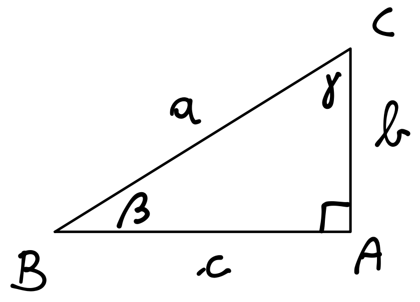
30/5/2018

TEOREMI DI TRIGONOMETRIA

SUI TRIANGOLI RETTANGOLI



$c = a \cos\beta$

$b = a \sin\beta$

PICCOLA CONSEGUENZA

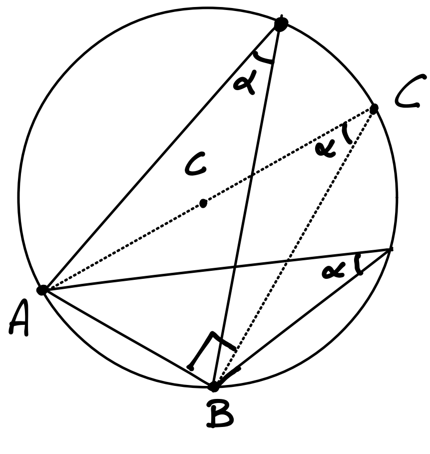
$\Downarrow$

$$\frac{b}{c} = \frac{\cancel{a}\sin\beta}{\cancel{a}\cos\beta} = \tan\beta$$

$\Downarrow$

$$b = c\tan\beta$$

TEOREMA DELLA CORDA



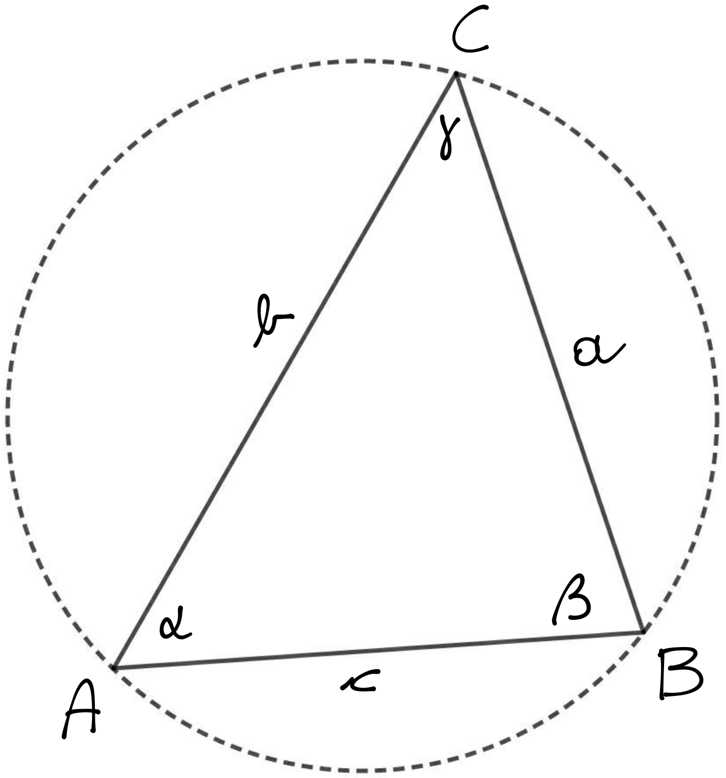
CIRCONFERENZA DI RAGGIO r

$$\overline{AB} = 2r \cdot \sin\alpha$$

α = uno qualsiasi degli angoli alla circonferenza che insistono sulla corda.

Tutti gli angoli alla circonferenza che insistono su AB hanno la stessa ampiezza α. Quindi considerando il diametro $2r$, il triangolo ABC è rettangolo e α è opposto ad AB, per cui $\overline{AB} = \overline{AC} \cdot \sin\alpha \Longrightarrow \overline{AB} = 2r\sin\alpha$

TEOREMA DEI SENI



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

r = raggio della circonferenza circoscritta ad AB

Per il TH. CORDA

$$a = 2r \cdot \sin\alpha$$
$$b = 2r \cdot \sin\beta$$
$$c = 2r \cdot \sin\gamma$$

$$\Downarrow$$

$$\frac{a}{\sin\alpha} = 2r \quad \frac{b}{\sin\beta} = 2r \quad \frac{c}{\sin\gamma} = 2r$$

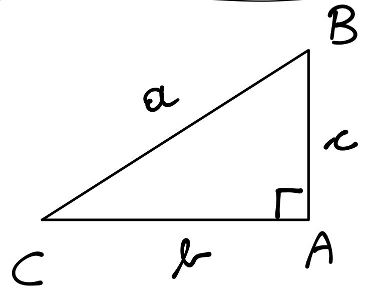
$$\Downarrow$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2r$$

