Интегралы часть 2.

8233

$$\int \cos(6x+1)dx$$

$t = 6x+1 \Rightarrow dt = 6dx \Rightarrow dx = \frac{1}{6}dt$

$$\int \cos(6x+1)dx = \int \cos t \cdot \tfrac{1}{6}dt = \frac{1}{6}\int \cos t \cdot dt = \frac{1}{6}\sin t + C =$$

$$= 1/6 \sin(6x+1) + C$$

8234

$$\int \frac{dx}{\sqrt[3]{(5x-2)^4}}, \quad t = 5x-2 \Rightarrow dt = 5dx \Rightarrow dx = 1/5\, dt$$

$$\int \frac{dx}{\sqrt[3]{(5x-2)^4}} = \frac{1}{5}\int \frac{1}{t^{4/3}}dt = \frac{1}{5}\cdot\frac{t^{-4/3+1}}{-4/3+1} + C = \frac{-3}{5\sqrt[3]{t}} + C = \frac{-3}{5\sqrt[3]{5x-2}} + C$$

8235

$$\int \frac{\sqrt{tg\,x}}{\cos^2 x}dx, \quad \frac{dt}{dx} = \frac{1}{\cos^2 x} \Rightarrow dx = \cos^2 x\, dt$$

$t = dx.$, $$\int \frac{\sqrt{tg\,x}}{\cos^2 x}dx = \int \frac{\sqrt{t}\cos^2 x\, dt}{\cos^2 x} = \int \sqrt{t}\, dt = \frac{2\sqrt{t^3}}{3} + C =$$

$$= \frac{2\sqrt{tg^3 x}}{3} + C$$

8236

$$\int \frac{e^x}{e^{2x}+9}dx, \quad t = e^x/3 \Rightarrow \frac{dt}{dx} = \frac{e^x}{3} \Rightarrow dx = 3e^{-x}dt$$

$$\int \frac{e^x}{e^{2x}+9}dx = \int \frac{3\,dt}{9t^2+9} = \frac{1}{3}\int \frac{dt}{t^2+1} = \frac{arctg\,t}{3} + C = \frac{arctg\, e^x/3}{3} + C$$

8237

$$\int \frac{x^5}{\sqrt{x^6+7}}dx, \quad t = x^6+7 \Rightarrow \frac{dt}{dx} = 6x^5 \Rightarrow dx = \frac{1}{6x^5}dt$$

$$\int \frac{x^5}{\sqrt{x^6+7}}dx = \frac{1}{6}\int \frac{1}{\sqrt{t}}dt = \frac{\sqrt{t}}{3} + C = \frac{\sqrt{x^6+7}}{3} + C$$

8238

$$\int \frac{dx}{\arccos x \cdot \sqrt{1-x^2}},\ t = \arccos x \Rightarrow \frac{dt}{dt} = -\frac{1}{\sqrt{1-x^2}} \Rightarrow dx = -\sqrt{1-x^2}\, dt$$

$$\int \frac{dx}{\arccos x \cdot \sqrt{1-x^2}} = \int \frac{-\sqrt{1-x^2}}{t \cdot \sqrt{1-x^2}}\, dt = \int \frac{-dt}{t} = \ln|t| + c =$$

$$= -\ln|\arccos x| + c.$$

8239

$$\int \frac{2x+3}{(x^2+3x-1)^4} dx,\ t = x^2+3x-1 \Rightarrow \frac{dt}{dx} = 2x+3 \Rightarrow dx = \frac{dt}{2x+3}$$

$$\int \frac{2x+3}{(x^2+3x-1)^4} dx = \int \frac{2x+3}{t^4(2x+3)}\, dt = \int \frac{dt}{t^4} = \frac{1}{3t^3} + C = \frac{1}{3(x^2+3x-1)^3} + C$$

8240

$$\int \cos^{11} 2x \cdot \sin 2x\, dx,\ t = \cos 2x \Rightarrow \frac{dt}{dx} = -2\sin 2x \Rightarrow dx = -\frac{dt}{2\sin 2x}$$

$$\int \cos^{11} 2x \cdot \sin 2x\, dx = \int \frac{t^{11} \sin 2x}{-2\sin 2x}\, dt = \int \frac{t^{11} dt}{-2} = -\frac{1}{2}\int t^{11} dt =$$

$$= -\frac{1}{2} \cdot \frac{t^{12}}{12} + C = -\frac{t^{12}}{24} + C = -\frac{\cos^{12} 2x}{24} + C$$

