Macroeconomics Assignment 2

ECON 20001

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Please note Q1 and Q2 time paths have been calculated in R while Q3 time path has been calculated in excel.

R link is at the bottom of this file

link

Question 1: Graphing [4 points]

Q1) Plot these ratios for Canada, Denmark, Ireland, and Taiwan over the period for which you have data. Does your data support the notion of convergence with the United States? Give a brief explanation to rationalize your finding. (2 points)

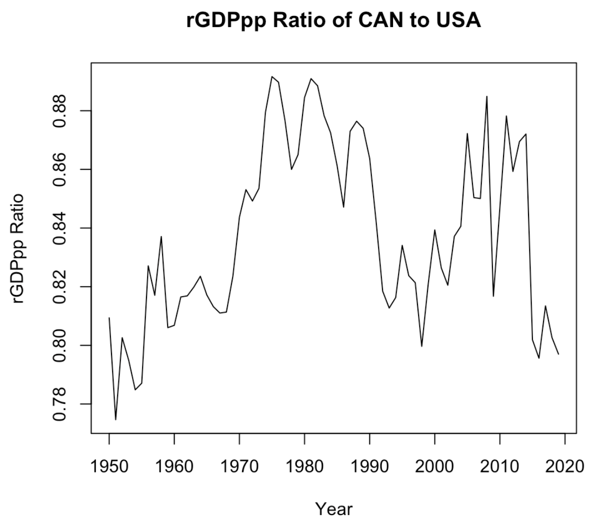


Figure 1

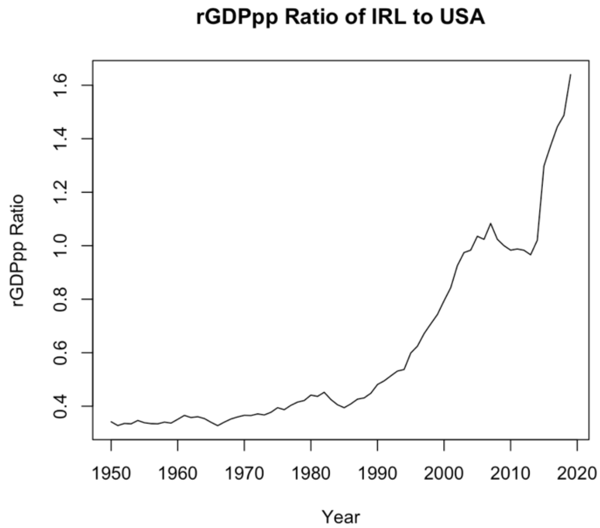


Figure 2

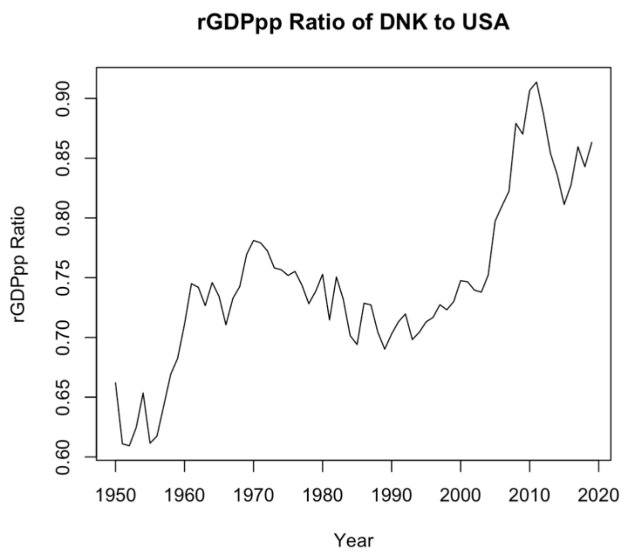


Figure 3

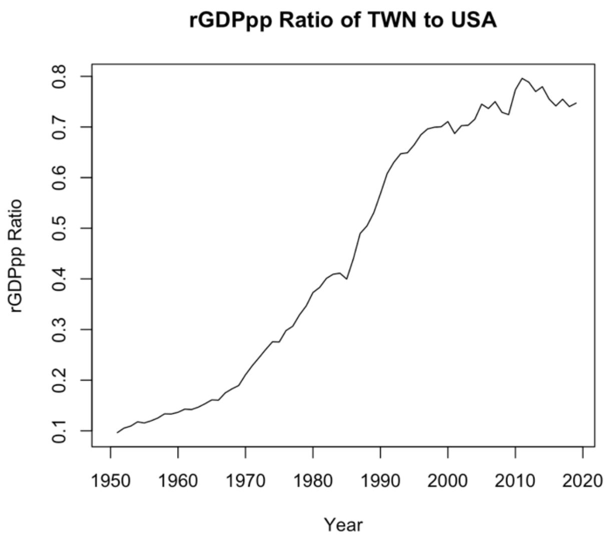
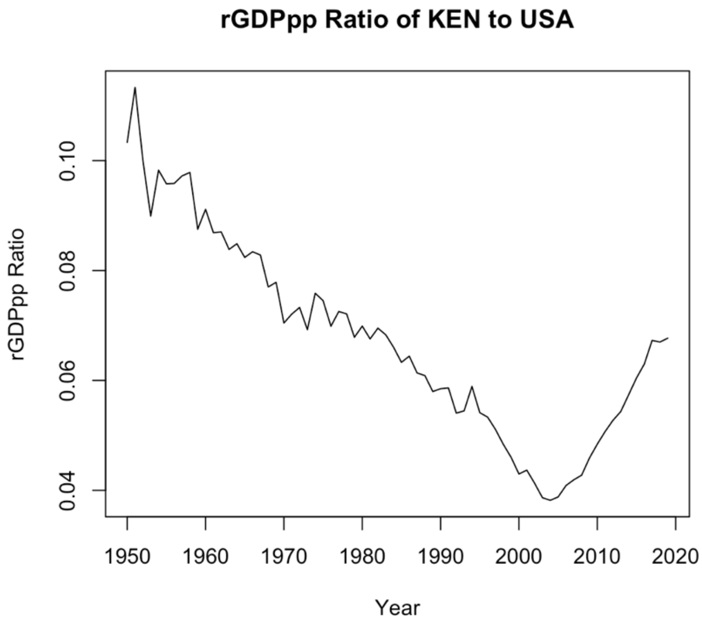
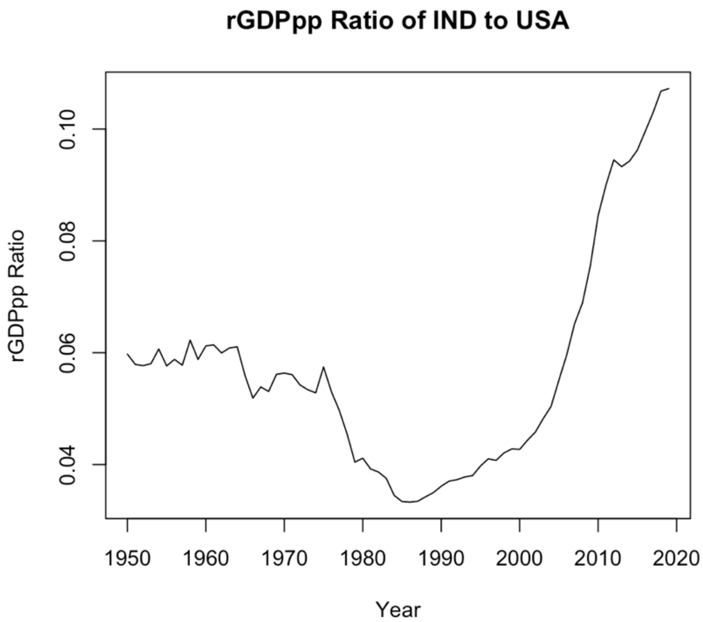