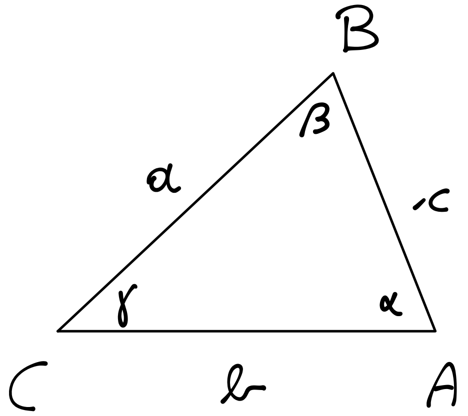
TEOREMA DEL COSENO (DI CARNOT)

↓

GENERALIZZAZIONE DEL TEOREMA DI PITAGORA



$$a^2 = b^2 + c^2$$ TEOREMA DI PITAGORA



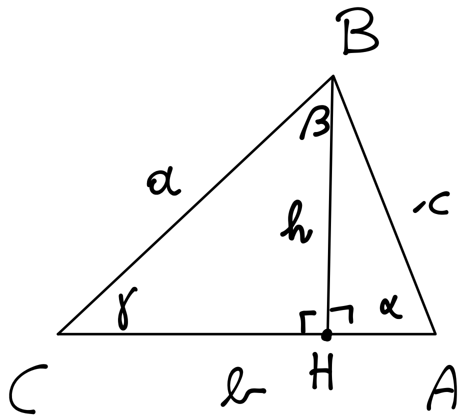
TH. DI CARNOT

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

DIMOSTRAZIONE



BCH e BHA sono triangoli rettangoli, quindi per loro vale il TH. DI PITAGORA

$$a^2 = h^2 + (b - \overline{HA})^2 =$$

$$= h^2 + (b - c\cos\alpha)^2 =$$

$$= (c\sin\alpha)^2 + (b - c\cos\alpha)^2 =$$

$$= c^2 \cdot \sin^2\alpha + b^2 + c^2\cos^2\alpha - 2bc\cos\alpha =$$

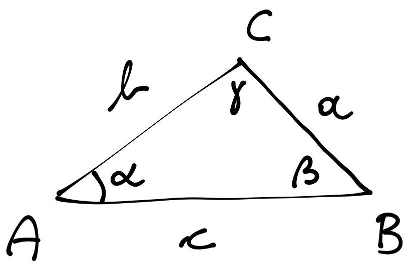
$$= c^2 \underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1} + b^2 - 2bc\cos\alpha =$$

$$= c^2 + b^2 - 2bc\cos\alpha$$

Q.E.D.
C.V.D.

RISOLVERE = trovare gli altri 2 lati e l'altro angolo

417 $c = 4\sqrt{2}$, $\alpha = 30°$, $\gamma = \frac{7}{12}\pi$. $\left[\beta = \frac{\pi}{4}; a = 4\sqrt{3} - 4; b = 4\sqrt{6} - 4\sqrt{2}\right]$



$$\gamma = \frac{7}{\cancel{12}_{\cancel{2}\;1}}\cancel{\pi} \times \frac{\cancel{180°}^{\cancel{30°}\;15}}{\cancel{\pi}} = 105°$$

$$\beta = 180° - 30° - 105° = 45° \rightarrow \frac{\pi}{4}$$

TH. SENI

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \Longrightarrow a = \frac{c}{\sin\gamma}\cdot\sin\alpha = \frac{4\sqrt{2}}{\frac{\sqrt{6}+\sqrt{2}}{4}}\cdot\frac{1}{2} =$$

$$\sin\gamma = \sin(105°) =$$
$$= \sin(60° + 45°) =$$
$$= \sin 60° \cos 45° + \cos 60° \sin 45° =$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$= \frac{\overset{8}{\cancel{16}}\sqrt{2}}{\cancel{2}(\sqrt{6}+\sqrt{2})}\cdot\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} =$$

$$= \frac{8(\sqrt{12}-2)}{6-2} =$$

$$= \frac{\overset{2}{\cancel{8}}(2\sqrt{3}-2)}{\cancel{4}} = \boxed{4\sqrt{3}-4}$$

TH. COSENO

$$b^2 = a^2 + c^2 - 2ac\cos\beta = (4\sqrt{3}-4)^2 + (4\sqrt{2})^2 - \cancel{2}(4\sqrt{3}-4)\overbrace{(4\sqrt{2})\frac{\sqrt{2}}{\cancel{2}}}^{8} =$$

$$= 48 + 16 - 32\sqrt{3} + 32 - 32\sqrt{3} + 32 =$$

$$= 128 - 64\sqrt{3} = 64(2-\sqrt{3})$$

$$b = 8\sqrt{2-\sqrt{3}} \simeq 4{,}1411\ldots$$