CS598PS – Machine Learning for Signal Processing

Sensor Arrays

20 October 2020

University of Illinois at Urbana-Champaign

Today's lecture

- Sensor arrays
- Beamforming and localization
- Array-based source separation

A question

- Why do you have two ears? (or eyes)

Sensor arrays

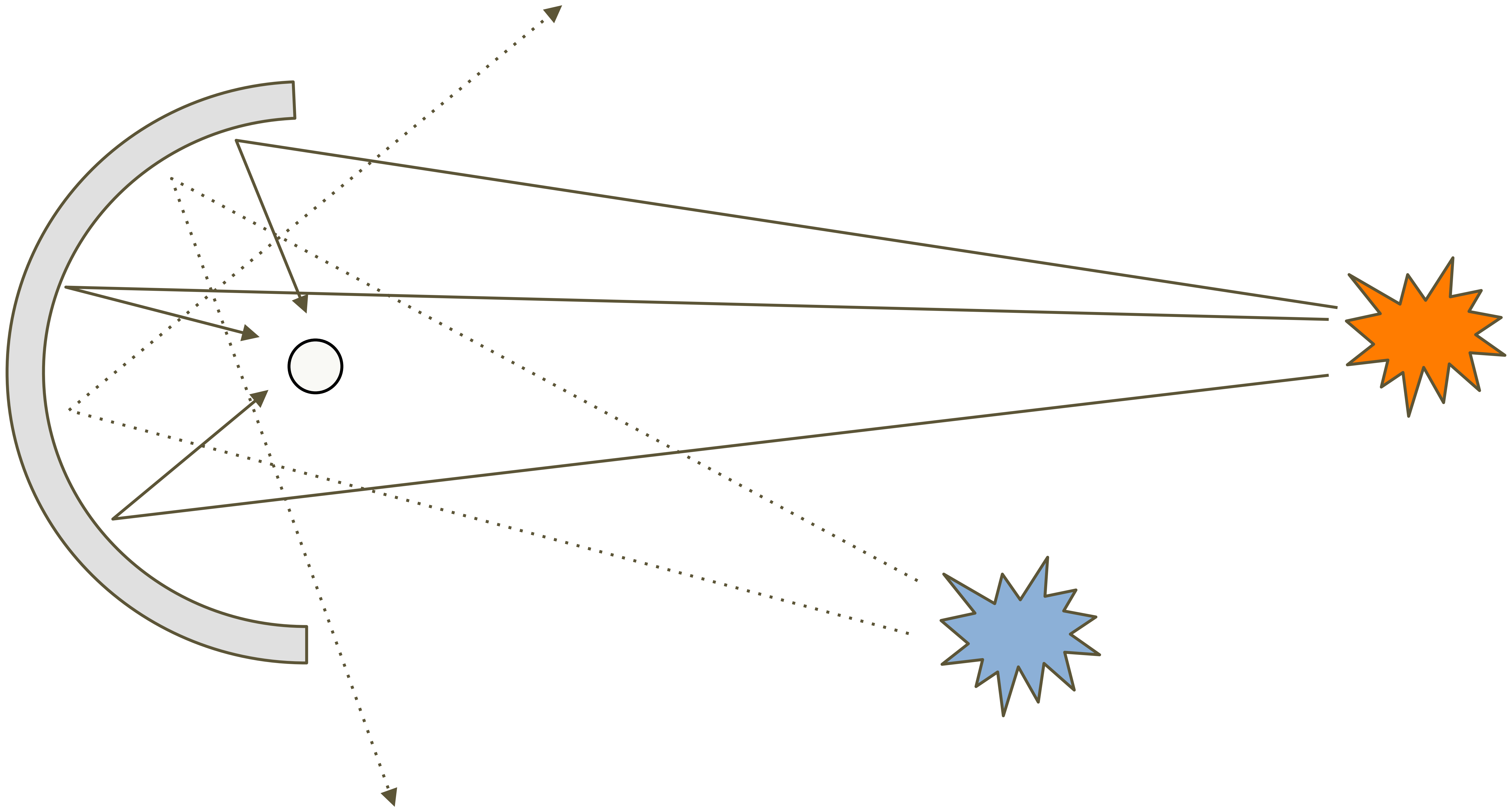
- Multiple sensors allow us to do more
  - They reveal the “spatial dimension”
- We can thus do things involving space
  - Focus on one direction only
  - Locate the emanating source of a signal

One more question

- Why does your ear have that strange shape on the outside?

Collecting from multiple sources

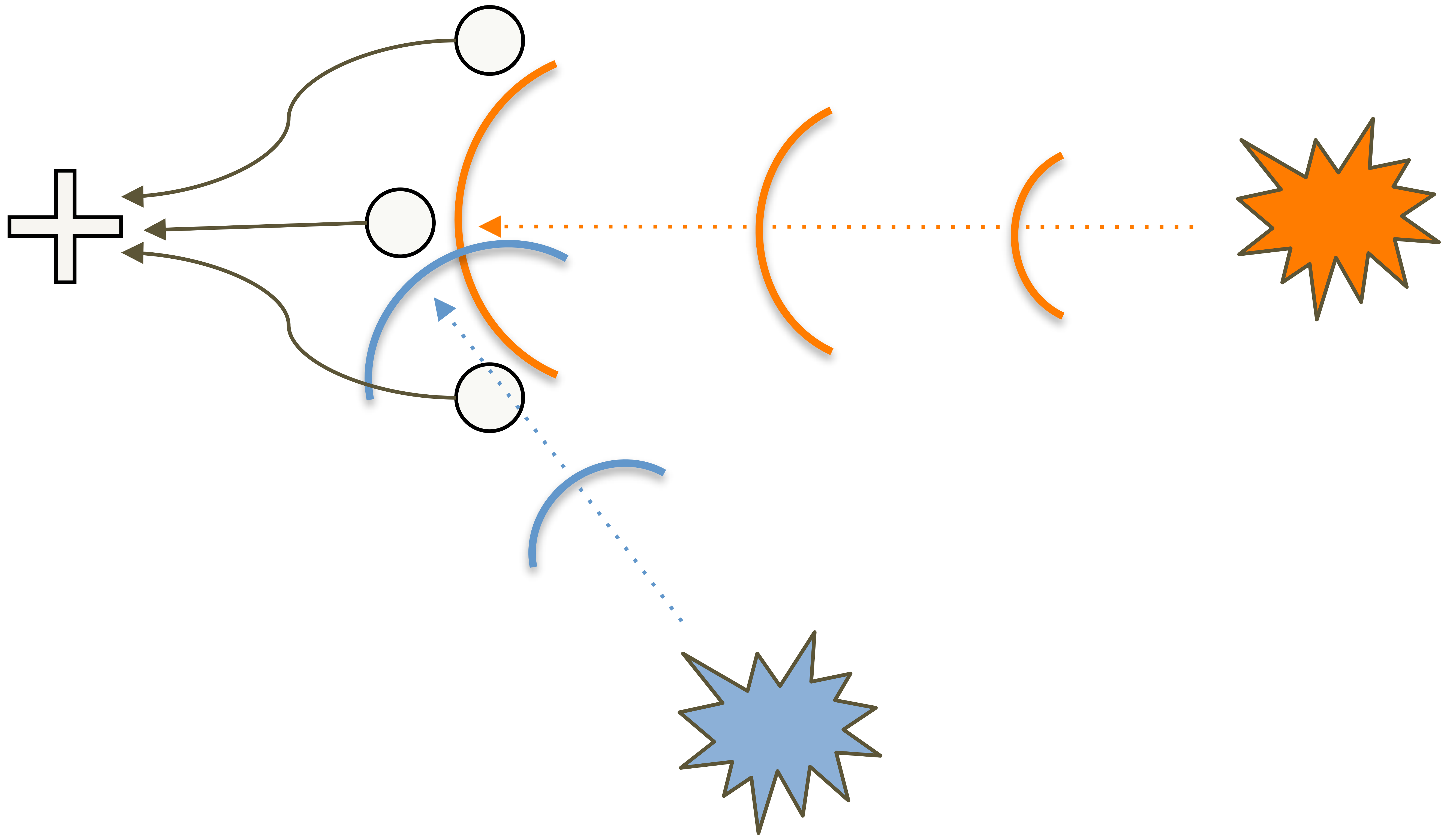
- A parabolic sensor
  - Focuses right ahead and ignores other directions



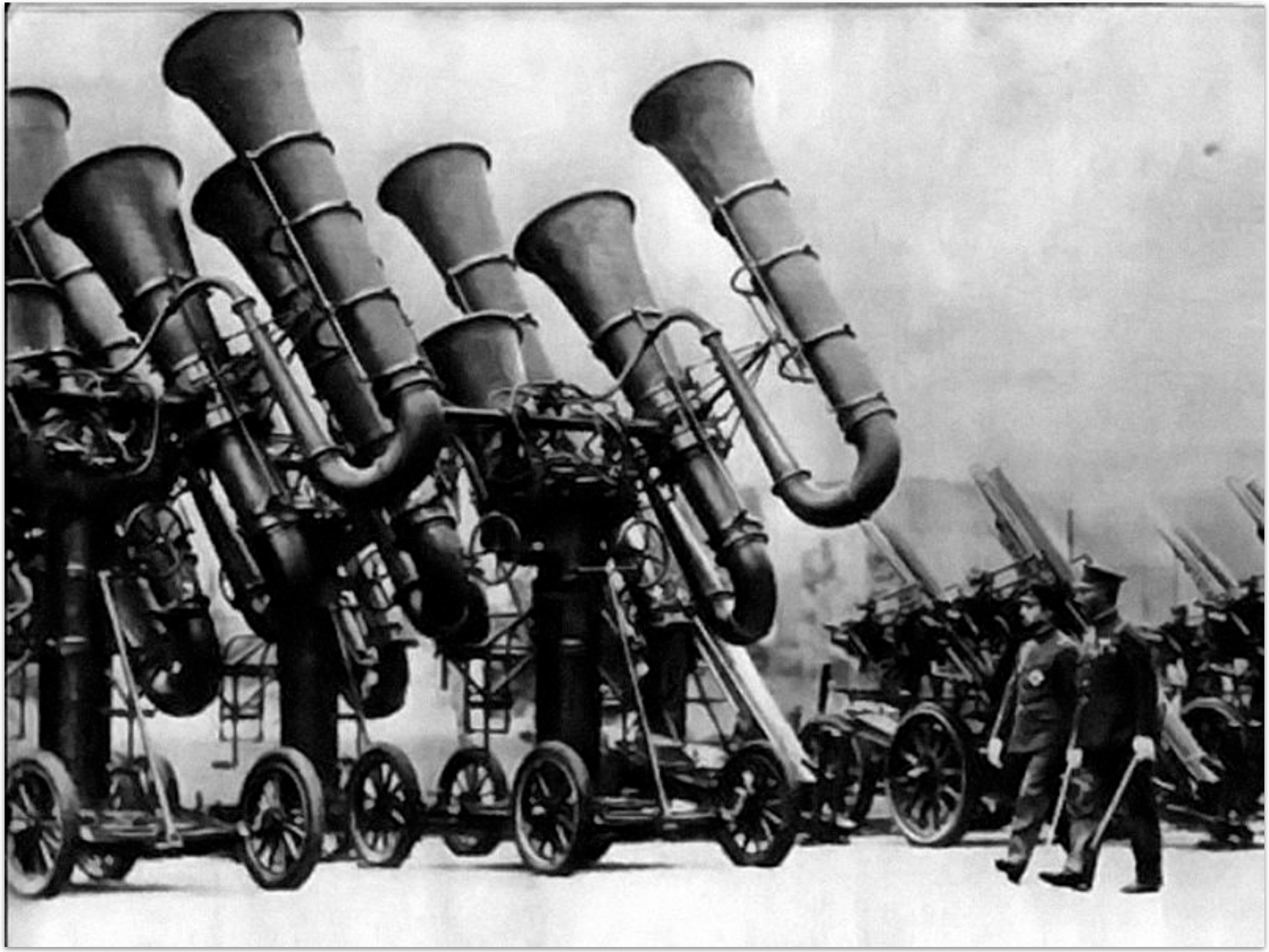
Known for a while

Discretizing the parabolic dish

- Emulating that process with a small number of discrete sensors

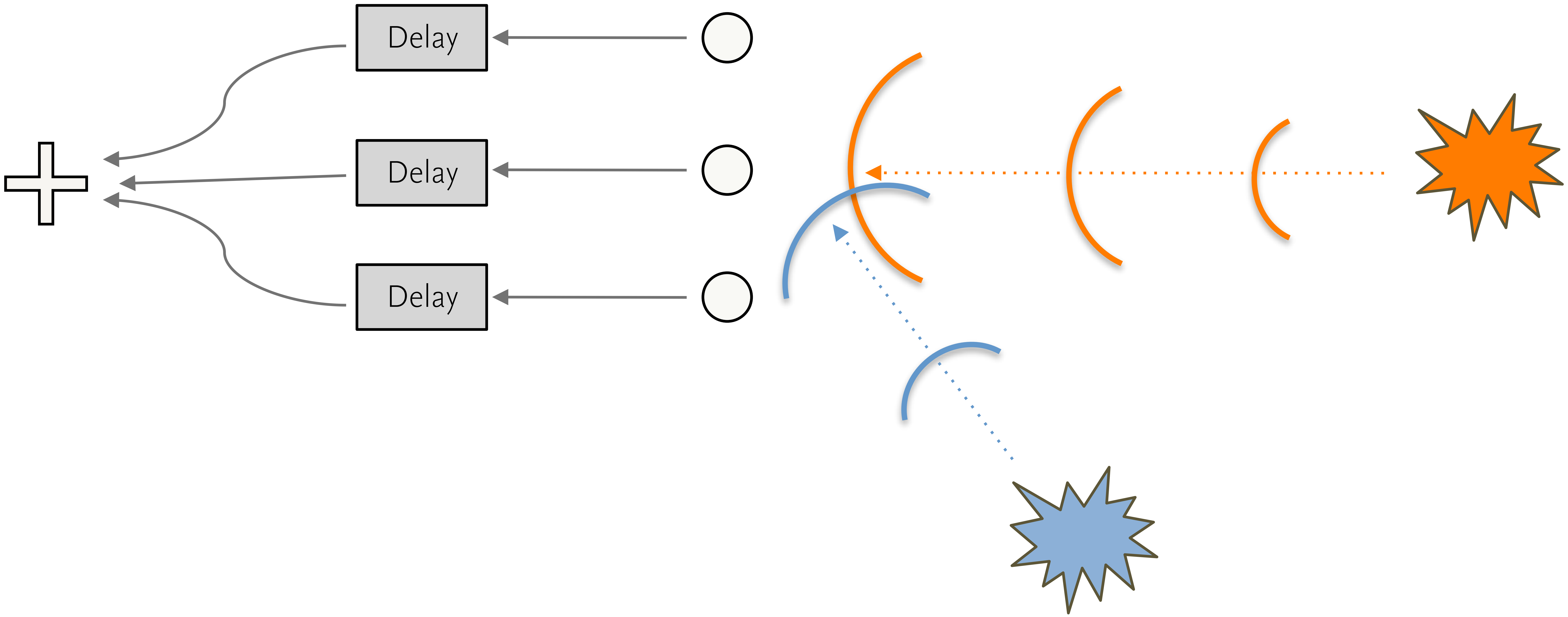


That’s been known for a while too



A more flexible setup

- Use adjustable delays for each sensor
  - We can now “steer” the array



The delay and sum beamformer

- Adjust sensor delays to focus on a source coming from a specific direction

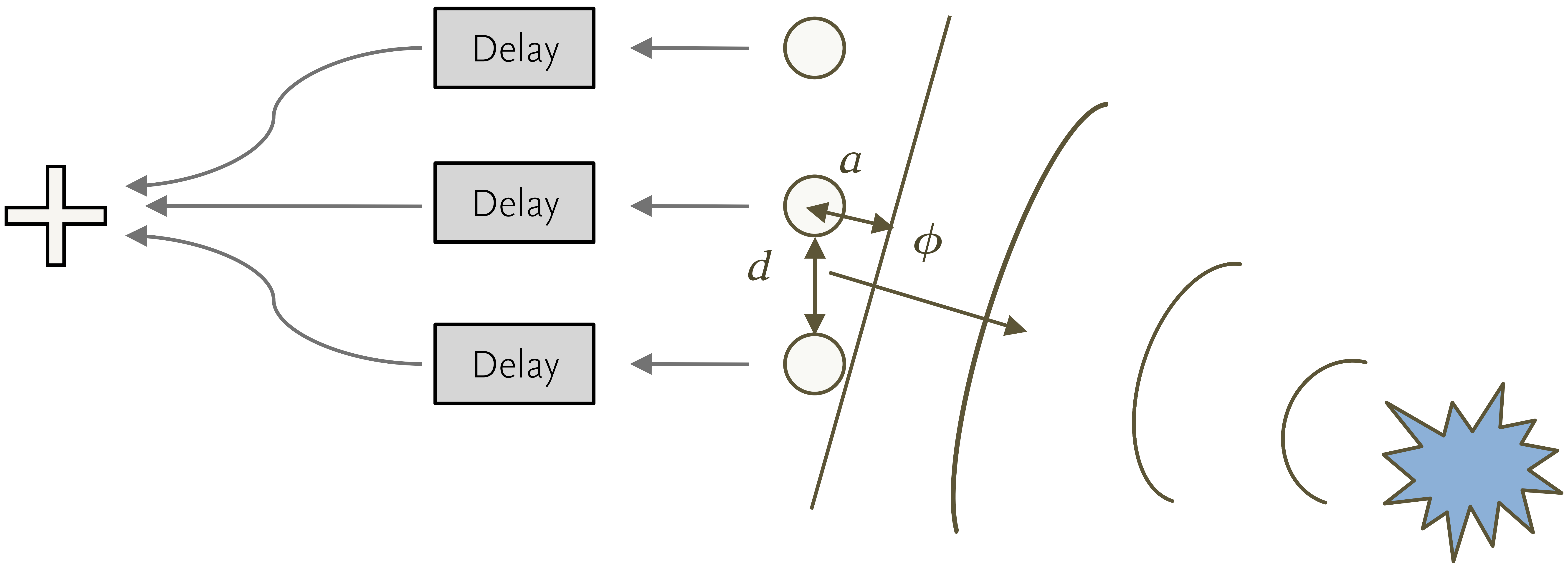
$$y(t) = \sum \tau_i * x_i(t)$$

- The delays depend on the array layout
  - We need to know the layout a priori

Uniform Linear Array (ULA)

- Far field assumption
  - “Wavefront is flat”

