24/3/2021

286 $\int \frac{\cos x}{9+\sin^2 x}\,dx =$ $\left[\frac{1}{3}\arctan\frac{\sin x}{3}+c\right]$

$$= \int \frac{\cos x}{9\left(1+\left(\frac{\sin x}{3}\right)^2\right)}\,dx = \frac{1}{9}\int \frac{\cos x}{1+\left(\frac{\sin x}{3}\right)^2}\,dx =$$

$$= \frac{1}{3}\int \frac{\frac{1}{3}\cos x}{1+\left(\frac{\sin x}{3}\right)^2}\,dx = \frac{1}{3}\int \left[\arctan\frac{\sin x}{3}\right]' dx =$$

$$= \boxed{\frac{1}{3}\arctan\frac{\sin x}{3}+c}$$

295 $\displaystyle\int \frac{1}{5+e^x}\,dx =$

$$I = \int \frac{1}{5+e^x}\,dx$$

$$= \int \frac{1+4+e^x-4-e^x}{5+e^x}\,dx =$$

$$= \int \frac{5+e^x}{5+e^x}\,dx - 4\int \frac{1}{5+e^x}\,dx - \int \frac{e^x}{5+e^x}\,dx =$$

$$= \int dx - 4\int \frac{1}{5+e^x}\,dx - \int \left[\ln(5+e^x)\right]'\,dx =$$

$$= x - 4\int \frac{1}{5+e^x}\,dx - \ln(5+e^x)$$

$$\Rightarrow \quad I = x - 4I - \ln(5+e^x) \qquad I = \int \frac{1}{5+e^x}\,dx$$

$$5I = x - \ln(5+e^x) + c$$

$$I = \frac{1}{5}x - \frac{1}{5}\ln(5+e^x) + c$$

$$\boxed{\int \frac{1}{5+e^x}\,dx = \frac{1}{5}x - \frac{1}{5}\ln(5+e^x) + c}$$

MODO ALTERNATIVO (AURORA)

$$\int \frac{1}{5+e^x}\,dx = \frac{1}{5}\int \frac{5}{5+e^x}\,dx = \frac{1}{5}\int \frac{5+e^x-e^x}{5+e^x}\,dx =$$

$$= \frac{1}{5}\left[\int \frac{5+e^x}{5+e^x}\,dx - \int \frac{e^x}{5+e^x}\,dx\right] =$$

$$= \frac{1}{5}\left[\int dx - \int \left[\ln(5+e^x)\right]'\,dx\right] = \boxed{\frac{1}{5}x - \frac{1}{5}\ln(5+e^x) + c}$$

311 $\displaystyle\int \frac{\cos x \cdot e^{\sqrt{\sin x}}}{2\sqrt{\sin x}}\,dx = \int \left(e^{\sqrt{\sin x}}\right)'\,dx = e^{\sqrt{\sin x}} + c$

$$\left(\sqrt{\sin x}\right)' = \frac{1}{2\sqrt{\sin x}}\cdot(\sin x)' =$$

$$= \frac{\cos x}{2\sqrt{\sin x}}$$

$$\left(e^{\sqrt{\sin x}}\right)' = e^{\sqrt{\sin x}}\cdot\left(\sqrt{\sin x}\right)'$$