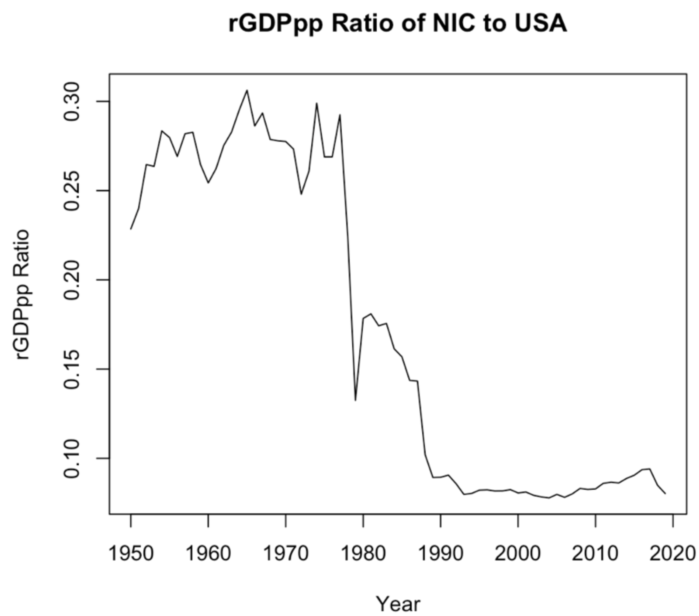
Figure 4

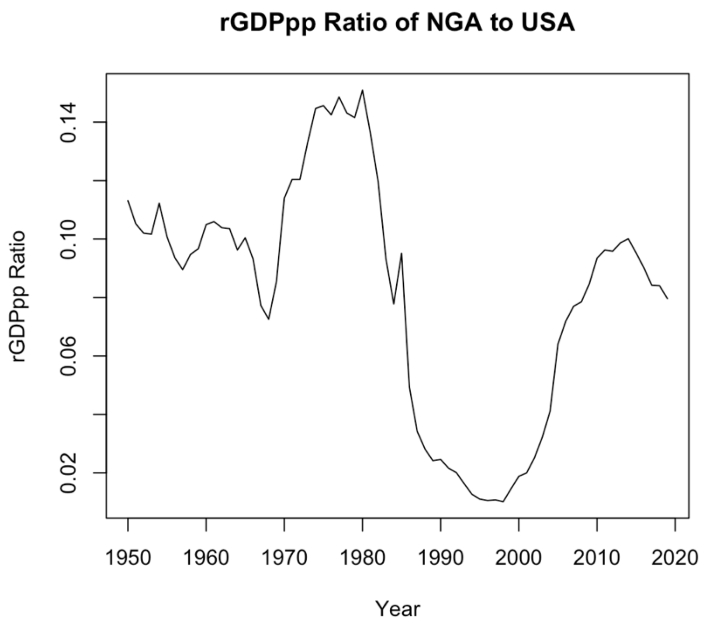
Convergence in the neoclassical Solow Swan model implies that countries with similar parameters, i.e. s, δ, gA and gN etc. will converge to a similar level of capital and output per worker as well as growth in the long term . We can see in figures 1 through 4 that these countries either all approach the USA in terms of output per capita or remain near the USA. For example Taiwan, over the course of 70 years increases its real GDP per person from 10% of US real GDP per person to ~ 80%, similarly Canada hovers around the 80% over the last 60 years. It is safe to say that this group of countries do support the notion of convergence, and this can be attributed to the fact that the countries are quite similar in that they are all quite wealthy, developed economies with similar parameters.

Q2) Plot these ratios for India, Kenya, Nicaragua, and Nigeria. Does this data support the notion of convergence with the United States? Give a brief explanation to rationalize your finding. (2 points)









On the other hand, this group of countries does not support the notion of convergence. We can see that they remain well below the US gdp per person, closer to the 10-25% range as opposed to the ~80% from the previous club of countries. This could be explained by the fact that these countries are mostly developing countries and therefore vastly different economies and thus parameters to the USA. Thus their long run level of output and capital per worker is different to the USA.

Question 2: Solow Growth Model [14 points]

Consider the Solow (neoclassical) growth model with aggregate production function:
$Y = K^{\alpha}(AN)^{1-\alpha}$. Each period lasts a year.

α	1/4	g_A	2%
δ	5%	g_N	2%
s	12%		

Table 1: Benchmark Parameter Values – Solow model

Q1) Using the parameter values in Table 1, calculate the steady-state values of capital per effective worker k ≡ K/AN, output per effective worker y ≡ Y/AN and consumption per effective worker c ≡ C/AN. Calculate the golden rule saving rate. Also calculate the growth rates of output per worker and consumption per worker along the balanced growth path. (2 points)

Equation 1

$$\begin{aligned}
&\textit{In steady state:}\\
sf(k^*) &= (\delta + g_A + g_N)k^*\\
y^* = f(k^*) &= {k^*}^{\alpha}\\
s{k^*}^{\alpha} &= (\delta + g_A + g_N)k^*\\
k^* &= \left(\frac{s}{(\delta + g_A + g_N)}\right)^{\frac{4}{3}}\\
k^* &\approx 1.468\\
y^* = {k^*}^{1/4} &= \left(\frac{s}{(0.09)}\right)^{\frac{1}{3}} = 1.101\\
c^* &= (1-s)y^* = 0.969\\
s_g = max(c^*) &= max\left((1-s)\left(\frac{s}{(0.09)}\right)^{\frac{1}{3}}\right)\\
s_g &= \frac{1}{4}
\end{aligned}$$

The golden rule savings rate is ¼. Both Output per worker and consumption per worker will grow at the rate of technological progress or gA which is 2%.

Q2) Suppose that the economy is initially in steady-state. In year t = 0 the saving rate increases from s = 0.12 to s = 0.17 (i.e., from 12% to 17%) while all other parameters have their benchmark values.

