11.5.7. $z = \sin x \sin y$, $d^2 z$ - ?

$z_x' = \cos x \sin y$, $z_y' = \sin x \cos y$, $z_{xy}'' = \cos x \cos y$

$z_{xx}'' = -\sin x \sin y$

$z_{yy}'' = -\sin x \sin y$

$d z^2 = -\sin x \sin y dx^2 + 2\cos x \cos y dx dy - \sin x \sin y dy^2$

11.5.8.

$z = 4x^3 + 3x^2 y + 3xy^2 - y^3$, $\frac{\partial^2 z}{\partial x^2}$ - ?

$z_x' = \frac{\partial z}{\partial x} = 12x^2 + 6xy + 3y^2$

$z_{xx}'' = \frac{\partial^2 z}{\partial x^2} = 24x + 6y$

11.5.9

$z = xy + \sin(x+y)$, $\frac{\partial^2 z}{\partial x^2}$ - ?

$z_x' = \frac{\partial z}{\partial x} = y + \cos(x+y)$

$z_{xx}'' = \frac{\partial^2 z}{\partial x^2} = -\sin(x+y)$

11.5.10. $z = \ln tg(x+y)$, $\frac{\partial^2 z}{\partial x \partial y}$ - ?

$z_x' = \frac{\partial z}{\partial x} = (\ln tg(x+y))'_x = \frac{1}{tg(x+y)} (tg(x+y))' (x+y)' =$

$\frac{1}{tg(x+y)\cos^2(x+y)}$

$z_{xy}'' = \frac{\partial^2 z}{\partial x \partial y} = \left(\frac{1}{tg(x+y)\cos^2(x+y)}\right)'_y =$

$= -\frac{(tg(x+y)\cos^2(x+y))'_y}{\cos^4(x+y) tg^2(x+y)} =$

$= \frac{(tg(x+y))'_y \cos^2(x+y) + tg(x+y)(\cos^2(x+y))'_y}{\cos^4(x+y) tg^2(x+y)} =$

$= \frac{-(x+y)'_y + tg(x+y) \cdot 2\cos(x+y)(\cos(x+y))'_y}{\cos^4(x+y) tg^2(x+y)} =$

$= \frac{1 + tg(x+y) 2\cos(x+y) \sin(x+y)}{\cos^4(x+y) tg^2(x+y)}$

11.5.11. $z = arctg \frac{x+y}{1-xy}$ $\frac{\partial^2 z}{\partial x \partial y}$ - ?

$$z'_x = \frac{\partial z}{\partial x} = \left(arctg \frac{x+y}{1-xy}\right)'_x = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \left(\frac{x+y}{1-xy}\right)'_x =$$

$$= \frac{1}{1+\frac{x^2+2xy+y^2}{1-2xy-x^2y^2}} \cdot \frac{(x+y)'_x(1-xy)-(x+y)(1-xy)'_x}{(1-xy)^2} =$$

$$= \frac{1}{1+\frac{x^2+2xy+y^2}{1-2xy-x^2y^2}} \cdot \frac{1-xy-(x+y)(-y)}{(1-xy)^2} =$$

$$= \frac{1-xy+xy+y^2}{(1-xy)^2+x^2+2xy+y^2} = \frac{1+y^2}{1-2xy+x^2y^2+x^2+2xy+y^2} =$$

$$= \frac{1+y^2}{1+x^2y^2+x^2+y^2} = \frac{1+y^2}{(1+y^2)(1+x^2)} = \frac{1}{1+x^2}$$

$$z''_{xy} = \frac{\partial^2 z}{\partial x \partial y} = \left(\frac{1}{1+x^2}\right)'_y = 0$$

11.5.12. $z = x^2 \ln(x+y)$. $\frac{\partial^2 z}{\partial x \partial y}$ - ?

$$z'_x = \frac{\partial z}{\partial x} = (x^2 \ln(x+y))'_x = (x^2)'_x \ln(x+y) + x^2(\ln(x+y))'_x =$$

$$= 2x \ln(x+y) + \frac{x^2}{x+y}$$

$$z''_{xy} = \frac{\partial^2 z}{\partial x \partial y} = \left(2x \ln(x+y) + \frac{x^2}{x+y}\right)'_y =$$

$$= \frac{2x}{x+y} - \frac{x^2}{x^2+2xy+y^2} = \frac{2x(x+y)-x^2}{(x+y)^2} = \frac{2x^2+2xy-x^2}{(x+y)^2} =$$

$$= \frac{x^2+2xy}{(x+y)^2} = \frac{x(x+2y)}{(x+y)^2}$$

11.5.13. $z = x\sin xy + y\cos xy,\ \frac{\partial^2 z}{\partial x^2} = ?$

$z'_x = \sin xy + (\sin xy)'_x \cdot x + y(-\sin xy)(xy)'_x =$

$= \sin xy + x\cos xy\,(xy)'_x - y^2\sin xy =$

$= \sin xy + xy\cos xy - y^2\sin xy =$

$z''_{xx} = \frac{\partial^2 z}{\partial x^2} = (\sin xy + xy\cos xy - y^2\sin xy)'_x =$