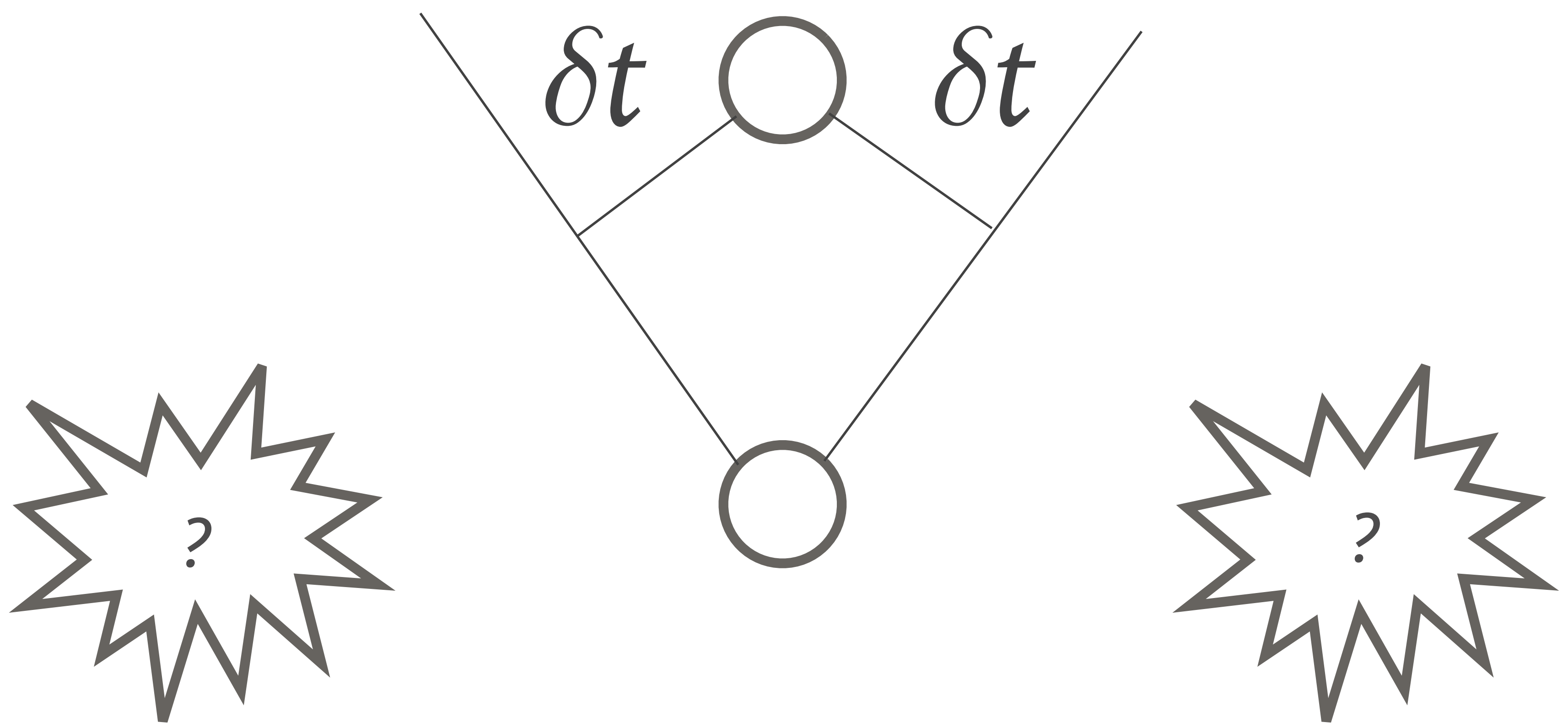
$$\tau_i(t) = \begin{cases} 1 & t = \dfrac{d\cos\phi}{C} \\ 0 & \text{otherwise} \end{cases}$$



Some problems

- Direction confusion

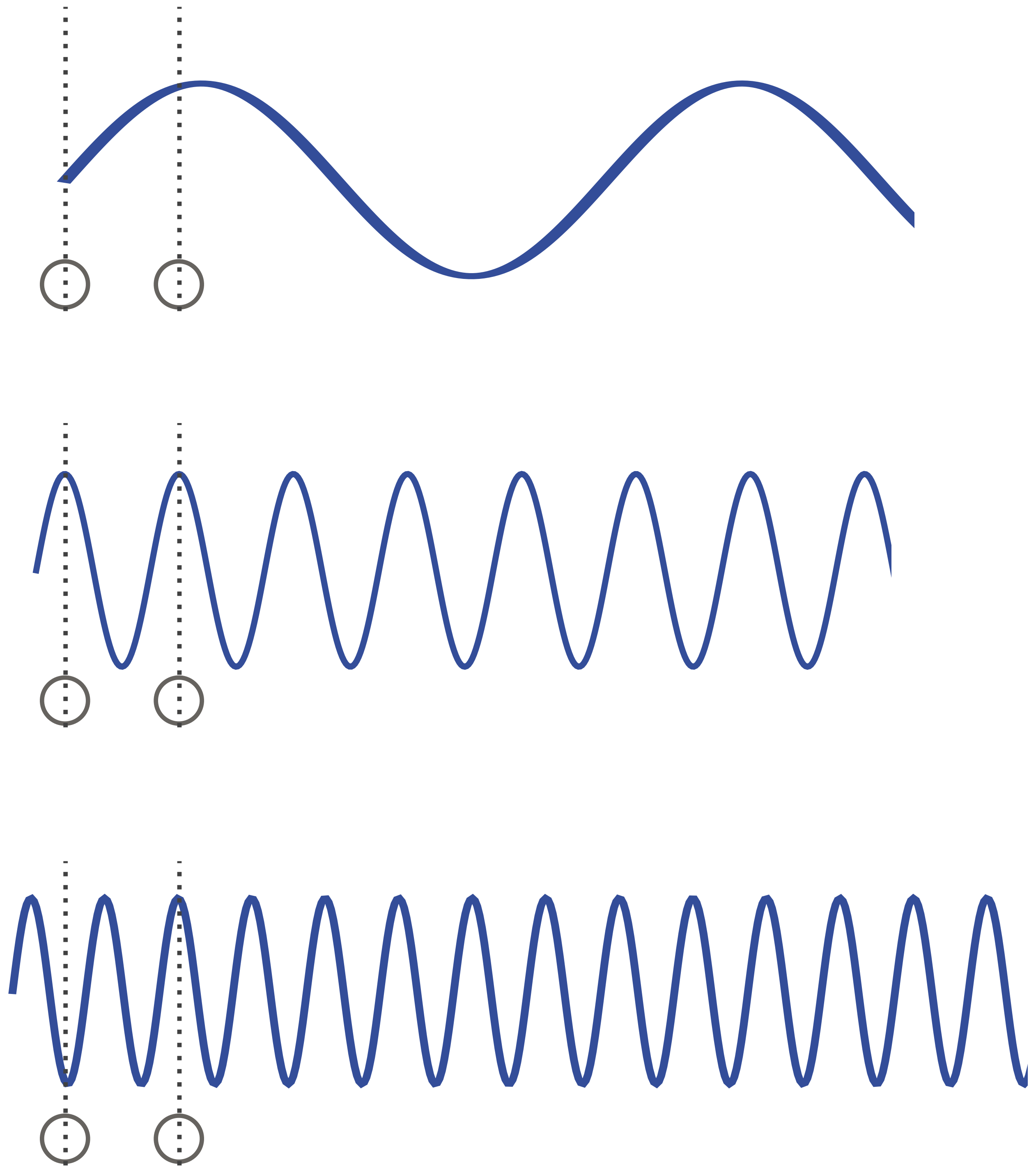
$$\delta t = \frac{d\cos\phi}{C} = \frac{d\cos(-\phi)}{C}$$

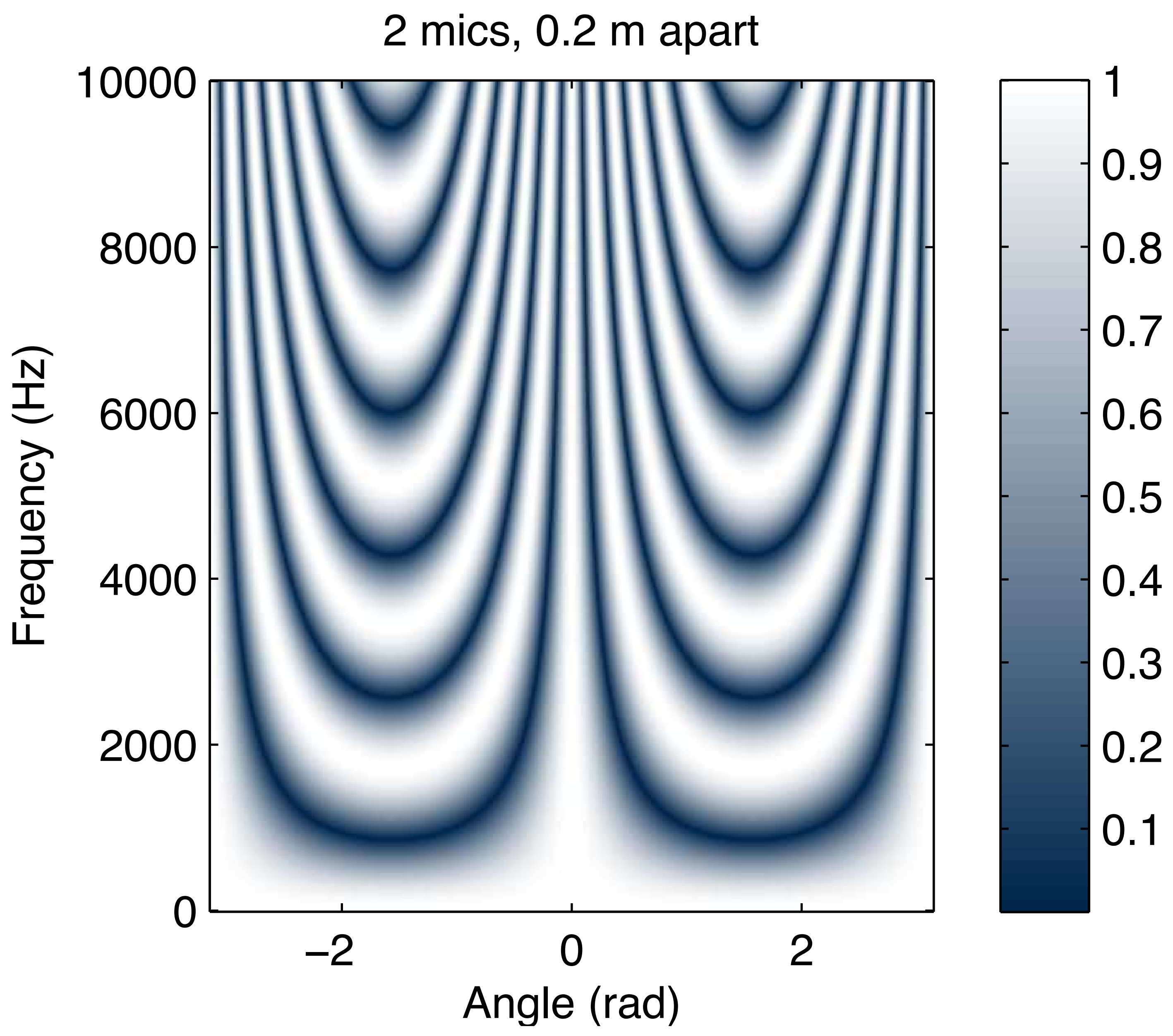


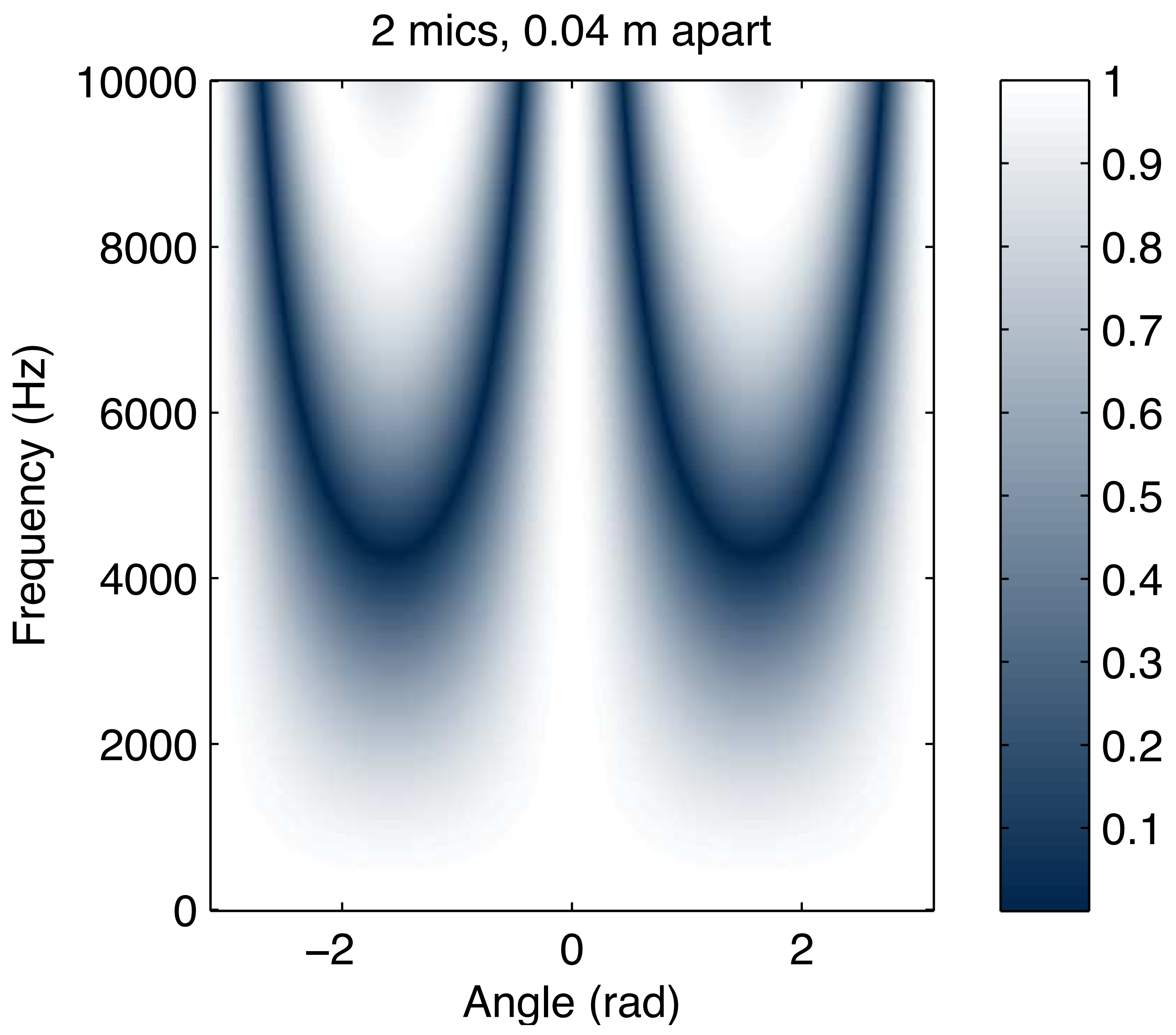
- Need at least $D+1$ sensors for D dimensions
- But regardless, the more the sensors the better!

The array response

- Spatial aliasing = ambiguities in high frequencies
  - Sensor distance must be less than half wavelength of highest freq



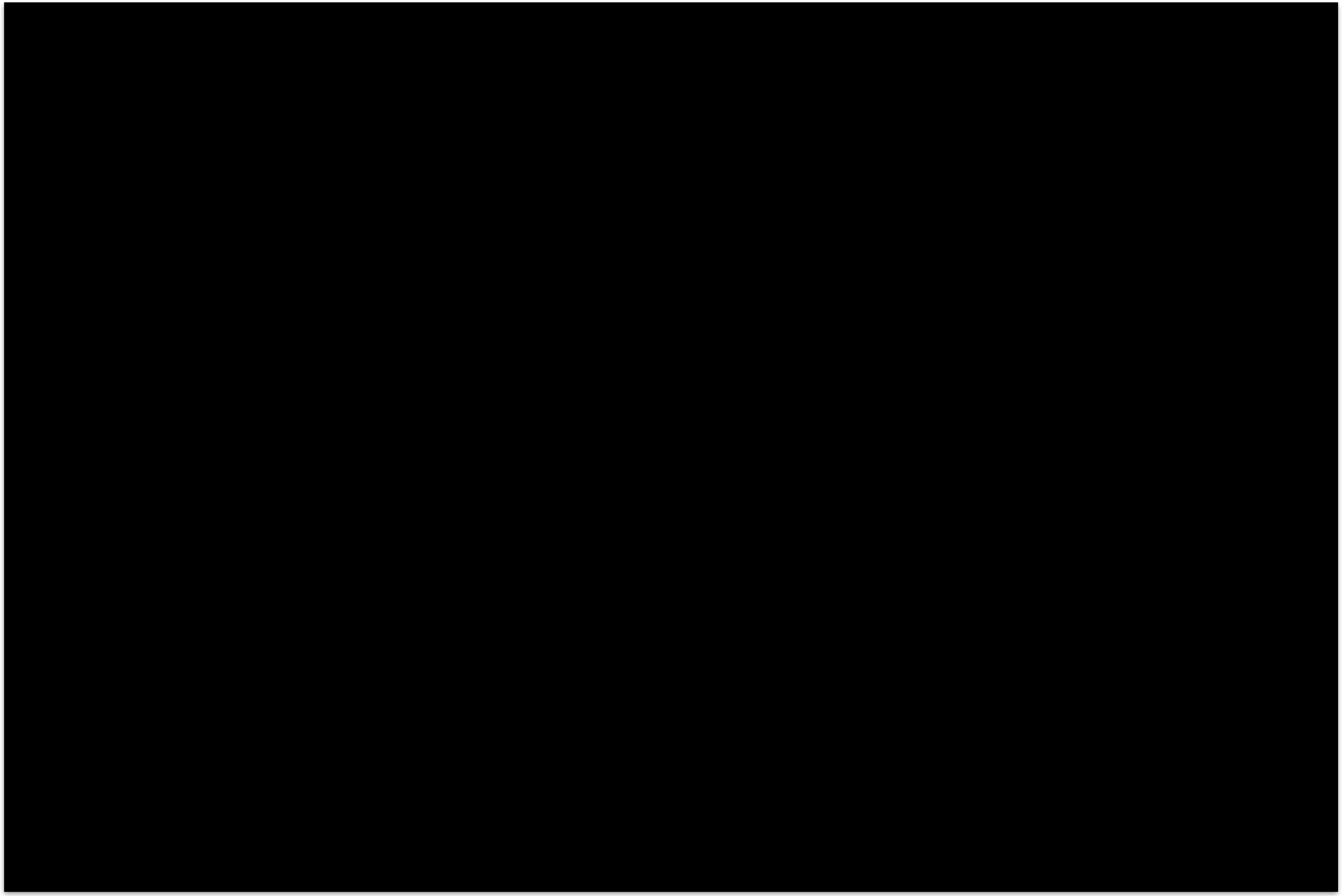




Many more things …

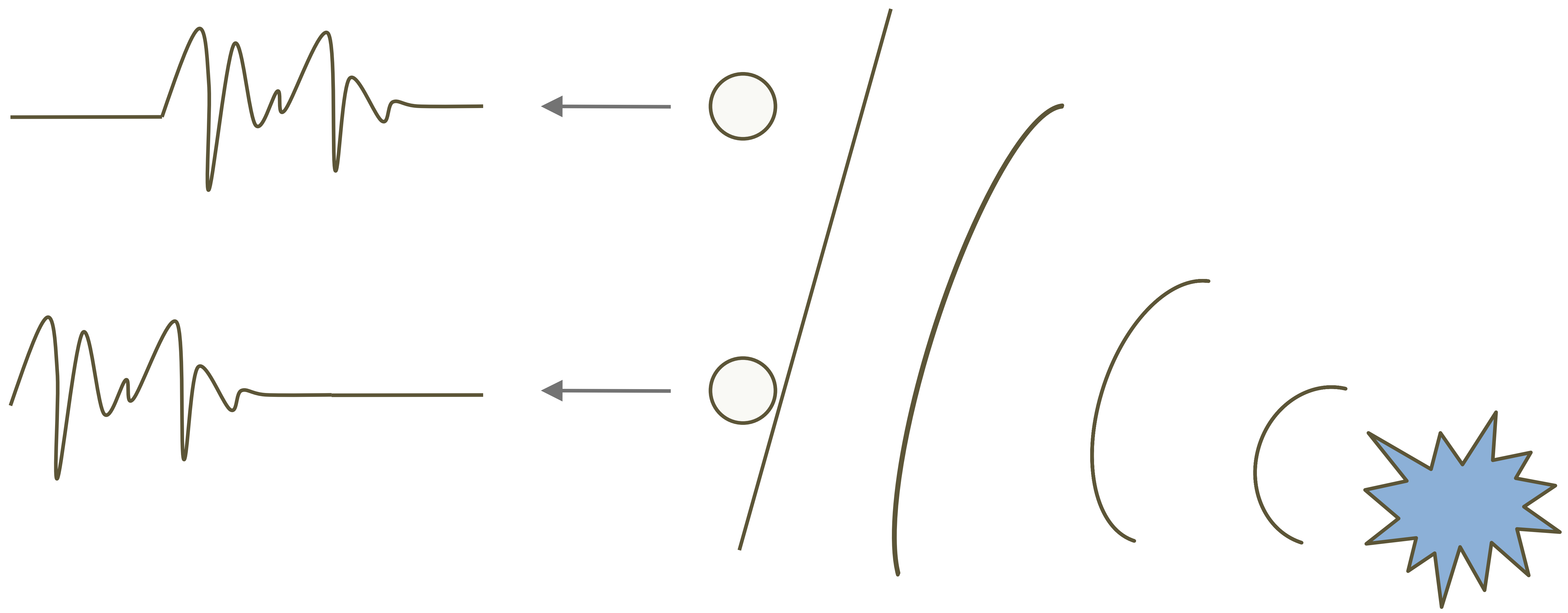
- Sensors might have non-uniform response
  - In direction or frequency
- Adaptive filters instead of delays
  - Suppress certain areas, boost others
- Rich literature in radio, sonar and audio

