

Задача 2.

7.2.4 $dy = ?$ $y = x^2 \ln x$

$$y' = (x^2 \ln x)' = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x = x(2 \ln x + 1)$$

$$\Rightarrow dy = y' dx = (x(2 \ln x + 1)) dx$$

7.2.5. $dy = ?$ $y = \frac{x-2}{x^2+1}$

$$y' = \left(\frac{x-2}{x^2+1} \right)' = \frac{1(x^2+1) - 2x \cdot (x-2)}{(x^2+1)^2} = \frac{-x^2+4x+1}{(x^2+1)^2}$$

$$\Rightarrow dy = y' dx = \left(\frac{-x^2+4x+1}{(x^2+1)^2} \right) dx$$

7.2.8. $\Delta y, dy = ?$, $y = x^2 + x - 5$, $x_0 = 0$, $\Delta x = 0,5$

$$\Delta y = y(x+\Delta x) - y(x) = (x+\Delta x)^2 + (x+\Delta x) - 5 - (x^2 + x - 5) = x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x - x^2 - x = \Delta x(2x+1) + (\Delta x)^2$$

$$\Delta y \Big|_{\substack{x_0=0 \\ \Delta x=0,5}} = 0,5(2 \cdot 0 + 1) + (0,5)^2 = 0,75$$

$$dy \Big|_{\substack{x_0=0 \\ \Delta x=0,5}} = 0,5(2 \cdot 0 + 1) = 0,5$$

7.2.10. $\sqrt[3]{26}$

$$x = 26 = 27 + (-1) \Rightarrow x_0 = 27, \Delta x = -1$$

$$\Rightarrow \sqrt[3]{26} \approx \sqrt[3]{27} + \frac{1}{3\sqrt[3]{27^2}} \cdot (-1) = 3 + \frac{1}{3 \cdot 9} \cdot (-1) = 2,96$$

7.2.12. $(1,02)^5$

$$F(x_0 + \Delta x) \approx F(x_0) + F'(x_0) \Delta x$$

$$(x_0 + \Delta x)^5 = (x_0)^5 + (x^5)'_{x_0} \Delta x$$

$$x = 1,02 = 1 + 0,02 \Rightarrow x_0 = 1, \Delta x = 0,02$$

$$(1,02)^5 = 1^5 + 5 \cdot 1^4 \cdot 0,02 = 1 + 0,1 = 1,1$$

7.2.14. $y = (x^2 + 1)^3$

$$1) dy = F'(x) dx = ((x^2 + 1)^3)' dx = 3(x^2 + 1)^2 (x^2 + 1)' dx = 6x(x^2 + 1)^2 dx$$

$$2) d^2y = d(dy) = d(6x(x^2 + 1)^2 dx) = (6x(x^2 + 1)^2)' dx^2 = (6x^5 + 12x^3 + 6x)' dx^2 = (30x^4 + 36x^2 + 6) dx^2 = 6(5x^4 + 6x^2 + 1) dx^2$$

7.2.15. $y = \sin^2 x$

$$1) dy = F'(x) dx = (\sin^2 x)' dx = 2 \sin x \cos x dx = \sin 2x dx$$

$$2) d^2y = d(dy) = d(\sin 2x dx) = (\sin 2x)' dx^2 = 2 \cos 2x dx^2$$