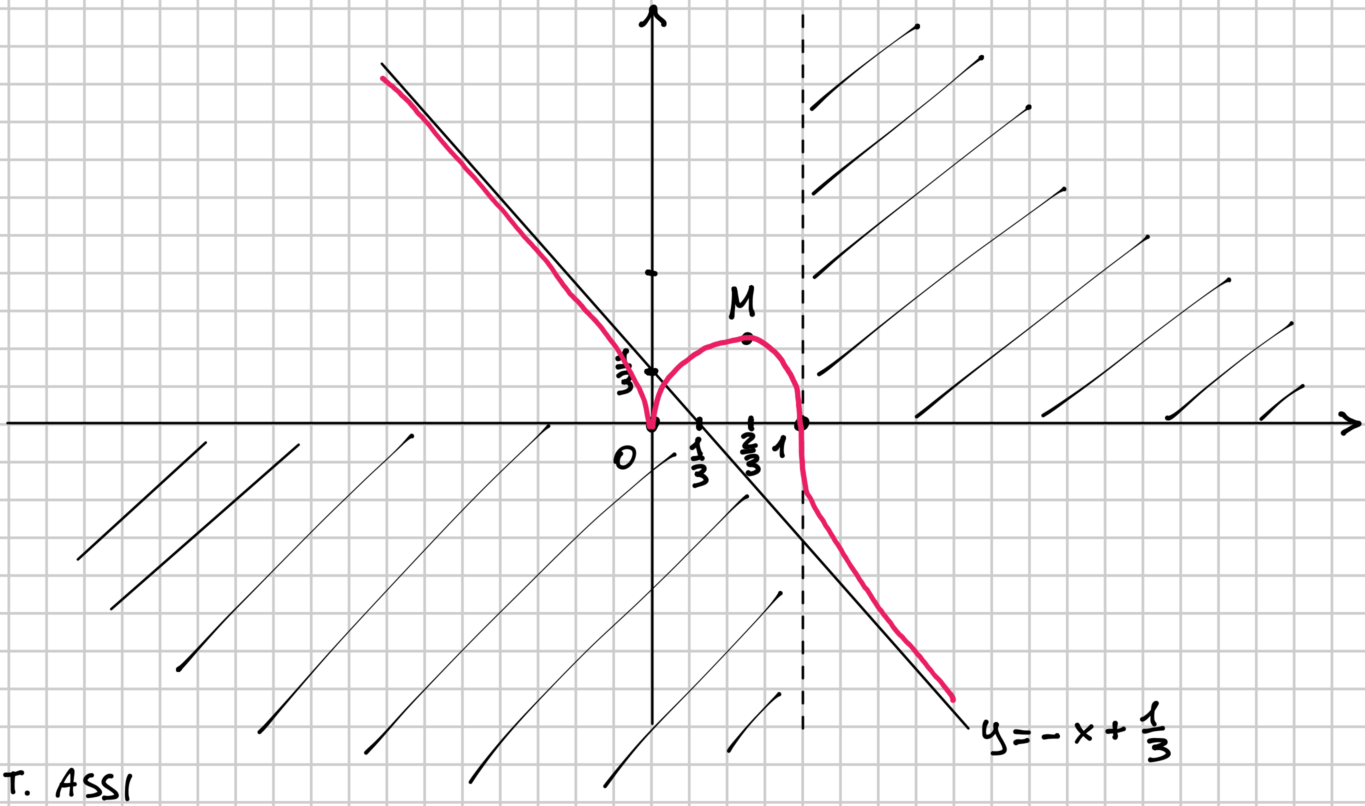
STUDIO DI FUNZIONE

127 $y = \sqrt[3]{x^2(1-x)}$

1) DOMINIO $\mathbb{R} = (-\infty, +\infty)$



2) INT. ASSI

$$\begin{cases} y = \sqrt[3]{x^2(1-x)} \\ y = 0 \end{cases} \Rightarrow x^2(1-x) = 0 \Rightarrow x = 1 \vee x = 0$$

