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MANUSCRIPT

Modeling criminality: the impact of emotions, norms and interaction structures

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Abstract Criminal behavior has been explained in the literature by rational or normative arguments. We propose a game theory framework of criminal behavior integrating both concepts. Specifically the modeling includes three factors, namely the gain from criminality, the adherence to a legal norm and social pressure from criminal peers. We show that criminality cannot be lower with increasing gain from criminality, lower adherence to the legal norm or higher social pressure from criminal peers. Finally, we observe by agent-based simulations that small local interaction structures lead to spatial segregation in criminality in the case where a polymorphic equilibrium is expected.

Keywords Criminality · Emotions · Norms · Network · Segregation · Agent-based modeling

1 Introduction

Criminal acts are explained by two types of arguments. On the one hand, the rational approach to crime is based on selfish motives relying on a cost-benefit analysis (Becker 1968). From an empirical point of view, there is some evidence that usual factors such as lower rates of unemployment, lower income inequalities, police crackdowns, more police officers and higher sanctions reduce the level of

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criminality (Bourguignon 2001; Bourguignon et al. 2003; Freeman 1999; Paternoster 2010). On the other hand, social interaction models (Glaeser and Scheinkman 2000; Glaeser et al. 2003; Brock and Durlauf 2001) that posit that individual behavior depends not only on the incentives faced by an individual but also on the behavior of peers, offer one promising way to explain the concentration of crime (Freeman 1999; Ludwig et al. 2001; Falk and Fischbacher 2002). In particular, some social scientists (Yar 2009; Cromby et al. 2010; Van Winden and Ashs 2009; Wenzel 2004) argue for a better understanding of criminality through aspects that recover both emotions and social norms in addition to the usual arguments on rationality. Individuals trade-off between rational aspects of criminality and moral concerns (Schwartz and Orleans 1967) or internalized sociological norms sustained by emotions such as shame, guilt or empathy (Van Winden and Ashs 2009) i.e. the degree by which a norm is followed is related both to an idiosyncratic emotion and to the number of individuals following the norm. In this paper, we posit two behavioral factors not directly related to a cost benefit argument. First, we consider a norm for legality but adherence to the norm measured by the guilt felt from legal offense depends on the proportion of people respecting the norm in one's own group. Second, we take into account the social pressure from criminal peers to commit illegal acts.

The first factor supposes that deviation from a norm provokes a discomfort that we call guilt, albeit at a different degree across individuals. Adding psychological aspects relating both to normative and emotional factors to monetary payoffs can explain cooperation rates above the equilibrium level in social dilemmas such as public good games (Dawes and Thaler 1988; Fehr and Fischbacher 2004; Gordon et al. 2005; Ledyard 1995; Waldeck 2013). Punishment is one mechanism that sustains cooperative behavior. Punishing a perceived unfair behavior activates a zone in the brain related to gains (De Quervain et al. 2004; Strobel et al. 2011) so that the cost of punishment is in some sense compensated by the satisfaction from the act of punishment. The mirror side of this process is that people may feel guilty about violating a norm and guilt is an idiosyncratic feeling triggered by perceiving oneself as being a bad person i.e. an internal emotional mechanism may sustain a cooperative behavior. The biosocial criminology (Beaver and Walsh 2011; Walsh 2011) also refers to the brain to explain criminality and the link between emotions created by the limbic system and the rational evaluation by the prefrontal cortex is a factor explaining antisocial behavior.[1] For example, Wenzel (2004) shows that social norms influence taxpaying behavior only if they are internalized through a process of identification with the group holding the norms. Another study by Mehlkop and Graeff (2010) on German tax fraud also shows the interaction effect between norm obedience and expected gains with more obedience to the law reducing the likelihood of fraud.

The second factor of our modeling relates to the fact that some criminal acts occur in groups, showing thereby a tendency to conform to a group behavior by social pressure from peers. Antisocial peer groups have emerged as one of the

[1] More generally, the neuroeconomic literature provides similar conclusions for social dilemmas (Zak 2004; Camerer et al. 2004).

strongest and most consistent predictors of adolescent delinquency (Faris and Ennett 2012; Haynie 2001). Peer pressure is correlated to the capacity of parents to control their own children, which indeed may be more difficult in a city suburb than in a residential area. Social control theory (Hirschi 2002) argues that adolescents who are tightly bonded to family, school, and peers are less likely to engage in delinquent acts. When bonded to delinquent peers, however, the bond is then toward delinquency (Faris and Ennett 2012). In this paper, social pressure is thus intended to capture the impact of neighborhood criminality on individual behavior.

Finally, a third aspect relates to the spatial or network structure of criminality. It has been shown that the variability in criminality across different geographical zones is not completely explained by the variance in social or economic factors (Glaeser et al. 1996). In a prisoner's dilemma, the existence of cooperative zones has been largely explained by the structure of network interactions (Nowak et al. 1994; Nowak and May 1992; Eshel et al. 1998; Epstein 1997; Cohen and Riolo 2001; Waldeck 2013). We study the impact of local interactions on the spatial variability of criminality. But more generally, the topology of the network is one factor creating the conditions for contagion and diffusion (Christakis and Fowler 2007; Ugander et al. 2012; Watts and Dodds 2007) and it should be part of the explanans of delinquent behavior.

The research questions are the following: first, how effective is punishment policy in reducing criminal behavior in this context? In particular, we study the impact of criminal gains, norm adherence and social pressure on the offense rate. Second, what is the impact of interaction structures on criminality? Section 2 of this article presents a highly stylized modeling of criminal behavior and shows the resulting equilibria in a global interaction context. Section 3 shows simulation results for the case where interactions are localized in a small neighborhood with the possibility of unhappy agents moving. In Sect. 4, extensions to the proposed modeling are discussed.

2 Modeling and equilibria

Each player has two actions: s/he can take a legal ($s = 1$) or an illegal action ($s = 0$). We take a dual cognitive—emotional approach in modeling a player's decision.

The modeling of the cost-benefit component

The cognitive (or rational) approach supposes that a player acts according to an expected utility principle.[2] The gain from each action is depicted in Fig. 1. The expected monetary gain from taking the legal action $s = 1$ depends on the benefits from employment and unemployment and on the probability u of being unemployed. Without loss of generality, we normalize the unemployment benefit

[2] Cognitive rationality would mean that players make a conscious cost-benefit analysis by taking into account both the consequences and the likelihood of different events. This does not mean that expected utility should be the appropriate framework as shown by prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992). Nevertheless, it may be a good first approximation in a stylized framework.

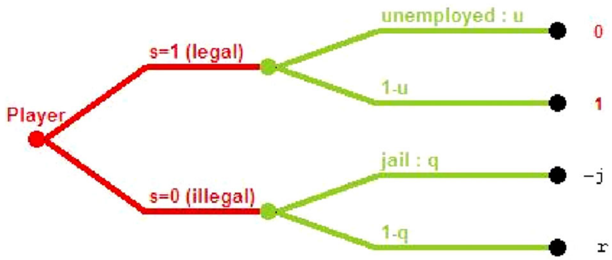


Fig. 1 Decision tree of an individual

to 0 and salary to 1. Assuming risk neutrality, expected monetary gains for $s = 1$ are then $G_L = 1 - u$. Gains from criminality depend on the probability of being caught q. j is the fine when caught and r is the monetary gain of a criminal who has not been caught. The expected monetary gain from undertaking an illegal action $s = 0$ is $G_I = -qj + (1 - q)r$.

Assumption 1 The relative gain G from a criminal action compared to remaining honest is then $G = G_I - G_L$. We suppose that G is the same for all individuals so that from an ex-ante point of view no economic heterogeneity exists which may explain spatial segregation in criminality.

The modeling of the social interaction components

We suppose that being honest ($s = 1$) is understood by individuals to be the norm to follow but that individuals adhere more or less strongly to the norm. The process by which this occurs is through an idiosyncratic emotion (Y), guilt, supported when deviating from the norm. Emotions and adherence to the norm may be influenced by the criminal justice system by encouraging and educating people toward norms which contribute to a general social and cultural climate discouraging violence (Salazar et al. 2003).

A second type of emotional feeling (X) relates to the social pressure exerted by criminals on honest individuals. We intend to model some conformity principle prevalent in gang behavior. X is interpreted as the idiosyncratic feeling of an honest individual from the pressure exerted by a criminal; it expresses the pressure to conform to criminal behavior and we think of this term as applying particularly well to youth behavior embedded in some social groups. The game in Table 1 represents the payoffs of player 1 when s/he faces a player 2. 'Real' payoffs are expressed by the expected gains G_L, G_I and the game in Table 1 without emotions (X, Y) is a game with a dominant strategy depending on the relative gain $G = G_I - G_L$.