I. **Calculate the new steady-state levels of capital per effective worker, output per effective worker and consumption per effective worker. Does long-run consumption per effective worker increase? Also calculate the long run growth rates of output per worker and consumption per worker? Do the long-run growth rates of output per worker and consumption per worker increase? Explain. (3 points)**

Equation 2

$$k^*_{s=0.17} = \left(\frac{0.17}{0.09}\right)^{\frac{4}{3}} = 2.33$$

$$y^* = k^{*\,1/4} = \left(\frac{0.17}{(0.09)}\right)^{\frac{1}{3}} = 1.24$$

$$c^* = (1-s)y^* = 1.03$$

The increase in the savings rate leads to an increase in the steady state level of capital, output and consumption per effective worker. While an increase in the savings rate will always lead to an increase in the steady state level of capital and output because it increases effective investment in capital, the change in consumption per effective worker depends on where the savings rate is relative to the golden savings rate. In this case because it was below Sg, an increase in the savings rate caused an increase in consumption per effective worker. The growth rate of output and consumption per worker do not increase because they depend only on productivity growth.

II. Calculate and plot the time-paths of (a) capital per effective worker, output per effective worker and consumption per effective worker for 100 years (t = 0, 1, ..., 100) and of (b) log output per worker and log consumption per worker. Describe and explain the short-run effect of the change in the saving rate on these variables. (3 points)

