23/1/2019

418 $x^6 + 7x^3 - 8 = 0$ $\left[-2, 1 \pm i\sqrt{3}, 1, \frac{-1 \pm i\sqrt{3}}{2}\right]$

$x^3 = t$

$t^2 + 7t - 8 = 0$

$(t+8)(t-1) = 0$ $\Rightarrow$ $x^3 = -8$ or $x^3 = 1$

$x^3 = -8 \Rightarrow$ trovare le 3 radici 3° di -8

$x^3 = 8(\cos\pi + i\sin\pi)$ $\qquad \rho = 8 \quad \vartheta = \pi$

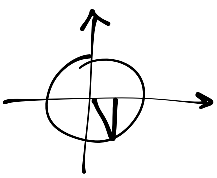
$x_0 = \sqrt[3]{8}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 + \sqrt{3}\,i$

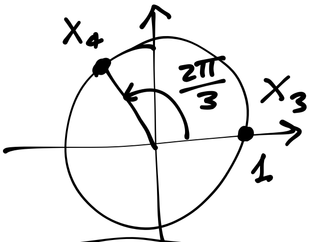
CONIUGATE

$x_1 = \sqrt[3]{8}\left(\cos\frac{\pi+2\pi}{3} + i\sin\frac{\pi+2\pi}{3}\right) = 2(-1 + i\cdot 0) = -2$

$x_2 = \sqrt[3]{8}\left(\cos\frac{\pi+4\pi}{3} + i\sin\frac{\pi+4\pi}{3}\right) =$

$= 2\left(\cos\frac{5}{3}\pi + i\sin\frac{5}{3}\pi\right) = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - \sqrt{3}\,i$





$x^3 = 1 \Rightarrow$ trovare radici dell'unità

$x_3 = 1$

$x_4 = 1\cdot\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$x_5 = 1\cdot\left(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

RISULTATO FINALE

$z = -2 \;\vee\; z = 1$

$\vee\; z = 1 \pm \sqrt{3}\,i$

$\vee\; z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

424 $x^2-(2+2i)x+2i-1=0$ $[i, 2+i]$

$$x = \frac{-b \pm r}{2a}$$

r = UNA DELLE DUE RADICI QUADRATE DI Δ

($\sqrt{\Delta}$ si usa solo se Δ è reale!)

$$\Delta = b^2 - 4ac = [-(2+2i)]^2 - 4(2i-1) =$$

$$= (2+2i)^2 - 8i + 4 = 4 + 4i^2 + \cancel{8i} - \cancel{8i} + 4 =$$

$$= 4 - 4 + 4 = 4 \leftarrow \text{è reale}$$

$$x = \frac{2+2i \pm 2}{2} = \begin{cases} \frac{2i}{2} = i \\ \frac{4+2i}{2} = 2+i \end{cases}$$

$$\boxed{x = i \lor x = 2+i}$$

425 $x^2 + \frac{(1+i)^2 - 11i}{3}x - 2 = 0$

$$x^2 + \frac{\cancel{1} - \cancel{1} + 2i - 11i}{3} x - 2 = 0$$

$$x^2 - 3i\,x - 2 = 0$$

$$\Delta = (-3i)^2 - 4\cdot(-2) = -9 + 8 = -1$$

$$x = \frac{3i \pm \sqrt{-1}}{2} = \frac{3i \pm i}{2} = \begin{cases} \frac{2i}{2} = i \\ \frac{4i}{2} = 2i \end{cases}$$

Risolvere in $\mathbb{C}$ l'equazione

$$z^2 + (2-i)z + 1 = 0$$

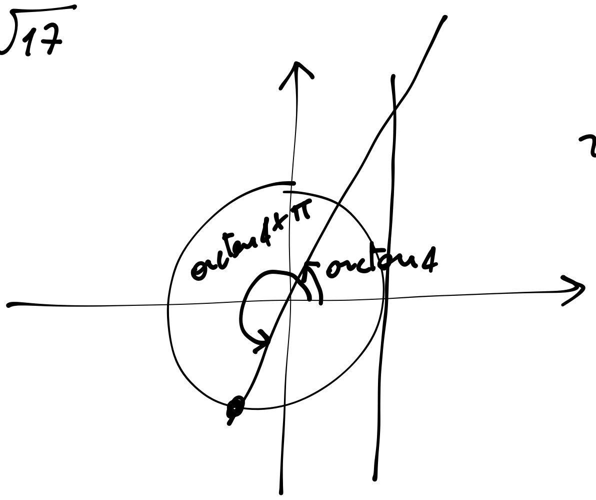
$$\Delta = (2-i)^2 - 4 = \cancel{4} - 1 - 4i - \cancel{4} = -1 - 4i$$

devo trovare una delle 2 radici (complesse) quadrate di Δ

$$\Delta = -1 - 4i = \sqrt{17}\left(-\frac{1}{\sqrt{17}} - \frac{4}{\sqrt{17}}i\right) \qquad \tan\vartheta = \frac{-\frac{4}{\sqrt{17}}}{-\frac{1}{\sqrt{17}}} = 4$$

$$\rho = \sqrt{1+16} = \sqrt{17}$$

$$\vartheta = \arctan 4 + \pi$$



$$\Delta = \sqrt{17}\left(\cos\vartheta + i\sin\vartheta\right) \qquad \text{con } \vartheta = \arctan 4 + \pi$$

le 2 radici di ϑ sono:

$$\Delta_0 = \sqrt[4]{17}\left(\cos\frac{\vartheta}{2} + i\sin\frac{\vartheta}{2}\right)$$

$$\Delta_1 = \sqrt[4]{17}\left(\cos\frac{\vartheta+2\pi}{2} + i\sin\frac{\vartheta+2\pi}{2}\right) =$$

$$= \sqrt[4]{17}\left(\cos\left(\frac{\vartheta}{2}+\pi\right) + i\sin\left(\frac{\vartheta}{2}+\pi\right)\right) = -\Delta_0$$

$$z = \frac{-2+i-\Delta_0}{2} \quad \vee \quad z = \frac{-2+i+\Delta_0}{2}$$

Le 2 soluzioni scritte esplicitamente sono

$$z = \frac{-2+i \pm \sqrt[4]{17}\left(-\sqrt{\frac{1}{2}-\frac{\sqrt{17}}{34}} + i\sqrt{\frac{1}{2}+\frac{\sqrt{17}}{34}}\right)}{2}$$

approssimativamente

$$z \simeq -1{,}6248 + 1{,}3002\,i \quad \vee \quad z \simeq -0{,}37519 - 0{,}30024\,i$$