$A(1,0) \quad O(0,0)$

3) SEGNO

$\sqrt[3]{x^2(1-x)} > 0 \qquad x^2(1-x) > 0 \qquad 1-x > 0 \qquad x < 1$

	0		1	
+	ϕ	+	ϕ	−

4) LIMITI

$$\lim_{x \to -\infty} \sqrt[3]{x^2(1-x)} = \sqrt[3]{+\infty \cdot (+\infty)} = +\infty$$

$$\lim_{x \to +\infty} \sqrt[3]{+\infty(-\infty)} = -\infty$$

Potrebbero esistere asintoti obliqui

5) RICERCA ASINTOTI OBL.

$$m = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\sqrt[3]{x^2(1-x)}}{x} = \lim_{x\to\pm\infty} \frac{\sqrt[3]{x^3\left(\frac{1}{x}-1\right)}}{x} =$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{x}\sqrt[3]{\frac{1}{x}-1}}{\cancel{x}} = \sqrt[3]{-1} = -1$$

$$q = \lim_{x\to\pm\infty} \left[f(x) - mx\right] = \lim_{x\to-\infty} \left[\sqrt[3]{x^2(1-x)} + x\right] = +\infty - \infty \quad \text{F.I.}$$

$$= \lim_{x\to\pm\infty} \left(\sqrt[3]{x^2(1-x)} + x\right) \cdot \frac{\sqrt[3]{x^4(1-x)^2} - x\sqrt[3]{x^2(1-x)} + x^2}{\sqrt[3]{x^4(1-x)^2} - x\sqrt[3]{x^2(1-x)} + x^2} =$$

$$= \lim_{x\to\pm\infty} \frac{x^2(1-x) + x^3}{\sqrt[3]{x^4(1+x^2-2x)} - x\sqrt[3]{x^2-x^3} + x^2} =$$

$$= \lim_{x\to\pm\infty} \frac{x^2 - \cancel{x^3} + \cancel{x^3}}{\sqrt[3]{x^4+x^6-2x^5} - x\sqrt[3]{x^3\left(\frac{1}{x}-1\right)} + x^2} =$$

$$= \lim_{x\to\pm\infty} \frac{x^2}{\sqrt[3]{x^6\left(\frac{1}{x^2}+1-\frac{2}{x}\right)} - x^2\sqrt[3]{\frac{1}{x}-1} + x^2} =$$

$$= \lim_{x\to\pm\infty} \frac{x^2}{x^2\sqrt[3]{\frac{1}{x^2}+1-\frac{2}{x}} - x^2\sqrt[3]{\frac{1}{x}-1} + x^2} =$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{x^2}}{\cancel{x^2}\left(\sqrt[3]{\frac{1}{x^2}+1-\frac{2}{x}} - \sqrt[3]{\frac{1}{x}-1} + 1\right)} = \frac{1}{1+1+1} = \frac{1}{3}$$

$y = -x + \frac{1}{3}$ AS. OBLIQUO PER $x \to \pm\infty$

6) STUDIO DERIVATA PRIMA

$$f(x) = \sqrt[3]{x^2(1-x)} = \left[x^2(1-x)\right]^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}\left[x^2(1-x)\right]^{-\frac{2}{3}} \cdot (2x - 3x^2) =$$

$$= \frac{1}{3} \frac{1}{\sqrt[3]{x^4(1-x)^2}} \cdot x(2-3x) = \frac{x(2-3x)}{3\sqrt[3]{x^4(1-x)^2}}$$

annullano il den.