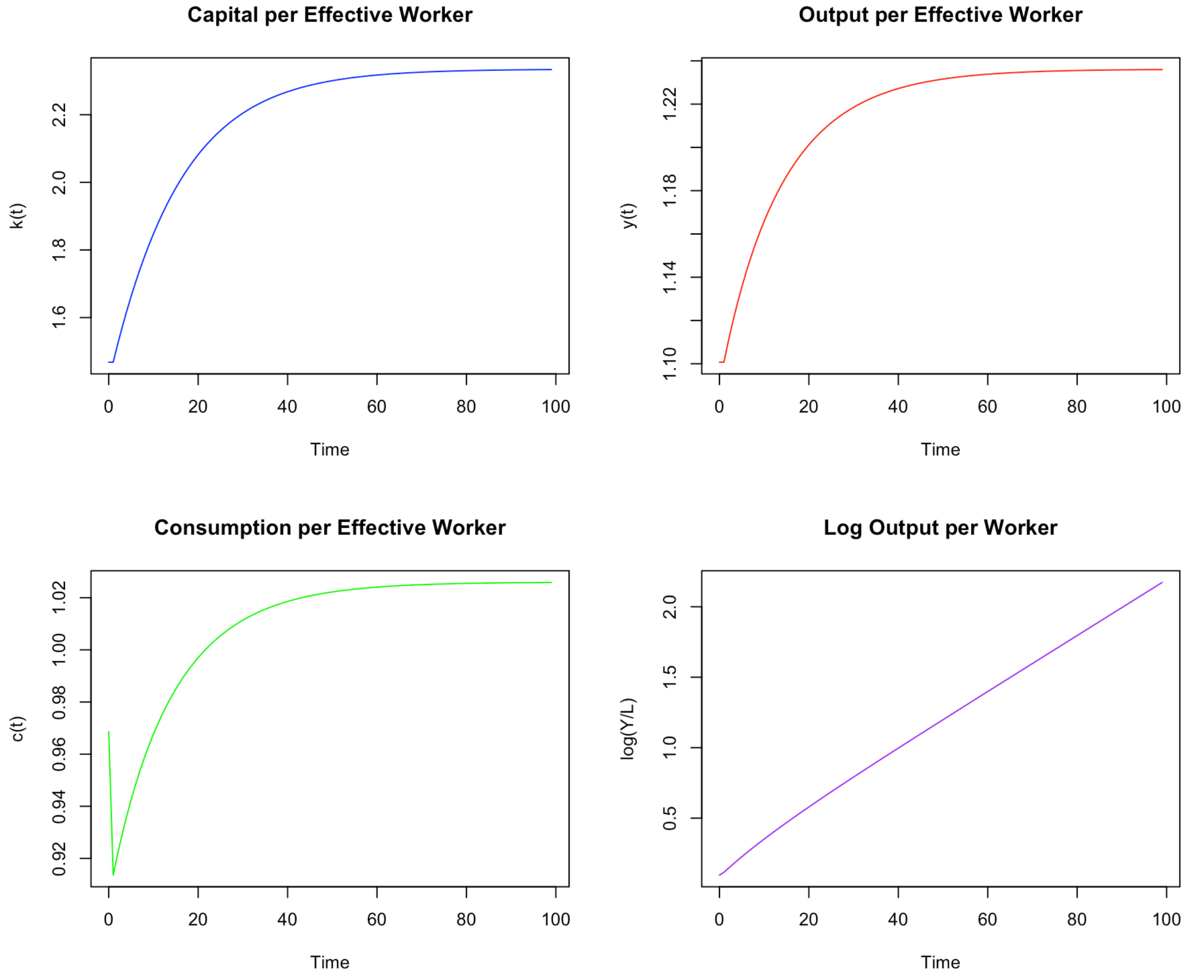


Figure 5

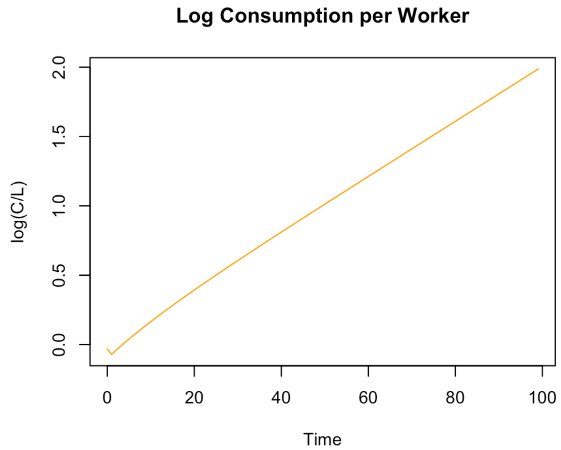


Figure 6

The following equation links the capital per effective worker between time periods. It suggests that the change in capital per effective worker is equal to the effective investment in capital per worker less the effective depreciation of capital per worker. This allows us to trace the time paths of kt and thus yt.

Equation 3

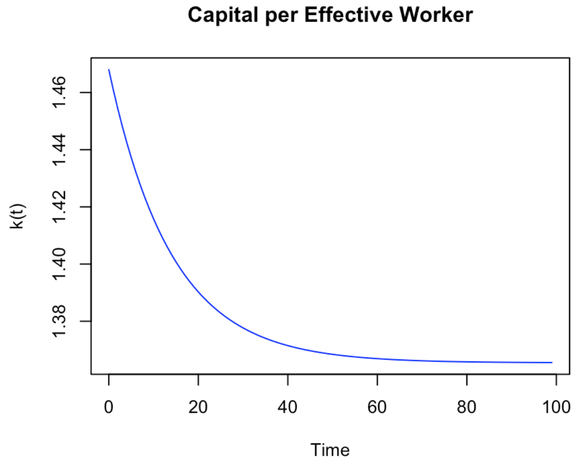
$$\Delta k_t = sf(k_t) - (\delta + g_A + g_N)k_t$$

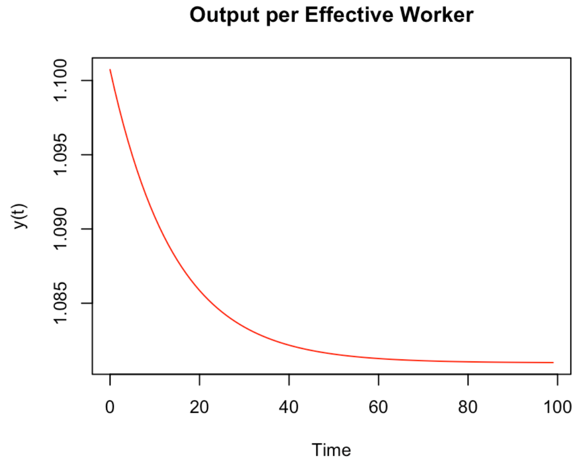
$$y^* = f\left(k^*\right) = {k^*}^{\alpha}$$