$= \cos xy\,(xy)'_x + y(x'_x\cos xy + x(\cos xy)'_x) =$

$= y\cos xy + y\cos xy - xy^2\sin xy -$

$- y^2\cos xy\,(xy)'_x = 2y\cos xy - xy^2\sin xy - y^3\cos xy =$

$= y\cos xy\,(2 - y^2) - xy^2\sin xy$

11.5.14. $z = \sin(x + \cos y),\ \frac{\partial^3 z}{\partial x^2 \partial y} - ?$

$z'_x = \frac{\partial z}{\partial x} = (\sin(x + \cos y))'_x =$

$= \cos(x + \cos y)(x + \cos y)'_x = \cos(x + \cos y)$

$z''_{xx} = \frac{\partial^2 z}{\partial x^2} = (\cos(x + \cos y))'_x = -\sin(x + \cos y)($

$x + \cos y)'_x = -\sin(x + \cos y).$

$z'''_{xxy} = \frac{\partial^3 z}{\partial x^2 \partial y} = (-\sin(x + \cos y))'_y =$

$= -\cos(x + \cos y)(x + \cos y)'_y = -\cos(x + \cos y)(-\sin y) =$

$= \sin y\cos(x + \cos y)$

11.5.15 $z = \ln\sqrt{x^2+y^2}$, $d^2 z - ?$

$$z'_x = (\ln\sqrt{x^2+y^2})'_x = \frac{1}{\sqrt{x^2+y^2}}(\sqrt{x^2+y^2})'_x =$$

$$= \frac{(x^2+y^2)'_x}{\sqrt{x^2+y^2}\cdot 2\sqrt{x^2+y^2}} = \frac{2x}{2(x^2+y^2)} = \frac{x}{x^2+y^2}$$

$$z'_y = (\ln\sqrt{x^2+y^2})'_y = \frac{y}{x^2+y^2}$$

$$z''_{xy} = \left(\frac{x}{x^2+y^2}\right)'_y = x\left(\frac{1}{x^2+y^2}\right)'_y = x\left(-\frac{(x^2+y^2)'_y}{(x^2+y^2)^2}\right)$$

$$= -x\frac{2y}{(x^2+y^2)^2}$$

$$z''_{xx} = \left(\frac{x}{x^2+y^2}\right)'_x = \frac{x x'(x^2+y^2) - x(x^2+y^2)'_x}{(x^2+y^2)^2} =$$

$$\frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$z''_{yy} = \left(\frac{y}{x^2+y^2}\right)'_y = \frac{yy'(x^2+y^2) - y(x^2+y^2)'_y}{(x^2+y^2)^2} =$$

$$= \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$d^2 z = \frac{y^2-x^2}{(x^2+y^2)^2}dx^2 + 2\left(-x\frac{2y}{x^2+y^2}\right)dxdy + \frac{x^2-y^2}{(x^2+y^2)^2}dy^2 =$$

$$= \frac{(y^2-x^2)dx^2 - 4xy\,dxdy + (x^2-y^2)dy^2}{(x^2+y^2)^2} =$$

$$= \frac{(y^2-x^2)(dx^2-dy^2) - 4xy\,dxdy}{(x^2+y^2)^2}$$

11.5.16 $z = \cos(x+y)$, d^2z - ?

$z'_x = \frac{dz}{dx} = (\cos(x+y))'_x = -\sin(x+y)\cdot(x+y)'_x = -\sin(x+y)$

$z'_y = \frac{dz}{dy} = (\cos(x+y))'_y = -\sin(x+y)\cdot(x+y)'_y = -\sin(x+y)$

$z''_{xx} = \frac{d^2z}{dx^2} = (-\sin(x+y))'_x = -\cos(x+y)(x+y)'_x = -\cos(x+y)$

$z''_{xy} = \frac{d^2z}{dxdy} = (-\sin(x+y))'_y = -\cos(x+y)(x+y)'_y =$
$= -\cos(x+y)$

$z''_{yy} = \frac{d^2z}{dy^2} = (-\sin(x+y))'_y = -\cos(x+y)\cdot(x+y)'_y =$
$= -\cos(x+y)$

$d^2z = -\cos(x+y)dx^2 + 2(-\cos(x+y))dxdy + (-\cos(x+y))dy^2 =$
$= -\cos(x+y)\,dx^2 - 2\cos(x+y)dxdy - \cos(x+y)\cdot dy^2 =$
$= -\cos(x+y)(dx^2 + 2dxdy + dy^2) = -\cos(x+y)(dx+dy)^2$

11.5.17. $z = x^2y - xy^2 + 7$, dz, d^2z - ?

