27/3/2021

387 $\int \frac{4}{1+\cos x}dx =$ $\left[4\tan\frac{x}{2}+c\right]$

$$= \int \frac{4}{1+\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\, dt =$$

$$= \int \frac{8}{1+\cancel{t^2}+1-\cancel{t^2}}\, dt =$$

$$= 4\int dt = 4t + c =$$

$$= \boxed{4\tan\frac{x}{2} + c}$$

FORMULE PARAMETRICHE

$$\sin x = \frac{2t}{1+t^2} \qquad t = \tan\frac{x}{2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\arctan t = \frac{x}{2}$$

$$x = 2\arctan t$$

$$\frac{dx}{dt} = \frac{2}{1+t^2} \Rightarrow dx = \frac{2}{1+t^2}dt$$

394 $\int \frac{\sin x + 3}{2\sin x}dx =$ $\left[\frac{1}{2}x+\frac{3}{2}\ln\left|\tan\frac{x}{2}\right|+c\right]$

$$= \int \frac{\sin x}{2\sin x}dx + \int \frac{3}{2\sin x}dx = \frac{1}{2}\int dx + \frac{3}{2}\int \frac{dx}{\sin x} =$$

$t = \tan\frac{x}{2}$

$$= \frac{1}{2}x + \frac{3}{2}\int \frac{\cancel{1+t^2}}{\cancel{2}t}\cdot\frac{\cancel{2}}{\cancel{1+t^2}}\,dt =$$

$x = 2\arctan t$

$$= \frac{1}{2}x + \frac{3}{2}\int \frac{1}{t}dt = \frac{1}{2}x + \frac{3}{2}\ln|t| + c =$$

$dx = \frac{2}{1+t^2}dt$

$$= \boxed{\frac{1}{2}x + \frac{3}{2}\ln\left|\tan\frac{x}{2}\right| + c}$$

$\sin x = \frac{2t}{1+t^2}$

398 $\int \sqrt{9-x^2}\,dx = \int \sqrt{3^2 - x^2}\,dx =$

$x = 3\sin t$

$t = \arcsin \frac{x}{3}$

$dx = 3\cos t\,dt$

$= \int \sqrt{3^2 - 3^2 \sin^2 t} \cdot 3\cos t\,dt =$

$= \int 3\overbrace{\sqrt{1-\sin^2 t}}^{\cos t} \cdot 3\cos t\,dt =$

$= 9\int \cos^2 t\,dt = 9\int \frac{1+\cos 2t}{2}\,dt =$

$\cos 2t = 2\cos^2 t - 1$

$\Downarrow$

$\cos^2 t = \frac{1+\cos 2t}{2}$

$= 9\int \frac{1}{2}\,dt + 9\int \frac{\cos 2t}{2}\,dt =$

$= \frac{9}{2}\int dt + \frac{9}{2}\int \cos 2t\,dt =$

$= \frac{9}{2}t + \frac{9}{4}\int (2\cos 2t)\,dt = \frac{9}{2}t + \frac{9}{4}\int (\sin 2t)'\,dt =$

$= \frac{9}{2}t + \frac{9}{4}\sin 2t + c = \frac{9}{2}\arcsin\frac{x}{3} + \frac{9}{4}\sin\left(2\cdot\arcsin\frac{x}{3}\right) + c =$

$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{9}{\cancel{4}_{2}}\cdot\cancel{2}\cdot\sin\left(\arcsin\frac{x}{3}\right)\cdot\cos\left(\arcsin\frac{x}{3}\right) + c =$

$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{\cancel{9}^{3}}{2}\cdot\frac{x}{\cancel{3}}\cdot\sqrt{1-\sin^2\left(\arcsin\frac{x}{3}\right)} + c$

$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{3}{2}x\sqrt{1-\frac{x^2}{9}} + c = \boxed{\frac{9}{2}\arcsin\frac{x}{3} + \frac{x}{2}\sqrt{9-x^2} + c}$

se faccio la derivata di questo viene $\sqrt{9-x^2}$

$$\boxed{\frac{9}{2}\arcsin\frac{x}{3}+\frac{x}{2}\sqrt{9-x^2}+c}$$

verifichiamolo:

DERIVATA

$$\frac{\cancel{9}^{\,3}}{2}\,\frac{1}{\sqrt{1-\frac{x^2}{9}}}\cdot\frac{1}{\cancel{3}}+\frac{1}{2}\left[\sqrt{9-x^2}+x\cdot\frac{1}{\cancel{2}\sqrt{9-x^2}}\cdot(-\cancel{2}x)\right]=$$

$$=\frac{3\cdot 3}{2\sqrt{9-x^2}}+\frac{1}{2}\left[\frac{9-x^2-x^2}{\sqrt{9-x^2}}\right]=$$

$$=\frac{9+9-2x^2}{2\sqrt{9-x^2}}=\frac{18-2x^2}{2\sqrt{9-x^2}}=\frac{\cancel{2}(9-x^2)}{\cancel{2}\sqrt{9-x^2}}\cdot\frac{\sqrt{9-x^2}}{\sqrt{9-x^2}}=$$

$$=\sqrt{9-x^2}$$