Table 1 Payoff matrix for player 1 with idiosyncratic emotions y^1 and x^1

		Player 2	
		$s^2 = 1$	$s^2 = 0$
Player 1	$s^1 = 1$	G_L	$G_L - x^1$
	$s^1 = 0$	$G_I - y^1$	G_I

Introducing idiosyncratic emotions, X and Y, changes the structure of the game and allows for a richer set of possible equilibria.

Assumption 2 For each individual i, Y^i is an i.i.d. random variable from a uniform distribution $[0, Y_M]$. For each individual i, X^i is an i.i.d. random variable from a Bernoulli distribution with $p = prob[X = X_M]$ and $1 - p = prob[X = 0]$. We suppose for simplicity that $X_M = Y_M$.[3]

Assumption 3 p is called social pressure and p may be a proxy for the degree of parental control. The higher p, the lower the parental control.

Social pressure p depends also on the quality of the living environment. In a suburb with high rises, it may be more difficult to exert effective parental control and p may then be higher than in residential areas.

Table 2 is best response equivalent to Table 1 i.e. Nash equilibria are the same. In addition, without changing the Nash analysis, we normalize the relative gain from criminality G and X^i, Y^i by Y_M. *Accordingly, Y^i follows a uniform distribution* [0, 1] *and X a Bernoulli distribution with $p = prob[X = 1]$ and G is interpreted as the relative gain normalized by the maximal emotion Y_M*. It is evident from Table 2 that an individual with $X = 1$ has a higher incentive to conform to criminal behavior than with $X = 0$. However, since Y measures adherence to honest behavior, the dominant conformity factor (guilt or social pressure) will depend on the probability that an individual i faces a criminal, that is on the social mix of the community in which the individual lives.

Payoffs for player i based on Table 2 can be written as

$$u^i(s^i, s^j | x^i, y^i) = s^i s^j (y^i - G) + (1 - s^i)(1 - s^j)(G + x^i) \tag{1}$$

An equilibrium is a pair of strategy $\left(s^{i^*}(x^i, y^i), s^{j^*}(x^j, y^j)\right)$ such that for each player i and every possible value (x^i, y^i), $s^{i^*}(x^i, y^i)$ maximizes $E_{(X^j, Y^j)} u^i(s^i, s^{j^*} | x^i, y^i)$.

Let η^j be the probability that $s^j = 1$. Player i's best response $BR^i(\eta^j)$ is given by the following condition:

$$BR^i(\eta^j) = 1 \text{ if } \eta^j y^i - (1 - \eta^j) x^i > G \tag{2}$$

$$BR^i(\eta^j) = 0 \text{ if } \eta^j y^i - (1 - \eta^j) x^i < G \tag{3}$$

$$BR^i(\eta^j) = [0, 1] \text{ if } \eta^j y^i - (1 - \eta^j) x^i = G \tag{4}$$

Given a probability of being honest η, a player with a couple of emotions (x, y) who stays honest (s = 1), satisfies Eq. 2. In a symmetric equilibrium the following must be true: the probability, that a player remains honest, corresponds to the probability that the joint distribution of emotions (X, Y) satisfies Eq. 2 i.e.:

[3] These assumptions are convenient for analytic tractability but allow a rich array of possible equilibrium configurations as shown in the following.

Table 2 Best response equivalent Payoff matrix for player 1

		Player 2	
		$s^2 = 1$	$s^2 = 0$
Player 1	$s^1 = 1$	$y^1 - G$	0
	$s^1 = 0$	0	$G + x^1$

$$\eta = \mathrm{Prob}[\eta \mathrm{Y} - (1-\eta)\mathrm{X} > \mathrm{G}] \tag{5}$$

Given the distribution of Y and X, for $\eta \neq 0$ Eq. 5 is equivalent to:

$$\eta = (1-p)[1 - F(G/\eta)] + p\left[1 - F\left(\frac{G + (1-\eta)}{\eta}\right)\right] \tag{6}$$

A Cournot best response dynamics is given by $\eta_t = h(\eta_{t-1})$ with $h(\eta)$ given by the right hand side of Eq. 6.

Let $G^* = p - 2 + \sqrt{p^2 - 6p + 5}$. The following theorem gives the Bayesian Nash equilibria of the game.

Theorem 4 Equilibria are given by[4]

- If $G \leq 0$ (criminality does not pay): an equilibrium with $\eta^* = 1$ always exists. A second equilibrium at $\eta^* = (1-p)$ coexists whenever $p \geq (1-G)/2$.
- $G \geq 0$: an equilibrium with $\eta^* = 0$ always exists. Stable polymorphic equilibria are given by:
- $\eta_1^* = \dfrac{(1-p) + \sqrt{(1-p)^2 - 4G(1-p)}}{2}$ if $G^* \leq G \leq (1-p)/4$.
- $\eta_2^* = \dfrac{(1+p) + \sqrt{(1+p)^2 - 4(G+p)}}{2}$ if
    - $G \leq G^*$ and $G > p$ or
    - $G \leq \frac{(1-p)^2}{4}$ and $G \leq p$

Note that for $G \leq 0$, an equilibrium with criminals may exist even if criminality does not pay. However the social pressure p exerted by criminals on honest individuals, must be large, i.e. $p \geq (1-G)/2 \geq 1/2$ which may be anomalously high. In addition, there is convergence to the equilibrium $(1-p)$ under Cournot best response dynamics if $\eta < \frac{(1+p) - \sqrt{(1+p)^2 - 4p(G+1)}}{2}$. Otherwise, convergence is to $\eta^* = 1$ (see Fig. 11 in Appendix 6).

Figure 2 shows the phase diagram for the parameter space (p, G) when $G \geq 0$. Note that $\eta_{+2}^* \geq \eta_{+1}^*$. A corollary of theorem 4 is that the coexistence of η_1^* and η_2^*

[4] Proofs of theorems in Appendix 6.

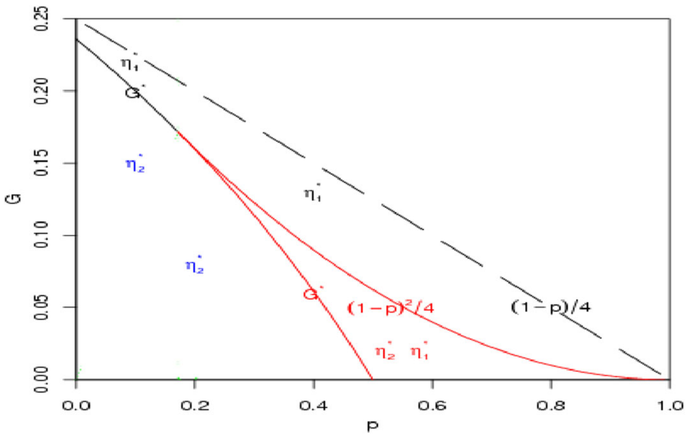


Fig. 2 Phase diagram for $G \geq 0$: η_1^* and η_2^*, resp. the *low* and *high* stable polymorphic equilibrium ($\eta^* = 0$ is an equilibrium for the whole space)

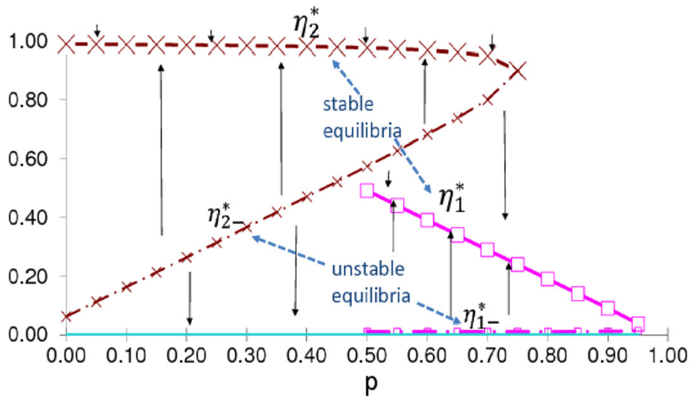


Fig. 3 An illustrative case of coexistence of η_1^* and η_2^* stable equilibrium for G = 0.01 with basin of attraction. Unstable equilibria are η_{1-}^* and η_{2-}^*. $\eta^* = 0$ is always a stable equilibrium for $G \geq 0$

equilibria occurs only for $G^* \leq G \leq \frac{(1-p)^2}{4}$ and $G \leq p$ (Fig. 2).

