Tugas Besar CPMK 2
Kalkulus

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October 26, 2020

Set
Let $U = \{2, 3, 4. \ldots, 20\}$, $A = \{x \in U \mid x \text{ is a prime number}\}$, $B = \{x \in U \mid x \text{ is an even number}\}$, and $C = \{x \in U \mid x \text{ is divisible by } 3\}$. Find

1. $(A \cup B) \cap C$
2. $(A \cap B) \cup (B \cap C)$
3. $(A \cup B)'$

Function
Given $f(x) = x^2 + 2$ and $g(x) = x + 1$, find

1. $(f \circ g)(x)$
2. $(f \circ g)(a^2)$
3. $(g \circ f)(\sqrt{a})$

Limit

1. Decide whether this following limit problem has limit:

$$\lim_{x \to \infty} \left(\frac{x^4 + x^2 + 6}{x^3 - x^2 + 1} \right)$$

   {Hint: $\infty - \infty \neq 0$, and $\frac{\infty}{\infty} \neq 1$}

2. Find the following limit:

$$\lim_{x \to 0} \left(\frac{x^4 - x^2}{x^3 + x^2} \right)$$

Differentiation

1. Using the definition of derivative, prove the following is correct:

$$\frac{\mathrm{d}}{\mathrm{d}x} \cos x = -\sin x$$

2. Solve this differentiation problem using some differentiation rules:

$$\frac{\mathrm{d}}{\mathrm{d}x} \frac{\sin\left(x^2 + 3x + 5\right)}{x^2 + 3x + 5}$$

Integration

1. Solve the following problem using the definition of definite integration:

$$\int_0^1 (x+5)\ \mathrm{d}x$$

2. Find the solution of the following definite integration:

$$\int_{-1}^{0} \left(x^4+2\right)\left(\frac{1}{x^2}+3\right)\ \mathrm{d}x$$

R Programming

Solve the following problems using R:

1. Set: Let

$$\begin{aligned} U &= \{1,2,3,\ldots,19\} \\ B &= \{x \in U \mid x \text{ is a prime number}\} \\ C &= \{x \in U \mid x \text{ is an even number}\} \\ A &= \{x \in U \mid x\ \leq 10\} \end{aligned}$$

   Find the following set : $(A \cup B \cup C)$

2. Function : Write the following equations into functions and then plot them
   (a) $f(x) = x^3 + x^2 - 6$
   (b) $f(x) = -x^2 - 6$

3. Limit:

$$\lim_{x\to 0}\left(\left(\frac{x^4-x^2}{x^3+x^2}\right)\left(\frac{\sin(x)}{x}\right)+\cos x\right)$$

4. Differentiation:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin x^2}{x^2}\,(x+1)\right)$$

5. Integration:

$$\int x^2 \cos\left(x^3+\pi\right)\sin\left(x^3+\pi\right)\mathrm{d}x$$