Extreme beamforming demo



Localization

- Determine delays that align with input
  - If you know the delays you can determine the angle of most likely incidence



Localization of a single source

- Cross-correlate sensor inputs

$$c_{i,j}(t) = x_i(t) * x_j(-t)$$

- Peak denotes relative delay between inputs
  - Convert delays to angle

$$\phi_{i,j} = \arcsin \frac{\tau_{i,j} C}{d_{i,j}}$$

  - But there's a problem

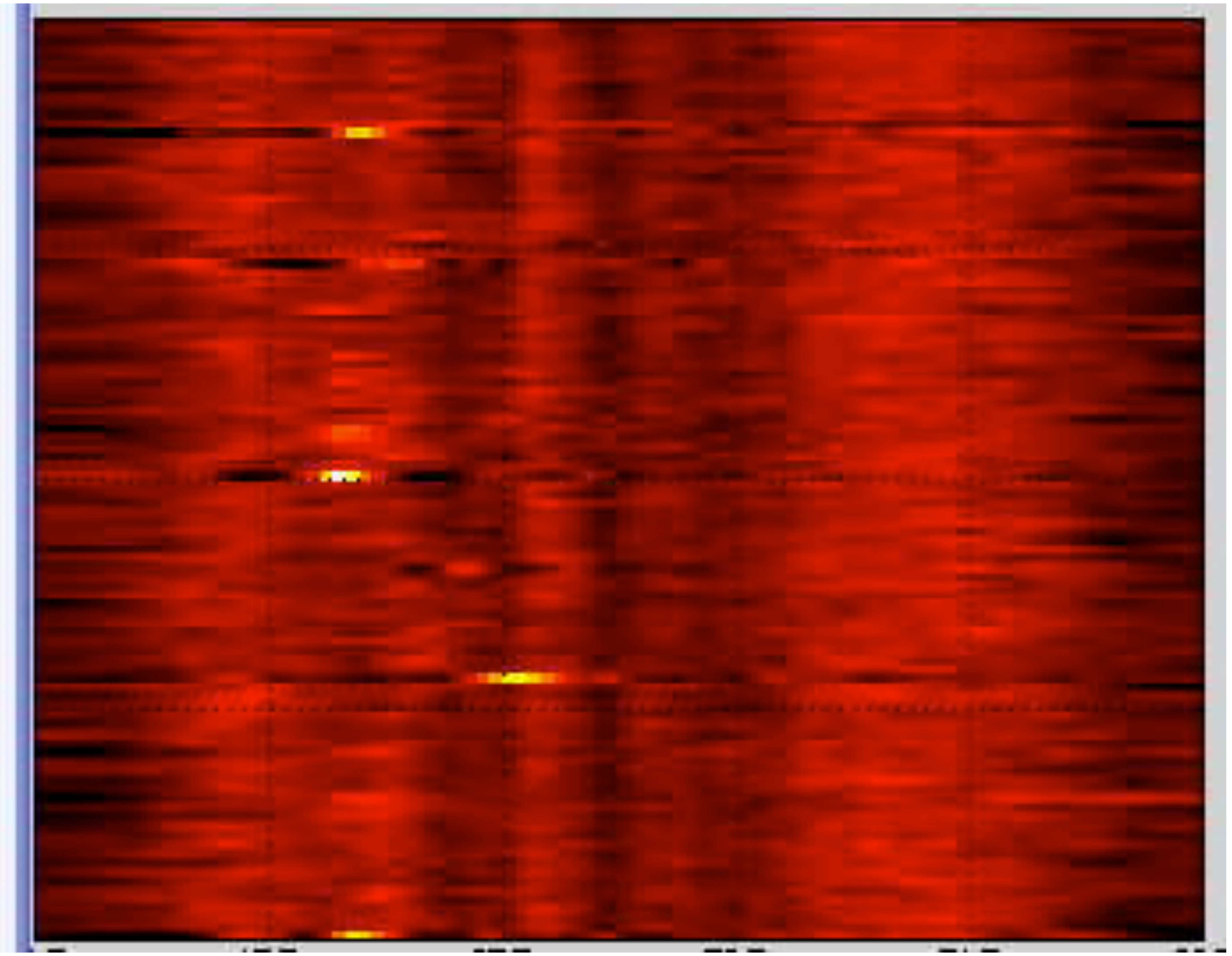
Localization of multiple sources

- If we have multiple sources estimation of one delay is not feasible
- Instead we can measure the energy from each location in the space we operate
  - Steer the array beam and measure signal
- This provides a spatial energy map

Multi-source demo

Situation video

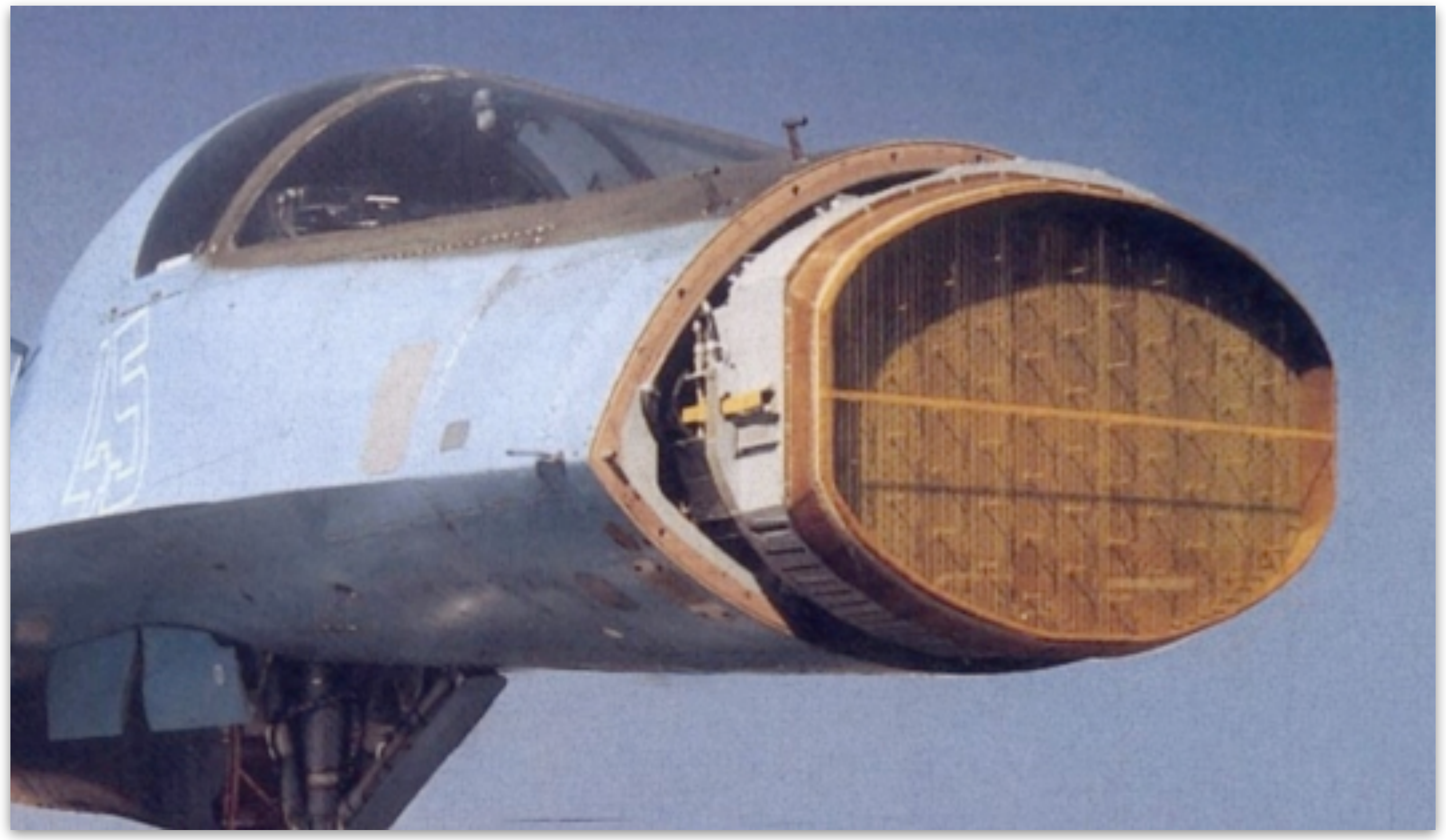
Energy over angle and time

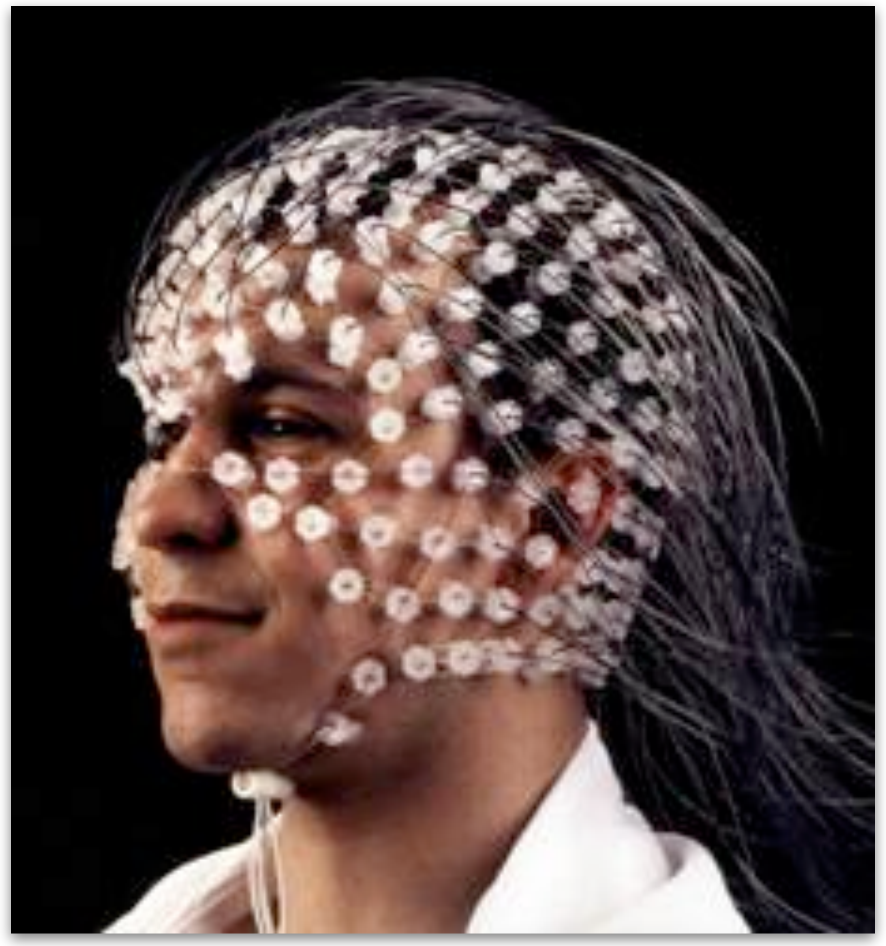


Time

Angle

Arrays are everywhere





One more

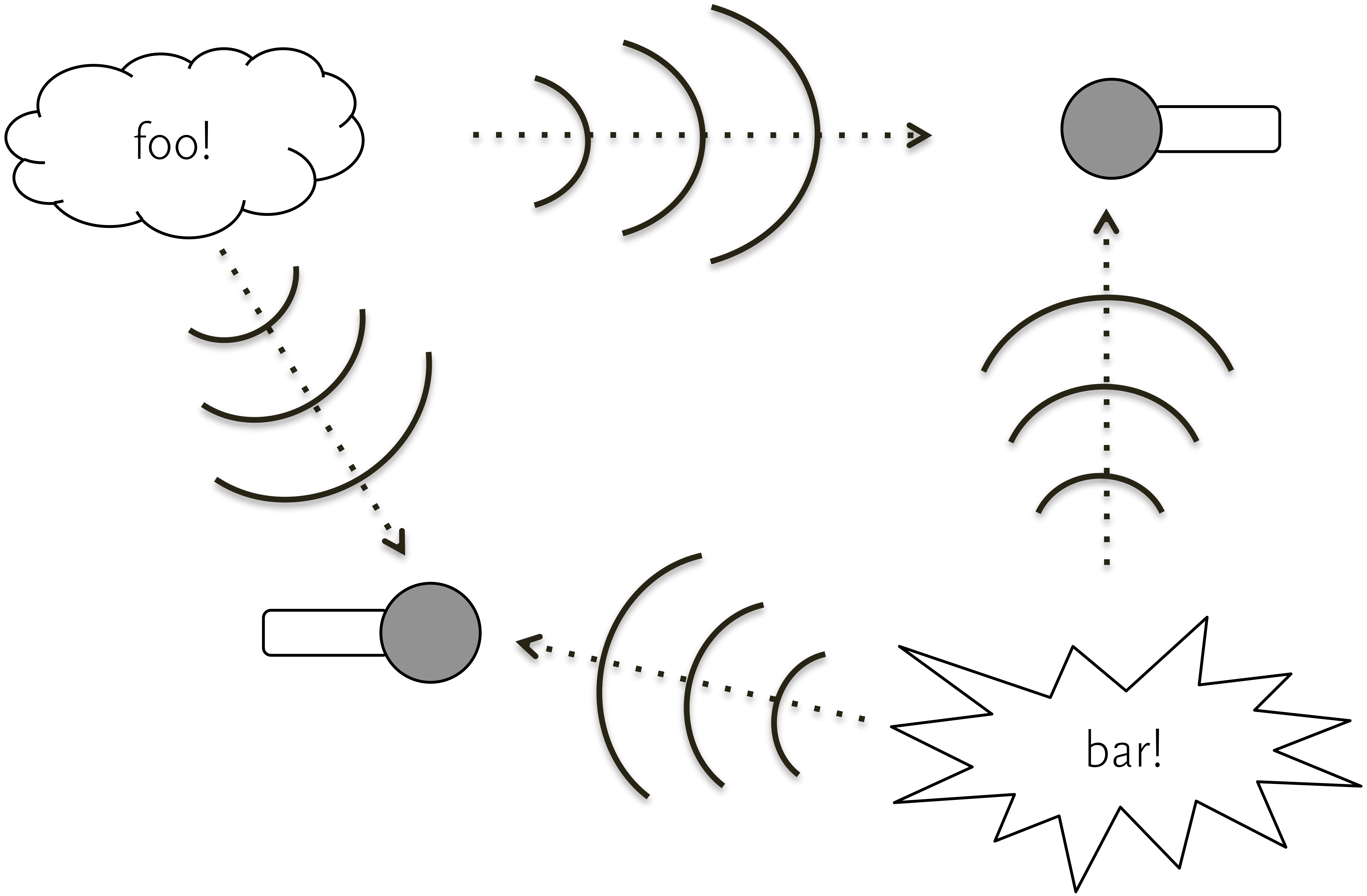
USS San Francisco – after hitting an underwater mountain in 2005

Using more machine learning

- Most array literature is DSP-based
  - Beamforming has its limitations

- Source separation and blind methods
  - Machine learning applications on arrays

A “simple” audio problem

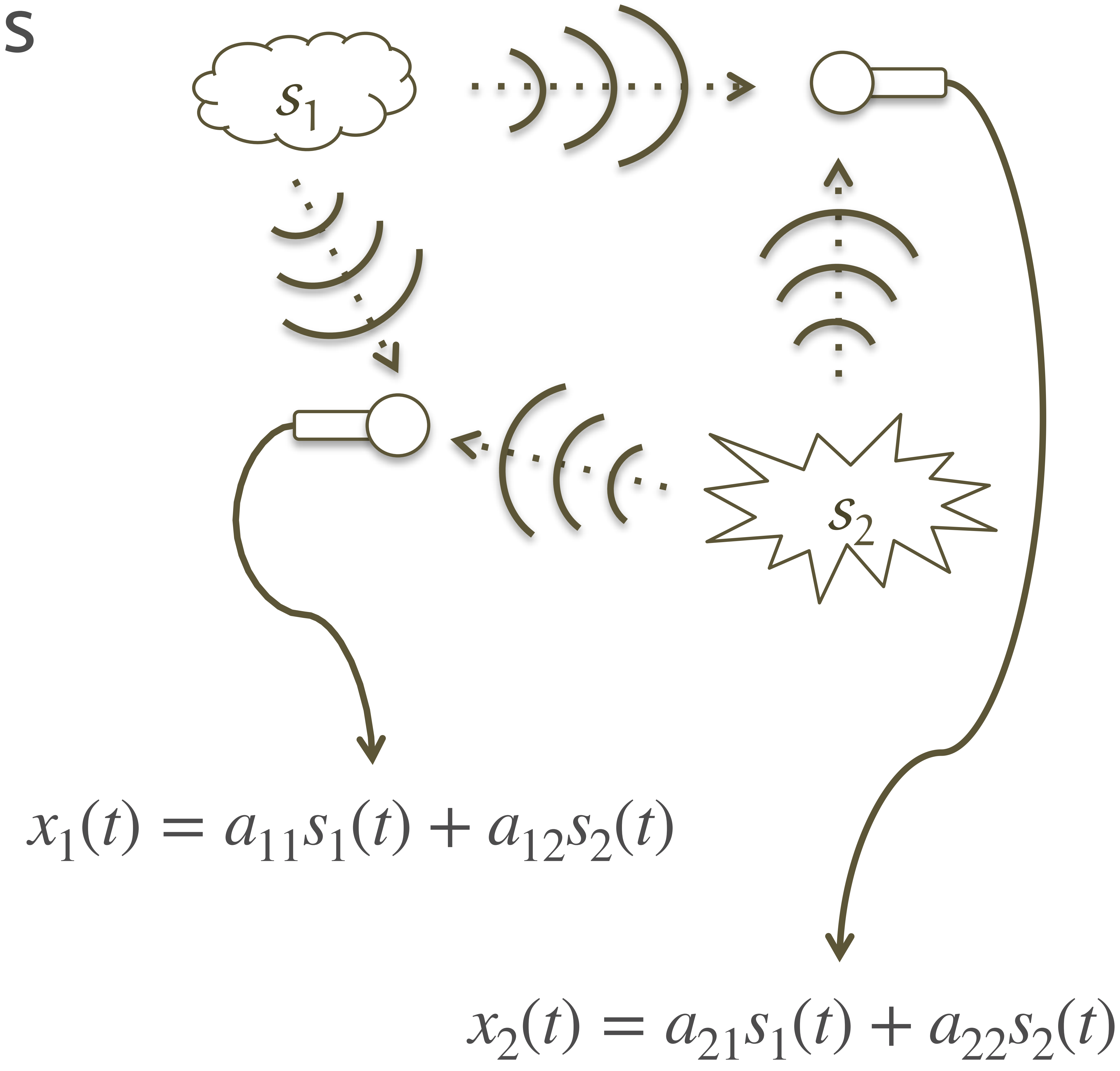


Formalizing the problem

- Each mic receives a mix of both sounds
  - Sound waves superimpose linearly
  - Ignoring propagation delays for now
- The simplified mixing model is:

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

- How do we solve this system and find the original signals $\mathbf{s}(t)$?



When can we solve this?