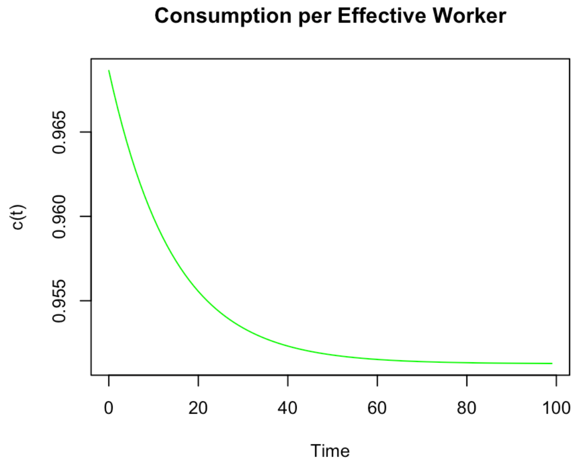
$$c^* = (1 - s)y^*$$

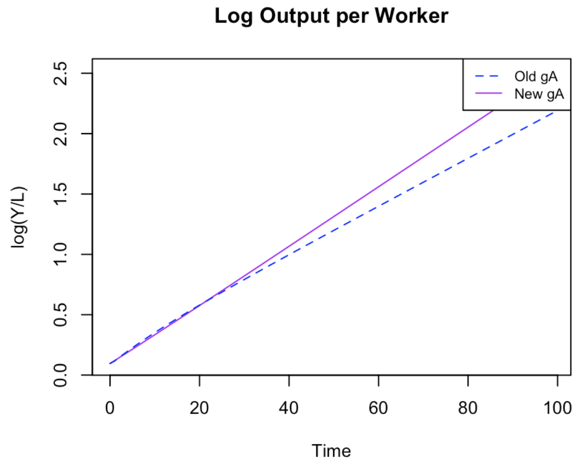
In order to find output and consumption per worker, the output and consumption per effective worker should be multiplied by the multifactor productivity value (A) for that period. This can be found by compounding the initial A value by gA (2%) for t periods. On impact of the savings rate change, the consumption per effective worker decreases because more income is being spent on savings (investment) instead of consumption. The following period output and capital per effective worker respond to this change by increasing and over many periods approach the new steady state which is higher than the original. Both capital and output per worker increase. While the economy is transitioning to the new steady state they grow faster than gA, but after converging their growth rate returns again to gA which is 2%.

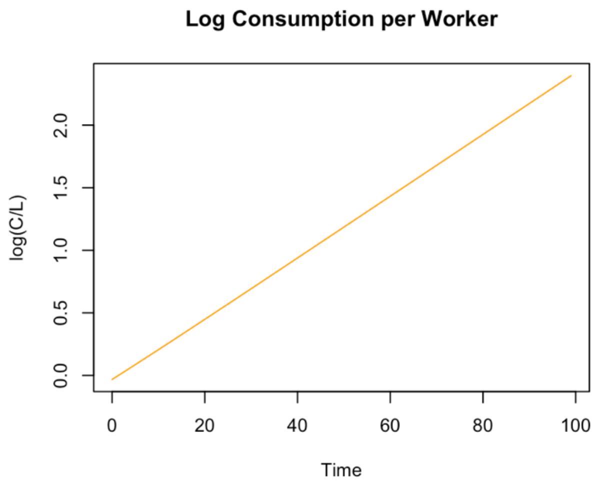
Q3) Compare the time-path of log output per worker from the increase in the saving rate in part (2) to the time path with the increase in the rate of technological progress. How many years pass before output per worker surpasses the level that would be obtained from the increase in the saving rate? What does this suggest about the relative importance of level versus growth rate effects? Explain. (3 points)







