Часть 3.

8319.

$$\int \frac{5dx}{x+\sqrt{2}} = 5\ln|x+\sqrt{x}|+C.$$

8320

$$\int \frac{40dx}{(x-0,5)^3} = \frac{4}{1-3}\cdot\frac{1}{(x-0,5)^2}+C = -\frac{2}{(x-0,5)^2}+C.$$

8321.

$$\int \frac{7dx}{(x+3)^6} = \frac{7}{1-6}\cdot\frac{1}{(x+6)^{6-1}}+C = \frac{7}{-5(x+3)^5}+C.$$

8322

$$\int \frac{dx}{(3x+2)^4} = \frac{1}{1-4}\cdot\frac{1}{(3x+2)^{4-1}}+C = -\frac{1}{9(3x+2)^3}+C.$$

8323.

$$\int \frac{dx}{x^2-4x+8} = \int \frac{dy}{y^2+4} = \frac{1}{2}\operatorname{arctg}\frac{y}{2}+C = \frac{1}{2}\operatorname{arctg}\frac{x-2}{2}+C.$$

8324.

$$\int \frac{dx}{x^2+x+1} = \int \frac{dy}{y^{2+3/4}} = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2y}{\sqrt{3}}+C = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C.$$

8325.

$$\int \frac{6x+1}{x^2-8x+25}dx = \int \frac{3(2x-8)+(1+24)}{x^2-8x+25}dx = 3\int \frac{(2x-8)dx}{x^2-8x+25}+25\int \frac{dx}{x^2-8x+25}$$

$$= 3\int \frac{(2x+8)dt}{t(2x-8)}+25\int \frac{dy}{y^2+9} = 3\int \frac{dt}{t}+25\int \frac{dy}{y^2+9} =$$

$$= 3\ln|t|+\frac{25}{3}\operatorname{arctg}\frac{y}{3} = 3\ln|x^2-8x+25|+\frac{25}{3}\operatorname{arctg}\frac{x-4}{3}$$

8326.

$$\int \frac{5x+2}{x^2+2x+10}\,dx = \int \frac{2,5(2x+2)+(2-5)}{x^2+2x+10}\,dx = 2,5\int \frac{2x+2}{x^2+2x+10}\,dx -$$

$$-3\int \frac{dx}{x^2+2x+10} = 2,5\int \frac{(2x+2)\,dx}{t(2x+2)} - 3\int \frac{dy}{y^2+9} = 2,5\int \frac{dt}{t} - 3\int \frac{dy}{y^2+9} =$$

$$= 2,5\ln|t| - 3\cdot\frac{1}{3}\operatorname{arctg}\frac{y}{3} + C = 2,5\ln|x^2+2x+10| - \operatorname{arctg}\frac{x+1}{3} + C$$

8327.

$$\int \frac{x+2}{x^2+3x+5}\,dx = \int \frac{0,5(2x+3)+(2-1,5)}{x^2+3x+5}\,dx =$$

$$= 0,5\int \frac{(2x+3)\,dx}{x^2+3x+5} + 0,5\int \frac{dx}{x^2+3x+5} = 0,5\int \frac{(2x+3)\,dx}{t(2x+3)} +$$

$$+ 0,5\int \frac{dy}{y^2+2,75} = 0,5\ln|t| + 0,5\frac{1}{\sqrt{2,75}}\operatorname{arctg}\frac{y}{\sqrt{2,75}} + C =$$

$$= 0,5\ln|x^2+3x+5| + \frac{0,5}{\sqrt{2,75}}\operatorname{arctg}\frac{x+1,5}{\sqrt{2,75}} + C.$$

8328.

$$\int \frac{2x-1}{5x^2+2x+1}\,dx = \int \frac{(2x-1)\,dx}{5(x^2+0,4x+0,2)} = \frac{1}{5}\int \frac{2x-1}{x^2+0,4x+0,2}\,dx =$$

$$= \frac{1}{5}\int \frac{1(2x+0,4)+(-1-0,4)}{x^2+0,4x+0,2}\,dx = \frac{1}{5}\int \frac{(2x+0,4)\,dx}{x^2+0,4x+0,2} - \frac{7}{25}\int \frac{dx}{x^2+0,4+0,2} =$$

$$= \frac{1}{5}\int \frac{(2x+0,4)\,dx}{t(2x+0,4)} - \frac{7}{25}\int \frac{dy}{y^2+0,16} = \frac{1}{5}\ln|t| - \frac{7}{25}\cdot\frac{1}{0,4}\operatorname{arctg}\frac{y}{0,4} + C$$

$$= \frac{\ln|x^2+0,4x+0,1|}{5} - 0,7\operatorname{arctg}\frac{x+0,2}{0,4} + C.$$

8329.