Figure 3 illustrates the coexistence of the two polymorphic equilibria and shows the basin of attraction of each equilibrium. More generally, Fig. 3 is representative of the case $G \geq 0$ with two stable polymorphic equilibria as depicted in Table 3. Table 4 shows convergence in the case $G \geq 0$ when only one polymorphic stable equilibrium exists.

Theorem 5 by performing comparative statics, the following properties hold in equilibrium:

– For a given G, increasing the social pressure p of criminals cannot increase η^* the rate of honesty in the society.

Table 3 Convergence in the multiple polymorphic equilibria case

Stable equilibria within circles								
0	$\leftarrow$	$\eta^*_{1,-}$	$\rightarrow$	$\eta^*_{\mathbf{1},+}$	$\leftarrow$	$\eta^*_{2,-}$	$\rightarrow$	$\eta^*_{\mathbf{2},+}$	$\leftarrow$	1

Table 4 Convergence when only one stable polymorphic equilibrium (either $\eta^*_{\mathbf{1},+}$ or $\eta^*_{\mathbf{2},+}$) exists

Stable equilibria within circles										
0	$\leftarrow$	$\eta^*_{1,-}$	$\leftrightarrows$	$\eta^*_{2,-}$	$\rightarrow$	$\eta^*_{\mathbf{1},+}$	$\leftrightarrows$	$\eta^*_{\mathbf{2},+}$	$\leftarrow$	1

– For a given p, increasing G, that is the relative gain from criminality, cannot increase the rate of honesty η^*.

The property that increasing social pressure p never increases honesty is depicted in Fig. 3 which is representative for the case $G \geq 0$ with coexistence of polymorphic equilibria: for p = 0 there are two equilibria, η_2^* and zero. The challenging case is when we start at η_2^*. Then increasing p decreases honesty (by the fact that $d\eta_2^*/dp < 0$) and for large p, eventually η_2^* disappears and there will be a drop to a lower equilibrium at η_1^* (with the property that $d\eta_1^*/dp < 0$) and finally to zero for $(1-p)/4 < G$.

The property that increasing G never increases the honesty rate is illustrated in Fig. 2 for $G \geq 0$: again if we start from an η_2^* equilibrium if it exists, jumps will be to a lower equilibrium η_1^* before ending up with an equilibrium with only criminals. With the additional properties that $d\eta_2^*/dG < 0$ and $d\eta_1^*/dG < 0$, honesty cannot increase when criminality pays more.

Theorem 5 is therefore an argument in favor of deterrence theory. By augmenting the probability of arrest or the severity of sentences (conditionally on those being perceived by individuals), criminality cannot increase and moreover in the case where criminality is profitable, that is $G > 0$, criminality will in fact decrease whenever the initial honesty rate is positive.

Moreover, since G is the relative gain normalized by the maximal guilt Y_M we have that increasing Y_M never decreases the rate of honesty. Note that a lower Y_M is equivalent to a stochastically dominated uniform distribution characterizing a population who adheres less to the norm. This is an argument in favor of policy encouraging the enforcement of social norms by discouraging violence and criminality (Salazar et al. 2003).

3 The impact of localized interactions

The equilibrium analysis for global interactions was shown in the preceding section. In this section, we intend to study the effect of localized interactions on criminality by the use of an agent based simulation model.[5] We follow by this the literature on criminality which points to the impact of neighborhood structures on criminality (Glaeser et al. 1996; Sampson et al. 1997). With localized interactions, η_i is now the proportion of honest agents in agent i's neighborhood with neighborhood sizes fixed at $k = 24$ or 44 or 100 depending on the treatment. The general procedure of the simulation model is described in section 5 and sketched in the following:

- In each period t, some agents choose an action $s = 0$ or $s = 1$. An agent i chooses the action with the maximum average utility where average utility is computed by taking the effective proportion $\eta_i(t-1)$ of honest agents in i's neighborhood in period $(t-1)$. That is, an agent behavior is summarized by a Cournot best response function to the behavior of individuals in a localized neighborhood during the preceding period.
- Once they made a choice, agents compute the happiness of being in their living environment. They are happy if criminality in their neighborhood is less than the tolerance rate. If agents are allowed to move, then, in each period, one unhappy and honest agent chooses a free place and moves in if the new place has a lower crime rate than the current one.
- A new period starts.

In addition to the localized interaction perspective, we want to study the effect of moving on criminality and segregation levels. In the "no move" case, we perform 50 runs for each of the following triple (gain G, social pressure p, neighborhood size k). The chosen parameters p, G are shown in Table 5 as well as the corresponding stable equilibrium. Since there are $4 \times 3 \times 3$ possible combinations of G, p, k, in all we have 1800 runs. In the following, H stands for high social pressure i.e. $p = 0.55$, L for low p i.e $p = 0.1$ and N for null p. Parameters for each run are fixed at: initial rate of honest agents 90 %, population size of 500 agents, total size of 625 cells meaning that there are 20 % of free cells to move on[6]; the time limit is 1500 periods; the number of agents choosing an action ($s = 0$ or $s = 1$) in each period is 20; we introduce some randomness in choice: with a probability of $\epsilon = 1$ % an agent's choice between $s = 0$ and $s = 1$ is random. Randomness allows to test the robustness of the results to small perturbations. Lastly, we control for initial conditions, i.e. the distribution of agents' emotions and actions on the lattice. For each (G, p, k), there are 50 runs corresponding to 50 different initial conditions. By changing one parameter for example G, the same 50 initial conditions, are applied to the new configuration G, p, k. In addition, we replicate 1800 runs by allowing

[5] The Simulations were done in Netlogo: http://ccl.northwestern.edu/netlogo/.

[6] This means that the effective neighborhood size is on average 80 % of k.

Table 5 Simulation parameters G, p with stable Equilibria in addition to other parameters

G	p		
	0(N)*	0.1(L)*	0.55(H)*
−0.05	**100**	**100**	**100**
0.05	**94.7**	**94.1**	**80**
0.1	**88.7**	**87**	**30**
0.2	**72.4**	**60**	**0**
Initial rate of honesty 90 %			
Population size = 500			
Number of cells 625			
Time limit: 1500 periods			
20 players making a choice s per period			
Error rate $\epsilon = 1$ %			
Neighborhood size k = 24, 44, 100			
50 runs for each combination k, G, p			
If move: one agent per period allowed to move			
If move: tolerance rate 10 %			
No boundary effects: the world wraps horizontally and vertically			

Expected Equilibria in bold fonts

* N for null, L for low, H for high pressure

agents to move; we control again for initial conditions to have the corresponding paired conditions to the "no move" case; if move is allowed, one unhappy agent, if there is one, is potentially moving per period. By analogy if each period represents one week, on average, approximately every agent can choose every 6 months between $s = 0$ and s

Figure 4 presents an illustrative case on the effect of the neighborhood size or the possibility of moving on the spatial allocation of criminality.

3.1 Effect of Neighborhood size in the "no moving" case

As a first step, we suppose that agents are constrained to stay in their living area even in the case of an unfavorable criminal environment.

The literature has shown that there is a spatial component in the degree of criminality. So, we need to define a measure that indicates whether criminality is evenly distributed or clustered across the space. Intuitively, a non segregated space is one where the distribution of criminals on the space would have been obtained by a random throwing of individuals on the lattice. A natural measure of non segregation would be a function τ defined as the sum over all agents of the number of criminal-honest pairs in a local neighborhood of sample size k. More precisely, for each agent i we compute the number of agents having a strategy different from i in a local neighborhood of size k (ignoring empty cells). We sum over all agents and

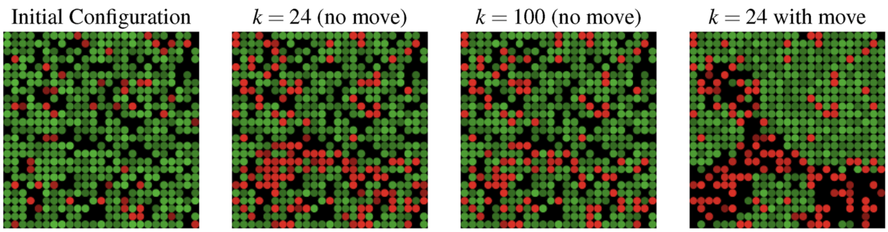


Fig. 4 An illustrative case: starting from the same initial condition (*left figure*), we compare different interaction structures, here $k = 4$, $k = 100$ as well as the possibility for an agent to move to a new place if unhappy. Expected equilibrium from global analysis was at 80. *Red* (*darker*) spots are criminals; *green* (*lighter*) spots are honest agents (*black cells* are free cells). Segregation is more pronounced either with $k = 24$ compared to $k = 100$ or when move is allowed. Parameters: $G = 0.1, p = 0.3$, others as in Table 5

divide by 2 to get the number of mixed pairs τ. The higher this index the lower segregation. Since k and η vary across and within a run for η, we need to define a unit free measure of segregation. Segregation arising by randomness with the same parameters k and η is not considered as abnormal segregation. A random configuration with parameters k, η and population size N would give an expected value of $\hat{\tau}$ = % of occupied cells $*k * \eta * (1 - \eta) * N$ of mixed pairs so that a normalized degree of segregation is $\tau_{norm} = (\hat{\tau} - \tau)/\hat{\tau}$. If τ_{norm} is near zero then segregation is not different from a random configuration. Higher values of τ_{norm} correspond to higher levels of segregation.[7]

Figure 5 shows that the proportion of honest people is similar for different neighborhood size k whereas segregation is a decreasing function of k with significant differences across different k. For $k = 100$ segregation is very low. It shows that local interactions may give rise to spatial inequalities.