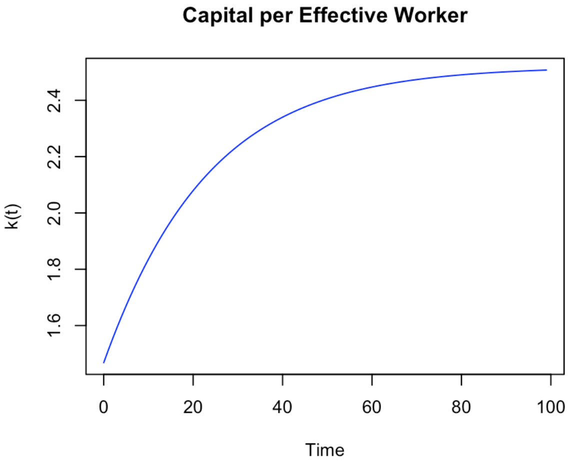


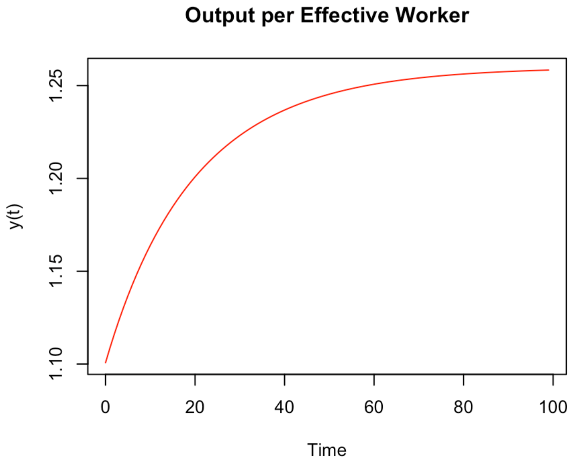


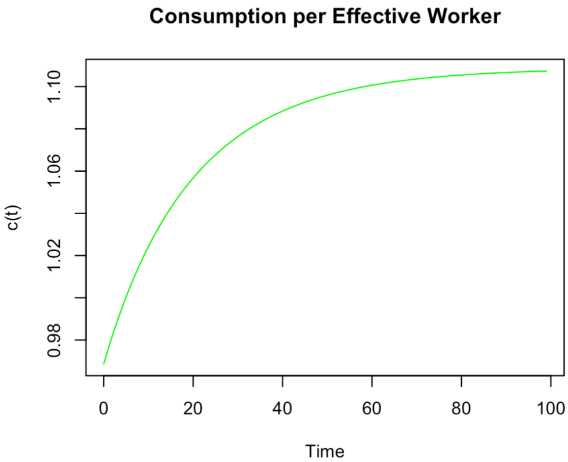
The increase in gA has no immediate impact effect on consumption, capital or output per effective worker because k is determined by the parameters in the previous period. From t=1, consumption, capital and output per worker begin to decrease. The new steady state capital and output per effective worker levels are lower because an increase in gA leads to an increase in depreciation per effective worker. The new levels are k* = 1.366 and y*=1.081. The levels of output and consumption per worker because these measures are not normalised by multifactor productivity and thus this growth is included. Again because the growth rate of C/N and Y/N depend only on gA, their growth rates increase to 2.5% and as such they are overtake the scenario where gA is 2% and 17% savings rate in about 23 years. Thus in the short run, level effects might seem more pronounced as changes in parameters like the savings rate lead to immediate changes in levels of capital, output, and potentially consumption per effective worker. In the long run, however, growth rate effects are paramount. A higher rate of technological progress enriches the growth prospects of an economy, leading to exponentially higher levels of output and consumption per worker over time, even if the initial level effects are adverse.

Q4) There is increasing concern about recent declines in the rate of population growth observed across a number of developed economies.

Compare the time-path of log output per worker from the increase in the saving rate in part (2) and the increase in rate of technological progress in part (3) to the time path with the decline in the rate of population growth. In light of your results, is there cause for concern if population growth rates would continue to decline? Explain. (you may assume that $N_0 = 1$ to facilitate your explanation). (3 points)







In the Solow-Swan model, a decline in the population growth rate gN impacts the steady-state level of capital and output per effective worker. A decrease in gN would mean less dilution of capital, leading to a higher steady-state level of capital per effective worker, which would also result in a higher steady-state level of output per effective worker k* and y* respectively. k* = 2.508 and y* = 1.258.

However, when assessing the overall impact on the economy, contrasting it with changes in savings rate and technological progress is crucial.

Implications on Time-Path of Log Output per Worker:
Decline in Population Growth Rate results in a higher steady-state level of capital and output per effective worker. The growth rate of Y/N and C/N is unaffected as gA is unaffected.

Increase in Savings Rate leads to higher investment and hence a higher steady-state level of capital and output per effective worker. The growth rate in output per worker remains the same as it is driven by the technological growth rate.

Increase in Technological Progress Rate gA enhances the long-term growth rates of output and consumption per worker. It leads to lower steady-state levels of capital and output per effective worker due to increased effective depreciation.

A decline in the population growth rate seems beneficial in the short term due to higher steady-state levels of capital and output per effective worker. In the long term there is no effect on the growth rates of Y/N and C/N because this is dependent only on gA.

<u>Question 3: Transition Dynamics of Growth Model [6 points]</u>

You will explore the dynamics of human capital and physical capital using the human capital accumulation model. The aggregate production function is: $Y = AK^{\alpha}H^{1-\alpha}$ and let the physical and human capital accumulation equations be given by,

$$K_{t+1} - K_t = s_K Y_t - \delta_K K_t,$$
$$H_{t+1} - H_t = s_H Y_t - \delta_H H_t$$

Q1) Define the human capital intensity (relative to physical capital) as $\boldsymbol{\phi_{t+1} = \frac{H_{t+1}}{K_{t+1}}}$ and solve for the transition dynamics of ϕ.

<u>We know from the Human Capital Intensity equation:</u>

$$\phi_{t+1} = \frac{H_{t+1}}{K_{t+1}}$$

$$= \frac{H_t + s_H Y_t - \delta_H H_t}{K_t + s_K Y_t - \delta_K K_t}$$

$$= \frac{\frac{H_t}{H_t} + s_H \frac{Y_t}{H_t} - \delta_H \frac{H_t}{H_t}}{\frac{K_t}{H_t} + s_K \frac{Y_t}{H_t} - \delta_K \frac{K_t}{H_t}}$$

$$= \frac{1 + s_H \frac{Y_t}{H_t} - \delta_H}{\frac{K_t}{H_t} + s_K \frac{Y_t}{H_t} - \delta_K \frac{K_t}{H_t}}$$

Using Production Function, rearrange and substitute it into the above:

$$Y = AK^{\alpha}H^{1-\alpha}$$

$$\frac{Y}{H} = AK^{\alpha}H^{-\alpha}$$

Knowing: $\phi_{t+1} = \frac{H_{t+1}}{K_{t+1}}$

$\frac{Y}{H} = A \times \frac{1}{\phi^{a}}$

Thus, upon substitution, the final Human Capital Intensity Equation is:

$$\phi_{t+1} = \frac{\phi_t + s_H A \phi_t{}^{1-\alpha} - \delta_H \phi_t}{1 + s_K A \phi_t{}^{1-\alpha} - \delta_K}$$