$$\int \frac{x-1}{(x^2+2x+3)^2}dx = \int \frac{0{,}5(2x+2)+(-1-1)}{(x^2+2x+3)^2}dx = 0{,}5\int \frac{(2x+2)dx}{(x^2+2x+3)^2} -$$

$$-2\int \frac{dx}{(x^2+2x+3)^2} = 0{,}5\int \frac{dt}{t^2} - 2\int \frac{dg}{(g^2+4)^2} =$$

$$= \frac{0{,}5}{-1}\cdot\frac{1}{x^2+2x+3} + 2\int \frac{dg}{(g^2+4)^2} =$$

$$= \frac{-0{,}5}{x^2+2x+3} + \frac{x+1}{8((x+1)^2+4)} + \frac{1}{16}\operatorname{arctg}\frac{x+1}{2}.$$

8330

$$\int \frac{dx+1}{(x^2+2x+5)^2}dx = \int \frac{1(2x+2)+(1-2)}{(x^2+2x+5)^2}dx = \int \frac{(2x+2)dx}{(x^2+2x+5)^2} - \int \frac{dx}{(x^2+2x+5)^2} =$$

$$= \int \frac{dt}{t^2} - \int \frac{dg}{(g^2+4)^2} = \frac{1}{x^2+2x+5} + \frac{1}{2\cdot 4}\frac{g}{g^2+4} + \frac{1}{4}\cdot\frac{4-3}{4-2}\int \frac{dg}{g^2+4} =$$

$$= -\frac{1}{x^2+2x+5} + \frac{x+1}{8((x+1)^2+4)} + \frac{1}{16}\operatorname{arctg}\frac{x+1}{2} + C$$

8331.

$$\int \frac{dx}{(x^2+1)^4} = \frac{1}{2\cdot 3\cdot 1}\cdot\frac{x}{(x^2+1)^3} + 1\cdot\frac{8-3}{8-2}\int \frac{dx}{(x^2+1)^3} =$$

$$= \frac{1}{6(x^2+1)^3} + \frac{5x}{24(x^2+1)^3} + \frac{15}{24}\int \frac{dx}{x^2+1} = \frac{1}{6(x^2+1)^3} + \frac{5x}{24(x^2+1)^2} +$$

$$+ \frac{5}{8}\operatorname{arctg} x + C.$$

8332.

$$\int\frac{(3x+2)dx}{(x^2-3x+3)^2}=\int\frac{1,5(2x-3)+(2+4,5)}{(x^2-3x+3)^2}dx=$$

$$=1,5\int\frac{(2x-3)}{(x^2-3x+3)^2}dx+6,5\int\frac{dx}{(x^2-3x+3)^2}=1,5\int\frac{dt}{t}+6,5\int\frac{dy}{(y^2+0,75)^2}=$$

$$=1,5\ln(x^2-3x+3)+\frac{6,5}{1,5(x^2-3x+3)}+\frac{13}{1,5\sqrt{3}}\operatorname{arctg}\frac{2(x-1,5)}{\sqrt{3}}+C$$

8.333.

$$\int\frac{2x-3}{(x-1)(x+2)}dx=\text{[crossed out]}=\int\frac{-dx}{3(x-1)}+\int\frac{7dx}{-3(x+2)}=$$

$$=\frac{1}{3}\int\frac{dx}{x-1}+\frac{7}{3}\int\frac{dx}{x+2}=-\frac{1}{3}\int\frac{dt}{t}+\frac{1}{3}\int\frac{dt}{t}=-\frac{1}{3}\ln|x-1|+\frac{7}{3}\ln$$

$|x+2|+C$

8334

$$\int\frac{x-4}{(x-2)(x-3)}dx=\frac{x-4}{(x-a)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3}=$$

$$=\int\frac{1,2}{x-2}dx-\int\frac{0,2}{x-3}dx=1,2\int\frac{dt}{t}-0,2\int\frac{0,2}{x-3}dt=$$

$$=1,2\ln|t|-0,2\ln|t|=1,2\ln|x-2|-0,2\ln|x-3|$$

8335

$$\int\frac{x\,dx}{x^2-4x-5}=\int\frac{x\,dx}{(x+1)(x-5)}=\frac{1}{6}\int\frac{dx}{x+1}+\frac{5}{6}\int\frac{dx}{x-5}=$$

$$=\frac{1}{6}\int\frac{dt}{t}+\frac{5}{6}\int\frac{dt}{t}=\frac{1}{6}\ln|t|+5/6\ln|t|+C=$$

$$=\frac{1}{6}\ln|x+1|+\frac{5}{6}\ln|x-5|+C.$$

8336.