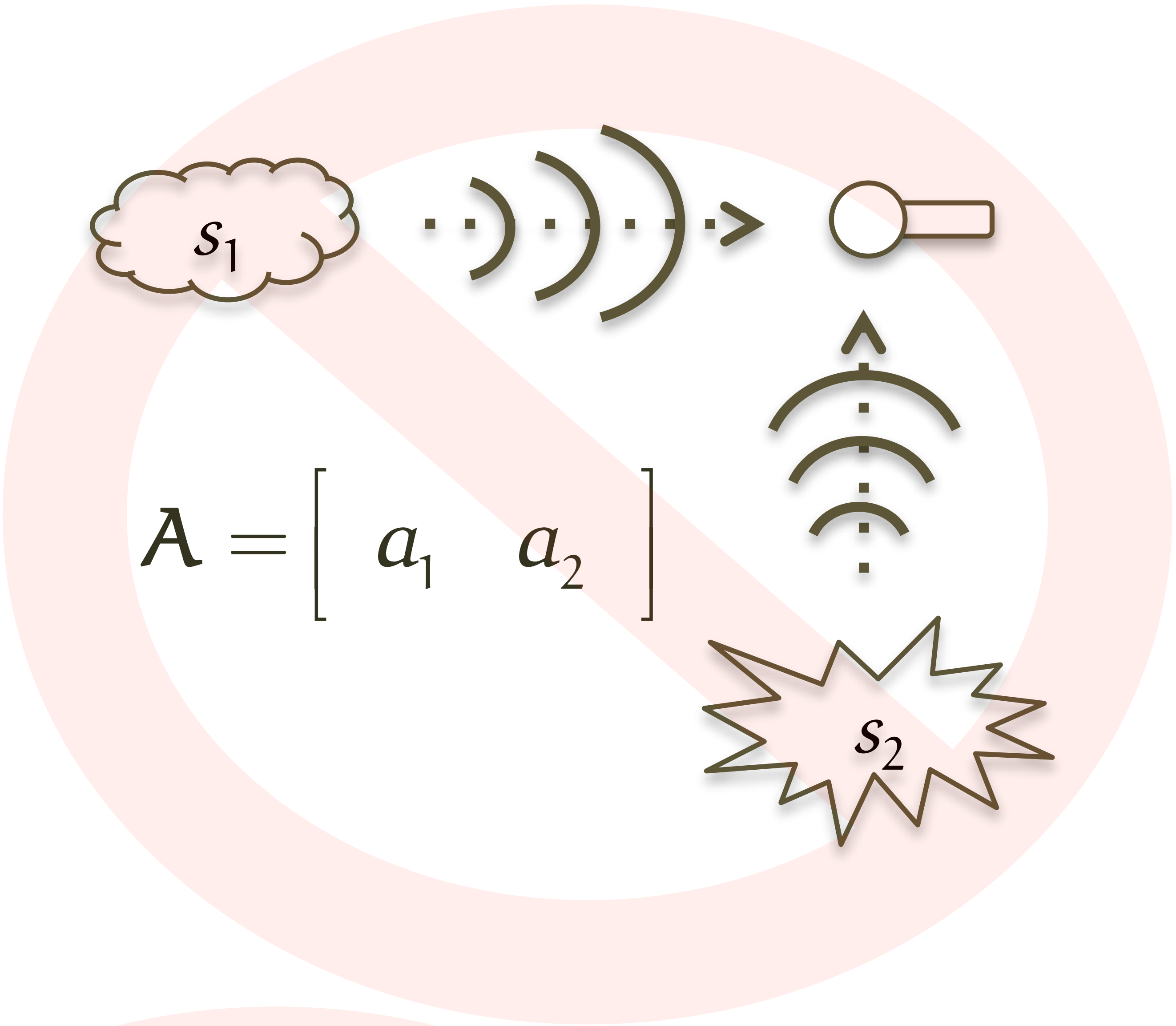
- The mixing equation is:

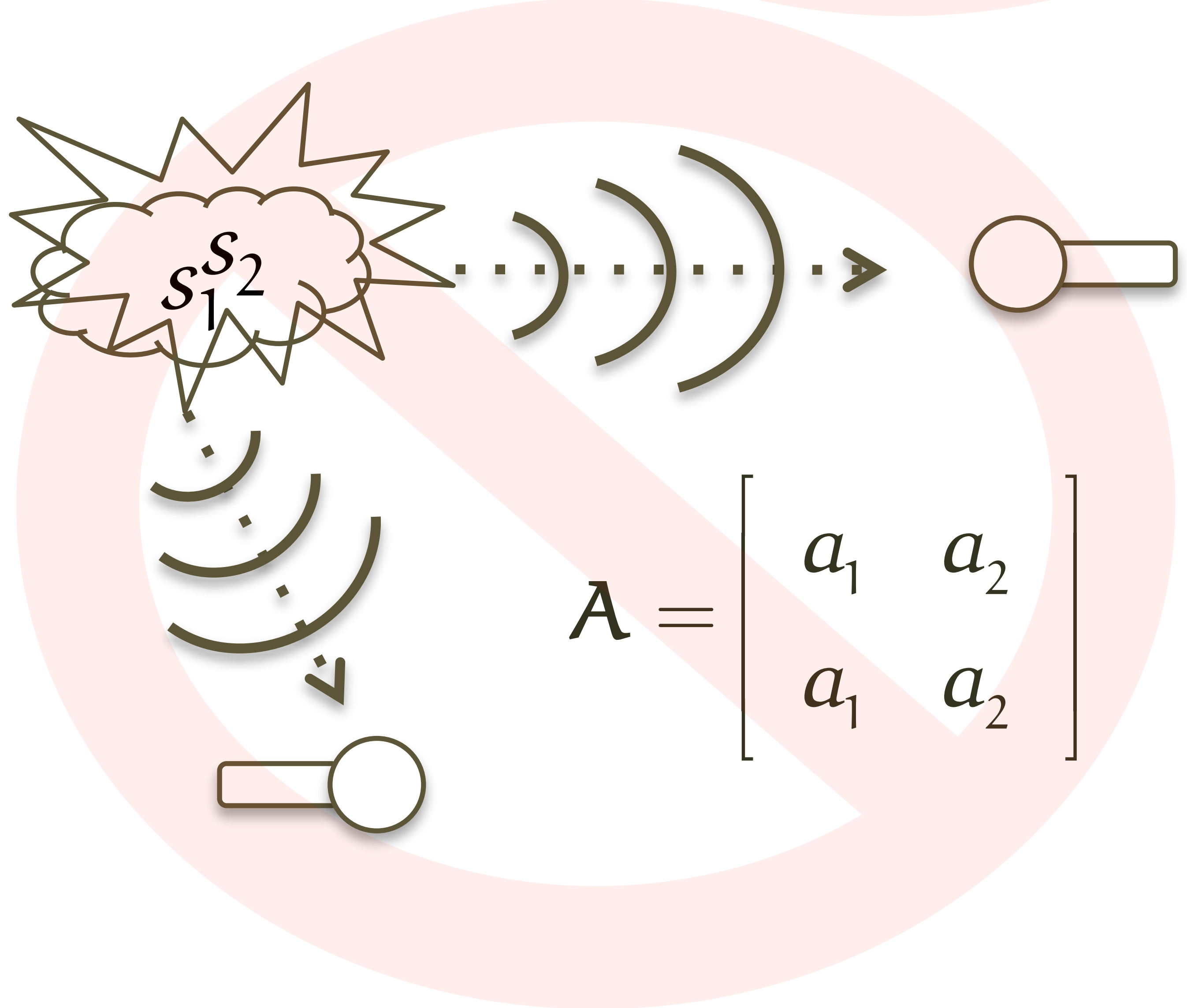
$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

- Our estimates of $\mathbf{s}(t)$ will be:

$$\hat{\mathbf{s}}(t) = \mathbf{A}^{-1} \cdot \mathbf{x}(t)$$

- To recover $\mathbf{s}(t)$, $\mathbf{A}$ must be invertible:
  - We need as many mics as sources
  - The mics/sources must not coincide
  - All sources must be audible
- Otherwise this is a different story …

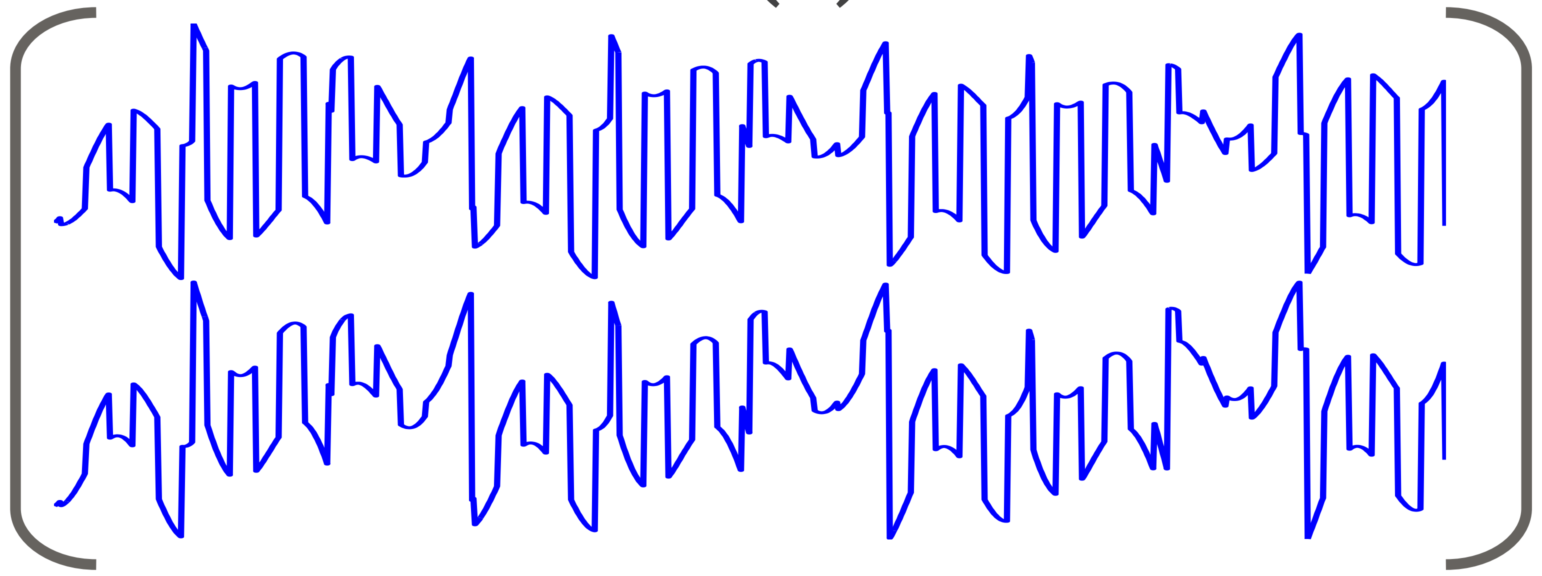




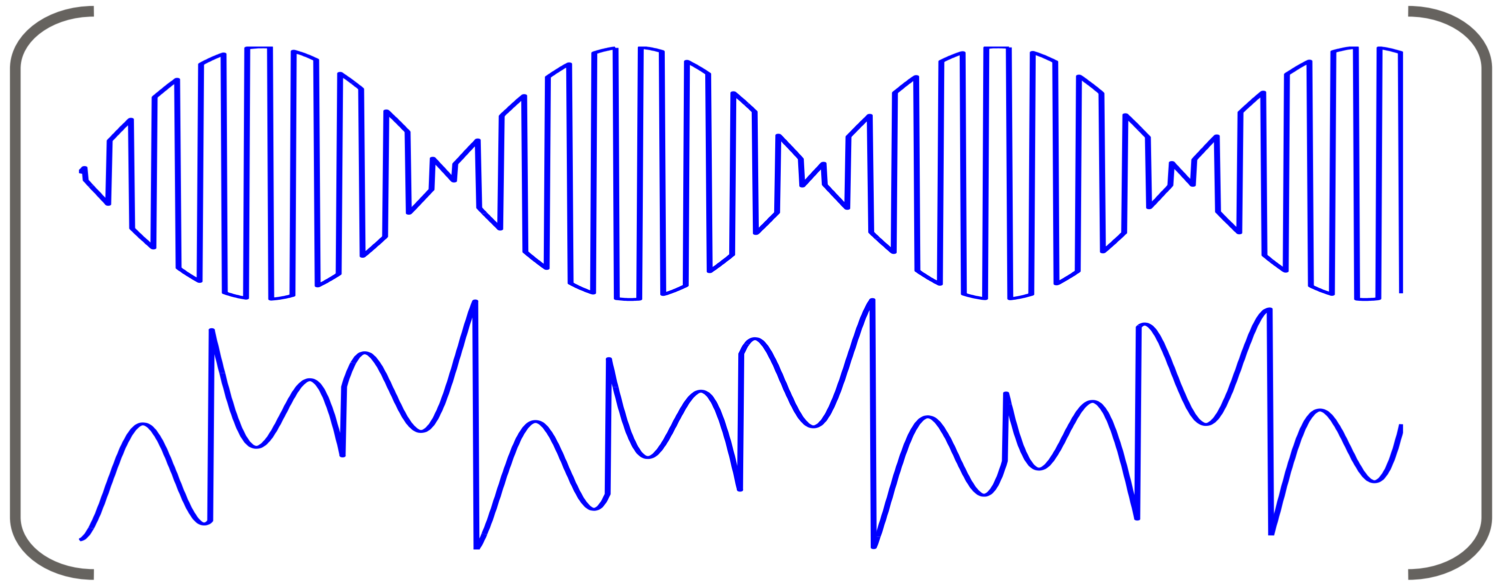
A simple example

- A simple invertible problem

$\mathbf{x}(t)$ $\quad$ $\mathbf{A}$ $\quad$ $\mathbf{s}(t)$



$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot$$



  - $\mathbf{s}(t)$ contains two structured waveforms
  - $\mathbf{A}$ is invertible (but we don't know it's values)
  - $\mathbf{x}(t)$ looks messy, doesn't reveal $\mathbf{s}(t)$ clearly
- Known as the Blind Source Separation problem (BSS)
  - *Blind* because we don't know anything about the original sources

What to look for

- We can only use $\mathbf{x}(t)$

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

- Is there a property we can take advantage of?
  - Yes! We know that different sounds are "different"

- The plan: Find a solution that enforces "differentness"

A first try

- Find $\mathbf{s}(t)$ by minimizing correlations
- Our estimate of $\mathbf{s}(t)$ is computed by: $\hat{\mathbf{s}}(t) = \mathbf{W} \cdot \mathbf{x}(t)$
  - If $\mathbf{W} \approx \mathbf{A}^{-1}$ then we have a good solution
- The goal is that the outputs become uncorrelated:

$$\left\langle \hat{s}_i(t) \cdot \hat{s}_j(t) \right\rangle = 0, \forall i \neq j$$

  - We assume here that our signals are zero mean
- So the problem to solve is: $\arg\min_{\mathbf{W}} \left\langle \sum_k w_{ik} x_k(t) \cdot \sum_k w_{jk} x_k(t) \right\rangle, \forall i \neq j$

Seen that before?

- This is PCA!

$$\begin{bmatrix} \hat{s}_1(t) & & \hat{s}_1(T) \\ \hat{s}_2(t) & \cdots & \hat{s}_3(T) \\ \hat{s}_3(t) & & \hat{s}_3(T) \end{bmatrix} = \mathbf{W} \cdot \begin{bmatrix} x_1(t) & & x_1(T) \\ x_2(t) & \cdots & x_2(T) \\ x_3(t) & & x_3(T) \end{bmatrix} \Rightarrow$$

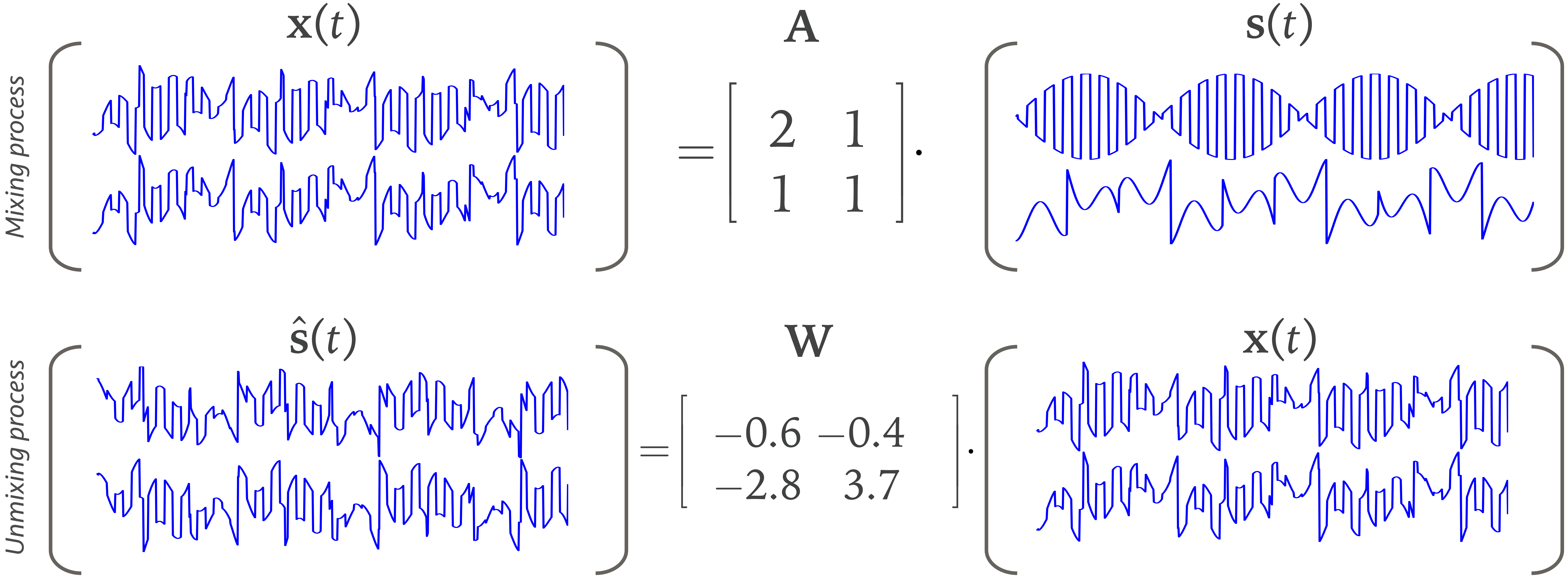
$$\begin{bmatrix} \hat{\mathbf{s}}(1), & \cdots, & \hat{\mathbf{s}}(T) \end{bmatrix} = \mathbf{W} \cdot \begin{bmatrix} \mathbf{x}(1), & \cdots, & \mathbf{x}(T) \end{bmatrix} \Rightarrow$$

$$\Rightarrow \hat{\mathbf{S}} = \mathbf{W} \cdot \mathbf{X}$$

- Easy to compute with a simple decomposition:

$$\mathbf{W} = \mathrm{eig}\left(\mathbf{X} \cdot \mathbf{X}^\top\right)$$

So how well does this work?



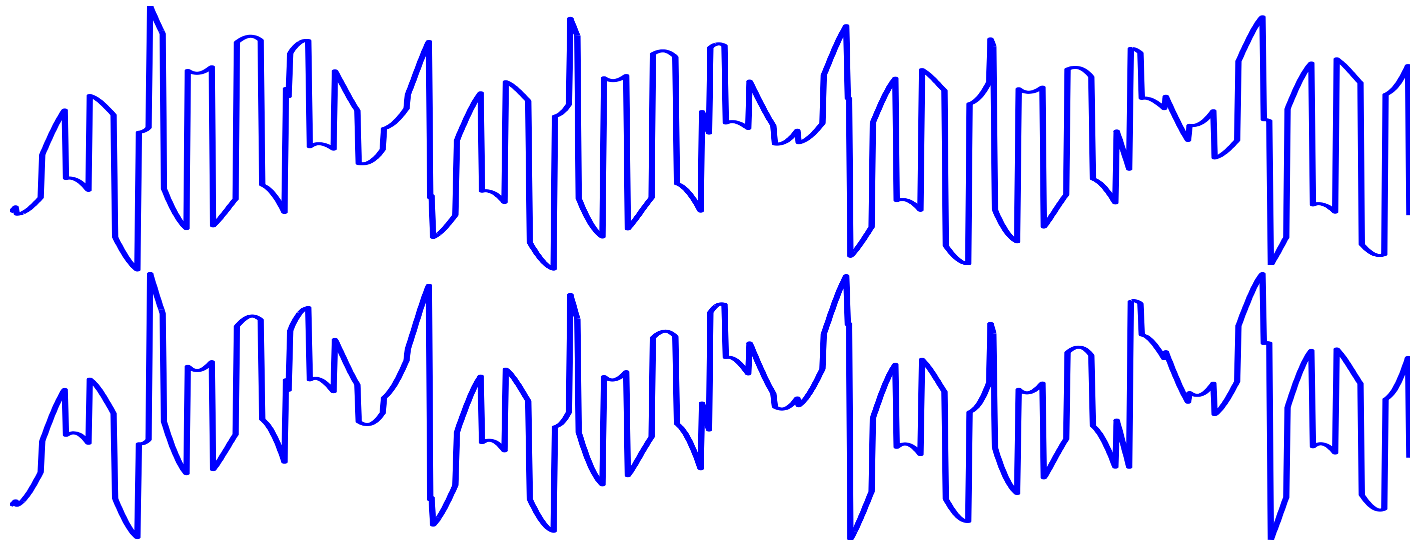
Uncorrelated outputs

Well, that was a waste of time ...

