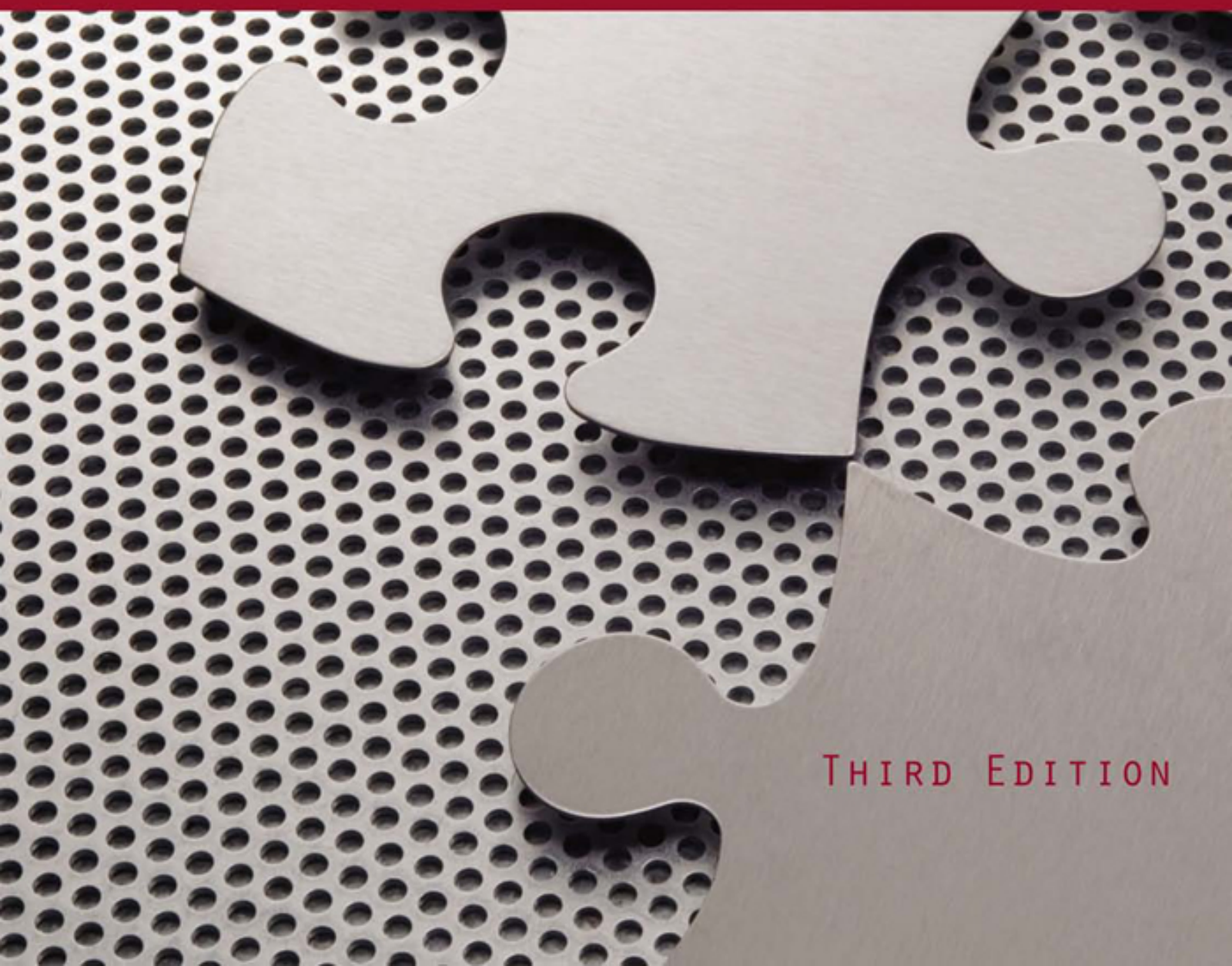
Programming Languages

Principles and Practice

Kenneth C. Louden & Kenneth A. Lambert



Third Edition

Programming Languages

Principles and Practice

Third Edition

Kenneth C. Louden
San Jose State University

Kenneth A. Lambert
Washington and Lee University

COURSE TECHNOLOGY
CENGAGE Learning™

Australia • Brazil • Japan • Korea • Mexico • Singapore • Spain • United Kingdom • United States

COURSE TECHNOLOGY
CENGAGE Learning™

Programming Languages: Principles and Practice, Third Edition
Kenneth C. Louden and Kenneth A. Lambert

Executive Editor: Marie Lee
Acquisitions Editor: Brandi Shailer
Senior Product Manager: Alyssa Pratt
Development Editor: Ann Shaffer
Editorial Assistant: Jacqueline Lacaire
Associate Marketing Manager: Shanna Shelton
Content Project Manager: Jennifer Feltri
Art Director: Faith Brosnan
Print Buyer: Julio Esperas
Cover Designer: Saizon Design
Cover Photo: © Ocean/Corbis
Compositor: Integra
Copyeditor: Foxxe Editorial
Proofreader: Christine Clark
Indexer: Sharon Hilgenberg

Library of Congress Control Number: 2010939435

ISBN-13: 978-1-111-52941-3

ISBN-10: 1-111-52941-8

Course Technology
20 Channel Center Street
Boston, MA 02210
USA

Printed in the United States of America
1 2 3 4 5 6 7 17 16 15 14 13 12 11

This book is an introduction to the broad field of programming languages. It combines a general presentation of principles with considerable detail about many modern languages. Unlike many introductory texts, it contains significant material on implementation issues, the theoretical foundations of programming languages, and a large number of exercises. All of these features make this text a useful bridge to compiler courses and to the theoretical study of programming languages. However, it is a text specifically designed for an advanced undergraduate programming languages survey course that covers most of the programming languages requirements specified in the 2001 ACM/IEEE-CS Joint Curriculum Task Force Report, and the CS8 course of the 1978 ACM Curriculum.

Our goals in preparing this new edition are to bring the language-specific material in line with the changes in the popularity and use of programming languages since the publication of the second edition in 2003, to improve and expand the coverage in certain areas, and to improve the presentation and usefulness of the examples and exercises, while retaining as much of the original text and organization as possible. We are also mindful of the findings and recommendations of the ACM SIGPLAN Programming Language Curriculum Workshop [2008], which reaffirm the centrality of the study of programming languages in the computer science curriculum. We believe that the new edition of our book will help students to achieve the objectives and outcomes described in the report, which was compiled by the leading teachers in our field.

To complete this book, students do not have to know any one particular language. However, experience with at least one language is necessary. A certain degree of computational sophistication, such as that provided by a course in data structures (CS2) and a discrete mathematics course, is also expected. A course in computer organization, which provides some coverage of assembly language programming and virtual machines, would be useful but is not essential. Major languages used in this edition include C, C++, Smalltalk, Java, Ada, ML, Haskell, Scheme, and Prolog; many other languages are discussed more briefly.

Overview and Organization

In most cases, each chapter largely is independent of the others without artificially restricting the material in each. Cross references in the text allow the student or instructor to fill in any gaps that might arise even if a particular chapter or section is skipped.

Chapter 1 surveys the concepts studied in later chapters, provides an overview of the history of programming languages, and introduces the idea of abstraction and the concept of different language paradigms.

Chapter 2 provides an overview of language design criteria. Chapter 2 could serve well as a culminating chapter for the book, but we find it arouses interest in later topics when covered here.

Chapters 3, 4, and 5 concretely address three major language paradigms, beginning with the function-oriented paradigm in Chapter 3. Scheme, ML, and Haskell are covered in some detail. This chapter also introduces the lambda calculus. Chapter 4, on logic programming, offers an extended section on Prolog, and devotes another section to the functional logic language Curry. Chapter 5 deals with the object-oriented paradigm. We use Smalltalk to introduce the concepts in this chapter. Individual sections also feature Java and C++.

Chapter 6 treats syntax in some detail, including the use of BNF, EBNF, and syntax diagrams. A brief section treats recursive definitions (like BNF) as set equations to be solved, a technique that recurs periodically throughout the text. One section is devoted to recursive-descent parsing and the use of parsing tools. The final section of this chapter begins a multi-chapter case study that develops a parser for a small language similar to Ada.

Chapters 7, 8, 9, and 10 cover the central semantic issues of programming languages: declaration, allocation, evaluation; the symbol table and runtime environment as semantic functions; data types and type checking; procedure activation and parameter passing; and exceptions and exception handling.

Chapter 11 gives an overview of modules and abstract data types, including language mechanisms for equational, or algebraic, specification.

Chapter 12 introduces the three principal methods of formal semantics: operational, denotational, and axiomatic. This is somewhat unique among introductory texts in that it gives enough detail to provide a real flavor for the methods.

Chapter 13 discusses the major ways parallelism has been introduced into programming languages: coroutines, threads, semaphores, monitors, and message passing, with examples primarily from Java and Ada. Its final section surveys recent efforts to introduce parallelism into LISP and Prolog, and the use of message passing to support parallel programming in the functional language Erlang.

Use as a Text

Like any programming languages text, this one covers a great deal of material. It should be possible to cover all of it in a two-semester or two-quarter sequence. Alternatively, there are two other, very different ways of delivering this material. They could loosely be called the "principles" approach and the "paradigm" approach. Two suggested organizations of these approaches in a semester-long course are as follows:

The principles approach: Chapters 1, 2, 3, 6, 7, 8, 9, and 10.

The paradigm approach: Chapters 1, 2, 3, 4, 5, 6, 7, 8, and 13. If there is extra time, selected topics from the remaining chapters.

Summary of Changes between the Second and Third Editions

The most obvious change from the second edition is the shifting of the three chapters on non-imperative programming languages to a much earlier position in the book (from Chapters 10-12 to Chapters 3-5, with the chapter on object-oriented programming now coming after those on functional and logic

programming). As a consequence, the chapters on syntax and semantics now appear a bit later (Chapters 6-10 instead of 4-8). There are several reasons for this rearrangement:

1. By being exposed early to programming languages and paradigms that they may not have seen, students will gain perspective on the language and paradigm that they already have used, and thus become aware of their power and their limitations.
2. Students will have an opportunity to write programs in one or more new languages much earlier in the course, thus giving them an opportunity to become proficient in alternative styles of programming.
3. The practical experience with some interesting and powerful new languages early in the course will build students' motivation for examining the more theoretical topics explored later, in the chapters on syntax and semantics.

Additional significant changes are as follows:

- The material on the history of programming languages in Chapter 2 has been condensed and moved to Chapter 1, thus shortening the book by one chapter. A brief discussion of machine language and assembly language has also been added to this chapter.
- A case study on the design of Python, a popular general-purpose scripting language, now follows the case study on C++ in Chapter 2. The two case studies illustrate the tradeoffs that occur when designing new languages.
- The chapter on object-oriented programming is now the last of the three chapters on programming paradigms instead of the first one. The order of these chapters now reflects the increasing complexity of the underlying models of computation of each programming paradigm (functions, logic, objects).
- The section on Scheme in the chapter on functional programming has been substantially rewritten and expanded.
- Object-oriented programming in Chapter 5 is now introduced with Smalltalk rather than Java. This new order of presentation will allow students to learn how a language was cleanly built around object-oriented concepts, before they see the tradeoffs and compromises that designers had to make in designing Java and C++.
- The section on Java in the chapter on object-oriented programming has been updated to include a discussion of interfaces, generic collections, and iterators.
- The section on logical constraint languages in the chapter on logic programming has been replaced with a discussion of the functional logic language Curry.
- Beginning in Chapter 6, on syntax, and extending through the Chapters 7-10, on semantics, new end-of-chapter sections present a case study of a parser for a small language that resembles Ada. The design of this software is presented

incrementally, starting with a raw syntax analyzer and adding features to handle static semantic analysis, such as scope analysis and type checking. This new case study will give students extra practical experience with the concepts they learn in each of these chapters.

- A brief discussion of Erlang, a functional programming language that uses message passing to support concurrent processing, has been added to Chapter 13 on parallel programming.

Instructor and Student Resources

The following supplemental materials are available when this book is used in a classroom setting. All of the resources available with this book are provided to the instructor on a single CD-ROM, and most are also available at login.cengage.com.

- **Electronic Instructor's Manual**. The Instructor's Manual that accompanies this textbook includes additional instructional material to assist in class preparation, including items such as Sample Syllabi, Chapter Outlines, Technical Notes, Lecture Notes, Quick Quizzes, Teaching Tips, Discussion Topics, and Sample Midterm and Final Projects.
- **ExamView®**. This textbook is accompanied by ExamView, a powerful testing software package that allows instructors to create and administer printed, computer (LAN-based), and Internet exams. ExamView includes hundreds of questions that correspond to the topics covered in this text, enabling students to generate detailed study guides that include page references for further review. The computer-based and Internet testing components allow students to take exams at their computers, and also save the instructor time by grading each exam automatically.
- **PowerPoint Presentations**. This book comes with Microsoft PowerPoint slides for each chapter. These are included as a teaching aid for classroom presentation and can be made available to students on the network for chapter review or printed for classroom distribution. Instructors can add their own slides for additional topics they introduce to the class.
- **Solution Files**. Selected answers for many of the exercises at the end of each chapter may be found on the Instructor Resources CD-ROM, or at login.cengage.com. Many are programming exercises (most rather short) focusing on languages discussed in the text. Conceptual exercises range from short-answer questions that test understanding of the material to longer, essay-style exercises and challenging "thought" questions. A few moments' reflection should give the reader adequate insight into the potential difficulty of a particular exercise. Further knowledge can be gained by reading the on-line answers, which are treated as an extension of the text and sometimes provide additional information beyond that required to solve the problem. Occasionally

the answer to an exercise on a particular language requires the reader to consult a language reference manual or have knowledge of the language not specifically covered in the text. Complete program examples are available through *www.cengage.com*. The author's Web site, at *home.wlu.edu/~lambertk*, also contains links to free, downloadable translators for all the major languages discussed in the book, many of which were used to test the examples.

- **Distance Learning**. Course Technology is proud to present online test banks in WebCT and Blackboard, to provide the most complete and dynamic learning experience possible. Instructors are encouraged to make the most of the course, both online and offline. For more information on how to access your online test bank, contact your local Course Technology sales representative.

Acknowledgments

Ken Louden would like to thank all those persons too numerous to mention who, over the years, have emailed him with comments, corrections, and suggestions. He remains grateful to the many students in his CS 152 sections at San Jose State University for their direct and indirect contributions to the first and second editions, and to his colleagues at San Jose State, Michael Beeson, Cay Horstmann, and Vinh Phat, who read and commented on individual chapters in the first edition.

Ken Lambert would like to thank his colleagues at Washington and Lee University, Tom Whaley, Simon Levy, and Joshua Stough, and his students in Computer Science 312, for many productive discussions of programming language issues and problems. He also greatly appreciates the work of the reviewers of this edition: Karina Assiter, Wentworth Institute of Technology; Dave Musicant, Carleton College; Amar Raheja, California State Polytechnic University, Pomona; Christino Tamon, Clarkson University.

He would be grateful to receive reports of errors and any other comments from readers at lambertk@wlu.edu.

Ken Lambert offers special thanks to all the people at Course Technology who helped make the third edition a reality, including Brandi Shailer, Acquisitions Editor; Alyssa Pratt, Senior Product Manager; Ann Shaffer, Development Editor; Jennifer Feltri, Content Project Manager. Also, thanks to Amrin Sahay, of Integra Software Services, for overseeing the process of transforming the manuscript into the printed book. Many thanks to Chris Scriver, MQA Project Leader, for ably overseeing the quality assurance testing, as well as to Teresa Storch and Serge Palladino, quality assurance testers, for their many helpful suggestions and corrections.

Finally, both authors would like to thank their wives and children for their love and support.

Notes and References

The report of the ACM SIGPLAN Programming Language Curriculum Workshop appears in *SIGPLAN Notices*, Volume 43, Number 11, November, 2008.

CHAPTER

Introduction

CHAPTER 1

How we communicate influences how we think, and vice versa. Similarly, how we program computers influences how we think about computation, and vice versa. Over the last several decades, programmers have, collectively, accumulated a great deal of experience in the design and use of programming languages. Although we still don't completely understand all aspects of the design of programming languages, the basic principles and concepts now belong to the fundamental body of knowledge of computer science. A study of these principles is as essential to the programmer and computer scientist as the knowledge of a particular programming language such as C or Java. Without this knowledge it is impossible to gain the needed perspective and insight into the effect that programming languages and their design have on the way that we solve problems with computers and the ways that we think about computers and computation.

It is the goal of this text to introduce the major principles and concepts underlying programming languages. Although this book does not give a survey of programming languages, specific languages are used as examples and illustrations of major concepts. These languages include C, C++, Ada, Java, Python, Haskell, Scheme, and Prolog, among others. You do not need to be familiar with all of these languages in order to understand the language concepts being illustrated. However, you should be experienced with at least one programming language and have some general knowledge of data structures, algorithms, and computational processes.

In this chapter, we will introduce the basic notions of programming languages and outline some of the basic concepts. Figure 1.1 shows a rough timeline for the creation of several of the major programming languages that appear in our discussion. Note that some of the languages are embedded in a family tree, indicating their evolutionary relationships.

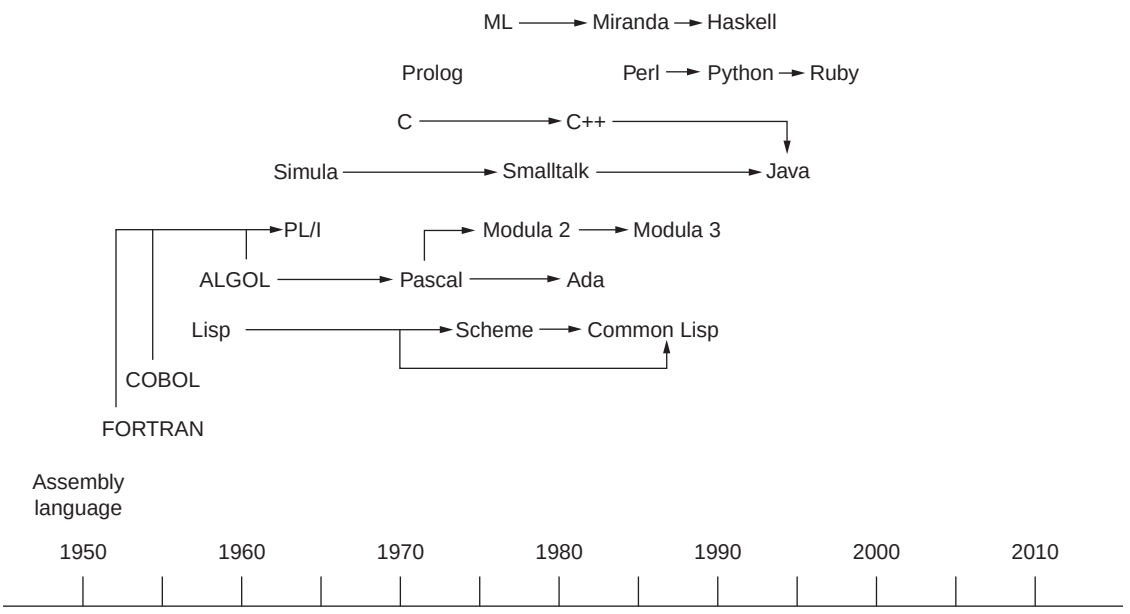


Figure 1.1 A programming language timeline

1.1 The Origins of Programming Languages

A definition often advanced for a programming language is "a notation for communicating to a computer what we want it to do," but this definition is inadequate. Before the middle of the 1940s, computer operators "hardwired" their programs. That is, they set switches to adjust the internal wiring of a computer to perform the requested tasks. This effectively communicated to the computer what computations were desired, but programming, if it could be called that, consisted of the expensive and error-prone activity of taking down the hardware to restructure it. This section examines the origins and emergence of programming languages, which allowed computer users to solve problems without having to become hardware engineers.

1.1.1 Machine Language and the First Stored Programs

A major advance in computer design occurred in the late 1940s, when John von Neumann had the idea that a computer should be permanently hardwired with a small set of general-purpose operations [Schneider and Gersting, 2010]. The operator could then input into the computer a series of binary codes that would organize the basic hardware operations to solve more-specific problems. Instead of turning off the computer to reconfigure its circuits, the operator could flip switches to enter these codes, expressed in **machine language**, into computer memory. At this point, computer operators became the first true programmers, who developed software—the machine code—to solve problems with computers.

Figure 1.2 shows the code for a short machine language program for the LC-3 machine architecture [Patt and Patel, 2003].

```
0010001000000100
0010010000000100
0001011001000010
0011011000000011
1111000000100101
0000000000000101
0000000000000110
0000000000000000
```

Figure 1.2 A machine language program

In this program, each line of code contains 16 bits or binary digits. A line of 16 bits represents either a single machine language instruction or a single data value. The last three lines of code happen to represent data values—the integers 5, 6, and 0—using 16-bit twos complement notation. The first five lines of code represent program instructions. Program execution begins with the first line of code, which is fetched from memory, decoded (interpreted), and executed. Control then moves to the next line of code, and the process is repeated, until a special halt instruction is reached.

To decode or interpret an instruction, the programmer (and the hardware) must recognize the first 4 bits of the line of code as an **opcode**, which indicates the type of operation to be performed. The remaining 12 bits contain codes for the instruction's operands. The operand codes are either the numbers of machine registers or relate to the addresses of other data or instructions stored in memory. For example, the first instruction, `0010001000000100`, contains an opcode and codes for two operands. The opcode `0010` says, "copy a number from a memory location to a machine register" (machine registers are high-speed memory cells that hold data for arithmetic and logic computations). The number of the register, `001`, is found in the next 3 bits. The remaining 9 bits represent an integer offset from the address of the next instruction. During instruction execution, the machine adds this integer offset to the next instruction's address to obtain the address of the current instruction's second operand (remember that both instructions and data are stored in memory). In this case, the machine adds the binary number 100 (4 in binary) to the number 1 (the address of the next instruction) to obtain the binary number 101 (5 in binary), which is the address of the sixth line of code. The bits in this line of code, in turn, represent the number to be copied into the register.

We said earlier that execution stops when a halt instruction is reached. In our program example, that instruction is the fifth line of code, `1111000000100101`. The halt instruction prevents the machine from continuing to execute the lines of code below it, which represent data values rather than instructions for the program.

As you might expect, machine language programming is not for the meek. Despite the improvement on the earlier method of reconfiguring the hardware, programmers were still faced with the tedious and error-prone tasks of manually translating their designs for solutions to binary machine code and loading this code into computer memory.

1.1.2 Assembly Language, Symbolic Codes, and Software Tools

The early programmers realized that it would be a tremendous help to use mnemonic symbols for the instruction codes and memory locations, so they developed **assembly language** for this purpose.

This type of language relies on software tools to automate some of the tasks of the programmer. A program called an **assembler** translates the symbolic assembly language code to binary machine code. For example, let's say that the first instruction in the program of Figure 1.2 reads:

```
LD R1, FIRST
```

in assembly language. The mnemonic symbol `LD` (short for "load") translates to the binary opcode `0010` seen in line 1 of Figure 1.2. The symbols `R1` and `FIRST` translate to the register number `001` and the data address offset `000000100`, respectively. After translation, another program, called a **loader**, automatically loads the machine code for this instruction into computer memory.

Programmers also used a pair of new input devices—a keypunch machine to type their assembly language codes and a card reader to read the resulting punched cards into memory for the assembler. These two devices were the forerunners of today's software text editors. These new hardware and software tools made it much easier for programmers to develop and modify their programs. For example, to insert a new line of code between two existing lines of code, the programmer now could put a new card into the keypunch, enter the code, and insert the card into the stack of cards at the appropriate position. The assembler and loader would then update all of the address references in the program, a task that machine language programmers once had to perform manually. Moreover, the assembler was able to catch some errors, such as incorrect instruction formats and incorrect address calculations, which could not be discovered until run time in the pre-assembler era.

Figure 1.3 shows the machine language program of Figure 1.2 after it has been "disassembled" into the LC-3 assembly language. It is now possible for a human being to read what the program does. The program adds the numbers in the variables `FIRST` and `SECOND` and stores the result in the variable `SUM`. In this code, the symbolic labels `FIRST`, `SECOND`, and `SUM` name memory locations containing data, the labels `R1`, `R2`, and `R3` name machine registers, and the labels `LD`, `ADD`, `ST`, and `HALT` name opcodes. The program is also commented (the text following each semicolon) to clarify what it does for the human reader.

```
.ORIG x3000      ; Address (in hexadecimal) of the first instruction
LD  R1, FIRST    ; Copy the number in memory location FIRST to register R1
LD  R2, SECOND   ; Copy the number in memory location SECOND to register R2
ADD R3, R2, R1   ; Add the numbers in R1 and R2 and place the sum in
                 ; register R3
ST  R3, SUM      ; Copy the number in R3 to memory location SUM
HALT             ; Halt the program
FIRST  .FILL #5  ; Location FIRST contains decimal 5
SECOND .FILL #6  ; Location SECOND contains decimal 6
SUM    .BLKW #1  ; Location SUM (contains 0 by default)
.END             ; End of program
```

Figure 1.3 An assembly language program that adds two numbers

Although the use of mnemonic symbols represents an advance on binary machine codes, assembly language still has some shortcomings. The assembly language code in Figure 1.3 allows the programmer to represent the abstract mathematical idea, "Let `FIRST` be 5, `SECOND` be 6, and `SUM` be `FIRST + SECOND`" as a sequence of human-readable machine instructions. Many of these instructions must move

data from variables/memory locations to machine registers and back again, however; assembly language lacks the more powerful abstraction capability of conventional mathematical notation. An **abstraction** is a notation or way of expressing ideas that makes them concise, simple, and easy for the human mind to grasp. The philosopher/mathematician A. N. Whitehead emphasized the power of abstract notation in 1911: "By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems. . . . Civilization advances by extending the number of important operations which we can perform without thinking about them." In the case of assembly language, the programmer must still do the hard work of translating the abstract ideas of a problem domain to the concrete and machine-dependent notation of a program.

A second major shortcoming of assembly language is due to the fact that each particular type of computer hardware architecture has its own machine language instruction set, and thus requires its own dialect of assembly language. Therefore, any assembly language program has to be rewritten to port it to different types of machines.

The first assembly languages appeared in the 1950s. They are still used today, whenever very low-level system tools must be written, or whenever code segments must be optimized by hand for efficiency. You will likely have exposure to assembly language programming if you take a course in computer organization, where the concepts and principles of machine architecture are explored.

1.1.3 FORTRAN and Algebraic Notation

Unlike assembly language, high-level languages, such as C, Java, and Python, support notations closer to the abstractions, such as algebraic expressions, used in mathematics and science. For example, the following code segment in C or Java is equivalent to the assembly language program for adding two numbers shown earlier:

```
int first = 5;
int second = 6;
int sum = first + second;
```

One of the precursors of these high-level languages was FORTRAN, an acronym for FORmula TRANslation language. John Backus developed FORTRAN in the early 1950s for a particular type of IBM computer. In some respects, early FORTRAN code was similar to assembly language. It reflected the architecture of a particular type of machine and lacked the structured control statements and data structures of later high-level languages. However, FORTRAN did appeal to scientists and engineers, who enjoyed its support for algebraic notation and floating-point numbers. The language has undergone numerous revisions in the last few decades, and now supports many of the features that are associated with other languages descending from its original version.

1.1.4 The ALGOL Family: Structured Abstractions and Machine Independence

Soon after FORTRAN was introduced, programmers realized that languages with still higher levels of abstraction would improve their ability to write concise, understandable instructions. Moreover, they wished to write these high-level instructions for different machine architectures with no changes. In the late 1950s, an international committee of computer scientists (which included John Backus) agreed on

a definition of a new language whose purpose was to satisfy both of these requirements. This language became ALGOL (an acronym for ALGOrithmic Language). Its first incarnation, ALGOL-60, was released in 1960.

ALGOL provided first of all a standard notation for computer scientists to *publish algorithms in journals*. As such, the language included notations for structured control statements for sequencing (`begin-end` blocks), loops (the `for` loop), and selection (the `if` and `if-else` statements). These types of statements have appeared in more or less the same form in every high-level language since. Likewise, elegant notations for expressing data of different numeric types (integer and float) as well as the array data structure were available. Finally, support for procedures, including recursive procedures, was provided. These structured abstractions, and more, are explored in detail later in this chapter and in later chapters of this book.

The ALGOL committee also achieved machine independence for program execution on computers by requiring that each type of hardware provide an ALGOL **compiler**. This program translated standard ALGOL programs to the machine code of a particular machine.

ALGOL was one of the first programming languages to receive a formal specification or definition. Its published report included a grammar that defined its features, both for the programmer who used it and for the compiler writer who translated it.

A very large number of high-level languages are descended from ALGOL. Niklaus Wirth created one of the early ones, Pascal, as a language for teaching programming in the 1970s. Another, Ada, was developed in the 1980s as a language for embedded applications for the U.S. Department of Defense. The designers of ALGOL's descendants typically added features for further structuring data and large units of code, as well as support for controlling access to these resources within a large program.

1.1.5 Computation Without the von Neumann Architecture

Although programs written in high-level languages became independent of particular makes and models of computers, these languages still echoed, at a higher level of abstraction, the underlying architecture of the von Neumann model of a machine. This model consists of an area of memory where both programs and data are stored and a separate central processing unit that sequentially executes instructions fetched from memory. Most modern programming languages still retain the flavor of this single processor model of computation. For the first five decades of computing (from 1950 to 2000), the improvements in processor speed (as expressed in Moore's Law, which states that hardware speeds increase by a factor of 2 every 18 months) and the increasing abstraction in programming languages supported the conversion of the industrial age into the information age. However, this steady progress in language abstraction and hardware performance eventually ran into separate roadblocks.

On the hardware side, engineers began, around the year 2005, to reach the limits of the improvements predicted by Moore's Law. Over the years, they had increased processor performance by shortening the distance between processor components, but as components were packed more tightly onto a processor chip, the amount of heat generated during execution increased. Engineers mitigated this problem by factoring some computations, such as floating-point arithmetic and graphics/image processing, out to dedicated processors, such as the math coprocessor introduced in the 1980s and the graphics processor first released in the 1990s. Within the last few years, most desktop and laptop computers have been built with multicore architectures. A multicore architecture divides the central processing unit (CPU) into two or more general-purpose processors, each with its own specialized memory, as well as memory that is shared among them. Although each "core" in a multicore processor is slower than the

CPU of a traditional single-processor machine, their collaboration to carry out computations in parallel can potentially break the roadblock presented by the limits of Moore's Law.

On the language side, despite the efforts of designers to provide higher levels of abstraction for von Neumann computing, two problems remained. First, the model of computation, which relied upon changes to the values of variables, continued to make very large programs difficult to debug and correct. Second, the single-processor model of computation, which assumes a sequence of instructions that share a single processor and memory space, cannot be easily mapped to the new hardware architectures, whose multiple CPUs execute in parallel. The solution to these problems is the insight that programming languages need not be based on any particular model of hardware, but need only support models of computation suitable for various styles of problem solving.

The mathematician Alonzo Church developed one such model of computation in the late 1930s. This model, called the **lambda calculus**, was based on the theory of recursive functions. In the late 1950s, John McCarthy, a computer scientist at M.I.T. and later at Stanford, created the programming language Lisp to construct programs using the functional model of computation. Although a Lisp interpreter translated Lisp code to machine code that actually ran on a von Neumann machine (the only kind of machine available at that time), there was nothing about the Lisp notation that entailed a von Neumann model of computation. We shall explore how this is the case in detail in later chapters. Meanwhile, researchers have developed languages modeled on other non–von Neumann models of computing. One such model is formal logic with automatic theorem proving. Another involves the interaction of objects via message passing. We examine these models of computing, which lend themselves to parallel processing, and the languages that implement them in later chapters.

1.2 Abstractions in Programming Languages

We have noted the essential role that abstraction plays in making programs easier for people to read. In this section, we briefly describe common abstractions that programming languages provide to express computation and give an indication of where they are studied in more detail in subsequent chapters. Programming language abstractions fall into two general categories: **data abstraction** and **control abstraction**. Data abstractions simplify for human users the behavior and attributes of data, such as numbers, character strings, and search trees. Control abstractions simplify properties of the transfer of control, that is, the modification of the execution path of a program based on the situation at hand. Examples of control abstractions are loops, conditional statements, and procedure calls.

Abstractions also are categorized in terms of **levels**, which can be viewed as measures of the amount of information contained (and hidden) in the abstraction. **Basic abstractions** collect the most localized machine information. **Structured abstractions** collect intermediate information about the structure of a program. **Unit abstractions** collect large-scale information in a program.

In the following sections, we classify common abstractions according to these levels of abstraction, for both data abstraction and control abstraction.

1.2.1 Data: Basic Abstractions

Basic data abstractions in programming languages hide the internal representation of common data values in a computer. For example, integer data values are often stored in a computer using a two's complement representation. On some machines, the integer value -64 is an abstraction of the 16-bit twos complement

value 1111111111000000. Similarly, a real or floating-point data value is usually provided, which hides the IEEE single- or double-precision machine representation of such numbers. These values are also called "primitive" or "atomic," because the programmer cannot normally access the component parts or bits of their internal representation [Patt and Patel, 2003].

Another basic data abstraction is the use of symbolic names to hide locations in computer memory that contain data values. Such named locations are called **variables**. The kind of data value is also given a name and is called a **data type**. Data types of basic data values are usually given names that are variations of their corresponding mathematical values, such as `int`, `double`, and `float`. Variables are given names and data types using a **declaration**, such as the Pascal:

```
var x : integer;
```

or the equivalent C declaration:

```
int x;
```

In this example, `x` is established as the name of a variable and is given the data type integer.

Finally, standard operations, such as addition and multiplication, on basic data types are also provided. Data types are studied in Chapter 8 and declarations in Chapter 7.

1.2.2 Data: Structured Abstractions

The **data structure** is the principal method for collecting related data values into a single unit. For example, an employee record may consist of a name, address, phone number, and salary, each of which may be a different data type, but together represent the employee's information as a whole.

Another example is that of a group of items, all of which have the same data type and which need to be kept together for purposes of sorting or searching. A typical data structure provided by programming languages is the **array**, which collects data into a sequence of individually indexed items. Variables can name a data structure in a declaration, as in the C:

```
int a[10];
```

which establishes the variable `a` as the name of an array of ten integer values.

Yet another example is the text file, which is an abstraction that represents a sequence of characters for transfer to and from an external storage device. A text file's structure is independent of the type of storage medium, which can be a magnetic disk, an optical disc (CD or DVD), a solid-state device (flash stick), or even the keyboard and console window.

Like primitive data values, a data structure is an abstraction that hides a group of component parts, allowing the programmer to view them as one thing. Unlike primitive data values, however, data structures provide the programmer with the means of constructing them from their component parts (which can include other data structures as well as primitive values) and also the means of accessing and modifying these components. The different ways of creating and using structured types are examined in Chapter 8.

1.2.3 Data: Unit Abstractions

In a large program, it is useful and even necessary to group related data and operations on these data together, either in separate files or in separate language structures within a file. Typically, such abstractions include access conventions and restrictions that support **information hiding**. These mechanisms

vary widely from language to language, but they allow the programmer to define new data types (data and operations) that hide information in much the same manner as the basic data types of the language. Thus, the unit abstraction is often associated with the concept of an **abstract data type**, broadly defined as a set of data values and the operations on those values. Its main characteristic is the separation of an interface (the set of operations available to the user) from its implementation (the internal representation of data values and operations). Examples of large-scale unit abstractions include the **module** of ML, Haskell, and Python and the **package** of Lisp, Ada, and Java. Another, smaller-scale example of a unit abstraction is the **class** mechanism of object-oriented languages. In this text, we study modules and abstract data types in Chapter 11, whereas classes (and their relation to abstract data types) are studied in Chapter 5.

An additional property of a unit data abstraction that has become increasingly important is its **reusability**—the ability to reuse the data abstraction in different programs, thus saving the cost of writing abstractions from scratch for each program. Typically, such data abstractions represent **components** (operationally complete pieces of a program or user interface) and are entered into a library of available components. As such, unit data abstractions become the basis for language **library** mechanisms (the library mechanism itself, as well as certain standard libraries, may or may not be part of the language itself). The combination of units (their **interoperability**) is enhanced by providing standard conventions for their interfaces. Many interface standards have been developed, either independently of the programming language, or sometimes tied to a specific language. Most of these apply to the class structure of object-oriented languages, since classes have proven to be more flexible for reuse than most other language structures (see the next section and Chapter 5).

When programmers are given a new software resource to use, they typically study its **application programming interface (API)**. An API gives the programmer only enough information about the resource's classes, methods, functions, and performance characteristics to be able to use that resource effectively. An example of an API is Java's Swing Toolkit for graphical user interfaces, as defined in the package `javax.swing`. The set of APIs of a modern programming language, such as Java or Python, is usually organized for easy reference in a set of Web pages called a **doc**. When Java or Python programmers develop a new library or package of code, they create the API for that resource using a software tool specifically designed to generate a doc.

1.2.4 Control: Basic Abstractions

Typical basic control abstractions are those statements in a language that combine a few machine instructions into an abstract statement that is easier to understand than the machine instructions. We have already mentioned the algebraic notation of the arithmetic and assignment expressions, as, for example:

```
SUM = FIRST + SECOND
```

This code fetches the values of the variables `FIRST` and `SECOND`, adds these values, and stores the result in the location named by `SUM`. This type of control is examined in Chapters 7 and 9.

The term **syntactic sugar** is used to refer to any mechanism that allows the programmer to replace a complex notation with a simpler, shorthand notation. For example, the extended assignment operation `x += 10` is shorthand for the equivalent but slightly more complex expression `x = x + 10`, in C, Java, and Python.

1.2.5 Control: Structured Abstractions

Structured control abstractions divide a program into groups of instructions that are nested within tests that govern their execution. They, thus, help the programmer to express the logic of the primary control structures of sequencing, selection, and iteration (loops). At the machine level, the processor executes a sequence of instructions simply by advancing a program counter through the instructions' memory addresses. Selection and iteration are accomplished by the use of **branch instructions** to memory locations other than the next one. To illustrate these ideas, Figure 1.4 shows an LC-3 assembly language code segment that computes the sum of the absolute values of 10 integers in an array named `LIST`. Comments have been added to aid the reader.

```
       LEA  R1, LIST      ; Load the base address of the array (the first cell)
       AND  R2, R2, #0    ; Set the sum to 0
       AND  R3, R3, #0    ; Set the counter to 10 (to count down)
       ADD  R3, R3, #10
WHILE  LDR  R4, R1, #0    ; Top of the loop: load the datum from the current
                          ; array cell
       BRZP INC           ; If it's >= 0, skip next two steps
       NOT  R4, R4        ; It was < 0, so negate it using twos complement
                          ; operations
       ADD  R4, R4, #1
INC    ADD  R2, R2, R4    ; Increment the sum
       ADD  R1, R1, #1    ; Increment the address to move to the next array
                          ; cell
       ADD  R3, R3, #-1   ; Decrement the counter
       BRP  WHILE         ; Goto the top of the loop if the counter > 0
       ST   R2, SUM       ; Store the sum in memory
```

Figure 1.4 An array-based loop in assembly language

If the comments were not included, even a competent LC-3 programmer probably would not be able to tell at a glance what this algorithm does. Compare this assembly language code with the use of the structured `if` and `for` statements in the functionally equivalent C++ or Java code in Figure 1.5.

```
int sum = 0;
for (int i = 0; i < 10; i++){
    int data = list[i];
    if (data < 0)
        data = -data;
    sum += data;
}
```

Figure 1.5 An array-based loop in C++ or Java

Structured selection and loop mechanisms are studied in Chapter 9.

Another structured form of iteration is provided by an **iterator**. Typically found in object-oriented languages, an iterator is an object that is associated with a collection, such as an array, a list, a set, or a tree. The programmer opens an iterator on a collection and then visits all of its elements by running the iterator's methods in the context of a loop. For example, the following Java code segment uses an iterator to print the contents of a list, called `exampleList`, of strings:

```
Iterator<String> iter = exampleList.iterator()
while (iter.hasNext())
    System.out.println(iter.next());
```

The iterator-based traversal of a collection is such a common loop pattern that some languages, such as Java, provide syntactic sugar for it, in the form of an **enhanced `for` loop**:

```
for (String s : exampleList)
    System.out.println(s);
```

We can use this type of loop to further simplify the Java code for computing the sum of either an array or a list of integers, as follows:

```
int sum = 0;
for (int data : list){
    if (data < 0)
        data = -data;
    sum += data;
}
```

Iterators are covered in detail in Chapter 5.

Another powerful mechanism for structuring control is the **procedure**, sometimes also called a **subprogram** or **subroutine**. This allows a programmer to consider a sequence of actions as a single action that can be called or invoked from many other points in a program. Procedural abstraction involves two things. First, a procedure must be defined by giving it a name and associating with it the actions that are to be performed. This is called **procedure declaration**, and it is similar to variable and type declaration, mentioned earlier. Second, the procedure must actually be called at the point where the actions are to be performed. This is sometimes also referred to as procedure **invocation** or procedure **activation**.

As an example, consider the sample code fragment that computes the greatest common divisor of integers `u` and `v`. We can make this into a procedure in Ada with the procedure declaration as given in Figure 1.6.

```
procedure gcd(u, v: in integer; x: out integer) is
    y, t, z: integer;
    begin
        z := u;
        y := v;
        loop
            exit when y = 0;
```

Figure 1.6 An Ada `gcd` procedure *(continues)*

(continued)

```
            t := y;
            y := z mod y;
            z := t;
         end loop;
         x := z;
      end gcd;
```

Figure 1.6 An Ada `gcd` procedure

In this code, we see the procedure header in the first line. Here `u`, `v`, and `x` are **parameters** to the procedure—that is, things that can change from call to call. This procedure can now be **called** by simply naming it and supplying appropriate **actual parameters** or **arguments**, as in:

```
gcd (8, 18, d);
```

which gives `d` the value 2. (The parameter `x` is given the `out` label in line 1 to indicate that its value is computed by the procedure itself and will change the value of the corresponding actual parameter of the caller.)

The system implementation of a procedure call is a more complex mechanism than selection or looping, since it requires the storing of information about the condition of the program at the point of the call and the way the called procedure operates. Such information is stored in a **runtime environment**. Procedure calls, parameters, and runtime environments are all studied in Chapter 10.

An abstraction mechanism closely related to procedures is the **function**, which can be viewed simply as a procedure that returns a value or result to its caller. For example, the Ada code for the `gcd` procedure in Figure 1.6 can more appropriately be written as a function as given in Figure 1.7. Note that the `gcd` function uses a recursive strategy to eliminate the loop that appeared in the earlier version. The use of recursion further exploits the abstraction mechanism of the subroutine to simplify the code.

```
function gcd(u, v: in integer) return integer is
    begin
        if v = 0
            return u;
        else
            return gcd(v, u mod v);
        end if;
    end gcd;
```

Figure 1.7 An Ada `gcd` function

The importance of functions is much greater than the correspondence to procedures implies, since functions can be written in such a way that they correspond more closely to the mathematical abstraction of a function. Thus, unlike procedures, functions can be understood independently of the von Neumann concept of a computer or runtime environment. Moreover, functions can be combined into higher-level abstractions known as **higher-order functions**. Such functions are capable of accepting other functions as arguments and returning functions as values. An example of a higher-order function is a **map**.

This function expects another function and a collection, usually a list, as arguments. The map builds and returns a list of the results of applying the argument function to each element in the argument list. The next example shows how the `map` function is used in Scheme, a dialect of Lisp, to build a list of the absolute values of the numbers in another list. The first argument to `map` is the function `abs`, which returns the absolute value of its argument. The second argument to `map` is a list constructed by the function `list`.

```
(map abs (list 33 -10 66 88 -4))              ; Returns (33 10 66 88 4)
```

Another higher-order function is named `reduce`. Like `map`, `reduce` expects another function and a list as arguments. However, unlike `map`, this function boils the values in the list down to a single value by repeatedly applying its argument function to these values. For example, the following function call uses both `map` and `reduce` to simplify the computation of the sum of the absolute values of a list of numbers:

```
(reduce + (map abs (list 33 -10 66 88 -4))     ; Returns 201
```

In this code, the `list` function first builds a list of numbers. This list is then fed with the `abs` function to the `map` function, which returns a list of absolute values. This list, in turn, is passed with the `+` function (meaning add two numbers) to the `reduce` function. The `reduce` function uses `+` to essentially add up all the list's numbers and return the result.

The extensive use of functions is the basis of the functional programming paradigm and the functional languages mentioned later in this chapter, and is discussed in detail in Chapter 3.

1.2.6 Control: Unit Abstractions

Control can also be abstracted to include a collection of procedures that provide logically related services to other parts of a program and that form a **unit**, or stand-alone, part of the program. For example, a data management program may require the computation of statistical indices for stored data, such as mean, median, and standard deviation. The procedures that provide these operations can be collected into a program unit that can be translated separately and used by other parts of the program through a carefully controlled interface. This allows the program to be understood as a whole without needing to know the details of the services provided by the unit.

Note that what we have just described is essentially the same as a unit-level data abstraction, and is usually implemented using the same kind of module or package language mechanism. The only difference is that here the focus is on the operations rather than the data, but the goals of reusability and library building remain the same.

One kind of control abstraction that is difficult to fit into any one abstraction level is that of parallel programming mechanisms. Many modern computers have several processors or processing elements and are capable of processing different pieces of data simultaneously. A number of programming languages include mechanisms that allow for the parallel execution of parts of programs, as well as providing for synchronization and communication among such program parts. Java has mechanisms for declaring **threads** (separately executed control paths within the Java system) and **processes** (other programs executing outside the Java system). Ada provides the **task** mechanism for parallel execution. Ada's tasks are essentially a unit abstraction, whereas Java's threads and processes are classes and so are structured abstractions, albeit part of the standard `java.lang` package. Other languages provide different levels of parallel abstractions, even down to the statement level. Parallel programming mechanisms are surveyed in Chapter 13.

1.3 Computational Paradigms

Programming languages began by imitating and abstracting the operations of a computer. It is not surprising that the kind of computer for which they were written had a significant effect on their design. In most cases, the computer in question was the von Neumann model mentioned in Section 1.1: a single central processing unit that sequentially executes instructions that operate on values stored in memory. These are typical features of a language based on the von Neumann model: variables represent memory locations, and assignment allows the program to operate on these memory locations.

A programming language that is characterized by these three properties—the sequential execution of instructions, the use of variables representing memory locations, and the use of assignment to change the values of variables—is called an **imperative** language, because its primary feature is a sequence of statements that represent commands, or imperatives.

Most programming languages today are imperative, but, as we mentioned earlier, it is not necessary for a programming language to describe computation in this way. Indeed, the requirement that computation be described as a sequence of instructions, each operating on a single piece of data, is sometimes referred to as the **von Neumann bottleneck**. This bottleneck restricts the ability of a language to provide either parallel computation, that is, computation that can be applied to many different pieces of data simultaneously, or nondeterministic computation, computation that does not depend on order.[1] Thus, it is reasonable to ask if there are ways to describe computation that are less dependent on the von Neumann model of a computer. Indeed there are, and these will be described shortly. Imperative programming languages actually represent only one **paradigm**, or pattern, for programming languages.

Two alternative paradigms for describing computation come from mathematics. The **functional** paradigm is based on the abstract notion of a function as studied in the lambda calculus. The **logic** paradigm is based on symbolic logic. Each of these will be the subject of a subsequent chapter. The importance of these paradigms is their correspondence to mathematical foundations, which allows them to describe program behavior abstractly and precisely. This, in turn, makes it much easier to determine if a program will execute correctly (even without a complete theoretical analysis), and makes it possible to write concise code for highly complex tasks.

A fourth programming paradigm, the **object-oriented** paradigm, has acquired enormous importance over the last 20 years. Object-oriented languages allow programmers to write reusable code that operates in a way that mimics the behavior of objects in the real world; as a result, programmers can use their natural intuition about the world to understand the behavior of a program and construct appropriate code. In a sense, the object-oriented paradigm is an extension of the imperative paradigm, in that it relies primarily on the same sequential execution with a changing set of memory locations, particularly in the implementation of objects. The difference is that the resulting programs consist of a large number of very small pieces whose interactions are carefully controlled and yet easily changed. Moreover, at a higher level of abstraction, the interaction among objects via message passing can map nicely to the collaboration of parallel processors, each with its own area of memory. The object-oriented paradigm has essentially become a new standard, much as the imperative paradigm was in the past, and so will feature prominently throughout this book.

Later in this book, an entire chapter is devoted to each of these paradigms.

[1]Parallel and nondeterministic computations are related concepts; see Chapter 13.

1.4 Language Definition

Documentation for the early programming languages was written in an informal way, in ordinary English. However, as we saw earlier in this chapter, programmers soon became aware of the need for more precise descriptions of a language, to the point of needing formal definitions of the kind found in mathematics. For example, without a clear notion of the meaning of programming language constructs, a programmer has no clear idea of what computation is actually being performed. Moreover, it should be possible to reason mathematically about programs, and to do this requires formal verification or proof of the behavior of a program. Without a formal definition of a language this is impossible.

But there are other compelling reasons for the need for a formal definition. We have already mentioned the need for machine or implementation independence. The best way to achieve this is through standardization, which requires an independent and precise language definition that is universally accepted. Standards organizations such as ANSI (American National Standards Institute) and ISO (International Organization for Standardization) have published definitions for many languages, including C, C++, Ada, Common Lisp, and Prolog.

A further reason for a formal definition is that, inevitably in the programming process, difficult questions arise about program behavior and interaction. Programmers need an adequate way to answer such questions besides the often-used trial-and-error process: it can happen that such questions need to be answered already at the design stage and may result in major design changes.

Finally, the requirements of a formal definition ensure discipline when a language is being designed. Often a language designer will not realize the consequences of design decisions until he or she is required to produce a clear definition.

Language definition can be loosely divided into two parts: **syntax**, or structure, and **semantics**, or meaning. We discuss each of these categories in turn.

1.4.1 Language Syntax

The syntax of a programming language is in many ways like the grammar of a natural language. It is the description of the ways different parts of the language may be combined to form phrases and, ultimately, sentences. As an example, the syntax of the `if` statement in C may be described in words as follows:

> **PROPERTY:** An `if` statement consists of the word "if" followed by an expression inside parentheses, followed by a statement, followed by an optional `else` part consisting of the word "else" and another statement.

The description of language syntax is one of the areas where formal definitions have gained acceptance, and the syntax of all languages is now given using a **grammar**. For example, a grammar rule for the C `if` statement can be written as follows:

```
<if-statement> ::= if (<expression>) <statement>
                   [else <statement>]
```

or (using special characters and formatting):

if-statement → `if` (*expression*) *statement*
[`else` *statement*]

The **lexical structure** of a programming language is the structure of the language's words, which are usually called **tokens**. Thus, lexical structure is similar to spelling in a natural language. In the example of a C `if` statement, the words `if` and `else` are tokens. Other tokens in programming languages include identifiers (or names), symbols for operations, such as `+` and `*` and special punctuation symbols such as the semicolon (`;`) and the period (`.`).

In this book, we shall consider syntax and lexical structure together; a more detailed study can be found in Chapter 6.

1.4.2 Language Semantics

Syntax represents only the surface structure of a language and, thus, is only a small part of a language definition. The semantics, or meaning, of a language is much more complex and difficult to describe precisely. The first difficulty is that "meaning" can be defined in many different ways. Typically, describing the meaning of a piece of code involves describing the effects of executing the code, but there is no standard way to do this. Moreover, the meaning of a particular mechanism may involve interactions with other mechanisms in the language, so that a comprehensive description of its meaning in all contexts may become extremely complex.

To continue with our example of the C `if` statement, its semantics may be described in words as follows (adapted from Kernighan and Richie [1988]):

> An `if` statement is executed by first evaluating its expression, which must have an arithmetic or pointer type, including all side effects, and if it compares unequal to 0, the statement following the expression is executed. If there is an `else` part, and the expression is 0, the statement following the "else" is executed.

This description itself points out some of the difficulty in specifying semantics, even for a simple mechanism such as the `if` statement. The description makes no mention of what happens if the condition evaluates to 0, but there is no `else` part (presumably nothing happens; that is, the program continues at the point after the `if` statement). Another important question is whether the `if` statement is "safe" in the sense that there are no other language mechanisms that may permit the statements inside an `if` statement to be executed without the corresponding evaluation of the `if` expression. If so, then the if-statement provides adequate protection from errors during execution, such as division by zero:

```
if (x != 0) y = 1 / x;
```

Otherwise, additional protection mechanisms may be necessary (or at least the programmer must be aware of the possibility of circumventing the if expression).

The alternative to this informal description of semantics is to use a formal method. However, no generally accepted method, analogous to the use of context-free grammars for syntax, exists here either. Indeed, it is still not customary for a formal definition of the semantics of a programming language to be given at all. Nevertheless, several notational systems for formal definitions have been developed and are increasingly in use. These include **operational semantics**, **denotational semantics**, and **axiomatic semantics**.

Language semantics are implicit in many of the chapters of this book, but semantic issues are more specifically addressed in Chapters 7 and 11. Chapter 12 discusses formal methods of semantic definition, including operational, denotational, and axiomatic semantics.

1.5 Language Translation

For a programming language to be useful, it must have a **translator**—that is, a program that accepts other programs written in the language in question and that either executes them directly or transforms them into a form suitable for execution. A translator that executes a program directly is called an **interpreter**, while a translator that produces an equivalent program in a form suitable for execution is called a **compiler**.

As shown in Figure 1-8, interpretation is a one-step process, in which both the program and the input are provided to the interpreter, and the output is obtained.

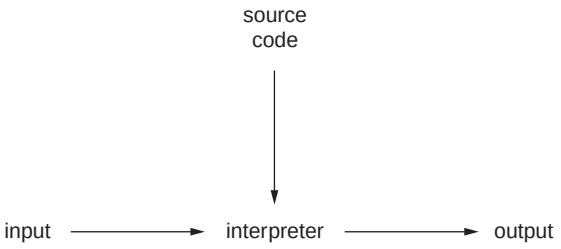


Figure 1.8 The interpretation process

An interpreter can be viewed as a simulator for a machine whose "machine language" is the language being translated.

Compilation, on the other hand, is at least a two-step process: the original program (or **source program**) is input to the compiler, and a new program (or **target program**) is output from the compiler. This target program may then be executed, if it is in a form suitable for direct execution (i.e., in machine language). More commonly, the target language is assembly language, and the target program must be translated by an **assembler** into an object program, and then **linked** with other object programs, and **loaded** into appropriate memory locations before it can be executed. Sometimes the target language is even another programming language, in which case a compiler for that language must be used to obtain an executable object program.

Alternatively, the target language is a form of low-level code known as **byte code**. After a compiler translates a program's source code to byte code, the byte code version of the program is executed by an interpreter. This interpreter, called a **virtual machine**, is written differently for different hardware architectures, whereas the byte code, like the source language, is machine-independent. Languages such as Java and Python compile to byte code and execute on virtual machines, whereas languages such as C and C++ compile to native machine code and execute directly on hardware.

The compilation process can be visualized as shown in Figure 1.9.

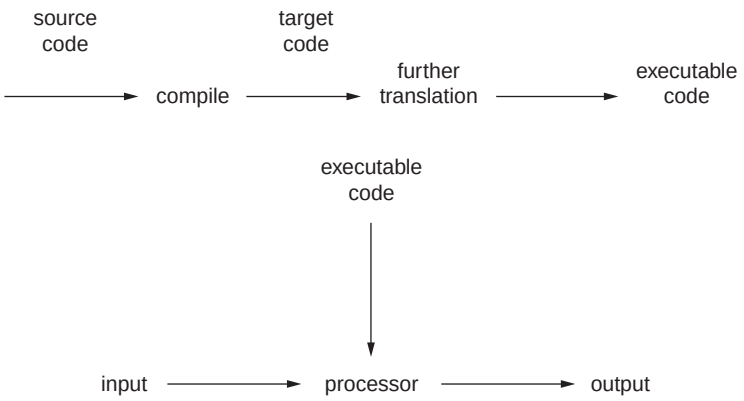


Figure 1.9 The compilation process

It is important to keep in mind that a language and the translator for that language are two different things. It *is* possible for a language to be defined by the behavior of a particular interpreter or compiler (a so-called **definitional** translator), but this is not common (and may even be problematic, in view of the need for a formal definition, as discussed in the last section). More often, a language definition exists independently, and a translator may or may not adhere closely to the language definition (one hopes the former). When writing programs one must always be aware of those features and properties that depend on a specific translator and are not part of the language definition. There are significant advantages to be gained from avoiding nonstandard features as much as possible.

A complete discussion of language translation can be found in compiler texts, but we will examine the basic front end of this process in Chapters 6–10.

1.6 The Future of Programming Languages

In the 1960s, some computer scientists dreamed of a single universal programming language that would meet the needs of all computer users. Attempts to design and implement such a language, however, resulted in frustration and failure. In the late 1970s and early 1980s, a different dream emerged—a dream that programming languages themselves would become obsolete, that new **specification languages** would be developed that would allow computer users to just say what they wanted to a system that would then find out how to implement the requirements. A succinct exposition of this view is contained in Winograd [1979]:

> Just as high-level languages enabled the programmer to escape from the intricacies of a machine's order code, higher level programming systems can provide help in understanding and manipulating complex systems and components. We need to shift our attention away from the detailed specification of algorithms, towards the description of the properties of the packages and objects with which we build. A new generation of programming tools will be based on the attitude that what we say in a programming system should be primarily declarative, not imperative: the fundamental use of a programming system is not in creating sequences of instructions for accomplishing tasks (or carrying out algorithms), but in expressing and manipulating descriptions of computational processes and the objects on which they are carried out. (Ibid., p. 393)

In a sense, Winograd is just describing what logic programming languages attempt to do. As you will see in Chapter 4, however, even though these languages can be used for quick prototyping, programmers still need to specify algorithms step by step when efficiency is needed. Little progress has been made in designing systems that can on their own construct algorithms to accomplish a set of given requirements.

Programming has, thus, not become obsolete. In a sense it has become even more important, since it now can occur at so many different levels, from assembly language to specification language. And with the development of faster, cheaper, and easier-to-use computers, there is a tremendous demand for more and better programs to solve a variety of problems.

What's the future of programming language design? Predicting the future is notoriously difficult, but it is still possible to extrapolate from recent trends. Two of the most interesting perspectives on the evolution of programming languages in the last 20 years come from a pair of second-generation Lisp programmers, Richard Gabriel and Paul Graham.

In his essay "The End of History and the Last Programming Language" [Gabriel 1996], Gabriel is puzzled by the fact that very high-level, mathematically elegant languages such as Lisp have not caught on in industry, whereas less elegant and even semantically unsafe languages such as C and C++ have become the standard. His explanation is that the popularity of a programming language is much more a function of the context of its use than of any of its intrinsic properties. To illustrate this point, he likens the spread of C in the programming community to that of a virus. The simple footprint of the C compiler and runtime environment and its connection to the UNIX operating system has allowed it to spread rapidly to many hardware platforms. Its conventional syntax and lack of mathematical elegance have appealed to a very wide range of programmers, many of whom may not necessarily have much mathematical sophistication. For these reasons, Gabriel concludes that C will be the ultimate survivor among programming languages well into the future.

Graham, writing a decade later in his book *Hacker and Painters* [Graham 2004] sees a different trend developing. He believes that major recent languages, such as Java, Python, and Ruby, have added features that move them further away from C and closer to Lisp. However, like C in an earlier era, each of these languages has quickly migrated into new technology areas, such as Web-based client/server applications and mobile devices. What then of Lisp itself? Like most writers on programming languages, Graham classifies them on a continuum, from fairly low level (C) to fairly high level (Java, Python, Ruby). But he then asks two interesting questions: If there is a range of language levels, which languages are at the highest level? And if there is a language at the highest level, and it still exists, why wouldn't people prefer to write their programs in it? Not surprisingly, Graham claims that Lisp, after 50 years, always has been and still is the highest-level language. He then argues, in a similar manner to Gabriel, that Lisp's virtues have been recognized only by the best programmers and by the designers of the aforementioned recent languages. However, Graham believes that the future of Lisp may lie in the rapid development of server-side applications.

Figure 1.10 shows some statistics on the relative popularity of programming languages since 2000. The statistics, which include the number of posts on these languages on comp.lang newsgroups for the years 2009, 2003, and 2000, lend some support to Graham's and Gabriel's analyses. (Comp newsgroups, originally formed on Usenet, provide a forum for discussing issues in technology, computing, and programming.)

```
    Mar 2009 (100d)       Feb 2003 (133 d)     Jan 2000 (365d)
    news.tuwien.ac.at     news.individual.net  tele.dk
    posts language        posts language       posts language
 1 14110 python           59814 java           229034 java
 2 13268 c                44242 c++            114769 basic
 3  9554 c++              27054 c              113001 perl
 4  9057 ruby             24438 python         102261 c++
 5  9054 java             23590 perl           79139 javascript
 6  5981 lisp             18993 javascript     70135 c
```

Figure 1.10 Popularity of programming languages (source: *www.complang.tuwien.ac.at/anton/comp.lang-statistics/*)

One thing is clear. As long as new computer technologies arise, there will be room for new languages and new ideas, and the study of programming languages will remain as fascinating and exciting as it is today.

Exercises

1.1 Explain why von Neumann's idea of storing a program in computer memory represented an advance for the operators of computers.

1.2 State a difficulty that machine language programmers faced when **(a)** translating their ideas into machine code, and **(b)** loading their code by hand into computer memory.

1.3 List at least three ways in which the use of assembly language represented an improvement for programmers over machine language.

1.4 An abstraction allows programmers to say more with less in their code. Justify this statement with two examples.

1.5 ALGOL was one of the first programming languages to achieve machine independence, but not independence from the von Neumann model of computation. Explain how this is so.

1.6 The languages Scheme, C++, Java, and Python have an integer data type and a string data type. Explain how values of these types are abstractions of more complex data elements, using at least one of these languages as an example.

1.7 Explain the difference between a data structure and an abstract data type (ADT), using at least two examples.

1.8 Define a recursive factorial function in any of the following languages (or in any language for which you have a translator): **(a)** Scheme, **(b)** C++, **(c)** Java, **(d)** Ada, or **(e)** Python.

1.9 Assembly language uses branch instructions to implement loops and selection statements. Explain why a `for` loop and an `if` statement in high-level languages represent an improvement on this assembly language technique.

1.10 What is the difference between the use of an index-based loop and the use of an iterator with an array? Give an example to support your answer.

1.11 List three reasons one would package code in a procedure or function to solve a problem.

1.12 What role do parameters play in the definition and use of procedures and functions?

1.13 In what sense does recursion provide additional abstraction capability to function definitions? Give an example to support your answer.
1.14 Explain what the `map` function does in a functional language. How does it provide additional abstraction capability in a programming language?
1.15 Which three properties characterize imperative programming languages?
1.16 How do the three properties in your answer to question 1.15 reflect the von Neumann model of computing?
1.17 Give two examples of lexical errors in a program, using the language of your choice.
1.18 Give two examples of syntax errors in a program, using the language of your choice.
1.19 Give two examples of semantic errors in a program, using the language of your choice.
1.20 Give one example of a logic error in a program, using the language of your choice.
1.21 Java and Python programs are translated to byte code that runs on a virtual machine. Discuss the advantages and disadvantages of this implementation strategy, as opposed to that of C++, whose programs translate to machine code.

Notes and References

The quote from A. N. Whitehead in Section 1.1 is in Whitehead [1911]. An early description of the von Neumann architecture and the use of a program stored as data to control the execution of a computer is in Burks, Goldstine, and von Neumann [1947]. A gentle introduction to the von Neumann architecture, and the evolution of computer hardware and programming languages is in Schneider and Gersting [2010].

References for the major programming languages used or mentioned in this text are as follows. The LC-3 machine architecture, instruction set, and assembly language are discussed in Patt and Patel [2003]. The history of FORTRAN is given in Backus [1981]; of Algol60 in Naur [1981] and Perlis [1981]; of Lisp in McCarthy [1981], Steele and Gabriel [1996], and Graham [2002]; of COBOL in Sammet [1981]; of Simula67 in Nygaard and Dahl [1981]; of BASIC in Kurtz [1981]; of PL/I in Radin [1981]; of SNOBOL in Griswold [1981]; of APL in Falkoff and Iverson [1981]; of Pascal in Wirth [1996]; of C in Ritchie [1996]; of C++ in Stroustrup [1994] [1996]; of Smalltalk in Kay [1996]; of Ada in Whitaker [1996]; of Prolog in Colmerauer and Roussel [1996]; of Algol68 in Lindsey [1996]; and of CLU in Liskov [1996]. A reference for the C programming language is Kernighan and Ritchie [1988]. The latest C standard is ISO 9899 [1999]. C++ is described in Stroustrup [1994] [1997], and Ellis and Stroustrup [1990]; an introductory text is Lambert and Nance [2001]; the international standard for C++ is ISO 14882-1 [1998]. Java is described in many books, including Horstmann [2006] and Lambert and Osborne [2010]; the Java language specification is given in Gosling, Joy, Steele, and Bracha [2005]. Ada exists in three versions: The original is sometimes called Ada83, and is described by its reference manual (ANSI-1815A [1983]); newer versions are Ada95 and Ada2005[2], and are described by their international standard (ISO 8652 [1995, 2007]). A standard text for Ada is Barnes [2006]. An introductory text on Python is Lambert [2010]. Common Lisp is presented in Graham [1996] and Seibel [2005]. Scheme is described in Dybvig [1996] and Abelson and Sussman [1996]; a language definition can be found in

[2]Since Ada95/2005 is an extension of Ada83, we will indicate only those features that are specifically Ada95/2005 when they are not part of Ada83.

Abelson et al. [1998]. Haskell is covered in Hudak [2000] and Thompson [1999]. The ML functional language (related to Haskell) is covered in Paulson [1996] and Ullman [1997]. The standard reference for Prolog is Clocksin and Mellish [1994]. The logic paradigm is discussed in Kowalski [1979], and the functional paradigm in Backus [1978] and Hudak [1989]. Smalltalk is presented in Lambert and Osborne [1997]. Ruby is described in Flanagan and Matsumoto [2008] and in Black [2009]. Erlang is discussed in Armstrong [2007].

Language translation techniques are described in Aho, Lam, Sethi, and Ullman [2006] and Louden [1997].

Richard Gabriel's essay on the last programming language appears in Gabriel [1996], which also includes a number of interesting essays on design patterns. Paul Graham's essay on high-level languages appears in Graham [2004], where he also discusses the similarities between the best programmers and great artists.

CHAPTER

Language Design Criteria

CHAPTER 2

What is good programming language design? By what criteria do we judge it? Chapter 1 emphasized human readability and mechanisms for abstraction and complexity control as key requirements for a modern programming language. Judging a language by these criteria is difficult, however, because the success or failure of a language often depends on complex interactions among many language mechanisms. Defining the "success" or "failure" of a programming language is also complex; for now, let's say that a language is successful if it satisfies any or all of the following criteria:

1. Achieves the goals of its designers
2. Attains widespread use in an application area
3. Serves as a model for other languages that are themselves successful

Practical matters not directly connected to language definition also have a major effect on the success or failure of a language. These include the availability, price, and quality of translators. Politics, geography, timing, and markets also have an effect. The C programming language has been a success at least partially because of the success of the UNIX operating system, which supported its use. COBOL, though chiefly ignored by the computer science community, continues as a significant language because of its use in industry, and because of the large number of legacy applications (old applications that continue to be maintained). The language Ada achieved immediate influence because of its required use in certain U.S. Defense Department projects. Java and Python have achieved importance through the growth of the Internet and the free distribution of these languages and their programming environments. The Smalltalk language never came into widespread use, but most successful object-oriented languages borrowed a large number of features from it.

Languages succeed for as many different reasons as they fail. Some language designers argue that an individual or small group of individuals have a better chance of creating a successful language because they can impose a uniform design concept. This was true, for example, with Pascal, C, C++, APL, SNOBOL, and LISP, but languages designed by committees, such as COBOL, Algol, and Ada, have also been successful.

When creating a new language, it's essential to decide on an overall goal for the language, and then keep that goal in mind throughout the entire design process. This is particularly important for special-purpose languages, such as database languages, graphics languages, and real-time languages, because the particular abstractions for the target application area must be built into the language design. However, it is true for general-purpose languages as well. For example, the designers of FORTRAN focused on efficient execution, whereas the designers of COBOL set out to provide an English-like nontechnical readability. Algol60 was designed to provide a block-structured language for describing algorithms and

Pascal was designed to provide a simple instructional language to promote top-down design. Finally, the designer of C++ focused on the users' needs for greater abstraction while preserving efficiency and compatibility with C.

Nevertheless, it is still extremely difficult to describe good programming language design. Even noted computer scientists and successful language designers offer conflicting advice. Niklaus Wirth, the designer of Pascal, advises that simplicity is paramount (Wirth [1974]). C. A. R. Hoare, a prominent computer scientist and co-designer of a number of languages, emphasizes the design of individual language constructs (Hoare [1973]). Bjarne Stroustrup, the designer of C++, notes that a language cannot be merely a collection of "neat" features (Stroustrup [1994], page 7). Fred Brooks, a computer science pioneer, maintains that language design is similar to any other design problem, such as designing a building (Brooks [1996]).

In this chapter, we introduce some general design criteria and present a set of more detailed principles as potential aids to the language designer and ultimately the language user. We also give some specific examples to emphasize possible good and bad choices, with the understanding that there often is no general agreement on these issues.

2.1 Historical Overview

In the early days of programming, machines were extremely slow and memory was scarce. Program speed and memory usage were, therefore, the prime concerns. Also, some programmers still did not trust compilers to produce efficient executable code (code that required the fewest number of machine instructions and the smallest amount of memory). Thus, one principal design criterion really mattered: **efficiency of execution**. For example, FORTRAN was specifically designed to allow the programmer to generate compact code that executed quickly. Indeed, with the exception of algebraic expressions, early FORTRAN code more or less directly mapped to machine code, thus minimizing the amount of translation that the compiler would have to perform. Judging by today's standards, creating a high-level programming language that required the programmer to write code nearly as complicated as machine code might seem counterproductive. After all, the whole point of a high-level programming language is to make life easier for the programmer. In the early days of programming, however, **writability**—the quality of a language that enables a programmer to use it to express a computation clearly, correctly, concisely, and quickly—was always subservient to efficiency. Moreover, at the time that FORTRAN was developed, programmers were less concerned about creating programs that were easy for people to read and write, because programs at that time tended to be short, written by one or a few programmers, and rarely revised or updated except by their creators.

By the time COBOL and Algol60 came on the scene, in the 1960s, languages were judged by other criteria than simply the efficiency of the compiled code. For example, Algol60 was designed to be suitable for expressing algorithms in a logically clear and concise way—in other words, unlike FORTRAN, it was designed for easy reading and writing by people. To achieve this design goal, Algol60's designers incorporated block structure, structured control statements, a more structured array type, and recursion. These features of the language were very effective. For example, C. A. R. Hoare understood how to express his QUICKSORT algorithm clearly only after learning Algol60.

COBOL's designers attempted to improve the readability of programs by trying to make them look like ordinary written English. In fact, the designers did not achieve their goal. Readers were not able to

easily understand the logic or behavior of COBOL programs. They tended to be so long and verbose that they were harder to read than programs written in more formalized code. But human readability was, perhaps for the first time, a clearly stated design goal.

In the 1970s and early 1980s, language designers placed a greater emphasis on simplicity and abstraction, as exhibited by Pascal, C, Euclid, CLU, Modula-2, and Ada. Reliability also became an important design goal. To make their languages more reliable, designers introduced mathematical definitions for language constructs and added mechanisms to allow a translator to partially prove the correctness of a program as it performed the translation. However, such program verification systems had limited success, primarily because they necessitated a much more complex language design and translator, and made programming in the language more difficult than it would be otherwise. However, these efforts did lead to one important related development, strong data typing, which has since become standard in most languages.

In the 1980s and 1990s, language designers continued to strive for logical or mathematical precision. In fact, some attempted to make logic into a programming language itself. Interest in functional languages has also been rekindled with the development of ML and Haskell and the continued popularity of Lisp/Scheme.

However, the most influential design criterion of the last 25 years has come from the object-oriented approach to abstraction. As the popularity of the object-oriented languages C++, Java, and Python soared, language designers became ever more focused on abstraction mechanisms that support the modeling of real-word objects, the use of libraries to extend language mechanisms to accomplish specific tasks, and the use of object-oriented techniques to increase the flexibility and reuse of existing code.

Thus, we see that design goals have changed through the years, as a response both to experience with previous language designs and to the changing nature of the problems addressed by computer science. Still, readability, abstraction, and complexity control remain central to nearly every design decision.

Despite the importance of readability, programmers still want their code to be efficient. Today's programs process enormous data objects (think movies and Web searches) and must run on miniature computers (think smart phones and tablets). In the next section, we explore the continuing relevance of this criterion to language design.

2.2 Efficiency

Language designers nearly always claim that their new languages support efficient programs, but what does that really mean? Language designers usually think of the efficiency of the target code first. That is, they strive for a language design that allows a translator to generate efficient executable code. For example, a designer interested in efficient executable code might focus on statically typed variables, because the data type of such a variable need not be checked at runtime. Consider the following Java code segment, which declares and initializes the variables `i` and `s` and then uses them in later computations.

```
int i = 10;
String s = "My information";

// Do something with i and s
```

Because the data types of these two variables are known at compile time, the compiler can guarantee that only integer and string operations will be performed on them. Thus, the runtime system need not pause to check the types of these values before executing the operations.

In contrast, the equivalent code in Python simply assigns values to typeless variables:

```
i = 10
s = "My information"

# Do something with i and s
```

The absence of a type specification at compile time forces the runtime system to check the type of a Python variable's value before executing any operations on it. This causes execution to proceed more slowly than it would if the language were statically typed.

As another example, the early dialects of FORTRAN supported static storage allocation only. This meant that the memory requirements for all data declarations and subroutine calls had to be known at compile time. The number of positions in an array had to be declared as a constant, and a subroutine could not be recursive (a nonrecursive subroutine needs only one block of memory or activation record for its single activation, whereas a recursive routine requires potentially many activation records, whose number will vary with the number of recursive calls). This restriction allowed the memory for the program to be formatted just once, at load time, thus saving processing time as well as memory. In contrast, most modern languages require dynamic memory allocation at runtime, both for recursive subroutines and for arrays whose sizes cannot be determined until runtime. This support mechanism, whether it takes the form of a system stack or a system heap (to be discussed in Chapters 7 and 10), can incur substantial costs in memory and processing time.

Another view of efficiency is **programmer efficiency**: How quickly and easily can a person read and write a program in a particular language? A programmer's efficiency is greatly affected by a language's **expressiveness**: How easy is it to express complex processes and structures? Or, to put it another way: How easily can the design in the programmer's head be mapped to actual program code? This is clearly related to the language's abstraction mechanisms. The structured control statements of Algol and its successors are wonderful examples of this kind of expressiveness. If the programmer can describe making a choice or repeating a series of steps in plain English, the translation (by hand) of this thought process to the appropriate `if` statement or `while` loop is almost automatic.

The conciseness of the syntax also contributes to a language's programming efficiency. Languages that require a complex syntax are often considered less efficient in this regard. For designers especially concerned with programmer efficiency, Python is an ideal language. Its syntax is extremely concise. For example, unlike most languages, which use statement terminators such as the semicolon and block delimiters such as the curly braces in control statements, Python uses just indentation and the colon. Figure 2.1 shows equivalent multiway `if` statements in Python and C to illustrate this difference.

C

```
if (x > 0){
    numSolns = 2;
    r1 = sqrt (x);
    r2 = - r1;
}
else if (x == 0){
    numSolns = 1;
    r1 = 0.0;
}
else
    numSolns = 0;
```

Python

```
if x > 0.0:
    numSolns = 2
    r1 = sqrt(x)
    r2 = -r1
elif x = 0.0:
    numSolns = 1
    r1 = 0.0
else:
    numSolns = 0
```

Figure 2.1 Comparing the syntax of multiway `if` statements in C and Python

The absence of explicit data types in variable declarations in some languages allows for more concise code, and the support for recursion and dynamic data structures in most languages provides an extra layer of abstraction between the programmer and the machine. Of course, an exclusive focus on programmer efficiency can compromise other language principles, such as efficiency of execution and reliability.

Indeed, reliability can be viewed as an efficiency issue itself. A program that is not reliable can incur many extra costs—modifications required to isolate or remove the erroneous behavior, extra testing time, plus the time required to correct the effects of the erroneous behavior. If the program is unreliable, it may even result in a complete waste of the development and coding time. This kind of inefficiency is a resource consumption issue in software engineering. In this sense, programmer efficiency also depends on the ease with which errors can be found and corrected and new features added. Viewed in this way, the ease of initially writing code is a less important part of efficiency. Software engineers estimate that 90% of their time is spent on debugging and maintenance, and only 10% on the original coding of a program. Thus, maintainability may ultimately be the most important index of programming language efficiency.

Among the features of a programming language that help to make programs readable and maintainable, probably the most important is the concept of regularity. We turn to this in the next section.

2.3 Regularity

Regularity is a somewhat ill-defined quality. Generally, however, it refers to how well the features of a language are integrated. Greater regularity implies fewer unusual restrictions on the use of particular constructs, fewer strange interactions between constructs, and fewer surprises in general in the way language features behave. Programmers usually take the regularity of a language for granted, until some feature causes a program to behave in a manner that astonishes them. For this reason, languages that satisfy the criterion of regularity are said to adhere to a principle of least astonishment.

Often regularity is subdivided into three concepts that are more well-defined: generality, orthogonal design, and uniformity. A language achieves **generality** by avoiding special cases in the availability or use of constructs and by combining closely related constructs into a single more general one. **Orthogonal** is a term borrowed from mathematics, where it refers to lines that are perpendicular. More generally, it is sometimes used to refer to two things that travel in independent directions, or that

function in completely separate spheres. In computer science, an **orthogonal design** allows language constructs to be combined in any meaningful way, with no unexpected restrictions or behavior arising as a result of the interaction of the constructs, or the context of use. The term **uniformity** refers to a design in which similar things look similar and have similar meanings and, inversely, in which different things look different.

These dimensions of regularity are explained with examples in the following sections. As you will see, the distinctions between the three are sometimes more a matter of viewpoint than actual substance, and you can always classify a feature or construct as irregular if it lacks just one of these qualities.

2.3.1 Generality

A language with generality avoids special cases wherever possible. The following examples illustrate this point.

Procedures and functions Function and procedure definitions can be nested in other function and procedure definitions in Pascal. They can also be passed as parameters to other functions and procedures. However, Pascal functions and procedures cannot be assigned to variables or stored in data structures. In C, one can create pointers to functions to treat them as data, but one cannot nest function definitions. By contrast, Python and most functional languages, such as Scheme and Haskell, have a completely general way of treating functions (nested functions definitions and functions as first-class data objects).

Operators In C, two structures or arrays cannot be directly compared using the equality operator ==, but must be compared element by element. Thus, the equality operator lacks generality. This restriction has been removed in Smalltalk, Ada, Python, and (partially) in C++ and Java. More generally, many languages have no facility for extending the use of predefined operators (like == or +) to new data types. Some languages, however, such as Haskell, permit overloading of existing operators and allow new operators to be created by the user. In such languages, operators can be viewed as having achieved complete generality.

Constants In Pascal, the values assigned to constants may not be computed by expressions, while in Modula-2, these computing expressions may not include function calls. Ada, however, has a completely general constant declaration facility.

2.3.2 Orthogonality

In a language that is truly orthogonal, language constructs do not behave differently in different contexts. Thus, restrictions that are context dependent are considered nonorthogonal, while restrictions that apply regardless of context exhibit a mere lack of generality. Here are some examples of lack of orthogonality:

Function return types In Pascal, functions can return only scalar or pointer types as values. In C and C++, values of all data types, except array types, can be returned from a function (indeed, arrays are treated in C and C++ differently from all other types). In Ada, Python, and most functional languages, this lack of orthogonality is removed.

Placement of variable declarations In C, local variables can only be defined at the beginning of a block (compound statement),[1] while in C++ variable definitions can occur at any point inside a block (but, of course, before any uses).

Primitive and reference types In Java, values of scalar types (`char`, `int`, `float`, and so forth) are not objects, but all other values are objects. Scalar types are called **primitive types**, whereas object types are called **reference types**. Primitive types use **value semantics**, meaning that a value is copied during assignment. Reference types use **reference semantics**, meaning that assignment produces two references to the same object. Also, in Java, collections of objects are treated differently from collections of primitive types. Among Java's collections, only arrays can contain primitive values. Primitive values must be placed in special wrapper objects before insertion into other types of collections. In Smalltalk and Python, by contrast, all values are objects and all types are reference types. Thus, these languages use reference semantics only, and collections of objects are treated in an orthogonal manner.

Orthogonality was a major design goal of Algol68, and it remains the best example of a language where constructs can be combined in all meaningful ways.

2.3.3 Uniformity

The term **uniformity** refers to the consistency of appearance and behavior of language constructs. A language is uniform when similar things look similar or behave in a similar manner, but lacks uniformity when dissimilar things actually look similar or behave similarly when they should not. Here are some examples of the lack of uniformity.

The extra semicolon In C++, a semicolon is necessary after a class definition, but forbidden after a function definition:

```
class A { ... } ; // semicolon required
int f () { ... } // semicolon forbidden
```

The reason the semicolon must appear after a class definition is that the programmer can include a list of variable names before the semicolon, if so desired, as is the case with `struct` declarations in C.

Using assignment to return a value In Pascal, return statements in functions look like assignments to variables. This is a case where different things should look different, but instead look confusingly alike. For example:

```
function f : boolean;
    begin
    ...
    f := true;
    end;
```

Most other languages use a dedicated `return` statement for returning values from functions.

[1]The ISO C Standard (ISO 9899 [1999]) removed this restriction.

2.3.4 Causes of Irregularities

Why do languages exhibit such irregularities at all? Surely the language designers did not intentionally set out to create strange restrictions, interactions, and confusions. Indeed, many irregularities are case studies in the difficulties of language design.

Take, for example, the problem with the semicolon in the C++ class declaration, noted as a lack of uniformity above. Since the designers of C++ attempted to deviate from C as little as possible, this irregularity was an essential byproduct of the need to be compatible with C. The lack of generality of functions in C and Pascal was similarly unavoidable, since both of these languages opt for a simple stack-based runtime environment (as explained in Chapter 10). Without some restriction on functions, a more general environment would have been required, and that would in turn have compromised the simplicity and efficiency of implementations. The irregularity of primitive types and reference types in Java is also the result of the designer's concern with efficiency.

On the other hand, it is worth noting that too great a focus on a particular goal, such as generality or orthogonality, can itself be dangerous. A case in point is Algol68. While this language was largely successful in meeting the goals of generality and orthogonality, many language designers believe that these qualities led to an unnecessarily obscure and complex language.

So how do you tell if a lack of regularity is reasonable? Consider the basic design goals of the language, and then think about how they would be compromised if the irregularity were removed. If you cannot justify a lack of regularity in this way, then it is probably a design flaw.

2.4 Security

As we have seen, the reliability of a programming language can be seriously affected if restrictions are not imposed on certain features. In Pascal, for instance, pointers are specifically restricted to reduce security problems, while in C they are much less restricted and, thus, much more prone to misuse and error. In Java, by contrast, pointers have been eliminated altogether (they are implicit in all object allocation), but at some cost of a more complicated runtime environment.

Security is closely related to reliability. A language designed with an eye toward security both discourages programming errors and allows errors to be discovered and reported. A concern for security led language designers to introduce types, type checking, and variable declarations into programming languages. The idea was to "maximize the number of errors that could not be made" by the programmer (Hoare [1981]).

However, an exclusive focus on security can compromise both the expressiveness and conciseness of a language, and typically forces the programmer to laboriously specify as many things as possible in the actual code. For example, LISP and Python programmers often believe that static type-checking and variable declarations make it difficult to program complex operations or provide generic utilities that work for a wide variety of data. On the other hand, in industrial, commercial, and defense applications, there is often a need to provide additional language features for security. Perhaps the real issue is how to design languages that are secure and that also allow for maximum expressiveness and generality. Examples of an advance in this direction are the languages ML and Haskell, which are functional in approach, allow multityped objects, do not require declarations, and yet perform static type-checking.

Strong typing, whether static or dynamic, is only one component of safety. Several modern languages, such as Python, Lisp, and Java, go well beyond this and are considered to be **semantically safe**.

That is, these languages prevent a programmer from compiling or executing any statements or expressions that violate the definition of the language. By contrast, languages such as C and C++ are not semantically safe. For example, an array index out of bounds error causes either a compile-time error or a runtime error in Python, Lisp, and Java, whereas the same error can go undetected in C and C++. Likewise, in languages such as C and C++, the programmer's failure to recycle dynamic storage can cause memory leaks, whereas in Python, Lisp, and Java, automatic garbage collection prevents this type of situation from ever occurring.

2.5 Extensibility

An **extensible** language allows the user to add features to it. Most languages are extensible at least to some degree. For example, almost all languages allow the programmer to define new data types and new operations (functions or procedures). Some languages allow the programmer to include these new resources in a larger program unit, such as a package or module. The new data types and operations have the same syntax and behavior as the built-in ones.

The designers of modern languages such as Java and Python also extend the built-in features of the language by providing new releases on a regular basis. These language extensions typically are backward-compatible with programs written in earlier versions of the language, so as not to disturb legacy code; occasionally, however, some earlier features become "deprecated," meaning that they may not be supported in future releases of the language.

Very few languages allow the programmer to add new syntax and semantics to the language itself. One example is LISP (see Graham [2002]). Not only can Lisp programmers add new functions, classes, data types, and packages to the base language, they can also extend Lisp's syntax and semantics via a macro facility. A **macro** specifies the syntax of a piece of code that expands to other, standard Lisp code when the interpreter or compiler encounters the first piece of code in a program. The expanded code may then be evaluated according to new semantic rules. For example, Common Lisp includes a quite general `do` loop for iteration but no simple `while` loop. That's no problem for the Lisp programmer, who simply defines a macro that tells the interpreter what to do when it encounters a `while` loop in a program. Specifically, the interpreter or compiler uses the macro to generate the code for an equivalent `do` loop wherever a `while` loop occurs in a program. Figure 2.2 shows LISP code segments for a `do` loop and an equivalent `while` loop that compute the greatest common divisor of two integers `a` and `b`.

```
(do ()                                  (while (> b 0)
    ((= 0 b))                             (let ((temp b))
  (let ((temp b))                           (setf b (mod a b))
    (setf b (mod a b))                      (setf a temp)))
    (setf a temp)))
```

Figure 2.2 A Lisp `do` loop and a Lisp `while` loop for computing the greatest common divisor of `a` and `b`

Most readers will understand what the `while` loop in Figure 2.2 does without knowing Lisp, for it is very similar in structure to the `while` loop of C++, Java, or Python. However, the `do` loop is more complicated. The empty parentheses on the first line omit the variable initializers and updates that would normally go there. The Boolean expression on the second line denotes a termination condition rather

than a continuation condition. The rest of the code is the body of the loop. The `while` loop in Figure 2.3 would clearly be a simpler and better fit for this problem, but it does not already exist in Lisp.

Figure 2.3 shows the code for a macro definition that would enable Lisp to translate a `while` loop to an equivalent `do` loop.

```
(defmacro while (condition &rest body)
  '(do ()
       ((not ,condition))
       ,@body))
```

Figure 2.3 A Lisp macro that defines how to translate a Lisp `while` loop to a Lisp `do` loop

This code extends the syntax of the language by adding a new keyword, `while`, to it and telling the interpreter/compiler what to do with the rest of the code following the keyword in the expression. In this case, an expression that includes the keyword `do` and its appropriate remaining elements is substituted for the `while` loop's code during interpretation or compilation. LISP, thus, allows programmers to become designers of their own little languages when the syntax and semantics of the base language do not quite suit their purposes. Graham [2004] and Seibel [2005] provide excellent discussions of this powerful capability.

We now conclude this chapter with two case studies, where we briefly examine the design goals and criteria of two language developers, Bjarne Stroustup [1994; 1996] and Guido van Rossum [2003]. They are primarily responsible for the popular languages C++ and Python, respectively.

2.6 C++: An Object-Oriented Extension of C

Bjarne Stroustrup's interest in designing a new programming language came from his experience as a graduate student at Cambridge University, England, in the 1970s. His research focused on a simulator for software running on a distributed system (a simulator is a program that models the behavior of a real-world object; in this case, the simulator pretended to be an actual set of distributed computers). Stroustrup first wrote this simulator in Simula67, because he found its abstraction mechanisms (primarily the class construct) to be ideal for expressing the conceptual organization of his particular simulator. In addition, he found the strong type-checking of Simula to be of considerable help in correcting conceptual flaws in the program design. He also found that the larger his program grew, the more helpful these features became.

Unfortunately, he also found that his large program was hard to compile and link in a reasonable time, and so slow as to be virtually impossible to run. The compilation and linking problems were annoying, but the runtime problems were catastrophic, since he was unable to obtain the runtime data necessary to complete his PhD thesis. After some study, he concluded that the runtime inefficiencies were an inherent part of Simula67, and that he would have to abandon that language in order to complete the project. He then rewrote his entire program in BCPL (a low-level predecessor of C), which was a language totally unsuited for the proper expression of the abstractions in his program. This effort was such a painful experience that he felt he should never again attempt such a project with the languages that were then in existence.

But, in 1979, as a new employee at Bell Labs in New Jersey, Stroustrup was faced with a similar task: simulating a UNIX kernel distributed over a local area network. Realizing that a new programming language was necessary, he decided to base it on C, with which he had become familiar at Bell Labs (considered the home of C and UNIX), and to add the class construct from Simula67 that he had found so useful.

Stroustrup chose C because of its flexibility, efficiency, availability, and portability (Stroustrup [1994], page 43). These qualities also fit nicely with the following design goals of the new language (ibid., page 23):

1. Support for good program development in the form of classes, inheritance, and strong type-checking
2. Efficient execution on the order of C or BCPL
3. Highly portable, easily implemented, and easily interfaced with other tools

2.6.1 C++: First Implementations

Stroustrup's first implementation of the new language came in 1979–80, in the form of a preprocessor, named Cpre, which transformed the code into ordinary C. The language itself, called **C with Classes**, added features that met most of the three goals listed above. Stroustrup called it a "medium success" (ibid., page 63). However, it did not include several important features, such as dynamic binding of methods (or virtual functions, as explained Chapters 5 and 9), type parameters (or templates, as explained in Chapters 5 and 9), or general overloading (as explained in Chapter 7). Stroustrup decided to expand the language in these and other directions, and to replace the preprocessor by a more sophisticated true compiler (which still generated C code as its target, for portability). The language that emerged in 1985 was called C++. Its compiler, **Cfront**, is still the basis for many compilers today. Stroustrup also expanded on the basic goals of the language development effort, to explicitly include the following goals:

1. C compatibility should be maintained as far as practical, but should not be an end in itself ("there would be no *gratuitous* incompatibilities," ibid., page 101).
2. The language should undergo incremental development based firmly in practical experience. That is, C++ should undergo an evolution driven by real programming problems, not theoretical arguments, while at the same time avoiding "featurism" (adding cool features just because it is possible).
3. Any added feature must entail zero cost in runtime efficiency, or, at the very least, zero cost to programs that do not use the new feature (the "zero-overhead" rule, ibid., page 121).
4. The language should not force any one style of programming on a programmer; that is, C++ should be a "multiparadigm" language.
5. The language should maintain and strengthen its stronger type-checking (unlike C). ("No implicit violations of the static type system," ibid., page 201.)
6. The language should be learnable in stages; that is, it should be possible to program in C++ using some of its features, without knowing anything about other, unused, features.
7. The language should maintain its compatibility with other systems and languages.

At this stage in its development, the language included dynamic binding, function and operator overloading, and improved type-checking, but not type parameters, exceptions, or multiple inheritance. These would all come later.

2.6.2 C++: Growth

In late 1985, Stroustrup published the first edition of his book on C++. In the meantime, Cpre and Cfront had already been distributed for educational purposes essentially at no cost, and interest in the language had begun to grow in industry. Thus, a first commercial implementation was made available in 1986. In 1987, the first conference specifically on C++ was organized by USENIX (the UNIX users' association). By October 1988, Stroustrup estimates that there were about 15,000 users of the language, with PC versions of C++ compilers appearing that same year. Between October 1979, when Stroustrup first began using Cpre himself and October 1991, Stroustrup estimates that the number of C++ users doubled every seven and a half months. The success of the language indicated to Stroustrup and others that a concerted attempt at creating a standard language definition was necessary, including possible extensions that had not yet been implemented.

2.6.3 C++: Standardization

Because C++ was rapidly growing in use, was continuing to evolve, and had a number of different implementations, the standardization presented a problem. Moreover, because of the growing user pool, there was some pressure to undertake and complete the effort as quickly as possible. At the same time, Stroustrup felt that the evolution of the language was not yet complete and should be given some more time.

Nevertheless, Stroustrup produced a reference manual in 1989 (the *Annotated Reference Manual*, or *ARM*; Ellis and Stroustrup [1990]) that included all of C++ up to that point, including proposals for new exception-handling and template (type parameter) mechanisms (multiple inheritance had already been added to the language and Cfront that same year). In 1990 and 1991, respectively, ANSI and ISO standards committees were convened; these soon combined their efforts and accepted the ARM as the "base document" for the standardization effort.

Then began a long process of clarifying the existing language, producing precise descriptions of its features, and adding features and mechanisms (mostly of a minor nature after the *ARM*) whose utility was agreed upon. In 1994, a major development was the addition of a standard library of containers and algorithms (the Standard Template Library, or STL). The standards committees produced two drafts of proposed standards in April 1995 and October 1996 before adopting a proposed standard in November 1997. This proposal became the actual ANSI/ISO standard in August 1998.

2.6.4 C++: Retrospective

Why was C++ such a success? First of all, it came on the scene just as interest in object-oriented techniques was exploding. Its straightforward syntax, based on C, was tied to no particular operating environment. The semantics of C++ incurred no performance penalty. These qualities allowed C++ to bring object-oriented features into the mainstream of computing. The popularity of C++ was also enhanced by its flexibility, its hybrid nature, and the willingness of its designer to extend its features based on practical experience. (Stroustrup lists Algol68, CLU, Ada, and ML as major influences after C and Simula67).

Is it possible to point to any "mistakes" in the design of C++? Many people consider C++ to be, like Ada and PL/I before it, a "kitchen-sink" language, with too many features and too many ways of doing similar things. Stroustrup justifies the size of C++ as follows (Stroustrup [1994], page 199):

The fundamental reason for the size of C++ is that it supports more than one way of writing programs. . . . This flexibility is naturally distasteful to people who believe that there is exactly one right way of doing things. . . . C++ is part of a trend towards greater language complexity to deal with the even greater complexity of the programming tasks attempted.

Stroustrup does, however, list one fact in the "biggest mistake" category (ibid., page 200)—namely, that C++ was first defined and implemented without any standard library (even without strings). This was, of course, corrected with the 1998 C++ standard. However, Java has shown that new languages must include an extensive library with graphics, user interfaces, networking, and concurrency. That is perhaps the biggest challenge for the C++ community in the future. Stroustrup himself comments (ibid., page 208): "In the future, I expect to see the major activity and progress shift from the language proper . . . to the tools, environments, libraries, applications, etc., that depend on it and build on it."

2.7 Python: A General-Purpose Scripting Language

Guido van Rossum was part of a team at Centrum voor Wiskunde en Informatica (CWI) in The Netherlands that worked on the design and implementation of the programming language ABC in the mid-1980s. The target users of ABC were scientists and engineers who were not necessarily trained in programming or software development. In 1986, van Rossum brought his ideas from working on ABC to another project at CWI, called Amoeba. Ameoba was a distributed operating system in need of a scripting language. Naming this new language Python, van Rossum developed a translator and virtual machine for it. Another of his goals was to allow Python to act as a bridge between systems languages such as C and shell or scripting languages such as Perl. Consequently, he set out to make Python's syntax simpler and cleaner than that of ABC and other scripting languages. He also planned to include a set of powerful built-in data types, such as strings, lists, tuples (a type of immutable list), and dictionaries. The dictionary, essentially a set of key/value pairs implemented via hashing, was modeled on a similar structure in Smalltalk but not present in other languages in the 1980s. The dictionary, which was also later implemented by Java's map classes, proved especially useful for representing collections of objects organized by content or association rather than by position.

2.7.1 Python: Simplicity, Regularity, and Extensibility

Python is easy for nonprogrammers to learn and use in part because it is based on a small but powerful set of primitive operations and data types that can be easily extended. The novice first learns this small set of resources and then extends them by either defining new resources or importing resources from libraries defined by others who work in the relevant domain. However, unlike in some other languages, each new unit of abstraction, whether it is the function, the class, or the module, retains a simple, regular syntax. These features reduce the cognitive overload on the programmer, whether a novice or an expert. Guido van Rossum believed that lay people (that is, professionals who are nonprogrammers, but also students who just want to learn programming) should be able to get their programs up and

running quickly; a simple, regular syntax and a set of powerful data types and libraries has helped to achieve that design goal.

2.7.2 Python: Interactivity and Portability

A principal pragmatic consideration in the design of Python was the need to create a language for users who do not typically write large systems, but instead write short programs to experiment and try out their ideas. Guido van Rossum saw that a development cycle that provides immediate feedback with minimal overhead for input/output operations would be essential for these users. Thus, Python can be run in two modes: expressions or statements can be run in a Python shell for maximum interactivity, or they can be composed into longer scripts that are saved in files to be run from a terminal command prompt. The difference in interactivity here is just one of degree; in either case, a programmer can try out ideas very quickly and then add the results to an existing body of code. This experimental style in fact supports the iterative growth of reasonably large-scale systems, which can be developed, tested, and integrated in a bottom-up manner.

Another consideration in the design of Python was the diversity of the intended audience for the new language. The ideal language for this group of programmers would run on any hardware platform and support any application area. These goals are accomplished in two ways. First, the Python compiler translates source code to machine-independent byte code. A Python virtual machine (PVM) or runtime environment is provided for each major hardware platform currently in use, so the source code and the byte code can migrate to new platforms without changes. Second, application-specific libraries support programs that must access databases, networks, the Web, graphical user interfaces, and other resources and technologies. Over the years, many of these libraries have become integral parts of new Python releases, with new ones being added on a regular basis.

2.7.3 Python: Dynamic Typing vs. Finger Typing

In our discussion thus far, we have seen that Python appears to combine the elements of simple, regular syntax found in Lisp (although without the prefix, fully parenthesized expressions), Lisp's interactivity and extensibility, and the support of portability, object orientation, and modern application libraries found in Java. All of these features serve to bridge the gap, as van Rossum intended, between a systems language like C and a shell language like Perl. However, there is one other language element that, in van Rossum's opinion, makes the difference between fast, easy program development and slow, tedious coding. Most conventional languages, such as Ada, C, C++, and Java, are statically typed. In these languages, the programmer must specify (and actually type on the keyboard) the data type of each new variable, parameter, function return value, and component nested within a data structure. According to van Rossum, this restriction might have a place in developing safe, production-quality system software. In the context of writing short, experimental programs, however, static typing inevitably results in needless **finger typing** by the programmer. This excess work requirement, in turn, hampers programmer productivity. Instead of helping to realize the goal of saying more with less, static typing forces the programmer to say less with more.

Guido van Rossum's solution to this problem was to incorporate in Python the dynamic typing mechanism found in Lisp and Smalltalk. According to this scheme, all variables are untyped: any variable can name any thing, but all things or values have a type. The checking of their types does occur

but is deferred until runtime. In the case of Python, the PVM examines the types of the operands before running any primitive operation on them. If a type error is encountered, an exception is always raised. Thus, it is not correct to say that Python and other dynamically typed languages are weakly typed; the types of all values are checked at runtime.

The real benefit of deferring type checking until runtime is that there is less overhead for the programmer, who can get a code segment up and running much faster. The absence of manifest or explicit types in Python programs also helps to reduce their size; 20,000 lines of C++ code might be replaced by 5,000 lines of Python code. According to van Rossum, when you can say more with less, your code is not only easier to develop but also easier to maintain.

2.7.4 Python: Retrospective

Certainly van Rossum did not intend Python to replace languages such as C or C++ in developing large or time-critical systems. Because Python does not translate to native machine code and must be type-checked at runtime, it would not be suitable for time-critical applications. Likewise, the absence of static type checking can also be a liability in the testing and verification of a large software system. For example, the fact that no type errors happen to be caught during execution does not imply that they do not exist. Thorough testing of all possible branches of a program is required in order to verify that no type errors exist.

Python has penetrated into many of the application areas that Java took by storm in the late 1990s and in the first decade of the present century. Python's design goal of ease of use for a novice or nonprogrammer has largely been achieved. The language is now in widespread use, and van Rossum notes that it has made some inroads as an introductory teaching language in schools, colleges, and universities. The support for starting simple, getting immediate results, and extending solutions to any imaginable application area continues to be the major source of Python's appeal.

Exercises

2.1 Provide examples of one feature (in a language of your choice) that promotes and one feature that violates each of the following design principles:

- Efficiency
- Extensibility
- Regularity
- Security

2.2 Provide one example each of orthogonality, generality, and uniformity in the language of your choice.

2.3 Ada includes a `loop . . . exit` construct. PL/I also includes a similar `loop . . . break` construct. Is there a similar construct in Python or Java? Are these constructs examples of any of the design principles?

2.4 In Java, integers can be assigned to real variables, but not vice versa. What design principle does this violate? In C, this restriction does not exist. What design principle does this violate?

2.5 Choose a feature from a programming language of your choice that you think should be removed. Why should the feature be removed? What problems might arise as a result of the removal?

2.6 Choose a feature that you think should be added to a programming language of your choice. Why should the feature be added? What needs to be stated about the feature to specify completely its behavior and interaction with other features?

2.7 In Ada, the `end` reserved word must be qualified by the kind of block that it ends: `if ... then ... end if`, `loop ... end loop`, and so on. In Python, begin/end blocks of code are marked by indentation. Discuss the effect these conventions have on readability, writability, and security.

2.8 Should a language require the declaration of variables? Languages such as Lisp and Python allow variable names to be used without declarations, while C, Java, and Ada require all variables to be declared. Discuss the requirement that variables should be declared from the point of view of readability, writability, efficiency, and security.

2.9 Languages such as Lisp and Python are dynamically typed, whereas languages such as C++ and Java are statically typed. Discuss the costs and benefits of each kind of typing.

2.10 The semicolon was used as an example of a nonuniformity in C++. Discuss the use of the semicolon in C. Is its use entirely uniform?

2.11 Two opposing views on comments in programs could be stated as follows:

(a) A program should always contain elaborate comments to make it readable and maintainable.

(b) A program should be as much as possible self-documenting, with comments added sparingly only where the code itself might be unclear.

Discuss these two viewpoints from the point of view of language design. What design features might aid one viewpoint but not the other? What might aid both?

2.12 Here are two more opposing statements:

(a) A program should never run with error checking turned off. Turning off error checking after a program has been tested is like throwing away the life preserver when graduating from the pool to the ocean.

(b) A program should always have error checking turned off after the testing stage. Keeping error checking on is like keeping training wheels on a bike after you've entered a race.

Discuss (a) and (b) from the point of view of language design.

2.13 Discuss the following two views of language design:

(a) Language design is compiler construction.

(b) Language design is software engineering.

2.14 Two contrasting viewpoints on the declaration of comments in a programming language are represented by Ada and C: In Ada, comments begin with adjacent hyphens and end with the end of a line:

```
-- this is an Ada comment
```

In C, comments begin with `/*` and proceed to a matching `*/`:

```
/* this is a C comment */
```

Compare these two comment features with respect to readability, writability, and reliability. C++ added a comment convention (two forward slashes) similar to that of Ada. Why didn't C++ use exactly the Ada convention?

2.15 The principle of locality maintains that variable declarations should come as close as possible to their use in a program. What language design features promote or discourage this principle? How well do Python, C, C++, and Java promote this principle?

2.16 A language with a good symbolic runtime debugger is often easier to learn and use than a language that lacks this feature. Compare the language systems you know with regard to the existence and/or quality of a debugger. Can you think of any language constructs that would aid or discourage the use of a debugger?

2.17 Many human factors affect language use, from psychology to human-machine interaction (or ergonomics). For example, programming errors made by people are far from random. Ideally, a language's design should attempt to prevent the most common errors. How could most common errors be prevented by language design?

2.18 Most programming languages now use the free format pioneered by Algol60, in which statements can begin anywhere, but must end, not with the end of a line of text, but with an explicit end symbol, such as a semicolon. By contrast, Python and a few other languages use fixed format, in which statements begin in a particular column and end at the end of the line of code, unless continuation marks are provided. Discuss the effect of fixed or free format on readability, writability, and security.

2.19 Here is a quote from D. L. Parnas [1985]: "Because of the very large improvements in productivity that were noted when compiler languages were introduced, many continue to look for another improvement by introducing better languages. Better notation always helps, but we cannot expect new languages to provide the same magnitude of improvement that we got from the first introduction of such languages. . . . We should seek simplifications in programming languages, but we cannot expect that this will make a big difference." Discuss.

2.20 A possible additional language design principle is learnability—that is, the ability of programmers to learn to use the language quickly and effectively. Describe a situation in which learnability may be an important requirement for a programming language to have. Describe ways in which a language designer can improve the learnability of a language.

2.21 In most language implementations, the integer data type has a fixed size, which means that the maximum size of an integer is machine dependent. In some languages like Scheme, however, integers may be of any size, and so become machine independent. Discuss the advantages and disadvantages of making such "infinite-precision" integers a requirement of a language definition. Is it possible to also implement "infinite-precision" real numbers? Can real numbers be made machine independent? Discuss.

2.22 Fred Brooks [1996] lists five basic design principles:

1. Design, don't hack.
2. Study other designs.
3. Design top-down.
4. Know the application area.
5. Iterate.
   (a) Explain what each of these means in terms of programming language design.
   (b) Describe to what extent the C++ design effort (Section 2.5) appears to meet each of the above design criteria.

2.23 Here is another quote from Stroustrup (Stroustrup [1994], page 45):

> . . . language design is not just design from first principles, but an art that requires experience, experiments, and sound engineering tradeoffs. Adding a major feature or concept to a language should not be a leap of faith but a deliberate action based on experience and fitting into a framework of other features and ideas of how the resulting language can be used.

Compare this view of language design with the list of five principles in the previous exercise.

2.24 In Chapter 1, we discussed how the von Neumann architecture affected the design of programming languages. It is also possible for a programming language design to affect the design of machine architecture as well. Describe one or more examples of this.

Notes and References

Horowitz [1984] gives a list of programming language design criteria similar to the ones in this chapter, which he partially attributes to Barbara Liskov. For an extensive historical perspective on language design, see Wegner [1976] (this paper and the papers by Wirth [1974] and Hoare [1973] mentioned in this chapter are all reprinted in Horowitz [1987]). The rationale for the design of Ada is discussed in Ichbiah et al. [1979], where many design principles are discussed. Many design issues in modern languages are discussed in Bergin and Gibson [1996], especially the articles on Pascal (Wirth [1996]), C (Ritchie [1996]), C++ (Stroustrup [1996]), Lisp (Steele and Gabriel [1996]), CLU (Liskov [1996]), Algol68 (Lindsey [1996]), and Prolog (Colmerauer and Roussel [1996]). Another article in that collection (Brooks [1996]) gives general advice on language design. Stroustrup [1994] is a significantly expanded version of the C++ paper in that collection and gives a detailed analysis of design issues in C++. See also Hoare [1981] for some wry comments on designing languages, compilers, and computer systems in general. Gabriel [1996] and Graham [2004] contain provocative discussions of recent language design. The proceedings of the third ACM SIGPLAN conference on the history of programming languages (HOPL III) [2007] contain further material on recent language design.

CHAPTER

Functional Programming

CHAPTER 3

Chapter 1 mentioned several different styles of programming, such as functional programming, logic programming, and object-oriented programming. Different languages have evolved to support each style of programming. Each type of language in turn rests on a distinct model of computation, which is different from the underlying von Neumann model reflected by most programming languages and most hardware platforms. In the next three chapters, we explore each of these alternative styles of programming, the types of languages that support them, and their underlying models of computation. In so doing, we will come to a better understanding of power and limits of languages that reflect the von Neumann model as well.

We begin with functional programming. Unlike other styles of programming, functional programming provides a uniform view of programs as functions, the treatment of functions as data, and the prevention of side effects. Generally speaking, a functional programming language has simpler semantics and a simpler model of computation than an imperative language. These advantages make functional languages popular for rapid prototyping, artificial intelligence, mathematical proof systems, and logic applications.

Until recently, the major drawback to functional languages was inefficient execution. Because of their semantics, such languages at first were interpreted rather than compiled, with a resulting substantial loss in execution speed. In the last 20 years, however, advances in compilation techniques for functional languages, plus advances in interpreter technology in cases where compilation is unsuitable or unavailable, have made functional languages very attractive for general programming. Because functional programs lend themselves very well to parallel execution, functional languages have ironically acquired an efficiency advantage over imperative languages in the present era of multicore hardware architectures (see Chapter 13). Additionally, modern functional languages such as those discussed in this chapter include mature application libraries, such as windowing systems, graphics packages, and networking capabilities. If speed is a special concern for part of a program, you can link a functional program to compiled code from other languages such as C. Thus, there is no reason today not to choose a functional language for implementing complex systems—even Web applications. A functional language is a particularly good choice in situations where development time is short and you need to use clear, concise code with predictable behavior.

Still, although functional languages have been around since the 1950s, they have never become mainstream languages. In fact, with the rise of object orientation, programmers have been even less inclined to use functional languages. Why is that? One possible reason is that programmers typically learn to program using an imperative or object-oriented language, so that functional programming seems inherently foreign and intimidating. Another is that object-oriented languages provide a strong organizing principle for structuring code and controlling complexity in ways that mirror the everyday experience of

real objects. Functional programming, too, has strong mechanisms for controlling complexity and structuring code, but they are more abstract and mathematical and, thus, not as easily mastered by the majority of programmers.

Even if you never expect to write "real" applications in a functional language, you should take the time to become familiar with functional programming. Functional methods, such as recursion, functional abstraction, and higher-order functions, have influenced and become part of many programming languages. They should be part of your arsenal of programming techniques.

In this chapter, we review the concept of a function, focusing on how functions can be used to structure programs. We give a brief introduction to the basic principles of functional programming. We then survey three modern functional languages—Scheme, ML, and Haskell—and discuss some of their properties. We conclude with a short introduction to lambda calculus, the underlying mathematical model for functional languages.

3.1 Programs as Functions

A program is a description of a specific computation. If we ignore the details of the computation—the "how" of the computation—and focus on the result being computed—the "what" of the computation—then a program becomes a virtual black box that transforms input into output. From this point of view, a program is essentially equivalent to a mathematical function.

DEFINITION: A ***function*** is a rule that associates to each x from some set X of values a unique y from a set Y of values. In mathematical terminology, if f is the name of the function, we write:

$$y = f(x)$$

or

$$f: X \rightarrow Y$$

The set X is called the **domain** of f, while the set Y is called the **range** of f. The x in $f(x)$, which represents any value from X, is called the **independent variable**, while the y from the set Y, defined by the equation $y = f(x)$, is called the **dependent** variable. Sometimes f is not defined for all x in X, in which case it is called a **partial function** (and a function that is defined for all x in X is called **total**).

Programs, procedures, and functions in a programming language can all be represented by the mathematical concept of a function. In the case of a program, x represents the input and y represents the output. In the case of a procedure or function, x represents the parameters and y represents the returned values. In either case, we can refer to x as "input" and y as "output." Thus, the functional view of programming makes no distinction between a program, a procedure, and a function. It always makes a distinction, however, between input and output values.

In programming languages, we must also distinguish between **function definition** and **function application**. A function definition describes how a value is to be computed using formal

parameters. A function application is a call to a defined function using actual parameters, or the values that the formal parameters assume for a particular computation. Note that, in mathematics, we don't always clearly distinguish between a parameter and a variable. Instead, the term **independent variable** is often used for parameters. For example, in mathematics, we write:

$$\text{square}(x) = x * x$$

for the definition of the squaring function. Then we frequently apply the function to a variable x representing an actual value:

$$\text{Let } x \text{ be such that square}(x) + 2 \ldots$$

In fact, one major difference between imperative programming and functional programming is in how each style of programming treats the concept of a variable. In mathematics, variables always stand for actual values, while in imperative programming languages, variables refer to memory locations that store values. Assignment allows these memory locations to be reset with new values. In mathematics, there are no concepts of memory location and assignment, so that a statement such as

$$x = x + 1$$

makes no sense. Functional programming takes a mathematical approach to the concept of a variable. In functional programming, variables are bound to values, not to memory locations. Once a variable is bound to a value, that variable's value cannot change. This also eliminates assignment, which can reset the value of a variable, as an available operation.

Despite the fact that most functional programming languages retain some notion of assignment, it is possible to create a **pure functional program**—that is, one that takes a strictly mathematical approach to variables.

The lack of assignment in functional programming makes loops impossible, because a loop requires a control variable whose value is reset as the loop executes. So, how do we write repeated operations in functional form? Recursion. For example, we can define a greatest common divisor calculation in C using both imperative and functional form, as shown in Figure 3.1.

```
void gcd( int u, int v, int* x)
{ int y, t, z;
  z = u ; y = v;
  while (y != 0)
  { t = y;
    y = z % y;
    z = t;
  }
  *x = z;
}
```

(a) Imperative version using a loop

```
int gcd( int u,  int v)
{ if (v == 0) return u;
  else return gcd(v, u % v);
}
```

(b) Functional version with recursion

Figure 3.1 C code for a greatest common divisor calculation

The second form (Figure 3.1(b)) is close to the mathematical (that is, recursive) definition of the function as

$$\gcd(u, v) = \begin{cases} u & \text{if } v = 0 \\ \gcd(v, u \bmod v) & \text{otherwise} \end{cases}$$

Another consequence of the lack of assignment is that we can have no notion of the internal state of a function. The value of any function depends only on the values of its arguments (and possibly nonlocal variables, which we examine later). For example, the square root of a given number never changes, because the value of the square root function depends only on the value of its argument. The balance of a bank account changes whenever a deposit is made, however, because this value depends not only on the amount deposited but also on the current balance (the internal state of the account).

The value of any function also cannot depend on the order of evaluation of its arguments. This is one reason for using functional programming for concurrent applications. The property whereby a function's value depends only on the values of its arguments (and nonlocal variables) is called **referential transparency**.[1] For example, the `gcd` function is referentially transparent because its value depends only on the value of its arguments. On the other hand, a function `rand`, which returns a pseudo random value, cannot be referentially transparent, because it depends on the state of the machine (and previous calls to itself). Indeed, a referentially transparent function with no parameters must always return the same value and, thus, is no different from a constant. In fact, a functional language is free to consider a function of no parameters to not be a function at all.[2] Referential transparency and the lack of assignment make the semantics of functional programs particularly straightforward. There is no notion of state, since there is no concept of memory locations with changing values. The runtime environment associates names to values only (not memory locations), and once a name enters the environment, its value can never change. These functional programming semantics are sometimes referred to as **value semantics**, to distinguish them from the more usual storage semantics or pointer semantics. Indeed, the lack of local state in functional programming makes it in a sense the opposite of object-oriented programming, where computation proceeds by changing the local state of objects.

Finally, in functional programming we must be able to manipulate functions in arbitrary ways, without arbitrary restrictions—in other words, functions must be general language objects. In particular, functions must be viewed as values themselves, which can be computed by other functions and which can also be parameters to functions. In other words, in functional programming, functions are **first-class data values**.

As an example, one of the essential operations on functions is **composition**. Composition is itself a function that takes two functions as parameters and produces another function as its returned value. Functions like this—that have parameters that are themselves functions or that produce a result that is a function, or both—are called **higher-order functions**.

Mathematically, the composition operator "o" is defined as follows: If f: $X \to Y$ and g: $Y \to Z$, then $g\ o\ f$: $X \to Z$ is given by $(g\ o\ f)(x) = g(f(x))$.

[1] A slightly different but equivalent definition, called the substitution rule, is given in Chapter 10.

[2] Both ML and Haskell (languages discussed later in this chapter) take this approach; Scheme does not: In Scheme, a parameterless function is different from a constant.

We can summarize the qualities of functional programming languages and functional programs as follows:

1. All procedures are functions and clearly distinguish incoming values (parameters) from outgoing values (results).
2. In pure functional programming, there are no assignments. Once a variable is bound to a value, it behaves like a constant.
3. In pure functional programming, there are no loops. Instead, loops are replaced by recursive calls.
4. The value of a function depends only on the value of its parameters and not on the order of evaluation or the execution path that led to the call.
5. Functions are first-class data values.

3.2 Scheme: A Dialect of Lisp

In the late 1950s and early 1960s, a team at MIT led by John McCarthy developed the first language that contained many of the features of modern functional languages. Based on ideas from mathematics, in particular the lambda calculus of Alonzo Church, it was called Lisp (for LISt Processing) because its basic data structure is a list. Lisp, which first existed as an interpreter on an IBM 704, incorporated a number of features that, strictly speaking, are not aspects of functional programming per se, but that were closely associated with functional languages because of the enormous influence of Lisp. These include:

1. The uniform representation of programs and data using a single general data structure—the list.
2. The definition of the language using an interpreter written in the language itself—called a **metacircular interpreter**.
3. The automatic management of all memory by the runtime system.

Unfortunately, no single standard evolved for the Lisp language, and many different Lisp systems have been created over the years. The original version of Lisp and many of its successors lacked a uniform treatment of functions as first-class data values. Also, they used dynamic scoping for nonlocal references. However, two dialects of Lisp that use static scoping and give a more uniform treatment of functions have become standard. The first, Common Lisp, was developed by a committee in the early 1980s. The second, Scheme, was developed by a group at MIT in the mid-1970s. In the following discussion, we use the definition of the Scheme dialect of Lisp given in Abelson et al. [1998], which is the current standard at the time of this writing.

3.2.1 The Elements of Scheme

All programs and data in Scheme are considered expressions. There are two types of expressions: atoms and parenthesized sequences of expressions. Atoms are like the literal constants and identifiers of an imperative language; they include numbers, characters, strings, names, functions, and a few other constructs we will not mention here. A parenthesized expression is simply a sequence of zero or

more expressions separated by spaces and surrounded by parentheses. Thus, the syntax of Scheme is particularly simple:[3]

```
expression → atom | '(' {expression} ')'

atom → number | string | symbol | character | boolean
```

This syntax is expressed in a notation called **extended Backus-Naur form**, which we will discuss in detail in Chapter 6. For now, note the use of the symbols in the rules shown in Table 3.1.

Table 3.1 Symbols used in an extended Backus-Naur form grammar

Symbol	Use
→	Means "is defined as"
|	Indicates an alternative
{}	Enclose an item that may be seen zero or more times
' '	Enclose a literal item

Thus, the first rule says that an expression is either an atom or a left parenthesis, followed by zero or more expressions, followed by a right parenthesis. The second rule defines what an atom can be. There are five options shown here, but there are actually several more in the language.

When parenthesized expressions are viewed as data, we call them lists. Some examples of Scheme expressions are shown in Table 3.2.

Table 3.2 Some Scheme expressions

Expression	What It Represents
`42`	an integer value
`"hello"`	a string
`#T`	the Boolean value "true"
`#\a`	the character 'a'
`(2.1 2.2 3.1)`	a list of real numbers
`a`	a symbol
`hello`	another symbol
`(+ 2 3)`	a list consisting of the symbol + and two integers
`(* (+ 2 3) (/ 6 2))`	a list consisting of a symbol followed by two lists

The meaning of a Scheme expression is given by an **evaluation rule**. The standard evaluation rule for Scheme expressions is as follows:

1. Atomic literals, such as numbers, characters, and strings, evaluate to themselves.
2. Symbols other than keywords are treated as identifiers or variables that are looked up in the current environment and replaced by the values found there.

[3]The current Scheme definition (R6RS, Sperber et al. [2009]) gives a more explicit syntactic description that includes some built-in functions and that specifies the precise position of definitions; however, the rules given here still express the general structure of a Scheme program.

An environment in Scheme is essentially a symbol table that associates identifiers with values.

3. A parenthesized expression or list is evaluated in one of two ways:
   a. If the first item in the expression is a Scheme keyword, such as `if` or `let`, a special rule is applied to evaluate the rest of the expression. Scheme expressions that begin with keywords are called **special forms**, and are used to control the flow of execution. We will have more to say about special forms in a moment.
   b. Otherwise, the parenthesized expression is a function application. Each expression within the parentheses is evaluated recursively in some unspecified order; the first expression among them must evaluate to a function. This function is then applied to remaining values, which are its arguments.

We can apply these rules to the sample Scheme expressions as follows: `42`, `"hello"`, `#T`, and `#\a` evaluate to themselves; `a` and `hello` are looked up in the environment and their values returned; `(+ 2 3)` is evaluated by looking up the value of + in the environment—it returns a function value, namely, the addition function, which is predefined—and then applying the addition function to the values of 2 and 3, which are 2 and 3 (since constants evaluate to themselves). Thus, the value 5 is returned. Similarly, `(* (+ 2 3) (/ 6 2))` evaluates to 15. The expression `(2.1 2.2 3.1)`, on the other hand, cannot be evaluated successfully, because its first expression `2.1` is a constant that is not a function. This expression does not represent a Scheme special form or function application and results in an error if evaluation is attempted.

The Scheme evaluation rule implies that all expressions in Scheme must be written in prefix form. It also implies that the value of a function (as an object) is clearly distinguished from a call to the function: The function value is represented by the first expression in an application, while a function call is surrounded by parentheses. Thus, a Scheme interpreter would show behavior similar to the following:

```
> +
#[PROCEDURE: +]

> (+ 2 3)
5              ; a call to the + procedure with arguments 2 and 3
```

Note that a Scheme end-of-line comment begins with a semicolon.

A comparison of some expressions in C and Scheme is given in Figure 3.2.

C	Scheme
`3 + 4 * 5`	`(+ 3 (* 4 5))`
`(a == b) && (a != 0)`	`(and (= a b) (not (= a 0)))`
`gcd(10,35)`	`(gcd 10 35)`
`gcd`	`gcd`
`getchar()`	`(read-char)`

Figure 3.2 Some expressions in C and Scheme

The Scheme evaluation rule represents **applicative order evaluation**. All subexpressions are evaluated first so that a corresponding expression tree is evaluated from leaves to root. Thus, the Scheme expression `(* (+ 2 3) (+ 4 5))` is evaluated by first evaluating the two additions and then evaluating the resultant expression `(* 5 9)`, as indicated by a bottom-up traversal of the expression tree shown in Figure 3.3.

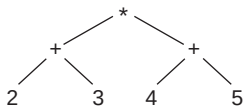


Figure 3.3 Expression tree for Scheme expression

This corresponds to the evaluation of `(2 + 3) * (4 + 5)` in a language like C, where expressions are written in infix form.

Since data and programs in Scheme follow the same syntax, a problem arises when data are represented directly in a program. As noted above, we can represent a list of numbers in Scheme by writing it inside parentheses, with spaces separating the numbers, as in:

```
(2.1 2.2 3.1)
```

However, this list will generate an error if given to the interpreter directly, because Scheme will try to evaluate it as a function call. Thus, the need arises to prevent the evaluation of a list and to consider it to be a list literal (much as simple atomic literals evaluate to themselves). This is accomplished by a special form whose sole purpose is to prevent evaluation. This form uses the Scheme keyword `quote`. The special rule for evaluating a `quote` special form is to simply return the expression following `quote` without evaluating it:

```
> (2.1 2.2 3.1)
Error: the object 2.1 is not a procedure
> (quote (2.1 2.2 3.1))
(2.1 2.2 3.1)
> (quote scheme)
scheme
```

Note that the last expression returns a Scheme **symbol**. Symbols actually form a distinct data type in Scheme. When they are evaluated, symbols are treated as identifiers or names that refer to values. When they are quoted, symbols are taken literally as they are. They can be added to lists or compared for equality to other symbols. The inclusion of symbols as an explicit data type makes Scheme an excellent language for processing any kind of symbolic information, including Scheme programs themselves. We shall have more to say about symbolic information processing later in this section.

The quote special form is used so often that there is a special shorthand notation for it—the single quotation mark or apostrophe:

```
> '(2.1 2.2 3.1)
(2.1 2.2 3.1)
```

Because all Scheme constructs are expressions, we need expressions that govern the control of execution. Loops are provided by recursive call, but selection must be given by special forms. The basic forms that do this are the `if` form, which is like an `if-else` construct, and the `cond` form, which is like an `if-elsif` construct. (Note that `cond` stands for **conditional expression**.)

```
(if (= a 0) 0         ; if a = 0 then return 0
    (/ 1 a))          ; else return 1/a

(cond((= a 0) 0)      ; if a=0 then return 0
     ((= a 1) 1)      ; elsif a=1 then return 1
     (else (/ 1 a)))  ; else return 1/a
```

The semantics of the expression `(if` *exp*1 *exp*2 *exp*3`)` dictate that *exp*1 is evaluated first; if the value of *exp*1 is the Boolean value false (`#f`), then *exp*3 is evaluated and its value returned by the `if` expression; otherwise, *exp*2 is evaluated and its value is returned. Also, *exp*3 may be absent; in that case, if *exp*1 evaluates to false, the value of the expression is undefined.

Similarly, the semantics of `(cond` *exp*1 . . . *expn*`)` dictate that each *expi* must be a pair of expressions: *expi* = (*fst snd*). Each expression *expi* is considered in order, and the first part of it is evaluated. If *fst* evaluates to `#T` (true), then *snd* is evaluated, and its value is returned by the `cond` expression. If none of the conditions evaluates to `#T`, then the expression in the `else` clause is evaluated and returned as the value of the `cond` expression (i.e., the keyword `else` in a `cond` can be thought of as always evaluating to `#T`). If there is no `else` clause (the `else` clause is optional) and none of the conditions evaluates to `#T`, then the result of the `cond` is undefined.

Notice that neither the `if` nor the `cond` special form obey the standard evaluation rule for Scheme expressions. If they did, all of their arguments would be evaluated each time, regardless of their values. This would then render them useless as control mechanisms. Instead, the arguments to such special forms are **delayed** until the appropriate moment. Scheme function applications use pass by value, while special forms in Scheme and Lisp use delayed evaluation. Delayed evaluation, an important issue in functional programming, is discussed further in Section 3.4.

Another important special form is the `let`, which allows variable names to be bound to values within an expression:

```
> (let ((a 2) (b 3)) (+ a b))
5
```

The first expression in a `let` is a **binding list**, which associates names to values. In this binding list of two bindings, `a` is given the value 2 and `b` the value 3. The names `a` and `b` can then be used for their values in the second expression, which is the body of the `let` and whose value is the returned value of the `let`. Note that `let` provides a local environment and scope for a set of variable names, in much the same manner as temporary variable declarations in block-structured languages such as Java and Python. The values of these variables can be accessed within the `let` form, but not outside it.

To develop a Scheme program, we must also have a way of creating functions. This is done using the `lambda` special form (the term `lambda` comes from Church's lambda calculus). A `lambda` expression has the following form:

```
(lambda param-list body)
```

When Scheme evaluates a `lambda` form, it creates a function with the specifed formal parameters and a body of code to be evaluated when that function is applied. For example, a function to compute the area of a circle is:

```
> (lambda (radius) (* 3.14 (* radius radius)))
#<procedure>
```

This function can be applied to an argument by wrapping the function and its argument in another set of parentheses:

```
> ((lambda (radius) (* 3.14 (* radius radius))) 10)
314.0
```

Because it can be useful to refer to functions by names, we can bind a name, such as `circlearea`, to a `lambda` within a `let`, and apply that name to an argument in the body of the `let`:

```
> (let ((circlearea (lambda (radius) (* 3.14 (* radius radius)))))
    (circlearea 10))
314.0
```

Unfortunately, a `let` cannot be used to define recursive functions, since `let` bindings cannot refer to themselves or each other. Thus, the following code will generate an error:

```
(let ((factorial (lambda (n) (if (= n 0) 1 (* n (factorial (- n 1)))))))
  (factorial 10))
```

For such situations there is a different special form, called `letrec`, which works just like a `let`, except that it allows arbitrary recursive references within the binding list:

```
(letrec ((factorial (lambda (n) (if (= n 0) 1 (* n (factorial (- n 1)))))))
  (factorial 10))
```

The `let` and `letrec` forms introduce variables that are visible within the scope and for the lifetime of the `let` or `letrec`. To make a new, global binding of a variable visible in the top-level environment of Scheme, we can use the `define` special form, as follows:

```
> (define circlearea (lambda (radius) (* 3.14 (* radius radius))))
> (define x 5)
> (circlearea 10)
314.0
> (circlearea x)
78.5
>
```

3.2.2 Dynamic Type Checking

The semantics of Scheme includes dynamic or latent type checking. This means two things: first, only values, not variables, have data types, and second, the types of values are not checked until it is absolutely necessary to do so at runtime. About the only time this happens automatically is right before a so-called primitive function, such as the one named by +, is applied to its arguments. If either of these

arguments is not a number, Scheme halts execution with an error message. By contrast, the types of arguments to programmer-defined functions are not automatically checked. For example, the `square` function defined here passes on a potential type error to the `*` function:

```
> (define square (lambda (n) (* n n)))
> (square "hello")
. . *: expects type <number> as 1st argument, given: "hello"
```

The error is indeed caught automatically, but perhaps not at the point where it is most useful. The Scheme programmer can check the type of any value by using any of the built-in type recognition functions, such as `number?` and `procedure?`, in the appropriate conditional expressions. However, forcing the programmer to write code for type checking slows down both the programmer's productivity and the code's execution speed. Later in this chapter, we show how other functional languages, such as ML and Haskell, solve this problem by introducing static type checking.