$z'_x = \frac{dz}{dx} = (x^2y - xy^2 + 7)'_x = 2xy - y^2$

$z'_y = \frac{dz}{dy} = (x^2y - xy^2 + 7)'_y = x^2 - 2xy$

$z''_{xx} = \frac{d^2z}{dx^2} = (2xy - y^2)'_x = 2y.$

$z''_{xy} = \frac{d^2z}{dxdy} = (2xy - y^2)'_y = 2x - 2y$

$z''_{yy} = \frac{d^2z}{dy^2} = (x^2 - 2xy)'_y = -2x$

$dz = (2xy - y^2)dx + (x^2 - 2xy)dy$

$d^2z = 2y\cdot dx^2 + 2(2x - 2y)dxdy + (-2x)dy^2 = 2ydx^2 + 4(x-y)dxdy -$
$-2xdy^2$

11.5.18. $z = xy - \frac{y}{x}$, dz, d^2z - ?

$z'_x = \frac{dz}{dx} = (xy - \frac{y}{x})'_x = y + \frac{y}{x^2}$

$z'_y = \frac{dz}{dy} = (xy - \frac{y}{x})'_y = x - \frac{1}{x}$

$z''_{xx} = (y - \frac{y}{x^2})'_x = -\frac{2y}{x^2}$

$z''_{yy} = (x - \frac{1}{x})'_y = 0$

$dz = (y + \frac{y}{x^2})dx + (x - \frac{1}{x})dy$

$d^2z = -\frac{2y}{x^2}dx^2 + 2(1 + \frac{1}{x^2})dxdy.$

11.5.19. $z = (x^2+y^2)^3$ dz, d^2z - ?

$z'_x = \frac{dz}{dx} = ((x^2+y^2)^3)'_x = 3(x^2+y^2)^2 \cdot (x^2+y^2)'_x = 6x(x^2+y^2)^2$

$z'_y = \frac{dz}{dx} = ((x^2+y^2)^3)'_y = 3(x^2+y^2)^2 \cdot (x^2+y^2)'_y =$

$= 6y(x^2+y^2)^2.$

$z''_{xx} = (6x(x^2+y^2)^2)'_x = (6x)'_x(x^2+y^2)^2 + 6x((x^2+y^2)^2)'_x =$

$= 6(x^2+y^2)^2 + 12x(x^2+y^2)(x^2+y^2)'_x = 6(x^2+y^2)^2 + 12x(x^2+y^2)2x =$

$= 6(x^2+y^2)^2 + 24x(x^2+y^2) = 6(x^2+y^2)(x^2+y^2+4x)$

$z''_{xy} = (6x(x^2+y^2)^2)'_y = 2(x^2+y^2)(x^2+y^2)'_y = 4y(x^2+y^2)$

$z''_{yy} = (6y(x^2+y^2)^2)'_y = (6y)'_y \cdot (x^2+y^2)^2 + 6y \cdot ((x^2+y^2)^2)'_y =$

$= 6(x^2+y^2)^2 + 12y(x^2+y^2)(x^2+y^2)'_y = 6(x^2+y^2)^2 + 24y$

$(x^2+y^2) = 6(x^2+y^2)(x^2+y^2+4y)$

$dz = 6x(x^2+y^2)^2dx + 6y(x^2+y^2)^2dy = 6(x^2+y^2)(xdx + y \cdot dy)$

$d^2z = 6(x^2+y^2)(x^2+y^2+4x)dx^2 + 8y(x^2+y^2)dxdy + 6(x^2+y^2)$

$(x^2+y^2+4y)dy^2$

11.5.20. $z=(\sin x)^{\cos y}$, dz; d^2z - ?

