Часть 5.

8519.

$$\int \frac{dx}{\cos x} = \ln\left| tg\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$$

8520

$$\int \frac{dx}{1-\sin x} = \int \frac{1}{1-\frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} = \int \frac{1}{\frac{1+t^2-2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} = \int \frac{1+t^2}{(t-1)^2} \cdot \frac{2dt}{1+t^2} =$$

$$= 2\int \frac{dt}{(t-1)^2} = 2\int \frac{d(t-1)}{(t-1)^2} = 2\frac{(t-1)^{-2}}{-2+1} + C = -\frac{2}{tg\frac{x}{2}-1} + C$$

8521.

$$\int \frac{dx}{5+4\sin x} = \int \frac{1+t^2}{5+5t^2+8t} \cdot \frac{2dt}{1+t^2} = \int \frac{2dt}{5t^2+2\cdot\sqrt{5}\cdot t\cdot\frac{4}{\sqrt{5}}+\frac{16}{5}+\frac{9}{5}} =$$

$$= 2\int \frac{dt}{\left(\sqrt{5}t+\frac{4}{\sqrt{5}}\right)^2+\left(\frac{3}{\sqrt{5}}\right)^2} = \frac{2}{\sqrt{5}} \cdot \frac{1}{\frac{3}{\sqrt{5}}} \cdot arctg\left(\frac{\sqrt{5}}{3}\cdot\left(\sqrt{5}\cdot t+\frac{4}{\sqrt{5}}\right)\right) + C =$$

$$= \frac{2}{3}\, arctg\left(\frac{5t+4}{3}\right) + C = \frac{2}{3}\, arctg\left(\frac{5tg\left(\frac{x}{2}\right)+4}{3}\right) + C$$

8522.

$$\int \frac{2-\sin x}{2+\cos x}dx = \int \frac{2+2t^2-2t}{2+2t^2-1-t^2} \cdot \frac{2dt}{1+t^2} = 4\int \frac{t^2-t+1}{(t^2+3)(t^2+1)} =$$

$$= 4\left(\frac{1}{2}\int\frac{t+2}{t^2+3}dt - \frac{1}{2}\int\frac{t\,dt}{t^2+1}\right) = 4\left(\frac{1}{2}\int\frac{t}{t^2+3}dt + \int\frac{dt}{t^2+3} - \frac{1}{2}\int\frac{dt\,t}{t^2+1}\right) =$$

$$= \int\frac{du}{u} - \int\frac{dv}{v} + 4\int\frac{dt}{t^2+3} = (\ln|u| - \ln|v|) + 4\cdot\frac{1}{\sqrt{3}}\cdot arctg\frac{t}{\sqrt{3}} + C$$

$$= \ln\left|\frac{t^2+3}{t^2+1}\right| + \frac{4}{\sqrt{3}}\cdot arctg\frac{t}{\sqrt{3}} + C = \ln\left|1+\frac{2}{\left(tg\frac{x}{2}\right)^2+1}\right| + \frac{4}{\sqrt{3}}\cdot arctg\left(\frac{tg\frac{x}{2}}{\sqrt{3}}\right) + C =$$

$$= \frac{2}{\frac{1}{\cos^2\frac{x}{2}}} = 2\cos^2\frac{x}{2} = 2\cdot\frac{1+\cos x}{2} = \ln(2+\cos x) + \frac{4}{\sqrt{3}}\cdot arctg\left(\frac{tg\frac{x}{2}}{\sqrt{3}}\right) + C.$$

8523

$$\int\frac{dx}{2\sin x-\cos x+5}=\int\frac{1}{\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}+5}\cdot\frac{2dt}{1+t^2}=$$

$$=\int\frac{dt}{6t^2+4t+4}=\int\frac{dt}{3t^2+2\cdot\sqrt{3}t\cdot\frac{1}{\sqrt{3}}+\frac{1}{3}+\frac{5}{3}}=\int\frac{dt}{(\sqrt{3}t+\frac{1}{\sqrt{3}})^2+(\frac{\sqrt{5}}{\sqrt{3}})^2}=$$

$$=\frac{1}{\sqrt{5}}\cdot arctg\left(\frac{3t+1}{\sqrt{5}}\right)+C=\frac{1}{\sqrt{5}}\,arctg\left(\frac{3tg(\frac{x}{2})+1}{\sqrt{5}}\right)+C.$$

