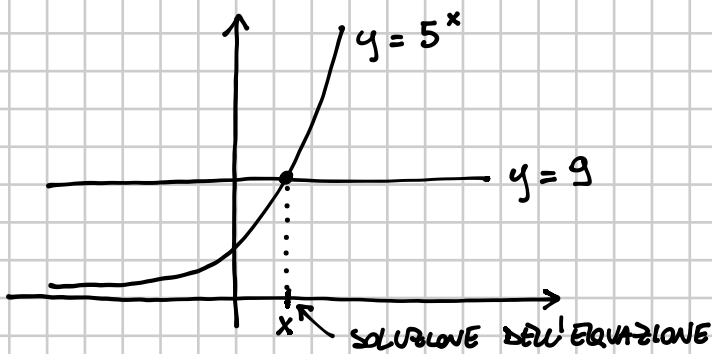


551

$$5^x = 9$$

$$x = \log_5 9 = \frac{\log 9}{\log 5}$$

**556**

$$3 \cdot 2^x + 2^{x+1} = 19$$

$$\left[\frac{\log 19 - \log 5}{\log 2} \right]$$

$$3 \cdot 2^x + 2 \cdot 2^x = 19$$

$$5 \cdot 2^x = 19$$

$$2^x = \frac{19}{5}$$

$$x = \log_2 \frac{19}{5} = \frac{\log \frac{19}{5}}{\log 2} = \frac{\log 19 - \log 5}{\log 2}$$

$$3^{x+1} - 2 \cdot 3^x + 3^{x+2} = 5^{x-1}$$

$$\left[\frac{2\log 5 + \log 2}{\log 5 - \log 3} \right]$$

$$3^x \cdot 3 - 2 \cdot 3^x + 3^x \cdot 3^2 = 5^x \cdot 5^{-1}$$

$$3^x(3 - 2 + 9) = \frac{1}{5} \cdot 5^x$$

$$10 \cdot 3^x = \frac{1}{5} \cdot 5^x \Rightarrow \frac{3^x}{5^x} = \frac{1}{50} \quad \left(\frac{3}{5}\right)^x = \frac{1}{50}$$

$$x = \log_{\frac{3}{5}} \frac{1}{50} = \frac{\log \frac{1}{50}}{\log \frac{3}{5}} = \frac{\overbrace{\log 1}^0 - \log 50}{\log 3 - \log 5} = \frac{-\log 2 \cdot 5^2}{\log 3 - \log 5} =$$

$$= \frac{\log 2 + \log 5^2}{\log 5 - \log 3} = \frac{\log 2 + 2 \log 5}{\log 5 - \log 3}$$

604

$$\frac{2}{25^x - 1} + \frac{3}{4} = \frac{2}{5^x - 1}$$

$$(5^x)^2 - 1$$

[log₅3]

$$\text{C.E. } 5^x \neq 1 \Rightarrow x \neq 0$$

$$\frac{2}{(5^x - 1)(5^x + 1)} + \frac{3}{4} = \frac{2}{5^x - 1}$$

$$5^x = t$$

$$\frac{2}{(t-1)(t+1)} + \frac{3}{4} = \frac{2}{t-1}$$

$$\frac{8 + 3(t-1)(t+1)}{4(t-1)(t+1)} = \frac{8(t+1)}{4(t-1)(t+1)}$$

$$\cancel{8} + 3t^2 - 3 = 8t + \cancel{8}$$

$$3t^2 - 8t - 3 = 0$$

$$\frac{\Delta}{4} = 16 + 9 = 25$$

$$t = \frac{4 \pm 5}{3} = \begin{cases} -\frac{1}{3} \\ 3 \end{cases}$$

$$5^x = -\frac{1}{3}$$

IMPOSSIBLE

$$5^x = 3$$

⇓

$$x = \log_5 3$$

605

$$4^x + 10^x = 25^x$$

$$\left[\log_{\frac{2}{5}} \frac{\sqrt{5}-1}{2} \right]$$

$$(2^x)^2 + 2^x \cdot 5^x = (5^x)^2$$

$$\frac{(2^x)^2}{(5^x)^2} + \frac{2^x \cdot \cancel{5^x}}{(5^x)^2} = 1$$

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(5^x)^2

$$\left(\frac{2^x}{5^x} \right)^2 + \frac{2^x}{5^x} - 1 = 0$$

$$\left[\left(\frac{2}{5} \right)^x \right]^2 + \left(\frac{2}{5} \right)^x - 1 = 0$$

$$t = \left(\frac{2}{5} \right)^x$$

$$t^2 + t - 1 = 0$$

$$t = \frac{-1 \pm \sqrt{5}}{2} = \begin{cases} \frac{-1 - \sqrt{5}}{2} & \text{non acc. (perché negativo)} \\ \frac{\sqrt{5} - 1}{2} \end{cases}$$

$$\left(\frac{2}{5} \right)^x = \frac{\sqrt{5} - 1}{2}$$

$$x = \log_{\frac{2}{5}} \frac{\sqrt{5} - 1}{2}$$

628

$$\sqrt{5^{x-1}} < 9 \cdot 3^{2x}$$

$$\left[x > \frac{\log 5 + 4 \log 3}{\log 5 - 4 \log 3} \right]$$

$$5^{\frac{x-1}{2}} < 9 \cdot 3^{2x}$$

$$5^{\frac{x}{2}} \cdot 5^{-\frac{1}{2}} < 9 \cdot (3^2)^x$$

$$\left(5^{\frac{1}{2}} \right)^x \cdot 5^{-\frac{1}{2}} < 9 \cdot (3^2)^x$$

$$\left(\frac{\sqrt{5}}{3^2} \right)^x < 9 \cdot 5^{\frac{1}{2}}$$

$$\left(\frac{\sqrt{5}}{9} \right)^x < 9\sqrt{5}$$

applico il $\log_{\frac{\sqrt{5}}{9}}$
e inverto la dis.

$$x > \log_{\frac{\sqrt{5}}{9}} 9\sqrt{5} = \text{perché } \frac{\sqrt{5}}{9} < 1$$

$$= \frac{\log 9\sqrt{5}}{\log \frac{\sqrt{5}}{9}} = \frac{\log 9 + \log \sqrt{5}}{\log \sqrt{5} - \log 9} =$$

$$= \frac{2 \log 3 + \frac{1}{2} \log 5}{\frac{1}{2} \log 5 - 2 \log 3} = \frac{4 \log 3 + \log 5}{\log 5 - 4 \log 3}$$

$$x > \frac{4 \log 3 + \log 5}{\log 5 - 4 \log 3}$$