More precisely, if we analyze honesty and segregation rates for different levels of (G, p), the left Fig. 6 shows that qualitatively speaking, for each (G, p), there is no huge difference in the average rate of honesty between the different treatments k ($k = 24, 44, 100$). But the right Fig. 6 shows an impact of k on segregation: a lower k produces higher segregation when (G, p) are such that polymorphic equilibria are expected.

To go beyond the visual impression, we performed a paired Wilcoxon test. Table 6 shows the Wilcoxon test of equality in means between treatments $k = 24$ and $k = 100$ for each level G, p both for honesty and segregation rates.

First, there is no significant difference between $k = 24$ and $k = 100$, both in the honesty rate and segregation, for monomorphic equilibria, if we exclude the $-0.05H$ case. That is, when a monomorphic equilibrium is expected, only the case $-0.05H$ shows a statistical higher segregation level for $k = 24$ compared to $k = 100$, showing that large social pressure combined to a low k can lead to segregated criminality even if criminality does not pay ($G = -0.05$). Differences

[7] See Grauwin et al. (2012) for a justification and an application to Schelling's model (Schelling 1971) and Waldeck (2013) for an application to a prisoner's dilemma.

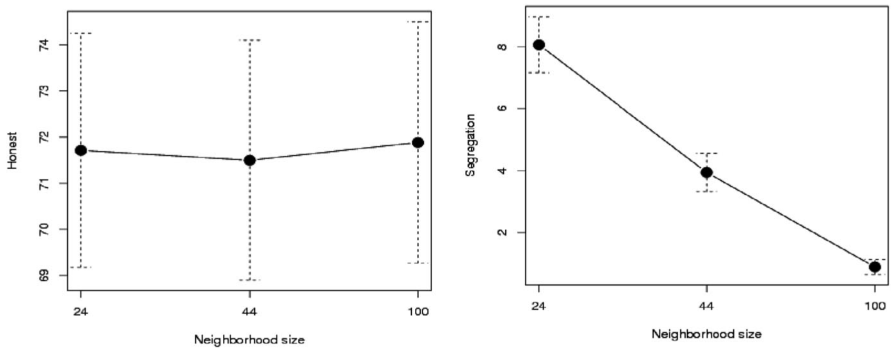


Fig. 5 For each k: honesty (*left figure*) and segregation rates (*right figure*). Each point is the average over 600 runs (i.e. for 3 p × 4 G× 50 runs) with confidence interval (CI) 95 %

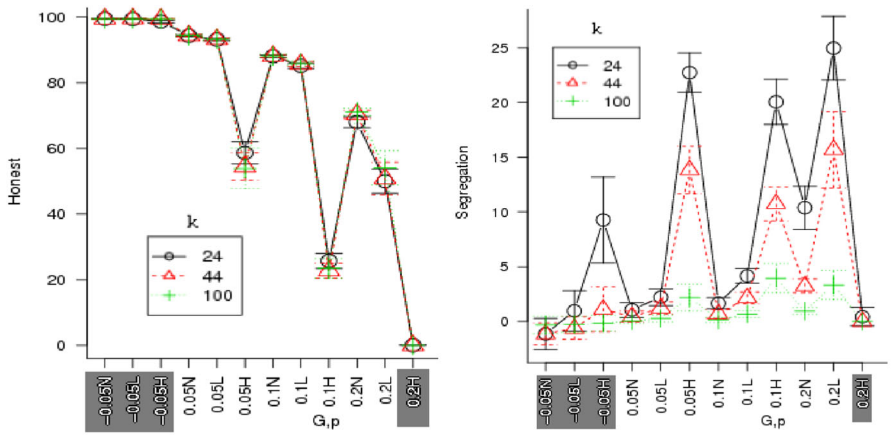


Fig. 6 The *left figure* represents the honesty rate, depending on the size of the neighborhood *k*, for different value of G and p as given in Table 5. The *right figure* shows the corresponding segregation level. The cases where monomorphic equilibria are expected are highlighted: (−0.05*N*, −0.05*L*, −0.05*H* with 100 % of honest individuals expected and 0.2*H* with 0 % of honest agents). Each point is an average over 50 runs with CI 95 %

appear for polymorphic equilibria: the equality in honesty rates can generally be rejected at a 5 % level.

Result 6 Moreover, segregation is always more pronounced for $k = 24$ compared to $k = 100$ in all cases where a polymorphic equilibrium is expected showing that segregation may emerge as a result of the interaction structure.

Moreover, right Fig. 7 shows that segregation levels are different for different k whatever the size of social pressure p (all p-values of a Wilcoxon test with independent samples <0.01) although there is no difference in honesty rates between the different treatments k whatever the level of social pressure p (all p-values >0.1).

Table 6 PAIRED Wilcoxon test for difference in mean between k = 100 and 24

	H0: no difference in Honesty between $k = 100$ and 24 p-value Wilcoxon	H0: no difference in Segregation rate between $k = 100$ and 24 p-value Wilcoxon
−0.05N	1	0.3747
−0.05L	0.05001	0.198
−0.05H	**5.675e−06**	**4.982e−05**
0.05N	0.1175	**0.002767**
0.05L	**0.01709**	**1.089e−06**
0.05H	**0.02302**	**7.79e−10**
0.1N	**0.03747**	**1.457e−06**
0.1L	**2.447e−06**	**1.425e−09**
0.1H	0.08312	**7.79e−10**
0.2N	**4.552e−09**	**8.279e−10**
0.2L	**0.0002632**	**7.79e−10**
0.2H	1	0.3222

In bold fonts, p-value <5 %

For a Student test, the same conclusion applies except for the 0.1*H* honesty rate for which the p-value is = 0.08 i.e. no significant difference

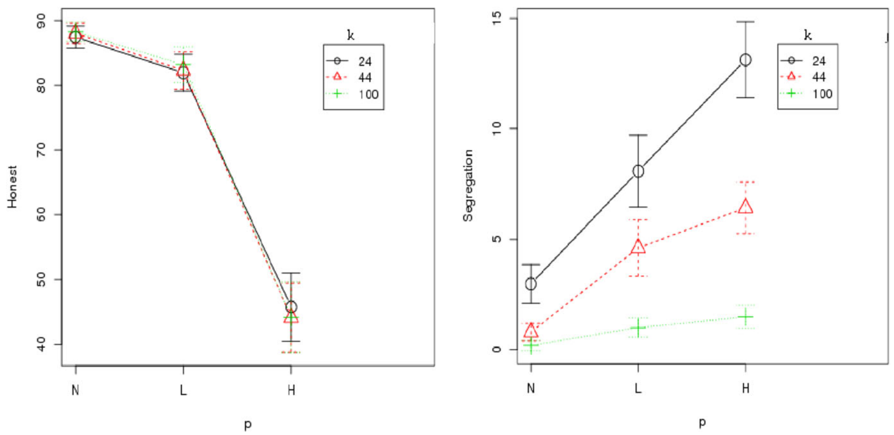


Fig. 7 The *left figure* shows that the neighborhood size *k* makes no significant difference in honesty rates but a huge difference in segregation (*right figure*). Each point is an average over 4 (levels of G) × 50 runs with CI 95 %

The higher social pressure, the higher the segregation effect of narrow neighborhoods.

Result 7 There is a cross effect of p and k on segregation, that is the higher p and the lower k the more segregated patterns will be. Thus the smaller the neighborhood and the higher social pressure, the more pronounced spatial heterogeneity in criminality will be.

3.2 Move allowed

In this section we allow unhappy agents to move. We define honest agents to be happy if the rate of criminality in their neighborhood is less than their tolerance for criminality. They are unhappy otherwise. We suppose that criminals are always happy and do not move.