$x \neq 0 \quad x \neq 1$

CONTROLLO IN $x=0$

$$\lim_{x\to 0} \frac{x(2-3x)}{3\sqrt[3]{x^4(1-x)^2}} = \overset{\text{F.I.}}{\frac{0}{0}} = \lim_{x\to 0} \frac{x(2-3x)}{3\sqrt[3]{x^4(1+x^2-2x)}} =$$

$$= \lim_{x\to 0} \frac{x(2-3x)}{3\sqrt[3]{\underbrace{x^6+x^4-2x^5}_{x^3(x^3+x-2x^2)}}} = \lim_{x\to 0} \frac{\cancel{x}(2-3x)}{3\cancel{x}\sqrt[3]{x^3+x-2x^2}} = \frac{2}{0} = \infty$$

il segno dipende da $x \to 0^{\pm}$

l'argomento $x^3+x-2x^2 \to 0^+$ per $x \to 0^+$

$\to 0^-$ per $x \to 0^-$

infatti $\underset{\downarrow\, 0^{\pm}}{x}\underbrace{(x^2+1-2x)}_{\downarrow\, 1}$

$\frac{2}{0^+} = +\infty$ per $x \to 0^+$

$\frac{2}{0^-} = -\infty$ per $x \to 0^-$

$f'_+(0) = +\infty$

$f'_-(0) = -\infty$

$x=0$ È UNA CUSPIDE

CONTROLLO IN $x=1$

$$\lim_{x\to 1} \frac{x(2-3x)}{3\sqrt[3]{x^4(1-x)^2}} = \frac{-1}{0^+} = -\infty$$

$x=1$ FLESSO A TANGENTE VERTICALE

INDIPENDENTEMENTE DA $x \to 1^{\pm}$

$$f'(x)=\frac{x(2-3x)}{3\sqrt[3]{x^4(1-x)^2}} \qquad x\neq 0 \quad x\neq 1$$

$$f'(x)=0 \Rightarrow \quad 2-3x=0 \qquad x=\frac{2}{3} \quad \text{PUNTO STAZIONARIO}$$

$$f'(x)>0 \qquad \frac{x(2-3x)}{\underbrace{3\sqrt[3]{x^4(1-x)^2}}_{>0\ \forall x\neq 0,\ x\neq 1}}>0 \Rightarrow \quad x(2-3x)>0$$

$x>0$

$2-3x>0 \Rightarrow x<\frac{2}{3}$

	0		$\frac{2}{3}$		1	
$-$		$+$		$+$		$+$
$+$		$+$	0	$-$		$-$
$-$		$+$		$-$		$-$
$\searrow$	CUSPIDE	$\nearrow$	MAX	$\searrow$	FLESSO	$\searrow$

$$f\left(\frac{2}{3}\right)=\sqrt[3]{\left(\frac{2}{3}\right)^2\left(1-\frac{2}{3}\right)}=\sqrt[3]{\frac{4}{9}\cdot\frac{1}{3}}=\sqrt[3]{\frac{4}{27}}=\frac{\sqrt[3]{4}}{3}\simeq 0{,}53$$

$$M\left(\frac{2}{3},\frac{\sqrt[3]{4}}{3}\right)$$

7) STUDIO DERIVATA SECONDA

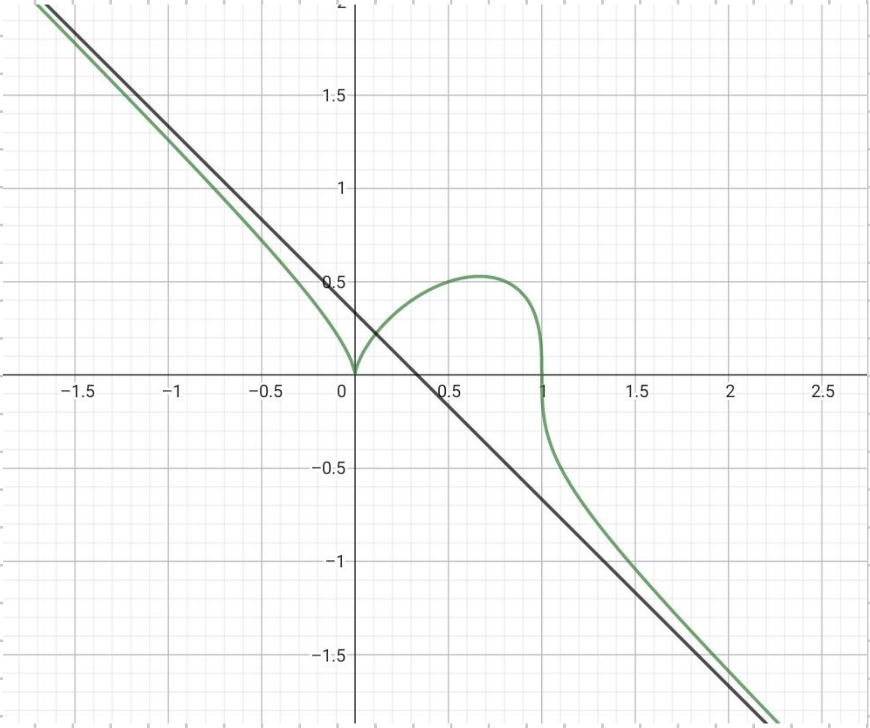
$$\frac{d}{dx}\left(\frac{2x-3x^2}{3\sqrt[3]{x^4(1-x)^2}}\right)=\frac{2}{9(x-1)\underbrace{\sqrt[3]{x^4(1-x)^2}}_{>0}} \qquad x\neq 0 \quad x\neq 1$$