What went wrong?

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

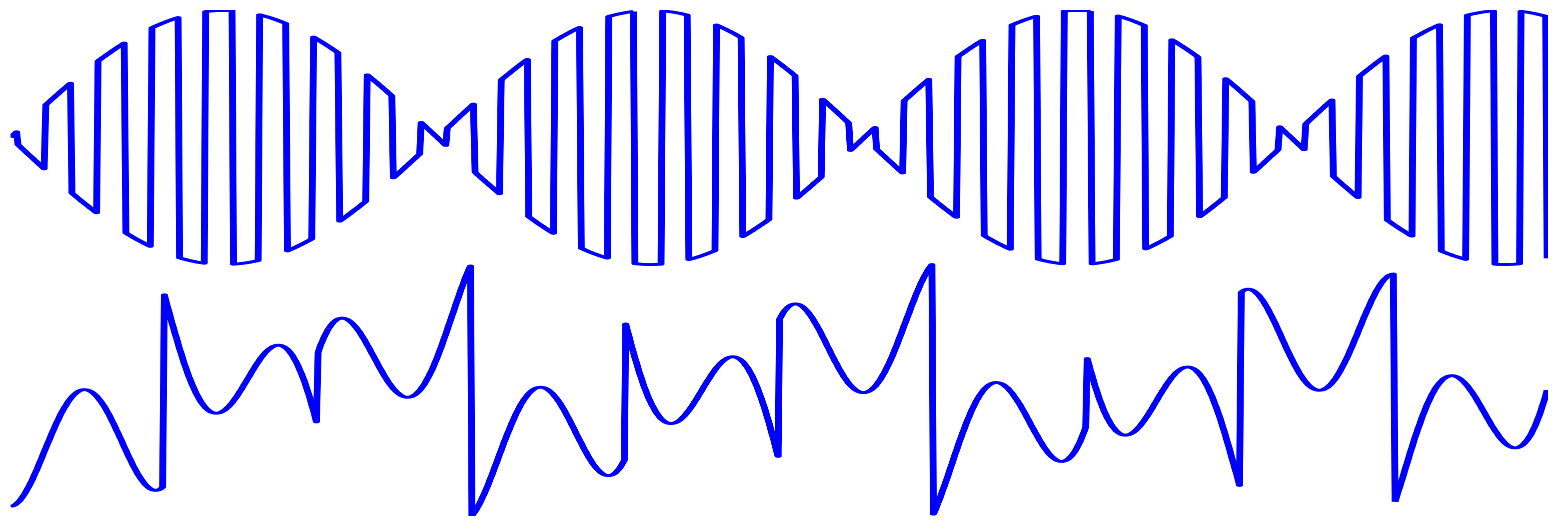
$\mathbf{x}(t)$

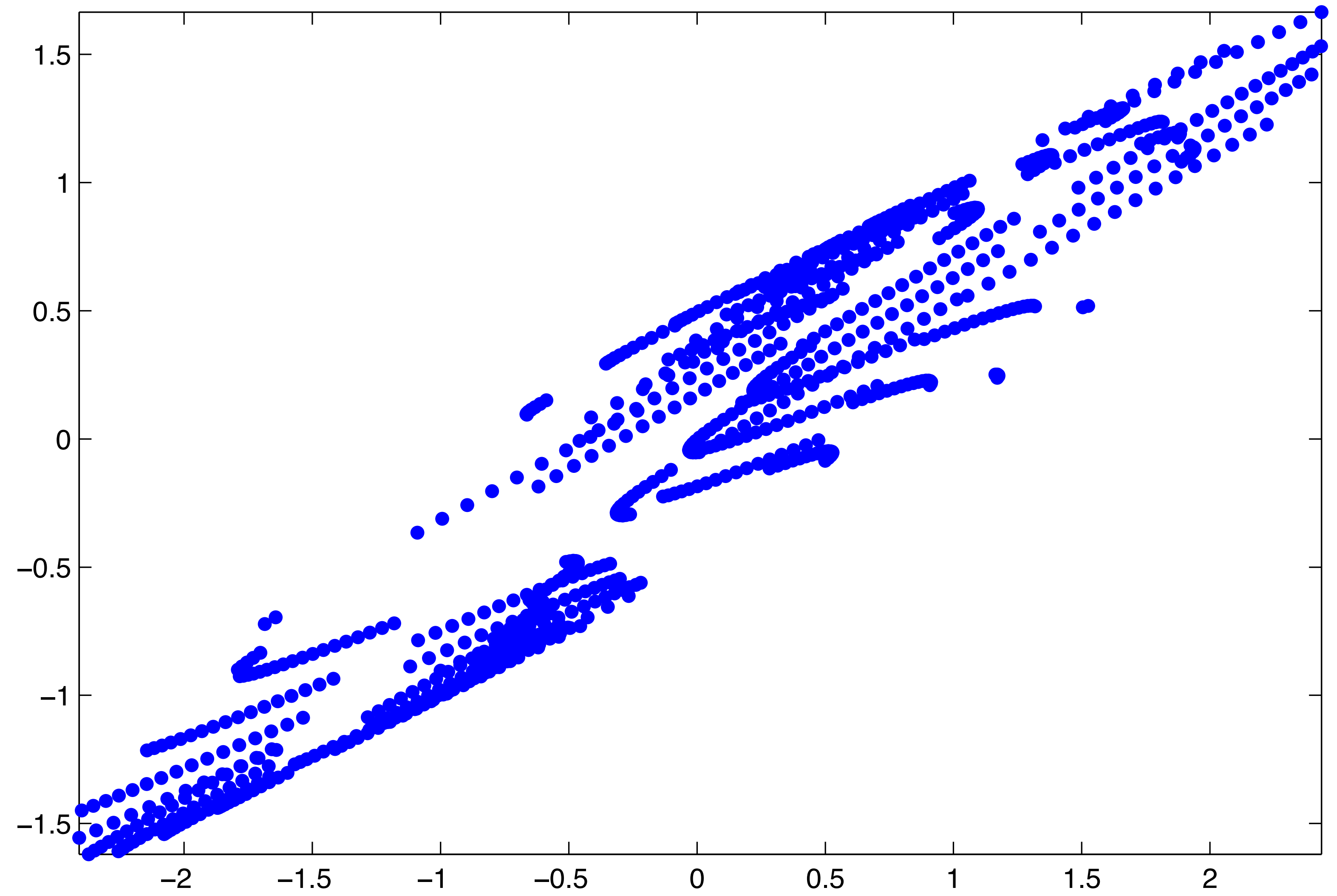


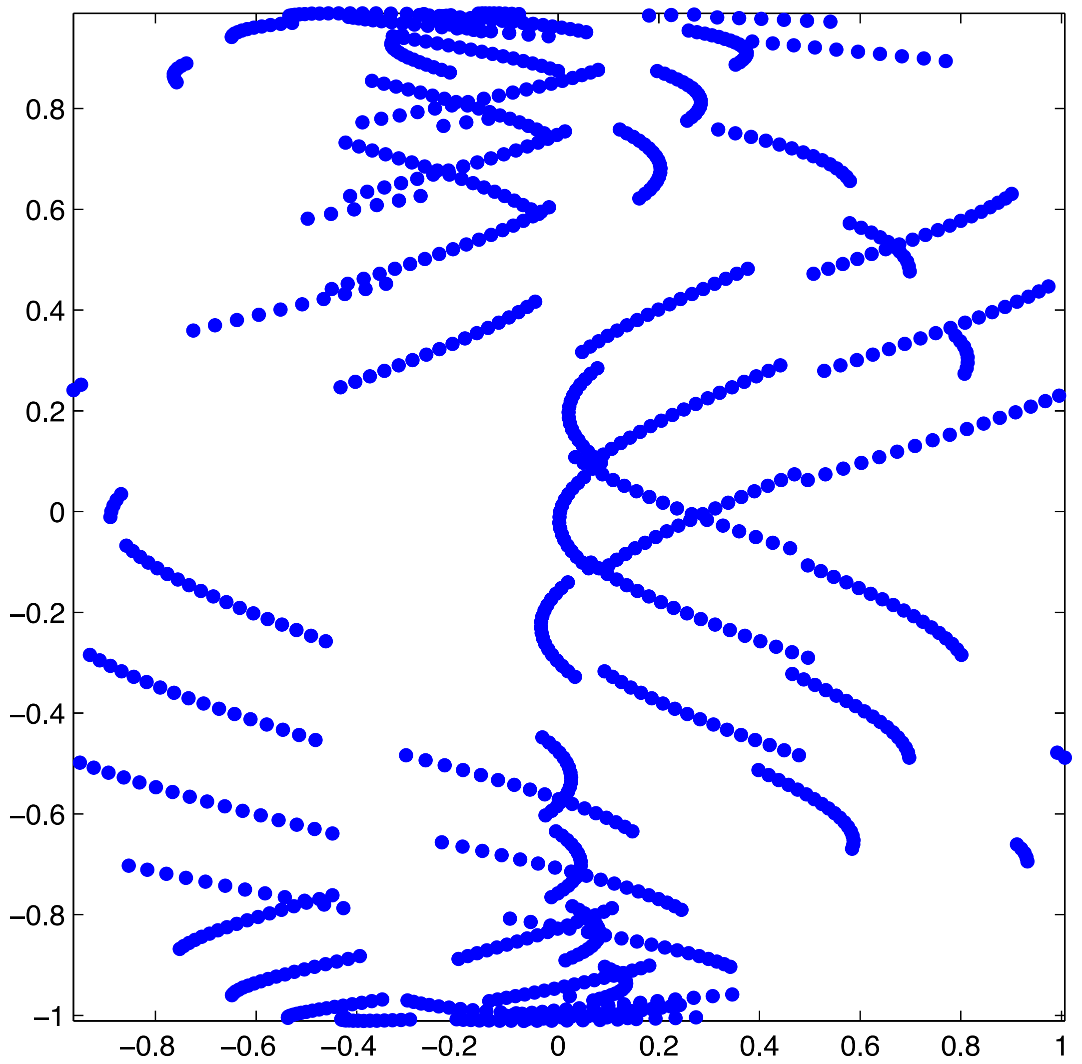
$\mathbf{A}$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot$$

$\mathbf{s}(t)$

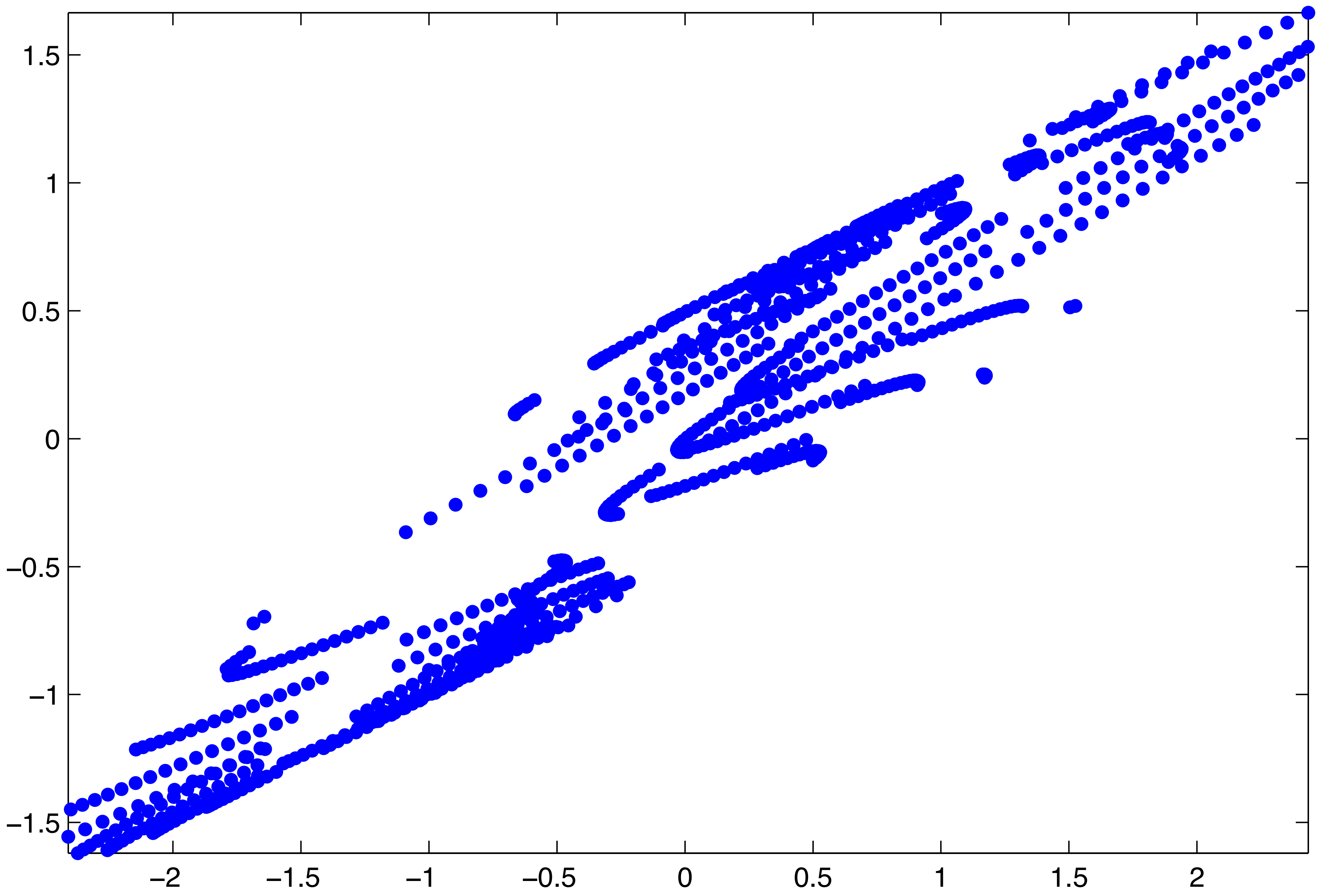




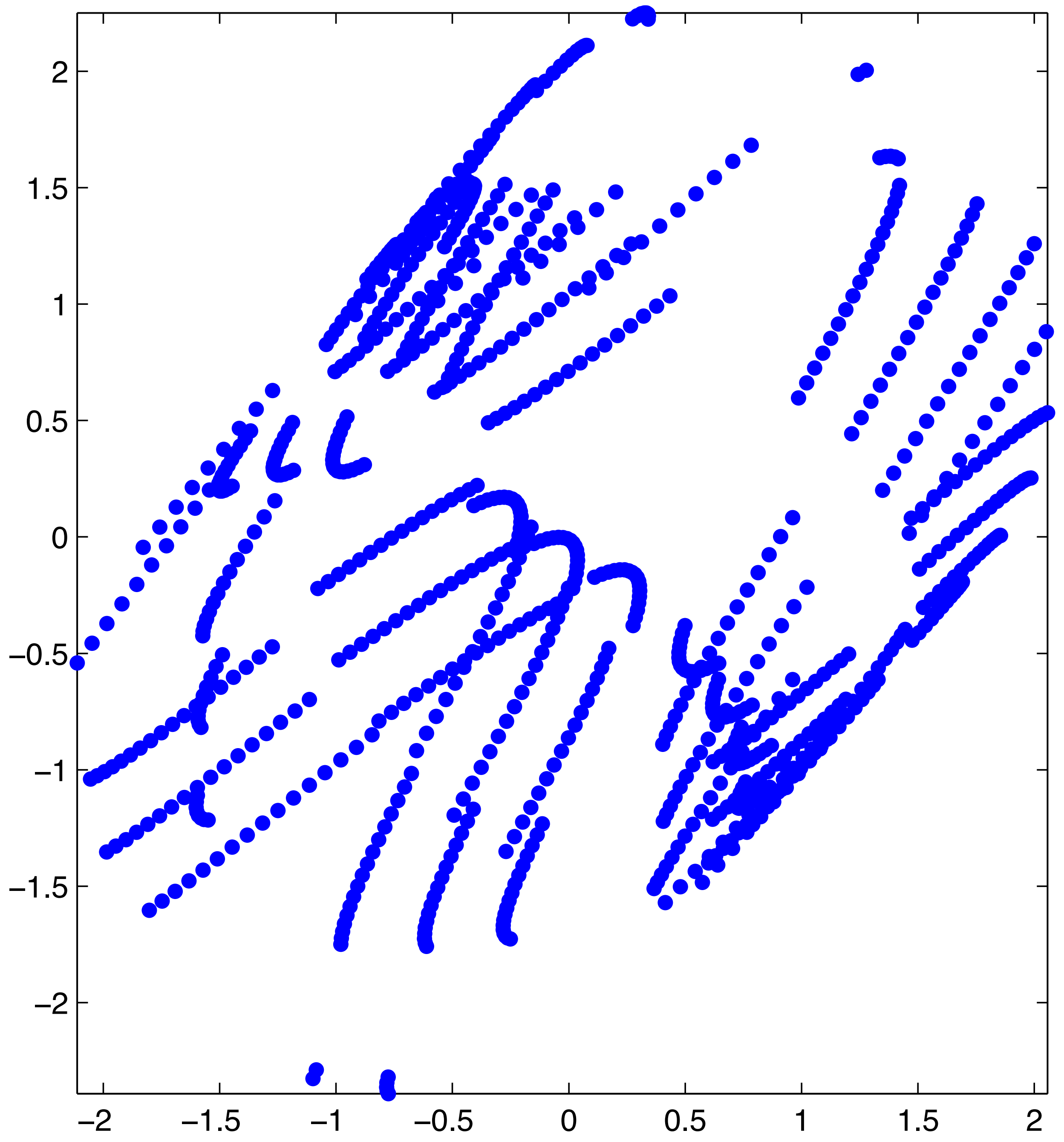


PCA does the wrong thing

What are the mixture eigenvectors?



Scaling/rotating to get decorrelation



What's wrong?

- Our data is not Gaussian
  - Been there before, right?

- We shouldn't use decorrelation, we need "statistical independence"
  - We should use <u>independent</u> component analysis!

