Часть 4.

8412

$$\int \frac{dx}{x+\sqrt[3]{x^2}} = \int \frac{3t^2dt}{t^3+t^2} = 3\int \frac{t^2dt}{t^2(t+1)} = 3\int \frac{dt}{t+1} =$$

$$= 3\ln|t+1|+C = 3\ln|\sqrt[3]{x}+1|+C$$

8413

$$\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}\,dx = \int \frac{t^2\cdot 4t^3dt}{1+t^3} = \int \frac{4t^5}{1+t^3}dt = 4\int \frac{t^5}{1+t^3}dt =$$

$$= 4\cdot\int \frac{1}{3}\,\frac{(u-1)}{u}\,du = \frac{4}{3}\int du - \frac{4}{3}\int \frac{1}{u}\,du = \frac{4}{3}u - \frac{4}{3}\ln|u|+C =$$

$$= \frac{4}{3}(1+t^3) - \frac{4}{3}\ln|1+t^3|+C = \frac{4}{3}\cdot\sqrt[4]{x^3} - \frac{4}{3}\ln|1+\sqrt[4]{x^3}|+C.$$

8414.

$$\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1-\sqrt[3]{x})}\,dx = \int \frac{t^6+t^4+t}{t^6-t^8}\cdot 6t^5dt =$$

$$= 6\int \frac{t(t^5+t^3+1)}{t(1-t^2)}\,dt = 6\int \frac{t^5+t^3+1}{1-t^2}\,dt =$$

$$= 6\int \frac{t^5+t^3+1}{t^2-1}\,dt = -6\int \frac{(t^2-1)(t^3+2t)+2t+1}{t^2-1} = -6\int \frac{(2t+1)\,dt}{t^2-1}$$

$$-6\int t^3dt - 12\int t\,dt = -12\int \frac{t\,dt}{t^2-1} - 6\int \frac{dt}{t^2-1} - 6\int t^3dt - 12\int t\,dt =$$

$$= -6\int \frac{du}{u} - 6\int \frac{dt}{t^2-1} - 6\int t^3dt - 12\int dt = -6\ln|u| -$$

$$- 6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| - 6\cdot\frac{t^4}{4} - 12\frac{t^2}{2}+C = -6\ln|\sqrt[3]{x}-1| - 3\ln$$

$$\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| - \frac{3}{2}\sqrt[3]{x^2} - 6\sqrt[3]{x}+C = -6\ln|\sqrt[3]{x}-1| - 3\ln|\sqrt[6]{x}-1| +$$

$$+ 3\ln|\sqrt[6]{x}+1| - \frac{3}{2}\sqrt[3]{x^2} - 6\sqrt[3]{x}+C = -9\ln|\sqrt[6]{x}-1| + 3\ln$$

$$|\sqrt[6]{x}+1| - \frac{3}{2}\sqrt[3]{x^2} - 6\sqrt[3]{x}+C.$$

8415.

$$\int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}} = 6\int \frac{t^3 t^5}{t^4(t^2-1)}dt = 6\int \frac{t^4}{t^2-1}dt =$$

$$= 6\int \frac{(t^2-1)(t^2+1)+1}{t^2-1}dt = 6\int \frac{dt}{t^2-1} + 6\int t^2 dt + 6\int dt =$$

$$= 6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + 6\frac{t^3}{3} + 6t + C = 3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| + 2\sqrt{x} + 6\sqrt[6]{x} + C.$$

8416.

$$\int \frac{\sqrt{x}\,dx}{1+\sqrt{x}} = 2\int \frac{t\cdot t\,dt}{1+t} = 2\int \frac{t^2-1+1}{t+1}dt = 2\int \frac{(t-1)(t+1)}{t+1}dt +$$

$$+ 2\int \frac{dt}{t+1} = 2\int t\,dt - 2\int dt + 2\int \frac{dt}{t+1} = 2\cdot\frac{t^2}{2} - 2t + 2\ln|t+1| + C =$$

$$= x - 2\sqrt{x} + 2\ln(\sqrt{x}+1) + C.$$

8417

$$\int \frac{\sqrt{x}\,dx}{1-\sqrt[3]{x}} = 6\int \frac{t^8}{1-t^2}dt = -6\int \frac{t^8}{t^2-1}dt = -6\int \frac{(t^2-1)(t^6+t^4+t^2+1)+1}{t^2-1}dt =$$

$$= -6\int \frac{dt}{t^2-1} - 6\int t^6 dt - 6\int t^4 dt - 6\int t^2 dt - 6\int dt =$$

$$= -3\ln\left|\frac{t-1}{t+1}\right| - 6\cdot\frac{t^7}{7} - 6\cdot\frac{t^5}{5} - 6\cdot\frac{t^3}{3} - 6t + C =$$

$$= -3\ln\left|\frac{x^{1/6}-1}{x^{1/6}+1}\right| - \frac{6}{7}\cdot x^{7/6} - \frac{6}{5}\cdot x^{5/6} - 2\cdot x^{1/2} - 6x^{1/6} + C.$$

