Civil and Environmental Engineering (CEE) 291D:
Homework # 2

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Handed out: 02/26/2025.
Due: 03/31/2025 at 9:59am (on bCourses)

Collaboration policy: You can work in study groups of two or three (no more than (3)) or on your own.

Each student must submit their own original paper in bCourses and all written work must be your own.

Submission policy: Submission must be done via bCourses.
Each paper must begin with the student's name and the name of the students they worked with (if any).

Example: Homework #2: firstname1, lastname1
Group: firstname2, lastname2 and firstname3, lastname3

Each uploaded paper and code must be named as follows:
S25-CE291D-hw2-lastname-firstname.pdf /S25-CE291D-hw2-lastname-firstname.py or S25-CE291D-hw2-lastname-firstname.ipynb

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IMPORTANT: We will not rerun the codes that you submit. So anything that needs to be considered for grading should be included in the PDF that you upload in bCourses, including figures and results.

Failure to follow these instructions will result in the paper not being graded.

Problem statement

The goal of this problem set is to use data-driven method to simulate, identify and deduce dynamical systems

Task 1: Using Neural Networks to solve systems of equations (20 points)

1. Download the python notebook HW2-KS.ipynb . The notebook contains the python code to solve the Kuramoto-Sivashinsky equation:

$$u_t = -uu_x - u_{xx} - u_{xxxx} \tag{KS}$$

2. Train a Neural Network to advance the solution of the system. Hint: use the provided python code to generate input and output of you network. (10 points)

3. Compare your evolution trajectories for your neural network with the solution of the dynamical system provided in the code with different initial conditions. (10 points)

Task 2: Dynamic Mode Decomposition (30 points)

1. Download the data "hw2-DMD-X0.csv" and "hw2-DMD-X1.csv". These datasets include the snapshots matrices (X, X') for a 2D flow on a channel.

2. Pre-process the data by creating the two matrices X1 and X2 using the code below

```
X1 = pd.read_csv('hw2-DMD-X0.csv', header=None)
X1 = X1.applymap(lambda s: complex(s.replace('i', 'j'))).values # This will give
    you the first snapshot matrix X
X2 = pd.read_csv('hw2-DMD-X1.csv', header=None)
X2 = X2.applymap(lambda s: complex(s.replace('i', 'j'))).values # This will give
    you the second snapshot matrix X'
```

3. Find the different modes of the system using Dynamic Mode Decomposition (DMD) (20 points)

4. Forecast the evolution of the system using the modes found with DMD (10 points)

Task 3: Identification of nonlinear systems (30 points)

1. Download the datasets for this task "hw2-ID-X.csv" and "hw2-ID-Xprime.csv"

2. Using the method seen in class for identification of nonlinear systems, find the governing equations for the datasets. The datasets represent the time series of a state variable $x(t) = [x_1(t), x_2(t)]$ and the timeseries of the derivative in time $\dot{x}(t) = [\dot{x}_1(t), \dot{x}_2(t)]$. (30 points)

Task 4: Linear control theory(20 points)

1. Determine whether the system below is controllable and/or observable (10 points)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mathbf{u}$$

$$y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \mathbf{x} + \mathbf{u}$$

2. Consider the system:

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{u}$$

   Find a feedback control law of the form $u = -K\mathbf{x}$ such that the system is stable (10 points)