3.2.3 Tail and Non-Tail Recursion

Because of the runtime overhead required to support procedure calls, loops are always preferable to recursion in imperative languages. However, the functional style of programming encourages recursion. How, then, can we avoid the inefficiency associated with recursion? In languages like Scheme, the answer is to make any recursive steps the last steps in any function. In other words, make your functions **tail recursive**. A Scheme compiler is required to translate such tail recursive functions to code that executes as a loop, with no additional overhead for function calls other than the top-level call. To see how this is so, consider the two definitions of the factorial function in Figure 3.4.

Non-Tail Recursive `factorial`

```
> (define factorial
    (lambda (n)
      (if (= n 1)
          1
          (* n (factorial (- n 1))))))
> (factorial 6)
720
```

Tail Recursive `factorial`

```
> (define factorial
    (lambda (n result)
      (if (= n 1)
          result
          (factorial (- n 1) (* n
                                result)))))
> (factorial 6 1)
720
```

Figure 3.4 Tail recursive and non-tail recursive functions

Note that *after* each recursive call of the non-tail recursive `factorial`, the value returned by that call must be multiplied by `n`, which was the argument to the previous call. Thus, a runtime stack is required to track the value of this argument for each call as the recursion unwinds. This version of `factorial`, thus, entails a linear growth of memory, a substantial performance hit when compared with a nonrecursive version that uses a loop. However, in the case of the tail recursive version, all the work of computing values is done when the two arguments are evaluated *before* each recursive call. The value of `n` is decremented,

as before. Another argument, `result`, is used to accumulate intermediate products on the way down through the recursive calls. The value of `result` is always 1 on the top-level call, and is multiplied by the current value of `n` before each recursive call. Because no work remains to be done after each recursive call, no runtime stack is really necessary to remember the arguments of previous calls. Thus, the Scheme compiler can translate the tail recursive `factorial` to something like this imaginary code, which uses a loop containing the special forms `begin`, `while`, and `set!`:

```
> (define factorial
    (lambda (n result)
      (begin
        (while (> n 1)
               (set! result (result * n))
               (set! n (- n 1)))
        result)))
```

The `begin` special form forces left-to-right execution of a sequence of expressions and returns the value of the last one. The `set!` special form resets the value of a variable. The `while` special form does not exist in Scheme, but it can be defined with a macro similar to the one discussed in Chapter 2. This of course looks like, and is, imperative code that no bona fide Scheme programmer would ever write. The point is that the programmer can adhere to a recursive, functional style without incurring a performance hit, if the language includes a smart compiler that can generate similar code when optimizing tail recursion.

3.2.4 Data Structures in Scheme

In Scheme, the basic data structure is the list. Although Scheme also supports structured types for vectors (one-dimensional arrays) and strings, a list can represent a sequence, a record, or any other structure. For example, the following is a list representation of a binary search tree:

```
("horse" ("cow" () ("dog" () ()))
("zebra" ("yak" () ()) () ))
```

A node in this tree is a list of three items `(name left right)`, where `name` is a string, and `left` and `right` are the child trees, which are also lists. Thus, the given list represents the tree of Figure 3.5.

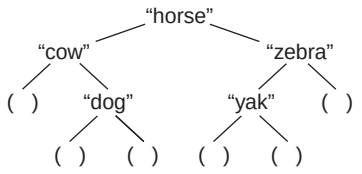


Figure 3.5 A binary search tree containing string data

To use lists effectively we must have an adequate set of functions that operate on lists. Scheme has many predefined list functions, but the basic functions that are common to all Lisp systems are the selector functions `car` and `cdr`, which access the head and the tail of a list, and the constructor function `cons`, which adds a new head to an existing list. Thus, if `L` is the list `(1 2 3)`, then:

```
> (car L)
1
> (cdr L)
(2 3)
> (cons 4 L)
(4 1 2 3)
```

The names `car` and `cdr` are a historical accident. The first machine on which Lisp was implemented was an IBM 704. In that first implementation, addresses or pointers were used to represent lists in such a way that the head of a list was the "Contents of the Address Register," or car, and the tail of a list was the "Contents of the Decrement Register," or cdr. This historical accident persists partially because of another accident: On account of the single letter difference in the names of the operations, repeated applications of both can be combined by combining the letters "a" and "d" between the "c" and the "r." Thus, `(car (cdr L))` becomes `(cadr L)`, `(cdr (cdr L))` becomes `(cddr L)`, and `(car (cdr (cdr L)))` becomes `(caddr L)`. For clarity, however, we will avoid these abbreviations.

The view of a list as a pair of values represented by the `car` and the `cdr` has also continued to be useful as a representation or visualization of a list. According to this view, a list `L` is a pointer to a "box" of two pointers, one to its `car` and the other to its `cdr`. See Figure 3.6.

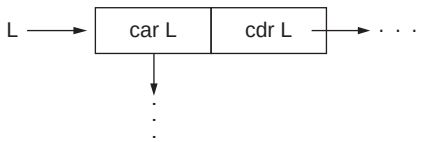


Figure 3.6 Visualizing a list with box and pointer notation

This **box and pointer notation** for a simple list such as `(1 2 3)` is shown in Figure 3.7.

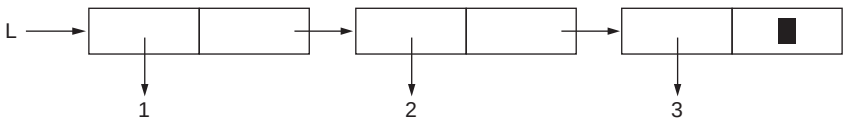


Figure 3.7 Box and pointer notation for the list `(1 2 3)`

The symbol black rectangle in the box at the end stands for the empty list `()`. A more complex example of a list structure is shown in Figure 3.8.

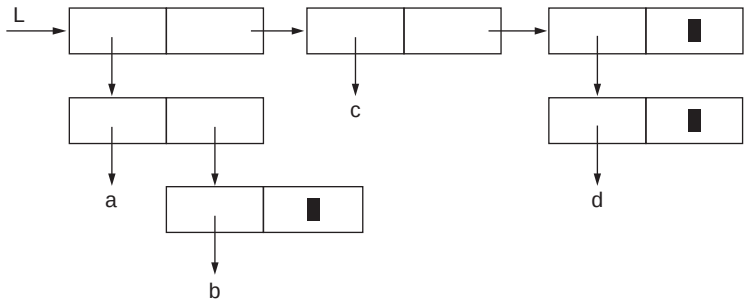


Figure 3.8 Box and pointer notation for the list `L = ((a b) c (d))`

A box and pointer diagram is useful for interpreting the list operations: The `car` operation represents following the first pointer, while the `cdr` operation represents following the second pointer. Thus, for the `L` of Figure 3.8, `(car (car L)) = a`, `(cdr (car L)) = (b)`, and `(car (cdr (cdr L))) = (d)`.

All the basic list manipulation operations can be written as functions using the primitives[4] `car`, `cdr`, `cons`, and `null?`. The last function is a predicate that returns true if its list argument is empty, or false otherwise. For example, an `append` operation that returns the appended list of two lists can be written as follows:

```
(define append (lambda (L M)
  (if (null? L) M
     (cons (car L) (append (cdr L) M)))))
```

and a `reverse` operation is as follows:

```
(define reverse (lambda (L)
  (if (null? L) '()
     (append (reverse (cdr L)) (list (car L))))))
```

In the `reverse` function, we use the primitive function `list`, which makes a list of one or more items:

```
> (list 2 3 4)
(2 3 4)
> (list '(a b) '(c d))
((a b) (c d))
```

In fact, in the `reverse` function, we could have used `(cons (car L) '())` instead of `(list (car L))`, since `(list a)` is the same as `(cons a '())`.[5]

[4]A primitive is a built-in procedure that is implemented at a lower level. Most built-in procedures required by the Scheme standard are primitives, so we will not emphasize this distinction.

[5]Actually, `append` and `reverse` are also really built-in primitives, although as we have seen, they could be implemented in Scheme itself.

Finally, we offer the example of the use of `car` and `cdr` to access the elements of a binary search tree defined as a list with structure `(data leftchild rightchild)`:

```
(define (leftchild B) (car ( cdr B)))
(define (rightchild B) (car (cdr (cdr B))))
(define (data B) (car B))
```

Now we can write a tree traversal procedure `print-tree` that prints out the data in the tree as follows:

```
(define print-tree (lambda (B)
 (cond ((null? B) '() )
       (else (print-tree (leftchild B))
             (display (data B))
             (newline)
             (print-tree (rightchild B)))))
```

3.2.5 Programming Techniques in Scheme

Programming in Scheme, as in any functional language, relies on recursion to perform loops and other repetitive operations. One standard technique for applying repeated operations to a list is to "cdr down and cons up"—meaning that we apply the operation recursively to the tail of a list and then collect the result with the `cons` operator by constructing a new list with the current result. One example is the `append` procedure code of the previous section. Another example is a procedure to square all the members in a list of numbers:

```
(define square-list (lambda (L)
   (if (null? L) '()
       (cons (* (car L) (car L)) (square-list (cdr L))))))
```

An example of a loop that is not applied to a list is a procedure that simply prints out the squares of integers from a lower bound to an upper bound:

```
(define print-squares (lambda (low high)
  (cond ((> low high) '())
       (else (display (* low low))
                (newline)
                (print-squares (+ 1 low) high)))))
```

A call to `(print-squares 1 100)` will generate the desired result.

In the `print-squares` example, we had to use the extra parameter `low` to control the recursion, just as a loop index controls repetition. Earlier we noted that, in such cases, tail-recursive procedures are preferred because of the relative ease with which translators can optimize them into actual loops. In fact, the `print-squares` function is tail recursive.

In the last section, we also demonstrated the technique of using an accumulating parameter to turn a non-tail-recursive procedure into a tail-recursive one. As an example in Scheme, we apply this technique

to the `square-list` function by defining a helping procedure `square-list1` with an extra parameter to accumulate the intermediate result:

```
(define square-list1 (lambda (L list-so-far)
  (if (null? L) list-so-far
      (square-list1 (cdr L)
            (append list-so-far (list (* (car L) (car L)))))))
```

Now we define `square-list` as a call to `square-list1` with `()` as its first accumulated value:

```
(define square-list (lambda (L) (square-list1 L '())))
```

Similarly, for `reverse` one defines `reverse1` and `reverse` as follows:

```
(define reverse1 (lambda (L list-so-far)
  (if (null? L) list-so-far
      (reverse1 (cdr L) (cons (car L) list-so-far)))))

(define reverse (lambda (L) (reverse1 L '())))
```

3.2.6 Higher-Order Functions

Since functions are first-class values in Scheme, we can write functions that take other functions as parameters and functions that return functions as values. Such functions are called **higher-order functions**.

To give a simple example of a function with a function parameter, we can write a function `map`[6] that applies another function to all the elements in a list and then pass it the `square` function to get the `square-list` example:

```
(define map (lambda (f L)
  (if (null? L) '()
      (cons (f (car L)) (map f (cdr L))))))

(define square (lambda (x) (* x x))

(define square-list (lambda (L) (map square L))
```

Here is an example of a function that has a function parameter and also returns a function value:

```
(define make-double (lambda (f)
  (lambda (x) (f x x)))
```

This function assumes that `f` is a function with two parameters and creates the function that repeats the parameter `x` in a call to `f`. The function value `doublefn` returned by `make-double` is created by a

[6]The map function is in fact a standard Scheme function, with behavior somewhat more general than what we describe here.

local `define` and then written at the end to make it the returned value of `make-double`. Note that `x` is not a parameter to `make-double` itself but to the function created by `make-double`.

We can use `make-double` to get both the `square` function and the `double` function (a function that doubles its numerical value):

```
(define square (make-double *))
(define double (make-double +))
```

The symbols "`*`" and "`+`" are the names for the multiplication and addition functions. Their values—which are function values—are passed to the `make-double` procedure, which in turn returns function values that are assigned to the names `square` and `double`. Note that we are using here the simple form of the `define`, which just assigns computed values rather than defining a function with a given body. Just as

```
(define a 2)
```

assigns the value 2 to the name `a`, the expression

```
(define square (make-double *))
```

assigns the function value returned by `make-double` to the name `square`.

As noted at the end of Section 3.2, this ability of functional languages, including Scheme, to return function values from higher-order functions means that the runtime environment of functional languages is more complicated than the stack-based environment of a standard block-structured imperative language. Returning the memory used by functions to free storage requires the use of automatic memory management techniques, such as **garbage collection**, which are discussed in Section 10.5.

3.2.7 Static (Lexical) Scoping

Early dialects of Lisp were dynamically scoped, whereas more modern dialects, such as Scheme and Common Lisp, are statically scoped. The term **static scope,** or **lexical scope**, refers to the area of a program text in which a declaration of a variable is visible. Thus, in static scoping, the meaning or value of a given variable can be determined just by reading the text of the program. In a dynamically scoped language, by contrast, the meaning of a variable depends on the runtime context.

The concept of scope is important in block-structured languages, where it is possible to nest the declarations of variables. Consider, for example, the following Scheme code segment, which nests a `let` within a `let`:

```
> (let ((a 2) (b 3))
    (let ((a (+ a b)))
      (+ a b)))
8
```

The scope rule states that the scope of a variable extends to the end of the block in which it is declared, including any nested blocks, unless that variable is redeclared within a nested block. Thus, in this example, the value of the first instance of `(+ a b)` is 5, whereas the value of the second instance of `(+ a b)` is 8.

This straightforward application of the scope rule becomes more complicated when we include functions with free variables. A **free variable** is a variable referenced within a function that is not also a formal parameter to that function and that is not bound within a nested function. A **bound variable** is a variable within a function that is also a formal parameter to that function. Here is an example of the definition and use of a function that contains a free variable and a bound variable:

```
> (let ((pi 3.14))
    (let ((circlearea (lambda (radius)
                        (* pi (* radius radius)))))
      (let ((pi 3.1416))
        (circlearea 10))))
314.0
```

The `circlearea` function contains one bound variable, `radius`, and one free variable, `pi`. The bound variable `radius` is also the function's formal parameter, and it picks up its value in the context of its function's call, when it is bound to the value 10. You can also see that the value of the free variable `pi` must have been 3.14 when the function referenced it during this application. Thus, the free variable `pi` picks up and retains its value in the outer `let`, which is the context of the function's definition, rather than in the inner `let`, which is the context of the function's call. By contrast, if Scheme were dynamically scoped, like older Lisps, the value of `pi` used in the function application would be the one given it by the inner `let`, or the context of the function's call. The application would then have produced the value 314.16. In that case, the function's free variable would have the same semantics as its formal parameter. That is, in dynamic scoping, the values of free variables and formal parameters vary with the context of the function's call.

Lexical scoping, because it fixes the meaning of free variables in one place in the code, generally makes a program easier to read and verify than dynamic scoping.

3.2.8 Symbolic Information Processing and Metalinguistic Power

Earlier in this section, we showed that Scheme programs and Scheme data can have the same syntax. For example, viewed as data, the expression `(f x y)` is just a list of three symbols, which can be accessed and manipulated with list operations. When viewed as a program, this expression is the application of a function named by the variable `f` to its argument values named by the variables `x` and `y`. The capacity to build, manipulate, and transform lists of symbols and then to evaluate them as programs gives the Scheme programmer the power to create new languages for special purposes. Abelson and Sussman [1996] call this type of power **metalinguistic** and give several examples of its use. Among them are the creation of a constraint language for expressing relations rather than functions, and the construction of a language that supports logical inference.

The creation of these little languages requires the programmer to specify the syntax of their expressions and their semantics as realized in new interpreters, all written in the base language Scheme. While building a new interpreter might sound like a much more daunting task than simply defining new functions or data types, the job is made surprisingly simple by many of the features of Scheme discussed thus far in this section.

To give the flavor of this idea, let's examine how a Scheme programmer might process a `let` expression if Scheme did not include `let` as a built-in special form. Recall that `let` allows the interpreter to evaluate an expression within a new environment created by binding a set of temporary variables to their values. A close inspection, however, shows that the `let` form is actually just syntactic sugar for the application of a `lambda` form to its arguments. Figure 3.9 shows examples of a `let` expression and a `lambda` application that have the same semantics.

```
(let ((a 3) (b 4))            ((lambda (a b) (* a b)) 3 4)
  (* a b))
```

Figure 3.9 `let` as syntactic sugar for the application of `lambda`

As you can see, the temporary variables of the `let` become the formal parameters of the `lambda`, and their initial values become the arguments of the `lambda`'s application. The body of the `let` becomes the body of the `lambda`.

The process of interpreting a `let` form that is not built-in involves two steps. First, we treat the `let` form as a datum (a list) to be transformed into another datum (another list) that represents the equivalent `lambda` application. Second, we evaluate the new representation. If we assume that there exists a function, `let-to-app`, that performs the required syntactic transformation, then we can apply the Scheme function `eval` to the result to evaluate it, as follows:

```
>  (eval (let-to-app '(let ((a 3) (b 4))(* a b))))
12
```

Note that we quote the argument of `let-to-app`, the `let` expression, to treat it as data (a list of elements). We then use `eval` to force evaluation of the result of `let-to-app`, which is another list of elements representing the application of the `lambda`.

The remaining task is to define the function `let-to-app`, which transforms any `let` expression to the corresponding `lambda` application. Our implementation makes use of several list-processing functions and the quote of a symbol. The comments indicate the phrases of code extracted from the `let` form and added to the list representing the application under construction:

```
(define let-to-app (lambda (let-form)
             (append (list                          ; begin the app
                     (list 'lambda                  ; begin the lambda
                         (map car (cadr let-form)) ; formal parameters
                          (caddr let-form)))        ; lambda body
                     (map cadr let-form)))))        ; argument expressions
```

Alternatively, we could define the syntactic forms of our new language (if they are not already built in) using a macro facility as mentioned in Chapter 2. Then the programmer using the new forms would not have to quote them and explicitly evaluate them with `eval`. Upon encountering the new forms, the Scheme compiler would automatically perform the required syntactic transformations and generate code that is evaluated in the usual manner at runtime.

3.3 ML: Functional Programming with Static Typing

As we have noted previously, ML (for MetaLanguage) is a functional programming language that is quite different from the dialects of Lisp, such as Scheme. First, ML has a more Algol-like syntax, which avoids the use of many parentheses. Second, it is statically typed, so that the type of every expression is determined before execution, and types can be checked for consistency. While traditional Lisp programmers may dislike the constraints of a static type system, ML's static type system offers significant advantages. For one thing, it makes the language more secure, because more errors can be found prior to execution. This is especially important in instructional settings and also helps ensure good software engineering. Also, ML's static type system improves efficiency because: (1) it makes it possible to predetermine size and allocation requirements, and (2) it makes checking types at runtime unnecessary. Additionally, ML's static type system, which you will study in detail in Chapter 8, is extremely flexible. It does not insist, as with C or Ada, that *all* types be declared by the programmer, and it also allows for parametric polymorphism, where types can contain type variables that can range over the set of all types.

ML was first developed in the late 1970s as part of a larger system for proving the correctness of programs, known as the Edinburgh Logic for Computable Functions (LCF) system, developed by a team led by Robin Milner. Milner also helped develop the strong type inference system, based on pattern matching, that is now called Hindley-Milner type checking. This form of type checking is a key feature of both ML and Haskell (discussed later in this chapter). During a revision of the ML language during the 1980s, features of the earlier language were combined with the HOPE language, developed by Rod Burstall, also at Edinburgh. This new language was named Standard ML, or SML. Another minor revision to the language occurred in 1997, and the current standard is referred to as SML97 or just ML97. We use that revision in this text.

In this section, we will first review the basics of ML, including comparisons with Scheme. We will then provide an introduction to ML data structures and programming techniques. (Some ML data structure information will be covered in Chapter 8.)

3.3.1 The Elements of ML

In ML, the basic program is, as in Scheme, a function declaration, as in:

```
> fun fact n = if n = 0 then 1 else n * fact(n - 1);
val fact = fn: int -> int
```

The reserved word `fun` in ML introduces a function declaration. The identifier immediately after `fun` is the name of the function, and the names of the parameters follow, up to the equal sign. After the equal sign is the body of the function.

The ML system responds to a declaration by returning the data type of the value defined. In this example, `fact` is a function, so ML responds that the value of fact is `fn`, with type `int -> int`, which means that `fact` is a function from integers (its domain) to integers (its range). Indeed, we could have given type declarations for the type of `fact` and the type of its parameter as follows:

```
> fun fact (n: int): int = if n = 0 then 1
else n * fact (n - 1);
val fact = fn: int -> int
```

Once a function has been declared it can be called as follows:

```
> fact 5;
val it = 120 : int
```

ML responds with the returned value and its type (`it` is the name of the current expression under evaluation). ML has essentially the same evaluation rule as Scheme: `fact` must evaluate to a function, then `5` is evaluated, and its type must agree with the parameter type of the function. The function is then called and its returned value printed together with its type. ML, however, does not need a quote function as in Scheme to prevent evaluation, because data are distinct from programs. In ML, parentheses are almost completely unnecessary, because the system can determine the meaning of items based solely on their position.

One can also define values in ML by using the `val` keyword:

```
> val Pi = 3.14159;
val Pi = 3.14159 : real
```

In ML, arithmetic operators are written as infix operators as they are in C or Ada. This differs from the uniform prefix notation of Lisp. As a result, operator precedence and associativity are an issue in ML. Of course, ML adheres to the standard mathematical conventions for the usual arithmetic operators. For example, 2 + 3 * 4 means 2 + (3 * 4).[7] In ML, it is also possible to turn infix operators into prefix operators using the `op` keyword, so that 2 + 3 * 4 can be written as:

```
> op + (2 , op * ( 3,4));
val it = 14 : int
```

Note here that the binary arithmetic operators take *pairs* of integers as their arguments. These pairs are elements of the Cartesian product type, or **tuple type** `int * int`:

```
> (2,3);
val it = (2,3) : int * int
> op +;
val it = fn : int * int -> int
```

Lists are an important feature of ML, as they are in almost every functional language. However, they are not as universal as they are in Lisp, because in ML programs are not themselves lists. A list in ML is indicated by square brackets, with elements separated by commas. For example, a list of the integers 1, 2, and 3 is represented as `[1, 2, 3]`:

```
> [1,2,3];
val it = [1,2,3] : int list
```

[7]On the other hand, the unary minus or negation operator, which is usually written in mathematics with the same - symbol as subtraction, is written using a tilde in ML: ~5 is negative 5.

ML determines the type of this list to be `int list`. ML's strong typing dictates that lists may only contain elements that all have the same type. Here is an example structure that violates this restriction by mixing types of elements in a list:

```
> [1,2,3.1];
Error: operator and operand don't agree [literal]
  operator domain: int * int list
  operand:         int * real list
  in expression:
    2 :: 3.1 :: nil
```

If we want to mix data of different types, we must use a **tuple**:

```
> (1,2,3.1);
val it = (1,2,3.1) : int * int * real
```

The error message generated in the earlier example also gives us a hint as to how ML constructs lists, which is similar to the way Scheme constructs lists. That is, the operator `::` corresponds to `cons` in Scheme, and constructs a list out of an element (which becomes the head of the list) and a previously constructed list (which becomes the tail of the list). For example:

```
> op :: ;
val it = fn : 'a * 'a list -> 'a list
> 2 :: [3,4];
val it = [2,3,4] : int list
```

Note that the type of the `::` operator contains a **type variable** `'a`, because `::` can be applied to a list containing values of any type. Thus, `::` is a function from any type `'a` and any list of the same type `'a` to a list of the same type `'a`.

In ML, every list is constructed by a series of applications of the `::` operator:

```
> 1 :: 2 :: 3 :: [];
val it = [1,2,3] : int list
```

Here `[]` is the empty list, which can also be written as the keyword `nil`.

ML has operations corresponding to Scheme's `car` and `cdr` operators as well, except that they are called `hd` and `tl` (for **head** and **tail**). For example:

```
> hd [1,2,3];
val it = 1 : int
> tl [1,2,3];
val it = [2,3] : int list
```

In fact, these functions are used much less in ML than their equivalents in Scheme, because ML's pattern-matching ability makes them unnecessary. In ML, if we want to identify the head and tail of a list `L`, we simply need to write `h::t` for the list. ML will then extract `h` as the head and `t` as the tail by matching the pattern against the list `L`. If `L` is empty, the pattern will not match, and the following warning will be generated:

```
> val h::t = [1,2,3];
Warning: binding not exhaustive
         h :: t = ...
val h = 1 : int
val t = [2,3] : int list
```

Pattern matching is more typically used in function definitions and in `case` expressions. For example, a list `append` function in ML can be defined as follows:[8]

```
> fun append ([] , L) = L
   |  append (h :: t , L) = h :: append (t , L);
val append = fn : 'a list * 'a list -> 'a list
```

This is equivalent to the following definition using a `case` expression:

```
> fun append (x,y) =
       case x of
         []     => y |
         (h::t) => h :: append (t,y);
```

Pattern matching can be used in virtually all function definitions where a case analysis is called for, and it can eliminate most uses of `if` expressions. For example, the recursive factorial function defined previously can also be written using pattern matching:

```
fun fact 0 = 1 | fact n = n * fact (n - 1);
```

Here the second pattern—the variable `n`—matches anything, but its position as the second pattern implies that it must not be zero; if it were, the first pattern (the integer literal `0`) would have matched.

Patterns in ML can also contain wildcards, written as the underscore character. Wildcards are patterns that match anything, but whose contents we don't care about. For example, we could define our own version of the `hd` function as follows:

```
fun hd (h::_) = h;
```

In fact, this definition will generate a warning message ("match nonexhaustive"), since we have not included a definition of what `hd` is to do in the case of the empty list. A better definition is the following, which is essentially exactly what the predefined `hd` does:

```
fun hd (h::_) = h | hd [] = raise Empty;
```

[8] In fact, ML has a built-in infix append operator @: [1,2,3] @ [4,5,6] = [1,2,3,4,5,6].

ML's strong typing, while secure and flexible, does impose important constraints on what is an acceptable program. A typical example is a square function, which we define here for real (floating point) values:

```
> fun square x: real = x * x;
val square = fn : real -> real
```

An error will now be generated if we try to call this function with an integer, since ML (like ADA) will not automatically convert an integer into a real:

```
> square 2;
Error: operator and operand don't agree [literal]
  operator domain: real
  operand:         int
  in expression:
    square 2
```

Instead, we need to manually convert the integer using a conversion function. In the following example, the function has the same name as the data type we are converting to:

```
> square (real 2);
val it = 4.0 : real
```

We might think that we could define a similar function with the same name for integers:

```
> fun square x: int = x * x;
val square = fn : int -> int
```

but ML does not allow overloading. Thus, this definition will just replace the previous one. We might also try defining a generic version of square by not specifying the type of the parameter:

```
> fun square x = x * x;
val square = fn : int -> int
```

In this case, however, ML simply assumes that we meant `x` to be an integer.[9] The problem is that, while the built-in arithmetic functions are overloaded for arithmetic types like `int` and `real`, we cannot make use of that fact to define any overloaded user-defined functions. For the most part, this is not a problem in the language Haskell (as explained later in this chapter).

A related issue is that certain values in ML do not contain enough type information for ML to properly type check an expression before it is evaluated. The main example of this is the empty list `[]`, which has type `'a list`. If ML is to evaluate a polymorphic function on the empty list, ML will, in certain cases, indicate that not enough information is known about the type of `[]`:

```
> rev [];
 type vars not generalized because of value
restriction are instantiated to dummy types (X1,X2,...)
val it = [] : ?.X1 list
```

[9]Older versions of ML made this function definition into an error.

Here `rev` is the built-in reverse function that reverses a list, and has type `' a list -> 'a list`. However, when **rev** is applied to the empty list, ML wants to instantiate the type variable `' a` to a specific type, and it cannot because not enough information is available. Nevertheless, ML compensates by supplying a "dummy" type, and gets the right answer, despite the warning message. If we wish, supplying a specific type avoids the problem:

```
- rev []:int list;
val it = [] : int list
```

A further unusual type-checking phenomenon is that ML makes a strong distinction between types that can be compared for equality and types that cannot. In particular, real numbers cannot be compared for equality:

```
> 2.1 = 2.1;
Error: operator and operand don't agree [equality type required]
  operator domain: ''Z * ''Z
  operand:         real * real
  in expression:
    2.1 = 2.1
```

When a polymorphic function definition involves an equality comparison, the type variable(s) can only range over the **equality types**, which are written with two quotes rather than a single quote:

```
> fun constList [] = true
        | constList [a] = true
        | constList (a::b::L) = a = b andalso constList (b::L);
val constList = fn : ''a list -> bool
```

Indeed, the equality function itself can only be applied to equality types:

```
> op =;
val it = fn : ''a * ''a -> bool
```

This somewhat awkward state of affairs has an elegant solution in Haskell, as you will see in the next section.

We conclude this section with a brief description of simple input and output in ML, and write an ML version of the Euclid program written in Scheme earlier in the chapter.

ML's version of the library package is the **structure** (see Chapter 11). It includes several standard predefined resources that are useful in performing input and output. The principle I/O functions are in the `TextIO` structure, and the simplest of these are the `inputLine` and `output` functions[10]:

[10]We are ignoring the simpler print function in the interest of uniformity.

```
> open TextIO; (* dereference the TextIO structure *)
... (* ML interpreter lists contents of TextIO *)
> output(stdOut,inputLine(stdIn));
             (* reads and writes a line of text *)
Hello, world!
Hello, world!
val it = () : unit
>
```

Note that the returned value of an `output` operation is the value `()`, which is of type `unit`. This is similar to the `void` type of C, but in ML every function must return *some* value, so the `unit` type has exactly one value `()` that represents "no actual value."[11]

Now, these two I/O functions only read and write strings. To convert between strings and numbers, we must use the `toString` and `fromString` functions from the utility structures for the appropriate types (for example, the `Int` structure for `int`s):

```
> Int.toString;
val it = fn : int -> string
> Int.fromString;
val it = fn : string -> int option
```

This presents yet another small hurdle. The `fromString` function may fail to extract any number, in which case it returns a value of the `option` type. This is defined as follows:

```
datatype 'a option = NONE | SOME of 'a;
```

Finally, when performing a series of input and output operations, it is convenient to collect these operations in an **expression sequence** (like the `begin` construct of Scheme), which is a semicolon-separated sequence of expressions surrounded by parentheses, and whose value is the value of the last expression listed:

```
> fun printstuff () =
       ( output(stdOut,"Hello\n");
         output(stdOut,"World!\n")
       );
val printstuff = fn : unit -> unit
> printstuff ();
Hello
World!
val it = () : unit
```

[11] `Unit` can also be used to imitate parameterless functions in ML, since ML requires every function to have at least one parameter.

Now we are ready for the `euclid` program in ML[12]:

```
> fun gcd (u,v) = if v = 0 then u
                        else gcd (v, u mod v) ;
val gcd = fn : int * int -> int
> fun euclid () =
     (  output(stdOut, "Enter two integers:\n");
        flushOut(stdOut);
        let val u = Int.fromString(inputLine(stdIn));
            val v = Int.fromString(inputLine(stdIn))
in
        case (u,v) of
        (SOME x, SOME y) =>
              ( output(stdOut,"The gcd of ");
                output(stdOut,Int.toString(x));
                output(stdOut," and ");
                output(stdOut,Int.toString(y));
                output(stdOut," is ");
                output(stdOut,Int.toString(gcd(x,y)));
                output(stdOut,"\n")
              ) |
         _ => output(stdOut, "Bad input.\n")
       end
     )
val euclid = fn : unit -> unit
> euclid ();
Enter two integers:
15
10
The gcd of 15 and 10 is 5
val it = () : unit
```

3.3.2 Data Structures in ML

Unlike Scheme, ML has a rich set of data types, from enumerated types to records to lists. You have already seen lists and tuples (Cartesian products) above. In Chapter 8, you will see additional examples of type structures in ML. We will preview some of those and examine some additional structures briefly here. While Scheme relies primarily on lists to simulate other data structures, ML has many options for user-defined data types, similar to C++ or Ada.

[12]There is one further quirk to be mentioned: We must use the `flushOut` function after the prompt string to make sure its printing is not delayed until after the input.

First, it is possible to give synonyms to existing data types using the keyword `type`:

```
> type Salary = real;
type Salary = real
> 35000.00: Salary;
val it = 35000.0 : Salary
> type Coords = real * real;
type Coords = real * real
> (2.1,3.2):Coords;
val it = (2.1,3.2) : Coords
```

ML also has new user-defined data types, introduced by the `datatype` keyword. For example, an enumerated type can be defined as follows:

```
> datatype Direction = North | East | South | West;
```

The vertical bar is used for alternative values in the declaration and the names, such as `North` and `East`, are called **value constructors** or **data constructors**. Data constructors can be used as patterns in declarations or case expressions, as in the following function definition:

```
> fun heading North = 0.0 |
      heading East = 90.0 |
      heading South = 180.0 |
      heading West = 270.0 ;
val heading = fn : Direction -> real
```

Recursive types such as binary search trees can also be declared using a `datatype` declaration:

```
> datatype 'a BST = Nil | Node of 'a * 'a BST * 'a BST;
```

Note that this `datatype` is parameterized by the type of data stored in the tree. We can now construct the tree of Figure 3.5 as follows:

```
> val tree = Node("horse",
            Node( "cow", Nil, Node("dog",Nil,Nil) ),
            Node("zebra",Node("yak",Nil,Nil), Nil));
val tree = Node ("horse",Node ("cow",Nil,Node #),Node ("zebra",Node #,Nil))
  : string BST
```

We can define `data`, `leftchild`, and `rightchild` functions on `BST`s by pattern matching. For example,

```
> fun leftchild (Node(data,left,right)) = left
       | leftchild Nil = raise Empty;
val leftchild = fn : 'a BST -> 'a BST
```

A traversal function for binary search trees that prints out the string nodes is as follows:

```
> fun print_tree Nil = () |
      print_tree (Node (data,left,right)) =
             ( print_tree left;
               output(stdOut,data);
               output(stdOut,"\n");
               print_tree right);
val print_tree = fn : vector BST -> unit
> print_tree tree;
cow
dog
horse
yak
zebra
val it = () : unit
```

3.3.3 Higher-Order Functions and Currying in ML

Higher-order functions and expressions whose values are functions are as useful in ML as in Scheme.

ML's keyword for a function expression, `fn`, is used with a special arrow (an equals followed by a greater than) to indicate the body of the function. For example:

```
> fn x => x * x;
val it = fn : int -> int
> (fn x => x * x) 4;
val it = 16 : int
```

A `fn` expression can be used to build anonymous functions and function return values. Indeed a `fun` definition is, as in Scheme, just syntactic sugar for the use of a `fn` expression:

```
fun square x = x * x;
```

is equivalent to:

```
val square = fn x => x * x;
```

One difference between ML and Scheme is that, if we use an ML `fn` expression to declare a recursive function, we must add the `rec` keyword in ML (similar to a Scheme `letrec`):

```
val rec fact = fn n => if n = 0 then 1 else n * fact (n - 1);
```

Function expressions can be used to create function return values as well as function arguments, as, for example, a `make_double` function that repeats an argument to a two-parameter function:

```
> fun make_double f = fn x => f (x,x);
val make_double = fn : ('a * 'a -> 'b) -> 'a -> 'b
> val square = make_double (op * );
val square = fn : int -> int
> val double = make_double (op +);
val double = fn : int -> int
> square 3;
val it = 9 : int
> double 3;
val it = 6 : int
```

ML has a number of built-in higher-order functions, including function composition, given by the letter `o` (lowercase letter "O"). For example:

```
> val double_square = double o square;
val double_square = fn : int -> int
> double_square 3;
val it = 18 : int
```

Higher-order functions and expressions whose values are functions are made even more useful in ML through a process known as currying. A function of multiple parameters that can be viewed as a higher-order function of a single parameter that returns a function of the remaining parameters is said to be **curried**, after the logician Haskell B. Curry. For example, the following definition of a `map` function is curried, because the parameters can be supplied separately[13] (or together):

```
> fun map f [] = [] | map f (h::t) = (f h):: map f t;
val map = fn : ('a -> 'b) -> 'a list -> 'b list
> val square_list = map square;
val square_list = fn : int list -> int list
> square_list [1,2,3];
val it = [1,4,9] : int list
> map (fn x => x + x) [1,2,3];
val it = [2,4,6] : int list
```

[13]The `map` function as defined here is in fact predefined in ML.

Note that in ML we have the choice of using a tuple to get an "uncurried" version of a function, or two separate parameters to get a curried version:

```
(* uncurried version of a gcd function: *)
> fun gcd (u,v) = if v = 0 then u else gcd (v , u mod v);
val gcd = fn : int * int -> int
(* curried version of a gcd function: *)
> fun gcd u v = if v = 0 then u else gcd v (u mod v);
val gcd = fn : int -> int -> int
```

In fact, in ML, the uncurried version of a function using a tuple is really just a function of a single parameter as well, except that its parameter is a tuple.

We call a language **fully curried** if function definitions are automatically treated as curried, and if all multiparameter built-in functions (such as the arithmetic operators) are curried. ML is not fully curried according to this definition, since all built-in binary operators are defined as taking tuples:

```
> op +;
val it = fn : int * int -> int
> op ::;
val it = fn : 'a * 'a list -> 'a list
> op @;
val it = fn : 'a list * 'a list -> 'a list
```

We could define curried versions of these functions, but they must be used as prefix functions rather than operators:

```
> fun plus x y = x + y;
val plus = fn : int -> int -> int
> plus 2 3;
val it = 5 : int
> val plus2 = plus 2;
val plus2 = fn : int -> int
> plus2 3;
val it = 5 : int
> fun append L1 L2 = L1 @ L2;
val append = fn : 'a list -> 'a list -> 'a list
> val append_to_one = append [1];
val append_to_one = fn : int list -> int list
> append_to_one [2,3];
val it = [1,2,3] : int list
```

3.4 Delayed Evaluation

An important problem that arises in the design and use of functional languages is the distinction between ordinary function applications and special forms. As we have already noted, in a language with an applicative order evaluation rule, such as Scheme and ML, all parameters to user-defined functions are evaluated at the time of a call, even though it may not be necessary to do so—or even wrong to do so.

Typical examples that do not use applicative order evaluation are the Boolean special forms `and` and `or`, as well as the `if` special form. In the case of the `and` special form, the Scheme expression `(and a b)` need not evaluate the parameter `b` if `a` evaluates to false. This is called short-circuit evaluation of Boolean expressions, and it is an example of the usefulness of delayed evaluation. In the case of the `if` special form, it is not a case of just usefulness, but of necessity; for the expression `(if a b c)` in Scheme to have the proper semantics, the evaluation of `b` and `c` must be delayed until the result of `a` is known, and based on that, either `b` or `c` is evaluated, but never both. For this reason, an `if` special form cannot be defined as a standard function in Scheme or ML. It also means that Scheme and ML must distinguish between two kinds of forms, those that use standard evaluation (function applications) and those that do not (the special forms).

A possible argument for restricting function evaluation to applicative order evaluation is that the semantics (and the implementation) are simpler. Consider the case of an expression in which one subexpression may have an undefined value, such as in the Scheme expression `(and (= 1 0) (=1 (/ 1 0)))` or its C equivalent `(1 == 0) && (1 == 1 / 0)`. In this case, delayed evaluation can lead to a well-defined result, even though subexpressions or parameters may be undefined. Functions with this property are called **nonstrict**, and languages with the property that functions are strict are easier to implement. In essence, strict languages satisfy a strong form of the GIGO principle (garbage in, garbage out), in that they will consistently fail to produce results when given incomplete or malformed input. Strictness also has an important simplifying effect on formal semantics (as discussed in Chapter 12).

Nevertheless, as you have seen, nonstrictness can be a desirable property. Indeed, the argument in favor of nonstrictness is essentially identical to the argument against it. Nonstrict functions can produce results even in the presence of incomplete or malformed input—as long as there is enough information to make the result reasonable. It is interesting to note that the language Algol60 included delayed evaluation in its pass by name parameter passing convention, as will be discussed in detail in Chapter 10. According to pass by name evaluation, a parameter is evaluated only when it is actually used in the code of the called procedure. Thus, the function:

```
function p(x: boolean; y: integer): integer;
begin
  if x then p := 1
  else p := y;
end;
```

will, using pass by name evaluation, return the value 1 when called as `p(true, 1 div 0)`, since `y` is never reached in the code of `p`, and so the value of `y`—the undefined expression `1 div 0`—will never be computed.

In a language that has function values, it is possible to delay the evaluation of a parameter by putting it inside a function "shell" (a function with no parameters). For example, in C, we can achieve the same effect as pass by name in the previous example by writing:

```
typedef int (*IntProc) ();
int divByZero ()
{  return 1 / 0;
}
int p(int x, IntProc y)
{ if (x) return 1;
  else return y();
}
```

and calling `p(1,divByZero)`. (Sometimes such "shell" procedures as `divByZero` are referred to as **pass by name thunks**, or just thunks, for somewhat obscure historical reasons.) In Scheme and ML, this process of surrounding parameters with function shells is even easier, since the `lambda` and `fn` function value constructors can be used directly, as in the following Scheme definition:

```
(define (p x y) (if x 1 (y)))
```

which can be called as follows:

```
(p #T (lambda () (/ 1 0)))
```

Note that the code of `p` must change to reflect the function parameter `y`, which must be surrounded by parentheses to force a call to `y`. Otherwise the function itself and not its value will be returned.

Indeed, Scheme has two special forms that do precisely what we have been describing. The special form `delay` delays the evaluation of its arguments and returns an object that can be thought of as a `lambda` "shell," or **promise** to evaluate its arguments. The corresponding special form `force` causes its parameter, which must be a delayed object, to be evaluated. Thus, the function `p` would be written as:

```
(define ( p x y) (if x 1 (force y)))
```

and called as:

```
(p #T (delay (/ 1 0)))
```

Inefficiency results from delayed evaluation when the same delayed expression is repeatedly evaluated. For example, in the delayed version of the `square` function,

```
(define (delayed-square x) (* (force x) (force x)))
```

the parameter `x` will be evaluated twice in the body of `delayed-square`, which is inefficient if `x` is a complicated expression. In fact, Scheme has an improvement to pass by name built into the `force` function. Scheme uses a **memoization** process, where the value of a delayed object is stored the first time the object is forced, and then subsequent calls to `force` simply return the stored value rather than recomputing it. (This kind of parameter evaluation is sometimes referred to as **pass by need**.)

Pass by name delayed evaluation is helpful in that it allows parameters to remain uncomputed if not needed and permits special forms similar to `if` and `cond` to be defined as ordinary functions. However, pass by name is not able to handle more complex situations where only parts of each parameter are

needed in a computation. Consider the following simple example of a `take` procedure that returns the first `n` items of a list:

```
(define (take n L)
   (if (= n 0) '()
      (cons (car L) (take (- n 1) (cdr L)))))
```

If we write a version in which the computation of `L` is delayed,

```
(define (take n L)
  (if (= n 0 ) '()
      (cons ( car (force L)) (take (- n 1)
                           (cdr (force L))))))
```

then a call `(take 1 (delay L))` will force the evaluation of the entire list `L`, even though we are interested in the very first element only. This can be disastrously inefficient if `L` happens to be a very long list produced by a list generation procedure such as:

```
(define (intlist m n)
  (if (> m n) '() (cons m (intlist (+ 1 m) n))))
```

Now a call `(take 1 (delay (intlist 2 100)))` will still construct the entire list `(2..100)` before taking the first element to produce the list `( 2 )`. What is needed is a `delay` in the second parameter to `cons` in `intlist` as well:

```
(define (intlist m n)
(if (> m n) '() (cons m (delay (intlist (+ 1 m) n)))))
```

so that taking `(cdr (cons m (delay ...)))` returns a delayed object to the recursive call to `take`. Thus, we have the following sequence of events in this computation, where each delayed object is represented by the computation it "promises" to perform enclosed in quotes:

1. The call `(take 1 (delay (intlist 2 100)))` causes the delayed object `(intlist 2 100)` to be passed as `L` to:

   ```
   (cons (car (force L)) (take (- 1 1) (cdr (force L))))
   ```

2. The first call to `(force L)` in the `cons` causes `L` to be evaluated to:

   ```
   (cons 2 ((delay (intlist (+ 1 2) 100))))
   ```

   which causes the construction of the list with head 2 and tail the delayed object `(intlist (+ 1 2) 100)`.

3. The `car` of `(force L)` returns 2 from the `cons`, leaving the following expression to be evaluated:

   ```
   (cons 2 (take (- 1 1) (cdr (force L))))
   ```

4. The call to `(take (- 1 1) (cdr (force L)))` causes `(- 1 1)` to be evaluated to 0, and the `cdr` of `(force L)` to be evaluated to the delayed object `(intlist (+ 1 2) 100)` as constructed in Step 2. The expression

`(take 0 (intlist (+ 1 2) 100))` is then evaluated and returns the empty list `'()` without forcing the evaluation of `(intlist (+ 1 2) 100)`.

5. The result of `(cons 2 '())` is finally evaluated, returning the list `(2)`. Thus, the rest of the integer list is never computed.

The scenario we have just described, together with memoization, is called **lazy evaluation**. This mechanism of evaluating expressions can be achieved in a functional language without explicit calls to `delay` and `force` by requiring the runtime environment to adhere to the following rules:

1. All arguments to user-defined functions are delayed.
2. All bindings of local names in `let` and `letrec` expressions are delayed.
3. All arguments to constructor functions (such as `cons`) are delayed.
4. All arguments to other predefined functions, such as the arithmetic functions "1," "*," and so on, are forced.
5. All function-valued arguments are forced.
6. All conditions in selection forms such as `if` and `cond` are forced.

According to these rules, operations on long lists can only compute as much of the list as is necessary. Lazy evaluation also makes it possible to include potentially infinite lists in languages, because only a part of the list would ever be computed at any one time. Lists that obey lazy evaluation rules are often called **streams**. This name is suggestive of the infinite nature of lists. A stream can be thought of as a partially computed list whose remaining elements can continue to be computed up to any desired number. Streams are an important mechanism in functional programming. To eliminate side effects completely, input and output must be introduced into functional languages as streams. Note that both Scheme and ML have ad hoc stream constructions (besides the manual `delay` and `force` procedures mentioned earlier).[14]

The primary example of a functional language with lazy evaluation is Haskell (again, named after Haskell B. Curry). Lazy evaluation supports a style of functional programming known as **generator-filter programming**. The programmer can separate a computation into procedures that generate streams and other procedures that take streams as arguments, without worrying about the efficiency of each step. Procedures that generate streams are called **generators**, and procedures that modify streams are called **filters**. For example, the `intlist` procedure in a previous Scheme example is a generator, and the `take` procedure is a filter.

A famous problem in functional programming that requires generator-filter programming is the **same-fringe** problem for lists. Two lists have the same fringe if they contain the same non-null atoms in the same order, or, to put it another way, when they are written as trees, their non-null leaves taken in left-to-right order are the same. For example, the lists `((2 (3)) 4)` and `(2 (3 4 ()))` have the same fringe. To determine whether two lists have the same fringe, we must **flatten** them to just lists of their atoms:

[14]Haskell has replaced I/O streams with a more general construct called a monad, which we also do not study here.

```
(define (flatten L)
  (cond ((null? L) '())
        ((list? L) (append (flatten (car L)) (flatten    (cdr L))))
        (else (cons L '()) )))
```

In the case of both lists in the previous paragraph, `flatten` returns the list `(2 3 4)`. The `flatten` function can be viewed as a filter, which then can be used as the input to the `samefringe` procedure, as follows:

```
(define (samefringe? L M) (equal? (flatten L)
                                  (flatten M)))
```

The problem with this computation is that, under the usual Scheme evaluation rule, `flatten` will produce complete lists of the elements of `L` and `M`, even if `L` and `M` differ already in their first fringe elements. Lazy evaluation, on the other hand, will compute only enough of the flattened lists as necessary before their elements disagree. Only when the lists actually do have the same fringe must the entire lists be processed.

A similar situation arises when using the sieve of Eratosthenes algorithm to compute prime numbers. In this method, a list of consecutive integers from 2 is generated, and each successive remaining number in the list is used to cancel out its multiples in the list. Thus, beginning with the list `(2 3 4 5 6 7 8 9 10 11)`, we first cancel multiples of 2, leaving the list `(2 3 5 7 9 11)`. We then cancel the multiples of 3, giving `(2 3 5 7 11)`, and now we cancel all remaining multiples of 5, 7, and 11 (of which there are none) to obtain the list of primes from 2 to 11 as `(2 3 5 7 11)`. Each cancellation step can be viewed as a filter on the list of the previous cancellation step. We will see in the next section how both the sieve of Eratosthenes problem and the fringe comparison problem can be solved in Haskell.

Why don't all functional languages use delayed evaluation? For one thing, it complicates the semantics of the language. In practical terms, this translates into an increased complexity in the runtime environment that must maintain the evaluation rules 1–6 listed earlier. Also, because of the interleaving of `delays` and `forces`, it is difficult to write programs with desired side effects, in particular, programs with variables that change as computation proceeds. In a way, this is similar to the synchronization problem for shared memory in parallel processing (studied in Chapter 13). Indeed, delayed evaluation has been described as a form of parallelism, with `delay` as a form of process suspension, and `force` a kind of process continuation. The principal point is that side effects, in particular assignment, do not mix well with lazy evaluation. This is in part the reason that Haskell is purely functional, with no variables or assignment, and why Scheme and ML are strict languages, with some ad hoc stream and delayed evaluation facilities.

3.5 Haskell—A Fully Curried Lazy Language with Overloading

Haskell is a pure functional language whose development began in the late 1980s, primarily at Yale University and the University of Glasgow. Its current standard is Haskell98, which we use in our discussion here. Haskell builds on and extends a series of purely functional lazy languages developed in the late 1970s and 1980s by David A. Turner, culminating in the Miranda programming language. Haskell also

contains a number of novel features, including function overloading and a mechanism called **monads** for dealing with side effects such as I/O (always a problem for pure functional languages). For more on the monad or I/O features, see the Notes and References for this chapter. In the following discussion, we focus on Haskell's lazy evaluation and overloading.

3.5.1 Elements of Haskell

Haskell's syntax is very similar to that of ML, with some notable differences. Haskell reduces the amount of syntax to an absolute minimum using internal program clues, including a **layout rule** that uses indentation and line formatting to resolve ambiguities. As a result, it is rare for a Haskell program to require a semicolon or bracketing of any kind. Indeed, no special syntax for definitions is required, and no vertical bar is used for functions defined using pattern matching. Here are a few examples of function definitions in Haskell:

```
fact 0 = 1
fact n = n * fact (n - 1)
square x = x * x
gcd1 u v = if v == 0 then u else gcd1 v (mod u v)
reverse1 [] = []
reverse1 (h:t) = reverse1 t ++ [h]
```

In these definitions we note the following differences between Haskell and ML. First, Haskell does not allow the redefinition of functions that are already predefined. (However, it is easy to change what is actually predefined.) Thus, we append a `1` to the functions `reverse` and `gcd` in the preceding code, since they are already predefined. Next, we note that the `cons` operator for constructing lists is written as a single colon. Types, on the other hand, are given using a double colon, which is exactly the reverse of ML. Pattern matching, as already mentioned, does not require the use of the `.` symbol. Finally, list concatenation in Haskell is given by the `++` operator.

Haskell is a fully curried language. All predefined binary operators are curried. In a construct called a **section** in Haskell, a binary operator can be partially applied to either argument using parentheses. Thus,

```
plus2 = (2 +)
```

defines a function that adds 2 to its argument on the left, while:

```
times3 = (* 3)
```

defines a function that multiplies 3 times its argument on the right:

```
> plus2 3
5
> times3 4
12
```

Infix functions can also be turned into prefix functions by surrounding them with parentheses. No `op` keyword is required. For example:

```
> (+) 2 3
5
> (*) 4 5
20
```

Haskell has anonymous functions or `lambda` forms, with the backslash representing the `lambda`:

```
> (\x -> x * x ) 3
9
```

3.5.2 Higher-Order Functions and List Comprehensions

Haskell includes many predefined higher-order functions, such as `map`. These are all in curried form (as noted above), so they can be partially applied to construct further higher-order functions. For example, the `map` function has type:

```
(a -> b) -> [a] -> [b]
```

A `square_list` function can be defined as follows, leaving off the list parameter by currying:

```
square_list = map (\x -> x * x)
```

As another example, the function composition operator in Haskell is denoted by the dot (`.`):

```
> ((*3) . (2+)) 5
21
```

Haskell, like ML, has built-in lists and tuples, as well as type synonyms and user-defined polymorphic types:

```
type ListFn a = [a] -> [a]
type Salary = Float
type Pair a b = (a,b)
data BST a = Nil | Node a (BST a) (BST a)
```

Note that type variables in Haskell (such as `a` and `b` in the preceding definitions) are written without the quote of ML, and are also written after the data type name, rather than before. The keyword `data` replaces the ML keyword `datatype`, and new user-defined types are written in a slightly different syntax without the `of` keyword. Type and constructor names in Haskell are also required to be uppercase, while function and value names must be lowercase.

Functions on new data types can use data constructors as patterns, as in ML:

```
flatten:: BST a -> [a]
flatten Nil = []
flatten (Node val left right) =
         (flatten left) ++ [val] ++ (flatten right)
```

Note that the definition is preceded by the type of the `flatten` function. The type specifier is permitted, although not required. A type specifier can be used to resolve ambiguities or restrict the type of the function beyond what Hindley-Milner type checking would infer. The use of this option is similar to the way ML allows the types of the parameters and returned types to be specified within the definition itself, something not allowed in Haskell98.

Haskell also has a special notation for operations that are applied over lists, called **list comprehensions**. List comprehensions are designed to look like set definitions in mathematics. For example, if we wanted to square a list of integers, we could write the following:

```
square_list lis = [ x * x | x <- lis]
```

This is really just syntactic sugar for `map (\x -> x * x) lis`, but it can make programming with lists even easier and more transparent. The notation can also be extended to include Boolean conditions, such as:

```
square_list_positive lis = [ x * x | x <- lis, x > 0]
```

which is also syntactic sugar for `map (\x -> x * x) (filter (>0) lis)`, where `filter` is a predefined list function defined as follows:

```
filter:: (a -> Bool) -> [a] -> [a]
filter pred [] = []
filter pred (h:t) =
   if pred h then h : filter pred t else filter pred t
```

As another example of the use of list comprehensions, a highly compact definition for quicksort can be given (using the first element of a list as the pivot):

```
qsort [] = []
qsort (h:t) = qsort [x | x <- t, x <= h] ++
                    [h] ++ qsort [x | x <- t, x > h]
```

3.5.3 Lazy Evaluation and Infinite Lists

Because Haskell is a lazy language, no value is computed unless it is actually needed for the computation of a program. Thus, given the definition:

```
f x = 2
```

any call to `f` will never evaluate its argument, since it is not needed for the result. Similarly, the following function will behave exactly like the built-in `if-then-else` expression:

```
my_if x y z = if x then y else z
```

Lazy evaluation also means that lists in Haskell are the same as streams, and are capable of being potentially infinite. Thus, we could define an infinite list of ones as follows, without danger of getting into an immediate infinite loop:

```
ones = 1 : ones
```

Haskell has, in fact, several shorthand notations for infinite lists. For example, the notation `[n..]` stands for the list of integers beginning with `n`. This same list can also be written as `[n, n+1..]`, and the list of ones can also be defined as `[1,1..]`. As a final example of this notation, the list of even positive integers can be written as `[2,4..]`.

Given a potentially infinite list, it is of course impossible to display or process the entire list. Thus, functions that partially compute the list become useful. Two such functions are `take` and `drop`; `take` extracts the first `n` items from a list, and `drop` discards the first `n` items (the underscore is the wildcard pattern, as in ML):

```
take 0 _ = []
take _ [] = []
take n (h:t) = h : take (n - 1) t
drop 0 lis = lis
drop _ [] = []
drop n (_:t) = drop (n - 1) t
```

For example, `take 5 (drop 4 [2..])` gives the result `[6,7,8,9,10]`.

Using lazy evaluation, it is possible to get an extremely compact representation of the Sieve of Eratosthenes introduced earlier in the discussion on ML:

```
sieve (p : lis) = p : sieve [n | n <- lis , mod n p /= 0]
primes = sieve [2..]
```

With this definition of the (infinite) list of primes, `take` can be used to compute any desired number of primes:

```
> take 100 primes
[2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,
73,79,83,89,97,101,103,107,109,113,127,131,137,139,149,
151,157,163,167,173,179,181,191,193,197,199,211,223,
227,229,233,239,241,251,257,263,269,271,277,281,283,
293,307,311,313,317,331,337,347,349,353,359,367,373,
379,383,389,397,401,409,419,421,431,433,439,443,449,
457,461,463,467,479,487,491,499,503,509,521,523,541]
```

3.5.4 Type Classes and Overloaded Functions

The last topic we discuss in this short overview of Haskell is its unique approach to overloading. Recall ML functions cannot be overloaded, and this either led to unresolved ambiguities or strong assumptions that ML had to make to avoid them. In particular, we could not define an arithmetic function such as `square` that would work for both integers and real numbers. Haskell has no such problem. Indeed, we can define a `square` function as usual:

```
square x = x * x
```

and then use it on any numeric value:

```
> square 2
4
> square 3.5
12.25
```

The type of `square` gives us a hint as to how Haskell does this:

```
square :: Num a => a -> a
```

This type is called a `qualified type`. It says essentially that `square` can be applied to any type `a` as long as `a` is a `Num`. But what is a `Num`? It is not a type, but a **type class**. That is, it is a set of types that all define certain functions. Here is a possible, though incomplete, definition of the `Num` type class:

```
class Num a where
    (+), (-), (*)  :: a -> a -> a
    negate         :: a -> a
    abs            :: a -> a
```

A type class definition states the names and types, also called **signatures**, of the functions that every type belonging to it must define. Thus, type classes are a little like Java interfaces. By itself, of course, this definition does nothing. For any types to belong to this class, we must give an **instance definition**, in which actual working definitions for each of the required functions are provided. For example, in the standard library, the data types `Int`, `Float`, and `Double` are all instances of `Num`. A (partial) instance definition for `Int` in the standard library looks as follows:

```
instance Num Int where
    (+)         = primPlusInt
    (-)         = primMinusInt
    negate      = primNegInt
    (*)         = primMulInt
    abs         = absReal
```

The functions to the right of the equal signs are all built-in definitions that are hidden inside the standard library modules.

Haskell determines the type of the `square` function by looking for the symbol * in a class definition (because the body of `square` depends on the availability of *) and applying the appropriate qualification. Then, the `square` function can be applied to any value whose type is an instance of the `Num` class.

There is, however, more structure to type classes than this simple description suggests. In the case of `Int`, for example, the `Num` class does not describe all of the functions that need to be available, such as equality tests and order functions like <. Nor does the `Int` instance include the functions `div` and `mod`. The `Num` class above does not even require any division operation. Additional type classes do require these functions. The class `Eq` requires the equality operator ==, the class `Ord` requires the less-than operator <, and the class `Integral` requires the `div` and `mod` operations. To provide full capability, `Int` should be defined to be an instance of all of these type classes. To accomplish this, many of the type classes themselves are defined to be part of other type classes, so that any instances are forced to be instances of these other classes as well. This dependency of type classes on other type classes is called **type class inheritance**. Type inheritance relies upon a hierarchy of type classes. Two type classes at the base of this hierarchy are the `Eq` and `Show` classes. The class `Show` is used to establish the existence of a `show` function, which is used to display values of member types. All predefined Haskell types are instances of the `Show` class. The class `Eq` establishes the ability of two values of a member type to be compared using the == operator. Most types are instances of `Eq`. However, function types are not, because the equality of functions cannot be defined in a way that would provide useful information.

Here is a complete definition of the `Eq` class from the Haskell standard library:

```
class Eq a where
    (==), (/=) :: a -> a -> Bool
    x == y     = not (x/=y)
    x /= y     = not (x==y)
```

This class definition shows an additional feature: the use of default definitions for the required functions. Such default definitions are typically based on other required functions. In the preceding code, the default definition of /= (not equal) depends on the definition of == being available, and the default definition of == depends on the definition of /= being available. This apparent circularity means that, for a type to be declared an instance of `Eq`, only one of the two required functions needs to be defined, with the other implicitly defined by default. For example, `Int` is defined to be an instance of `Eq` as follows:

```
instance Eq Int where (==) = primEqInt
```

To establish the inheritance requirements for each type class, Haskell uses a notation similar to type qualification. For example, the definition of the `Num` class in the Haskell standard library begins as follows:

```
class (Eq a, Show a) => Num a where ...
```

And the definition of the `Ord` class is as follows (with the default function definitions removed):

```
class (Eq a) => Ord a where
    compare                 :: a -> a -> Ordering
    (<), (<=), (>=), (>)    :: a -> a -> Bool
    max, min                :: a -> a -> a
```

The numeric class hierarchy of Haskell is shown in Figure 3.10, along with sample functions that are required by some of the type classes.

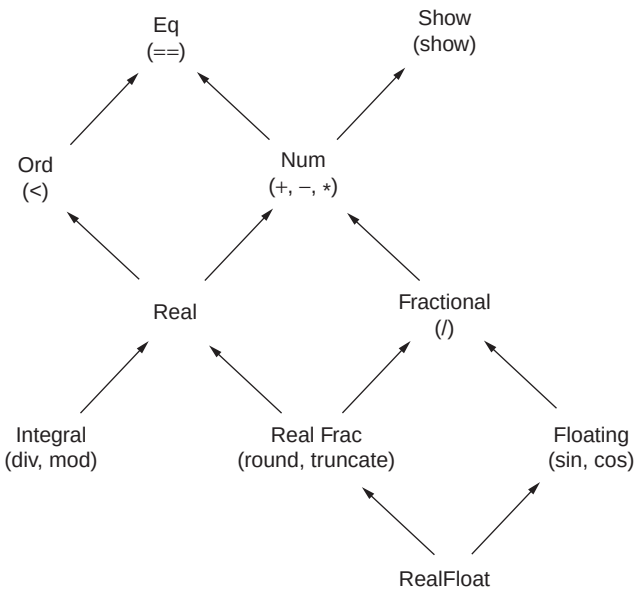


Figure 3.10 The numeric type class hierarchy in Haskell, with sample functions required by some of the classes in parentheses

The following is a final example of the use of type classes in Haskell. When a program defines a new data type, such as the `BST` type defined previously,

```
data BST a = Nil | Node a (BST a) (BST a)
```

then such trees cannot in general either be displayed or compared for equality, since we have not defined `BST` to be an instance of the `Show` or `Eq` type classes. For example, if we define:

```
tree = Node "horse"
(Node "cow" Nil (Node "dog" Nil Nil))
           (Node "zebra" (Node "yak" Nil Nil) Nil)
```

then a Haskell interpreter will respond as follows when we try to display or compare `tree`:[15]

```
> tree
ERROR: Cannot find "show" function for:
*** Expression : tree
*** Of type    : Tree [Char]
> tree == tree
ERROR: Illegal Haskell 98 class constraint in inferred type
*** Expression : tree == tree
*** Type       : Eq (BST [Char]) => Bool
```

In order to make a user-defined data type useful, we typically must define it to be an instance of `Eq` and `Show`. Note that `Show` requires a function `show` that converts the data type in question into a `String` so that it can be displayed. For example:

```
instance Eq a => Eq (BST a)
    where
    Nil == Nil = True
    Node x a b == Node y c d = x == y && a == c && b == d
    _ == _ = False

instance Show a => Show (BST a) where show = ...
```

Note in particular how the definition of equality for a `BST` depends on the membership of the type parameter `a` in `Eq`, because otherwise the equality of the stored data cannot be tested.

It is often necessary to define a new data type to be a member of certain classes like `Eq`, `Show`, and `Ord`, so Haskell offers a way to automatically generate these definitions. Haskell simply assumes that the "natural" definitions are desired. A data type is shown by printing it out as a string, and equality is defined component-wise. To accomplish this, a `deriving` clause is added to the data definition:

```
data BST a = Nil | Node a (BST a) (BST a)
             deriving (Show,Eq)
```

With this definition of `BST`, Haskell has no trouble displaying a `BST` or comparing two `BST`s for equality:

```
> tree
Node "horse" (Node "cow" Nil (Node "dog" Nil Nil))
  (Node "zebra" (Node "yak" Nil Nil) Nil)
> tree == tree
True
```

[15]This is the output from the HUGS Haskell interpreter—see the Notes and References.

3.6 The Mathematics of Functional Programming: Lambda Calculus

Lambda calculus was invented by Alonzo Church in the 1930s as a mathematical formalism for expressing computation by functions, similar to the way a Turing machine is a formalism for expressing computation by a computer. In fact, lambda calculus can be used as a model for (purely) functional programming languages in much the same way that Turing machines can be used as models for imperative programming languages. Thus, it is an important result for functional programming that lambda calculus as a description of computation is equivalent to Turing machines. This implies the result that a purely functional programming language, with no variables and no assignment, and with an `if-then-else` construct and recursion, is Turing complete. This means that computation performed by a Turing machine can be described by such a language.

Lambda calculus provides a particularly simple and clear view of computation. It is useful for anyone interested in functional programming to have at least some knowledge of lambda calculus, because many functional languages, including Lisp, ML, and Haskell, were based on lambda calculus. Therefore, we offer this section as a basic introduction to the subject. For those with more than a superficial interest in functional programming, we recommend consulting one of the texts listed at the end of the chapter.

The essential construct of lambda calculus is the **lambda abstraction**:

$$(\lambda\, x\, .\, 1\ 1\ x)$$

This can be interpreted exactly as the lambda expression:

```
(lambda (x) (+ 1 x))
```

in Scheme, namely, as representing an unnamed function of the parameter `x` that adds 1 to `x`. Note that, like Scheme, expressions such as (1 1 *x*) are written in prefix form.

The basic operation of lambda calculus is the **application** of expressions such as the lambda abstraction. The expression

$$(\lambda\, x\, .\, +\ 1\ x)\ 2$$

represents the application of the function that adds 1 to *x* to the constant 2. Lambda calculus expresses this by providing a **reduction rule** that permits 2 to be substituted for *x* in the lambda (and removing the lambda), yielding the desired value:

$$(\lambda\, x\, .\, +\ 1\ x)\ 2 \Rightarrow (+\ 1\ 2) \Rightarrow 3$$

The syntax for lambda calculus is as follows:

$$\begin{aligned} exp &\rightarrow constant \\ &\mid variable \end{aligned}$$

| (*exp exp*)

| (λ *variable* . *exp*)

Constants in this grammar are numbers like 0 or 1 and certain predefined functions like 1 and *. Variables are names like *x* and *y*. The third rule for expressions represents function application (*f x*) as noted earlier. The fourth rule gives lambda abstractions. Note that only functions of a single variable are allowed, but since lambda expressions provide for higher-order functions, this is not a significant restriction, as the notion of currying, discussed earlier in this chapter, allows for functions of several variables to be interpreted simply as functions of a single variable returning functions of the remaining variables. Thus, lambda calculus as defined here is fully curried.

Variables in lambda calculus are not like variables in a programming language—they do not occupy memory, because lambda calculus has no concept of memory. Lambda calculus variables correspond instead to function parameters as in purely functional programming. The set of constants and the set of variables are not specified by the grammar. Thus, it is more correct to speak of many **lambda calculi**. Each specification of the set of constants and the set of variables describes a particular lambda calculus. Lambda calculus without constants is called pure lambda calculus.

Parentheses are included in the rules for function application and lambda abstraction to avoid ambiguity, but by convention may be left out, in case no ambiguity results. Thus, according to our grammar, we should have written (λ*x*. ((+ 1) *x*)) for (λ*x*. + 1 *x*) and ((λ*x*. ((+ 1) *x*) 2) for (λ*x*. + 1 *x*) 2. However, no ambiguity results from the way they were written.

The variable *x* in the expression (λ*x*.*E*) is said to be **bound** by the lambda. The scope of the binding is the expression *E*. An occurrence of a variable outside the scope of any binding of it by a lambda is a **free occurrence**. An occurrence that is not free is a **bound** occurrence. Thus, in the expression (λ*x*. *E*), all occurrences of *x* in *E* are bound. For example, in the expression

(λ*x*. + *y x*)

x is bound and *y* is free. If we try to apply the function specified by this lambda abstraction, we substitute for *x*, but we have no way of determining the value of *y*:

(λ*x*. + *y x*) 2 ⇒ (+ *y* 2)

The variable *y* is like a nonlocal reference in the function—its value must be specified by an external environment.

Different occurrences of a variable can be bound by different lambdas, and some occurrences of a variable may be bound, while others are free. In the following expression:

(λ*x*. + ((λ*y*. ((λ*x*. * *x y*) 2)) *x*) *y*)

the occurrence of *x* after the * is bound by a different lambda than the outer *x*. Also, the first occurrence of *y* is bound, while the second occurrence is free. In fact, applying the functions defined by the lambdas gives the following reduction:

$$(\lambda x.\ +\ ((\lambda y.\ ((\lambda x.\ *\ x\ y)\ 2))\ x)\ y) \Rightarrow$$

$$(\lambda x.\ +\ ((\lambda y.\ (*\ 2\ y))\ x)\ y) \Rightarrow$$

$$(\lambda x.\ +\ (*\ 2\ x)\ y)$$

When reducing expressions that contain multiple bindings with the same name, care must be taken not to confuse the scopes of the bindings—sometimes renaming is necessary (discussed shortly).

We can view lambda calculus as modeling functional programming by considering a lambda abstraction as a function definition and the juxtaposition of two expressions as function application. As a model, however, lambda calculus is extremely general. For example, nothing prevents us in lambda calculus from applying a function to itself or even defining a function that takes a function parameter and applies it to itself: $(x\ x)$ and $(\lambda x.\ x\ x)$ are legal expressions in lambda calculus. A more restrictive form of lambda calculus, called the **typed lambda calculus**, includes the notion of data type from programming languages, thus reducing the set of expressions that are allowed. We will not consider the typed lambda calculus further.

Since lambda calculus is so general, however, very precise rules must be given for transforming expressions, such as substituting values for bound variables. These rules have historical names, such as "alpha-conversion" and "beta-conversion." We will give a short overview to show the kinds of operations that can occur on lambda expressions.

The primary method for transforming lambda expressions is by **substitution**, or **function application**. We have seen a number of examples of this already: $((\lambda x.\ +\ x\ 1)\ 2)$ is equivalent to $(\ +\ 2\ 1)$, substituting 2 for *x* and eliminating the lambda. This is exactly like a call in a programming language to the function defined by the lambda abstraction, with 2 substituted for *x* as a value parameter. Historically, this process has been called **beta-reduction** in lambda calculus. One can also view this process in reverse, where $(+\ 2\ 1)$ becomes $((\lambda x.\ +\ x\ 1)\ 2)$, in which case it is called **beta-abstraction**. **Beta-conversion** refers to either beta-reduction or beta-abstraction and establishes the equivalence of the two expressions. Formally, beta-conversion says that $((\lambda x.E)\ F)$ is equivalent to $E[F/x]$, where $E[F/x]$ is *E* with all free occurrences of *x* in *E* replaced by *F*.

Care must be taken in beta-conversion when *F* contains variable names that occur in *E*. Consider the expression:

$$((\lambda x.(\ \lambda y.\ +\ x\ y))\ y)$$

The first occurrence of *y* is bound, while the second is free. If we were to replace *x* by *y* blindly, we would get the incorrect reduction $(\lambda y.\ +\ y\ y)$. This is called the **name capture** problem. What we have to do is change the name of *y* in the inner lambda abstraction so that the free occurrence will not conflict, as follows:

$$((\lambda x.(\ \lambda y.\ +\ x\ y))\ y) \Rightarrow ((\lambda x.(\ \lambda z.\ +\ x\ z))\ y) \Rightarrow (\lambda z.\ +\ y\ z)$$

Name change is another available conversion process in lambda calculus, called **alpha-conversion**: $(\lambda x.E)$ is equivalent to $(\lambda y.E[y/x])$, where as before, $E[y/x]$ stands for the expression E with all free occurrences of x replaced by y. (Note that if y is a variable that already occurs in E there can be trouble similar to the substitution problem just discussed.)

Finally, there is a conversion that allows for the elimination of "redundant" lambda abstractions. This conversion is called **eta-conversion**. A simple example is the expression $(\lambda x.\ (+\ 1\ x))$. Since 1 is curried, the expression (+ 1) is a function of one argument, and the expression (+ 1 x) can be viewed as the application of the function + 1 to x. Thus, $(\lambda x.\ (+\ 1\ x))$ is equivalent to the function (+ 1) without the lambda abstraction. In general, $(\lambda x.\ (E\ x))$ is equivalent to E by eta-conversion, as long as E contains no free occurrences of x. A further example of eta-conversion is:

$$(\lambda x.\ (\lambda y.(+\ x\ y))) \Rightarrow (\lambda x.\ (+\ x)) \Rightarrow 1$$

which is to say that the first expression is just another notation for the 1 function.

It is worth noting that eta-reduction is helpful in simplifying curried definitions in functional languages. For example, the ML definition

```
fun square_list lis = map (fn x => x * x) lis ;
```

can be simplified by eta-reduction to

```
val square_list = map (fn x => x * x) ;
```

The order in which beta-reductions are applied to a lambda expression can have an effect on the final result obtained, just as different evaluation orders can make a difference in programming languages. In particular, it is possible to distinguish applicative order evaluation (or pass by value) from normal order evaluation (or pass by name). For example, in the following expression:

$$((\lambda x.\ *\ x\ x)\ (+\ 2\ 3))$$

we could either use applicative order, replacing (1 2 3) by its value and then applying beta-reduction, as in

$$((\lambda x.\ *\ x\ x)\ (+\ 2\ 3)) \Rightarrow ((\lambda x.\ *\ x\ x)\ 5) \Rightarrow (*\ 5\ 5) \Rightarrow 25$$

or we could apply beta-reduction first and then evaluate, giving normal order evaluation:

$$((\lambda x.\ *\ x\ x)\ (\ +\ 2\ 3)) \Rightarrow (*\ (+\ 2\ 3)\ (+\ 2\ 3)) \Rightarrow (*\ 5\ 5) \Rightarrow 25$$

Normal order evaluation is, as we have noted several times, a kind of **delayed** evaluation, since the evaluation of expressions is done only after substitution.

Normal order evaluation can have a different result than applicative order, as we have also previously noted. A striking example is when parameter evaluation gives an undefined result. If the value of an expression does not depend on the value of the parameter, then normal order will still compute the correct value, while applicative order will also give an undefined result.

For example, consider the lambda expression $((\lambda x.\ x\ x)\ (\lambda x.\ x\ x))$. Beta-reduction on this expression results in the same expression all over again. Thus, beta-reduction goes into an "infinite loop," attempting to reduce this expression. If we use this expression as a parameter to a constant expression, as in:

$$((\lambda y.\ 2)\ ((\lambda x.\ x\ x)\ (\lambda x.\ x\ x)))$$

then using applicative order, we get an undefined result, while using normal order we get the value 2, since it does not depend on the value of the parameter y.

Functions that can return a value even when parameters are undefined are said to be **nonstrict**, while functions that are always undefined when a parameter is undefined are **strict**. In lambda calculus notation, the symbol $\perp$ (pronounced "bottom") represents an undefined value, and a function f is strict if it is always true that $(f\ \perp) = \perp$. Thus, applicative order evaluation is strict, while normal order evaluation is nonstrict.

Normal order reduction is significant in lambda calculus in that many expressions can be reduced, using normal order, to a unique normal form that cannot be reduced further. This is a consequence of the famous **Church-Rosser theorem**, which states that reduction sequences are essentially independent of the order in which they are performed. The equivalence of two expressions can then be determined by reducing them using normal order and comparing their normal forms, if they exist. This has practical applications for translators of functional languages, too, since a translator can use normal forms to replace one function by an equivalent but more efficient function.

Finally, in this brief overview of lambda calculus, we wish to show how lambda calculus can model recursive function definitions in exactly the way we have treated them in the previous section. Consider the following definition of the factorial function:

$$\text{fact} = (\lambda n.\ (\text{if}\ (=\ n\ 0)\ 1\ (*\ n\ (\text{fact}\ (-\ n\ 1)))))$$

We remove the recursion by creating a new lambda abstraction as follows:

$$\text{fact} = (\lambda F.\ \lambda n.\ (\text{if}\ (=\ n\ 0)\ 1\ (*\ n\ (F\ (-\ n\ 1))))))\ \text{fact}$$

In other words, the right-hand side can be viewed as a lambda abstraction, which, when applied to fact, gives fact back again.

If we write this abstraction as

$$H = (\lambda F.\ \lambda n.\ (\text{if}\ (=\ n\ 0)\ 1\ (*\ n\ (F\ (-\ n\ 1)))))$$

then the equation becomes

$$\text{fact} = H\ \text{fact}$$

or that fact is a **fixed point** of H. We could construct a least-fixed-point solution by building up a set representation for the function. However, in the lambda calculus, lambda expressions are primitive; that is, we cannot use sets to model them. Thus, to define a recursive function fact in the lambda calculus, we

need a function Y for constructing a fixed point of the lambda expression H. Y needs, therefore, to have the property that $Y\ H$ = fact or $Y\ H = H\ (Y\ H)$. Such a Y can indeed be defined by the following lambda expression:

$$Y = (\lambda\ h.\ (\lambda\ x.\ h\ (x\ x))\ (\lambda\ x.\ h\ (x\ x))).$$

Y is called a **fixed-point combinator**, a combinator being any lambda expression that contains only bound variables. Such a Y actually constructs a solution that is in some sense the "smallest." Thus, one can refer to the **least-fixed-point semantics** of recursive functions in lambda calculus, as well as in the ordinary set-based theory of functions.

Exercises

3.1 The following C function computes the power ab, where a is a floating-point number and b is a (non-negative) integer:

```
double power(double a, int b){
   int i;
   double temp = 1.0;
   for (i = 1; i <= b; i++) temp *= a;
   return temp;
   }
```

(a) Rewrite this procedure in functional form.
(b) Rewrite your answer to **(a)** using an accumulating parameter to make it tail recursive.

3.2 A function that returns the maximum value of a 0-ended list of integers input from a user is given by the following C function:

```
int readMax(){
   int max, x;
   scanf("%d",&x);
   if (x != 0){
      max = x;
      while (x != 0){
         scanf("%d",&x);
         if (x > max) max = x;
      }
      return max;
   }
}
```

Rewrite this procedure as much as possible in functional style using tail recursion (the `scanf` procedure prevents a complete removal of variables).

3.3 Recursive sorts are easier to write in functional style than others. Two recursive sorts are Quicksort and Mergesort. Write functional versions of **(a)** Quicksort; **(b)** Mergesort in an imperative language of your choice (e.g., C, Ada, C++, Python), using an array of integers as your basic data structure.

3.4 It is possible to write nonrecursive programs that implement recursive algorithms using a stack, but at the cost of extra source code complexity (a necessary overhead of using a nonrecursive language like FORTRAN). Write nonrecursive versions of **(a)** Quicksort; **(b)** Mergesort. Discuss the difficulties of the added complexity. **(c)** Which of the two sorts is easier to implement nonrecursively? Why?

3.5 Which of the following functions are referentially transparent? Explain your reasoning.

(a) The factorial function

(b) A function that returns a number from the keyboard

(c) A function `p` that counts the number of times it is called

3.6 Is a function that has no side effects referentially transparent? Explain.

3.7 The binomial coefficients are a frequent computational task in computer science. They are defined as follows for $n >= 0$, $0 <= k <= n$ (! is factorial and $0! = 1$):

$$B(n, k) = \frac{n!}{(n - k)!\, k!}$$

(a) Write a procedure using a loop to compute $B(n, k)$. Test your program on $B(10, 5)$.

(b) Use the following recurrence and the fact that $B(n, 0) = 1$ and $B(n, n) = 1$ to write a functional procedure to compute $B(n, k)$:

$$B(n, k) = B(n - 1, k - 1) + B(n - 1, k)$$

Test your program on $B(10, 5)$.

(c) Can you write a more efficient program than your program of **(b)** and still preserve the functional style? (*Hint*: Think of pass by need memoization as discussed in Section 3.5.)

3.8 Complete the following functional programming exercises using Scheme, ML, or Haskell:

(a) Write a tail-recursive function to compute the length of an arbitrary list.

(b) Write a function that computes the maximum and minimum of a list of integers.

(c) Write a function that collects integers from the user until a 0 is encountered and returns them in a list in the order they were input (Scheme or ML only).

(d) Use the functions you wrote for exercises (b) and (c) to write a program to input a 0-ended list of integers, print the list in the order entered, and print the maximum and minimum of the list. For this exercise, use only Scheme or ML.

(e) Write Quicksort for a list of integers (Scheme or ML only).

(f) Write Mergesort for a list of integers.

(g) A Pythagorean triple is a tuple of integers (x, y, z) such that $x * x + y * y = z * z$. Write a function with a parameter n to print all Pythagorean triples such that $1 \le x \le y \le z \le n$.

(h) Write a higher-order function `twice` that takes as a parameter a function of one argument and returns a function that represents the application of that function to its argument twice. Given the usual definition of the `square` function, what function is `(twice (twice square))`?

(i) Write a higher-order function `inc_n` that takes an integer `n` as a parameter and returns an *n*th increment function, which increments its parameter by `n`. Thus, in Scheme syntax, `((inc_n 3) 2)` = 5 and `((inc_n -2) 3)` = 1.

3.9 Suppose that you want to use a list data structure to implement a `Set` data type. Write `insert` and `member` operations in **(a)** Scheme; **(b)** ML; **(c)** Haskell. Make sure that in ML and Haskell your `Set` data type is distinct from a list. **(d)** In Haskell, define your `Set` type to be an instance of classes `Eq` and `Show`.

3.10 Given the definition of a binary search tree in Haskell as given in the text:

```
data BST a = Nil | Node a (BST a) (BST a)
```

write a `show` function `(show:: (BST a) -> String)` as it would be generated by a Haskell `deriving Show` clause.

3.11 Many functions discussed in this chapter are not completely tail recursive, but are almost tail recursive. In these functions, the recursive call comes just before an arithmetic operation, which is the last operation in the function. For example, factorial and length of a list are almost tail recursive in this sense. Describe a general pattern for turning an almost tail-recursive function into a loop, in the same way that a tail-recursive function can be so transformed, as described in Section 3.1. Can a translator recognize almost tail recursion as easily as tail recursion? How might it do so?

3.12 Draw box and pointer diagrams for the following Scheme lists:

```
((((a))))
(1 (2 (3 4)) (5))
((a ( )) ((c) (d) b) e)
```

3.13 Use the box and pointer diagrams of the last exercise to compute the following for each Scheme list for which they make sense:

```
(car (car L))
(car (cdr L))
(car (cdr (cdr (cdr L))))
(cdr (car (cdr (cdr L))))
```

3.14 Represent the following elements as `car/cdr` expressions of the lists of Exercise 3.12:
of the first list
of the second list
of the third list

3.15 Scheme's basic data structure is actually a little more general than the lists described in this chapter. Indeed, since the `car` of a list can be a list or an atom, it is also possible for the `cdr` to be an atom as well as a list. In the case when the `cdr` is not a list, the resulting structure is called a **pair** or **S-expression** instead of a list and is written `(a . b)`, where `a` is the `car` and `b` is the `cdr`. Thus, `(cons 2 3) = (2 . 3)`, and `(cdr '(2 . 3)) = 3`. The list `(2 3 4)` can then be written in pair notation as `(2 . (3 . (4 . ())))`. Discuss any advantages you can think of to this more general data structure.

3.16 Write a Scheme list representation for the binary search tree shown in Figure 3.11.

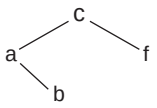


Figure 3.11 Binary search tree for Exercise 3.16

3.17 Write an `insert` procedure in Scheme for the binary search tree data structure described in Section 3.3.2.

3.18 Try to write a tail-recursive version of the `append` function in Scheme. What problems do you encounter? Can you think of a list representation that would make this easier?

3.19 **(a)** Give an algebraic specification for the ADT `List` with the following operations, and with properties as in Scheme: `car`, `cdr`, `cons`, `null?`, `makenull`.

(b) Write a C++ or Java class, or an Ada package to implement the ADT of (a).

(c) Use your implementation of the `List` ADT in (b) to write a "tiny Scheme" interpreter, that is, an interpreter that has only the ADT list functions as available operations and only has numbers as atoms.

3.20 In Section 3.2.6 we defined the `make-double` higher-order function in Scheme. What functions are returned by `(make-double - )` and `(make-double / )`?

3.21 Scheme does not have a built-in `filter` function. Like `map`, `filter` is a higher-order function that expects a function and a list as arguments and returns a list of results, but the function argument to `filter` is a predicate or Boolean function of one argument. `filter` uses the predicate to test each element in the list and returns a list of just those elements that pass the test. Define a `filter` function in Scheme.

3.22 **(a)** Give an example to show that Scheme does not curry its functions.

(b) Write a Scheme higher-order function that takes a function of two parameters as its parameter and returns a curried version of the function.

3.23 List the ML types for all the functions in Exercise 3.8.

3.24 Write an ML function that determines the length of any list. Show how an ML system can determine the type of the length function using pattern matching.

3.25 List the Haskell types for all the functions in Exercise 3.8.

3.26 Write Haskell list comprehensions for the following lists: **(a)** all integers that are squares, **(b)** all Pythagorean triples (see Exercise 3.8), and **(c)** all perfect numbers (a perfect number is the sum of all of its proper factors).

3.27 **(a)** Write functions to test whether ML and Scheme use short-circuit evaluation for Boolean expressions.

(b) Why do we not need to test Haskell to answer this question for that language?

3.28 When a function is defined using pattern matching in ML, the text mentions that an ML system may complain if cases are left out of the definition. Such missing cases imply the function is partial. Discuss the advantages and disadvantages of requiring all functions to be total in a programming language.

3.29 The `fact` function defined in this chapter for ML and Scheme is in fact partial, yet an ML translator will not discover this. Why?

3.30 ML and Haskell do not have general list structures like Scheme, but require the elements in a list to have all the same data type. Why is this? What data structure in an imperative language do such lists imitate?

3.31 Write a Scheme program to show that the Scheme procedures `force` and `delay` actually use pass by need memoization.

3.32 **(a)** Write a sieve of Eratosthenes in Scheme or ML using generators and filters, similar to the Haskell version in Section 3.5.

(b) Rewrite the Scheme version to use `force` and `delay`.

3.33 Rewrite the Scheme `intlist` function in Section 3.4 so that it takes only a lower bound as a parameter and produces a stream of integers from that point on: `(intlist 5) = (5 6 7 8 ...)`.

3.34 Haskell list comprehensions are actually compact expressions for generator-filter programs as noted in the text. For example, the list comprehension

```
evens = [ n | n <- [2..], mod n 2 == 0]
```

is equivalent to a generator procedure that produces the stream of integers beginning with 2 (represented by `[2..]`) and sends its output to the selector procedure of the predicate `mod n 2 = 0` that passes on the list whose elements satisfy the predicate. Write a Scheme procedure that uses `force` and `delay` to produce the stream of even integers in a similar generator-filter style.

3.35 Rewrite the `flatten` Scheme procedure of Section 3.4 to use `force` and `delay`.

3.36 (From Abelson and Sussman [1996]) Define a delayed version of the `map` function from Section 3.2.6 as follows:

```
(define (delayed-map f L)
  (if (null? L) '()
      (delay (cons (f (car (force L)))
                   (delayed-map f (cdr (force L)))))))
```

Now define a `show` procedure that prints a value and returns the same value:

```
define (show x) (display x) (newline) x)
```

Finally, define a delayed list as follows:

```
(define L
  (delayed-map show (delay (intlist 1 100))))
```

where `intlist` is the delayed version from Section 3.4. What will the Scheme interpreter print when given the following expressions to evaluate in the order given (`take` is also the delayed version in Section 3.4):

```
> (take 5 L)
... some output here
> (take 7 L)
... some more output here
```

(*Hint*: The answer depends on whether Scheme uses pass by need or not; see Exercise 3.31.)

3.37 Write lambda calculus expressions for the higher-order functions `twice` and `inc_n`. (See Exercise 3.8.)

3.38 Assume square = ($\lambda x.$ * x x) in lambda calculus. Show the steps in an applicative order and normal order reduction of the expression (twice (twice square)). (See the previous exercise.)

3.39 Give applicative and normal order reductions for the following lambda expressions. State which steps use which conversions and which variables are bound and which are free.

(a) ($\lambda x.$ (($\lambda y.$(* 2 y)) (+ x y))) y

(b) ($\lambda x.$ $\lambda y.$ (x y)) ($\lambda z.$ (z y))

3.40 It is a surprising fact in lambda calculus that lists can be expressed as lambda abstractions. Indeed, the list constructor cons can be written as ($\lambda x.$ $\lambda y.$ $\lambda f.$ f x y). With this definition one can write car as ($\lambda z.$ z ($\lambda x.$ $\lambda y.$ x)) and cdr as ($\lambda z.$ z ($\lambda x.$ $\lambda y.$ y)). Show that using these definitions the usual formulas (car (cons a b)) = a and (cdr (cons a b)) = b are true.

3.41 (From Abelson and Sussman [1996]) It is also possible to express the integers as lambda abstractions:

zero = $\lambda f.$ $\lambda x.x$

one = $\lambda f.$ $\lambda x.$(f x)

two = $\lambda f.$ $\lambda x.$(f (f x))

. . .

These are called Church numbers, after Alonzo Church.

(a) Given the following definition of the successor function:

$$\text{successor} = \lambda n.\lambda f.\ \lambda x.(f\ ((n\ f)\ x))$$

show that (successor zero) 5 one and (successor one) 5 two.

(b) Generalize (a) to any Church number.

(c) Define addition and multiplication for Church numbers.

(d) Write out an implementation of your lambda expressions in (c) as procedures in a functional language, and write an output procedure that shows they are correct.

3.42 Use the lambda expression H = ($\lambda F.$ $\lambda n.$ (if (= n 0) 1 (* n (F (- n 1))))) and the property that the fact function is a fixed point of H to show that *fact* 1 = 1.

3.43 We noted that the fixed-point combinator Y can itself be written as the lambda abstraction ($\lambda h.$ ($\lambda x.$ h (x x)) ($\lambda x.$ h (x x))). Show that this expression for Y does indeed give it the property that Y H = H (Y H), for any H.

3.44 Consider writing the fixed-point combinator in Scheme, ML, or Haskell by using the fixed-point property y h = h (y h) as the definition.

(a) What is the type of y in ML or Haskell?

(b) Using the definition of h for the factorial function:

$$h\ g\ n = \text{if } n = 0 \text{ then } 1 \text{ else } n * g(n - 1)$$

does y h actually produce the correct factorial function in Scheme, ML, or Haskell? Explain.

Notes and References

There are many books on functional programming. We list only a few here that are directly related to the languages and issues discussed in the chapter. For an overview of functional programming languages, including historical perspectives, from a somewhat different viewpoint from this chapter, see Hudak [1989] and MacLennan [1990]. Reade [1989] is a comprehensive text on functional programming, with many examples from ML. Abelson and Sussman [1996] is also a good reference for much of this chapter, as well as for the Scheme language (Section 3.2). Friedman et al. [1996] is an advanced reference for Scheme. The (current) definition of Scheme is published in Sperber, Dybvig, Flatt, and van Stratten [2009]. The Scheme R6RS report can be found online at *http://www.r6rs.org/final/html/r6rs/r6rs.html.*

Functional programming with the ML language (Section 3.3) is treated in Paulson [1996] and Ullman [1998]. Milner et al. [1997] gives the definition of Standard ML97, and Milner and Tofte [1991] provides a description of some of the theoretical issues surrounding the design of ML; see also Mitchell and Harper [1988].

The Haskell language (Section 3.5) is presented in Hudak [2000], Thompson [1999], and Bird et al. [1998]. The influential predecessor language Miranda is described in Turner [1986], with underlying ideas presented in Turner [1982].

General techniques for functional programming are studied in Okasaki [1999] and Lapalme and Rabhi [1999]. Implementation techniques for functional languages are studied in Peyton Jones [1987] and Appel [1992].

Delayed evaluation is studied in Henderson [1980]; parts of Section 3.4 are adapted from that reference.

Interest in functional programming increased dramatically following the ACM Turing Award lecture of Backus [1978], in which he describes a general framework for functional programming, called FP. A significant modern version of Lisp not discussed in this chapter is Common Lisp, which is described in Steele [1982], Graham [1996], and Seibel [2005] and defined in Steele [1984]. The ANSI Standard for Common Lisp is ANSI X3.226 [1994].

The lambda calculus (Section 3.6) began with Church [1941] and is studied in Curry and Feys [1958], Barendregt [1982], and Hankin [1995]. Overviews of lambda calculus are presented in Peyton Jones [1987], where the use of normal forms in compiling Miranda is discussed; parts of the presentation in Section 3.6 are patterned after that text. Gunter [1992] and Sethi [1995] also survey the lambda calculus and study a statically typed version called the typed lambda calculus. Paulson [1996] describes an interpreter for lambda calculus. A history of the lambda calculus and the Church-Rosser theorem is given in Rosser [1982]. Currying is apparently originally due not to Curry but to Schönfinkel [1924].

Traditionally, functional programming has been thought to be highly inefficient because of its reliance on function call and dynamic memory management. Advances in algorithms, such as generational garbage collection, and advances in translation techniques make this less true. Some references dealing with these issues are Steele [1977] and Gabriel [1985]. Using recursive calls to do functional programming in an imperative language is also less expensive than one might imagine, and the techniques in Section 3.1 are usable in practice. An example of this is discussed in Louden [1987].

CHAPTER

Logic Programming

CHAPTER 4

Logic, the science of reasoning and proof, has existed since the time of the ancient Greek philosophers. Mathematical, or symbolic, logic, the theory of mathematical proofs, began with the work of George Boole and Augustus De Morgan in the middle of the 1800s. Since then, logic has become a major mathematical discipline and has played a significant role in the mathematics of the twentieth century, particularly with respect to the famous incompleteness theorems of Kurt Gödel. (See the Notes and References.)

Logic is closely associated with computers and programming languages in a number of ways. First, computer circuits are designed with the help of Boolean algebra, and Boolean expressions and data are almost universally used in programming languages to control the actions of a program. Second, logical statements are used to describe **axiomatic semantics**, or the semantics of programming language constructs. Logical statements can also be used as formal specifications for the required behavior of programs, and together with the axiomatic semantics of the language, they can be used to prove the correctness of a program in a purely mathematical way.

In the opposite direction, computers are used as tools to implement the principles of mathematical logic, with programmers creating programs that can construct proofs of mathematical theorems using the principles of logic. These programs, known as **automatic deduction systems** or **automatic theorem provers**, turn proofs into computation. Experience with such programs in the 1960s and 1970s led to the major realization that the reverse is also true: Computation can be viewed as a kind of proof. Thus, logical statements, or at least a restricted form of them, can be viewed as a programming language and executed on a computer, given a sophisticated enough interpretive system. This work, primarily by Robinson, Colmerauer, and Kowalski (see the Notes and References), led to the programming language **Prolog**. The development of efficient Prolog interpreters in the late 1970s, particularly at the University of Edinburgh, Scotland, resulted in a tremendous increase in interest in logic programming systems.

Then, in 1981, Prolog achieved almost instant fame when the Japanese government announced that it would serve as the base language for the Fifth Generation Project, whose goal was the development of advanced computer systems incorporating reasoning techniques and human language understanding. The project ended somewhat inconclusively in 1992. Nevertheless, a significant amount of research and development of logic programming systems occurred as a result. Prolog remains today the most significant example of a logic programming language.

In the following sections, we will first give a brief introduction to mathematical logic and the way logic can be used as a programming language. Next, we turn our attention to a description of Prolog and the techniques of writing Prolog programs. We also give a brief description of the principles behind the operation of a Prolog system, and some of the weaknesses of these systems. Finally, we describe some of the additional work on equational and constraint logic systems.

4.1 Logic and Logic Programs

Before you can learn about logic programming, you need to know a little bit about mathematical logic. The kind of logic used in logic programming is the **first-order predicate calculus**, which is a way of formally expressing **logical statements**, that is, statements that are either true or false.

EXAMPLE 1

The following English statements are logical statements:

0 is a natural number.
2 is a natural number.
For all x, if x is a natural number, then so is the successor of x.
21 is a natural number.

A translation into predicate calculus is as follows:

```
natural(0).
natural(2).
For all x, natural(x) → natural(successor (x)).
natural(21).
```

Among these logical statements, the first and third statements can be viewed as **axioms** for the natural numbers: statements that are assumed to be true and from which all true statements about natural numbers can be **proved**. Indeed, the second statement can be proved from these axioms, since `2 = successor(successor(0)) and natural(0) → natural (successor(0)) → natural (successor(successor (0)))`. The fourth statement, on the other hand, cannot be proved from the axioms and so can be assumed to be false.

First-order predicate calculus classifies the different parts of such statements as follows:

1. *Constants*. These are usually numbers or names. Sometimes they are called atoms, since they cannot be broken down into subparts. In Example 1, 0 is a constant.
2. *Predicates*. These are names for functions that are true or false, like Boolean functions in a program. Predicates can take a number of arguments. In Example 1, the predicate `natural` takes one argument.
3. *Functions*. First-order predicate calculus distinguishes between functions that are true or false—these are the predicates—and all other functions, like `successor` in Example 1, which represent non-Boolean values.
4. *Variables That Stand for as yet Unspecified Quantities*. In Example 1, `x` is a variable.
5. *Connectives*. These include the operations `and`, `or`, and `not`, just like the operations on Boolean data in programming languages. Additional connectives in predicate calculus are implication, signified by $\rightarrow$, and equivalence, indicated by $\leftrightarrow$. These are not really new operations: $a \rightarrow b$ means that b is true whenever a is true. This is equivalent to the statement b or not a. Also $a \leftrightarrow b$ means the same as $(a \rightarrow b)$ and $(b \rightarrow a)$.

6. *Quantifiers*. These are operations that introduce variables. In Example 1, `for all` is the quantifier for `x`; it is called the **universal quantifier**. The statement:

   ```
   For all x, natural(x) → natural(successor(x))
   ```

   means that for every `x` in the universe, if `x` is a natural number, then the successor of `x` is also a natural number. A universal quantifier is used to state that a relationship among predicates is true for all things in the universe named by the variable `x`.

   There is also the **existential quantifier**, `there exists`, as in the following statement:

   ```
   there exists x, natural(x).
   ```

   This statement means that there exists an `x` such that `x` is a natural number. An existential quantifier is used to state that a predicate is true of at least one thing in the universe, indicated by the variable `x`.

   A variable introduced by a quantifier is said to be **bound** by the quantifier. It is possible for variables also to be **free**, that is, not bound by any quantifier.

7. *Punctuation Symbols*. These include left and right parentheses, the comma, and the period. (Strictly speaking, the period isn't necessary, but we include it since most logic programming systems use it.) Parentheses are used to enclose arguments and also to group operations. Parentheses can be left out in accordance with common conventions about the precedence of connectives, which are usually assumed to be the same as in most programming languages. (Thus, the connectives in order of decreasing precedence are `not`, `and`, `or`, $\rightarrow$, and $\leftrightarrow$.)

In predicate calculus, arguments to predicates and functions can only be **terms**—that is, combinations of variables, constants, and functions. Terms cannot contain predicates, quantifiers, or connectives. In Example 1, terms that appear include constants 0, 21, 2, the variable `x`, and the term `successor(x)`, consisting of the function `successor` with argument `x`. Some examples of additional terms that can be written are `successor(0)` and `successor(successor(successor(x)))`.

EXAMPLE 2

The following are logical statements in English:

A horse is a mammal.
A human is a mammal.
Mammals have four legs and no arms, or two legs and two arms.
A horse has no arms.
A human has arms.
A human has no legs.

A possible translation of these statements into first-order predicate calculus is as follows:

```
mammal(horse).
mammal(human).
for all x, mammal(x) →
  legs(x, 4) and arms(x, 0) or legs(x, 2) and arms(x, 2).
arms(horse, 0).
not arms(human, 0).
legs(human, 0).
```

In this example, the constants are the integers 0, 2, and 4 and the names horse and human. The predicates are `mammal`, `arms`, and `legs`. The only variable is *x*, and there are no functions.

As in the previous example, we might consider the first five statements to be axioms—statements defining true relationships. Then, as we shall see shortly, `arms(human, 2)` becomes provable from the axioms. It is also possible to prove that `legs(human, 2)` is true, so the last statement is false, given the axioms.

Note that we have used precedence to leave out many parentheses in the preceding statements, so that, for example, when we write

```
legs(x, 4) and arms(x, 0) or legs(x, 2) and arms(x, 2)
```

we mean the following:

```
(legs(x,4) and arms(x,0)) or (legs(x,2) and arms(x,2)).
```

In addition to the seven classes of symbols described, first-order predicate calculus has **inference rules**, which are ways of deriving or proving new statements from a given set of statements.

EXAMPLE 3

A typical inference rule is the following:

From the statements $a \rightarrow b$ and $b \rightarrow c$, one can derive the statement $a \rightarrow c$, or written more formally,

$$\frac{(a \rightarrow b) \text{ and } (b \rightarrow c)}{a \rightarrow c}$$

Inference rules allow us to construct the set of all statements that can be derived, or proved, from a given set of statements: These are statements that are always true whenever the original statements are true. For example, the first five statements about mammals in Example 2 allow us to derive the following statements:

```
legs(horse,4).
legs(human,2).
arms (human, 2).
```

Stated in the language of logic, these three statements become **theorems** derived from the first five statements or axioms of Example 2 . Notice that proving these statements from the given statements can be viewed as the computation of the number of arms and legs of a horse or a human. Thus, the

set of statements in Example 2 can be viewed as representing the potential computation of all logical consequences of these statements.

This is the essence of logic programming: A collection of statements is assumed to be axioms, and from them a desired fact is derived by the application of inference rules in some automated way. Thus, we can state the following definition:

A ***logic programming language*** is a notational system for writing logical statements together with specified algorithms for implementing inference rules.

The set of logical statements that are taken to be axioms can be viewed as the **logic program**, and the statement or statements that are to be derived can be viewed as the input that initiates the computation. Such inputs are also provided by the programmer and are called **queries** or **goals**. For example, given the set of axioms of Example 2, if we wanted to know how many legs a human has, we would provide the following query:

Does there exist a *y* such that *y* is the number of legs of a human?
or, in predicate calculus,

```
there exists y, legs(human,y)?
```

and the system would respond with something like:

```
yes: y 5 2
```

For this reason, logic programming systems are sometimes referred to as **deductive databases**, databases consisting of a set of statements and a deduction system that can respond to queries. Notice that these are different from ordinary databases, since they contain not only facts such as `mammal(human)` or `natural(0)` but also more complex statements such as `natural(x)` → `natural(successor(x))`, and the system can answer not only queries about facts but also queries involving such implications.

In a pure logic programming system, nothing is said about how a particular statement might be derived from a given set of statements. The specific path or sequence of steps that an automatic deduction system chooses to derive a statement is the **control problem** for a logic programming system. The original statements represent the logic of the computation, while the deductive system provides the control by which a new statement is derived. This property of logic programming systems led Kowalski to state the logic programming paradigm as the pseudoequation:

$$\text{algorithm} = \text{logic} + \text{control}$$

as a contrast to Niklaus Wirth's expression of imperative programming as:

$$\text{algorithms} = \text{data structures} + \text{programs}$$

(See the Notes and References.) Kowalski's principle points out a further feature of logic programming: Since logic programs do not express the control, operations (in theory at least) can be carried out in any order or simultaneously. Thus, logic programming languages are natural candidates for parallelism.

Unfortunately, automated deduction systems have difficulty handling all of first-order predicate calculus. First, there are too many ways of expressing the same statements, and second, there are too many inference rules. As a result, most logic programming systems restrict themselves to a particular subset of predicate calculus, called Horn clauses, which we will study briefly next.

4.2 Horn Clauses

A **Horn clause** (named after its inventor Alfred Horn) is a statement of the form:

$$a_1 \text{ and } a_2 \text{ and } a_3 \ldots \text{ and } a_n \rightarrow b$$

where the a_i are only allowed to be simple statements involving no connectives. Thus, there are no *or* connectives and no quantifiers in Horn clauses. The preceding Horn clause says that a_1 through a_n imply b, or that b is true if all the a_i are true. b is called the **head** of the clause, and the $a_1 \ldots, a_n$ is the **body** of the clause. In the Horn clause, the number of a_i's may be 0, in which case the Horn clause has the form:

$$\rightarrow b$$

Such a clause means that b is always true. In other words, b is an axiom and is usually written without the connective $\rightarrow$. Such clauses are sometimes also called **facts**.

Horn clauses can be used to express most, but not all, logical statements. Indeed, one algorithm that is beyond the scope of this book can perform a reasonable translation from predicate calculus statements to Horn clauses. (See the Notes and References.) The basic idea is to remove *or* connectives by writing separate clauses, and to treat the lack of quantifiers by assuming that variables appearing in the head of a clause are universally quantified, while variables appearing in the body of a clause (but not in the head) are existentially quantified.

EXAMPLE 4

The following statements from Example 1 are written in first-order predicate calculus:

```
natural(0).
for all x, natural(x) → natural (successor (x)).
```

These can be very simply translated into Horn clauses by dropping the quantifier:

```
natural(0).
natural(x) → natural (successor(x)).
```

EXAMPLE 5

Consider the logical description for the Euclidian algorithm to compute the greatest common divisor of two positive integers u and v:
The gcd of u and 0 is u.
The gcd of u and v, if v is not 0, is the same as the gcd of v and the remainder of dividing v into u.
Translating this into first-order predicate calculus gives:

```
for all u, gcd(u, 0, u).
for all u, for all v, for all w,
not zero(v) and gcd(v, u mod v, w) → gcd(u, v, w).
```

(Remember that `gcd(u, v, w)` is a predicate expressing that w is the gcd of u and v.)
To translate these statements into Horn clauses, we need again only drop the quantifiers:

```
gcd(u, 0, u).
not zero(v) and gcd(v, u mod v, w) → gcd(u, v, w).
```

EXAMPLE 6

The foregoing examples contain only universally quantified variables. To see how an existentially quantified variable in the body may also be handled, consider the following statement:

> *x* is a grandparent of *y* if *x* is the parent of
> someone who is the parent of *y*.

Translating this into predicate calculus, we get

```
for all x, for all y, (there exists z, parent(x, z) and parent(z, y))
→ grandparent (x, y).
```

As a Horn clause this is expressed simply as:

```
parent(x, z) and parent(z, y) → grandparent(x, y).
```

EXAMPLE 7

To see how connectives are handled, consider the following statement:

> For all *x*, if *x* is a mammal then *x* has two or four legs.

Translating in predicate calculus, we get:

```
for all x, mammal(x) → legs(x, 2) or legs(x, 4).
```

This may be approximated by the following Horn clauses:

```
mammal(x) and not legs(x, 2) → legs(x, 4).
mammal(x) and not legs(x, 4) → legs(x, 2).
```

In general, the more connectives that appear to the right of a $\rightarrow$ connective in a statement, the harder it is to translate into a set of Horn clauses; see Exercise 4.8.

Horn clauses are of particular interest to automatic deduction systems such as logic programming systems, because they can be given a **procedural interpretation**. If we write a Horn clause in reverse order

$$b \leftarrow a_1 \text{ and } a_2 \text{ and } a_3 \ldots \text{ and } a_n$$

we can view this as a definition of procedure *b*: the body of *b* is given by the body of the clause, namely the operations indicated by the a_i's. This is very similar to the way context-free grammar rules are interpreted as procedure definitions in recursive descent parsing, as we shall see in Chapter 6. There is more than a passing similarity here, since logic programs can be used to directly construct parsers. (See the Notes and References.) In fact, the parsing of natural language was one of the motivations for the original development of Prolog. The major difference between parsing and logic programming is that in pure logic programming, the order in which the a_i's are called is not specified.

Nevertheless, most logic programming systems are deterministic in that they perform the calls in a certain prespecified order, usually left to right, which is exactly the order indicated by context-free grammar rules. The particular kind of grammar rules used in Prolog programs are called **definite clause grammars**.

Horn clauses can also be viewed as **specifications** of procedures rather than strictly as implementations. For example, we could view the following Horn clause as a specification of a sort procedure:

```
sort(x, y) ← permutation(x, y) and sorted(y).
```

This says that (assuming that `x` and `y` are lists of things) a sort procedure transforms list `x` into a sorted list `y` such that `y` is a permutation of `x`. To complete the specification, we must, of course, supply

specifications for what it means for a list to be sorted and for a list to be a permutation of another list. The important point is that we may think of the Horn clause as not necessarily supplying the algorithm by which y is found, given an x, but only the properties such a y must have.

With the foregoing procedural interpretation in mind, most logic programming systems not only write Horn clauses backward but also drop the *and* connectives between the a_i, separating them with commas instead. Thus, the greatest common divisor clauses in Example 5 would appear as follows:

```
gcd(u, 0, u).
gcd(u, v, w) ← not zero(v), gcd(v, u mod v, w).
```

This is beginning to look suspiciously like a more standard programming language expression for the gcd, such as

```
gcd(u, v) = if v = 0 then u else gcd(v, u mod v).
```

From now on we will write Horn clauses in this form.

Now let's consider the issue of variable scope in a procedural interpretation of Horn clauses. As with procedure definitions in a block-structured language, the assumption is that all variables are local to each call of the procedure. Indeed, variables used in the head can be viewed as parameters, while variables used only in the body can be viewed as local, temporary variables. This is only an approximate description, as the algorithms used to implement the execution of Horn clauses treat variables in a more general way, as you will see in Section 4.3.

We haven't yet seen how queries or goal statements can be expressed as Horn clauses. In fact, a query is exactly the opposite of a fact—a Horn clause with no head:

```
mammal(human) ←.   — a fact
mammal(human).     — a query or goal
```

A Horn clause without a head could also include a sequence of queries separated by commas:

```
← mammal(x), legs(x, y).
```

Why queries correspond to Horn clauses without heads should become clear when you understand the inference rule that logic programming systems apply to derive new statements from a set of given statements, namely, the resolution rule or principle, studied next.

4.3 Resolution and Unification

Resolution is an inference rule for Horn clauses that is especially efficient. Resolution says that if we have two Horn clauses, and we can match the head of the first Horn clause with one of the statements in the body of the second clause, then the first clause can be used to replace its head in the second clause by its body. In symbols, if we have Horn clauses:

$$a \leftarrow a_1, \ldots, a_n.$$
$$b \leftarrow b_1, \ldots, b_m.$$

and b_i matches a, then we can infer the clause:

$$b \leftarrow b_1, \ldots, b_i\text{-}1, a_1, \ldots, a_n, b_{i+1}, \ldots, b_m.$$

The simplest example of this occurs when the body of the Horn clauses contains only single statements, such as in:

$$b \leftarrow a.$$

and:

$$c \leftarrow b.$$

In this case, resolution says that we may infer the following clause:

$$c \leftarrow a.$$

This is precisely the inference rule that we gave in Example 3 in Section 4.1.

Another way of looking at resolution is to combine left-hand and right-hand sides of both Horn clauses and then cancel those statements that match on both sides. Thus, for the simplest example,

$$b \leftarrow a.$$

and:

$$c \leftarrow b.$$

give:

$$b, c \leftarrow a, b.$$

and canceling the b,

$$\not{b}, c \leftarrow a, \not{b}$$

gives:

$$c \leftarrow a.$$

Now you can see how a logic programming system can treat a goal or list of goals as a Horn clause without a head. The system attempts to apply resolution by matching one of the goals in the body of the headless clause with the head of a known clause. It then replaces the matched goal with the body of that clause, creating a new list of goals, which it continues to modify in the same way. The new goals are called **subgoals**. In symbols, if we have the goal:

$$\leftarrow a.$$

and the clause $a \leftarrow a_1, \ldots, a_n$, then resolution replaces the original goal a with the subgoals:

$$\leftarrow a_1, \ldots, a_n.$$

If the system succeeds eventually in eliminating all goals—thus deriving the empty Horn clause—then the original statement has been proved.

EXAMPLE 8

The simplest case is when the goal is already a known fact, such as:

```
mammal(human).
```

and one asks whether a human is a mammal:

```
← mammal(human).
```

Using resolution, the system combines the two Horn clauses into:

```
mammal(human) ← mammal(human).
```

and then cancels both sides to obtain:

```
←.
```

Thus, the system has found that indeed a human is a mammal and would respond to the query with "Yes." ■

EXAMPLE 9

A slightly more complicated example is the following. Given the rules and facts:

```
legs(x, 2) ← mammal(x), arms(x, 2).
legs(x, 4) ← mammal(x), arms(x, 0).
mammal(horse).
arms(horse, 0).
```

if we supply the query:

```
← legs(horse,4).
```

then applying resolution using the second rule, we get:

```
legs(x, 4) ← mammal(x), arms(x, 0), legs(horse, 4).
```

Now, to cancel the statements involving the predicate `legs` from each side, we have to match the variable `x` to `horse`, so we replace *x* by horse everywhere in the statement:

```
legs(horse, 4) ← mammal(horse), arms(horse, 0), legs(horse, 4).
```

and cancel to get the subgoals:

```
← mammal(horse), arms(horse, 0).
```

Now we apply resolution twice more, using the facts `mammal(horse)` and `arms(horse, 0)` and cancel:

```
mammal(horse) ← mammal(horse), arms(horse, 0).
arms(horse, 0).
arms(horse, 0) ← arms(horse, 0).
```

Since we have arrived at the empty statement, our original query is true. ■

Example 9 demonstrates an additional requirement when we apply resolution to derive goals. To match statements that contain variables, we must set the variables equal to terms so that the statements become identical and can be canceled from both sides. This process of pattern matching to make statements identical is called **unification**, and variables that are set equal to patterns are said to be **instantiated**. Thus, to implement resolution effectively, we must also provide an algorithm for unification. A very general unification algorithm does exist and was mentioned in Chapter 3 in relation to the Hindley-Milner type inference. Most logic programming systems, however, use a slightly weaker algorithm, which we discuss in Section 4.4.

EXAMPLE 10

To show in more detail the kind of unification that takes place in a logic programming language, consider the greatest common divisor problem (Euclid's algorithm, Example 5):

```
gcd(u, 0, u).
gcd(u, v, w) ← not zero(v), gcd(v, u mod v, w).
```

Now given the goal:

```
← gcd(15, 10, x).
```

resolution fails using the first clause (10 does not match 0), so using the second clause and unifying `gcd(u, v, w)` with `gcd(15, 10, x)` gives:

```
gcd(15, 10, x) ← not zero(10), gcd(10, 15 mod 10, x), gcd(15, 10, x).
```

Assuming that the system knows that `zero(10)` is false, so that `not zero(10)` is true, and simplifying `15 mod 10` to 5, we cancel `gcd (15, 10, x)` from both sides and obtain the subgoal:

```
← gcd(10, 5, x).
```

Note that this is just another goal to be resolved like the original goal. This we do by unification as before, obtaining

```
gcd(10, 5, x) ← not zero(5), gcd(5, 10 mod 5, x), gcd(10, 5, x).
```

and so get the further subgoal:

```
← gcd(5, 0, x).
```

Now this matches the first rule,

```
gcd(u, 0, u).
```

so instantiating `x` to 5 results in the empty statement, and the system will reply with something like:

```
Yes: x = 5
```

Resolution and unification have performed the computation of the greatest common divisor of 10 and 15, using Euclid's algorithm! ■

Now let's consider another problem that must be solved before a working resolution implementation can be achieved. To achieve efficient execution, a logic programming system must apply a fixed algorithm that specifies: (1) the order in which the system attempts to resolve a list of goals, and (2) the order in which clauses are used to resolve goals. As an example of (1), consider the goal:

```
← legs(horse, 4).
```

In Example 9, this led to the two subgoals:

```
← mammal(horse), arms(horse, 0).
```

A logic programming system must now decide whether to try to resolve mammal(horse) first or arms(horse,0) first. In this example, both choices lead to the same answer. However, as you will see in the examples that follow, the order can have a significant effect on the answers found.

The order in which clauses are used can also have a major effect on the result of applying resolution. For example, given the Horn clauses:

```
ancestor(x, y) ← parent(x, z), ancestor(z, y).
ancestor(x, x).
parent(amy, bob).
```

if we provide the query:

```
← ancestor(x, bob).
```

there are two possible answers: `x = bob` and `x = amy`. If the assertion `ancestor(x, x)` is used, the solution `x = bob` will be found. If the clause `ancestor(x, y) ← parent(x, z), ancestor(z, y)` is used, the solution `x = amy` will be found. Which one is found first, or even if any is found, can depend on the order in which the clauses are used, as well as the order in which goals are resolved.

Logic programming systems using Horn clauses and resolution with prespecified orders for (1) and (2), therefore, violate the basic principle that such systems set out to achieve: that a programmer need worry only about the logic itself, while the control (the methods used to produce an answer) can be ignored. Instead, a programmer must always be aware of the way the system produces answers. It is possible to design systems that will always produce all the answers implied by the logic, but they are (so far) too inefficient to be used as programming systems.

4.4 The Language Prolog

Prolog is the most widely used logic programming language. It uses Horn clauses and implements resolution via a strictly linear depth-first strategy and a unification algorithm described in more detail shortly. Although an ISO standard for Prolog now exists, for many years there was no standard, and implementations contained many variations in the details of their syntax, semantics, built-in functions, and libraries. This is still the case, but the most widely used implementation (the so-called **Edinburgh Prolog** developed in the late 1970s and early 1980s at the University of Edinburgh) was somewhat of a de facto standard and indeed was used as the basis for the ISO standard. We use its notation for the clauses and goals in our examples below. We discuss the essential features of Prolog in the sections that follow.

4.4.1 Notation and Data Structures

Prolog uses notation almost identical to that developed earlier for Horn clauses, except that the implication arrow, ←, is replaced by a colon followed by a dash (or minus sign) (`:-`). Thus, the sample `ancestor` program from the previous section would be written in Prolog syntax as follows:

```
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
ancestor(X, X).
parent(amy, bob).
```

and Example 4 would be written as follows:

```
natural(0).
natural(successor(X)) :- natural(X).
```

Note that the variables `X` and `Y` are written in uppercase. Prolog distinguishes variables from constants and names of predicates and functions by using uppercase for variables and lowercase for constants and names. It is also possible in most Prolog systems to denote a variable by writing an underscore before the name, as in `ancestor(_x,_x)`.

In addition to the comma connective that stands for `and`, Prolog uses the semicolon for `or`. However, the semicolon is rarely used in programming, since it is not a standard part of Horn clause logic.

Basic data structures are terms like `parent(X, Z)` or `successor(successor(0))`. Prolog also includes the list as a basic data structure. A Prolog list is written using square brackets (like ML and Haskell, with the list consisting of items `x`, `y`, and `z` written as `[x,y,z]`. Lists may contain terms or variables.

It is also possible to specify the head and tail of a list using a vertical bar. For example, `[H|T]` means that `H` is the first item in the list and `T` is the tail of the list. As in ML and Haskell, this type of notation can be used to extract the components of a list via pattern matching. Thus, if `[H|T] = [1,2,3]`, then `H = 1` and `T = [2,3]`. It is also possible to write as many terms as one wishes before the bar. For example, `[X,Y|Z] = [1,2,3]` gives `X = 1`, `Y = 2`, and `Z = [3]`. The empty list is denoted by `[]` and does not have a first element.

Prolog has a number of standard predicates that are always built in, such as `not`, `=`, and the I/O operations `read`, `write`, and `nl` (for newline). One anomaly in Prolog is that the less than or equal to operator is usually written =< instead of <= (perhaps because the latter is too easily confused with implication).

4.4.2 Execution in Prolog

Prolog compilers exist, but most systems are run as interpreters. A Prolog program consists of a set of Horn clauses in Prolog syntax, which is usually entered from a file and stored in a dynamically maintained database of clauses. Once a set of clauses has been entered into the database, goals can be entered either from a file or from the keyboard to begin execution. Thus, once a Prolog system has begun to execute, it will provide the user with a prompt for a query, such as

```
?-_
```

Note that, while Prolog syntax is now standardized for clauses and queries, the responses from a Prolog interpreter (including the prompt as shown above) are *not* standardized; we give these responses in a plausible notation, but different ISO-compliant interpreters may behave slightly differently.

EXAMPLE 11

If the clauses

```
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
ancestor(X, X).
parent(amy, bob).
```

have been entered into the database, then the following queries would cause the indicated response:

```
?- ancestor(amy,bob).
yes.

?- ancestor(bob,amy).
no.

?- ancestor(X,bob).
X = amy ->_
```

In the last query there are two answers. Most Prolog systems will find one answer and then wait for a user prompt before printing more answers. If the user supplies a semicolon at the underscore (meaning `or`), then Prolog continues to find more answers:

```
?- ancestor(X,bob).
X = amy ->;
X = bob

?-_
```

A carriage return usually cancels the continued search. ■

4.4.3 Arithmetic

Prolog has built-in arithmetic operations and an arithmetic evaluator. Arithmetic terms can be written either in the usual infix notation or as terms in prefix notation: `3 + 4` and `+(3, 4)` mean the same thing. However, Prolog cannot tell when to consider an arithmetic term as a term itself (that is, strictly as data), or when to evaluate it. Thus,

```
?- write(3 + 5).
3 + 5
```

To force the evaluation of an arithmetic term, a new operation is required: the built-in predicate `is`. Thus, to get Prolog to evaluate `3 + 5`, we need to write:

```
?- X is 3 + 5, write(X).
X = 8
```

A further consequence of this is that two arithmetic terms may not be equal as terms, even though they have the same value[1]:

```
?- 3 + 4 = 4 + 3.
no
```

To get equality of values, we must force evaluation using `is`, for example, by writing the predicate:

```
valequal(Term1, Term2) :-
        X is Term1, Y is Term2, X = Y.
```

We would then get:

```
?- valequal(3 + 4, 4 + 3).
yes
```

Now you can see how to write Euclid's algorithm for the greatest common divisor from Example 10 in Prolog. We wrote this algorithm in generic Horn clauses as:

```
gcd(u, 0, u).
gcd(u, v, w) ← not zero(v), gcd(v, u mod v, w).
```

In Prolog this translates to:

```
gcd(U, 0, U).
gcd(U, V, W) :-
    not(V = 0) , R is U mod V, gcd(V, R, W).
```

The middle statement `R is U mod V` is required to force evaluation of the `mod` operation.

4.4.4 Unification

Unification is the process by which variables are instantiated, or allocated memory and assigned values, so that patterns match during resolution. It is also the process of making two terms the same in some sense. The basic expression whose semantics is determined by unification is equality: In Prolog the goal `s = t` attempts to unify the terms `s` and `t`. It succeeds if unification succeeds and fails otherwise. Thus, we can study unification in Prolog by experimenting with the effect of equality:

```
?- me = me.
yes

?- me = you.
no

?- me = X.
X = me

?- f(a, X) = f(Y, b).
X = b
Y = a
```

(continues)

[1]We write a space between the 3 and the period in the following goal, since 3. could be interpreted as the floating-point number 3.0.

(continued)

```
?- f(X) = g(X).
no

?- f(X) = f(a, b).
no

?- f(a, g(X)) = f(Y, b).
no

?- f(a, g(X)) = f(Y, g(b)).
X = b
Y = a
```

From these experiments, we can formulate the following unification algorithm for Prolog:

1. A constant unifies only with itself: me = me succeeds but me = you fails.
2. A variable that is uninstantiated unifies with anything and becomes instantiated to that thing.
3. A structured term (i.e., a function applied to arguments) unifies with another term only if it has the same function name and the same number of arguments, and the arguments can be unified recursively. Thus, `f(a, X)` unifies with `f(Y, b)` by instantiating `X` to `b` and `Y` to `a`.

A variation on case 2 is when two uninstantiated variables are unified:

```
?- X = Y.
X = _23
Y = _23
```

The number printed on the right-hand side—in this case, `23`—will differ from system to system and indicates an internal memory location set aside for that variable. Thus, unification causes uninstantiated variables to share memory—that is, to become aliases of each other.

We can use unification in Prolog to get very short expressions for many operations. As examples, let us develop Prolog programs for the list operations **append** and **reverse**. We note again that because Prolog allows a list to be represented by a term such as `[X|Y]`, where `X` and `Y` are variables, there is no need for a built-in function to return the head of a list or the tail of a list: just setting `[X|Y] = [1, 2, 3]` with uninstantiated variables `X` and `Y` returns the head as `X` and the tail as `Y` by unification. Similarly, there is no need for a list constructor like the cons operation of LISP. If we want to add 0, say, to the list `[ 1, 2, 3 ]`, we simply write `[ 0 | [1,2,3] ]`; for example,

```
?- X = [0|[1, 2, 3]].
X = [0, 1, 2, 3]
```

However, we could, if we wanted, write the following clause:

```
cons(X, Y, L) :- L = [X|Y].
```

and then use it to compute heads, tails, and constructed lists, depending on how we instantiate the variables on the left-hand side:

```
?- cons(0, [1, 2, 3], A).
A = [0, 1, 2,3]

?- cons(X, Y, [1, 2, 3]).
X = 1
Y = [2,3]
```

Thus, we can use variables in a term as either input or output parameters, and Prolog clauses can be "run" backward as well as forward—something the procedural interpretation of Horn clauses did not tell us!

Indeed, unification can be used to shorten clauses such as cons further. Since the = operator is there only to force unification, we can let resolution cause the unification automatically by writing the patterns to be unified directly in the parameters of the head. Thus, the following definition of cons has precisely the same meaning as the previous definition:

```
cons(X, Y, [X|Y]).
```

The appearance of the pattern `[X|Y]` in place of the third variable automatically unifies it with a variable used in that place in a goal. This process could be referred to as **pattern-directed invocation**. The functional languages ML and Haskell studied in Chapter 3 have a similar mechanism, and both the runtime systems and the type checkers of these two languages use forms of unification similar to Prolog's.

Now let's write an append procedure:

```
append(X, Y, Z) :- X = [], Y = Z.
append(X,Y,Z)   :-
                  X = [A|B], Z = [A|W], append(B, Y, W).
```

The first clause states that appending any list to the empty list just gives that list. The second clause states that appending a list whose head is `A` and tail is `B` to a list `Y` gives a list whose head is also `A` and whose tail is `B` with `Y` appended.

Rewriting this using pattern-directed invocation, we get the following extremely concise form:

```
append([], Y, Y).
append([A|B], Y, [A|W]) :- append(B, Y, W).
```

This `append` can also be run backward and can even find all the ways to append two lists to get a specified list:

```
?- append(X, Y, [1, 2]).
X = []
Y = [1, 2] ->;

X = [1]
Y = [2] ->;

X = [1,2].
Y = []
```

Prolog does this by first using the first clause and matching X to the empty list `[]` and `Y` to the final list. It then continues the search for solutions by using the second clause, matching `X` to `[A|B]` and setting up the subgoal `append(B, Y, W)` with `W = [2]`. This begins the search over again with `B` in place of `X`, so B is first matched with `[]` and `Y` with `[2]`, giving `X = [1|[]] = [1]`. B is then matched with a new `[A|B]` by the second clause, and so on.