8418

$$\int \frac{\sqrt{x+2}}{x}dx = \int \frac{t \cdot 2t}{t^2-2}dt = 2\int\frac{t^2}{t^2-2}dt = 2\int\frac{t^2-2}{t^2-2}dt + 2\int\frac{2}{t^2-2}dt =$$

$$= 2\int\frac{t^2}{t^2-2}dt = 2\int\frac{t^2-2}{t^2-2}dt + 2\int\frac{2}{t^2-2}dt = 2\int dt + 4\int\frac{dt}{t^2-2} = 2t + 4\cdot$$

$$\frac{1}{2\sqrt2} \ln\left|\frac{t-\sqrt2}{t+\sqrt2}\right| + C = 2\sqrt{x+2} + \sqrt2 \ln\left|\frac{\sqrt{x+2}-\sqrt2}{\sqrt{x+2}+\sqrt2}\right| + C.$$

84.19.

$$\int\frac{x\,dx}{\sqrt{x+1}+\sqrt[3]{x+1}} = \int\frac{(t^6-1)\,6t^5\,dt}{t^3+t^2} = 6\int\frac{t^9+t^3-t^3}{t+1}dt =$$

$$= 6\int(t^8-t^7+t^6-t^5+t^4-t^3)dt = 6\int t^8dt - 6\int t^7dt + 6\int t^6dt -$$

$$-6\int t^5dt + 6\int t^4dt - 6\int t^3dt = 6\cdot\frac{t^9}{9} - 6\cdot\frac{t^8}{8} + 6\cdot\frac{t^7}{7} - 6\cdot\frac{t^6}{6} + 6\cdot\frac{t^5}{5} -$$

$$-6\cdot\frac{t^4}{4} + C = 6\cdot\frac{(\sqrt[6]{x+1})^9}{9} - \frac{6(\sqrt[6]{x+1})^8}{8} + 6\cdot\frac{(\sqrt[6]{x+1})^7}{7} -$$

$$-6\cdot\frac{(\sqrt[6]{x+1})^6}{6} + \frac{6(\sqrt[6]{x+1})^5}{5} - 6\frac{(\sqrt[6]{x+1})^4}{4} + C =$$

$$= \frac{2}{3}(x+1)^{\frac32} - \frac34(x+1)^{4/3} + \frac67(x+1)^{7/6} - x + 1 + \frac65(x+1)^{5/6} - \frac32(x+1)^{\frac23} + C.$$

8420

$$\int\frac{dx}{(x+1)^{3/2}+(x+1)^{1/2}} = 2\int\frac{t\,dt}{t^3+t} = 2\int\frac{dt}{t^2+1} = 2\cdot1\cdot\operatorname{arctg}\frac{t}{1} + C =$$

$$= 2\operatorname{arctg}\sqrt{x+1} + C.$$

8421.

$$\int \frac{\sqrt{1+x}+1}{\sqrt{1+x}-1}\,dx = 2\int \frac{t^2+t}{t-1}\,dt = 2\int \frac{(t-1)(t+2)+2}{t-1}\,dt =$$

$$= 2\int t\,dt + 4\int dt + 4\int \frac{dt}{t-1} = t^2+4t+4\cdot \ln|t-1| = x+1+\sqrt{x+1}\cdot 4 +$$
$$+4\cdot \ln|\sqrt{x+1}-1|+C = x+4\sqrt{x+1}+4\ln|\sqrt{x+1}-1|+C.$$

8422.

$$\int \frac{x-1}{\sqrt{2x-1}}\,dx = \int \frac{t^2-1}{2t}\,t\,dt = \frac{1}{2}\int t^2 dt - \frac{1}{2}\int dt = \frac{1}{2}\cdot\frac{t^3}{3} - \frac{1}{2}t + C =$$
$$= \frac{\sqrt{(2x-1)^3}}{6} - \frac{\sqrt{2x-1}}{2} + C = \frac{(x-2)\sqrt{2x-1}}{3} + C$$