ICA for array mixtures

- Given the input:

$$\hat{\mathbf{s}}(t) = \mathbf{W} \cdot \mathbf{x}(t)$$

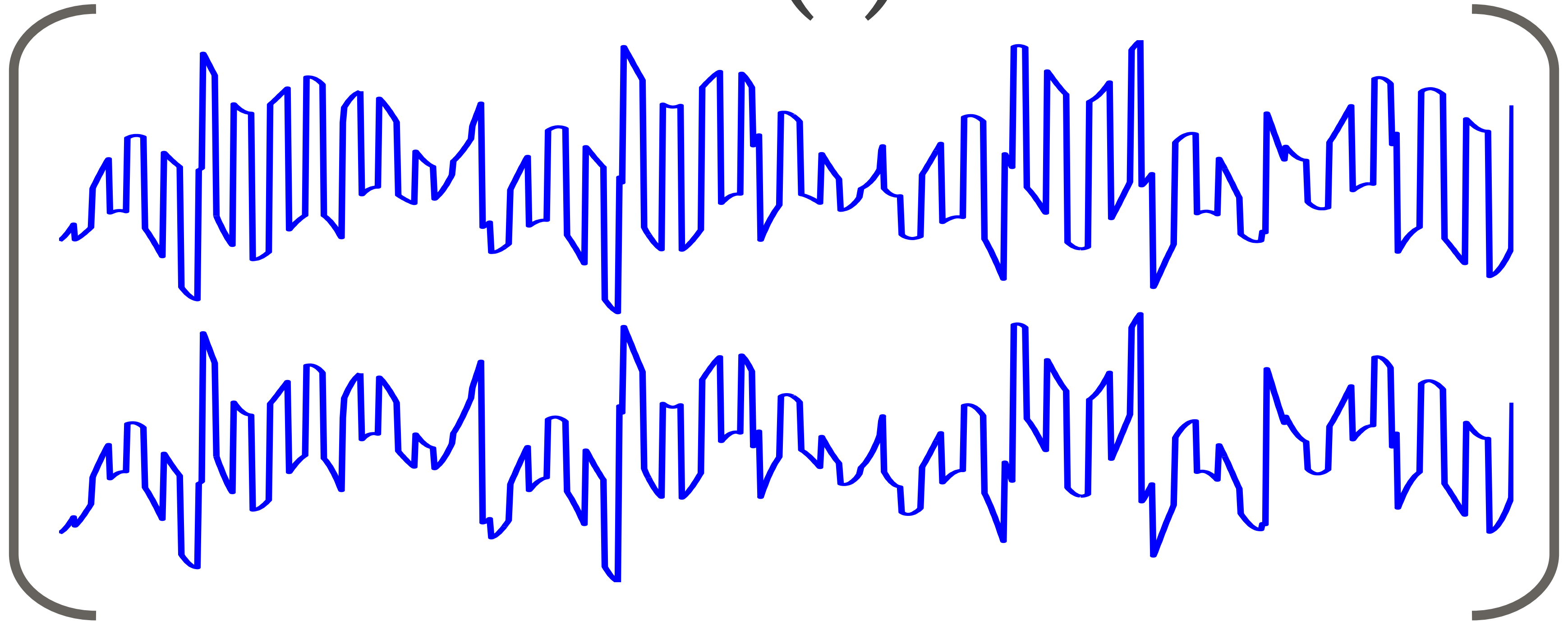
- Find $\mathbf{W}$ so that the transformed data has maximally statistically independent outputs

- So far in ICA we looked at the features ($\mathbf{W}$), now we care about the transformed data itself ($\mathbf{s}$)

Trying this on our dataset

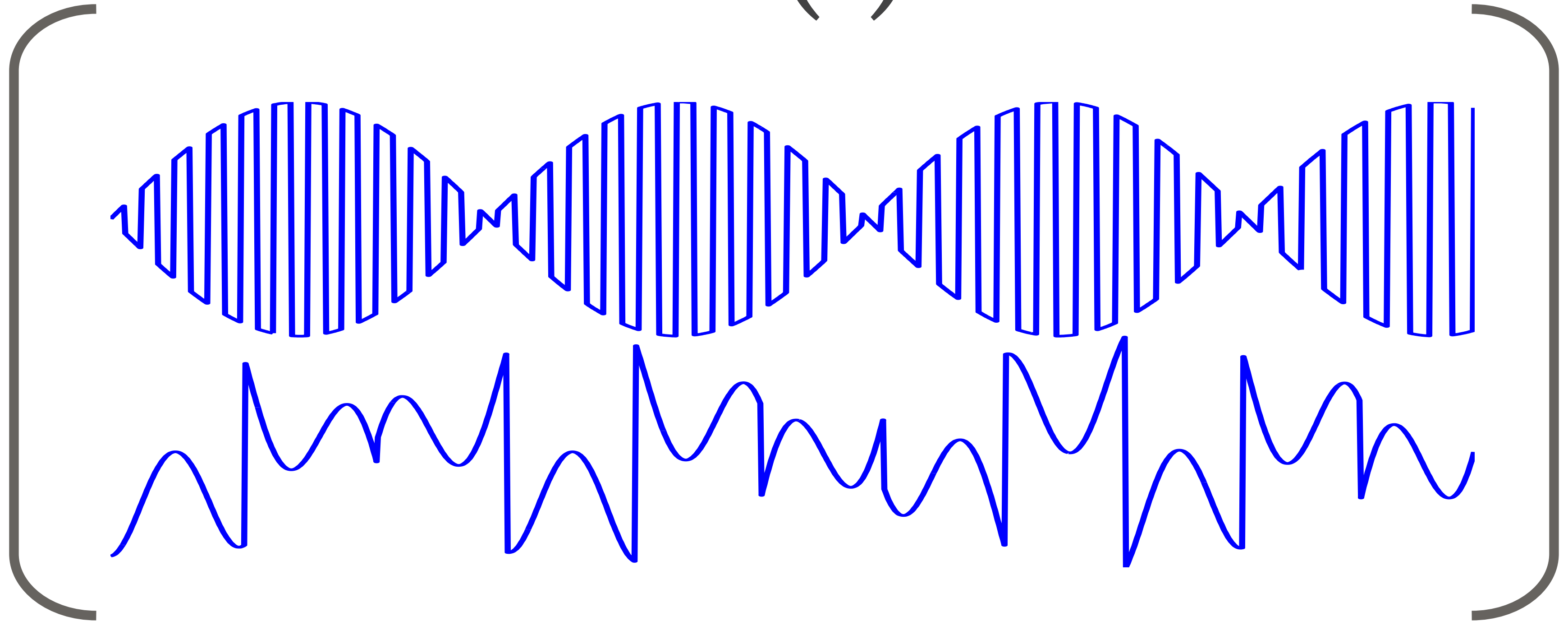
- We can now separated the mixture!

$\mathbf{x}(t)$

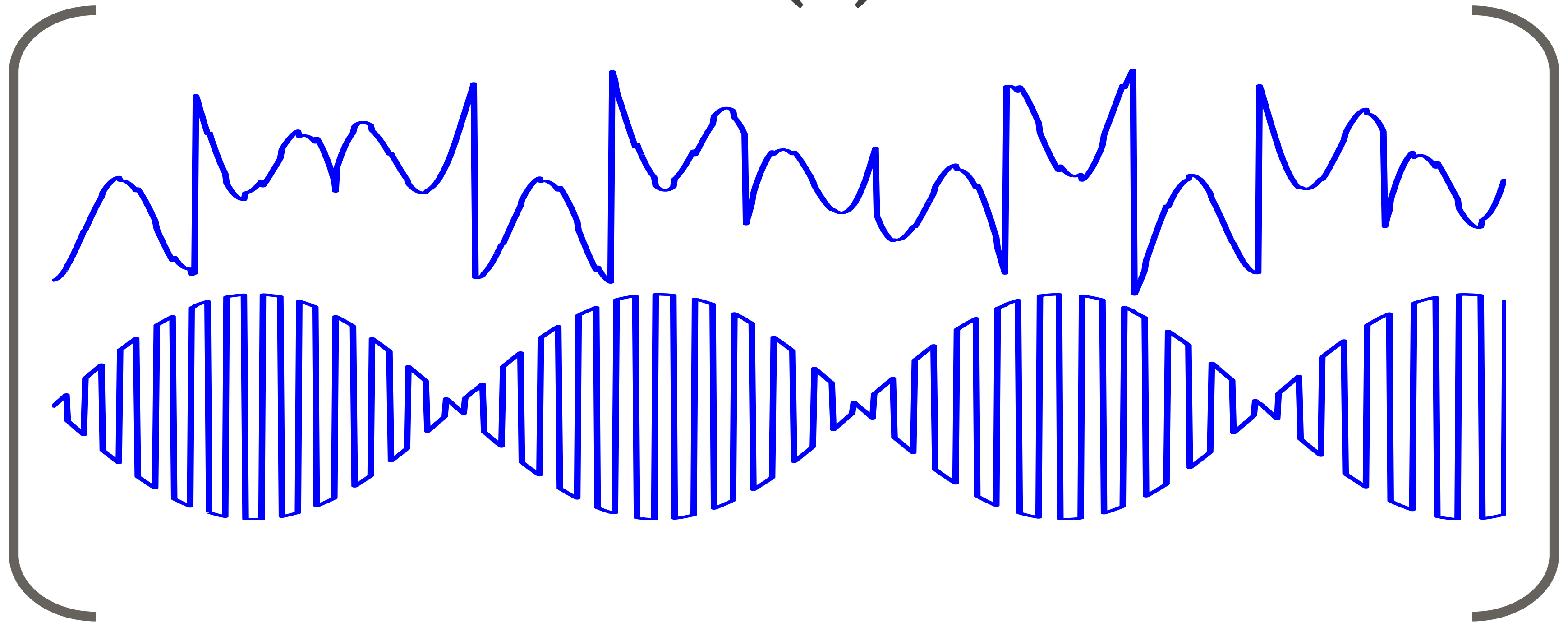


$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot$$

$\mathbf{s}(t)$

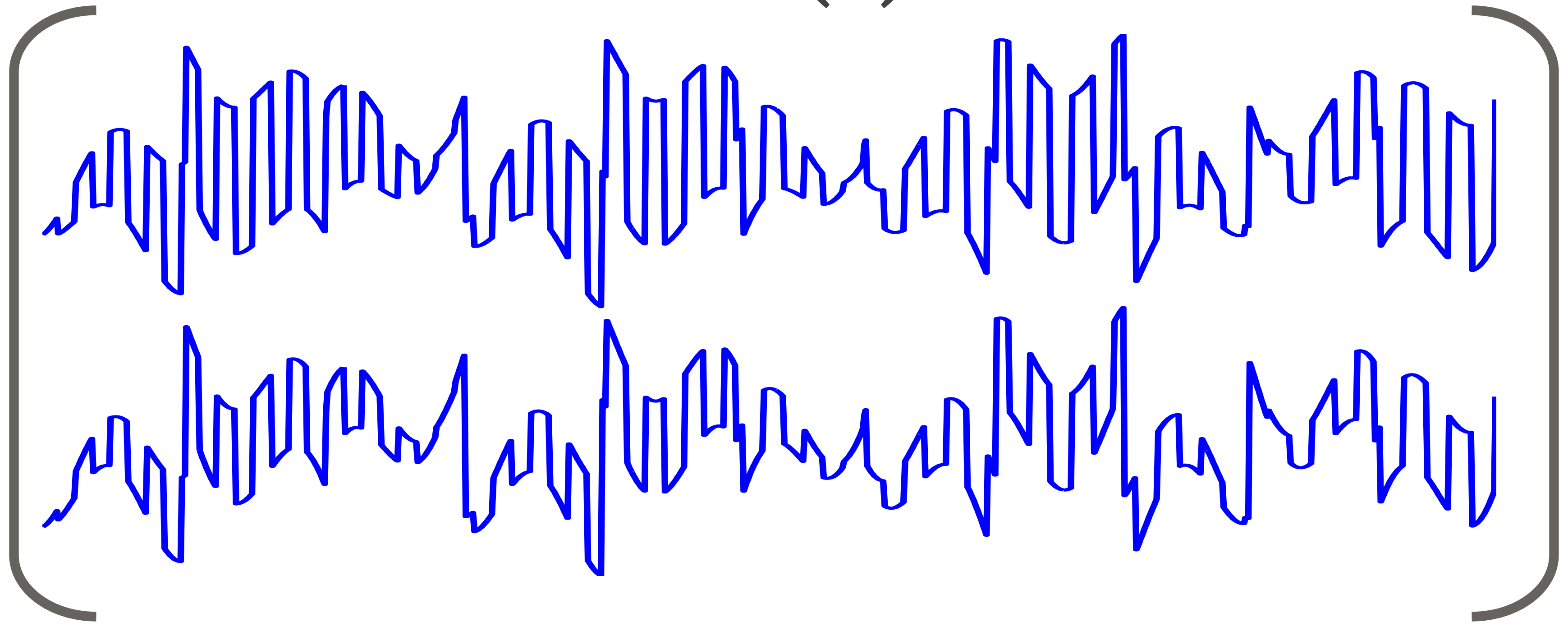


$\hat{\mathbf{s}}(t)$



$$= \begin{bmatrix} -1.3 & 2.7 \\ 2.5 & -2.5 \end{bmatrix} \cdot$$

$\mathbf{x}(t)$



The matrices are labelled $\mathbf{A}$ and $\mathbf{W}$ respectively.

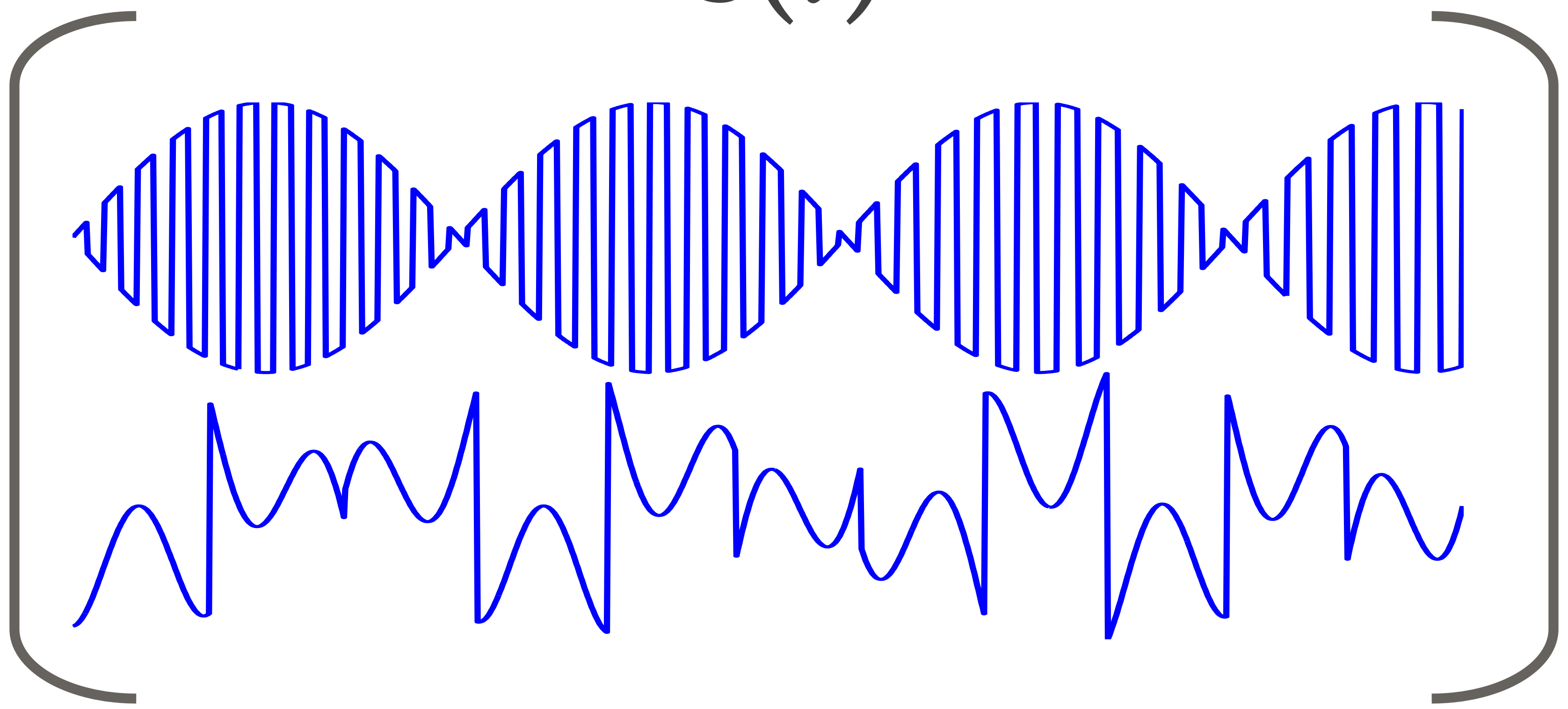
A couple of issues (not bad ones—yet)

- Permutation invariance
  - Order of outputs is random
- Scaling invariance
  - Scale is also random
- ICA will actually recover:

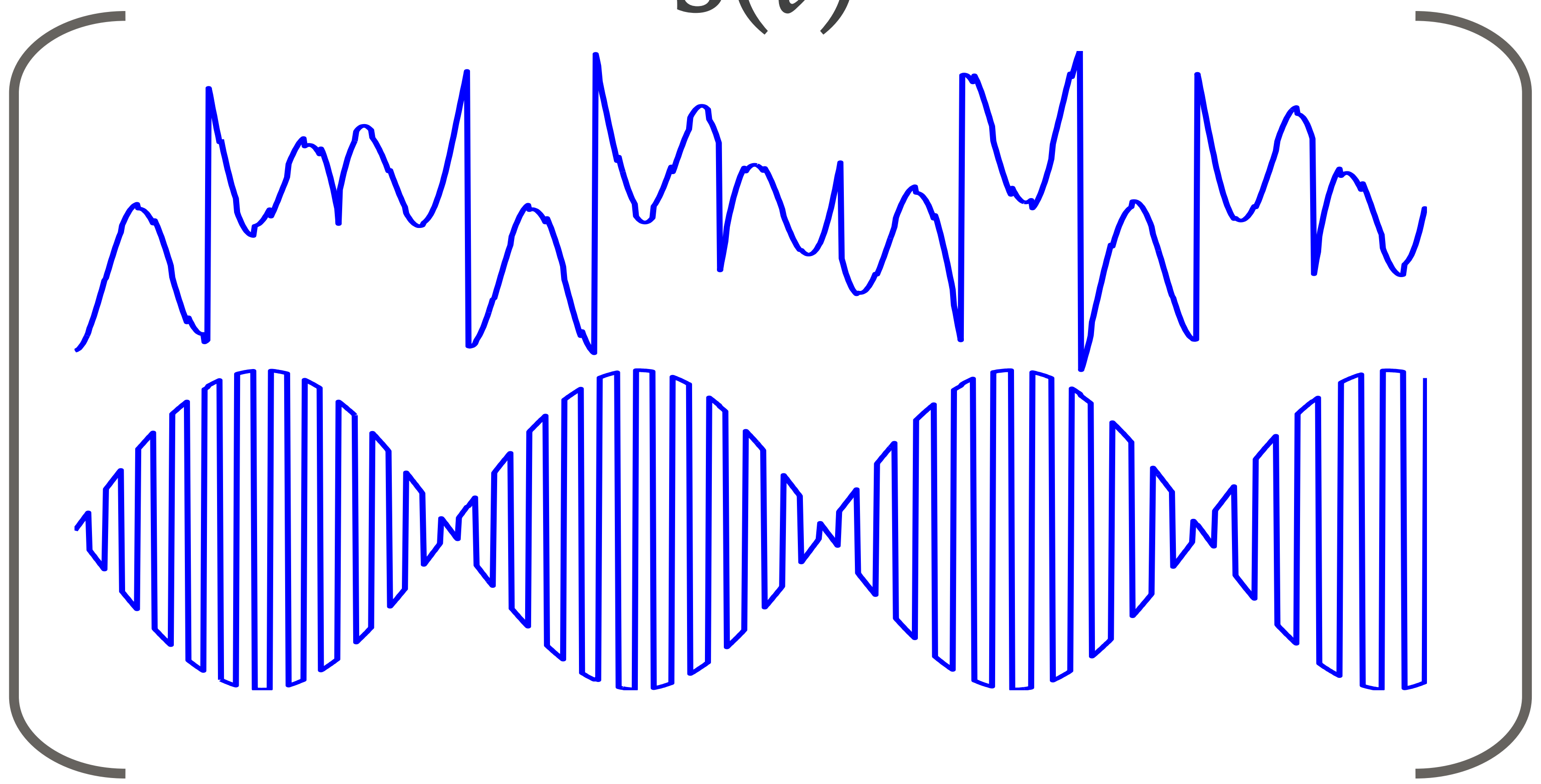
$$\hat{\mathbf{s}}(t) = \mathbf{D} \cdot \mathbf{P} \cdot \mathbf{s}(t)$$