Finally, in this section, we give a Prolog definition for the reverse of a list:

```
reverse([],[]).
reverse([H|T], L) :- reverse(T, L1),
                     append(L1, [H], L).
```

Figure 4.1 collects the three principal Prolog examples of this section, for ease of reference.

```
gcd(U, 0, U).
gcd(U, V, W) :- not(V = 0) , R is U mod V, gcd(V, R, W).

append([], Y, Y).
append([A|B], Y, [A|W]) :- append(B, Y, W).

reverse([], []).
reverse([H|T], L) :- reverse(T, L1),
                     append(L1, [H], L).
```

Figure 4.1 Prolog clauses for `gcd`, `append`, and `reverse`

4.4.5 Prolog's Search Strategy

Prolog applies resolution in a strictly linear fashion, replacing goals left to right and considering clauses in the database in top-to-bottom order. Subgoals are also considered immediately once they are set up. Thus, this search strategy results in a depth-first search on a tree of possible choices.

For example, consider the following clauses:

(1) `ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).`

(2) `ancestor(X, X).`

(3) `parent(amy, bob).`

Given the goal ancestor(X,bob), Prolog's search strategy is left to right and depth first on the tree of subgoals shown in Figure 4.2. Edges in that figure are labeled by the number of the clause above used by Prolog for resolution, and instantiations of variables are written in curly brackets.

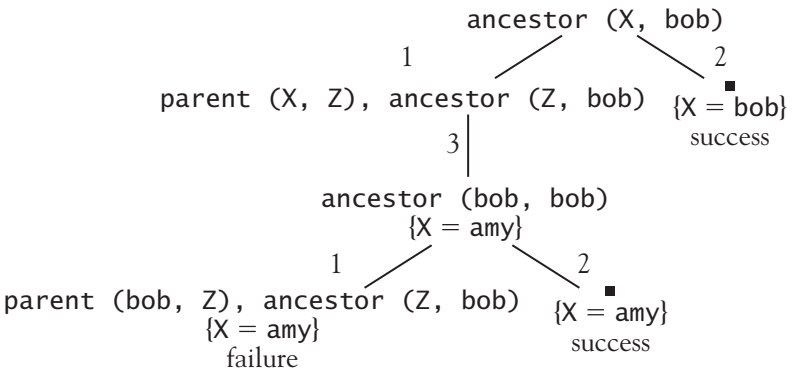


Figure 4.2 A Prolog search tree showing subgoals, clauses used for resolution, and variable instantiations

Leaf nodes in this tree occur either when no match is found for the leftmost clause or when all clauses have been eliminated, thus indicating success. Whenever failure occurs, or the user indicates a continued search with a semicolon, Prolog **backtracks** up the tree to find further paths to a leaf, releasing instantiations of variables as it does so. Thus, in the tree shown, after the solution `X = amy` is found, if backtracking is initiated, this instantiation of `X` will be released, and the new path to a leaf with `X = bob` will be found.

This depth-first strategy is extremely efficient, because it can be implemented in a stack-based or recursive fashion. However, it also means that solutions may not be found if the search tree has branches of infinite depth. For example, suppose we had written the clauses in a slightly different order:

(1) `ancestor(X, Y) :- ancestor(Z, Y), parent(X, Z).`

(2) `ancestor(X, X).`

(3) `parent(amy, bob).`

This causes Prolog to go into an infinite loop, attempting to satisfy `ancestor(Z, Y)`, continually reusing the first clause. Of course, this is a result of the left-recursive way the first clause was written and the fact that no other clauses precede it. In true logic programming, the order of the clauses should not matter. Indeed, a logic programming system that adopts breadth-first search instead of depth-first search will always find solutions if there are any. Unfortunately, breadth-first search is far more expensive than depth-first search, so few logic programming systems use it. Prolog always uses depth-first search.

4.4.6 Loops and Control Structures

We can use the depth-first search with backtracking of Prolog to perform loops and repetitive searches. What we must do is force backtracking even when a solution is found. We do this with the built-in predicate `fail`. As an example, we can get Prolog to print all solutions to a goal such as `append`, with no need to give semicolons to the system as follows. Define the predicate:

```
printpieces(L) :-append(X, Y, L),
                write(X),
                write(Y),
                nl,
                fail.
```

Now we get the following behavior:

```
?- printpieces([1, 2]).
[][1,2]
[1][2]
[1,2] []
no
```

Backtracking on failure forces Prolog to write all solutions at once.

We can also use this technique to get repetitive computations. For example, the following clauses generate all integers ≥ 0 as solutions to the goal `num(X)`:

(1) `num(0).`
(2) `num(X) :- num(Y), X is Y + 1.`

The search tree is displayed in Figure 4.3. In this tree, it has an infinite branch to the right. (The different uses of Y from clause (2) in the tree are indicated by adding quotes to the variable name.)

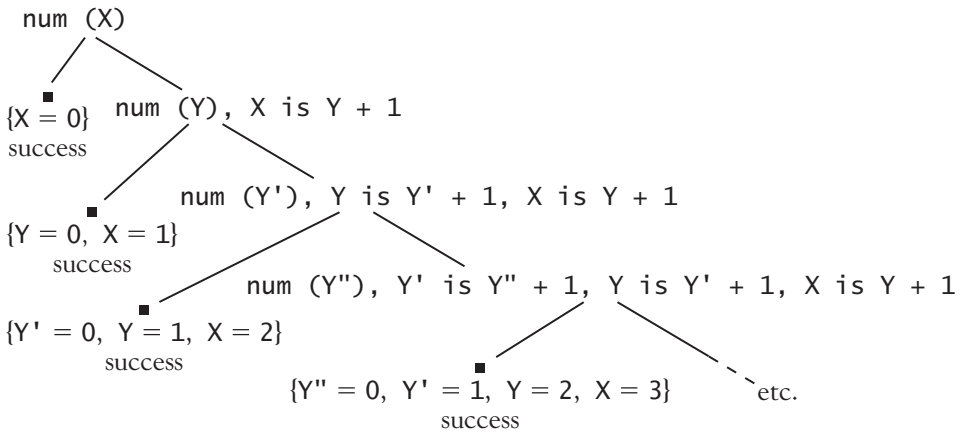


Figure 4.3 An infinite Prolog search tree showing repetitive computations

Now we could try to generate the integers from 1 to 10, say, by writing:

```
writenum(l, J) :- num(X),
                  I =< X,
                  X =< J,
                  write(X),
                  nl,
                  fail.
```

and giving the goal `writenum(1,10)`. Unfortunately, this will go into an infinite loop after `X = 10`, generating ever-larger integers `X`, even though `X =< 10` will never succeed.

What is needed is some way of stopping the search from continuing through the whole tree. Prolog has an operator to do this: the **cut**, usually written as an exclamation point. The cut freezes a choice when it is encountered. If a cut is reached on backtracking, the search of the subtrees of the parent node of the node containing the cut stops, and the search continues with the grandparent node. In effect, the cut prunes the search tree of all other siblings to the right of the node containing the cut.

EXAMPLE 12

Consider the tree of Figure 4.2. If we rewrite the clauses using the cut as follows:

```
(1) ancestor(X, Y):- parent(X, Z), !, ancestor(Z, Y).
(2) ancestor(X, X).
(3) parent(amy, bob).
```

then only the solution `x = amy` will be found, since the branch containing `X = bob` will be pruned from the search, as shown in Figure 4.4.

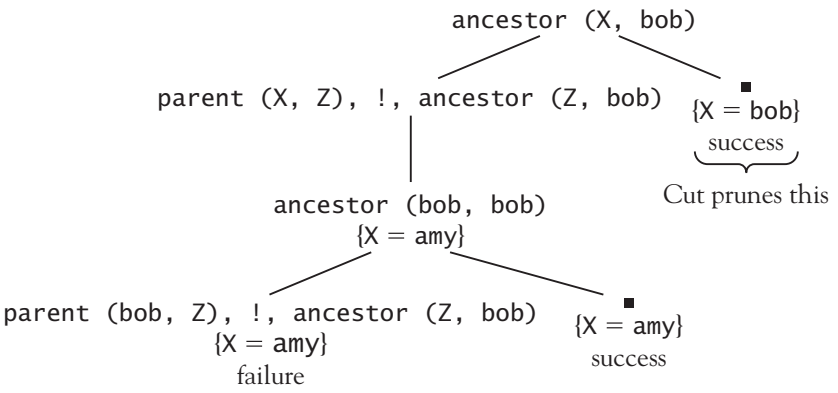


Figure 4.4 Consequences of the cut for the search tree of Figure 4.2

On the other hand, if we place the cut as follows,

```
(1) ancestor(X, Y) :- !, parent(X, Z), ancestor(Z, Y).
(2) ancestor(X, X).
(3) parent(amy, bob).
```

then no solutions at all will be found, as shown in Figure 4.5.

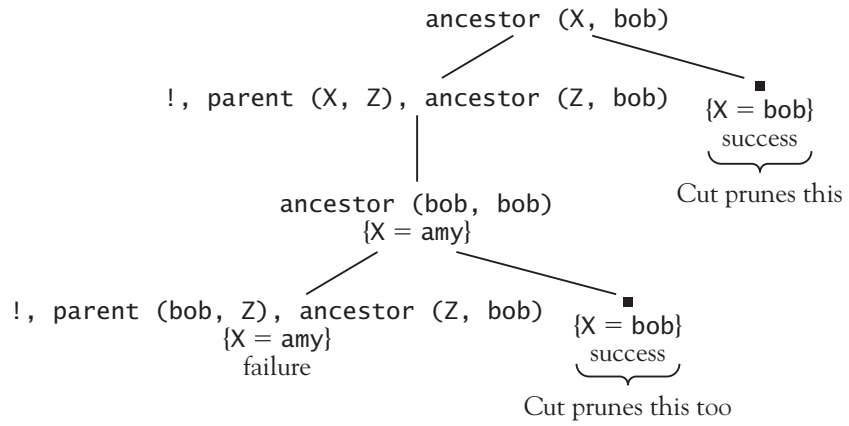


Figure 4.5 A further use of the cut prunes all solutions from Figure 4.2

Finally, if we place the cut as follows,

(1) `ancestor(X, Y) : - parent(X, Z), ancestor(Z, Y).`
(2) `ancestor(X, X) : - !.`
(3) `parent(amy, bob).`

both solutions will still be found, since the right subtree of `ancestor(X, bob)` is not pruned (in fact nothing at all is pruned in this example).

The cut can be used as an efficiency mechanism to reduce the number of branches in the search tree that need to be followed. The cut also allows us to solve the problem of the infinite loop in the program to print the numbers between `I` and `J`. The following is one solution:

```
num(0).
num(X) : - num(Y), X is Y + 1.
writenum(I, J) :- num(X),
                  I =< X,
                  X =< J,
                  write(X), nl,
                  X = J, !,
                  fail.
```

In this code, `X = J` will succeed when the upper-bound `J` is reached, and then the cut will cause backtracking to fail, halting the search for new values of `X`.

The cut can also be used to imitate `if-else` constructs in imperative and functional languages. To write a clause such as:

$$D = \text{if } A \text{ then } B \text{ else } C$$

we write the following Prolog:

```
D :- A, !, B.
D :- C.
```

Note that we could have achieved almost the same result without the cut,

```
D :- A, B.
D :- not(A), C.
```

but this is subtly different, since A is executed twice. Of course, if A has no side effects, then the two forms are equivalent. Nevertheless, the cut does improve the efficiency.

Finally, we give in Figure 4.6 a longer program that uses most of the features discussed so far. It is a program to compute primes using the sieve of Eratosthenes and is adapted from an example in Clocksin and Mellish [1994]).

```
primes(Limit, Ps) :- integers(2, Limit, Is),
                     sieve(Is, Ps).
integers(Low, High, [Low|Rest]) :-
           Low =< High, !, M is Low+1,
           integers(M, High, Rest).
integers(Low, High, []).
sieve([], []).
sieve([I|Is], [I|Ps]) :- remove(I, Is, New),
                         sieve(New, Ps).
remove(P, [], []).
remove(P, [I|Is], [I|Nis]) :-
      not(0 is I mod P), !, remove(P, Is, Nis).
remove(P, [I|Is], Nis) :-
      0 is I mod P, !, remove(P, Is, Nis).
```

Figure 4.6 The Sieve of Eratosthenes in Prolog (adapted from Clocksin and Mellish [1994])

4.5 Problems with Logic Programming

The original goal of logic programming was to make programming into a specification activity—that is, to allow the programmer to specify only the properties of a solution and to let the language implementation provide the actual method for computing the solution from its properties. This is what is meant by **declarative programming**, which takes the perspective that a program describes *what* a solution to a given problem is, not *how* that problem is solved. Logic programming languages, and Prolog in particular, have only partially met this goal. Instead, the nature of the algorithms used by logic programming systems, such as resolution, unification, and depth-first search, have introduced many pitfalls into the specifics of writing programs. As a programmer, you must be aware of them to write efficient, or even correct, programs. As we have already seen, it is also necessary to view a Prolog program as a set of procedure calls, which must be placed in a certain sequence in order to avoid infinite loops or inefficient execution. Moreover, the Prolog programmer must also occasionally take an even lower-level perspective of a program, as when exploiting the underlying backtracking mechanism to implement a cut/fail loop.

In this section, we discuss some of the more subtle problems relating to the adequacy of Prolog for representing logic. These include the occur-check problem in unification, problems with negation (the `not` operator), the limitations of Horn clauses in logic, and the need for control information in a logic program.

4.5.1 The Occur-Check Problem in Unification

The unification algorithm used by Prolog is actually incorrect: When unifying a variable with a term, Prolog does not check whether the variable itself occurs in the term it is being instantiated to. This is known as the occur-check problem. The simplest example is expressed by the following clause:

```
is_own_successor :- X = successor(X).
```

This will be true if there exists an `X` for which `X` is its own successor. However, even in the absence of any other clauses for `successor`, Prolog still answers `yes`. In other words, any `X` will do! That this is incorrect becomes apparent only if we make Prolog try to print such an `X`, as with:

```
is_own_successor(X) :- X = successor(X).
```

Now Prolog will respond with an infinite loop:

```
?- is_own_successor(X).
X = successor(successor(successor(successor(
    successor(successor(successor(successor(
    successor(successor(successor(successor(
    successor(successor(successor(successor(
    successor(successor(successor(successor(
    successor(successor(successor(successor(
    successor(successor(successor(successor(
    successor(successor(successor(successor .....
```

This occurs because unification has constructed `X` as a circular structure indicated by the picture shown in Figure 4-7.

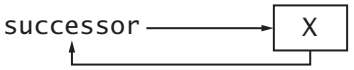


Figure 4-7: Circular structure created by unification

X is finite, but the attempt to print `X` is infinite. Thus, what should be logically false now becomes a programming error.

Why doesn't Prolog's unification contain a check for such occurrences? The answer is that unification without the occur-check can be relatively easily implemented in an efficient way, while efficient algorithms that include the occur-check are more complex. (For a discussion, see Lloyd [1984, p. 23].)

4.5.2 Negation as Failure

All logic programming systems have the following basic property: something that cannot be proved to be true is assumed to be false. This is called the **closed-world assumption**. We have already seen an example of this in Example 1, where we noted that, since `natural(21)` cannot be proved from the axioms for the predicate `natural`, it is assumed to be false. This property of logic programming, together with the way negation is implemented, results in further surprises in the behavior of logic programs.

How can one implement the `not` operator in logic programming? The straightforward answer is that the goal `not(X)` succeeds whenever the goal `X` fails. This is what is meant by **negation as failure**. As a simple example, consider the following program consisting of one clause:

```
parent(amy,bob).
```

If we now ask:

```
?- not(mother(amy, bob)).
```

the answer is `yes`, since the system does not know that `amy` is female, and that female parents are mothers. If we were to add these facts to our program, `not(mother(amy, bob))` would no longer be true.

The foregoing property of logic programs—that adding information to a system can reduce the number of things that can be proved—is called **nonmonotonic reasoning** and is a consequence of the closed-world assumption. Nonmonotonic reasoning has been the subject of considerable study in logic programming and artificial intelligence. (See the Notes and References.)

A related problem is that failure causes instantiations of variables to be released by backtracking, so that after failure, a variable may no longer have an appropriate value. Thus, for example, `not(not(X))` does not have the same result as `X` itself (we assume the fact `human(bob)` in this example):

```
?- human(X).
X = bob

?- not(not(human(X))).
X = _23
```

The goal `not(not(human(X)))` succeeds because `not(human(X))` fails, but when `not(human(X))` fails, the instantiation of `X` to bob is released, causing `X` to be printed as an uninstantiated variable.

A similar situation is shown by the following:

```
?- X = 0, not(X = 1).
X = 0

?- not (X = 1), X = 0.
no
```

The second pair of goals fails because `X` is instantiated to `1` to make `X = 1` succeed, and then `not (X = 1)` fails. The goal `X = 0` is never reached.

4.5.3 Horn Clauses Do Not Express All of Logic

Not every logical statement can be turned into Horn clauses. In particular, statements involving quantifiers may not be expressible in Horn clause form. A simple example is the following:

$$p(a) \text{ and (there exists } x, \text{ not}(p(x))).$$

We can certainly express the truth of $p(a)$ as a Horn clause, but the second statement cannot be written in Horn clause form. If we try to do so in Prolog, we might try something like:

```
p(a).
not(p(b)).
```

but the second statement will result in an error (trying to redefine the not operator). Perhaps the best approximation would be simply the statement `p(a)`. Then, the closed-world assumption will force `not(p(X))` to be true for all `X` not equal to `a`, but this is really the logical equivalent of:

$$p(a) \text{ and (for all } x, \text{ not}(x = a) \rightarrow \text{not}(p(a))).$$

which is not the same as the original statement.

4.5.4 Control Information in Logic Programming

We have already talked about the cut as a useful, even essential, explicit control mechanism in Prolog. Because of its depth-first search strategy, and its linear processing of goals and statements, however, Prolog programs also contain implicit information on control that can easily cause programs to fail. One example is the ancestor program used earlier:

```
ancestor(X, Y) : - parent(X, Z), ancestor(Z, Y).
ancestor(X, X).
parent(amy, bob).
```

If we accidentally wrote the right-hand side of the first clause backward, as:

```
ancestor(X, Y) :- ancestor(Z, Y), parent(X, Z).
```

we would immediately get into an infinite loop: Prolog will try to instantiate the variable `Z` to make `ancestor` true, causing an infinite descent in the search tree using the first clause. On the other hand, if we changed the order of the clauses to:

```
ancestor(X, X).
ancestor(X, Y) :- ancestor(Z, Y), parent(X, Z).
parent(amy, bob).
```

the search would now find both solutions `X = amy` and `X = bob` to the query `ancestor(amy, X)`, but would still go into an infinite loop searching for further (nonexistent) solutions. A similar situation exists if we write the clauses for the program to generate natural numbers in reverse order:

```
num(X) : - num(Y), X is Y + 1 .
num(0).
```

Now the goal `num(X)` will go into an infinite loop. Nothing at all will be generated.

A more complicated logical question is the representation of algorithmic logic using Horn clauses. We mentioned in Section 4.3 that a specification for a sorting procedure could be given as follows, for two lists `S` and `T`:

```
sort(S, T) :- permutation(S, T), sorted(T).
```

We can now give specifications for the meaning of permutation and sorted in Prolog syntax, in Figure 4.8.

```
sorted([]).
sorted([X]).
sorted([X, Y|Z]) :- X =< Y, sorted([Y|Z]).

permutation([], []).
permutation(X, [Y|Z]) :- append(U, [Y|V], X),
                         append(U, V, W),
                         permutation(W, Z).
```

Figure 4.8 The definitions of the `permutation` and `sorted` properties in Prolog

This represents a mathematical definition for what it means for a list of numbers to be sorted in increasing order. But as a program, it represents almost the slowest sort in the world. Permutations of the unsorted list are generated until one of them happens to be sorted!

In the best of all possible worlds, one would want a logic programming system to accept the mathematical definition of a property and find an efficient algorithm to compute it. Of course, in Prolog or any other logic programming system, we not only provide specifications in our programs, but we must also provide algorithmic control information. Thus, in the `gcd` program of Example 10, we specified explicitly the steps used by Euclid's algorithm instead of providing a mathematical definition of the greatest common divisor. Similarly, to get a reasonably efficient sorting program in Prolog, we must specify the actual steps an algorithm must take to produce the result. For example, a `quicksort` program in Prolog is given in Figure 4.9.

```
qsort([], []).
qsort([H|T], S) :- partition(H, T, L, R),
                   qsort(L, L1),
                   qsort(R, R1),
                   append(L1, [H|R1], S).
partition(P, [A|X], [A|Y], Z) :- A < P,
                                 partition(P, X, Y, Z).
partition(P, [A|X], Y, [A|Z]) :- A >= P,
                                 partition(P, X, Y, Z).
partition(P, [], [], []).
```

Figure 4.9 A `quicksort` program in Prolog (adapted from Clocksin and Mellish [1994])

In this program, the standard method of partitioning is shown by the clauses for the predicate `partition`, and the two recursive calls to `qsort` in the second clause are readily evident. Thus, in specifying sequential algorithms such as sorting, Prolog is not so different from imperative or functional programming as one might think.

There is, however, one useful aspect to being able to execute a specification for a property, even if it is not acceptable as an algorithm for computing that property: It lets us test whether the specification is correct. Prolog has in fact been used successfully as a basis for running such specifications during the design of a program. (See the Notes and References.)

4.6 Curry: A Functional Logic Language

In this chapter and the previous one, we have examined two styles or paradigms of programming, the functional and the logical, and some languages that support them. By using these paradigms instead of the more conventional imperative paradigm, a programmer can more or less write an executable specification of a program—that is, describe what a program does to solve a problem and leave much of the implementation, or how the solution will be carried out, to the underlying interpreter of the language. Thus, such languages support a **declarative style** of programming, as distinct from the more conventional procedural style associated with imperative languages. Each paradigm, whether functional or logical, nevertheless brings something different to the declarative enterprise. In a functional language, a program is a set of function definitions, which specify rules for operating on data to transform it into other data. In a logic language, a program is a set of rules and facts from which a proof is constructed for a solution to a problem. We have also seen that each of these versions of the declarative paradigm has some specific disadvantages. Logic programming often forces the programmer to abandon the declarative perspective when solving some problems, such as numeric calculations, that require a functional perspective, in which a set of inputs is mapped to a single output. Functional programming imposes the opposite restriction on the programmer. A function, which deterministically maps a given set of arguments to a single value, is not well suited for problems that involve determining a set of values that a variable can have that satisfy certain constraints and that typically are computed in a nondeterministic fashion.

In this section, we give a short overview of the language Curry, which brings together the advantages of functional and logic programming in a single language. Our discussion is an adaptation, with examples, of the work reported by Antoy and Hanus [2010].

4.6.1 Functional Programming in Curry

Curry is an extension of the programming language Haskell discussed in Chapter 3. Both languages are named after the mathematician Haskell Curry. Curry retains the syntax and semantics of Haskell for functional programming, and adds new syntax and semantics for logic programming. As in Haskell, a functional program in Curry consists of a set of function definitions that describe transformations of data values into other data values. Pattern matching, case analysis, recursion, and strong, polymorphic typing are critical aspects of the language. As you saw in Chapter 3, function definitions are sets of equations, whose interpretation involves replacing the pattern on the left side of the = symbol with the pattern on the right side, after replacing the variables in the patterns with the actual parameters. Curry also uses lazy evaluation, whereby the values of actual parameters are computed only when needed.

4.6.2 Adding Nondeterminism, Conditions, and Backtracking

A pure functional language supports only deterministic computation. This means that the application of a function to a given set of arguments always produces the same value. However, some problems, such as flipping a coin, are underspecified, in that their solutions come from a set of values that need only satisfy some constraints. In the case of a coin toss, a definite value is returned, but it will be either heads or tails. Curry supports such nondeterminism by allowing a set of equations for a function to be tried in no particular order. For example, the choice operator `?` is defined with two equations:

```
x ? y = x
x ? y = y
```

When `?` is applied to some operands, Curry, unlike Haskell, does not automatically try the first equation that appears in the definition, which would always return the first operand. Moreover, if one equation fails during execution, another equation in the set is then tried, although that won't happen in this particular example. Thus, a nondeterministic function for flipping a coin can be defined using `?`:

```
flipCoin = 0 ? 1
```

In Section 4.5.4, we showed some Prolog rules for sorting a list that generated permutations of a list until one of them satisfied the constraint of being sorted. This sorting strategy was clear and simple but suffered the woeful inefficiency of generating permutations of the list until one of them satisfied the `sorted` predicate. A similar, but more efficient, strategy can be used to define functions to sort a list in Curry.

Working top-down, we define two functions, `sorted` and `permutation`, such that the application:

```
sorted (permutation xs)
```

returns a sorted list of the elements in `xs`. Obviously, if the list returned by a call of `permutation` is not sorted, the computation must somehow backtrack and try another call of the function, until it produces a sorted list. The strategy used here exploits Curry's support for nondeterminism and backtracking to produce a simple implementation.

The function `sorted` expects a list as an argument and returns a sorted list of the same elements. The first two equations in its definition specify the results for an empty list and a list of one element:

```
sorted []  = []
sorted [x] = [x]
```

The third equation handles the case of a list of two or more elements. In this case, the equation includes a **condition** (specified by the code to the right of the `|` symbol) that permits evaluation of its right side only if the first element in the list is less than or equal to the second element:

```
sorted (x:y:ys) | x <= y
                = x : sorted(y:ys)
```

As you can see, this function will succeed and return a sorted list just when it receives a permutation of the list that happens to be sorted. However, it fails as soon as the first two elements of the list argument are out of order.

The function `permutation` inserts the first element of a nonempty list argument into a permutation of the rest of that list:

```
permutation []     = []
permutation (x:xs) = insert x (permutation xs)
```

The function `insert` places an element at an arbitrary position in a list. The function is defined nondeterministically for nonempty lists, in the following pair of equations:

```
insert x ys     = x : ys
insert x (y:ys) = y : insert x ys
```

This code is simple and elegant enough, but two questions remain to be answered. First, how does Curry back up and call for another permutation of the list if the `sorted` function fails? Second, why is this version of the sorting strategy more efficient than the Prolog version in Section 4.5.4?

To answer the first question, let's assume that the function `sorted`, at some point, fails. This means that the first two elements of its argument, a list permutation, have been found to be out of order. In Curry, control then returns to the definition of `sorted`, to try an equation whose formal parameter pattern matches that of the actual parameter. This pattern-matching step requires another call of the function `permutation`, however, to determine whether the actual parameter is a list of zero, one, or two or more elements. Also, because `permutation` is at bottom nondeterministic, it stands a good chance of returning a different ordering of elements in the list than on its previous call. Even if two consecutive identical lists are examined, this process of generating and testing will continue until a sorted list is returned.

The answer to the second question, which concerns the inefficiency of generating permutations in this strategy, lies in understanding the advantage of lazy evaluation. Recall that Curry, like Haskell, evaluates only enough information in a function's parameters to be able to perform the required computations at each step. As you saw in the discussion of list processing in Section 3.4, this means that functions that supply lists to other functions need only provide the front-end portions that are needed for immediate computations. This allows for the representation of infinite lists and a generate-and-filter model of processing. In the present case, the `sorted` function need only receive from the call of the `permutation` function at most the first two elements of a list, in order to be in a position to fail or continue demanding more. The fact that the Curry-based strategy can fail early gives it an efficiency advantage over the Prolog-based strategy, which must complete each permutation of a list before it can be tested.

4.6.3 Adding Logical Variables and Unification

Two other elements of logic programming, logical variables and unification, give Curry the ability to solve equations with unknown or partial information. This process involves viewing some variables as free—not in the sense of the free variables considered in Chapter 3, which are always bound at the time they are referenced within a function—but free in the sense that they can be instantiated in a manner that satisfies a set of equations that includes them. Curry uses the symbol `=:=` instead of the symbol `=` to specify an equation that should be solved in this manner, rather than an equation that defines a function application. For example, the equation:

```
zs ++ [2] =:= [1, 2]
```

uses the list concatenation operator ++ to solve for the variable zs. It does so by instantiating the variable zs with the only value, [1], that makes this equation true. Lazy evaluation is put to work again with free variables, which are instantiated only when their values are needed to apply a function. Thus, to evaluate the previous equation, Curry chooses a value for zs that allows one of the equations for the definition of ++:

```
[]     ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
```

to be applied. This value will be either [], for the first equation, or a value of the form (x:xs), for the second one. The first rule does not succeed, because [] ++ [2] does not equal [1, 2]. However, the second equation does succeed, with the bindings of 1 for x, [] for xs, and [2] for ys.

As in Prolog, logical variables can remain uninstantiated if not enough information is available. They can also be used to obtain multiple solutions to the same equation, as in:

```
first ++ second =:= [1, 2]
```

Finally, logical variables can be used in the definition of new functions. For example, the equation:

```
zs ++ [e] =:= [1, 2, 3]
```

is satisfied for zs = [1, 2] and e = 3. Using this information, we can define a function last that extracts the last element in a nonempty list, as follows:

```
last xs | zs ++ [e] =:= xs
        = e
        where zs, e free
```

This equation includes a condition with an equation that contains two free variables, as well as the argument to the function. The outer equation essentially generalizes the inner equation for any nonempty list.

Exercises

4.1 A standard method for analyzing logical statements is the truth table: Assigning truth values to elementary statements allows us to determine the truth value of a compound statement, and statements with the same truth tables are logically equivalent. Thus the statement "*p* and not *p*" is equivalent to "false" by the following truth table:

p	not *p*	*p* and not *p*	false
false	true	false	false
true	false	false	false

Use truth tables to show that $p \rightarrow q$ is equivalent to (not *p*) or *q* (remember that $p \rightarrow q$ is false only if *p* is true and *q* is false).

4.2 A **tautology** is a statement that is always true, no matter what the truth values of its components. Use a truth table to show that false $\rightarrow p$ is a tautology for any statement *p*.

4.3 A **refutation system** is a logical system that proves a statement by assuming it is false and deriving a contradiction. Show that Horn clause logic with resolution is a refutation system. (*Hint*: The empty clause is assumed to be false, so a goal $\leftarrow a$ is equivalent to $a \rightarrow$ false. Show that this is equivalent to not(a).)

4.4 Write the following statements in the first-order predicate calculus:
If it is raining or snowing, then there is precipitation.
If it is freezing and there is precipitation, then it is snowing.
If it is not freezing and there is precipitation, then it is raining.
It is snowing.

4.5 Write the statements in Exercise 4.4 as Prolog clauses, in the order given. What answer does Prolog give when given the query "Is it freezing?" The query "Is it raining?" Why? Can you rearrange the clauses so that Prolog can give better answers?

4.6 Write the following mathematical definition of the greatest common divisor of two numbers in first-order predicate calculus: the gcd of u and v is that number x such that x divides both u and v, and, given any other number y such that y divides u and v, then y divides x.

4.7 Translate the definition of the gcd in Exercise 4.6 into Prolog. Compare its efficiency to Euclid's gcd algorithm as given in Figure 4.1.

4.8 Write the following statement as Prolog clauses: Mammals have four legs and no arms, or two arms and two legs.

4.9 Add the statement that a horse is a mammal and that a horse has no arms to the clauses of Exercise 4.8. Can Prolog derive that a horse has four legs? Explain.

4.10 Write Prolog clauses to express the following relationships, given the parent relationship: grandparent, sibling, cousin.

4.11 Write a Prolog program to find the last item in a list.

4.12 Write a Prolog program to find the maximum and minimum of a list of numbers.

4.13 Write a Prolog program that reads numbers from the standard input until a 0 is entered, creates a list of the numbers entered (not including the 0), and then prints the list in the order entered, one number per line. (read(X) can be used to read integer X from the input if the integer is entered on its own line and is terminated by a period.)

4.14 Write a Prolog program that will sort a list of integers according to the mergesort algorithm.

4.15 Compare the Prolog program for quicksort in Figure 4.9 with the Haskell version in Section 3.5. What are the differences? What are the similarities?

4.16 Prolog shares some features with functional languages like Scheme, ML, and Haskell, studied in Chapter 3. Describe two major similarities. Describe two major differences.

4.17 In Prolog it is possible to think of certain clauses as representing tail recursion, in which the final term of a clause is a recursive reference, and a cut is written just before the final term. For example, the gcd clause

```
gcd(U, V, W) :- not(V=0), R is U mod V, !,
                gcd(V, R, W).
```

can be viewed as being tail recursive. Explain why this is so.

4.18 Write a Prolog program to print all Pythagorean triples (x, y, z) such that $1 \leq x$, $y \leq z \leq 100$. ((x, y, z) is a Pythagorean triple if $x * x + y * y = z * z$.)

4.19 Write Prolog clauses for a member predicate: member(X,L) succeeds if X is a member of the list L (thus member(2,[2,3]) succeeds but member(1,[2,3]) fails). What happens if you use your clauses to answer the query member(X,[2,3])? The query member(2,L)? Draw search trees of subgoals as in Section 4.4.5 to explain your answers.

4.20 Rewrite the factorial Prolog program

```
fact(0, 1).
fact(N, R) :-
    N > 0, N1 is N - 1, fact(N1, R1), R is N * R1.
```

to make it tail recursive (see Exercise 4.17).

4.21 Draw search trees of subgoals and explain Prolog's responses based on the trees for the following goals (see Figure 4.1):

(a) `gcd(15, 10, X).`
(b) `append(X, Y, [1, 2]).`

4.22 Rewrite the Prolog clauses for Euclid's algorithm (gcd) in Figure 4.1 to use the cut instead of the test `not(V = 0)`. How much does this improve the efficiency of the program? Redraw the search tree of Exercise 4.21(a) to show how the cut prunes the tree.

4.23 Explain using a search tree why the cut in the following program has no effect on the solutions found to the query ancestor(X,bob). Does the cut improve the efficiency at all? Why or why not?

```
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
ancestor(X, X) :- !.
parent(amy, bob).
```

4.24 If we used cuts to improve the efficiency of the Prolog append program, we would write:

```
append([], Y, Y) :- !.
append([A|B], Y, [A|W]) :- append(B, Y, W).
```

Now given the goal `append(X, Y, [1, 2])` Prolog only responds with the solution `X = [], Y = [1, 2]`. Explain using a search tree.

4.25 Rewrite the sieve of Eratosthenes Prolog program of Figure 4.6 to remove all the cuts. Compare the efficiency of the resulting program to the original.

4.26 Explain the difference in Prolog between the following two definitions of the sibling relationship:

```
sibling1(X, Y) :- not(X = Y), parent(Z, X), parent(Z, Y).
sibling2(X, Y) :- parent(Z, X), parent(Z, Y), not(X = Y).
```

Which definition is better? Why?

4.27 Given the following Prolog clauses:

```
ancestor(X, Y) :- ancestor(Z, Y), parent(X, Z).
ancestor(X, X).
parent(amy, bob).
```

explain, using a search tree of subgoals, why Prolog fails to answer when given the goal ancestor(X,bob).

4.28 Given the following Prolog clauses:

```
ancestor(X, X).
ancestor(X, Y) :- ancestor(Z, Y), parent(X, Z).
parent(amy, bob).
```

explain Prolog's response to the query ancestor(amy,X) using a search tree of subgoals.

4.29 What is wrong with the following Prolog specification for a sort procedure?

```
sort(S, T) :- sorted(T), permutation(S, T).
```

Why?

4.30 Given only the following Prolog clause:

```
human(bob).
```

Prolog will respond as follows:

```
?- human(X).
X = bob

?- not(human(X)).
no
```

Why did Prolog respond to the last goal as it did? Is Prolog saying that there are no X that are not human, that is, that all X are human? Why?

4.31 The following is a possible implementation of a `for` loop construct in Prolog, similar to the statement `for(I = L; I <= H;I++)` in C:

```
for(I, I, I) : - !.
for(I, I, H) .
for(I, L, H) :- L1 is L + 1, for(I, L1, H).
```

Using this definition, we can repeat operations over `I`, such as printing all integers between `L` and `H` as follows:

```
printint(L, H) :- for(I, L, H), write(I), nl, fail.
```

Explain how backtracking causes the for predicate to behave like a loop.

4.32 In the brief explanation of the difference between free and bound variables in Section 4.1, the details about potential reuse of names were ignored. For example, in the statement:

$$a(x) \rightarrow \text{there exists } x,\ b(x)$$

the x in b is bound, while the x in a is free. Thus, the **scope of a binding** must be properly defined to refer to a subset of the uses of the bound variable. Develop scope rules for bindings from which it can be determined which uses of variables are free and which are bound.

4.33 Compare the definition of free and bound variables in logical statements with the definition of free and bound variables in the lambda calculus (Chapter 3). How are they similar? How are they different?

4.34 The occur-check was left out of Prolog because simple algorithms are inefficient. Describe using the append clauses of Figure 4.1 why the occur-check can be inefficient.

4.35 We have seen that Prolog can have difficulties with infinite data sets, such as that produced by the clauses

```
int(0).
int(X) :- int(Y), X is Y + 1 .
```

and with self-referential terms such as:

```
X = [1|X]
```

which causes an occur-check problem. In Haskell (Chapter 3), infinite data constructions, such as the foregoing, are possible using delayed evaluation. Does delayed evaluation make sense in Prolog? Can you think of any other ways infinite data structures might be dealt with?

4.36 Kowalski [1988] makes the following statement about his pseudoequation A(lgorithm) = L(ogic) + C(ontrol): "With Prolog we have a fixed C and can improve A only by improving L. Logic programming is concerned with the possibility of changing both L and C." Explain why he says this. Explain why it is not completely true that Prolog cannot change C.

4.37 In Chapter 3, we described a unification method used by ML and Haskell to perform type inference. Compare this unification with the unification of Prolog. Are there any differences?

4.38 Unification can be described as a substitution process that substitutes more specific values for variables so that equality of two expressions is achieved. For example, in Prolog the lists [1|Y] = [X] can be unified by substituting 1 for X and [] for Y. It is possible for different substitutions to cause equality. For instance, if [1|Y] is unified with X, one could set X = [1] and Y = [], or one could set X = [1|Y] and leave Y uninstantiated. This latter substitution is **more general** than the first because the additional substitution Y = [] transforms the first into the second. A substitution is a **most general unifier** of two expressions if it is more general than every other substitution that unifies the two expressions. Why does the unification algorithm of Prolog produce a most general unifier? Would it be useful for a logic programming language to have a unification algorithm that does not produce a most general unifier? Why?

Notes and References

Logic programming arose out of work on proof techniques and automated deduction. The resolution principle was developed by Robinson [1965], and Horn clauses were invented by Horn [1951]. Kowalski [1979b] gives a general introduction to logic and the algorithms used in logic programming, such as unification and resolution. Lloyd [1984] gives a mathematical treatment of general unification techniques and negation as failure.

The early history of the development of Prolog is discussed in a pair of companion articles by Kowalski [1988] and Cohen [1988]. Both mention the importance of the first interpreters developed by Alain Colmerauer and Phillipe Roussel in Marseilles and the later systems written by David Warren, Fernando Pereira, and Luis Pereira in Edinburgh. For a further perspective, see Colmerauer and Roussel [1996].

For a perspective and overview of the Japanese Fifth Generation Project (1981–1992), see Shapiro and Warren [1993].

Kowalski [1979a] introduced the famous formulation of Prolog's basic strategy as "Algorithm = Logic + Control" and gives a detailed account of how computation can be viewed as theorem proving, or "controlled deduction." (Niklaus Wirth's formulation of imperative programming "Algorithms + Data Structures = Programs" is the title of Wirth [1976] and is discussed in Chapters 1 and 8.)

Clocksin and Mellish [1994] is a basic reference for Prolog programming. Some of the examples in this chapter are based on examples in that text. Other references for programming techniques in Prolog are Sterling and Shapiro [1986], Malpas [1987], and Bratko [2000]. A Prolog standard was adopted by the ISO and ANSI in 1995 (ISO 13211-1 [1995]).

Davis [1982] describes how Prolog can be used as a runnable specification in the design of a software system. Warren [1980] shows how Prolog can be used in language translation and compilers. Clocksin and Mellish [1994] provide a chapter on the use of Prolog to parse grammars and a chapter on the relation of logic programming to logic, in particular the translation of statements in predicate calculus into Horn clauses. For an approach to infinite data structures in Prolog (Exercise 4.35), see Colmerauer [1982].

Nonmonotonic reasoning, mentioned in Section 4.5.2, has been the subject of considerable ongoing research; see for example Minker [2000] or Antoniou and Williams [1997].

CHAPTER

Object-Oriented Programming

CHAPTER 5

The history of object-oriented programming languages begins in the 1960s with the Simula project, an attempt to design a programming language that extended Algol60 in a way suitable for performing computer simulations of real-world situations. One of the central goals of this project was to incorporate into the language the notion of an object, which, similar to a real-world object, is an entity with certain properties that control its ability to react to events in predefined ways. According to this way of looking at programming, a program consists of a set of objects, which can vary dynamically, and which execute by acting and reacting to each other, in much the same way that a real-world process proceeds by the interaction of real-world objects. These ideas were incorporated into the general-purpose language Simula67, at the Norwegian Computing Center in Oslo, by Ole-Johan Dahl and Kristen Nygaard.

By the 1970s, the programming world was ready to incorporate some elements of Simula67 into new languages. Its influence was twofold. It was, first of all, an important factor in the development of abstract data type mechanisms, which you will study in Chapter 11. Second, and perhaps more importantly, it was crucial to the development of the object paradigm itself, in which a program is considered to be a collection of interacting independent objects. This is most evident in the development of the Dynabook Project, which culminated in the language Smalltalk-80, the first language to incorporate the object paradigm in a thorough and consistent way.

Beginning in the mid-1980s, interest in **object-oriented programming** exploded, just like interest in structured programming and top-down design in the early 1970s. Language designers focused on both object-oriented programming's utility as a language paradigm and its usefulness as a methodology for program design. These days, almost every language has some form of structured constructs, such as the `if-then-else` statement and procedural abstraction, and it is well known that it is possible to apply structured programming principles even in a relatively unstructured language like FORTRAN. Similarly, the ideas of object-oriented programming can be applied, at least to a certain extent, in non-object-oriented languages. Nevertheless, the real power of the technique comes only in a language with true object-oriented constructs, and over the last three decades object-oriented programming has proven itself to be an extremely effective mechanism for promoting code reuse and modifiability. It is now the dominant paradigm for large software projects, despite certain drawbacks in a few areas, such as its ability to express abstract data types.

In the following sections, we will review the major concepts of object-oriented programming, using Smalltalk as our primary example. The concepts we will cover include the notions of object and class as a pattern for objects, inheritance of operations as a tool for code reuse and the maintenance of control over dependencies, and the dynamic nature of operations as an essential feature of reuse. We will also study two other languages as major examples of the object-oriented paradigm: Java and C++. Finally, we will look briefly at some issues of object-oriented design and techniques for the implementation of object-oriented language features.

5.1 Software Reuse and Independence

Object-oriented programming languages satisfy three important needs in software design: the need to reuse software components as much as possible, the need to modify program behavior with minimal changes to existing code, and the need to maintain the independence of different components. Abstract data type mechanisms can increase the independence of software components by separating interfaces from implementations and by controlling dependencies. In theory, abstract data type mechanisms should also provide for the reuse of software components. In practice, however, each new programming problem tends to require a slightly different version of the services provided by a module. Thus, the ideal solution varies the services a module offers to clients, while at the same time retaining control over access to those services. In this section, we explore the principal ways that the services of software components can be varied. These can then be used to evaluate the versatility and effectiveness of an object-oriented mechanism in the subsequent sections.

There are four basic ways that a software component can be modified for reuse: extension of the data or operations, redefinition of one or more of the operations, abstraction, and polymorphism:

- *Extension of the data or operations*. As an example of this kind of modification, consider a queue with the following operations: `create`, `enqueue`, `dequeue`, `front`, and `empty`. In a particular application, it may be necessary to extend the operations of a queue so that elements can be removed from the rear of the queue and added to the front of the queue. Such a data structure is called a double-ended queue or deque, with new operations `addfront` and `deleterear`. These two new operations need to be added to the basic queue structure without necessarily changing the underlying implementation data. As another example of modification by extension, a window is defined on a computer screen as a rectangle specified by its four corners, with operations that may include `translate`, `resize`, `display`, and `erase`. A text window can be defined as a window with some added text to be displayed. Thus, a text window extends a window by adding data, without necessarily changing the operations to be performed.
- *Redefinition of one or more of the operations*. Even if the operations on a new type of data remain essentially the same as those on existing types, it may be necessary to redefine some of them to accommodate new behavior. For example, a text window may have the same operations on it as a general-purpose window, but the display operation needs redefinition in order to display the text in the window as well as the window itself. Similarly, if a square is obtained from a rectangle, an area or perimeter function may need to be redefined to take into account the reduced data needed in the computation.
- In several areas in computing, the basic structure of each application is so similar to others that software developers have begun using **application frameworks**. An application framework is a collection of related software resources, usually in object-oriented form, that is used

by software developers through redefinition and reuse to provide specific services for applications. Examples of application frameworks include the **Swing** windowing toolkit in Java and **Microsoft Foundation Classes** in C++. Both of these frameworks allow graphical user interfaces to be constructed, modified, and reused with a minimum of effort.

- *Abstraction, or the collection of similar operations from two different components into a new component*. For example, a circle and a rectangle are both objects that have position and that can be translated and displayed. These properties can be combined into an abstract object called a figure, which has the common features of circles, rectangles, triangles, and so on. Specific examples of a figure are then forced to have each of these common properties.
- *Polymorphism, or the extension of the type of data that operations can apply to*. You have already seen examples of this in previous chapters as two kinds of polymorphism: overloading and parameterized types. Extending the types that an operation applies to can also be viewed as an example of abstraction, where common operations from different types are abstracted and gathered together. A good example is a `toString` function, which should be applicable to any object as long as it has a textual representation.

Design for reuse is not the only goal of object-oriented languages. Restricting access to internal details of software components is another. This requirement is necessary to ensure that clients use components in a way that is independent of the implementation details of the component and that any changes to the implementation of a component will have only a local effect.

The two goals of restricting access and allowing modifiability for reuse can be mutually incompatible. For example, to add operations to a queue, one needs access to the internal implementation details of the queue's data. On the other hand, access restriction can also be complementary to the need for modification. Access restriction can force operations to be defined in an abstract way, which may make it easier to modify their implementation without changing the definition.

Mechanisms for restricting access to internal details go by several names. Some authors call them **encapsulation mechanisms**, while others refer to them as **information-hiding mechanisms**. We now examine how these concepts are realized in a pure object-oriented language, Smalltalk.

5.2 Smalltalk

Smalltalk arose out of a research project, called the Dynabook Project, begun by Alan Kay at Xerox Corporation's Palo Alto Research Center in the early 1970s and continued by Adele Goldberg and Daniel Ingalls. The Dynabook, in theory at least, was conceived as the prototype of today's laptop and tablet computers. The system would include a WIMP (windows, icons, mouse, pull-down menus) user interface and be networked to other computers for group activity. Ideally, it would include a program development environment capable of modeling all of the interactions among these entities. The vision of this research

group was ahead of its time; its realization had to await the turn of the present century, when hardware costs and performance made it a realistic basis for contemporary computing.

Smalltalk was influenced by both Simula and Lisp. A series of stages marked its initial development, with its principal early versions being Smalltalk-72, Smalltalk-76, and Smalltalk-80. By the late 1980s, ordinary desktop computers had become powerful enough to support the memory-intensive Smalltalk programming environment. A company named Digitalk released Smalltalk/V, which ran on both Windows and Macintosh platforms. Smalltalk achieved an ANSI standard in 1998, and several open source and low-cost commercial versions are currently available.

Of all the object-oriented languages, Smalltalk has the most consistent approach to the object-oriented paradigm. In Smalltalk, almost every language entity is an object, including constants (such as numbers and characters) and classes themselves. Thus, Smalltalk can be said to be **purely** object oriented in essentially the same sense that languages such as Haskell can be called purely functional.

Smalltalk borrowed from Lisp the mechanisms of garbage collection (automatic recycling of unused dynamic storage at runtime) and dynamic typing (the checking of the types of operands in expressions at runtime). However, Smalltalk was innovative not only from the language point of view but also in that, from the beginning, it was envisioned as a complete program development environment for a personal workstation. The user interface was also novel in that it included a windowing system with menus and a mouse, long before such systems became common for personal computers.

Smalltalk as a language is interactive and dynamically oriented. Programmers can experiment by entering expressions and statements in an interactive interpreter. All classes and objects are created by interaction with the system, using a set of browser windows. New classes are compiled as they are entered into a class dictionary, and the interaction of objects is also managed by the system. The Smalltalk system comes with a large hierarchy of preexisting classes, and each new class must be a subclass of an existing class. In the following subsections, we explore the syntax of Smalltalk expressions and statements, the definition of new classes, and the fundamental object-oriented concepts of message passing, encapsulation, inheritance, and polymorphism.

5.2.1 Basic Elements of Smalltalk: Classes, Objects, Messages, and Control

Every object in the real world has a set of properties and behaviors. For example, a car has properties that include the number of wheels, the number of doors, and mileage rating. Its behaviors include moving forward, moving in reverse, and turning left or right. Likewise, every object in Smalltalk (that is, every data value and every class or data type), whether it is built-in or created by the programmer, has a set of properties and behaviors. In a Smalltalk program, objects get other objects to do things by sending them messages. A **message** is a request for a service. The object that receives a message (sometimes called a **receiver**) performs its service by invoking an appropriate **method**. This method may in turn send messages to other objects to accomplish its task. As you will see shortly, a method is like a procedure or function, and is defined in the receiver object's class or one of its ancestor classes in a class hierarchy. The **sender** of a message may also supply data with it, in the form of parameters or arguments. When the receiver's method call is completed, it may return data (another object) to the sender. Thus, sending

a message to an object is a bit like calling a function in a functional language. However, some messages, called **mutators**, may result in a change of state in the receiver object, whereas a function call simply returns a value to the caller. The process of sending and receiving messages is also called **message passing**. The set of messages that an object recognizes is sometimes called its **interface**.

The syntax of Smalltalk message passing is quite simple but somewhat unconventional. The object receiving the message is written first, followed by the message name (also called a **selector**) and any arguments. The elements of an expression containing objects and messages are typically read and evaluated from left to right. For example, we can create a new set object by sending the message `new` to Smalltalk's built-in `Set` class, as follows:

```
Set new   "Returns a new set object"
```

Note that a class name is capitalized and a message name is not. Smalltalk comments are enclosed in double quotation marks. Any of the code examples in this subsection can be evaluated by entering them in a Smalltalk transcript window and selecting the **Show it** option from the **Smalltalk** menu. We use the Smalltalk/V syntax throughout this discussion.

We can examine the number of elements in a new set by sending the `size` message to this object:

```
Set new size     "Returns 0"
```

In this code, the `Set` class receives the `new` message, which returns an instance of `Set`, as before. This object in turn receives the `size` message, which returns an integer object. The message `new` is also called a **class message**, because it is sent to a class, whereas the message `size` is called an **instance message**, because it is sent to an instance of a class. These messages are also examples of **unary messages**, in that they expect no arguments.

Messages that expect arguments are called **keyword messages**. For example, we can see whether a new set contains a given element by sending it the `includes:` message. This message expects a single argument, the object that is the target of the search. In the next example, a set is asked if it includes the string 'Hello':

```
Set new includes: 'Hello'           "Returns false"
```

Note that the name of a keyword message ends in a colon (`:`). If there is more than one argument, another keyword must precede each argument. The arguments to a keyword message are evaluated before the message is sent. The following example adds a string to a new set object:

```
Set new add: 'Ken'           "This set now includes 'Ken'"
```

The `add:` message is an example of a **mutator**, which changes the state of the receiver object.

A slightly more complicated example creates a new array of 10 elements and stores the value 66 at the first position in the array. The code first sends the class message `new:` to the `Array` class to instantiate an array of 10 positions (each of which contains the default value `nil`). This array then receives the instance message `at:put:` to store an integer at the array's first position.

```
(Array new: 10) at: 1 put: 66   "Stores 66 at the first position in the array"
```

The keyword message `at:put:` expects two arguments. The first one is the index of an object in the array. The second one is the object to be placed at that index. Unary messages have a higher precedence than keyword messages, and parentheses can be used for clarity or to override precedence. Note that the

last example requires parentheses to distinguish the two keyword messages. Without the parentheses, the interpreter would treat `new:at:put` as a class message for the `Array` class. This would generate an unrecognized message error.

Smalltalk also provides a small set of **binary messages** that allow the programmer to write arithmetic and comparison expressions using conventional infix notation. For example, each of the expressions:

```
3 + 4           "Returns 7"
3 < 4           "Returns true"
```

is actually syntactic sugar for a keyword message that is sent to the first integer object (the receiver). The second integer object is the argument to the message. Binary messages have a lower precedence than keyword messages.

As in many other languages, the programmer can use variables to refer to objects. They can also use sequences of statements to organize code. The next code segment assigns a new array of five positions to a temporary variable and then sends five messages in separate statements to reset its contents:

```
| array |
array := Array new: 5.     "Create an array of 5 positions"
array at: 1 put: 10.       "Run a sequence of 5 replacements"
array at: 2 put: 20.
array at: 3 put: 30.
array at: 4 put: 40.
array at: 5 put: 50
```

Temporary variables are introduced between vertical bars and are not capitalized. Statements are separated by a period. As in Lisp, Smalltalk variables have no assigned data type; any variable can name any thing. Note also that Smalltalk's assignment operator, `:=`, is written as in Pascal and Ada.

When a sequence of messages is sent to the same object, we can use a semicolon to cascade the messages:

```
| array |
array := Array new: 5.    "Create an array of 5 positions"
array at: 1 put: 10;      "Run a sequence of 5 replacements, cascaded"
      at: 2 put: 20;
      at: 3 put: 30;
      at: 4 put: 40;
      at: 5 put: 50
```

As in many other languages (Lisp, Java, and Python), Smalltalk's variables use **reference semantics** rather than **value semantics**. In reference semantics, a variable *refers* to an object, whereas in value semantics (see languages like Ada and C++), a variable *is* or *contains* an object. Thus, an assignment of one Smalltalk variable to another does not transfer a copy of the object itself, as in Ada or C++, but just establishes another reference (essentially an alias) to the same object. Copying requires an explicit instantiation of a new object with the same attributes as the original. Smalltalk includes two operators,

= for equality and == for object identity, to distinguish the two relations. They are illustrated in the next code segment, which uses Smalltalk's `#( )` notation for a literal array:

```
| array alias clone |
array := #(0 0).    "Create an array containing two integers"
alias := array.     "Establish a second reference to the same object"
clone := #(0 0).    "Create a copy of the first array object"
array == alias.     "true, they refer to the exact same object"
array = alias.      "true, because they are =="
array = clone.      "true, because they refer to distinct objects with"
                       "the same properties"
array == clone         "false, because they refer to distinct objects"
clone at: 1 put: 1. "The second array object now contains 1 and 0"
array = clone          "false, because the two array objects no longer"
                       "have the same properties"
```

The sharing of objects in reference semantics enhances the efficiency of object-oriented programs, at the cost of possible safety problems because of aliasing. This is not a problem for functional languages, because they allow no side effects on objects.

Ideally, the code to initialize array elements should run as a loop, which would work for an array of any size. The next code segment shows a `to:do` loop that sets the object at each position in an array to a new value. This loop is similar to an index-based `for` loop in other languages.

```
| array |
array := Array new: 100.      "Instantiate an array of 100 positions"
1 to: array size do: [:i |    "Index-based loop sets the element at each i"
      array at: i put: i * 10]
```

The message `to:do:` is sent to an integer, which represents the lower bound of the iteration. The argument associated with the keyword `to:` is also an integer, representing the iteration's upper bound (in this case, the size of the array). The argument associated with the keyword `do:` is a Smalltalk **block**. A block, which is enclosed in brackets,`[]`, is an object that contains the code that executes on each pass through the loop. Blocks are similar to `lambda` forms in Scheme, in that they can contain arguments in the form of **block variables**. The `to:do:` block in our example contains the block variable `i`, which receives each value in the range between the lower bound and the upper bound during the execution of the loop. On each pass through the `to:do:` loop, Smalltalk sends the block the `value:` message, with the next number in the series as an argument. The remaining code in a block can be any sequence of statements. As you can see, even control statements are expressed in terms of message passing in Smalltalk.

Other types of control statements, such as `if/else` statements and `while` loops, are also expressed in terms of keyword messages in Smalltalk. The next code segment uses the `ifTrue:ifFalse:` message to set the array cells at the odd positions to 1 and at the even positions to 2:

```
| array |
array := Array new: 100.      "Instantiate an array of 100 positions"
1 to: array size do: [:i |    "Treat odd and even positions differently"
   i odd
      ifTrue: [array at: i put: 1]
      ifFalse: [array at: i put: 2]]
```

The receiver of the `ifTrue:ifFalse:` message must be a Boolean object. The two arguments are blocks of code that express the alternative courses of action in the `if/else` logic. The receiver object evaluates just one of the two blocks, based on its own state (either true or false). Note that the scopes of the temporary variable `array` and the block variable `i` in this example include any nested blocks.

Because the loop message `to:do:` is sent to a number, the code that implements its method is found in the `Number` class. We will discuss defining methods for classes shortly, but here is a preview, which shows that `to:do:` is really just syntactic sugar for a more cumbersome, count-controlled `whileTrue:` loop:

```
to: aNumber do: aBlock
   "Evaluate the one argument block aBlock for the
    numbers between the receiver and the argument
    aNumber where each number is the previous number
    plus 1."
  | index |
  index := self.
  [index <= aNumber]
      whileTrue: [
          aBlock value: index.
          index := index + 1]
```

In the context of this method definition, the symbol `self` refers to the receiver of the message, which in this case is a number. The `whileTrue:` message is associated with two blocks. Unlike the `ifTrue:ifFalse:` message, whose receiver object is evaluated just once, the `whileTrue:` message must have a block as its receiver object. Smalltalk evaluates this block, which contains a Boolean expression, one or more times, until it returns `false`. Each time the first block evaluates to `true`, the second block (`whileTrue:`'s argument), which represents the body of the loop, is also evaluated. The code in this second block consists of two statements. The first statement evaluates `aBlock` (the argument of the `to:do:` method being defined here) by sending it the `value:` message. During the course of the loop, the argument to this message (`index`) picks up the value of each number in the series between `self` and `aNumber`. The second statement increments `index` to achieve this effect.

The final example code segment in this subsection shows how to print the contents of an array to the transcript window. The code uses a `do:` loop to traverse the array. Here we are interested not in replacing elements at index positions within the array but in just visiting the elements themselves. The `do:` loop visits each of the array's elements in sequence, from the first to the last. On each pass through the loop, the block argument is evaluated with the block variable bound to the current element.

```
array do: [:element |      "Print to transcript each element followed by a return"
   Transcript nextPutAll: element printString; cr]
```

The global variable `Transcript` refers to Smalltalk's transcript window object. The message `nextPutAll` expects a string as an argument, so we use the message `printString` to obtain the string representation of the element before sending it to the transcript with this message. The message `cr` asks the transcript to display a return character.

Smalltalk includes many types of collection classes and different types of messages for performing iterations on collections, such as maps and filters, similar to those of functional languages. We discuss some of these operations, also called **iterators**, shortly.

As you can see from these brief examples, most Smalltalk code consists of creating objects of the appropriate classes and sending them messages. The objects themselves handle most of the algorithmic details. The rest of software development then consists in choosing the right classes for the right jobs and learning their interfaces. When such classes do not already exist, the Smalltalk programmer creates new classes that customize existing classes. To see how this is done, we now explore two categories of classes, magnitudes and collections, in Smalltalk's massive built-in class hierarchy.

5.2.2 The Magnitude Hierarchy

A class provides a definition of the properties and behavior of a set of objects. In Smalltalk, the built-in classes are organized in a tree-like hierarchy, with a single class, called `Object`, at the root. Each class below `Object` inherits any properties and behavior defined in any classes that are its ancestors in the hierarchy. The classes in the hierarchy usually descend from the more general to the more specific, with each new class adding to and/or modifying the behavior and/or the properties of those already available in the classes above it. **Inheritance**, thus, supports the reuse of structure and behavior. **Polymorphism**, or the use of the same names for messages requesting similar services from different classes, is also a form of code reuse. In this section, we examine a portion of Smalltalk's magnitude classes, which are shown in the class diagram in Figure 5.1.

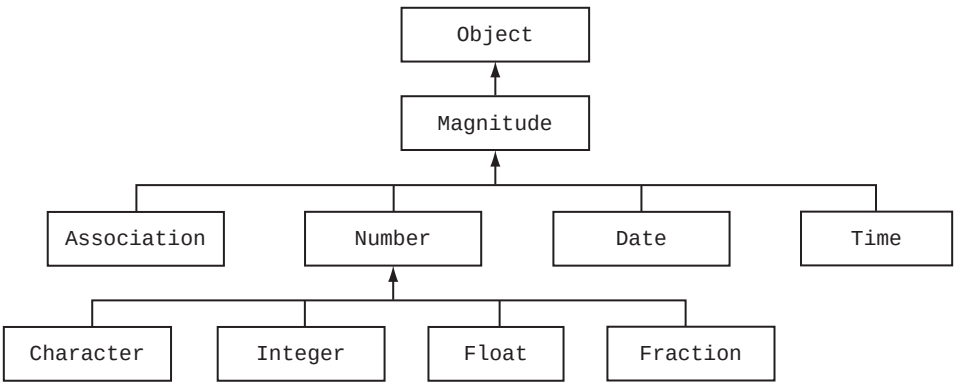


Figure 5.1 Smalltalk's magnitude classes

As you can see from Figure 5.2, the magnitude classes include common types of numbers found in other languages, such as `Integer`, `Float`, and `Fraction`, as well as some other types related to numeric quantities, such as `Time` and `Date`. These classes are sometimes called **concrete classes**,

because their instances are objects that applications normally create and manipulate. By contrast, two other classes in this hierarchy, `Magnitude` and `Number`, are sometimes called **abstract classes**. They are not normally instantiated, but instead serve as repositories of common properties and behaviors for the classes below them in the hierarchy.

We now explore the magnitude class hierarchy in Smalltalk's class browser, to see how classes and their methods are defined and also to learn how the designers of the language already leverage inheritance and polymorphism. (Recall that Smalltalk was conceived not just as a language but also as a programming environment.) Figure 5.2 shows a class browser with the list of magnitude classes visible in the leftmost pane of the window.

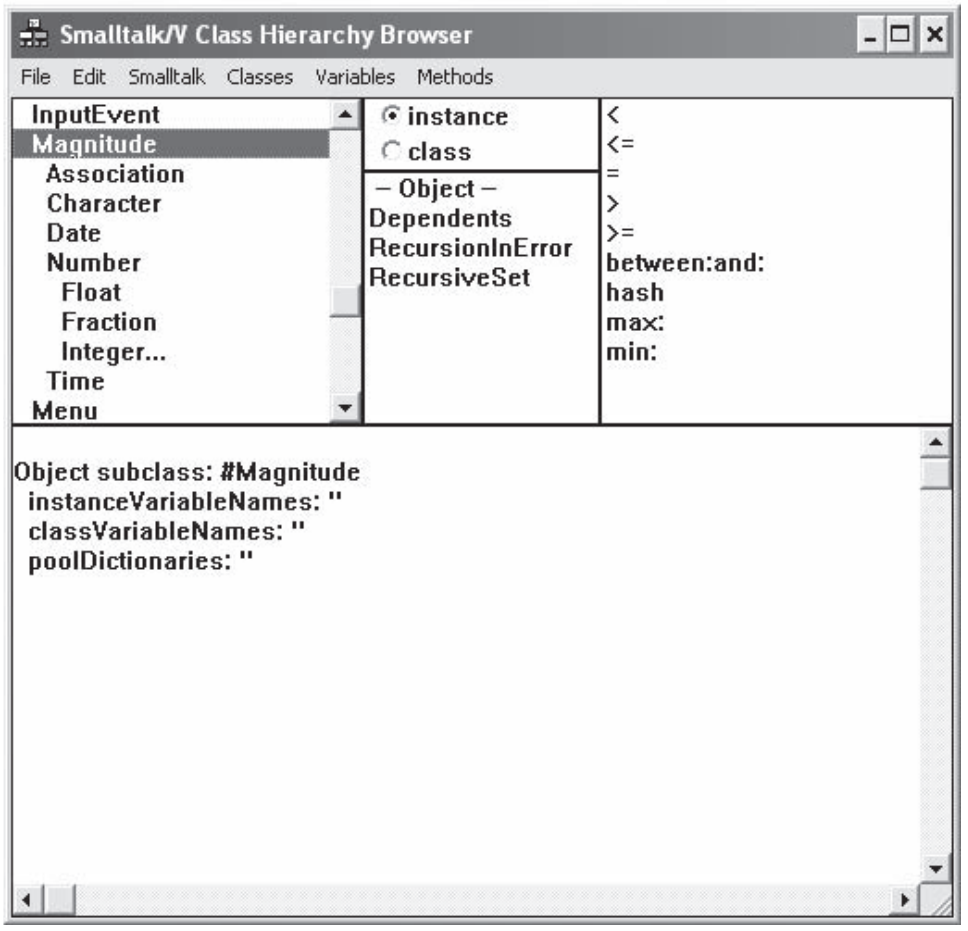


Figure 5.2 A class browser open on the magnitude classes

Note that the most abstract class, `Magnitude`, has been selected. Its instance messages are listed in the rightmost pane. These include the comparison operators and several other messages. These are the most basic messages that any object of a more specific type of magnitude should also recognize. In the bottom pane of Figure 5.2 is code that states that `Magnitude` is a subclass of `Object`; this implies that any message defined in the `Object` class will also be recognized by objects of any magnitude type (or

any other type in Smalltalk). This code also shows that the `Magnitude` class defines no instance variables, class variables, or pool dictionaries, thus leaving it to subclasses to include them if they are needed. Instance variables correspond to storage for properties that belong to each individual object when it is created. Class variables and pool dictionaries correspond to storage for properties that all objects of a class share in common.

Figure 5.3 shows the same browser window after the `max:` message has been selected in the message list.

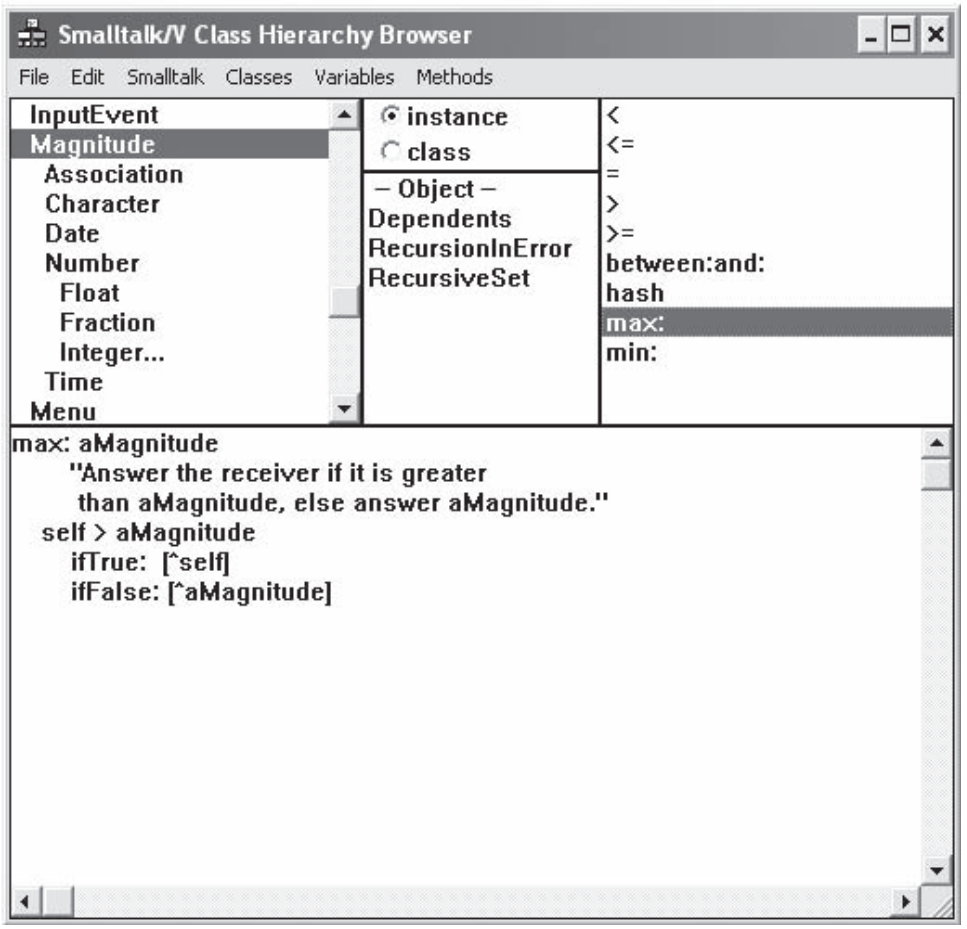


Figure 5.3 The definition of the `max:` method in the `Magnitude` class

When the programmer selects the `max:` message, the method definition corresponding to the message appears in the bottom pane. According to the method header and comment, this method expects another magnitude object as its one argument. The method compares the value of `self`, which refers to the receiver object, to the argument object, and returns the larger of the two objects. (The return operator in Smalltalk is `^`, which has the lowest precedence of any operator.) When referenced in documentation, the methods of a particular class are often prefixed with the class name, to distinguish them from methods with the same name defined in another class. Thus, the method just examined is identified as `Magnitude>>>max:`.

If we then select the > operator in the message list, we see the following code:

```
> aMagnitude
    "Answer true if the receiver is greater
     than aMagnitude, else answer false."
  ^self implementedBySubclass
```

While the code for the implementation of `max:` is straightforward, the code for the > operator seems unusual, to say the least. It does not tell us how the two magnitudes will be compared, but appears to pass the buck to an unspecified subclass. Nevertheless, most of the power of object-oriented inheritance and polymorphism is exemplified in this code. We now explain why.

The method `Magnitude>>>max:` is finished and complete; no subclasses, either built-in or new, will ever have to implement it, as long as they implement the > operator. How does this work? When the expression `x max: y` is run with two fractions, for example, Smalltalk searches first in the class of the receiver object (`Fraction`) for the method corresponding to this message. Not finding that method in `Fraction`, Smalltalk moves up to its parent class, `Number`, where the search fails again. A matching method is finally found in the `Magnitude` class, which is the parent class of `Number`. When the method `Magnitude>>>max:` is then invoked, the symbols `self` and `aMagnitude` are in fact still bound to two fraction objects. Consequently, the search process for the method corresponding to the > operator begins with the `Fraction` class once again. Smalltalk will find the appropriate method definition for comparing two fraction objects in that class, and all will be well.

Before we examine how to define a new concrete magnitude class, let's view some code in the `Fraction` class for a role model. The code for this class shows, among other things, instance variables for the fraction's numerator and denominator, instance methods that return or modify these values, the > operator for fractions, and a class method to instantiate fractions. The class methods can be viewed by selecting the **Class** radio button in the middle pane of the browser. Here is the code for these resources:

```
Number subclass: #Fraction
  instanceVariableNames:
    'numerator denominator '
  classVariableNames: ''
  poolDictionaries: ''

numerator: n denominator: d
    "(Class method.) Answer an instance of class
     Fraction and initialize the numerator and denominator
     instance variables to n and d respectively."
  ^self basicNew numerator: n denominator: d

numerator: n denominator: d
    "(Instance method.) Answer the receiver.
     The numerator and denominator of the receiver
     are set to the n and d arguments respectively."
```

(continues)

(continued)

```
    numerator := n.
    denominator := d.
    ^self

  denominator
      "Answer the denominator of the receiver."
    ^denominator

  numerator
      "Answer the numerator of the receiver."
    ^numerator

  > aFraction
      "Answer true if the receiver is greater
       than aFraction, else answer false."
    ^(numerator * aFraction denominator) > (denominator * aFraction numerator)

  printOn: aStream
      "Append the ASCII representation of
       the receiver to aStream."
    numerator printOn: aStream.
    aStream nextPut: $/.
    denominator printOn: aStream
```

Note that the numerator and the denominator of the receiver object in the > method can be accessed directly by mentioning the instance variable names. However, the numerator and denominator of the parameter object can only be accessed by sending it the corresponding messages. Note also that the > method in the `Fraction` class uses the > operator once again (polymorphically), this time with the numbers that represent the products of the numerators and denominators. The same holds true for the `printOn:` method, which is run again with the numerator and the denominator. Finally, the class includes two methods named `numerator:denominator:`. One of these is a class method that is used to instantiate a `Fraction`. The other is an instance method that resets the values of an existing `Fraction` object's instance variables to new values. In another interesting example of polymorphism, the class method calls the instance method with the same name!

Now we can write code in the transcript window to create two fractions and find the larger of the two:

```
| oneHalf twoThirds |
oneHalf := Fraction numerator: 1 denominator: 2.    "Create two fractions"
twoThirds := Fraction numerator: 2 denominator: 3.
(oneHalf max: twoThirds) printString                   "Returns '2/3'"
```

We have examined two definitions of the > operator, one abstract (in the `Magnitude` class, with implementation deferred) and the other concrete (in the `Fraction` class, with implementation

realized). They are polymorphic, because they have the same name but are defined differently in different classes. The instance of > in the `Magnitude` class is only there to support the use of comparisons in `Magnitude>>>max:`, whereas the instance of > in the `Fraction` class is there to realize that behavior specifically for fractions. Likewise, Smalltalk locates the `printString` method for fractions in the `Object` class, and this method in turn sends the `printOn:` message to the fraction object to build its string representation. When a message is sent to an object, Smalltalk binds the message name to the appropriate method found during the search process described earlier. **Dynamic** or **runtime binding** of messages to methods based on the class of the receiver object is probably the most important key to organizing code for reuse in object-oriented systems.

Now that we have seen how Smalltalk's magnitude classes fit together, we are ready to add one of our own. Smalltalk has no built-in class for complex numbers, so we define a new one, `Complex`, as a subclass of `Number`. Doing so will give us the benefit of inheriting all of the implemented methods in `Number` and `Magnitude`, and also require us to implement several of the methods marked as implemented by a subclass. Before we do so, however, let's anticipate some of the behavior of our new class by imagining the following session, which creates two complex numbers and displays their product in the transcript window:

```
| c1 c2 c3 |
c1 := Complex realPart: 3.0 imaginaryPart: 2.5.
c2 := Complex realPart: 6.0 imaginaryPart: 4.5.
c3 := c1 * c2.
Transcript nextPutAll: (c3 printString); cr
 69.75+96.0i
```

We start by selecting the `Number` class and then selecting the **Add Subclass** option in the **Classes** menu. This allows the programmer to name the new class and begin entering information about its instance variables in the template shown in the bottom pane. There are two instance variables, `realPart` and `imaginaryPart`. After adding them, we compile the code by selecting menu option **File/Save**. Next, we select the **Class** radio button in the middle pane and the menu option **Methods/New Method** to define a class method to instantiate complex numbers. The code for this method, entered in the method template in the bottom pane, is similar to the code for the class method in the `Fraction` class shown earlier. Next, we define two instance methods with the same names as the instance variables. These methods should return the values of those variables, respectively. As you have seen, users of a Smalltalk object other than `self` cannot access its instance variables without sending a message to that object.

The only job left is to fill in the code for the remaining methods that should be implemented by this class. This can be done incrementally, by adding individual methods, compiling them, and testing them. The complete set of methods is listed in the `Magnitude` and `Number` classes. While there are a daunting number of these methods, many of them are already implemented in terms of more basic operations, such as arithmetic and comparisons. After we provide these, we might be 90% finished. Code reuse provides a big win here!

We conclude this subsection by presenting the code for the properties of the `Complex` class, the types of methods shown earlier for the `Fraction` class, and the > and the * operators, which first appear in the `Magnitude` and `Number` classes, respectively.

```
Number subclass: #Complex
  instanceVariableNames:
    'realPart imaginaryPart '
  classVariableNames: ''
  poolDictionaries: ''

realPart: r imaginaryPart: i
    "(Class method.) Answer an instance of class Complex
     and initialize the realPart and imaginaryPart
     instance variables to r and i respectively."
  ^self basicNew realPart: r imaginaryPart: i

realPart: r imaginaryPart: i
    "(Instance method.) Answer the receiver. The
     realPart and imaginaryPart of the receiver are
     set to the r and i arguments respectively."
  realPart := r.
  imaginaryPart := i.
  ^self

imaginaryPart
    "Answer the imaginary part."
  ^imaginaryPart

realPart
    "Answer the real part."
  ^realPart

> aNumber
    "Answer true if receiver is greater than aNumber or false otherwise."
  ^(self realPart > aNumber) realPart or: [
      (self realPart = aNumber realPart) and: [
          self imaginaryPart > aNumber imaginaryPart]]

* aNumber
    "Answer the result of multiplying
     the receiver by aNumber."
  | rp ip |
  rp := self realPart * aNumber realPart –
        self imaginaryPart * aNumber imaginaryPart.
  ip := self realPart * aNumber imaginaryPart +
        self imaginaryPart * aNumber realPart.
  ^self class realPart: rp imaginaryPart: ip
```

(continues)

(continued)

```
printOn: aStream
    "Append the ASCII representation of
     the receiver to aStream."
  realPart printOn: aStream.
  aStream nextPut: $+.
  imaginaryPart printOn: aStream.
  aStream nextPut: $i
```

5.2.3 The Collection Hierarchy

Collections are containers whose elements are organized in a specific manner. The different types of organization include linear, sorted, hierarchical, graph, and unordered. Linear collections include the familiar examples of strings, arrays, lists, stacks, and queues. Unordered collections include bags, sets, and dictionaries (also known as map collections in Java). Graphs and trees are also collections, as are sorted collections, whose elements have a natural ordering (each element is less than or equal to its successor).

Built-in collections in imperative languages, such as FORTRAN, C, Pascal, and Ada, have historically been limited to arrays and, if the application programmer is lucky, strings. The programmer must build the other types of collections using the array data structure or by defining linked nodes. Functional languages like Lisp include the list, which is essentially a linked structure, as the basic collection type.

Smalltalk provides the standard for object-oriented languages by including a large set of collection types. They are organized, for maximum code reuse, in a collection class hierarchy. Figure 5.4 shows a class diagram of the most important Smalltalk collection classes.

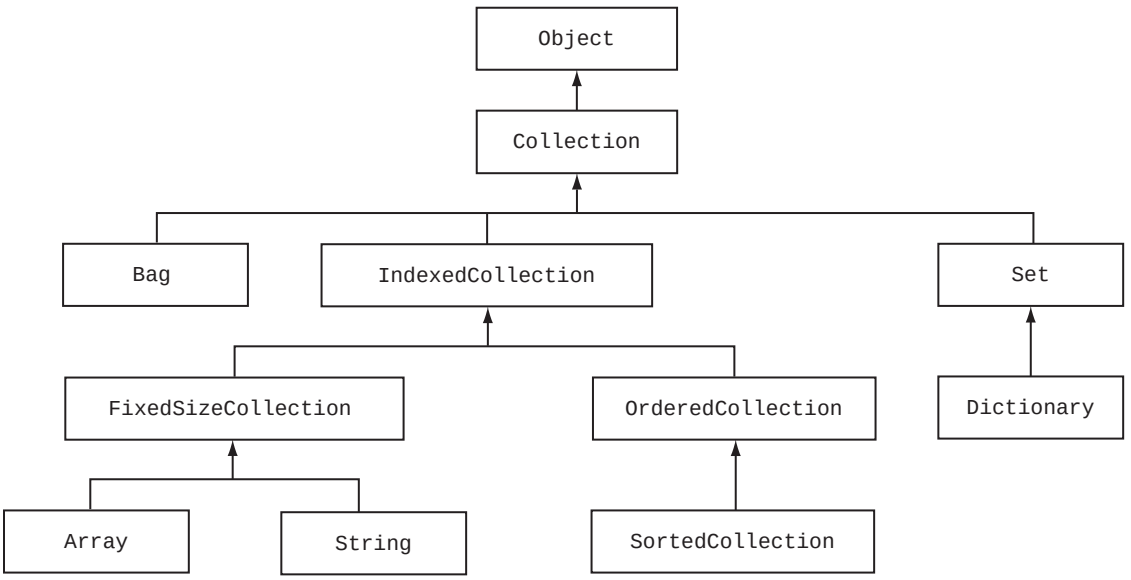


Figure 5.4 The Smalltalk collection class hierarchy

As you have seen in the case of the magnitude hierarchy, the classes in the collection hierarchy are split into abstract classes for organizing common structure and behavior and concrete classes that are instantiated in applications. The most abstract class is `Collection`, which naturally includes methods that should apply to any type of collection. The two most interesting types of methods in this class are the iterators and the methods to convert one type of collection to another type. We now examine the use and implementation of several of these.

The most basic iterator is `do:`. As you saw earlier, this iterator implements a loop that allows the programmer to traverse or visit all of the elements in a collection. Because the manner in which this is done will vary with the organization of the collection, `do:` is implemented by subclasses. However, once the subclass implementation of `do:` is set, the `Collection` class can provide complete implementations of all of the other basic types of iterators, which are listed in Table 5.1.

Table 5.1 The basic types of iterators in Smalltalk's `Collection` class

Message	Iterator Type	What It Does
`do:aBlock`	Simple element-based traversal	Visits each element in the receiver collection and evaluates a block whose argument is each element.
`collect:aBlock`	Map	Returns a collection of the results of evaluating a block with each element in the receiver collection.
`select:aBlock`	Filter in	Returns a collection of the objects in the receiver collection that cause a block to return true.
`reject:aBlock`	Filter out	Returns a collection of the objects in the receiver collection that do not cause a block to return true.
`inject:anObject into:aBlock`	Reduce or fold	Applies a two-argument block to pairs of objects, starting with `anObject` and the first element in the collection. Repeatedly evaluates the block with the result of its previous evaluation and the next element in the collection. Returns the result of the last evaluation.

Because they are defined in the `Collection` class, Smalltalk iterators are highly polymorphic. They can work with any types of collections, not just lists, as in functional languages like Lisp. The following series of examples shows the use of the different Smalltalk iterators with strings.

```
'Hi there' collect: [ :ch |
   ch asUpperCase]                              "Returns 'HI THERE'"

'Hi there' select: [ :ch |
   ch asVowel]                                  "Returns 'iee'"

'Hi there' reject: [ :ch |
   ch = Space]                                  "Returns 'Hithere'"

'Hi there' inject: Space into: [ :ch1 :ch2 |
   ch1, ch2]                                    "Returns ' H i t h e r e'"
```

The two iterators `select:` and `inject:into:` can be used to generate the sum of the odd numbers in a collection named `col`, as follows:

```
(col select: [ :n | n odd])
   inject: 0 into: [ :n1 :n2 |
      n1 + n2]                          "Returns the sum of the odd numbers in col"
```

The implementations of Smalltalk iterators rely on the methods `do:` and `add:`, both of which are implemented by subclasses of `Collection`. Here is the code for one of the iterators, `collect:`, which is the polymorphic equivalent of a map in functional languages:

```
collect: aBlock
   "For each element in the receiver, evaluate aBlock with
    that element as the argument. Answer a new collection
    containing the results as its elements from the aBlock
    evaluations."
  | answer |
  answer := self species new.
  self do: [ :element |
      answer add: (aBlock value: element)].
  ^answer
```

Note that this code creates a new instance of the same class as the receiver object. To this new collection object are added the results of evaluating a block with each element of the receiver collection. The new collection is then returned. A similar `do:` loop pattern is used in the implementations of the other iterators.

Another interesting set of methods in the `Collection` class allows the programmer to convert any type of collection, including newly added ones, to many of the built-in types of collections. Most of these methods, such as `asBag` and `asSet`, create an instance of the specified type of collection and send it the `addAll:` message with the receiver object as an argument. The `addAll:` method, which is also included in the `Collection` class, runs a `do:` loop on its parameter object to add each of its elements to the receiver with the receiver's `add:` method. The `add:` method is, naturally, implemented by a subclass. Here is the code for the `Collection` methods `asBag` and `addAll:`.

```
asBag
   "Answer a Bag containing the elements of the receiver."
  ^(Bag new)
      addAll: self;
      yourself

addAll: aCollection
   "Answer aCollection. Add each element of
    aCollection to the elements of the receiver."
  aCollection do: [ :element | self add: element].
  ^aCollection
```

Smalltalk has no built-in classes for stacks, queues, trees, or graphs, but we can easily see how to add them. Let's develop this idea for queues. There are two common implementations of queues, one based on an array and the other based on linked nodes. An array-based queue class could be a subclass of the `Array` class or the `OrderedCollection` class (which is like the array-based `ArrayList` class in Java). However, subclassing one of these classes for queues would be a poor design choice, because users of such queues would then have at their disposal operations, such as `at:put:`, which would violate the spirit of restricting access to the front or rear of a queue collection. A better choice would be to subclass the `Collection` class, whose operations, such as the iterators and the type conversion methods, provide some positive benefits without the negative consequences. The `ArrayQueue` methods would include not just the standard ones such as `enqueue`, `dequeue`, `size`, and `isEmpty` but also `do:` and `add:`, which are needed to invest queues with the functionality of many of the `Collection` methods. As for properties, the `ArrayQueue` class would contain either an array or an ordered collection to hold its data elements. The completion of this version of a queue in Smalltalk is left as an exercise.

The other major implementation, `LinkedQueue`, is also a subclass of `Collection`. It has the same interface, or set of methods, as `ArrayQueue`. However, the elements in a `LinkedQueue` are contained in a sequence of linked nodes. Linked nodes come in many flavors, among them singly linked, doubly linked, and variations on each of these with a circular link back to the first node or a special header node. To support any of these options, we need a separate class that contains a data element and one or more links. For a simple singly linked structure, the `Association` class in Smalltalk's magnitude hierarchy already fills the bill.

Two instance variables that point to the first and last nodes in the linked structure enable the implementation to quickly locate the front and rear elements in the queue, respectively. The size of the queue is also tracked with a separate instance variable. Following is a listing of the properties and methods of a `LinkedQueue` class in Smalltalk:

```
Collection subclass: #LinkedQueue
  instanceVariableNames:
    'front rear size '
  classVariableNames: ''
  poolDictionaries: ''

new
   "Answer an instance of the receiver, with front and
    rear pointers as empty links and size of 0."
  ^super new initialize

initialize
   "Answer this object. Set the instance variables to initial values."
  | newNode |
  front := nil.
  rear := nil.
  size := 0.
  ^self
```

(continues)

(continued)

```
enqueue: anObject
   "Answer anObject. Add anObject to the rear of the receiver."
  | newNode |
  newNode := Association key: anObject value: nil.
  rear notNil ifTrue: [rear value: newNode].
  rear := newNode.
  front isNil ifTrue: [front := newNode].
  size := size + 1.
  ^anObject

dequeue
   "Answer the object at the front of the receiver.
    Also removes this object from the receiver."
  | oldNode |
  oldNode := front.
  front := front value.
  front isNil ifTrue: [rear := nil].
  size := size – 1.
  ^oldNode key

size
   "Answer the size of the receiver."
  ^size

add: anObject
   "A synonym for enqueue."
  ^self enqueue: anObject

do: aBlock
   "Basic iterator, evaluates aBlock on each object
    in the receiver."
  | probe |
  probe := front.
  [probe notNil] whileTrue: [
    aBlock value: probe key.
     probe := probe value]
```

Note that the class method `new` and the instance method `initialize` collaborate to instantiate a `LinkedQueue`. The method `LinkedQueue>>>new` sends the message `new` to `super`, which refers to one of `LinkedQueue`'s superclasses, to obtain a new object. This object then receives the message `initialize`, which sets the instance variables in it. The reason that we need a separate instance method to do this is that the class method `new` cannot access any instance variables.

To examine the behavior and state of a linked queue, we evaluate the following code segment in the transcript window:

```
| q |
q := LinkedQueue new.
q addAll: #(1 2 3 4 5).     "Copy data from an array literal."
q dequeue.
q enqueue: 'Ken'.
q inspect
```

Smalltalk's `inspect` message opens an inspector window on the receiver object. The programmer can then browse the values of the object's instance variables in this window. Figure 5.5 shows the inspector window for our linked queue.

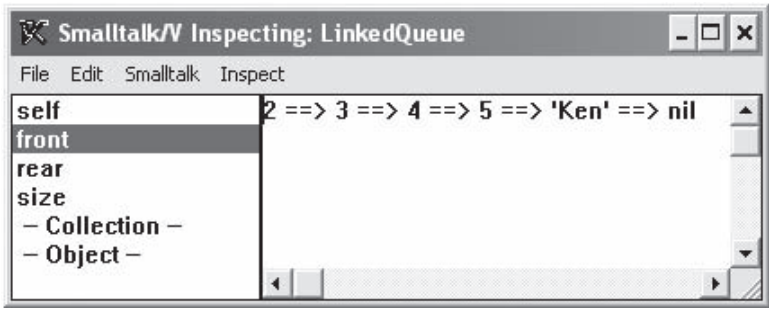


Figure 5.5 A Smalltalk inspector window on a `LinkedQueue` object

Note that the state of the instance variable `front` actually looks like a linked structure. The programmer can browse into the objects contained in the structure by double-clicking each one.

Smalltalk programmers have found the language and its programming environment a joy to work with. However, in the early 1990s, just as Smalltalk was coming into use on ordinary computers, two languages caught on and stole any thunder that Smalltalk might have built for object-oriented programming. These languages are Java and C++, which we explore in the next two sections.

5.3 Java

James Gosling and his team at Sun Microsystems originally intended Java as a programming language for systems embedded in appliances such as microwave ovens. Their initial emphasis was on portability and a small footprint for code, as captured in the slogan, "write once, run anywhere." To achieve this goal, Java programs would compile to machine-independent byte code, and each target platform would provide its own Java Virtual Machine or interpreter for this byte code. It turned out that this vision of software development and distribution was ideally suited for many of the developments and advances in computing in the last 20 years, including networks, the Web, and modern developments like handheld, mobile devices.

Java's designers also realized that the language for this sort of application development should be fully object oriented, and, to appeal to the widest range of programmers, have a conventional syntax such

as that of C. Finally, the language should come with a large set of libraries to support applications in all areas of modern computing.

Java's portability, small code footprint, conventional syntax, and support for compile-time type checking gave it an immediate advantage over Smalltalk. Nevertheless, it adopted many of the features of Smalltalk, including runtime garbage collection and the use of references instead of explicit pointers, which make Java a safer language than C or C++. Moreover, unlike C++, Java is not a hybrid of object-oriented and procedural languages, but is purely object oriented, with one exception. In Java, the scalar data types (also called **primitive types**) are not objects, for reasons of efficiency.

In this section, we give an overview of the features of Java that support object-oriented programming. In the process, you will see that the language not only allows the programmer to write Smalltalk code in C-like syntax but also represents some advances on the Smalltalk way of developing programs.

5.3.1 Basic Elements of Java: Classes, Objects, and Methods

As in Smalltalk, a Java application instantiates the appropriate classes and gets the resulting objects to do things by calling methods on them (note the change in terminology, compared to Smalltalk's "sending messages to them"). The only exceptions to this are the use of the scalar types, such as `int`, `float`, and `char`, which appear as operands in expressions or are passed as arguments to methods or returned as their values. Classes such as `String` are already available in the standard `java.lang` package, or may be imported from other packages, such as `java.util` or `javax.swing`. Programmer-defined classes can also be placed in their own packages for import in any applications.

The syntax of variable declaration, object instantiation, and method calls in Java closely resembles that of variable declarations and function calls in C. Variable declaration and object instantiation use the following syntax:

```
<class name> <variable name> = new <class name>(<argument-1, . ., argument-n>);
```

and a method call, as in Smalltalk, places the object to the left of the method name:

```
<object>.<method name>(<argument-1, . ., argument-n>)
```

For example, let's assume that someone has defined a new class called `Complex` for complex numbers and placed it in a package called `numbers`. The following short application creates two complex numbers, adds them, and displays the results in the terminal window:

```
import numbers.Complex;

public class TestComplex{

    static public void main(String[] args){
        Complex c1 = new Complex(3.5, 4.6);
        Complex c2 = new Complex(2.0, 2.0);
        Complex sum = c1.add(c2);
        System.out.println(sum);
    }
}
```

This application consists of an import statement, a class definition, and a single method definition within that class. The method is `static`, which means that it is a **class method** and can be run as `TextComplex.main(<array of strings>)`. This is exactly what the Java Virtual Machine does automatically when the application is launched. The JVM places in the `args` array any command-line arguments that might be present at the launch. Our `main` method ignores this argument and performs as expected.

Now let's develop a small portion of the new `Complex` class for representing complex numbers. Because numbers are not objects in Java, we cannot subclass under a magnitude hierarchy that will give us built-in functionality. Thus, we build the `Complex` class from scratch. Here is a listing with an explanation following:

```
package numbers;

public class Complex extends Object{

    // Default constructor
    public Complex(){
        this(0, 0);
    }

    // Parameterized constructor
    public Complex(double realPart, double imaginaryPart){
        re = realPart;
        im = imaginaryPart;
    }

    public double realPart(){
        return re;
    }

    public double imaginaryPart(){
        return im;
    }

    public Complex add(Complex c){
        return new Complex(this.realPart() + c.realPart(),
                           this.imaginaryPart() + c.imaginaryPart());
    }

    public Complex multiply(Complex c){
        return new Complex(this.realPart() * c.realPart() –
                           this.imaginaryPart() * c.imaginaryPart(),
                           this.realPart() * c.imaginaryPart() +
                           this.imaginaryPart() * c.realPart());
}
```

(continues)

(continued)

```
    public String toString(){
        return this.realPart() + "+" + this.imaginaryPart() + "i";
    }

    // Instance variables
    private double re, im;
}
```

The module begins by declaring that this resource belongs to a Java package named `numbers`. The class header shows that `Complex` extends Java's `Object` class. This extension could have been omitted because it happens by default, but is added for emphasis. The instance variables for `Complex` class are `re` and `im`. They are declared with `private` access, meaning that they are visible and accessible only within the class definition. This is how data encapsulation is enforced in Java. The two **accessor methods** `realPart` and `imaginaryPart` are defined with `public` access. This allows programmers who use complex numbers to view but not modify their internal state. Note that these methods are also called on the receiver object referenced by the keyword `this` and the parameter object referenced by `c`, within the methods `add`, `multiply`, and `toString`. This might seem like overkill, in view of the fact that the instance variables of both objects could have been referenced directly in these methods (using `this.re` or just `re` for the receiver's variable and `c.re` for the parameter's variable). However, the use of the two accessor methods within the class definition places an abstraction barrier between the data and their use, which allows the programmer to change how the data are represented without disturbing other code (more on this point shortly).

The method `toString` is the only polymorphic method defined in the `Complex` class. Its definition overrides the one inherited from the `Object` class. The purpose of `toString` in Java is to return a string representation of the receiver object (in a manner similar to Smalltalk's `printOn:` and `printString`). As defined in `Object`, the default behavior of `toString` is to return the name of the receiver's class. The definition of `toString` in the `Complex` class returns information about the instance, not the class. Some Java methods, such as `System.out.println` and the string concatenation operator `+`, automatically run an object's `toString` method to obtain its text.

In addition to the instance variables and methods, the definition of `Complex` contains two overloaded **constructors**. These are like methods, except that they have the same name as the class and have no return value. Constructors specify initial values for the instance variables and can perform other operations required when an object is first constructed (hence the name). The second constructor has two parameters that provide initial values for the instance variables. The first constructor takes no parameters (it is a so-called **default constructor**) and initializes both `re` and `im` to 0. Note that the first constructor accomplishes its task by calling the second one, using the keyword `this` with the appropriate arguments. This process, called **constructor chaining**, is another form of code reuse.

As mentioned earlier, the use of private access to instance variables and the use of accessor methods even within the class definition allow us to change the representation of the data without disturbing other

code. We could rewrite the class `Complex` to use polar coordinates without changing any of the methods, other than `realPart`, `imaginaryPart`, and one constructor, as shown in the following code:

```
package numbers;

public class Complex extends Object{

    public Complex(){
        this(0, 0);
}

    public Complex(double realPart, double imaginaryPart){
        radius = Math.sqrt(realpart * realpart + imaginaryPart * imaginaryPart);
        angle = Math.atan2(imaginaryPart, realpart);
    }

    public double realPart(){
        return radius * Math.cos(angle);
    }

    public double imaginaryPart(){
        return radius * Math.sin(angle);
    }

    public Complex add(Complex c){
        return new Complex(this.realPart() + c.realPart(),
                           this.imaginaryPart() + c.imaginaryPart());
    }

    public Complex multiply(Complex c){
        return new Complex(this.realPart() * c.realPart() –
                           this.imaginaryPart() * c.imaginaryPart(),
                           this.realPart() * c.imaginaryPart() +
                           this.imaginaryPart() * c.realPart());
    }

    public String toString(){
        return this.realPart() + "i" + this.imaginaryPart();
    }

    private double radius, angle;
    }
```

Like Smalltalk, Java uses reference semantics for variables that name objects. Classes are, thus, also called **reference types**. Variables that name values of primitive types, such as `int`, `float`, and `char`, have value semantics in Java. The equality operator == used with primitive types can also be used

with reference types, but in that context, it means object identity, as it does in Smalltalk. Java includes an `equals` method in the `Object` class which defaults to the behavior of the == operator but can be overridden in subclasses to implement a comparison of two distinct objects for the same properties.

Methods for the objects `z` and `w` can be invoked after instantiation by using the usual dot notation. For example,

```
z = z.add(w);
```

adds `w` to `z`, creating a new `Complex` object in the process, and assigns this new object to `z` (throwing away the object previously pointed to by z). It is also possible to nest operations so that, for example,

```
z = z.add(w).multiply(z);
```

performs the computation `z = (z + w) * z`. In the process, a temporary object (we'll call it `t`) is created to hold the value of `z + w`, and a new object is then created to hold the value `t + z`. This object is then referenced by `z`, and `z`'s old object is thrown away. This has essentially all the advantages of abstraction, and with overloading, can make the type `Complex` look more or less like a built-in type. Unfortunately, Java does not allow operator overloading, but C++ does, as discussed later in this chapter.

The disadvantages to this mechanism are two-fold. First, without automatic garbage collection, serious space leaks become difficult to avoid. For this reason, most object-oriented languages require automatic garbage collection (Smalltalk and Java, for example, but not C++). Second, this view of binary operations is not symmetric, since the left-hand operand is singled out as the target of the method call. In a mixed-paradigm language like C++, such operators could be implemented instead as ordinary functions or as overloaded operators in method definitions. Alternatively, some object-oriented languages, such as the Common Lisp Object System (CLOS), allow **multimethods**, in which more than one object can be the target of a method call.

5.3.2 The Java Collection Framework: Interfaces, Inheritance, and Polymorphism

As mentioned earlier, Java includes a large number of libraries or packages with classes to support many kinds of applications. Some of these sets of classes are related in what one might call a framework. One such framework, which is similar in many ways to the Smalltalk collection hierarchy, is the Java collection framework, as defined in the package `java.util`. In this section, we explore Java's support in this framework for inheritance, polymorphism, and distinguishing concrete and abstract classes. We also examine how two other features not found in Smalltalk—parameterized types and interfaces—support the organization of code in a statically typed object-oriented language like Java.

Conceptually, an **interface** is a set of operations on objects of a given type. Interfaces serve as the glue that connects components in software systems. A resource's interface should give its users only the information they need to use it and its implementers only enough information to implement it. Java codifies the concept of an interface with a special syntax that allows the programmer to list a type name and a set of public method headers. Java interfaces can belong to packages and can be compiled in advance of any other code. An interface can then be realized or implemented by any class that includes all of the methods specified in that interface.

Java's collection framework is structured at the top level in terms of a set of interfaces. Figure 5.6 shows a diagram of some of them.

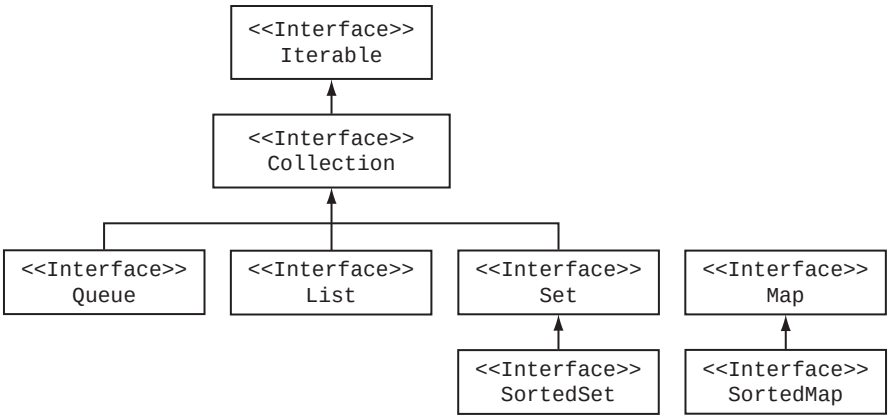


Figure 5.6 Some Java collection interfaces

The interfaces appear in a tree-like hierarchy, much like classes. The more general interfaces are at the top. Each interface below the root extends its parent interface above it. Thus, for example, the `List` interface includes not only the methods specific to lists but also implicitly those in the `Collection` and `Iterable` interfaces. The position of the `Iterable` interface at the root of this tree ensures that every collection must include an `iterator` method. We will have much more to say about iterators in a moment. Java's designers thought it best not to include the `Map` interfaces in the hierarchy (see Exercise 5.8).

The next code segment shows the code for some of the methods specified in the `Collection` interface:

```
public Interface Collection<E> extends Iterable<E>{
    boolean   add(E element);
    boolean   addAll(Collection<E> c);
    void      clear();
    boolean   contains(E element);
    boolean   containsAll(Collection<E> c);
    boolean   equals(Object o);
    boolean   isEmpty();
    boolean   remove(Element E);
    boolean   removeAll(Collection<E> c);
    boolean   retainAll(Collection<E> c);
    int       size();
}
```

You will recall some similar methods from our discussion of Smalltalk's `Collection` class. Note also the presence of notations such as `<E>` and `E` in the interface and method headers. These notations represent **type parameters**. You saw type parameters used with the functional language Haskell in Chapter 4. Java is statically typed, and all data types, including the types of the elements in collections, must be explicitly specified at compile time. Collections with type parameters are also

called **generic collections**, as distinct from the nongeneric, **raw collections** that appeared in early versions of Java. A generic collection exploits parametric polymorphism, which allows collections of different element types to rely on a single definition of the collection type.

Java collections can contain elements of any reference types. The complete type (including the actual element type) of a given collection variable must be specified when that variable is declared. For example, the following code declares two variables, one to reference a list of strings and the other to reference a set of integers:

```
List<String> listOfStrings;
Set<Integer> setOfInts;
```

Note that the type parameters for the element types of `List` and `Set` have been replaced by the actual element type names, `String` and `Integer`, respectively. This prevents the programmer from adding anything other than a string to the list and anything other than an integer to the set. (Java automatically wraps `int` values in `Integer` objects when they are added to a collection of integers, and automatically unwraps them when they are returned from a collection.)

The classes that implement the collection interfaces also form a hierarchy. Figure 5.7 shows some of these. The dashed arrow in the diagram indicates that a class implements an interface. A solid arrow indicates that a class extends another class and an interface extends another interface. If a class implements an interface, any of its subclasses also (implicitly) implement that interface.

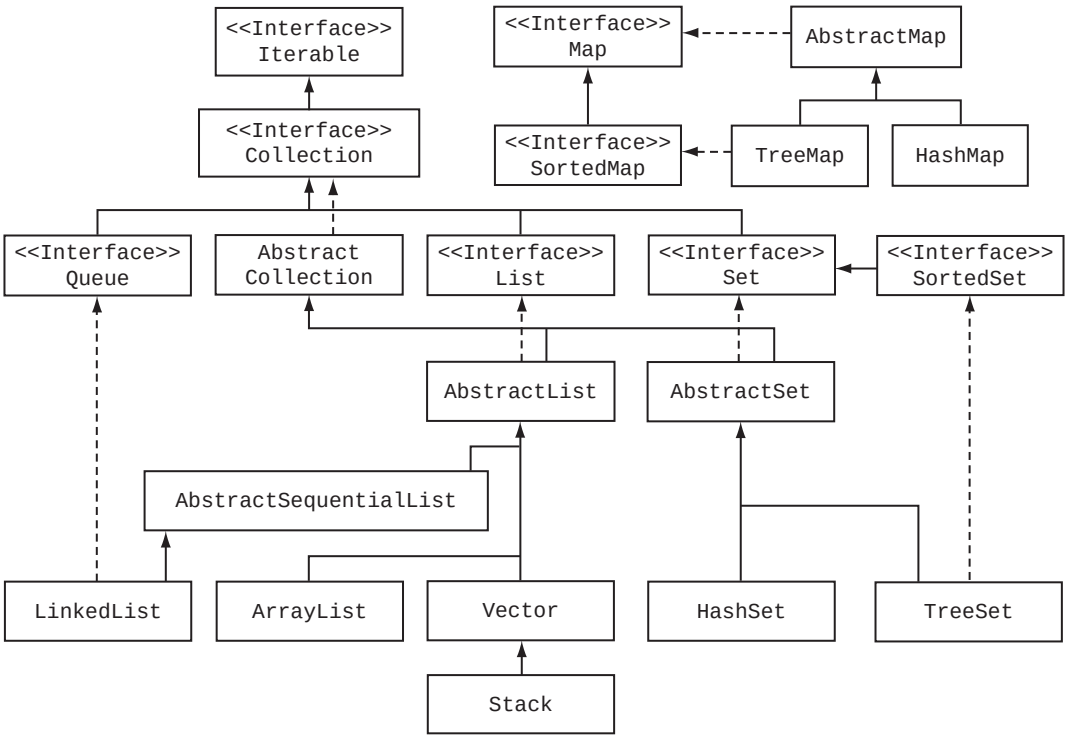


Figure 5.7 Some Java collection classes and their interfaces

Some of these classes are obviously abstract, even without the hints offered by their names. These classes implement common properties and behavior for their subclasses. The classes on the frontier of the tree are concrete and thus can be instantiated. Note also that some classes, such as `LinkedList`, implement more than one interface. Thus, a linked list can behave either as a list or as a queue. For example, the following code segment declares variables for a list of strings, a queue of floats, and a list of characters. The code also initializes each of the variables to new list objects. The first two are linked lists, and the third is an array list.

```
List<String>    listOfStrings = new LinkedList<String>();
Queue<Float>    queueOfFloats = new LinkedList<Float>();
List<Character> listOfChars   = new ArrayList<Character>();
```

From the compiler's (or the programmer's) perspective, the lists of strings and characters in this code are just lists. That is, the same methods can be called on the two `List` variables even though the two lists they reference have different implementations and different element types. However, because the variable `queueOfFloats` has been declared to be of type `Queue<Float>`, the linked list that it refers to will only respond to the methods specified in the `Queue` interface. Any other `List` method used with this variable will be flagged as a syntax error. Thus, the danger that a list object might violate the spirit of a queue, which we saw earlier in our discussion of inheritance in Smalltalk, is avoided with these declarations.

By contrast, the design decision to make the Java `Stack` class a subclass of `Vector`, which implements the `List` interface (see Figure 5.7), is not the most fortunate one. In this case, the spirit of a stack could be violated, because a programmer might use any of the list operations on it.

To remedy this design flaw in Java, let's develop a new type of stack class called `LinkedStack`. This implementation of a stack will behave like a pure, restricted stack, but allow users to run some of Java's `Collection` methods with stacks as well. Our new class will include the standard stack methods `push`, `pop`, and `peek`, as well as the `Collection` methods `size`, `add`, and `iterator`. By defining `LinkedStack` as a subclass of the `AbstractCollection` class, we will get a good deal of additional behavior for free. Figure 5.8 shows a diagram of our proposed subframework.

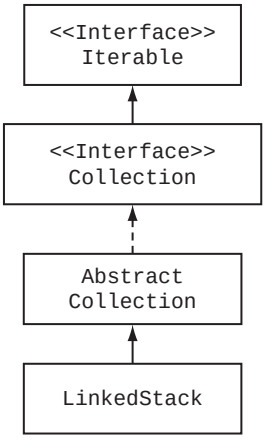


Figure 5.8 Adding `LinkedStack` to the collection framework

Our implementation of `LinkedStack` closely resembles that of our `LinkedQueue` class in Smalltalk. We use a singly linked sequence of nodes to hold the elements, with a single external pointer, called `top`, that points to the first node. An integer used to track the number of elements, named `size`, is the only other instance variable. The `Node` class is not defined externally but is nested as a **private inner class** within the `LinkedStack` class. We postpone the development of the iterator for stacks for a moment. Here is the code for the basic `LinkedStack` class:

```
import java.util.AbstractCollection;
import java.util.Iterator;

public class LinkedStack<E> extends AbstractCollection<E>{

    // Constructor
    public LinkedStack(){
        top = null;
        size = 0;
    }

    public E peek(){
        return top.data;
    }

    public E pop(){
        E data = top.data;
        top = top.next;
        size -= 1;
        return data;
    }

    public void push(E data){
        top = new Node<E>(data, top);
        size += 1;
    }

    // The next three methods are required by AbstractCollection
    public boolean add(E data){
        push(data);
        return true;
    }

    public int size(){
        return size;
    }
```

(continues)

(continued)

```
    // Just a stub for now; will be completed shortly
    public Iterator<E> iterator(){
        return null;
    }

    // Instance variables for LinkedStack
    private Node<E> top;
    private int size;
    // Node class for LinkedStack
    private class Node<E>{

        // Constructor for Node
        private Node(E data, Node<E> next){
            this.data = data;
            this.next = next;
        }

        // Instance variables for Node
        private E data;
        private Node<E> next;
    }
}
```

The header of the `LinkedStack` class includes the type variable notation `<E>` after both `LinkedStack` and `AbstractCollection` (as explained shortly, the return type of the `iterator` method, `Iterator<E>`). The nested `Node` class uses the same notation for the element type. Likewise, all uses of the `Node` type in variable declarations must include the same notation. The type variable `E` is used directly without the angle brackets for the type of elements in variable and parameter declarations and for the return type of methods.

The nested `Node` class is defined as `private`, because no classes outside of `LinkedStack` need to use it. Its constructor and instance variables are also defined as `private`. The `Node` class includes no accessor or mutator methods for its `data` and `next` fields; it is intended as a simple container class whose fields can safely be referenced directly using the dot notation within the `LinkedStack` implementation. Thus, `Node` includes just a constructor and two instance variables. The `Node` class is recursive; one of its instance variables, a reference to the next node, is also of type `Node<E>`. Java has no explicit pointer type, so links are always implemented as references. The `null` value, which indicates an empty link, can be assigned to a variable of any reference type.

Before we develop the `iterator` method for `LinkedStack`, a few words must be said about Java iterators. The `iterator` method is the only method specified in the `java.util.Iterable` interface. This method returns a new object of type `Iterator` when run on a collection object. The collection object is also called the **backing store** for the iterator object. For example, one could create a `LinkedStack` of strings and then open an iterator object on it as follows:

```
LinkedStack<String> stack = new LinkedStack<String>();
Iterator<String> iter = stack.iterator();
```

The relationship between the stack and iterator objects is shown in Figure 5.9.

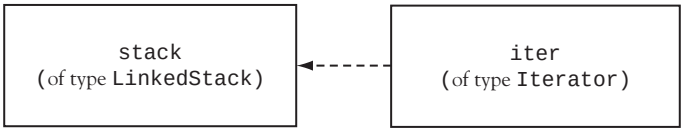


Figure 5.9 An iterator opened on a stack object

Note that the `Iterator` type is also parameterized for the same element type as its backing store.

The `Iterator` type is an interface, which specifies the three methods shown in the following code segment:

```
public Interface Iterator<E>{

    public boolean hasNext();  // True if more elements, false otherwise
    public E next();           // Returns the current element and advances
    public void remove();      // Removes the current element from the store
}
```

Returning to our earlier code segment, where we created an iterator object named `iter`, we can use the `Iterator` methods `hasNext` and `next` in a `while` loop to visit each of the elements in the iterator's backing store (the linked stack). The next code segment simply prints these elements:

```
while (iter.hasNext())
    System.out.println(iter.next());
```

Java's enhanced `for` loop is syntactic sugar for the iterator-based `while` loop just shown. Although the next code segment appears to loop through the stack, the code using the iterator with a `while` loop is actually at work behind the scene:

```
for (String s : stack)
    System.out.println(s);
```

The only prerequisites for the use of an enhanced `for` loop are that the collection class implements the `Iterable` interface and that it also implements an `iterator` method.

To implement an `iterator` method, the `LinkedStack` class must define and instantiate a class that implements the `Iterator` interface. We name this class `StackIterator` and instantiate it in the `iterator` method, as follows:

```
public Iterator<E> iterator(){
    return new StackIterator<E>(top);
}
```

Note that the `StackIterator` object receives a reference to the top of the stack as an argument when it is created.

The `StackIterator` class is another nested, `private` class. It maintains a single instance variable called `probe`, which will be used to visit each node in the stack's linked structure. This variable is

initially set to `top`, which was passed to the `StackIterator` object at instantiation (note that `top` and `probe` now refer to the same object, or `null` if the stack is empty). The method `hasNext` returns `false` when `probe` is `null`. This indicates that the end of the stack has been reached. The method `next` saves a reference to the element in the current node, advances the `probe` reference to the next node, and returns the element. The method `remove` must also be included in the `StackIterator` class but should not be supported for stacks. Therefore, this method throws an `UnsupportedOperationException`. Here is the code for the `StackIterator` class:

```
private class StackIterator<E> implements Iterator<E>{

    // Constructor
    private StackIterator(Node<E> top){
        probe = top;
    }

    public E next(){
        E data = probe.data;
        probe = probe.next;
        return data;
    }

    public boolean hasNext(){
        return probe != null;
    }

    public void remove(){
        throw new UnsupportedOperationException();
    }

    // Instance variable
    private Node<E> probe;
}
```

We claimed earlier that interfaces form the glue that holds software components together. In Java, the interfaces `Iterable` and `Iterator` collaborate to support the use of a simple `for` loop over the elements of any collection class. Recall that in Smalltalk, the methods `do:`, `size`, and `add:` are implemented by all `Collection` subclasses. These classes then inherit from `Collection` a number of high-level methods, such as `addAll:` and other types of iterators. The Java collection framework is set up to provide similar benefits. If a collection extends `AbstractionCollection`, it must implement the methods `iterator`, `size`, and `add`. These methods are declared as `abstract` in the `AbstractCollection` class, meaning that their implementation is deferred to (and required at compile time by) subclasses. However, they are also used in `AbstractCollection` to implement

the other `Collection` methods, such as `addAll` and `contains` (similar to Smalltalk's `includes:`). The following snippet of the `AbstractCollection` class shows how this is done:

```
abstract public class AbstractCollection<E> implements Collection<E>{

    // Methods to be implemented by subclasses
    abstract public int size();
    abstract public boolean add(E element);
    abstract public Iterator<E> iterator();

    // Methods implemented here but inherited by subclasses
    public boolean addAll(Collection<E> c){
        boolean added = true;
        for (E element : c)
            added = this.add(element) && added;
        return added;
    }

    public boolean contains(Object target){
        for (E element : this)
            if (! target.equals(element)) return false;
        return true;
    }

    // Other methods
}
```

As you can see, the programmer can use not only a `for` loop with a linked stack but also the `Collection` method `addAll` to copy elements from a list or a set to a linked stack, the `Collection` method `contains` to search a linked stack for a given element, and so forth. The collection hierarchy does not include built-in map, filter, and reducing iterators, but we show how to define these later in this chapter.

5.3.3 Dynamic Binding and Static Binding

As you have seen, Java is pure object-oriented language with a C-like syntax and static typing. Part of Java's success in the programming community, as compared with Smalltalk, is attributable to the latter two qualities. Additionally, the design of Java can allow some of the runtime cost (and most of the error recovery) of method lookup to be shifted from runtime to compile time. The Java compiler statically determines which implementation of a method to use when the receiver object's actual class can be determined at compile time. At that point, all of the information about the class's methods and those of its ancestor classes is available. For example, for the method call:

```
aCollection.addAll(aCollection)
```

the Java compiler can determine that the method `addAll` is implemented in the `AbstractCollection` class, and thus generate the code for a call of that particular implementation of the method. This process

is known as **static binding**, in contrast to the dynamic binding that you saw implemented in Smalltalk. Unfortunately, the Java compiler will not actually generate code for static binding unless the method being called has been defined as `final` or `static`. A final method is one that cannot be overridden in a subclass, and a static method is one that is called on a class rather than an instance. In both of these cases, there can be no possible call down to the class of the receiver object, so the method calls are statically bound.

By contrast, when the compiler gets to the call of `this.add(element)` in the `addAll` method's code, it cannot determine, at compile time, which concrete version of the `add` method to call. The symbol `this` could refer to a stack, a list, or any other subtype of `Collection`. So, the actual type of this object cannot be determined until runtime. In general, calls "from above" to methods implemented by subclasses must always be handled by dynamic binding at runtime.

Although the residue of dynamic binding in Java incurs a runtime performance hit, Java's implementation of method lookup, which uses a jump table rather than a search of an inheritance tree, still gives Java an efficiency edge over Smalltalk. Moreover, the compile-time requirement that abstract methods must be implemented by subclasses introduces an aspect of safety (compile-time error checking) that is absent from Smalltalk. In this example, the compiler guarantees that at least there will be an `add` method to call at runtime, once the JVM has determined the actual type of its receiver object.

5.3.4 Defining Map, Filter, and Reduce in Java

Map, filter, and reduce are higher-order functions in a functional language, and built-in collection methods in Smalltalk. As you saw in Section 5.2, these Smalltalk methods are also polymorphic, in that they operate on collections of any type, not just lists. Although Java has only a basic iterator, it is possible to define map, filter, and reduce methods that are similar in behavior to their Smalltalk counterparts. Doing so will serve to illustrate some important points about interfaces, classes, generics, and static typing in Java.

The `map`, `filter`, and `reduce` methods can be defined as static methods in a special class named `Iterators` (similar to Java's `Math` and `Collections` classes). The `map` and `filter` methods expect an input collection as an argument and return an output collection as a value. The programmer can pass to these methods any collection parameter that implements the `Collection` interface. The output collection is also defined to be of type `Collection`, but the actual object returned will be of the same concrete class (with perhaps a different element type) as the input collection. Thus, if we choose to convert a list (or set) of integers to a list of strings, we can send in any list (or set) of integers and get back a list (or set) of strings of the same concrete class (for example, `ArrayList` or `HashSet`), masquerading as a collection. This collection then can be cast down to its actual class if desired. Likewise, the `reduce` method takes a collection as an argument, but returns an object of the collection's element type as its result. Table 5.2 shows the rough syntactic form of each method. Note that the element type parameters of the collections are specified, but the exact syntax of the operation parameter is yet to be determined.

Table 5.2 The rough syntax of `map`, `filter`, and `reduce`

`public static<E, R> Collection<R> map(<an operation>, Collection<E> c)`
`public static<E> Collection<E> filter(<an operation>, Collection<E> c)`
`public static<E> E reduce(E baseElement, <an operation>, Collection<E> c)`

The only problem still to be solved is how to represent the operation that is passed as the remaining argument to these methods. In a functional language, this operation is a function of one or two arguments, either predefined or constructed on the fly as a `lambda` form. In Smalltalk, this operation is a block of code with one or two arguments. In Java, we can define the operation as a special type of object that recognizes a method that will be called within the higher-order method's implementation.

There are actually three types of operations:

1. As used in `map`, the operation is a method of one argument that transforms a collection element into another value (perhaps of a different type). Let's call this operation `R transform(E element)` and include it in a special interface called `MapStrategy`.
2. As used in `filter`, the operation is a method of one argument that returns a Boolean value. We call this method `boolean accept(E element)` and include it in the interface `FilterStrategy`.
3. As used in `reduce`, the operation is a method of two arguments that returns an object of the same type. We call this method `E combine(E first, E second)` and include it in the interface `ReduceStrategy`.

Here is the code for the three interfaces that specify the three types of operations for any mapping, filtering, or reducing process:

```
package iterators;

public interface MapStrategy<E, R>{
    // Transforms an element of type E into a result of type R
    public R transform(E element);
}

package iterators;

public interface FilterStrategy<E>{
    // Returns true if the element is accepted or false otherwise.
    public boolean accept(E element);
}

package iterators;

public interface ReduceStrategy<E>{
    // Combines two elements of type E into a result of type E.
    public E combine(E first, E second);
}
```

The method headers in Table 5.2 can now be completed by including the type of the strategy/object passed to each method. Table 5.3 shows these changes.

Table 5.3 The finished syntax of `map`, `filter`, and `reduce`

```
public static<E, R> Collection<R> map(MapStrategy<E, R> strategy,
                                      Collection<E> c)
```

```
public static<E> Collection<E> filter(FilterStrategy< E> strategy,
                                      Collection<E> c)
```

```
public static<E> E reduce(E baseElement,
                          ReduceStrategy<E> strategy,
                          Collection<E> c)
```

The use of these three strategy interfaces tells the implementer of the iterators that she can expect an object that recognizes the appropriate strategy method (`transform`, `accept`, or `combine`). Likewise, they tell the user of the iterators that he need only supply an object of a class that implements one of these interfaces to use them correctly. We now present the implementation of the iterators, followed by an example application.

As mentioned earlier, the home of the three iterator methods is a new class named `Iterators`. The simplest method is `reduce`, which combines a collection of objects into a single object. Here is the code:

```
public static<E> E reduce(E baseElement,
                          ReduceStrategy<E> strategy,
                          Collection <E> c){
    for (E element : c)
        baseElement = strategy.combine(baseElement, element);
    return baseElement;
}
```

As you can see, the implementer has no knowledge of the particular operation being used to combine the results, other than that its target object (here named `strategy`) implements the `ReduceStrategy` interface. In this respect, the method call of `strategy.combine` is like the call of a formal function parameter in a higher-order function.

The `map` method is slightly more complicated, because it must create a new collection of the same type as the input collection. Fortunately, the Java methods `getClass()` and `newInstance()` come to our rescue. The first method returns the class of an object, and the second method returns a new instance of this class, using the class's default constructor. The code must be embedded in Java's `try/catch` exception-handling syntax for the error checking required by these methods. The code for `map` follows. The implementation of `filter` is left as an exercise.

```
public static<E, R> Collection<R> map(MapStrategy<E, R> strategy,
                                      Collection<E> c){
    try{
        Collection<R> results = c.getClass().newInstance();
        for (E element : c)
```

(continues)

(continued)

```
            results.add(strategy.transform(element));
        return results;
    }catch(Exception e){
        System.out.println(e);
        return null;
    }
}
```

As mentioned earlier, the user of one of our new iterators is responsible for supplying a collection and an instance of the corresponding strategy interface. The programmer could define a new class that implements `MapStrategy`, `FilterStrategy`, or `ReduceStrategy`, but there is no need to go to that trouble. Java allows the programmer to create an instance of an anonymous class at the point of use, in much the same manner as functional programmers create `lambda` forms. The syntax for this is:

```
new <interface name>(){
    <method implementations>
}
```

Table 5.4 shows the `MapStrategy` interface and an example instantiation. The instantiation transforms an integer value into a double value by taking the square root of the integer.

Table 5.4 The `MapStrategy` interface and an instantiation

Interface	Instantiation

`public interface MapStrategy<E, R>{`
`  public R transform(E element);`
`}`
	`new MapStrategy<Integer, Double>(){`
	`  public Double transform(Integer i){`
	`  return Math.sqrt(i);`
	`  }`
	`}`

The next code segment shows a complete application that takes our new iterators for a test drive. Note that two different particular mapping strategies are used, and then one filtering strategy and one reducing strategy. The same code could be used on a set of integers or on any other collection class (like the `LinkedStack` explored in Section 5.3.2) of integers.

```
import iterators.*;
import java.util.*;

/*
Tests the map, filter, and reduce iterators with array lists.
Creates strategy objects as anonymous instances of the appropriate interfaces.
*/

public class TestIterators{

    static public void main(String[] args){
```

(continues)

(continued)

```
        // The list contains [1 2 3 4 5 6 7 8]
        List<Integer> list = new ArrayList<Integer>();
        for (int i = 1; i <= 8; i++)
            list.add(i);

        // Compute the square roots
        Collection<Double> squareRoots =
            Iterators.map(new MapStrategy<Integer, Double>(){
                public Double transform(Integer i){
                    return Math.sqrt(i);
                }
            }, list);
        System.out.println(squareRoots);

        // Convert to string representations
        Collection<String> stringReps =
            Iterators.map(new MapStrategy<Integer, String>(){
                public String transform(Integer i){
                    return i + "";
                }
            }, list);
        System.out.println(stringReps);

        // Keep the even numbers
        Collection<Integer> evens =
            Iterators.filter(new FilterStrategy<Integer>(){
                public boolean accept(Integer i){
                    return i % 2 == 0;
                }
            }, list);
        System.out.println(evens);

        // Sum the numbers
        int sum = Iterators.reduce(0, new ReduceStrategy<Integer>(){
            public Integer combine(Integer first, Integer second){
                return first + second;
            }
        }, list);
        System.out.println(sum);
    }
}
```

How do our Java versions of map, filter, and reduce stack up against their counterparts in functional languages and in Smalltalk? The functional versions are simple, because they are functions that accept other functions as arguments. There is nothing new to learn here. However, map, filter, and reduce are limited to list collections. The Smalltalk versions are more general, because they are polymorphic over any collections. The syntax of a Smalltalk block is really no more complicated than that of a lambda form. The syntax of the Java versions is slightly more complicated, but much of that complexity can be viewed as boilerplate for the essential equivalent of a lambda form or anonymous block of code. The real benefit of the Java versions, as compared to those of Smalltalk, lies in the static type checking. As always, type errors (the use of a noncollection object or a nonstrategy object) are caught early, at compile time, and all the errors are always caught. This makes the use of map, filter, and reduce virtually foolproof, if a bit cumbersome, in Java.

5.4 C++

C++ was originally developed by Bjarne Stroustrup at AT&T Bell Labs (where C was developed) as an extension of C with Simula-like classes, but in the late 1980s and 1990s it grew well beyond its origins into a language containing a large number of features, some object oriented, some not. It is a compromise language in that it contains most of the C language as a subset, as Simula contains Algol60 as a subset. C++ was also designed to be an efficient, practical language, developing out of the needs of its users. As such it became the primary object-oriented language of the early 1990s and remains extremely popular for non-Web applications. Since the first reference manual was published in 1986, many new features have been added, including multiple inheritance, templates, operator overloading, and exceptions. The language has now stabilized since the adoption of an international standard in 1998 (C++98) and as revised in 2003 (C++03). Work is in progress on a new standard (C++0x), whose publication is due near the time this book will be printed.

5.4.1 Basic Elements of C++: Classes, Data Members, and Member Functions

C++ contains class and object declarations similar to those of Java. Instance variables and methods are both called **members** of a class: Instance variables are referred to as **data members**, and methods are referred to as **member functions**. Subclasses are referred to as **derived classes** and superclasses are called **base classes**. One basic difference between C++ and most other object-oriented languages, including Java, is that objects are not automatically pointers or references. Indeed, except for inheritance and dynamic binding of member functions, the `class` data type in C++ is essentially identical to the `struct` (or record) data type of C.

Similar to Java, three levels of protection are provided for class members in C++: public, private, and protected. **Public** members are those accessible to client code, as well as to derived classes. **Protected** members are inaccessible to client code but are still accessible to derived classes. **Private** members are accessible neither to clients nor to derived classes. In a `class` declaration, the default protection is private, while in a `struct` declaration, the default is public. Additionally, the keywords `private`, `public`, and `protected` establish blocks in class declarations, rather than apply only to individual member declarations as in Java:

```
class A{

// A C++ class

    public:

    // all public members here

    protected:

    // all protected members here

    private:

    // all private members here

};
```

(Note the final semicolon after the closing bracket, which is a vestige of C syntax; a list of variable names could appear before this semicolon, with the preceding class definition as their data type.)

In C++, initialization of objects is given as in Java by **constructors** that have the same name as that of the class. Constructors are automatically called when an object is allocated space in memory, and actual parameters to the constructor are evaluated at that time. Thus, constructors are similar to Java, except that in C++ objects need not be references, and so a constructor can be automatically called as part of a declaration, as well as in a `new` expression. Unlike Java, C++ does not have required built-in garbage collection, and so C++ also includes **destructors**, which like constructors have the same name as the class, but which are preceded by the tilde symbol "~" (sometimes used in logic to mean "not"). Destructors are automatically called when an object is deallocated (either by going out of scope or through the use of the `delete` operator), but they must be provided by the programmer and so represent a manual approach to memory deallocation.

A class declaration in C++ does not always contain the implementation code for all member functions. Member functions can be implemented outside the declaration by using the **scope resolution operator** indicated by a double colon "`::`" after the class name. It gets its name from the fact that it causes the scope of the class declaration to be reentered. Member functions that are given implementations in a class declaration are automatically assumed to be **inline** (that is, a compiler may replace the function call by the actual code for the function).