8241

$$\int \frac{7^{\sqrt{x}}}{\sqrt{x}} dx,\ t = \sqrt{x} \Rightarrow \frac{dt}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\, dt$$

$$\int \frac{7^{\sqrt{x}}}{\sqrt{x}} dx = \int \frac{7^t 2\sqrt{x}}{t}\, dt = \int \frac{7^t 2t}{t}\, dt = \frac{1}{2}\int 7^t dt = \frac{1}{2} \cdot \frac{7^t}{\ln 7} + C =$$

$$= \frac{7}{2\ln 7} + C$$

8242

$\int \frac{e^{1/x}}{x^2} dx, \ t = e^{1/x} \Rightarrow \frac{dt}{dx} = \frac{-e^{1/x}}{x^2} \Rightarrow dx = \frac{x^2}{-e^{1/x}} dt$

$\int \frac{e^{1/x}}{x^2} dx = \int \frac{t \cdot x^2}{-e^{1/x} \cdot x^2} dt = \int \frac{t}{-e^{1/x}} dt =$

$= -\frac{1}{e^{1/x}} \int t dt = -\frac{1}{e^{1/x}} \cdot \frac{t^2 + c}{2} = \frac{t^2}{2e^{1/x}} + C =$

$= \frac{(e^{1/x})^2}{2e^{1/x}} + C = -\frac{e^{1/x}}{2} + C$

8243

$\int \frac{\ln 5x}{x} dx; \ t = \ln 5x; \Rightarrow \frac{dt}{dt} = \frac{1}{x} \Rightarrow dx = x dt$

$\int \frac{\ln 5x}{x} dx = \int \frac{t \cdot x dt}{x} = \int t \, dt = \frac{t^2}{t} dt + c = \frac{\ln^2(5x)}{2} + C$

8244

$\int ctg x \, dx, \ t = \sin x \Rightarrow \frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

$\int \frac{\cos x}{\sin x} dx = \int \frac{\cos x}{t \cos x} dt = \int \frac{dt}{t} = \ln|t| + C = \ln|\sin x| + c$

8245

$\int 4x \cdot \sqrt[3]{x^2+8} \, dx, \ t = x^2 + 8 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow dx = \frac{dt}{2x}$

$\int 4x \sqrt[3]{x^2+8} \, dx = \int 4x \cdot \sqrt[3]{t} \frac{dt}{2x} = \int 2\sqrt[3]{t} dt = \frac{1}{2} \int t^{1/3} dt =$

$= \frac{3\sqrt[3]{t^4}}{8} + C = \frac{3\sqrt[3]{(x^2+8)^4}}{8} + c$

8246

$\int \frac{\cos x}{\sin^2 x} dx,\ t=\sin x, \Rightarrow \frac{dt}{dx}=\cos x \Rightarrow dx = \frac{dt}{\cos x}$

$\int \frac{\cos x}{\sin^2 x} dx = \int \frac{\cos x}{t^2 \cdot \cos x} dt = \int \frac{dt}{t^2} = \frac{t^{-1}}{-1} + C = -\frac{1}{t} + C = -\frac{1}{\sin x} + C$

8247

$\int tg\, 2x\, dx,\ t = 2x, \Rightarrow \frac{dt}{dx} = x \Rightarrow dx = \frac{dt}{2.}$

$\int tg\, 2x\, dx = \int \frac{tg\, t}{2} dt = \frac{1}{2}\int tg\, t\, dt = \frac{1}{2}\int \frac{\sin}{\cos t} dt = a\cos t$

$\frac{da}{dt} = -\sin t \Rightarrow a = \frac{da}{-\sin t}$

$\frac{1}{2}\int \frac{\sin t}{\cos t} dt = \frac{1}{2} \frac{\sin t\, da}{-a \sin t} = -\frac{1}{2}\int \frac{da}{a} + C =$

$= -\frac{1}{2} \ln|a| + C = -\frac{1}{2} \ln|\cos t| + C = -\frac{1}{2} \ln(\cos(2x)) + C$

8248.

$\int \frac{x}{x^4+1} dx,\ t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow dx = \frac{dt}{2x}$

$\int \frac{x}{x^4+1} dx = \int \frac{\sqrt{t}\, dt}{(t^2+1)2x} = \int \frac{\sqrt{t}\, dt}{(t^2+1)2\sqrt{t}} = \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2} \operatorname{arctg} t + C$

$= \frac{\operatorname{arctg} x^2}{2} + C$

8249

$\int e^{-x} \cdot x^2 dx,\ t = e^{-x^3} \Rightarrow \frac{dt}{dx} = -3x^2 e^{-x^3} \Rightarrow dx = \frac{dt}{-3x^2 e^{-x^3}}$