(CON WOLFRAM ALPHA)

il segno della derivata seconda dipende solo da $x-1$

$$f''(x)>0 \Rightarrow \quad x>1$$

		0		1	
f''	$-$		$-$		$+$
	$\cap$		$\cap$	FLESSO TANG. VERTICALE	$\cup$



2
1.5
1
0.5
−0.5
−1
−1.5
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5

127 Si consideri l'equazione:

$$4x^3 - 14x^2 + 20x - 5 = 0.$$

Si dimostri che essa per $0 < x < 1$ ha un'unica radice reale e se ne calcoli un valore approssimato con due cifre decimali esatte.

(*Esame di Stato, Liceo scientifico, Corso sperimentale, Sessione suppletiva*, 2013, *quesito* 8)

[0,31]

$f(x) = 4x^3 - 14x^2 + 20x - 5$

f è CONTINUA

e DERIVABILE in $\mathbb{R}$

TH. ZERI $\quad f(0) = -5 \qquad f(1) = 4 - 14 + 20 - 5 = 5$

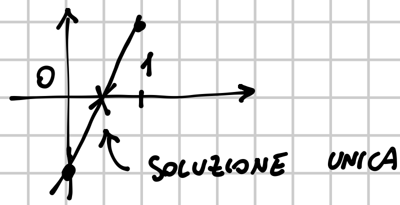
$f(0)$ e $f(1)$ hanno segni opposti $\Rightarrow \exists$ una soluzione in $(0, 1)$

$f'(x) = 12x^2 - 28x + 20$

$f'(x) > 0 \Rightarrow 12x^2 - 28x + 20 > 0$

$3x^2 - 7x + 5 > 0 \qquad \Delta = 49 - 60 < 0$

$f'(x) > 0 \ \forall x \in \mathbb{R} \Rightarrow f$ è strett. crescente e INIETTIVA



$f(0) < 0 \qquad f(1) > 0$

$0 \qquad\qquad 1$

$x_1 = \dfrac{0 + 1}{2} = \dfrac{1}{2} \qquad f\left(\dfrac{1}{2}\right) = \ldots = 2 > 0$

