Тест 4.

1.1.3.5 ~~$z = \frac{x^2y^2}{x^2y^2-(x-y)^2}$, $M_0(2;2)$, $\Delta x = 0,2$, $\Delta y = 0,1$.~~

$x_0 = 2$, $y_0 = 1$ $z = 3x^2 + xy - y^2 + 1$, $M_0(2;1)$, $\Delta x = 0,1$ $\Delta y = 0,2$.

$x_0 + \Delta x = 2,1$ $y_0 + \Delta y = 1,2$

$z(M_0) = z(2;1) = 3 \cdot 2^2 + 2 \cdot 1 - 1^2 + 1 = 14$

$z(x_0 + \Delta x; y_0) = z(2,1; 1) = 3 \cdot 2,1^2 + 2,1 \cdot 1 - 1^2 + 1 = 13,23 + 2,1 - 1 = 15,33$

$z(x_0; y_0 + \Delta y) = z(2; 1,2) = 3 \cdot 2^2 + 2 \cdot 1,2 - 1,2^2 + 1 = 13,96$

$z(x_0 + \Delta x; y_0 + \Delta y) = z(2,1; 1,2) = 3 \cdot 2,1^2 + 2,1 \cdot 1,2 - 1,2^2 + 1 = 15,31$

1.1.3.3. $z = \frac{x^2y^2}{x^2y^2-(x-y)^2}$, $M_0(2;1)$ $\Delta x = 0,2$ $\Delta y = 0,1$

$x_0 = 2$; $y_0 = 2$, $\Delta x + x_0 = 1,8$ $y_0 + \Delta y = 2,1$

$z(x_0 y_0) = z(M_0) = z(2;2) = \frac{16}{16} = 1$

$z(x_0 + \Delta x; y_0) = z(1,8; 2) = 1,00309598$

$z(x_0; y_0 + \Delta y) = z(2; 2,1) = 1,00056721$

$z(x_0 + \Delta x; y_0 + \Delta y) = z(1,8; 2,1) = 1,00633879$

$\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0; y_0) = 1,00309598 - 1 = 0,03$.

$\Delta_y z = z(x_0; y_0 + \Delta y) - z(x_0; y_0) = 0,0006$.

$\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) = 0,0063$.

1.1.3.4.

$z = \frac{(x^2+y^2)^2}{xy}$, $M_0(1;1)$, $\Delta x = -0{,}1$, $\Delta y = -0{,}1$

$x_0 = 1$; $y_0 = 1$

$x_0 + \Delta x = 0{,}9$ $\quad y_0 + \Delta y = 0{,}9$.

$z(M_0) = z(1;1) = 4$

$z(x_0 + \Delta x; y_0) = z(0{,}9; 1) = 4{,}04$.

$z(x_0; y_0 + \Delta y) = z(1; 0{,}9) = 4{,}09$.

$z(x_0 + \Delta x; y_0 + \Delta y) = z(0{,}9; 0{,}9) = 4$.

$\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0; y_0) = 0{,}4$.

$\Delta_y z = z(x_0; y_0 + \Delta y) = z(x_0 y_0) = 0{,}4$.

$\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) = 0$.

1.1.36. $z = 3x^2 + xy - y^2 + 1$; $M_0(2;1)$, $\Delta x = 0{,}01$, $\Delta y = 0{,}02$.

$x_0 = 2$; $y_0 = 1$

$x_0 + \Delta x = 2{,}01$ $\quad y_0 + y\Delta = 1{,}02$.

$z(M_0) = z(2;1) = 14$.

$z(x_0 + \Delta x; y_0 + y\Delta) = z(2{,}01; 1{,}02) = 14{,}1301$.

$\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) = 0{,}1301$

1.1.3.7 $z = x^2 - xy + y^2$, $M_0(2;1)$ $M_1(2,1; 1,2)$.

$x_0 = 2$ $y_0 = 1$

$x_0 + \Delta x = 2,1$ $y_0 + \Delta y = 1,2$.

$\Delta z = z(M_1) - z(M_0)$

$z(M_1) = (2,1)^2 - 2,1 \cdot 1,2 + 1,2^2 = 3,33$

$z(M_0) = 2^2 - 2 \cdot 1 + 1^2 = 3$

$\Delta z = 3,33 - 3 = 0,33$

1.1.3.8.