$z'_x = \frac{dz}{dx} = ((\sin x)^{\cos y})'_x = \cos y (\sin x)^{\cos y - 1} \cdot \cos x =$

$= \cos y \cdot \frac{(\sin x)^{\cos y}}{\sin x} \cos x = \cos y \cdot ctg x (\sin x)^{\cos y}$

$z'_y = \frac{dz}{dy} = ((\sin x)^{\cos y})'_y = (\sin x)^{\cos y} \ln(\sin x) \cdot (-\sin y)$

$z''_{xx} = (\cos y \cdot ctg x \cdot (\sin x)^{\cos y})'_x = \cos y ((ctg x)'_x (\sin x)^{\cos y}$

$+ ctg x ((\sin x)^{\cos y})'_x) = \cos y (-\frac{1}{\sin^2 x} \cdot \sin x^{\cos y}$

$+ ctg^2 x \cdot \cos y \cdot \sin x^{\cos y}) = \cos y \cdot \sin x^{\cos y}$

$(-\frac{1}{\sin^2 x} + ctg^2 x \cos y)$

$z''_{xy} = (\cos y \cdot ctg x (\sin x)^{\cos y})'_y = ctg x ((\cos y)'_y \cdot (\sin x)^{\cos y}$

$+ \cos y ((\sin x)^{\cos y})'_y) = ctg x ((-\sin y)(\sin x)^{\cos y} + \cos y$

$(\sin x)^{\cos y} \ln(\sin x) \cdot (-\sin y))$

$ctg x (-\sin y)(\sin x)^{\cos y} (1 + \cos y \cdot \ln(\sin x))$

$z''_{yy} = ((\sin x)^{\cos y} \cdot \ln(\sin x)(-\sin y))'_y =$

$= \ln(\sin x)(((\sin x)^{\cos y})'_y \cdot (-\sin y) + (\sin x)^{\cos y} (-\sin y)'_y) =$

$= \ln(\sin x)((\sin x)^{\cos y}(\ln(\sin x) \cdot \sin^2 y + \sin x^{\cos y}(-\cos y)) =$

$= \ln(\sin x)(\sin x)^{\cos y}(\ln(\sin x) \cdot \sin^2 y - \cos y)$

$dz = \cos y \cdot ctg x (\sin x)^{\cos y} dx + (\sin x)^{\cos y} \ln(\sin x)$

$(-\sin y) dy = \sin x^{\cos y}(\cos y \cdot ctg x dx - \ln(\sin x) \sin y dy)$

$d^2z = \cos y \cdot \sin x^{\cos y}(-\frac{1}{\sin^2 x} + ctg^2 x \cos y) dx^2$

$- 2 ctg x \cdot \sin y (\sin x)^{\cos y}(1 + \cos y \cdot \ln \sin x) dx dy +$

$+ \ln(\sin x)(\sin x)^{\cos y}(\ln(\sin x) \cdot \sin^2 y - \cos y) dy^2$

11.5.21. $z = x - 3\sin y$, dz, d^2z - ?

$z'_x = \frac{dz}{dx} = (x - 3\sin y)'_x = 1$

$z'_y = \frac{dz}{dy} = (x - 3\sin y)'_y = -3\cos y$

$z''_{xx} = (1)'_x = 0$ $z''_{yy} = (-3\cos y)'_y = 3\sin y$

$z''_{xy} = (1)'_y = 0$ $dz = dx - 3\cos y\, dy$

$d^2z = 3\sin\ dy^2$

11.5.22. $z = \ln\sqrt{x^2+y}$, dz, d^2z - ?

$z'_x = \frac{dz}{dx} = (\ln\sqrt{x^2+y})'_x = \frac{(\sqrt{x^2+y})'_x}{\sqrt{x^2+y}} = \frac{(x^2+y)'_x}{2(x^2+y)} =$

$= \frac{2x}{2(x^2+y)} = \frac{x}{x^2+y}$

$z'_y = (\ln\sqrt{x^2+y})'_y = \frac{1}{2(x^2+y)}$

$z''_{xx} = \left(\frac{x}{x^2+y}\right)'_x = \frac{x'_x(x^2+y) - x(x^2+y^2)'_x}{(x^2+y)^2} =$

$= \frac{x^2+y-2x^2}{(x^2+y)^2} = \frac{-x^2+y}{(x^2+y)^2}$

$z''_{xy} = \left(\frac{x}{x^2+y}\right)'_y = x\left(\frac{1}{x^2+y}\right)'_y = \frac{-x}{(x^2+y)^2}$

$z''_{yy} = \left(\frac{1}{2(x^2+y)}\right)'_y = \frac{-1}{2(x^2+y)^2}$

$dz = \frac{x\cdot dx}{x^2+y} + \frac{dy}{2(x^2+y)} = \frac{2x\,dx + dy}{2(x^2+y)}$

$d^2z = \frac{-x^2+y}{(x^2+y^2)} = dx^2 - \frac{4x}{(x^2+y^2)}dx\,dy - \frac{dy}{2(x^2+y)^2} =$

$= \frac{2(-x^2+y)dx^2 - 4x\,dx\,dy - dy^2}{2(x^2+y)^2}$