Часть 3.

7.3.20.

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)=\lim_{x\to 0}\left(\frac{\sin x-x}{x\sin x}\right)=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(\sin x-x)'}{(x\sin x)'}=$$

$$=\lim_{x\to 0}\frac{\cos x-1}{\sin x+x\cos x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{-\sin x}{\cos x+\cos x-x\sin x}=\frac{0}{2}=0$$

7.3.21.

$$\lim_{x\to\infty}x(e^{1/x}-1)=\lim_{x\to\infty}\frac{e^{1/x}-1}{1/x}=\left[\frac{0}{0}\right]=\lim_{x\to\infty}\frac{(e^{1/x}-1)'}{(1/x)'}=$$

$$=\lim_{x\to\infty}\frac{e^{1/x}(-\frac{1}{x^2})}{-1/x^2}=\lim_{x\to\infty}e^{1/x}=e^0=1$$

7.3.22.

$$\lim_{x\to 1}\left(\frac{1}{1-x^3}-\frac{1}{1-x^2}\right)=\lim_{x\to 1}\frac{(1-x^2)-(1-x^3)}{(1-x^3)(1-x^2)}=\lim_{x\to 1}\frac{x^3-x^2}{(1-x^3)(1-x^2)}=$$

$$=\left[\frac{0}{0}\right]=\lim_{x\to 1}\frac{3x^2-2x}{-3x^2(1-x^2)+(1-x^3)(-2x)}=\lim_{x\to 1}\frac{3x^2-2x}{-3x^2+3x^4-2x+2x^4}=$$

$$=\lim_{x\to 1}\frac{3x^2-2x}{5x^4-3x^2-2x}=\frac{1}{0}=\infty$$

7.3.24.

$$\lim_{x\to 0}x^{tg\,x}$$

$$\lim_{x\to 0}\ln(x^{tg\,x})=\lim_{x\to 0}tg\,x\ln x=\lim_{x\to 0}\frac{\ln x}{\frac{1}{tg\,x}}=\left[\frac{\infty}{\infty}\right]=$$

$$=\lim_{x\to 0}\frac{1}{x}:\left(\frac{-1}{tg^2x}\cdot\frac{1}{\cos^2x}\right)=\lim_{x\to 0}\frac{1}{x}\cdot\frac{\sin^2x}{-1}=\lim_{x\to 0}\frac{\sin^2x}{-x}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{2\sin x\cos x}{-1}=\lim_{x\to 0}\frac{\sin 2x}{-1}=0\Rightarrow\lim_{x\to 0}y=1$$

7.3.25. $\lim_{x\to 0}(\cos 2x)^{1/x^2}$

$$\lim_{x\to 0}\ln(\cos 2x^{1/x^2})=\lim_{x\to 0}\frac{1}{x^2}\ln\cos 2x=\lim_{x\to 0}\frac{\ln\cos 2x}{x^2}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{(\ln\cos 2x)'}{(x^2)'}=\lim_{x\to 0}\frac{-2\sin 2x}{2x\cos 2x}=\lim_{x\to 0}\frac{-tg\,2x}{x}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{-2}{\cos^2 2x}=\frac{-2}{1}=-2\Rightarrow\lim_{x\to 0}y=e^{-2}.$$

7.3.26. $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{x^2}$

$\lim\limits_{x\to 0}\ln\left(\left(\frac{1}{x}\right)^{x^2}\right) = \lim\limits_{x\to 0} x^2 \ln\frac{1}{x} = \lim\limits_{x\to 0}\frac{\ln\frac{1}{x}}{1/x^2} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to 0}$

$\frac{(\ln 1/x)'}{(1/x^2)'} = \lim\limits_{x\to 0}\frac{-x}{-2/x^3} = \lim\limits_{x\to 0}\frac{-x^4}{2} = 0 \Rightarrow \lim\limits_{x\to 0} y = 1$

7.3.27. $\lim\limits_{x\to 0} x^{\frac{1}{1+\ln x}}$

$\lim\limits_{x\to 0}\ln\left(x^{\frac{1}{1+\ln x}}\right) = \lim\limits_{x\to 0}\frac{1}{1+\ln x}\ln x = \lim\limits_{x\to 0}\frac{\ln x}{1+\ln x} = \left[\frac{\infty}{\infty}\right] =$