8423

$$\int \frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}} = -2\int \frac{t^3\,dt}{t^2-t} = -2\int (t+1)\,dt - \int \frac{dt}{t-1} =$$

$$= -2\cdot\frac{t^2}{2} - 2t - 2\ln|t-1| + C = -t^2 - 2t - 2\ln|t-1| + C =$$
$$= -\sqrt{1-2x} - 2\sqrt[4]{1-2x} - 2\ln|\sqrt[4]{1-2x}-1| + C.$$

8924.

$$\int \frac{1}{(2-x)^2}\cdot\sqrt{\frac{2-x}{2+x}}\,dx = \int \frac{1}{\left(2-\frac{2-2t^2}{t^2+1}\right)^2}\cdot t\cdot\frac{(-8t)}{(t^2+1)^2}\,dt =$$
$$= \int \frac{(t^2+1)^2}{16t^4}\cdot\frac{t(-8t)}{(t^2+1)^2}\,dt = \int -\frac{dt}{2t^2} = -\frac{1}{2}\int t^{-2}dt =$$
$$= -\frac{1}{2}\cdot\frac{t^{-2+1}}{-2+1} + C = \frac{1}{2t} + C = \frac{\sqrt{2+x}}{2\sqrt{2-x}} + C$$

8425.

$$\int\frac{dx}{\sqrt{(x-1)^3(x-2)}}=\int\frac{\sqrt{x-1}\,dx}{\sqrt{(x-1)^4}\cdot\sqrt{x-2}}=\int\frac{dx}{(x-1)^2\cdot\sqrt{\frac{x-2}{x-1}}}=$$

$$=\int\frac{1}{\left(\frac{t^2-2}{t^2-1}-1\right)^2\cdot t}\cdot\frac{2t}{(t^2-1)^2}dt=$$

$$=\int\frac{(t^2-1)^2}{t}\cdot\frac{2t}{(t^2-1)^2}dt=\int 2dt=2\int dt=2t+C=2\sqrt{\frac{x-2}{x-1}}+C.$$

8426

$$\int\frac{dx}{\sqrt[3]{(x-1)^2(x+1)^4}}=\int\frac{dx}{(x+1)\sqrt[3]{\frac{x+1}{x-1}}}=\int\frac{1}{\left(\frac{t^3+1}{t^3-1}-1\right)t}\cdot\frac{-6t^2}{(t^3-1)^2}dt=$$

$$=-3\int\frac{t}{t^3-1}dt=-3\int\frac{t}{(t-1)(t^2+t+1)}dt=-3\int\frac{1}{3(t-1)}dt-$$

$$-3\int\frac{-t+1}{3(t^2+t+1)}dt=-\int\frac{1}{t-1}dt+\int\frac{t-1}{t^2+t+1}dt=$$

$$=-\int\frac{dt}{t-1}+\frac{1}{2}\int\frac{du}{u}-\frac{3}{2}\int\frac{du}{y^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\ln|t-1|+\frac{1}{2}\ln|u|-$$

$$-\frac{3}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{2y}{\sqrt{3}}+C=-\ln\left|\sqrt[3]{\frac{x+1}{x-1}}-1\right|+$$

$$+\frac{1}{2}\ln\left|\sqrt[3]{\left(\frac{x+1}{x-1}\right)^2}+\sqrt[3]{\frac{x+1}{x-1}}+1\right|-\sqrt{3}\operatorname{arctg}\left(\frac{2\sqrt[3]{\frac{x+1}{x-1}}+1}{\sqrt{3}}\right)$$

8427

$$\int\frac{dx}{(1-x)\sqrt{1-x^2}}=\int\frac{dx}{(1-x)\sqrt{(1-x)(1+x)}}=\int\frac{dx}{(1-x)^2\cdot\sqrt{\frac{1+x}{1-x}}}=$$

$$=\int\frac{1}{\left(1-\frac{t^2-1}{t^2+1}\right)^2 t}\cdot\frac{4t}{(t^2+1)^2}dt=\int\frac{(t^2+1)^2}{4t}\cdot\frac{4t}{(t^2+1)^2}dt=$$

$$=\int dt=t+C=\sqrt{\frac{1+x}{1-x}}+C$$

8428.