Per trovare un'approssimazione della soluzione usiamo il metodo di BISEZIONE

$x_1 = 0 \qquad x_2 = 1 \qquad$ 0 è approssimazione per difetto, 1 per eccesso

$f(0) = -5 \qquad f(1) = 5 \qquad$ ERRORE $= 1 - 0 = 1$

x_1	x_2	$f(x_1)$	$f(x_2)$	$\lvert x_1 - x_2 \rvert$	$\frac{x_1+x_2}{2}$	$f\left(\frac{x_1+x_2}{2}\right)$
0	1	-5	5	1	$\frac{1}{2}$	2
0	$\frac{1}{2}$	-5	2	$0{,}5$	$\frac{1}{4}$	$-\frac{13}{16}$
$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{13}{16}$	2	$0{,}25$	$\frac{3}{8}$	$\frac{95}{128}$
$\frac{1}{4}$	$\frac{3}{8}$	$-\frac{13}{16}$	$\frac{95}{128}$	$0{,}125$	$\frac{5}{16}$	$\frac{5}{1024}$
$\frac{1}{4}$	$\frac{5}{16}$	$-\frac{13}{16}$	$\frac{5}{1024}$	$0{,}0625$	$\frac{9}{32}$	$-\frac{3223}{8192}$
$\frac{9}{32}$	$\frac{5}{16}$	$-\frac{3223}{8192}$	$\frac{5}{1024}$	$0{,}03125$	$\frac{19}{64}$	$-\frac{12565}{65536}$
$\frac{19}{64}$	$\frac{5}{16}$	$-\frac{12565}{65536}$	$\frac{5}{1024}$	$0{,}015625$	$\frac{39}{128}$	$-\frac{48649}{524288}$
$\frac{39}{128}$	$\frac{5}{16}$	$-\frac{48649}{524288}$	$\frac{5}{1024}$	$0{,}0078125$	$\frac{79}{256}$	$-\frac{183697}{4194304}$
$\frac{79}{256}$	$\frac{5}{16}$	$-\frac{183697}{4194304}$	$\frac{5}{1024}$	$0{,}00390625$	$\frac{159}{512}$	$-\frac{651553}{33554432}$
$\frac{159}{512}$ $0{,}3105\ldots$	$\frac{5}{16}$ $0{,}3125$			$0{,}0019\ldots$		

STOP

la soluzione è compresa fra $0{,}3105\ldots$ e $0{,}3125$, quindi l'approssimazione con 2 cifre decimali esatte è $\boxed{0{,}31}$

CON EXCEL

N	x_1	x_2	f(x1)	f(x2)	x2-x1	(x1+x2)/2	f((x1+x2)/2)
1	0	1	-5	5	1	0.5	2
2	0	0.5	-5	2	0.5	0.25	-0.8125
3	0.25	0.5	-0.8125	2	0.25	0.375	0.7421875
4	0.25	0.375	-0.8125	0.7421875	0.125	0.3125	0.004882813
5	0.25	0.3125	-0.8125	0.004882813	0.0625	0.28125	-0.393432617
6	0.28125	0.3125	-0.393432617	0.004882813	0.03125	0.296875	-0.191726685
7	0.296875	0.3125	-0.191726685	0.004882813	0.015625	0.3046875	-0.092790604
8	0.3046875	0.3125	-0.092790604	0.004882813	0.0078125	0.30859375	-0.043796778
9	0.30859375	0.3125	-0.043796778	0.004882813	0.00390625	0.310546875	-0.019417793
10	0.310546875	0.3125	-0.019417793	0.004882813	0.001953125	0.311523438	-0.007257704
11	0.311523438	0.3125	-0.007257704	0.004882813	0.000976563	0.312011719	-0.001185
12	0.312011719	0.3125	-0.001185	0.004882813	0.000488281	0.312255859	0.001849517
13	0.312011719	0.312255859	-0.001185	0.001849517	0.000244141	0.312133789	0.000332411
14	0.312011719	0.312133789	-0.001185	0.000332411	0.00012207	0.312072754	-0.000426256
15	0.312072754	0.312133789	-0.000426256	0.000332411	6.10352E-05	0.312103271	-4.69131E-05
16	0.312103271	0.312133789	-4.69131E-05	0.000332411	3.05176E-05	0.31211853	0.000142751
17	0.312103271	0.31211853	-4.69131E-05	0.000142751	1.52588E-05	0.312110901	4.79198E-05
18	0.312103271	0.312110901	-4.69131E-05	4.79198E-05	7.62939E-06	0.312107086	5.03512E-07
19	0.312103271	0.312107086	-4.69131E-05	5.03512E-07	3.8147E-06	0.312105179	-2.32047E-05
20	0.312105179	0.312107086	-2.32047E-05	5.03512E-07	1.90735E-06	0.312106133	-1.13506E-05
21	0.312106133	0.312107086	-1.13506E-05	5.03512E-07	9.53674E-07	0.312106609	-5.42354E-06