$\int e^{-x^3} \cdot x^2 dx = \int \frac{t \cdot x^2}{-3x^2 \cdot t} dt = -\frac{1}{3}\int dt = -\frac{1}{3} t + C = \frac{-e^{-x^3}}{3} + C$

8250

$\int \frac{x^2}{\sqrt{x^6-4}}dx,\ t=x^3/2 \Rightarrow \frac{dt}{dx}=\frac{3x^2}{2} \Rightarrow dx=\frac{2dt}{3x^2}$

$\int \frac{x^2}{\sqrt{x^6-4t}}dx=\int \frac{x^2\cdot 2dt}{\sqrt{4t^2-4}\cdot 3x^2}=\frac{2}{3}\int\frac{dt}{\sqrt{4t^2-4}}=\frac{2}{3}\ln|2t+\sqrt{4t^2-4}|+C=$

$=\frac{2}{3}\ln|x^3+\sqrt{x^6-4}|+C$

8251

$\int(8\cos\frac{x}{3}-5)^2\sin\frac{x}{3}dx,\ t=8\cos\frac{x}{3}-5$

$\Rightarrow \frac{dt}{dx}=-\frac{8}{3}\sin\frac{x}{3} \Rightarrow dx=\frac{dt}{-8/3\sin x/3}$

$\int(8\cos\frac{x}{3}-5)^2\sin\frac{x}{3}dx=\int t^2\sin\frac{x}{3}\frac{dt}{-\frac{8}{3}\sin\frac{x}{3}}=\int-\frac{3}{8}t^2dt=$

$=-\frac{3}{8}\int t^2dt=-\frac{3}{8}\cdot\frac{t^3}{3}+C=-\frac{t^3}{8}+C=\frac{(8\cos\frac{x}{3}-5)^3}{8}+C$

8252.

$\int\frac{3x^2-2x+7}{\sqrt{x^3-x^2+7x-2}}dx,\ t=x^3-x^2+7x-2 \Rightarrow \frac{dt}{dx}=3x^2-2x+7 \Rightarrow dx=\frac{dt}{3x^2-2x+7}$

$\int\frac{3x^2-2x+7}{\sqrt{x^3-x^2+7-2}}dx=\int\frac{3x^2-2x+7}{\sqrt{t}(3x^2-2x+7)}dt=\int\frac{dt}{\sqrt{t}}=\int t^{-1/2}dt=\frac{t^{1/2}}{1/2}+C=$

$=2\sqrt{t}+C=2\sqrt{x^3-x^2+7x-2}+C$

8253

$\int x(2x+1)^{35}dx,\ t=2x+1 \Rightarrow \frac{dt}{dx}=2 \Rightarrow dx=\frac{dt}{2}$

$\int x(2x+1)^{35}dx=\int xt^{35}\frac{dt}{2}=\int\frac{t-1}{2}\cdot t^{35}\cdot\frac{dt}{2}=\frac{1}{4}\int(t-1)\cdot t^{35}dt=\frac{1}{4}\int(t^{36}-t^{35})dt=$

$=\frac{1}{4}(\int t^{36}dt-\int t^{35}dt)=\frac{1}{4}\left(\frac{t^{37}}{37}-\frac{t^{36}}{36}\right)+C=\frac{t^{37}}{148}-\frac{t^{36}}{144}+C=\frac{(2x+1)^{37}}{148}-\frac{(2x+1)^{36}}{144}+C$

8254

$\int (x-2)\sqrt{x+4}\,dx,\ t=x+4 \Rightarrow \frac{dt}{dx}=1 \Rightarrow dx=dt.$

$\int (x-2)\sqrt{x+4}\,dx = \int (t+h)\sqrt{t}\,dt = \int (t^{3/2}+2t^{1/2})\,dt =$

$= \int t^{3/2}dt + 2\int t^{1/2}dt = \frac{2t^{5/2}}{5} + \frac{4}{3}\cdot t^{3/2} + C = \frac{2\sqrt{t^5}}{5} + \frac{4\sqrt{t^3}}{3} + C =$

$= \frac{2\sqrt{(x+4)^5}}{5} + \frac{4\sqrt{(x+4)^3}}{3} + C$

8255.

$\int \frac{3\sqrt{x}-2\cos 1/x^2}{x^3}\,dx = \int \frac{3\sqrt{x}\,dx}{x^3} - \int \frac{2\cos 1/x^2}{x^3}\,dx =$

$= 3\int \frac{1}{x^{5/2}} - 2\int \frac{\cos 1/x^2}{x^3}\,dx = 3\left(-\frac{2}{3x^{3/2}}\right) - 2\int \frac{\cos t\cdot(x)^3\,dt}{x^3\cdot 2} =$

$= -\frac{2}{\sqrt{x^3}} + \int \cos t\,dt = -\frac{2}{\sqrt{x^3}} + \sin t + C = -\frac{2}{\sqrt{x^3}} + \sin 1/x^2 + C$

8256.