Q2) Consider a balanced growth path where physical capital and human capital grow at the same growth rate. Assume the parameter values are as in Table 2, solve for the human capital intensity ϕ^* > 0 along the balanced growth. (2 points)

δ_K	6%	s_K	15%
δ_H	2%	s_H	10%
α	1/2	A	1

Table 2: Parameter Values for (2)

The Balanced Growth path insinuates:

$$\frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{K_t}$$

$$\frac{H_t}{K_t} = \frac{H_{t+1}}{K_t}$$

$$\phi_t = \phi_{t+1} = \phi^*$$

<u>Substituting in ϕ^* in all appropriate areas of the Human capital intensity equation:</u>

$$\phi^* = \frac{\phi^* + s_H A(\phi^*)^{1-\alpha} - \delta_H \phi^*}{1 + s_K A(\phi^*)^{1-\alpha} - \delta_K}$$

<u>Using values in Table 2, the Human Capital intensity (ϕ^*) is:</u>

$$\phi^* = \frac{\phi^* + 0.1 \times 1(\phi^*)^{1-0.5} - 0.02(\phi^*)}{1 + 0.15 \times 1(\phi^*)^{1-0.5} - \delta_K}$$

$$1 + 0.15\phi^{*0.5} - 0.06 = 1 + 0.1\phi^{*-0.5} - 0.02$$

$$1 + 0.15\phi^{*0.5} - 0.06 = 1 + 0.1\phi^{*-0.5} - 0.02$$

$$0.15\phi - 0.04\phi^{0.5} - 0.1 = 0$$

<u>Taking $\phi^{0.5} = t$:</u>

$$0.15t^2 - 0.04t - 0.1 = 0$$

<u>Solving via Quadratic function:</u>

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = 0.961, -0.694$$

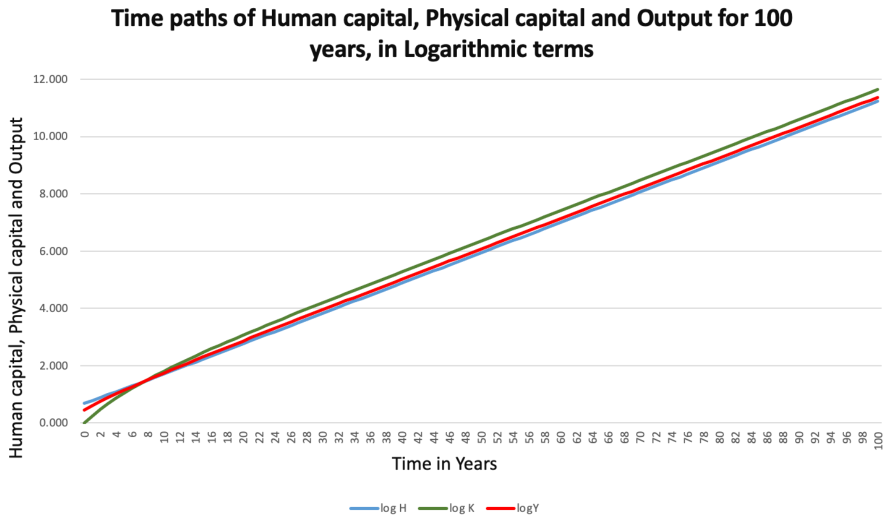
<u>Given $\phi^{0.5} = t$:</u>

$$\phi = (0.961)^2 \ \ and \ (-0.694)^2$$

$$= 0.923$$

$$= 0.482$$

Q3) Describe the dynamics of human capital and physical capital. What are the growth rates of human capital, physical capital and output in the long run? Has the ratio of human capital to physical capital converged to a steady state? If so, what is the steady state ratio of human capital to physical capital? (3 points)



With reference to the graph above, it is evident that over time both human capital and physical capital are displaying an increasing trend, this is due to the capital accumulation that occurs over time, contributing to positive overall growth. Originally, human capital grew quicker than physical capital as the economy initially started at a higher H, with $H_0 = 2$ and $K_0 = 1$. However, as seen over the graph trajectory, in the long term these values have converged to a steady state of 0.66, seen by the calculations of the ratio of human capital to physical capital provided in the Excel sheet.

The growth rates of human capital, physical capital and output in the long run are calculated from the below equation:

$$g_Y = \frac{lnY_t - lnY_0}{T}$$

- g_H = 0.1054 = 10.54%
- g_K = 0.1165 = 11.65%
- g_Y = 0.1091 = 10.91%

All in all, seen through the close parallel lines, these variables share similar growth rates.

R github link:

link