5}{x^2}=0\\ x<1 \wedge x\neq 0\end{cases}\Rightarrow x^2=5 \quad x=\pm\sqrt5$$

$(x=+\sqrt5 \text{ N.A.})$

$\Downarrow$

$x=-\sqrt5$

SEGNO DI f'

$f'(x)>0$

$$\begin{cases}\dfrac{-x^2+4x-3}{(2-x)^2}>0\\ x>1 \quad x\neq 2\end{cases}\Rightarrow x^2-4x+3<0$$

$1<x<3 \wedge x\neq 2$

$$\begin{cases}1-\dfrac{5}{x^2}>0\\ x<1 \wedge x\neq 0\end{cases}\Rightarrow \frac{5}{x^2}<1 \quad 5<x^2$$

$x^2>5$

$$\begin{cases}x<-\sqrt5 \vee x>\sqrt5\\ x<1 \wedge x\neq 0\end{cases}$$

$\Downarrow$

$x<-\sqrt5$

		$-\sqrt{5}$		0		1		2		3	
f'	$+$	0	$-$	$\nexists$	$-$	$\nexists$	$+$	$\nexists$	$+$	0	$-$
	$\nearrow$		$\searrow$		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$
		MAX				MIN (P.TO ANGOLOSO)				MAX	

$$f'(x)=\begin{cases}\dfrac{-x^2+4x-3}{(2-x)^2} & \text{se} \quad x>1 \wedge x\neq 2\\[2ex] 1-\dfrac{5}{x^2} & \text{se} \quad x<1 \wedge x\neq 0\end{cases}$$

$$f''(x)=\begin{cases}\dfrac{(-2x+4)(2-x)^2+2(2-x)(-x^2+4x-3)}{(2-x)^4}=(*)\\[2ex] \dfrac{10}{x^3}\end{cases}$$

$$(*)=\frac{(-2x+4)(4+x^2-4x)+(4-2x)(-x^2+4x-3)}{(2-x)^4}=$$

$$=\frac{-8x-\cancel{2x^3}+\cancel{8x^2}+16+\cancel{4x^2}-\cancel{16x}-\cancel{4x^2}+\cancel{16x}-12+\cancel{2x^3}-\cancel{8x^2}+6x}{(2-x)^4}=$$

$$=\frac{4-2x}{(2-x)^4}$$

$$f''(x)=\begin{cases}\dfrac{4-2x}{(2-x)^4} & x>1 \wedge x\neq 2\\[2ex] \dfrac{10}{x^3} & x<1 \wedge x\neq 0\end{cases}$$

NON CI SONO ZERI DI f''

$$f''(x) = \begin{cases} \dfrac{4-2x}{(2-x)^4} & x>1 \wedge x \neq 2 \\[2ex] \dfrac{10}{x^3} & x<1 \wedge x \neq 0 \end{cases}$$

SEGNO DI f''

$$\begin{cases} \dfrac{4-2x}{(2-x)^4} > 0 \Rightarrow 4-2x>0 \quad x<2 \\[2ex] x>1 \wedge x\neq 2 \end{cases} \Rightarrow 1<x<2$$

$$\begin{cases} \dfrac{10}{x^3} > 0 \\[2ex] x<1 \wedge x\neq 0 \end{cases} \qquad \begin{cases} x>0 \\[2ex] x<1 \wedge x\neq 0 \end{cases} \Rightarrow 0<x<1$$

		0		1		2	
f''	$-$	$\nexists$	$+$	$\nexists$	$+$	$\nexists$	$-$
	$\frown$		$\smile$		$\smile$		$\frown$

f ha la concavità verso il basso in $]-\infty, 0[$ e in $]2, +\infty[$

f ha la concavità verso l'alto in $]0, 2[$ (1 punto angoloso)

Non ci sono punti di flesso