$\int \frac{7x+2}{\sqrt{x^2+10}}\,dx = \int \frac{7x}{\sqrt{x^2+10}}\,dx + 2\int \frac{dx}{\sqrt{x^2+10}} = 7\int \frac{dt\cdot x}{\sqrt{t}\cdot 2x} + 2\int \frac{dt}{\sqrt{t}\cdot 2x} =$

$= \frac{7}{2}\int t^{-1/2}\,dt + 2\int \frac{dt}{\sqrt{t}\cdot\sqrt{t-10}} = \frac{7}{2}\,\frac{\sqrt{t}}{1/2} + 2\int \frac{dt}{\sqrt{t^2-10}} =$

$= 7\sqrt{t} + 2\ln\left|t+\sqrt{t^2-10}\right| + C =$

$= 7\cdot\sqrt{x^2+10} + 2\ln\left|x^2+10+\sqrt{(x^2+10)^2-10}\right| + C$

8257.

$\int \frac{dx}{e^x+e^{-x}} = \int \frac{dx}{e^x} + \int \frac{dx}{e^{-x}} = \int e^{-x}dx + \int e^{x}dx = \int -e^{-t}dt + e^{t} = -\int e^{t}dt + e^{t} =$

$= -e^{t} + e^{x} + C = -e^{-x} + e^{x} + C = -\frac{1}{e^x} + e^x + C$

8258.

$$\int \frac{x+8}{x^2+3}dx = \int \frac{x}{x^2+3}dx + 8\int \frac{dx}{x^2+3} =$$

$$\int \frac{x}{x^2+3}dx = \int \frac{x \cdot dt}{t \cdot 2x} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|x^2+3| + C$$

$$\int \frac{dx}{x^2+3} = \int \frac{\sqrt{3}\,dr}{3r^2+3} = \int \frac{\sqrt{3}\,dr}{3(r^2+1)} = \frac{1}{\sqrt{3}}\int \frac{dr}{r^2+1} = \frac{1}{\sqrt{3}}\,\mathrm{arctg}\, r + C =$$

$$= \frac{1}{\sqrt{3}} \cdot \mathrm{arctg}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$\Rightarrow \frac{1}{2}\ln(x^2+3) + \frac{8}{\sqrt{3}}\,\mathrm{arctg}\left(\frac{x}{\sqrt{3}}\right) + C$$

8259.

$$\int \frac{x+4\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx = \int \frac{x}{\sqrt{1-x^2}}dx + 4\int \frac{\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx =$$

$$\int \frac{x}{\sqrt{1-x^2}}dx = \int \frac{x\,dt}{\sqrt{t}\,(-2x)} = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\sqrt{t} + C = -\sqrt{1-x^2} + C$$

$$\int \frac{\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx = r = \arcsin x \Rightarrow \frac{dr}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow dx = \sqrt{1-x^2}\,dr =$$

$$= \int \frac{\sqrt{r}\cdot dr\sqrt{1-x^2}}{\sqrt{1-x^2}} = \int \sqrt{r}\,dr = \frac{2\sqrt{r^3}}{3} + C = \frac{2\sqrt{\arcsin^3 x}}{3} + C$$

$$\Rightarrow -\sqrt{1-x^2} + \frac{8\cdot\sqrt{\arcsin^3 x}}{3} + C$$

8260.

$$\int \frac{1-6x}{(x+1)(x-1)}dx = \int \frac{1-6x}{x^2-1}dx = \int \frac{dx}{x^2-1} - 6\int \frac{x}{x^2-1}dx =$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 6\int \frac{x}{x^2-1}dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 6\int \frac{x\,dt}{t\cdot 2x} = \frac{1}{2}\ln$$

$$\left|\frac{x-1}{x+1}\right| - 3\int \frac{dt}{t} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 3\ln|t| + C = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| -$$

$$- 3\ln|x^2-1| + C$$

8261.

$$\int (\cos^2x - \sin^2x)\sqrt[3]{1+\sin 2x}\,dx = \int \cos 2x\cdot\sqrt[3]{1+\sin 2x}\,dx =$$

$$= \int \frac{\cos 2x\cdot\sqrt[3]{t}\,dt}{2\cos 2x} = \frac{1}{2}\cdot\int \sqrt[3]{t}\,dt = \frac{1}{2}\cdot\frac{3t^{4/3}}{4} + C =$$

$$= \frac{3\sqrt[3]{(\sin 2x+1)^4}}{8} + C$$

8262.