As a first example in C++, we offer a partial definition of a complex number class:

```
class Complex{

    public:

    // Constructor
    Complex(double r = 0, double i = 0)
    : re(r), im(i) { }
```

(continues)

(continued)

```
    double imaginarypart(){
        return im;
    }

    Complex operator-(){ // unary minus
        return Complex( -re, -im);
    }

    Complex operator+(Complex y){
        return Complex(re + y.realpart(),
                       im + y.imaginarypart());
    }

    Complex operator-(Complex y){ // subtraction
        return Complex(re - y.realpart(),
                       im - y.imaginarypart());
    }

    Complex operator*(Complex y){
    return Complex(re * y.realpart() - im * y.imaginarypart(),
                   im * y.realpart() + re * y.imaginarypart());
    }

    private:

    double re, im;
};
```

This example uses operator overloading in C++, so that complex arithmetic can be written exactly like built-in arithmetic, for example:

```
Complex z(1,2);
Complex w(-1,1);
Complex x = z + w;
```

The syntax for instance variable initialization in a C++ constructor is somewhat unusual. The instance variable names are listed after a colon in a comma-separated list between the constructor declaration and body, with the initial values in parentheses after each instance name. The reason for this notation in C++ is efficiency: C++ requires that all instance variables be initialized *prior* to the execution of the body of any constructor. Thus, initialization would be performed *twice* if it were expressed as regular code inside the body of the constructor (once by the system and once by the actual code). That is, the `Complex` constructor declaration in this example:

```
Complex(double r = 0, double i = 0)
: re(r), im(i) { }
```

means that `re` is initialized to the value of `r` and `im` is initialized to the value of `i`. In other words, the above code has essentially the same meaning as:

```
Complex(double r = 0, double i = 0){
    re = r; im = i;
}
```

The constructor is also defined with default values given for its parameters:

```
Complex(double r = 0, double i = 0)
```

This allows `Complex` objects to be created with 0, 1, or 2 parameters, and avoids the need to create different overloaded constructors:

```
Complex i(0,1); // i constructed as (0,1)
Complex y; // y constructed as default (0,0)
Complex x(1); // x constructed as (1,0)
```

5.4.2 Using a Template Class to Implement a Generic Collection

You saw how a generic collection class is defined in Java in Section 5.3.2. A similar feature, called **template classes**, is used to define generic collections in C++. Although the standard template library (STL) of C++ includes several built-in collection classes, let's develop a `LinkedStack` class to see how a C++ template class works.

Our C++ `LinkedStack` class has the same basic interface as the Java version discussed in Section 5.3.2. Here is a short C++ program that exercises some of its capabilities:

```
#include <iostream>
using std::cout;
using std::endl;
#include "LinkedStack.c"

int main(){
    int number;
    LinkedStack<int> stack;
    for (number = 1; number <= 5; number++)
        stack.push(number);
    cout << "The size is " << stack.size() << endl;
    while (stack.size() != 0)
        cout << stack.pop() << " ";
    cout << endl;
}
```

As you can see, this code declares a stack type in a similar manner to the Java version, although in this case the `int` type can be specified directly as the stack's element type. However, unlike in Java, a new `LinkedStack` object is automatically created and assigned to the `stack` variable upon the mere mention of that variable in the declaration. Otherwise, the usage of `LinkedStack` is exactly the same as in the Java example.

Our Java version of `LinkedStack` exploited inheritance and polymorphism by extending an `AbstractionCollection` class, which provided higher-level functionality and an interface to other collection classes. Lacking this class in C++, we define `LinkedStack` as a top-level class. Our C++ version includes a constructor, a destructor (more on this in a moment), the public stack instance methods, a nested node type, and a helper method to allocate and initialize a new node. Here is the code for the class:

```
template <class E> class LinkedStack{

    public:

    // Constructor
    LinkedStack()
    : top(0), mySize(0) {}

    // Destructor
    ~LinkedStack(){
        while (size() > 0)
            pop();
    }

    E peek(){
        return top.data;
    }

    E pop(){
        E element = top->data;
        node * garbage = top;
        top = top->next;
        mySize -= 1;
        delete garbage;     // Must return node storage to the heap
        return element;
    }

    void push(E element){
        top = getNode(element, top);
        mySize += 1;
    }

    int size(){
        return mySize;
    }

    private:

    // The node type for LinkedStack
    struct node{
```

(continues)

(continued)

```
        E data;
        node* next;
    };

    // Data members for LinkedStack
    node* top;
    int mySize;

    // Helper method to initialize a new node
    node* getNode(E data, node * next){
        node * newNode = new node;
        newNode->data = data;
        newNode->next = next;
        return newNode;
    }

};
```

The notation `template <class E>` in the class header introduces the type parameter `E` for the stack's element type. This parameter will be filled in with an actual C++ type in the client program's code at compile time. The constructor and public instance methods are similar to those of the `Complex` class discussed earlier.

However, because the `LinkedStack` class utilizes dynamic storage for its nodes, it should also include a destructor. The job of this method is to return each node to the system heap when the `LinkedStack` object is no longer accessible from the client program (for example, when a function that creates a temporary stack returns from its call). The destructor accomplishes this by popping the stack until it is empty. The `pop` method, in turn, manually returns the node just removed from the linked structure to the system heap, by using the `delete` operator. Because C++ has no automatic garbage collection, failure to recycle storage in this manner constitutes a memory leak, which can lead to memory problems in production situations.

Note also that the `node` type is a C-style `struct`, but could have been defined as a C++ class instead. Because a node is just a private container for a datum and a link, the use of a `struct` and a private method `getNode` that builds and returns a pointer to the initialized node works just as well.

Finally, the use of explicit C-style pointers rather than references illustrates another difference between Java and C++. Even if we had defined a `Node` class in C++, we would still have had to use the `*` operator to indicate a link to the next node and a pointer variable to the node itself.

5.4.3 Static Binding, Dynamic Binding, and Virtual Functions

Dynamic binding of member functions is provided as an option in C++, but it is not the default. All member functions are statically bound, at compile time. Only functions defined using the keyword `virtual` are candidates for dynamic binding. For example, if we want to redefine an `area` function for a square derived from a rectangle, we would do so as follows in C++:

```
class Rectangle{
    public:

    virtual double area(){
        return length * width;
    }

    ...

    private:

    double length,width;

};

class Square : public Rectangle{

    public:

    double area(){         // redefines rectangle::area dynamically
        return width * width;
    }

  ...
};
```

Of course, what we really want in this example is an abstract `area` function in an abstract class `ClosedFigure`. This can be achieved in C++ by the use of a so-called **pure virtual declaration**:

```
class ClosedFigure{         // an abstract class

    public:

    virtual double area() = 0; // pure virtual

    ...
};

class Rectangle : public ClosedFigure{

    public:

    double area(){
       return length * width; }
```

(continues)

(continued)

```
    ...

    private:

    double length,width;
};

class Circle : public ClosedFigure{

    public:

    double area(){
        return PI*radius*radius; }

    ...

    private

    double radius;
};
```

The 0 in a virtual function declaration indicates that the function is null at that point, that is, has no body. Thus, the function cannot be called, hence is abstract, and also renders the containing class abstract. Pure virtual functions must be declared `virtual`, since they must be dynamically bound during execution and overriden in a derived class.

Note in the preceding code that the overriding definitions of area in `Circle` and `Rectangle` do not repeat the keyword `virtual` (although it is permissible to do so). This is because, once a function is declared as `virtual`, it remains so in all derived classes in C++.

Returning to the issue of dynamic binding in C++, it is not sufficient to declare methods as `virtual` to enable dynamic binding; in addition, objects themselves must be either dynamically allocated or otherwise accessed through a reference. To see this in action, consider the following code:

```
(1) class A
(2) {
(3) public:
(4)   void p()
(5)   { cout << "A::p\n" ;
(6)   }

(7)   virtual void q()
(8)   { cout << "A::q\n" ;
(9)   }
```

(continues)

(continued)

```
(10)   void f()
(11)   { p();
(12)     q();
(13)   }
(14) };

(15) class B : public A
(16) {
(17) public:
(18)   void p()
(19)   { cout << "B::p\n" ;
(20)   }
(21)   void q()
(22)   { cout << "B::q\n" ;
(23)   }
(24) };

(25) int main()
(26) { A a;
(27)   B b;
(28)   a.f();
(29)   b.f();
(30)   a = b;
(31)   a.f();
(32) }
```

The somewhat surprising output from this program is:

```
A::p
A::q
A::p
B::q
A::p
A::q
```

In the call `b.f()` (line 29) the inherited function `A.f` is called. Inside this function in `A` are calls to `p` and `q` (lines 11 and 12). Since `p` is not virtual, dynamic binding does not apply, and this call is resolved at compile time to `A::p()`. On the other hand, the call to `q` *is* virtual, and the implicit object parameter is used to dynamically resolve the call during execution. Thus, the call to `q` is implicitly a call `this->q()`, and in the call `b.f()`, `this` is the address of `b`, so `B::q` appears on the fourth line of the above output. On the other hand, the assignment `a = b` (line 30) simply copies the data of `b` to `a`, and does not change the address of `a` or the `this` pointer in the call to `a.f()` (line 31). Thus, `A::p` and `A::q` are again called.

If, on the other hand, we had changed the main program above to:

```
(33) int main(){
(34)     A* a = new A;
(35)     B* b = new B;
(36)     a->f();
(37)     b->f();
(38)     a = b;
(39)     a->f();
(40)     return 0;
(41) }
```

we would see the output:

```
A::p
A::q
A::p
B::q
A::p
B::q
```

since now the assignment `a = b` (line 38) copies a pointer and changes this pointer in the call to `a->f()` to point to `b`'s object (which is a `B`) instead of the previous object pointed to by `a` (which was an `A`).

C++ offers multiple inheritance using a comma-separated list of base classes, as in

```
class C : public A, private B {...};
```

Multiple inheritance in C++ ordinarily creates *separate* copies of each class on an inheritance path. For example, the declarations:

```
class A {...};
class B : public A {...};
class C : public A {...};
class D : public B, public C {...};
```

provide any object of class `D` with *two* separate copies of objects of class `A`, and so create the inheritance graph shown in Figure 5.10.

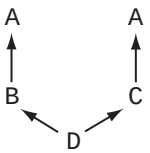


Figure 5.10: Inheritance graph showing multiple inheritance

This is an example of **repeated inheritance**. To get a single copy of an `A` in class `D`, one must declare the inheritance using the `virtual` keyword:

```
class A {...};
class B : virtual public A {...};
class C : virtual public A {...};
class D : public B, public C {...};
```

This achieves the inheritance graph shown in Figure 5.11.

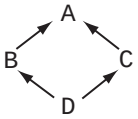


Figure 5.11: Inheritance graph showing shared inheritance

This is an example of **shared inheritance**. Resolution of method conflicts created by multiple (shared) inheritance is explored in Exercise 5.16.

5.5 Design Issues in Object-Oriented Languages

In this section, we survey a few of the issues surrounding the introduction of object-oriented features into a programming language and summarize the different approaches taken in the three languages we have studied.

Object-oriented features represent at their heart dynamic rather than static capabilities (such as the dynamic binding of methods to objects), so one aspect of the design of object-oriented languages is to introduce features in such a way as to reduce the runtime penalty of the extra flexibility. In a dynamically oriented (and usually interpreted) language like Smalltalk, this is less important, but in languages like C++, and to a certain extent in Java, the runtime penalty of their object-oriented features over the non-object-oriented features of their cousins (such as C and Algol) is an important aspect of their overall design. In particular, C++ was designed with efficiency as a primary goal.

One feature promoting efficiency has already been mentioned in the section on C++: Member functions whose implementations are included in the class definition are automatically inline functions in C++. This avoids the penalty of function call during execution.

Other issues that arise in connection with the efficient design of object-oriented languages involve the proper organization of the runtime environment as well as the ability of a translator to discover optimizations. Discussion of such implementation issues is delayed until the next section.

We note also that this chapter discusses design issues only as they relate to overall language design, not program design. Designing object-oriented programs in a way that takes maximum advantage of an object-oriented language involves complex software engineering issues and is outside the scope of this book. Several of the references at the end of the chapter contain more information on object-oriented software design.

5.5.1 Classes vs. Types

The introduction of classes into a language with data types means that classes must be incorporated in some way into the type system. There are several possibilities, including the following three:

1. Classes could be specifically excluded from type checking. Objects would then become typeless entities, whose class membership would be maintained separately from the type system—generally at runtime using a tagging mechanism. This method of adding classes to an existing language is the simplest and has been used in a few languages, such as Objective-C (see the references at the end of the chapter).
2. Classes could become type constructors, thus making class declarations a part of the language type system. This is the solution adopted by C++ and a number of other languages that add object-oriented facilities to an existing language. In C++ a class is just a different form of record, or structure, type. In this case, the assignment compatibility of objects of descendant classes to objects of ancestor classes must be incorporated into the type-checking algorithms. Specifically, if `x` is an object of a descendant class of the class of object `y`, then type checking must permit the assignment `y = x` but must flag the assignment `x = y` as an error. This complicates the type-checking algorithm.
3. Classes can also simply *become* the type system. That is, all other structured types are excluded from the language. This is the approach of many languages designed from the beginning as object-oriented, such as Smalltalk. This is also almost true of Java, where the class and the array are the only two structured types; however, there are still the basic types such as `boolean`, `int`, `double`, `char`, and their variants, and type checking must also involve these types. Using classes as the basis for the type system means that, under static typing, the validity of all object assignments can be checked prior to execution, even though exact class membership is not known until execution time.[1]

5.5.2 Classes vs. Modules

Classes provide a versatile mechanism for organizing code that also encompasses some of the desired features of abstract data type mechanisms: Classes provide a mechanism for specifying interfaces, for localizing code that applies to specific collections of data, and for protecting the implementation through the use of private data. Unfortunately, except in Java, classes do not allow the clean separation of implementation from interface, nor do classes protect the implementation from exposure to client code. Thus, code that uses a class becomes dependent on the particular implementation of the class even if it cannot make use of the details of that implementation.

Another problem with the use of classes alone to structure programs is that classes are only marginally suited for controlling the importing and exporting of names in a fine-grained way. In simple cases, a class can be used as a namespace or scope for a collection of utilities. For example, the Java `Math` class

[1] There are always situations, such as downcasting, where exact class membership must be tested in some way. We mention the Java `instanceof` operation. C++ has a related `dynamic_cast` operation that tests class membership while performing a cast during execution.

consists entirely of static constants and methods and is, therefore, really a namespace masquerading as a class. However, it is not possible to import names from other classes and reexport a selection of them without some effort, so classes do not function well as static namespaces or modules.

For this reason, object-oriented languages often include module mechanisms that are independent of the object-oriented mechanisms. The principle example is C++; it has the namespace mechanism, which is orthogonal to the class mechanism. Java, too, has a package mechanism and interfaces that allow classes to be collected into groups. When defined in a package, classes can be given special names by which their contents can be accessed (typically these names are also associated with a directory structure).

Sometimes object-oriented mechanisms are more closely related to a module structure. For instance, Ada95 introduced a number of object-oriented mechanisms (which we have not studied in this text), and the semantics of declarations using these mechanisms often depend on their relationship to the Ada packages in which they are defined.

Thus, the issue of what kind of module mechanism fits best in an object-oriented setting, and how best to integrate the two notions of class and module, remains, to a certain extent, open.

C++ also has a mechanism that allows a class to open up details of its implementation to another class or function: The other class or function is listed by the class as a `friend`. For example, in a C++ implementation of a complex data type, the problem arises that a binary operation such as addition acquires an unusual syntax when written as a method of class `Complex`, namely,

```
x.operator+(y)
```

(Fortunately, C++ does allow the substitute syntax `x + y`.) An alternative solution, and the only one in some earlier C++ implementations), is to declare the + operation outside the class itself as a `friend` (so that it can access the internal data of each `Complex` object) as follows:

```
class Complex
{
private:
  double re,im;
public:
  friend Complex operator+(Complex,Complex);
  ...
};
Complex operator+(Complex x, Complex y)
{ return Complex (x.re+y.re,x.im+y.im); }
```

5.5.3 Inheritance vs. Polymorphism

There are four basic kinds of polymorphism:

1. Parametric polymorphism, in which type parameters may remain unspecified in declarations. ML type variables, Ada generic packages, C++ templates, and Java generic interfaces and classes are examples of parametric polymorphism.
2. Overloading, or ad hoc polymorphism, in which different function or method declarations share the same name and are disambiguated through the use of the types of the parameters in each declaration (and sometimes the types of the returned values). Overloading is available in C++, Java, Haskell, and Ada, but not C or ML.
3. Subtype polymorphism, in which all the operations of one type can be applied to another type.
4. Inheritance, which is a kind of subtype polymorphism. Inheritance can also be viewed as a kind of overloading on the type of the implicit object parameter (the `this` pointer). However, this form of overloading is restricted in that it overloads only on this one parameter. Thus, most object-oriented languages also provide overloading on the explicit parameters of methods.

The problem with the mixture of inheritance and overloading is that it does not account for binary (or *n*-ary) methods that may need overloading based on class membership in two or more parameters. This is called the **double-dispatch** or **multi-dispatch** problem.

We noted a good example of this problem earlier in this chapter when defining the `Complex` class. Take, for example, complex addition, which can be written in Java as `x.add(y)`, for `x` and `y` `Complex` objects. This can be written even more appropriately as `x.operator+(y)` or `x + y` in C++. Using overloading, this can even be extended to the case where `y` is no longer `Complex` but might be a `double` or an `int`. How might we define `Complex` addition if `x` is a `double` and `y` is a `Complex`? In Java this cannot be done, and in C++ it cannot be done if the `+` operator is defined as a method of the `Complex` class. The way C++ solves this problem is to define a separate, free (overloaded) `+` function taking two `Complex` parameters (see the previous subsection) and then to provide conversions from `int` and `double` to `Complex` (which actually can come simply from the constructors alone).[2]

This has the decidedly unpleasant effect of splitting up the definition of complex numbers into a class definition and a number of free function definitions.

Several attempts have been made to design an object-oriented language that solves this problem via so-called **multimethods**: methods that can belong to more than one class and whose overloaded dispatch can be based on the class membership of several parameters. One of the more successful attempts is the **Common Lisp Object System (CLOS)**. You can consult the references at the end of this chapter for details.

While overloading and inheritance are related concepts, and, except for the multiple dispatch problem, appear to mesh well in languages such as Java and C++, parametric polymorphism is more

[2] Constructors taking a single parameter in C++ are called copy constructors. Such constructors provide automatic conversions in ways that we have not studied here; see the C++ references at the end of the chapter.

difficult to integrate, and C++ and Java remain somewhat solitary examples of languages that attempt an integration of the two concepts. As we have noted, Smalltalk and many other object-oriented languages imitate parametric polymorphism using the based-object approach, but this removes the security of static typing from code that uses this form of polymorphism (a similar style of programming is possible in C++ using `void*` pointers, which also circumvents static type checking).

5.6 Implementation Issues in Object-Oriented Languages

In this section, we discuss a few of the issues encountered when implementing an object-oriented language. We also present several methods that allow object-oriented features to be implemented in an efficient way. Since these are primarily issues of efficient code generation by a compiler, they are less applicable to a dynamic, primarily interpreted language such as Smalltalk.

5.6.1 Implementation of Objects and Methods

Typically, objects are implemented exactly as record structures would be in C or Ada, with instance variables representing data fields in the structure. For example, an object of the following class (using Java syntax),

```
class Point{
    ...
    public void moveTo(double dx, double dy){
        x += dx;
        y += dy;
    }
    ...
    private double x,y;
}
```

could be allocated space for its instance variables `x` and `y` just like the structure:

```
struct{
    double x;
    double y;
};
```

by allocating space for `x` and `y` sequentially in memory, as shown in Figure 5.12.

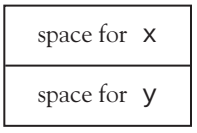


Figure 5.12: Space allocated for a C struct

An object of a subclass can be allocated as an extension of the preceding data object, with new instance variables allocated space at the end of the record. For example, given the declaration (again in Java syntax):

```
class ColoredPoint extends Point{

    ...

    private Color color;
}
```

an object of class `ColoredPoint` could be allocated as shown in Figure 5.13.

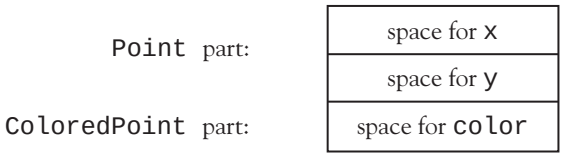


Figure 5.13: Allocation of space for an object of class `ColoredPoint`

Now the instance variables `x` and `y` inherited by any `ColoredPoint` object from `Point` can be found at the same offset from the beginning of its allocated space as for any point object. Thus, the location of `x` and `y` can be determined statically, even without knowing the exact class of an object; you just need to know that it belongs to a descendant class of `Point`.

Methods can also be implemented as ordinary functions. Methods do differ from functions in two basic ways. First, a method can directly access the data of the current object of the class. For example, the `moveTo` method of class `Point` changes the internal state of a point by assigning to the instance variables `x` and `y`. To implement this mechanism, we simply view a method as a function with an implicit extra parameter, which is a pointer that is set to the current instance at each call. For example, the `moveTo` method of class `Point` given can be implicitly declared as follows:

```
procedure moveTo(Point* p, double dx, double dy);
```

In this case, each call `p.moveTo(dx,dy)` would be interpreted by the translator as `moveTo(p,dx,dy)`. Note that, as before, the instance variables of the object pointed to by `p` can be found by static offset from `p`, regardless of the actual class membership of the object at the time of execution.

A more significant problem arises with the dynamic binding of methods during execution, since this depends on the class membership of the current object when the call is made. This problem is discussed next.

5.6.2 Inheritance and Dynamic Binding

In the implementation of objects that we have described, only space for instance variables is allocated with each object; allocation of methods is not provided for. This works as long as methods are completely equivalent to functions, and the location of each is known at compile time, as is the case for ordinary functions in an imperative language.

A problem arises with this implementation when dynamic binding is used for methods, since the precise method to use for a call is not known except during execution. A possible solution is to keep all dynamically bound methods[3] as extra fields directly in the structures allocated for each object. Runtime information on the class structure can still be maintained efficiently using a statically constructed representation of the class hierarchy, typically some kind of table, with an extra pointer to give access to the appropriate position in the table. If it is possible to create classes dynamically, then this table must be dynamically maintained, but it is still possible to make this reasonably efficient using hash tables.

5.6.3 Allocation and Initialization

Object-oriented languages such as C++ can maintain a runtime environment in the traditional stack/heap fashion of C, as will be described in Chapter 10. Within such an environment, it is possible to allocate objects on either the stack or the heap. However, because objects are generally dynamic in nature, it is more common for them to be allocated on the heap than the stack. Indeed, that is the approach adopted in Java and Smalltalk.

C++, by contrast, permits an object to be allocated either directly on the stack or as a pointer. In the case of stack allocation, the compiler is responsible for the automatic generation of constructor calls on entry into a block and destructor calls on exit, while in the case of heap allocation of pointers, explicit calls to `new` and `delete` cause constructor and destructor calls to be scheduled. This means that C++ can maintain exactly the same runtime environment as C, but with the added burden to the compiler that it must schedule the constructor and destructor calls, as well as create and maintain locations for temporary objects.

Smalltalk and Java, on the other hand, have no explicit deallocation routines, but require the use of a garbage collector to return inaccessible storage to the heap. The execution overhead and added runtime complexity of such a requirement are significant, but techniques such as generational garbage collection can be used to reduce the penalty. See Chapter 10 for a further discussion.

During the execution of initialization code, it is necessary to execute the initialization code of an ancestor class before current initializations are performed. A simple example of this arises when instance variables defined in an ancestor need to be allocated space before they can be used in the initialization of a descendant. In C++ and Java, the calls to initialization code of ancestors are scheduled automatically by a compiler. In C++, where multiple inheritance can create several different paths from a descendant class to an ancestor class, initialization code is performed according to the topological sort of the dependency graph determined by the order in which parent classes are listed in each derived class declaration.

In addition to the question of the order of initializations, especially in a language with multiple inheritance, problems can arise related to the existence of appropriate constructors when they are to be supplied automatically. For example, in Java, when there is no default constructor for a superclass, an explicit constructor call must be supplied in all constructors of subclasses.

[3] Recall that *all* methods are (potentially) dynamically bound in Java and Smalltalk, but not in C++.

Exercises

5.1 Obtain an implementation of Smalltalk and enter the `Complex` class discussed in Section 5.2, and then add methods to add and subtract complex numbers (you just compute the sums and differences of the real parts and the imaginary parts, respectively).

5.2 Smalltalk's `Integer` class includes a `gcd:` method that uses a loop to compute the greatest common divisor of two integers. Add a new method named `recursiveGcd:` to the `Integer` class. This method should compute the greatest common divisor of two integers using a recursive strategy. Note that Smalltalk uses the `\\` operator for integer remainder.

5.3 Add an `ArrayQueue` class to Smalltalk's `Collection` hierarchy. This class should have the same functionality as the `LinkedQueue` class developed in Section 5.2. However, the new class should use an `OrderedCollection` object to contain the elements.

5.4 Implement a `LinkedPriorityQueue` class in Smalltalk. The objects added to this type of queue must recognize the <, =, and > operators. The objects are ranked in the queue from smallest (at the front) to largest (at the rear). If two objects are equal, they are inserted in standard FIFO queue order. Your implementation should make maximum use of the `LinkedQueue` class developed in Section 5.2.

5.5 Compare and contrast the types of problems for which functional programming languages and object-oriented programming languages are best suited, using at least two example problems.

5.6 The designers of Java distinguish the primitive types (scalars) from the reference types (all other types of values) in the language. Discuss the costs and benefits to the programmer of this decision.

5.7 In the design of collection classes, some collections are subclasses of other collections (the "is-a" relation), whereas other collections contain or use other collections in their implementations (the "has-a" relation). Pick an example of each type of design and explain why the design decision is preferable to the alternative.

5.8 In Smalltalk, the `Dictionary` class is a subclass of the `Set` class, whereas in Java, the `Map` classes are not included in the `Collection` class hierarchy as subclasses. Compare and contrast the costs and benefits of these two design decisions.

5.9 Describe the difference between instance methods and class methods in Smalltalk and give an example of each.

5.10 A class variable is visible to and shared by all instances of a class. How would such a variable be used in an application?

5.11 What is a constructor in Java? How can constructors be overloaded to support code reuse?

5.12 A static method in Java is similar to a class method in Smalltalk (see Section 5.3.4 for an example). Define a static method `getInstance` in the `Complex` class developed in Section 5.3. This method expects two floating-point numbers as arguments and returns a new complex number containing these values. Then, write a code segment that shows how this method might be used.

5.13 Explain the difference between the == and = operators in Smalltalk. How do they illustrate reference semantics?

5.14 Describe the difference between abstract classes and concrete classes, giving an example of each.

5.15 Explain why the methods `equals` and `toString` are defined in Java's `Object` class.

5.16 Explain why a programmer might include the Java methods `equals` and `toString` when defining a new class.

5.17 Complete the implementation of the Java `filter` method in the `Iterators` class discussed in Section 5.3.4, and then run the example application (after adding the other resources developed in that section).

5.18 Describe how dynamic binding of methods in an object-oriented language works, with a specific example from Smalltalk, Java, or C++.

5.19 When is static binding of methods possible in an object-oriented language? Give specific examples from Java and C++.

5.20 Compare and contrast how data are encapsulated and information is hidden in Smalltalk, Java, and C++.

5.21 Explain the difference between a class and an interface in Java, using at least one example.

5.22 Study the `Comparable` interface in Sun's Java documentation, and modify the `Complex` class developed in Section 5.3 so that it implements this interface. Explain why this move will enable a programmer to sort arrays or lists of `Complex` numbers using methods in Java's `Arrays` or `Collections` classes.

5.23 Java and C++ support generic collections with type parameters, whereas Smalltalk does not. Does that fact place the Smalltalk programmer at a disadvantage? Why or why not?

5.24 The class diagrams of Figures 5.6 and 5.7 show Java's `List` interface and its implementing classes. In Section 5.3, we developed a `LinkedStack` class as a subclass of `AbstractCollection`. An alternative implementation, called `ArrayStack`, includes the same methods as `LinkedStack`. Draw a class diagram that places these two classes and a new interface called `TrueStack` in the appropriate places in the hierarchy. (*Hints*: The `TrueStack` interface should extend the `Collection` interface. The stack classes should extend the `AbstractCollection` class and also implement the `TrueStack` interface.)

5.25 Modify the `LinkedStack` class and implement the `ArrayStack` class and the `TrueStack` interface mentioned in the previous exercise.

5.26 List the types of iterators supported in Smalltalk and explain how they work.

5.27 The enhanced `for` loop in Java depends on an iterator. Explain how this works.

5.28 Maps and filters exist in functional languages and in Smalltalk. Explain how these software constructs work in both contexts.

5.29 If `x` is an object of class `A`, and `y` is an object of a class `B` that is a subclass of `A`, after the assignment `x = y`, why is `x.m()` illegal when `m` is a method of `B` but not `A`?

5.30 Describe the two meanings of the keyword **virtual** in C++ discussed in the text.

5.31 What is the difference between a type and a class?

5.32 Some have claimed that deep inheritance hierarchies make system maintenance more difficult, not easier. Explain how that could be true.

5.33 A `print` operation is a good candidate for a dynamically bound method. Why?

5.34 Given the following code in Java:

```
class A{

    public void p(){
        System.out.println("A.p");
    }

    public void q(){
        System.out.println("A.q");
    }

    public void r(){
        p();
        q();
    }
}

class B extends A{

    public void p(){
        System.out.println("B.p");
    }
}

class C extends B{

    public void q(){
        System.out.println("C.q");
    }

    public void r(){
        q();
        p();
    }
}
...
A a = new B();
a.r();
a = new C();
a.r();
```

What does this code print? Why?

5.35 Given the following code in C++:

```
class A{

    public:

    virtual void p(){
        cout << "A.p" << endl;
    }

    void q(){
        cout << "A.q" << endl ;
    }

    virtual void r(){
        p();
        q();
    }
};

class B : public A{

    public:

    void p(){
        cout << "B.p" << endl;
    }
};

class C : public B{

    public:

    void q(){
        cout << "C.q" << endl;
    }

    void r(){
        q();
        p();
    }
};
```

(continues)

(continued)

```
...
A a;
C c;
a = c;
a.r();
A* ap = new B;
ap->r();
ap = new C;
ap->r();
```

What does this code print? Why?

5.36 Explain in detail why multiple interface inheritance in Java does not suffer from the same problems as multiple class inheritance in C++.

5.37 Meyer [1997] distinguishes overloading from parametric polymorphism as follows (Ibid., p. 38): "Overloading is a facility for client programmers: it makes it possible to write the same client code when using different implementations of a data structure provided by different modules. Genericity [parameterization] is for module implementors: it allows them to write the same module code to describe all instances of the same implementation of a data structure, applied to various types of objects." Discuss this view of polymorphism.

Notes and References

A good general reference for object-oriented programming techniques is Meyer [1997], which uses the interesting object-oriented language Eiffel, which you have not studied in this chapter. Budd [1997] and Booch [1994] describe object-oriented principles in a language-independent way. Bruce [2002] provides a principled examination of types and classes. Java, of course, has many references, but two that emphasize object-oriented principles are Arnold et. al [2000] and Horstmann [2006]. C++ is described in Stroustrup [1997] and Lippman and Lajoie [1998]. Books on Smalltalk proper include Lalonde [1994], Lewis [1995], Sharp [1997], and Lambert and Osborne [1997].

Other interesting object-oriented languages that are not studied here are described in the following references. The Dylan language is described in Feinberg et al. [1996]. Eiffel is a carefully designed language with a Pascal-like syntax (Meyer [1997]). Modula-3 is a Modula-2 derivative with object-oriented extensions (Cardelli et al. [1989], Nelson [1991], and Cardelli et al. [1992]). Objective C is an extension of C with object-oriented features designed by Cox [1984, 1986]. An object-oriented extension of Pascal is described in Tesler [1985]. Object-oriented versions of LISP include CLOS (Gabriel, White, and Bobrow [1991]; Bobrow et al. [1988]); Loops (Bobrow and Stetik [1983]); and Flavors (Moon [1986]).

Information on the use of C++ templates and their interaction with object-oriented techniques can be found in Alexandrescu [2001], Koenig and Moo [1996], Josuttis [1999], and Stroustrup [1997].

Many research papers, as well as reports on object-oriented programming techniques, have appeared in OOPSLA [1986ff]. An interesting application of object-oriented techniques to the sieve of Eratosthenes in C++ is given in Sethi [1996].

CHAPTER

Syntax

CHAPTER 6

Syntax is the structure of a language. In Chapter 1, we noted the difference between the syntax and semantics of a programming language. In the early days of programming languages, both the syntax and semantics of a language were described by lengthy English explanations and many examples. While the semantics of a language are still usually described in English, one of the great advances in programming languages has been the development of a formal system for describing syntax that is now almost universally in use. In the 1950s, Noam Chomsky developed the idea of context-free grammars, and John Backus, with contributions by Peter Naur, developed a notational system for describing these grammars that was used for the first time to describe the syntax of Algol60. These **Backus-Naur forms—BNFs** for short—have subsequently been used in the definition of many programming languages, including C, Java, and Ada. Indeed, every modern programmer and computer scientist needs to know how to read, interpret, and apply BNF descriptions of language syntax. BNFs occur with minor textual variations in three basic forms: original BNF; extended BNF (EBNF), popularized by Niklaus Wirth (and very briefly used to describe the syntax of Scheme in Chapter 3); and syntax diagrams.

In Section 6.1, we briefly look at lexical structures, which are the building blocks on which syntax is often based. We also explore the process of scanning, or recognizing, lexical structure. In Section 6.2, we introduce context-free grammars and their description in BNF. In Section 6.3, we describe the representation of syntactic structure using trees. In Section 6.4, we consider a few of the issues that arise in constructing BNFs for a programming language. EBNFs and syntax diagrams are introduced in Section 6.5. In Section 6.6, we discuss the basic technique of recursive-descent parsing and its close relationship to EBNF and syntax diagrams and briefly look at YACC/Bison, a standard tool for analyzing grammars and constructing parsers. In Section 6.7, we examine the sometimes hazy boundaries among the lexical, syntactic, and semantic structures of a programming language. Finally, Section 6.8 puts some of these ideas to work by developing a syntax analyzer for a small programming language.

6.1 Lexical Structure of Programming Languages

The **lexical structure** of a programming language is the structure of its **tokens**, or words. A language's lexical structure and its syntactic structure are two different things, but they are closely related. Typically, during its **scanning** phase, a translator collects sequences of characters from the input program and forms them into tokens. During its **parsing** phase, the translator then processes these tokens, thereby determining the program's syntactic structure.

Tokens generally fall into several distinct categories. Typical token categories include the following:

- **reserved words**, sometimes called **keywords**, such as `if` and `while`
- **literals** or **constants**, such as `42` (a numeric literal) or `"hello"` (a string literal)
- **special symbols**, such as "`;`", "`<=`", or "`+`"
- **identifiers**, such as `x24, monthly_balance`, or `putchar`

Reserved words are so named because an identifier cannot have the same character string as a reserved word. Thus, in C, the following variable declaration is illegal because `if` is a reserved word:

```
double if;
```

Sometimes programmers are confused about the difference between reserved words and **predefined identifiers**, or identifiers that have been given an initial meaning for all programs in the language but that are capable of redefinition. (To add to the confusion, sometimes these identifiers are also called keywords.) Examples of predefined identifiers are all standard data types in Ada, such as `integer` and `boolean`, and in Python, such as `round` and `print`. See Section 6.7 for more on this phenomenon.

In some older languages identifiers had a fixed maximum size, while in most contemporary languages identifiers can have arbitrary length. Occasionally, even when arbitrary-length identifiers are allowed, only the first six or eight characters are guaranteed to be significant. (You can see how this can be a source of confusion for programmers.)

A problem arises in determining the end of a variable-length token such as an identifier and in distinguishing identifiers from reserved words. As an example, the sequence of characters:

```
doif
```

in a program could be either the two reserved words `do` and `if` or the identifier `doif`. Similarly, the string `x12` could be a single identifier or the identifier `x` and the numeric constant 12. To eliminate this ambiguity, it is a standard convention to use the **principle of longest substring** in determining tokens (sometimes called the principle of "maximum munch"). The principle works like this: At each point, the longest possible string of nonblank characters is collected into a single token. This means that `doif` and `x12` are always identifiers. It also means that intervening characters, even blanks, can make a difference. Thus, in most languages:

```
do if
```

is not an identifier; instead, it is the two reserved words `do` and `if`.

The format of a program can affect the way tokens are recognized. For example, as you just saw, the principle of longest substring requires that certain tokens be separated by **token delimiters** or **white space**. The end of a line of text can be doubly meaningful because it can be white space, but it can also mean the end of a structural entity. Indentation can also be used in a programming language to determine structure. In fact, Python's simple syntax rests on the significance of indentation.

A **free-format** language is one in which format has no effect on the program structure (other than to satisfy the principle of longest substring). Most modern languages are free format, but a few have significant format restrictions. Rarely is a language **fixed format**, in which all tokens must occur in prespecified locations on the page.

Tokens in a programming language are often described in English, but they can also be described formally by **regular expressions**, which are descriptions of patterns of characters. Regular expressions have three basic operations: concatenation, repetition, and choice or selection. Repetition is indicated by the use of an asterisk after the item to be repeated, choice is indicated by a vertical bar between the items from which the selection is to be made, and concatenation is given simply by sequencing the items without an explicit operation. Parentheses are also often included to allow for the grouping of operations. For example, `(a|b)*c` is a regular expression indicating 0 or more repetitions of either the characters `a` or `b` (choice), followed by a single character `c` (concatenation); strings that match this regular expression include `ababaac`, `aac`, `babbc`, and just `c` itself, but not `aaaab` or `bca`.

Regular expression notation is often extended by additional operations and special characters to make them easier to write. For example, square brackets with a hyphen indicate a range of characters, `+` indicates one or more repetitions, `?` indicates an optional item, and a period indicates any character. With these notations, we can write compact regular expressions for even fairly complex tokens. For example,

```
[0-9]+
```

is a regular expression for simple integer constants consisting of one or more digits (characters between 0 and 9). Also, the regular expression:

```
[0-9]+(\.[0-9]+)?
```

describes simple (unsigned) floating-point literals consisting of one or more digits followed by an optional fractional part consisting of a decimal point (with a backslash, or escape, character in front of it to remove the special meaning of a period as matching any character), followed again by one or more digits.

Many utilities use regular expressions in text searches. This is true of most modern text editors, as well as various file search utilities, such as UNIX `grep` ("global regular expression print"). The UNIX `lex` utility can also be used to automatically turn a regular expression description of the tokens of a language into a scanner. (A successor to `lex`, called `flex`, or fast `lex`, is freely available and runs on most operating systems.)

While a study of `lex/flex` and the use of regular expressions to construct scanners is beyond the scope of this text, in simple cases, scanners can be constructed by hand relatively easily. To give you a sense of such a scanner construction, we present C code for a simple scanner in Figure 6.1. There are six tokens in this language (represented as constants of an enumeration type, lines 4–5): integer constants consisting of one or more digits; the special characters `+`, `*`, `(`, and `)`; and the end of line token (represented in C or Java as the special character `'\n'`, or newline). Such a language may be used to specify a simple integer arithmetic expression with addition and multiplication on a single line of input, such as

```
(2 + 34) * 42
```

Blanks are to be skipped, and any character (such as `a` or # or `-`) that is not specified is to be considered an error (thus, the need for the additional `ERROR` token on line 5).

Before closing this section and moving on to the study of syntax and parsing, we note the following additional features of the code in Figure 6.1. First, the code contains a `main` function (lines 31–47) to test drive the scanner (which is represented by the `getToken` function, lines 8–30). Given a line of input, the `main` function simply calls `getToken` repeatedly and prints each token as it is recognized; a number is also printed with its numeric value, and an error is printed with the offending character. Second, we note

that the scanner communicates the additional numeric and character information to the main program via the global variables `numVal` and `currChar` (lines 6 and 7). A more sophisticated design would eliminate these globals and package this information into a data structure containing all token information; see Exercise 6.7.

```
(1)  #include <ctype.h>
(2)  #include <stdio.h>

(3)  /* the tokens as an enumerated type */
(4)  typedef enum {TT_PLUS, TT_TIMES, TT_PAREN, TT_RPAREN,
(5)               TT_EOL,TT_NUMBER,TT_ERROR} TokenType;

(6)  int numVal;   /* computed numeric value of a NUMBER token */
(7)  int currChar; /* current character */

(8)  TokenType getToken(){
(9)      while (currChar == ' ')         /* skip blanks */
(10)         currChar = getchar();
(11)     if (isdigit(currChar)){         /* recognize a NUMBER token */
(12)         numval = 0;
(13)         while (isdigit(currChar)){
(14)             /* compute numeric value */
(15)             numval = 10 * numval + currChar - '0';
(16)             currChar = getchar();
(17)         }
(18)         return TT_NUMBER;
(19)     }
(20)     else{                           /* recognize a special symbol */
(21)         switch (currChar){
(22)             case '(':  return TT_LPAREN; break;
(23)             case ')':  return TT_RPAREN; break;
(24)             case '+':  return TT_PLUS; break;
(25)             case '*':  return TT_TIMES; break;
(26)             case '\n': return TT_EOL; break;
(27)             default:   return TT_ERROR; break;
(28)         }
(29)     }
(30) }
(31) main(){
(32)     TokenType token;
(33)     currChar = getchar();
(34)     do{
(35)         token = getToken();
(36)         switch (token){
(37)             case TT_PLUS:   printf("TT_PLUS\n"); break;
```

Figure 6.1 A scanner for simple integer arithmetic expressions (*continues*)

(*continued*)

```
(38)             case TT_TIMES:  printf("TT_TIMES\n"); break;
(39)             case TT_LPAREN: printf("TT_LPAREN\n"); break;
(40)             case TT_RPAREN: printf("TT_RPAREN\n"); break;
(41)             case TT_EOL:    printf("TT_EOL\n"); break;
(42)             case TT_NUMBER: printf("TT_NUMBER: %d\n", numval); break;
(43)             case TT_ERROR:  printf("TT_ERROR: %c\n", curr_char); break;
(44)         }
(45)     } while (token != TT_EOL);
(46)     return 0;
(47) }
```

Figure 6.1 A scanner for simple integer arithmetic expressions

Given the input line:

```
* + ( ) 42 # 345
```

the program of Figure 6.1 produces the following output:

```
TT_TIMES
TT_PLUS
TT_LPAREN
TT_RPAREN
TT_NUMBER: 42
TT_ERROR: #
TT_NUMBER: 345
TT_EOL
```

Note that no order for the tokens is specified by the scanner. Defining and recognizing an appropriate order is the subject of syntax and parsing, studied in the remainder of this chapter.

6.2 Context-Free Grammars and BNFs

We begin our description of grammars and BNFs with the example in Figure 6.2 (the numbers are for reference purposes).

(1) *sentence* → *noun-phrase verb-phrase* .
(2) *noun-phrase* → *article noun*
(3) *article* → `a | the`
(4) *noun* → `girl | dog`
(5) *verb-phrase* → *verb noun-phrase*
(6) *verb* → `sees | pets`

Figure 6.2 A grammar for simple english sentences

In English, simple sentences consist of a noun phrase and a verb phrase followed by a period. We express this as the grammar rule 1 in Figure 6.2. Rules 2 through 6 describe in turn the structure of noun phrases, verb phrases, and other substructures that appear in previous rules. Each of these grammar rules consists of a name in italics (the name of the phrase being described), followed by an arrow (which can be read as "consists of" or "is defined as,"), followed by a sequence of other names and symbols. The italics serve to distinguish the names of the phrases from the actual words, or tokens, that may appear in the language (which, in our examples, are also indicated in a different typeface). For instance, we would not wish to confuse the keyword `program` in Pascal with the program structure itself:

program → `program` . . .

Typically, of course, we would prefer also to use different names to distinguish phrases from tokens.

The symbol "→" is a **metasymbol** that serves to separate the left-hand side from the right-hand side of a rule. The vertical bar "|" is also a metasymbol and means "or" or choice (similar to its use in regular expressions). Thus, rule 6 above states that a verb is either the word "`sees`" or the word "`pets.`" Sometimes a metasymbol is also an actual symbol in a language. In that case the symbol can be surrounded by quoation marks to distinguish it from the metasymbol, or the metasymbol can be written in a different typeface, or both. Often it is a good idea to do this for special symbols like punctuation marks, even when they are not also metasymbols. For example, rule 1 has a period in it. While a period is not part of any metasymbol described, it can easily be mistaken for one. Hence, it might be better to write the rule as follows:

sentence → *noun-phrase verb-phrase* '.'

In this case, then, the quoation marks also become metasymbols.

Some notations also rely on metasymbols (such as angle brackets) rather than italics or fonts to distinguish phrases from tokens, which can also be useful in situations where formatting is not available (such as in text files or handwritten text). In that case, the arrow is also often replaced by a metasymbol that is also pure text (such as two colons and an equal sign). For example, we might see rule 1 of Figure 6.2 written as follows:

<sentence> ::= <noun-phrase> <verb-phrase> "."

Here the angle brackets also become metasymbols, and double quotation marks have been used instead of single quotation marks to mark the period as a token.

Indeed, there are many different, often personal, preferences in the notational conventions used in writing grammar rules. There is also an ISO standard format for BNF notation (ISO 14977 [1996]), which is almost universally ignored, perhaps because, by the time of its adoption (1996), other conventions such as the above were already entrenched. For example, the grammar rule for a sentence would appear in standard ISO form as:

sentence = noun phrase , verb phrase "." ;

Any legal sentence according to the foregoing grammar can be constructed as follows: We start with the symbol *sentence* (the **start symbol** for the grammar) and proceed to replace left-hand sides by choices of right-hand sides in the foregoing rules. This process is called a **derivation** in the language. Thus, we could construct, or derive, the sentence "`the girl sees a dog.`" as in Figure 6.3, where each step is annotated by the number of the rule from Figure 6.2 that is used in that step.

sentence ⇒ *noun-phrase verb-phrase* . (rule 1)
⇒ *article noun verb-phrase* . (rule 2)
⇒ `the` *noun verb-phrase* . (rule 3)
⇒ `the girl` *verb-phrase* . (rule 4)
⇒ `the girl` *verb noun-phrase* . (rule 5)
⇒ `the girl sees` *noun-phrase* . (rule 6)
⇒ `the girl sees` *article noun* . (rule 2)
⇒ `the girl sees a` *noun* . (rule 3)
⇒ `the girl sees a dog` . (rule 4)

Figure 6.3 A derivation using the grammar of Figure 6.2

Conversely, we could start with the sentence "`the girl sees a dog.`" and work backward through the derivation of Figure 6.3 to arrive at *sentence* and so have shown that the sentence is legal in the language.

The simple grammar of Figure 6.2 already exhibits most of the properties of programming language grammars. Note that you can't assume that just because sentence is legal that it actually makes sense in a real-life context. For example, "`the dog pets the girl.`" is a nonsensical, yet legal sentence. Note that this example also contains a subtle error: Articles that appear at the beginning of sentences should be capitalized. This type of property, known as a positional property, is hard to represent using context-free grammars. Also, we have written the tokens in the foregoing sentence with spaces between the tokens, but the grammar does not specify whether such spaces are necessary (we might have easily written "`thegirlseesadog`."). This question is left to a scanner, which consumes white-space characters and recognizes the individual tokens or words, as noted in the previous section. (See also Exercise 6.9.) Finally, the grammar also does not specify additional requirements on input format, such as the fact that a sentence should be followed by some kind of termination symbol (such as a newline or end of file marker). Occasionally, we need to make this latter condition explicit in a grammar, writing a rule such as:

$$input \rightarrow sentence\ \$$$

to indicate that a sentence is to be followed by some kind of end marker. The marker is indicated by the dollar sign, and may or may not need to be generated explicitly.

Let's take a moment to formulate some informal definitions for the topics discussed so far. A **context-free grammar** consists of a series of grammar rules. These rules, in turn, consist of a left-hand side that is a single phrase structure name, followed by the metasymbol "→", followed by a right-hand side consisting of a sequence of items that can be symbols or other phrase structure names. The names for phrase structures (like *sentence*) are called **nonterminals**, since they are broken down into further phrase structures. The words or token symbols are also called **terminals**, since they cannot be broken down. Grammar rules are also called **productions**, since they "produce" the strings of the language using derivations. Productions are in **Backus-Naur form** if they are as given using only the metasymbols "→" and "|". (Sometimes parentheses are also allowed to group things together.)

A context-free grammar also has a special nonterminal called the **start symbol**, as noted above. This nonterminal stands for the entire, top-level phrase being defined (such as a sentence or a program),

and is the symbol that all derivations begin with. A context-free grammar defines a language, called the **language of the grammar**. This language is the set of all strings of terminals for which there exists a derivation beginning with the start symbol and ending with the string of terminals.

Figure 6.2 includes seven terminals ("`girl`," "`dog`," "`sees`," "`pets`," "`the`," "`a`," and "`.`"), six nonterminals, and six productions. The language defined by this grammar is also finite, meaning that there are only finitely many sentences that can be derived in the language (see Exercise 6.8), but in general languages defined by grammars are not finite.

Typically there are as many productions in a context-free grammar as there are nonterminals, although one could eliminate the "`|`" metasymbol by writing each choice separately, such as:

noun → `girl`
noun → `dog`

in which case each nonterminal would correspond to as many productions as there are choices.

Why is such a grammar **context-free**? The simple reason is that the nonterminals appear singly on the left-hand sides of productions. This means that each nonterminal can be replaced by any right-hand side choice, no matter where the nonterminal might appear. In other words, there is no **context** under which only certain replacements can occur. For example, in the grammar just discussed, it makes sense to use the verb "pets" only when the subject is "girl"; this can be thought of as a context sensitivity. We can write out context-sensitive grammars by allowing context strings to appear on left-hand sides of the grammar rules. Some authors consider context sensitivities to be purely syntactic issues. We shall adopt the view, however, that anything not expressible using context-free grammars is a semantic, not a syntactic, issue. (Even some things that *are* expressible as context-free grammars are often better left to semantic descriptions, since they involve writing many extra productions; see Exercise 6.30.)

As an example of a context sensitivity, recall that, in the preceding grammar, articles at the beginning of sentences should be capitalized. One way of doing this is to rewrite the first rule as:

sentence → *beginning noun-phrase verb-phrase* '.'

and then add the context-sensitive rule:

beginning article → `The | A`

Now the derivation would look as follows:

sentence ⇒ *beginning noun-phrase verb-phrase* . (new rule 1)
⇒ *beginning article noun verb-phrase* . (rule 2)
⇒ `The` *noun verb-phrase* . (new context-sensitive rule)
⇒ . . .

Context-free grammars have been studied extensively by formal language theorists and are now so well understood that it is natural to express the syntax of any programming language in BNF form. Indeed, doing so makes it easier to write translators for the language, since the parsing stage can be automated, as you will learn in Section 6.6.

A typical simple example of the use of a context-free grammar in a programming language is the description of simple integer arithmetic expressions with addition and multiplication given in Figure 6.4.

```
expr → expr + expr | expr * expr | ( expr ) | number
number → number digit | digit
digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Figure 6.4 A simple integer arithmetic expression grammar

Note the recursive nature of the rules: An expression can be the sum or product of two expressions, each of which can be further sums or products. Eventually, of course, this process must stop by selecting the *number* alternative, or we would never arrive at a string of terminals.

Note also that the recursion in the rule for *number* is used to generate a repeated sequence of digits. For example, the number 234 is constructed as in Figure 6.5.

```
number ⇒ number digit
       ⇒ number digit digit
       ⇒ digit digit digit
       ⇒ 2 digit digit
       ⇒ 23 digit
       ⇒ 234
```

Figure 6.5 A derivation for the *number* 234 using the grammar of Figure 6.4

This process could actually be performed by a scanner, as we have noted in the previous section, since it deals only with sequences of characters. Indeed, the grammar above has as its terminals only single character tokens such as + and 9, and the question of white space is ignored. We will return to this question in Section 6.7, but for simplicity we ignore scanning questions for the moment.

As a more complex example of a grammar in BNF, Figure 6.6 shows the beginning of a BNF description for the C language.

```
translation-unit → external-declaration
    | translation-unit external-declaration

external-declaration → function-definition | declaration

function-definition → declaration-specifiers declarator
                          declaration-list compound-statement
    | declaration-specifiers declarator compound-statement
    | declarator declaration-list compound-statement
    | declarator compound-statement

declaration → declaration-specifiers ';'
    | declaration-specifiers init-declarator-list ';'

init-declarator-list → init-declarator
```

Figure 6.6 Partial BNFs for C (adapted from Kernighan and Ritchie [1988]) (*continues*)

(*continued*)

```
                | init-declarator-list ',' init-declarator

init-declarator → declarator | declarator '=' initializer
declarator → pointer direct-declarator | direct-declarator

pointer → '*' type-qualifier-list pointer | '*' type-qualifier-list
        | '*' pointer | '*'

direct-declarator → ID
        | '(' declarator ')' | direct_declarator '[' ']'
        | direct_declarator '[' constant_expression ']'
        | direct_declarator '(' parameter_type_list ')'
        | direct_declarator '(' identifier_list ')'
        | direct_declarator '(' ')'
...
...
```

Figure 6.6 Partial BNFs for C (adapted from Kernighan and Ritchie [1988])

An alternative way of explaining how BNF rules construct the strings of a language is as follows. Given a grammar rule such as:

$$expr \rightarrow expr + expr \mid number$$

(abstracted from Figure 6.4), let **E** be the set of strings representing expressions, and let **N** represent the set of strings representing numbers. The preceding grammar rule can be viewed as a **set equation**:

$$\mathbf{E} = \mathbf{E} + \mathbf{E} \cup \mathbf{N}$$

where **E** 1 **E** is the set constructed by concatenating all strings from **E** with the "+" symbol and then all strings from E and "∪" make up set union. Assuming that the set **N** has already been constructed, this represents a recursive equation for the set **E**. *The smallest set* **E** *satisfying this equation* can be taken to be the set defined by the grammar rule. Intuitively, this consists of the set:

$$\mathbf{N} \cup \mathbf{N} + \mathbf{N} \cup \mathbf{N} + \mathbf{N} + \mathbf{N} \cup \mathbf{N} + \mathbf{N} + \mathbf{N} + \mathbf{N} \cup \ldots$$

In Exercise 6.53, you will have the chance to prove that this is the smallest set satisfying the given equation. Solutions to recursive equations appear frequently in formal descriptions of programming languages, and are a major object of study in the theory of programming languages.

6.3 Parse Trees and Abstract Syntax Trees

Syntax establishes structure, not meaning, but the meaning of a sentence (or program) is related to its syntax. For example, in English a sentence has a subject and a predicate. These are semantic concepts, since the subject (the "actor") and the predicate (the "action") determine the meaning of the sentence. A subject generally comes at the beginning of a sentence and is given by a noun phrase. Thus, in the syntax of an English sentence, a noun phrase is placed first and is subsequently associated with a subject. Similarly, in the grammar for expressions, when we write:

$$expr \rightarrow expr + expr$$

we expect to add the values of the two right-hand expressions to get the value of the left-hand expression. This process of associating the semantics of a construct to its syntactic structure is called **syntax-directed semantics**. We must, therefore, construct the syntax so that it reflects the semantics we will eventually attach to it as much as possible. (Syntax-directed semantics could just as easily have been called semantics-directed syntax.)

To make use of the syntactic structure of a program to determine its semantics, we must have a way of expressing this structure as determined by a derivation. A standard method for doing this is with a **parse tree**, which is a graphical depiction of the replacement process in a derivation. For example, the parse tree for the sentence "`the girl sees a dog.`" is shown in Figure 6.7.

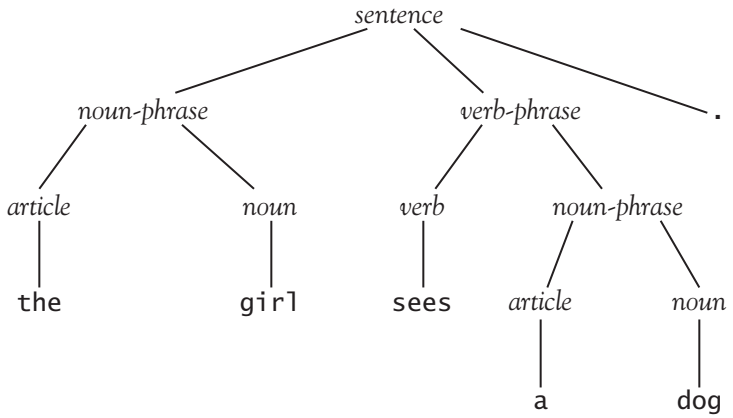


Figure 6.7: Parse tree for the sentence "`the girl sees a dog.`"

Similarly, the parse tree for the number `234` in the expression grammar is shown in Figure 6.8.

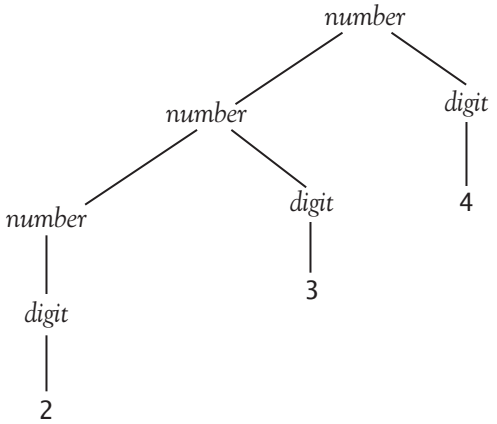


Figure 6.8: Parse tree for the number `234`

A parse tree is labeled by nonterminals at interior nodes (nodes with at least one child) and terminals at leaves (nodes with no children). The structure of a parse tree is completely specified by the grammar rules of the language and a derivation of a particular sequence of terminals. The grammar rules specify

the structure of each interior node, and the derivation specifies which interior nodes are constructed. For example, the tree for the number `234` is specified by the grammar rules for *number* and the derivation of `234` as given in the previous section. Indeed, the first two steps of the derivation are:

$$\begin{aligned} \textit{number} &\Rightarrow \textit{number digit} && \text{(step 1)} \\ &\Rightarrow \textit{number digit digit} && \text{(step 2)} \end{aligned}$$

and correspond to the construction of the two children of the root (step 1) and the two children of the left child of the root (step 2). Not surprisingly, there are exactly as many steps in the derivation of `234` as there are interior nodes in the parse tree.

All the terminals and nonterminals in a derivation are included in the parse tree. However, not all the terminals and nonterminals are necessary to determine completely the syntactic structure of an expression or sentence. For example, the structure of the number `234` can be completely determined from the tree shown in Figure 6.9.

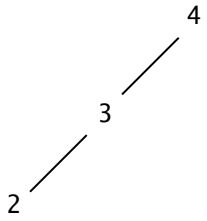


Figure 6.9: Parse tree for determining structure of the number `234`

As another example, Figure 6.10 shows how a parse tree for `3 + 4 * 5` can be condensed.

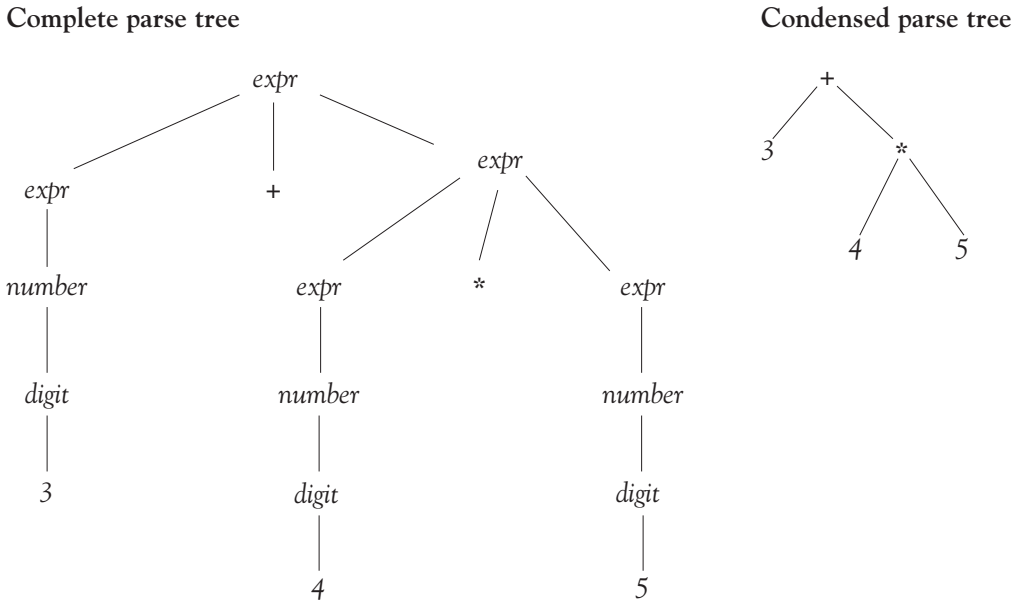


Figure 6.10: Condensing parse tree for `3 + 4 * 5`

Such trees are called **abstract syntax trees** or just **syntax trees**, because they abstract the essential structure of the parse tree. Abstract syntax trees may also do away with terminals that are redundant once the structure of the tree is determined. For example, the grammar rule:

if-statement → `if` (*expression*) *statement* `else` *statement*

gives rise to the parse tree and abstract syntax tree shown in Figure 6.11.

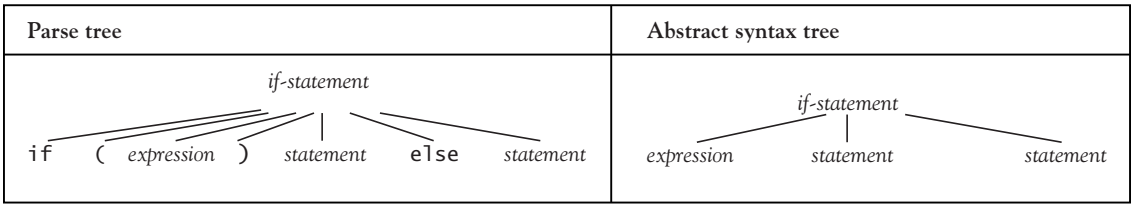


Figure 6.11: Parse tree and abstract syntax tree for grammar rule *if-statement* → `if` (*expression*) *statement* `else` *statement*

It is possible to write out rules for abstract syntax in a similar way to the BNF rules for ordinary syntax. For example, the abstract syntax for an *if-statement* corresponding to the previous BNF rule could be written simply as:

if-statement → *expression statement statement*

Of course, this can also be a little confusing, since this is not what we would see in an actual program. For this reason, abstract syntax is typically of less interest to the programmer and is often left unspecified. Sometimes ordinary syntax is **concrete syntax** to distinguish it from abstract syntax. Of course, abstract syntax is of great importance to the language designer and translator writer, since it is the abstract, not the concrete, syntax that expresses a language's essential structure, and a translator will often construct a syntax tree rather than a full parse tree because it is more concise. In this text, rather than specifying both concrete and abstract syntax, we will focus on the concrete syntax and infer typical abstract structures for syntax trees directly from the concrete syntax.

6.4 Ambiguity, Associativity, and Precedence

Two different derivations can lead to the same parse tree or syntax tree: In the derivation for `234` in Figure 6.5, we could have chosen to replace the *digit* nonterminals first:

number ⇒ *number digit*
⇒ *number* `4`
⇒ *number digit* `4`
⇒ *number* `34`
. . .

but we would still have had the same parse tree (and syntax tree). However, different derivations can also lead to different parse trees. For example, if we construct `3 + 4 * 5` from the expression grammar of Figure 6.4, we can use the derivation:

$$
\begin{aligned}
expr &\Rightarrow expr + expr \\
&\Rightarrow expr + expr * expr \\
&\qquad \text{(replace the second } expr \text{ with } expr * expr\text{)} \\
&\Rightarrow number + expr * expr \\
&\ldots
\end{aligned}
$$

or the derivation:

$$
\begin{aligned}
expr &\Rightarrow expr * expr \\
&\Rightarrow expr + expr * expr \\
&\qquad \text{(replace the first } expr \text{ with } expr + expr\text{)} \\
&\Rightarrow number + expr * expr \\
&\ldots
\end{aligned}
$$

These, in turn, lead to the two distinct parse trees shown in Figure 6.12. (Dashed lines in the remaining trees in this chapter indicate missing nodes in the parse trees that are irrelevant to the current discussion.) This derivation can also lead to the two distinct abstract syntax trees shown in Figure 6.13.

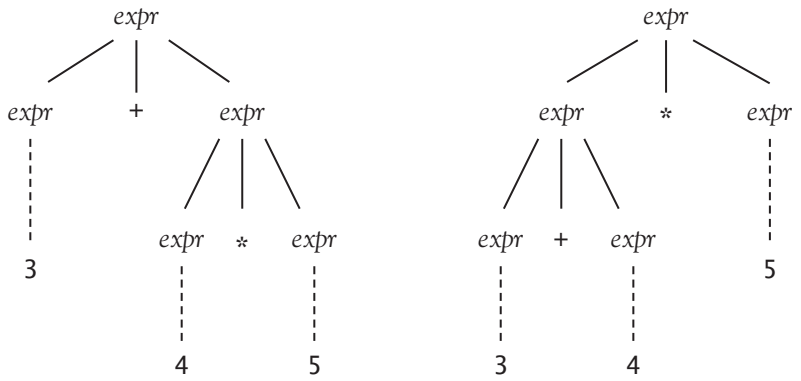


Figure 6.12: Two parse trees for `3 + 4 * 5`

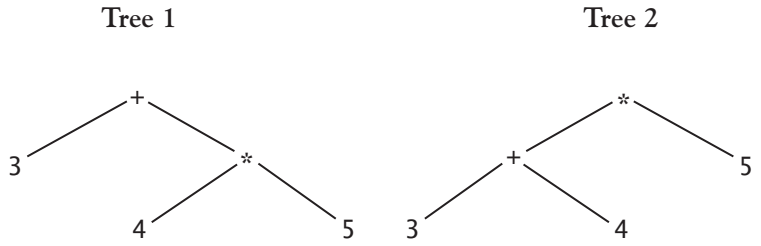


Figure 6.13 Two abstract syntax trees for `3 + 4 * 5`, indicating the ambiguity of the grammar of Figure 6.4

A grammar such as that in Figure 6.4, for which two distinct parse (or syntax) trees are possible for the same string, is **ambiguous**. Ambiguity can also be expressed directly in terms of derivations. Even though many derivations may in general correspond to the same parse tree, certain special derivations that are constructed in a special order can only correspond to unique parse trees. One kind of

derivation that has this property is a **leftmost derivation**, where the leftmost remaining nonterminal is singled out for replacement at each step. For example, the derivations in Figures 6.3 and 6.5 are leftmost, but the derivation at the beginning of this section (deriving the same string as Figure 6.5) is not. Each parse tree has a unique leftmost derivation, which can be constructed by a preorder traversal of the tree. Thus, the ambiguity of a grammar can also be tested by searching for two different leftmost derivations of the same string. If such leftmost derivations exist, then the grammar must be ambiguous, since each such derivation must correspond to a unique parse tree. For example, the ambiguity of the grammar of Figure 6.4 is also demonstrated by the two leftmost derivations for the string `3 + 4 * 5` as given in Figure 6.14.

Leftmost Derivation 1 (Corresponding to Tree 1 of Figure 6.13)	**Leftmost Derivation 2 (Corresponding to Tree 2 of Figure 6.13)**
expr ⇒ *expr* + *expr*	expr ⇒ expr * *expr*
⇒ *number* + *expr*	⇒ *expr* + *expr* * *expr*
⇒ *digit* + *expr*	⇒ *number* + *expr* * *expr*
⇒ 3 + *expr*	⇒ . . . (etc.)
⇒ 3 + *expr* * *expr*	
⇒ 3 + *number* * *expr*	
⇒ . . . (etc.)	

Figure 6.14 Two leftmost derivations for `3 + 4 * 5`, indicating the ambiguity of the grammar of Figure 6.4

Ambiguous grammars present difficulties, since no clear structure is expressed. To be useful, either the grammar must be revised to remove the ambiguity or a **disambiguating rule** must be stated to establish which structure is meant.

Which of the two parse trees (or leftmost derivations) is the correct one for the expression `3 + 4 * 5`? If we think of the semantics to be attached to the expression, we can understand what this decision means. The first syntax tree of Figure 6.13 implies that the multiplication operator * is to be applied to the 4 and 5 (resulting in the value 20), and this result is added to 3 to get 23. The second syntax tree, on the other hand, says to add 3 and 4 first (getting 7) and then multiply by 5 to get 35. Thus, the operations are applied in a different order, and the resulting semantics are quite different.

If we take the usual meaning of the expression 3 + 4 * 5 from mathematics, we would choose the first tree of Figure 6.13 over the second, since multiplication has precedence over addition. This is the usual choice in programming languages, although some languages (such as APL) make a different choice. How could we express the fact that multiplication should have precedence over addition? We could state a disambiguating rule separately from the grammar, or we could revise the grammar. The usual way to revise the grammar is to write a new grammar rule (called a "term") that establishes a "precedence cascade" to force the matching of the "*" at a lower point in the parse tree:

$$expr \rightarrow expr + expr \mid term$$
$$term \rightarrow term * \text{term} \mid (\ expr\ ) \mid number$$

We have not completely solved the ambiguity problem, however, because the rule for an *expr* still allows us to parse `3 + 4 + 5` as either `(3 + 4) + 5` or `3 + (4 + 5)`. In other words, we can make addition either **right-** or **left-associative**, as shown in Figure 6.15.

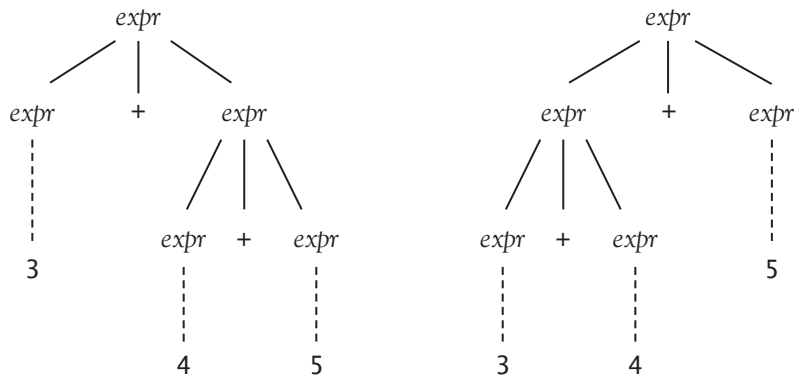


Figure 6.15: Addition as either right- or left-associative

In the case of addition, this does not affect the result, but if we were to include subtraction, it surely would: 8 - 4 - 2 = 2 if "-" is left-associative, but 8 - 4 - 2 = 6 if "-" is right-associative. What is needed is to replace the rule:

$$expr \rightarrow expr + expr$$

with either:

$$expr \rightarrow expr + term$$

or:

$$expr \rightarrow term + expr$$

The first rule is **left-recursive**, while the second rule is **right-recursive**. A left-recursive rule for an operation causes it to left-associate, as in the parse tree shown in Figure 6.16. Similarly, a right-recursive rule causes it to right-associate, as also shown in Figure 6.16.

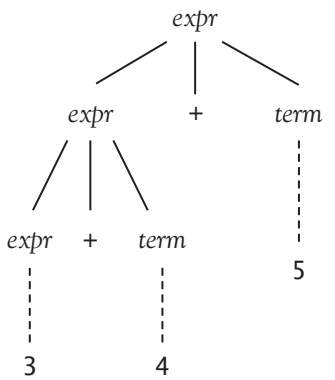
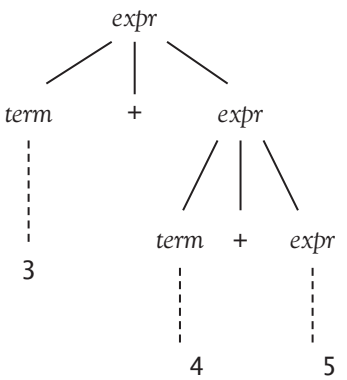
A left-recursive rule for an operation causes it to left-associate.	**A right-recursive rule for an operation causes it to right-associate.**
	

Figure 6.16: Parse trees showing results of left- and right-recursive rules

The revised grammar for simple integer arithmetic expressions that expresses both precedence and associativity is given in Figure 6.17.

```
expr → expr + term | term
term → term * factor | factor
factor → ( expr ) | number
number → number digit | digit
digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Figure 6.17 Revised grammar for simple integer arithmetic expressions

The grammar of Figure 6.17 is now unambiguous (a proof of this requires more advanced techniques from parsing theory). Moreover, the syntax trees generated by this grammar correspond to the semantics of the arithmetic operations as they are usually defined. Sometimes the process of rewriting a grammar to eliminate ambiguity causes the grammar to become extremely complex, and in such cases we prefer to state a disambiguating rule (see the if-statement discussion in Chapter 9).

6.5 EBNFs and Syntax Diagrams

We noted earlier that the grammar rule:

$$number \rightarrow number\ digit \mid digit$$

generates a number as a sequence of digits:

$$\begin{aligned} number &\Rightarrow number\ digit \\ &\Rightarrow number\ digit\ digit \\ &\Rightarrow number\ digit\ digit\ digit \\ &\Rightarrow digit\ \ldots\ digit \\ &\ldots \\ &\Rightarrow \text{(arbitrary repetitions of digit)} \end{aligned}$$

Similarly, the rule:

$$expr \rightarrow expr + term \mid term$$

generates an expression as a sequence of terms separated by "+'s":

$$\begin{aligned} expr &\Rightarrow expr + term \\ &\Rightarrow expr + term + term \\ &\Rightarrow expr + term + term + term \\ &\ldots \\ &\Rightarrow term + \ldots + term \end{aligned}$$

This situation occurs so frequently that a special notation for such grammar rules is adopted that expresses more clearly the repetitive nature of their structures:

$$number \rightarrow digit\ \{digit\}$$

and:

$$expr \rightarrow term\ \{+\ term\}$$

In this notation, the curly brackets `{ }` stand for "zero or more repetitions of." Thus, the rules express that a number is a sequence of one or more digits, and an expression is a term followed by zero or more repetitions of a + and another term. In this notation, the curly brackets have become new metasymbols, and this notation is called **extended Backus-Naur form**, or **EBNF** for short.

This new notation obscures the left associativity of the + operator that is expressed by the left recursion in the original rule in BNF. We can get around this by simply assuming that any operator involved in a curly bracket repetition is left-associative. Indeed, if an operator were right-associative, the corresponding grammar rule would be right-recursive. Right-recursive rules are usually not written using curly brackets (the reason for this is explained more fully in the next section). The problem of expressing right associativity points out a flaw in EBNF grammars: Parse trees and syntax trees cannot be written directly from the grammar. Therefore, we will always use BNF notation to write parse trees.

A second common situation is for a structure to have an optional part, such as the optional else-part of an if-statement in C, which is expressed in BNF notation for C as follows:

$$\begin{aligned} \textit{if-statement} \rightarrow\ & \texttt{if}\ (\ \textit{expression}\ )\ \textit{statement}\ \mid \\ & \texttt{if}\ (\ \textit{expression}\ )\ \textit{statement}\ \texttt{else}\ \textit{statement} \end{aligned}$$

This is written more simply and expressively in EBNF, as follows:

$$\textit{if-statement} \rightarrow \texttt{if}\ (\ \textit{expression}\ )\ \textit{statement}\ [\ \texttt{else}\ \textit{statement}\ ]$$

where the square brackets "[ ]" are new metasymbols indicating optional parts of the structure.

Another example is the grammar rule for *function-definition* in the BNF grammar for C; see Figure 6.6:

$$\begin{aligned} \textit{function-definition} \rightarrow\ & \textit{declaration-specifiers declarator} \\ & \textit{declaration-list compound-statement} \\ & \mid \textit{declaration-specifiers declarator compound-statement} \\ & \mid \textit{declarator declaration-list compound-statement} \\ & \mid \textit{declarator compound-statement} \end{aligned}$$

Using EBNF, this can be much more simply expressed as:

$$\begin{aligned} \textit{function-definition} \rightarrow\ & [\ \textit{declaration-specifiers}\ ]\ \textit{declarator} \\ & [\ \textit{declaration-list}\ ]\ \textit{compound-statement} \end{aligned}$$

Right-associative (binary) operators can also be written using these new metasymbols. For example, if "@" is a right-associative operator with BNF:

$$expr \rightarrow term\ @\ expr \mid term$$

then this rule can be rewritten in EBNF as follows:

$$expr \rightarrow term\ [\ @\ expr\ ]$$

For completeness, we write out the grammar of Figure 6.3 for simple integer arithmetic expressions in EBNF in Figure 6.18.

```
expr → term { + term }
term → factor { * factor }
factor → ( expr ) | number
number → digit { digit }
digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Figure 6.18 EBNF rules for simple integer arithmetic expressions

A sometimes-useful graphical representation for a grammar rule is the **syntax diagram**, which indicates the sequence of terminals and nonterminals encountered in the right-hand side of the rule. For example, syntax diagrams for *noun-phrase* and *article* of our simple English grammar presented in Section 6.2 would be drawn as shown in Figure 6.19. Figure 6.20 shows these two rules condensed into one.

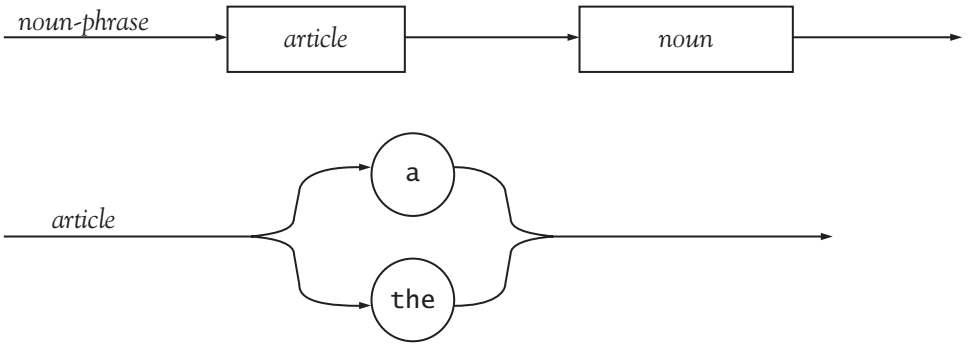


Figure 6.19: Syntax diagrams for *noun-phrase* and *article* of the simple English grammar presented in Section 6.2

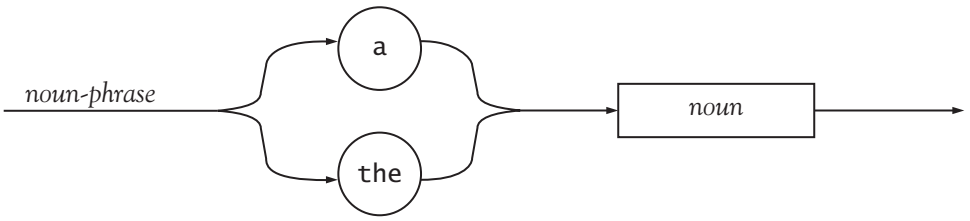


Figure 6.20: Condensed version of diagrams shown in Figure 6.19

Syntax diagrams use circles or ovals for terminals and squares or rectangles for nonterminals, connecting them with lines and arrows to indicate appropriate sequencing. Syntax diagrams can also condense several grammar rules into one diagram.

Syntax diagrams for the expression grammar in Figure 6.18 are given in Figure 6.21. Note the use of loops in the diagrams to express the repetition given by the curly brackets in the EBNFs.

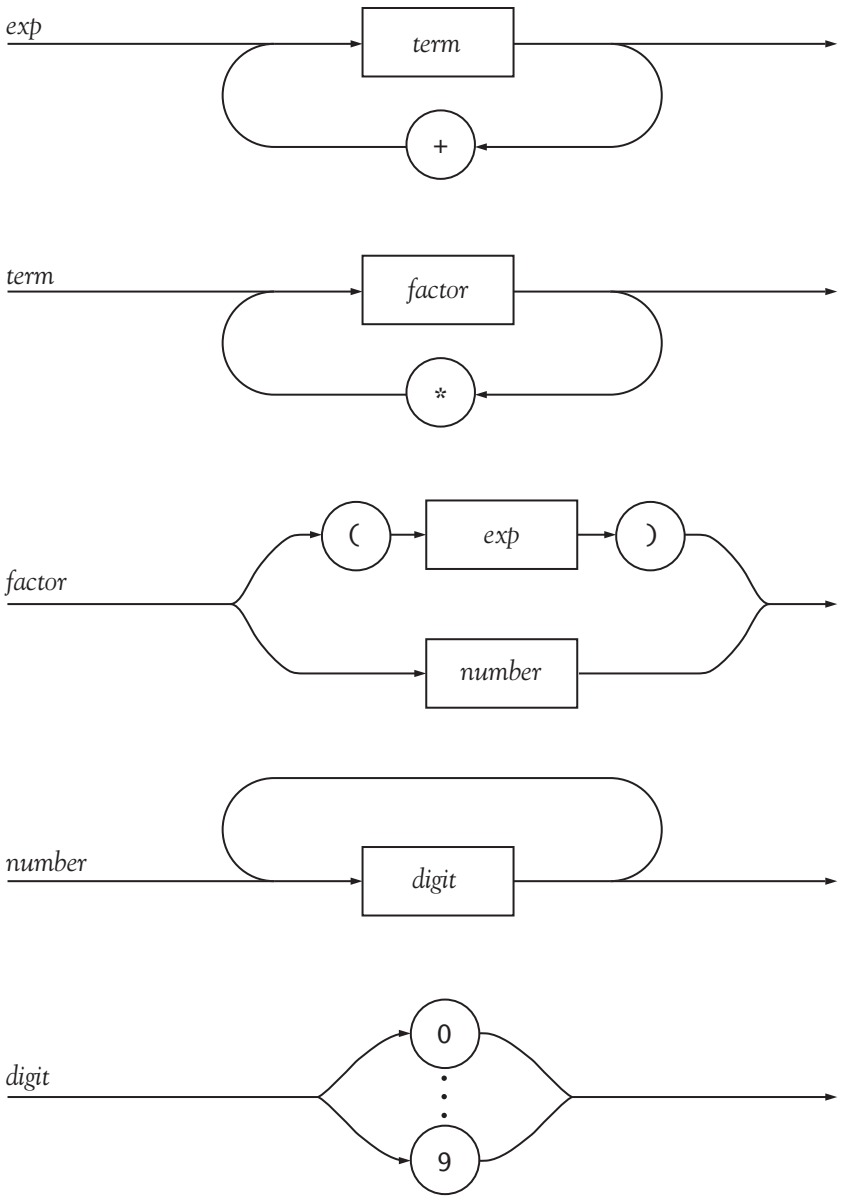


Figure 6.21: Syntax diagrams for a simple integer expression grammar

As an example of how to express the square brackets [ ] in syntax diagrams, Figure 6.22 shows the syntax diagram for the *if-statement* in C.

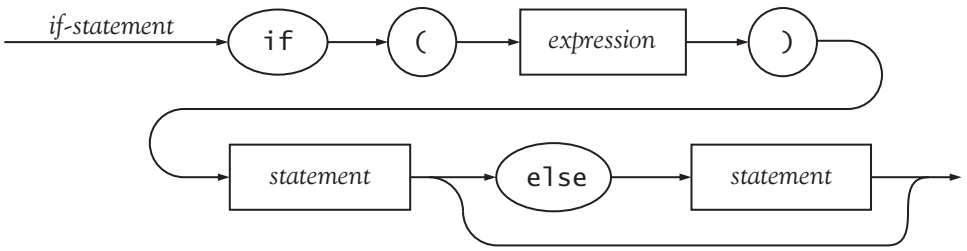


Figure 6.22 Syntax diagram for the *if-statement* in C

Syntax diagrams are always written from the EBNF notation, not the BNF notation, for reasons that are made clear in the next section. Thus, the diagram for *expr* shown in Figure 6.23 would be incorrect.

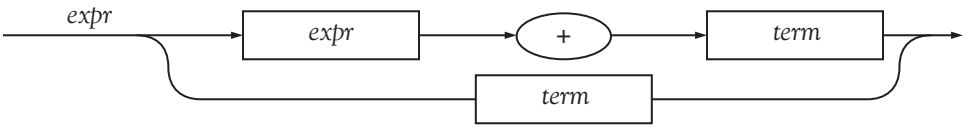


Figure 6.23: Incorrect syntax diagram for *expr*

Syntax diagrams are visually appealing but take up a great deal of space. As more programmers have become familiar with rules written directly in BNF or EBNF, syntax diagrams have been used less frequently, and now are seen much more rarely than in the past. They remain useful, however, as flow charts for writing parsing procedures, a topic to which we now turn.

6.6 Parsing Techniques and Tools

A grammar written in BNF, EBNF, or as syntax diagrams describes the strings of tokens that are syntactically legal in a programming language. The grammar, thus, also implicitly describes the actions that a parser must take to parse a string of tokens correctly and to construct, either implicitly or explicitly, a derivation or parse tree for the string. The simplest form of a parser is a **recognizer**—a program that accepts or rejects strings, based on whether they are legal strings in the language. More general parsers build parse trees (or, more likely, abstract syntax trees) and carry out other operations, such as calculating values for expressions.

Given a grammar in one of the forms we have discussed, how does it correspond to the actions of a parser? One method of parsing attempts to match an input with the right-hand sides of the grammar rules. When a match occurs, the right-hand side is replaced by, or **reduced** to, the nonterminal on the left. Such parsers are **bottom-up** parsers, since they construct derivations and parse trees from the leaves to the root. They are sometimes also called **shift-reduce** parsers, since they shift tokens onto a stack prior to reducing strings to nonterminals. In the other major parsing method, **top-down parsing**, nonterminals are expanded to match incoming tokens and directly construct a derivation. Both top-down and bottom-up parsing can be automated; that is, a program, called a **parser generator**, can be written that will automatically translate a BNF description into a parser. Since bottom-up parsing is somewhat more powerful than top-down parsing, it is usually the preferred method for parser generators (also called **compiler compilers**). One such parser generator in wide use is YACC (short for "yet another compiler compiler"), or its freeware version Bison, which we will study later in this section.

However, is an older, very effective method for constructing a parser from a grammar that is still in use. This method, which is called **recursive-descent parsing**, turns the nonterminals into a group of mutually recursive procedures whose actions are based on the right-hand sides of the BNFs.

The right-hand sides are interpreted in the procedures as follows: tokens are matched directly with input tokens as constructed by a scanner. Nonterminals are interpreted as calls to the procedures corresponding to the nonterminals.

As an example, in our simplified English grammar, procedures for *sentence, noun-phrase*, and *article* would be written as follows (in C-like pseudocode):

```
void sentence(){
    nounPhrase();
    verbPhrase();
}

void nounPhrase(){
    article();
    noun();
}

void article(){
    if (token == "a") match("a", "a expected");
    else if (token == "the") match("the", "the expected");
    else error("article expected");
}
```

In this pseudocode, we use strings to imitate tokens, and a global variable called `token` to hold the current token as constructed by a scanner. The `getToken` procedure of the scanner is called whenever a new token is desired. A `match` of a token corresponds to a successful test for that token, followed by a call to `getToken`:

```
void match(TokenType expected, char* message){
    if (token == expected) getToken();
    else error(message);
}
```

Each parsing procedure starts with the variable `token` already holding the first token that should be expected. Thus, a call to `getToken` must precede the first call to the `sentence` procedure. Likewise, each parsing procedure leaves behind the next token in the input stream when it is finished. The `error` procedure prints an error message and aborts the parse. (Note that errors need only be detected when actual tokens are expected, as in the procedure `article`. We could have checked for "`a`" or "`the`" in the `sentence` procedure, but in this scheme it is unnecessary.)

If we apply this process to the BNF description of Figure 6.17, we encounter a problem with the left-recursive rules such as the one for an expression:

$$expr \longrightarrow expr + term \mid term$$

If we were to try to write this rule as a recursive-descent procedure, two fatal difficulties arise. First, the first choice in this rule (*expr* → *expr* + *term*) would have the procedure immediately calling itself recursively, which would result in an infinite recursive loop. Second, there is no way to decide which of the two choices to take (*expr* → *expr* + *term or expr* → *term*) until a + is seen (or not) much later in the input. The problem is caused by the existence of the left recursion in the first alternative of the grammar rule, since the right-recursive rule:

$$expr \rightarrow term + expr \mid term$$

has only a very modest problem and can be turned into a parsing procedure by simply calling `term()` and then testing for a `"+"`:

```
void expr(){
    term();
    if (token == "+"){
        match("+", "+ expected");
        expr();
    }
}
```

Unfortunately, this grammar rule (and the associated parsing procedure) make + into a right-associative operator. How can we remove the problem caused by the left recursion, while at the same time retaining the left associative behavior of the + operation? The solution is implicit in the EBNF description of this same grammar rule, which expresses the recursion as a loop:

$$expr \rightarrow term \{ + term\}$$

This form of the grammar rule for *expr* corresponds to the recursive-descent code:

```
void expr(){
    term();
    while (token == "+"){
        match("+", "+ expected");
        term();
        /* perform left associative operations here */
    }
}
```

While this form of the grammar does suppress the explicit left associativity of +, the corresponding code *does* provide the opportunity for enforcing left associativity by timing whatever operations are to be performed (tree building or expression calculation) right after the second call to `term()` (at the end of the body of the loop, as indicated by the comment).

Thus, the curly brackets in EBNF represent **left recursion removal** by the use of a loop. While there are more general mechanisms than this for left recursion removal, this simple use of EBNF usually suffices in practice.

Right-recursive rules on the other hand, as we have already noted, present no such problem in recursive-descent parsing. However, the code that we write for a right-recursive rule such as:

$$expr \rightarrow term \ @ \ expr \mid term$$

(we change the name of the operation to @ to avoid confusion with +) also corresponds to the use of square brackets in EBNF to express this optional structure:

$$expr \rightarrow term \ [\ @ \ expr \ ]$$

This process is called **left factoring**, since it is as if the *term* in both alternatives of the grammar rule is "factored out," leaving either @ *expr* or nothing (thus making @ *expr* optional).

Another typical left-factoring situation arises when we have an if-statement with an optional else-part:

$$\begin{aligned} \textit{if-statement} \rightarrow \ & \texttt{if} \ (\ \textit{expression} \ ) \ \textit{statement} \mid \\ & \texttt{if} \ (\ \textit{expression} \ ) \ \textit{statement} \ \texttt{else} \ \textit{statement} \end{aligned}$$

This cannot be translated directly into code, since both alternatives begin with the same prefix. As in the previous example, in EBNF this is written with square brackets, and the common parts of the alternatives are "factored out":

$$\textit{if-statement} \rightarrow \texttt{if} \ (\ \textit{expression} \ ) \ \textit{statement} \ [\ \texttt{else} \ \textit{statement} \ ]$$

This corresponds directly to the recursive-descent code to parse an *if-statement*:

```
void ifStatement(){
    match("if", "if expected");
    match("(", "( expected");
    expression();
    match(")", ") expected");
    statement();
    if (token == "else"){
        match("else", "else expected");
        statement();
    }
}
```

Thus, in both left-recursive and left-factoring situations, EBNF rules or syntax diagrams correspond naturally to the code of a recursive-descent parser. This actually is one of the main reasons for their use.

As a demonstration of the power and ease of recursive-descent parsing, Figure 6.24 provides the complete code in C for an integer calculator based on the grammar of Figure 6.18. This program accepts a single line of input containing a simple integer arithmetic expression consisting of numbers, the operators + and *, and parentheses, and prints the computed value of the expression, or an error message if the expression is not syntactically correct. As in the grammar, the calculator assumes all tokens are characters, and it does not permit spaces to appear in an expression. To make it easy to print the computed value, and also to make sure only a single expression is entered in the input line,

a new grammar rule is added to the grammar of Figure 6.18 (and the start symbol is changed to *command*):

$$command \rightarrow expr \texttt{'\textbackslash n'}$$

In effect, this grammar rule makes the newline character (`'\n'`) into the marker for the end of the input (just like the $ token discussed earlier in this chapter).

Finally, we note that the timing of the computation of the results in each procedure ensures that the operators + and * are left-associative.

```
(1)  #include <ctype.h>
(2)  #include <stdlib.h>
(3)  #include <stdio.h>

(4)  int token; /* holds the current input character for the parse */

(5)  /* declarations to allow arbitrary recursion */
(6)  void command();
(7)  int expr();
(8)  int term();
(9)  int factor();
(10) int number();
(11) int digit();

(12) void error(char* message){
(13)     printf("parse error: %s\n", message);
(14)     exit(1);
(15) }

(16) void getToken(){
(17)     /* tokens are characters */
(18)     token = getchar();
(19) }
(20) void match(char c, char* message){
(21)     if (token == c) getToken();
(22)     else error(message);
(23) }

(24) void command(){
(25)     /* command -> expr '\n' */
(26)     int result = expr();
(27)     if (token == '\n') /* end the parse and print the result */
(28)         printf("The result is: %d\n",result);
(29)     else error("tokens after end of expression");
(30) }
```

Figure 6.24 A calculator for simple integer arithmetic expressions using recursive-descent parsing (*continues*)

(*continued*)

```
(31) int expr(){
(32)     /* expr -> term { '+' term } */
(33)     int result = term();
(34)     while (token == '+'){
(35)         match('+', "+ expected");
(36)         result += term();
(37)     }
(38)     return result;
(39) }

(40) int term(){
(41)     /* term -> factor { '*' factor } */
(42)     int result = factor();
(43)     while (token == '*'){
(44)         match('*', "* expected");
(45)         result *= factor();
(46)     }
(47)     return result;
(48) }

(49) int factor(){
(50)     /* factor -> '(' expr ')' | number */
(51)     int result;
(52)     if (token == '('){
(53)         match('(', "( expected");
(54)         result = expr();
(55)         match(')', ") expected");
(56)     }
(57)     else
(58)         result = number();
(59)     return result;
(60) }

(61) int number(){
(62)     /* number -> digit { digit } */
(63)     int result = digit();
(64)     while (isdigit(token))
(65)     /* the value of a number with a new trailing digit
(66)        is its previous value shifted by a decimal place
(67)        plus the value of the new digit
(68)     */
(69)        result = 10 * result + digit();
(70)     return result;
(71) }
```

Figure 6.24 A calculator for simple integer arithmetic expressions using recursive-descent parsing (*continues*)

(*continued*)

```
(72) int digit(){
(73)     /* digit -> '0' | '1' | '2' | '3' | '4'
(74)                | '5' | '6' | '7' | '8' | '9' */
(75)     int result;
(76)     if (isdigit(token)){
(77)         /* the numeric value of a digit character
(78)         is the difference between its ascii value and the
(79)         ascii value of the character '0'
(80)         */
(81)         result = token - '0';
(82)         match(token, "( expected");
(83)     }
(84)     else
(85)         error("digit expected");
(86)     return result;
(87) }

(88) void parse(){
(89)     getToken(); /* get the first token */
(90)     command(); /* call the parsing procedure for the start symbol */
(91) }

(92) int main(){
(93)     parse();
(94)     return 0;
(95) }
```

Figure 6.24 A calculator for simple integer arithmetic expressions using recursive-descent parsing

In this method for converting a grammar into a parser, the resulting parser bases its actions only on the next available token in the input stream (stored in the `token` variable in the code). This use of a single token to direct a parse is called **single-symbol lookahead**. A parser that commits itself to a particular action based only on this lookahead is called a **predictive parser** (sometimes laboriously referred to as a top-down parser with single-symbol lookahead and no backtracking).

Predictive parsers require that the grammar to be parsed satisfies certain conditions so that this decision-making process will work. The first condition is the ability to choose among several alternatives in a grammar rule. Suppose that a nonterminal A has the following choices:

$$A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$$

(where each α_i stands for a string of tokens and nonterminals). To decide which choice to use, the tokens that begin each of the α_i must be different. Given a string a of tokens and nonterminals, we define **First**(α) to be the set of tokens that can begin the string α. For example, given the grammar rules:

$$\begin{aligned} \textit{factor} &\rightarrow \texttt{(}\ \textit{expr}\ \texttt{)} \mid \textit{number} \\ \textit{number} &\rightarrow \textit{digit}\ \{\ \textit{digit}\ \} \\ \textit{digit} &\rightarrow \texttt{0} \mid \texttt{1} \mid \texttt{...} \mid \texttt{9} \end{aligned}$$

we have:

$$\text{First}(\ \texttt{(}\ \textit{expr}\ \texttt{)}\ ) = \{\ \texttt{(}\ \}$$

$$\text{First}(\ \textit{number}\ ) = \text{First}(\ \textit{digit}\ )$$

$$= \text{First}(\texttt{0}) \cup \text{First}(\texttt{1}) \cup \ldots \text{First}(\texttt{9})$$

$$= \{\ 0,...,9\ \}$$

and then:

$$\text{First}(\ \textit{factor}\ ) = \text{First}(\ \texttt{(}\ \textit{expr}\ \texttt{)}\ ) \cup \text{First}(\ \textit{number}\ )$$
$$= \{\ \texttt{(}, \texttt{0}, \texttt{1}, \ldots, \texttt{9}\ \}$$

The requirement that a predictive parser be able to distinguish between choices in a grammar rule can be stated in terms of First sets, as follows. Given the grammar rule:

$$A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$$

the First sets of any two choices must not have any tokens in common; that is,

$$\text{First}(\alpha_i) \cap \text{First}(\alpha_j) = \emptyset \text{ for all } i \neq j$$
(Ø denotes the empty set)

In the example of the grammar rule for a factor:

$$\textit{factor} \rightarrow \texttt{(}\ \textit{expr}\ \texttt{)} \mid \textit{number}$$

this condition for predictive parsing is satisfied, since:

$$\text{First}(\ \texttt{(}\ \textit{expr}\ \texttt{)}\ ) \cap \text{First}(\ \textit{number}\ ) = \{\ \texttt{(}\ \} \cap \{\ \texttt{0, . . .,9}\ \} = \emptyset$$

A second condition for predictive parsing arises when structures are optional. For example, to parse the grammar rule

$$\textit{expr} \rightarrow \textit{term}\ [\ \texttt{@}\ \textit{expr}\ ]$$

we must test for the presence of the token "@" before we can be sure that the optional part is actually present:

```
void expr(){
    term();
    if (token == '@'){
        match('@', "@ expected");
        expr();
    }
}
```

However, if the token @ can also come after an expression, this test is insufficient. If @ is the next token in the input, it may be the start of the optional "@ *expr*" part, or it may be a token that comes after

the whole expression. Thus, the second requirement for predictive parsing is that, for any optional part, no token beginning the optional part can also come after the optional part.

We can formalize this condition in the following way. For any string *A* of tokens and nonterminals that appears on the right-hand side of a grammar rule, define **Follow**(a) to be the set of tokens that can follow a. The second condition for predictive parsing can now be stated in terms of Follow sets. Given a grammar rule in EBNF with an optional a,

$$A \rightarrow \beta\ [\alpha]\ \sigma$$

we must have:

$$\text{First}(\alpha) \cap \text{Follow}(\alpha) = \emptyset$$

As an example of the computation of Follow sets, consider the grammar of Figure 6.18. Follow sets for the nonterminals can be computed as follows. From the rule:

$$\textit{factor} \rightarrow (\ \textit{expr}\ )$$

we get that the token "`)`" is in Follow(*expr*). Since *expr* can also comprise a whole expression, *expr* may be followed by the end of the input, which we indicate by the special symbol $. Thus, Follow(*expr*) 5 { `)`, $ }. From the rule:

$$\textit{expr} \rightarrow \textit{term}\ \{\ +\ \textit{term}\}$$

we obtain that "+" is in Follow(*term*). Since a *term* also appears as the last thing in an *expr*, anything that follows an *expr* can also follow a *term*. Thus,

$$\text{Follow}(\textit{term}) = \{\ ),\ \$,\ +\ \}$$

Finally, Follow(*factor*) = { `)`, $, +, `*` } by a similar computation.

Note that the grammar of Figure 6.18 automatically satisfies the second condition for predictive parsing, since there are no optional structures inside square brackets. A more instructive example comes from Figure 6.6 (a small part of a BNF grammar for C), where the grammar rule for a *declaration* in EBNF is written as:

$$\textit{declaration} \rightarrow \textit{declaration-specifiers}\ [\ \textit{init-declarator-list}\ ]\ \text{`;'}$$

By tracing through the rules for *init-declarator-list* in Figure 6.6, we can determine that First(*init-declarator-list*) = { `ID, *, (` } and Follow(*init-declarator-list*) 5 { `;`, `,` }, so that:

$$\text{First}(\textit{init-declarator-list}) \cap \text{Follow}(\textit{init-declarator-list}) = \emptyset$$

thus satisfying the second condition for predictive parsing in this case also.

The computation of First and Follow sets can also be useful in recursive-descent parsing to obtain appropriate tests; see Exercise 6.49.

We mentioned at the beginning of this section that the process of converting grammar rules into a parser can be performed by a program known as a parser generator, or compiler-compiler. A parser generator takes a version of BNF or EBNF rules as its input and then generates an output that is a parser program in some language. This program can then be compiled to provide an executable parser. Of course, providing only a grammar to the parser generator will result in a recognizer.

To get the parser to construct a syntax tree or perform other operations, we must provide operations or actions to be performed that are associated with each grammar rule, that is, a syntax-directed scheme.

We mentioned earlier that YACC, which is available on most UNIX systems, is a widely used parser generator. Its freeware version, Bison, is available for many different systems (including all versions of UNIX). YACC was written by Steve Johnson in the mid-1970s; it generates a C program that uses a bottom-up algorithm to parse the grammar. The format of a YACC specification is as follows:

```
%{ /* code to insert at the beginning of the parser */
%}
/* other YACC definitions, if necessary */
%%
/* grammar and associated actions */
%%
/* auxiliary procedures */
```

The grammar is given in a BNF-like form, with associated actions written in C. Figure 6.25 shows a complete YACC/Bison description of a simple integer expression interpreter using the grammar of Figure 6.17 (Section 6.4). Note that it uses the original left-recursive BNF grammar—bottom-up parsers are not bothered by left recursion. The actions provided with the rules are essentially the same as in the recursive-descent calculator of Figure 6.24 (and the C program generated by YACC behaves exactly as that program from the user's perspective). The YACC/Bison convention for constructing values from a parse is that the value of the left-hand side of a production is indicated by the symbol "$$," while the value of the *n*th symbol of the right-hand side is given by `$n`. (These values are always integers unless specified otherwise.) For example, on line 7 of Figure 6.25, `$1 + $3` represents the sum of the first and third symbols of the right-hand side of the rule:

```
expr : expr '+' term
```

so that the value of `expr` and `term` on the right are added. Note the different conventions for metasymbols, nonterminals, and tokens.

```
(1)  %{
(2)  #include <stdio.h>
(3)  %}

(4)  %%
(5)  command : expr '\n' { printf("The result is: %d\n",$1); }
(6)          ;
```

Figure 6.25 YACC/Bison specification of a calculator for simple integer arithmetic expressions (*continues*)

(*continued*)

```
(7)  expr     : expr '+' term { $$ = $1 + $3; }
(8)           u term { $$ = $1; }
(9)           ;
(10) term     : term '*' factor { $$ = $1 * $3; }
(11)          u factor { $$ = $1; }
(12)          ;

(13) factor : number { $$ = $1; }
(14)        u '(' expr ')' { $$ = $2; }
(15)        ;

(16) number : number digit { $$ = 10 * $1 + $2; }
(17)        u digit { $$ = $1; }
(18)        ;

(19) digit  : '0' { $$ = 0; }
(20)        u '1' { $$ = 1; }
(21)        u '2' { $$ = 2; }
(22)        u '3' { $$ = 3; }
(23)        u '4' { $$ = 4; }
(24)        u '5' { $$ = 5; }
(25)        u '6' { $$ = 6; }
(26)        u '7' { $$ = 7; }
(27)        u '8' { $$ = 8; }
(28)        u '9' { $$ = 9; }
(29)        ;

(30) %%
(31) main()
(32) { yyparse();
(33)   return 0;
(34) }

(35) int yylex(void)
(36) { static int done = 0; /* flag to end parse */
(37)   int c;
(38)   if (done) return 0; /* stop parse */
(39)   c = getchar();
(40)   if (c == '\n') done = 1; /* next call will end parse */
(41)   return c;
(42) }

(43) int yyerror(char *s)
(44) /* allows for printing error message */
(45) {    printf("%s\n",s);
(46) }
```

Figure 6.25 YACC/Bison specification of a calculator for simple integer arithmetic expressions

YACC/Bison generates a procedure `yyparse` from the grammar, so we have to provide a `main` procedure that calls `yyparse` (lines 31–34). YACC/Bison also assumes that tokens (aside from individual characters) are recognized by a scanner procedure called `yylex` similar to the `getToken` procedure of Figure 6.24. Thus, we provide a `yylex` in C that reads and returns a character (lines 35–42). One difference, however, between `yylex` and `getToken` is that YACC/Bison will only end a parse when `yylex` returns 0 (in the recursive-descent version, we could write special code that ended the parse on a newline character). Thus, `yylex` includes code to remember when a newline is reached and to return 0 on the next call after a newline (allowing the generated parser to match the newline directly before finishing; see Exercise 6.28). It would also be possible to generate `yylex` automatically from a **scanner generator** such as LEX/Flex, but we will not study this here. (For more on this, see the Notes and References at the end of this chapter.) Finally, an error procedure `yyerror` is needed to print error messages.

YACC/Bison is useful not only to the translator writer, but also to the language designer. Given a grammar, it provides a description of possible problems and ambiguities. However, to read this information we must have a more specific knowledge of bottom-up parsing techniques. The interested reader may consult the Notes and References at the end of this chapter.

6.7 Lexics vs. Syntax vs. Semantics

A context-free grammar typically includes a description of the tokens of a language by including the strings of characters that form the tokens in the grammar rules. For example, in the partial English grammar introduced in Section 6.2, the tokens are the English words "a," "the," "girl," "dog," "sees," and "pets," plus the "." character. In the simple integer expression grammar of Section 6.3, the tokens are the arithmetic symbols "1" and "*," the parentheses "(" and ")," and the digits 0 through 9. Specific details of formatting, such as the white-space conventions mentioned in Section 6.1, are left to the scanner and need to be stated as lexical conventions separate from the grammar.

Some typical token categories, such as literals or constants and identifiers, are not fixed sequences of characters in themselves, but are built up out of a fixed set of characters, such as the digits 0..9. These token categories can have their structure defined by the grammar, as for example the grammar rules for *number* and *digit* in the expression grammar. However, it is possible and even desirable to use a scanner to recognize these structures, since the full recursive power of a parser is not necessary, and the scanner can recognize them by a simple repetitive operation. Using a scanner makes the recognition of numbers and identifiers faster and simpler, and it reduces the size of the parser and the number of grammar rules.

To express the fact that a number in the expression grammar should be a token rather than represented by a nonterminal, we can rewrite the grammar of Figure 6.18 as in Figure 6.26.

expr → *term* { + *term*}
term → *factor* { * *factor* }
factor → (*expr*) | NUMBER

Figure 6.26 Numbers as tokens in simple integer arithmetic

By making the string NUMBER uppercase in the new grammar, we are saying that it is not a token to be recognized literally, but one whose structure is determined by the scanner. The structure of such tokens must then be specified as part of the lexical conventions of the language. As noted in Section 6.1, the structure of such tokens can be specified using regular expressions. However, many language designers choose to include the structure of such tokens as part of the grammar, with the understanding that an implementor may include these in the scanner instead of the parser. Thus, a description of a language using BNF, EBNF, or syntax diagrams may include not only the syntax but most of the lexical structure (or **lexics**) of a programming language as well. The boundary, therefore, between syntactic and lexical structure is not always clearly drawn, but depends on the point of view of the designer and implementor.

The same is true for syntax and semantics. We have been taking the approach that syntax is anything that can be defined with a context-free grammar and semantics is anything that cannot. However, many authors include properties that we would call semantic as syntactic properties of a language. Examples include such rules as declaration before use for variables and no redeclaration of identifiers within a procedure. These are rules that are context sensitive and cannot be written as context-free rules. Hence we prefer to think of them as semantic rather than syntactic rules.

Another conflict between syntax and semantics arises when languages require certain strings to be **predefined identifiers** rather than **reserved words**. Recall from Section 6.1 that reserved words are fixed strings of characters that are tokens themselves and that cannot be used as identifiers. By contrast, a predefined identifier is a fixed string that is given a predefined meaning in a language, but this meaning can be changed by redefining it within a program. As we have noted, the strings `if`, `while`, and `do` are reserved words in C, Java, and Ada. However, in Ada, all of the basic data type names, such as `integer` and `boolean`, are predefined identifiers that can be redefined. For example, the following declaration in Ada is perfectly legal:

```
integer: boolean;
```

After this declaration, the name `integer` has become a variable of type `boolean`. (You're not alone if you find this confusing.)

C, C++, and Java, on the other hand, make the basic types such as `int`, `double`, and `bool` (in C++) or `boolean` (in Java) reserved words that cannot be redefined. This is probably a better approach, since these names have a fixed semantics in every program that we would hardly want to change even if we could.

Finally, syntax and semantics can become interdependent in languages when semantic information must be used to distinguish ambiguous parsing situations. A simple situation where this occurs is in C, where type names and variable names must be distinguishable during parsing. Consider, for example, the C expression:

```
(x)-y
```

If `x` is a type name, this is a cast of the value `-y` to type `x`. However, if `x` is a variable name, then this is subtraction of `y` from `x`. Thus, a parser must have context information on identifiers available to disambiguate such situations. One might view this as a design flaw in the grammar of C. (Java has the same problem, but not Ada.) However, it is a relatively easy problem to overcome in practice, as you will see in later chapters.

6.8 Case Study: Building a Syntax Analyzer for TinyAda

We conclude this chapter by developing a parser for a small language that is similar in some ways to the Ada programming language. This little language, called TinyAda, is large enough to illustrate the syntactic features of many high-level languages, yet small enough to require a medium-sized programming project. In this section, we inaugurate the project by discussing a simple syntax analyzer. The next few chapters revisit the project to add code to analyze some semantics as well.

An EBNF grammar for TinyAda is shown in Figure 6.27. Note the semantic qualifiers, such as <type>, enclosed in angle brackets. You can ignore these for now, but we will return to them in the next few chapters.

```
In the following grammar

    =    means "is defined as"
    " "  enclose literal items
    [ ]  enclose items which may be omitted
    { }  enclose items which may appear zero or more times
    |    indicates a choice
    ( )  are used for grouping required choices
    < >  enclose semantic qualifications

subprogramBody =
         subprogramSpecification "is"
         declarativePart
         "begin" sequenceOfStatements
         "end" [ <procedure>identifier ] ";"

declarativePart = { basicDeclaration }

basicDeclaration = objectDeclaration | numberDeclaration
         | typeDeclaration | subprogramBody

objectDeclaration =
         identifierList ":" typeDefinition ";"

numberDeclaration =
         identifierList ":" "constant" ":=" <static>expression ";"

identifierList = identifier { "," identifier }

typeDeclaration = "type" identifier "is" typeDefinition ";"

typeDefinition = enumerationTypeDefinition | arrayTypeDefinition
         | range | <type>name

range = "range " simpleExpression ".." simpleExpression
```

Figure 6.27 An EBNF grammar for TinyAda (*continues*)

(*continued*)

```
index = range | <type>name

enumerationTypeDefinition = "(" identifierList ")"

arrayTypeDefinition = "array" "(" index { "," index } ")" "of" <type>name

subprogramSpecification = "procedure" identifier [ formalPart ]

formalPart = "(" parameterSpecification { ";" parameterSpecification } ")"

parameterSpecification = identifierList ":" mode <type>name

mode = [ "in" ] | "in" "out" | "out"

sequenceOfStatements = statement { statement }

statement = simpleStatement | compoundStatement

simpleStatement = nullStatement | assignmentStatement
         | procedureCallStatement | exitStatement

compoundStatement = ifStatement | loopStatement

nullStatement = "null" ";"

assignmentStatement = <variable>name ":=" expression ";"

ifStatement =
         "if" condition "then" sequenceOfStatements
         { "elsif" condition "then" sequenceOfStatements }
         [ "else" sequenceOfStatements ]
         "end" "if" ";"

loopStatement =
         [ iterationScheme ] "loop" sequenceOfStatements "end" "loop" ";"

iterationScheme = "while" condition

exitStatement = "exit" [ "when" condition ] ";"

procedureCallStatement = <procedure>name [ actualParameterPart ] ";"

actualParameterPart = "(" expression { "," expression } ")"
```

Figure 6.27 An EBNF grammar for TinyAda (*continues*)

(*continued*)

```
condition = <boolean>expression

expression = relation { "and" relation } | { "or" relation }

relation = simpleExpression [ relationalOperator simpleExpression ]

simpleExpression =
         [ unaryAddingOperator ] term { binaryAddingOperator term }

term = factor { multiplyingOperator factor }

factor = primary [ "**" primary ] | "not" primary

primary = numericLiteral | stringLiteral | name | "(" expression ")"

name = identifier [ indexedComponent ]

indexedComponent = "(" expression { "," expression } ")"

relationalOperator = "=" | "/=" | "<" | "<=" | ">" | ">="

binaryAddingOperator = "+" | "-"

unaryAddingOperator = "+" | "-"

multiplyingOperator = "*" | "/" | "mod"
```

Figure 6.27 An EBNF grammar for TinyAda

As you can see from the grammar, TinyAda includes several kinds of declarations (constant, variable, type, and procedure), statements (assignment, selection, loop, and procedure call), and expressions. The top-level phrase in this little language, called `subprogramBody`, is a procedure declaration, which might serve the same role for a TinyAda program as the `main` function of the C language does in the execution of a program. Note also that the rules for declarations, statements, and expressions are indirectly recursive, allowing for nested declarations, statements, and expressions.

Figure 6.28 shows a short TinyAda program, which includes two procedure declarations and several constant declarations, type declarations, variable declarations, statements, and expressions.

```
procedure TEST is

    COLUMN_MAX : constant := 10;
    ROW_MAX : constant := COLUMN_MAX;
    type COLUMN_INDEX is range 1..COLUMN_MAX;
    type ROW_INDEX is range 1..ROW_MAX;
    type MATRIX is array(COLUMN_INDEX, ROW_INDEX) of INTEGER;
```

Figure 6.28 A TinyAda program (*continues*)

(*continued*)

```
A : MATRIX;
I : INTEGER;

procedure INIT_MATRIX(X : in INTEGER; Y : out MATRIX) is

     I, J : INTEGER;

     begin
     I := 1;
     while I <= COLUMN_MAX loop
          J := 1;
          while J <= ROW_MAX loop
               Y(I, J) := X;
               J := J + 1;
               end loop;
          I := I + 1;
          end loop;
     end INIT_MATRIX;

begin
I := 1;
INIT_MATRIX(I, A);
end TEST;
```

Figure 6.28 A TinyAda program

Although TinyAda is not case sensitive, we use lowercase for reserved words and uppercase for identifiers in program examples.

The initial version of our parser is known as a **parsing shell**. The purpose of this program is to apply the grammar rules to a source program and report any syntax or lexical errors. The parser is a mere shell, because it simply checks whether the tokens in each phrase are of the correct types, without performing any additional analysis. In later chapters, we show how to fill in this shell with additional mechanisms for semantic analysis.

The complete program consists of a scanner and a parser, but it is structured in terms of four cooperating components, each of which is implemented as a Java class. Here are their names, roles, and responsibilities:

1. The `Token` class represents tokens in the language. For simple syntax analysis, a token object need only include a code for the token's type, such as was used in earlier examples in this chapter. However, a token can include other attributes, such as an identifier name, a constant value, and a data type, as we will see in later chapters.
2. The `Chario` class (short for character I/O) converts the source program's text into a stream of characters for the scanner, thus enabling the scanner to focus on lexical analysis rather than low-level text processing. The `Chario` class also handles the output of any error messages.

3. The `Scanner` class recognizes and generates tokens in a stream of characters and returns these tokens to the parser. The `Scanner` class also detects any lexical errors.
4. The `Parser` class uses a recursive descent strategy to recognize phrases in a stream of tokens. Unlike the scanner, which continues to generate tokens after lexical errors, the parser halts execution upon encountering the first syntax error in a source program.

The flow of data among these classes is shown in Figure 6.29.

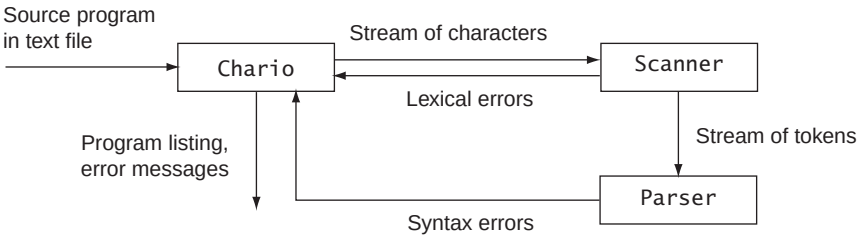


Figure 6.29 Data flow in the TinyAda syntax analyzer

Complete source code for the `Token`, `Chario`, and `Scanner` classes and a partially completed `Parser` class can be found on the book's Web site. Also available are two user interfaces for the program, one that is terminal-based and one that is GUI-based. If you want to execute a completed version of the parser, executable `jar` files for both versions are available as well. In this section, we focus on the development of several parsing methods, and leave the rest as exercises for the reader.

The `Parser` object receives tokens from the `Scanner` object and sends error messages to the `Chario` object. At program startup, the parser's constructor receives these two objects as parameters and saves references to them in instance variables. The parser also includes instance variables for the current token in the input stream and for several sets of tokens. These sets include some of TinyAda's operator symbols and the tokens that begin various declarations and statements in the language (described earlier in this chapter as First Sets). Figure 6.30 shows the Java code for the parser's instance variable declarations and their initialization.

```
public class Parser extends Object{

    private Chario chario;
    private Scanner scanner;
    private Token token;

    private Set<Integer> addingOperator,
                         multiplyingOperator,
                         relationalOperator,
                         basicDeclarationHandles,
                         statementHandles;
```

Figure 6.30 The constructor and data for the Parser class (*continues*)

(*continued*)

```
    public Parser(Chario c, Scanner s){
        chario = c;
        scanner = s;
        initHandles();
        token = scanner.nextToken();
    }

    private void initHandles(){
        addingOperator = new HashSet<Integer>();
        addingOperator.add(Token.PLUS);
        addingOperator.add(Token.MINUS);
        multiplyingOperator = new HashSet<Integer>();
        multiplyingOperator.add(Token.MUL);
        multiplyingOperator.add(Token.DIV);
        multiplyingOperator.add(Token.MOD);
        relationalOperator = new HashSet<Integer>();
        relationalOperator.add(Token.EQ);
        relationalOperator.add(Token.NE);
        relationalOperator.add(Token.LE);
        relationalOperator.add(Token.GE);
        relationalOperator.add(Token.LT);
        relationalOperator.add(Token.GT);
        basicDeclarationHandles = new HashSet<Integer>();
        basicDeclarationHandles.add(Token.TYPE);
        basicDeclarationHandles.add(Token.ID);
        basicDeclarationHandles.add(Token.PROC);
        statementHandles = new HashSet<Integer>();
        statementHandles.add(Token.EXIT);
        statementHandles.add(Token.ID);
        statementHandles.add(Token.IF);
        statementHandles.add(Token.LOOP);
        statementHandles.add(Token.NULL);
        statementHandles.add(Token.WHILE);
    }

    // Other parsing methods go here
}
```

Figure 6.30 The constructor and data for the Parser class

After the main application program instantiates the parser, it runs the method `parse` on this object to commence the syntax analysis. This method will either return normally if the source program has no syntax errors, or throw an exception when the first syntax error is encountered. The utility methods `accept` and `fatalError` collaborate to check for an expected token and throw an exception if an error occurs. Here is the code for these three `Parser` methods:

```
public void parse(){
    subprogramBody();
    accept(Token.EOF, "extra symbols after logical end of program");
}

private void accept(int expected, String errorMessage){
    if (token.code != expected)
       fatalError(errorMessage);
    token = scanner.nextToken();
}

private void fatalError(String errorMessage){
    chario.putError(errorMessage);
    throw new RuntimeException("Fatal error");
}
```

Note that the `accept` method calls the `Scanner` method `nextToken` to advance the scan to the next token if the current token's code is the expected one. The `fatalError` method calls the `Chario` method `putError` to print an error message before throwing the exception. Finally note that the `parse` method calls the `subprogramBody` method, which corresponds to the top-level rule in the TinyAda grammar. After processing a complete source program, this method calls the `accept` method to ensure that the end of the source file has been reached.

All that remains to implement the parser is to translate the grammar rules into parsing methods. We mentioned earlier that some coders find it easier first to translate the EBNF rules to syntax diagrams, which then can serve as flow charts for the parsing methods. This option is left as an exercise for the reader. Before we explore the implementation of several methods, we need to make several more points about their design:

1. With few exceptions, each parsing method is responsible for analyzing a phrase in the source program that corresponds to exactly one rule in the grammar. Occasionally, when the first few tokens in a phrase appear in two rules (see `numberDeclaration` and `objectDeclaration` in the grammar), it will be convenient to merge the processing for both rules into a single method.
2. Each parsing method assumes that the parser's `token` variable refers to the first token to be processed in that parsing method's grammar rule. This is why, before parsing starts, the `Scanner` method `nextToken` must be run once, to obtain the first token for the parsing method `subprogramBody`.
3. Each parsing method leaves behind the next token following the phrase just processed. The `accept` method is designed to accomplish that most of the time.
4. Each parsing method expects no parameters and returns no value. With the exception of the current token, the parsing methods do not need to share any information, because the rules for simple syntax analysis are context-free. In later chapters, we will enable the parsing methods to communicate in order to share the context-sensitive information needed for semantic analysis.

To give the flavor of the coding process, let's develop several methods from the rules. The first one, `subprogramBody`, processes the phrase defined by the top-level rule. The EBNF rule appears in a comment above the method header.

```
/*
subprogramBody =
         subprogramSpecification "is"
         declarativePart
         "begin" sequenceOfStatements
         "end" [ <procedure>identifier ] ";"
*/
private void subprogramBody(){
    subprogramSpecification();
    accept(Token.IS, "'is' expected");
    declarativePart();
    accept(Token.BEGIN, "'begin' expected");
    sequenceOfStatements();
    accept(Token.END, "'end' expected");
    if (token.code == Token.ID)
         token = scanner.nextToken();
    accept(Token.SEMI, "semicolon expected");
}
```

As you can see, this method usually calls the `accept` method to process a terminal symbol (or token) and calls another parsing method to process a nonterminal symbol (or phrase). There is one `if` statement, which processes the optional identifier enclosed in `[]` symbols in the EBNF.

Our next two methods collaborate to process basic declarations.

```
/*
declarativePart = { basicDeclaration }
*/
private void declarativePart(){
    while (basicDeclarationHandles.contains(token))
        basicDeclaration();
}

/*
basicDeclaration = objectDeclaration | numberDeclaration
                 | typeDeclaration | subprogramBody
*/
private void basicDeclaration(){
    switch (token.code){
        case Token.ID:
            numberOrObjectDeclaration();
            break;
```

(continues)

(*continued*)

```
            case Token.TYPE:
                typeDeclaration();
                break;
             case Token.PROC:
                subprogramBody();
                break;
            default: fatalError("error in declaration part");
        }
    }
```

A declarative part consists of zero or more basic declarations. The `declarativePart` method must, therefore, call the `basicDeclaration` method within a loop. This loop runs as long as the current token is a member of the set of basic declaration handles, which are the four different types of tokens that can begin a basic declaration. The `basicDeclaration` method then decides which type of basic declaration to process, based on the handle or current token. The actual processing of each declaration is, naturally, a matter of passing the buck on to another parsing method.

The last parsing method that we show here belongs to the processing of expressions. A simple expression consists of an optional unary adding operator followed by a term. This term is then followed by zero or more pairs of binary adding operators and terms.