  - Where $\mathbf{D}$ is diagonal and $\mathbf{P}$ is a permutation matrix

$\mathbf{s}(t)$



$\hat{\mathbf{s}}(t)$

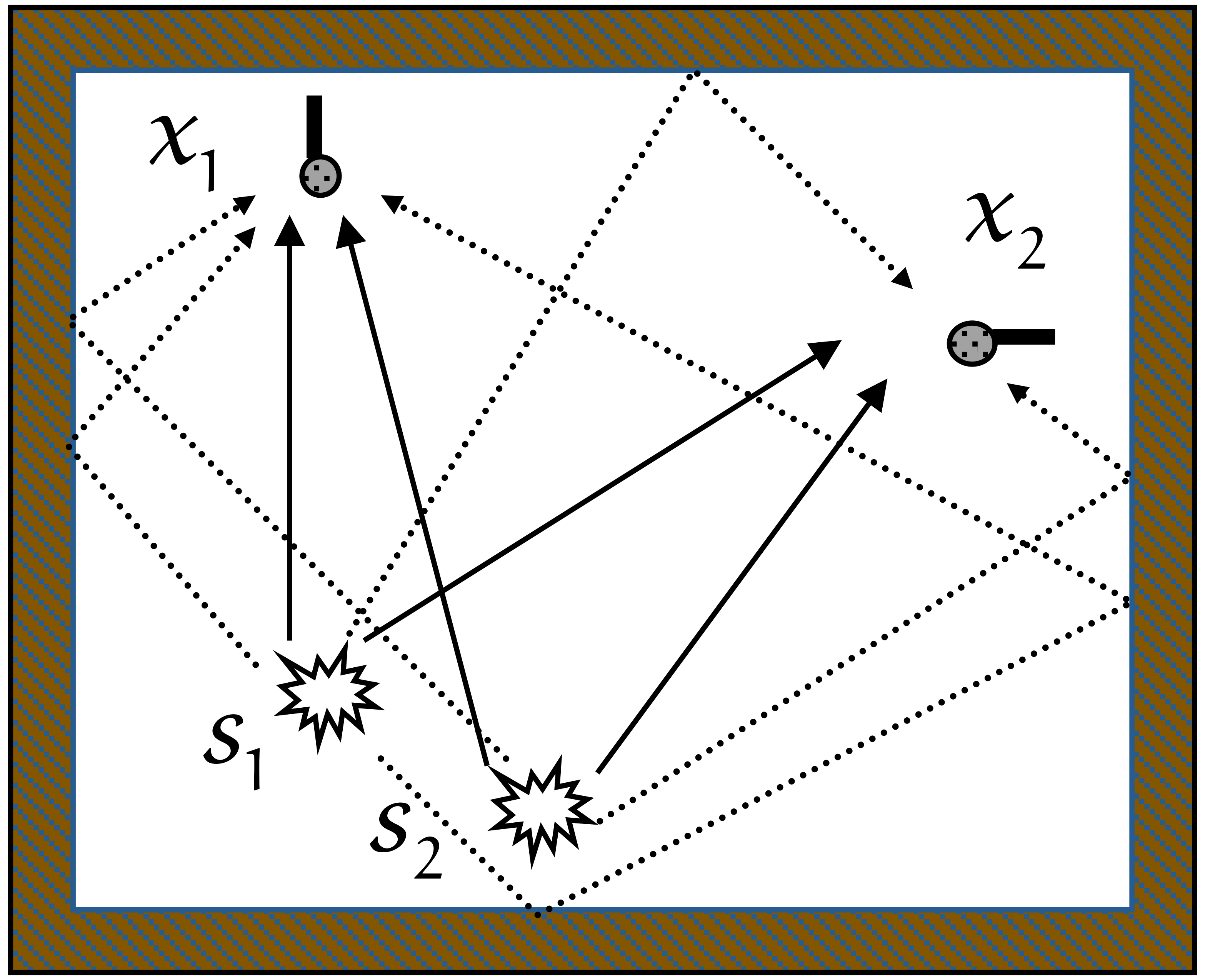


Widely applicable algorithm

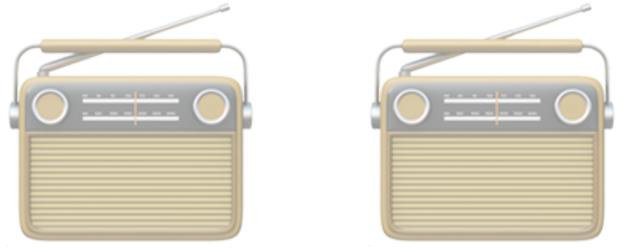
- Also used heavily in bio signal analysis
  - Separating mother/fetal heartbeat
  - Finding “sources” in EEG recordings
- Popular in communications
  - Decouple mixed transmissions when using array antennas
  - Separate hyperspectral images from satellites
- Discover chemical makeups
  - E.g. find petroleum elements in crude mixtures

Instantaneous mixing limitations

- Sounds don't mix instantaneously
- There are multiple effects
  - Room reflections
  - Sensor response
  - Propagation delays
  - Propagation and reflection filtering
- Most can be seen as filters
- We must model a convolutive mixture



Unsuccessful separation

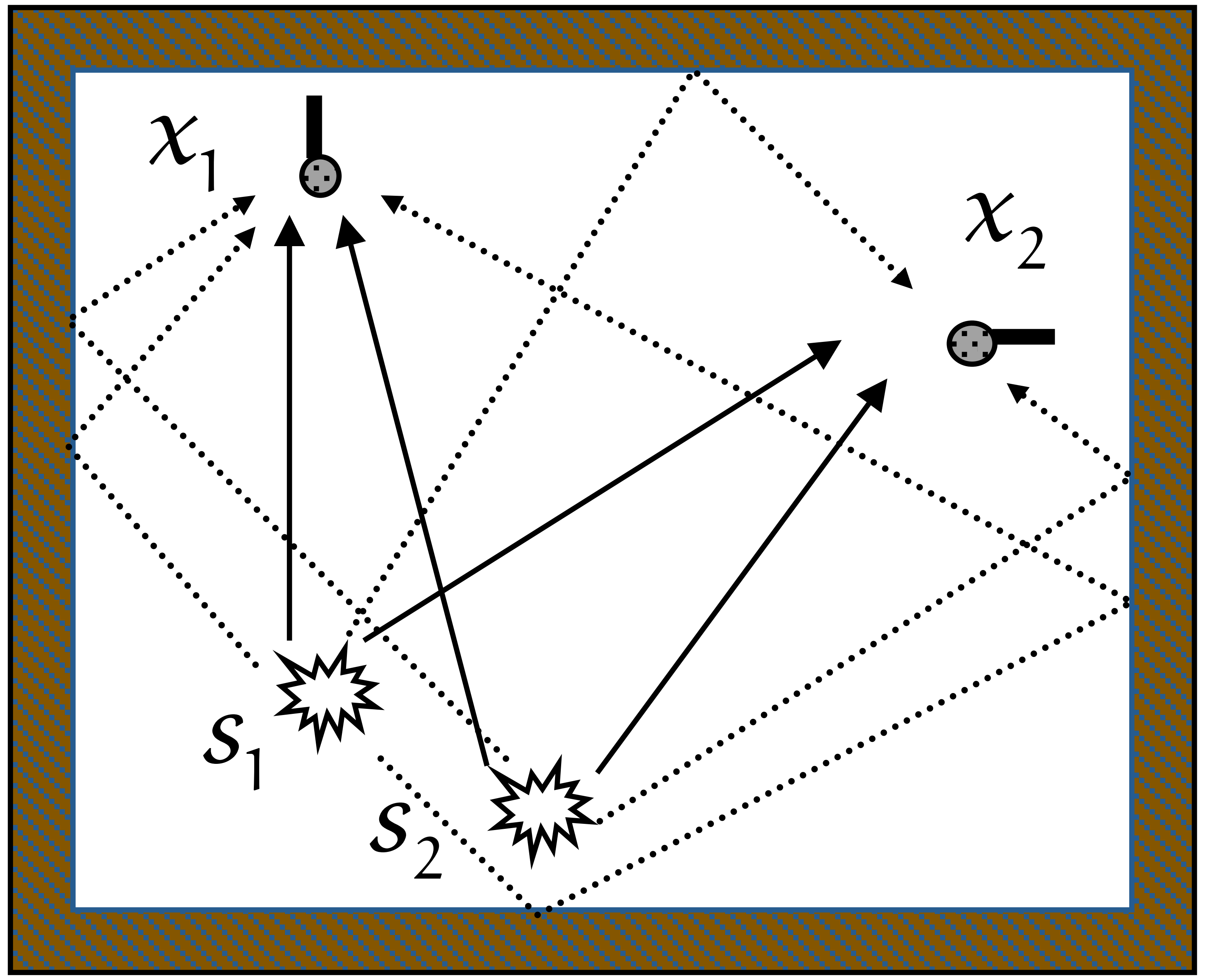


Convolutive mixing

- Instead of *instantaneous mixing*:

$$x_i(t) = \sum_{j=1} a_{ij} s_j(t)$$

- We now have *convolutive mixing*: $$x_i(t) = \sum_j \sum_k a_{ij}(k) s_j(t-k)$$
    - The mixing filters $a_{ij}(k)$ capture all the mixing effects in this model
- How do we do ICA now?



FIR matrix algebra

- Matrices with FIR filters as elements

$$\underline{\mathbf{A}} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix}$$

$$\underline{a}_{ij} = \begin{bmatrix} a_{ij}(0) & \cdots & a_{ij}(k-1) \end{bmatrix}$$

- FIR matrix multiplication performs convolution and accumulation

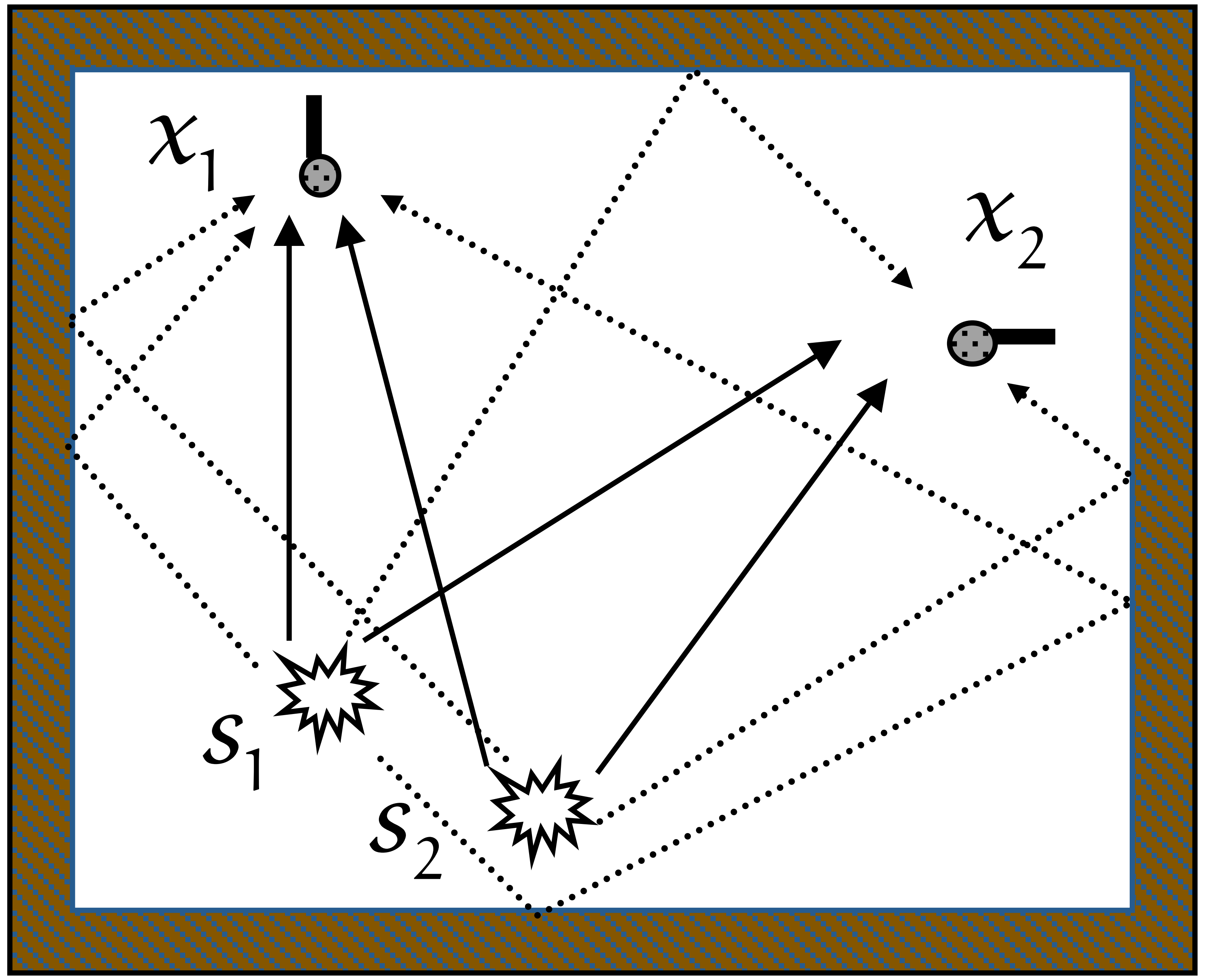
$$\underline{\mathbf{A}} \cdot \underline{\mathbf{b}} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \cdot \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} = \begin{bmatrix} \underline{a}_{11} * \underline{b}_1 + \underline{a}_{12} * \underline{b}_2 \\ \underline{a}_{21} * \underline{b}_1 + \underline{a}_{22} * \underline{b}_2 \end{bmatrix}$$

Back to convolutive mixing

- Rewrite convolutive mixing as:

$$x_i(t) = \sum_j \sum_k a_{ij}(k) s_j(t-k) \Rightarrow$$

$$\Rightarrow \underline{\mathbf{x}}(t) = \underline{\mathbf{A}} \cdot \underline{\mathbf{s}}(t) = \begin{bmatrix} a_{11} * s_1(t) + a_{12} * s_2(t) \\ a_{21} * s_1(t) + a_{22} * s_2(t) \end{bmatrix}$$



- Tidier formulation!
- We can use the FIR matrix abstraction to solve this problem

The easy solution

- Straightforward translation of instantaneous learning rules using FIR matrices, e.g.:

$$\Delta \underline{\mathbf{W}} \propto \left(\mathbf{I} - f(\underline{\mathbf{W}} \cdot \underline{\mathbf{x}}) \cdot (\underline{\mathbf{W}} \cdot \underline{\mathbf{x}})^{\top}\right) \cdot \underline{\mathbf{W}}$$

- Not so easy with algebraic approaches
  - FIR tensors, FIR eigendecompositions, etc ...
- Multiple other (and more rigorous/better behaved) approaches have been developed

Complications with this approach

- Required convolutions are expensive
  - Real-room filters are long
  - FIR products can become very time consuming
- Convergence is hard to achieve
  - Huge parameter space
  - Tightly interwoven parameter relationships
- A slow optimization nightmare!

FIR matrix algebra, part II

- FIR matrices have frequency domain versions:

$$\underline{\mathbf{A}} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \xrightarrow{\textit{frequency domain}} \hat{\underline{\mathbf{A}}} = \begin{bmatrix} \hat{\underline{a}}_{11} & \hat{\underline{a}}_{12} \\ \hat{\underline{a}}_{21} & \hat{\underline{a}}_{22} \end{bmatrix}$$

$$\hat{\underline{a}}_{ij} = \mathrm{DFT}\left[\underline{a}_{ij}\right]$$

- And their products are simpler:

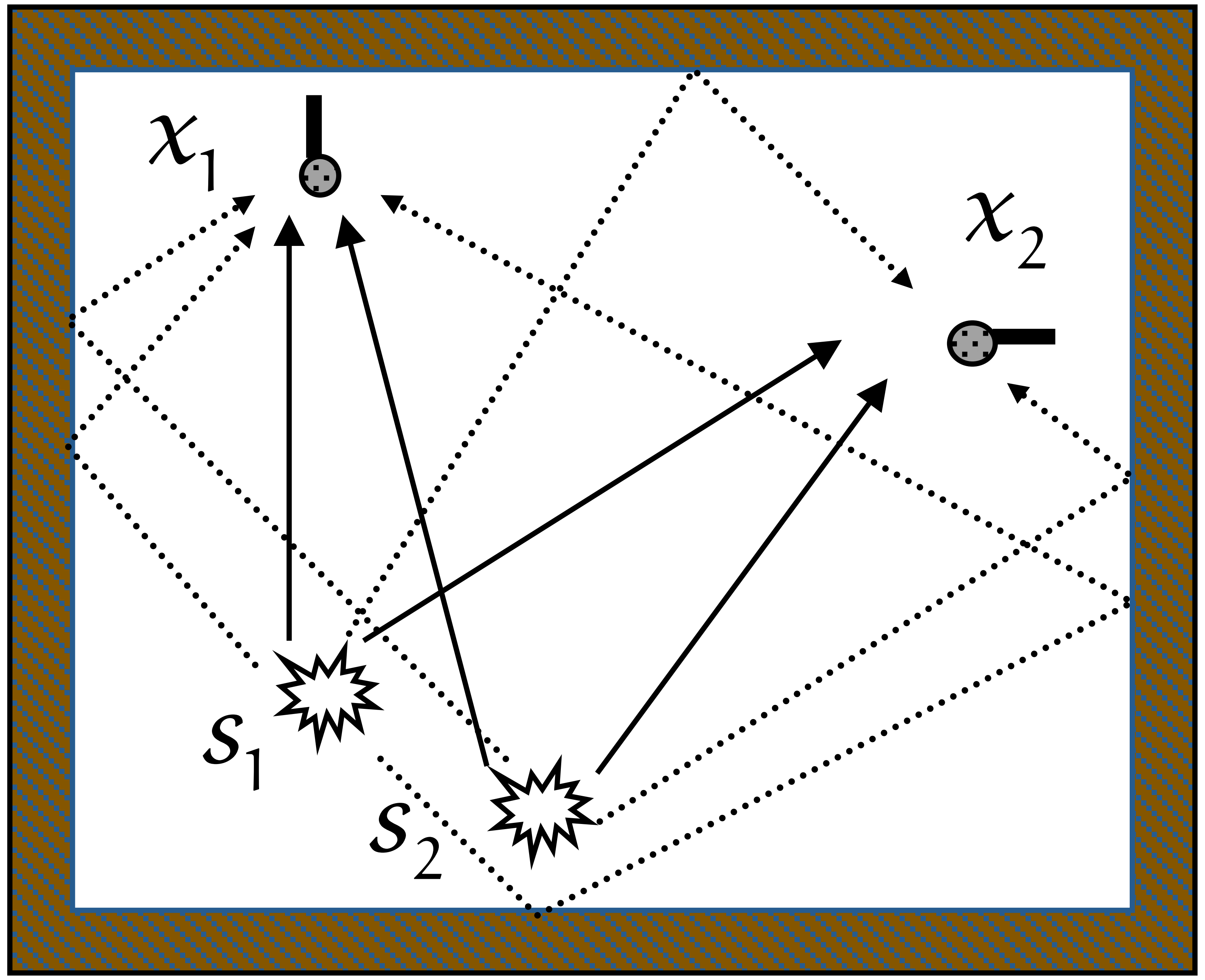
$$\hat{\underline{\mathbf{A}}} \cdot \hat{\underline{\mathbf{b}}} = \begin{bmatrix} \hat{\underline{a}}_{11} \circ \hat{\underline{b}}_1 + \hat{\underline{a}}_{12} \circ \hat{\underline{b}}_2 \\ \hat{\underline{a}}_{21} \circ \hat{\underline{b}}_1 + \hat{\underline{a}}_{22} \circ \hat{\underline{b}}_2 \end{bmatrix}$$

$$\hat{\underline{a}} \circ \hat{\underline{b}} = \begin{bmatrix} a(0) \cdot b(0) & \cdots & a(k-1) \cdot b(k-1) \end{bmatrix}$$

Yet another formulation

- We now model the mixing in the frequency domain instead:

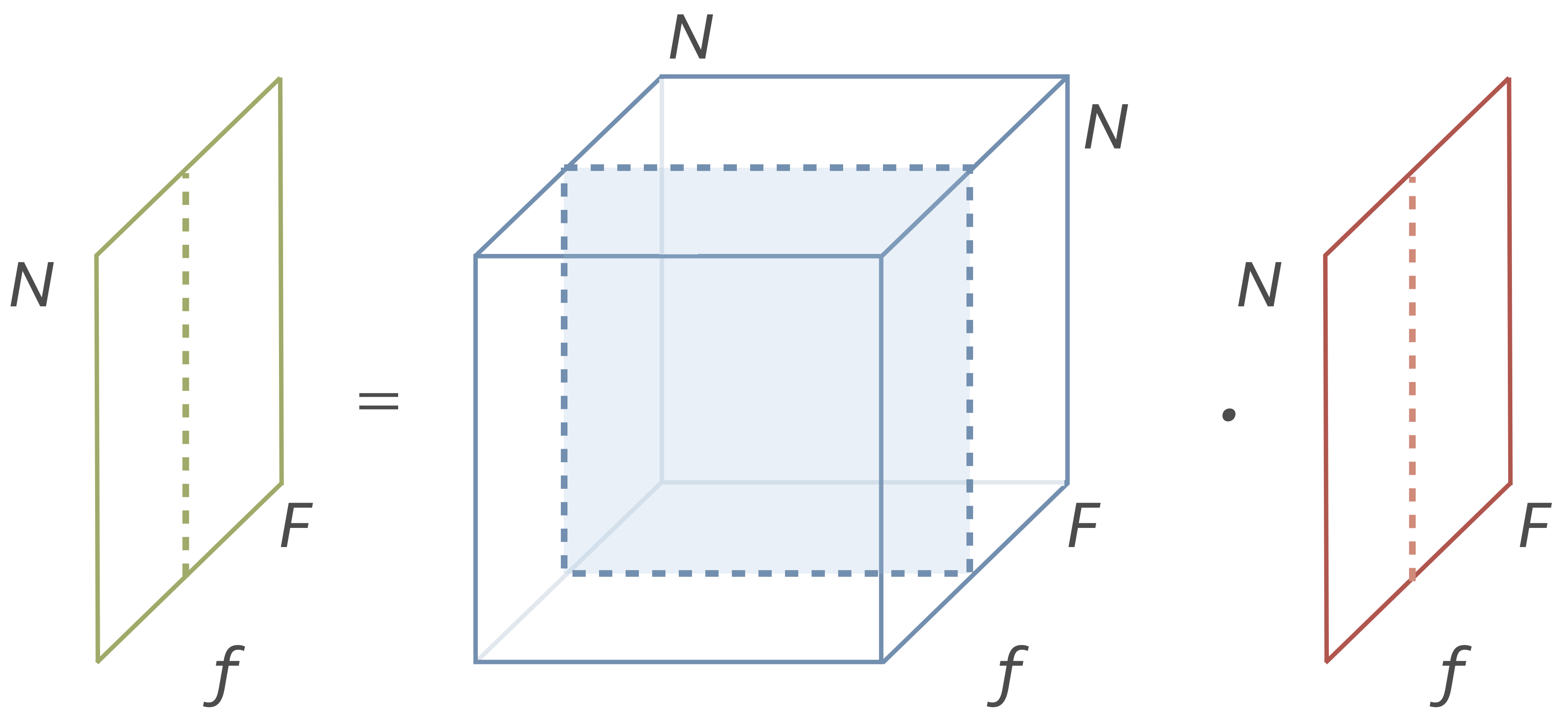
$$\hat{\underline{\mathbf{X}}} = \hat{\underline{\mathbf{A}}} \cdot \hat{\underline{\mathbf{S}}}$$



- Compact form as before
  - Makes the notation manageable
  - But also has an advantage!