8524.

$$\frac{1+\sin x}{(1+\cos x)\sin x}\,dx=\int\frac{1+\frac{2t}{1+t^2}}{(1+\frac{1+t^2}{1-t^2})\cdot\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int\frac{(t+1)^2\cdot(1+t^2)}{4t}\cdot\frac{2dt}{1+t^2}=$$

$$=\frac{1}{2}\int\frac{t^2+2t+1}{t}\,dt=\frac{1}{4}tg^2\frac{x}{2}+tg\frac{x}{2}+\frac{1}{2}\ln|tg\frac{x}{2}|+C.$$

8525.

$$\int\frac{dx}{3\sin^2x-\cos^2x+4}=\int\frac{1}{\frac{3t^2}{1+t^2}-\frac{3}{1+t^2}+\frac{4+4t^2}{1+t^2}}\cdot\frac{dt}{1+t^2}=$$

$$=\frac{1}{9}\int\frac{dt}{t^2+(\frac{1}{3})^2}=\frac{1}{9}\cdot\frac{1}{\frac{1}{3}}\cdot arctg\left(\frac{t}{\frac{1}{3}}\right)+C=\frac{1}{3}arctg(3tg\,x)+C.$$

8526

$$\int\frac{dx}{4\sin^2x+9\cos^2x}=\int\frac{1}{\frac{4t^2}{1+t^2}+\frac{9}{1+t^2}}\cdot\frac{dt}{1+t^2}=\int\frac{dt}{4t^2+9}=\frac{1}{4}\int\frac{dt}{t^2+(\frac{3}{2})^2}=$$

$$=\frac{1}{4}\cdot\frac{1}{\frac{3}{2}}\cdot arctg\left(\frac{t}{\frac{3}{2}}\right)+C=\frac{1}{6}\,arctg\left(\frac{2tgx}{3}\right)+C.$$

8527

$$\int\frac{dx}{1+3\cos x}=\int\frac{1}{1+\frac{3}{1+t^2}}\cdot\frac{dt}{1+t^2}=\int\frac{t^2+1}{t^2+4}\cdot\frac{dt}{t^2+1}=\frac{1}{2}\,arctg\frac{t}{2}+C=$$

$$=\frac{1}{2}\,arctg\left(\frac{tgx}{2}\right)+C.$$

8528.

$$\int \frac{dx}{\sin^4 x} = \int \frac{1}{\frac{t^4}{(1+t^2)^2}} \cdot \frac{dt}{1+t^2} = \int \frac{(1+t^2)\,dt}{t^4} =$$

$$= \frac{t^{-3}}{-3} + \frac{t^{-1}}{-1} + C = -\frac{1}{3}ctg^3 x - ctg\, x + C$$

8529.

$$\int \sin^5 x\,dx = \int (\sqrt{1-t^2})^5 \cdot \frac{-dt}{\sqrt{1-t^2}} = -\int (t^4 - 2t^2 + 1)\,dt =$$

$$= -\frac{\cos^5 x}{5} + \frac{2\cos^3 x}{3} - \cos x + C$$

8530

$$\int \sin^4 x \cdot \cos^5 x\,dx = \int t^4 (\sqrt{1-t^2})^5 \cdot \frac{dt}{\sqrt{1-t^2}} =$$

$$= \int t^4 (-1+t^2)^2\,dt = \frac{\sin^9 x}{9} - \frac{2\sin^7 x}{7} + \frac{\sin^5 x}{5} + C$$

8531.

$$\int \frac{\sin 2x\,dx}{\cos^7 x} = \int \frac{2\cos x \sin x}{\cos^7 x}\,dx = 2\int \frac{\sin x}{\cos^6 x}\,dx =$$

$$= 2\int \frac{\sqrt{1-t^2}}{t^6} \cdot \frac{-dt}{\sqrt{1-t^2}} = \frac{2}{5t^5} + C = \frac{2}{5\cos^5 x} + C$$

8532.