```
    /*
    simpleExpression =
             [ unaryAddingOperator ] term { binaryAddingOperator term }
    */
    private void simpleExpression(){
        if (addingOperator.contains(token))
            token = scanner.nextToken();
        term();
        while (addingOperator.contains(token)){
            token = scanner.nextToken();
            term();
        }
    }
```

Once again, an option in the grammar translates to an `if` statement in the method. In this case, there are actually two options (the two unary adding operators), but the code is simplified by testing the current token for membership in a set of tokens. The same technique is used to control the entry into the `while` loop to process the zero or more items following the first term. Note also that after each optional token is found, it is consumed from the input stream by calling the `Scanner` method `nextToken`. This will guarantee that the parsing method `term` sees the appropriate token when it begins its processing.

The development of the remaining parsing methods is left as an exercise for the reader. Although there is a large number of these methods, their structure is simple and fairly regular. You can write them in a top-down fashion, and use stubs for many of the methods if you want to test your code before

completing them all. If you do that, be sure to use example TinyAda programs that exercise only the methods you have completed. You should then test any completed methods with TinyAda programs that exercise their error-detection capabilities.

Exercises

6.1 **(a)** The C programming language distinguishes character constants from string constants by using single quotation marks for characters and double quotation marks for strings. Thus, `'c'` is the character c, while `"c"` is a string of length 1 consisting of the single character c. Why do you think this distinction is made? Is it useful?

(b) Python, on the other hand, uses double quotation marks for both characters and strings (thus, `"c"` is either a character or string, depending on context). Discuss the advantages and disadvantages of these two approaches.

6.2 Devise a test in one or more of the following languages to determine whether comments are considered white space: **(a)** C, **(b)** Java, **(c)** Ada, and **(d)** Python. What is the result of performing your test?

6.3 Discuss the pros and cons of ignoring or requiring "white space" (i.e., blanks, end-of-lines, and tabs) when recognizing tokens.

6.4 Many programming languages (like C) do not allow nested comments. Why is this requirement made? Modula-2 and a few other languages do allow nested comments. Why is this useful?

6.5 **(a)** Describe the strings that are represented by the regular expression:

```
[0-9]+((E|e)(\+|\-)?[0-9]+)?
```

(b) Write a regular expression for C identifiers consisting of letters, digits, and the underscore character '_', and starting with a letter or underscore.

6.6 **(a)** Numeric constants may be signed or unsigned in a programming language (e.g., `123` is an unsigned integer constant, but `-123` is a signed integer constant). However, there is a problem associated with signed constants. Describe it. (*Hint:* Consider expressions involving subtraction.)

(b) Can you think of a solution to the problem of part (a)?

6.7 Rewrite the scanner program of Figure 6.1 by packaging the code for scanning an integer literal as a function.

6.8 In the simplified English grammar of Figure 6.2, there are only finitely many legal sentences. How many are there? Why?

6.9 **(a)** Suppose that white space was completely ignored (as in FORTRAN) in the grammar of Figure 6.2, so that sentences could be written as, for example, `"thegirlseesadog."` Can this grammar still be parsed? Explain.

(b) Answer the previous question again, assuming that the nouns `theorist` and `orator` were added to the grammar.

6.10 Translate the BNF rules of Figure 6.6 into EBNF.

6.11 Add subtraction and division to the **(a)** BNF, **(b)** EBNF, and **(c)** syntax diagrams of simple integer arithmetic expressions (Figures 6.9, 6.10, and 6.11). Be sure to give them the appropriate precedences.

6.12 Add the integer remainder and power operations to (a) the arithmetic BNF or (b) EBNF of Figures 6.9 or 6.10. Use `%` for the remainder operation and `^` for the power operation. Recall that the remainder operation is left-associative and has the same precedence as multiplication, but that power is right-associative (and has greater precedence than multiplication, so `2 ^ 2 ^ 3` = 256, not 64).

6.13 Unary minuses can be added in several ways to the arithmetic expression grammar of Figure 6.17 or Figure 6.18. Revise the BNF and EBNF for each of the cases that follow so that it satisfies the stated rule:

(a) At most, one unary minus is allowed in each expression, and it must come at the beginning of an expression, so `-2 + 3` is legal (and equals 1) and `-2 + (-3)` is legal, but `-2 + -3` is not.

(b) At most, one unary minus is allowed before a number or left parenthesis, so `-2 + -3` and `-2 * -3` are legal, but `--2` and `-2 + --3` are not.

(c) Arbitrarily many unary minuses are allowed before numbers and left parentheses, so everything above is legal.

6.14 Using the grammar of Figure 6.17, draw parse trees and abstract syntax trees for the arithmetic expressions:

(a) `((2))`
(b) `3 + 4 * 5 + 6 * 7`
(c) `3 * 4 + 5 * 6 + 7`
(d) `3 * (4 + 5) * (6 + 7)`
(e) `(2 + (3 + (4 + 5)))`

6.15 Finish writing the pseudocode for a recursive-descent recognizer for the English grammar of Section 6.2 that was begun in Section 6.6.

6.16 Translate the pseudocode of the previous exercise into a working recognizer program in C or another programming language of your choice. (*Hint:* This will require a scanner to separate the text into token strings.)

6.17 Revise the program of the previous exercise so that it randomly *generates* legal sentences in the grammar. (This will require the use of a random number generator.)

6.18 Modify the recursive-descent calculator program of Figure 6.24 to use the grammar of Figure 6.26 and the scanner of Figure 6.1 (so that the calculator also skips blanks appropriately).

6.19 Add subtraction and division to either `(a)` the calculator program of Figure 6.24, or `(b)` your answer to the previous exercise.

6.20 Add the remainder and power operations as described in Exercise 6.12 to **(a)** the program of Figure 6.24, **(b)** your answer to Exercise 6.18, or **(c)** your answer to Exercise 6.19.

6.21 Rewrite the program of Figure 6.24 or your answer to any of the previous three exercises (except for the remainder operation) to produce floating-point answers, as well as accept floating-point constants using the following revised grammar rules (in EBNF):

. . . (rules for *expr* and *term* as before)
factor → `(` *expr* `)` | *decimal-number*
decimal-number → *number* [`.` *number*]
. . . (rules for *number* and *digit* as before)

6.22 To eliminate the left recursion in the simple integer arithmetic grammar, one might be tempted to write:

$$expr \rightarrow term + term$$

Why is this wrong?

6.23 Modify the YACC/Bison program of Figure 6.25 to use the grammar of Figure 6.26 and the scanner of Figure 6.1 (so that the calculator also skips blanks appropriately). This will require some additions to the YACC definition and changes to the scanner, as follows. First, YACC already has a global variable similar to the `numval` variable of Figure 6.1, with the name `yylval`, so `numval` must be replaced by this variable. Second, YACC must define the tokens itself, rather than use an `enum` such as in Figure 6.1; this is done by placing a definition as follows in the YACC definition:

```
%{ /* code to insert at the beginning of the parser */
%}
#token NUMBER PLUS TIMES ...
%%
etc....
```

6.24 Add subtraction and division to either **(a)** the YACC/Bison program of Figure 6.25, or **(b)** your answer to the previous exercise.

6.25 Add the remainder and power operations as described in Exercise 6.12 to **(a)** the YACC/Bison program of Figure 6.25, **(b)** your answer to Exercise 6.23, or **(c)** your answer to Exercise 6.24.

6.26 Repeat Exercise 6.21 for the YACC/Bison program of Figure 6.25. This will require that the type of the parsing result be redefined to `double`, and this requires the insertion of a new definition into the YACC definition as follows:

```
%{
#define YYSTYPE double
...
%}
...
%%
etc....
```

6.27 **(a)** Explain why the code for a command in Figure 6.24 (lines 23–29) does not call `match('\n')`.
(b) Explain what the behavior of the code of Figure 6.24 *would* be if command *did* call `match('\n')`.

6.28 The YACC/Bison code of Figure 6.25 could be simplified by using a newline to end the parse, as in the code of Figure 6.24. We would simply change the grammar rule for `command` (Figure 6.25, line 5) to remove the newline:

```
command : expr { printf("The result is: %d\n",$1); }
```

and change the `yylex` procedure (Figure 6.25, lines 35–42) to:

```
int yylex(void){
    int c;
    c = getchar();
    if (c == '\n') return 0; /* newline will end parse */
    return c;
}
```

Unfortunately, this changes the behavior of the YACC/Bison parser so that it is no longer the same as that of the recursive-descent version.

(a) Describe the different behavior. (*Hint:* Consider a missing operator, as in "`3 4`".)

(b) Explain why you think this different behavior occurs.

6.29 Capitalization of articles at the beginning of a sentence was viewed as a context sensitivity in the text. However, in the simple English grammar of Figure 6.2, capitalization can be achieved by adding only context-free rules. How would you do this?

6.30 It was stated in this chapter that it is not possible to include in the BNF description the rule that there should be no redeclaration of variables. Strictly speaking, this is not true if only finitely many variable names are allowed. Describe how one could include the requirement that there be no redeclaration of variables in the grammar for a language, if only finitely many identifiers are allowed. Why isn't it a good idea?

6.31 The text notes that it is more efficient to let a scanner recognize a structure such as an unsigned integer constant, which is just a repetition of digits. However, an expression is also just a repetition of terms and pluses:

$$expr \rightarrow term \ \{ + term \}$$

Why can't a scanner recognize all expressions?

6.32 Write a BNF description for a statement-sequence as a sequence of statements separated by semicolons. (Assume that statements are defined elsewhere.) Then, translate your BNF into EBNF and/or syntax diagrams.

6.33 Write a BNF description for a statement-sequence as a sequence of statements in which each statement is terminated by a semicolon. (Assume that statements are defined elsewhere.) Then, translate your BNF into EBNF and/or syntax diagrams.

6.34 C uses the semicolon as a statement terminator (as in the previous exercise) but also allows a statement to be empty (that is, consisting of only a semicolon), so that the following is a legal C program:

```
main(){
    ;;;;;;;;
    return 0;
}
```

Discuss the advantages and disadvantages of this.

6.35 **(a)** List the predefined identifiers in Python and describe their meanings.
(b) Does C have predefined identifiers? Explain.
(c) Does Ada have predefined identifiers? Explain.

6.36 **(a)** One measure of the complexity of a language is the number of reserved words in its syntax. Compare the number of reserved words in C, C++, Ada, and Java.
(b) One could argue that the preceding comparison is misleading because of the use of predefined identifiers in Pascal and standard (predefined) libraries in C++, Java, and Ada. Discuss the pros and cons of adding the number of predefined identifiers or the number of library identifiers into the comparison.

6.37 Here is a legal Pascal program:

```
program yecch;
    var true, false: boolean;
    begin
    true := 1 = 0;
    false := true;
    ...
    ...
    end.
```

What values do `true` and `false` have in this program? What design principle does this violate? Is this program possible in Ada? In C? In C++? In Java?

6.38 Is it possible to have a language without any reserved words? Discuss.

6.39 Some languages use format to distinguish the beginning and ending of structures, as follows:

```
if (x == 0)
    (* all indented statements here are part of
    the if *)
else
    (* all indented statements here are part of
    the else *)
(* statements that are not indented are outside the else *)
```

Discuss the advantages and disadvantages of this for **(a)** writing programs in the language, and **(b)** writing a translator. (This rule, sometimes called the **Offside rule**, is used in Haskell; see Landin [1966].)

6.40 A number is defined in the grammar of Figure 6.17 using a left-recursive rule. However, it could also be defined using a right-recursive rule:

$$number \rightarrow digit\ number \mid digit$$

Which is better, or does it matter? Why?

6.41 In Exercise 6.21, a decimal constant was defined using the rule:

$$decimal\text{-}number \rightarrow number\ [\ .\ number\ ]$$

This causes a problem when computing the value of the fractional part of a decimal number (to the right of the decimal point), because of the left-recursive rule for a *number*.

(a) Describe the problem. Is this problem as significant when a scanner recognizes a decimal number as it is when a parser does the recognition?

(b) A solution to the problem of part (a) is to write the fractional number as a right-recursive rule, as in the previous exercise. Write out BNF rules expressing this solution, and explain why it is better.

(c) Implement your solution of part (b) in the code of Exercise 6.21 or Exercise 6.26.

6.42 Given the following BNF:

$$expr \rightarrow \texttt{(}\ list\ \texttt{)} \mid \texttt{a}$$
$$list \rightarrow list\ \texttt{,}\ expr \mid expr$$

(a) Write EBNF rules and/or syntax diagrams for the language.

(b) Draw the parse tree for `((a, a), a, (a))`.

(c) Write a recursive-descent recognizer for the language.

6.43 One way of defining the abstract syntax for the expressions of Figure 6.17 is as follows:

$$expr \rightarrow expr + expr \mid expr * expr \mid number$$
$$number \rightarrow digit\ \{\ digit\ \}$$
$$digit \rightarrow \texttt{0} \mid \texttt{1} \mid \texttt{2} \mid \texttt{3} \mid \texttt{4} \mid \texttt{5} \mid \texttt{6} \mid \texttt{7} \mid \texttt{8} \mid \texttt{9}$$

(a) Why are there no precedences or associativities expressed in this grammar?

(b) Why are there no parentheses in this grammar?

(c) The rule for *number* is written in EBNF in this grammar, yet we stated in the text that this obscures the structure of the parse tree. Why is the use of EBNF legitimate here? Why did we not use EBNF in the rule for *expr*?

6.44 Write data type definitions in C, C++, or Java for an *expr* abstract syntax tree, as given in the text and the previous exercise.

6.45 Some languages, like ML and Haskell, allow the abstract syntax tree to be defined in a way that is virtually identical to syntax rules. Write data type definitions in ML or Haskell that follow the abstract syntax for *expr* described in Exercise 6.43.

6.46 Compute First and Follow sets for the nonterminals of the grammar of Exercise 6.42.

6.47 Show that any left-recursive grammar rule does not satisfy the first condition for predictive parsing.

6.48 Show that the following grammar does not satisfy the second rule of predictive parsing:

$$stmt \rightarrow if\text{-}stmt \mid \texttt{other}$$
$$if\text{-}stmt \rightarrow \texttt{if}\ stmt\ [\texttt{else}\ stmt\ ]$$

6.49 Given the following grammar in EBNF:

$$expr \rightarrow \texttt{(}\ list\ \texttt{)} \mid \texttt{a}$$
$$list \rightarrow expr\ [\ list\ ]$$

(a) Show that the two conditions for predictive parsing are satisfied.

(b) Write a recursive-descent recognizer for the language.

6.50 In Section 6.7, the definition of Follow sets was somewhat nonstandard. More commonly, Follow sets are defined for nonterminals only rather than for strings, and optional structures are accommodated by allowing a nonterminal to become empty. Thus, the grammar rule:

if-statement → `if (` *expression* `)` *statement*
| `if (` *expression* `)` *statement* `else` *statement*

is replaced by the two rules:

if-statement → `if (` *expression* `)` *statement else-part*
else-part → `else` *statement* | ε

where the symbol ε (Greek epsilon) is a new metasymbol standing for the empty string. A nonterminal *A* can then be recognized as optional if it either becomes ´ directly, or there is a derivation beginning with *A* that derives ´. In this case, we add ´ to First(*A*). Rewrite the second condition for predictive parsing using this convention, and apply this technique to the grammars of Exercises 6.42 and 6.49.

6.51 The second condition for predictive parsing, involving Follow sets, is actually much stronger than it needs to be. In fact, for every optional construct β:

$$A \rightarrow \alpha\ [\ \beta\ ]\ \gamma$$

a parser only needs to be able to tell when β is present and when it isn't. State a condition that will do this without using Follow sets.

6.52 The second condition for predictive parsing is suspect (even more than the previous exercise indicates) because there is a natural disambiguating rule in those cases where it is not satisfied. What is it? Explain its effect when parsing the grammar of Exercise 6.48.

6.53 Given the following grammar in BNF:

string → *string string* | `a`

this corresponds to the set equation:

$$\mathbf{S} = \mathbf{SS} \cup \{\mathtt{a}\}$$

Show that the set $\mathbf{S}_0 = \{\mathtt{a}, \mathtt{aa}, \mathtt{aaa}, \mathtt{aaaa}, \ldots\}$ is the smallest set satisfying the equation. (*Hint:* First show that $\mathbf{S}_0$ does satisfy the equation by showing set inclusion in both directions. Then, using induction on the length of strings in $\mathbf{S}_0$, show that, given any set **S'** that satisfies the equation, $\mathbf{S}_0$ must be contained in **S'**.)

6.54 According to Wirth [1976], data structures based on syntax diagrams can be used by a "generic" recursive-descent parser that will parse any set of grammar rules that satisfy the two conditions for predictive parsing. A suitable data structure is given by the following C declarations:

```
typedef int TokenKind;
typedef struct RuleRec* RulePtr;
typedef struct RuleRec{
    RulePtr next, other;
```

(continues)

(continued)

```
        int isToken;
        union{
            TokenKind name;
            RulePtr rule;
        } token_or_rule;
    } RuleRec;
```

The `next` field is used to point to the next item in the grammar rule, and the `other` field is used to point to choices given by the | metasymbol. Thus, the data structure for the grammar rule:

$$factor \rightarrow (\ expr\ )\ |\ number$$

from Figure 6.18 would look like the one shown in Figure 6.31. The fields of this data structure are shown in Figure 6.32.

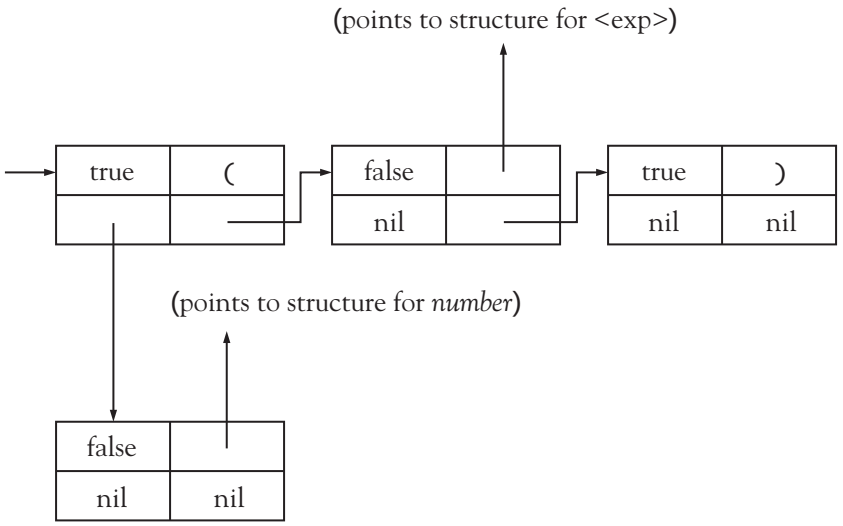


Figure 6.31: Data structure for the grammar rule *factor* → (*expr*) | *number*

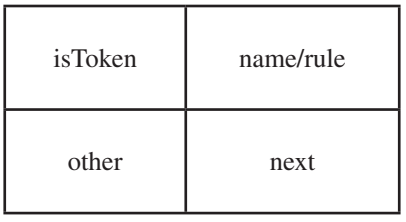


Figure 6.32: Fields of the data structure shown in Figure 6.31

(a) Draw the data structures for the rest of the grammar rules in Figure 6.18. (*Hint:* For repetitive and optional structures you will need a special token to represent the empty string.)

(b) Write a generic parse procedure that uses these data structures to recognize an input string, assuming the existence of *getToken* and *error* procedures.

(c) Write a parser-generator that reads EBNF rules (either from a file or standard input) and generates the preceding data structures.

6.55 Draw syntax diagrams that express the rules in the EBNF grammar for TinyAda of Figure 6.15.

6.56 Complete the implementation of the TinyAda parser discussed in Section 6.8.

Notes and References

Many of the topics discussed in this chapter are treated in more detail in Aho, Sethi, and Ullman [1986] and Louden [2004]. An overview similar to this chapter, but with a little more depth, can be found in Louden [1997b]. EBNF and syntax diagrams are discussed in Wirth [1976] and in Horowitz [1984]. Context-free grammars and regular expressions are explained in Hopcroft and Ullman [1979] and Sipser [1997]. The original description of context-free grammars is in Chomsky [1956]. An early use of BNF in the description of Algo160 is in Naur [1963a]. A description of general algorithms to compute First and Follow sets, and to perform left-recursion removal, are found in Aho, Sethi, and Ullman [1986], Louden [1997], and Wirth [1976]. Our description of Follow sets is for EBNF grammars and is slightly nonstandard; see Exercise 6.50. YACC is described in more detail in Johnson [1975] and LEX in Lesk [1975]. A BNF description of C can be found in Kernighan and Ritchie [1988], for C++ in Stroustrup [1997], for Java in Gosling et al. [2000], for Pascal in Cooper [1983], and for Ada in Horowitz [1987]. All of these BNF descriptions can be found in many other places as well.

CHAPTER

Basic Semantics

CHAPTER 7

In Chapter 1, we made the distinction between the syntax of a programming language (what the language constructs look like) and its semantics (what the language constructs actually do). In Chapter 6, you saw how the syntactic structure of a language can be precisely specified using context-free grammar rules in Backus-Naur form (BNF). In this chapter, we will introduce the semantics of programming languages in more detail.

Specifying the semantics of a programming language is a more difficult task than specifying its syntax. This is not surprising, given that semantics has to do with meaning, which can be subtle and open to interpretation. There are several ways to specify semantics:

1. *Language reference manual*. This is the most common way to specify semantics. The expanding use of English descriptions has resulted in clearer and more precise reference manuals, but they still suffer from the lack of precision inherent in natural language descriptions and also may have omissions and ambiguities.
2. *Defining translator*. The advantage of specifying semantics by defining a translator is that questions about a language can be answered by experiment (as in chemistry or physics). A drawback is that questions about program behavior cannot be answered in advance—we must execute a program to discover what it does. Another drawback is that bugs and machine dependencies in the translator become parts of the language semantics, possibly unintentionally. Also, the translator may not be portable to all machines and may not be generally available.
3. *Formal definition.* Formal, mathematical methods are precise, but they are also complex and abstract, and require study to understand. Different formal methods are available, with the choice of method depending on its intended use. Perhaps the best formal method to use for the description of the translation and execution of programs is denotational semantics, which describes semantics using a series of functions.

In this chapter, we will specify semantics by using a hybrid of the type of informal description you might find in a manual combined with the simplified functions used in denotational descriptions. We will provide abstractions of the operations that occur during the translation and execution of programs in general, whatever the language, but we will concentrate on the details of Algol-like languages, such as C, C++, Java, Pascal, and Ada. We conclude this chapter with a case study on the semantic analysis of programs in the TinyAda language. More formal methods of describing semantics are studied in Chapter 12.

7.1 Attributes, Binding, and Semantic Functions

A fundamental abstraction mechanism in any programming language is the use of **names**, or **identifiers**, to denote language entities or constructs. In most languages, variables, procedures, and constants can have names assigned by the programmer. A fundamental step in describing the semantics of a language is to describe the conventions that determine the meaning of each name used in a program.

In addition to names, a description of the semantics of most programming languages includes the concepts of **location** and **value**. Values are any storable quantities, such as the integers, the reals, or even array values consisting of a sequence of the values stored at each index of the array. Locations are places values can be stored. Locations are like addresses in the memory of a specific computer, but we can think of them more abstractly than that. To simplify, we can think of locations as being numbered by integers starting at 0 and going up to some maximum location number.

The meaning of a name is determined by the properties, or **attributes**, associated with the name. For example, the C declaration[1]

```
const int n = 5;
```

makes `n` into an integer constant with value 5. That is, it associates to the name `n` the data type attribute "integer constant" and the value attribute 5. (In C, the constant attribute—that `n` can never change its value—is part of the data type; in other languages this may be different.) The C declaration

```
int x;
```

associates the attribute "variable" and the data type "integer" to the name `x`. The C declaration

```
double f(int n){
...
}
```

associates the attribute "function" and the following additional attributes to the name `f`:

1. The number, names, and data types of its parameters (in this case, one parameter with name `n` and data type "integer")
2. The data type of its returned value (in this case, "double")
3. The body of code to be executed when `f` is called (in this case, we have not written this code but just indicated it with three dots)

Declarations are not the only language constructs that can associate attributes to names. For example, the assignment:

```
x = 2;
```

associates the new attribute "value 2" to the variable `x`. And, if `y` is a pointer variable[2] declared as:

```
int* y;
```

the C++ statement

```
y = new int;
```

[1] In C, C++, Pascal, and Ada, some declarations are called "definitions"; the distinctions will be explained later.
[2] Pointers are discussed more fully in Sections 8.5 and 8.7.

allocates memory for an integer variable (that is, associates a location attribute to it) and assigns this location to `*y`, that is, associates a new value attribute to `y`.

The process of associating an attribute with a name is called **binding**. In some languages, constructs that cause values to be bound to names (such as the C constant declaration earlier) are in fact called bindings rather than declarations. For example, in the ML code

```
let val x = 2; val y = 3 in x + y
```

this is a `let` expression which binds the value 2 to `x` and the value 3 to `y` (these are called let-bindings).

In purely functional languages such as Haskell, a value can be bound to a name only once. Thus, in these languages there is no need for a separate location binding, to store a value that can be reset later in the program by an assignment statement.

An attribute can be classified according to the time during the translation/execution process when it is computed and bound to a name. This is called the **binding time** of the attribute. Binding times can be classified into two general categories: **static binding** and **dynamic binding**. Static binding occurs prior to execution, while dynamic binding occurs during execution. An attribute that is bound statically is a static attribute, while an attribute that is bound dynamically is a dynamic attribute.

Languages differ substantially in which attributes are bound statically and which are bound dynamically. Often, functional languages have more dynamic binding than imperative languages. Binding times can also depend on the translator. Interpreters, for example, can translate and execute code simultaneously and so can establish most bindings dynamically, while compilers can perform many bindings statically. To make the discussion of attributes and binding independent of such translator issues, we usually refer to the binding time of an attribute as the earliest time that the language rules permit the attribute to be bound. Thus, an attribute that *could* be statically bound, but that is dynamically bound by a translator, is still referred to as a static attribute.

As examples of binding times, consider the previous examples of attributes. In the declaration

```
const int n = 2;
```

the value 2 is bound statically to the name `n`, and in the declaration

```
int x;
```

the data type "integer" is bound statically to the name `x`. A similar statement holds for the function declaration.

On the other hand, the assignment `x = 2` binds the value 2 dynamically to `x` when the assignment statement is executed. And the C++ statement

```
y = new int;
```

dynamically binds a storage location to `*y` and assigns that location as the value of `y`.

Binding times can be further refined into subcategories of dynamic and static binding. A static attribute can be bound during parsing or semantic analysis (translation time), during the linking of the program with libraries (link time), or during the loading of the program for execution (load time). For example, the body of an externally defined function will not be bound until link time, and the location of a global variable is bound at load time, since its location does not change during the execution of the program. Similarly, a dynamic attribute can be bound at different times during execution—for example, on entry or exit from a procedure, or on entry or exit from the entire program.

Names can be bound to attributes even prior to translation time. Predefined identifiers, such as the Pascal data types `boolean` and `char`, have their meanings (and hence attributes) specified by the

language definition. Data type `boolean`, for example, is specified as having the two values `true` and `false`. Some predefined identifiers, such as data type `integer` and constant `maxint`, have their attributes specified by the language definition *and* by the implementation. The language definition specifies that data type `integer` has values consisting of a subset of the integers and that `maxint` is a constant, while the implementation specifies the value of `maxint` and the actual range of data type `integer`.[3]

Thus, we have the following possible binding times for attributes of names:

- Language definition time
- Language implementation time
- Translation time ("compile-time")
- Link time
- Load time
- Execution time ("runtime")

All binding times in this list, except for the last, represent static binding. (Dynamic binding times during execution were examined in Chapters 3 and 5.)

Bindings must be maintained by a translator so that appropriate meanings are given to names during translation and execution. A translator does this by creating a data structure to maintain the information. Since we are not interested in the details of this data structure, but only its properties, we can think of it abstractly as a function that expresses the binding of attributes to names. This function is a fundamental part of language semantics and is usually called the **symbol table**. Mathematically, the symbol table is a function from names to attributes, which we could write as SymbolTable : Names → Attributes, or more graphically as shown in Figure 7.1.

Figure 7.1 Mapping names to attributes in a symbol table

This function will change as translation and/or execution proceeds to reflect additions and deletions of bindings within the program being translated and/or executed.

Viewed in this way, the parsing phase of translation includes three types of analysis:

1. Lexical analysis, which determines whether a string of characters represents a token in the vocabulary of the language
2. Syntax analysis, which determines whether a sequence of tokens represents a phrase in the context-free grammar of the language
3. Static semantic analysis, which is concerned with establishing the attributes of names in declarations and ensuring that the use of these names in references conforms to their declared attributes

[3] This assumes that a program does not redefine the meanings of these names in a declaration. Redefinition is a possibility since these predefined identifiers are not reserved words.

During the execution of a compiled program, attributes such as locations and values must also be maintained. A compiler generates code that maintains these attributes in data structures during execution. The memory allocation part of this process—that is, the binding of names to storage locations is usually considered separately and is called the **environment**, as shown in Figure 7.2.

Figure 7.2 Mapping names to locations in an environment

Finally, the bindings of storage locations to values is called the **memory**, since it abstracts the memory of an actual computer (sometimes it is also called the **store** or the **state**). See Figure 7.3.

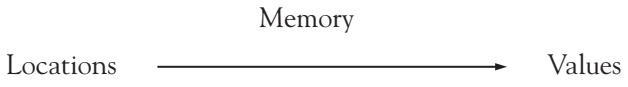


Figure 7.3 Mapping locations to values in a memory

7.2 Declarations, Blocks, and Scope

Declarations are, as you have seen, a principal method for establishing bindings. Bindings can be determined by a declaration either **implicitly** or **explicitly**. For example, the C declaration

```
int x;
```

establishes the data type of `x` explicitly using the keyword `int`, but the exact location of `x` during execution is only bound implicitly and, indeed, could be either static or dynamic, depending on the location of this declaration in the program. Similarly, the value of `x` is either implicitly zero or undefined, depending on the location of the declaration. On the other hand, the declaration

```
int x = 0;
```

explicitly binds 0 as the initial value of `x`.

Not only can bindings be implicit in a declaration, but the entire declaration itself may be implicit. For example, in some languages simply using the name of the variable causes it to be declared, although the first use usually takes the form of an assignment statement that binds a value to the name.

Sometimes a language will have different names for declarations that bind certain attributes but not others. For example, in C and C++, declarations that bind *all* potential attributes are called **definitions**, while declarations that only *partially* specify attributes are called simply declarations. For example, the function declaration (or **prototype**)

```
double f(int x);
```

specifies only the data type of the function `f` (i.e., the types of its parameters and return value) but does not specify the code to implement `f`. Similarly,

```
struct x;
```

specifies an **incomplete type** in C or C++, so this declaration is also not a definition. (Such specialized declarations are used to resolve recursive definitions and in cases where the complete definition can only be found at link time—see subsequent remarks in this chapter.)

In dynamically typed languages such as Lisp, Smalltalk, and Python, variable names typically have only a value binding, with the type attributes belonging to the associated values.

Declarations commonly appear in particular language constructs. One such standard language construct, typically called a **block**, consists of a sequence of declarations followed by a sequence of statements. Within a block, the sequence of statements following a declaration are surrounded by syntactic markers such as braces or begin-end pairs. In C, blocks are called **compound statements** and appear as the body of functions in function definitions, and also anywhere an ordinary program statement could appear. Thus,

```
void p (){
    double r, z; /* the block of p */
    ...
    { int x, y; /* another, nested, block */
      x = 2;
      y = 0;
      x += 1;
    }
    ...
}
```

establishes two blocks, one of which represents the body of `p` and the other nested inside the body of `p`.

In addition to declarations associated with blocks, C also has an "external" or "global" set of declarations outside any compound statement:

```
int x;
double y;
/* these are external to all functions
 and so global */

main()
{ int i, j; /* these are associated with the block of main only */
  ...
}
```

Declarations that are associated with a specific block are also called **local**, while declarations in surrounding blocks (or global declarations) are called **nonlocal** declarations. For example, in the previous definition of the procedure `p`, variables `r` and `z` are local to `p` but are nonlocal from within the second block nested inside `p`.

The notion of blocks containing declarations began with Algol60, and all Algol descendants exhibit this **block structure** in various ways. For example, in Ada, a block is formed by a `begin-end` pair preceded by the keyword `declare`:

```
declare x: integer;
        y: boolean;
begin
  x := 2;
  y := true;
  x := x + 1;
  ...
end;
```

In block-structured languages, blocks can be nested to an arbitrary depth, and names can be redeclared within nested blocks. Instead of identifying each new declared name with a unique integer (counting from 0), we can associate with each declared name a **level number** and an **offset**. This pair of numbers is called the name's **lexical address**. Names declared in the outermost level have the level number 0. The level number increases as we move into each nested block. The offsets of the names declared within each nested block are then numbered beginning with 0. Thus, in the following Ada code segment, the declarations of `x` and `y` in the outer block have the lexical addresses (0, 0) and (0, 1), whereas the same names in the nested block have the lexical addresses (1, 0) and (1, 1).

```
declare x: integer;
        y: boolean;
begin
  x := 2;
  y := true;
  x := x + 1;
  declare x: integer;
          y: boolean;
  begin
    x := 3;
    y := false;
    x := x * 6;
  end;
end;
```

Other language constructs besides blocks are important sources of declarations. For example, in C, a `struct` definition is composed of local variable (or **member**) declarations, which are within it:

```
struct A{
    int x; /* local to A */
    double y;
```

(continues)

(continued)

```
    struct{
        int* x; /* nested member declarations */
        char y;
    } z;
};
```

Note how the syntax with braces is almost, but not quite, that of a block. There is an extra semicolon at the end.

Similarly, in object-oriented languages, the **class** is an important source of declarations. Indeed, in pure object-oriented languages such as Java and Smalltalk, the class is the *only* declaration that does not itself need to be inside another class declaration. In other words, in pure object-oriented languages the class is the *primary* source of declarations. Consider the following example from Java (keeping in mind that in Java, functions are called methods and members are called fields):

```
/* Java example of class declaration */

public class IntWithGcd{
    ...
    /* local declaration of intValue method */
    public int intValue(){
        return value;
    }

    /* local declaration of gcd method */
    public int gcd( int v ){
        ...
    }

    /* local declaration of value field */
    private int value;
}
```

Finally, declarations can also be collected into larger groups as a way of organizing programs and providing special behavior. **Ada packages** and **tasks** belong to this category, as do **Java packages** (rather different from Ada packages), ML, Haskell, and Python **modules**, and C++ **namespaces**. These will be treated in more detail in Chapter 11.

Declarations bind various attributes to names, depending on the kind of declaration. The **scope of a binding** is the region of the program over which the binding is maintained. We can also refer to the scope of a declaration if all the bindings established by the declaration (or at least all of the bindings we are interested in at a particular moment) have identical scopes. Sometimes, by abuse of language, we refer to the scope of a name, but this is dangerous, since the same name may be involved in several different declarations, each with a different scope. For example, the following C code contains two declarations of the name `x`, with different meanings and different scopes:

```
void p(){
    int x;
    ...
}

void q(){
    char x;
    ...
}
```

In a block-structured language like C (and most modern languages), where blocks can be nested, the scope of a binding is limited to the block in which its associated declaration appears (and other blocks contained within it). Such a scope rule is called **lexical scope** because it follows the structure of the blocks as they appear in the written code.

A simple example of scope in C is given in Figure 7.4.

```
(1)  int x;

(2)  void p(){
(3)      char y;
(4)      ...
(5)  } /* p */

(6)  void q(){
(7)      double z;
(8)      ...
(9)  } /* q */

(10) main(){
(11)     int w[10];
(12)     ...
(13) }
```

Figure 7.4 Simple C program demonstrating scope

In Figure 7.4, the declarations of variable `x` (line 1) and procedures `p` (lines 2–5), `q` (lines 6–9), and `main` (lines 10–13) are global. The declarations of `y` (line 3), `z` (line 7), and `w` (line 11), on the other hand, are associated with the blocks of procedures `p`, `q`, and `main`, respectively. They are local to these functions, and their declarations are valid only for those functions. C has the further rule that the scope of a declaration begins at the point of the declaration itself. This is the so-called **declaration before use** rule: The scope of a declaration extends from the point just after the declaration to the end of the block in which it is located.[4]

[4] C's scope rules are actually a bit more complicated, and we discuss a few of these complications further on in the chapter.

We repeat the example of Figure 7.4 in Figure 7.5, drawing brackets to indicate the scope of each declaration.

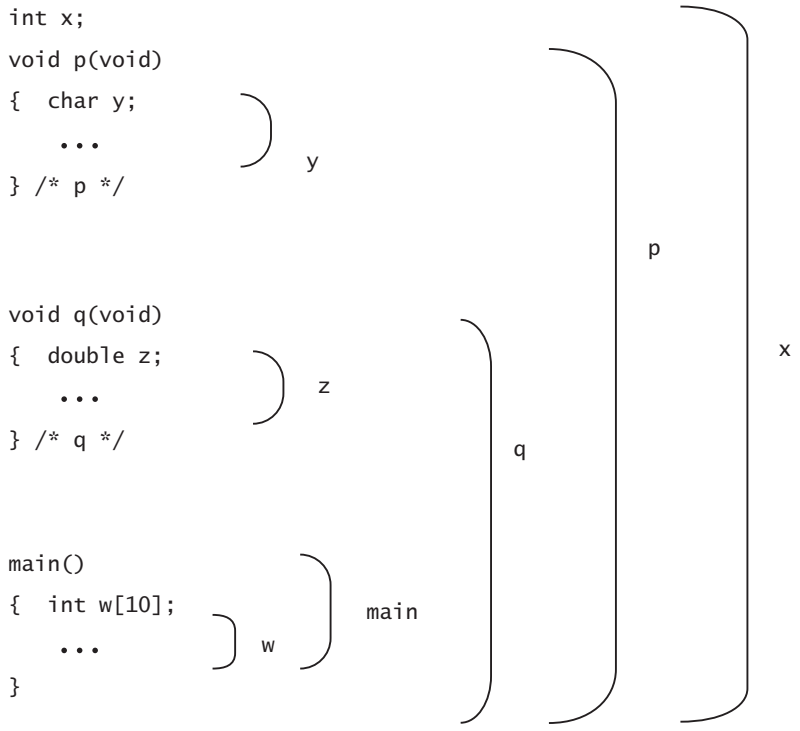


Figure 7.5 C Program from Figure 7.4 with brackets indicating scope

Blocks in other Algol-like languages establish similar scope rules. For example, in the following Ada code,

```
B1: declare
     x: integer;
     y: boolean;
    begin
     x := 2;
     y := false;
    B2: declare
         a, b: integer;
        begin
          if y then a := x;
          else b := x;
          end if;
        end B2;
     ...
    end B1;
```

the scope of the declarations of x and y is all of block B1 (including block B2), while the scope of the declarations of a and b is block B2 only.

One feature of block structure is that declarations in nested blocks take precedence over previous declarations. Consider the following example:

```
(1)  int x;

(2) void p(){
(3)     char x;
(4)     x = 'a'; /* assigns to char x */
(5)     ...
(6) }

(7) main(){
(8)     x = 2; /* assigns to global x */
(9)     ...
(10) }
```

The declaration of x in p (line 3) takes precedence over the global declaration of x (line 1) in the body of p. Thus, the global integer x cannot be accessed from within p. The global declaration of x is said to have a **scope hole** inside p, and the local declaration of x is said to **shadow** its global declaration. For this reason, a distinction is sometimes made between the scope and the visibility of a declaration. **Visibility** includes only those regions of a program where the bindings of a declaration apply, while scope includes scope holes (since the bindings still exist but are hidden from view). In C++, the **scope resolution operator** :: (double colon) can be used to access such hidden declarations (as long as they are global). For example:

```
int x;

void p(){
char x;
  x = 'a'; // assigns to char x
  ::x = 42; // assigns to global int x
  ...
}

main(){
x = 2; // assigns to global x
  ...
}
```

This demonstrates the fact that operators and declaration modifiers can alter the accessibility and scope of declarations—a widespread phenomenon in programming languages. For example, Ada allows

any named enclosing scope to be accessed using a dot notation similar to record structure access, so the above C++ behavior can also be achieved in Ada, as follows:

```
B1: declare
      a: integer;
      y: boolean;
    begin
      a := 2;
      y := false;
    B2: declare
          a, b: integer; -- local a now hides a in B1
        begin
          if y then a := B1.a; -- select the a in B1
          else b := B1.a;
          end if;
        end B2;
      ...
    end B1;
```

This is called **visibility by selection** in Ada.

C++ also uses the scope resolution operator to enter the scope of a class from the "outside" to provide missing definitions, since C++ only requires declarations, not complete definitions, within a class declaration:

```
/* C++ example of class declaration similar to previous Java example */
class IntWithGcd{

    public:
    /* local declaration of intValue function - code for intValue() is not
     provided */
    int intValue();

    int gcd ( int v ); /* local declaration of gcd */

    private:
    int value; /* local declaration of value member */
};

/* use scope resolution to give actual definition of intValue */
int IntWithGcd::intValue(){
    return value;
}
```

One further example in this vein is the way additional keyword modifiers in a declaration can alter the scope of the declaration. In C, global variable declarations can actually be accessed across files, using the `extern` keyword:

File 1:

```
extern int x; /* uses x from somewhere else */
...
```

File 2:

```
int x; /* a global x that can be accessed by other files */
...
```

Now if File 1 and File 2 are compiled separately and then linked together, the external `x` in File 1 will be identified by the linker with the `x` provided in File 2. If, on the other hand, when writing File 2, the programmer wishes to restrict the scope of `x` to File 2 alone and, thus, prevent external references, the programmer may use the `static` keyword:

File 2:

```
/* a global x that can only be seen within this file */
static int x;
...
```

Now an attempt to link File 1 with File 2 will result in an error: undefined reference to `x`. This is a primitive version of the kind of scope-modifying access control that classes and modules provide, which is discussed in detail in Chapters 5 and 11.

Scope rules need also to be constructed in such a way that recursive, or self-referential, declarations are possible when they make sense. Since functions must be allowed to be recursive, this means that the declaration of a function name has scope beginning *before* the block of the function body is entered:

```
int factorial (int n){
    /* scope of factorial begins here */
    /* factorial can be called here */
    ...
}
```

On the other hand, recursive variable declarations don't, in general, make sense:

```
int x = x + 1; /* ??? */
```

This declaration should probably be an error, and indeed Java and Ada flag this as an error, even though the scope of the `x` begins at the = sign. In Ada, this is true because self-references in variable declarations are specifically outlawed. In Java, this is due to the fact that the self-reference is flagged as an uninitialized use. C and C++, however, do not flag this as an error if `x` is a local variable[5], but simply add one to the (probably random) uninitialized value of `x`.

[5] Global variables in C and C++ cannot have such an initializer expression, since their initial values must be compile-time quantities; see Section 7.6.2.

A special scoping situation, related to recursion, occurs in class declarations in object-oriented languages. Local declarations inside a class declaration generally have scope that extends backwards to include the entire class (thus, partially suspending the declaration before use rule). For example, in the previous `IntWithGcd` example in Java, the local data `value` was referenced in the class declaration even before it was defined. This rule exists so that the order of declarations within a class does not matter. Translators for such languages must generally perform two passes to parse a program. In the first pass, the parser enters all of the declared names into a symbol table. In the second pass, the parser looks up each reference to a name in the table to verify that a declaration exists.

The way the symbol table processes declarations must correspond to the scope of each declaration. Thus, different scope rules require different behavior by, and even different structure within, the symbol table. In the next section, we examine the basic ways that a symbol table can help in analyzing scope in a block-structured language.

7.3 The Symbol Table

A symbol table is like a dictionary: It must support the insertion, lookup, and deletion[6] of names with associated attributes, representing the bindings in declarations. A symbol can be implemented with any number of data structures to allow for efficient access, such as hash tables and binary search trees. However, the maintenance of scope information in a lexically scoped language with block structure requires that declarations be processed in a stacklike fashion. That is, on entry into a block, all declarations of that block are processed and the corresponding bindings added to the symbol table. Then, on exit from the block, the bindings provided by the declarations are removed, restoring any previous bindings that may have existed. This process is called **scope analysis**. Without restricting our view of a symbol table to any particular data structure, we may nevertheless view the symbol table schematically as a collection of names, each of which has a stack of declarations associated with it, such that the declaration on top of the stack is the one whose scope is currently active. To see how this works, consider the C program of Figure 7.6.

```
(1) int x;
(2) char y;

(3) void p(){
(4)     double x;
(5)     ...
(6)     { int y[10];
(7)       ...
(8)     }
(9)     ...
(10) }

(11) void q(){
(12)     int y;
```

Figure 7.6 C program demonstrating symbol table structure *(continues)*

[6] Deletion, of course, may not mean complete erasure from the symbol table, but simply marking it as no longer active, since bindings may need to be revisited even after a scope is exited by the translator.

(continued)

```
(13)     ...
(14) }

(15) main(){
(16)     char x;
(17)     ...
(18) }
```

Figure 7.6 C program demonstrating symbol table structure

The names in this program are x, y, p, q, and main, but x and y are each associated with three different declarations with different scopes. Right after the processing of the variable declaration at the beginning of the body of p (line 5 of figure 7.6), the symbol table can be represented as in Figure 7.7. (Since all function definitions are global in C, we do not indicate this explicitly in the attributes of Figure 7.7 and subsequent figures.)

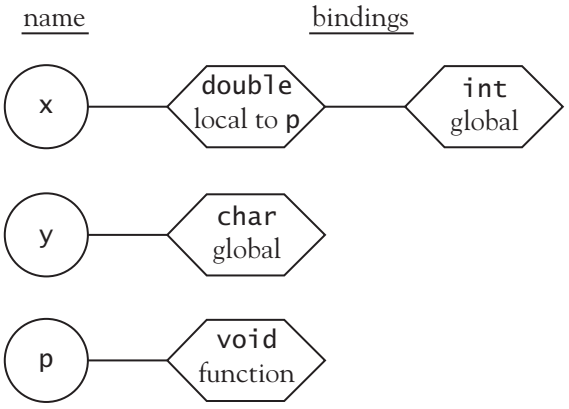


Figure 7.7 Symbol table structure at line 5 of Figure 7.6

After the processing of the declaration of the nested block in p, with the local declaration of y (i.e., at line 7 of Figure 7.6), the symbol table is as shown in Figure 7.8.

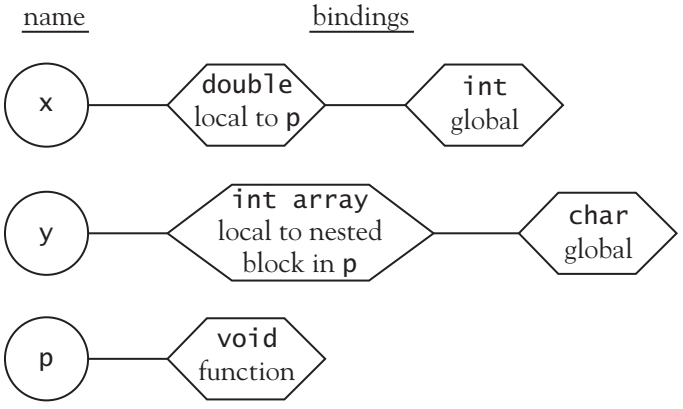


Figure 7.8 Symbol table structure at line 7 of Figure 7.6

Then, when the processing of p has finished (line 10 of Figure 7.6), the symbol table becomes as shown in Figure 7.9 (with the local declaration of y popped from the stack of y declarations at line 8 and then the local declaration of x popped from the x stack at line 10).

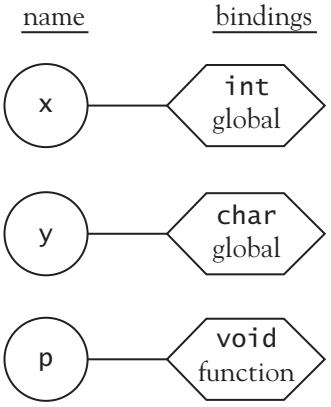


Figure 7.9 Symbol table structure at line 10 of Figure 7.6

After q is entered (line 13), the symbol table becomes like the one shown in Figure 7.10.

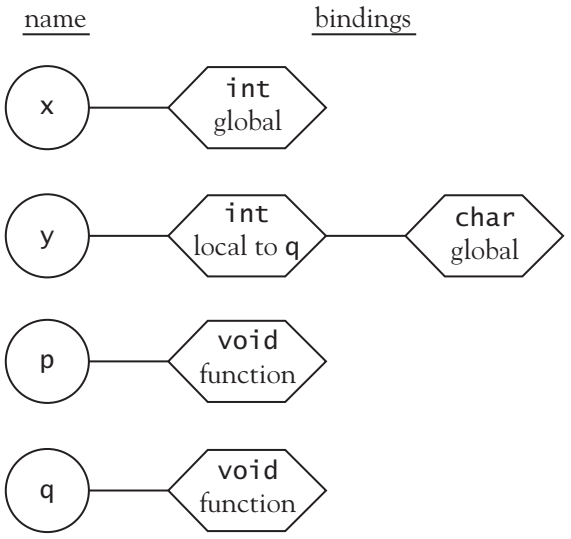


Figure 7.10 Symbol table structure at line 13 of Figure 7.6

When q exits (line 14), the symbol table is as shown in Figure 7.11.

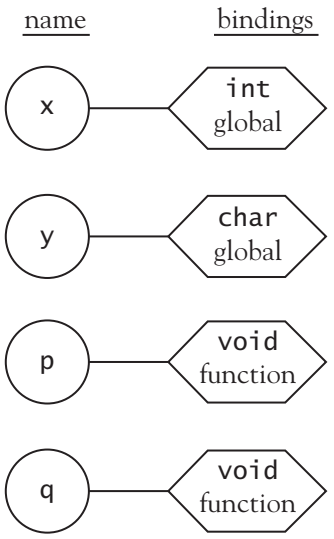


Figure 7.11 Symbol table structure at line 14 of Figure 7.6

Finally, after main is entered (line 17), the symbol table is as shown in Figure 7.12.

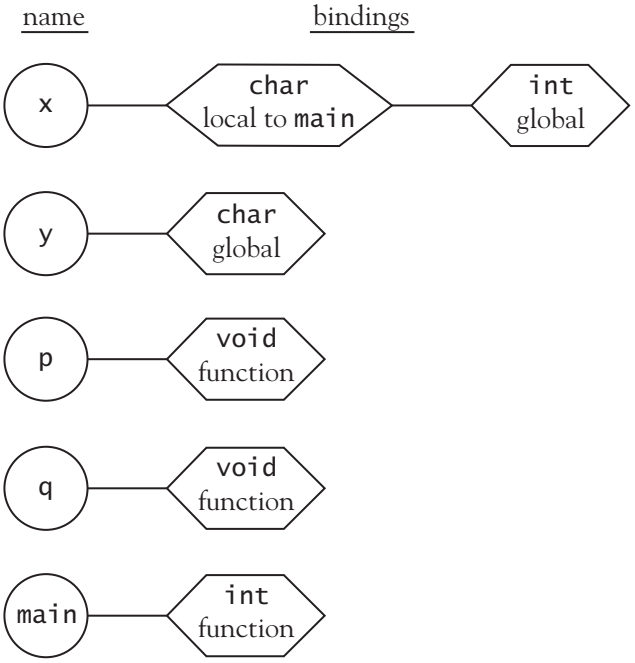


Figure 7.12 Symbol table structure at line 17 of Figure 7.6

Note that this process maintains the appropriate scope information, including the scope holes for the global declaration of x inside p and the global declaration of y inside q.

This representation of the symbol table assumes that the symbol table processes the declarations statically—that is, prior to execution. This is the case if the symbol table is managed by a compiler, and the bindings of the declarations are all static. However, if the symbol table is managed dynamically, that is, during execution, then declarations are processed as they are encountered along an execution path through the program. This results in a different scope rule, which is usually called **dynamic scoping**, and our previous lexical scoping rule is sometimes called **static scoping**.

To show the difference between static and dynamic scoping, let us add some code to the example of Figure 7.6, so that an execution path is established, and also give some values to the variables, so that some output can be generated to emphasize this distinction. The revised example is given in Figure 7.13.

```
(1) #include <stdio.h>

(2) int x = 1;
(3) char y = 'a';
(4) void p(){
(5)     double x = 2.5;
(6)     printf("%c\n",y);
(7)     { int y[10];
(8)     }
(9) }

(10) void q(){
(11)     int y = 42;
(12)     printf("%d\n",x);
(13)     p();
(14) }

(15) main(){
(16)     char x = 'b';
(17)     q();
(18)     return 0;
(19) }
```

Figure 7.13 C program of Figure 7.6 with added code

Now consider what would happen if the symbol table for the code of Figure 7.13 were constructed dynamically as execution proceeds. First, execution begins with `main`, and all the global declarations must be processed before `main` begins execution, since `main` must know about all the declarations that occur before it. Thus, the symbol table at the beginning of the execution (line 17) is as shown in Figure 7.14.

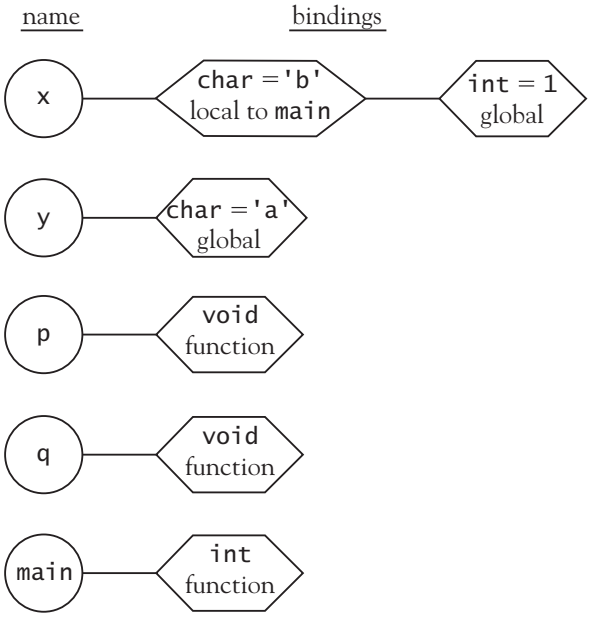


Figure 7.14 Symbol table structure at line 17 of Figure 7.13 using dynamic scope

Note that this is the same symbol table used when `main` is processed statically (Figure 7.12), except that we have now added value attributes to the picture. Note we have yet to process the bodies of any of the other functions.

Now `main` proceeds to call `q`, and we begin processing the body of `q`. The symbol table on entry into `q` (line 12) is then as shown in Figure 7.15.

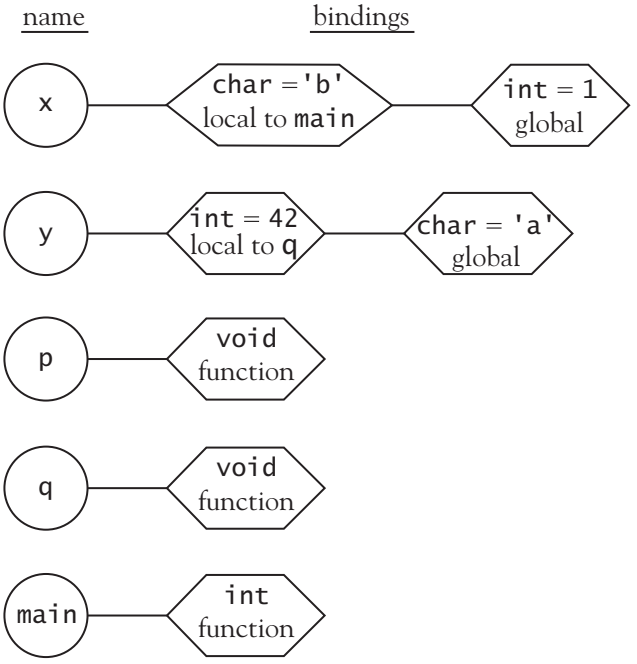


Figure 7.15 Symbol table structure at line 12 of Figure 7.13 using dynamic scope

This is now quite different from the symbol table on entry into q using static processing (Figure 7.10). Note also that *each* call to q may have a different symbol table on entry, depending on the execution path to that call, whereas when using lexical scoping, each procedure has only one symbol table associated with its entry. (Since there is only one call to q in this example, this doesn't happen in this case.)

To continue with the example, q now calls p, and the symbol table becomes like the one shown in Figure 7.16.

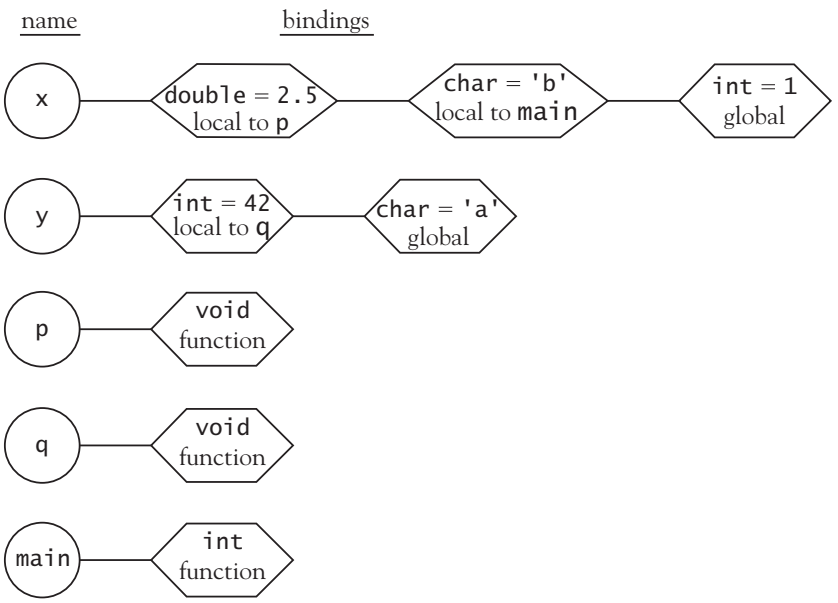


Figure 7.16 Symbol table structure at line 6 of Figure 7.13 using dynamic scope

Now consider how dynamic scoping will affect the semantics of this program and produce different output. First, note that the actual output of this program (using lexical scoping, which is the standard for most languages, including C) is:

```
1
a
```

since in the first `printf` statement (line 6), the reference is to the global `y`, regardless of the execution path, and in the second `printf` statement (line 12) the reference is to the global `x`, again regardless of the execution path, and the values of these variables are 'a' and 1 throughout the program.

However, using dynamic scoping,[7] the nonlocal references to `y` and `x` inside `p` and `q`, respectively, can change, depending on the execution path. In this program, the `printf` statement at line 12 of Figure 7.13 is reached with the symbol table as in Figure 7.15 (with a new declaration of `x` as the character "b" inside `main`). Thus, the `printf` statement will print this character interpreted as an integer (because of the format string `"%d\n"`) as the ASCII value 98. Second, the `printf` reference to `y` inside `p` (line 6 of Figure 7.13) is now the integer `y` with value 42 defined inside `q`, as shown in Figure 7.16. This value will now be interpreted as a character (ASCII 42 = '*'), and the program will print the following:

```
98
*
```

[7] Of course, since C uses lexical scope, we cannot actually execute this program this way. We are using C syntax only as a convenience.

This example brings up many of the issues that make dynamic scoping problematic, and why few languages use it. The first, and perhaps foremost problem, is that under dynamic scoping, when a nonlocal name is used in an expression or statement, the declaration that applies to that name cannot be determined by simply reading the program. Instead, the program must be executed, or its execution traced by hand, to find the applicable declaration (and different executions may lead to different results). Therefore, the semantics of a function can change radically as execution proceeds. You saw this in the example above, where the reference to `y` in the `printf` statement on line 6 of Figure 7.13 cannot be known until execution time (if we wanted, we could have rewritten this example to make the `y` reference actually depend on user input). Thus, the semantics of `p` change during execution.

The example demonstrates another serious problem: Since nonlocal variable references cannot be predicted prior to execution, neither can the data types of these variables. In the example, the reference to `y` in the `printf` statement on line 6 of Figure 7.13 is assumed to be a character (from the data type of the global `y`), and we therefore write a character format directive `"%c"` in the format string of the `printf` function. However, with dynamic scoping, the variable `y` could have any type when `p` is executed. That means this formatting directive is likely to be wrong. In the example, we were lucky that C has very liberal conversion rules between two basic data types such as `char` and `int`. However, imagine what would happen if the global `y` or the `y` local to `q` were a `double` or even a user-defined structure—we still are likely not to get a runtime error in C, but predicting the output can be difficult (and machine dependent!). In other, stricter languages like Ada or Pascal, this would definitely lead to a runtime error—not at all what we would like. Thus, static binding of data types (static typing) and dynamic scoping are inherently incompatible. (See Chapter 8 for more on this issue.)

Nevertheless, dynamic scoping remains a possible option for highly dynamic, interpreted languages when we do not expect programs to be extremely large. The reason is that the runtime environment is made considerably simpler by using dynamic scoping in an interpreter. Indeed, at the end of Section 7.1, we noted that the environment in an interpreter includes the symbol table, so it may seem that it is impossible to maintain lexical scope in an interpreter, since, by definition, the symbol table is maintained dynamically. This is not the case. However, maintaining lexical scope dynamically does require some extra structure and bookkeeping (see Chapter 10). Thus, languages like APL, Snobol, and Perl (all interpreted languages with relatively small expected programs) have traditionally opted for dynamic scoping.[8]

Early dialects of Lisp, too, were dynamically scoped. The inventor of Lisp, John McCarthy, has stated that he felt this was a bug in the initial implementation of Lisp in 1958 (McCarthy [1981], page 180). However, as mentioned in Chapter 3, the popular Scheme dialect of Lisp has used lexical scoping from its inception (Steele and Gabriel [1996], page 249), and Common Lisp (Steele [1982] [1984]) offers lexical scoping as its default option.

Aside from the question of lexical versus dynamic scope, there is significant additional complexity surrounding the structure and behavior of the symbol table that we have not yet discussed. Indeed, the symbol table as we have discussed it so far—a single table for an entire program, with insertions on entry

[8] Recent versions of Perl now offer lexical scoping.

into a scope and deletions on exit—is appropriate only for the simplest languages, such as C and Pascal, with a strict declaration before use requirement, and where scopes cannot be reaccessed once they have been exited during processing by a translator.

Actually, even this description does not fully cover all situations in C and Pascal. Consider, for example, `struct` declarations in C (or `record` declarations in Pascal) as given in Figure 7.17. Clearly, each of the two `struct` declarations in this code (lines 1–5 and 7–11 of Figure 7.17) must contain the further declarations of the data fields within each `struct`, and these declarations must be accessible (using the "dot" notation of member selection) whenever the `struct` variables themselves (`x` and `y`) are in scope. This means two things: (1) A `struct` declaration actually contains a local symbol table itself as an attribute (which contains the member declarations), and (2) this local symbol table cannot be deleted until the `struct` variable itself is deleted from the "global" symbol table of the program.

```
(1)  struct{
(2)      int a;
(3)      char b;
(4)      double c;
(5)  } x = {1, 'a',2.5};

(6)  void p(){
(7)      struct{
(8)          double a;
(9)          int b;
(10)         char c;
(11)     } y = {1.2,2, 'b'};
(12)     printf("%d, %c, %g\n",x.a,x.b,x.c);
(13)     printf("%f, %d, %c\n",y.a,y.b,y.c);
(14) }

(15) main(){
(16)     p();
(17)     return 0;
(18) }
```

Figure 7.17 Code example illustrating scope of local declarations in a C `struct`

Thus, inside `p` (line 12 of Figure 7.17), the symbol table for the preceding code might look as shown in Figure 7.18, with the symbol table attribute labeled as "symtab").

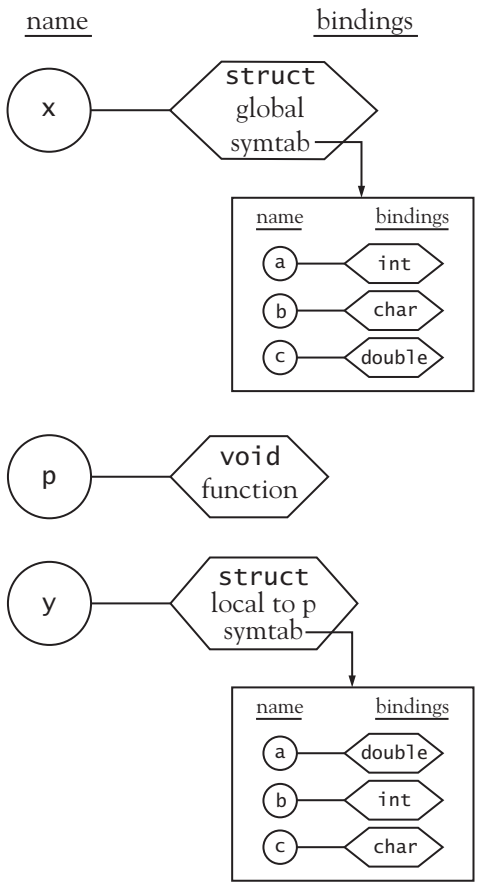


Figure 7.18 Representation of the symbol table structure at line 12 of the program of Figure 7.17, showing local `struct` symbol tables

Any scoping structure that can be referenced directly in a language must also have its own symbol table. Examples include all named scopes in Ada; classes, structs, and namespaces in C++; and classes and packages in Java. Thus, a more typical structure for the symbol table of a program in any of these languages is to have a table for each scope. When each new scope is entered during the parsing of a source program, a new symbol table for it is pushed onto a stack of symbol tables. The names declared in this scope are then entered into this table. If a duplicate name is encountered at this point, the name has already been declared in the same scope and an error is returned. When a reference to a name is encountered, a search for it begins in the table at the top of the stack. If an entry for the name is not found in this table, the search continues in the next table below it, and so on, until an entry is found or the bottom of the stack is reached. If the bottom of the stack is reached, the name is undeclared and an error is returned.

As a second example of this phenomenon, we rewrite the C example of Figure 7.13 into Ada as shown in Figure 7.19.

```
(1) with Text_IO; use Text_IO;
(2) with Ada.Integer_Text_IO; use Ada.Integer_Text_IO;

(3) procedure ex is
(4)   x: integer := 1;
(5)   y: character := 'a';

(6)   procedure p is
(7)   x: float := 2.5;
(8)   begin
(9)     put(y); new_line;
(10)  A: declare
(11)       y: array (1..10) of integer;
(12)    begin
(13)      y(1) := 2;
(14)      put(y(1)); new_line;
(15)      put(ex.y); new_line;
(16)    end A;
(17)  end p;

(18)  procedure q is
(19)    y: integer := 42;
(20)  begin
(21)    put(x); new_line;
(22)    p;
(23)  end q;

(24)   begin
(25)   declare
(26)     x: character := 'b';
(27)   begin
(28)     q;
(29)     put(ex.x); new_line;
(30)   end;
(31) end ex;
```

Figure 7.19 Ada code corresponding to Figure 7.13

A possible organization for the symbol table of this program after entry into block A inside p (line 12 of Figure 7.19) is shown in Figure 7.20 (we ignore the question of how to represent the imported packages). Note in Figure 7.20 that we no longer have to mention which specific scope each declaration belongs to, since membership in a particular symbol table already gives us that information. Also, note that if a lookup in an inner table fails, the lookup must proceed with the next outermost table. Thus, as indicated in the diagram with dotted arrows, there must be links from each inner scope to the next outer scope.

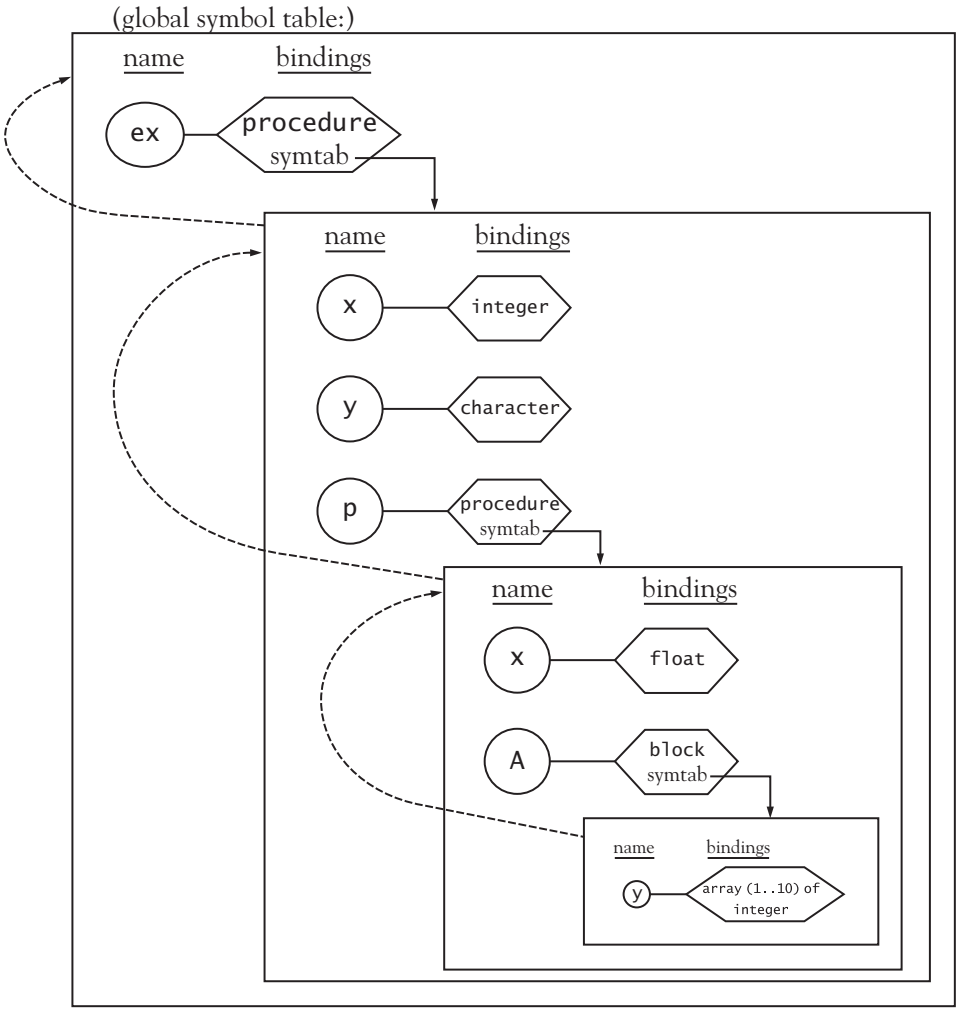


Figure 7.20 Symbol table structure at line 12 of Figure 7.19

Consider, for example, how the lookup of `ex.y` in line 15 of Figure 7.19 would proceed from within block `A` in function `p`. First, `ex` would be looked up in the symbol table of `A`. Since this lookup fails, the next outermost symbol table would be consulted (the symbol table of `p`) by following the dotted line. This lookup also fails, and the next symbol table—that of `ex`—is consulted. This fails too, and finally the outermost symbol table is arrived at, which has only the entry for `ex`. This is the entry one we want. With the entry found, the lookup of `y` proceeds (the `ex.` in `ex.y` is stripped off for this lookup). The character variable `y` is found. This is the variable that is meant in line 15.

A final question about symbol tables and scope is to what extent the same name can be reused for different functions, variables, and other entities within the same scope. Such reuse, which is called overloading, is discussed, along with related issues, in the next section.

7.4 Name Resolution and Overloading

An important question about declarations and the operation of the symbol table in a programming language is to what extent the same name can be used to refer to different things in a program. One might at first consider that this should not be allowed, as it can lead to serious confusion and unreadability. However, consider, for example, the addition operation denoted by a + sign, which is typically a built-in binary infix[9] operator in most languages and whose result is the sum of its two operands. In virtually all languages, this simple + sign refers to at least two (and often more) completely different operations: integer addition and floating-point addition (typically denoted in assembly language by two different symbols ADDI and ADDF, or similar names). The + operator is then said to be **overloaded**. Clearly this reuse of the same symbol does not cause confusion, since these operations are closely related (and are even "the same" in some mathematical sense). Just as clearly, it would be a major annoyance if a programming language required programmers to use two different symbols for these operations (say +% for integer addition and +# for floating-point addition).

How does a translator disambiguate (distinguish between) these two uses of the "+" symbol?

It does so by looking at the data types of the operands. For example, in C, if we write `2+3`, we mean integer addition, and if we write `2.1+3.2`, we mean floating-point addition. Of course, `2+3.2` is still potentially ambiguous, and the rules of a language must deal in some way with this case (most languages automatically convert 2 to 2.0, but Ada says this is an error).

It is, therefore, a major advantage for a language to allow overloading of operators and function names based in some way on data types. C++ and Ada allow extensive overloading of both function names and operators. Java also allows overloading, but only on function names, not operators.[10] The functional language Haskell also allows overloading of both functions and operators, and even allows new operators to be defined (which is not allowed in C++ or Ada). Several versions of FORTRAN (Fortran90/95 and later) also allow limited forms of overloading. We discuss overloading here using C++, Ada, and Java (which have similar, but not identical, mechanisms). Overloading in Haskell was described briefly in Chapter 3. See the Notes and References for overloading in Fortran 90/95.

The basic method for allowing overloaded functions is to expand the operation of the lookup operation in the symbol table to perform lookups based not only on the name of a function but also on the number of its parameters and their data types. This process of choosing a unique function among many with the same name, which represents an expansion of the symbol table capabilities, is called **overload resolution**.

To see how overload resolution works, we consider the simple but useful example of a `max` function for numbers. This function can be defined for integers, doubles, and all other numeric types, and may have two, three, or four (or even more) parameters. For our purposes, we define only three of the possible `max` functions (in C++ syntax)[11] in Figure 7.21.

[9] That is, written between its operands.

[10] At least, not in the current version of Java—a proposal exists to allow operator overloading in future versions.

[11] These functions can be easily turned into Ada code, but Java does not have freestanding functions; to write this example in Java, we would write these functions as static methods inside the class containing the `main` function.

```
int max(int x, int y){ // max #1
    return x > y ? x : y;
}

double max(double x, double y){ // max #2
    return x > y ? x : y;
}

int max(int x, int y, int z){ // max #3
    return x > y ? (x > z ? x : z) : (y > z ? y : z);
```

Figure 7.21 Three overloaded `max` functions in C++

We will refer to these as `max` #1, `max` #2, and `max` #3, respectively.

Now consider the following calls:

```
max(2,3); // calls max #1
max(2.1,3.2); // calls max #2
max(1,3,2); // calls max #3
```

The symbol table can easily determine the appropriate `max` function for each of these calls from the information contained in each call (the **calling context**)—it just needs to count the number of parameters and then look (in the two-parameter case) to see if the parameter values are integers or doubles.

Difficulties do arise in determining which of several overloaded definitions should be used when several different definitions of an overloaded function are equally likely in a particular calling context under the rules of the language. Consider, for example, the following call:

```
max(2.1,3); // which max?
```

Here the answer depends on the language rules for converting a value of one type to another; see Chapter 8. In C++, for example, this call is **ambiguous**: C++ allows conversions both from integer to double and from double to integer (automatic truncation). Thus, the preceding call could be converted to either:

```
max(2,3); // max #1
```

or

```
max(2.1,3.0); // max #2
```

and the language rules do not say which conversion should be preferred. In Ada, too, the call `max(2.1,3)` is illegal, but for a different reason: No automatic conversions at all are allowed in Ada. In Ada, all parameter types must match a definition exactly. On the other hand, this call is perfectly legitimate in Java since Java permits conversions that do not lose information. Thus, Java would convert 3 to 3.0 and call `max` #2 since the other possible call involves converting 2.1 to 2 and would result in information loss (the fractional part would disappear). Note also that the call:

```
max(2.1,4,3);
```

is legal in C++ (and results in 2.1 being truncated to 2), but it is not legal in either Ada or Java. On the other hand, suppose we add the definitions of Figure 7.22 to those of Figure 7.21.

```
double max(int x, double y){ // max #4
    return x > y ? (double) x : y;
}

double max(double x, int y){ // max #5
    return x > y ? x : (double) y;
}
```

Figure 7.22 Two more overloaded `max` functions in C++ (see Figure 7.21)

Then, the calls `max(2.1,3)` and `max(2,3.2)` become legal in C++ and Ada, since now there is an exact match for each call. Of course, these extra definitions are unnecessary (but not harmful) in Java.

Thus, automatic conversions as they exist in C++ and Java significantly complicate the process of overload resolution (but less so in Java than C++ because of Java's more restrictive conversion policy).

An additional issue in overload resolution is to what extent additional information in a calling context, besides the number and types of the parameter values, can be used. Ada, for example, allows the return type and even the names of the parameters in a definition to be used. Consider the Ada program in Figure 7.23.

```
(1)  procedure overload is

(2)  function max(x: integer; y: integer) -- max #1
(3)     return integer is
(4)  begin
(5)     if x > y then return x;
(6)     else return y;
(7)     end if;
(8)  end max;

(9)  function max(x: integer; y: integer) -- max #2
(10)    return float is
(11) begin
(12)    if x > y then return float(x);
(13)    else return float(y);
(14)    end if;
(15) end max;

(16) a: integer;
(17) b: float;
(18) begin -- max_test
(19)    a := max(2,3); -- call to max # 1
(20)    b := max(2,3); -- call to max # 2
(21) end overload;
```

Figure 7.23 An example of `max` function overloading in Ada

Because line 19 of Figure 7.23 assigns the result of the `max` call to `integer a`, while line 20 assigns the result to `float b`, Ada can still resolve the overloading because there are no automatic conversions. C++ and Java, on the other hand, ignore the return type (if they didn't, the rules for overload resolution would become even more complex, with more room for ambiguity), so this program would be an error in either of those languages.

Both Ada and C++ (but not Java) also allow built-in operators to be extended by overloading. For example, suppose that we have a data structure in C++:

```
typedef struct { int i; double d; } IntDouble;
```

We can then define a "+" and a "<" (less than) operation on `IntDouble` as follows:

```
bool operator < (IntDouble x, IntDouble y){
    return x.i < y.i && x.d < y.d;
}

IntDouble operator + (IntDouble x, IntDouble y){
    IntDouble z;
    z.i = x.i + y.i;
    z.d = x.d + y.d;
    return z;
}
```

Now code such as:

```
IntDouble x, y;
...
if (x < y) x = x + y;
else y = x + y;
```

is possible. A complete, runnable example of this C++ code is given in Figure 7.24. A corresponding Ada example is given in Figure 7.25.

```
#include <iostream>

using namespace std;

typedef struct { int i; double d; } IntDouble;

bool operator < (IntDouble x, IntDouble y){
    return x.i < y.i && x.d < y.d;
}
```

Figure 7.24 A complete C++ program illustrating operator overloading *(continues)*

(continued)

```
IntDouble operator + (IntDouble x, IntDouble y){
    IntDouble z;
    z.i = x.i + y.i;
    z.d = x.d + y.d;
    return z;
}

int main(){
    IntDouble x = {1,2.1}, y = {5,3.4};
    if (x < y) x = x + y;
    else y = x + y;
    cout << x.i << " " << x.d << endl;
    return 0;
}
```

Figure 7.24 A complete C++ program illustrating operator overloading

```
with Text_IO; use Text_IO;
with Ada.Integer_Text_IO;
use Ada.Integer_Text_IO;
with Ada.Float_Text_IO;
use Ada.Float_Text_IO;

procedure opover is

type IntDouble is
record
  i: Integer;
  d: Float;
end record;

function "<" (x,y: IntDouble) return Boolean is
begin
  return x.i < y.i and x.d < y.d;
end "<";

function "+" (x,y: IntDouble) return IntDouble is
  z: IntDouble;
begin
  z.i := x.i + y.i;
  z.d := x.d + y.d;
  return z;
```

Figure 7.25 A complete Ada program illustrating operator overloading analogous to the C++ program of Figure 7.24 *(continues)*

(continued)

```
end "+";
x, y: IntDouble;

begin
  x := (1,2.1);
  y := (5,3.4);
  if (x < y) then x := x + y;
  else y := x + y;
  end if;
  put(x.i); put(" "); put(x.d); new_line;
end opover;
```

Figure 7.25 A complete Ada program illustrating operator overloading analogous to the C++ program of Figure 7.24

Of course, when overloading a built-in operator, we must accept the syntactic properties of the operator. That is, we cannot change their associativity or precedence. Indeed, there is no *semantic* difference between operators and functions, only the *syntactic* difference that operators are written in infix form, while function calls are always written in prefix form. In fact, in Ada, all operators also have a prefix form: `"+"(3,2)` means the same as `3 + 2`. C++ also allows prefix form for operators but only for overloaded ones applied to user-defined types:

```
// x and y are IntDoubles as above
x = operator + (x, y);
```

In Ada one can even redefine the built-in operators using overloading, while in C++ this is prohibited.

In Ada the operators that are allowed to be overloaded are those that traditionally have been viewed as infix or prefix mathematical operators. In C++, however, a much wider class of operations are viewed as overloadable operators. For example, the subscript operation can be viewed as a binary operator (with the "[" part of the operator symbol written infix and the "]" part of the operator symbol written postfix). These operations in C++ can only be overriden using object-oriented constructs (in the parlance of object-oriented programming, these overloaded functions must be "member functions" defined in a C++ class). Indeed, as you learned in Chapter 5, further opportunities for overloading are provided by object-oriented techniques.

Up to this point, we have discussed only overloading of functions—by far the most important case. Now let's consider an additional potential for overloading: reusing the same name for things of completely different kinds. For example, could we use the same name to indicate both a data type and a variable, or both a variable and a function? In most languages, this is not permitted, and with good reason: It can be extremely confusing, and programs have little to gain by using such overloading (as opposed to function or operator overloading, which is very useful).

Occasionally, however, such overloading comes in handy as a way of limiting the number of names that one actually has to think about in a program. Take, for example, a typical C definition of a recursive type:

```
struct A{
    int data;
    struct A * next;
};
```

This is "old-style" C in which the data type is given by the `struct` name `A`, but the keyword `struct` must also always appear. Using a `typedef` one can remove the need for repeating the `struct`:

```
typedef struct A A;
struct A{
    int data;
    A * next;
};
```

Notice that we have used the same name, `A`, both for the `struct` name and the `typedef` name. This is legal in C, and it is useful in this case, since it reduces the number of names that we need to remember.[12] This implies that `struct` names in C occupy a different symbol table than other names, so the principle of unique names for types (and variables) can be preserved.[13] Some languages have extreme forms of this principle, with different symbol tables for each of the major kinds of definitions, so that one could, for example, use the same name for a type, a function, and a variable (why this might be useful is another question).

Java is a good example of this extreme form of name overloading, where the code of Figure 7.26 is perfectly acceptable to a Java compiler.[14]

```
class A
{  A A(A A)
   { A:
      for(;;)
      { if (A.A(A) == A) break A; }
      return A;
  }
}
```

Figure 7.26 Java class definition showing overloading of the same name for different language constructs (adapted from Arnold, Gosling, and Holmes [2000], p. 153)

Figure 7.26 includes a definition of a class, a method (a function), a parameter, and a label, all of which share the name `A`. In Exercise 7.20, you will have the chance to explain which instances of a name represent definitions and which represent references.

[12] In C++, the `typedef` is unnecessary, since the definition of `struct A` automatically creates a `typedef` named `A`.
[13] In fact, C implementations often simply add a prefix like `struct_` to a `struct` name and use the same symbol table, so the `struct` name `A` becomes `struct_A` and remains unique.
[14] In Java, the different symbol tables for different kinds of definitions are called **namespaces**, not to be confused with the namespaces of C++.

7.5 Allocation, Lifetimes, and the Environment

Having considered the symbol table in some detail, we need also to study the environment, which maintains the bindings of names to locations. We will introduce the basics of environments without functions here, as a comparison to the symbol table, but defer the study of environments in the presence of functions and procedures until Chapter 10.

Depending on the language, the environment may be constructed statically (at load time), dynamically (at execution time), or with a mixture of the two. A language that uses a completely static environment is FORTRAN—all locations are bound statically. A language that uses a completely dynamic environment is LISP—all locations are bound during execution. C, C++, Ada, Java, and other Algol-style languages are in the middle—some allocation is performed statically, while other allocation is performed dynamically.

Not all names in a program are bound to locations. In a compiled language, names of constants and data types may represent purely compile-time quantities that have no existence at load or execution time. For example, the C global constant declaration

```
const int MAX = 10;
```

can be used by a compiler to replace all uses of `MAX` by the value 10. The name `MAX` is never allocated a location and indeed has disappeared altogether from the program when it executes.

Declarations are used to construct the environment as well as the symbol table. In a compiler, the declarations are used to indicate what allocation code the compiler is to generate as the declaration is processed. In an interpreter, the symbol table and the environment are combined, so attribute binding by a declaration in an interpreter includes the binding of locations.

Typically, in a block-structured language, global variables can be allocated statically, since their meanings are fixed throughout the program. Local variables, however, are allocated dynamically when execution reaches the block in question. In the last section you saw that, in a block-structured language, the symbol table uses a stack-like mechanism to maintain the bindings of the declarations. Similarly, the environment in a block-structured language binds locations to local variables in a stack-based fashion. To see how this works, consider the following C program fragment of Figure 7.27 with nested blocks.

```
(1) A: {  int x;
(2)       char y;
(3)       ...
(4)    B: {  double x;
(5)          int a;
(6)          ...
(7)       } /* end B */
(8)    C: {  char y;
(9)          int b;
(10)         ...
(11)      D: {  int x;
(12)            double y;
(13)            ...
```

Figure 7.27 A C program with nested blocks to demonstrate allocation by the environment *(continues)*

(continued)

```
(14)          } /* end D */
(15)          ...
(16)       } /* end C */
(17)       ...
(18)    } /* end A */
```

Figure 7.27 A C program with nested blocks to demonstrate allocation by the environment

When the code is executed and a block is entered, memory for the variables declared at the beginning of each block is allocated. When a block is exited, this memory is deallocated. If we view the environment as a linear sequence of storage locations, with locations allocated from the top in descending order, then the environment at line 3 of Figure 7.27 (after the entry into A) looks as shown in Figure 7.28 (ignoring the size of memory allocated for each variable).

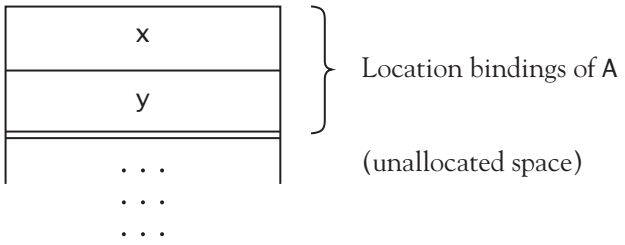


Figure 7.28 The environment at line 3 of Figure 7.27 after the entry into A

The environment after entry into B is shown in Figure 7.29.

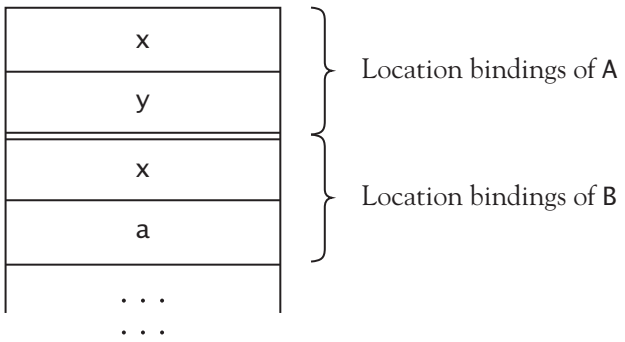


Figure 7.29 The environment at line 6 of Figure 7.27 after the entry into B

On exit from block B (line 7) the environment returns to the environment as it existed just after the entry into A. Then, when block C is entered (lines 8–9), memory for the variables of C is allocated and the environment is as shown in Figure 7.30.

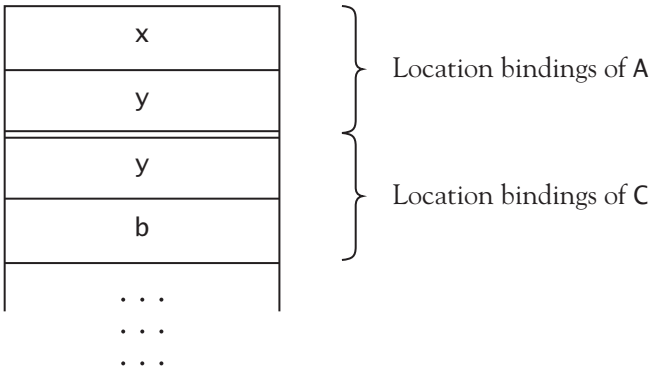


Figure 7.30 The environment at line 10 of Figure 7.27 after the entry into C

Notice that the variables y and b of block C are now allocated the same memory space that previously was allocated to the variables x and a of block B. This is okay, since we are now outside the scope of those variables, and they will never again be referenced.

Finally, on entry into block D (lines 11–12), the environment is as shown in Figure 7.31.

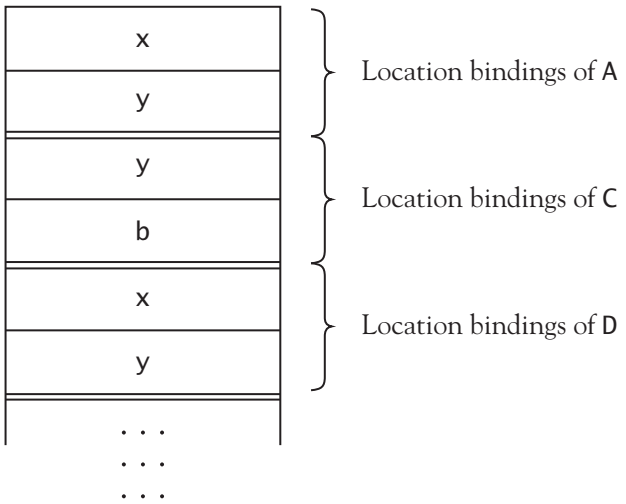


Figure 7.31 The environment at line 1 of Figure 7.27 after the entry into D

On exit from each block the location bindings of that block are successively deallocated, until, just prior to exit from block A, we have again recovered the original environment of A. In this way, the environment behaves like a stack. (Traditionally, environments are drawn upside down from the usual way stacks are depicted.)

This behavior of the environment in allocating and deallocating space for blocks is relatively simple. Procedure and function blocks are more complicated. Consider the following procedure declaration in C syntax:

```
void p(){
    int x;
    double y;
      ...
} /* p */
```

During execution, this declaration will not itself trigger an execution of the block of p, and the local variables x and y of p will not be allocated at the point of the declaration. Instead, memory for the variables x and y will be allocated only when p is called. Also, each time p is called, new memory for the local variables will be allocated. Thus, every call to p results in a region of memory being allocated in the environment. We refer to each call to p as an **activation** of p and the corresponding region of allocated memory as an **activation record**. A more complete description of the structure of activation records, and the information necessary to maintain them, is postponed to the discussion of procedures in Chapter 10.

It should be clear from these examples that, in a block-structured language with lexical scope, the same name may be associated with several different locations (though only one of these can be accessed at any one time). For instance, in the environment of the previous nested blocks example, the name x is bound to two different locations during the execution of block D (lines 11–14), and the name y is bound to three different locations (though only the x and y of D are accessible at that time). We must therefore distinguish among a name, an allocated location, and the declaration that causes them to be bound. We will call the allocated location an object. That is, an object is an area of storage that is allocated in the environment as a result of the processing of a declaration. According to this definition, variables and procedures in C are associated with objects, but global constants and data types are not (since type and global constant declarations do not result in storage allocation).[15] **The lifetime** or **extent** of an object is the duration of its allocation in the environment. The lifetimes of objects can extend beyond the region of a program where they may be accessed. For example, in the previous block example, the declaration of integer x in block A defines an object whose lifetime extends through block B, even though the declaration has a scope hole in B, and the object is not accessible from inside B. Similarly, it is possible for the reverse to happen: As explained later in this chapter in Section 7.7, an object can be accessible beyond its lifetime.

When pointers are available in a language, a further extension of the structure of the environment is necessary. A **pointer** is an object whose stored value is a reference to another object.

In C, the processing of the declaration

```
int* x;
```

by the environment causes the allocation of a pointer variable x, but *not* the allocation of an object to which x points. Indeed, x may have an undefined value, which could be any arbitrary location in memory. To permit the initialization of pointers that do not point to an allocated object, and to allow a program to determine whether a pointer variable points to allocated memory, C allows the integer 0 to be used as an

[15] This notion of object is not the same as that in object-oriented programming languages; see Chapter 5.

address that cannot be the address of any allocated object, and various C libraries give the name `NULL` to 0, so that one can write in C:

```
int* x = NULL;
```

Other languages generally reserve a special symbol for this address (`null` in Java and Ada, `nil` in Pascal), so that it cannot be used as an identifier.[16] With this initialization, `x` can be tested to see if it points to an allocated object:

```
if (x != NULL) *x = 2;
```

For `x` to point to an allocated object, we must manually allocate it by the use of an allocation routine. Again, C is unusual in that there is no built-in allocation routine specified by the language, but the `stdlib` library module contains several functions to allocate and deallocate memory. The most commonly used of these are the `malloc` (for *m*emory *alloc*ation) and `free` functions. Thus,

```
x = (int*) malloc(sizeof(int));
```

allocates a new integer variable and at the same time assigns its location to be the value of `x`. The `malloc` function returns the location it allocates and must be given the size of the data for which it is to allocate space. This can be given in implementation-independent form using the built-in `sizeof` function, which is given a data type and returns its (implementation-dependent) size. The `malloc` function must also be cast to the data type of the variable to which its result is being assigned, by putting the data type in parentheses before the function, since the address it returns is an *anonymous* pointer (a `void*` in C). (Casts and anonymous pointers are explained in Chapter 8.)

Once `x` has been assigned the address of an allocated integer variable, this new integer object can be accessed by using the expression `*x`. The variable `x` is said to be **dereferenced** using the unary * operator.[17] We can then assign integer values to `*x` and refer to those values as we would with an ordinary variable, as in:

```
*x = 2;
printf("%d\n",*x);
```

`*x` can be also be deallocated by calling the `free` procedure, as in:

```
free(x);
```

(Note that dereferencing is not used here, although it is `*x` that is being freed, not `x` itself.)

C++ simplifies the dynamic allocation of C by adding built-in operators `new` and `delete`, which are reserved names. Thus, the previous code in C++ would become:

```
int* x = new int; // C++
*x = 2;
cout << *x << endl; // output in C++
delete x;
```

[16] Though technically, `null` is a literal, in practice there is little difference between this literal use and that of a reserved word.
[17] While this use of * is made to look similar to the way `x` is defined, this is an entirely different use of the * symbol from its purpose in a declaration.

Note the special syntax in this code—`new` and `delete` are used as unary operators, rather than functions, since no parentheses are required. Pascal is similar to C++, only the `delete` procedure is called `dispose`.

To allow for arbitrary allocation and deallocation using `new` and `delete` (or `malloc` and `free`), the environment must have an area in memory from which locations can be allocated in response to calls to `new`, and to which locations can be returned in response to calls to `delete`. Such an area is traditionally called a **heap** (although it has nothing to do with the heap data structure). Allocation on the heap is usually referred to as **dynamic allocation**, even though allocation of local variables is also dynamic, as we have seen. To distinguish these two forms of dynamic allocation, allocation of local variables according to the stack-based scheme described earlier is sometimes called **stack-based** or **automatic**, since it occurs automatically under control of the runtime system. (A more appropriate term for pointer allocation using `new` and `delete` might be *manual* allocation, since it occurs under programmer control.)

In a typical implementation of the environment, the stack (for automatic allocation) and the heap (for dynamic allocation) are kept in different sections of memory, and global variables are also allocated in a separate, static area. Although these three memory sections could be placed anywhere in memory, a common strategy is to place the three adjacent to each other, with the global area first, the stack next, and the heap last, with the heap and stack growing in opposite directions (to avoid stack/heap collisions in case there is no fixed boundary between them). This is usually depicted as shown in Figure 7.32.

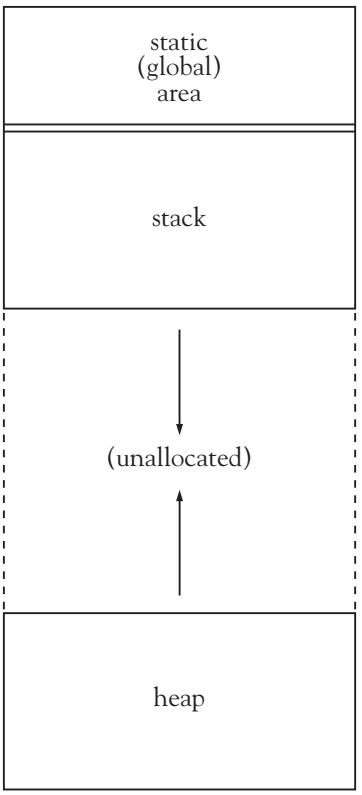


Figure 7.32 Structure of a typical environment with a stack and a heap

Note that, although the heap is shown as growing in only one direction, storage can be released anywhere in the allocated part of the heap, and that storage needs to be reusable. Thus, a simple stack protocol will not work for the heap. Chapter 10 discusses this question in more detail.

A further complication in the management of the heap is that many languages require that heap deallocation be managed automatically (like the stack, except that allocation may still be under user control). For example, virtually all functional languages require that the heap be completely automatic, with no allocation or deallocation under programmer control. Java, on the other hand, places allocation, but not deallocation, under programmer control. Java also further restricts heap allocation to objects only, that is, variables of reference types. Thus, the previous example would look as follows in Java:

```
// in Java must use class Integer, not int
// also, cannot allocate without assigning a value
Integer x = new Integer(2);
// print the actual integer value of x, i.e. 2
System.out.println(x);
// no delete operation allowed
```

Note how this code also does not have any explicit dereferencing (such as `*x` in C); Java forbids the use of such dereferencing—indeed, it is not even part of Java's syntax, as there is no "dereference" operator.

Languages like Java and the functional languages, such as Scheme, ML, and Haskell, do not allow much programmer control over heap allocation and deallocation, nor explicit pointer manipulation, such as dereferencing. The reason for this is that these are all inherently unsafe operations and can introduce seriously faulty runtime behavior that is very difficult to analyze and fix (as any C or C++ programmer can attest). Moreover, such faulty program behavior can, if extreme enough, compromise the entire operating system of a computer and cause an entire system or network to fail, or, worse, to act in malicious ways. For example, one entry point for computer viruses has traditionally been the unsafe access to addresses allowed by explicit pointer references in programs. This inherent lack of safety for heap allocation is studied further in Section 7.7.

One final, somewhat unusual, example of a language mechanism for heap allocation is that of Ada. Ada also has a `new` operation but no corresponding `delete` operation, similar to Java. However, Ada allows a `delete` operation to be user-defined using a standard language utility called `Ada.Unchecked_Deallocation`. This makes it more difficult for the programmer to use, as well as alerting anyone reading the code that this code is potentially unsafe, while at the same time allowing explicit programmer control of this feature as in C/C++. Figure 7.33 shows the preceding Java example rewritten in Ada (in its own block, to provide for the necessary declarations).

```
(1)  declare
(2)    type Intptr is access Integer;
(3)    procedure delete_int is
(4)       new Ada.Unchecked_Deallocation(Integer,Intptr);
(5)        -- generic procedure instantiation in Ada
```

Figure 7.33 Ada code showing manual dynamic deallocation *(continues)*

(continued)

```
-- see Chapter 8
(6)    x: Intptr := new Integer;
(7)  begin
(8)    x.all := 2;
(9)    put(x.all); new_line;
(10)    delete_int(x);
(11) end;
```

Figure 7.33 Ada code showing manual dynamic deallocation

Note how the keyword `access` is used in Figure 7.33 (line 2) to indicate a pointer, and the dereference operator (line 8 of Figure 7.33) is `.all`. This comes from the expectation of Ada (as with Java) that heap allocation will be used primarily for record (class) structures. The `.all` refers to *all fields* (in our case, there are no fields, only an integer, but one must still use the dereferencer). Unlike Java, Ada does not require `x` to be a record, however.

To summarize, in a block-structured language with heap allocation, there are three kinds of allocation in the environment: static (for global variables), automatic (for local variables), and dynamic (for heap allocation). These categories are also referred to as the **storage class** of the variable. Some languages, such as C, allow a declaration to specify a storage class as well as a data type. Typically, this is used in C to change the allocation of a local variable to static:

```
int f(){
static int x;
...
}
```

Now `x` is allocated only once, and it has the same meaning (and value) in all calls to `f`. This combination of local scope with global lifetime can be used to preserve local state across calls to `f`, while at the same time preventing code outside the function from changing this state. For example, Figure 7.34 shows the code for a function in C that returns the number of times it has been called, together with some interesting main program code.

```
(1)  int p(){
(2)     static int p_count = 0;
(3)     /* initialized only once - not each call! */
(4)     p_count += 1;
(5)     return p_count;
(6)  }

(7)  main(){
(8)      int i;
```

Figure 7.34 C program demonstrating the use of a local static variable *(continues)*

(continued)

```
(9)      for (i = 0; i < 10; i++){
(10)     if (p() % 3) printf("%d\n",p());
(11)     }
(12)     return 0;
(13) }
```

Figure 7.34 C program demonstrating the use of a local static variable

7.6 Variables and Constants

Although references to variables and constants look the same in many programming languages, their roles and semantics in a program are very different. This section clarifies and distinguishes their basic semantics.

7.6.1 Variables

A **variable** is an object whose stored value can change during execution. A variable can be thought of as being completely specified by its attributes, which include its name, its location, its value, and other attributes such as data type and size of memory storage. A schematic representation of a variable can be drawn as shown in Figure 7.35.

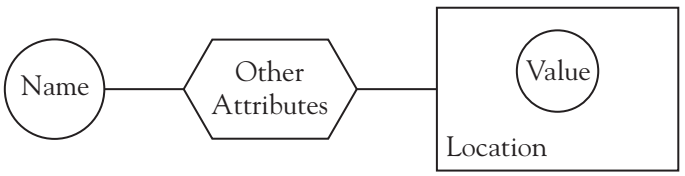


Figure 7.35 Schematic representation of a variable and its attributes

This picture singles out the name, location, and value of a variable as being its principal attributes. Often we will want to concentrate on these alone, and then we picture a variable in the diagram shown in Figure 7.36, which is known as a **box-and-circle diagram**.

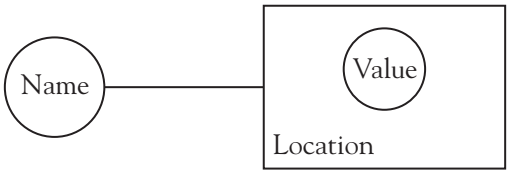


Figure 7.36 Schematic representation of a variable, its value, and its location

In a box-and-circle diagram, the line between the name and the location box represents the binding of the name to the location by the environment. The circle inside the box represents the value bound by the memory—that is, the value stored at that location.

The principal way a variable changes its value is through the **assignment** statement `x = e`, where `x` is a variable name and *e* is an expression. The semantics of this statement are that *e* is evaluated to a value, which is then copied into the location of `x`. If *e* is a variable name, say, `y`, then the assignment (in C syntax):

```
x = y
```

can be viewed as shown in Figure 7.37, with the double arrow indicating copying.

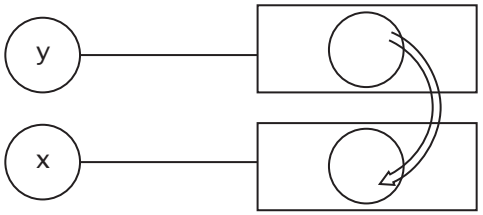


Figure 7.37 Assignment with copying of values

Since a variable has both a location and a value stored at that location, it is important to distinguish clearly between the two. However, this distinction is obscured in the assignment statement, in which `y` on the right-hand side stands for the value of `y` and `x` on the left-hand side stands for the location of `x`. For this reason, the value stored in the location of a variable is sometimes called its **r-value** (for right-hand side value), while the location of a variable is its **l-value** (for left-hand side value). A few languages make this distinction explicit. ML is one modern language that does so. For example, in ML, variables are explicitly thought of as locations, or references to values. If `x` is an integer variable in ML, then its type is "reference to integer," and to obtain its value we must write `!x`, which dereferences `x` to produce its value. Thus, to increment `x` we must write in ML:

```
x := !x + 1;
```

and the assignment of the value of the variable `y` to `x` would be written in ML as:

```
x := !y;
```

In C, in addition to the standard automatic dereferencing of l-values to r-values, there is an explicit **address of operator** `&` that turns a reference into a pointer. This allows the address of a variable to be explicitly fetched as a pointer (which can then be dereferenced using the standard * operator on pointers). Thus, given the C declaration

```
int x;
```

`&x` is a pointer to `x` (the "address" of `x`) and `*&x` is again `x` itself (both as an l-value and an r-value; see Exercise 7.24). Also, C allows the mixing of expressions with assignments, where both the r-value and l-value of a variable are involved. Thus, to increment the integer `x` by 1, one can write either `x = x + 1` or `x += 1` (indicating that `x` is both added to 1 and reassigned). The mixing of r-values, l-values, and pointers in C can lead to confusing and unsafe situations. For example,

```
x + 1 = 2;
```

is illegal (`x + 1` is not an l-value), but

```
*(&x + 1) = 2;
```

is legal. In this case, `&x` is a pointer, to which 1 is added using address arithmetic, and then 2 is stored at the resulting address; as you can imagine, this is unsafe!

These features of C—the mixing of references and values, the use of the `&` operator, and the distinction between a reference and a pointer—all make C vulnerable to extremely obscure and even dangerous or improper use of variables, many of which cannot easily be caught either during translation or execution. They are, however, a consequence of C's design goal as a fast, unsafe language for systems programming.

Ada95 also has an "address-of" operator: If `x` has an l-value, then `x'access` is a pointer to `x` (in Ada pointer types are called **access types**).[18] Ada95 applies strict rules, however, that limit the use of these types in ways that make it much more difficult to misuse them than in C. This is discussed in further detail in Chapter 10.

In some languages, assignment has a different meaning. Locations are copied instead of values. In this form of assignment, called **assignment by sharing**, case `x = y` has the result of binding the location of `y` to `x` instead of its value, as shown in Figure 7.38.

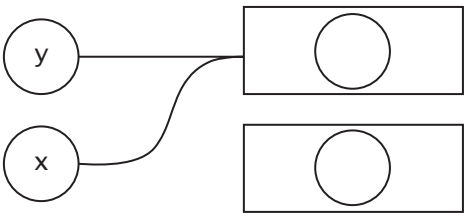


Figure 7.38 The result of assignment by sharing

An alternative, called **assignment by cloning**, is to allocate a new location, copy the value of `y`, and then bind `x` to the new location, as shown in Figure 7.39.[19]

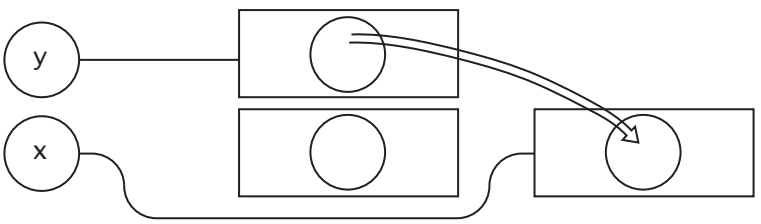


Figure 7.39 The result of assignment by cloning

In both cases this interpretation of assignment is sometimes referred to as **pointer semantics** or **reference semantics** to distinguish it from the more usual semantics, which is sometimes referred to

[18] The apostrophe is used in Ada to fetch various attributes of program entities; thus, `x'access` fetches the `access` attribute of the variable `x`, that is, the address of `x` as a pointer.
[19] A *clone* is an exact copy of an object in memory.

as **storage semantics** or **value semantics**. Java, for example, uses assignment by sharing for all object variables, but not for simple data; see Section 7.7 for an example. Other languages that use reference semantics are Smalltalk, LISP, and Python.

Figure 7.40 shows assignment by sharing, with the name x being associated directly to a new location (that of y). This is difficult to achieve for a compiler, since the symbol table commonly does not exist during execution. Standard implementations of assignment by sharing (such as in Java) use pointers and implicit dereferencing. In this view of assignment by sharing, x and y are implicitly pointers (with implicit automatically allocated pointer values).

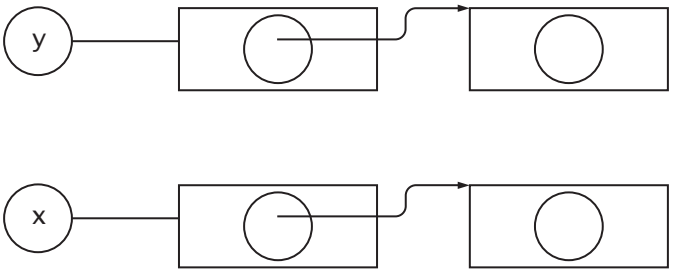


Figure 7.40 Variables as implicit references to objects

Then the assignment x = y has the effect shown in Figure 7.41. Note that this is the same as if we wrote x = y for pointer variables x and y in C.

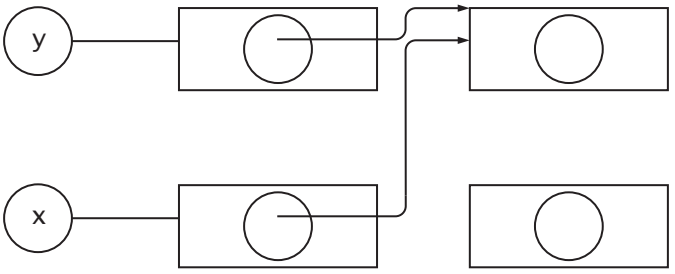


Figure 7.41 Assignment by sharing of references

However, in the absence of address-of and dereference operators (& and * in C), this implementation is indistinguishable from the original picture. (See Exercise 7.25 and Section 7.7.1.)

7.6.2 Constants

A **constant** is a language entity that has a fixed value for the duration of its existence in a program. As shown in Figure 7.42, a constant is like a variable, except that it has no location attribute, but a value only.

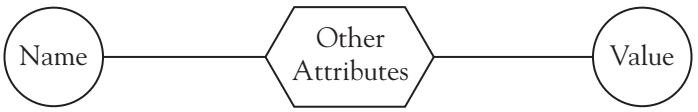


Figure 7.42 A constant and its attributes

We sometimes say that a constant has **value semantics** instead of the storage semantics of a variable. This does not mean that a constant is not stored in memory. It is possible for a constant to have a value that is known only at execution time. In this case, its value must be stored in memory, but, unlike a variable, once this value is computed, it cannot change, and the location of the constant cannot be explicitly referred to by a program.

This notion of a constant is **symbolic**; that is, a constant is essentially a name for a value. Sometimes representations of values, like the sequence of digits 42 or the representation of a character such as "a," are called constants. To distinguish them from the constants in a constant declaration, we sometimes refer to these representations of values as **literals**.

Many languages—especially the functional languages—emphasize the use of constants over variables because of their much simpler semantics: Each constant name stands for only one value, and, once the value is created, it remains the same regardless of the position or use of the name within the code. Some languages, such as Haskell, even prohibit variables altogether (the way some languages prohibit goto's) and rely entirely on constants for computations. (For more on this, see Chapter 3.)

As we have noted, constants can be static or dynamic. A static constant is one whose value can be computed prior to execution, while a dynamic constant has a value that can be computed only during execution. Static constants may also be of two kinds: Some constants can be computed at translation time (compile time), while others can be computed at load time (like the static location of a global variable), or right at the beginning of the execution of the program. This distinction is important, since a compile-time constant can, in principle, be used by a compiler to improve the efficiency of a program and need not actually occupy memory, while a load-time or dynamic constant must be computed either on startup or as execution proceeds and must be stored in memory. Unfortunately, the word "static" is used sometimes to refer to compile-time constants and sometimes to load-time constants. We shall try to be precise in the following description by referring to static translation-time constants as **compile-time constants** (even though the language may be interpreted rather than compiled), and we shall restrict the use of the term **static constant** to load-time constants. We can also make a distinction between general constants and **manifest constants**: A manifest constant is a name for a literal, while a constant can be more general.

Consider, for example, the following C code:

```
#include <stdio.h>
#include <time.h>

const int a = 2;
const int b = 27+2*2;
/* warning - illegal C code! */
const int c = (int) time(0);

int f( int x){
    const int d = x+1;
    return b+c;
}
...
```

In this code, a and b are compile-time constants (a is a manifest constant), while c is a static (load-time) constant, and d is a dynamic constant. In C, the const attribute can be applied to any variable in a program and simply indicates that the value of a variable, once set, cannot be changed; other criteria determine whether a variable is static or not (such as the global scope in which a, b, and c are defined above). In C, however, load-time constants cannot be defined because of a separate rule restricting initial values of global variables to a narrow subset of compile-time expressions. Thus, in C, we cannot even write:

```
const int a = 2;
int b = 27 + a * a; /* also illegal in C */
```

in the preceding program, because the initial value of b cannot be computed from the constant a. Instead, it must be computed from literals only (with a few small exceptions). However, in C++, which removes these restrictions, the preceding program is legal.

Ada is similar to C++ in that a constant declaration such as:

```
time : constant integer := integer(seconds(clock));
```

may appear anywhere in a program (and is static if global, and dynamic if local).

Java is similar to C++ and Ada, with two exceptions. First, in Java, a constant is indicated by using the keyword final (indicating that it gets only one, or a "final" value). Second, to get a static constant, one must use the keyword static (Java is structured so that there are no global variables as such). For example, the complete Java program in Figure 7.43 prints out the date and time at the moment when the program begins to run.

```
import java.util.Date;

class PrintDate{
    public static void main(String[] args){
        System.out.println(now);
    }
    static final Date now = new Date();
}
```

Figure 7.43 Java program demonstrating a load-time (static final) constant variable

One issue often suppressed or overlooked in language discussions is that function definitions in virtually all languages are definitions of constants whose values are functions. Indeed, a C definition such as:

```
int gcd( int u, int v){
    if (v == 0) return u;
    else return gcd(v, u % v);
}
```

defines the name gcd to be a (compile-time) constant whose value is a function with two integer parameters, returning an integer result, and whose operation is given by the code of its body. Notice how this

definition differs from that of a function *variable* in C, to which we could assign the value of `gcd`, as given in Figure 7.44.

```
(1) int (*fun_var)(int,int) = gcd;

(2) main(){
(3)     printf("%d\n", fun_var(15,10));
(4)     return 0;
(5) }
```

Figure 7.44 C code showing the use of a function variable

C is a little inconsistent in the code of Figure 7.44, in that we must define a function variable like `fun_var` as a pointer[20], but when assigning it a value and calling it, use of pointer syntax is not required. C also has no way of writing a function *literal* (that is, a function value, without giving it a name such as `gcd`). As we would expect, functional languages do a much better job of clarifying the distinction between function constants, function variables, and function literals (also called anonymous functions). For example, in ML, a function literal (in this case the integer square function) can be written as:

```
fn(x:int) => x * x
```

The square function can also be defined as a function constant (a constant is a `val` in ML) as:

```
val square = fn(x:int) => x * x;
```

as well as in more traditional C-like syntax:

```
fun square (x: int) = x * x;
```

An unnamed function literal can also be used directly in expressions without ever giving it a name. For example,

```
(fn(x:int) => x * x) 2;
```

evaluates the (unnamed) "square" function, passes it the value 2, and returns the value 4:

```
val it = 4 : int
```

Similar syntax is possible in Scheme and Haskell (see Chapter 3).

7.7 Aliases, Dangling References, and Garbage

This section describes several of the problems that arise with the naming and dynamic allocation conventions of programming languages, particularly the conventional block-structured languages C, C++, and Ada. As a programmer, you can learn how to avoid these problematic situations. As a language designer, you can build solutions to these problems into your language, so that the programmer doesn't have to be concerned with them at all (Java programmers are good examples of this). A few of these language design solutions are discussed in this section.

[20] This is necessary because without the pointer syntax this would be a function declaration or prototype.

7.7.1 Aliases

An **alias** occurs when the same object is bound to two different names at the same time. Aliasing can occur in several ways. One is during procedure call and is studied in Chapter 10. Another is through the use of pointer variables. A simple example in C is given by the following code:

```
(1) int *x, *y;
(2) x = (int *) malloc(sizeof(int));
(3) *x = 1;
(4) y = x;    /* *x and *y now aliases */
(5) *y = 2;
(6) printf("%d\n",*x);
```

After the assignment of x to y (line 4), *y and *x both refer to the same variable, and the preceding code prints 2. We can see this clearly if we record the effect of the preceding code in our box-and-circle diagrams of Section 7.6, as follows.

After the declarations (line 1), both x and y have been allocated in the environment, but the values of both are undefined. We indicate that in Figure 7.45 by shading in the circles indicating values.

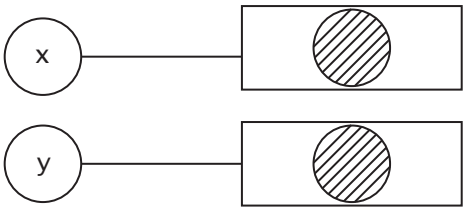


Figure 7.45 Allocation of storage for pointers x and y

After line 2, *x has been allocated, and x has been assigned a value equal to the location of *x, but *x is still undefined. See Figure 7.46.

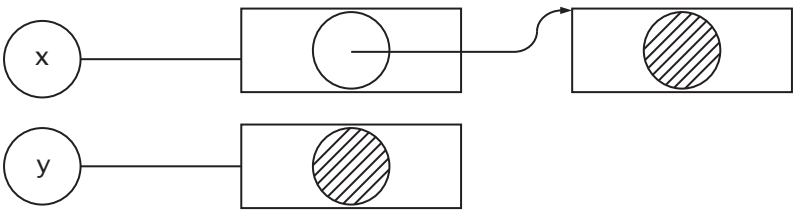


Figure 7.46 Allocation of storage for *x

After line 3, the situation is as shown in Figure 7.47.

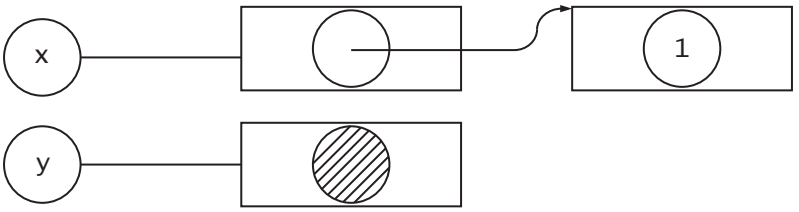


Figure 7.47 Result of *x = 1

Line 4 now copies the value of x to y, and so makes *y and *x aliases of each other. (Note that x and y are not aliases of each other.) See Figure 7.48.

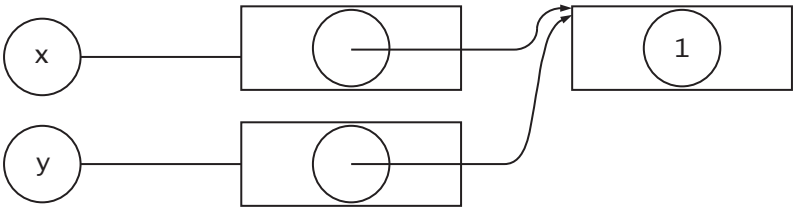


Figure 7.48 Result of y = x

Finally, line 5 results in the diagram shown in Figure 7.49.

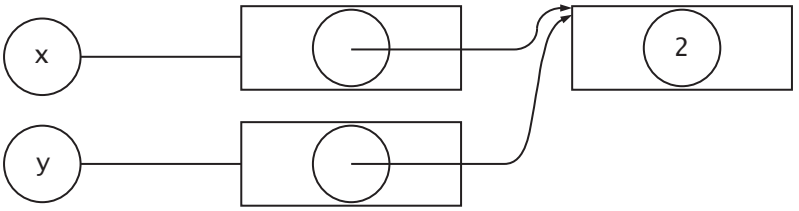


Figure 7.49 Result of *y = 2

Aliases present a problem in that they cause potentially harmful **side effects**. For our purposes, we define a side effect of a statement to be any change in the value of a variable that persists beyond the execution of the statement.[21] According to this definition, side effects are not all harmful, since an assignment is explicitly intended to cause one. However, side effects that are changes to variables whose names do not directly appear in the statement are potentially harmful in that the side effect cannot be determined from the written code. In the preceding example, the assignment `*y = 2` also changed `*x`, even though no hint of that change appears in the statement that causes it (line 5). We must instead read the earlier statements in this code to discover that this is happening.

Aliasing due to pointer assignment is difficult to control and is one of the reasons that programming with pointers is so difficult. One language that does attempt to limit aliasing, not only by pointers, but throughout the language, is Euclid. (For more on this, see the Notes and References at the end of this chapter.)

[21] Other definitions of side effect exist; see Exercise 7.23.

A third way that aliases can be created is through assignment by sharing, as described in the previous section, since assignment by sharing implicitly uses pointers. Java is a major example of this kind of aliasing. Consider the following Java program:

```
(1) class ArrTest{
(2)     public static void main(String[] args){
(3)         int[] x = {1,2,3};
(4)         int[] y = x;
(5)         x[0] = 42;
(6)         System.out.println(y[0]);
(7)     }
(8) }
```

In line 3, an array named `x` is created with size 3, such that `x[0]` = 1, `x[1]` = 2, and `x[2]` = 3. In line 4, another array `y` is defined and initialized with `x`. In line 5, `x[0]` is changed to 42, and in line 6, `y[0]` is printed, which is now also 42, because of assignment by sharing.

Because of this aliasing problem in Java, Java has a mechanism for explicitly **cloning** any object, so that aliases are not created by assignment. For example, if we replace line 4 in the preceding code with:

```
int[] y = (int[]) x.clone();
```

then the value printed in line 6 is 1, as we might reasonably expect.

7.7.2 Dangling References

Dangling references are a second problem that can arise from the use of pointers. A dangling reference is a location that has been deallocated from the environment but that can still be accessed by a program. In other words, a dangling reference occurs if an object can be accessed beyond its lifetime in the environment.

A simple example of a dangling reference is a pointer that points to a deallocated object. In C, the use of the `free` procedure can cause a dangling reference, as follows:

```
int *x , *y;
...
x = (int *) malloc(sizeof(int));
...
*x = 2;
...
y = x; /* *y and *x now aliases */
free(x); /* *y now a dangling reference */
...
printf("%d\n",*y); /* illegal reference */
```

In C, it is also possible for dangling references to result from the automatic deallocation of local variables when the block of the local declaration is exited. This is because, as noted previously, C has the address of operator & that allows the location of any variable to be assigned to a pointer variable. Consider the following C code:

```
(1) { int * x;
(2)   { int y;
(3)     y = 2;
(4)     x = &y;
(5)   }
(6)   /* *x is now a dangling reference */
(7) }
```

At line 5, when we exit the block in which `y` is declared, the variable `x` contains the location of `y` and the variable `*x` is an alias of `y`. However, in the standard stack-based environment we described in Section 7.5, `y` was deallocated on exiting from the block. A similar example is the following C code:

```
int * dangle(){
    int x;
    return &x;
}
```

Whenever function `dangle` is called, it returns the location of its local automatic variable `x`, which has just been deallocated. Thus, after any assignment such as `y = dangle()` the variable `*y` will be a dangling reference.

Ada does not permit this kind of dangling reference, since it has no equivalent to the & operator of C. Ada *does* allow the first kind of dangling reference, but only if the program uses the `Unchecked_Deallocation` package (see Section 7.5, Figure 7.33).

Java does not allow dangling references at all, since there are no explicit pointers in the language, no address of operator, and no memory deallocation operators such as `free` or `delete`.

7.7.3 Garbage

One easy way to eliminate the dangling reference problem is to prevent any deallocation from the environment. This causes the third problem that we discuss in this section, namely, garbage. Garbage is memory that has been allocated in the environment but that has become inaccessible to the program.

A typical way for garbage to occur in C is to fail to call `free` before reassigning a pointer variable:

```
int *x;
...
x = (int *) malloc(sizeof(int));
x = 0;
```

At the end of this code, the location allocated to `*x` by the call to `malloc` is now garbage, since `x` now contains the null pointer, and there is no way to access the previously allocated object. A similar situation occurs when execution leaves the region of the program in which `x` itself is allocated, as in:

```
void p(){
    int *x;
    x = (int *) malloc(sizeof(int));
    *x = 2;
}
```

When procedure `p` is exited, the variable `x` is deallocated and `*x` is no longer accessible by the program. A similar situation occurs in nested blocks.

Garbage is a problem in program execution because it is wasted memory. However, an argument can be made that programs that produce garbage are less seriously flawed than programs that contain dangling references. A program that is internally correct but that produces garbage may fail to run because it runs out of memory. That is, if it does not exceed available memory, it will produce correct results (or at least not be incorrect because of the failure to deallocate inaccessible memory). A program that accesses dangling references, on the other hand, may run but produce incorrect results, may corrupt other programs in memory, or may cause runtime errors that are hard to locate.

For this reason it is useful to remove the need to deallocate memory explicitly from the programmer (which, if done incorrectly, can cause dangling references), while at the same time automatically reclaiming garbage for further use. Language systems that automatically reclaim garbage are said to perform **garbage collection**.

We should note that the stack-based management of memory in the environment of a block-structured language can already be called a kind of simple garbage collection: When the scope of an automatic variable declaration is exited, the environment reclaims the location allocated to that variable by popping from the stack the memory allocated for the variable.

Historically, functional language systems, particularly LISP systems, pioneered garbage collection as a method for managing the runtime deallocation of memory. Indeed, in LISP, all allocation as well as deallocation is performed automatically. As a result, as we have previously noted, functional languages have no explicit pointers or operations like `malloc` and `free`. Object-oriented language systems also often rely on garbage collectors for the reclamation of memory during program execution. This is the case for Smalltalk and Java. C++ is the notable exception.[22]

Language design is a key factor in what kind of runtime environment is necessary for the correct execution of programs. Nevertheless, the language design itself may not explicitly state what kind of memory allocation is required. For example, the definition of Algol60 introduced block structure, and so implicitly advocated the use of a stack-based environment, without explicitly describing or requiring it. Definitions of LISP have also not mentioned garbage collection, even though a typical LISP system cannot execute correctly without it.

[22] The C++ Standard does allow for nonstandard heap managers, however, and the `new` and `delete` operators can be overloaded (for example, `delete` may be redefined to do nothing at all). However, all such redefinitions are nonstandard and currently not well supported by typical C++ implementations.

One way for the designer of an Algol-like language to indicate the need for automatic garbage collection would be to include in the language definition a `new` procedure for pointers but fail to include a corresponding `free` or `delete` procedure. Java takes this approach, as we have noted, and so does Ada, except that Ada allows the manual introduction of such a procedure using `Unchecked_Deallocation`. A further quirk of Ada is that, even with the use of `Unchecked_Deallocation`, a garbage collector may be called if a pointer is not correctly deallocated. Since one of Ada's key design goals was to be used in real-time situations where control of the speed of execution of a program is critical, Ada provides a further option to turn off any existing garbage collectors for variables of given data types:

```
-- Intptr defined as in Figure 7.33
Pragma CONTROLLED(Intptr);
```

7.8 Case Study: Initial Static Semantic Analysis of TinyAda

Section 6.8 in the previous chapter introduced a syntax analyzer for the little language TinyAda. That program took the form of a simple parsing shell that pulled tokens from a scanner until a syntax error was detected or the source program was recognized as syntactically correct. In this section, we discuss extending the parsing shell to perform some semantic analysis. In particular, we focus on designing and using tools for scope analysis and for restricting the use of identifiers in a program. This will involve a focus on two of the attributes of an identifier discussed earlier in this chapter: its name and the role it plays in a program (constant, variable, type, or procedure).

7.8.1 Scope Analysis

TinyAda is lexically or statically scoped. Here are the scope rules of the language:

1. All identifiers must be declared before they are used or referenced.
2. There may be at most one declaration of a given identifier in a single block.
3. In TinyAda, a new block begins with the formal parameter specifications of a procedure and extends to the reserved word `end` belonging to that procedure.
4. The visibility of a declared identifier extends into nested blocks, unless that identifier is redeclared in a nested block.
5. Identifiers are not case sensitive.

TinyAda has five built-in, or predefined, identifiers. They are the data type names `integer`, `char`, and `boolean`, and the Boolean constants `true` and `false`. Obviously, these identifiers must already be visible in a top-level scope before a source program is parsed. The static nesting level of this scope is, therefore, 0. Also entered into the level 0 scope is the name of the single top-level procedure. Nested within the level 0 scope, at level 1, is a new scope that contains the procedure's formal parameters, if any, and any identifiers introduced in the procedure's basic declarations. The introduction of new scopes for the names declared in nested procedures follows the same pattern.

As mentioned earlier in this chapter, scope analysis involves entering information into a symbol table when identifier declarations are encountered, and looking up this information in the symbol table

when identifier references are encountered. To handle the nested scopes in a TinyAda source program, the parser can use a stack of symbol tables. This stack is initialized to empty at program startup. Before parsing commences, a new table for scope level 0 is pushed onto the stack, and the information for the built-in identifiers is entered into it. This process is repeated for the programmer-defined identifiers when each new scope is entered. When a scope is exited, at the end of a procedure declaration, the table at the top of the stack is popped. When an identifier reference is encountered, the parser searches for that identifier's information in the topmost table in the stack, proceeding down through its tables until the information is found or the bottom of the stack is reached. The parser can now detect two scope errors. First, if a newly declared identifier is already present in the top-level table, the error is an identifier already declared in the same block. Second, if an identifier is referenced in the source program and cannot be found anywhere in the stack of tables, the error is an undeclared identifier.

We define two new classes to support scope analysis. The first class, called `SymbolEntry`, holds information about an identifier, such as its name and other attributes to be considered later in semantic analysis. The second class, called `SymbolTable`, manages the stack of scopes just discussed. This class handles the entry into and exit from a scope, the entry and lookup of identifiers, and the error detection process. The interface for the `SymbolTable` class is shown in Table 7.1. Its implementation is available on this book's Web site.

Table 7.1 The interface for the `SymbolTable` class

`SymbolTable` Method	What It Does
`SymbolTable(Chario c)`	Creates an empty stack of tables, with a reference to a `Chario` object for the output of error messages.
`void enterScope()`	Pushes a new table onto the stack.
`void exitScope();`	Pops a table from the stack and prints its contents.
`SymbolEntry enterSymbol(String name);`	If `name` is not already present, inserts an entry for it into the table and returns that entry; otherwise, prints an error message and returns an empty entry.
`SymbolEntry findSymbol(String name);`	If `name` is already present, returns its entry; otherwise, prints an error message and returns an empty entry.

Note that the parser does not halt when a scope error occurs. It is generally much easier to recover from semantic errors than from syntax errors.

The parser defines a new method for creating the symbol table and loading it with TinyAda's built-in identifiers, as follows:

```
private void initTable(){
   table = new SymbolTable(chario);   // table is a new instance variable
   table.enterScope();
   table.enterSymbol("BOOLEAN");
   table.enterSymbol("CHAR");
   table.enterSymbol("INTEGER");
   table.enterSymbol("TRUE");
   table.enterSymbol("FALSE");
}
```

To facilitate the processing of identifiers, the `Parser` class defines two other new methods, `enterId` and `findId`. These methods are now called in all of the places in the grammar rules where the parser expects to consume identifiers. Each method first checks the token's code to determine that it is in fact an identifier, and, if it is not, registers a fatal error (this is no change from the raw syntax analyzer of Chapter 6). Otherwise, the method enters or looks up the identifier in the symbol table and then consumes the token. Here is the code for these two `Parser` methods:

```
private SymbolEntry enterId(){
   SymbolEntry entry = null;
   if (token.code == Token.ID)
      entry = table.enterSymbol(token.string);
   else
      fatalError("identifier expected");
   token = scanner.nextToken();
   return entry;
}

private SymbolEntry findId(){
   SymbolEntry entry = null;;
   if (token.code == Token.ID)
      entry = table.findSymbol(token.string);
   else
      fatalError("identifier expected");
   token = scanner.nextToken();
   return entry;
}
```

We conclude this subsection with some examples of the use of each new scope analysis method. The `Parser` methods `subprogramBody` and `subprogramSpecification` collaborate to process a TinyAda procedure declaration. Toward the end of `subprogramBody`, the parser must exit the local scope of the TinyAda procedure. The parser might also encounter an optional procedure identifier at the end of this phrase, so the identifier must be found in the outer scope just restored. Here is the revised code for `subprogramBody`:

```
/*
subprogramBody =
         subprogramSpecification "is"
         declarativePart
         "begin" sequenceOfStatements
         "end" [ <procedure>identifier ] ";"
*/
private void subprogramBody(){
   subprogramSpecification();
```

(continues)

(continued)

```
      accept(Token.IS, "'is' expected");
      declarativePart();
      accept(Token.BEGIN, "'begin' expected");
      sequenceOfStatements();
      accept(Token.END, "'end' expected");
      table.exitScope();                    // The nested scope ends here
      if (token.code == Token.ID)           // Look up procedure's ID
         findId();
      accept(Token.SEMI, "semicolon expected");
   }
```

Our last example method, `subprogramSpecification`, processes a TinyAda procedure header. This method should see the declaration of the procedure name, which is entered into the current scope. The method might also see formal parameter declarations, so it enters a new scope, as shown in the next definition.