$$\int \frac{e^{tgx} - 7\sin x + 5\sin 2x}{\cos^2x}dx = \int \frac{e^{tgx}}{\cos^2x}dx - 7\int \frac{\sin x\,dx}{\cos^2x} + 5\int \frac{\sin 2x\,dx}{\cos^2x}$$

$$\int \frac{e^{tgx}}{\cos^2x}dx = \int \frac{e^t\,dt\cdot\cos^2x}{\cos^2x} = \int e^t\,dt = e^t + C = e^{tgx} + C$$

$$\int \frac{\sin x}{\cos^2x}dx = \int \frac{\sin x\,dt}{t(-2\cos x\sin x)} = -\frac{1}{2}\int \frac{dt}{t\sqrt{t}} = \frac{1}{2}\int t^{-3/2}dt = \frac{1}{2}\cdot\frac{t^{-1/2}}{-1/2} + C =$$

$$= \frac{1}{\sqrt{t}} + C = \frac{1}{\sqrt{\cos^2x}} + C = \frac{1}{\cos x} + C$$

$$\int \frac{\sin 2x}{\cos^2x}dx = \int \frac{\sin 2x}{t\cdot(-\sin 2x)}dt = 2\int \frac{dt}{t} = 2\cdot\ln|t| + C = 2\ln|\cos^2x| + C.$$

$$\Rightarrow e^{tgx} - \frac{7}{\cos x} - 10\ln|\cos x| + C.$$

8263.

$$\int \sqrt{16-x^2}\,dx = \int \sqrt{16-16\sin^2 t}\cdot 4dt\cos t =$$

$$= 4\cdot\int 4\cos t\cdot\cos t\,dt = 16\int\cos^2 t\,dt =$$

$$= 16\int \frac{\cos 2t+1}{2}\,dt = 8\cdot\int\cos 2t\,dt + 8\int \cancel{t}dt =$$

$$= 8\int\cos r\cdot\frac{dr}{2} + 8t + C = 4\int\cos r\cdot dr + 8t + C = \cos r + 8\arcsin\frac{t}{4} + C =$$

$$= 4\sin 2t + 8\arcsin\frac{x}{4} + C = 4\sin 2(\arcsin\frac{x}{4}) + 8\arcsin\frac{x}{4} + C =$$

$$= 2x\sqrt{1-\frac{x^2}{16}} + 8\arcsin\frac{x}{4} + C.$$

8264.

$$\int\frac{dx}{1+\sqrt{x}} = \int\frac{2t-2}{1+t-1}\,dt = 2\int\frac{t-1}{t}\,dt = 2\int\frac{t}{t}\,dt - 2\cdot\int\frac{dt}{t} =$$

$$= 2t - 2\ln|t| + C = 2(\sqrt{x}+1) - 2\ln|\sqrt{x}+1| + C = 2(\sqrt{x}+1) - 2\ln(\sqrt{x}+1) + C.$$

8265.

$$\int x\sqrt{x+3}\,dx = \int(t-3)\sqrt{t}\,dt = \int(t\sqrt{t} - 3\sqrt{t})\,dt = \int t^{3/2}dt - 3\int t^{1/2}dt =$$

$$= \frac{2t^{5/2}}{5} - 3\cdot\frac{2t^{3/2}}{3} + C = \frac{2\cdot(x+3)^{5/2}}{5} - 2(x+3)^{3/2} + C =$$

$$= \frac{2\sqrt{(x+3)^5}}{5} - 2\sqrt{(x+3)^3} + C.$$

8266.

$$\int\frac{dx}{(x+1)\sqrt{x}} = \int\frac{2\sqrt{x}\,dt}{(t^2+1)\sqrt{x}} = 2\int\frac{dt}{t^2+1} = 2\,\mathrm{arctg}\,t + C =$$

$$= 2\,\mathrm{arctg}\sqrt{x} + C$$

8267.

$$\int \frac{x\,dx}{\sqrt{1-x}} = \int \frac{-(1-t)\,dt}{\sqrt{1-1+t}} = -\int \frac{(1-t)\,dt}{\sqrt{t}} = -\left(\int \frac{dt}{\sqrt{t}} - \int \frac{t\,dt}{\sqrt{t}}\right) =$$

$$= -\int \frac{dt}{\sqrt{t}} + \int \sqrt{t}\,dt = -2\sqrt{t} + \frac{2\sqrt{t^3}}{3} + C =$$

$$= -2\sqrt{1-x} + \frac{2\sqrt{(1-x)^3}}{3} + C$$

8268.