$= \lim\limits_{x\to 0}\frac{(\ln x)'}{(1+\ln x)'} = \lim\limits_{x\to 0}\frac{1/x}{1/x} = x/x = 1 \Rightarrow \lim\limits_{x\to 0} y = e$

7.3.30. $P(x) = x^5 - 3x^4 + 7x + 2,\ x_0 = 2.$

$P(2) = 32 - 48 + 14 + 2 = 0$

$P'_x = 5x^4 - 12x^3 + 7,\ P'(2) = -9$

$P''_{xx} = 20x^3 - 36x^2,\ P''(2) = 16$

$P'''_{xxx} = 60x^2 - 72x,\ P'''(2) = 96$

$P^{(4)}_{x} = 120x - 72,\ P^{(4)}(2) = 168.$

$P^{(5)}_x = 120,\ P^{(5)}(2) = 120$

$P^{(6)}_x = 120' = 0,\ P^{(6)}(20) = 0$

$P(x) = 0 + \frac{-9}{1!}(x-2) + \frac{16}{2!}(x-2)^2 + \frac{96}{3!}(x-2)^3 + \frac{168}{4!}(x-2)^4 + \frac{120}{5!}(x-2)^5 = -9(x-2) + 8(x-2)^2 + 16(x-2)^3 + 7(x-2)^4 + (x-2)^5.$

7.3.32.

$$f(x) = 2^x, \ x_0 = \log_2 3$$

$$f(\log_2 3) = 2^{\log_2 3} = 3$$

$$f'_x = (2^x)' = 2^x \ln 2$$

$$f'(\log_2 3) = 2^{\log_2 3} \ln 2 = 3\ln 2$$

$$f''_{xx} = (2^x \ln 2)' = \ln 2 (2^x)' = \ln^2 2 \cdot 2^x$$

$$f''(\log_2 3) = \ln^2 \cdot 2^{\log_2 3} = 3\ln^2 2$$

$$f'''_{xxx} = (\ln^2 2 \cdot 2^x)' = \ln^2 2 (2^x)' = \ln^3 2 \cdot 2^x$$

$$f'''(\log_2 3) = \ln^3 2 \cdot 2^{\log_2 3} = 3\ln^3 2$$

$$\Rightarrow f^{(n)}(\log_2 3) = 3(\ln 2)^n$$

$$f(x) = 3 + \frac{3\ln 2}{1!}(x - \log_2 3) + \frac{3\ln^2 2}{2!}(x - \log_2 3)^2 + \frac{3\ln^3 2}{3!}(x - \log_2 3)^3 + \ldots + \frac{3\ln 2}{n!}(x - \log_2 3)^n + o(x - \log_2 3)^n$$

7.3.34. $f(x) = e^{2+x}, \ k = 4.$

$$f(x_0) = e^{2-0} = e^2$$

$$f'_x = e^{2-x}$$

$$f'(0) = -e^2$$

$$f''_x = e^{2-x}$$

$$f''(0) = e^2$$

$$f'''_x = -e^{2-x}$$

$$f'''(0) = -e^2$$

$$f^{(4)}_x = e^{2-x}$$

$$f^{(4)}(0) = e^2$$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}x + \frac{f''(x_0)}{2!}x^2 + \frac{f'''(x_0)}{3!}x^3 + \frac{f^{(4)}(x_0)}{4!}x^4 + o(x^4)$$

7.3.35. $f(x) = \arcsin x,\ k=3$

$f(x_0) = \arcsin 0 = 0$

$f'_x = (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$f'(0) = 1$

$f''_x = \frac{x}{\sqrt{(1-x^2)^3}}$

$f'''_x = (1-x^2)^{-2/3} + \frac{4x^2}{3}(1-x^2)^{-5/3}$

$f'''(0) = 1$

$f''(0) = 0$

$f(x) = 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{1}{3!}x^3 + o(x^3) = x + \frac{x^3}{6} + o(x^3)$