$z = \lg(x^2 + y^2)$ $M_0(2;1)$ $M_1(2,1; 0,9)$

$\Delta z = z(M_1) - z(M_0) = \lg(2,1^2 + 0,9^2) - \lg(2^2 + 1^2) = \lg 1,044$.

1.1.3.13. $U = x^4 \cos^2 y - y^2 \sin^3 x^5$

$U'_x = (x^4 \cos^2 y - y^2 \sin^3 x^5)'_x = 4x^3 \cos^2 y - 15 y^2 x^4 \sin^2 x^5 \cos x^5$

$U'_y = (x^4 \cos^2 y - y^2 \sin^3 x^5)'_y = -2 \cos y \sin y \cdot x^4 - 4y \sin^3 x^5 =$

$= -\sin 2y \cdot x^4 - 4y \sin^3 x^5$

1.1.3.14 $z = x^2 \cos 2xy - 2x^2 y \sin xy - y^2 \cos(x+y)$

$z'_y = -2x^2 \sin 2xy - 2y \sin(x+y) - y^2 \cos(x+y)$

1.1.3.15. $u = x^y \cdot (xy)^z + z^{xy}$

$u'_x = y x^{y-1} + z x^{z-1} y^z + z^{xy} \ln z$

$u'_y = x^y \ln x + x^z \cdot z\, y^{z-1} + z^{xy} \ln z$

1.1. 3.19. $1,04^{2,03}$.

$f(x;y) = x^y$, $x = 1,04$; $y = 2,03$.

$x_0 = 1$ $y_0 = 2$ $f(1;2) = 1$

$\Delta x = 0,04$ $\Delta y = 0,03$.

$f'_x = (x^y)'_x = y \cdot x^{y-1}$

$f'_y = x^y \cdot \ln x$

$f'_x(1;2) = 2 \cdot 1^{2-1} = 2$

$f'_y(1;2) = 1^2 \cdot \ln 1 = 0$

$f(1,04;2,03) = f(1;2) + f'_x(1;2) \cdot \Delta x + f'_y(1;2) \cdot \Delta y = 1,08$.

11.3.20 $\sqrt{(1,04)^2 + (3,01)^2}$

$f(x;y) = \sqrt{x^2+y^2}$, $x = 1,04$; $y = 3,01$

$x_0 = 1$ $y_0 = 3$ $f(1;3) = \sqrt{10}$

$\Delta x = 0,04$ $\Delta y = 0,01$

$f'_x(\sqrt{x^2+y^2})'_x = \frac{x}{\sqrt{x^2+y^2}}$, $f'_y(\sqrt{x^2+y^2})'_y = \frac{y}{\sqrt{x^2+y^2}}$

$f'_y(1;3) = \frac{3}{\sqrt{10}}$

$f'_x(1;3) = \frac{3}{\sqrt{10}}$

$f(1,03;3,01) = \sqrt{10} + \frac{1}{\sqrt{10}} \cdot 0,01 + \frac{3}{\sqrt{10}} \cdot 0,01 = 3,186$

1.1.3.21. $\sin 28^\circ \cdot \cos 61^\circ$

$f(x;y) = \sin x \cdot \cos y$

$x_0 = 30^\circ = \frac{\pi}{6}$ $\quad y_0 = 60^\circ = \frac{\pi}{3}$

$\Delta x = -\pi/30$ $\quad \Delta y = \frac{\pi}{160}$

$f(x_0; y_0) = f\left(\frac{\pi}{6}; \frac{\pi}{3}\right) = \sin\frac{\pi}{6} \cdot \cos\frac{\pi}{3} = \frac{1}{4}$

$f_x'\left(\frac{\pi}{6}; \frac{\pi}{3}\right) = \cos\frac{\pi}{6} \cos\frac{\pi}{3} = \frac{\sqrt{3}}{4}$

$f_y'\left(\frac{\pi}{6}; \frac{\pi}{3}\right) = -\sin\frac{\pi}{6} \cdot \sin\frac{\pi}{3} = -\frac{\sqrt{3}}{4}$

$f(28^\circ; 61^\circ) = \frac{1}{4} + \frac{\sqrt{3}}{4} \cdot \left(-\frac{\pi}{90}\right) + \left(-\frac{\sqrt{3}}{4}\right) \cdot \frac{\pi}{180} = 0{,}0227.$