$$\int \frac{x^2\,dx}{\sqrt{1-x^2}} = \int \frac{\sin^2 t \cos t}{\cos t}\,dt = \int \sin^2 t\,dt = \int \frac{1-\cos 2t}{2}\,dt =$$

$$= \int \frac{1}{2}\,dt - \int \frac{\cos 2t}{2}\,dt = \frac{1}{2}\int dt - \frac{1}{2}\int \cos 2t\,dt =$$

$$= \frac{t}{2} - \frac{1}{2}\int \cos r \cdot \frac{1}{2}\,dr = \frac{t}{2} - \frac{1}{4}\int \cos r\,dr = \frac{t}{2} - \frac{\sin r}{4} + C =$$

$$= \frac{t}{2} - \frac{\sin 2t}{4} + C = \frac{\arcsin x}{2} - \frac{2x\sqrt{1-x^2}}{4} + C =$$

$$= \frac{\arcsin x - x\sqrt{1-x^2}}{2}$$

8269.

$$\int x \ln x\,dx = \frac{\ln x \cdot x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx = \frac{\ln x \cdot x^2}{2} - \frac{1}{2}\int x\,dx =$$

$$= \frac{\ln x \cdot x^2}{2} - \frac{x^2}{4} + C = \frac{2\ln x \cdot x^2 - x^2}{4} + C = \frac{x^2(2\ln x - 1)}{4} + C$$

8270.

$$\int (2x+3)\cos x\,dx = (2x+3)\sin x - \int 2\sin x\,dx =$$
$$= (2x+3)\sin x + 2\cos x + C.$$

8271.

$$\int x\,\mathrm{sh}\,5x\,dx = \frac{x\cdot \mathrm{ch}\,5x}{5} - \int \frac{\mathrm{ch}\,5x}{5}dx = \frac{x\,\mathrm{ch}\,5x}{5} -$$
$$- \frac{1}{5}\int \mathrm{ch}\,5x\,dx = \frac{x\,\mathrm{ch}\,5x}{5} - \frac{1}{5}\int \mathrm{ch}\,t\,\frac{dt}{5} = \frac{x\,\mathrm{ch}\,5x}{5} - \frac{1}{25}\mathrm{ch}\,t + C =$$
$$= \frac{x\,\mathrm{ch}\,5x}{5} - \frac{1}{25}\mathrm{sh}\,5x + C = \frac{5x\,\mathrm{ch}\,5x - \mathrm{sh}\,5x}{25} + C$$

8272.

$$\int \frac{x\cos x\,dx}{\sin^3 x} = -\frac{x}{2\sin^2 x} - \int -\frac{dx}{2\sin^2 x} = -\frac{x}{2\sin^2 x} + \frac{1}{2}\int \frac{dx}{\sin^2 x} =$$
$$= -\frac{x}{2\sin^2 x} - \frac{\mathrm{ctg}\,x}{2} + C$$

8273.

$$\int x^2 \ln x\,dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3x}dx = \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2\,dx =$$
$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C = \frac{x^3(3\ln x - 1)}{9} + C.$$

8274.

$$\int (x^2-4x+1)e^{-x}dx = -e^{-x}(x^2-4x+1) - \int -e^{-x}(2x-4)dx =$$
$$= -e^{-x}(x^2-4x+1) + 2\int e^{-x}(x-2)dx = -e^{-x}(x^2-4x+1) - 2e^{-x}(x-2) +$$
$$+ 2\int e^{-x}dx = -e^{-x}(x^2-4x+1+2x-4) - 2\int e^{t}dt = -e^{-x}(x^2-2x-3) - 2e^{t} + C =$$
$$= -e^{-x}(x^2-2x-3) - 2e^{-x} + C = -e^{-x}(x^2-2x-3+2) + C = -e^{-x}(x^2-2x-1) + C.$$

8275.

$\int x^3 e^x dx = x^3 e^x - \int 3x^2 e^x dx = x^3 e^x - 3\int x^2 e^x dx =$

$= x^3 e^x - 3(x^2 e^x - 2\int x e^x dx) = x^3 e^x - 3(x^2 e^x - 2(x e^x - \int e^x dx)) =$

$= x^3 e^x - 3(x^2 e^x - 2x e^x + 2e^x) + C = x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C =$

$= e^x(x^3 - 3x^2 + 6x - 6) + C$

8276.