$$\int \frac{2x^2-11}{x^2-x-6}\,dx = \int \frac{x^2+x+6}{x^2+x+6}\,dx - \int \frac{2x+23}{x^2+x-6}\,dx =$$

$$= 2x - \int \frac{2x+23}{x^2+x-6}\,dx = 2x - \int \frac{2x+1+22}{x^2+x+6}\,dx =$$

$$= 2x - \int \frac{(2x+1)\,dt}{t(2x+1)} - 22\int \frac{dx}{x^2+x-6} = 2x - \ln|t| + C - 22\int \frac{dy}{y^2-\frac{25}{4}} =$$

$$= 2x - \ln|x^2+x-6| - 22\cdot\frac{1}{2\cdot\frac{5}{2}}\ln\left|\frac{y-5/2}{y+5/2}\right| + C =$$

$$= 2x - \ln|(x^2+x-6) - 4{,}4\ln\left|\frac{x-2}{x+3}\right| + C.$$

8337

$$\int \frac{-3x^2+x+19}{(x-4)(x-2)(x+1)}\,dx = -1{,}3\int \frac{dx}{x-4} - 2{,}5\frac{dx}{x-2} + 0{,}8\int \frac{dx}{x+1} =$$

$$= -1{,}3\int \frac{dt}{t} - 2{,}5\int \frac{dt}{t} + 0{,}8\int \frac{dt}{t} = -1{,}3\ln|(x-4) -$$

$$-2{,}5\ln|x-2| + 0{,}8\ln|x+1| + C$$

8338

$$\int \frac{x-1}{(x+1)(x^2-4)}dx = \int \frac{2/3}{x+1}dx + \int \frac{-2/3\cdot x+5/3}{x^2-4} =$$

$$= \frac{2}{3}\int \frac{dx}{x+1} + \int \frac{-2/3\cdot x+5/3}{x^2-4} = \frac{2}{3}\int \frac{dt}{t} + \int \frac{-3\cdot 2x+5/3}{x^2-4}dx =$$

$$= \frac{2}{3}\ln|t|+C - 3\int \frac{2x}{x^2-4}dx + \frac{5}{3}\int \frac{dx}{x^2-4} = \frac{2}{3}\ln|x+1| - 3\int \frac{2+dt}{t+2x} +$$

$$+ \frac{5}{3}\cdot\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C = \frac{2}{3}\ln|x-1| - 3\ln|t| + \frac{5}{12}\ln\left|\frac{x-2}{x+2}\right| + C =$$

$$= \frac{2}{3}\ln|x-1| - 3\ln|x^2-4| + \frac{5}{12}\ln\left|\frac{x-2}{x+2}\right| + C.$$

8335.

$$\int \frac{(x^2+2)dx}{(x-1)(x+1)^2} = \text{[illegible]} = \int \frac{0,5dx}{x-1} + \int \frac{0,5dx}{x+1} + \int \frac{-2}{(x+1)^2}dx =$$

$$= -0,5\int \frac{dt}{t} + 0,5\int \frac{dt}{t} - 2\int \frac{dt}{t} =$$

$$= -0,5\ln|x-1| + 0,5\ln|x+1| - \frac{2}{x+1} + C$$

8340.

$$\int \frac{2x+3}{(x-2)^3}dx = \frac{2x+3}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$

$$2x+3 = A(x^2-4x+4) + B(x-2) + C$$

$$2x+3 = Ax^2 - 4Ax + 4A + Bx - 2B + C$$

$$2x+3 = Ax^2 + x(-4A+B) + 4A - 2B + C.$$

$$\int 0\cdot dx + 2\int \frac{dx}{(x-2)^2} + 7\int \frac{dx}{(x-2)^2} =$$

$$= 2\int \frac{dt}{t^2} + 7\int t^{-3}dt = -\frac{2}{7} + \frac{7t^2}{-2} + C =$$

$$= \frac{2}{x-2} - \frac{3,5}{(x-2)^2} + C,$$

8341.

$$\int \frac{x^4\,dx}{(x^2-1)(x+2)} = \int \frac{x^4\,dx}{x^3+2x^2-x-2} =$$

$$= \int \frac{(x-2)(x^3+2x^2-x-2)+5x^2-4}{x^3+2x^2-x-2}\,dx =$$

$$= \int (x-2)\,dx + \int \frac{5x^2-4}{(x-1)(x+2)}\,dx =$$

$$= \int 5\,dx - 2\int 6\,dx + \int \frac{-1/3\,x + 2/3}{x^2-1} + \frac{16}{3}\int \frac{dx}{x+2} =$$

$$= \frac{x^2}{2} - 2x - \frac{1}{6}\int \frac{2x\,dx}{x^2-1} + \frac{2}{3}\int \frac{dx}{x^2-1} + \frac{16}{3}\ln|x+2| + C =$$

$$= \frac{x^2-4x}{2} - \frac{1}{6}\int \frac{d(x^2-1)}{x^2-1} + \frac{2}{3}\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + \frac{16}{3}\ln|x+2| + C =$$