The tolerance rate for honest agents is fixed at 10 % for all runs. Unhappy honest agents will move only if the criminality rate in their new neighborhood is below the old one. Allowing unhappy honest agents to move to a new place with a lower criminality rate will necessarily increase segregation since it will favor clustering of zones according to honesty. This is shown in right Fig. 8 which shows that segregation is always higher in the "move" case in comparison to the "no move" case whenever the predicted equilibrium was polymorphic.

Although there is a huge difference in segregation whenever a polymorphic equilibrium was expected, there is no such sharp difference in honesty rates between the "move" and "no move" case (left Fig. 8).

Result 8 Allowing agents to move does not induce a fundamental qualitative change in honesty rates while leading to large segregation levels in the polymorphic equilibria configurations.

3.3 Does higher G or p lead to higher criminality?

Theorem 5 shows that increasing G never increases the rate of honest individuals. Does the result hold with local interactions both for the move and no move cases? Fig. 9 shows that this is indeed the case. Increasing gains from criminality from -0.05 to 0.2 decreases the honesty rate. The results are shown for neighborhood size $k = 24$ and again there is a close parallel in the honesty rate between the no

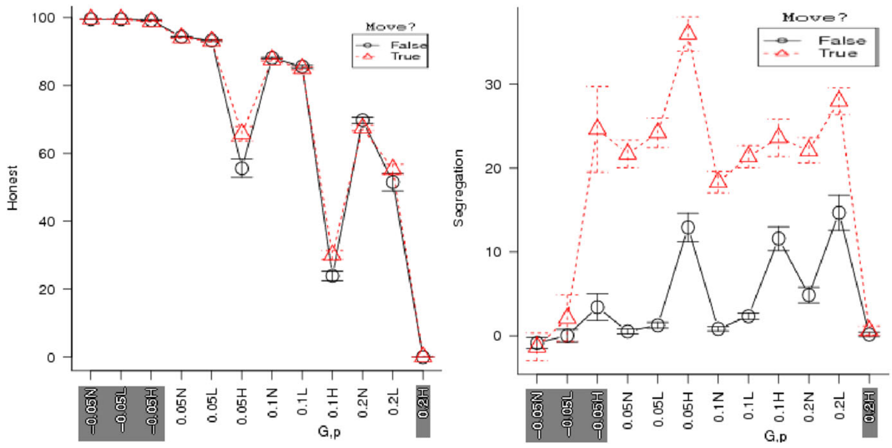


Fig. 8 *Left figure* % of honest agents. *Right figure* segregation. The monomorphic equilibria are highlighted. Figures show averages and CI over 50 × 3 (neighborhood size) runs

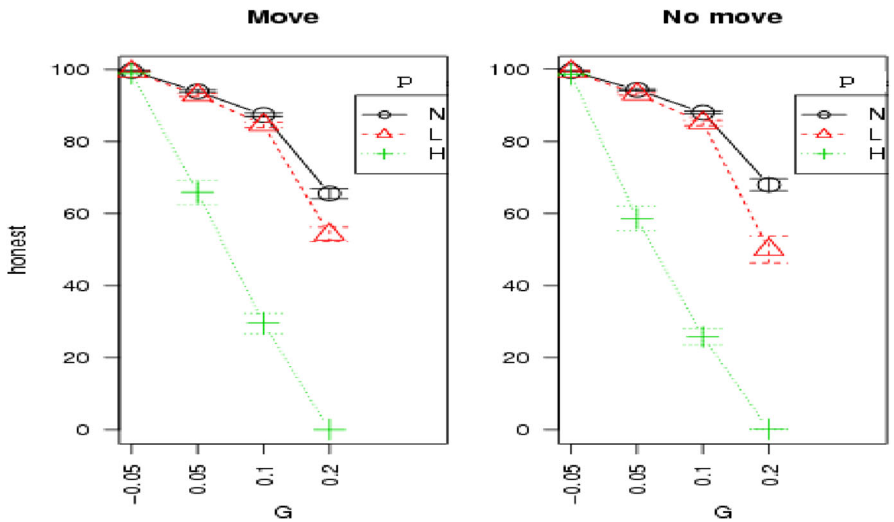


Fig. 9 Honesty rates for different levels of G and p in the case of neighborhood size $k = 24$: each point represents an average over 50 runs with 95 % CI

move and move curves. The same figures (not shown here) are obtained for $k = 44$ and $k = 100$. Performing a Wilcoxon paired test shows that the hypothesis of equality of mean can be rejected (all p-values of order E$-$10) for the alternative that the honesty rate is strictly decreasing with increasing G.

Moreover, Fig. 9 also shows that raising p does not increase the rate of honesty both in the "no move" or "move" case. A paired Wilcox test shows in fact that increasing social pressure decreases honesty in all cases but $G = -0.05$ for treatments $p = N$ to $p = L$. In this later case, we cannot reject equality of mean both in the move and no move cases (p-value >0.1).

Result 9 Globally, all the results for local interactions are conform to theorem 5: honesty is decreasing both with G and with social pressure when criminality pays.

Moreover, a sensibility analysis shows that result 9 extends to other initial configurations with an initial proportion of $\eta = (30\ \%, 60\ \%)$ instead of $\eta = 90\ \%$.

4 Conclusion and future directions

There is evidence from different academic fields in behavioral and cognitive sciences that in addition to classical economic considerations, there are also internal reward mechanisms influencing decisions. We have shown that integrating such internal mechanisms into a framework of rational choice leads to interesting insights: first, polymorphic equilibria appear, depending on the relative gain from criminality and social pressure. These polymorphic equilibria exist even if the economic situation of each individual is the same. Second, coexistence of equilibria implies path dependency. We could converge to a high or a low equilibrium depending on the initial situation i.e. the initial crime rate. Third, the structure of interactions, i.e. the size of the relevant neighborhood, is one factor explaining segregation, i.e. the existence of crime clusters. Economic inequalities could add to the segregation between different zones, but they are not necessary. The high correlation between individual behavior and the behavior in the neighborhood is due to the structure of the interaction network. Additionally, segregation may increase by population movement without changing fundamentally the level of honesty. But this may change the perception of criminality across space. Finally, our modeling suggests that on average, subjects have no less chance to be criminals the more criminals there are around them (Akers and Jennings 2009): this is due to the fact that guilt from violating the law is proportional to the number of honest individuals. Experimental evidence suggests that this is behaviorally meaningful: for example the contribution in public goods games increases with the increase in contributions from others. Moreover in the lab, the more people steal, the more others steal (Falk and Fischbacher 2002). That is, normative aspects should not be neglected for controlling criminality. Acceptance and internalisation of norms or parental control may be important aspects for low criminality. It suggests that a society, where controls by peers are more important, may be safer but at the expense of reduced freedom.[8] In addition, we have shown that increasing the gain from criminality never lowers criminality. This implies that increasing punishment mechanism, such as the three-strikes rules (Chen 2008), may indeed be effective in struggling with criminality.

The modeling may be improved in a few directions. First there is missing a clear co-evolution process between the intensity of police control and the level of criminality.[9] However, we could argue that in the short term, the quantity of resources available to the police is rather constrained and changing only at a low pace. But future research should explore the interplay between the game theoretical model proposed in this paper and the means of enforcing the law in the mid or long term. Not only should we ask about the importance of these means and their effect on crime deterrence but we should also question the impact of their spatial allocation. A more realistic modeling should also introduce heterogeneity in economic conditions and non monetary payoffs like social status. This could be easily done by extending the game tree in Fig. 1 to symbolic payoffs in addition to heterogeneous monetary payoffs, especially if social status is correlated to monetary payoffs. An individual with a high social status could also be an opinion leader in the sense of a person with a high number of easily influenced connected people. In this paper, the size of the neighborhood is fixed so that there is no opinion leader in this sense. But as Watts and Dodds (2007) show, the diffusion process is not so much driven by opinion leaders as by a critical mass of easily influenced individuals

[8] A related game theoretical paper by Short et al. (2010) shows that another type of peer control is the strategy of informing the police when a crime has been committed: the presence of individuals employing the "informant" strategy is a key to the emergence of systems where honesty dominates.