$\int \frac{\arccos x}{\sqrt{1+x}} dx = \int \arccos x (1+x)^{-\frac{1}{2}} dx =$

$= \arccos x \cdot 2\sqrt{x+1} - \int \frac{2\sqrt{x+1}}{-\sqrt{1-x^2}} dx = 2\sqrt{x+1} \arccos x + 2\int \frac{dx}{\sqrt{1-x}} =$

$= 2\sqrt{x+1} \arccos x + 2\int \frac{-dt}{\sqrt{t}} = 2\sqrt{x+1} \arccos x - 4\sqrt{t} + C =$

$= 2\sqrt{x+1} \arccos x - 4\sqrt{1-x} + C.$

8277.

$\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}} dx = \int \arcsin\sqrt{x} \cdot (1-x)^{-\frac{1}{2}} dx =$

$= -2\sqrt{1-x} \arcsin\sqrt{x} - \int \frac{-2\sqrt{1-x}}{2\sqrt{x}\sqrt{1-x}} dx = -2\sqrt{1-x} \arcsin\sqrt{x} + \int \frac{dx}{\sqrt{x}} =$

$= -2\sqrt{1-x} \arcsin\sqrt{x} + 2\sqrt{x} + C = 2(\sqrt{x} - \sqrt{1-x} \arcsin\sqrt{x}) + C$

8279.

$\int \cos(\ln x) dx = \int \cos t \, e^t dt = e^t \cos t - \int -\sin t \, e^t dt =$

$= e^t \cos t - (e^t \sin t) - \int e^t(-\cos t) dt)$

$\int \cos t \, e^t dt = e^t \cdot \cos t + e^t \sin t - \int e^t \cos t \, dt$

$2\int e^t \cos t \, dt = e^t \cdot (\cos t + \sin t)$

$\int e^t (\cos t \, dt) = \frac{e^t(\cos t + \sin t)}{2}$

$\int \cos \ln x \, dx = \frac{x(\cos(\ln x) + \sin(\ln x))}{2}.$

8280.

$$\int e^{3x}\cdot\cos^2 x\,dx = \frac{e^{3x}\cdot\cos^2 x}{3} - \int\frac{-2\cos x\sin x\cdot e^{3x}}{3}dx =$$

$$= \frac{e^{3x}\cdot\cos^2 x}{3} + \frac{1}{3}\int \sin 2x\cdot e^{3x}dx = \frac{e^{3x}\cdot\cos^2 x}{3} + \frac{1}{3}\left(\frac{e^{3x}\sin 2x}{3}\right.$$

$$\left.-\int\frac{e^{3x}\cdot 2\cos 2x}{3}dx\right) = \frac{e^{3x}\cdot\cos^2 x}{3} + \frac{e^{3x}\sin 2x}{9} - \frac{1}{3}\cdot\frac{2}{3}\cdot$$

$$\cdot\int e^{3x}\cdot\cos 2x\,dx = \frac{e^{3x}\cdot\cos^2 x}{3} + \frac{e^{3x}\sin 2x}{9} - \frac{2}{9}\left(\frac{e^{3x}\cos 2x}{3}\right. -$$

$$\left.-\int\frac{e^{3x}\cdot(-2\sin 2x)dx}{3}\right) =$$

8281

$$\int e^{\sqrt{x}}dx = \int e^t\cdot 2t\cdot dt = 2\int e^t\cdot t\,dt = 2\cdot(te^t - \int e^t dt) =$$

$$= 2t\cdot e^t - 2e^t + C = 2e^t(t-1) + C = 2e^{\sqrt{x}}(\sqrt{x}-1) + C.$$

8282.

$$\int\frac{x\,dx}{\cos^2 x} = x\,tg\,x - \int tg\,x\,dx = x\,tg\,x + \ln|\cos x| + C.$$

8283

$$\int x^3\cdot e^{x^2}dx = \int\frac{x\,t\,e^t\,dt}{2x} = \frac{1}{2}\int t\cdot e^t dt = \frac{1}{2}(t\cdot e^t - \int e^t dt) =$$

$$= \frac{1}{2}t\cdot e^t - \frac{1}{2}e^t + C = \frac{e^t(t-1)}{2} + C = \frac{e^{x^2}(x^2-1)}{2} + C.$$

8285

$\int \sin 2x \ln \sin x\,dx = \sin^2 x \ln \sin x - \int \frac{\cos x \sin^2 x}{\sin x}dx =$

$= \sin^2 x \ln \sin x - \int \cos x \sin x\,dx = \sin^2 x \ln \sin x -$

$- (\cos^2 x - \int \sin x \cos x\,dx) = \sin^2 x \ln \sin x - \int \cos x \sin x\,dx =$