```
   /*
   subprogramSpecification = "procedure" identifier [ formalPart ]
   */
   private void subprogramSpecification(){
      accept(Token.PROC, "'procedure' expected");
      enterId();
      table.enterScope();                  // Nested scope begins here
      if (token.code == Token.L_PAR)
         formalPart();
   }
```

The scope entered just before the call of `formalPart` remains in effect for the other local declarations further down in the procedure declaration. The remaining modifications to the parsing methods are left as an exercise for the reader.

Note that the uses of the methods `enterId` and `findId` in the example throw away the `SymbolEntry` object returned by these methods. The parser does not really need this object for scope analysis. However, we will show how to use it to perform further semantic checks in the following section and in later chapters.

7.8.2 Identifier Role Analysis

In most high-level languages, an identifier names some entity, such as a variable, a constant, or an entire data type. We call this attribute of an identifier its **role**. You should not confuse an identifier's role with its value or its data type, both of which are additional attributes. Indeed, the roles of TinyAda identifiers appear among the semantic qualifiers (in angle brackets) in the EBNF grammar of Section 6.8. The role of an identifier imposes certain restrictions on its use. For example, only a variable or a parameter

identifier can appear on the left side of an assignment statement, and only a type identifier can appear as the element type of an array. If identifiers with any other roles are found in these positions, they should be flagged as semantic errors.

How does an identifier acquire a role to begin with? It does so in its declaration, where the parser has enough contextual information to determine it. But then how can the parser check for this role much later, when the identifier is used? The identifier's role is saved with it in the symbol table, where it can be accessed when needed. Just as in scope analysis, role analysis uses the symbol table to share contextual information about identifiers among otherwise independent parsing methods.[23]

Role analysis requires modifications to the `SymbolEntry` class and the `Parser` class. A `SymbolEntry` object now includes a role field as well as a name field. The possible values for a role are defined as constants in this class. They include `CONST`, `VAR`, `TYPE`, `PROC`, and `PARAM`, with the obvious meanings. In addition to this new attribute and its possible values, the `SymbolEntry` class defines two new methods, `setRole` and `append`. We will see how these are used in a moment.

The parser sets the appropriate role of a `SymbolEntry` object when its identifier is declared. For example, the method `typeDeclaration` introduces a new type name in a TinyAda program. Thus, after the name is entered into the symbol table, the role `TYPE` is entered into the name's `SymbolEntry`, as follows:

```
/*
typeDeclaration = "type" identifier "is" typeDefinition ";"
*/
private void typeDeclaration(){
      accept(Token.TYPE, "'type' expected");
      SymbolEntry entry = enterId();
      entry.setRole(SymbolEntry.TYPE);      // Establish the ID's role as TYPE
      accept(Token.IS, "'is' expected");
      typeDefinition();
      accept(Token.SEMI, "semicolon expected");
}
```

Other type names in a TinyAda program include the built-in data types, which are entered into the symbol table when the table is created.

To access and check the role of an identifier, the parser just looks in its `SymbolEntry` after looking up the identifier in the symbol table. If the found role does not match the expected role, the parser prints an error message and continues processing. Occasionally, a phrase can allow identifiers with different roles. For example, variables, parameters, and constants can appear in expressions, but not type names or procedure names.

The utility method `acceptRole`, which takes a `SymbolEntry`, an error message, and either a role or a set of roles as arguments, performs the required role checking. Here is the code for this method, and

[23] Note that parsers for real-world languages such as Ada typically do not rely on the symbol table to perform static semantic analysis beyond scope analysis. Instead, they generate a parse tree during syntax analysis and then perform traversals of that tree to install and access semantic information for further processing. This type of strategy is the subject of courses in compiler construction.

its use in the parsing method `index`. Note that the `acceptRole` method is overloaded for a single role and for a set of roles.

```
private void acceptRole(SymbolEntry s, int expected, String errorMessage){
   if (s.role != SymbolEntry.NONE && s.role != expected)
      chario.putError(errorMessage);
}

private void acceptRole(SymbolEntry s, Set<Integer> expected,
                        String errorMessage){
   if (s.role != SymbolEntry.NONE && ! (expected.contains(s.role)))
      chario.putError(errorMessage);
}

/*
index = range | <type>name
*/
private void index(){
   if (token.code == Token.RANGE)
      range();
   else if (token.code == Token.ID){
      SymbolEntry entry = findId();
      acceptRole(entry, SymbolEntry.TYPE, "must be a type name");
   }
   else
      fatalError("error in index type");
}
```

The `acceptRole` methods test the entry's role for the value `SymbolEntry.NONE`. This will be the role if the identifier was not even found in the symbol table. Thus, no role error should be reported in this case.

Some TinyAda declarations, such as `index`, introduce a single identifier, whereas others, such as `parameterSpecification` and `enumerationTypeDefinition`, introduce a list of identifiers. To ease the entry of attributes, all of which are the same for a list of identifiers, the `SymbolEntry` class now includes a `next` field, which can serve as a link to the entry for the next identifier in a list. To build this list, the parser obtains the entry for the first identifier in the list and runs the `SymbolEntry` method `append` to append the entry of each remaining identifier to the list. The `SymbolEntry` method `setRole` can then set the same role in the entire list of entries. The parsing methods `enumerationTypeDefinition` and `identiferList` show how this mechanism works. The `identifierList` method now returns a `SymbolEntry` object, which is actually the head of a list of `SymbolEntry` objects. The `enumerationTypeDefinition` method receives this object and sets its role, just as if it represented a single identifier. Here is the code for these two parsing methods:

```
/*
enumerationTypeDefinition = "(" identifierList ")"
*/
private void enumerationTypeDefinition(){
   accept(Token.L_PAR, '(' expected);
   SymbolEntry list = identifierList();   // Obtain the list of IDs
   list.setRole(SymbolRef.CONST);         // All of the IDs are now constants
   accept(Token.R_PAR, "')' expected");
}

/*
identifierList = identifier { "," identifier }
*/
private SymbolEntry identifierList(){
   SymbolEntry list = enterId();
   while (token.code == Token.COMMA){
      token = scanner.nextToken();
      list.append(enterId());             // Grow the list of entries
   }
   return list;
}
```

The remaining modifications to the parsing methods for role analysis are left as an exercise for the reader. Still other attributes of identifiers will be used to enforce further semantic restrictions in later chapters.

Exercises

7.1 For the languages C and Java, give as precise binding times as you can for the following attributes. Explain your answers.

(a) The maximum number of significant digits in a real number
(b) The meaning of *char*
(c) The size of an array variable
(d) The size of an array parameter
(e) The location of a local variable
(f) The value of a constant
(g) The location of a function

7.2 Discuss the relative advantages of early binding and late binding.

7.3 In FORTRAN, global variables are created using a `COMMON` statement, but it is not required that the global variables have the same name in each subroutine or function. Thus, in the code

```
FUNCTION F
COMMON N
   ...
END

SUBROUTINE S
COMMON M
   ...
END
```

variable `N` in function `F` and variable `M` in subroutine `S` share the same memory, so they behave as different names for the same global variable. Compare this method of creating global variables to that of C. How is it better? How is it worse?

7.4 Discuss the advantages and disadvantages of C's declaration before use rule with C++'s relaxation of that rule to extend the scope of a declaration inside a class declaration to the entire class, regardless of its position in the declaration.

7.5 C++ relaxes C's rule that all declarations in a block must come at the beginning of the block. In C++, declarations can appear anywhere a statement can appear. Discuss this feature of C++ from the point of view of **(a)** the principle of locality; **(b)** language complexity; **(c)** implementation complexity.

7.6 Describe the bindings performed by a C `extern` declaration. Why is such a declaration necessary?

7.7 As noted in this chapter, C and C++ make a distinction between a **declaration** and a **definition**. Which of the following are declarations only and which are definitions?

(a) `char * name;`
(b) `typedef int * IntPtr;`
(c) `struct rec;`
(d) `int gcd(int,int);`
(e) `double y;`
(f) `extern int z;`

7.8 Using the first organization of the symbol table described in the text (a single simple table), show the symbol table for the following C program at the three points indicated by the comments **(a)** using lexical scope and **(b)** using dynamic scope. What does the program print using each kind of scope rule?

```
#include <stdio.h>

int a,b;

int p(){int a, p;
  /* point 1 */
```

(continues)

(continued)

```
  a = 0; b = 1; p = 2;
  return p;
}

void print(){
printf("%d\n%d\n",a,b);
}

void q (){
int b;
   /* point 2 */
   a = 3; b = 4;
   print();
}

main(){
/* point 3 */
   a = p();
   q();
}
```

7.9 Using the second organization of the symbol table described in the text (a stack of tables), show the symbol table for the following Ada program at the three points indicated by the comments **(a)** using lexical scope and **(b)** using dynamic scope. What does the program print using each kind of scope rule?

```
procedure scope2 is
a, b: integer;
function p return integer is
a: integer;
begin -- point 1
  a := 0; b := 1; return 2;
end p;

procedure print is
begin -- point 2
  put(a); new_line; put(b); new_line; put(p);
    new_line;
end print;

procedure q is
b, p: integer;
begin -- point 3
  a := 3; b := 4; p := 5; print;
end q;
```

(continues)

(continued)

```
  begin
    a := p;
    q;
  end scope2;
```

7.10 Describe the problem with dynamic scoping.

7.11 Sometimes the symbol table is called the "static environment." Does that seem right to you? Why or why not?

7.12 Is extent the same as dynamic scope? Why or why not?

7.13 An alternative organization for a simple symbol table in a block-structured language to that described in the text is to have a stack of names, rather than a stack of symbol tables. All declarations of all names are pushed onto this stack as they are encountered and popped when their scopes are exited. Then, given a name, its currently valid declaration is the first one found for that name in a top-down search of the stack.

(a) Redo Exercise 7.8 with this organization for the symbol table.

(b) Sometimes this organization of the symbol table is called "deep binding," while the organization described in the text is called "shallow binding." Why are these terms appropriate? Should one of these organizations be required by the semantics of a programming language? Explain.

7.14 The following program prints two integer values; the first value typically is garbage (or possibly 0, if you are executing inside a debugger or other controlled environment), but the second value might be 2. Explain why.

```
#include <stdio.h>

void p(void){
int y;
  printf("%d\n", y);
  y = 2;
}
main(){
p(); p();
  return 0;
}
```

7.15 The following program prints two integer values; the first value typically is garbage (or possibly 0, if you are executing inside a debugger or other controlled environment), but the second value might be 2. Explain why.

```
#include <stdio.h>
main()
{
  { int x;
```

(continues)

(continued)

```
    printf("%d\n",x);
    x = 2;
  }
  { int y;
    printf("%d\n",y);
  }
  return 0;
}
```

7.16 Explain the difference between aliasing and side effects.

7.17 Suppose that the following function declarations (in C++ syntax) are available in a program:

```
(1) int pow(int, int);
(2) double pow(double,double);
(3) double pow(int, double);
```

and suppose that the following code calls the `pow function`:

```
int x;
double y;
x = pow(2,3);
y = pow(2,3);
x = pow(2,3.2);
y = pow(2,3.2);
x = pow(2.1,3);
y = pow(2.1,3);
x = pow(2.1,3.2);
y = pow(2.1,3.2);
```

Given the language **(a)** C++; **(b)** Java; or **(c)** Ada, write down the number of the `pow` function called in each of the eight calls, or write "illegal" if a call cannot be resolved in the language chosen, or if a data type conversion cannot be made.

7.18 Pointers present some special problems for overload resolution. Consider the following C++ code:

```
void f( int* x) { ... }

void f( char* x) { ... }

int main()
{ ...
  f(0);
  ...
}
```

What is wrong with this code? How might you fix it? Can this problem occur in Java? In Ada?

7.19 Default arguments in function definitions, such as:

```
void print( int x, int base = 10);
```

also present a problem for overload resolution. Describe the problem, and give any rules you can think of that might help resolve ambiguities. Are default arguments reasonable in the presence of overloading? Explain.

7.20 Figure 7.27 of the text illustrates Java namespace overloading, which allows the programmer to use the same name for different kinds of definitions, such as classes, methods (functions), parameters, and labels. In the code of Figure 7.27, there are 12 appearances of the name `A`. Describe which of these appearances represent definitions, and specify the kind of definition for each. Then list the other appearances that represent uses of each definition.

7.21 The text mentions that the lookup operation in the symbol table must be enhanced to allow for overloading. The insert operation must also be enhanced.

(a) Describe in detail how both operations should behave using a standard interface for a dictionary data structure.

(b) Should the symbol table itself attempt to resolve overloaded names, or should it leave that job to other utilities in a translator? Discuss.

7.22 Many programming languages (including C, C++, Java, and Ada) prohibit the redeclaration of variable names in the same scope. Discuss the reasons for this rule. Could not variables be overloaded the same way functions are in a language like C++ or Ada?

7.23 A common definition of "side effect" is a change to a nonlocal variable or to the input or output made by a function or procedure. Compare this definition of "side effect" to the one in the text.

7.24 **(a)** Which of the following C expressions are l-values? Which are not? Why? (Assume `x` is an `int` variable and `y` is an `int*` variable.)

```
(1) x + 2
(2) &x
(3) *&x
(4) &x + 2
(5) *(&x + 2)
(6) &*y
```

(b) Is it possible for a C expression to be an l-value but not an r-value? Explain.

(c) Is `&(&z)` ever legal in C? Explain.

7.25 Show how to use an address-of operator such as `&` in C to discover whether a translator is implementing assignment by sharing or assignment by cloning, as described in Section 7.6.1.

7.26 Given the following C program, draw box-and-circle diagrams of the variables after each of the two assignments to `**x` (lines 11 and 15). Which variables are aliases of each other at each of those points? What does the program print?

```
(1) #include <stdio.h>

(2) main(){
(3)   int **x;
```

(continues)

(continued)

```
(4)   int *y;
(5)   int z;
(6)   x = (int**) malloc(sizeof(int*));
(7)   y = (int*) malloc(sizeof(int));
(8)   z = 1;
(9)   *y = 2;
(10)  *x = y;
(11)  **x = z;
(12)  printf("%d\n",*y);
(13)  z = 3;
(14)  printf("%d\n",*y);
(15)  **x = 4;
(16)  printf("%d\n",z);
(17)  return 0;
(18) }
```

7.27 Explain the reason for the two calls to `malloc` (lines 6 and 7) in the previous exercise. What would happen if line 6 were left out? Line 7?

7.28 Repeat Exercise 7.26 for the following code:

```
(1) #include <stdio.h>

(2) main()
(3) { int **x;
(4)   int *y;
(5)   int z;
(6)   x = &y;

(7)   y = &z;
(8)   z = 1;
(9)   *y = 2;
(10)  *x = y;
(11)  **x = z;
(12)  printf("%d\n",*y);
(13)  z = 3;
(14)  printf("%d\n",*y);
(15)  **x = 4;
(16)  printf("%d\n",z);
(17)  return 0;
(18) }
```

7.29 A **single-assignment variable** is a variable whose value can be computed at any point but that can be assigned to only once, so that it must remain constant once it is computed. Discuss the usefulness of this generalization of the idea of constant compared to constant declarations in C, C++, Java, or Ada. How does this concept relate to that of a dynamic constant, as discussed in Section 7.6?

7.30 In Ada, an object is defined as follows: "An object is an entity that contains a value of a given type." This means that there are two kinds of objects: constants and variables. Compare this notion of an object with the one discussed in Section 7.6.

7.31 Why is the following C code illegal?

```
{ int x;
  &x = (int *) malloc(sizeof(int));
  ...
}
```

7.32 Why is the following C code illegal?

```
{ int x[3];
  x = (int *) malloc(3*sizeof(int));
  ...
}
```

7.33 Here is a legal C program with a function variable:

```
(1)  #include <stdio.h>

(2)  int gcd( int u, int v){
(3)     if (v == 0) return u;
(4)     else return gcd(v, u % v);
(5)  }
(6)  int (*fun_var)(int,int) = &gcd;

(7)  main(){
(8)     printf("%d\n", (*fun_var)(15,10));
(9)     return 0;
(10) }
```

Compare lines 6 and 8 of this code to the equivalent code of Figure 7.45 (lines 1 and 3). Why is this version of the code permitted? Why is it not required?

7.34 Make a list of the rules in the EBNF of TinyAda (Section 6.8) where identifiers are declared, and modify the code for your parser so that their names are entered into the symbol table when these declarations are encountered.

7.35 Make a list of rules in the EBNF of TinyAda where identifiers are referenced, and modify the relevant parsing methods to look up these identifiers in the symbol table.

7.36 Make a list of the rules in the EBNF of TinyAda (Section 6.8) where identifier roles are introduced in declarations, and modify the code for your parser so that this information is recorded in symbol entries when these declarations are encountered.

7.37 Make a list of rules in the EBNF of TinyAda where identifiers are referenced, and modify the code to accept the roles of these identifiers, using appropriate error messages.

Notes and References

Most of the concepts in this chapter were pioneered in the design of Algol60 (Naur [1963a]), except for pointer allocation. Pointer allocation was, however, a part of the design of Algol68 (Tanenbaum [1976]). Some of the consequences of the design decisions of Algol60, including the introduction of recursion, may not have been fully understood at the time they were made (Naur [1981]), but certainly by 1964 the full implementations of Algol60 used most of the techniques of today's translators (Randell and Russell [1964]). For more detail on these techniques, consult Chapter 10 and a compiler text such as Louden [1997] or Aho, Sethi, and Ullman [1986].

Structured programming, which makes use of the block structure in an Algol-like language, became popular some time later than the appearance of the concept in language design (Dahl, Dijkstra, and Hoare [1972]). For an interesting perspective on block structure, see Hanson [1981].

The distinction that C and C++ make between definition and declaration is explained in more detail in Stroustrup [1997] and Kernighan and Ritchie [1988]. The effect of a declaration during execution, including its use in allocation, is called **elaboration** in Ada and is explained in Cohen [1996].

The term "object" has almost as many definitions as there are programming languages. For definitions that differ from the one used in this chapter, see Exercise 7.30 and Chapter 6. The notions of symbol table, environment, and memory also change somewhat from language to language. For a more formal, albeit simple, example of an environment function, see Chapter 12. For a more detailed discussion of the theoretical representation of environments, see Meyer [1990]. For a more detailed description of environments in the presence of procedure calls, see Chapter 10.

Overloading in C++ is discussed in detail in Stroustrup [1997]; overloading in Java in Arnold, Gosling, and Holmes [2000] and Gosling, Joy, Steele, and Bracha [2000]; overloading in Ada in Cohen [1996]. As an aside, overloading rules and ambiguity resolution occupy a significant portion of the C++ language standard (more than 50 pages, including one entire chapter).

Aliases and the design of the programming language Euclid, which attempts to remove all aliasing, are discussed in Popek et al. [1977] (see also Lampson et al. [1981]). Techniques for garbage collection and automatic storage management have historically been so inefficient that their use in imperative languages has been resisted. With modern advances that has changed, and many modern object-oriented languages require it (Java, Python, Ruby) and do not suffer greatly from execution inefficiency. Functional languages, too, have improved dramatically in efficiency while maintaining fully automatic dynamic allocation. For an amusing anecdote on the early use of garbage collection, see McCarthy [1981, p. 183]. Garbage collection techniques are studied in Chapter 10.

For a perspective on the problem of dynamic scoping and static typing, see Lewis et al. [2000].

CHAPTER

Data Types

CHAPTER 8

Every program uses data, either explicitly or implicitly, to arrive at a result. Data in a program are collected into data structures, which are manipulated by control structures that represent an algorithm. This is clearly expressed by the following pseudoequation (an equation that isn't mathematically exact but expresses the underlying principles),

$$\text{algorithms} = \text{data structures} + \text{programs}$$

which comes from the title of a book by Niklaus Wirth (Wirth [1976]). How a programming language expresses data and control largely determines how programs are written in the language. The present chapter studies the concept of data type as the basic concept underlying the representation of data in programming languages, while the chapters that follow study control.

Data in its most primitive form inside a computer is just a collection of bits. A language designer could take this view and build up all data from it. In essence, the designer could create a virtual machine (that is, a simulation of hardware—though not necessarily the actual hardware being used) as part of the definition of the language. However, such a language would not provide the kinds of abstraction necessary for large or even moderately sized programs. Thus, most programming languages include a set of simple data entities, such as integers, reals, and Booleans, as well as mechanisms for constructing new data entities from these. Such abstractions are an essential mechanism in programming languages and contribute to achieving the design goals of readability, writability, reliability, and machine independence. However, we should also be aware of the pitfalls to which such abstraction can lead.

One potential problem is that machine dependencies are often part of the implementation of these abstractions, and the language definition may not address these because they are hidden in the basic abstractions. An example of this is the **finitude** of all data in a computer, which is masked by the abstractions. For example, when we speak of integer data we often think of integers in the mathematical sense as an infinite set: $\ldots, -2, -1, 0, 1, 2, \ldots$, but in a computer's hardware there is always a largest and smallest integer. Sometimes a programming language's definition ignores this fact, thus making the language machine dependent.[1]

A similar situation arises with the precision of real numbers and the behavior of real arithmetic operations. This is a difficult problem for the language designer to address, since simple data types and arithmetic are usually built into hardware. A positive development was the establishment in 1985 by the Institute of Electrical and Electronics Engineers (IEEE) of a floating-point standard that attempts

[1]In a few languages, particularly functional languages, integers can be arbitrarily large and integer arithmetic is implemented in software; while this removes machine dependencies, it usually means arithmetic is too slow for computationally intensive algorithms.

to reduce the dependency of real number operations on the hardware (and encourages hardware manufacturers to ensure that their processors comply with the standard). C++, Ada, and Java all rely on some form of this standard to make floating-point operations more machine-independent. Indeed, in an attempt to reduce machine dependencies to a minimum, the designers of Java and Ada built into their standards strong requirements for all arithmetic operations. C++, too, has certain minimal requirements as part of its standard, but it is less strict than Ada and Java.

An even more significant issue regarding data types is the disagreement among language designers on the extent to which type information should be made explicit in a programming language and be used to verify program correctness prior to execution. Typically, the line of demarcation is clearest between those who emphasize maximal flexibility in the use of data types and who advocate no explicit typing or translation-time type verification, and those who emphasize maximum restrictiveness and call for strict implementation of translation-time type checking. A good example of a language with no explicit types or translation-time typing is Scheme,[2] while Ada is a good example of a very strictly type-checked language (sometimes referred to as a **strongly typed** language). There are many reasons to have some form of static (i.e., translation-time) type checking, however:

1. Static type information allows compilers to allocate memory efficiently and generate machine code that efficiently manipulates data, so **execution efficiency** is enhanced.
2. Static types can be used by a compiler to reduce the amount of code that needs to be compiled (particularly in a recompilation), thus improving **translation efficiency**.
3. Static type checking allows many standard programming errors to be caught early, improving **writability**, or efficiency of program coding.
4. Static type checking improves the **security** and **reliability** of a program by reducing the number of execution errors that can occur.
5. Explicit types in a programming language enhance **readability** by documenting data design, allowing programmers to understand the role of each data structure in a program, and what can and cannot be done with data items.
6. Explicit types can be used to **remove ambiguities** in programs. A prime example of this was discussed in the previous chapter (Section 7.4): Type information can be used to resolve overloading.
7. Explicit types combined with static type checking can be used by programmers as a **design tool**, so that incorrect design decisions show up as translation-time errors.
8. Finally, static typing of interfaces in large programs enhances the development of large programs by verifying **interface consistency** and **correctness**.

[2]To say that Scheme has no explicit types and no translation-time type checking is not to say that Scheme has no types. Indeed, every data value in Scheme has a type, and extensive checking is performed on the type of each value during execution.

Many language designers considered these arguments so compelling that they developed languages in which virtually every programming step required the exact specification in the source code of the data types in use and their properties. This led programmers and other language designers to complain (with some justification) that type systems were being used as mechanisms to force programmers into a rigid discipline that was actually *reducing* writability and good program design.[3]

Since these arguments were first made in the 1960s and 1970s, many advances have been made in making static typing more flexible, while at the same time preserving all or most of the previously listed advantages to static typing. Thus, most modern languages (except for special-purpose languages such as scripting and query languages) use some form of static typing, while using techniques that allow for a great deal of flexibility.

In this chapter, we first describe the primary notion of **data type** (the basic abstraction mechanism, common to virtually all languages), and the principal types and type constructors available in most languages. After discussing these basic principles of data types, we will describe the mechanisms of static type checking and type inference, and discuss some of the modern methods for providing flexibility in a type system, while still promoting correctness and security. Other mechanisms that provide additional flexibility and security come from modules, and will be studied in later chapters.

8.1 Data Types and Type Information

Program data can be classified according to their **types**. At the most basic level, virtually every data value expressible in a programming language has an implicit type. For example, in C the value 21 is of type `int`, 3.14159 is of type `double`, and `"hello"` is of type "array of `char`".[4] In Chapters 6 and 7, you saw that the types of variables are often explicitly associated with variables by a declaration, such as:

```
int x;
```

which assigns data type `int` to the variable `x`. In a declaration like this, a type is just a name (in this case the keyword `int`), which carries with it some properties, such as the kinds of values that can be stored, and the way these values are represented internally.

Some declarations implicitly associate a type with a name, such as the Pascal declaration

```
const PI = 3.14159;
```

which implicitly gives the constant `PI` the data type `real`, or the ML declaration

```
val x = 2;
```

which implicitly gives the constant `x` the data type `int`.

Since the internal representation is a system-dependent feature, from the abstract point of view, we can consider a type name to represent the possible values that a variable of that type can hold. Even more

[3]Bjarne Stroustrup, for example (Stroustrup [1994], p. 20), has written that he found Pascal's type system to be a "straightjacket that caused more problems than it solved by forcing me to warp my designs to suit an implementation-oriented artifact."

[4]This type is not expressible directly using C keywords, though the C++ type `const char*` could be used. In reality, we should distinguish the type from its keyword (or words) in a language, but we shall continue to use the simple expedient of using keywords to denote types whenever possible.

abstractly, we can consider the type name to be essentially identical to the set of values it represents, and we can state the following:

DEFINITION:1. A ***data type*** is a set of values.

Thus, the declaration of `x` as an `int` says that the value of `x` must be in the set of integers as defined by the language (and the implementation), or, in mathematical terminology (using $\in$ for membership) the declaration

```
int x;
```

means the same as:

$$\text{value of } x \in \textit{Integers}$$

where *Integers* is the set of integers as defined by the language and implementation. In Java, for example, this set is always the set of integers from to 22,147,483,648 to 2,147,483,647, since integers are always stored in 32-bit two's-complement form.

A data type as a set can be specified in many ways: It can be explicitly listed or enumerated, it can be given as a subrange of otherwise known values, or it can be borrowed from mathematics, in which case the finiteness of the set in an actual implementation may be left vague or ignored.[5] Set operations can also be applied to get new sets out of old (see below).

A set of values generally also has a group of operations that can be applied to the values. These operations are often not mentioned explicitly with the type, but are part of its definition. Examples include the arithmetic operations on integers or reals, the subscript operation `[ ]` on arrays, and the member selector operator on structure or record types. These operations also have specific properties that may or may not be explicitly stated [e.g., `(x + 1) - 1 = x` or `x + y = y + x`]. Thus, to be even more precise, we revise our first definition to include explicitly the operations, as in the following definition:

DEFINITION:2. A ***data type*** is a set of values, together with a set of operations on those values having certain properties.

In this sense, a data type is actually a mathematical algebra, but we will not pursue this view to any great extent here. (For a brief look at algebras, see the mathematics of abstract data types in Chapter 11.)

As we have already noted, a language translator can make use of the set of algebraic properties in a number of ways to assure that data and operations are used correctly in a program. For example, given a statement such as

```
z = x / y;
```

a translator can determine whether `x` and `y` have the same (or related) types, and if that type has a division operator defined for its values (thus also specifying which division operator is meant, resolving any

[5]In C, these limits are defined in the standard header files `limits.h` and `float.h`. In Java, the classes associated with each of the basic data types record these limits (for example, `java.lang.Integer.MAX_VALUE` 5 2147483647).

overloading). For example, in C and Java, if x and y have type int, then integer division is meant (with the remainder thrown away) and the result of the division also has type int, and if x and y have type double, then floating-point division is inferred.[6] Similarly, a translator can determine if the data type of z is appropriate to have the result of the division copied into it. In C, the type of z can be any numeric type (and any truncation is automatically applied), while in Java, z can only be a numeric type that can hold the entire result (without loss of precision).

The process a translator goes through to determine whether the type information in a program is consistent is called **type checking**. In the preceding example, type checking verifies that variables x, y, and z are used correctly in the given statement.

Type checking also uses rules for inferring types of language constructs from the available type information. In the example above, a type must be attached to the expression x / y so that its type may be compared to that of z (in fact, x / y may have type int or double, depending on the types of x and y). The process of attaching types to such expressions is called **type inference**. Type inference may be viewed as a separate operation performed during type checking, or it may be considered to be a part of type checking itself.

Given a group of basic types like int, double, and char, every language offers a number of ways to construct more complex types out of the basic types; these mechanisms are called **type constructors**, and the types created are called **user-defined types**. For example, one of the most common type constructors is the **array**, and the definition

```
int a[10];
```

creates in C (or C++) a variable whose type is "array of int" (there is no actual array keyword in C), and whose size is specified to be 10. This same declaration in Ada appears as:

```
a: array (1..10) of integer;
```

Thus, the "array" constructor takes a base type (int or integer in the above example), and a size or range indication, and constructs a new data type. This construction can be interpreted as implicitly constructing a new set of values, which can be described mathematically, giving insight into the way the values can be manipulated and represented.

New types created with type constructors do not automatically get names. Names are, however, extremely important, not only to document the use of new types but also for type checking (as we will describe in Section 8.6), and for the creation of recursive types (Section 8.3.5). Names for new types are created using a **type declaration** (called a **type definition** in some languages). For example, the variable a, created above as an array of 10 integers, has a type that has no name (an **anonymous type**). To give this type a name, we use a typedef in C:

```
typedef int Array_of_ten_integers[10];
Array_of_ten_integers a;
```

or a type declaration in Ada:

```
type Array_of_ten_integers is array (1..10) of integer;
a: Array_of_ten_integers;
```

[6]Ada, on the other hand, has different symbols for these two operations: div is integer division, while / is reserved for floating-point division.

With the definition of new user-defined types comes a new problem. During type checking a translator must often compare two types to determine if they are the same, even though they may be user-defined types coming from different declarations (possibly anonymous, or with different names). Each language with type declarations has rules for doing this, called **type equivalence** algorithms. The methods used for constructing types, the type equivalence algorithm, and the type inference and type correctness rules are collectively referred to as a **type system**.

If a programming language definition specifies a complete type system that can be applied statically and that guarantees that all (unintentional) data-corrupting errors in a program will be detected at the earliest possible point, then the language is said to be **strongly typed**. Essentially, this means that all type errors are detected at translation time, with the exception of a few errors that can only be checked during execution (such as array subscript bounds). In these cases, code is introduced to produce a runtime error. Strong typing ensures that most **unsafe programs** (i.e., programs with data-corrupting errors) will be rejected at translation time, and those unsafe programs that are not rejected at translation time will cause an execution error prior to any data-corrupting actions. Thus, no unsafe program can cause data errors in a strongly typed language. Unfortunately, strongly typed languages often reject safe programs as well as unsafe programs, because of the strict nature of their type rules. (That is, the set of **legal programs**—those accepted by a translator—is a *proper* subset of the set of safe programs.) Strongly typed languages also place an additional burden on the programmer in that appropriate type information must generally be explicitly provided, in order for type checking to work properly. The main challenge in the design of a type system is to minimize both the number of illegal safe programs and the amount of extra type information that the programmer must supply, while still retaining the property that all unsafe programs are illegal.

Ada is a strongly typed language that, nevertheless, has a fairly rigid type system, thus placing a considerable burden on the programmer. ML and Haskell are also strongly typed, but they place less of a burden on the programmer, and with fewer illegal but safe programs. Indeed, ML has a completely formalized type system in which all properties of legal programs are mathematically provable. Pascal and its related languages (such as Modula-2) are usually also considered to be strongly typed, even though they include a few loopholes. C has even more loopholes and so is sometimes called a **weakly typed language**. C++ has attempted to close some of the most serious type of loopholes of C, but for compatibility reasons it still is not completely strongly typed.

Languages without static type systems are usually called **untyped languages** (or **dynamically typed languages**). Such languages include Scheme and other dialects of Lisp, Smalltalk, and most scripting languages such as Perl, Python, and Ruby. Note, however, that an untyped language does not necessarily allow programs to corrupt data—this just means that all safety checking is performed at execution time. For example, in Scheme *all* unsafe programs will generate runtime errors, and no safe programs are illegal. This is an enviable situation from the point of view of strong typing, but one that comes with a significant cost. (All type errors cause unpleasant runtime errors, and the code used to generate these errors causes slower execution times.)

In the next section, we study the basic types that are the building blocks of all other types in a language. In Section 8.3, we study basic type constructors and their corresponding set interpretations. In Section 8.4, we give an overview of typical type classifications and nomenclature. In Section 8.5, we study type equivalence algorithms, and in Section 8.6, type-checking rules. Methods for allowing the programmer to override type checking are examined in Section 8.7. Sections 8.8 and 8.9 give an overview of **polymorphism**, in which names may have multiple types, while still permitting static type checking. Section 8.10 concludes the chapter with a discussion of type analysis in a parser for TinyAda.

8.2 Simple Types

Algol-like languages (C, Ada, Pascal), including the object-oriented ones (C++, Java), all classify data types according to a relatively standard basic scheme, with minor variations. Unfortunately, the names used in different language definitions are often different, even though the concepts are the same. We will attempt to use a generic name scheme in this section and then point out differences among some of the foregoing languages in the next.

Every language comes with a set of **predefined types** from which all other types are constructed. These are generally specified using either keywords (such as `int` or `double` in C++ or Java) or predefined identifiers (such as `String` or `Process` in Java). Sometimes variations on basic types are also predefined, such as `short`, `long`, `long double`, `unsigned int`, and so forth that typically give special properties to numeric types.

Predefined types are primarily **simple types**: types that have no other structure than their inherent arithmetic or sequential structure. All the foregoing types except `String` and `Process` are simple. However, there are simple types that are not predefined: **enumerated types** and **subrange types** are also simple types.

Enumerated types are sets whose elements are named and listed explicitly. A typical example (in C) is

```
enum Color {Red, Green, Blue};
```

or (the equivalent in Ada):

```
type Color_Type is (Red, Green, Blue);
```

or (the equivalent in ML):

```
datatype Color_Type = Red | Green | Blue;
```

In Ada and ML, enumerations are defined in a type declaration, and are truly new types. In most languages (but not ML), enumerations are **ordered**, in that the order in which the values are listed is important. Also, most languages include a predefined **successor** and **predecessor** operation for any enumerated type. Finally, in most languages, no assumptions are made about how the listed values are represented internally, and the only possible value that can be printed is the value name itself. As a runnable example, the short program in Figure 8.1 demonstrates Ada's enumerated type mechanism, which comes complete with symbolic I/O capability so that the actual defined names can be printed. In particular, see line 6, which allows enumeration values to be printed.

```
(1)  with Text_IO; use Text_IO;
(2)  with Ada.Integer_Text_IO; use Ada.Integer_Text_IO;

(3)  procedure Enum is
(4)    type Color_Type is (Red,Green,Blue);
(5)    -- define Color_IO so that Color_Type values can be -- printed
(6)    package Color_IO is new Enumeration_IO(Color_Type);
(7)    use Color_IO;
```

Figure 8.1 An Ada program demonstrating the use of an enumerated type *(continues)*

(continued)

```
(8)    x : Color_Type := Green;
(9)  begin
(10)   x := Color_Type'Succ(x); -- x is now Blue
(11)   x := Color_Type'Pred(x); -- x is now Green
(12)   put(x); -- prints GREEN
(13)   new_line;
(14) end Enum;
```

Figure 8.1 An Ada program demonstrating the use of an enumerated type

By contrast, in C an `enum` declaration defines a type "`enum . . .`," but the values are all taken to be names of integers and are automatically assigned the values 0, 1, and so forth, unless the user initializes the `enum` values to other integers; thus, the C `enum` mechanism is really just a shorthand for defining a series of related integer constants, except that a compiler could use less memory for an `enum` variable. The short program in Figure 8.2, similar to the earlier Ada program, shows the use of C `enums`. Note the fact that the programmer can also control the actual values in an `enum` (line 3 of Figure 8.2), even to the point of creating overlapping values.

```
(1)  #include <stdio.h>

(2)  enum Color {Red,Green,Blue};
(3)   enum NewColor {NewRed = 3, NewGreen = 2, NewBlue = 2};

(4)  main(){
(5)    enum Color x = Green; /* x is actually 1 */
(6)    enum NewColor y = NewBlue; /* y is actually 2 */
(7)    x++; /* x is now 2, or Blue */
(8)    y--; /* y is now 1 -- not even in the enum */
(9)    printf("%d\n",x); /* prints 2 */
(10)    printf("%d\n",y); /* prints 1 */
(11)   return 0;
(12) }
```

Figure 8.2 A C program demonstrating the use of an enumerated type, similar to Figure 8.1

Java omits the `enum` construction altogether.[7]

Subrange types are contiguous subsets of simple types specified by giving a least and greatest element, as in the following Ada declaration:

```
type Digit_Type is range 0..9;
```

The type `Digit_Type` is a completely new type that shares the values 0 through 9 with other integer types, and also retains arithmetic operations, too, insofar as they make sense. This is useful if we want

[7]Java does have an `Enumeration` interface, but it is a different concept.

to distinguish this type in a program from other integer types, and if we want to minimize storage and generate runtime checks to make sure the values are always in the correct range. Languages in the C family (C, C++, Java) do not have subrange types, since the same effect can be achieved manually by writing the appropriate-sized integer type (to save storage), and by writing explicit checks for the values, such as in the following Java example:

```
byte digit; // digit can contain −128..127
...
if (digit > 9 || digit < 0) throw new DigitException();
```

Subrange types are still useful, though, because they cause such code to be generated automatically, and because type checking need not assume that subranges are interchangeable with other integer types.

Typically subranges are defined by giving the first and last element from another type for which, like the integers, every value has a next element and a previous element. Such types are called **ordinal types** because of the discrete order that exists on the set. All numeric integer types in every language are ordinal types, and enumerations and subranges are also typically ordinal types. Ordinal types always have comparison operators (like < and >=), and often also have successor and predecessor operations (or ++ and -- operations). Not all types with comparison operators are ordinal types, however: real numbers are ordered (i.e., 3.98, 3.99), but there is no successor or predecessor operation. Thus, a subrange declaration such as:

```
type Unit_Interval is range 0.0..1.0;
```

is usually illegal in most languages. Ada is a significant exception.[8]

Allocation schemes for the simple types are usually conditioned by the underlying hardware, since the implementation of these types typically relies on hardware for efficiency. However, as noted previously, languages are calling more and more for standard representations such as the IEEE 754 standard, which also requires certain properties of the operators applied to floating-point types. Some languages, like Java, also require certain properties of the integer types. If the hardware does not conform, then the language implementor must supply code to force compliance. For example, here is a description of the Java requirements for integers, taken from Arnold, Gosling, and Holmes [2000], page 156:

Integer arithmetic is modular two's complement arithmetic—that is, if a value exceeds the range of its type (`int` or `long`), it is reduced modulo the range. So integer arithmetic never overflows or underflows, but only wraps.

Integer division truncates toward zero (7/2 is 3 and 27/2 is 23). For integer types, division and remainder obey the rule:

```
(x/y)*y + x%y == x
```

So 7%2 is 1 and 27%2 is 21. Dividing by zero or remainder by zero is invalid for integer arithmetic and throws an `ArithmeticException`.

Character arithmetic is integer arithmetic after the `char` is implicitly converted to `int`.

[8] In Ada, however, the number of digits of precision must also be specified, so we must write `type Unit_Interval is digits 8 range 0.0 .. 1.0;`.

8.3 Type Constructors

Since data types are sets, set operations can be used to construct new types out of existing ones. Such operations include Cartesian product, union, powerset, function set, and subset.

When applied to types, these set operations are called **type constructors**. In a programming language, all types are constructed out of the predefined types using type constructors. In the previous section, we have already seen a limited form of one of these constructors—the subset construction—in subrange types. There are also type constructors that do not correspond to mathematical set constructions, with the principal example being pointer types, which involve a notion of storage not present in mathematical sets. There are also some set operations that do not correspond to type constructors, such as intersection (for reasons to be explained). In this section, we will catalog and give examples of the common type constructors.

8.3.1 Cartesian Product

Given two sets U and V, we can form the Cartesian or cross product consisting of all ordered pairs of elements from U and V:

$$U \times V = \{(u, v) \mid u \text{ is in } U \text{ and } v \text{ is in } V\}$$

Cartesian products come with projection functions $p_1: U \times V \to U$ and $p_2: U \times V \to V$, where $p_1((u, v)) = u$ and $p_2((u, v)) = v$. This construction extends to more than two sets. Thus $U \times V \times W = \{(u, v, w) \mid u \text{ in } U, v \text{ in } V, w \text{ in } W\}$. There are as many projection functions as there are components.

In many languages the Cartesian product type constructor is available as the **record** or **structure construction**. For example, in C the declaration

```
struct IntCharReal{
   int i;
   char c;
   double r;
};
```

constructs the Cartesian product type `int` × `char` × `double`.

In Ada this same declaration appears as:

```
type IntCharReal is record
   i: integer;
   c: character;
   r: float;
end record;
```

However, there is a difference between a Cartesian product and a record structure: In a record structure, the components have names, whereas in a product they are referred to by position. The projections in a record structure are given by the **component selector operation** (or **structure member operation**): If `x` is a variable of type `IntCharReal`, then `x.i` is the projection of `x` to the integers. Some authors, therefore,

consider record structure types to be different from Cartesian product types. Indeed, most languages consider component names to be part of the type defined by a record structure. Thus,

```
struct IntCharReal{
   int j;
   char ch;
   double d;
};
```

can be considered different from the `struct` previously defined, even though they represent the same Cartesian product set.

Some languages have a purer form of record structure type that is essentially identical to the Cartesian product, where they are often called **tuples**. For example, in ML, we can define `IntCharReal` as:

```
type IntCharReal = int * char * real;
```

Note that here the asterisk takes the place of the ×, which is not a keyboard character. All values of this type are then written as tuples, such as `(2,#"a",3.14)` or `(42,#"z",1.1)`.[9] The projection functions are then written as `#1`, `#2`, and so forth (instead of the mathematical notation p_1, p_2, etc.), so that `#3 (2,#"a",3.14)` = 3.14.

A data type found in object-oriented languages that is related to structures is the **class**. Classes, however, include functions that act on the data in addition to the data components themselves, and these functions have special properties that were studied in Chapter 5 (such functions are called **member functions** or **methods**). In fact, in C++, a `struct` is really another kind of class, as was noted in that chapter. The `class` data type is closer to our second definition of data type, which includes functions that act on the data. We will explore how functions can be associated with data later in this chapter and in subsequent chapters.

8.3.2 Union

A second construction, the union of two types, is formed by taking the set theoretic union of their sets of values. Union types come in two varieties: discriminated unions and undiscriminated unions. A union is **discriminated** if a **tag** or **discriminator** is added to the union to distinguish which type the element is—that is, which set it came from. Discriminated unions are similar to disjoint unions in mathematics. Undiscriminated unions lack the tag, and assumptions must be made about the type of any particular value (in fact, the existence of undiscriminated unions in a language makes the type system *unsafe*, in the sense discussed in Section 8.2).

In C (and C++) the `union` type constructor constructs undiscriminated unions:

```
union IntOrReal{
   int i;
   double r;
};
```

Note that, as with `structs`, there are names for the different components (`i` and `r`). These are necessary because their use tells the translator which of the types the raw bits stored in the union should be

[9]Characters in ML use double quotation marks preceded by a # sign.

interpreted as; thus, if x is a variable of type `union IntOrReal`, then `x.i` is always interpreted as an `int`, and `x.r` is always interpreted as a `double`. These component names should not be confused with a discriminant, which is a separate component that indicates which data type the value *really* is, as opposed to which type we may think it is. A discriminant can be imitated in C as follows:

```
enum Disc {IsInt,IsReal};
struct IntOrReal{
   enum Disc which;
   union{
      int i;
      double r;
   } val;
};
```

and could be used as follows:

```
IntOrReal x;
x.which = IsReal;
x.val.r = 2.3;
...
if (x.which == IsInt) printf("%d\n",x.val.i);
else printf("%g\n",x.val.r);
```

Of course, this is still unsafe, since it is up to the programmer to generate the appropriate test code (and the components can also be assigned individually and arbitrarily).

Ada has a completely safe union mechanism, called a **variant record**. The above C code written in Ada would look as follows:

```
type Disc is (IsInt, IsReal);
type IntOrReal (which: Disc) is
record
   case which is
      when IsInt =>  i: integer;
      when IsReal => r: float;
   end case;
end record;
...
x: IntOrReal := (IsReal,2.3);
```

Note how in Ada the IntOrReal variable x must be assigned both the discriminant and a corresponding appropriate value at the same time—it is this feature that guarantees safety. Also, if the code tries to access the wrong variant during execution (which cannot be predicted at translation time):

```
put(x.i); -- but x.which = IsReal at this point
```

then a CONSTRAINT_ERROR is generated and the program halts execution.

Functional languages that are strongly typed also have a safe union mechanism that extends the syntax of enumerations to include data fields. Recall from Section 8.2 that ML declares an enumeration as in the following example (using vertical bar for "or"):

```
datatype Color_Type = Red | Green | Blue;
```

This syntax can be extended to get an IntOrReal union type as follows:

```
datatype IntOrReal = IsInt of int | IsReal of real;
```

Now the "tags" IsInt and IsReal are used directly as discriminants to determine the kind of value in each IntOrReal (rather than as member or field names). For example,

```
val x = IsReal(2.3);
```

creates a value with a real number (and stores it in constant x), and to access a value of type IntOrReal we must use code that tests which kind of value we have. For example, the following code for the printIntOrReal function uses a case expression and **pattern matching** (i.e., writing out a sample format for the translator, which then fills in the values if the actual value matches the format):

```
fun printInt x =
   (print("int: "); print(Int.toString x); print("\n"));

fun printReal x =
   (print("real: "); print(Real.toString x); print("\n"))

fun printIntOrReal x =
   case x of
      IsInt(i) => printInt i |
      IsReal(r) =>printReal r ;
```

The printIntOrReal function can now be used as follows:

```
printIntOrReal(x); (* prints "real: 2.3" *)
printIntOrReal (IsInt 42); (* prints "int: 42" *)
```

The tags IsInt and IsReal in ML are called **data constructors**, since they construct data of each kind within a union; they are similar to the object constructors in an object-oriented language. (Data constructors and the pattern matching that goes with them were studied in Chapter 3.)

Unions can be useful in reducing memory allocation requirements for structures when different data items are not needed simultaneously. This is because unions are generally allocated space equal to the maximum of the space needed for individual components, and these components are stored in overlapping regions of memory. Thus, a variable of type IntOrReal (without the discriminant) might be allocated 8 bytes. The first four would be used for integer variants, and all 8 would be used for a real variant. If a discriminant field is added, 9 bytes would be needed, although the number would be probably rounded to 10 or 12.

Unions, however, are not needed in object-oriented languages, since a better design is to use inheritance to represent different nonoverlapping data requirements (see Chapter 5). Thus, Java does not have unions. C++ does retain unions, presumably primarily for compatibility with C.

Even in C, however, unions cause a small but significant problem. Typically, unions are used as a variant part of a `struct`. This requires that an extra level of member selection must be used, as in the `struct IntOrReal` definition above, where we had to use `x.val.i` and `x.val.r` to address the overlapping data. C++ offers a new option here—the **anonymous union** within a `struct` declaration, and we could write `struct IntOrReal` in C++ as follows:

```
enum Disc {IsInt,IsReal};
struct IntOrReal{   // C++ only!
   enum Disc which;
   union{
      int i;
      double r;
   }; // no member name here
};
```

Now we can address the data simply as `x.i` and `x.r`. Of course, Ada already has this built into the syntax of the variant record.

8.3.3 Subset

In mathematics, a subset can be specified by giving a rule to distinguish its elements, such as pos_int = $\{x \mid x$ is an integer and $x > 0\}$. Similar rules can be given in programming languages to establish new types that are subsets of known types. Ada, for example, has a very general **subtype** mechanism. By specifying a lower and upper bound, subranges of ordinal types can be declared in Ada, as for example:

```
subtype IntDigit_Type is integer range 0..9;
```

Compare this with the simple type defined in Section 8.2:

```
type Digit_Type is range 0..9;
```

which is a completely new type, *not* a subtype of `integer`.

Variant parts of records can also be fixed using a subtype. For example, consider the Ada declaration of `IntOrReal` in the preceding section a subset type can be declared that fixes the variant part (which then must have the specified value):

```
subtype IRInt is IntOrReal(IsInt);
subtype IRReal is IntOrReal(IsReal);
```

Such subset types **inherit** operations from their parent types. Most languages, however, do not allow the programmer to specify which operations are inherited and which are not. Instead, operations are automatically or implicitly inherited. In Ada, for example, subtypes inherit all the operations of the parent type. It would be nice to be able to exclude operations that do not make sense for a subset type; for example, unary minus makes little sense for a value of type `IntDigit_Type`.

An alternative view of the subtype relation is to *define* it in terms of sharing operations. That is, a type S is a subtype of a type T if and only if all of the operations on values of type T can also be applied to values of type S. With this definition, a subset may not be a subtype, and a subtype may not be a subset. This view, however, is a little more abstract than the approach we take in this chapter.

Inheritance in object-oriented languages can also be viewed as a subtype mechanism, in the same sense of sharing operations as just described, with a great deal more control over which operations are inherited.

8.3.4 Arrays and Functions

The set of all functions $f: U \rightarrow V$ can give rise to a new type in two ways: as an **array type** or as a **function type**. When U is an ordinal type, the function f can be thought of as an **array** with **index type** U and **component type** V: if i is in U, then $f(i)$ is the ith component of the array, and the whole function can be represented by the sequence or tuple of its values $(f(low), \ldots, f(high))$, where *low* is the smallest element in U and *high* is the largest. (For this reason, array types are sometimes referred to as **sequence types**.) In C, C++, and Java the index set is always a positive integer range beginning at zero, while in Ada any ordinal type can be used as an index set.

Typically, array types can be defined either with or without sizes, but to define a *variable* of an array type it's typically necessary to specify its size at translation time, since arrays are normally allocated statically or on the stack (and thus a translator needs to know the size).[10] For example, in C we can define array types as follows:

```
typedef int TenIntArray [10];
typedef int IntArray [];
```

and we can now define variables as follows:

```
TenIntArray x;
int y[5];
int z[] = {1,2,3,4};
IntArray w = {1,2};
```

Note that each of these variables has its size determined statically, either by the array type itself or by the initial value list: `x` has size 10; `y`, 5; `z`, 4; and `w`, 2. Also, it is illegal to define a variable of type `IntArray` without giving an initial value list:

```
IntArray w; /* illegal C! */
```

Indeed, in C the size of an array cannot even be a computed constant—it must be a literal:

```
const int Size = 5;
int x[Size]; /* illegal C, ok in C++ (see Section 8.6.2) */
int x[Size*Size] /* illegal C, ok in C++ */
```

And, of course, any attempt to dynamically define an array size is illegal (in both C and C++):[11]

[10] Arrays can be allocated dynamically on the stack, but this complicates the runtime environment; see Chapter 10.
[11] This restriction has been removed in the 1999 C Standard.

```
int f(int size) {
   int a[size]; /* illegal */
   ...
}
```

C does allow arrays without specified size to be parameters to functions (these parameters are essentially pointers—see later in this section):

```
int array_max (int a[], int size){
   int temp, i;
   assert(size > 0);
   temp = a[0];
   for (i = 1; i < size; i++)
      if (a[i] > temp) temp = a[i];
   return temp;
}
```

Note in this code that the size of the array a had to be passed as an additional parameter. The size of an array is not part of the array (or of an array type) in C (or C++).

Java takes a rather different approach to arrays. Like C, Java array indexes must be nonnegative integers starting at 0. However, unlike C, Java arrays are always dynamically (heap) allocated. Also, unlike C, in Java the size of an array can be specified completely dynamically (but once specified cannot change unless reallocated). While the size is not part of the array type, the size *is* part of the information stored when an array is allocated (and is called its **length**). Arrays can also be defined using C-like syntax, or in an alternate form that associates the "array" property more directly with the component type. Figure 8.3 contains an example of some array declarations and array code in Java packaged into a runnable example.

```
import java.io.*;
import java.util.Scanner;

public class ArrayTest{

   static private int array_max(int[] a){     // note location of []
      int temp;
      temp = a[0];
      // length is part of a
      for (int i = 1; i < a.length; i++)
         if (a[i] > temp) temp = a[i];
      return temp;
   }
```

Figure 8.3 A Java program demonstrating the use of arrays *(continues)*

(continued)

```
    public static void main (String args[]){   // this placement of [] also
                                               // allowed
       System.out.print("Input a positive integer: ");
       Scanner reader = new Scanner(System.in));
       int size = reader.nextInt();
       int[] a = new int[size] ; // Dynamic array allocation
       for (int i = 0; i < a.length; i++) a[i] = i;
       System.out.println(array_max(a));
    }
 }
```

Figure 8.3 A Java program demonstrating the use of arrays

Ada, like C, allows array types to be declared without a size (so-called **unconstrained arrays**) but requires that a size be given when array variables (but not parameters) are declared. For example, the declaration

```
type IntToInt is array (integer range <>) of integer;
```

creates an array type from a subrange of integers to integers. The brackets "<>" indicate that the precise subrange is left unspecified. When a variable is declared, however, a range must be given:

```
table : IntToInt(-10..10);
```

Unlike C (but like Java), array ranges in Ada can be given dynamically. Unlike Java, Ada does not allocate arrays on the heap, however (see Chapter 10). Figure 8.4 represents a translation of the Java program of Figure 8.3, showing how arrays are used in Ada.

```
with Text_IO; use Text_IO;
with Ada.Integer_Text_IO;
use Ada.Integer_Text_IO;

procedure ArrTest is
   type IntToInt is array (INTEGER range <>) of INTEGER;

   function array_max(a: IntToInt) return integer is
      temp: integer;
      begin
         temp := a(a'first);     -- first subscript value for a
                                 -- a'range = set of legal subscripts
         for i in a'range loop
            if a(i) > temp then
```

Figure 8.4 An Ada program demonstrating the use of arrays, equivalent to the Java program of Figure 8.3 *(continues)*

(continued)

```
                temp := a(i);
             end if;
          end loop;
          return temp;
       end array_max;

    size: integer;

    begin
       put_line("Input a positive integer: ");
       get(size);
       declare
          x: IntToInt(1..size);      -- dynamically sized array
          max: integer;
          begin
             for i in x'range loop   -- x'range = 1..size
                x(i) := i;
             end loop;
             put(array_max(x));
             new_line;
          end;
    end ArrTest;
```

Figure 8.4 An Ada program demonstrating the use of arrays, equivalent to the Java program of Figure 8.3

Multidimensional arrays are also possible, with C/C++ and Java allowing arrays of arrays to be declared:

```
int x[10][20]; /* C code */
int[][] x = new int [10][20]; // Java code
```

Ada and Fortran have separate multidimensional array constructs as in the following Ada declaration:

```
type Matrix_Type is
   array (1..10,−10..10) of integer;
```

In Ada this type is viewed as different from the type:

```
type Matrix_Type is
   array (1..10) of array (−10..10) of integer;
```

Variables of the first type must be subscripted as `x(i,j)`, while variables of the second type must be subscripted as in `x(i)(j)`. (Note that square brackets are never used in Ada.)

Arrays are perhaps the most widely used type constructor, since their implementation can be made extremely efficient. Space is allocated sequentially in memory, and indexing is performed by an offset

calculation from the starting address of the array. In the case of multidimensional arrays, allocation is still linear, and a decision must be made about which index to use first in the allocation scheme. If `x` is defined (in Ada) as follows:

```
x: array (1..10,-10..10) of integer;
```

then `x` can be stored as `x[1,-10], x[1,-9], x[1,-8], . . . , x[1,10], x[2,-10], x[2,-9]`, and so on (**row-major form**) or as `x[1,-10], x[2,-10], x[3,-10], . . . , x[10,-10], x[1,-9], x[2,-9]`, and so on (**column-major form**). Note that if the indices can be supplied separately (from left to right), as in the C declaration

```
int x[10][20];
```

then row-major form *must* be used. Only if all indices must be specified together can column-major form be used (in FORTRAN this is the case, and column-major form is generally used).

Multidimensional arrays have a further quirk in C/C++ because of the fact that the size of the array is not part of the array and is not passed to functions (see the previous `array_max` function in C). If a parameter is a multidimensional array, then the size of all the dimensions except the first must be specified in the parameter declaration:

```
int array_max (int a[][20] , int size)
     /* size of second dimension required ! */
```

The reason is that, using row-major form, the compiler must be able to compute the distance in memory between `a[i]` and `a[i+1]`, and this is only possible if the size of subsequent dimensions is known at compile time. This is not an issue in Java or Ada, since the size of an array is carried with an array value dynamically.

Functional languages usually do not supply an array type, since arrays are specifically designed for imperative rather than functional programming. Some functional languages have imperative constructs, though, and those that do often supply some sort of array mechanism. Scheme has, for example, a **vector** type, and some versions of ML have an array module. Typically, functional languages use the **list** in place of the array, as discussed in Chapter 3.

General function and procedure types can also be created in some languages. For example, in C we can define a function type from integers to integers as follows:

```
typedef int (*IntFunction)(int);
```

and we can use this type to define either variables or parameters:

```
int square(int x) { return x*x; }
IntFunction f = square;
int evaluate(IntFunction g, int value)
{   return g(value); }
...
printf("%d\n", evaluate(f,3)); /* prints 9 */
```

Note that, as we remarked in the previous chapter, C requires that we define function variables, types, and parameters using pointer notation, but then does not require that we perform any dereferencing (this is to distinguish function definitions from function types).

Of course, functional languages have very general mechanisms of this kind as well. For example, in ML we can define a function type, such as the one in the previous code segment, in the following form:

```
type IntFunction = int -> int;
```

and we can use it in a similar way to the previous C code:

```
fun square (x: int) = x * x;
val f = square;
fun evaluate (g: IntFunction, value: int) = g value;
...
evaluate(f,3); (* evaluates to 9 *)
```

In Ada95, we can also write function parameters and types. Here is the `evaluate` function example in Ada95:

```
type IntFunction is
          access function (x:integer) return integer;
-- "access" means pointer in Ada

function square (x: integer) return integer is
   begin
      return x * x;
   end square;

function evaluate (g: IntFunction; value: integer) return integer is
   begin
      return g(value);
   end evaluate;

f: IntFunction := square'access;
-- note use of access attribute to get pointer to square
...
evaluate(f,3); -- evaluates to 9
```

By contrast, most pure object-oriented languages, such as Java and Smalltalk, have no function variables or parameters. These languages focus on objects rather than functions.

The format and space allocation of function variables and parameters depend on the size of the address needed to point to the code representing the function and on the runtime environment required by the language. See Chapter 10 for more details.

8.3.5 Pointers and Recursive Types

A type constructor that does not correspond to a set operation is the **reference** or **pointer** constructor, which constructs the set of all addresses that refer to a specified type. In C the declaration

```
typedef int* IntPtr;
```

constructs the type of all addresses where integers are stored. If `x` is a variable of type `IntPtr`, then it can be **dereferenced** to obtain a value of type integer: `*x = 10` assigns the integer value 10 to the location given by `x`. Of course, `x` must have previously been assigned a valid address; this is usually accomplished dynamically by a call to an allocation function such as `malloc` in C. (Pointer variables were discussed in Chapter 7.)

The same declaration as the foregoing in Ada is:

```
type IntPtr is access integer;
```

Pointers are implicit in languages that perform automatic memory management. For example, in Java all objects are implicitly pointers that are allocated explicitly (using the `new` operator like C++) but deallocated automatically by a garbage collector. Functional languages like Scheme, ML, or Haskell also use implicit pointers for data but do both the allocation and deallocation automatically, so that there is no syntax for pointers (and no `new` operation).

Sometimes languages make a distinction between pointers and **references**, a reference being the address of an object under the control of the system, which cannot be used as a value or operated on in any way (although it may be copied), while a pointer can be used as a value and manipulated by the programmer (including possibly using arithmetic operators on it). In this sense, pointers in Java are actually references, because, except for copying and the `new` operation, they are under the control of the system. C++ is perhaps the only language where both pointers and references exist together, although in a somewhat uneasy relationship. Reference types are created in C++ by a **postfix & operator**, not to be confused with the *prefix* address-of & operator, which returns a pointer. For example:

```
double r = 2.3;
double& s = r; // s is a reference to r – C++ only!
               // so they share memory
s += 1; // r and s are now both 3.3
cout << r << endl; // prints 3.3
```

This can also be implemented in C++ (also valid in C) using pointers:

```
double r = 2.3;
double* p = &r; // p has value = address of r
*p += 1; // r is 3.3
cout << r << endl; // prints 3.3
```

References in C++ are essentially constant pointers that are dereferenced every time they are used (Stroustrup [1997], p. 98). For example, in the preceding code, `s += 1` is the same as `r += 1` (`s` is dereferenced first), while `p +=1` increments the *pointer* value of `p`, so that `p` is now pointing to a different location (actually 8 bytes higher, if a `double` occupies 8 bytes).

A further complicating factor in C++ and C is that arrays are implicitly constant pointers to their first component. Thus, we can write code in C or C++ as follows:

```
int a[] = {1,2,3,4,5}; /* a[0] = 1, etc. */
int* p = a; /* p points to first component of a */
printf("%d\n", *p); /* prints value of a[0], so 1 */
printf("%d\n", *(p+2)); /* prints value of a[2]= 3 */
printf("%d\n", *(a+2)); /* also prints 3 */
printf("%d\n", 2[a]); /* also prints 3! */
```

Indeed, `a[2]` in C is just a shorthand notation for `a + 2`, and, by the commutativity of addition, we can equivalently write `2[a]`!

Pointers are most useful in the creation of **recursive types**: a type that uses itself in its declaration. Recursive types are extremely important in data structures and algorithms, since they naturally correspond to recursive algorithms, and represent data whose size and structure is not known in advance, but may change as computation proceeds. Two typical examples are lists and binary trees. Consider the following C-like declaration of lists of characters:

```
struct CharList{
   char data;
   struct CharList next; /* not legal C! */
};
```

There is no reason in principle why such a recursive definition should be illegal; recursive functions have a similar structure. However, a close look at this declaration indicates a problem: Any such data must contain an infinite number of characters! In the analogy with recursive functions, this is like a recursive function without a "base case," that is, a test to stop the recursion:

```
int factorial (int n){
   return n * factorial (n – 1);       /* oops */
}
```

This function lacks the test for small `n` and results in an infinite number of calls to `factorial` (at least until memory is exhausted). We could try to remove this problem in the definition of `CharList` by providing a base case using a union:

```
union CharList{
   enum { nothing } emptyCharList; /* no data */
   struct{
      char data;
      union CharList next; /* still illegal */
   } charListNode;
};
```

Of course, this is still "wishful thinking" in that, at the line noted, the code is still illegal C. However, consider this data type as an abstract definition of what it means to be a `CharList` as a set, and then it actually makes sense, giving the following recursive equation for a `CharList`:

```
CharList = {nothing} < char 3 CharList
```

where < is union and 3 is Cartesian product. In Section 6.2.1, we described how BNF rules could be interpreted as recursive set equations that define a set by taking the smallest solution (or **least fixed point**). The current situation is entirely analogous, and one can show that the least fixed point solution of the preceding equation is:

```
{nothing} < char < (char 3 char) <
                  (char 3 char 3 char) <...
```

That is, a list is either the empty list or a list consisting of one character or a list consisting of two characters, and so on; see Exercise 8.7. Indeed, ML allows the definition of `CharList` to be written virtually the same as the above illegal C:

```
datatype CharList
   = EmptyCharList | CharListNode of char * CharList ;
```

Why is this still illegal C? The answer lies in C's data allocation strategy, which requires that each data type have a fixed maximum size determined at translation time. Unfortunately, a variable of type `CharList` has no fixed size and cannot be allocated prior to execution. This is an insurmountable obstacle to defining such a data type in a language without a fully dynamic runtime environment; see Chapter 10. The solution adopted in most imperative languages is to use pointers to allow manual dynamic allocation. Thus, in C, the direct use of recursion in type declarations is prohibited, but indirect recursive declarations through pointers is allowed, as in the following (now legal) C declarations:

```
struct CharListNode{
   char data;
   struct CharListNode* next; /* now legal */
};
typedef struct CharListNode* CharList;
```

With these declarations, each individual element in a `CharListNode` now has a fixed size, and they can be strung together to form a list of arbitrary size. Note that a union is no longer necessary, because we represent the empty list with a null pointer (the special address 0 in C). Nonempty `CharLists` are constructed using manual allocation. For example, the list containing the single character `'a'` can be constructed as follows:

```
CharList cl
   = (CharList) malloc(sizeof(struct CharListNode));
(*cl).data = 'a'; /* can also write cl->data = 'a'; */
(*cl).next = 0; /* can also write cl->next = 0; */
```

This can then be changed to a list of two characters as follows:

```
(*cl).next
     = (CharList) malloc(sizeof(struct CharListNode));
(*(*cl).next).data = 'b'; /*or cl->next->data = 'b'; */
(*(*cl).next).next = 0;  /* or cl->next->next = 0; */
```

8.3.6 Data Types and the Environment

At a number of points in this section, when the structure of a data type required space to be allocated dynamically, we have referred the reader to the discussion in Chapter 10 on environments,. This type of storage allocation is the case for pointer types, recursive types, and general function types. In their most general forms, these types require fully dynamic environments with automatic allocation and deallocation ("garbage collection") as exemplified by the functional languages and the more dynamic object-oriented languages (such as Smalltalk). More traditional languages, such as C++ and Ada, carefully restrict these types so that a heap (a dynamic address space under programmer control) suffices, in addition to a traditional stack-based approach to nested blocks (including functions), as discussed in Chapter 7. While it would make sense to discuss these general environment issues at this point and relate them directly to data structures, we opt to delay a full discussion until Chapter 10, where all environment issues, including those of procedure and function calls, can be addressed.

More generally, the requirements of recursive data and recursive functions present a duality. In a language with very general function constructs (such as the functional languages of Chapter 3), recursive data can be modeled entirely by functions. Similarly, object-oriented languages (Chapter 5) can model recursive functions entirely by recursive data objects or polymorphic methods. While we do not focus on this duality in detail in this book, specific examples can be found in Chapters 3 and 5 and their exercises.

8.4 Type Nomenclature in Sample Languages

Although we have presented the general scheme of type mechanisms in Sections 8.2 and 8.3, various language definitions use different and confusing terminology to define similar things. In this section, we give a brief description of the differences among three of the languages used in previous examples: C, Java, and Ada.

8.4.1 C

An overview of C data types is given in Figure 8.5. The simple types are called **basic types** in C, and types that are constructed using type constructors are called **derived types**. The basic types include: the `void` type (in a sense the simplest of all types, whose set of values is empty); the **numeric types**; the **integral types,** which are ordinal; and the **floating types**. There are three kinds of floating types and 12 possible kinds of integral types. Among these there are four basic kinds, all of which can also have either `signed` or `unsigned` attributes given them (indicated in Figure 8.5 by listing the four basic types with `signed` and `unsigned` in parentheses above them). Of course, not all of the 12 possible integral types are distinct. For example, `signed int` is the same as `int`. Other duplicates are possible as well.

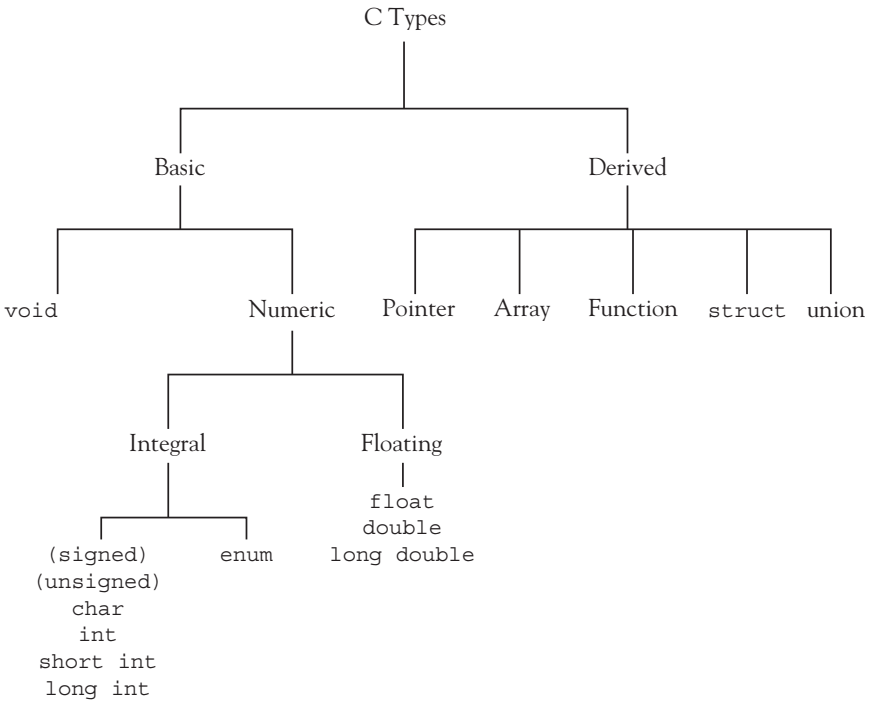


Figure 8.5 The type structure of C

8.4.2 Java

The Java type structure is shown in Figure 8.6. In Java the simple types are called **primitive types**, and types that are constructed using type constructors are called **reference types** (since they are all implicitly pointers or references). The primitive types are divided into the single type `boolean` (which

is *not* a numeric or ordinal type) and the numeric types, which split as in C into the integral (ordinal) and floating-point types (five integral and two floating point). There are only three type constructors in Java: the array (with no keyword as in C), the **class**, and the **interface**. The `class` and `interface` constructors were described in Chapter 5.

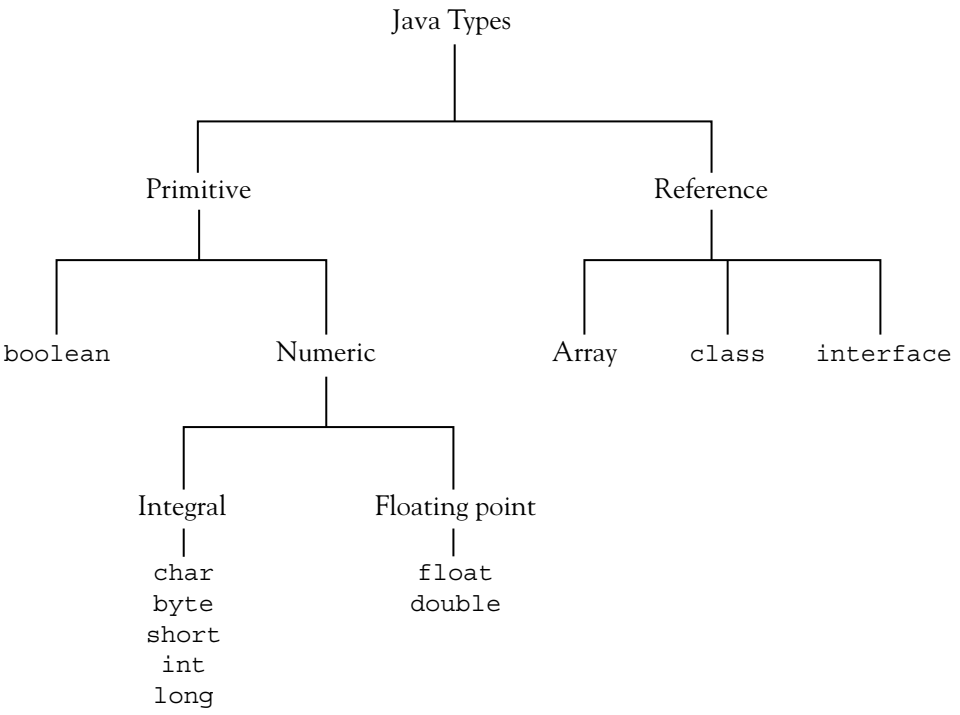


Figure 8.6 The type structure of Java

8.4.3 Ada

Ada has a rich set of types, a condensed overview of which is given in Figure 8.7. Simple types, called **scalar types** in Ada, are split into several overlapping categories. Ordinal types are called **discrete** types in Ada. **Numeric types** comprise the **real** and **integer** types. Pointer types are called `access` types. Array and record types are called **composite types**.

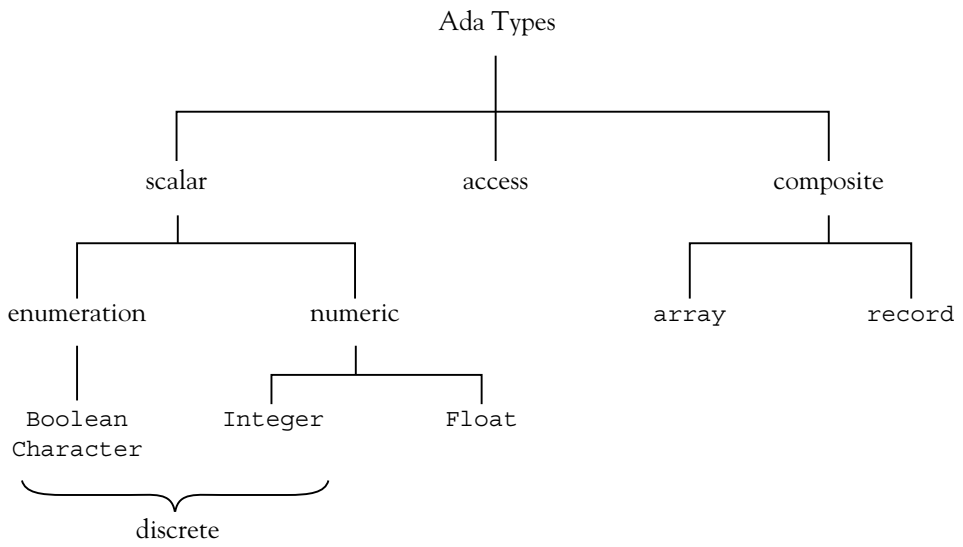


Figure 8.7 The type structure of Ada (somewhat simplified)

8.5 Type Equivalence

A major question involved in the application of types to type checking has to do with **type equivalence**: When are two types the same? One way of trying to answer this question is to compare the sets of values simply as sets. Two sets are the same if they contain the same values: For example, any type defined as the Cartesian product *A* X *B* is the same as any other type defined in the same way. On the other hand, if we assume that type *B* is not the same as type *A*, then a type defined as *A* X *B* is not the same as a type defined as *B* X *A*, since *A* X *B* contains the pairs (*a, b*) but *B* X *A* contains the pairs (*b, a*). This view of type equivalence is that two data types are the same if they have the same structure: They are built in exactly the same way using the same type constructors from the same simple types. This form of type equivalence is called **structural equivalence** and is one of the principal forms of type equivalence in programming languages.

Example

In a C-like syntax, the types `struct Rec1` and `struct Rec2` defined as follows are structurally equivalent, but `struct Rec1` and `struct Rec3` are not (the `char` and `int` fields are reversed in the definition of `struct Rec3`):

```
struct Rec1{
   char x;
   int y;
   char z[10];
};
```

(continues)

(continued)

```
struct Rec2{
   char x;
   int y;
   char z[10];
};

struct Rec3{
   int y;
   char x;
   char z[10];
};
```

Structural equivalence is relatively easy to implement (except for recursive types, as explained later in this chapter). What's more, it provides all the information needed to perform error checking and storage allocation. It is used in such languages as Algol60, Algol68, FORTRAN, COBOL, and a few modern languages such as Modula-3. It is also used selectively in such languages as C, Java, and ML, as explained later in this chapter. To check structural equivalence, a translator may represent types as trees and check equivalence recursively on subtrees (see Exercise 8.36). Questions still arise, however, in determining how much information is included in a type under the application of a type constructor. For example, are the two types `A1` and `A2` defined by

```
typedef int A1[10];
typedef int A2[20];
```

structurally equivalent? Perhaps yes, if the size of the index set is not part of an array type; otherwise, no. A similar question arises with respect to member names of structures. If structures are taken to be just Cartesian products, then, for example, the two structures

```
struct RecA{
   char x;
   int y;
};
```

and

```
struct RecB{
   char a;
   int b;
};
```

should be structurally equivalent; typically, however, they are not equivalent, because variables of the different structures would have to use different names to access the member data.

A complicating factor is the use of **type names** in declarations. As noted previously, type expressions in declarations may or may not be given explicit names. For example, in C, variable declarations may be given using **anonymous types** (type constructors applied without giving them names), but names can also be given right in `struct`s and `union`s, or by using a `typedef`. For example, consider the C code:

```
struct RecA{
   char x;
   int y;
} a;

typedef struct RecA RecA;

typedef struct{
   char x;
   int y;
} RecB;

RecB b;

struct{
   char x;
   int y;
} c;
```

Variable `a` has a data type with two names: `struct RecA` and `RecA` (as given by the `typedef`). Variable `b`'s type has only the name `RecB` (the `struct` name was left blank). And variable `c`'s type has no name at all! (Actually, `c`'s type still has a name, but it is internal and cannot be referred to by the programmer). Of course, the types `struct RecA`, `RecA`, `RecB`, and `c`'s anonymous type are all structurally equivalent.

Structural equivalence in the presence of type names remains relatively simple—simply replace each name by its associated type expression in its declaration—*except* for recursive types, where this rule would lead to an infinite loop (recall that a major reason for introducing type names is to allow for the declaration of recursive types). Consider the following example (a variation on an example used previously):

```
typedef struct CharListNode* CharList;
typedef struct CharListNode2* CharList2;

struct CharListNode{
   char data;
   CharList next;
};

struct CharListNode2{
   char data;
   CharList2 next;
};
```

Clearly `CharList` and `CharList2` are structurally equivalent, but a type checker that simply replaces type names by definitions will get into an infinite loop trying to verify it! The secret is to *assume* that `CharList` and `CharList2` are structurally equivalent to start with. It then follows easily that `CharListNode` and `CharListNode2` are structurally equivalent, and then that `CharList` and `CharList2` are indeed themselves equivalent. Of course, this seems like a circular argument, and it must be done carefully to give correct results. The details can be found in Louden [1997].

To avoid this problem, a different, much stricter, type equivalence algorithm was developed that focuses on the type names themselves. Two types are the same only if they have the same name. For obvious reasons, this is called **name equivalence**.

Example

In the following C declarations,

```
struct RecA{
   char x;
   int y;
};

typedef struct RecA RecA;

struct RecA a;
RecA b;
struct RecA c;
struct{
   char x;
   int y;
} d;
```

all of the variables `a`, `b`, `c`, and `d` are structurally equivalent. However, `a` and `c` are name equivalent (and not name equivalent to `b` or `d`), while `b` and `d` are not name equivalent to any other variable. Similarly, given the declarations

```
typedef  int Ar1[10];
typedef Ar1 Ar2;
typedef int Age;
```

types `Ar1` and `Ar2` are structurally equivalent but not name equivalent, and `Age` and `int` are structurally equivalent but not name equivalent. ■

Name equivalence in its purest form is even easier to implement than structural equivalence, as long as we *force* every type to have an explicit name (this can actually be a good idea, since it *documents* every type with an explicit name): Two types are the same only if they are the same name, and two variables are type equivalent only if their declarations use exactly the same type name. We can also invent **aliases** for types (e.g., `Age` above is an alias for `int`), and the type checker forces the programmer to keep uses of aliases distinct (this is also a very good *design tool*).

The situation becomes slightly more complex if we allow variable or function declarations to contain new types (i.e., type constructors) rather than existing type names only. Consider, for example, the following declarations:

```
struct{
   char x;
   int y;
} d,e;
```

Are `d` and `e` name equivalent? Here there are no visible type names from which to form a conclusion, so a name equivalence algorithm could say either yes or no. A language might include a rule that a combined declaration as this is equivalent to separate declarations:

```
struct{
   char x;
   int y;
} d;

struct{
   char x;
   int y;
} e;
```

In this case, new internal names are generated for each new `struct`, so `d` and `e` are clearly *not* name equivalent). On the other hand, using a single `struct` in a combined declaration could be viewed as constructing only *one* internal name, in which case `d` and `e` *are* equivalent.

Ada is one of the few languages to implement a very pure form of name equivalence. The only time Ada allows type constructors to be used in variable declarations is with the array constructor:

```
a: array (1..10) of integer;
```

The following is illegal Ada:

```
a: record
   x: integer;
   y: character;
end record;
```

Instead, we must write:

```
type IntChar is record
   x: integer;
   y: character;
end record;
a: IntChar;
```

Thus, ambiguity is avoided in Ada by requiring type names in variable and function declarations in virtually all cases. Ada views simultaneous array variable declarations without type names as having separate, inequivalent types.

Note that one small special rule is made in Ada for aliases and subtypes. According to this rule, writing:

```
type Age is integer;
```

is illegal. The Ada compiler wants to know whether we want an actual new type or just an alias that should be considered equivalent to the old type. Here we must write either:

```
type Age is new integer;
     -- Age is a completely new type
```

or:

```
subtype Age is integer;
    -- Age is just an alias for integer
```

As noted previously, the `subtype` designation is also used to create subset types, which are also not new types, but indications for runtime value checking.

C uses a form of type equivalence that falls between name and structural equivalence, and which can be loosely described as "name equivalence for `structs` and `unions`, structural equivalence for everything else." What is really meant here is that applying the `struct` or `union` type constructor creates a new, nonequivalent, type. Applying any other type constructor, or using a `typedef`, does not create a new type but one that is equivalent to every other type with the same structure (taking into account the special rule for `structs` and `unions`).

Example

```
struct A{
   char x;
   int y;
};

struct B{
   char x;
   int y;
};

typedef struct A C;
typedef C* P;
typedef struct B * Q;
typedef struct A * R;
typedef int S[10];
```

(continues)

(continued)

```
typedef int T[5];
typedef int Age;
typedef int (*F)(int);
typedef Age (*G)(Age);
```

Types `struct A` and `C` are equivalent, but they are not equivalent to `struct B`; types `P` and `R` are equivalent, but not to `Q`; types `S` and `T` are equivalent; types `int` and `Age` are equivalent, as are function types `F` and `G`.

Pascal adopts a similar rule to C, except that almost all type constructors, including arrays, pointers, and enumerations, lead to new, inequivalent types.[12] However, new names for existing type names are, as in C's `typedef`s, equivalent to the original types. (Sometimes this rule is referred to as **declaration equivalence**.) Thus, in:

```
type
   IntPtr = ^integer;
   Age = integer;
var
   x: IntPtr;
   y: ^integer;
   i: Age;
   a: integer;
```

`x` and `y` are not equivalent in Pascal, but `i` and `a` are.

Java has a particularly simple approach to type equivalence. First, there are no `typedef`s, so naming questions are minimized. Second, `class` and `interface` declarations implicitly create new type names (the same as the class/interface names), and name equivalence is used for these types. The only complication is that arrays (which cannot have type names) use structural equivalence, with special rules for establishing base type equivalence (we do not further discuss Java arrays in this text—see the Notes and References for more information).

Last, we mention the type equivalence rule used in ML. ML has *two* type declarations—`datatype` and `type`, but only the former constructs a new type, while the latter only constructs aliases of existing types (like the `typedef` of C). For example,

```
type Age = int;
datatype NewAge = NewAge int;
```

Now `Age` is equivalent to `int`, but `NewAge` is a new type not equivalent to `int`. Indeed, we reused the name `NewAge` to also stand for a data constructor for the `NewAge` type, so that `(NewAge 2)` is a value of type `NewAge`, while 2 is a value of type `int` (or `Age`).

One issue we have omitted from this discussion of type equivalence is the interaction between the type equivalence algorithm and the type-checking algorithm. As explained earlier, type equivalence

[12]Subrange types in Pascal are implemented as runtime checks, not new types.

comes up in type checking and can be given a somewhat independent answer. In fact, some type equivalence questions may never arise because of the type-checking rules, and so we might never be able to write code in a language that would answer a particular type equivalence question. In such cases, we might sensibly adopt an operational view and simply disregard the question of type equivalence for these particular cases. An example of this is discussed in Exercise 8.25.

8.6 Type Checking

Type checking, as explained at the beginning of the chapter, is the process by which a translator verifies that all constructs in a program make sense in terms of the types of its constants, variables, procedures, and other entities. It involves the application of a type equivalence algorithm to expressions and statements, with the type-checking algorithm varying the use of the type equivalence algorithm to suit the context. Thus, a strict type equivalence algorithm such as name equivalence could be relaxed by the type checker if the situation warrants.

Type checking can be divided into **dynamic** and **static** checking. If type information is maintained and checked at runtime, the checking is dynamic. Interpreters by definition perform dynamic type checking. But compilers can also generate code that maintains type attributes during runtime in a table or as type tags in an environment. A Lisp compiler, for example, would do this. Dynamic type checking is required when the types of objects can only be determined at runtime.

The alternative to dynamic typing is static typing: The types of expressions and objects are determined from the text of the program, and type checking is performed by the translator before execution. In a strongly typed language, all type errors must be caught before runtime, so these languages must be statically typed, and type errors are reported as compilation error messages that prevent execution. However, a language definition may leave unspecified whether dynamic or static typing is to be used.

Example 1

C compilers apply static type checking during translation, but C is not really strongly typed since many type inconsistencies do not cause compilation errors but are automatically removed through the generation of conversion code, either with or without a warning message. Most modern compilers, however, have error level settings that do provide stronger typing if it is desired. C++ also adds stronger type checking to C, but also mainly in the form of compiler warnings rather than errors (for compatibility with C). Thus, in C++ (and to a certain extent also in C), many type errors appear only as warnings and do not prevent execution. Thus, ignoring warnings can be a "dangerous folly" (Stroustrup [1994], p. 42). ■

Example 2

The Scheme dialect of Lisp (see Chapter 3) is a dynamically typed language, but types *are* rigorously checked, with all type errors causing program termination. There are no types in declarations, and there are no explicit type names. Variables and other symbols have no predeclared types but take on the type of the value they possess at each moment of execution. Thus, types in Scheme must be kept as explicit attributes of values. Type checking consists of generating errors for functions requiring certain values to perform their operations. For example, `car` and `cdr` require their operands to be lists: `(car 2)` generates an error. Types can be checked explicitly by the programmer, however, using predefined test functions.

Types in Scheme include lists, symbols, atoms, and numbers. Predefined test functions include `number?` and `symbol?`. (Such test functions are called **predicates** and always end in a question mark.)

Example 3

Ada is a strongly typed language, and all type errors cause compilation error messages. However, even in Ada, certain errors, like range errors in array subscripting, cannot be caught prior to execution, since the value of a subscript is not in general known until runtime. However, the Ada standard guarantees that all such errors will cause exceptions and, if these exceptions are not caught and handled by the program itself, program termination. Typically, such runtime type-checking errors result in the raising of the predefined exception `Constraint_Error`.

An essential part of type checking is **type inference**, where the types of expressions are inferred from the types of their subexpressions. Type-checking rules (that is, when constructs are type correct) and type inference rules are often intermingled. For example, an expression $e1 + e2$ might be declared type correct if $e1$ and $e2$ have the same type, and that type has a "1" operation (type checking), and the result type of the expression is the type of $e1$ and $e2$ (type inference). This is the rule in Ada, for example. In other languages this rule may be softened to include cases where the type of one subexpression is automatically convertible to the type of the other expression.

As another example of a type-checking rule, in a function call, the types of the actual parameters or arguments must match the types of the formal parameters (type checking), and the result type of the call is the result type of the function (type inference).

Type-checking and type inference rules have a close interaction with the type equivalence algorithm. For example, the C declaration

```
void p ( struct { int i; double r; } x ){
...
}
```

is an error under C's type equivalence algorithm, since no actual parameter can have the type of the formal parameter `x`, and so a type mismatch will be declared on every call to `p`. As a result, a C compiler will usually issue a warning here (although, strictly speaking, this is legal C).[13] Similar situations occur in Pascal and Ada.

The process of type inference and type checking in statically typed languages is aided by explicit declarations of the types of variables, functions, and other objects. For example, if `x` and `y` are variables, the correctness and type of the expression `x + y` is difficult to determine prior to execution unless the types of `x` and `y` have been explicitly stated in a declaration. However, explicit type declarations are not an absolute requirement for static typing: The languages ML and Haskell perform static type checking but do not require types to be declared. Instead, types are inferred from context using an inference mechanism that is more powerful than what we have described (it will be described in Section 8.8).

Type inference and correctness rules are often one of the most complex parts of the semantics of a language. Nonorthogonalities are hard to avoid in imperative languages such as C. In the remainder of this section, we will discuss major issues and problems in the rules of a type system.

[13]This has been raised to the level of an error in C++.

8.6.1 Type Compatibility

Sometimes it is useful to relax type correctness rules so that the types of components need not be precisely the same according to the type equivalence algorithm. For example, we noted earlier that the expression *e*1 + *e*2 may still make sense even if the types of *e*1 and *e*2 are different. In such a situation, two different types that still may be correct when combined in certain ways are often called **compatible** types. In Ada, any two subranges of the same base type are compatible (of course, this can result in errors, as we shall see in Section 8.6.3.). In C and Java, all numeric types are compatible (and conversions are performed such that as much information as possible is retained).

A related term, **assignment compatibility**, is often used for the type correctness of the assignment *e*1 + *e*2 (which may be different from other compatibility rules because of the special nature of assignment). Initially, this statement may be judged type correct when `x` and *e* have the same type. However, this ignores a major difference: The left-hand side must be an **l-value** or **reference**, while the right-hand side must be an **r-value**. Many languages solve this problem by requiring the left-hand side to be a variable name, whose address is taken as the l-value, and by automatically **dereferencing** variable names on the right-hand side to get their r-values. In ML this is made more explicit by saying that the assignment is type correct if the type of the left-hand side (which may be an arbitrary expression) is `ref t` (a reference to a value of type `t`), and the type of the right-hand side is `t`. ML also requires explicit dereferencing of variables used on the right-hand side:

```
val x = ref 2; (* type of x is ref int *)
x = !x + 1; (* x dereferenced using ! *)
```

As with ordinary compatibility, assignment compatibility can be expanded to include cases where both sides do not have the same type. For example, in Java the assignment `x` = *e* is legal when *e* is a numeric type whose value can be converted to the type of `x` without loss of information (for example, `char` to `int`, or `int` to `long`, or `long` to `double`, etc.). On the other hand, if *e* is a floating-point type and *x* is an integral type, then this is a type error in Java (thus, assignment compatibility is different from the compatibility of other arithmetic operators). Such assignment compatibilities may or may not involve the conversion of the underlying values to a different format; see Section 8.7.

8.6.2 Implicit Types

As noted at the beginning of this chapter, types of basic entities such as constants and variables may not be given explicitly in a declaration. In this case, the type must be inferred by the translator, either from context information or from standard rules. We say that such types are **implicit**, since they are not explicitly stated in a program, though the rules that specify their types must be explicit in the language definition.

As an example of such a situation in C, variables are implicitly integers if no type is given,[14] and functions implicitly return an integer value if no return type is given:

```
x; /* implicitly integer */

f(int x) /* implicitly returns an int */
{ return x + 1; }
```

[14]This has been made illegal by the 1999 ISO C Standard.

As another example, in Pascal named constants are implicitly typed by the literals they represent:

```
const
   PI = 3.14156; (* implicitly real *)
   Answer = 42; (* implicitly integer *)
```

Indeed, literals are the major example of implicitly typed entities in all languages. For example, `123456789` is implicitly an `int` in C, unless it would overflow the space allocated for an `int`, in which case it is a `long int` (or perhaps even an `unsigned long int`, if necessary). On the other hand, `1234567L` is a `long int` by virtue of the "L" suffix. Similarly, any sequence of characters surrounded by double quotation marks such as `"Hello"` (a "C string") is implicitly a character array of whatever size is necessary to hold the characters plus the delimiting null character that ends every such literal. Thus, `"Hello"` is of type `char[6]`.

When subranges are available as independent types in a language (such as Ada), new problems arise in that literals must now be viewed as potentially of several different types. For instance, if we define:

```
type Digit_Type is range 0..9;
```

then the number 0 could be an `integer` or a `Digit_Type`. Usually in such cases the context of use is enough to tell a translator what type to choose. Alternatively, one can regard 0 as immediately convertible from `integer` to `Digit_Type`.

8.6.3 Overlapping Types and Multiply-Typed Values

As just noted, types may overlap in that two types may contain values in common. The `Digit_Type` example above shows how subtyping and subranges can create types whose sets of values overlap. Normally, it is preferable for types to be disjoint as sets, so that every expressible value belongs to a unique type, and no ambiguities or context sensitivities arise in determining the type of a value. However, enforcing this rule absolutely would be far too restrictive and would, in fact, eliminate one of the major features of object-oriented programming—the ability to create subtypes through inheritance that refine existing types yet retain their membership in the more general type. Even at the most basic level of predefined numeric types, however, overlapping values are difficult to avoid. For instance, in C the two types `unsigned int` and `int` have substantial overlap, and in Java, a small integer could be a `short`, an `int`, or a `long`. Thus, rules such as the following are necessary (Arnold, Gosling, and Holmes [2000], p. 143):

> Integer constants are long if they end in `L` or `l`, such as `29L`; `L` is preferred over `l` because `l` (lowercase `L`) can easily be confused with `1` (the digit one). Otherwise, integer constants are assumed to be of type `int`. If an int literal is directly assigned to a `short`, and its value is within the valid range for a `short`, the integer literal is treated as if it were a `short` literal. A similar allowance is made for integer literals assigned to `byte` variables.

Also, as already noted, subranges and subtypes are the source of range errors during execution that cannot be predicted at translation time and, thus, a source of program failure that type systems were in part designed to prevent. For example, the Ada declarations

```
subtype SubDigit is integer range 0..9;
subtype NegDigit is integer range −9..−1;
```

create two subranges of integers that are in fact disjoint, but the language rules allow code such as:

```
x: SubDigit;
y: NegDigit;
...
x := y;
```

without a compile-time error. Of course, this code cannot execute without generating a `CONSTRAINT_ERROR` and halting the program. Similar results hold for other languages that allow subranges, like Pascal and Modula-2.

Sometimes, however, overlapping types and multiply-typed values can be of very direct benefit in simplifying code. For example, in C the literal `0` is not only a value of every integral type but is also a value of every pointer type, and represents the null pointer that points to no object. In Java, too, there is a single literal value `null` that is, in essence, a value of every reference type.[15]

8.6.4 Shared Operations

Each type is associated, usually implicitly, with a set of operations. Often these operations are shared among several types or have the same name as other operations that may be different. For example, the operator + can be real addition, integer addition, or set union. Such operators are said to be **overloaded**, since the same name is used for different operations. In the case of an overloaded operator, a translator must decide which operation is meant based on the types of its operands. In Chapter 7 we studied in some detail how this can be done using the symbol table. In that discussion, we assumed the argument types of an overloaded operator are disjoint as sets, but problems can arise if they are not, or if subtypes are present with different meanings for these operations. Of course, there should be no ambiguity for built-in operations, since the language rules should make clear which operation is applied. For example, in Java, if two operands are of integral type, then integer addition is *always* applied (and the result is an `int`) regardless of the type of the operands, *except* when one of the operands has type `long`, in which case `long` addition is performed (and the result is of type `long`). Thus, in the following Java code, 40000 is printed (since the result is an `int`, not a `short`):

```
short x = 20000;
System.out.println(x + x);
```

[15]This is not quite accurate, according to the Java Language specification (Gosling et al. [2000], p. 32): "There is also a special null type, the type of the expression `null`, which has no name. Because the null type has no name, it is impossible to declare a variable of the null type or to cast to the null type. The null reference is the only possible value of an expression of null type. The null reference can always be cast to any reference type. In practice, the programmer can ignore the null type and just pretend that `null` is merely a special literal that can be of any reference type."

On the other hand,

```
x = x + x;
```

is now illegal in Java—it requires a cast (see next section):

```
x = (short) (x + x);
```

Thus, there is no such thing as `short` addition in Java, nor does Java allow other kinds of arithmetic on any integral types other than `int` and `long`.

Ada, as usual, allows much finer control over what operations are applied to data types, as long as these are indeed new types and not simply subranges. (Subranges simply inherit the operations of their base types.) Consider, for example, the following Ada code:

```
type Digit_Type is range 0..9;

function "+"(x,y: Digit_Type) return Digit_Type is
   a: integer := integer(x);
   b: integer := integer(y);
begin
   return Digit_Type((a+b) mod 10);
end "+";
```

This defines a new addition operation on `Digit_Type` that wraps its result so that it is always a digit. Now, given the code

```
a: Digit_Type := 5+7;
```

it may seem surprising, but Ada knows enough here to call the new `"+"` function for `Digit_Type`, rather than use standard integer addition, and a gets the value 2 (otherwise a `Constraint_Error` would occur here). The reason is (as noted in Chapter 7) that since there are no implicit conversions in Ada, Ada can use the result type of a function call, as well as the parameter argument types, to resolve overloading, and since the result type is `Digit_Type`, the user-defined `"+"` must be the function indicated.

C++ does not allow this kind of overloading. Instead, operator overloading must involve new, user-defined types. For example, the following definition:

```
int * operator+(int* a, int* b){ // illegal in C++
   int * x = new int;
   *x = *a+*b;
   return x;
}
```

is illegal in C++, as would be any attempt to redefine the arithmetic operators on any built-in type.

8.7 Type Conversion

In every programming language, it is necessary to convert one type to another under certain circumstances. Such **type conversion** can be built into the type system so that conversions are performed automatically, as in a number of examples discussed previously. For example, in the following C code:

```
int x = 3;
...
x = 2.3 + x / 2;
```

at the end of this code x still has the value 3: `x / 2` is integer division, with result 1, then 1 is converted to the `double` value 1.0 and added to 2.3 to obtain 3.3, and then 3.3 is truncated to 3 when it is assigned to x.

In this example, two automatic, or **implicit conversions** were inserted by the translator. The first is the conversion of the `int` result of the division to `double` (1 → 1.0) before the addition .The second is the conversion of the `double` result of the addition to an `int` (3.3 → 3) before the assignment to x. Such implicit conversions are sometimes also called **coercions**. The conversion from `int` to `double` is an example of a **widening** conversion, where the target data type can hold all of the information being converted without loss of data, while the conversion from `double` to `int` is a **narrowing** conversion that may involve a loss of data.

Implicit conversion has the benefit of making it unnecessary for the programmer to write extra code to perform an obvious conversion. However, implicit conversion has drawbacks as well. For one thing, it can weaken type checking so that errors may not be caught. This compromises the strong typing and the reliability of the programming language. Implicit conversions can also cause unexpected behavior in that the programmer may expect a conversion to be done one way, while the translator actually performs a different conversion. A famous example in (early) PL/I is the expression

```
1/3 + 15
```

which was converted to the value 5.33333333333333 (on a machine with 15-digit precision): The leading 1 was lost by overflow because the language rules state that the precision of the fractional value must be maintained.

An alternative to implicit conversion is **explicit conversion**, in which conversion directives are written right into the code. Such conversion code is typically called a **cast** and is commonly written in one of two varieties of syntax. The first variety is used in C and Java and consists of writing the desired result type inside parentheses before the expression to be converted. Thus, in C we can write the previous implicit conversion example as explicit casts in the following form:

```
x = (int) (2.3 + (double) (x / 2));
```

The other variety of cast syntax is to use function call syntax, with the result type used as the function name and the value to be converted as the parameter argument. C++ and Ada use this form. Thus, the preceding conversions would be written in C++ as:

```
x = int( 2.3 + double( x / 2 ) );
```

C++ also uses a newer form for casts, which will be discussed shortly.

The advantange to using casts is that the conversions being performed are documented precisely in the code, with less likelihood of unexpected behavior, or of human readers misunderstanding the code. For this reason, a few languages prohibit implicit conversions altogether, forcing programmers to write out casts in all cases. Ada is such a language. Additionally, eliminating implicit conversions makes it easier for the translator (and the reader) to resolve overloading. For example, in C++ if we have two function declarations:

```
double max (int, double);
double max (double,int);
```

then the call `max(2,3)` is ambiguous because of the possible implicit conversions from `int` to `double` (which could be done either on the first or second parameter). If implicit conversions were not allowed, then the programmer would be forced to specify which conversion is meant. (A second example of this phenomenon was just seen in Ada in the previous section.)

A middle ground between the two extremes of eliminating all implicit conversions and allowing conversions that could potentially be errors is to allow only those implicit conversions that are guaranteed not to corrupt data. For example, Java follows this principle and only permits widening implicit conversions for arithmetic types. C++, too, adopts a more cautious approach than C toward narrowing conversions, generating warning messages in all cases except the `int` to `char` narrowing implicit conversion.[16]

Even explicit casts need to be restricted in various ways by languages. Often, casts are restricted to simple types, or even to just the arithmetic types. If casts are permitted for structured types, then clearly a restriction must be that the types have identical sizes in memory. The translation then merely **reinterprets** the memory as a different type, without changing the bit representation of the data. This is particularly true for pointers, which are generally simply reinterpreted as pointing to a value of a different type. This kind of explicit conversion also allows for certain functions, such as memory allocators and deallocators, to be defined using a "generic pointer." For example, in C the `malloc` and `free` functions are declared using the generic or **anonymous pointer type** `void*`:

```
void* malloc (size_t );
void free( void*);
```

and casts may be used to convert from any pointer type[17] to `void*` and back again:

```
int* x = (int*) malloc(sizeof(int));
...
free( (void*)x );
```

Actually, C allows both of these conversions to be implicit, while C++ allows implicit conversions from pointers to `void*`, but not vice versa.

Object-oriented languages also have certain special conversion requirements, since inheritance can be interpreted as a subtyping mechanism, and conversions from subtypes to supertypes and back are necessary in some cases. This kind of conversion was discussed in more detail in Chapter 5. However, we mention here that C++, because of its mixture of object-oriented and nonobject-oriented features, as well as its compatibility with C, identifies four different kinds of casts, and has invented new syntax for each of these casts: `static_cast`, `dynamic_cast`, `reinterpret_cast`, and `const_cast`. The first, `static_cast`, corresponds to the casts we have been describing here[18] (static, since the compiler interprets the cast, generates code if necessary, and the runtime system is not involved). Thus, the previous explicit cast example would be written using the C++ standard as follows:

```
x = static_cast<int>(2.3 + static_cast<double>(x / 2));
```

An alternative to casts is to allow predefined or library functions to perform conversions. For example, in Ada casts between integers and characters are not permitted. Instead, two predefined

[16]Stroustrup [1994], pp. 41–43, discusses why warnings are not generated in this case.
[17]Not "pointers to functions," however.
[18]The `reinterpret_cast` is not used for `void*` conversions, but for more dangerous conversions such as `char*` to `int*` or `int` to `char*`.

functions (called **attribute functions** of the `character` data type) allow conversions between characters and their associated ASCII values:

```
character'pos('a') -- returns 97, the ASCII value of 'a'
character'val(97) -- returns the character 'a'
```

Conversions between strings and numbers are also often handled by special conversion functions. For example, in Java the `Integer` class in the `java.lang` library contains the conversion functions `toString`, which converts an `int` to a `String`, and `parseInt`, which converts a `String` to an int:

```
String s = Integer.toString(12345); // s becomes "12345"
int i = Integer.parseInt("54321"); // i becomes 54321
```

A final mechanism for converting values from one type to another is provided by a loophole in the strong typing of some languages. Undiscriminated unions can hold values of different types, and without a discriminant or tag (or without runtime checking of such tags) a translator cannot distinguish values of one type from another, thus permitting indiscriminate reinterpretation of values. For example, the C++ declaration

```
union{
   char c;
   bool b;
} x;
```

and the statements

```
x.b = true;
cout << static_cast<int>(x.c) << endl;
```

would allow us to see the internal value used to represent the Boolean value `true` (in fact, the C++ standard says this should always be 1).

8.8 Polymorphic Type Checking

Most statically typed languages require that explicit type information be given for all names in each declaration, with certain exceptions about implicitly typed literals, constants, and variables noted previously. However, it is also possible to use an extension of type inference to determine the types of names in a declaration *without explicitly giving those types*. Indeed, a translator can either: (1) collect information on *uses* of a name, and infer from the set of all uses a probable type that is the most general type for which all uses are correct, or (2) declare a type error because some of the uses are incompatible with others. In this section, we will give an overview of this kind of type inference and type checking, which is called **Hindley-Milner type checking** for its coinventors (Hindley [1969], Milner [1978]). Hindley-Milner type checking is a major feature of the strongly typed functional languages ML and Haskell.

First, consider how a conventional type checker works. For purposes of discussion, consider the expression (in C syntax) `a[i] + i`. For a normal type checker to work, both `a` and `i` must be declared, `a` as an array of integers, and `i` as an integer, and then the result of the expression is an integer.

Syntax trees show this in more detail. The type checker starts out with a tree such as the one shown in Figure 8.8.

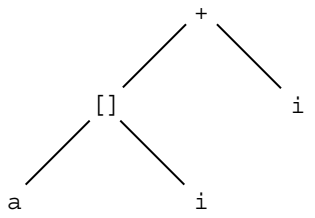


Figure 8.8 Syntax tree for a[i] + i

First, the types of the names (the leaf nodes) are filled in from the declarations, as shown in Figure 8.9.

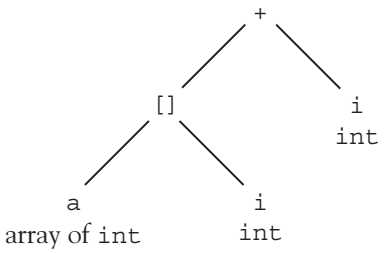


Figure 8.9 Types of the names filled in from the declarations

The type checker then checks the subscript node (labeled `[]`); the left operand must be an array, and the right operand must be an `int`; indeed, they are, so the operation type checks correctly. Then the inferred type of the subscript node is the component type of the array, which is `int`, so this type is added to the tree, as shown in Figure 8.10.

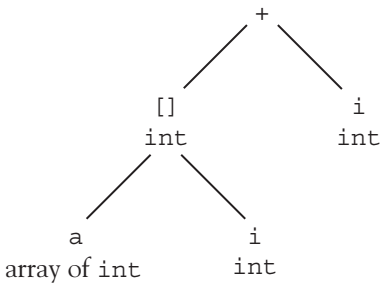


Figure 8.10 Inferring the type of the subscript node

Finally, the + node is type checked; the two operands must have the same type (we assume no implicit conversions here), and this type must have a + operation. Indeed, they are both `int`, so the + operation is type correct. Thus, the result is the type of the operands, which is `int`, as shown in Figure 8.11.

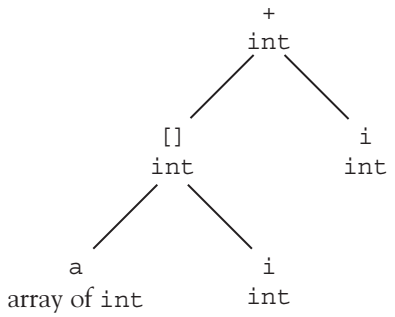


Figure 8.11 Inferring the type of the + node

Could the type checker have come to any other conclusion about the types of the five nodes in this tree? No, not even if the declarations of `a` and `i` were missing. Here's how it would do it.

First, the type checker would assign **type variables** to all the names for which it did not already have types. Thus, if `a` and `i` do not yet have types, we need to assign some type variable names to them. Since these are internal names that should not be confused with program identifiers, let us use a typical convention and assign the Greek letters α and β as the type variables for `a` and `i`. See Figure 8.12.

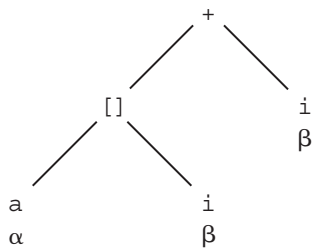


Figure 8.12 Assigning type variable to names without types

Note in particular that both occurrences of `i` have the same type variable β.

Now the type checker visits the subscript node, and infers that, for this to be correct, the type of `a` must actually be an array (in fact, array of γ, where γ[19] is another type variable, since we don't yet have any information about the component type of `a`). Also, the type checker infers that the type of `i` must be `int` (we assume that the language only allows `int` subscripts in this example). Thus, β is replaced by `int` in the entire tree, and the situation looks like Figure 8.13.

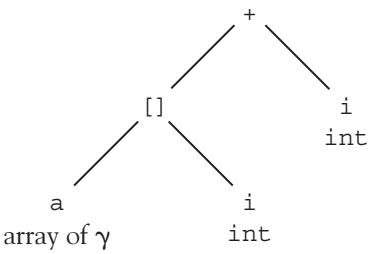


Figure 8.13 Inferring the type of the variable a

[19]Greek letter gamma.

Finally, the type checker concludes that, with these assignments to the type variables, the subscript node is type correct and has the type γ (the component type of a). See Figure 8.14.

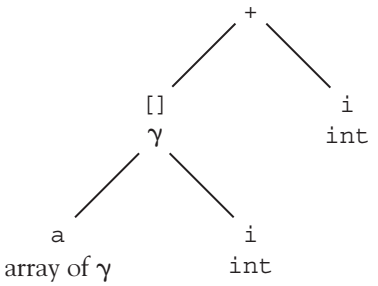


Figure 8.14 Inferring the type of the subscript node

Now the type checker visits the + node, and concludes that, for it to be type correct, γ must be `int` (the two operands of + must have the same type). Thus γ is replaced by `int` *everywhere* it appears, and the result of the + operation is also `int`. Note that we have arrived at the same typing of this expression as before, without using any prior knowledge about the types of a and i. See Figure 8.15.

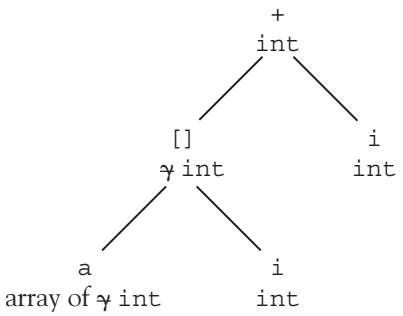


Figure 8.15 Substituting int for the type variable

This is the basic form of operation of Hindley-Milner type checking.

We note two things about this form of type checking. First, as we have indicated, once a type variable is replaced by an actual type (or a more specialized form of a type), then *all* instances of that variable must be updated to that same new value for the type variable. This process is called **instantiation** of type variables, and it can be achieved using various forms of indirection into a table of type expressions that is very similar to a symbol table. The second thing to note is that when new type information becomes available, the type expressions for variables can change form in various ways. For example, in the previous example α became "array of γ," β became `int`, and then γ became `int` as well. This process can become even more complex. For example, we might have two type expressions "array of α" and "array of β" that need to be the same for type checking to succeed. In that case, we must have α == β, and so β must be changed to α everywhere it occurs (or vice versa). This process is called **unification**; it is a kind of pattern matching that appears frequently in programming languages: Versions of it exist in several contexts in functional languages like ML and Haskell (to handle type-checking and

pattern-matching code), logic languages like Prolog (to handle variables and to direct execution), and even C++ compilers (to handle template instantiation). These other uses will be discussed elsewhere in this book. Unification involves essentially three cases (here described in terms of types):

1. Any type variable unifies with any type expression (and is instantiated to that expression).
2. Any two type constants (that is, specific types like `int` or `double`) unify only if they are the same type.
3. Any two type constructions (that is, applications of type constructors like "array" or `struct`) unify only if they are applications of the same type constructor and all of their component types also (recursively) unify.

For example, unifying type variable β with type expression "array of α" is case 1 above, and β is instantiated to "array of α." Unifying `int` to `int` is an example of case 2. Unifying "array of α" with "array of β" is case 3, and we must recurse on the component types of each expressions (which are α and β, and thus unify by case 1). On the other hand, by case 2, `int` cannot unify with "array of α," and by case 3, "array of `int`" cannot unify with "array of `char`."

Hindley-Milner type checking can be extremely useful in that it simplifies tremendously the amount of type information that the programmer must write down in code.[20] But if this were the only advantage it would not be an overwhelming one (see previous footnote). In fact, Hindley-Milner type checking gives an enormous advantage over simple type checking in that *types can remain as general as possible*, while still being strongly checked for consistency.

Consider, for example, the following expression (again in C syntax):

```
a[i] = b[i]
```

This assigns the value of `b[i]` to `a[i]` (and returns that value). Suppose again that we know nothing in advance about the types of `a`, `b`, and `i`. Hindley-Milner type checking will then establish that `i` must be an `int`, `a` must be an "array of α," and `b` must be an "array of β," and then (because of the assignment, and assuming no implicit conversions), α == β. Thus, type checking concludes with the types of `a` and `b` narrowed to "array of α," but α is still a completely unconstrained type variable that could be *any* type. This expression is said to be **polymorphic**, and Hindley-Milner type checking implicitly implements **polymorphic type checking**—that is, we get such polymorphic type checking "for free" as an automatic consequence of the implicit type variables introduced by the algorithm.

In fact, there are several different interpretations for the word "polymorphic" that we need to separate here. Polymorphic is a Greek word meaning "of many forms." In programming languages, it applies to names that (simultaneously) can have more than one type. We have already seen situations in which names can have several different types simultaneously—namely, the **overloaded functions**. So overloading is one kind of polymorphism. However, overloaded functions have only a finite (usually small) set of types, each one given by a different declaration. The kind of polymorphism we are seeing here is different: The type "array of α" is actually a set of *infinitely many* types, depending on the (infinitely many) possible instantiations of the type variable α. This

[20]Of course, this must be balanced with the advantage of documenting the desired types directly in the code: It is often useful to write explicit type information even when unnecessary in a language like ML, for documentation purposes.

type of polymorphism is called **parametric polymophism** because α is essentially a *type parameter* that can be replaced by any type expression. In fact, so far in this section, you have seen only **implicit parametric polymorphism**, since the type parameters (represented by Hindley-Milner type variables) are *implicitly introduced* by the type checker, rather than being explicitly written by the programmer. (In the next section, we will discuss **explicit parametric polymorphism**.) To distinguish overloading more clearly from parametric polymorphism, it is sometimes called **ad hoc polymorphism**. *Ad hoc* means "to this" in Latin and refers to anything that is done with a specialized goal or purpose in mind. Indeed, overloading is always represented by a set of distinct declarations, each with a special purpose. Finally, a third form of polymorphism occurs in object-oriented languages: Objects that share a common ancestor can also either share or redefine the operations that exist for that ancestor. Some authors refer to this as **pure polymorphism**. However, in keeping with modern terminology, we shall call it **subtype polymorphism**, since it is closely related to the view of subtypes as sharing operations (see Section 8.3.3). Subtype polymorphism was discussed in Chapter 5.

Finally, a language that exhibits no polymorphism, or an expression or function that has a unique, fixed type, is said to be **monomorphic** (Greek for "having one form").

The previous example of a polymorphic expression was somewhat too simple to show the true power of parametric polymorphism and Hindley-Milner type checking. Polymorphic functions are the real goal of this kind of polymorphism, so consider an example that appeared in the discussion of overloading in Chapter 7—namely, that of a `max` function such as:

```
int max (int x, int y){
   return x > y ? x : y;
}
```

We note two things about this function. First, the body for this function is the same, regardless of the type (`double` or any other arithmetic type could be substituted for `int` without changing the body). Second, the body *does* depend on the existence of a greater-than operator > for each type used in overloading this function, and the > function itself is overloaded for a number of types. Thus, to *really* remove the dependency of this function on the type of its parameters, we should add a new parameter representing the greater-than function:[21]

```
int max (int x, int y, int (*gt)(int a,int b) ) {
   return gt(x,y) ? x : y;
}
```

Now let us consider a version of this function that does not specify the type of `x`, `y`, or `gt`. We could write this in C-like syntax as:

```
max (x, y, gt) {
   return gt(x,y) ? x : y;
}
```

[21] Actually, leaving the dependency on the > function is also possible; it is called **constrained parametric polymorphism** and will be briefly discussed in the next section.

or, to use the syntax of ML (where this is legal code):

```
fun max (x, y, gt) = if gt(x,y) then x else y;
```

Let us invent a syntax tree for this definition and assign type variables to the identifiers, as shown in Figure 8.16.

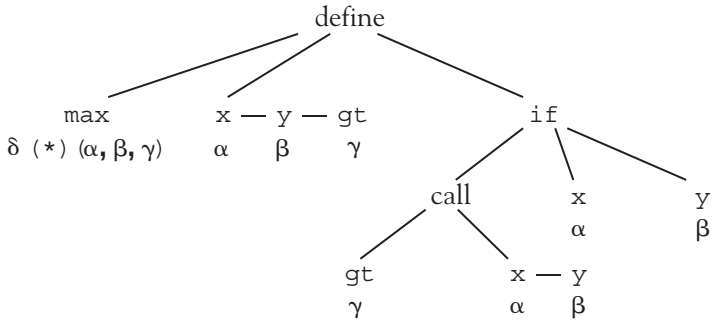


Figure 8.16 A syntax tree for the function max

Note that we are using C notation for the type of the function `max`, which is a function of three parameters with types α, β, γ, and returns a value of type δ.

We now begin type checking, and visit the call node. This tells us that `gt` is actually a function of two parameters of types α and β, and returns a type ε (which is as yet unknown). Thus, γ = ε (*)(α, β) and we get the tree shown in Figure 8.17.

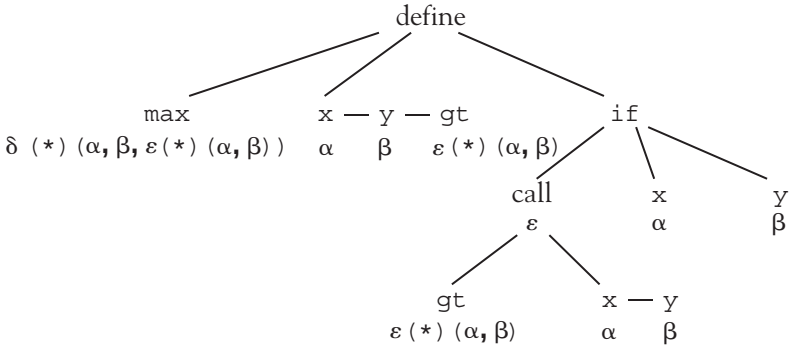


Figure 8.17 Substituting for a type variable

Note that the result of the call has type ε, the return type of the `gt` function.

Now type checking continues with the `if` node. Typically, such a node requires that the test expression return a Boolean value (which doesn't exist in C), and that the types of the "then" part and the "else" part must be the same. (C will actually perform implicit conversions in a `?:` expression, if necessary.) We will assume the stronger requirements (and the existence of a `bool` type), and we get that α = β = δ and ε = `bool`, with the tree shown in Figure 8.18.

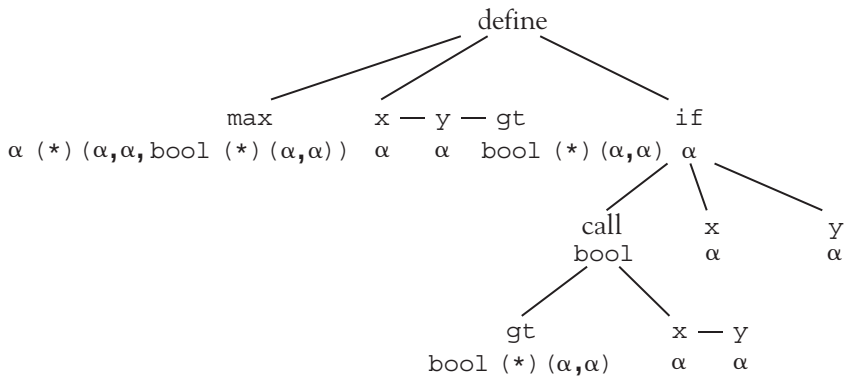


Figure 8.18 Further substitutions

($\delta = \alpha$ because the result type of the if expression is now α, and this is the type of the body of the function; hence the return type of the body.) Thus, `max` has been determined to have type α `(*)(`α, α, `bool (*)(`α, α`))`, where α is *any* type—in other words, a type parameter that can have any actual type substituted for it. In ML this type is printed as:

```
'a * 'a * ('a * 'a -> bool) -> 'a
```

where `'a` is used instead of the type variable α, a function type is indicated by the arrow `->`, and the multiple parameters are represented by a tuple (or Cartesian product) type[22] (the Cartesian product `A` $\times$ `B` in ML is printed as `A*B` and values of this type are written as tuples `(a, b)` with $\alpha \in$ `A` and `b` $\in$ `B`).

With this typing for the function `max`, we can use `max` in any situation where the actual types unify with the polymorphic type. For example, if we provide the following definitions in ML:

```
fun gti (x:int,y) = x > y;
fun gtr (x:real,y) = x > y;
fun gtp ((x,y),(z,w)) = gti (x,z);
```

we can call the `max` function as follows:

```
max(3,2,gti); (* returns 3 *)
max(2.1,3.2,gtr); (* returns 3.2 *)
max((2,"hello"),(1,"hi"),gtp); (* returns (2,"hello") *)
```

Note that α `(*)(`α, α, `bool (*)(`α, α`))` really is the most general type possible for the `max` function with the given implementation (called its **principal type**), and that each call to `max` **specializes** this principle type to a monomorphic type, which may also implicitly specialize the types of the parameters. For example, the type of the `gtp` function is `bool (*) ((int,` α`),(int,` β`))` (or `(int * 'a) * (int * 'b) -> bool` in ML syntax), with two type variables for the types of `y` and `w`, since these two variables are not used in the body of `gtp`, so each can be of any type. Now, in the call to `max` above, we still could not use two different types for `y` and `w`, since the `max` function insists on the same type for the first two parameters:

[22]In fact, we have written the parameters (`x, y, gt`) in tuple notation in the definition. It is possible to write them differently, resulting in what is called a Curried function. This was discussed in Chapter 4.

