Часть 6.

9127

$$\int_1^{e^2}\frac{\ln^3 x}{3x}\,dx = \int_0^2\frac{t^3\,dt}{3} = \frac{1}{3}\int_0^2 t^3\,dt = \frac{1}{3}\cdot\frac{t^4}{4}\Big|_0^2 =$$

$$= \frac{4}{3}$$

9128

$$\int_{\pi}^{2\pi}\frac{x+\cos x}{x^2+2\sin x}\,dx = \int_{\pi^2}^{4\pi^2}\frac{\frac{1}{2}dt}{t} = \frac{1}{2}\int_{\pi^2}^{4\pi^2}\frac{dt}{t} = \frac{1}{2}\ln|t|\Big|_{\pi^2}^{4\pi^2} =$$

$$= \frac{1}{2}\cdot 2\cdot\ln(2\pi) - \frac{1}{2}\cdot 2\cdot\ln(\pi) = \ln(2\pi) - \ln(\pi) =$$

$$= \ln\left(\frac{2\pi}{\pi}\right) = \ln(2)$$

9129.

$$\int_0^1\frac{4\,\mathrm{arctg}\,x - x}{1+x^2}\,dx = 4\int_0^1\frac{\mathrm{arctg}\,x}{1+x^2}\,dx - \int_0^1\frac{dx}{1+x^2} =$$

$$= 4\int_0^{\frac{\pi}{4}} t\,dt - \frac{1}{2}\int_1^2\frac{du}{u} = 4\cdot\frac{t^2}{2}\Big|_0^{\pi/4} - \frac{1}{2}\ln|u|\Big|_1^2 =$$

$$= \left(2\cdot\left(\frac{\pi}{4}\right)^2 - 2\cdot 0\right) - \frac{1}{2}\left(\ln|2| - \ln|1|\right) = 2\cdot\frac{\pi^2}{16} - \frac{1}{2}\ln(2) =$$

$$= \frac{\pi^2}{8} - \frac{\ln(2)}{2}.$$

9132.

$$\int_0^1\frac{dx}{\sqrt{x^4+x^2+1}} = \int_0^1\frac{x\,dx}{\sqrt{x^4+2\cdot\frac{1}{2}\cdot x^2+\frac{1}{4}+\frac{3}{4}}} = \int_0^1\frac{x\,dx}{\sqrt{(x^2+\frac{1}{2})^2+\frac{3}{4}}} = \frac{1}{2}\int_{\frac{1}{2}}^{1\frac{1}{2}}\frac{dt}{\sqrt{t^2+\frac{3}{4}}} =$$

$$= \frac{1}{2}\ln\left|t+\sqrt{t^2+\frac{3}{4}}\right|\Big|_{\frac{1}{2}}^{1\frac{1}{2}} = \frac{1}{2}\left(\ln\left(\frac{3}{2}+\sqrt{3}\right) - \ln\left(\frac{1}{2}+1\right)\right) =$$

$$= \frac{1}{2}\ln\left(\frac{\frac{3+2\sqrt{3}}{2}}{\frac{3}{2}}\right) = \frac{1}{2}\ln\left(\frac{3+2\sqrt{3}}{3}\right)$$

9134.

$$\int_0^{\frac{\pi}{3}} tgx\cdot \ln(\cos x)\,dx = \int_0^{-\ln 2} -t\,dt = \int_{-\ln 2}^{0} t\,dt =$$

$$= \frac{t^2}{2}\Big|_{-\ln 2}^{0} = \frac{0^2}{2} - \frac{(-\ln 2)^2}{2} = -\frac{\ln^2(2)}{2}$$

9166.

$$\int_0^{\frac{\pi}{2}} \frac{5\,dx}{1+\cos x} = \int_0^1 \frac{5}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2\,dt}{1+t^2} = \int_0^1 \frac{5(1+t^2)}{1+t^2+1-t^2}\cdot\frac{2\,dt}{1+t^2} =$$

$$= \int_0^1 \frac{5\cdot 2\,dt}{2} = 5\int_0^1 dt = 5t\Big|_0^1 = 5.$$

9168.

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x+\sin x} = \int_0^1 \frac{1}{1+\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}}\cdot\frac{2\,dt}{1+t^2} = \int_0^1 \frac{1+t^2}{2+2t}\cdot\frac{2\,dt}{1+t^2} =$$

$$= \int_0^1 \frac{dt}{1+t} = \ln|1+1| - \ln|0+1| = \ln(2) - 0 = \ln(2).$$

9170.

$$\int_1^2 3x(1-x)^{17}dx = \int_0^{-1} 3(1-t)t^{17}(-1)\,dt = 3\int_{-1}^{0}(t^{17}-t^{18})\,dt =$$

$$= 3\int_{-1}^{0} t^{17}dt - 3\int_{-1}^{0} t^{18}dt = 3\cdot\frac{t^{18}}{18}\Big|_{-1}^{0} - 3\cdot\frac{t^{19}}{19}\Big|_{-1}^{0} =$$

$$= -\frac{37}{114}.$$

9.1106.

$$\int_0^{\sqrt{3}} \frac{x^2}{(1+x^2)^2}dx = \int \frac{x}{(1+x^2)^2}dx = \frac{1}{2}\int \frac{dt}{t^2} = -\frac{x}{2+2x^2}\Big|_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{dx}{2+2x^2} =$$

$$= \left(-\frac{1}{2}\cdot\frac{x}{1+x^2} - \frac{1}{2}\operatorname{arctg}x\right)\Big|_0^{\sqrt{3}} = \left(\frac{1}{2}\cdot\frac{\pi}{3} - \frac{\sqrt{3}}{8}\right) - (0-0) =$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8} = \frac{4\pi - 3\sqrt{3}}{24}.$$

9.1107.

$$\int_0^{\pi^2/4} \sin\sqrt{x}\,dx = \int_0^{\frac{\pi}{2}} 2t\sin(t)\,dt = 2\int_0^{\frac{\pi}{2}} t\sin(t)\,dt =$$

$$= 2t\cdot(-\cos t)\Big|_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} \cos t\,dt = -2t\cos t\Big|_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} \cos t\,dt =$$

$$= \left(-2\,\frac{\pi}{2}\cdot 0 + 2\cdot 1\right) - (0 + 2\cdot 0) = (0+2) - (0+0) =$$

$$= 2.$$

9.1108.

$$\int_0^{9} e^{\sqrt{x}}dx = \int_0^3 2te^t dt = 2\int_0^3 te^t dt = 2te^t\Big|_0^3 - 2\int_0^3 e^t dt =$$

$$= (2te^t - 2e^t)\Big|_0^3 = (2\cdot 3\cdot e^3 - 2e^3) - (2\cdot 0\cdot e^0 - 2\cdot e^0) =$$

$$= 4e^3 + 2.$$