$$\int\frac{dx}{x(1+\sqrt[3]{x})^3}=\int x^{-1}(1+x^{1/3})^{-3}dx=\int t^{-3}(1+t)^{-3}\cdot 3t^2dt=$$

$$=\int t^{-1}(t^3+3t^2+3t+1)^{-3}dt=3\int(t^4+3t^3+3t^2+t)^{-1}dt=$$

$$=3\int\frac{dt}{t(t+1)^3}=3\int\frac{dt}{t}-3\int\frac{dt}{t+1}-3\int\frac{dt}{(t+1)^2}-3\int\frac{dt}{(t+1)^3}=$$

$$=3\ln|t|-3\ln|t+1|-\frac{3(t+1)^{-1}}{-1}-\frac{3(t+1)^{-2}}{-2}+C=$$

$$=3\ln(x^{1/3}+1)^{-2}+C=\ln|x|-3\ln(\sqrt[3]{x}+1)+\frac{3}{\sqrt[3]{x}+1}+\frac{3}{2(\sqrt[3]{x}+1)^2}+C$$

8429

$$\int x^3\sqrt{1+x^2}dx=\int x^3\cdot(1+x^2)^{\frac{1}{2}}dx=\int\sqrt{(t^2-1)^3}\,t\cdot\frac{t}{\sqrt{t^2-1}}dt=$$

$$=\int(t^2-1)\cdot t^2dt=\int t^4dt-\int t^2dt=\frac{t^5}{5}-\frac{t^3}{3}+C=\frac{1}{5}(\sqrt{1+x^2})^5-\frac{1}{3}$$

$$(\sqrt{1+x^2})^3+C=\frac{(1+2x^2+x^4)\sqrt{1+x^2}}{5}-\frac{(1+x^2)\sqrt{1+x^2}}{3}+C$$

8430.

$$\int \frac{dx}{x^{11}\sqrt{x^4+1}} = \int x^{-11}\cdot(x^4+1)^{-\frac{1}{2}}dx = \int \sqrt[4]{(t^2-1)^4}\cdot\sqrt{\frac{t^2-1}{t^2}}\cdot(-1)\cdot\frac{t\,dt}{2\sqrt[4]{t^2-1}(t^2-1)}$$

$$= -\frac{1}{2}\int (t^2-1)^2\cdot\sqrt[4]{(t^2-1)^3}\cdot\frac{\sqrt[4]{(t^2-1)^2}}{t}\cdot\frac{t\,dt}{\sqrt[4]{t^2-1}\cdot(t^2-1)} =$$

$$= -\frac{1}{2}\int (t^2-1)^2dt = -\frac{1}{2}\int t^4dt + \frac{1}{2}\int 2t^2dt - \frac{1}{2}\int dt =$$

$$= -\frac{1}{2}\cdot\frac{t^5}{5} + \frac{t^3}{3} - \frac{1}{2}t + C = -\frac{(x^4+1)^2\cdot\sqrt{x^4+1}}{10x^{10}} + \frac{(x^4+1)\sqrt{x^4+1}}{3x^6} -$$

$$-\frac{\sqrt{x^4+1}}{2x^2} + C = -\frac{(3x^8+6x^4+3)\sqrt{x^4+1}}{30x^{10}} + \frac{(10x^8+10x^4)\sqrt{x^4+1}}{30x^{10}} =$$

$$-\frac{15x^8\sqrt{x^4+1}}{30x^{10}} + C = -\frac{(8x^8-4x^4+3)\sqrt{x^4+1}}{30x^{10}} + C.$$

8431.

$$\int\frac{dx}{\sqrt{x}(1-\sqrt{x})^2} = \int x^{-\frac{1}{2}}(1-x^{\frac{1}{2}})^{-2}dx = \int\frac{2t\,dt}{t(1-t)^2} = 2\int\frac{dt}{(t-1)^2} =$$

$$= 2\int (t-1)^{-2}d(t-1) = 2\cdot\frac{(t-1)^{-1}}{-1} + C = -\frac{2}{t-1} + C =$$

$$= \frac{2}{1-\sqrt{x}} + C.$$

8434

$$\int \sqrt{x}(1+\sqrt{x})^3 dx = \int \sqrt{x}(1+3\sqrt{x}+3x+x\sqrt{x})dx =$$

$$= \int (x^{\frac{1}{2}}+3x+3x^{3/2}+x^2)dx = \int x^{\frac{1}{2}}dx + 3\int x\,dx + 3\int x^{\frac{3}{2}}dx +$$

$$+ \int x^2 dx = \frac{x^{3/2}}{3/2} + 3\cdot\frac{x^2}{2} + 3\cdot\frac{x^{5/2}}{5/2} + \frac{x^3}{3} + C =$$

$$= \frac{2x\sqrt{x}}{3} + \frac{3x^2}{2} + \frac{6x^2\sqrt{x}}{5} + \frac{x^3}{3} + C.$$