[9] In this respect, Tsebelis (1990) proposes a theoretical game between police and criminals where he shows that the equilibrium play between criminals and police is in mixed strategy. But some experimental evidence of the game proposed by Tsebelis (1990) shows that contrary to the equilibrium prediction of Tsebelis' game, higher punishment may deter criminality (Rauhut 2009) so that something in the proposed game structure must be missing.

which in our paper are those with low guilt or sensible to social pressure. Diffusion of criminality is then influenced by the size of the neighborhood but the primary effect is on spatial heterogeneity of criminality and no so much on the attained level of criminality. Nevertheless a test on how different network structures impact on criminality (Haynie 2001) should be a useful way toward a more reliable modeling. Agent based modeling is in this sense a new useful tool for experimenting the impact of different parameters on criminality and criminologists should seriously take this instrument into account for policy modeling. The proposed model should only be seen as a first step toward this direction.

5 Pseudo-code for the agent-based model

- At initialization: set the % of free cells; set the neighborhood size k, the tolerance rate and the population characteristics according to assumptions 1, 2, 3.
- The general procedure for one period is the following:
  - Each period t, some agents choose a strategy ($s = 0$ or $s = 1$). With probability $1 - \epsilon$, they choose the strategy maximizing the average payoff in Table 1 given the proportion η in their neighborhood at period $t-1$. Once chosen, a list of all agents' strategies is updated as well as a list of the state of happiness for each agent $\rightarrow$ Procedure Update-happiness
  - If move is allowed, choose one unhappy agent: move the agent to a new place if it has a lower crime rate than the initial one. Then update the happiness of all agents $\rightarrow$ Procedure Update-happiness.
  - Update all global variables, including the proportion of honest agents, the proportion of happy agents and the segregation level.
  - Start a new period

- The Sub-Procedure Update-Happiness
  - Update two lists of length "number of agents": one list is the criminality rate in each agent's neighborhood; the second is the state of happiness (0 or 1) of each agent (Happy if the crime rate in the neighborhood is less than the tolerance rate).
  - Compute the proportion of happy agents.

- The sub-Procedure for updating global variables: Compute the new rate of honest agents. Compute the segregation rate.

To compute the segregation level:

- Given k and the honesty rate, compute the expected number of mixed pairs i.e. k $\times$ (percentage of occupied cells) $\times$ (percentage of honest agents) $\times$ (1 $-$ percentage of honest agents) $\times$ population size

- For each agent: count the number of agents with a different strategy from myself in my neighborhood of size k.
- Sum over all agents and divide by 2 to get the number of mixed pairs.
- Set segregation 100 × (1 − (mixed-pairs divided by expected-mixed-pairs)).

Appendix

Proof of Theorem 4

The best response of player i satisfies:

$$BR^i(\eta^j) = 1 \text{if} \eta^j y^i - (1 - \eta^j) x^i > G \tag{7}$$

1. Monomorphic equilibra (one strategy played): given the best response function 7, one easily shows that
   - $\eta^* = 0$ is an equilibrium for $G \geq 0$ (criminality pays more than legality).
   - $\eta^* = 1$ is an equilibrium for $G \leq 0$.

   The rest of the proof proceeds as follow, for each case:

   (a) either $G \geq 0$
   (b) or $G \leq 0$:

First	Find the *polymorphic equilibria*.
Second	Find the *stable equibria* for a Cournot best response dynamics.
Third	*Phase diagram* with respect to parameters G and p in the case $G \geq 0$.

2. Polymorphic equilibria i.e. $0 < \eta^* < 1$. A polymorphic symmetric equilibrium satisfies Eq. 8 given by:

$$\eta = (1 - p)[1 - F(G/\eta)] + p\left[1 - F\left(\frac{G + (1 - \eta)}{\eta}\right)\right] \tag{8}$$

   with F the cumulative uniform distribution on $[0, 1]$.

   (a) CASE $G \geq 0$: Criminality pays more than legality.

If $G \geq 1$ then $\eta^* = 0$ is the unique equilibrium.

Suppose that $0 \leq G < 1$ in the following. Depending on the value of G we have three cases

- If $\eta \leq G$, Eq. 8 gives $\eta^* = 0$ as the unique equilibrium

– If $G < \eta \leq \frac{(G+1)}{2}$. Resolving Eq. 8 leads to the following equilibria:

$$\eta^*_{1,\pm} = \frac{(1-p) \pm \sqrt{(1-p)^2 - 4G(1-p)}}{2} \tag{9}$$

– Lastly, if $\eta \geq \frac{(G+1)}{2}$, Eq. 8 leads to

$$\eta^*_{2,\pm} = \frac{(1+p) \pm \sqrt{(1+p)^2 - 4(G+p)}}{2} \tag{10}$$

Stability of equilibria

– Case $G < \eta \leq (G+1)/2$ with equilibria $\eta^*_{1,\pm}$. Let $\Delta = (1-p)^2 - 4G(1-p)$. Denoting the rate of cooperation at time t by η_t, we have the following Cournot best response dynamics:

$$\eta_t = (1-p)(1 - G/\eta_{t-1}) \tag{11}$$

if we note $h(\eta) = (1-p)(1 - G/\eta)$, the stable equilibrium η^*, with $\eta^* = \eta^*_{1,+}$ or $\eta^* = \eta^*_{1,-}$, (see Eq. 9) satisfies

$$|\frac{dh(\eta^*)}{d\eta}| < 1 \tag{12a}$$

$$\Leftrightarrow 4(1-p)G < (1-p)^2 \pm 2(1-p)\sqrt{\Delta} + \Delta \tag{12b}$$

$$\Leftrightarrow \mp(1-p)\sqrt{\Delta} < \Delta \tag{12c}$$

The first case with $-(1-p)\sqrt{\Delta} < \Delta$ is always true if $\Delta \geq 0$ (the condition for the existence of an $\eta^*_{1,\pm}$ equilibrium) and thus *equilibrium* $\eta^*_{1,+}$ *is stable*. The second case $(1-p)\sqrt{\Delta} < \Delta$ is equivalent to $(1-p) - \sqrt{\Delta} < 0$ i.e $\eta^*_{1,-} < 0$ which is impossible.

– Case $\eta \geq (G+1)/2$ with equilibria $\eta^*_{2,\pm}$ (see Eq. 10). Denote $\Delta = (1+p)^2 - 4(G+p)$. Denoting the rate of cooperation at time t by η_t, we have

$$\eta_t = 1 - (G + p - p\eta_{t-1})/\eta_{t-1} \tag{13}$$

if we denote $h(\eta) = 1 - (G + p - p\eta)/\eta$, the stable point η^* satisfies

$$|\frac{dh(\eta^*)}{d\eta}| < 1 \tag{14a}$$

$$\Leftrightarrow 4(p+G) < (1+p)^2 \pm 2(1+p)\sqrt{\Delta} + \Delta \tag{14b}$$

$$\Leftrightarrow \mp(1+p)\sqrt{\Delta} < \Delta \tag{14c}$$

The first case with $-(1+p)\sqrt{\Delta} < \Delta$ is always true if $\Delta \geq 0$ and thus *equilibrium* $\eta^*_{2,+}$ *is stable*. The second case $(1+p)\sqrt{\Delta} < \Delta$ is equivalent to $(1+p) - \sqrt{\Delta} < 0$ i.e $\eta^*_{2,-} < 0$ which is impossible. In addition one can prove that $\eta^*_{1,-} \leq \eta^*_{2,-}$ (with strict inequality for $p > 0$) and that $\eta^*_{1,+} \leq \eta^*_{2,+}$.

To sum up, the Cournot dynamics is given by the right hand side of Eq. 8 and depending on the size of G, can be simplified and written as in Eqs. 11 and 13: this is shown in Table 7 and each possible scenario, where at least one polymorphic equilibrium exists, is depicted in Fig. 10:

- Note that by Table 7, η_t is increasing when $\eta \in [\eta^*_{2,-}\eta^*_{2,+}]$ or $\eta \in [\eta^*_{2,-}\eta^*_{2,+}]$ and decreasing otherwise. In the case of coexistence of two polymorphic equilibria $\eta^*_{1,+}$ and $\eta^*_{2,+}$, we have then two cases to consider for convergence, depending on whether $\eta^*_{1,+} \gtreqless \eta^*_{2,-}$. Now $\eta^*_{1,+} \geq \eta^*_{2,-}$ is always false. If it was true, then one would have by table 8 both that η_t is decreasing for $\eta_t < \eta^*_{2,-}$ and increasing for $\eta_t > \eta^*_{1,-}$ (noting that $\eta^*_{1,-} \leq \eta^*_{2,-}$ is always true) so that when $\eta^*_{1,-} < \eta_t < \eta^*_{2,-}$ one motion would be increasing to $\eta^*_{1,+}$ and another decreasing to 0, which is impossible. So that we have the convergence properties given in Tables 3 and 4.