$$\int \frac{\sin^4 x}{\cos x}\,dx = \int \frac{t^4}{\sqrt{1-t^2}} \cdot \frac{dt}{\sqrt{1-t^2}} = \int \frac{(t^2-1)(t^2+1)+1}{t^2-1}\,dt =$$

$$= -\int t^2\,dt - \int dt - \int \frac{dt}{t^2-1} = -\frac{t^3}{3} - t - \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C =$$

$$= -\frac{\sin^3 x}{3} - \sin x - \frac{1}{2}\ln\left|\frac{\sin x - 1}{\sin x + 1}\right| + C$$

8535

$\int \sin^4 x \cdot \cos^4 x\,dx = \int (\sin x \cdot \cos x)^4 dx = \frac{1}{16}\int \sin^4 2x\,dx =$

$= \frac{1}{64}\int (1-\cos 4x)^2 dx = \frac{1}{64} - \frac{1}{32}\int \cos 4x\,dx + \frac{1}{64}\int \cos^2 4x\,dx =$

$= \frac{1}{64}\int dx - \frac{1}{32}\int \cos 4x\,dx + \frac{1}{128}\int dx + \frac{1}{128}\int \cos 8x\,dx =$

$= \frac{\sin 8x}{1024} - \frac{\sin 4x}{128} + \frac{3x}{128} + C = \frac{\sin 8x - 8\sin 4x + 24x}{1024} + C$

8536

$\int \sin x \cdot \sin 3x\,dx = \frac{1}{2}\int \cos 2x\,dx - \frac{1}{2}\int \cos 4x\,dx = \frac{1}{2}\cdot\frac{1}{2}\cdot \sin 2x - \frac{1}{2}\cdot\frac{1}{4}$

$\sin 4x + C = \frac{\sin 2x}{4} - \frac{\sin 4x}{8} + C.$

8537

$\int \sin\frac{x}{12} \cos\frac{x}{3}\,dx = \int \cos\frac{4x}{12} \cdot \sin\frac{x}{12}\,dx =$

$= \frac{1}{2}\int \sin\frac{5x}{12}\,dx - \frac{1}{2}\int \sin\frac{x}{4}\,dx = -\frac{1}{2}\cdot\frac{1}{\frac{5}{12}}\cdot\cos\frac{5x}{12} - \frac{1}{2}\cdot\frac{1}{\frac{1}{4}}\cdot(-1)\cos\frac{x}{4} + C =$

$= 2\cos\left(\frac{x}{4}\right) - \frac{6}{5}\cos\left(\frac{5x}{12}\right) + C$

8538

$\int \cos x \cdot \cos 3x\,dx = \int \cos 3x \cdot \cos x\,dx =$

$= \frac{1}{2}\int \cos 4x\,dx + \frac{1}{2}\int \cos 2x\,dx =$

$= \frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x + C.$

8541.

$\int tg^4\frac{x}{2}\,dx = \int tg^2\frac{x}{2}\cdot\left(\frac{1}{\cos\frac{x}{2}} - 1\right)dx = \int \frac{tg^2\frac{x}{2}}{\cos^2\frac{x}{2}}\,dx -$

$- \int \frac{dx}{\cos\frac{x}{2}} + \int dx = 2\int t^2\,dt - \int \frac{dx}{\cos\frac{x}{2}} + \int dx =$

$= 2\cdot\frac{t^3}{3} - \frac{1}{1/2}\cdot tg\frac{x}{2} + x + C = \frac{2\,tg^3\frac{x}{2}}{3} - 2\,tg\frac{x}{2} + x + C.$

8542.

$\int tg^7 x\,dx = \int tg^5 x\left(\frac{1}{\cos^2 x} - 1\right)dx = \int \frac{tg^5 x}{\cos^2 x}\,dx -$

$- \int \frac{tg^3 x}{\cos^2 x}\,dx + \int tg^3 x\,dx = \int \frac{tg^5 x}{\cos^2 x}\,dx - \int \frac{tg^3 x}{\cos^2 x}\,dx +$

$+ \int \frac{tg\,x}{\cos^2 x}\,dx - \int tg\,x\,dx = \int t^5\,dt - \int t^3\,dt + \int t\,dt - \int tg\,x\,dx =$

$= \frac{t^6}{6} - \frac{t^4}{4} + \frac{t^2}{2} + \ln|\cos x| + C = \frac{tg^6 x}{6} - \frac{tg^4 x}{4} + \frac{tg^2 x}{2} + \ln|\cos x| + C.$