8435.

$$\int \sqrt[3]{x^3-4}\cdot x^2 dx = \int x^2\cdot(x^3-4)^{\frac{1}{3}}dx = \int \sqrt[3]{(t^3+4)^2}\cdot t\cdot\frac{t^2}{3\sqrt[3]{(t^3+4)^2}}dt =$$

$$= \int t^3 dt = \frac{t^4}{4} + C = \frac{(x^3-4)\sqrt[3]{x^3-4}}{4} + C.$$

8436.

$$\int \frac{dx}{\sqrt{1-2x-x^2}} = \int \frac{dx}{\sqrt{2-(x+1)^2}} = \int \frac{dp}{\sqrt{(\sqrt{2})^2-p^2}} = \arcsin\frac{p}{\sqrt{2}} + C =$$

$$= \arcsin\left(\frac{x+1}{\sqrt{2}}\right) + C.$$

8440.

$$\int \frac{\sqrt{1-x^2}}{x}dx = \int x^{-1}(1-x^2)^{\frac{1}{2}}dx = \int \frac{t}{\sqrt{1-t^2}}\cdot(-1)\cdot\frac{t\,dt}{\sqrt{1-t^2}} =$$

$$= -\int \frac{t^2dt}{1-t^2} = \int \frac{t^2dt}{t^2-1} = \int \frac{(t^2-1)+1}{t^2-1}dt = \int dt + \int\frac{dt}{t^2-1} =$$

$$= t + \frac{1}{2}\cdot \ln\left|\frac{t-1}{t+1}\right| + C = \sqrt{1-x^2} + \frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + C$$

8441.

$$\int \sqrt{4-x^2}dx = \int \sqrt{4-(2\sin t)^2}\cdot 2\cos(t)dt = 2\int\sqrt{4-4\sin t}\cdot\cos t\,dt = 4\int\sqrt{1-\sin t}\cdot\cos t\,dt = 4\int\cos t\cdot\cos t\,dt = 4\int\cos^2 t\,dt =$$

$$= 4\int\frac{1+\cos 2t}{2}dt = 2\int(1+\cos 2t)dt = 2\int dt + 2\int\cos 2t\,dt =$$

$$= 2t + \sin 2t + C = 2\arcsin\frac{x}{2} + \sin\left(2\arcsin\frac{x}{2}\right) +$$

$$+ C = 2\arcsin\frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + C$$

8442.

$$\int x\cdot\sqrt[5]{x-2}\,dx=\int x^{1}(x-2)^{\frac{1}{5}}dx=\int(t^{5}+2)\cdot t\cdot 5t^{4}dt=5\int t^{10}dt+10$$

$$\int t^{5}dt=5\cdot\frac{t^{11}}{11}+10\cdot\frac{t^{6}}{6}+C=\frac{5\sqrt[5]{(x-2)^{11}}}{11}+5\cdot\frac{\sqrt[5]{(x-2)^{6}}}{3}+C=$$

$$=\frac{15\cdot(x-2)(x-2)^{6/5}}{33}+\frac{55(x-2)^{6/5}\cdot 6}{33}=\frac{5\cdot(x-2)^{\frac{6}{5}}(3x+5)+C}{33}$$

8443.

$$\int\frac{dx}{x^{2}\sqrt{x^{2}+1}}=\int x^{2}(x^{2}+1)^{-\frac{1}{2}}dx=\int\frac{t^{2}-1}{\sqrt{\frac{1}{t^{2}-1}+1}}(-1)\cdot\frac{t\,dt}{\sqrt{t^{2}-1}(t^{2}-1)}=$$

$$=-\int\frac{(t^{2}-1)\sqrt{t^{2}-1}}{t}\cdot\frac{t\,dt}{\sqrt{t^{2}-1}(t^{2}-1)}=$$

$$=-\int dt=-t+C=-\sqrt{t^{2}+1}+C=-\sqrt{\frac{1}{x^{2}}+1}+C=$$

$$=-\frac{\sqrt{x^{2}+1}}{x}+C.$$