Phase diagram in the case $G \geq 0$

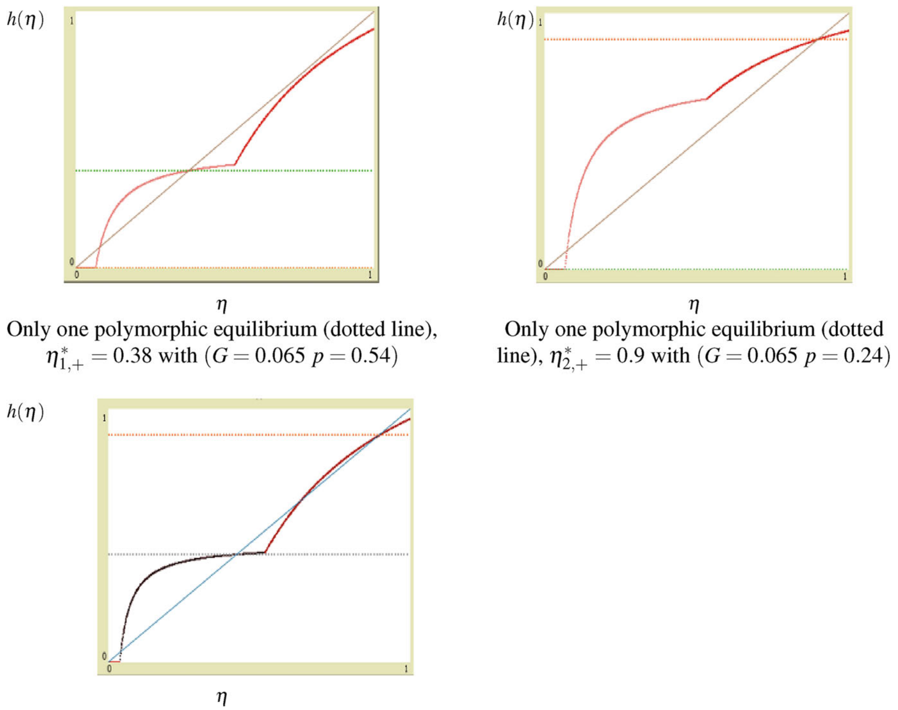
Let us show the conditions in terms of p, G under which the stable $\eta^*_1 = \frac{(1-p)+\sqrt{(1-p)^2-4G(1-p)}}{2}$ equilibrium and $\eta^*_2 = \frac{(1+p)+\sqrt{(1+p)^2-4(G+p)}}{2}$ are defined.

First, we prove that η^*_1 exists iff $G^* \leq G \leq (1-p)/4$ with $G^* = p - 2 + \sqrt{p^2 - 6p + 5}$. The proof goes as follow:

The equilibrium η^*_1 is defined if $G < \eta^*_1 \leq (G+1)/2$ and $\Delta_1 = (1-p)^2 - 4G(1-p) \geq 0$ i.e. the discriminant is positive. This last condition is equivalent to $G \leq (1-p)/4$. Condition $G < \eta^*_1$ is equivalent to $G < \frac{1-p+\sqrt{\Delta_1}}{2}$. Together, the last two conditions require that $G \leq Min\{(1-p)/4; \frac{1-p+\sqrt{\Delta_1}}{2}\}$ with a strict inequality for

Table 7 Dynamics

η is					
between	0	G		$(G+1)/2$	
$h(\eta) =$	0		$(1-p)(1-G/\eta_{t-1})$		$1+p-(G+p)/\eta_{t-1}$
Equilibrium	0		$\eta^*_{1,-}\ \eta^*_{1,+}$		$\eta^*_{2,-}\ \eta^*_{2,+}$
$h(\eta > \eta$:					
when			$\eta \in \left[\eta^*_{1,-}\eta^*_{1,+}\right]$		$\eta \in \left[\eta^*_{2,-}\eta^*_{2,+}\right]$



Only one polymorphic equilibrium (dotted line), $\eta^*_{1,+} = 0.38$ with $(G = 0.065\ p = 0.54)$

Only one polymorphic equilibrium (dotted line), $\eta^*_{2,+} = 0.9$ with $(G = 0.065\ p = 0.24)$

Coexistence of two polymorphic equilibria $\eta^*_{1,+} = 0.427$, $\eta^*_{2,+} = 0.902$ for $(p = 0.533\ G = 0.036)$

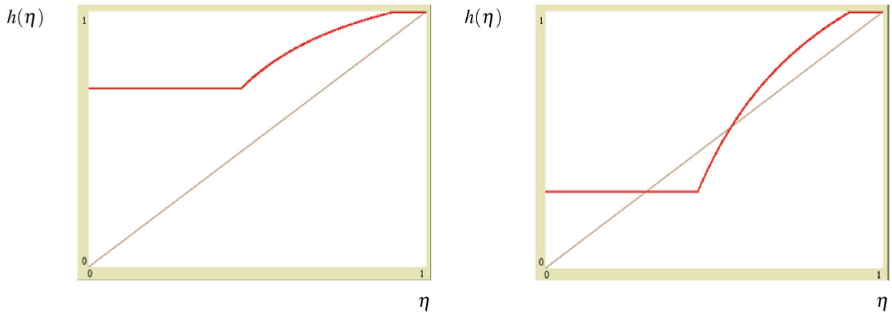
Fig. 10 Dynamics: $\eta_t = h(\eta_{t-1})$. The *red curve* depicts the function $h(\eta)$. The *dot line* represent the stable equilibrium

Table 8 The impossible case with multiple polymorphic equilibria: $\eta^*_{1,+} \geq \eta^*_{2,-}$

Stable equilibria within circles										
0	$\leftarrow$	$\eta^*_{1,-}$	$\leftrightarrows$	$\eta^*_{2,-}$	$\rightarrow$	$\boldsymbol{\eta^*_{1,+}}$	$\leftrightarrows$	$\boldsymbol{\eta^*_{2,+}}$	$\leftarrow$	1

$\frac{1-p+\sqrt{\Delta_1}}{2}$. Now $(1-p)/4 < \frac{1-p+\sqrt{\Delta_1}}{2}$ whenever $\Delta_1 \geq 0$ so that $G \leq (1-p)/4$ is sufficient. The condition $\eta^*_1 \leq (G+1)/2$ is equivalent to $G^2 + 2(2-p)G + 2p - 1 \geq 0$ Two roots of the equality are $G^*_{\pm} = p - 2 \pm \sqrt{p^2 - 6p + 5}$. Only G^*_+ is positive. Now $\frac{dG^*_+}{dp}$ is strictly negative. G^*_+ decreases from $-2+\sqrt{5} > 0$ to -1. The sign of $G^2 + 2(2-p)G + 2p - 1$ is negative between both roots $G^*_{\pm}$ and positive elsewhere so that $G^2 + 2(2-p)G + 2p - 1 \geq 0$ is equivalent to $G \geq G^*_+$. Summing up, if we denote G^*_+ by G^*, then η^*_1 exists iff $G^* \leq G \leq (1-p)/4$. Note that $G^* \leq (1-p)/4$ is always true.

Second, we prove that η^*_2 is defined for $G \leq Min\{G^*; (\frac{1-p}{2})^2\}$ for $G > p$ and $G \leq (\frac{1-p}{2})^2$ for $G \leq p$. The proof goes as follows: the equilibrium η^*_2 is defined if



Only an equilibrium at 1 for $(G = -0.1 p = 0.3)$

A second equilibrium at $p = 0.3$ for $(G = -0.1, p = 0.7)$

Fig. 11 Dynamics: $\eta_t = h(\eta_{t-1})$. The *red curve* depicts the function $h(\eta)$

$\eta_2^* \geq (G+1)/2$ and $\Delta_2 = (1+p)^2 - 4(G+p) \geq 0$ i.e. the discriminant must be positive. This last condition gives $G \leq (\frac{1-p}{2})^2$. $\eta_2^* \geq (G+1)/2$ is equivalent to $G - p \leq \sqrt{\Delta_2}$. If $G \leq p$ this condition is always true provided that the discriminant is non negative. If $G > p$ then $G - p \leq \sqrt{\Delta_2}$ is equivalent to $G^2 + 2(2-p)G + 2p - 1 \leq 0$. This is equivalent to $G_-^* \leq G \leq G_+^*$. Given that $G_-^* < 0$ and denoting G_+^* by G^*, we sum up all the conditions: $G \leq Min\{G^*; (\frac{1-p}{2})^2\}$ for $G > p$ and $G \leq (\frac{1-p}{2})^2$ for $G \leq p$. Now, simple computations show that $G^* \leq (\frac{1-p}{2})^2$ is always true with equality satisfied at $p = 3 - 2\sqrt{2}$. That is $G \leq G^*$ for $G > p$ and $G \leq (\frac{1-p}{2})^2$ for $G \leq p$.

