Волошин В.О. ИВТ 2.1

$$\begin{cases} 2x_1 + 3x_2 - 4x_3 + 2x_4 + 5x_5 = 9 \\ -x_1 + 2x_2 - 3x_3 - x_4 - 4x_5 = -1 \\ x_1 - 2x_2 + 2x_3 + x_4 + 3x_5 = 3 \\ 0 + x_2 + 2x_3 - x_4 + 2x_5 = -1 \\ 0 + 7x_2 + x_3 + 0 + x_5 = 2 \end{cases}$$

$A \cdot X = B$

1 способ (приведение матрицы к верхней Δ-ой)

$$\left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ -1 & 2 & -3 & -1 & -4 & -1 \\ 1 & -2 & 2 & 1 & 3 & 3 \\ 0 & 1 & 2 & -1 & 2 & -1 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \overset{II-(-\frac{1}{2})\cdot I}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 1 & -2 & 2 & 1 & 3 & 3 \\ 0 & 1 & 2 & -1 & 2 & -1 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \overset{III-\frac{1}{2}\cdot I}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 0 & -3/2 & 4 & 0 & 1/2 & -3/2 \\ 0 & 1 & 2 & -1 & 2 & -1 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right)$$

$$\overset{III-(-1)\cdot II}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 & 2 & -1 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \overset{IV-\frac{2}{7}\cdot II}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 0 & 0 & 24/7 & -1 & 17/7 & -2 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \overset{V-2\cdot II}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 0 & 0 & 24/7 & -1 & 17/2 & -2 \\ 0 & 0 & 11 & 0 & 4 & -5 \end{array}\right)$$

$$\overset{IV-\left(\frac{-24}{7}\right)\cdot III}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 0 & 0 & 0 & -1 & -1 & 34/7 \\ 0 & 0 & 11 & 0 & 4 & -5 \end{array}\right) \overset{V-(-11)\cdot IV}{\sim} \left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ 0 & 7/2 & -5 & 0 & -3/2 & 7/2 \\ 0 & 0 & -1 & 0 & -1 & 2 \\ 0 & 0 & 0 & -1 & -1 & 34/7 \\ 0 & 0 & 0 & 0 & -7 & 17 \end{array}\right)$$

Корни:

$-7x_5 = 17 \Rightarrow x_5 = -\frac{17}{7}$

$-x_4 = \frac{34}{7} + \frac{-17}{7} = \frac{17}{7} = \frac{-17}{7}$

$-x_3 = 2 + -\frac{17}{7} = \frac{-3}{7} = \frac{3}{7}$

$\frac{7}{2}x_2 = \frac{7}{2} + 5 \cdot \frac{3}{7} + \frac{3}{2} \cdot \frac{-17}{7} = 2 = \frac{4}{7}$

$2x_1 = 9 - 3 \cdot \frac{4}{7} + 4 \cdot \frac{3}{7} - 2 \cdot \frac{-17}{7} - 5 \cdot \frac{-17}{7} = 26 = 13$

Ответ: $X_1 = 13, X_2 = 4/7, X_3 = 3/7, X_4 = -17/7, X_5 = -17/7$

II способ (приведем матрицу к линейной з-ч)

$$\left(\begin{array}{ccccc|c} 2 & 3 & -4 & 2 & 5 & 9 \\ -1 & 2 & -3 & -1 & -4 & -1 \\ 1 & -2 & 2 & 1 & 3 & 3 \\ 0 & 1 & 2 & -1 & 2 & -1 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} \text{I} - 5\text{V} \\ \text{II} + 4\text{V} \\ \text{III} - 3\text{V} \\ \text{IV} - 2\text{V} \end{array} \left(\begin{array}{ccccc|c} 2 & -32 & -9 & 2 & 0 & -1 \\ -1 & 30 & 1 & -1 & 0 & 7 \\ 1 & -23 & -1 & 1 & 0 & -4 \\ 0 & -13 & 0 & -1 & 0 & -5 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \sim \begin{array}{l} \text{I} + 2 \cdot \text{V} \\ \text{II} - \text{IV} \\ \text{III} + \text{IV} \end{array}$$

$$\sim \left(\begin{array}{ccccc|c} 2 & -58 & -9 & 0 & 0 & 3 \\ -1 & 43 & 1 & 0 & 0 & 12 \\ 1 & -36 & -1 & 0 & 0 & -9 \\ 0 & -13 & 0 & -1 & 0 & -5 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} \text{I} + 9 \cdot \text{II} \\ \text{III} + \text{III} \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} -7 & 379 & 0 & 0 & 0 & 111 \\ 0 & 7 & 0 & 0 & 0 & 3 \\ 1 & -36 & -1 & 0 & 0 & -9 \\ 0 & -13 & 0 & -1 & 0 & -5 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} \text{I} + 47 \cdot \text{II} \\ \text{III} \end{array} \sim \left(\begin{array}{ccccc|c} -7 & 0 & 0 & 0 & 0 & -30 \\ 0 & 7 & 0 & 0 & 0 & 3 \\ 1 & -36 & -1 & 0 & 0 & -9 \\ 0 & -13 & 0 & -1 & 0 & -5 \\ 0 & 7 & 1 & 0 & 1 & 2 \end{array}\right)$$