Peeking a little deeper

- Looking at slices of the FIR matrices in $\underline{\hat{\mathbf{X}}} = \underline{\hat{\mathbf{A}}} \cdot \underline{\hat{\mathbf{S}}}$

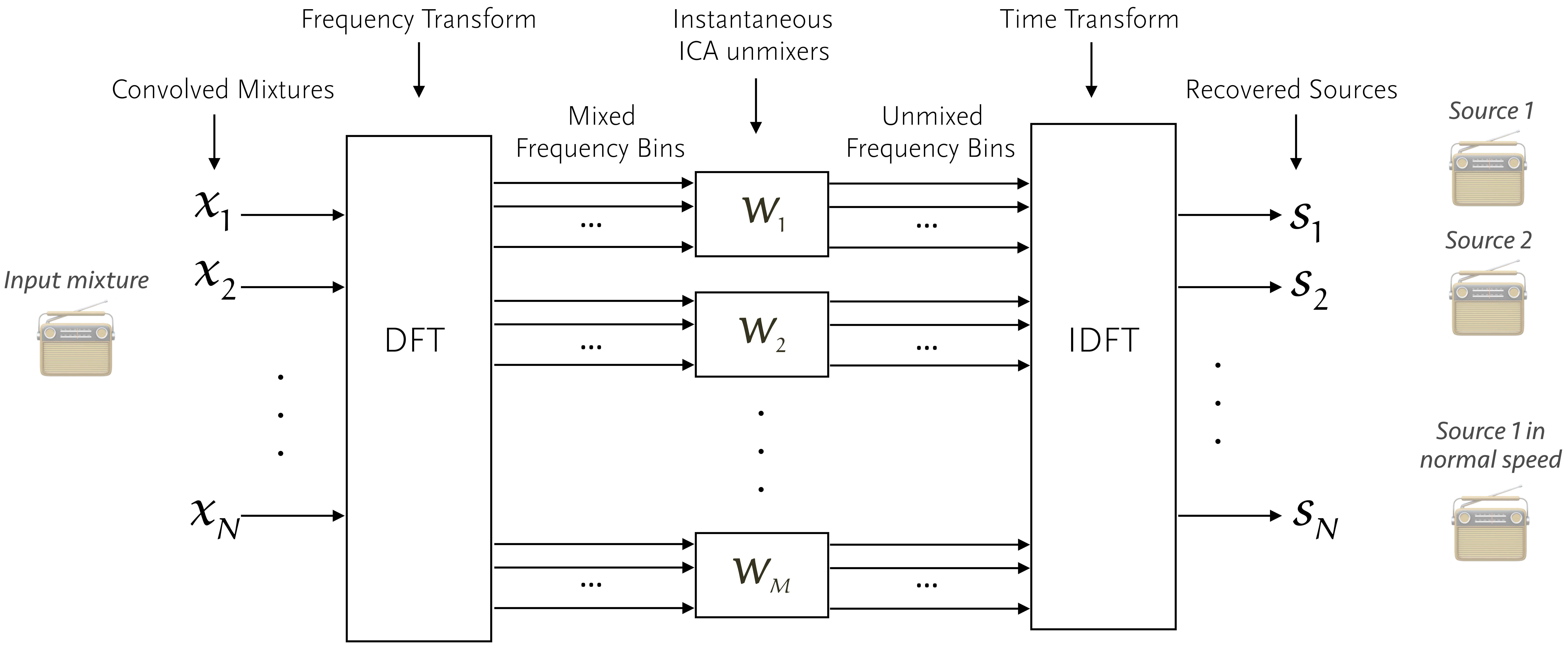


- Each slice is:

$$\underline{\hat{\mathbf{X}}}^{(f)} = \begin{bmatrix} x_1^{(f)} \\ x_2^{(f)} \end{bmatrix} = \begin{bmatrix} a_{1,1}^{(f)} & a_{1,2}^{(f)} \\ a_{2,1}^{(f)} & a_{2,2}^{(f)} \end{bmatrix} \cdot \begin{bmatrix} s_1^{(f)} \\ s_2^{(f)} \end{bmatrix}$$

Hey, that's an instantaneous mixture!

Overall flowgraph



Some complications …

- Permutation issues
  - We don't know which source will end up in each narrowband output …
  - Resulting output can have separated narrowband elements from both sounds!
- Scaling issues
  - Narrowband outputs can be scaled arbitrarily
  - This results in spectrally colored outputs

Original source

Colored source

Extracted source with permutations

Scaling issue

- One simple fix is to normalize the separating matrices:

$$\mathbf{W}_f^{norm} = \mathbf{W}_f^{orig} \cdot \left| \mathbf{W}_f^{orig} \right|^{1/N}$$

  - Results into more reasonable scaling

- More sophisticated approaches exist but this is not a major problem

- Some spectral coloration is unavoidable

Original Input

Colored output

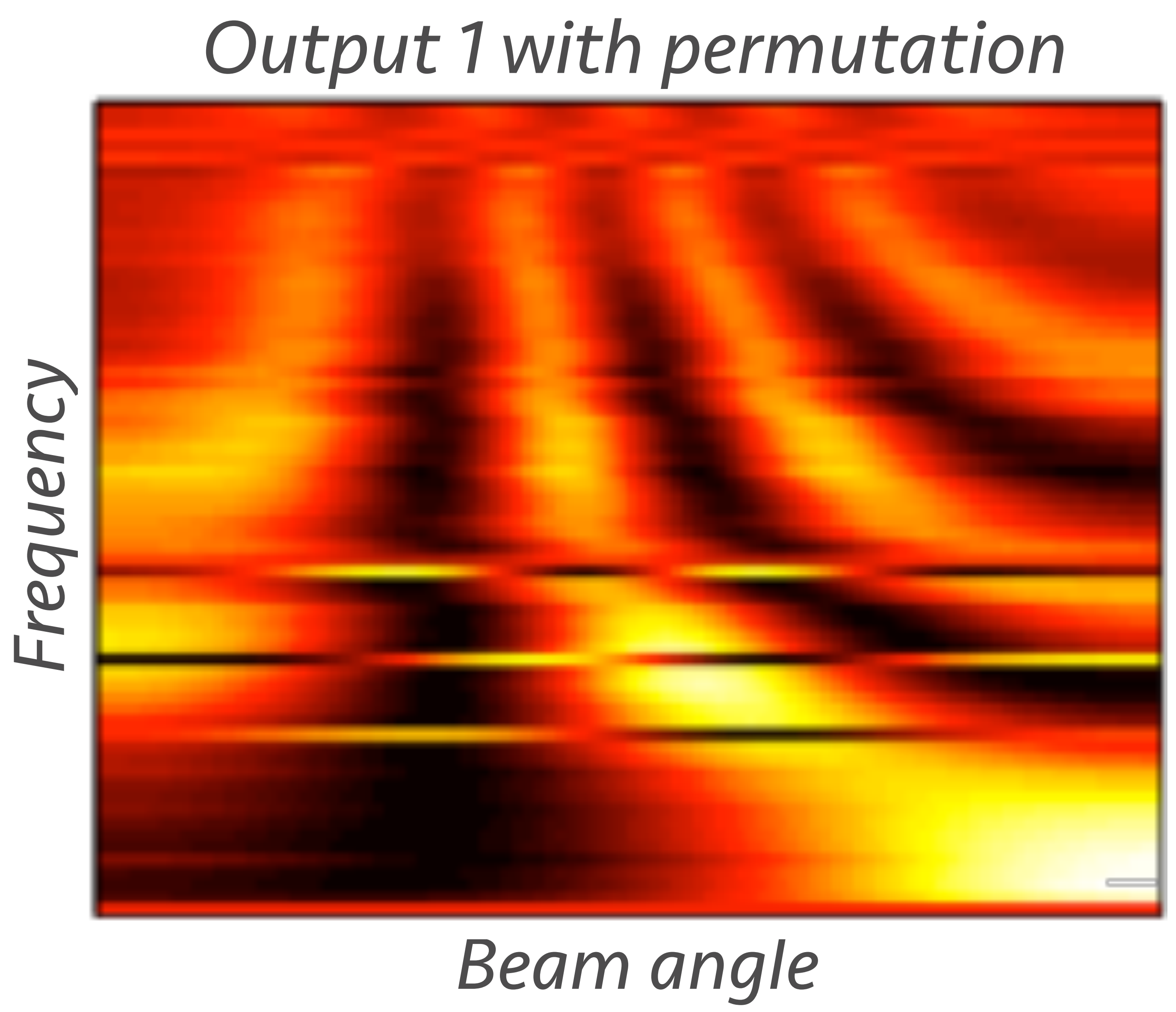
Adjusted output

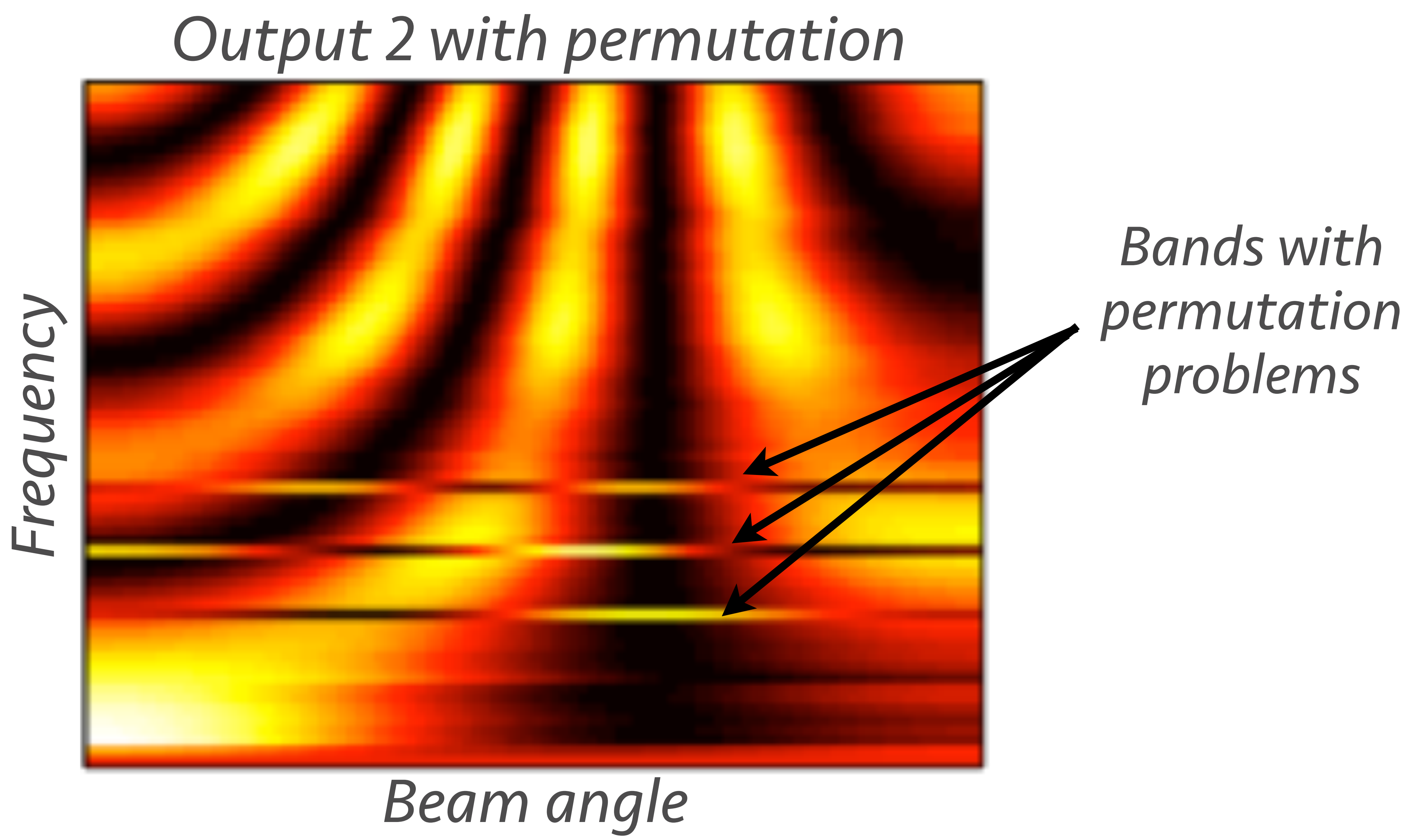
The permutation problem

- Continuity of unmixing matrices
  - Adjacent unmixing matrices tend to be a similar
  - We can permute/bias them accordingly (doesn't work that great)
- Smoothness of spectral output
  - Narrowband components from each source tend to modulate the same way
  - Permute unmixing matrices to ensure adjacent narrowband output are similarly modulated (works ok)
- The above can fail miserably for more than two sources!
  - Combinatorial explosion!

Beamforming and ICA

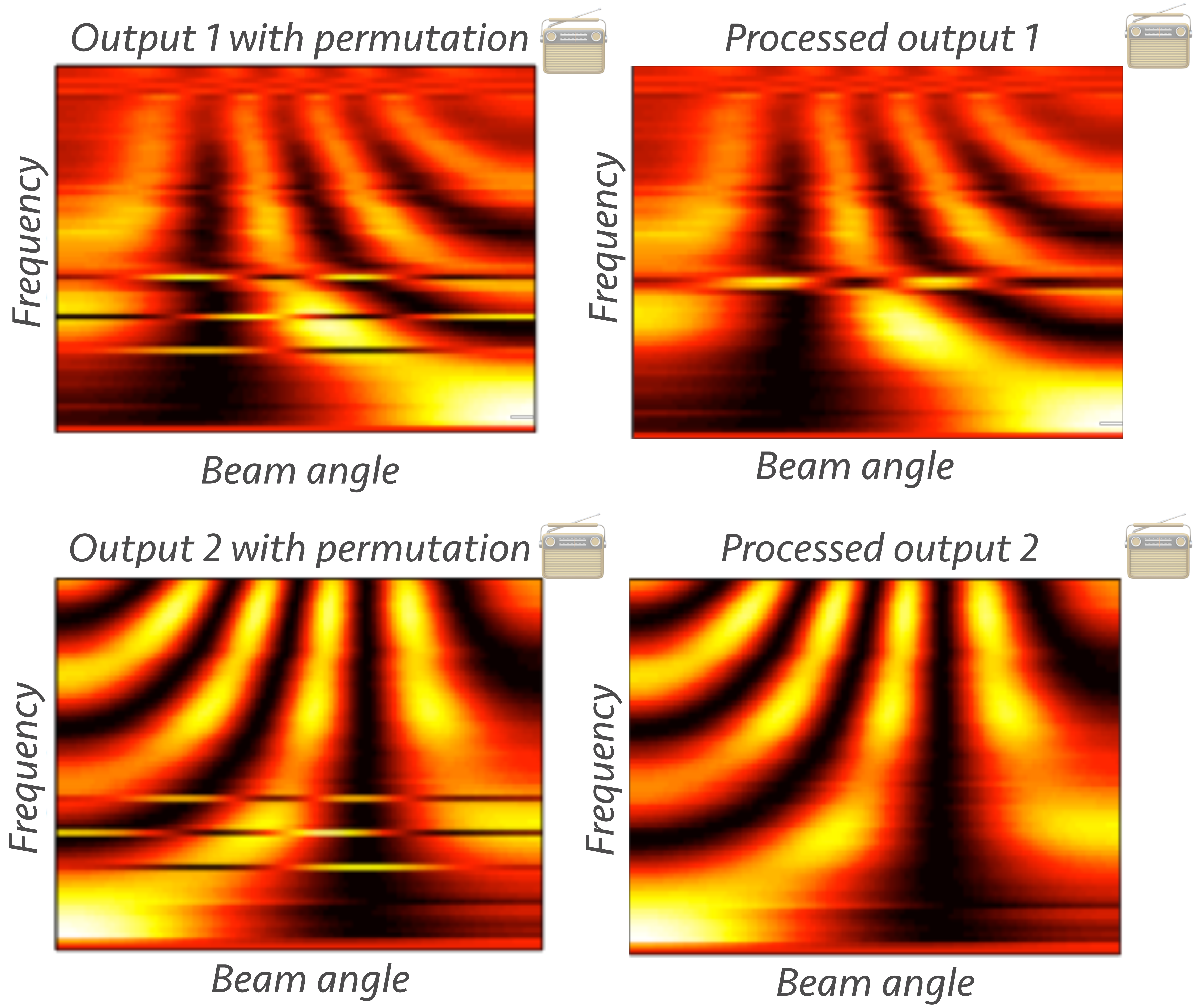
- If we know the placement of the sensors we can obtain the spatial response of the ICA solution
- ICA places nulls to cancel out interfering sources
    - Just as in the instantaneous case we cancel out sources
- We can visualize the permutation problem now
    - Out of place bands





Beamforming to resolve permutations

- Spatial information can be used to resolve permutations
  - Find permutations that preserve zeros or smooth out the responses
- Works fine, although can be flaky if the array response is not clean



The *N*-input *N*-output problem

- ICA, in any formulation, inverts a square mixing matrix
  - Implies that we have the same number of input as outputs
  - E.g. in a street with 30 sources we need at least 30 mics!
- Solutions exist for M ins, N outs where $M > N$
- If $N > M$ we can only beamform
  - In some cases extra sources can be treated as noise
  - This can be very restrictive
    - But we'll see ways out of it in the next lecture

Recap

- Sensor arrays
- Beamforming and localization
- Array source separation
  - FIR Matrix algebra

Next lecture

- Underconstrained source separation
- Single-channel source separation

Reading

- Beamforming and localization
  - http://onlinelibrary.wiley.com/book/10.1002/9780470994443 (uiuc access only)

- Blind source separation and ICA
  - http://paris.cs.illinois.edu/pubs/smaragdis-neurocomp.pdf
  - https://ieeexplore.ieee.org/abstract/document/1323089