$= \sin^2 x \ln \sin x + \cos^2 x + \int \sin x \cos x\,dx$

$2\int \sin x \cos x\,dx = -\cos^2 x$

$\int \sin x \cos x\,dx = \frac{-\cos^2 x}{2}$

$\int \sin 2x \ln \sin x\,dx = \sin^2 x \ln \sin x + \frac{\cos^2 x}{2} + C$

8288

$\int \arcsin^2 x\,dx = \int t^2 \sqrt{1-x^2}\,dt = \int t^2 \cos t\,dt =$

$= t \sin t - \int 2t \sin t\,dt = t^2 \sin t - 2\int t \sin t\,dt =$

$= t^2 \sin t - 2(-t\cos t - \int \cos t\,dt) =$

$= t^2 \sin t + 2t\cos t - 2\int \cos t\,dt = t^2 \sin t + 2t\cos t - 2\sin t + C =$

$= x \arcsin^2 + 2\arcsin x \cos(\arcsin x) - 2x + C =$

$= x \arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C.$

8290.

$\int \mathrm{arctg}\sqrt{x} = x\,\mathrm{arctg}\sqrt{x} - \int \frac{\sqrt{x}\,dx}{2(x+1)} = x\,\mathrm{arctg}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}\,dx}{x+1} =$

$= x\,\mathrm{arctg}\sqrt{x} - \frac{1}{2}\int \frac{t \cdot 2t\,dt}{t^2+1} = x\,\mathrm{arctg}\sqrt{x} - \int \frac{t^2\,dt}{t^2+1} =$

$= x\,\mathrm{arctg}\sqrt{x} - \int \left(\frac{t^2+1}{t^2+1} - \frac{1}{t^2+1}\right)dt = x\,\mathrm{arctg}\sqrt{x} - \int dt + \int \frac{dt}{t^2+1} =$

$= x\,\mathrm{arctg}\sqrt{x} - t + \mathrm{arcctg}\,t + C = x\,\mathrm{arcctg}\sqrt{x} - \sqrt{x} + \mathrm{arctg}\sqrt{x} + C.$

$= \mathrm{arctg}\sqrt{x}(x+1) - \sqrt{x} + C.$

8292.

$$\int \frac{\ln^2 x\,dx}{x\sqrt{3-\ln x}} = \int \frac{-(3-t)^2 x\,dt}{x\sqrt{t}} = -\int \frac{(t-3)^2 dt}{\sqrt{t}} =$$

$$= -\int \frac{t^2-6t+9}{\sqrt{t}}dt = -\int t^{\frac{3}{2}}dt + 6\int \sqrt{t}\,dt + 9\int \frac{dt}{\sqrt{t}} =$$

$$= -\frac{2t^{5/2}}{5} + \frac{12t^{3/2}}{3} - 18\sqrt{t} + C = \frac{-2\sqrt{(3-\ln x)^5}}{5} +$$

$$+ 4\sqrt{(3-\ln x)^3} - 18\sqrt{3-\ln x} + C$$

8293.

$$\int \frac{e^{\operatorname{arctg} x} + 8x\,dx}{1+x^2} = \int \frac{e^{\operatorname{arctg}}dx}{1+x^2} + 8\int \frac{x\,dx}{x^2+1} =$$

$$= \int \frac{e^{\operatorname{arctg} x}dx}{x^2+1} = \int \frac{e^t(x^2+1)dt}{x^2+1} = \int e^t dt = e^t + C = e^{\operatorname{arctg} x} + C.$$

$$\int \frac{x\,dx}{x^2+1} = \int \frac{x\,dt}{t\cdot 2x} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln(x^2+1) + C =$$

$$= \frac{1}{2}\ln(x^2+1) + C.$$

$$\Rightarrow e^{\operatorname{arctg} x} + 4\ln(x^2+1) + C.$$

8294.

$$\int \frac{3x + 5\sin\left(\frac{1}{e^x}\right)}{e^x}dx = \int e^{-x}(3x + 5\sin(e^{-x}))dx =$$

$$= \int e^{-x}\cdot 3x\,dx \cdot \int e^{-x}\cdot 5\sin(e^{-x})dx = 3\int xe^{-x}dx +$$

$$+ 5\int e^{-x}\sin(e^{-x})dx =$$

$$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx = -xe^{-x} - \int e^t dt = -xe^{-x} - e^t + C = -$$

$$- xe^{-x} - e^{-x} + C = e^{-x}(-x-1) + C.$$

$$\int e^{-x}\sin(e^{-x})dx = \int \frac{t\sin t\,dt}{-t} = -\int \sin t\,dt = \cos t + C = \cos(e^{-x}) + C$$

$$\Rightarrow 3e^{-x}(-x-1) + 5\cos(e^{-x}) + C.$$