$$= \frac{x(x-4)}{2} - \frac{1}{6}\ln|x^2-1| + \frac{2}{6}\ln\left|\frac{x-1}{x+1}\right| + \frac{16}{3}\ln(x+2) + C =$$

$$= \frac{3x(x-4) - \ln|x-1| - \ln|x+1| + 2\ln|x-1| - 2\ln|x+1| + 32\ln|x+2|}{6} =$$

$$= \frac{3x(x-4) + \ln|x-1| - 3\ln|x+1| + 32\ln|x+2|}{6} + C$$

8344

$$\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{3}\int \frac{dx}{x^2+1} - \frac{1}{3}\int \frac{dx}{x^2+4} =$$

$$= \frac{1}{3}\operatorname{arctg} x - \frac{1}{6}\operatorname{arctg}\frac{x}{8} + C$$

8346

$$\int \frac{x^4+x^3+x^2+x+1}{(x^2+1)^2 x}\,dx = \int \frac{dx}{x^2+1} + \int \frac{-x}{(x^2+1)^2}\,dx + \int \frac{dx}{x} =$$

$$= \int \frac{dx}{x^2+1} - 0{,}5\int \frac{2x\,dx}{(x^2+1)^2} + \int \frac{dx}{(x^2+1)^2} + \int \frac{dx}{x} =$$

$$= \int \frac{dx}{x^2+1} - 0{,}5\int \frac{2x\,dx}{t^2\,dx} + \int \frac{dx}{(x^2+1)^2} + \int \frac{dx}{x} =$$

$$\int \frac{dx}{x^2-1} - 0{,}5\int \frac{dt}{t} + \frac{1}{2}\cdot\frac{x}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1} - \frac{dx}{x} =$$

$$= \operatorname{arctg} x + \frac{0{,}5}{x^2+1} + \frac{x}{2(x^2+1)} + 0{,}5\operatorname{arctg} x + \ln|x|$$

5348.

$$\int \frac{3x+5}{x(x^2+1)^2}\,dx = 5\int \frac{dx}{x} - 5\int \frac{x}{x^2-1}\,dx + \int \frac{5x+3\,dx}{(x^2+1)^2} =$$

$$= 5\ln|x| - 5\int \frac{t\,dt}{t\,2x} + \int \frac{5x+3}{(x^2+1)^2}\,dx + C =$$

$$= 5\ln|x| - 2{,}5\ln(t) + \int \frac{5x+3}{(t^2+1)^2}\,dx + C =$$

$$= 5\ln|x| - 2{,}5\ln(x^2+1) + 2{,}5\int \frac{2x\,dx}{(x^2+1)^2} + 3\int \frac{dx}{(x^2+1)^2} =$$

$$= 5\ln|x| - 2{,}5\ln(x^2+1) + 2{,}5\int \frac{dt}{t^2} + 3\left(\frac{x}{2(x^2+1)} + \frac{1}{2}\int \frac{dx}{x^2+1}\right) =$$

$$= 5\ln|x| - 2{,}5\ln|x^2+1) - \frac{2{,}5}{(x^2+1)^2} + \frac{1{,}5}{x^2+1} + 1{,}5\operatorname{arctg} x + C$$

8352.

$$\int \frac{1+e^x}{(1-e^{2x})e^x}dx = \int \frac{t+1}{t^2}dt + \int \frac{t+1}{1-t^2}dt =$$

$$= \int \frac{dt}{t} + \int \frac{dt}{t^2} + \int \frac{(t+1)dt}{(1-t)(t+1)} = \ln|t| - \frac{1}{t} + \ln|1-t| + C =$$

$$= \ln|e^x| - \frac{1}{e^x} + \ln|1-e^x| + C.$$

8353.

$$\int \frac{\cos x\,dx}{(\sin x)(\sin x+2)} = \frac{1}{3}\int \frac{dt}{t-1} - \frac{1}{3}\int \frac{dt}{t+2} = \frac{1}{3}\ln|t-1| -$$

$$- \frac{1}{3}\ln|t+2| + C = \frac{1}{3}\ln|\sin x - 1| - \frac{1}{3}\ln|\sin x + 2| + C.$$

8354.

$$\int \frac{\sin^4 x\,dx}{\cos x} = \frac{1}{2}\int \frac{(1-a)(-a-1)+1}{1-a}da =$$

$$= \frac{1}{2}\int(-a-1)da + \frac{1}{2}\int \frac{da}{1-a} = -\frac{1}{2}\int a\,da - \frac{1}{2}\int da +$$

$$+ \frac{1}{2}\int \frac{da}{1-a} = \frac{-0,5\cdot a^2}{2} - \frac{a}{2} + \frac{1}{2}\ln|1-a| + C =$$

$$= 0,25\sin^4 x - 0,5\sin^2 x + 0,5\ln(\cos^2 x) + C.$$