```
max((2,"hello"),(1,2),gtp); (* illegal! *)
```

so in any call to `max` the `gtp` function is specialized to the type `bool(*) ((int,α),(int,α))`.

An extension of this principle is that any polymorphically typed object that is passed into a function as a parameter must have a fixed specialization for the duration of the function. Thus, if `gtp` were called again within the `max` function, it would still have to retain the above specialization. The same is not true for nonlocal references to polymorphic functions. For example, the following code is legal ML:

```
fun ident x = x;
fun makepair (x,y) = (ident x,ident y);
makepair(2,3.2);
 (* ok -- ident has two different types, one for each call *)
```

However, making `ident` into a parameter results in a type error:

```
fun makepair2 (x,y,f) = (f x, f y);
makepair2(2,3.2,ident);
      (* type error -- ident cannot have general type *)
```

This restriction on polymorphic types in Hindley-Milner type systems is called **let-bound polymorphism**.

Let-bound polymorphism complicates Hindley-Milner type checking because it requires that type variables be divided into two separate categories. The first category is the one in use during type checking, whose value is everywhere the same and is universally changed with each instantiation/specialization. The second is part of a polymorphic type in a nonlocal reference, which can be instantiated differently at each use. Sometimes these latter type variables are called **generic** type variables. Citations for further detail on this issue can be found in the Notes and References.

An additional problem with Hindley-Milner type checking arises when an attempt is made to unify a type variable with a type expression that contains the variable itself. Consider the following function definition (this time in ML syntax):

```
fun f (g) = g (f);
```

Let's examine what happens to this definition during type checking. First, `g` will be assigned the type variable α, and `f` will be assigned the type $\alpha \rightarrow \beta$ (a function of one parameter of type α returning a value of type β, using ML-like syntax for function types). Then the body of `f` will be examined, and it indicates that `g` is a function of one parameter whose type must be the type of `f`, and whose return type is also the return type of `f`. Thus, α (the type of `g`) should be set equal to $(\alpha \rightarrow \beta) \rightarrow \beta$. However, trying to establish the equation $\alpha = (\alpha \rightarrow \beta) \rightarrow \beta$ will cause an infinite recursion, because α will contain itself as a component. Thus, before any unification of a type variable with a type expression is attempted, the type checker must make sure that the type variable is not itself a part of the type expression it is to become equal to. This is called the **occur-check**, and it must be a part of the unification algorithm for type checking to operate correctly (and declare such a situation a type error). Unfortunately, this is a difficult check to perform in an efficient way, and so in some language contexts where unification is used (such as in versions of Prolog) the occur-check is omitted. However, for Hindley-Milner type checking to work properly, the occur-check is essential.

Finally, we want to mention the issue of translating code with polymorphic types. Consider the previous example of a polymorphic `max` function. Clearly the values `x` and `y` of arbitrary type will need to be copied from location to location by the code, since `gt` is to be called with arguments `x` and `y`, and then either `x` or `y` is to be returned as the result. However, without knowing the type of `x` (and `y`), a translator cannot determine the size of these values, which must be known in order to make copies. How can a translator effectively deal with the problem?

There are several solutions. Two standard ones are as follows:

1. *Expansion*. Examine all the calls of the `max` function, and generate a copy of the code for each different type used in these calls, using the appropriate size for each type.
2. *Boxing and tagging*. Fix a size for all data that can contain any scalar value such as integers, floating-point numbers, and pointers; add a bit field that tags a type as either scalar or not, with a size field if not. All structured types then are represented by a pointer and a size, and copied indirectly, while all scalar types are copied directly.

In essence, solution 1 makes the code look as if separate, overloaded functions had been defined by the programmer for each type (but with the advantage that the compiler generates the different versions, not the programmer). This is efficient but can cause the size of the target code to grow very large (sometimes called **code bloat**), unless the translator can use target machine properties to combine the code for different types. Solution 2 avoids code bloat but at the cost of many indirections, which can substantially affect performance. Of course, this indirection may be necessary anyway for purposes of memory management, but there is also the increase in data size required by the tag and size fields, even for simple data.

8.9 Explicit Polymorphism

Implicit parametric polymorphism is fine for defining polymorphic functions, but it doesn't help us if we want to define polymorphic data structures. Suppose, for example, that we want to define a stack that can contain any kind of data (here implemented as a linked list):

```
typedef struct StackNode{
   ?? data;
   struct StackNode * next;
} * Stack;
```

Here the double question mark stands for the type of the data, which must be written in for this to be a legal declaration. Thus, to define such a data type, it is impossible to make the polymorphic type *implicit*; instead, we must write the type variable *explicitly* using appropriate syntax. This is called **explicit parametric polymorphism**.

Explicit parametric polymorphism is allowed in ML, of course. It would be foolish to have implicit parametric polymorphism for functions without explicit parametric polymorphism for data structures.

The same notation is used for type parameters as in the types for implicitly polymorphic functions: `'a`, `'b`, `'c` can be used in data structure declarations to indicate arbitrary types. Here is a `Stack` declaration in ML that imitates the above C declaration, but with an explicit type parameter:

```
datatype 'a Stack = EmptyStack
                  | Stack of 'a * ('a Stack);
```

First, note that this is unusual syntax, in that the type parameter `'a` is written *before* the type name (`Stack`) rather than after it (presumably in imitation of what one does without the parameter, which is to define various stacks such as `IntStack`, `CharStack`, `StringStack`, etc.). Second, this declaration incorporates the effect of the pointer in C by expressing a stack as a union of the empty stack with a stack containing a pair of components—the first the data (of type `'a`), and the second another stack (the tail or rest of the original stack). We can now write values of type `Stack` as follows:

```
val empty = EmptyStack; (* empty has type 'a Stack *)
val x = Stack(3, EmptyStack); (*x has type int Stack *)
```

Note that `EmptyStack` has no values, so is still of polymorphic type, while `x`, which contains the single value 3, is specialized by this value to `int Stack`.

Explicit parametric polymorphism creates no special new problems for Hindley-Milner type checking. It can operate in the same way on explicitly parameterized data as it did on implicitly parameterized functions. Indeed, polymorphic functions can be declared using an explicitly parameterized type, as for instance:

```
fun top (Stack p) = #1(p);
```

which (partially) defines `top` as a function `'a Stack -> 'a` by using the projection function `#1` on the pair `p` to extract its first component, which is the data. Thus, for the `x` previously defined, `top x` = 3.

Explicitly parameterized polymorphic data types fit nicely into languages with Hindley-Milner polymorphic type checking, but in fact they are nothing more than a mechanism for creating **user-defined type constructors**.

Recall that a type constructor, like array or `struct`, is a *function* from types to types, in that types are supplied as parameter values (these are the component types), and a new type is returned. For example, the array constructor in C, denoted by the brackets `[ ]`, takes a component type (`int`, say) as a parameter and constructs a new type (array of `int`). This construction can be expressed directly in C as a `typedef`, where the syntactic ordering of the type parameter and return type is different from that of a function but entirely analogous. See Figure 8.19.

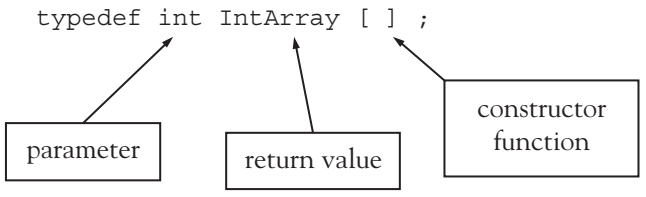


Figure 8.19 The components of a type definition in C

A type declaration in ML using the type constructor `Stack` is essentially the same kind of construction, as shown in Figure 8.20.

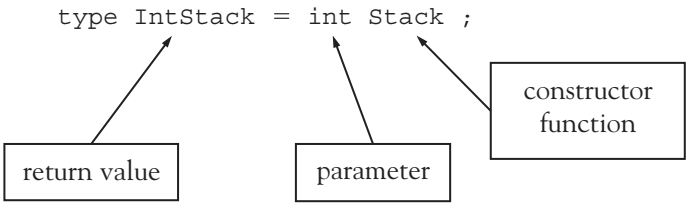


Figure 8.20: The components of a type definition in ML

When comparing an explicitly parameterized polymorphic type construction such as `Stack` in ML to equivalent constructs in C or other languages, we do need to remain aware of the differences among the different uses of the name `Stack` in a declaration like:

```
datatype 'a Stack = EmptyStack
                  | Stack of 'a * ('a Stack);
```

The first occurrence of `Stack` (to the left of the = sign) defines the `Stack` type constructor, while the second occurrence of `Stack` (after the `|`) defines the *data* or *value constructor* `Stack` (the third occurrence of `Stack` is a recursive *use* of the type constructor meaning of `Stack`). Value constructors correspond precisely to the constructors of an object-oriented language; reusing the same name for the value and type constructor is standard practice in a number of languages. In C++ and Java, for example, constructors always have the same name as their associated class (and type). See Chapter 5 for more detail.

C++ is, in fact, an example of a language with explicit parametric polymorphism, but without the associated implicit Hindley-Milner type checking. The mechanism for explicit polymorphism is the **template**, which we introduced in Chapter 5. A template can be used either with functions or the `class` or `struct` type constructor (thus, unlike ML,[23] both functions and data structures have explicit type parameters).

Here, for example, is a version of the `Stack` class introduced in Chapter 5 using an explicit C++ type parameter `T`:[24]

```
template <typename T>
struct StackNode{
   T data;
   StackNode<T> * next;
};
template <typename T>
struct Stack{
   StackNode<T> * theStack;
};
```

[23]Actually, ML functions *can* be written with explicit type parameters (e.g., `ident (x: 'a) = x`), but it is unusual to do so, since Hindley-Milner type checking introduces them automatically.

[24]The keyword `class` was used instead of `typename` in older versions of C++; the new keyword much more clearly denotes the actual meaning of T as a type parameter.

Notice how we must also use a `struct` definition in C++ to get a `Stack` type that is a pointer to `StackNode`—there are no template `typedef`s in C++.

Now we can define an explicitly parameterized `top` function equivalent to the previous ML definition as follows:

```
template <typename T>
T top (Stack<T> s){
   return s.theStack->data;
}
```

We can use these definitions in some sample code as follows:

```
Stack<int> s;
s.theStack = new StackNode<int>;
s.theStack->data = 3;
s.theStack->next = 0;

int t = top(s); // t is now 3
```

Note that the call to `top(s)` does not require that the type parameter of `top` be explicitly given—the type parameter is inferred to be `int` by the compiler from the type of `s`. On the other hand, whenever a `Stack` variable is defined, an actual (nonparameterized) type must be written as the type parameter. Writing a definition such as:

```
Stack s; // illegal C++
```

in C++ is not allowed.

An explicitly parameterized `max` function can also be written in C++ that is equivalent to the implicitly parameterized ML version:

```
template <typename T>
T max (T x, T y, bool (*gt)(T, T)){
   return gt(x,y) ? x : y ;
}
```

To use this function, we can define a function such as:

```
bool gti(int x, int y){
   return x > y;
}
```

and then we can call the `max` function as follows:

```
int larger = max(2,3,gti); // larger is now 3
```

In fact, we can also define a max function that depends implicitly on the existence of a > operator for its type parameter T:

```
template <typename T>
T max(T x, T y){
return x > y ? x : y ;

}
```

We can call this function using values of any type for which the > operator exists:

```
int larger_i = max(2,3); // larger_i is now 3
double larger_d = max(3.1,2.9); // larger_d is now 3.1
Stack<int> s,t,u;
u = max(s,t);
     // Error! operator > not defined for Stack types
```

This form of parametric polymorphism is called **(implicitly) constrained parametric polymorphism**, because it implicitly applies a constraint to the type parameter T—namely, that only a type with the > operator defined for its values can be substituted for the type parameter T. The Java collection classes provide a similar capability, as discussed in Chapter 5. This is a form of parametric polymorphism that ML does not have.[25]

Ada also allows explicitly parameterized functions and, somewhat indirectly, also explicitly parameterized types. The essential difference between Ada and C++ or ML is that Ada allows parameterization only for program **units**, which are separately compiled program segments that can either be a subprogram (i.e., a function or a procedure) or an Ada **package**. An Ada package is essentially a **module**, and in Chapter 11 we discuss modules and their relation to data types. Here we will only briefly sketch how parametric polymorphism can be achieved using Ada units; see Chapter 11 for more detail on Ada units.

An Ada unit that includes parameters is called a **generic unit**. Type parameters must be specified as private, indicating that no specific information is available about which types will be used with the unit. Here is a **package specification** in Ada that implements the same Stack structure example as before, parameterized by a type T:

```
generic
   type T is private;
package Stacks is
   type StackNode;
   type Stack is access StackNode;
   type StackNode is
   record
```

(continues)

[25]A functional language that *does* have implicitly constrained parametric polymorphism is Haskell (see Chapter 4).

(continued)

```
      data: T;
      next: Stack;
   end record;
   function top( s: Stack ) return T ;
end Stacks;
```

Note that the `Stack` data type and the package name are not the same, and that the `top` function is also declared (but not implemented) here, since it is parameterized on the same type `T`. This package specification needs an associated **package body** that provides the actual implementation of the `top` function:

```
package body Stacks is
   function top( s: Stack ) return T is
   begin
     return s.data;
   end top;
end Stacks;
```

Now this Stacks package can be used by a program as follows:

```
with Stacks; -- import the Stacks package
...
package IntStacks is new Stacks(integer);
     -- instantiate the parameterized package
     -- with type integer

use IntStacks;
   -- without this, we must use the package name
   -- before all of its components, e.g.
   -- IntStacks.Stack,
   -- IntStacks.StackNode, IntStacks.pop, etc.

s : Stack := new StackNode;
t : integer;
...
s.data := 3;
s.next := null;
t := top(s); -- t is now 3
```

Explicitly parameterized polymorphic functions can also be defined in Ada using generic units, but since functions are themselves compilation units, a package is not required (unless we wanted to define more than one function at a time in the same place). For example, here is the specification of the `max` function of previous examples in Ada:

```
generic
   type T is private;
   with function gt(x, y : T) return boolean;
function max (x, y : T) return T;
```

Notice that, rather than making the `gt` function a parameter to the `max` function itself, we have included it as a "generic" parameter to the unit; this is **explicitly constrained parametric polymorphism**, which, unlike the implicitly constrained polymorphism of C++, makes explicit that the `max` function depends on the `gt` function, even though it is not in the list of parameters to the function call itself.

The `max` function unit requires also a body, where the actual code for the `max` function is written:

```
-- body of max unit
function max (x,y:T) return T is
begin
if gt(x,y) then return x;
   else return y;
   end if;
end max;
```

Now the `max` function can be used by an Ada program as follows:

```
with max; -- import the max unit
...
function maxint is new max(integer,">");
-- instantiate the max function for integers,
-- using the ">" function as the gt function.

i: integer;
...
i := maxint(1,2); -- i is now 2
```

As is generally true with Ada, everything must be done explicitly in the code. That is, polymorphism must always be explicit, parameterization must make all dependencies explicit, and instantiation must be made explicit. This is true, even to the extent of requiring a new name—e.g., `maxint` above.

8.10 Case Study: Type Checking in TinyAda

In Section 7.8, we added code for static semantic analysis to a parser for TinyAda. In this code, identifiers are checked to ensure that they are declared before they are used and that they are not declared more than once in the same block. Also, the role of an identifier, as a constant, variable, procedure, or type name, is recorded when it is declared. This information is used to ensure that the identifier is being

referenced in an appropriate context. In this section, we develop and partially implement a design for type checking in the parser. As before, this will involve adding new information to the symbol table when an identifier is declared, and looking up that information to enforce semantic restrictions when the identifier is referenced.

8.10.1 Type Compatibility, Type Equivalence, and Type Descriptors

For a statically typed language like TinyAda, the parser must check the type of any identifier when it is referenced. Many of these checks occur when the identifier is an operand in an expression. For example, in the TinyAda expression `X + 1`, the variable `X` must be of type `INTEGER` or a subrange type of `INTEGER`, whereas in the expression `not (A or B)`, the variables `A` and `B` must be of type `BOOLEAN`. The types of identifiers allowed in operands of expressions, thus, must be the types required by the operators. Moreover, because operands can also be any expressions, each expression also must have a type. For example, the subexpression `A or B` in the expression `not (A or B)` must be of type `BOOLEAN`.

Another contextual factor that restricts the types of identifiers and expressions is their placement in assignment statements or as actual parameters to a procedure call. The name on the left side of an assignment statement must be type-compatible with the expression on its right. The expression or name passed as an actual parameter to a procedure call must be type-compatible with the corresponding formal parameter in the procedure's declaration.

Finally, the types of certain elements of declarations are also restricted. For example, the index types of array type definitions must be the ordinal types `BOOLEAN` or `CHAR` or a subrange of `INTEGER`.

TinyAda uses a loose form of name equivalence to determine whether or not two identifiers (or expressions) are type-compatible. In the cases of arrays and enumerations, this means that two identifiers are type-compatible if and only if they were declared using the same type name in their declarations. In the case of the built-in types `INTEGER`, `CHAR`, and `BOOLEAN` and their programmer-defined subrange types, two identifiers are type-compatible if and only if their supertypes are name-equivalent. For example, the supertype of the type name `INDEX` in the following declaration is `INTEGER`:

```
type INDEX is range 1..10;
```

The supertype of `INTEGER` is itself; therefore, a variable of type `INDEX` is type-compatible with a variable of type `INTEGER`, because their supertypes are the same. From these basic facts about type compatibility and type equivalence, the rules for type checking in the TinyAda parser can be derived.

The primary data structure used to represent type attributes is called a **type descriptor**. A type descriptor for a given type is entered into the symbol table when the corresponding type name is introduced. This happens at program startup for the built-in type names `BOOLEAN`, `CHAR`, and `INTEGER`, and later when new type declarations are encountered. When identifier references or expressions are encountered and their types are checked, the parser either compares the type descriptors of two operands or compares a given type descriptor against an expected descriptor. When strict name equivalence is required (for arrays or enumerations), the two type descriptors must be the exact same object. Otherwise (for the built-in ordinal types and their subranges), the two type descriptors must have the exact same supertype descriptor. These rules lead naturally to the implementation of several type-checking methods that appear later in this section.

8.10.2 The Design and Use of Type Descriptor Classes

Conceptually, a type descriptor is like a variant record, which contains different attributes depending on the category of data type being described. Here is the information contained in the type descriptors for each category of data type in TinyAda.

1. Each type descriptor includes a field called a **type form**, whose possible values are `ARRAY`, `ENUM`, `SUBRANGE`, and `NONE`. This field serves to identify the category of the data type and allows the user to access its specific attributes.
2. The array type descriptor includes attributes for the index types and the element type. These attributes are also type descriptors.
3. The enumeration type descriptor includes a list of symbol entries for the enumerated constant names of that type.
4. The type descriptors for subrange types (including `BOOLEAN`, `CHAR`, and `INTEGER`) include the values of the lower bound and upper bound, as well as a type descriptor for the supertype.

There is no variant record structure in Java to represent this information, but we can model it nicely with a `TypeDescriptor` class and three subclasses, `ArrayDescriptor`, `SubrangeDescriptor`, and `EnumDescriptor`. The top-level class includes the type form, which is automatically set when an instance of any subclass is created. The subclasses include the information just listed, with the representation of records and enumerated types merged into one class. Each class also includes methods to set its attributes. The user can access these directly as public fields. The complete implementation of these classes is available on the book's Web site.

At program startup, the parser now creates type descriptors for the three built-in types. These descriptors are declared as instance variables of the parser, for easy access and because they will be the unique supertypes of all subrange types. We define a new method to initialize these objects and update another method to attach them as types to the built-in identifiers of TinyAda. Here is the code for these changes:

```
// Built-in type descriptors, globally accessible within the parser
private SubrangeDescriptor BOOL_TYPE, CHAR_TYPE, INT_TYPE;

// Sets up the type information for the built-in type descriptors
private void initTypeDescriptors(){
   BOOL_TYPE = new SubrangeDescriptor();
   BOOL_TYPE.setLowerAndUpper(0, 1);
   BOOL_TYPE.setSuperType(BOOL_TYPE);
   CHAR_TYPE = new SubrangeDescriptor();
   CHAR_TYPE.setLowerAndUpper(0, 127);
   CHAR_TYPE.setSuperType(CHAR_TYPE);
   INT_TYPE = new SubrangeDescriptor();
   INT_TYPE.setLowerAndUpper(Integer.MIN_VALUE, Integer.MAX_VALUE);
```

(continues)

(continued)

```
      INT_TYPE.setSuperType(INT_TYPE);
   }

   // Attach type information to the built-in identifiers
   private void initTable(){
      table = new SymbolTable(chario);
      table.enterScope();
      SymbolEntry entry = table.enterSymbol("BOOLEAN");
      entry.setRole(SymbolEntry.TYPE);
      entry.setType(BOOL_TYPE);
      entry = table.enterSymbol("CHAR");
      entry.setRole(SymbolEntry.TYPE);
      entry.setType(CHAR_TYPE);
      entry = table.enterSymbol("INTEGER");
      entry.setRole(SymbolEntry.TYPE);
      entry.setType(INT_TYPE);
      entry = table.enterSymbol("TRUE");
      entry.setRole(SymbolEntry.CONST);
      entry.setType(BOOL_TYPE);
      entry = table.enterSymbol("FALSE");
      entry.setRole(SymbolEntry.CONST);
      entry.setType(BOOL_TYPE);
   }
```

The parser also defines two utility methods to check types. The first, `matchTypes`, compares two type descriptors for compatibility using name equivalence. The second, `acceptTypeForm`, compares a type descriptor's type form to an expected type form. Each method outputs an error message if a type error occurs, as shown in the following code:

```
   // Compares two types for compatibility using name equivalence
   private void matchTypes(TypeDescriptor t1, TypeDescriptor t2,
                           String errorMessage){
      // Need two valid type descriptors to check for an error
      if (t1.form == TypeDescriptor.NONE || t2.form == TypeDescriptor.NONE)
       return;
      // Exact name equivalence: the two descriptors are identical
      if (t1 == t2) return;
      // To check two subranges, the supertypes must be identical
      if (t1.form == TypeDescriptor.SUBRANGE && t2.form == TypeDescriptor.
       SUBRANGE){
         if (((SubrangeDescriptor)t1).superType != ((SubrangeDescriptor)t2).
          superType)
            chario.putError(errorMessage);
      }
      else
```

(continues)

(continued)

```
        // Otherwise, at least one record, array, or enumeration does not match
        chario.putError(errorMessage);
  }

  // Checks the form of a type against an expected form
  private void acceptTypeForm(TypeDescriptor t, int typeForm, String
   errorMessage){
     if (t.form == TypeDescriptor.NONE) return;
     if (t.form != typeForm)
        chario.putError(errorMessage);
  }
```

Note that the formal type descriptor parameters of these two methods are declared to be of the class `TypeDescriptor`, which allows the methods to receive objects of any of the type descriptor subclasses as actual parameters. Both methods then check type forms to determine which actions to take. If the form is `NONE`, a semantic error has already occurred somewhere else, so the methods quit to prevent a ripple effect of error messages. Otherwise, further checking occurs. In the case of `matchTypes`, Java's == operator is used to detect the object identity of the two type descriptors. This can be done without knowing which classes these objects actually belong to. However, if the two descriptors are not identical, the types will be compatible only if they are both subranges and their supertypes are identical. After determining that the form of the two descriptors is `SUBRANGE`, the `TypeDescriptor` objects must be cast down to the `SubrangeDescriptor` class before accessing their supertype fields. Fortunately, this type of downcasting will rarely occur again in the parser; the use of `TypeDescriptor` as a parameter type and a method return type will provide very clean, high-level communication channels between the parsing methods.

8.10.3 Entering Type Information in Declarations

Type information must now be entered wherever identifiers are declared in a source program. We noted these locations in Section 8.8, where the parser enters identifier role information. The type information comes either from type identifiers (in number or object declarations, parameter specifications, and record component declarations) or from a type definition. In the case of a type identifier, the type descriptor is already available and is simply accessed in the identifier's symbol entry. In the case of a type definition, a new type might be created. Therefore, the `typeDefinition` method should return to its caller the `TypeDescriptor` for the type just processed. The following code shows the implementation of this method and its use in the method `TypeDeclaration`, which declares a new type name:

```
  /*
  typeDeclaration = "type" identifier "is" typeDefinition ";"
  */
  private void typeDeclaration(){
     accept(Token.TYPE, "'type' expected");
     SymbolEntry entry = enterId();
     entry.setRole(SymbolEntry.TYPE);
```

(continues)

(continued)

```
      accept(Token.IS, "'is' expected");
      entry.setType(typeDefinition());            // Pick up the type
       descriptor here
      accept(Token.SEMI, "semicolon expected");
   }

   /*
   typeDefinition = enumerationTypeDefinition | arrayTypeDefinition
                  | range | <type>name
   */
   private TypeDescriptor typeDefinition(){
      TypeDescriptor t = new TypeDescriptor();
      switch (token.code){
         case Token.ARRAY:
            t = arrayTypeDefinition();
            break;
         case Token.L_PAR:
            t = enumerationTypeDefinition();
            break;
         case Token.RANGE:
            t = range();
            break;
         case Token.ID:
            SymbolEntry entry = findId();
            acceptRole(entry, SymbolEntry.TYPE, "type name expected");
            t = entry.type;
            break;
         default: fatalError("error in type definition");
      }
      return t;
   }
```

Note how each lower-level parsing method called in the `typeDefinition` method also returns a type descriptor, which is essentially like a buck being passed back up the call tree of the parsing process. We see the buck stopping at one point, when the type definition is just another identifier, whose type descriptor is accessed directly. The strategy of passing a type descriptor up from lower-level parsing methods is also used in processing expressions, as we will see shortly.

The next example shows how a new type is actually created. An enumeration type consists of a new set of constant identifiers. Here is the code for the method `enumerationTypeDefinition`:

```
   /*
   enumerationTypeDefinition = "(" identifierList ")"
   */
   private TypeDescriptor enumerationTypeDefinition(){
      accept(Token.L_PAR, "'(' expected");
```

(continues)

(continued)

```
      SymbolEntry list = identifierList();
      list.setRole(SymbolEntry.CONST);
      EnumDescriptor t = new EnumDescriptor();
      t.setIdentifiers(list);
      list.setType(t);
      accept(Token.R_PAR, "')' expected");
      return t;
   }
```

The method now returns a type descriptor. The three new lines of code in the middle of the method

- Instantiate a type descriptor for an enumerated type.
- Set its `identifiers` attribute to the list of identifiers just parsed.
- Set the type of these identifiers to the type descriptor just created.

The processing of array and subrange type definitions is similar and left as an exercise for the reader.

8.10.4 Checking Types of Operands in Expressions

A quick glance at the rules for TinyAda expressions gives some hints as to how their types should be checked. Working top-down, a condition must be a Boolean expression, so clearly the `expression` method must return a type descriptor to be checked. Here is the new code for the `condition` method:

```
   /*
   condition = <boolean>expression
   */
   private void condition(){
      TypeDescriptor t = expression();
      matchTypes(t, BOOL_TYPE, "Boolean expression expected");
   }
```

The `expression` method in turn might see two or more relations separated by Boolean operators. Thus, the `relation` method must also return a type descriptor. If a Boolean operator is detected in the `expression` method, then the types of the two relations must each be matched against the parser's `BOOL_TYPE` variable.

The coder of the parser must read each syntax rule for expressions carefully and add the appropriate code for type checking to the corresponding method. As an example, here is the code for the method `factor`, which shows the checking of some operands as well as the return of the appropriate type descriptor:

```
/*
factor = primary [ "**" primary ] | "not" primary
*/
private TypeDescriptor factor(){
   TypeDescriptor t1;
   if (token.code == Token.NOT){
      token = scanner.nextToken();
      t1 = primary();
      matchTypes(t1, BOOL_TYPE, "Boolean expression expected");
   }
   else{
      t1 = primary();
      if (token.code == Token.EXPO){
         matchTypes(t1, INT_TYPE, "integer expression expected");
         token = scanner.nextToken();
         TypeDescriptor t2 = primary();
         matchTypes(t2, INT_TYPE, "integer expression expected");
      }
   }
   return t1;
}
```

This method has the following three options for returning a type descriptor:

1. There is no operator in the factor, so the type descriptor `t1` obtained from the first call of `primary` is returned directly to the caller.
2. The operator is a leading `not`. Then the type descriptor `t1` is returned, after matching it to `BOOL_TYPE`.
3. The operator is a binary `**`. Then the type descriptor `t1` is returned, after matching it and the type descriptor `t2`, obtained from the second call to `primary`, to `INT_TYPE`.

As you can see, the type of every operand must be checked, and great care must be taken to ensure that the right type descriptor is returned. The modification of the remaining methods for parsing expressions is left as an exercise for the reader.

8.10.5 Processing Names: Indexed Component References and Procedure Calls

The syntax of TinyAda indexed component references and procedure calls is the same if the procedure expects at least one parameter. As shown in the previous code segment, the method `name` now must distinguish these two types of phrases, based on the role of the leading identifier. The code for processing the trailing phrase, which consists of a parenthesized list of expressions, can now be split into two distinct parsing methods, `actualParameterPart` and `indexedComponent`.

The method `actualParameterPart` receives the procedure name's symbol entry as a parameter. The entry contains the information for the procedure's formal parameters, which was entered earlier when the procedure was declared. As it loops through the list of expressions, this method should match the type of each expression to the type of the formal parameter in that position. Also, the number of expressions and formal parameters must be the same.

The method `indexedComponent` receives the array identifier's entry as a parameter. This entry's type descriptor contains the information for the array's index types and element type, which was entered earlier when that array type was defined. This method uses the same strategy to match expressions to declared index types as we saw for matching the types of formal and actual parameters in the method `actualParameterPart`. However, `indexedComponent` must also return a new symbol entry that contains the array's element type, as well as the role of the array identifier. This will be the information returned by the `name` method for role and type analysis farther up the call tree. The completion of these two methods, as well as the rest of the code for type analysis in the TinyAda parser, is left as an exercise for the reader.

8.10.6 Completing Static Semantic Analysis

Two other types of semantic restrictions can be imposed during parsing. One restriction involves the checking of **parameter modes**. TinyAda has three parameter modes: input only (signified by the keyword `in`), output only (signified by the keyword `out`), and input/output (signified by the keywords `in out`). These modes are used to control the flow of information to and from procedures. For example, only a variable or a parameter with the same mode can be passed as an actual parameter where the formal parameter has been declared as `out` or `in out`. Within a procedure's code, an `out` parameter cannot appear in an expression, and an `in` parameter cannot be the target of an assignment statement.

The other restriction recognizes a distinction between static expressions and other expressions. A static expression is one whose value can be computed at compile time. Only static expressions can be used in number declarations and range type definitions. We will discuss the enforcement of these two types of restrictions in later chapters.

Exercises

8.1 Compare the flexibility of the dynamic typing of Lisp with the reliability of the static typing of Ada, ML, C++, or Java. Think of as many arguments as you can both for and against static typing as an aid to program design.

8.2 Look up the implementation details on the representation of data by your C/C++ compiler (or other non-Java compiler) that you are using and compare them to the sketches of typical implementations given in the text. Where do they differ? Where are they the same?

8.3 **(a)** The Boolean data type may or may not have an order relation, and it may or may not be convertible to integral types, depending on the language. Compare the treatment of the Boolean type in at least two of the following languages with regard to these two issues: Java, C++, Ada, ML.

(b) Is it useful for Booleans to be ordered? Convertible to integers?

8.4 **(a)** Given the C declarations

```
struct{
   int i;
   double j;
} x, y;
struct{
   int i;
   double j;
} z;
```

the assignment `x = z` generates a compilation error, but the assignment `x = y` does not. Why?

(b) Give two different ways to fix the code in part (a) so that `x = z` works. Which way is better and why?

8.5 Suppose that we have two C arrays:

```
int x[10];
int y[10];
```

Why won't the assignment `x = y` compile in C? Can the declarations be fixed so the assignment will work?

8.6 Given the following variable declarations in C/C++:

```
enum { one } x;
enum { two } y;
```

the assignment `x = y` is fine in C, but generates a compiler error in C++. Explain.

8.7 Given the following function variable declarations in C/C++:

```
int (*f)(int);
int (*g)(int);
int (*h)(char);
```

the assignment `f = g` is fine in C and C++, but the assignment `h = g` generates a compiler warning in C and a compiler error in C++. Explain.

8.8 Show that the set:

```
{emptylist} ∪ char ∪ (char × char) ∪
(char × char × char) ∪...
```

is the smallest solution to the set equation:

```
CharList = {emptylist} ∪ char × CharList
```

(See Section 8.3.5 and Exercise 6.53.)

8.9 Consider the set equation

$$\chi \times \text{char} = \chi$$

(a) Show that any set X satisfying this equation must be infinite.
(b) Show that any element of a set satisfying this equation must contain an infinite number of characters.
(c) Is there a smallest solution to this equation?

8.10 Consider the following declaration in C syntax:

```
struct CharTree{
   char data;
   struct CharTree left, right;
};
```

(a) Rewrite `CharTree` as a union, similar to the `union CharList` declaration of Section 8.3.5.
(b) Write a recursive set equation for your declaration in part (a).
(c) Describe a set that is the smallest solution to your equation of part (b).
(d) Prove that your set in part (c) is the smallest solution.
(e) Rewrite this as a valid C declaration.

8.11 Here are some type declarations in C:

```
typedef struct Rec1 * Ptr1;
typedef struct Rec2 * Ptr2;
struct Rec1{
int data;
   Ptr2 next;
};
struct Rec2{
   double data;
   Ptr2 next;
};
```

Should these be allowed? Are they allowed? Why or why not?

8.12 **(a)** What is the difference between a subtype and a derived type in Ada?
(b) What C type declaration is the following Ada declaration equivalent to?

```
subtype New_Int is integer;
```

(c) What C type declaration is the following Ada declaration equivalent to?

```
type New_Int is new integer;
```

8.13 In Ada the equality test (`x = y` in Ada syntax) is legal for all types, as long as `x` and `y` have the same type. In C, however, the equality test (`x == y` in C syntax) is only permitted for variables of a few types; however, the types of `x` and `y` need not always be the same.
(a) Describe the types of `x` and `y` for which the C comparison `x == y` is legal.
(b) Why does the C language not allow equality tests for variables of certain types?
(c) Why does the C language allow comparisons between certain values of different types?

8.14 Describe the type correctness and inference rules in C for the conditional expression:

```
e1 ? e2 : e3
```

Must `e2` and `e3` have the same type? If they have the same type, can it be any type?

8.15 Suppose we used the following C++ code that attempts to avoid the use of a `static_cast` to print out the internal value of `true` (see the code at the end of Section 8.7):

```
union{
   int i;
   bool b;
} x;
...
x.b = true;
cout << x.i << endl;
```

Why is this wrong? (*Hint:* Try the assignment `x.i=20000` before `x.b=true.`)

8.16 Programmers often forget that numeric data types such as `int` are only approximations of mathematical number systems. For example, try the following two divisions in C or C++ or Java on your system, and explain the result: `-2147483648 / -1; -2147483647 / -1.`

8.17 The chapter makes no mention of the initialization problem for variables. Describe the rules for variable initialization in C, Ada, ML, or other similar language. In particular, are variables of all types initialized when a program starts running? If so, how is the initial value constructed?

8.18 Here are some type and variable declarations in C syntax:

```
typedef char* Table1;
typedef char* Table2;

Table1 x,y;
Table2 z;
Table2 w;
```

State which variables are type equivalent under **(a)** structural equivalence, **(b)** name equivalence, and **(c)** the actual C equivalence algorithm. Be sure to identify those cases that are ambiguous from the information at hand.

8.19 Given the declarations

```
int x[10];
int y[5];
```

are `x` and `y` type equivalent in C?

8.20 Given the following C declaration:

```
const PI = 3.14159;
```

if we then use the constant `PI` in a geometry formula (like `A = PI*r*r`), we get the wrong answer. Why?

8.21 Given the following Java declaration:

```
short i = 2;
```

the Java compiler generates an error for the statement:

```
i = i + i;
```

Why?

8.22 Here are some type and variable declarations in C syntax:

```
typedef struct{
   int x;
   char y;
} Rec1;
typedef Rec1 Rec2;
typedef struct{
   int x;
   char y;
} Rec3;
Rec1 a,b;
Rec2 c;
Rec3 d;
```

State which variables are type equivalent under **(a)** structural equivalence, **(b)** name equivalence, and **(c)** the actual C equivalence algorithm. Be sure to identify those cases that are ambiguous from the information at hand.

8.23 In C++ the equality test == can be applied to arrays, but it tests the wrong thing. Also, the assignment operator = cannot be applied to arrays at all. Explain.

8.24 Can equality and assignment be applied to Ada arrays? Do they test the right thing?

8.25 By the type rules stated in the text, the array constructor in C does not construct a new type; that is, structural equivalence is applied to arrays. How could you write code to test this? (*Hint:* Can we use an equality test? Are there any other ways to test compatibility?)

8.26 Can a union in C be used to convert `int`s to `double`s and vice versa? To Convert `long`s to `float`s? Why or why not?

8.27 In a language in which "/" can mean either integer or real division and that allows coercions between integers and reals, the expression `I + J / K` can be interpreted in two different ways. Describe how this can happen.

8.28 Show how Hindley-Milner type checking determines that the following function (in ML syntax) takes an integer parameter and returns an integer result (i.e., is of type `int -> int` in ML terminology):

```
fun fac n = if n = 0 then 1 else n * fac (n-1);
```

8.29 Here is a function in ML syntax:

```
fun f (g,x) = g (g(x));
```

(a) What polymorphic type does this function have?
(b) Show how Hindley-Milner type checking infers the type of this function.
(c) Rewrite this function as a C++ template function.

8.30 Write a polymorphic `swap` function that swaps the values of two variables of the same type in:
(a) C++
(b) ML

8.31 Write a polymorphic `max` function in C++ that takes an array of values of any type and returns the maximum value in the array, assuming the existence of a ">" operation.

8.32 Repeat the previous exercise, but add a parameter for a `gt` function that takes the place of the ">" operation.

8.33 Can both functions of the previous two exercises exist (and be used) simultaneously in a C++ program? Explain.

8.34 Explicit parametric polymorphism need not be restricted to a single type parameter. For example, in C++ one can write:

```
template <typename First, typename Second>
struct Pair{
   First first;
   Second second;
};
```

Write a C++ template function `makePair` that takes two parameters of different types and returns a `Pair` containing its two values.

8.35 Write the `Pair` data structure of the previous exercise as an Ada generic package, and write a similar `makePair` function in Ada.

8.36 Is C++ parametric polymorphism let-bound? (*Hint:* See the functions `makepair` and `makepair2` near the end of Section 8.8.

8.37 Does the occur-check problem occur in C++? Explain.

8.38 A problem related to let-bound polymorphism in ML is that of **polymorphic recursion**, where a function calls itself recursively, but on different types at different points in the code. For example, consider the following function (in ML syntax):

```
fun f x = if x = x then true else (f false);
```

What type does this function have in ML? Is this the most general type it could have? Show how ML arrived at its answer.

8.39 Does the polymorphic recursion problem of the previous exercise exist in C++? Explain.

8.40 Data types can be kept as part of the syntax tree of a program and checked for structural equivalence by a simple recursive algorithm. For example, the type

```
struct{
   double x;
   int y[10];
};
```

might be kept as the tree shown in Figure 8.21.

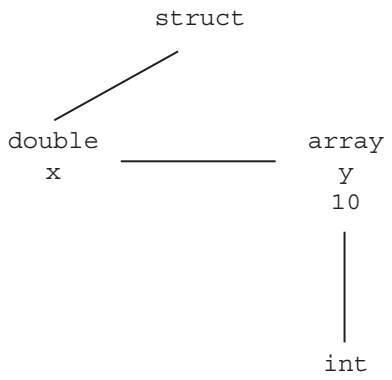


Figure 8.21: A syntax tree for the struct definition

Describe a tree node structure that could be used to express the types of C or a similar language as trees. Write out a `TypeEqual` function in pseudocode that would check structural equivalence on these trees.

8.41 How could the trees of the previous exercise represent recursive types? Modify your `TypeEqual` function to take recursive types into account.

8.42 The language C uses structural type equivalence for arrays and pointers, but declaration equivalence for structs and unions. Why do you think the language designers did this?

8.43 Compare the BNFs for type declarations in C (Java, Ada) with the type tree of Figure 8.5 (8.6, 8.7). To what extent is the tree a direct reflection of the syntax?

8.44 What new type constructors does C++ add to those of C? Add these to the type tree of Figure 8.5. (*Hint:* Use the BNFs for C++ as motivation.)

8.45 The type tree for Ada (Figure 8.7) is somewhat simplified, with a number of constructors omitted. Add the missing constructors. Are there any missing simple types?

8.46 What operations would you consider necessary for a string data type? How well do arrays of characters support these operations?

8.47 Given the following C++ declarations,

```
double* p = new double(2);
void* q;
int* r;
```

which of the following assignments does the compiler complain about?

```
q = p;
p = q;
r = p;
p = r;
r = q;
r = (int*)q;
r = static_cast<int*>(q);
r = static_cast<int*>(p);
r = reinterpret_cast<int*>(p);
```

Try to explain the behavior of the compiler. Will `*r` ever have the value 2 after one of the assignments to `r`? Why?

8.48 In Java, the following local variable declaration results in a compiler error:

```
float x = 2.1;
```

Why? Find two ways of removing this error.

8.49 Should the equality test be available for floating-point types (explain)? Is it available in Java? Ada? ML?

8.50 Should the equality test be available for function types (explain)? Is it available in C? Ada? ML?

8.51 The conversion rules for an arithmetic expression such as $e1 + e2$ in C are stated in Kernighan and Ritchie [1978] as follows (these are the pre-ANSI rules, which are simpler than the ANSI rules, but have the same result in many cases):

> First, any operands of type `char` or `short` are converted to `int`, and any of type `float` are converted to `double`. Then, if either operand is `double`, the other is converted to `double` and that is the type of the result.
>
> Otherwise, if either operand is `long`, the other is converted to `long` and that is the type of the result.
>
> Otherwise, if either operand is `unsigned`, the other is converted to `unsigned` and that is the type of the result.
>
> Otherwise, both operands must be `int`, and that is the type of the result.

Given the following expression in C, assuming `x` is an `unsigned` with value 1, describe the type conversions that occur during evaluation and the type of the resulting value. What is the resulting value?

```
'0' + 1.0 * (-1 + x)
```

8.52 In C there is no Boolean data type. Instead, comparisons such as `a == b` and `a <= b` return the integer value 0 for false and 1 for true. On the other hand, the `if` statement in C considers any nonzero value of its condition to be equivalent to true. Is there an advantage to doing this? Why not allow the condition `a <= b` to return any nonzero value if it is true?

8.53 Complete the entry of type information for the TinyAda parser discussed in Section 8.10. You should modify the type definition methods and the parameter declaration methods. Be sure to add type checking to these methods where relevant. Note that the `SymbolEntry` class and the `TypeDescriptor` class include two methods for setting formal parameter lists and lists of array index types, respectively. Test your parser with example TinyAda programs that exercise all of the modified methods.

8.54 Complete the modifications for type analysis to the TinyAda parsing methods that process expressions. Be sure to add type checking to these methods where relevant. Test your parser with example TinyAda programs that exercise all of the modified methods.

8.55 Complete the modifications for type analysis to the TinyAda parsing methods that process indexed variable references. Note that the `ArrayDescriptor` class includes the method `getIndexes`, which returns an iterator on the array's index types.

8.56 Complete the modifications for type analysis to the TinyAda parsing methods that process actual parameters of procedure calls. Note that the `SymbolEntry` class includes the method `getParameters`, which returns an iterator on the procedure's formal parameters.

8.57 Complete the modifications for type analysis to the TinyAda parsing methods that process names and assignment or procedure call statements.

Notes and References

Data type systems in Algol-like languages such as Pascal, C, and Ada seem to suffer from excessive complexity and many special cases. In part, this is because a type system is trying to balance two opposing design goals: expressiveness and security. That is, a type system tries to make it possible to catch as many errors as possible while at the same time allowing the programmer the flexibility to create and use as many types as are necessary for the natural expression of an algorithm. ML (as well as Haskell) escapes from this complexity somewhat by using structural equivalence for predefined structures but name equivalence for user-defined types; however, this does weaken the effectiveness of type names in distinguishing different uses of the same structure. An interesting variant on this same idea is Modula-3, an object-oriented modification of Modula-2 that uses structural equivalence for ordinary data and name equivalence for objects (Cardelli et al. [1989a,b]; Nelson [1991]). Ada has perhaps the most consistent system, albeit containing many different ideas and hence extremely complex.

One frustrating aspect of the study of type systems is that language reference manuals rarely state the underlying type-checking algorithms explicitly. Instead, the algorithms must be inferred from a long list of special rules, a time-consuming investigative task. There are also few references that give detailed overviews of language type systems. Two that do have considerable detail are Cleaveland [1986] and Bishop [1986]. A lucid description of Pascal's type system can be found in Cooper [1983]. A very detailed description of Ada's type system can be found in Cohen [1996]. Java's type system is described in Gosling et al. [2000], C's type system in Kernighan and Ritchie [1988], C++'s type system

in Stroustrup [1997], and ML's type system in Ullman [1998] and Harper and Mitchell [1993]; Haskell's in Thompson [1999].

For a study of type polymorphism and overloading, see Cardelli and Wegner [1985]. The Hindley-Milner algorithm is clearly presented in Cardelli [1987], together with Modula-2 code that implements it for a small sample language; let-bound polymorphism and the occur-check are also discussed. Let-bound polymorphism is also described in Ullman [1998], Section 5.3, where "generic" type variables are called "generalizable." C++ templates are amply described in Stroustrup [1997]. Ada "generics" are described in Cohen [1996]. Polymorphic recursion (Exercise 9.38) is described in Henglein [1993].

The IEEE floating-point standard, mentioned in Section 8.2, appears in IEEE [1985].

CHAPTER

Control I—Expressions and Statements

CHAPTER 9

In Chapter 8, we discussed basic and structured abstraction of data through the use of data types. In this chapter, we discuss the basic and structured abstraction of control through the use of expressions and statements. Structured control through the use of procedures and functions, and the organization of run-time environments, will be studied in the next chapter. As noted in Chapter 1, unit-level data and control converge to the same kinds of language mechanisms, and they will be studied in the chapter on modules.

Expressions represent the most fundamental computation mechanism in programs. An **expression**, in its pure, mathematical, form, returns a value and produces no side effect. That is, it does not change program memory. By contrast, a **statement** is executed for its side effects and returns no value. Many languages, however, do not make such a clear distinction and allow expressions to contain side effects. In functional languages—sometimes also called **expression languages**—virtually all language constructs are expressions. Even in nonfunctional languages, expressions play a significant role, and in some languages, such as C (which could be called an **expression-oriented language**), expressions make up a much larger portion of the language than statements. Expressions are also the parts of programming languages that are closest in appearance to mathematics. In the absence of side effects, program expressions have semantics that are very similar to those of mathematical expressions, which leads to semantic simplicity and precision. Unfortunately, in the presence of side effects, expressions can behave in ways that are very different from their mathematical counterparts. This can be a source of confusion and error. In fact, the semantics of expressions with side effects have a significant control component. The way that such expressions are evaluated, including the order in which subexpressions are computed, can have a major impact on their meaning, something that is never true for mathematical expressions. That is why we discuss them in this chapter, along with more explicit forms of control.

Explicit control structures first appeared in programming languages as `GOTO`s, which are simple imitations of the jump statements of assembly language, transferring control directly, or after a test, to a new location in the program. With Algol60 came the improvement of **structured control**, in which control statements transfer control to and from sequences of statements that are (at least in principle) **single-entry**, **single-exit**, that is, those that enter from the beginning and exit from the end. Examples of such single-entry, single-exit constructs are the **blocks** of Algol60, Algol68, C, and Ada, which may also include declarations, and which you studied in Chapter 7.[1]

Structured programming led to an enormous improvement in the readability and reliability of programs. It's no surprise, then, that structured control constructs are part of most major languages today. Some languages do away with `GOTO`s altogether, although a (now somewhat muted) debate continues to this day on the utility of `GOTO`s within the context of structured programming.

[1]All these languages contain certain relaxations of the single-entry, single-exit property of these constructs, but the principle of controlling entry and exit points remains.

In this chapter, we will first discuss expressions and the variety of control questions that can arise during their evaluation. We then discuss structured control mechanisms (the assignment statement was already studied in Chapter 7), after which we review the `GOTO` issue. In the last section of this chapter, we begin a discussion of exception handling, which is essentially a preemptive control mechanism, causing the explicit control structure of a program to be disturbed. In that section, we discuss the appearance and operation of exceptions in several languages; implementation issues are postponed to the next chapter, since they involve the runtime environment.

9.1 Expressions

As you saw in Chapters 6 and 7, basic expressions are literals (manifest constants) and identifiers. More complex expressions are built up recursively from basic expressions by the application of operators and functions, sometimes involving grouping symbols such as parentheses. For example, in the simple arithmetic expression 3 + 4 * 5, the + **operator** is applied to its two **operands**, consisting of the integer literal 3 and the (sub)expression 4 * 5, which in its turn represents the application of the * operator to its operands, consisting of the integer literals 4 and 5. Operators can take one or more operands: An operator that takes one operand is called a **unary operator**; an operator that takes two is a **binary operator**. Operators can be written in **infix**, **postfix**, or **prefix** notation, corresponding to an inorder, postorder, or preorder traversal of the syntax tree of the expression. For example, as you learned in Chapter 6, the infix expression 3 + 4 * 5 with the syntax tree shown in Figure 9.1 is written in postfix form as 3 4 5 * + and in prefix form as + 3 * 4 5.

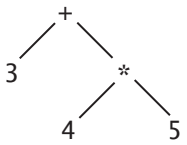


Figure 9.1 Syntax tree for infix expression 3 + 4 * 5

The advantage of postfix and prefix forms is that they do not require parentheses to express the order in which operators are applied. Thus, operator precedence is not required to disambiguate an unparenthesized expression. For instance, (3 + 4) * 5 is written in postfix form as 3 4 + 5 * and in prefix form as * + 3 4 5. Associativity of operators is also expressed directly in postfix and prefix form without the need for a rule. For example, the postfix expression 3 4 5 + + right associates and 3 4 + 5 + left associates the infix expression 3 + 4 + 5.

Many languages make a distinction between operators and functions. Operators are predefined and (if binary) written in infix form, with specified associativity and precedence rules, whereas functions, which can be either predefined or user-defined, are written in prefix form and the operands are viewed as **arguments** or **actual parameters** to calls of the functions. For example, in C, if we wrote the expression 3 + 4 * 5 with user-defined addition and multiplication operations, it would appear as `add(3,mul(4,5))`. In fact, the distinction between operators and functions is arbitrary, since they are equivalent concepts. Historically, however, the distinction *was* significant, since built-in operators were implemented using highly optimized **inline code** (code for the function body that is inserted directly at

the point where the function would be called),[2] while function calls required the building of sometimes time-consuming **activations** (described in the next chapter). Modern translators, however, often inline even user-defined functions; in any case, the performance penalty of activations has largely disappeared in modern architectures.

Programming languages, too, have increasingly allowed programmers to overload the built-in operators and even to define new infix operators with arbitrary associativity and precedence, as you saw in Sections 3.4 and 5.4. Some languages even allow all operators to be written in either prefix or infix form. For example, 3 + 4 * 5 can be written just as well in Ada as `"+"(3,"*"(4,5))`. A few languages use either the prefix or postfix form exclusively, for both predefined and user-defined operations. For example, stack-based languages such as PostScript and FORTH use postfix notation exclusively, while Lisp uses prefix notation. Lisp also requires expressions to be **fully parenthesized**; that is, all operators and operands must be enclosed in parentheses. This is because LISP operators can take variable numbers of arguments as operands. Thus, 3 + 4 * 5 and (3 + 4) * 5 would look as follows in LISP:

```
(+ 3 (* 4 5))
(* (+ 3 4) 5)
```

Each programming language has rules for evaluating expressions. According to one common evaluation rule, all operands are evaluated first and then operators are applied to them. This rule, called **applicative order evaluation**, or sometimes **strict** evaluation, is the most common rule in programming languages. It corresponds to a bottom-up evaluation of the values at nodes of the syntax tree representing an expression. For example, the expression (3 + 4) * (5 – 6) is represented by the syntax tree shown in Figure 9.2.

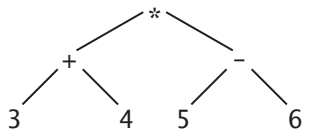


Figure 9.2 Syntax tree for the expression (3 + 4) * (5 – 6)

In applicative order evaluation, first the "+" and "–" nodes are evaluated to 7 and –1, respectively, and then the "*" is applied to get –7.

Let us also consider how this would appear in terms of user-defined functions, written in prefix form in C syntax:

```
mul(add(3,4),sub(5,6))
```

Applicative order says evaluate the arguments first. Thus, calls to `add (3,4)` and `sub(5,6)` are made, which are also evaluated using applicative order. Then, the calls are replaced by their returned values, which are 7 and –1. Finally, the call

```
mul(7,-1)
```

[2]Code inlining is an important optimization technique for translators; indeed, C++ has an `inline` keyword to allow the programmer to specifically request inlining, if possible; see Chapter 6. Inlining of operators and nonrecursive functions with no nonlocal references is relatively easy, but in the most general case inlining may be difficult or impossible. For an example of the difficulties of inlining, see Exercise 11.14.

is made. Note that this process corresponds to a bottom-up "reduction" of the syntax tree to a value. First the + and - nodes are replaced by their values 7 and −1, and finally the root * node is replaced by the value −7, which is the result of the expression.

This raises the question of the order in which the subexpressions (3 + 4) and (5 – 6) are computed, or the order in which the calls `plus(3,4)` and `minus(5,6)` are made. A natural order is left to right, corresponding to a left-to-right traversal of the syntax tree. However, many languages explicitly state that there is no specified order for the evaluation of arguments to user-defined functions. The same is often true for predefined operators as well.

There are several reasons for this. One is that different machines may have different requirements for the structure of calls to procedures and functions. Another is that a translator may attempt to rearrange the order of computation so that it is more efficient. For example, consider the expression (3 + 4) * (3 + 4). A translator may discover that the same subexpression, namely, 3 + 4, is used twice and will evaluate it only once. And in evaluating a call such as `max(3,4+5)`, a translator may evaluate 4 + 5 before 3 because it is more efficient to evaluate more complicated expressions first.

If the evaluation of an expression causes no side effects, then the expression will yield the same result, regardless of the order of evaluation of its subexpressions. In the presence of side effects, however, the order of evaluation can make a difference. Consider, for example, the C program of Figure 9.3.

```
(1)  #include <stdio.h>

(2)  int x = 1;

(3)  int f(){
(4)     x += 1;
(5)     return x;
(6)  }

(7)  int p(int a, int b) {
(8)     return a + b;
(9)  }
(10) main(){
(11)    printf("%d\n",p(x,f()));
(12)    return 0;
(13) }
```

Figure 9.3 C Program showing evaluation order matters in the presence of side effects

If the arguments of the call to `p` on line 11 are evaluated left to right, this program will print 3. If the arguments are evaluated right to left, the program will print 4. The reason is that a call to the function `f` has a side effect: It changes the value of the global variable `x`.

In a language that explicitly states that the order of evaluation of expressions is unspecified, programs that depend on the order of evaluation for their results (such as the one above) are incorrect, even though they may have predictable behavior for one or more translators.

In spite of the problems side effects cause in expression evaluation, sometimes expressions are explicitly constructed with side effects in mind. For example, in C, assignment (a quintessential side effect) is an expression, not a statement: `x = y` not only assigns the value of `y` to `x` but also returns the copied value as its result.[3] Thus, assignments can be combined in expressions, so that

```
x = (y = z)
```

assigns the value of `z` to both `x` and `y`. Note that in languages with this kind of construction, assignment can be considered to be a binary operator similar to arithmetic operators. In that case its precedence is usually lower than other binary operators, and it is made right associative. Thus,

```
x = y = z + w
```

in C assigns the sum of `z` and `w` to both `x` and `y` (and returns that value as its result).

C and other expression languages also have a **sequence operator**, which allows several expressions to be combined into a single expression and evaluated sequentially. In C, the sequence operator is the comma operator, which has precedence lower than any other operator. Unlike most other operators in C, the order of evaluation is specified as left to right, and the value of the rightmost expression is the value returned by the entire expression. For example, given that `x` and `y` are integer variables with values 1 and 2, respectively, the C expression

```
x = (y+=1,x+=y,x+1)
```

returns the value 5 and leaves `x` with the value 5 and `y` with the value 3.

The evaluation of an expression can sometimes be performed even without the evaluation of all its subexpressions. An interesting case is that of the Boolean, or logical, expressions. For example, the Boolean expressions (in Ada or C++ syntax)

```
true or x
```

and

```
x or true
```

are true regardless of whether `x` is true or false. Similarly,

```
false and x
```

and

```
x and false
```

are clearly false regardless of the value of `x`. In a programming language, one can specify that Boolean expressions are to be evaluated in left-to-right order, up to the point where the truth value of the entire expression becomes known and then the evaluation stops. A programming language that has this rule is said to possess **short-circuit evaluation** of Boolean or logical expressions.

[3]When a type conversion occurs, the type of the copied value is that of `x`, not `y`.

Short-circuit evaluation has a number of benefits. One is that a test for the validity of an array index can be written in the same expression as a test of its subscripted value, as long as the range test occurs first:

```
if (i <= lastindex and a[i] >= x)  ... // C++ code
```

Without short-circuit evaluation, the test `i <= lastindex` will not prevent an error from occurring if `i > lastindex`, since the expression `a[i]` in the second part of the expression will still be computed. (This assumes, of course, that an out-of-range array subscript *will* produce an error.) Without short-circuit evaluation, the test must be written using nested `if`'s:

```
if (i <= lastindex) if (a[i] >= x) ...
```

Similarly, a test for a null pointer can be made in the same expression as a dereference of that pointer, using short-circuit evaluation:

```
if (p != 0 and p->data == x) ... // C++ code
```

Note that the order of the tests becomes important in short-circuit evaluation. Short-circuit evaluation will protect us from a runtime error if we write:

```
// ok:  y % x requires x != 0
if (x != 0 and y % x == 0) ...
```

but not if we write:

```
if (y % x == 0 and x != 0) ... // not ok!
```

Most languages require Boolean operators to be short-circuit. C/C++, Java, and ML all have short-circuit Boolean operators (but not Pascal!). In Ada, which offers both kinds, the keywords `and` and `or` are not short-circuit, but the operators (two keywords each) "`and then`" and "`or else`" *are* short-circuit. For example:

```
-- Ada short-circuit code:
if (i <= lastindex) and then (a(i) = x) then ...
```

While we are on the subject of Boolean operators, we note the unfortunate lack of agreement on the names of these operators. C and Java use the somewhat obscure notation `&&` for `and`, `||` for `or`, and `!` for `not`. C++ allows these same operators but also allows the more common keywords `and`, `or`, and `not`. ML's names for the binary logical operators are `andalso` and `orelse`, presumably to call attention to their short-circuit nature. (ML has another, completely separate use for the keyword `and`, and the keyword `or` does not exist in ML.) Other languages occasionally use imitations of the operations as they are written in mathematical logic. For example, `/\` (forward slash followed by backslash) is used for `and`, `\/` (the reverse of `/\`) is used for `or`, and ~ (tilde) is used for `not`.

Boolean expressions are not the only expressions whose subexpressions may not all be evaluated: Many languages have expressions that mimic control statements but return values; two of these are **if expressions** and **case expressions**.

An `if` (or `if-then-else`) operator is a **ternary operator**—that is, it has *three* operands. These operands are as follows: the conditional test expression (which is typically Boolean), the "then" expression (whose value is the result of the entire expression if the test expression is true), and the "else" expression (whose value is the result of the entire expression if the test expression is false). As a prefix function it would be written as:

```
if(test-exp,then-exp,else-exp)
```

but most languages use a **mix-fix** form that distributes parts of the syntax of the operator throughout the expression in traditional style. For example, in ML the if-expression appears as:

```
if e1 then e2 else e3
```

while in C the same expression appears as:

```
e1 ? e2 : e3
```

Note that an if-expression may not have an optional else-part (unlike an if-statement), since there would be no value to return if the test expression were to evaluate to false.

If-expressions *never* have all of their subexpressions evaluated. That is, `e1` (the test expression) is *always* evaluated first, but, based on that value, only one of `e2` and `e3` is ever evaluated. This is the only reason for the if-expression to exist at all.

ML and some other functional languages also have a **case expression**:

```
case e1 of
  a => e2 |
  b => e3 |
  c => e4 ;
```

which is more or less equivalent to a series of nested if-expressions: `if e1 = a then e2 else if e1 = b then e3...`. Case-expressions have some special properties, however, and were discussed in Chapter 3.

Interestingly, the short-circuit Boolean operations can be defined using if-expressions as follows:

```
e1 and e2 = if e1 then e2 else false (* short circuit and *)
e1 or e2 = if e1 then true else e2   (* short circuit or *)
```

Short-circuit Boolean and `if` operators are a special case of operators that **delay** evaluating their operands. The general situation is called **delayed evaluation**, or sometimes **nonstrict evaluation**. In the case of the short-circuit operators, `e1 and e2` and `e1 or e2`, both delay the evaluation of `e2` until `e1` is evaluated. Similarly, the `if` operator delays the evaluation of both `e2` and `e3` until `e1` is evaluated.

It is worth restating the fact that, in the absence of side effects (changes to variables in memory, input and output), the order of evaluation of expressions is immaterial to the final value of the expression.[4] In fact, in a language in which side effects do not exist or are carefully controlled, such as the pure

[4]As long as it actually *has* a value; see Chapter 8.

functional languages (Chapter 3), expressions in programs share an important property with mathematical expressions. We call this shared property the **substitution rule** (or sometimes **referential transparency**). According to this rule, any two expressions in a program that have the same value may be substituted for each other anywhere in the program. In other words, their values always remain equal, regardless of the context in which they are evaluated. For example, if `x` and `y` have the same value at one point in a referentially transparent program, then they *always* have the same value[5] and may be freely substituted for each other anywhere. (Note that this prohibits `x` and `y` from being variables in the programming language sense. Instead, they must be constants.)

Referential transparency allows for a very strong form of delayed evaluation to be used that has important theoretical and practical consequences. It is called **normal order evaluation**. A precise mathematical definition can be given in the context of the lambda calculus, as was explained briefly in Chapter 3. For our purposes in this section, normal order evaluation of an expression means that each operation (or function) begins its evaluation *before* its operands are evaluated, and each operand is evaluated only if it is needed for the calculation of the value of the operation.

As an example of normal order evaluation, consider the following functions (in C syntax):

```
int double(int x){
   return x + x;
}

int square(int x){
   return x * x;
}
```

Now consider the expression `square(double(2))`. Using normal order evaluation, the call to `square` is replaced by `double(2)*double(2)` without evaluating `double(2)`, and then `double(2)` is replaced by `2 + 2`, so that the following sequence of replacements occurs:

`square(double(2))` ⇒ `double(2)*double(2)` ⇒ `(2 + 2)*(2 + 2)`

Only after these replacements occur is the expression evaluated, typically in left-to-right order:

`(2 + 2)*(2 + 2)` ⇒ `4 * (2 + 2)` ⇒ `4 * 4` ⇒ `16`

Note how this differs from applicative order evaluation, which evaluates the call to `double` before the call to `square`, and would result in the following sequence of replacements (with evaluation intermingled):

`square(double(2))` ⇒ `square(2 + 2)` ⇒ `square(4)` ⇒ `4 * 4` ⇒ `16`

Normal order evaluation implements a kind of code inlining, in which the bodies of functions are substituted at their call sites *before* evaluation occurs; see footnote 2.

[5]As long as we are in the scope of a particular declaration of these variables. Obviously, different declarations may cause different values to be associated with these names.

In the absence of side effects, normal order evaluation does not change the semantics of a program. While it might seem inefficient (e.g., `2+2` gets evaluated twice instead of once in the previous example), it can be made efficient, and it has a number of good properties that make it useful, particularly for functional languages. As a simple example, if C used normal order evaluation, the C if-expression *`e1 ? e2 : e3`* can be written (for each data type) as an ordinary C function, instead of needing special syntax in the language:

```
int if_exp(int x, int y, int z){
   if (x) return y; else return z;
}
```

Here `y` and `z` are only evaluated if they are needed in the code of `if_exp`, which is the behavior that we want.

On the other hand, the presence of side effects can mean that normal order evaluation substantially changes the semantics of a program. Consider, for example, the C function:

```
int get_int(){
   int x;
   /* read an integer value into x from standard input */
   scanf("%d",&x);
   return x;
}
```

This function has a side effect (input).

Now consider what would happen to the expression `square(get_int())` using normal order evaluation: It would be expanded into `get_int()*get_int()`, and *two* integer values, rather than just one, would be read from standard input—a very different outcome from what we would expect.

Normal order evaluation appears as so-called **lazy evaluation** in the functional language Haskell, as was studied in detail in Chapter 3. Normal order evaluation appears also as a parameter passing technique for functions in Algol60, where it is known as **pass by name**; this will be studied briefly in the next chapter.

9.2 Conditional Statements and Guards

The most typical form of structured control is execution of a group of statements only under certain conditions. This involves making a Boolean, or logical, test before entering a sequence of statements. The well-known **if-statement** is the most common form of this construct. The various ways such a conditional can be introduced will be discussed shortly.

First, however, we want to describe a general form of conditional statement that encompasses all the various conditional constructs. This general form, known as the **guarded if** statement, was introduced by E. W. Dijkstra, and looks like this:

```
if B1 -> S1
|  B2 -> S2
|  B3 -> S3
       ...
|  Bn -> Sn
fi
```

The semantics of this statement are as follows: each `Bi`'s is a Boolean expression, called a **guard**, and each `Si`'s is a statement sequence. If one `Bi`'s evaluates to true, then the corresponding statement sequence `Si` is executed. If more than one `Bi`'s is true, then the one and only corresponding `Si`'s is selected for execution. If no `Bi` is true, then an error occurs.

There are several interesting features in this description. First, it does not say that the first `Bi` that evaluates to true is the one chosen. Thus, the guarded if introduces nondeterminism into programming, a feature that becomes very useful in concurrent programming (Chapter 13). Second, it leaves unspecified whether all the guards are evaluated. Thus, if the evaluation of a `Bi` has a side effect, the result of the execution of the guarded if may be unknown. Of course, the usual deterministic implementation of such a statement would sequentially evaluate the `Bi`'s until a true one is found, whence the corresponding `Si` is executed and control is transferred to the point following the guarded statement.

The two major ways that programming languages implement conditional statements like the guarded if are as if-statements and case-statements.

9.2.1 If-Statements

The basic form of the if-statement is as given in the extended Backus-Naur form (EBNF) rule for the if statement in C, with an optional "else" part:

if-statement → `if (` *expression* `)` *statement* [`else` *statement*]

where *statement* can be either a single statement or a sequence of statements surrounded by braces:

```
if (x > 0) y = 1/x;
else{
   x = 2;
   y = 1/z;
}
```

This form of the `if` (which also exists in Java and Pascal) is problematic, however, because it is ambiguous in the syntactic sense described in Chapter 6. Indeed, the statement

```
if (e1) if (e2) S1 else S2
```

has two different parse trees according to the BNF. These parse trees are shown in Figure 9.4.

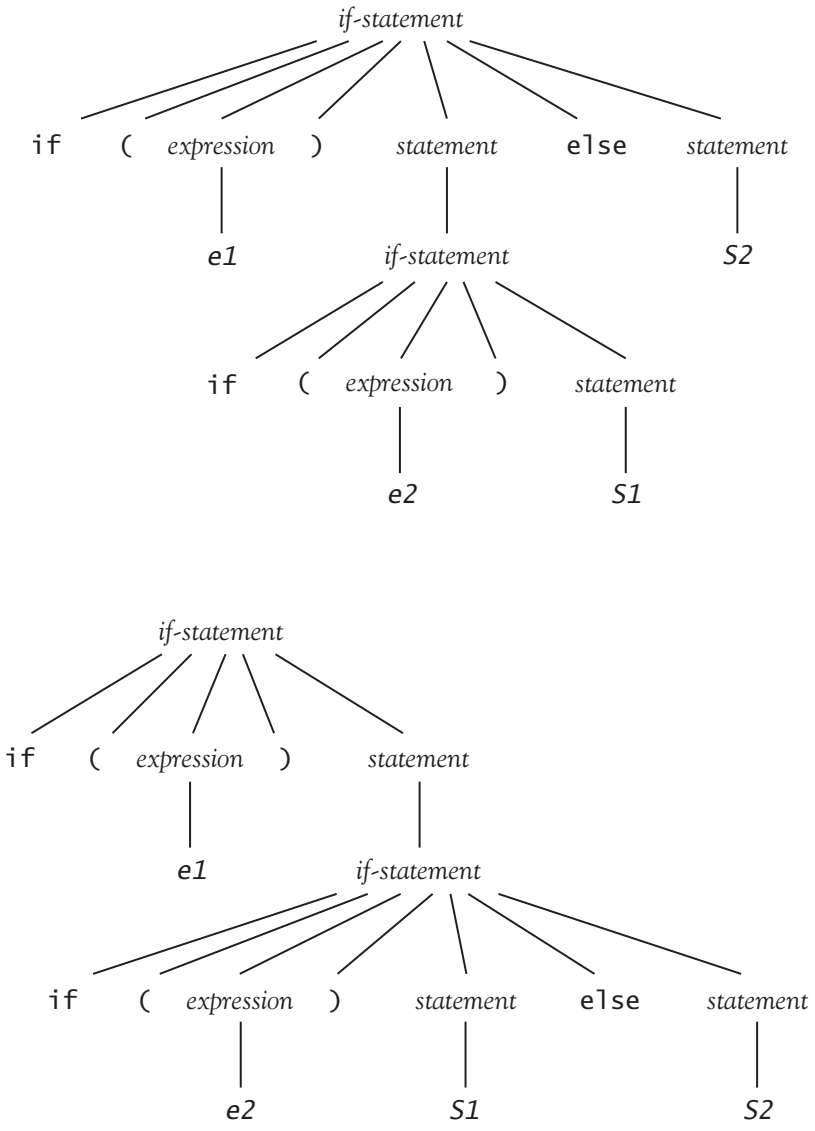


Figure 9.4 Two parse trees for the statement `if` (e1) `if` (e2) S1 `else` S2

This ambiguity is called the **dangling-else** problem. The syntax does not tell us whether an `else` after two if-statements should be associated with the first or second `if`. C and Pascal solve this problem by enforcing a **disambiguating rule**, which works as follows: The `else` is associated with the closest preceding `if` that does not already have an `else` part. This rule is also referred to as the **most closely nested rule** for if-statements. It states that the second parse tree is the correct one.

The fact that the dangling-else problem arises at all indicates an essential language design problem. This is true for two reasons. First, it makes us state a new rule to describe what is essentially a syntactic feature. Second, it makes the interpretation of the if-statement by the reader more difficult; that is, it

violates the readability design criterion. As an illustration, if we want actually to associate the `else` with the first `if` in the preceding statement, we would have to write either:

```
if ( e1 ) { if ( e2 ) S1 } else S2
```

or:

```
if ( e1 ) if ( e2 ) S1 else ; else S2
```

There are other ways to solve the dangling else than by using a disambiguating rule. In fact, it is possible to write out BNF rules that specify the association precisely, but these rules make the grammar somewhat more complex. (Java does this—see Exercises 9.28 and 9.29.) A better way is to use a **bracketing keyword** for the if-statement, such as the Ada rule:

if-statement → `if` *condition* `then` *sequence-of-statements*
`[else` *sequence-of-statements*`] end if ;`

The two bracketing keywords `end if` (together with the semicolon) close the if-statement and remove the ambiguity, since we must now write either:

```
if e1 then if e2 then S1 end if ; else S2 end if;
```

or:

```
if e1 then if e2 then S1 else S2 end if ; end if ;
```

to decide which `if` is associated with the `else` part. This also removes the necessity of using brackets (or a `begin - end` pair) to open a new sequence of statements. The if-statement can open its own sequence of statements and, thus, becomes fully structured:

```
if x > 0.0 then
   y := 1.0/x;
   done := true;
else
   x := 1.0;
   y := 1.0/z;
   done := false;
end if;
```

A similar approach was taken in the historical (but influential) language Algol68, which suggestively uses keywords spelled backward as bracketing keywords:

```
if e1 then S1 else S2 fi
```

Thus, the bracketing keyword for if-statements is `fi`. (One still sees this convention in theoretical language design publications—witness the guarded if of Dijkstra, described above.)

An extension of the if-statement simplifies things when there are many alternatives. Multiple `else`'s statements would have to be written in Ada as:

```
if e1 then S1
else if e2 then S2
else if e3 then S3
end if ; end if ; end if;
```

with many "`end if`"s piled up at the end to close all the if-statements. Instead, the "`else if`" is compacted into a new reserved word `elsif` that opens a new sequence of statements at the same level:

```
if e1 then S1
elsif e2 then S2
elsif e3 then S3
end if ;
```

An additional issue regarding the if-statement is the required type of controlling expression (written in the preceding examples variously as *`Bi`* or *`ei`*, indicating possibly Boolean or not). In Java and Ada (as well as Pascal), the test must always have the Boolean type. C has no Boolean type, and in C (as well as C++), the controlling expression can have either the integral type (see Figure 9.3) or the pointer type. The resulting value is then compared to 0 (the null pointer in the case of pointer type); unequal comparison is equivalent to true, equal comparison is equivalent to false. Thus, in C a test such as:

```
if (p != 0 && p->data == x) ...
```

can be written as:

```
if (p && p->data == x) ...
```

9.2.2 Case- and Switch-Statements

The case-statement (or switch-statement in C/C++/Java) was invented by C. A. R. Hoare as a special kind of guarded if, where the guards, instead of being Boolean expressions, are ordinal values that are selected by an ordinal expression. An example in C is given in Figure 9.5.

```
(1) switch (x - 1){
(2)    case 0:
(3)      y = 0;
(4)      z = 2;
(5)      break;
(6)    case 2:
(7)    case 3:
(8)    case 4:
(9)    case 5:
(10)     y = 3;
(11)     z = 1;
(12)     break;
```

Figure 9.5 An example of the use of the switch statement in C *(continues)*

(continued)

```
(13)   case 7:
(14)   case 9:
(15)     z = 10;
(16)     break;
(17)   default:
(18)     /* do nothing */
(19)     break;
(20) }
```

Figure 9.5 An example of the use of the switch statement in C

The semantics of this statement are to evaluate the controlling expression (`x - 1` on line 1 in Figure 9.5), and to transfer control to the point in the statement where the value is listed. These values must have integral types in C (and conversions occur, if necessary, to bring the types of the listed cases into the type of the controlling expression). No two listed cases may have the same value after conversion. Case values may be literals as in the above example, or they can be constant expressions as C defines them (that is, compile-time constant expressions with no identifiers; see Chapter 7). If the value of the controlling expression is not listed in a case label, then control is transferred to the `default` case (line 17 in Figure 9.5), if it exists. If not, control is transferred to the statement following the `switch` (that is, the switch statement **falls through**).

The structure of this statement in C has a number of somewhat novel features. First, the `case` labels are treated syntactically as ordinary labels, which allows for potentially bizarre placements; see Exercise 9.30. Second, without a `break` statement, execution falls through to the next case. This allows cases to be listed together without repeating code (such as the cases 2 through 5, lines 6–9 in Figure 9.5); it also results in incorrect execution if a `break` statement is left out.

Java has a `switch` statement that is virtually identical to that of C, except that it tightens up the positioning of the `case` labels to eliminate bizarre situations.

A somewhat more "standard" version of a case statement (from the historical point of view) is that of Ada. The C example of Figure 9.5 is written in Ada in Figure 9.6.

```
(1)  case x - 1 is
(2)    when 0 =>
(3)          y := 0;
(4)          z := 2;
(5)    when 2 .. 5 =>
(6)          y := 3;
(7)          z := 1;
(8)    when 7 | 9 =>
(9)          z := 10;
(10) when others =>
(11)   null;
(12) end case;
```

Figure 9.6 An example of the use of the case statement in Ada, corresponding to the C example of Figure 9.5

In Ada, case values can be grouped together, either by listing them individually, separated by vertical bars (as in line 8 of Figure 9.6), or by listing a range (such as 2 . . 5 in line 5 of Figure 9.6). Also, in Ada case values must not only be distinct but they must also be exhaustive. If a legal value is not listed and there is no default case (called `others` in Ada, as on line 10 of Figure 9.6), a compile-time error occurs (i.e., there is no fall through as in C). An implication of this is that the complete set of possible values for the controlling expression must be known at compile time. Thus, enumerations and small subranges are typically used to control case statements.

The functional language ML also has a `case` construct, but it is an expression that returns a value, rather than a statement (as is usual in functional languages). An example of a function in ML whose body is a case expression is given in Figure 9.7.

```
(1) fun casedemo x =
(2)   case x - 1 of
(3)     0 => 2 |
(4)     2 => 1 |
(5)     _ => 10
(6)   ;
```

Figure 9.7 An example of a case expression in ML

The syntax of a case expression in ML is similar to that of Ada, except that cases are separated by vertical bars (and there is no `when` keyword). Further, cases in an ML case expression are patterns to be matched, rather than ranges or lists of values. In Figure 9.7, the only patterns are the numbers 0, 2, and the **wildcard pattern** (the underscore in line 5 of Figure 9.7), which functions as a default case. For example, the function definition of Figure 9.7 would result in the following evaluations in ML:

```
- casedemo 1;
val it = 2 : int
- casedemo 9;
val it = 10 : int
```

ML permits the cases of a case expression to be not exhaustive, but generates a warning in that case (since no value can be returned if an unlisted case occurs, and a runtime error must occur):

```
val casedemo = fn : int -> int
- fun casedemo x =
=   case x - 1 of
=     0 => 2 |
=     2 => 1
=   ;
stdIn:18.3-20.11 Warning: match nonexhaustive
         0 => ...
         2 => ...

val casedemo = fn : int -> int
```

Pattern matching and the case expression in ML were discussed in Chapter 3.

9.3 Loops and Variations on WHILE

Loops and their use to perform repetitive operations, especially using arrays, have been one of the major features of computer programming since the beginning—computers were in a sense invented to make the task of performing repetitive operations easier and faster. A general form for a loop construct is given by the structure corresponding to Dijkstra's guarded if, namely, the **guarded do**:

```
do B1 - > S1
 | B2 - > S2
 | B3 - > S3
. . .
 | Bn -> Sn
od
```

This statement is repeated until all the *Bi*'s are false. At each step, one *Bi*'s is selected nondeterministically, and the corresponding *Si* is executed.

The most basic form of loop construct, which is essentially a guarded do with only one guard (thus eliminating the nondeterminism), is the while-loop of C/C++/Java,

```
while (e) S
```

or the while-loop of Ada,

```
while e loop S1 ... Sn end loop;
```

In these statements, *e* (the test expression, which must be Boolean in Ada and Java, but not C or C++) is evaluated first. If it is true (or nonzero), then the statement *S* (or statement sequence *S1 ... Sn*) is executed, then *e* is evaluated again, and so on. Note that if *e* is false (or zero) to begin with, then the code inside is never executed. Most languages have an alternative statement that ensures that the code of the loop is executed at least once. In C and Java, this is the `do` (or `do-while` statement):

```
do S while (e);
```

Of course, this is exactly equivalent to the following code,

```
S;
while (e) S
```

so the do-statement is "**syntactic sugar**" (a language construct that is completely expressible in terms of other constructs), but it does express a common situation.

The `while` and `do` constructs have the property that termination of the loop is explicitly specified only at the beginning (while-statement) or end (do-statement) of the loop. Often it is also convenient to exit from inside a loop at one or more intermediate points. For this reason, C (and Java) provide two options. The first option is a `break` statement, which can be used inside a loop (in addition to its use inside a `switch` statement) to exit the loop completely (and continue execution with the statement immediately following the loop). The second option is a `continue` statement that skips the remainder of the body of the loop, resuming execution with the next evaluation of the control expression. The `break` statement is called an `exit` statement in Ada. Ada has no `continue` statement.

Multiple termination points complicate the semantics of loops, so some languages (like Pascal) prohibit them. A typical idiom in C is to indicate the presence of internal termination points (which might otherwise be overlooked) by writing `1` (or `true` in C++ or Java) for the test in a while-loop:

```
while (1) /* always succeeds */
{ ...
  if ( ... ) break;
}
```

This same effect is achieved in Ada by simply omitting the `while` at the beginning of a loop:

```
loop S1... exit when e; ... Sn end loop;
```

A common special case of a looping construct is the **for-loop** of C/C++/Java:

```
for ( e1; e2; e3 ) S;
```

This is completely equivalent in C to:

```
e1;
while (e2)
{ S;
  e3;
}
```

The expression *e1* is the **initializer** of the for-loop, *e2* is the **test**, and *e3* is the **update**. Typically, a for-loop is used in situations where we want an index to run through a set of values from first to last, as in processing the elements of an array:

```
for (i = 0; i < size; i++)
       sum += a[i];
```

Note that the **index** variable `i` is declared outside of the for-loop. C++, Java, and the 1999 ISO C Standard offer the additional feature that a for-loop initializer may contain declarations, so that a loop index can be defined right in the loop:

```
for ( int i = 0; i < size; i++)
       sum += a[i];
```

The scope of such declarations is limited to the rest of the loop; it is as if the for-loop code were inside its own block:[6]

[6]Many C++ compilers as of this writing do not implement this correctly.

```
{  // for  loop in C++, Java, and 1999 ISO C
   e1; // may contain local declarations
   while (e2)
   {  S;
      e3;
   }
}
```

Many languages severely restrict the format of the for-loop (which, as noted, is just syntactic sugar in C), so that it can *only* be used for indexing situations. For example, in Ada the for-loop can *only* be written as a process controlled by a single local index variable, which is implicitly defined by the loop. Thus, the previous C for-loop would be written in Ada as:

```
for i in 0 .. size - 1 loop
      sum := sum + a(i);
end loop;
```

and `i` is implicitly defined and is local to the code of the loop. Even more, `i` is viewed as a *constant* within the body of the loop, so assignments to `i` are illegal (because this would disrupt the normal loop progression):

```
for i in 0 .. size - 1 loop
      sum := sum + a(i);
      if sum > 42 then i := size; -- illegal in Ada
      end if;
end loop;
```

This form of loop is often included in languages because it can be more effectively optimized than other loop constructs. For example, the control variable (and perhaps the final bound) can be put into registers, allowing extremely fast operations. Also, many processors have a single instruction that can increment a register, test it, and branch, so that the loop control and increment can occur in a single machine instruction. To gain this efficiency, however, many restrictions must be enforced, such as those for Ada mentioned above. Most of the restrictions involve the control variable `i`. Typical restrictions are the following:

- The value of `i` cannot be changed within the body of the loop.
- The value of `i` is undefined after the loop terminates.
- `i` must be of restricted type and may not be declared in certain ways, for example, as a parameter to a procedure, or as a record field, or perhaps it must even be a local variable.

Further questions about the behavior of loops include:

- Are the bounds evaluated only once? If so, then the bounds may not change after execution begins.

- If the lower bound is greater than the upper bound, is the loop executed at all? Most modern languages perform a bound test at the beginning rather than the end of the loop, so the loop behaves like a while-loop. Some older FORTRAN implementations, however, have loops that always execute at least once.
- Is the control variable value undefined even if an `exit` or `break` statement is used to leave the loop before termination? Some languages permit the value to be available on "abnormal termination," but not otherwise.
- What translator checks are performed on loop structures? Some language definitions do not insist that a translator catch assignments to the control variable, which can cause unpredictable results.

As you saw in Chapter 5, object-oriented languages provide a special type of object called an iterator for looping over the elements of a collection. At a minimum, an iterator includes methods for testing for the presence of an element in the sequence, moving to the next element, and accessing the element just visited. Such languages often use an iterator as the underlying mechanism of a `for-each` loop, which allows the programmer to access all of the elements in a collection with very little syntactic overhead. Figure 9.8 shows two functionally equivalent Java code segments that use an iterator to visit and print all the elements in a list of strings. The first one uses an iterator and its methods explicitly, whereas the second one uses the simpler `for` loop syntax.

```
Iterator iter<String> = list.iterator();
while (iter.hasNext())
   System.out.println(iter.next());

for (String s : list)
   System.out.println(s);
```

Figure 9.8 Two ways to use a Java iterator to process a list

9.4 The GOTO Controversy and Loop Exits

`Gotos` used to be the mainstay of a number of programming languages, such as Fortran77 and BASIC. For example, the Fortran77 code:

```
10 IF (A(I).EQ.0) GOTO 20
  ...
   I   = I + 1
   GOTO 10
20 CONTINUE
```

is the Fortran77 equivalent of the C:

```
while (a[i] != 0)  i++;
```

With the increasing use of structured control in the 1960s, as pioneered by Algol60, computer scientists began a debate about the usefulness of `goto`s, since they can so easily lead to unreadable **spaghetti code**, as in Figure 9.9.

```
   IF (X.GT.0) GOTO 10
   IF (X.LT.0) GOTO 20
   X = 1
   GOTO 30
10 X = X + 1
   GOTO 30
20 X = -X
   GOTO 10
30 CONTINUE
```

Figure 9.9 An example of spaghetti code in Fortran77

Then, in 1966, Böhm and Jacopini produced the theoretical result that `goto`s were in fact *completely unnecessary*, since *every* possible control structure could be expressed using structured control constructs (although of course with the possible addition of a significant number of extra tests). Finally, in 1968, the computer scientist E. W. Dijkstra published a famous note ("GOTO Statement Considered Harmful," Dijkstra [1968]) in which he forcefully argued that not only were `goto` statements unnecessary but that their use was damaging in many ways.

The `goto` statement is very close to actual machine code. As Dijkstra pointed out, its "unbridled" use can compromise even the most careful language design and lead to undecipherable programs. Dijkstra proposed that its use be severely controlled or even abolished. This unleashed one of the most persistent controversies in programming, which raged up until the late 1980s (and even occasionally causes outbursts to this day). At first, a large number of programmers simply refused to believe the theoretical result and considered programming without `goto`s to be impossible. However, the academic community began campaigning vigorously for Dijkstra's position, teaching `goto`-less programming whenever possible. Thanks to these efforts, along with the increasing influence of Pascal, which applies strong limits on the use of `goto`s, programmers gradually became used to the idea of only using `goto`s in rare situations in which the flow of control seems difficult to write in a natural way using standard loops and tests.

However, programmers continued to cling to the idea that the use of `goto`s *was* justified in certain cases, and that academicians were doing a disservice to the programming community by not teaching the "proper" use of `goto`s. This view was succinctly expressed in a letter in 1987 ("'Goto considered harmful' considered harmful," Rubin [1987]) which unleashed a new flurry of controversy over whether there was sufficient justification for the complete abolition of `goto` statements in high-level languages.

Nowadays, depending on the audience and point of view, this controversy may seem quaint. However, echoes of the goto controversy continue to be heard. For example, there is still some debate on the propriety of unstructured exits from loops. Proponents of structured exits argue that a loop should always have a single exit point, which is either at the beginning point of the loop (in the header of a `while` loop or a `for` loop) or at the end of the loop (in the footer of a `do-while` or `repeat-until` loop). In a language without a goto or similar construct, the syntax of high-level loop structures both supports and enforces this restriction. The opponents of this orthodoxy, typically programmers who use

C, C++, Java, or Python, argue that this restriction forces the programmer to code solutions to certain problems that are more complicated than they would be if unstructured exits were allowed. Two of these problems are the search of an array for a given element and the use of a sentinel-based loop to input and process data. To see the issues involved, we present Java code examples that solve each problem using the two competing approaches.

A typical search method expects an array of integers and a target integer as parameters. The method returns the index of the target integer if it's in the array, or −1 otherwise. Table 9.1 shows two versions of this method, using structured and unstructured exits from the loop.

Table 9.1 Search methods using structured and unstructured loop exits

Search Using Structured Loop Exit

```
int search(int array[], int target){
   boolean found = false;
   int index = 0;
   while (index < array.length && ! found)
      if (array[index] == target)
         found = true;
      else
         index++;
   if (found)
      return index;
   else
      return -1;
}
```

Search Using Unstructured Loop Exit

```
int search(int array[], int target){
   for (int index = 0; index < array.length;
    index++)
      if (array[index] == target)
         return index;
   return -1;
}
```

As you can see, the first method in Table 9.1 uses a `while` loop with two conditions, one of which tests for the end of the array. The other tests for the detection of the target item. This version of the method requires a separate Boolean flag, as well as a trailing `if-else` statement to return the appropriate value. The method adheres to the structured loop exit principle, but at the cost of some complexity in the code. The second version of the method, by contrast, uses an embedded `return` statement to exit a `for` loop immediately upon finding the target value. If the target is not found, the loop runs to its structured termination, and a default of −1 is returned below the loop. The unstructured loop exit, thus, eliminates the need for a Boolean flag and a trailing `if-else` statement.

A sentinel-based loop is often used in situations where a series of input values must be processed. Table 9.2 shows two versions of a Java method that accepts integers from a scanner object and processes them. The sentinel value to terminate the process is −1.

Table 9.2 Methods using structured and unstructured sentinel-based loop exits

Structured Loop Exit

```
void processInputs(Scanner s){
   int datum = s.nextInt();
   while (datum != -1){
      process(datum);
      datum = s.nextInt();
   }
}
```

Unstructured Loop Exit

```
void processInputs(Scanner s){
   while (true){
      int datum = s.nextInt();
      if (datum == -1)
         break;
      process(datum);
   }
}
```

The first method in Table 9.2 shows why this problem is often called the **loop and a half problem**. To maintain a structured loop exit, a separate priming input statement must read the first datum above the loop. Another input statement is run at the bottom of the loop before the datum is tested for the sentinel value once more. The second version of the method eliminates the redundant input statement by placing the single input statement within a `while (true)` loop. The input statement is immediately followed by an `if` statement that tests for the sentinel. A nested `break` statement exits the loop if the sentinel is reached. Although this code includes an extra `if` statement, there is now just a single input statement, and the loop control condition is as simple as it could be.

These examples will likely not end the GOTO debate; for an extended discussion, see Roberts [1995].

9.5 Exception Handling

So far, all the control mechanisms you have studied have been **explicit**. At the point where a transfer of control takes place, there is a syntactic indication of the transfer. For example, in a **while**-loop, the loop begins with the keyword `while`. In a procedure call, the called procedure with its arguments is named at the point of call. There are situations, however, where transfer of control is **implicit**, which means the transfer is set up at a different point in the program than that where the actual transfer takes place. At the point where the transfer actually occurs, there may be no syntactic indication that control will in fact be transferred.

Such a situation is **exception handling**, which is the control of error conditions or other unusual events during the execution of a program. Exception handling involves the declaration of both exceptions and exception handlers. An exception is any unexpected or infrequent event. When an exception occurs, it is said to be **raised** or **thrown**. Typical examples of exceptions include runtime errors, such as out-of-range array subscripts or division by zero. In interpreted languages, exceptions can also include static errors, such as syntax and type errors. (These errors are not exceptions for compiled languages, since a program containing them cannot be executed.) Exceptions need not be restricted to errors, however. An exception can be any unusual event, such as input failure or a timeout. An **exception handler** is a procedure or code sequence that is designed to be executed when a particular exception is raised and that is supposed to make it possible for normal execution to continue in some way. An exception handler is said to **handle** or **catch** an exception.

Exception handling was pioneered by the language PL/I in the 1960s and significantly advanced by CLU in the 1970s, with the major design questions eventually resolved in the 1980s and early 1990s. Today, virtually all major current languages, including C++, Java, Ada, Python, ML, and Common Lisp (but not C, Scheme, or Smalltalk) have built-in exception-handling mechanisms. Exception handling has, in particular, been integrated very well into object-oriented mechanisms in Python, Java, and C++, and into functional mechanisms in ML and Common Lisp. Also, languages that do not have built-in mechanisms sometimes have libraries available that provide them, or have other built-in ways of simulating them. Still, exception-handling techniques are often ignored when programming is taught and deserve more widespread use by working programmers.

In this chapter, we will use C++ as our main example for exception handling, with additional overviews of Ada and ML exceptions.

Exception handling is an attempt to imitate in a programming language the features of a hardware interrupt or error trap, in which the processor transfers control automatically to a location that is specified

in advance according to the kind of error or interrupt. It is reasonable to try to build such a feature into a language, since it is often unacceptable for a program to allow the underlying machine or operating system to take control. This usually means that the program is aborted, or "crashes." Programs that exhibit this behavior fail the test of **robustness**, which is part of the design criteria of security and reliability. A program must be able to recover from errors and continue execution.

Even in a language with a good exception mechanism, however, it is unreasonable to expect a program to be able to catch and handle every possible error that can occur. The reason is that too many possible failures can occur at too low a level—the failure of a hardware device, memory allocation problems, communication problems—all can lead to situations in which the underlying operating system may need to take drastic action to terminate a program, without the program being able to do anything about it. Such errors are sometimes referred to as **asynchronous** exceptions, since they can occur at any moment, not just in response to the execution of program code. By contrast, errors that a program can definitely catch are called **synchronous** exceptions and are exceptions that occur in direct response to actions by the program (such as trying to open a file, perform a particular calculation, or the like). User-defined exceptions can only be synchronous (since they must be raised directly in program code), but predefined or library exceptions may include a number of asynchronous exceptions, since the runtime environment for the language can cooperate with the operating system to allow a program to trap some asynchronous errors.

It is helpful in studying exception-handling mechanisms to recall how exceptions can be dealt with in a language without such facilities. Exception conditions in such languages have to be found before an error occurs, and this assumes it is possible to test for them in the language. One can then attempt to handle the error at the location where it occurs, as in the following C code:

```
if (y == 0)
                handleError("denominator in ratio is zero");
else
    ratio = x / y;
```

Or, if the error occurs in a procedure, one can either pass an error condition back to the caller, as in the C code

```
enum ErrorKind {OutOfInput, BadChar, Normal};
  ...
ErrorKind getNumber ( unsigned* result)
{ int ch = fgetc(input);
  if (ch == EOF) return OutOfInput;
  else if (! isdigit(ch)) return BadChar;
  /* continue to compute */
  ...
  *result = ... ;
  return Normal;
}
```

or, alternatively, one could pass an exception-handling procedure into the procedure as a parameter:

```
enum ErrorKind {OutOfInput, BadChar, Normal};
typedef void (*ErrorProc)(ErrorKind);
  ...
unsigned value;
  ...
void handler (ErrorKind error)
{
  ...
}
  ...
unsigned getNumber (ErrorProc handle)
{ unsigned result;
  int ch = fgetc(input);
  if (ch == EOF) handle(OutOfInput);
  else if (! isdigit(ch)) handle(BadChar);
  /* continue to compute */
  ...
  return result;
}
  ...
  ...
value = getNumber (handler);
```

Explicit exception testing makes a program more difficult to write, since the programmer must test in advance for all possible exceptional conditions. We would like to make this task easier by declaring exceptions in advance of their occurrence and specifying what a program is to do if an exception occurs. In designing such program behavior we must consider the following questions:

Exceptions. What exceptions are predefined in the language? Can they be disabled? Can user-defined exceptions be created? What is their scope?

Exception handlers. How are they defined? What is their scope? What default handlers are provided for predefined exceptions? Can they be replaced?

Control. How is control passed to a handler? Where does control pass after a handler is executed? What runtime environment remains after an error handler executes?

We will discuss each of these topics in turn with examples from C++, Ada, and ML.

9.5.1 Exceptions

Typically an exception occurrence in a program is represented by a data object that may be either predefined or user-defined. Not surprisingly, in a functional language this data object is a *value* of some type, while in a structured or object-oriented language it is a variable or object of some structured type. Typically, the *type* of an exception value or object is predefined, with exceptions themselves given in

a special declaration that creates a value or variable of the appropriate type. For example, in ML or Ada an exception is declared using the reserved word `exception`:

```
exception Trouble; (* a user-defined exception *)
exception Big_Trouble; (* another user-defined exception *)
```

Unusually, in C++ there is no special exception type, and thus no reserved word to declare them. Instead, any structured type (`struct` or `class`) may be used to represent an exception:[7]

```
struct Trouble {} trouble;
struct Big_Trouble {} big_trouble;
```

Now, these declarations as shown are minimal, since they only provide simple values or objects that represent the occurrence of an exception. Typically, we would want also some additional information to be attached to these exceptions, such as an error message (a string) and also perhaps a summary of the data or program state that led to the exception (such as a numeric value that led to an invalid computation, or an input character that was unexpected). It is easy to see how to do this in C++. We simply add data members to the defined data structure:

```
struct Trouble{
   string error_message;
   int wrong_value;
} trouble;
```

In ML this can also be done:

```
exception Trouble of string * int;
```

Unusually, in Ada this cannot be done directly. Exception objects are constants that contain no data.[8]

Exception declarations such as the above typically observe the same scope rules as other declarations in the language. Thus, in Ada, ML, and C++, lexical, or static, scope rules are observed for exception declarations. Since exceptions occur during runtime and, as you will see below, can cause execution to exit the scope of a particular exception declaration, it can happen that an exception cannot be referred to by name when it is handled. The handler's design must take this into account. Typically, one wishes to minimize the trouble that this causes by declaring user-defined exceptions globally in a program. Local exceptions may under certain circumstances make sense, however, to avoid creating a large number of superfluous global exceptions. Note also that, when data are included in an exception, we want to separate the exception *type* declaration from the exception *object/value* declaration, since in general different instances of the same exception type may exist simultaneously with different data, so the objects/values must be created as the exceptions occur. As long as the exception types themselves are global, no scope issues result, as you will see below. Thus, the C++ declaration above (which defines a variable as well as a type) should be amended to only declare a type:

[7]Since we do not discuss the object-oriented features of C++ until Chapter 11, we write C++ code that is as close to C as possible in this section, even though somewhat unidiomatic.

[8]Ada95 does add a library `Ada.Exceptions` that has mechanisms for passing information along with exceptions, but it is not as clean as adding data to the exception objects themselves.

```
struct Trouble{
   string error_message;
   int wrong_value;
} ; // declare exception object later
```

Most languages also provide some predefined exception values or types. For instance, in Ada there are the following predefined exceptions:

`Constraint_Error`: Caused by exceeding the bounds of a subrange or array; also caused by arithmetic overflow and division by zero.

`Program_Error`: This includes errors that occur during the dynamic processing of a declaration.

`Storage_Error`: This is caused by the failure of dynamic memory allocation.

`Tasking_Error`: This occurs during concurrency control.

In ML there are a number of similar predefined exceptions, such as `Div` (division by 0), `Overflow` (arithmetic), `Size` (memory allocation), and `Subscript` (array subscript out of bounds); additional predefined exceptions exist that have to do with specific program constructs in ML and that correspond roughly to Ada's `Program_Error`. There also is a predefined value constructor `Fail of string`, which allows a program to construct exception values containing an error message, without having to define a new exception type.

In C++, in keeping with usual practice, there are no predefined exception types or exceptions in the language proper. However, many standard library modules provide exceptions and exception mechanisms. Some of these standard exceptions are:

`bad_alloc`: Failure of call to `new`.

`bad_cast`: Failure of a `dynamic_cast` (see Chapter 5).

`out_of_range`: Caused by a checked subscript operation on library containers (the usual subscript operation is not checked).

`overflow_error`, `underflow_error`, `range_error`: Caused by math library functions and a few others.

We also mention below another standard exception, `bad_exception`, which can be useful in handling exceptions in C++.

9.5.2 Exception Handlers

In C++ exception handlers are associated with **try-catch** blocks, which can appear anywhere a statement can appear. A sketch of a C++ try-catch block appears in Figure 9.10.

```
(1)  try
(2)  { // to perform some processing
(3)   ...
(4)  }
(5)  catch (Trouble t)
(6)  { // handle the trouble, if possible
```

Figure 9.10 A C++ try-catch block *(continues)*

(continued)

```
(7)    displayMessage(t.error_message);
(8)    ...
(9)  }
(10) catch (Big_Trouble b)
(11) { // handle big trouble, if possible
(12)   ...
(13) }
(14) catch (...) // actually three dots here, not an ellipsis!
(15) { // handle any remaining uncaught exceptions
(16) }
```

Figure 9.10 A C++ try-catch block

Here the compound statement after the reserved word `try` is required (the curly brackets on lines 2 and 4 of Figure 9.10), and any number of `catch` blocks can follow the initial `try` block. Each `catch` block also requires a compound statement and is written as a function of a single parameter whose type is an exception type (which can be any class or structure type). The exception parameter may be consulted to retrieve appropriate exception information, as in line 7 of Figure 9.10. Note also in Figure 9.10 the catch-all catch block at the end that catches any remaining exceptions, as indicated by the three dots inside the parentheses on line 14.

The syntax of try-catch blocks in C++ is really overkill, since catch blocks could have been associated with any statement or block. Indeed, in Ada and ML that is exactly the case. For example, in Ada the reserved word `exception` is reused to introduce handlers. Handles can appear at the end of any block, as shown in Figure 9.11, which contains Ada code corresponding to the C++ code of Figure 9.10. Note also the similarity of the syntax of the `exception` list (lines 5–13 of Figure 9.11) to that of the Ada `case` statement.

```
(1)  begin
(2)    -- try to perform some processing
(3)    ...
(4)  exception
(5)    when Trouble =>
(6)      --handle trouble, if possible
(7)      displayMessage("trouble here!");
(8)      ...
(9)    when Big_Trouble =>
(10)     -- handle big trouble, if possible
(11)     ...
(12)   when others =>
(13)     -- handle any remaining uncaught exceptions
(14) end;
```

Figure 9.11 An Ada try-catch block corresponding to Figure 9.9

In ML, the reserved word `handle` can come at the end of any expression, and introduces the handlers for exceptions generated by that expression, as shown in Figure 9.12.

```
(1)  val try_to_stay_out_of_trouble =
(2)  (* try to compute some value *)
(3)  handle
(4)    Trouble (message,value) =>
(5)         ( displayMessage(message); ... ) |
(6)    Big_Trouble =>  ...  |
(7)    _ =>
(8)      (* handle any remaining uncaught exceptions *)
(9)      ...
(10) ;
```

Figure 9.12 An example of ML exception handling corresponding to Figures 9.10 and 9.11

Exceptions in ML are distinguished by pattern-matching/unification within the `handle` clause (see Chapters 3 and 8). The syntax is again similar to that of the `case` expression in ML (Section 9.2, Figure 9.7). In particular, the underscore character on line 7 of Figure 9.12 is again a wildcard pattern as in Figure 9.7 and has the same effect as the Ada `others` clause or the C++ `catch( ... )`. It will match any exception not previously handled.

The scope of handlers defined as above extends, of course, only to the statements/expression to which they are attached. If an exception reaches such a handler, it replaces whatever handlers may exist elsewhere, including predefined handlers.

Predefined handlers typically simply print a minimal error message, indicating the kind of exception, possibly along with some information on the program location where it occurred, and terminate the program. None of the three languages we are considering here require any additional behavior. Also, in Ada and ML there is no way to change the behavior of default handlers, except to write a handler as we have just indicated.

C++, on the other hand, offers a way to replace the default handler with a user-defined handler, using the `<exceptions>` standard library module. One does this by calling the `set_terminate` function, passing it an argument consisting of a `void` function taking no parameters. This function will then be called as the default handler for all exceptions not explicitly caught in the program code.

In Ada it is not possible to replace the default handling action with a different one, but it is possible to *disable* the default handler by telling the compiler not to include code to perform certain checks, such as bounds or range checking. This is done using the `suppress` pragma (compiler directive), as in:

```
pragma suppress(Overflow_Check, On => Float);
```

which turns off all overflow checking for floating-point numbers throughout this pragma's scope (such a pragma is viewed as a declaration with standard scope rules). This can mean that such an Ada program runs to completion and produces incorrect results without any error message being generated. Normally, this is precisely the kind of program behavior that Ada was designed to prevent. However, in those cases where maximum efficiency is needed and the programmers are certain of the impossibility of an exception occurring, this feature can be helpful.

9.5.3 Control

Finally, in this subsection we discuss how exceptions are reported and how control may be passed to a handler. Typically, a predefined or built-in exception is either automatically raised by the runtime system, or it can be manually raised by the program. User-defined exceptions can, of course, only be raised manually by the program.

C++ uses the reserved word `throw` and an exception object to raise an exception:

```
if (/* something goes wrong */)
{ Trouble t; // create a new Trouble var to hold info
  t.error_message = "Bad news!";
  t.wrong_value = current_item;
  throw t;
}
else if (/* something even worse happens */)
  throw big_trouble; // can use global var, since no info
```

Ada and ML both use the reserved word `raise`:

```
-- Ada code:
if -- something goes wrong
then
   raise Trouble; -- use Trouble as a constant
elsif -- something even worse happens
then
   raise Big_Trouble; -- use Big_Trouble as a constant
end if;

(* ML code: *)
if (* something goes wrong *)
then (* construct a Trouble value *)
   raise Trouble("Bad news!",current_item)
else if (* something even worse happens *)
   raise Big_Trouble (* Big_Trouble is a constant *)
else ... ;
```

When an exception is raised, typically the current computation is abandoned, and the runtime system begins a search for a handler. In Ada and C++, this search begins with the handler section of the block in which the exception was raised. If no handler is found, then the handler section of the next enclosing block is consulted, and so on (this process is called **propagating the exception**). If the runtime system reaches the outermost block of a function or procedure without finding a handler, then the call is exited to the caller, and the exception is again raised in the caller as though the call itself had raised the exception. This proceeds until either a handler is found, or the main program is exited, in which case the default handler is called. The process is similar in ML, with expressions and subexpressions replacing blocks.

The process of exiting back through function calls to the caller during the search for a handler is called **call unwinding** or **stack unwinding**, the stack being the call stack whose structure is described in detail in the next chapter.

Once a handler has been found and executed, there remains the question of where to continue execution. One choice is to return to the point at which the exception was first raised and begin execution again with that statement or expression. This is called the **resumption model** of exception handling, and it requires that, during the search for a handler and possible call unwinding, the original environment and call structure must be preserved and reestablished prior to the resumption of execution.

The alternative to the resumption model is to continue execution with the code immediately following the block or expression in which the handler that is executed was found. This is called the **termination model** of exception handling, since it essentially discards all the unwound calls and blocks until a handler is found.

Virtually all modern languages, including C++, ML, and Ada, use the termination model of exception handling. The reasons are not obvious, but experience has shown that termination is easier to implement and fits better into structured programming techniques, while resumption is rarely needed and can be sufficiently simulated by the termination model when it is needed.

Here is an example of how to simulate the resumption model using termination in C++. Suppose that a call to `new` failed because there was not enough free memory left, and we wished to call a garbage collector and then try again. We could write the code as follows:

```
while (true)
   try
   { x = new X; // try to allocate
     break; // if we get here, success!
   }
   catch (bad_alloc)
   { collect_garbage(); // can't exit yet!
     if ( /* still not enough memory */)
      // must give up to avoid infinite loop
      throw bad_alloc;
   }
```

Finally, we mention also that exception handling often carries substantial runtime overhead (see the next chapter for details). For that reason, and also because exceptions represent a not very structured control alternative, we want to avoid overusing exceptions to implement ordinary control situations, where simple tests would do the job. For example, given a binary search tree data structure (containing, e.g., integers), such as

```
struct Tree{
   int data;
   Tree * left;
   Tree * right;
};
```

we could write a search routine as in Figure 9.13.

```
void fnd (Tree* p, int i) // helper procedure
{ if (p != 0)
     if (i == p->data) throw p;
     else if (i <p->data) fnd(p->left,i);
     else fnd(p->right,i);
}

Tree * find (Tree* p, int i)
{  try
   { fnd(p,i);
   }
   catch(Tree* q)
   {  return q;
   }
   return 0;
}
```

Figure 9.13 A binary tree `find` function in C++ using exceptions (adapted from Stroustrup [1997], pp. 374–375)

However, this code is likely to run much more slowly than a more standard search routine that does not use exceptions. Also, more standard code is probably easier to understand.

9.6 Case Study: Computing the Values of Static Expressions in TinyAda

In languages such as Pascal and Ada, a symbolic constant has a value that can be determined at compile time. Pascal requires its symbolic constants to be defined as literals. Ada is more flexible, allowing the value of any expression not including a variable or a function call to serve as the value of a symbolic constant. Such expressions are called **static expressions**, to distinguish them from other types of expressions whose values cannot be known until run time. In this section, we examine how a parser for TinyAda can compute the values of static expressions.

9.6.1 The Syntax and Semantics of Static Expressions

Static expressions can appear in two types of TinyAda declarations. The first, a number declaration, defines a symbolic constant. The second, a range type definition, defines a new subrange type. The next code segment, which defines a type for rectangular matrices whose widths are twice that of their heights, shows these two syntactic forms in action:

```
ROW_MAX : constant := 10;
COLUMN_MAX : constant := ROW_MAX * 2;
type MATRIX_TYPE is range 1..ROW_MAX, range 1..COLUMN_MAX of BOOLEAN;
```

Note how the use of the expression ROW_MAX * 2 in the definition of COLUMN_MAX maintains a 2:1 aspect ratio of the matrix dimensions, allowing the programmer to vary their size by changing just the value of ROW_MAX.

The following grammar rules for the two types of declarations show the placements of the static expressions in them:

```
numberDeclaration = identifierList ":" "constant" ":=" <static>expression ";"

range = "range" <static>simpleExpression ".." <static>simpleExpression
```

Syntactically, static expressions look just like other expressions. Their semantics are also similar, in that the applications of the arithmetic, comparison, and logical operators produce the expected results. However, to ensure that these results can be computed at compile time, static expressions cannot include variable or parameter references.

In TinyAda, the values of static expressions ultimately come from literals in the token stream, or from the values of constant names, including the two built-in symbolic constants, TRUE and FALSE. The two types of literals include integer and character values. The scanner translates the different source code representations of these to a single internal form—base 10 integers for all of the TinyAda integer representations and ASCII values for the 128 character literals. The internal forms of the two Boolean values are, by convention, the integers 0 for FALSE and 1 for TRUE. The parser uses these internal representations to evaluate all static expressions, as you will see shortly.

9.6.2 Entering the Values of Symbolic Constants

Each symbolic constant now has a value attribute in its symbol entry record. The method setValue installs this value, which must be an integer, for a list of constant identifiers. For example, when symbol entries for the built-in names TRUE and FALSE are initialized, their values must be entered as well:

```
private void initTable(){
   ...
   entry = table.enterSymbol("TRUE");
   entry.setRole(SymbolEntry.CONST);
   entry.setType(BOOL_TYPE);
   entry.setValue(1);                    // TRUE = 1, internally
   entry = table.enterSymbol("FALSE");
   entry.setRole(SymbolEntry.CONST);
   entry.setType(BOOL_TYPE);
   entry.setValue(0);                    // FALSE = 0, internally
}
```

The method numberOrObjectDeclaration must also install a value for the list of constant identifiers just processed. Working top-down, we assume that the method that parses the TinyAda expression

on the right side of the := symbol also computes this value and returns it. However, there are three problems with simply reusing the current parsing method expression:

1. All expression methods already return a type descriptor, which is still needed for type checking and to set the type attributes of the constant identifiers and the subrange types.
2. These methods permit, through the current chain of calls to lower-level methods, the presence of variable and parameter names, as well as array indexed components.
3. Not all expressions are static, so some control mechanism must be provided to activate the computation of values when needed.

The first problem is easily solved by having each expression method return a symbol entry instead of a type descriptor. When the expression is static, the entry object can contain both the value and the type of the current expression. Each of these attributes is then available for use in further semantic processing.

The other two problems could be solved by passing a Boolean flag as a parameter to each expression method, to control the decisions about whether to compute a value and whether to accept only constant names. However, the additional logic and interface would complicate the implementation. Therefore, we adopt, at the cost of some redundant code, a strategy of defining a second set of methods that parse only static expressions.

Here is the code for numberOrObjectDeclaration, which shows a call of the new top-level method staticExpression:

```
/*
objectDeclaration = identifierList ":" typeDefinition ";"

numberDeclaration = identifierList ":" "constant" ":=" <static>expression ";"
*/
private void numberOrObjectDeclaration(){
   SymbolEntry list = identifierList();
   accept(Token.COLON, "':' expected");
   if (token.code == Token.CONST){
      list.setRole(SymbolEntry.CONST);
      token = scanner.nextToken();
      accept(Token.GETS, "':=' expected");
      SymbolEntry entry = staticExpression();  // Get the entry here and
      list.setType(entry.type);                // then extract the type
      list.setValue(entry.value);              // and the value.
   }
   else{
      list.setRole(SymbolEntry.VAR);
      list.setType(typeDefinition());
   }
   accept(Token.SEMI, "semicolon expected");
}
```

The method `range` receives the entries returned by the processing of two static simple expressions. The values in these entries must be installed as the lower and upper bounds of a new subrange type descriptor. The `range` method performs type checking on the simple expressions, as before, and now can check to see that the lower bound is less than or equal to the upper bound. The changes to the code for the method `range` are left as an exercise for the reader.

9.6.3 Looking Up the Values of Static Expressions

The value of the simplest form of a static expression is encountered in the new method `staticPrimary`. This value will be an integer or character literal in the token stream or the value of a constant identifier. The structure of this method is similar to that of its nonstatic counterpart, with two exceptions. First, the new method returns a symbol entry. Second, the new method simply looks up an identifier rather than calling the method `name`. Here is the code for the method `staticPrimary`:

```
/*
primary = numericLiteral | name | "(" expression ")"
*/
SymbolEntry staticPrimary(){
   SymbolEntry entry = new SymbolEntry();
   switch (token.code){
      case Token.INT:
         entry.type = INT_TYPE;
         entry.value = token.integer;
         token = scanner.nextToken();
         break;
      case Token.CHAR:
         entry.type = CHAR_TYPE;
         entry.value = token.integer;
         token = scanner.nextToken();
         break;
      case Token.ID:
         entry = findId();
         acceptRole(entry, SymbolEntry.CONST, "constant name expected");
         break;
      case Token.L_PAR:
         token = scanner.nextToken();
         entry = staticExpression();
         accept(Token.R_PAR, "')' expected");
         break;
      default: fatalError("error in primary");
   }
   return entry;
}
```

Note that the method must create a new symbol entry object to contain the value and type information of a character literal or an integer literal. Otherwise, the entry is either that of the identifier or the one returned by the nested call to `staticExpression`.

9.6.4 Computing the Values of Static Expressions

The other static expression methods also pass a symbol entry object back to their callers. If no operators are encountered, this entry comes from the processing of a single subexpression. Otherwise, we must deal with two or more symbol entries, each of which is the result of parsing an operand expression. The types of these entries are checked, as before. The current operator is then applied to the values of each pair of entries to compute a new value. This value, suitably wrapped in a new symbol entry object, is then either returned or used in further computations.

As an example of this processing, we show the code for the method `staticTerm`, which computes and returns the value of one or more factors separated by multiplying operators. Here is the code, followed by an explanation:

```
/*
term = factor { multiplyingOperator factor }
*/
private SymbolEntry staticTerm(){
   SymbolEntry s1 = staticFactor();
   if (multiplyingOperator.contains(token.code))
      matchTypes(s1.type, INT_TYPE, "integer expression expected");
   while (multiplyingOperator.contains(token.code)){
      Token op = token;
      token = scanner.nextToken();
      SymbolEntry s2 = staticFactor();
      matchTypes(s2.type, INT_TYPE, "integer expression expected");
      s1 = computeValue(s1.value, s2.value, op);
   }
   return s1;
}
```

As before, this method checks the types of the operands only if it sees an operator. Now, however, those types are buried in symbol entries. Moreover, the method must remember the particular operator token just seen, using the temporary variable `op`, so that it can be used to compute the term's value. Finally, the helper method `computeValue` is passed the values of the two operands and the operator as parameters. This method computes the new value, installs it in a new symbol entry object, and returns this object to the `term` method for further processing. The definitions of the `computeValue` method and the remaining methods for parsing static expressions are left as exercises for the reader.

Exercises

9.1 Rewrite the following infix expression in prefix and postfix form and draw the syntax tree:

```
(3 - 4) / 5 + 6 * 7
```

9.2 Write a BNF description of **(a)** postfix arithmetic expressions and **(b)** prefix arithmetic expressions.

9.3 Modify the recursive descent calculator of Figure 6.12 to translate infix expressions to postfix expressions.

9.4 In LISP the following unparenthesized prefix expression is ambiguous:

```
+ 3 * 4 5 6
```

Why? Give two possible parenthesized interpretations.

9.5 Examples were given in the text that show the usefulness of a short-circuit `and` operation. Give an example to show the usefulness of a short-circuit `or` operation.

9.6 Write a program to prove that short-circuit evaluation is used for the logical operators of C/C++.

9.7 Write a program to determine the order of evaluation of function arguments for your C/C++/Ada compiler.

9.8 Java specifies that all expressions, including arguments to calls, are evaluated from left to right. Why does Java do this, while C/C++ does not?

9.9 We noted in Chapter 5 that the + operator is left associative, so that in the expression $a + b + c$, the expression $a + b$ is computed and then added to c. Yet we stated in Section 9.1 that an expression $a + b$ may be computed by computing b before a. Is there a contradiction here? Does your answer also apply to the subtraction operator? Explain.

9.10 Suppose that we were to try to write a short-circuit version of `and` in C as the following function:

```
int and (int a, int b){
return a ? b : 0
}
```

(a) Why doesn't this work?

(b) Would it work if normal order evaluation were used? Why?

9.11 Describe one benefit of normal order evaluation. Describe one drawback.

9.12 Consider the expressions

```
(x != 0) and (y % x == 0)
```

and

```
(y % x == 0) and (x != 0)
```

In theory, both these expressions could have the value `false` if `x == 0`.

(a) Which of these expressions has a value in C?

(b) Which of these expressions has a value in Ada?

(c) Would it be possible for a programming language to require that both have values? Explain.

9.13 Describe the difference between the C/C++ operators `&&` and `||` and the operators `&` and `|`. Are these latter operators short-circuit? Why or why not?

9.14 Given the two functions (in C syntax):

```
int cube(int x){ return x*x*x; }
int sum(int x, int y, int z){ return x + y + z; }
```

describe the process of normal order evaluation of the expression `sum(cube(2),cube(3),cube(4))`, and compare it to applicative order evaluation. In particular, how many additions and multiplications are performed using each method?

9.15 A problem exists in normal order evaluation of recursive functions. Describe the problem using as an example the recursive factorial function

```
int factorial(int n)
{ if (n == 0) return 1; else return n * factorial ( n – 1 ); }
```

Propose a possible solution to the problem, and illustrate it with the above function, using the call `factorial(3)`.

9.16 Describe the process of evaluating the expression `factorial(5)` using normal order. When are the subtractions (such as 5 - 1) performed? When are the multiplications (such as 5 * 24) performed?

9.17 C insists that a sequence of statements be surrounded by braces `{ ... }` in structured statements such as while-statements:

while-stmt → `while (` *expression* `)` *statement*
statement → *compound-statement* | `...`
compound-statement → `{` [*declaration-list*] [*statement-list*] `}`
statement-list → *statement-list statement* | *statement*

Suppose that we eliminated the braces in compound statements and wrote the grammar as follows:

while-stmt → `while (` *expression* `)` *statement*
statement → *compound-statement* | `...`
compound-statement → [*declaration-list*] [*statement-list*]
statement-list → *statement-list statement* | *statement*

Show that this grammar is ambiguous. What can be done to correct it without going back to the C convention?

9.18 The `default` case in a `switch` statement in C/C++ corresponds in a way to the else-part of an if-statement. Is there a corresponding "dangling-default" ambiguity similar to the dangling-else ambiguity in C/C++? Explain.

9.19 Since the `default` case in a `switch` statement in C/C++ is similar to the else-part of an if-statement, why not use the reserved word `else` instead of the new keyword `default`? What design principles apply here?

9.20 Show how to imitate a while-statement in C/C++ with a do-statement.

9.21 Show how to imitate a for-statement in C/C++ with a do-statement.

9.22 Show how to imitate a while-statement in C/C++ with a for-statement.

9.23 Show how to imitate a do-statement in C/C++ with a for-statement.

9.24 You saw in this chapter (and in Exercise 8.14) that it makes sense to have an if-expression in a language.

(a) Does it make sense to have a while-expression; that is, can a while-expression return a value, and if so, what value?

(b) Does it make sense to have a case-or switch-expression; that is, can a case-expression return a value, and if so, what value?

(c) Does it make sense to have a do-or repeat-expression; that is, can a do-expression return a value, and if so, what value?

9.25 We noted in the text that, in a language that uses normal order evaluation, an if-expression can be written as an ordinary function. Can a while-expression be written in such a language? Explain.

9.26 Give examples of situations in which it would be appropriate to use a `break` statement and a `return` statement to exit loops.

9.27 An important difference between the semantics of the case-statement in C/C++ and Ada is that an unlisted case causes a runtime error in Ada, while in C/C++, execution "falls through" to the statement following the case-statement. Compare these two conventions with respect to the design principles of Chapter 2. Which principles apply?

9.28 (Aho, Sethi, and Ullman [1986]) The problem with the dangling else in C/C++ can be fixed by writing more complicated syntax rules. One attempt to do so might be as follows:

statement → `if (` *expression* `)` *statement* | *matched-statement*
matched-statement →
`if (` *expression* `)` *matched-statement* `else` *statement* | `...`

(a) Show that this grammar is still ambiguous.

(b) How can it be fixed so that the grammar expresses the most closely nested rule unambiguously?

9.29 Locate a grammar for the Java language, and describe how it fixes the dangling-else ambiguity. How many extra grammar rules does it use to do this?

9.30 Here are three possible switch-statements in C/C++/Java syntax:

```
int x = 2;
switch (x) x++;

int x = 2;
switch (x)
{ x ++; }

int x = 2;
switch (x)
{ case 1: if (x > 2)
  case 2:   x++;
  default: break;
}
```

(a) Which of these are legal in C/C++? For those that are legal, what is the value of x after each is executed?

(b) Which of these are legal in Java? For those that are legal, what is the value of x after each is executed?

9.31 Here is a legal switch-statement in C/C++:

```
int n = ...;
int q = (n + 3) / 4;
switch (n % 4)
{ case 0:  do {  n++;
  case 3:    n++;
  case 2:    n++;
  case 1:    n++;
 } while (--q > 0);
}
```

Suppose that n has the value **(a)** 0; **(b)** 1; **(c)** 5; **(d)** 8. What is the value of n in each case after the execution of the above switch-statement? Can you state a general rule for the value of n after this code executes? If so, state it. If not, explain why not.

9.32 Describe the effect of the FORTRAN spaghetti code of Figure 9.9, Section 9.4. Write C/C++ statements without GOTOs that are equivalent to the FORTRAN code.

9.33 Clark [1973] humorously describes a "come from" statement as a replacement for the GOTO statement:

```
10 J = 1
11 COME FROM 20
12 PRINT *, J
   STOP
13 COME FROM 10
20 J = J + 2
```

This sample program in FORTRAN syntax prints 3 and stops. Develop a description of the semantics of the COME FROM statement. What problems might a translator have generating code for such a statement?

9.34 C++ and Java loops are often written with empty bodies by placing all side effects into the tests, such as in the following two examples:

```
i = 0;
while (a[i++] != 0);


for (i = 0; a[i] != 0; i++);
```

(a) Are these loops equivalent?

(b) Are there any advantages or disadvantages to using this style of code? Explain.

9.35 Test each of the following C/C++ code fragments with a translator of your choice (or rewrite it into another language and test it) and explain its behavior:
(a)

```
int i, n = 3;
for (i = 1; i <= n; i++)
{ printf ("i = %d\n", i);
  n = 2;
}
```

(b)

```
int i;
for (i = 1; i <= 3; i++)
{ printf ("i = %d\n", i);
  i = 3;
}
```

9.36 Give an example in C++ or Ada of an exception that is propagated out of its scope.

9.37 In Section 9.5, three alternatives to simulating exceptions were discussed: (1) testing for an error condition before the error occurs; (2) passing a flag (an enumerated type) back as well as a value to indicate success or various failure outcomes; or (3) passing an error-handling function as a parameter. A fourth alternative is a variation on (2), where instead of passing two values back, one of which is a flag, we indicate an error by using an actual value, such as -1 or 999 that is not a legal value for the computation but is still a legal value for the result type. Discuss the advantages and disadvantages of this method compared to the use of an enumerated flag. What design principles from Chapter 2 apply?

9.38 Suppose that in a C++ `try` block you had some code that you wanted to execute on exit, regardless of whether an exception occurred or not. Where would you have to place this code? What if the code needed to be executed only if an exception did *not* occur?

9.39 **(a)** Rewrite the binary search tree `find` function of Figure 9.13, Section 9.5.3 to eliminate the use of an exception.
(b) Compare the running times of your version of `find` in part (a) with the version in the text.

9.40 Suppose that we wrote the following `try` block in C++:

```
try
{ // do something
}
catch (...) { cout << "general error!\n"; }
catch (range_error) { cout << "range error\n"; }
```

What is wrong with this code?

9.41 Install the values for the built-in constants `TRUE` and `FALSE` in the TinyAda Parser. You should then see these values displayed in the TinyAda program listing when you run the parser.

9.42 Add the helper method `computeValue` to the TinyAda parser. This method's signature is

```
SymbolEntry computeValue(int v1, int v2, Token op)
```

The method should apply the operator to the two values to compute a new value, and then create and return a new symbol entry object that contains this value and the role of constant. Remember to provide computations for all of the arithmetic, comparison, and logical operators. In the case of the unary operators, you should assume that the first value is 0 and the second value is the actual operand. Remember also that Boolean operands use the representations 0 for `FALSE` and 1 for `TRUE`.

9.43 Add the definitions of the methods for parsing static expressions to the TinyAda parser, then modify the definition of the method `numberOrObjectDeclaration` as shown in Section 9.6. Now you can test the parser with some TinyAda programs that use static expressions in number declarations.

9.44 Modify the method `range` in the TinyAda parser as described in Section 9.6, and test the parser with some TinyAda programs that define new subrange types.

Notes and References

Normal order evaluation and applicative order evaluation are described in Abelson and Sussman [1996] and Paulson [1996]. You will see normal order evaluation again in Chapter 10 in the pass by name parameter passing mechanism of Algol60, and you also saw it in Chapter 3 when discussing delayed evaluation and the lambda calculus.

Dijkstra's guarded `if` and guarded `do` commands are described in Dijkstra [1975]. Hoare describes his design of the case-statement in Hoare [1981]. The unusual use of a C switch-statement in Exercise 9.31 is called Duff's device in Gosling et al. [2000, page 289]. The dangling-else ambiguity is discussed in Louden [1997], Aho, Sethi, and Ullman [1986], and Gosling et al. [2000]; these references describe a number of different approaches to the problem.

Dijkstra's famous letter on the `GOTO` statement appears in Dijkstra [1968a]. A letter by Rubin [1987] rekindled the argument, which continued in the letters of the *Communications of the ACM* throughout most of 1987. Dijkstra's letter, however, was not the earliest mention of the problems of the `GOTO` statement. Naur [1963b] also comments on their drawbacks. For the disciplined use of `GOTO`s, see Knuth [1974]. For an amusing takeoff on the `GOTO` controversy, see Clark [1973] and Exercise 9.33. Roberts's discussion of nonstructured exits from loops and subroutines is in Roberts [1995].

An early seminal paper on exception handling is Goodenough [1975]. Exception handling in CLU, which also was a major influence on modern-day methods, is described and comparisons with other languages are given in Liskov and Snyder [1979]. See also Liskov et al. [1984]. Exception handling in Ada is discussed in Cohen [1996] and in Luckam and Polak [1980]. Exception handling in C++ is described in Stroustrup [1997]; Stroustrup [1994] gives a detailed view of the choices made in designing the exception handling of C++, including an extensive history of termination versus resumption semantics. Exception handling in ML is described in Ullman [1998], Paulson [1996], Milner and Tofte [1991], and Milner et al. [1997]. A control structure related to exceptions is the **continuation** of the Scheme dialect of LISP. For a description of this mechanism see Springer and Friedman [1989], and Friedman, Haynes, and Kohlbecker [1985].

CHAPTER

Control II—Procedures and Environments

CHAPTER 10

In the previous chapter, we discussed statement-level control structures and simple structured control mechanisms. In this chapter, we extend the study of blocks to the study of procedures and functions, which are blocks whose execution is deferred and whose interfaces are clearly specified.

Many languages make strong syntactic distinctions between procedures and functions. A case can be made for a significant semantic distinction as well. According to this argument, functions should (in an ideal world) produce a value only and have no side effects (thus sharing semantic properties with mathematical functions), while procedures produce no values and operate by producing side effects. Indeed, the distinction between procedures and functions is in essence the same as the difference between expressions and statements as discussed in the previous chapter. That is, procedure calls are statements, while function calls are expressions. However, most languages do not enforce semantic distinctions. Furthermore, functions can produce side effects as well as return values, whereas procedures may be written as functions in disguise—producing values through their parameters, while causing no other side effects. Thus, we shall not make a significant distinction between procedures and functions in this chapter, since in most languages their semantic properties are similar, even when their syntax is not.[1]

Procedures were first introduced when memory was scarce as a way of splitting a program into small, separately compiled pieces. It is not uncommon, even today, to see FORTRAN or C code in which every procedure is compiled separately. However, in modern use, separate compilation is more closely associated with modules (see the next chapter), while procedures are used as a mechanism to abstract a group of related operations into a single operation that can be used repeatedly without repeating the code. Procedures can also represent recursive processes that are not as easily represented by other control structures. They can imitate or replace loop operations (as noted in Chapter 1, and of particular interest in functional languages).

While virtually all programming languages have some notion of procedure or function, the generality of procedures varies tremendously from language to language. This generality has a major impact on the structure and complexity of the runtime environment needed for proper execution of programs. Fortran77 has a relatively simple notion of procedure as a static entity without recursion. This led directly to the notion of a static environment in which all memory allocation is performed prior to the start of execution. The idea of a recursive, dynamic procedure, pioneered in Algol60, resulted in the stack-based environments common in most imperative and object-oriented languages today. As we saw in Chapters 1 and 3, LISP and other functional languages generalized the notion of function to the point that functions

[1]C/C++ make virtually no syntactic difference between expressions and statements: Any expression can be turned into a statement by tacking on a semicolon (a so-called **expression statement**); the value of the expression is then simply thrown away.

are first-class data objects themselves—they can be dynamically created and used as values just like any other data structure. This led to even more dynamic environments that are not restricted to a stacklike structure. They also require dynamic memory management, including garbage collection. On the other hand, the basic notion of an **activation record** as the collection of data needed to maintain a single execution of a procedure, is common to virtually all runtime environments.

We begin this chapter with a review of the various syntactic ways of defining and calling procedures and functions. We continue with an investigation into the basic semantic properties of procedures, which represent a more complex notion of block structure than the one discussed in the previous chapter. We then study the major parameter-passing techniques, and the semantic behavior they generate. Next, we outline the structure of activation records and the three basic varieties of runtime environments, including mechanisms for implementing exception handling. We conclude with an overview of methods for maintaining dynamically allocated environments with garbage collection.

10.1 Procedure Definition and Activation

A **procedure** is a mechanism in a programming language for abstracting a group of actions or computations. The group of actions is called the **body** of the procedure, with the body of the procedure represented as a whole by the name of the procedure. A procedure is defined by providing a **specification** or **interface** and a body. The specification gives the name of the procedure, a list of the types and names of its **formal parameters**, and the type of its returned value, if any:

```
// C++ code
void intswap (int& x, int& y){ // specification
   int t = x;   // body
   x = y;       // body
   y = t;       // body
}
```

Procedure `intswap` swaps the values of its parameters `x` and `y`, using a local variable `t`.

In some languages, and in some situations, a procedure specification can be separated from its body, when the specification must be available in advance:

```
void intswap (int&, int&); // specification only
```

Note that this specification does not require the names of the formal parameters to be specified.[2] In C++ such a specification is called (confusingly) a *declaration*, while the complete definition (including the body) is called a *definition*. (In C, declarations are called *prototypes*.) Typically, even when a specification precedes a definition, it must be repeated with the body.

You **call**, or **activate**, a procedure by stating its name, together with **arguments** to the call, which correspond to its formal parameters:

```
intswap (a, b);
```

[2]They can of course be supplied if the programmer wishes, but they need not agree with names in other specifications or definitions.

A call to a procedure transfers control to the beginning of the body of the called procedure (the **callee**). When execution reaches the end of the body, control is returned to the **caller**. In some languages, control can be returned to the caller before reaching the end of the callee's body by using a **return-statement**:

```
// C++ code
void intswap (int& x, int& y){
   if (x == y) return;
   int t = x;
   x = y;
   y = t;
}
```

In some languages, such as FORTRAN, to call a procedure you must also include the keyword `CALL`, as in

```
CALL INTSWAP (A, B)
```

In FORTRAN, procedures are called **subroutines**.

As we have noted, some programming languages distinguish between procedures, which carry out their operations by changing their parameters or nonlocal variables, and **functions**, which appear in expressions and compute **returned values**. Functions may or may not also change their parameters and nonlocal variables. In C and C++, all procedures are implicitly functions; those that do not return values are declared `void` (such as the `swap` function above), while ordinary functions are declared to have the (return) type of the value that they return:

```
int max (int x, int y){
   return x > y ? x : y;
}
```

In Ada and FORTRAN, different keywords are used for procedures and functions:

```
-- Ada procedure
procedure swap ( x,y: in out integer) is
  t: integer;
begin
  if (x = y) then return;
  end if;
  t := x;
  x := y;
  y := t;
end swap;

-- Ada function
function max ( x,y: integer ) return integer is
begin
   if (x > y) then return x;
   else return y;
   end if;
end max;
```

Some languages allow only functions (that is, all procedures must return values). Functional languages in particular have this property; see Chapter 3.

Procedure and function declarations can be written in a form similar to constant declarations, using an equal sign, as in the following ML function declaration for a `swap` procedure:[3]

```
 (* ML code *)
fun swap (x,y) =
  let val t = !x
  in
     x := !y;
     y := t
  end;
```

The use of an equal sign to declare procedures is justified, because a procedure declaration gives the procedure name a meaning that remains constant during the execution of the program. We could say that a procedure declaration creates a **constant procedure value** and associates a symbolic name—the name of the procedure—with that value.

A procedure communicates with the rest of the program through its parameters and also through **nonlocal references**, that is, references to variables declared outside of its own body. The **scope rules** that establish the meanings of nonlocal references were introduced in Chapter 7.

10.2 Procedure Semantics

From the point of view of semantics, a procedure is a block whose declaration is separated from its execution. In Chapter 7, we saw examples of blocks in C and Ada that are not procedure blocks; these blocks are always executed immediately when they are encountered. For example, in C, blocks `A` and `B` in the following code are executed as they are encountered:

```
A:
{ int x,y;
   ...
  x = y * 10;
  B:
  { int i;
    i = x / 2;
    ...
  } /* end B */
} /* end A */
```

In Chapter 7, you saw that the **environment** determines the allocation of memory and maintains the meaning of names during execution. In a block-structured language, when a block is encountered during execution, it causes the allocation of local variables and other objects corresponding to the declarations

[3]This procedure in ML has type `'a ref * 'a ref -> unit` and is polymorphic; see Chapter 9. Note that its return type is `unit`, which is the equivalent of the C `void` type.

of the block. This memory allocated for the local objects of the block is called the **activation record** (or sometimes **stack frame**) of the block. The block is said to be **activated** as it executes under the bindings established by its activation record. As blocks are entered during execution, control passes from the activation of the surrounding block to the activation of the inner block. When the inner block exits, control passes back to the surrounding block. At this point, the activation record of the inner block is released, returning to the environment of the activation record of the surrounding block.

For example, in the preceding C code, during execution x and y are allocated in the activation record of block A, as shown in Figure 10.1.

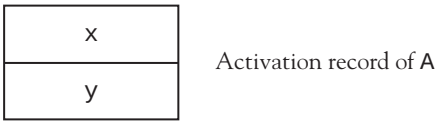


Figure 10.1 The activation record for block A

When block B is entered, space is allocated in the activation record of B. See Figure 10.2.

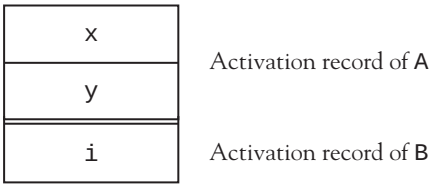


Figure 10.2 The activation records for blocks A and B

When B exits, the environment reverts to the activation record of A. Thus, the activation of B must retain some information about the activation from which it was entered.

In the preceding code, block B needs to access the variable x declared in block A. A reference to x inside B is a **nonlocal** reference, since x is not allocated in the activation record of B, but in the activation record of the surrounding block A. This, too, requires that B retain information about its surrounding activation.

Now consider what would happen if B were a procedure called from A instead of a block entered directly from A. Suppose, for the sake of a concrete example, that we have the following situation, which we give in C syntax:

```
(1)  int x;

(2)  void B(void)
(3)  { int i;
(4)    i = x / 2;
(5)    ...
(6)  } /* end B */
(7)  void A(void)
(8)  { int x, y;
```

(continues)

(continued)

```
(9)     ...
(10)    x = y * 10;
(11)    B();
(12) } /* end A */

(13) main()
(14) { A();
(15)   return 0;
(16) }
```

B is still entered from A (line 11), and the activation of B must retain some information about the activation of A so that control can return to A on exit from B, but there is now a difference in the way nonlocal references are resolved: Under the lexical scoping rule (see Chapter 7), the x in B (line 4) is the global x of the program (line 1),[4] not the x declared in A (line 8). In terms of activation records, we have the picture shown in Figure 10.3.

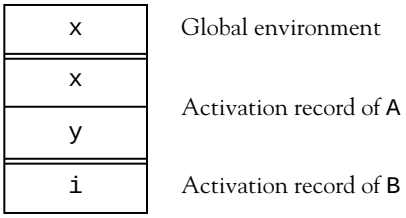


Figure 10.3 The activation records of the global environment, block A, and a procedure call of B

The activation of B must retain information about the global environment, since the nonlocal reference to x will be found there instead of in the activation record of A. This is because the global environment is the **defining environment** of B, while the activation record of A is the **calling environment** of B. (Sometimes the defining environment is called the **static environment** and the control environment the **dynamic environment**.) For blocks that are not procedures, the defining environment and the calling environment are always the same. By contrast, a procedure has different calling and defining environments. Indeed, a procedure can have any number of calling environments during which it will retain the same defining environment.

The actual structure of the environment that keeps track of defining and calling environments will be discussed in the next section. In this section, we are interested in the way an activation of a block **communicates** with the rest of the program.

Clearly, a nonprocedure block communicates with its surrounding block via nonlocal references: Lexical scoping allows it access to all the variables in the surrounding block that are not redeclared in its own declarations. By contrast, under lexical scoping, a procedure block can communicate only with its

[4]Technically, x is called an **external** variable, but, by the scope rules of C, such a variable has scope extending from its declaration to the end of the file in which it is declared; thus (ignoring separate compilation issues), it is global to the program.

defining block via references to nonlocal variables. It has no way of directly accessing the variables in its calling environment. In the sample C code, procedure `B` cannot directly access the local variable `x` of procedure `A`. Nevertheless, it may need the value of `x` in `A` to perform its computations.

The method of communication of a procedure with its calling environment is through **parameters**. A **parameter list** is declared along with the definition of the procedure, as in the `max` procedure of the previous section:

```
int max (int x, int y){
   return x > y ? x : y;
}
```

Here, `x` and `y` are parameters to the `max` function. They do not take on any value until `max` is called, when they are replaced by the arguments from the calling environment, as in:

```
z = max(a+b,10);
```

In this call to `max`, the parameter `x` is replaced by the argument `a + b`, and the parameter `y` is replaced by the argument 10. To emphasize the fact that parameters have no value until they are replaced by arguments, parameters are sometimes called **formal parameters**, while arguments are called **actual parameters**.

A strong case can be made that procedures should communicate with the rest of the program using *only* the parameters—that is, a procedure or function should *never* use or change a nonlocal variable. After all, such use or change represents a dependency (in the case of a use) or a side effect (in the case of a change) that is invisible from a reading of the procedure specification—it can only be determined from examining the body of the procedure (which could be very long, or even missing in the case of a library procedure). While this rule is generally a good one with regard to *variables* (e.g., a global variable should still be a parameter if it is changed), it is too much to expect with regard to *functions* and *constants*. In general, the body of a procedure will need to call other procedures in the program and library procedures, and may also need to use constants (such as PI). These are all nonlocal references, but we would not want to clutter up the parameter list with all such references (particularly since they are *all* essentially constants). Thus, nonlocal *uses* cannot be avoided.

Nevertheless, some procedures may depend only on parameters and fixed language features. These are said to be in **closed form**, since they contain no nonlocal dependencies. An example is the `max` function above, which has no dependencies on nonlocal definitions—it depends entirely on fixed language features and its parameters.

On the other hand, a polymorphic definition of the same function in C++ (using templates):

```
template <typename T>
T max (T x, T y)
{ return x > y ? x : y ; }
```

is *not* in closed form, since the ">" operator can be overloaded for non-built-in types. Thus, to write this function in closed form, we would have to include the ">" operation in the parameter list:

```
template <typename T>
T max (T x, T y, bool (*gt)(T,T))
{ return gt(x,y) ? x : y ;
}
```

(These examples were discussed in detail in Section 8.9.)

If we choose not to do this, then the semantics of this function can only be determined relative to its enclosing environment (its environment of definition assuming static scoping). The code of this function together with a representation of its defining environment is called a **closure**, because it can be used to resolve all outstanding nonlocal references relative to the body of the function. It is one of the major tasks of a runtime environment to compute closures for all functions in cases where they are needed.[5]

An additional issue affecting the semantics of a procedure is the nature of the correspondence between the parameters and the arguments during a call. Earlier we stated that parameters are replaced by the arguments during a call, but the nature of this replacement was not specified. In fact, this association is a binding that can be interpreted in many different ways. This interpretation is called the **parameter-passing mechanism**. In the next section, we discuss this topic in some detail.

10.3 Parameter-Passing Mechanisms

As noted at the end of the previous section, the nature of the bindings of arguments to parameters has a significant effect on the semantics of procedure calls. Languages differ substantially as to the kinds of parameter-passing mechanisms available and the range of permissible implementation effects that may occur. (You have already seen one of these implementation effects in Chapter 9, namely the effect different evaluation orders of the arguments can have in the presence of side effects.) Some languages (C, Java, functional languages) offer only one basic kind of parameter-passing mechanism, while others (C++) may offer two or more. In languages with one mechanism, it is also often possible to imitate other mechanisms by using indirection or other language features.

In this section, we will discuss four important parameter-passing mechanisms: pass by value, pass by reference, pass by value-result, and pass by name.[6] Some variations on these will be discussed in the exercises.

10.3.1 Pass by Value

In **pass by value**, by far the most common mechanism for parameter passing, the arguments are expressions that are evaluated at the time of the call, with the arguments' values becoming the values of the parameters during the execution of the procedure. In the simplest form of pass by value, value parameters behave as constant values during the execution of the procedure. Thus, we can think of pass by value as a process in which all the parameters in the body of the procedure are replaced by the corresponding argument values. For instance, we can think of the call `max (10, 2 + 3)` of the preceding `max` function as executing the body of `max` with `x` replaced by `10` and `y` replaced by `5`:

```
10 > 5 ? 10 : 5
```

[5]In this particular example, the reference to ">" can be resolved at compile time, and no closure needs to be computed, as we will see later in this chapter.

[6]Parameter-passing mechanisms are often referred to using the words "call by" rather than "pass by," as in "call by value," "call by reference," etc. We prefer "pass by" and use it consistently in this text.

This form of pass by value is usually the only parameter-passing mechanism in functional languages. Pass by value is also the default mechanism in C++ and Pascal, and is essentially the only parameter-passing mechanism in C and Java. However, in these languages a slightly different interpretation of pass by value is used: The parameters are viewed as local variables of the procedure, with initial values given by the values of the arguments in the call. Thus, in C, Java, and Pascal, value parameters may be assigned, just as with local variables (but cause no changes outside the procedure), while Ada `in` parameters may not be the targets of assignment statements (see the discussion of Ada parameters in Section 10.3.5).

Note that pass by value does not imply that changes cannot occur outside the procedure through the use of parameters. If the parameter has a pointer or reference type, then the value is an address and can be used to change memory outside the procedure. For example, the following C function definitely changes the value of the integer pointed to by the parameter `p`:

```
void init_p (int* p)
{ *p = 0; }
```

On the other hand, directly assigning to `p` does *not* change the argument outside the procedure:

```
void init_ptr (int* p)

{ p = (int*) malloc(sizeof(int)); /* error - has no effect! */
}
```

In addition, in some languages certain values are implicitly pointers or references. For example, in C, arrays are implicitly pointers (to the first location of the array); this means an array value parameter can always be used to change the values stored in the array:

```
void init_p_0 (int p[])
{ p[0] = 0; }
```

In Java, too, objects are implicitly pointers, so any object parameter can be used to change its data:

```
void append_1 (Vector v)
// adds an element to Vector v
{ v.addElement( new Integer(1)); }
```

However, as in C, direct assignments to parameters do not work either:

```
void make_new (Vector v)
{ v = new Vector(); /* error - has no effect! */ }
```

10.3.2 Pass by Reference

In pass by reference, an argument must be something like a variable with an allocated address. Instead of passing the value of the variable, pass by reference passes the location of the variable, so that the parameter becomes an **alias** for the argument, and any changes made to the parameter occur to the argument as well. In FORTRAN, pass by reference is the only parameter-passing mechanism. In C++ and Pascal, pass by reference (instead of the default pass by value) can be specified

by using an ampersand (&) after the data type in C++, and a var keyword before the variable name in Pascal:

```
void inc(int& x) // C++
{ x++ ; }

procedure inc(var x: integer); (* Pascal *)
begin
  x :=   x + 1;
end;
```

After a call to inc(a) the value of a has increased by 1, so that a side effect has occurred. Multiple aliasing is also possible, such as in the following example:

```
int a;

void yuck (int& x, int& y)
{ x = 2;
  y = 3;
  a = 4;
}
...
yuck (a, a);
```

Inside procedure yuck after the call, the identifiers x, y, and a all refer to the same variable, namely, the variable a.

As noted previously, C can achieve pass by reference by passing a reference or location explicitly as a pointer. C uses the operator & to indicate the location of a variable and the operator * to dereference a pointer. For example:

```
void inc (int* x) /*C imitation of pass by reference */
{ (*x)++; /* adds 1 to *x */ }
...
int a;
...
inc(&a); /* pass the address of a to the inc function */
```

This code has the same effect as the previous C++ or Pascal code for the inc procedure. Of course, there is the annoying necessity here of explicitly taking the address of the variable a, and then explicitly deferencing it again in the body of inc.

A similar effect can be achieved in ML by using reference types to imitate pass by reference. (In ML, as in C, only pass by value is available.) For example:

```
fun inc (x: int ref) = x := !x + 1;
```

One additional issue that must be resolved in a language with pass by reference is the response of the language to reference arguments that are not variables and, thus, do not have known addresses within the runtime environment. For example, given the C++ code

```
void inc(int& x)
{ x++; }
...
inc(2); // ??
```

how should the compiler respond to the call `inc(2)`? In FORTRAN, in which pass by reference is the *only* available parameter mechanism, this is syntactically correct code. The response of the compiler is to create a temporary integer location, initialize it with the value 2, and then apply the `inc` function. This actually mimics pass by value, since the change to the temporary location does not affect the rest of the program. In C++ and Pascal, however, this is an error. In these languages, reference arguments *must* be l-values—that is, they must have known addresses.

10.3.3 Pass by Value-Result

Pass by value-result achieves a similar result to pass by reference, except that no actual alias is established. Instead, in pass by value-result, the value of the argument is copied and used in the procedure, and then the final value of the parameter is copied back out to the location of the argument when the procedure exits. Thus, this method is sometimes known as **copy-in, copy-out**, or **copy-restore**.

Pass by value-result is only distinguishable from pass by reference in the presence of aliasing. Thus, in the following code (written for convenience in C syntax):

```
void p(int x, int y)
{ x++;
  y++;
}

main()
{ int a = 1;
  p(a,a);
  ...
}
```

`a` has value 3 after `p` is called if pass by reference is used, while `a` has the value 2 if pass by value-result is used.

Issues left unspecified by this mechanism, and possibly differing in different languages or implementations, are the order in which results are copied back to the arguments and whether the locations of the arguments are calculated only on entry and stored or whether they are recalculated on exit.

A further option is that a language can offer a **pass by result** mechanism as well, in which there is no incoming value, but only an outgoing one.

10.3.4 Pass by Name and Delayed Evaluation

Pass by name is the term used for this mechanism when it was introduced in Algol60. At the time it was intended as a kind of advanced inlining process for procedures, so that the semantics of procedures could be described simply by a form of textual replacement, rather than an appeal to environments and closures. (See Exercise 10.14.) It turned out to be essentially equivalent to the normal order delayed evaluation described in the previous chapter. It also turned out to be difficult to implement, and to have complex interactions with other language constructs, particularly arrays and assignment. Thus, it was rarely implemented and was dropped in all Algol60 descendants (AlgolW, Algol68, C, Pascal, etc.). Advances in delayed evaluation in functional languages, particularly pure functional languages such as Haskell (where interactions with side effects are avoided), have increased interest in this mechanism, however, and it is worthwhile to understand it as a basis for other delayed evaluation mechanisms, particularly the more efficient **lazy evaluation** studied in Chapter 3.

The idea of pass by name is that the argument is not evaluated until its actual use as a parameter in the called procedure. Thus, the *name* of the argument, or its textual representation at the point of call, replaces the name of the parameter to which it corresponds. As an example, in the C code

```
void inc(int x)
{ x++; }
```

if a call such as `inc(a[i])` is made, the effect is of evaluating `a[i]++`. Thus, if `i` were to change before the use of `x` inside `inc`, the result would be different from either pass by reference or pass by value-result:

```
int i;
int a[10];

void inc(int x)
{ i++;
  x++;
}

main()
{ i = 1;
  a[1] = 1;
  a[2] = 2;
  inc(a[i]);
  return 0;
}
```

This code has the result of setting `a[2]` to 3 and leaving `a[1]` unchanged.

Pass by name can be interpreted as follows. The text of an argument at the point of call is viewed as a function in its own right, which is evaluated every time the corresponding parameter name is reached in the code of the called procedure. However, the argument will always be evaluated in the environment

of the caller, while the procedure will be executed in its defining environment. To see how this works, consider the example in Figure 10.4.