Finally equilibria η_1^* and η_2^* coexist only if $G \geq G^*$ and this last condition is satisfied only in the case $G \leq p$ and $G^* \leq G \leq (\frac{1-p}{2})^2$.

(b) CASE $G \leq 0$: legality pays more than criminality. First, searching for POLYMORPHIC EQUILIBRIA.

Since $G \leq 0$, Eq. 8 leads to

$$\eta = 1 - pF\left[\frac{G + (1-\eta)}{\eta}\right] \tag{15}$$

First, if $G \leq -1$ then $\eta^* = 1$ is the unique equilibrium. Suppose that $G > -1$ in the following.

- If $\frac{G+(1-\eta)}{\eta} \geq 1$ i.e. $\eta \leq (G+1)/2$ Eq. 15 leads to $\eta^* = 1 - p$
- If $0 \leq \frac{G+(1-\eta)}{\eta} < 1$ i.e. $(G+1)/2 < \eta \leq (G+1)$ and Eq. 15 leads to

$$\eta_\pm^* = \frac{(1+p) \pm \sqrt{(1+p)^2 - 4p(G+1)}}{2} \tag{16}$$

Some simple calculations show that $\Delta_3 = (1+p)^2 - 4p(G+1) \geq 0$ and $\eta_+^* \geq 1$.

– If $\eta > (G+1)$ then $\eta^* = 1$ is the unique equilibrium.

To sum up, $\eta^* = 1$ is always an equilibrium; if $\eta \leq (G+1)/2$ then $\eta^* = 1-p$ is an additional equilibrium. If $(G+1)/2 < \eta \leq (G+1)$ then $\eta = \eta^*_-$ is another equilibrium.

STABILITY OF EQUILIBRIA: a Cournot best response dynamics is given by the following equations:

$$\eta_t = 1 - pF\left[\frac{G + (1-\eta_{t-1})}{\eta_{t-1}}\right] = h(\eta_{t-1}) \text{ } if \text{ } \eta \geq (G+1)/2 \tag{17a}$$

$$\eta_t = 1 - p \text{ } if \text{ } \eta < (G+1)/2 \tag{17b}$$

Stability requires that $|\frac{dh(\eta^*)}{d\eta}| < 1$. Simple calculations show that $|\frac{dh(\eta^*_+)}{d\eta}| < 1$ and $|\frac{dh(\eta^*_-)}{d\eta}| > 1$

Now let us note two things: first, if $p \geq (1-G)/2$ then an equilibrium at $(1-p)$ exists. Second, $\eta^*_- > (G+1)/2 \Leftrightarrow p > (1-G)/2$

– Case 1: $p < (1-G)/2$. No equilibria at $1-p$ exists. Note that $p < (1-G)/2 \Rightarrow \eta^*_- < (G+1)/2$.
  – If $\eta > \eta^*_-$ then the process converges to $\eta^* = 1$ ($\eta^*_+ > 1$ is the stable equilibrium).
  – If $\eta_t < \eta^*_-$ then $\eta_t < (G+1)/2$ so that the dynamics gives $\eta_{t+1} = 1-p$ which given that $p < (1-G)/2$ (that is $1-p > (1+G)/2 > \eta^*_-$) implies that $\eta_{t+1} > \eta^*_-$ and the process converges again to $\eta^* = 1$

  Case 2: $p \geq (1-G)/2$. Note that in this case $\eta^*_- \geq (G+1)/2$.
    – If $\eta > \eta^*_-$ then the process converges to $\eta = 1$.
    – If $\eta_t < \eta^*_-$ then η_{t+1} is lower than η_t and for some τ, $\eta_\tau < G+1)/2$ and the process converges to $1-p$.

  To sum up the case $G \leq 0$, the process converges to the equilibrium $1-p$ if and only if $p \geq (1-G)/2$ and $\eta < \eta^*_-$. Otherwise the process converges to 1.

Proof of Theorem 5

Part 1: Honesty never increases with social pressure

– case $G \geq 0$. Restating some properties: first, whenever an η^*_{+1} equilibrium exists then $\frac{\eta^*_{+1}}{dp} < 0$. Second, whenever an η^*_{+2} equilibrium exists then $\frac{\eta^*_{+2}}{dp} < 0$. Third, remembering that $G^*(p) = p - 2 + \sqrt{p^2 - 6p + 5}$, note that $G^* \leq (1-p)^2/4 \leq (1-p)/4$. Fourth, $\eta^*_{+2} \geq \eta^*_{+1}$ for all positive p and G. Fifth, $\eta^* = 0$ is always an equilibrium for $G \geq 0$. The proof consists of, starting from a value of $p = 0$, to increase p and to look that whenever we start from a polymorphic equilibrium we can only switch to a equilibrium with a lower level equilibrium of honest agents. Let us denote by $G^*(0)$ the value of G^* for p =0. We have that $\frac{dG^*(p)}{dp} < 0$. For the proof, we partition G and show that for each partition increasing p never decreases criminality.

- If $G \geq 0.25$, only the equilibrium $\eta^* = 0$ exists independently of p.
- if $G^*(0) \leq G \leq 0.25$. The η^*_{+1} equilibrium exists and an η^*_{+2} cannot exist for any p. If we are in an η^*_{+1} equilibrium, we know that η^*_{+1} is decreasing with p reaching the full defection equilibrium for some $p > 0$. Now to prove that an η^*_{+2} cannot exist for $G \geq G^*(0) \geq G^*(p)$: if p increases an η^*_{+2} would appear only in the case $G \leq p$ and $G \leq (1-p)^2/4$. This two conditions imply that $G \leq 3 - 2\sqrt{2}$ contradicting the fact that $G \geq G^*(0) = -2 + \sqrt{5}$.
- if $G \leq G^*(0)$. Only the η^*_{+2} equilibrium exists at p = 0. If p increases η^*_{+2} decreases. Now by increasing p, an η^*_{+1} equilibrium will appear either in coexistence with the η^*_{+2} equilibrium or as the unique polymorphic equilibrium. Since $\eta^*_{+2} \geq \eta^*_{+1}$ and η^*_{+2} is decreasing we can only switch to a lower equilibrium.

- The case with $G < 0$ has two equilibria, one defined for all G and p which is $\eta^* = 1$ and thus independent of p and one equal to $1-p$ which decreases whenever p increases. If $p = 0$ only the $\eta^* = 1$ equilibrium exists. For $p \geq (1-G)/2$ there is coexistence. Moreover the bifurcation point $\eta^*_- = \frac{(1+p)-\sqrt{(1+p)^2-4p(G+1)}}{2}$ increases with p rendering the $1-p$ equilibrium more likely. The proof of this last assertion follows by the fact that the sign of $\frac{d\eta^*_-}{dp}$ is the same as $\sqrt{(1+p)^2 - 4p(G+1)} - p + 2G + 1$. This last term can never be negative.

Part 2: Honesty never increases with criminal Gains

- Case $G \leq 0$: equilibria are independent of G in this case. For $G \leq -1$ only $\eta = 1$ is an equilibrium. If G increases an equilibrium at $1-p$ appears only when $G \geq 1-2p$. Moreover the bifurcation point $\eta = \frac{(1+p)-\sqrt{(1+p)^2-4p(G+1)}}{2}$ increases with G rendering the $1-p$ equilibrium more likely.
- Case $G \geq 0$: first, both $\frac{\eta^*_{+2}}{dG} < 0$ and $\frac{\eta^*_{+1}}{dG} < 0$ and $\eta^*_{+2} \geq \eta^*_{+1}$. Let p^* be such as $G^*(p^*) = 0$.
  - If $p < p^*$, $\eta^*_2 = 1$ is the equilibrium at $G = 0$. By increasing G we decrease η^*_2, switching to the lower equilibrium η^*_1 for $G > G^*$ and for $G > (1-p)/4$ to an equilibrium with only criminals.
  - if $p > p^*$ we have coexistence of η^*_1 and η^*_2. Either the equilibrium is at η^*_1 and by increasing G the level of the equilibrium decreases. Or it is at η^*_2 and

we decrease η_2^* by increasing G, switching to the lower equilibrium η_1^* for $G > (1-p)^2/4$ and for $G > (1-p)/4$ to an equilibrium with only criminals.

Moreover in the case $G > 0$, the unstable equilibria are increasing in G showing that the basin of attraction of each of the high equilibria reduces when G increases. Since this is also true for the overlapping zone with coexistence of equilibria, both the switch from η_2^* to η_1^* or from η_1^* to zero, become more probable the higher G.

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