```
(1)  #include <stdio.h>
(2)  int i;

(3)  int p(int y)
(4)  { int j = y;
(5)    i++;
(6)    return j+y;
(7)  }

(8)  void q(void)
(9)  { int j = 2;
(10)   i = 0;
(11)   printf("%d\n", p(i + j));
(12) }

(13) main()
(14) { q();
(15)   return 0;
(16) }
```

Figure 10.4 Pass by name example (in C syntax)

The argument `i + j` to the call to `p` from `q` is evaluated every time the parameter `y` is encountered inside `p`. The expression `i + j` is, however, evaluated as though it were still inside `q`, so on line 4 in `p` it produces the value 2. Then, on line 6, since `i` is now 1, it produces the value 3 (the `j` in the expression `i + j` is the `j` of `q`, so it hasn't changed, even though the `i` inside `p` has). Thus, if pass by name is used for the parameter `y` of `p` in the program, the program will print 5.

Historically, the interpretation of pass by name arguments as functions to be evaluated during the execution of the called procedure was expressed by referring to the arguments as **thunks**.[7] For example, the above C code could actually imitate pass by name using a function, except for the fact that it uses the local definition of `j` inside `q`. If we make `j` global, then the following C code will actually print 5, just as if pass by name were used in the previous code:

```
#include <stdio.h>
int i,j;

int i_plus_j(void)
{ return i+j; }
```

(continues)

[7]Presumably, the image was of little machines that "thunked" into place each time they were needed.

(continued)

```
int p(int (*y)(void))
{ int j = y();
  i++;
  return j+y();
}

void q(void)
{ j = 2;
  i = 0;
  printf("%d\n", p(i_plus_j));
}

main()
{ q();
  return 0;
}
```

Pass by name is problematic when side effects are desired. Consider the `intswap` procedure we have discussed before:

```
void intswap (int x, int y)
{ int t = x;
  x = y;
  y = t;
}
```

Suppose that pass by name is used for the parameters `x` and `y` and that we call this procedure as follows,

```
intswap(i,a[i])
```

where `i` is an integer index and `a` is an array of integers. The problem with this call is that it will function as the following code:

```
t = i;
i = a[i];
a[i] = t;
```

Note that by the time the address of `a[i]` is computed in the third line, `i` has been assigned the value of `a[i]` in the previous line. This will not assign `t` to the array `a` subscripted at the original `i`, unless `i = a[i]`.

It is also possible to write bizarre (but possibly useful) code in similar circumstances. One of the earliest examples of this is called **Jensen's device** after its inventor J. Jensen. Jensen's device uses pass by name to apply an operation to an entire array, as in the following example, in C syntax:

```
int sum (int a, int index, int size)
{ int temp = 0;
  for (index = 0; index < size; index++)  temp += a;
  return temp;
}
```

If `a` and `index` are pass by name parameters, then in the following code:

```
int x[10], i, xtotal;
...
xtotal = sum(x[i],i,10);
```

the call to `sum` computes the sum of all the elements `x[0]` through `x[9]`.

10.3.5 Parameter-Passing Mechanism versus Parameter Specification

One can fault these descriptions of parameter-passing mechanisms in that they are tied closely to the internal mechanics of the code that is used to implement them. While one can give somewhat more theoretical semantic descriptions of these mechanisms, all of the questions of interpretation that we have discussed still arise in code that contains side effects. One language that tries to address this issue is Ada. Ada has two notions of parameter communication, `in` parameters and `out` parameters. Any parameter can be declared `in`, `out`, or `in out` (i.e., both). The meaning of these keywords is exactly what you would expect: An `in` parameter specifies that the parameter represents an incoming value only; an `out` parameter specifies an outgoing value only; and an `in out` parameter specifies both an incoming and an outgoing value. (The `in` parameter is the default, and the keyword `in` can be omitted in this case.)

Any parameter implementation whatsoever can be used to achieve these results, as long as the appropriate values are communicated properly (an `in` value on entry and an `out` value on exit). Ada also declares that any program that violates the protocols established by these parameter specifications is **erroneous**. For example, an `in` parameter cannot legally be assigned a new value by a procedure, and the value of an `out` parameter cannot be legally used by the procedure. Thus, pass by reference could be used for an `in` parameter as well as an `out` parameter, or pass by value could be used for `in` parameters and copy out for `out` parameters.

Fortunately, a translator can prevent many violations of these parameter specifications. In one case, an `in` parameter cannot be assigned to or otherwise have its value changed, because it should act like a constant. In another case, an `out` parameter can only be assigned to, because its value should never be used. Unfortunately, `in out` parameters cannot be checked with this specificity. Moreover, in the presence of other side effects, as we have seen, different implementations (reference and copy in-copy out, for example) may have different results. Ada still calls such programs erroneous, but translators cannot in general check whether a program is erroneous under these conditions. Thus, the outcome of this specification effort is somewhat less than what we might have hoped for.

10.3.6 Type Checking of Parameters

In strongly typed languages, procedure calls must be checked so that the arguments agree in type and number with the parameters of the procedure. This means, first of all, that procedures may not have a variable number of parameters and that rules must be stated for the type compatibility between parameters and arguments. In the case of pass by reference, parameters usually must have the same type, but in the case of pass by value, this can be relaxed to assignment compatibility (Chapter 8), and can allow conversions, as is done in C, C++, and Java. Ada, of course, does not allow conversions.

10.4 Procedure Environments, Activations, and Allocation

In this section, we give more detail on how information can be computed and maintained in an environment during procedure calls. We already saw in Chapter 7 and in Section 10.2 that the environment for a block-structured language with lexical scope can be maintained in a stack-based fashion, with an activation record created on the environment stack when a block is entered and released when the block is exited. We also saw how variables declared locally in the block are allocated space in this activation record.

In this section, we explore how this same structure can be extended to procedure activations, in which the defining environment and the calling environment differ. We also survey the kinds of information necessary to maintain this environment correctly. We have already noted that some notion of **closure** is necessary in this case to resolve nonlocal references. A clear understanding of this **execution model** is often necessary to fully understand the behavior of programs, since the semantics of procedure calls are embedded in this model.

We also want to show that a completely stack-based environment is no longer adequate to deal with procedure variables and the dynamic creation of procedures, and that languages with these facilities, particularly functional languages, are forced to use a more complex fully dynamic environment with garbage collection.

But, first, by way of contrast, we give a little more detail about the fully static environment of FORTRAN, which is quite simple, yet completely adequate for that language. We emphasize that the structures that we present here are only meant to illustrate the general scheme. Actual details implemented by various translators may differ substantially from the general outlines given here.

10.4.1 Fully Static Environments

In a language like Fortran77,[8] all memory allocation can be performed at load time, and the locations of all variables are fixed for the duration of program execution. Function and procedure (or subroutine) definitions cannot be nested (that is, all procedures/functions are global, as in C),[9] and recursion is not allowed (unlike C).[10] Thus, all the information associated with a function or subroutine can be statically

[8]Fortran90 significantly extends the features of Fortran77, including allowing recursion and dynamically allocated variables.
[9]In Section 10.4.2, we will see why nested procedures present problems even for nonstatic environments.
[10]More precisely, recursive calls are permitted in Fortran77, but each new call overwrites the data of all previous calls, so recursion does not work. Procedures with this property are sometimes called **non-reentrant**.

allocated. Each procedure or function has a fixed activation record, which contains space for the local variables and parameters, and possibly the return address for proper return from calls. Global variables are defined by `COMMON` statements, and they are determined by pointers to a common area. Thus, the general form of the runtime environment for a FORTRAN program with subroutines `S1...Sn` is as shown in Figure 10.5.

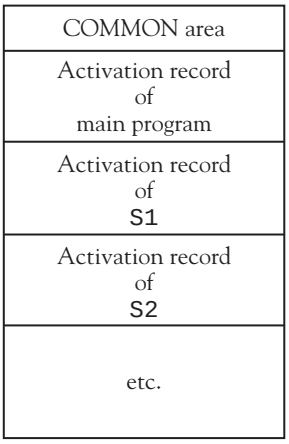


Figure 10.5 The runtime environment of a FORTRAN program with subprograms S1 and S2

Each activation record, moreover, is broken down into several areas, as shown in Figure 10.6.

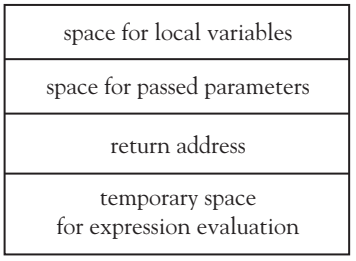


Figure 10.6 The component areas of an activation record

When a call to a procedure `S` occurs, the parameters are evaluated, and their locations (using pass by reference) are stored in the parameter space of the activation record of `S`. Then, the current instruction pointer is stored as the return address, and a jump is performed to the instruction pointer of `S`. When `S` exits, a jump is performed to the return address. If `S` is a function, special arrangements must be made for the returned value; often it is returned in a special register, or space can be reserved for it in the activation record of the function or the activation record of the caller. Note that all nonlocal references must be to the global `COMMON` area, so no closure is needed to resolve these references. Thus, no extra bookkeeping information is necessary in an activation record to handle nonlocal references.

As an example, consider the following FORTRAN program:

```
      REAL TABLE (10), MAXVAL
      READ *, TABLE (1), TABLE (2), TABLE (3)
      CALL LRGST (TABLE, 3, MAXVAL)
      PRINT *, MAXVAL
      END

      SUBROUTINE LRGST (A, SIZE, V)
      INTEGER SIZE
      REAL A (SIZE), V
      INTEGER K
      V = A (1)
      D0 10 K = 1, SIZE
      IF (A( K) GT. V) V = A (K)
10    CONTINUE
      RETURN
      END
```

The environment of this program would look as shown in Figure 10.7, where we have added pointers indicating location references that exist immediately after the call to `LRGST` from the main program.

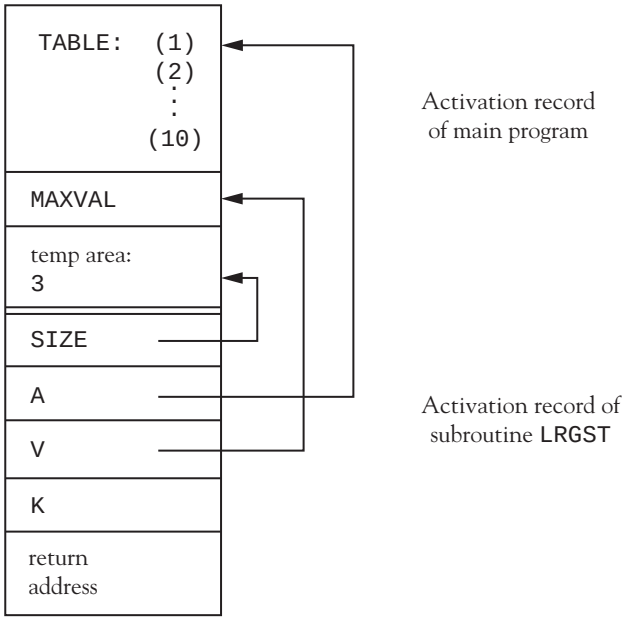


Figure 10.7 The runtime environment immediately after a call to the subroutine LRGST

10.4.2 Stack-Based Runtime Environments

In a block-structured language with recursion, such as C and other Algol-like languages, activations of procedure blocks cannot be allocated statically. Why? Because a procedure may be called again before its previous activation is exited, so a new activation must be created on each procedure entry. As you have seen, this can be done in a stack-based manner, with a new activation record created on the stack every time a block is entered and released on exit.

What information must be kept in the environment to manage a stack-based environment? As with the fully static environment of FORTRAN, an activation is necessary to allocate space for local variables, temporary space, and a return pointer. However, several additional pieces of information are required. First, a pointer to the current activation must be kept, since each procedure has no fixed location for its activation record, but the location of its activation record may vary as execution proceeds. This pointer to the current activation must be kept in a fixed location, usually a register, and it is called the **environment pointer** or **ep**, since it points to the current environment.

The second piece of information required in a stack-based environment is a pointer to the activation record of the block from which the current activation was entered. In the case of a procedure call, this is the activation of the caller. This piece of information is necessary because, when the current activation is exited, the current activation record needs to be removed (i.e., popped) from the stack of activation records. This means that the ep must be restored to point to the previous activation, a task that can be accomplished only if the old ep, which pointed to the previous activation record, is retained in the new activation record. This stored pointer to the previous activation record is called the **control link**, since it points to the activation record of the block from which control passed to the current block and to which control will return. Sometimes this control link is called the **dynamic link**.

A simple example is the following program, in C syntax:

```
void p(void)
{ ... }

void q(void)
{ ...
  p();
}

main()
{ q();
  ...
}
```

At the beginning of execution of the program, there is just an activation record for `main` and the ep points there (we ignore global data in this example). See Figure 10.8.

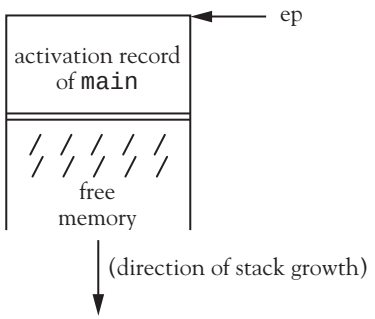


Figure 10.8 The runtime environment after `main` begins executing

After the call to q, an activation record for q has been added to the stack, the ep now points to the activation record of q, and q has stored the old ep as its control link. See Figure 10.9.

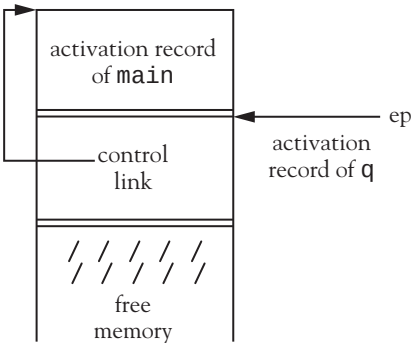


Figure 10.9 The runtime environment after q begins executing

When p is called inside q, a new frame is added for p. Thus, after the call to p from within q, the activation stack looks as shown in Figure 10.10.

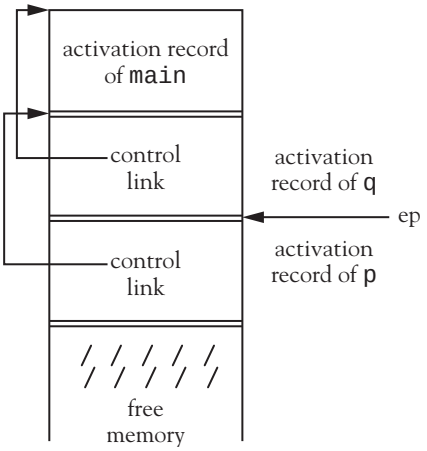


Figure10.10 The runtime environment after p is called within q

Now when p is exited, the activation record of p is returned to free memory, and the ep is restored from the control link of the activation record of p, so that it points to the activation record of q again. Similarly, when q is exited, the ep is restored to point to the original environment of the main program.

With this new requirement, the fields in each activation record need to contain the information in Figure 10.11.

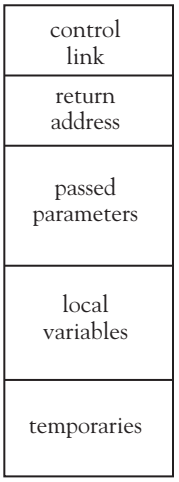


Figure 10.11 The component areas of an activation record with a control link

Consider now how the environment pointer can be used to find variable locations. Local variables are allocated in the current activation record, to which the ep points. Since the local variables are allocated as prescribed by the declarations of the block, and these declarations are static, each time the block is entered, the same kinds of variables are allocated in the same order. Thus, each variable can be allocated the same position in the activation record relative to the beginning of the record.[11] This position is called the **offset** of the local variable. Each local variable can be found using its fixed offset from the location to which the ep points.

Consider the following additions to our previous example:

```
int x;

void p( int y)
{ int i = x;
  char c;
  ...
}
```

(continues)

[11]More precisely, recursive calls are permitted in Fortran77, but each new call overwrites the data of all previous calls, so recursion does not work. Procedures with this property are sometimes called **non-reentrant**.

(continued)

```
void q ( int a)
{ int x;
  ...
  p(1);
}

main()
{ q(2);
  return 0;
}
```

Any activation record of p will have a format like that in Figure 10.12. Each time p is called, the parameter y and the local variables i and c will be found in the same place relative to the beginning of the activation record.

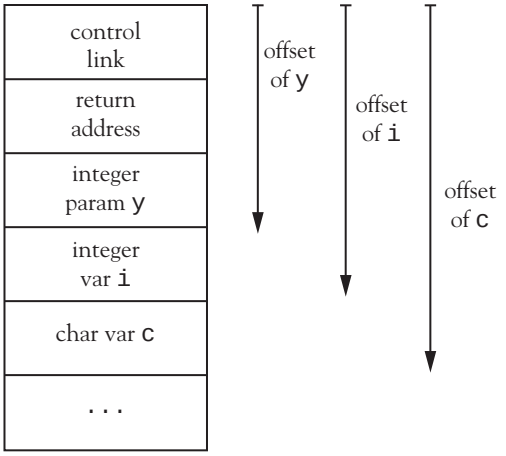


Figure 10.12 An activation record of p showing the offsets of the parameter and temporary variables

Now consider the case of nonlocal references, such as the reference to x in p indicated in the foregoing code. How is x found? Fortunately, in C as in FORTRAN, procedures cannot be nested, so all nonlocal references outside a procedure are actually global and are allocated statically. Thus, additional structures to represent closures are not required in the activation stack. Any nonlocal reference is found in the global area and can actually be resolved prior to execution.

However, many languages, including Pascal, Ada, and Modula-2, *do* permit nested procedures. With nested procedures, nonlocal references to local variables are permissible in a surrounding procedure scope. For example, see the following Ada code:

```
procedure q is
  x: integer;

  procedure p (y: integer) is
    i: integer := x;
  begin
    ...
  end p;

  procedure r is
  x: float;
  begin
    p(1);
    ...
  end r;

begin
  r;
end q;
```

Figure 10.13 shows what the activation stack looks like during the execution of p.

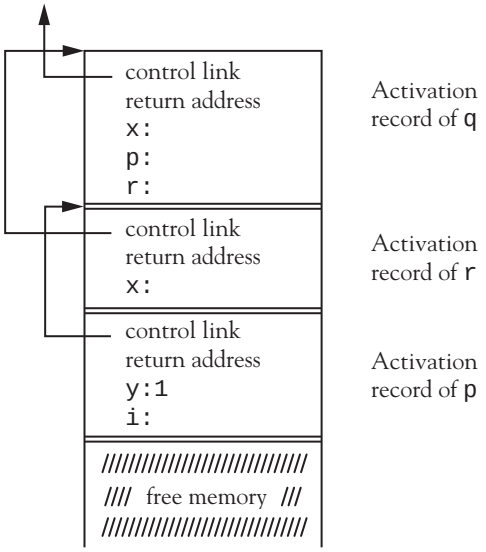


Figure 10.13 The runtime environment with activations of p, r, and q

How is it possible to find the nonlocal reference to the x of q from inside p? One idea would be to follow the control link to the activation record of r, but this would find the x local to r.[12] This would achieve dynamic scope rather than lexical scope. To achieve lexical scope, a procedure such as p must maintain a link to its **lexical** or **defining environment**. This link is called the **access link**, since it provides access to nonlocal variables. (Sometimes the access link is called the **static link**, since it is designed to implement lexical, or static, scope.) Since p is defined inside q, the access link of p is the ep that exists when p is defined, so the access link of p points to the activation of q. Now each activation record needs a new field, the access link field, and the complete picture of the environment for our example is as shown in Figure 10.14.

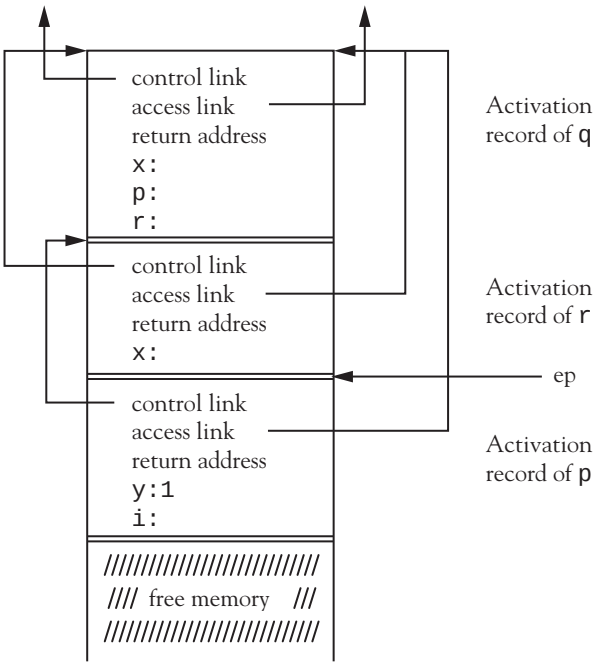


Figure 10.14 The runtime environment with activations of p, r, and q with access links

When blocks are deeply nested, it may be necessary to follow more than one access link to find a nonlocal reference. For example, in the Ada code

```
procedure ex is
  x: ... ;

  procedure p is
    ...
    procedure q is
    begin
       ... x ... ;
```

(continues)

[12]This would actually require that the full symbol table be maintained during execution, rather than replacing names by fixed offsets, since the offset (or even the existence) of a name in an arbitrary caller cannot be predicted to be at any fixed offset.

(continued)

```
    end q;
  begin -- p
                  ...
  end p;
begin -- ex
  ...
end ex;
```

to access x from inside q requires following the access link in the activation record of q to its defining environment, which is an activation record of p, and then following the access link of p to the global environment. This process is called **access chaining**, and the number of access links that must be followed corresponds to the difference in nesting levels, or **nesting depth**, between the accessing environment and the defining environment of the variable being accessed.

With this organization of the environment, the closure of a procedure—the code for the procedure, together with a mechanism for resolving nonlocal references—becomes significantly more complex, since every time a procedure is called, the defining environment of that procedure must be included as part of the activation record. Thus, a function or procedure in a language like Ada or Pascal must be represented not just by a pointer to the code for the procedure but also by a closure consisting of a pair of pointers. The first of these is a code or instruction pointer, which we denote by ip. The second one is the access link, or environment pointer of its defining environment, which we denote by ep. We write this closure as <ep, ip>, and we use it as the representation of procedures in the following discussion.

We finish this section with an example of an Ada program with nested procedures and a diagram of its environment at one point during execution (Figures 10.15 and 10.16). Note in the diagram of Figure 10.16 that the nested procedure show has two different closures, each corresponding to the two different activations of p in which show is defined.

```
(1)  with Text_IO; use Text_IO;
(2)  with Ada.Integer_Text_IO;
(3)  use Ada.Integer_Text_IO;

(4)  procedure lastex is

(5)    procedure p(n: integer) is

(6)      procedure show is
(7)      begin
(8)        if n > 0 then p(n - 1);

(9)        end if;

(10)       put(n);
(11)       new_line;
(12)     end show;
```

Figure 10.15 An Ada program with nested procedures *(continues)*

(continued)

```
(13)   begin -- p
(14)     show;
(15)   end p;
(16) begin -- lastex
(17)   p(1);
(18) end lastex;
```

Figure 10.15 An Ada program with nested procedures

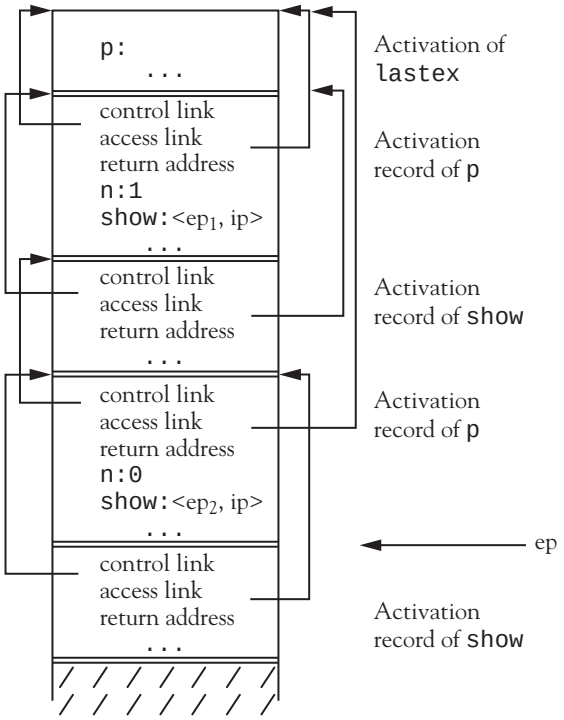


Figure 10.16 Environment of `lastex` during the second call to `show` (line 8 of Figure 10.15)

10.4.3 Dynamically Computed Procedures and Fully Dynamic Environments

The stack-based runtime environment of the type shown in Figure 10.16 is completely adequate for almost all block-structured languages with lexical scope. The use of <ep, ip> closures for procedures makes this environment suitable even for languages with parameters that are themselves procedures, as long as these parameters are value parameters: A procedure that is passed to another procedure is passed as a closure (an <ep, ip> pair), and when it is called, its access link is the ep part of its closure. Examples of languages for which this organization is necessary are Ada and Pascal.

A stack-based environment does have its limitations, however, which we already noted in Chapter 8. For instance, any procedure that can return a pointer to a local object, either by returned value or through a pass by reference parameter, will result in a **dangling reference** when the procedure is exited, since the activation record of the procedure will be deallocated from the stack. The simplest example of this occurs when the address of a local variable is returned, as for instance in the C code:

```
int * dangle(void)
{ int x;
  return &x;
}
```

An assignment `addr = dangle()` now causes `addr` to point to an unsafe location in the activation stack.

This situation cannot happen in Java, however, since the address of a local variable is unavailable. Ada95 also makes this an error by stating the **Access-type Lifetime Rule:** An attribute `x'access` yielding a result belonging to an access type `T` (i.e., a pointer type) is only allowed if `x` can remain in existence at least as long as `T`. Thus, the Ada code equivalent to the above C code

```
type IntPtr is access Integer;

function dangle return IntPtr is
  x: Integer;
begin
   return x'access;
end dangle;
```

is illegal because the `IntPtr` type definition occurs in an outer scope relative to `x` (as it must to allow the definition of `dangle`), and so violates the Access-type Lifetime Rule.

However, there are situations in which a compiler cannot check for this error statically (see the Notes and References). In Ada an exception will still occur during execution (`Program_Error`), but in C/C++ and a few other languages this error will not be caught either statically or dynamically. In practice, it is easy for programmers who understand the environment to avoid this error.

A more serious situation occurs if the language designer wishes to extend the expressiveness and flexibility of the language by allowing procedures to be dynamically created. That is, if the language designer allows procedures to be returned from other procedures via returned value or reference parameters. This kind of flexibility is usually desired in a functional language, and even in some object-oriented languages, such as Smalltalk, where methods can be created dynamically. In a language where this is the case, procedures become first-class values, which means no "arbitrary" restrictions apply to their use. In such a language, a stack-based environment cannot be used, since the closure of a locally defined procedure will have an ep that points to the current activation record. If that closure is available outside the activation of the procedure that created it, the ep will point to

an activation record that no longer exists. Any subsequent call to that procedure will have an incorrect access environment.

Consider the following example, in which we may reasonably want a procedure to create another procedure. We write this example in Ada95, since Ada95 has procedure types and parameters, and so can express the situation quite well (this example is adapted from Abelson and Sussman [1996]):

```
type WithdrawProc is
  access function (x:integer) return integer;

InsufficientFunds: exception;

function makeNewBalance (initBalance: integer)
   return WithDrawProc
is
   currentBalance: integer;

   function withdraw (amt: integer) return integer is
   begin
     if amt <= currentBalance then
       currentBalance := currentBalance - amt;
     else
       raise InsufficientFunds;
     end if;
     return currentBalance;
   end withdraw;

begin
   currentBalance := initBalance;
   return withdraw'access;
end makeNewBalance;
```

We now might want to make two different accounts from which to withdraw, one with an initial balance of 500 and the other with an initial balance of 100 dollars:

```
withdraw1, withdraw2: WithdrawProc;

withdraw1 := makeNewBalance(500);
withdraw2 := makeNewBalance(100);
```

The problem is that in a stack-based environment, the activation records in which both these functions were created have disappeared: Each time `makeNewBalance` returns, the local environment of `makeNewBalance` is released. For example,

```
newBalance1 := withdraw1(100);
newBalance2 := withdraw2(50);
```

should result in a value of 400 for `newBalance1` and a value of 50 for `newBalance2`, but if the two instances of the local `currentBalance` variable (with values 500 and 100) have disappeared from the environment, these calls will not work.

Pascal does not have this problem, since there are no procedure variables (procedures can only be value parameters). In C all procedures are global, so again this problem cannot arise. Ada considers this program to be similar to the dangling reference code, and calls this a violation of the Access-type Lifetime Rule. Thus, Ada will generate a compile-time error message in this case.

Nevertheless, we may want to be able to do things just like this in a language in which functions and procedures are first class values; that is, no nongeneralities or nonorthogonalities should exist for functions and procedures. Such a language, for example, is LISP. What kind of environment would we need to allow such constructs?

Now it is no longer possible for the activation record of procedure `makeNewBalance` to be removed from the environment, as long as there are references to any of its local objects. Such an environment is **fully dynamic** in that it deletes activation records only when they can no longer be reached from within the executing program. Such an environment must perform some kind of automatic reclamation of unreachable storage. Two standard methods of doing so are **reference counts** and **garbage collection**; these will be studied in Section 10.5.

This situation also means that the structure of the activations becomes treelike instead of stacklike. The control links to the calling environment no longer necessarily point to the immediately preceding activation. For example, in the two calls to `makeNewBalance`, the two activations remain, with their control links both pointing at the defining environment. See Figure 10.17.

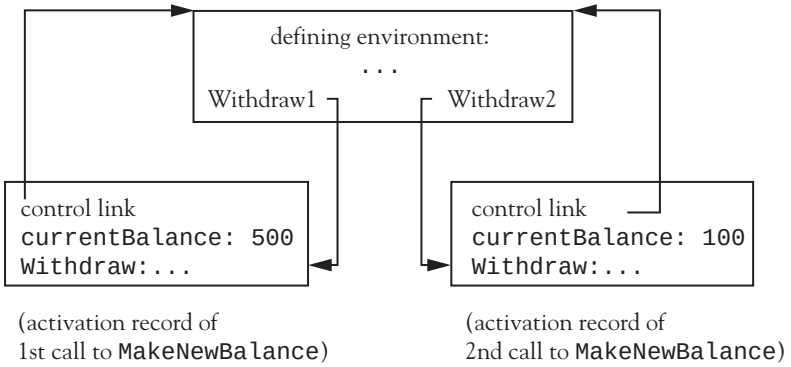


Figure 10.17 A tree-like runtime environment for a language with first-class procedures

Each activation of `makeNewBalance` can disappear only if `withdraw1` or `withdraw2` are reassigned or disappear themselves.

This is the model under which Scheme and other functional languages execute. In the next section, we will discuss issues involved in maintaining such environments.

10.5 Dynamic Memory Management

In a typical imperative language such as C, the automatic allocation and deallocation of storage occurs only for activation records on a stack. In this relatively easy process, space is allocated for an activation record when a procedure is called and deallocated when the procedure is exited. Explicit dynamic allocation and use of pointers is also available under manual programmer control using a **heap** of memory separate from the stack, as we described in Chapter 7. However, as we noted there, manual memory management on the heap suffers from a number of potential problems, including the creation of garbage and dangling references. Thus, languages with significant needs for heap storage, such as Java, are better off leaving nonstack dynamic storage to a memory manager that provides automatic garbage collection.

This means that any language that does not apply significant restrictions to the use of procedures and functions *must* provide automatic garbage collection, since the stack-based system of procedure call and return is no longer correct. Functional languages such as Scheme, ML, and Haskell, which have first-class function values, fall into this category, as do object-oriented languages such as Smalltalk and Java.

A very simple solution to the problem of managing dynamic memory is to *not* deallocate memory once it has been allocated. In other words, every call to a function creates a new activation record in memory that is not deallocated on exit. This method has two advantages: It is correct, and it is easy to implement. What's more, it actually can work for small programs. However, it is not really an option for most programs written in functional and object-oriented languages. After all, in these languages, processing occurs primarily by function or method call, many instances of which may be recursive. Moreover, the internal representation of data uses pointers and requires a substantial amount of indirection and memory overhead. Thus, if deallocation does not occur, memory is very quickly exhausted.

However, this method *has* been used in conjunction with virtual memory systems, since the address space for such systems is almost inexhaustible. This only defers the basic problem, however, since it causes the swap space of the operating system to become exhausted and can cause a serious degradation of performance because of page faults. These topics are beyond the scope of this book but at least serve to point out interplay between memory management and operating system issues.

Automatic memory management actually falls into two categories: the **reclamation** of previously allocated but no longer used storage, previously called **garbage collection**, and the **maintenance** of the free space available for allocation. Indeed, maintenance of free space is needed even for manual heap operations in languages such as C; it is also a little more straightforward than is reclamation, so we discuss it first.

10.5.1 Maintaining Free Space

Generally, a contiguous block of memory is provided by the operating system for the use of an executing program. The free space within this block is maintained by a list of free blocks. One way to do this is via a linked list, such as the circular list in the following figure, indicating the total available space, with allocated blocks shaded and free blocks blank. See Figure 10.18.

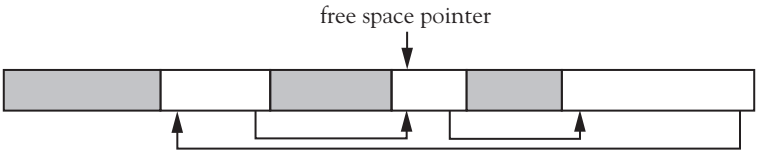


Figure 10.18 The free space represented as a linked list

When a block of a certain size needs to be allocated, the memory manager searches the list for a free block with enough space, and then adjusts the free list to remove the allocated space, as in the picture shown in Figure 10.19.

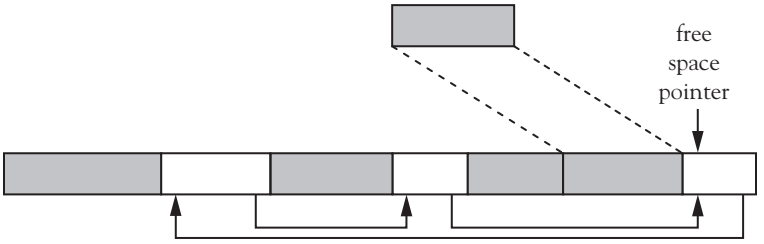


Figure 10.19 Allocating storage from the free space

When memory is reclaimed, blocks are returned to the free list. For example, if the light-shaded areas are storage to be reclaimed (as shown in Figure 10.20), then the new free list after reclamation would look as shown in Figure 10.21.

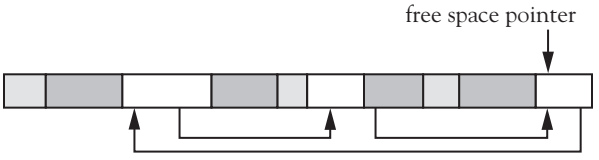


Figure 10.20 Returning storage to the free list

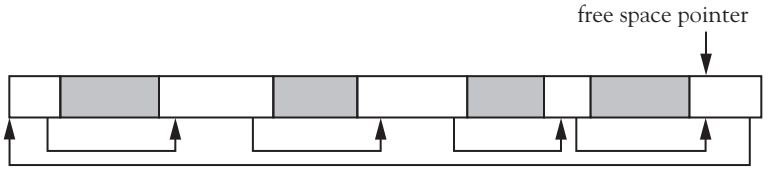


Figure 10.21 The free list after the return of storage

When blocks of memory are returned to the free list, they must be joined with immediately adjacent blocks to form the largest contiguous block of free memory. This process is called **coalescing**. In the preceding example, the small block freed in the middle of the memory area was coalesced with the free block adjacent to it on the right. Even with coalescing, however, a free list can become **fragmented**—that is, it can break into a number of small-sized blocks. When this occurs, it is possible for the allocation of a large block to fail, even though there is enough total space available to allocate it. To prevent this, memory must occasionally be **compacted** by moving all free blocks together and coalescing them into

one block. For example, the five blocks of free memory in the previous diagram could be compacted as shown in Figure 10.22.

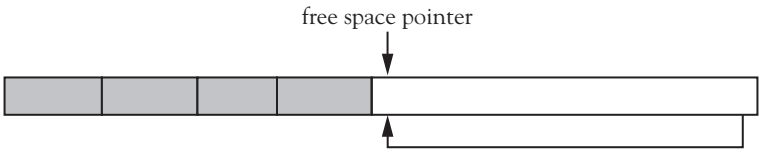


Figure 10.22 The free list after compaction

Storage compaction involves considerable overhead, since the locations of most of the allocated quantities will change and data structures and tables in the runtime environment will have to be modified to reflect these new locations.

10.5.2 Reclamation of Storage

Recognizing when a block of storage is no longer referenced, either directly or indirectly through pointers, is a much more difficult task than the maintenance of the free list itself. Historically, two main methods have been used: reference counting and mark and sweep.

Reference counting is considered an "eager" method of storage reclamation, in that it tries to reclaim space as soon as it is no longer referenced. Each block of allocated storage contains an extra count field, which stores the number of references to the block from other blocks. Each time a reference is changed, these reference counts must be updated. When the reference count drops to zero, the block can be returned to the free list. In theory this seems like a simple and elegant solution to the problem, but in fact it suffers from serious drawbacks. One obvious one is the extra memory that it takes to keep the reference counts themselves. Even more serious, the effort to maintain the counts can be fairly large. For example, when making an assignment, which we will model by a C-like pointer assignment `p = q`, first the old value of `p` must be followed, and its reference count decremented by one. If this reference count should happen to drop to zero, it is not sufficient simply to return it to free storage, since it may itself contain references to other storage blocks. Thus, reference counts must be decremented recursively. Finally, when `q` is copied to `p`, its reference count must be incremented. A pseudocode description of assignment would, therefore, look as follows:

```
void decrement(p)
{ p->refcount--;
  if (p->refcount == 0)
  { for all fields r of *p that are pointers do
       decrement(r);
    deallocate(*p);
  }
}
void assign(p,q)
{ decrement(p);
  p = q;
  q->refcount++;
}
```

However, the overhead to maintain reference counts is not the worst flaw of this scheme. Even more serious is that circular references can cause unreferenced memory to never be deallocated.

For example, consider a circular linked list such as the one shown in Figure 10.23.

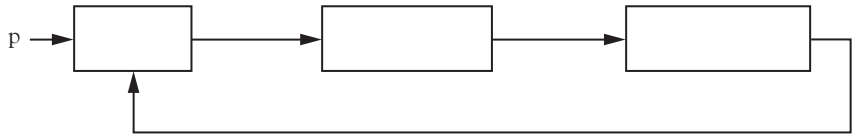


Figure 10.23 A circular linked list

If p is deallocated, the reference count of the block to which p points will drop from 2 to 1. The entire list will never be deallocated.

The standard alternative to reference counts is **mark and sweep**. Whereas reference counting is considered an eager method, mark and sweep is a "lazy" method, in that it puts off reclaiming any storage until the allocator runs out of space, at which point it looks for all storage that can be referenced and moves all unreferenced storage back to the free list. It does this in two passes. The first pass follows all pointers recursively, starting with the current environment or symbol table, and marks each block of storage reached. This process requires an extra bit of storage for the marking. A second pass then sweeps linearly through memory, returning unmarked blocks to the free list.

In method, unlike with reference counts, freeing blocks with circular references is not a problem. However, mark and sweep requires extra storage. It also suffers from another serious drawback: The double pass through memory causes a significant delay in processing, sometimes as much as a few seconds, each time the garbage collector is invoked, which can be every few minutes. This is clearly unacceptable for many applications involving interactive or immediate response. However, many modern LISP and Smalltalk implementations are still plagued by this issue.

One way around the problem is to split available memory into two halves and allocate storage only from one-half at a time. Then, during the marking pass, all reached blocks are immediately copied to the second half of storage not in use. In this method, which is often called **stop and copy**, no extra mark bit is required in storage, and only one pass is required. It also performs compaction automatically. Once all reachable blocks in the used area have been copied, the used and unused halves of memory are interchanged, and processing continues. However, this does little to improve processing delays during storage reclamation.

In the 1980s, a method was invented that reduces this delay significantly. Called **generational garbage collection**, it adds a permanent storage area to the reclamation scheme of the previous paragraph. Allocated objects that survive long enough are simply copied into permanent space and are never deallocated during subsequent storage reclamations. This means that the garbage collector needs to search only a very small section of memory for newer storage allocations, and the time for such a search is reduced to a fraction of a second. Of course, it is possible for permanent memory still to become exhausted with unreachable storage, but this is a much less severe problem than before, since temporary storage tends to disappear quickly, while storage that stays allocated for some time is small and tends to persist anyway. This process has been shown to work very well, especially with a virtual memory system.

10.6 Exception Handling and Environments

In the previous chapter, we described exception handling in some detail in terms of its syntax, semantics, and use. This section introduces some implementation issues, particularly as they affect the structure and behavior of the runtime environment. Further detail can be found in the sources listed in the Notes and References.

In principle, the operations of raising and handling exceptions are similar to procedure calls, and they can be implemented in similar ways. However, there are major differences as well. Primary among these are:

1. An activation cannot be created on the runtime stack to represent the raising of an exception, since this stack itself may be unwound as previously described when searching for a handler.
2. A handler must be found and called dynamically, rather than statically as with ordinary function calls.
3. The actions of a handler are based on the type of the exception, rather than the exception value itself; thus, exception type information must be retained during execution, contrary to standard practice for statically typed languages.[13]

The first difference can be easily overcome. When an exception is raised, no activation record is created. Instead, the exception object and its type information are placed in a known location, such as a register or static memory. A jump is then performed to generic code that looks for a handler. Exit code is called if a handler is not found. The return address for the successful handling of an exception must also be stored in a known location. Under the termination model, this address is the location following the block in which the exception occurred. Otherwise, it is the return address of the most recent call, if the block is a procedure.

The second difference is more problematic. Pointers to handlers must, at least in theory, be kept on some kind of stack. Each time code is entered that has an associated handler, a new handler pointer is pushed, and when that same code is exited, the pointer is popped again to reveal any previous handlers. If this stack is to be implemented directly, it must be maintained either on the heap or elsewhere in its own memory area (i.e., not on the runtime stack), and a pointer to the current stack top must be maintained, either in static memory or in a register.

The third difference is somewhat less problematic but may involve some complexity that we will not go into here. The main issue is how to record the needed type information (essentially the type names) without added overhead in the exception structures themselves. One possibility is to have some sort of lookup table.

Once these issues have been addressed, the implementation of handlers is relatively straightforward. The basic idea is to collect all handler code attached to a particular block into a single handler implemented as a switch statement, based on the type of the exception parameter received, with a

[13]Of course, languages with dynamic types, such as LISP and Smalltalk, already have type information available at runtime. C++ and Java also have mechanisms available for preserving and extracting type information when necessary at runtime; in C++ the mechanism is called RTTI (for Run Time Type Information); in Java it is called Reflection.

default case that pops the handler stack (adjusting the current return address) and, if necessary, pops the runtime stack before reraising the same exception. (Procedure blocks must at least have this latter default handler, even if no exceptions are explicitly handled.) For example, consider the following handler code:

```
void factor() throw (Unwind,InputError)
{ try
  {...}
  catch (UnexpectedChar u)
  {...}
  catch (Unwind u)
  {...}
  catch (NumberExpected n)
  {...}
}
```

The handler for this `try` block might be implemented as in the following pseudocode:

```
switch (exception.type)
{ case UnexpectedChar: ...; break;
  case Unwind : ...; break;
  case NumberExpected: ... ; break;
  default: pop the runtime stack and reraise the exception;
}
pop the runtime stack and return to caller;
```

The main problem with the implementation techniques described so far is that the maintenance of the handler stack causes a potentially significant runtime penalty, even for code that does not use exception handling. One would like to provide an alternative to the handler stack that can be statically generated and, thus, cost nothing for code that does not use exception handling (this was a major design goal for C++). Such an alternative is a sorted table of code addresses that records the available handlers. When an exception occurs, the exception code performs a lookup based on the code address where the exception occurred (a binary search of the address table, for example). If no handler is found, the block in which the exception occurred is exited and the exit address is used for a new lookup.

Of course, such an address table has its own problems. First, the table itself can be quite large, causing the memory use of the program to grow significantly (a classic time-space tradeoff). Second, when an exception does occur, there can be an even greater execution speed penalty because of multiple searches of the address table. In a typical C++ system, for example, handling an exception may be as much as two orders of magnitude slower than a simple jump such as a loop exit. Thus, it makes even more sense in such systems to avoid using exceptions for routine control flow, and save them for truly unusual events.

10.7 Case Study: Processing Parameter Modes in TinyAda

Being a parameter is one of the roles that an identifier can play in TinyAda. In the role analysis discussed in Section 7.8.2, the use of parameters was subjected to the same restrictions as the use of variables. They could appear in any expression or on the left side of an assignment statement. In Section 9.6, we saw that neither variables nor parameters can appear in static expressions. In the present chapter, we have seen how parameters can be further specified in terms of the roles they play in conveying information to and from procedures. These new roles are called parameter modes, and they allow us to apply a new set of constraints during semantic analysis, to which we now turn.

10.7.1 The Syntax and Semantics of Parameter Modes

The syntax for declaring TinyAda's parameter modes is specified in the two syntax rules for parameter specifications:

```
parameterSpecification = identifierList ":" mode <type>name

mode = [ "in" ] | "in" "out" | "out"
```

Parameter modes are marked by the single reserved words `in` and `out`, or by these two words in succession, or by no marker at all. The last option is a default, which has the same meaning as `in` only.

As discussed in Section 10.3.5, a parameter declared with the mode of `in` is intended for input to a procedure only. As such, it can appear in any expression to be referenced for its value, but it cannot be the target of an assignment statement. Also, an `in` parameter cannot be passed as an actual parameter to a procedure that is expecting an `out` or an `in out` parameter at that position in its formal parameter list. Inversely, an `out` parameter is intended for output from a procedure only, so it cannot be referenced for its value. Thus, an `out` parameter can only be the target of an assignment statement, or be passed as an actual parameter to a procedure that expects another `out` parameter in that position in its formal parameter list. Finally, an `in out` parameter can be used for both input and output in a procedure. Thus, it can appear anywhere in the body of a procedure that a variable can. Table 10.1 summarizes the static semantics of TinyAda's parameter modes.

Table 10.1 The static semantics of TinyAda's parameter modes

Parameter Mode	Role in a Procedure	Semantic Restrictions
`in`	Input only	May appear only in expressions, or only in the position of an `in` parameter when passed as an actual parameter to a procedure.
`out`	Output only	May appear only as the target of an assignment statement, or only in the position of an `out` parameter when passed as an actual parameter to a procedure.
`in out`	Input and output	May appear anywhere that a variable may appear.

10.7.2 Processing Parameter Declarations

To store information about a parameter's mode, the `SymbolEntry` class is modified to include a `mode` field, with the possible values `SymbolEntry.IN`, `SymbolEntry.OUT`, and `SymbolEntry.IN_OUT`. The method `setMode` is also provided to install a parameter mode into a list of parameter identifiers. Armed

with these changes, we can now recast the parsing method for `parameterSpecification` to recognize and enter parameter mode information, as follows:

```
/*
parameterSpecification = identifierList ":" mode <type>name
*/
private SymbolEntry parameterSpecification(){
   SymbolEntry list = identifierList();
   list.setRole(SymbolEntry.PARAM);
   accept(Token.COLON, "':' expected");
   if (token.code == Token.IN){
      token = scanner.nextToken();
      list.setMode(SymbolEntry.IN);
   }
   if (token.code == Token.OUT){
      token = scanner.nextToken();
      if (list.mode == SymbolEntry.IN)
         list.setMode(SymbolEntry.IN_OUT);
      else
         list.setMode(SymbolEntry.OUT);
   }
   if (list.mode == SymbolEntry.NONE)
      list.setMode(SymbolEntry.IN);
   SymbolEntry entry = findId();
   list.setType(entry.type);
   acceptRole(entry, SymbolEntry.TYPE, "type name expected");
   return list;
}
```

Note that the default value of the mode in the symbol entry is `SymbolEntry.NONE`, which will be the case if the parser does not see any of the explicit reserved words for modes in the token stream. In that case, the mode is reset to `SymbolEntry.IN`.

10.7.3 Processing Parameter References

Table 10.1 provides the guidelines for checking references to parameters in a TinyAda source program. First, let's consider parameter names that appear in expressions and assignment statements. A name in an expression is detected in the method `primary`. If that name is a parameter, then its mode cannot be `OUT`. Otherwise, the parameter name is accepted. A name that is the target of an assignment statement is detected in the method `assignmentOrCallStatement`. If that name is a parameter, then its mode cannot be `IN`. Otherwise, the parameter name is accepted.

The only other location where parameter modes must be checked is during the parsing of actual parameters to procedure calls. The method `actualParameterPart` was developed in Section 8.10.5 to match the number and types of actual parameters against those of the procedure's formal parameters. This method must now also examine the modes of the formal parameters in order to apply the new constraints listed in

Table 10.1. If the mode of the formal parameter is `IN`, then the method `expression` is called, as before, to parse the actual parameter. Otherwise, the mode of the formal parameter must be `OUT` or `IN_OUT`, so the method `name` is called to parse the actual parameter. Then, the name just processed must be a variable or a parameter. If the name is a parameter, its mode cannot be `IN`. Otherwise, the parameter name is accepted.

The modification of these parsing methods is left as an exercise for the reader.

Exercises

10.1 Beginning programmers often add a `return` statement at the end of each procedure:

```
void inc (int* x)
{ (*x)++;
  return;
}
```

The thinking is that, as with `switch` statements, there might be a "fall-through" if the `return` is not there. Could a function mechanism be designed that would allow this? Would it make sense to have such a mechanism?

10.2 What if we wrote an increment procedure in C as follows (removing the parentheses around `*x`):

```
void inc (int* x)
{ *x++;
}
```

Is this legal? If so, what does this procedure do? If not, why not?

10.3 Can a `swap` procedure that interchanges the values of its parameters be written in Java? Discuss.

10.4 **(a)** Consider the `intswap` C++ procedure of Section 10.1:

```
void intswap (int& x, int& y)
{ int t = x;
  x = y;
  y = t;
}
```

Is this procedure in closed form? Why or why not?

(b) Is the following polymorphic `swap` procedure in C++ in closed form (why or why not)?

```
template <typename T>
void swap (T& x, T& y)
{ T temp = x;
  x = y;
  y = temp;
}
```

10.5 Suppose we tried to write a C procedure that would initialize an array to a certain size as follows:

```
void init(int a[], int size)
{ a = (int*) malloc(size*sizeof(int));
}
```

What, if anything, is wrong with this? Explain.

10.6 Here are some function declarations in C:

```
void f(int x, double y);
void f(int a, double);
void f(int, double z);
void f(int y, double q)
{ if (q > y) printf("ok!\n");
  else printf("not ok!\n");
}
```

Would it be legal to include all of the declarations in the same code file? Explain.

10.7 Suppose that we tried to write a procedure in Java that would append "hello" to the end of a string:

```
class TestParams
{ public static void appendHello( String s)
  { s = s+"hello";
  }
  public static void main( String[] args)
  { String s = "Greetings! ";
    appendHello(s);
    System.out.println(s);
  }
}
```

Explain the behavior of this program. Can it be fixed?

10.8 Give the output of the following program (written in C syntax) using the four parameter-passing methods discussed in Section 10.3:

```
int i;
int a[2];

void p( int x, int y)
{ x++;
  i++;
  y++;
}
main()
{ a[0] = 1;
```

(continues)

(continued)

```
  a[1] = 1;
  i = 0;
  p(a[i],a[i]);
  printf("%d\n",a[0]);
  printf("%d\n",a[1]);
  return 0;
}
```

10.9 Give the output of the following program using the four parameter-passing methods of Section 10.3:

```
int i;
int a[3];

void swap( int x, int y)
{ x = x + y;
  y = x - y;
  x = x - y;
}

main()
{ i = 1;
  a[0] = 2;
  a[1] = 1;
  a[2] = 0;
  swap(i,a[i]);
  printf("%d %d %d %d\n", i, a[0], a[1], a[2]);
  swap(a[i],a[i]);
  printf("%d %d %d\n", a[0], a[1], a[2]);
  return 0;
}
```

10.10 FORTRAN has the convention that all parameter passing is by reference. Nevertheless, as described in the chapter, it is possible to call subroutines with nonvariable expressions as arguments, as in `CALL P(X,X+Y,2)`.

(a) Explain how one can pass a variable `X` by value in FORTRAN.

(b) Suppose that subroutine `P` is declared as follows:

```
SUBROUTINE P(A)
INTEGER A
PRINT *, A
A = A + 1
RETURN
END
```

and is called from the main program as follows:

```
CALL P(1)
```

In some FORTRAN systems, this will cause a runtime error. In others, no runtime error occurs, but if the subroutine is called again with 1 as its argument, it may print the value 2. Explain how both behaviors might occur.

10.11 In Ada the compiler checks to make sure that the value of an `in` parameter is not changed in a procedure, and then allows `in` parameters to be implemented using pass by reference. Why not insist on pass by value for `in` parameters, so that the compiler check can be eliminated (as in C and Java)?

10.12 Ada restricts the parameters in a function declaration to be `in` parameters. Why is this?

10.13 A variation on pass by name is **pass by text**, in which the arguments are evaluated in delayed fashion, just as in pass by name, but each argument is evaluated in the environment of the called procedure, rather than in the calling environment. Show that pass by text can have different results from pass by name.

10.14 The Algol60 definition states the following **substitution rule** for pass by name:

1. Any formal parameter … is replaced, throughout the procedure body, by the corresponding actual parameter, after enclosing this latter in parentheses wherever syntactically possible. Possible conflicts between identifiers inserted through this process and other identifiers already present within the procedure body will be avoided by suitable systematic changes of the formal or local identifiers involved.
2. Finally the procedure body, modified as above, is inserted in place of the procedure statement and executed. If the procedure is called from a place outside the scope of any nonlocal quantity of the procedure body the conflicts between the identifiers inserted through this process of body replacement and the identifiers whose declarations are valid at the place of the procedure statement or function designator will be avoided through suitable systematic changes of the latter identifiers (Naur [1963a], p. 12).
   (a) Give an example of an identifier conflict as described in rule 1.
   (b) Give an example of an identifier conflict as described in rule 2.
   (c) Carry out the replacement process described in these two rules on the program in Figure 10.4.

10.15 The following two exercises relate to Jensen's device and pass by name, as discussed in Section 10.3.4.

(a) Write a function that uses Jensen's device and pass by name to compute the scalar product of two vectors declared as integer arrays with the same number of elements.

(b) The following `sum` procedure (in C syntax) was used as an example of pass by name and Jensen's device at the end of Section 10.3.4:

```
int sum (int a, int index, int size)
{ int temp = 0;
  for (index = 0; index < size; index++) temp += a;
  return temp;
}
```

Rewrite this code using a function parameter for the parameter `a` to imitate pass by name, so that it actually executes like pass by name in C.

10.16 The text did not discuss the scope of parameter declarations inside function or procedure declarations.

(a) Describe the scope of the declaration of x in the following C procedure:

```
void p( int x)
{  int y, z;
   ...
}
```

(b) State whether the following C procedure declarations are legal, and give reasons:

```
void p( int p) { ... }
void q( int x) { int x; ... }
```

10.17 Draw the stack of activation records for the following C program after (a) the call to r on line 13 and (b) after the call to p on line 14. Show the control links, and fill in the local names in each activation record. (c) Describe how variables r and x are found on lines 4 and 5 during the execution of p.

```
(1)  int x;
(2)  void p(void)
(3)  { double r = 2;
(4)    printf("%g\n",r);
(5)    printf("%d\n",x);
(6)  }
(7)  void r(void)
(8)  { x = 1;
(9)    p();
(10) }
(11) void q(void)
(12) { double x = 3;
(13)   r();
(14)   p();
(15) }
(16) main()
(17) { p();
(18)   q();
(19)  return 0;
(20) }
```

10.18 Draw the stack of activation records for the following Ada program **(a)** after the first call to procedure `b`; **(b)** after the second call to procedure `b`. Show the control links *and* the access links for each activation record. **(c)** Indicate how `b` is found in procedure `c`.

```
procedure env is

  procedure a is

    procedure b is

      procedure c is
      begin
        b;
      end c;

    begin
      c;
    end b;

  begin
    b;
  end a;

begin
  a;
end env;
```

10.19 The following Ada program contains a function parameter.
(a) Draw the stack of activation records after the call to `g` in `p`. (b) What does the program print and why?

```
with Text_IO; use Text_IO;
with Ada.Integer_Text_IO;
use Ada.Integer_Text_IO;

procedure params is
  procedure q is
  type IntFunc is access function (n:integer) return integer;
  m: integer := 0;

  function f (n: integer) return integer is
  begin
    return m + n;
  end f;
  procedure p (g: IntFunc) is
```

(continues)

(continued)

```
        m: integer := 3;
      begin
        put(g(2)); new_line;
      end p;

      begin
        p(f' access);
      end q;

    begin
      q;
    end params;
```

10.20 Show that the access link in an activation record (the "static" link) isn't static.

10.21 Suppose that we tried to avoid adding an access link to each activation record in a language with nested procedures by devising a mechanism for counting our way through dynamic links until the appropriate activation was reached (dynamic links will always lead eventually to the correct lexical environment). Why can't this work?

10.22 FORTRAN allows the passing of variable-length arrays, as in the following examples:

```
SUBROUTINE P(A,I)
INTEGER I
REAL A(*)
...
SUBROUTINE Q(B,N)
INTEGER N,B(N)
...
```

Does this cause any problems for the activation records of `P` and `Q`, which in FORTRAN must have fixed size and location?

10.23 Some FORTRAN implementations use pass by value-result rather than pass by reference. Would this affect your answer to the previous exercise? Why?

10.24 In C, blocks that are not procedures, but have variable declarations, such as:

```
{ int i;
  for (i=1; i<n; i++) a[i] = i;
}
```

can be allocated an activation record just as procedures. What fields in the activation record are unused or redundant in this case? Can such blocks be implemented without a new activation record?

10.25 Suppose that we disallowed recursion in Ada. Would it be possible to construct a fully static environment for the language? Is the same true for C?

10.26 Describe how, in a language without procedure parameters, the environment pointer of a procedure closure does not need to be stored with the procedure, but can be computed when the procedure is called.

10.27 Suppose that we wanted to state a rule that would make the C dangling reference created by the following function illegal:

```
int* dangle(void)
{ int x;
  return &x;
}
```

Suppose that we decided to make the following statement: The address of a local variable cannot be a returned value and cannot be assigned to a nonlocal variable.

(a) Can this rule be checked statically?

(b) Does this rule solve the problem of dangling references created by the use of a stack-based environment?

10.28 The example program `params` of Exercise 10.19 remains valid from the point of view of a stack-based runtime environment if the definitions of procedure `g` and the `IntFunc` type are moved outside of procedure `q`, and yet Ada considers the new program to be erroneous.

(a) Draw the new runtime environment after the call to `g` in `p` that results from this change to show that it is still valid.

(b) Discuss the reasons that Ada declares an error in this program (see the access-type lifetime rule discussion in Section 10.4.3).

10.29 The following program in Ada syntax has a function that has another function as its returned value, and so needs a fully dynamic runtime environment as described in Section 10.4.3. Draw a picture of the environment when `g` is called inside `b`. What should this program print?

```
with Text_IO; use Text_IO;
with Ada.Integer_Text_IO;
use Ada.Integer_Text_IO;

procedure ret is
  type IntFunc is
        access function (n:integer) return integer;

  function a return IntFunc is
  m: integer;

    function addm( n: integer) return integer is
    begin
      return (n + m);
    end addm;
```

(continues)

(continued)

```
    begin
      m := 0;
      return addm'access;
    end a;

    procedure b(g: IntFunc) is
    begin
      put(g(2));
    end b;

  begin
    b(a);
  end ret;
```

10.30 Using a debugger, try to determine whether your C++ compiler uses static address tables to look up exception handlers. Describe what the debugger tells you is occurring when an exception is raised.

10.31 Write declarations in a language of your choice for the maintenance of a free list of memory as **(a)** a circular singly linked list, and **(b)** a noncircular doubly linked list. Write procedures to deallocate a block using both of these data structures, being sure to coalesce adjacent blocks. Compare the ease of coalescing blocks in each case.

10.32 Show how compaction of memory can be accomplished by using a table to maintain an extra level of indirection, so that when a block of storage is moved, only one pointer needs to be changed.

10.33 In Section 10.5, a method of mark and sweep garbage collection was described in which memory is split into two sections, only one of which is used for allocation between calls to the garbage collector. It was claimed in the text that this eliminates the need for extra space to mark memory that has been seen. Describe how this can be accomplished.

10.34 In the generational method of garbage collection, it is still necessary eventually to reclaim "permanently" allocated storage after long execution times. It has been suggested that this could be made to interfere the least with execution by scheduling it to be performed "offline," that is, during periods when the running program is waiting for input or other processing. For this to be effective, the garbage collector must have a good interface with the operating system, especially if virtual memory is in use. Describe as many operating system issues as you can think of in this approach.

10.35 Modify the method `assignmentOrCallStatement` in the TinyAda parser so that it enforces the constraints on parameter names that are the targets of assignment statements.

10.36 Modify the method `primary` in the TinyAda parser so that it enforces the constraints on parameter names that appear in expressions.

10.37 Modify the method `actualParameterPart` in the TinyAda parser so that it enforces the constraints on actual parameters that are passed as arguments to procedures.

Notes and References

Structures of runtime environments are described in more detail in Louden [1997] and Aho, Sethi, and Ullman [1986]. Closures of procedures with nonlocal references are related to closures of lambda expressions with free variables; see Section 10.7. The access-type lifetime rule in Ada is discussed in more detail in Cohen [1996].

Dynamic memory management is treated in more detail in Horowitz and Sahni [1984] and Aho, Hopcroft, and Ullman [1983]. Traditionally, languages that have first-class procedures and, thus, must use automatic memory management, have been thought to be highly inefficient for that reason. Advances in algorithms, such as generational garbage collection, and advances in translation techniques make this much less true. References dealing with efficiency issues are Steele [1977] and Gabriel [1985]; see also Louden [1987]. Overviews of garbage collection techniques can be found in Wilson [1992] or Cohen [1981]. Generational garbage collection was first described in Ungar [1984]. A generational garbage collector and runtime environment for the functional language ML is described in Appel [1992].

Exception handling implementation is discussed in Scott [2000] and Stroustrup [1994]. Details of two implementations can be found in Koenig and Stroustrup [1990] and Lenkov et al. [1992]. See also Lajoie [1994ab].

CHAPTER

Abstract Data Types and Modules

CHAPTER 11

In Chapter 9, we defined a data type as a set of values, along with certain operations on those values. In that discussion, we divided data types into two kinds: predefined and user-defined. Predefined types, such as `integer` and `real`, are designed to insulate the user of a language from the implementation of the data type, which is machine-dependent. These data types can be manipulated by a set of predefined operations, such as the arithmetic operations, whose implementation details are also hidden from the user. Their use is completely specified by predetermined semantics, which are either explicitly stated in the language definition or are implicitly well known (like the mathematical properties of the arithmetic operations).

A language's user-defined types, on the other hand, are built from data structures. A data structure is not viewed as a simple value, but rather as a combination of simple values organized in a particular manner. A data structure is created from the language's built-in data types and the data structure's type constructors. The internal organization of a data structure is visible to the user, and it does not come with any operations other than the accessing operations of the data structure itself (such as the field selection operation on a record structure). One can, of course, define functions to operate on a data structure. However, in contrast to the standard procedure or function definitions available in most programming languages, these user-defined functions are not directly associated with the data type, and the implementation details of the data type and the operations are visible throughout the program.

It would be very desirable to have a mechanism in a programming language to construct data types that would have as many of the characteristics of a built-in type as possible. Such a mechanism should provide the following:

1. A method for defining both a data type and operations on that type. The definitions should all be collected in one place, and the operations should be directly associated with the type. The definitions should not depend on any implementation details. The definitions of the operations should include a specification of their semantics.
2. A method for collecting the implementation details of the type and its operations in one place, and of restricting access to these details by programs that use the data type.

A data type constructed using a mechanism satisfying one or both of these criteria is often called an **abstract data type** (or **ADT** for short). Note, however, that such a type is not really more abstract than an ordinary built-in type. It is just a more comprehensive way of creating user-defined types than the usual type declaration mechanism of Algol-like languages.

Criteria 1 and 2 promote three important design goals for data types: modifiability, reusability, and security. Modifiability is enhanced by interfaces that are implementation-independent, since changes can be made to an implementation without affecting its use by the rest of the program. Reusability is enhanced by standard interfaces, since the code can be reused by or plugged into different programs. Security is enhanced by protecting the implementation details from arbitrary modification by other parts of a program.

Some language authors dispense with criteria 1 and 2 in favor of encapsulation and information hiding, which they consider the essential properties of an abstract data type mechanism. **Encapsulation** refers to: a) the collection of all definitions related to a data type in one location; and b) restricting the use of the type to the operations defined at that location. **Information hiding** refers to the separation of implementation details from these definitions and the suppression of these details in the use of the data type. Since encapsulation and information hiding are sometimes difficult to separate in practice, we will instead focus on criteria 1 and 2 as our basis for studying abstract data types.

Confusion can sometimes result from the failure to distinguish a **mechanism** for constructing types in a programming language that has the foregoing properties, with the **mathematical concept** of a type, which is a conceptual model for actual types. This second notion is sometimes also called an abstract data type, or abstract type. Such mathematical models are often given in terms of an **algebraic specification**, which can be used to create an actual type in a programming language using an abstract data type mechanism with the foregoing properties.

Even more confusion exists over the difference between abstract data types and **object-oriented programming**, the subject of Chapter 6. Object-oriented programming emphasizes the capability of language entities to control their own use during execution and to share operations in carefully controlled ways. In an object-oriented programming language, the primary language entity is the object—that is, something that occupies memory and has state. Objects are also active, however, in that they control access to their own memory and state. In this sense, an abstract data type mechanism lends itself to the object-oriented approach, since information about a type is localized and access to this information is controlled. However, abstract data type mechanisms do not provide the level of active control that represents true object-oriented programming. See Section 11.7 and Chapter 6 for more details.

The notion of an abstract data type is in fact independent of the language paradigm (functional, imperative, object-oriented) used to implement it, and languages from all three paradigms are used in this chapter as examples of different approaches to ADTs.

An ADT mechanism is often expressed in terms of a somewhat more general concept called a **module**, which is a collection of services that may or may not include a data type or types. In the sections that follow, we describe a method for specifying abstract data types and then introduce some standard abstract data type mechanisms in programming languages. Next, we describe some aspects of modules and separate compilation, which we compare to abstract data types, with examples from C, C++, and Java. Additional sections provide examples from Ada and ML. We also survey some of the limitations of these mechanisms. In the last section, we discuss the mathematics of abstract data types.

11.1 The Algebraic Specification of Abstract Data Types

In this section, we focus on one example of an abstract data type, the **complex** data type, which is not a built-in data type in most programming languages. Mathematically, complex numbers are well-known objects and are extremely useful in practice, since they represent solutions to algebraic equations. A complex number is usually represented as a pair of real numbers (x, y) in Cartesian coordinates, which are conventionally written as $x + iy$, where i is a symbol representing the complex number $\sqrt{-1}$. x is called the "real" part, and y is called the "imaginary" part. There are also other ways to represent complex numbers—for example, as polar coordinates (r,θ). In defining complex numbers, however, we shouldn't need to specify their representation, but only the operations that apply to them. These include the usual arithmetic operations +, −, *, and /. We also need a way to create a complex number from a real and imaginary part, as well as functions to extract the real and imaginary parts from an existing complex number.

A general specification of a data type must include the name of the type and the names of the operations, including a specification of their parameters and returned values. This is the **syntactic specification** of an abstract data type and is often also called the **signature** of the type. In a language-independent specification, it is appropriate to use the function notation of mathematics for the operations of the data type: Given a function f from set X to set Y, X is the domain, Y is the range, and we write f: $X \rightarrow Y$. Thus, a signature for the complex data type looks like this:

type complex **imports** real

operations:

+: complex × complex → complex
-: complex × complex → complex
*: complex × complex → complex
/: complex × complex → complex
-: complex → complex
makecomplex: real × real → complex
realpart: complex → real
imaginarypart: complex → real

Note that the dependence of the complex data type on an already existing data type, namely, real, is made explicit by the "imports real" clause. (Do not confuse this imports clause with the `import` statement in some languages.) Note also that the negative sign is used for both subtraction and negation, which are two different operations. Indeed, the usual names or symbols are used for all the arithmetic operations. Some means must eventually be used to distinguish these from the arithmetic operations on the integers and reals (or they must be allowed to be overloaded, as some languages provide).

The preceding specification lacks any notion of semantics, or the properties that the operations must actually possess. For example, $z * w$ might be defined to be always 0! In mathematics, the semantic properties of functions are often described by **equations** or **axioms**. In the case of arithmetic operations, examples of axioms are the associative, commutative, and distributive laws. In the equations that follow, x, y, and z are assumed to be variables of type complex; that is, they can take on any complex value.

$x + (y + z) = (x + y) + z$ (associativity of +)
$x * y = y * x$ (commutativity of *)
$x * (y + z) = x * y + x * z$ (distributivity of * over +)

Axioms such as these can be used to define the semantic properties of complex numbers, or the properties of the complex data type can be **derived** from those of the real data type by stating properties of the operations that lead back to properties of the real numbers. For example, complex addition can be based on real addition according to the following properties:

realpart(x + y) = realpart(x) + realpart(y)
imaginarypart(x + y) = imaginarypart(x) + imaginarypart(y)

The appropriate arithmetic properties of the complex numbers can then be proved from the corresponding properties for reals. A complete algebraic specification of type complex combines signature, variables, and equational axioms:[1]

type complex **imports** real

operations:

+: complex × complex → complex
=: complex × complex → complex
*: complex × complex → complex
/: complex × complex → complex
-: complex → complex
makecomplex : real × real → complex
realpart : complex → real
imaginarypart : complex → real

variables: x,y,z: complex; r,s: real

axioms:

realpart(makecomplex(r,s)) = r
imaginarypart(makecomplex(r,s)) = s
realpart(x + y) = realpart(x) + realpart(y)
imaginarypart(x + y) = imaginarypart(x) + imaginarypart(y)
realpart(x - y) = realpart(x) - realpart(y)
imaginarypart(x - y) = imaginarypart(x) - imaginarypart(y)
. . .
(more axioms)
. . .

Such a specification is called an **algebraic specification** of an abstract data type. It provides a concise specification of a data type and its associated operations. The accompanying equational semantics give a clear indication of implementation behavior, often containing enough information to allow coding directly from the equations. Finding an appropriate set of equations, however, can be a difficult task. The next part of this section gives some suggestions on how to find them.

Remember the difference in the foregoing specification between **equality** as used in the axioms and the **arrow** of the syntactic specification of the functions. Equality is of values returned by functions, while the arrows separate a function's domain and range.

[1]The variables section of a specification is a notational convenience that simply lists the names (and their types) over which the axioms are assumed to be universally quantified in the mathematical sense. Thus, the first axiom in this algebraic specification is actually "for all real r and s, realpart(makecomplex(r,s)) = s."

A second example of an algebraic specification of an abstract data type is the following specification of a queue:

type queue(element) **imports** boolean

operations:

createq: queue
enqueue: queue × element → queue
dequeue: queue → queue
frontq: queue → element
emptyq: queue → boolean

variables: *q*: queue; *x*: element

axioms:

emptyq(createq) = true
emptyq(enqueue(*q*,*x*)) = false
frontq(createq) = error
frontq(enqueue(*q*,*x*)) = if emptyq(*q*) then *x* else frontq(*q*)
dequeue(createq) = error
dequeue(enqueue(*q*,*x*)) = if emptyq(*q*) then *q* else enqueue(dequeue(*q*),*x*)

This specification exhibits several new features. First, the data type queue is **parameterized** by the data type element, which is left unspecified. Such a type parameter can be replaced by any type. You indicate it by placing its name inside parentheses, just as a function parameter would be. Second, the specification includes a constant, create, that is technically not a function. Such constants may also be a part of the algebraic specification; if desired, createq can be viewed as a function of no parameters that always returns the same value. Intuitively, we can see that createq is a new queue that has been initialized to empty. Third, we now include axioms that specify error values, such as

frontq(createq) = error

Such axioms can be called **error axioms**, and they provide limitations on the application of the operations. The actual error value of an error axiom is unspecified. Finally, the equations are specified using an if-then-else function, whose semantics are as follows:

if true then *a* else *b* = *a*
if false then *a* else *b* = *b*

Note also that the dequeue operation as specified does not return the front element of the queue, as in most implementations: It simply throws it away. Abstractly, this is simpler to handle and does not take away any functionality of the queue, since the frontq operation can extract the front element before a dequeue operation is performed.

The equations specifying the semantics of the operations in an algebraic specification of an abstract data type can be used as a specification of the properties of an implementation. It can also be used as a guide to the code for an implementation, and to prove specific properties about objects of the type.

For example, the following applications of axioms show that dequeuing from a queue with one element leaves an empty queue:

```
emptyq(dequeue(enqueue (createq, x)))
        = emptyq(createq)        (by the sixth axiom above)
        = true                   (by the first axiom)
```

Note that the operations and the axioms are specified in a mathematical form that includes no mention of memory or of assignment. Indeed, enqueue returns a newly constructed queue every time a new element is added. These specifications are in fact in purely functional form, with no variables and no assignment. (The variables in the specification are taken in the mathematical sense to be just names for values.) This is important, since mathematical properties, such as the semantic equations, are much easier to express with purely functional constructions. (See Chapter 4 for more on purely functional programming.) In practice, abstract data type implementations often replace the specified functional behavior with an equivalent imperative one using explicit memory allocation, as well as assignment or update operations. The question of how to infer that an imperative implementation is in some sense equivalent to a purely functional one, such as the above, is beyond the scope of our study here.

How do we find an appropriate axiom set for an algebraic specification? In general, this is a difficult question. We can, however, make some judgments about the kind and number of axioms necessary by looking at the syntax of the operations. An operation that creates a new object of the data type being defined is called a **constructor**, while an operation that retrieves previously constructed values is called an **inspector**. In the queue example, createq and enqueue are constructors, while frontq, dequeue, and emptyq are inspectors. Inspector operations can also be broken down into **predicates**, which return Boolean values, and **selectors**, which return non-Boolean values. Thus frontq and dequeue are selectors,[2] while emptyq is a predicate.

In general, we need one axiom for each combination of an inspector with a constructor. For the queue example, the axiom combinations are:

```
emptyq(createq)
emptyq(enqueue(q,x))
frontq(createq)
frontq(enqueue(q,x))
dequeue(createq)
dequeue(enqueue(q,x))
```

According to this scheme, there should be six rules in all, and that is in fact the case.

As a final example of an algebraic specification of an abstract data type, we give a specification for the stack abstract data type, which has many of the same features as the queue ADT:

type stack(element) **imports** boolean

operations:

createstk: stack
push: stack × element → stack
pop: stack → stack
top: stack → element
emptystk: stack → boolean

[2]Strictly speaking, dequeue is a non-primitive constructor. We call it a selector because it acts like one in this context.

variables: *s*: stack; *x*: element

axioms:

emptystk(createstk) = true
emptystk(push(*s*,*x*)) = false
top(createstk) = error
top(push(*s*,*x*)) = *x*
pop(createstk) = error
pop(push(*s*,*x*)) = *s*

In this case, the operations createstk and push are constructors, the pop and top operations are selectors, and the emptystk operation is a predicate. By our proposed scheme, therefore, there should again be six axioms.

11.2 Abstract Data Type Mechanisms and Modules

In this section, we explore some mechanisms for creating abstract data types and the manner in which modules represent an extension of these mechanisms.

11.2.1 Abstract Data Type Mechanisms

Some languages have a specific mechanism for expressing abstract data types. Such a mechanism must have a way of separating the specification or signature of the ADT (the name of the type being specified, and the names and types of the operations) from its implementation (a data structure implementing the type and a code body for each of the operations). Such a mechanism must also guarantee that any code outside the ADT definition cannot use details of the implementation but can operate on a value of the defined type only through the provided operations.

ML is an example of a language with a special ADT mechanism, called `abstype`. It is viewed as a kind of type definition mechanism. In Figure 11.1 it is used to specify a queue.

```
(1) abstype 'element Queue = Q of 'element list
(2) with
(3)  val createq = Q [];
(4)  fun enqueue (Q lis, elem) = Q (lis @ [elem]);
(5)  fun dequeue (Q lis) = Q (tl lis);
(6)  fun frontq (Q lis) = hd lis;
(7)  fun emptyq (Q []) = true | emptyq (Q (h::t)) = false;
(8) end;
```

Figure 11.1 A queue ADT as an ML `abstype`, implemented as an ordinary ML list

Here we have used an ordinary list in ML (`'element list`, line 1). Since an `abstype` must be a new type and not a type defined elsewhere, we also had to wrap `'element list` in a constructor, for which we have used the single letter `Q`;[3] this constructor is not visible outside the definition of the `abstype`. Recall also that lists are written in ML using square brackets `[...]`, `@` is the append operator (line 4), `h::t` is a pattern for the list with head `h` and tail `t` (line 7), `hd` is the head function on lists (line 6), and `tl` is the tail function on lists (line 5). (For more on the details of the actual type constructor specification `Q of 'element list` see Chapter 9.)

When this definition is processed by the ML translator, it responds with a description of the signature of the type:

```
type 'a Queue
val createq = - : 'a Queue
val enqueue = fn : 'a Queue * 'a -> 'a Queue
val dequeue = fn : 'a Queue -> 'a Queue
val frontq = fn : 'a Queue -> 'a
val emptyq = fn : 'a Queue -> bool
```

Since ML has parametric polymorphism, the `Queue` type can be parameterized by the type of the element that is to be stored in the queue, and we have done that with the type parameter `'element` (reported as `'a` by the ML system). Notice how ML refuses to specify the internal structure in this description, writing a dash for the actual representation of `createq`, and using only `'a Queue` to refer to the type. Thus, all internal details of the representation of `Queue`, including its constructors, are suppressed and remain private to the code of the `abstype` definition.

With the above definition, the `Queue` type can be used in ML as follows:

```
- val q = enqueue(createq,3);
val q = - : int Queue
- val q2 = enqueue(q,4);
val q2 = - : int Queue
- frontq q2;
val it = 3 : int
- val q3 = dequeue q2;
val q3 = - : int Queue
- frontq q3;
val it = 4 : int
```

[3]It is a common practice to use the same name as the data type, namely `Queue`, for this constructor, but we have not done so in an attempt to make the code easier to understand.

As a second example, a complex number ADT can be specified in ML as shown in Figure 11.2.

```
(1) abstype Complex = C of real * real
(2) with
(3)  fun makecomplex (x,y) = C (x,y);
(4)  fun realpart (C (r,i)) = r;
(5)  fun imaginarypart (C (r,i)) = i;
(6)  fun +: ( C (r1,i1), C (r2,i2) ) = C (r1+r2, i1+i2);
(7)  infix 6 +: ;
(8)  (* other operations *)
(9) end;
```

Figure 11.2 A complex number ADT as an ML `abstype`

ML allows user-defined operators, which are called **infix functions** in ML. Infix functions can use special symbols, but they cannot reuse the standard operator symbols, since ML does not allow user-defined overloading. Thus, in line 6 of Figure 11.2, we have defined the addition operator on complex numbers to have the name `+:` (a plus sign followed by a colon). Line 7 makes this into an infix operator (with left associativity) and gives it the precedence level 6, which is the ML precedence level for the built-in additive operators such as `+` and `-`. The `Complex` type can be used as follows:

```
- val z = makecomplex (1.0,2.0);
val z = - : Complex
- val w = makecomplex (2.0,~1.0); (* ~ is negation *)
val w = - : Complex
- val x = z +: w;
val x = - : Complex
- realpart x;
val it = 3.0 : real
- imaginarypart x;
val it = 1.0 : real
```

11.2.2 Modules

An ADT mechanism such as we have described in Section 11.2.1—essentially an extension of the definition of an ordinary data type—is completely adequate, even superior, as a way of implementing an abstract data type in a language with strong type abstractions such as ML. However, even in ML it is not the end of the story, since a pure ADT mechanism does not address the entire range of situations where an ADT-like abstraction mechanism is useful in a programming language.

Consider, for example, a mathematical function package consisting of the standard functions such as sine, cosine, exponential, and logarithmic functions. Because these functions are closely related, it makes sense to encapsulate their definitions and implementations and hide the implementation details (which could then be changed without changing client code that uses these functions). However, such

a package is not associated directly with a data type (or the data type, such as `double`, is built-in or defined elsewhere). Thus, such a package does not fit the format of an ADT mechanism as described in Section 11.2.1.

Similarly, as shown in Figure 11.3, a programming language compiler is a large program that is typically split into separate pieces corresponding to the phases described in Chapter 1.

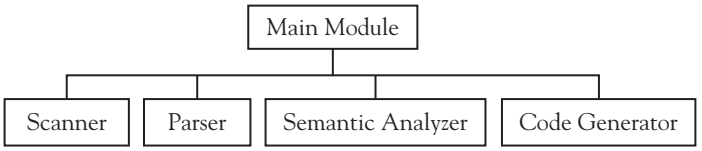


Figure 11.3 Parts of a programming language compiler

We would certainly wish to apply the principles of encapsulation and information hiding to each of the code pieces in this diagram. Here also, there is no special data type that can be associated directly with each phase. Instead, these examples demonstrate the need for an encapsulation mechanism that is viewed more generally as a provider of services, and that is given by the concept of a module:

DEFINITION: A ***module*** is a program unit with a public interface and a private implementation; all services that are available from a module are described in its public interface and are exported to other modules, and all services that are needed by a module must be imported from other modules.

As providers of services, modules can export any mix of data types, procedures, variables, and constants. Also, because modules have explicit (public) interfaces and separate (private) implementations, they are ideal mechansims to provide separate compilation and library facilities within a software development environment. Indeed, many languages, such as Ada and Java, tie the structure of modules to separate compilation issues (although the relationship is kept loose enough to allow for variation from system to system). Typically in a compiler-based system, the interface for each module in a library or program is kept in textual form, while the implementation may only be provided as object code and is only needed at the link step to produce a running program.

Thus, modules are an essential tool in program decomposition, complexity control, and the creation of libraries for code sharing and reuse. Additionally, modules assist in the control of **name proliferation**: large programs have large numbers of names, and mechanisms to control access to these names, as well as preventing name clashes, are an essential requirement of a modern programming language. Nested scopes are only the first step in such name control, and modules typically provide additional scope features that make the task of name control more manageable. Indeed, one view of a module is that its main purpose is to provide controlled scopes, exporting only those names that its interface requires and keeping hidden all others. At the same time, even the exported names should be **qualified** by the module name to avoid accidental name clashes when the same name is used by two different modules. Typically, this is done by using the same dot notation used for structures, so that feature `y` from module `X` has the name `X.y` when imported into client code. (Note that the ML `abstype` mechanism discussed previously does not have this essential name control feature.)

Finally, a module mechanism can document the dependencies of a module on other modules by requiring explicit import lists whenever code from other modules is used. These dependencies can be used by a compiler to automatically recompile out-of-date modules and to automatically link in separately compiled code.

In the following sections, we describe the mechanisms that several different languages (with very different approaches) offer to provide modular facilities.

11.3 Separate Compilation in C, C++ Namespaces, and Java Packages

We collect three different mechanisms in this section, because they are all less concerned with the full range of modular properties and are aimed primarily at separate compilation and name control.

11.3.1 Separate Compilation in C and C++

The first language we consider is C. C does not have any module mechanisms as such, but it does have separate compilation and name control features that can be used to at least simulate modules in a reasonably effective way. Consider, for example, a queue data structure in C. A typical organization would be to place type and function specifications in a so-called **header file** `queue.h`, as in Figure 11.5. Only type definitions and function declarations without bodies (called **prototypes** in C) go into this file. This file is used as a specification of the queue ADT by textually including it in client code as well as implementation code using the C preprocessor `#include` directive. Sample implementation code and client code are provided in Figures 11.6 and 11.7, respectively. Graphically, the separation of specification, implementation, and client code is as shown in Figure 11.4 (with the inclusion dependencies represented by the arrows).

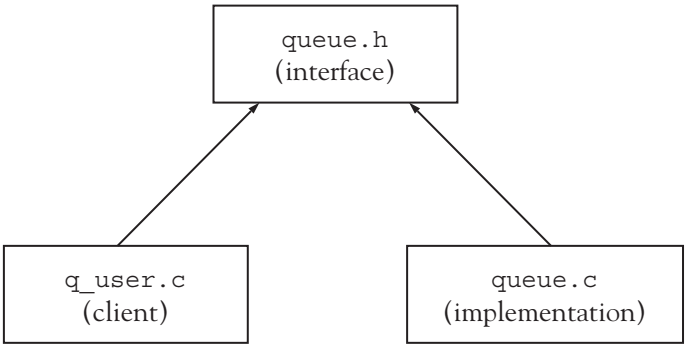


Figure 11.4 Separation of specification, implementation, and client code

```
(1) #ifndef QUEUE_H
(2) #define QUEUE_H

(3) struct Queuerep;
(4) typedef struct Queuerep * Queue;
```

Figure 11.5 A `queue.h` header file in C *(continues)*

(continued)

```
(5) Queue createq(void);
(6) Queue enqueue(Queue q, void* elem);
(7) void* frontq(Queue q);
(8) Queue dequeue(Queue q);
(9) int emptyq(Queue q);

(10) #endif
```

Figure 11.5 A `queue.h` header file in C

```
(1) #include "queue.h"
(2) #include <stdlib.h>
(3)  struct Queuerep
(4)  { void* data;
(5)    Queue next;
(6)  };

(7)  Queue createq()
(8)  { return 0;
(9)  }

(10) Queue enqueue(Queue q, void* elem)
(11) { Queue temp = malloc(sizeof(struct Queuerep));
(12)   temp->data = elem;
(13)   if (q)
(14)   { temp->next = q->next;
(15)     q->next = temp;
(16)     q = temp;
(17)   }
(18)   else
(19)   { q = temp;
(20)     q->next = temp;
(21)   }
(22)   return q;
(23) }

(24) void* frontq(Queue q)
(25) { return q->next->data;
(26) }

(27) Queue dequeue(Queue q)
(28) { Queue temp;
(29)   if (q == q->next)
```

Figure 11.6 A `queue.c` implementation file in C *(continues)*

(continued)

```
(30)   { temp = q;
(31)     q = 0;
(32)   }
(33)   else
(34)   { temp = q->next;
(35)     q->next = q->next->next;
(36)   }
(37)   free(temp);
(38)   return q;
(39) }

(40) int emptyq(Queue q)
(41) { return q == 0;
(42) }
```

Figure 11.6 A `queue.c` implementation file in C

```
(1)  #include <stdlib.h>
(2)  #include <stdio.h>
(3)  #include "queue.h"

(4)   main()
(5)   {  int *x = malloc(sizeof(int));
(6)      int *y = malloc(sizeof(int));
(7)      int *z;
(8)      Queue q = createq();
(9)      *x = 2;
(10)     *y = 3;
(11)     q = enqueue(q,x);
(12)     q = enqueue(q,y);
(13)     z = (int*) frontq(q);
(14)     printf("%d\n",*z); /* prints 2 */
(15)     q = dequeue(q);
(16)     z = (int*) frontq(q);
(17)     printf("%d\n",*z); /* prints 3 */
(18)     return 0;
(19)  }
```

Figure 11.7 A client code file `q_user.c` in C

We make the following comments about the code shown in Figures 11.5–11.7.

The header file `queue.h` exhibits the standard C convention of surrounding the code with an `#ifndef ... #endif` preprocessor block (Figure 11.5, lines 1, 2, and 10). This causes the symbol `QUEUE_H` (a conventional modification of the filename) to be defined the first time the file is included, and all repeated inclusions to be skipped. This is a necessary feature of header files, since there may be multiple paths through which a header file is included. Without it, the resulting repeated `Queue` type definition will cause a compilation error.

The definition of the `Queue` data type is hidden in the implementation by defining `Queue` to be a pointer type (Figure 11.5, line 4), and leaving the actual queue representation structure (`Queuerep`, line 3) as an **incomplete type**. If this were not done, then the entire `Queue` data structure would have to be declared in the header file (since the compiler would need to know its size) and, thus, exposed to clients. Clients would then be able to write code that depends on the implementation, thus destroying the desired implementation independence of the interface. On the other hand, making `Queue` into a pointer type has other drawbacks, since assignment and equality tests do not necessarily work as desired—see Section 11.7.

Ideally, a `Queue` data type needs be parameterized by the data type of the elements it stores, as you have seen. However, C does not have user polymorphism of any kind. Nevertheless, parametric polymorphism can be effectively simulated by the use of `void*` elements, which are "pointers to anything" in C and which can be used with casts to store any dynamically allocated object. Of course, there is the added burden of dynamically allocating all data and providing appropriate casts. Also, this code can be dangerous, because client code must always know the actual data type of the elements being stored.

The `queue.c` implementation file provides the actual definition for `Queuerep` (Figure 11.6, lines 3–6), and the code for the bodies of the interface functions. In the sample code, we have used a singly linked circular list with the queue itself represented by a pointer to the last element (the empty queue is represented by the null pointer 0).

The files `queue.c` and `q_user.c` can be separately compiled, and either can be changed without recompiling the other. For example, the queue implementation could be changed to a doubly linked list with pointers to the first and last elements (Exercise 11.9) without affecting any client code. Also, after the `queue.c` file is compiled, no further recompilation is needed (and the implementation can be supplied to clients as header and object file only). This makes the implementation and client code reasonably independent of each other.

This mechanism for implementing abstract data types works fairly well in practice; it is even better suited to the definition of modules that do not export types, such as a math module (see the standard C `math.h` file), where the complications of incomplete types and dynamic allocation do not arise. However, the effectiveness of this mechanism depends entirely on convention, since neither C compilers nor standard linkers enforce any of the protection rules normally associated with module or ADT mechanisms. For example, a client could fail to include the header file `queue.h`, but simply textually copy the definitions in `queue.h` directly into the `q_user.c` file (by cut and paste, for instance). Then the dependence of the client code on the queue code is not made explicit by the `#include` directive. Worse, if changes to the `queue.h` file were made (which would mean all client code should be recompiled), these changes would not automatically be imported into the client code, but neither the compiler nor the linker would report an error: not the compiler, since the client code would be consistent with the now-incorrect queue interface code, and not the linker, since the linker only ensures that some definition exists for every name, not that the definition is consistent with its uses. In fact, client code can circumvent the protection of the incomplete `Queuerec` data type itself by this same mechanism: If the actual structure of

`Queuerec` is known to the client, it can be copied into the client code and its structure used without any complaint on the part of the compiler or linker. This points out that this use of separate compilation is not a true module mechanism but a simulation, albeit a reasonably effective one.

We note also that, while the `#include` directives in a source code file do provide some documentation of dependencies, this information cannot be used by a compiler or linker because there are no language rules for the structure of header files or their relationship to implementation files. Thus, all C systems require that the user keep track of out-of-date source code and manually recompile and relink.[4]

One might suppose that C++ improves significantly on this mechanism, and in some ways it does. Using the class mechanism of object-oriented programming, C++ allows better control over access to the `Queue` data type. C++ also offers better control over names and name access through the `namespace` mechanism (next subsection), but C++ offers no additional features that would enhance the use of separate compilation to simulate modules.[5] In fact, such additional features would, as noted, place an extra burden on the linker to discover incompatible definitions and uses, and a strict design goal of C++ was that the standard C linker should continue to be used (Stroustrup [1994], page 120). Thus, C++ continues to use textual inclusion and header files to implement modules and libraries.

One might also imagine that the C++ template mechanism could be used to eliminate the use of `void*` to simulate parametric polymorphism in C. Unfortunately, templates in C++ do not mix well with separate compilation, since instantiations of template types require knowledge of the separately compiled template code by the C++ compiler. Thus, the use of templates would not only force the details of the `Queuerec` data type to be included in the header file, but, using most compilers, even the code for the function implementations would have to be put into the header file (and thus the entire contents of `queue.c` would have to be moved to `queue.h`!). In fact, the standard template library of C++ exhibits precisely this property. Virtually all the code is in the header files only. Fortunately, the use of access control within the class mechanism still allows some form of protection, but the situation is far from ideal. The use of templates in data types is, thus, most effective in conjunction with the class construct. See Chapter 6 for an example.

11.3.2 C++ Namespaces and Java Packages

The `namespace` mechanism in C++ provides effective support for the simulation of modules in C. The `namespace` mechanism allows the programmer to introduce a named scope explicitly, thus avoiding name clashes among separately compiled libraries. (However, the use of namespaces is not explicitly tied to separate compilation.)

Consider the previous example of a Queue ADT structure in C. A major problem with the implementation of this ADT is that other libraries are likely available in a programming environment that could well include a different implementation of a queue, but with the same name. Additionally, a programmer may want to include some code from that library as well as use the given Queue implementation. In C this is not possible. In C++ a namespace can be used to disambiguate such name clashes. Figure 11.8 lists the code for the `queue.h` file as it would look using a namespace in C++.

[4]The lack of such a module facility in C led directly to the construction of tools to manage compilation and linking issues, such as the UNIX `make` utility.

[5]Strictly speaking, the C++ standard does make it a violation of the "one definition rule" to have two different definitions of the same data structure that are not syntactically and semantically identical, but then allows linkers to fail to catch this error. See Stroustrup [1997], Section 11.2.3 for a discussion.

```
#ifndef QUEUE_H
#define QUEUE_H

namespace MyQueue
{ struct Queuerep;
  typedef Queuerep * Queue;
            // struct no longer needed in C++
  Queue createq();
  Queue enqueue(Queue q, void* elem);
  void* frontq(Queue q);
  Queue dequeue(Queue q);
  bool emptyq(Queue q);
}

#endif
```

Figure 11.8 The `queue.h` header file in C++ using a namespace

Inside the `queue.cpp` file, the definitions of the queue functions must now use the scope resolution operator, for example:

```
struct MyQueue::Queuerep
{ void* data;
  Queue next;
};
```

Alternatively, one could reenter the `MyQueue` namespace by surrounding the implementation code in `queue.cpp` with the same namespace declaration as in the header. (Namespaces can be reentered whenever one wishes to add definitions or implementations to a namespace.)

A client of `MyQueue` has three options:

1. After the appropriate `#include`, the user can simply refer to the `MyQueue` definitions using the qualified name (i.e., with `MyQueue::` before each name used from the `MyQueue` namespace).
2. Alternatively, the user can write a `using` declaration for each name used from the namespace and then use each such name without qualification.
3. Finally, the user can "unqualify" all the names in the namespace with a single `using namespace` declaration.

Consider Figure 11.9, which lists C++ client code for the above C++ queue code. This code exhibits all three of the above options to reference names in a namespace. A `using` declaration (option 2) is given on line 3, allowing the standard library function `endl` from `iostream` to be used without qualification. A `using namespace` declaration (option 3) is given on line 4, allowing all names in `MyQueue` to be

used without qualification, and the qualified name `std::cout` is used for the references to `cout` (lines 18 and 21).

```
(1)  #include <iostream>
(2)  #include "queue.h"

(3)  using std::endl;
(4)  using namespace MyQueue;

(5)  main()
(6)  {  int *x = new int;
(7)     int *y = new int;
(8)     int *z;
(9)     // explicit qualification unnecessary
(10)    // but permitted
(11)    Queue q = MyQueue::createq();
(12)    *x = 2;
(13)    *y = 3;
(14)    q = enqueue(q,x);
(15)    q = enqueue(q,y);
(16)    z = (int*) frontq(q);
(17)    // explicit qualification needed for cout
(18)    std::cout << *z << endl; // prints 2
(19)    q = dequeue(q);
(20)    z = (int*) frontq(q);
(21)    std::cout << *z << endl; // prints 3
(22) }
```

Figure 11.9 A client code file `q_user.cpp` in C++ with `using` declarations

Java, too, has a namespace-like mechanism, called the **package**. Since Java emphasizes object orientation and the class construct, packages are constructed as groups of related classes. Each separately compiled file in a Java program is allowed to have only one public class, and this class can be placed in a package by writing a package declaration as the first declaration in the source code file:

```
package myqueue;
```

Other Java code files can now refer to the data type `Queue` by the name `myqueue.Queue`, if the data type `Queue` is the public class in its file. The use of a Java `import` declaration also allows the programmer to dereference this name in a similar manner to the C++ `using` declaration:

```
import myqueue.Queue;
```

The use of an asterisk after the package name allows the programmer to import *all* names in a Java package:

```
import myqueue.*;
```

This is similar to the C++ `using namespace` declaration.

Note here that the Java `import` declaration does *not* correspond to the abstract notion of an import in the definition of a module in the previous section. Indeed, an import in the latter sense represents a dependency on external code, and it is more closely related to the C `#include` statement than to the Java `import` declaration. Code dependencies in Java are hidden by the fact that the compiler automatically searches for external code using the package name and, typically, a system search path (commonly called `CLASSPATH`). Package names can also contain extra periods, which are usually interpreted by Java as representing subdirectories. For example, the package `myqueue` might be best placed in a directory `mylibrary`, and then the package name would be `mylibrary.myqueue`. Using the fully qualified name of a library class allows any Java code to access any other public Java code that is locatable along the search path by the compiler, without use of an `import` declaration. Indeed, the compiler will also check for out-of-date source code files, and recompile all dependent files automatically. Thus, library dependencies can be well hidden in Java. Even the link step in Java does not need any dependency information, since linking is performed only just prior to execution by a utility called the **class loader**, and the class loader uses the same search mechanism as the compiler to find all dependent code automatically.

11.4 Ada Packages

Ada's module mechanism is the **package** (briefly discussed in Chapter 9), because the package is used in Ada not only to implement modules but also to implement parametric polymorphism.[6] An Ada package is divided into a package **specification** and a package **body**. A package specification is the public interface to the package, and it corresponds to the syntax or signature of an ADT (Section 11.1). Package specifications and package bodies also represent compilation units in Ada, and can be compiled separately.[7] Clients are also compiled separately and linked to compiled package bodies. A package specification in Ada for type `Complex` is given in Figure 11.10.

```
(1)  package ComplexNumbers is

(2)  type Complex is private;

(3)  function "+"(x,y: in Complex) return Complex;
(4)  function "-"(x,y: in Complex) return Complex;
(5)  function "*"(x,y: in Complex) return Complex;
(6)  function "/"(x,y: in Complex) return Complex;
(7)  function "-"(z: in Complex) return Complex;
(8)  function makeComplex (x,y: in Float) return Complex;
(9)  function realPart (z: in Complex) return Float;
```

Figure 11.10 A package specification for complex numbers in Ada *(continues)*

[6]Ada packages should not be confused with Java packages, which are a rather different concept; see the previous section.
[7]Some implementations may not require the compilation of a package specification, since the compiled version is essentially a symbol table with the same information as the specification file itself.

(continued)

```
(10) function imaginaryPart (z: in Complex) return Float;

(11) private
(12)    type Complex is
(13)    record
(14)       re, im: Float;
(15)    end record;
(16) end ComplexNumbers;
```

Figure 11.10 A package specification for complex numbers in Ada

This specification of a complex number ADT uses oveloaded operators in Ada. Additionally, this specification uses a **private** declaration section (lines 11–16). Any declarations given in this section are inaccessible to a client. Type names, however, can be given in the public part of a specification and designated as private (line 2). Note, however, that an actual type declaration must still be given in the private part of the specification. This violates both criteria for abstract data type mechanisms. Specifically, it violates criterion 1 in that the specification is still dependent on actual implementation details, and it violates criterion 2 in that the implementation details are divided between the specification and the implementation. However, the use of the private declarations at least prevents clients from making any use of the implementation dependency, although clients still must be recompiled if the implementation type changes. As in the C queue example of Section 11.2, this problem can be partially removed in Ada by using pointers, and a revised specification in Ada that removes the dependency of the specification on the implementation type (at the cost of introducing pointers) is as follows:

```
package ComplexNumbers is

type Complex is private;

-- functions as before

private
   type ComplexRecord;
   -- incomplete type defined in package body
   type Complex is access ComplexRecord;
   -- a pointer type
end ComplexNumbers;
```

A corresponding implementation for this somewhat improved package specification is given in Figure 11.11.

```
(1)  package body ComplexNumbers is

(2)  type ComplexRecord is record
(3)     re, im: Float;
(4)  end record;

(5)  function "+"(x,y: in Complex) return Complex is
(6)  t: Complex;
(7)  begin
(8)     t := new ComplexRecord;
(9)     t.re := x.re + y.re;
(10)    t.im := x.im + y.im;
(11)    return t;
(12) end "+";

(13) -- more operations here

(14) function makeComplex (x,y: in Float)
(15)       return Complex is
(16) begin
(17)    return new ComplexRecord'(re => x , im => y);
(18) end makeComplex;

(19) function realPart (z: in Complex) return Float is
(20) begin
(21)    return z.re;
(22) end realPart;
(23) function imaginaryPart (z: in Complex)
(24)    return Float is
(25) begin
(26)    return z.im;
(27) end imaginaryPart;
(28) end ComplexNumbers;
```

Figure 11.11 A package implementation for complex numbers in Ada using pointers

Note that pointers are automatically dereferenced in Ada by the dot notation. Also, note that fields can be assigned during allocation using the apostrophe attribute delimiter (the single quotation mark in `makeComplex`, line 17). A client program can use the `ComplexNumbers` package by including a `with` clause at the beginning of the program, similar to a C `#include` directive:

```
with ComplexNumbers;

procedure ComplexUser is
  z,w: ComplexNumbers.Complex;
begin
  z := ComplexNumbers.makeComplex(1.0,-1.0);
  ...
  w := ComplexNumbers."+"(z,z);
end ComplexUser;
```

Packages in Ada are automatically namespaces in the C++ sense, and all referenced names must be referred to using the package name as a qualifier (the notation, however, is identical to field dereference in records or structures). Ada also has a `use` declaration analogous to the `using` declaration of C++ that dereferences the package name automatically:

```
with ComplexNumbers;
use ComplexNumbers;
procedure ComplexUser is
  z,w: Complex;
  ...
begin
  z := makeComplex(1.0,-1.0);
  ...
  w := z + z;
  ...
end ComplexUser;
```

Notice that the infix form of the overloaded "+" operator can only be used when the package name is dereferenced.

One of the major uses for the package mechanism in Ada is to implement parameterized types as discussed in Chapter 9; parameterized packages are called **generic packages** in Ada. As an example, we give an implementation of a Queue ADT that is entirely analogous to the C implementation of Section 11.2 (a queue given by a circularly linked list with rear pointer). Figure 11.12 shows the package specification, Figure 11.13 shows the package implementation, and Figure 11.14 shows some sample client code.

```
(1)  generic
(2)    type T is private;
(3)  package Queues is
(4)    type Queue is private;
(5)    function createq return Queue;
(6)    function enqueue(q:Queue;elem:T) return Queue;
```

Figure 11.12 A parameterized queue ADT defined as an Ada generic package specification *(continues)*

(continued)

```
(7)    function frontq(q:Queue) return T;
(8)    function dequeue(q:Queue) return Queue;
(9)    function emptyq(q:Queue) return Boolean;
(10) private
(11)   type Queuerep;
(12)   type Queue is access Queuerep;
(13) end Queues;
```

Figure 11.12 A parameterized queue ADT defined as an Ada generic package specification

```
(1)  package body Queues is

(2)    type Queuerep is
(3)    record
(4)      data: T;
(5)      next: Queue;
(6)    end record;

(7)    function createq return Queue is
(8)    begin
(9)      return null;
(10)   end createq;

(11)   function enqueue(q:Queue;elem:T) return Queue is
(12)     temp: Queue;
(13)   begin
(14)     temp := new Queuerep;
(15)     temp.data := elem;
(16)     if (q /= null) then
(17)        temp.next := q.next;
(18)        q.next := temp;
(19)     else
(20)        temp.next := temp;
(21)     end if;
(22)     return temp;
(23)   end enqueue;
(24)   function frontq(q:Queue) return T is
(25)   begin
(26)     return q.next.data;
(27)   end frontq;
```

Figure 11.13 Generic package implementation in Ada for the `Queues` package specification of Figure 11.12 *(continues)*

(continued)

```
(28)   function dequeue(q:Queue) return Queue is
(29)   begin
(30)     if q = q.next then
(31)        return null;
(32)     else
(33)        q.next := q.next.next;
(34)        return q;
(35)     end if;
(36)   end dequeue;

(37)   function emptyq(q:Queue) return Boolean is
(38)   begin
(39)     return q = null;
(40)   end emptyq;

(41) end Queues;
```

Figure 11.13 Generic package implementation in Ada for the `Queues` package specification of Figure 11.12

```
(1)  with Ada.Text_IO; use Ada.Text_IO;

(2)  with Ada.Float_Text_IO;
(3)  use Ada.Float_Text_IO;

(4)  with Ada.Integer_Text_IO;
(5)  use Ada.Integer_Text_IO;

(6)  with Queues;

(7)  procedure Quser is

(8)  package IntQueues is new Queues(Integer);
(9)  use IntQueues;

(10) package FloatQueues is new Queues(Float);
(11) use FloatQueues;

(12) fq: FloatQueues.Queue := createq;
(13) iq: IntQueues.Queue := createq;

(14) begin
(15)   fq := enqueue(fq,3.1);
```

Figure 11.14 Sample Ada client code that uses the `Queues` generic package of Figures 11.12 and 11.13 *(continues)*

(continued)

```
(16)   fq := enqueue(fq,2.3);
(17)   iq := enqueue(iq,3);
(18)   iq := enqueue(iq,2);
(19)   put(frontq(iq)); -- prints 3
(20)   new_line;
(21)   fq := dequeue(fq);
(22)   put(frontq(fq)); -- prints 2.3
(23)   new_line;
(24) end Quser;
```

Figure 11.14 Sample Ada client code that uses the `Queues` generic package of Figures 11.12 and 11.13

We note the following about the code in Figure 11.14. First, lines 1–5 import and dereference standard Ada library IO functions. Line 6 imports the `Queues` package. Lines 8 and 10 create two different instantiations of the `Queues` generic package, one for integers and one for floating-point numbers; these are given the package names `IntQueues` and `FloatQueues`. These two packages are then dereferenced by the `use` declarations in lines 9 and 11. Note that this creates name ambiguities, since all of the names from each of these packages conflict with the names from the other package. Ada's overloading mechanism can, however, resolve most of these ambiguities, based on the data types of the parameters and results. Thus, it is still possible to use the simple names, such as `enqueue`, instead of the qualified names `IntQueues .enqueue` and `FloatQueues.enqueue`. However, it is impossible to disambiguate the names when the data type `Queue` is used in the definitions of `fq` and `iq` (lines 12 and 13). In this case, the qualified names `IntQueues.Queue and FloatQueues.Queue` are used to provide the necessary disambiguation.

11.5 Modules in ML

In Section 11.2, we demonstrated the ML `abstype` definition, which allows easy definition of abstract types with associated operations. ML also has a more general module facility, which consists of three mechanisms: **signatures**, **structures**, and **functors**. The signature mechanism is essentially an interface definition (it even adopts the name *signature* from the mathematical description of the syntax of an ADT as in Section 11.1). A structure is essentially an implementation of a signature; viewed another way, the signature is essentially the type of the structure. Functors are essentially functions from structures to structures, with the structure parameters having "types" given by signatures. Functors allow for the parameterization of structures by other structures. For more on functors, see Section 11.7.4.

Figure 11.15 shows an ML signature for a queue ADT. The `sig ... end` expression (lines 2 and 9) produces a value of type `signature` by listing the types of the identifiers being defined by the signature. This signature value is then stored as the name `QUEUE`. Since a signature is not a type as such, we must export the type `Queue` as one of the identifiers; this is done in line 3 with the declaration `type 'a Queue`, which identifies the name `Queue` as an exported type, parameterized by the type variable `'a`. Thus, this signature allows queues to contain elements of arbitrary type (but each queue can only contain elements of a single type). The type `Queue` may be either a type synonym for a previously existing type, or a new type; making it a previously existing type can result in a loss of protection, however (see Exercise 11.29); thus, in our examples we will always use a new type.

```
(1) signature QUEUE =
(2)    sig
(3)    type 'a Queue
(4)    val createq: 'a Queue
(5)    val enqueue: 'a Queue * 'a -> 'a Queue
(6)    val frontq: 'a Queue -> 'a
(7)    val dequeue: 'a Queue -> 'a Queue
(8)    val emptyq: 'a Queue -> bool
(9)    end;
```

Figure 11.15 A QUEUE signature for a queue ADT in ML

Figure 11.16 shows a structure `Queue1` that provides an implementation for the `QUEUE` signature. Using the `QUEUE` signature specification in the first line of this definition (after the colon but before the equal sign) restricts the public interface of this structure to only the features mentioned in the signature, and it requires the structure to implement every feature in the signature in such a way that it has exactly the type specified in the signature. Line 1 of Figure 11.16, thus, defines the name `Queue1` to have type `structure`, implementing the signature `QUEUE`. Lines 2 through 10 then provide the actual `structure` value, which is constructed by the `struct ... end` expression. This expression implements the `Queue` type in exactly the same way as in Figure 11.1, and the internal representation of the `Queue` as a list, including the constructor `Q`, is not accessible outside of the structure because it is not part of the `QUEUE` signature.

```
(1)  structure Queue1: QUEUE =
(2)     struct
(3)     datatype 'a Queue = Q of 'a list
(4)     val createq = Q [];
(5)     fun enqueue(Q lis, elem) = Q (lis @ [elem]);
(6)     fun frontq (Q lis) = hd lis;
(7)     fun dequeue (Q lis) = Q (tl lis);
(8)     fun emptyq (Q []) = true
(9)       | emptyq (Q (h::t)) = false;
(10)    end;
```

Figure 11.16 An ML structure `Queue1` implementing the `QUEUE` signature as an ordinary built-in list with wrapper

The structure `Queue1` can be used in ML as follows:

```
- val q = Queue1.enqueue(Queue1.createq,3);
val q = Q [3] : int Queue1.Queue
- Queue1.frontq q;
val it = 3 : int
- val q1 = Queue1.dequeue q;
val q1 = Q [] : int Queue1.Queue
- Queue1.emptyq q1;
val it = true : bool
```

Note that the internal representation of Queue is not suppressed in the ML interpreter responses as it is with the `abstype` specification of Section 11.2. Also, all names in `Queue1` must be qualified by the name of the structure, unlike the `abstype` specification. As with other languages, this qualification can be removed if desired; in ML the expression that does so is the `open` expression:

```
- open Queue1;
opening Queue1
  datatype 'a Queue = ...
  val createq : 'a Queue
  val enqueue : 'a Queue * 'a -> 'a Queue
  val frontq : 'a Queue -> 'a
  val dequeue : 'a Queue -> 'a Queue
  val emptyq : 'a Queue -> bool
```

The ML interpreter responds to the `open` expression by listing the signature that `Queue1` implements. The features of `Queue1` can now be used without qualification:

```
- val q = enqueue (createq,3);
val q = Q [3] : int Queue
- frontq q;
val it = 3 : int
- val q1 = dequeue q;
val q1 = Q [] : int Queue
- emptyq q1;
val it = true : bool
```

Figure 11.17 shows a second implementation of the `QUEUE` signature, this time using a user-defined linked list instead of the built-in ML list structure. In the definition of the `Queue` data type (lines 3 and 4), we have used two constructors: `Createq` and `Enqueue`. These constructors perform essentially the same operations as the functions `createq` and `enqueue`, so we have given them the same names (except for the uppercase first letter). With these names, the definitions of the functions `frontq`, `dequeue`, and `emptyq` look virtually identical to the equational axiomatic specification for the Queue ADT given in Section 11.1.

```
(1)  structure Queue2: QUEUE =
(2)     struct
(3)     datatype 'a Queue = Createq
(4)                       | Enqueue of 'a Queue * 'a ;
(5)     val createq = Createq;
(6)     fun enqueue(q,elem) = Enqueue (q,elem);
(7)     fun frontq (Enqueue(Createq,elem)) = elem
(8)     | frontq (Enqueue(q,elem)) = frontq q;
(9)     fun dequeue (Enqueue(Createq,elem)) = Createq
```

Figure 11.17 An ML structure `Queue2` implementing the `QUEUE` signature as a user-defined linked list *(continues)*

(continued)

```
(10)                  | dequeue (Enqueue(q,elem))
(11)                          = Enqueue(dequeue q, elem);
(12)    fun emptyq Createq = true | emptyq _ = false;
(13)    end;
```

Figure 11.17 An ML structure `Queue2` implementing the `QUEUE` signature as a user-defined linked list

The `Queue2` structure of Figure 11.17 can be used exactly as the `Queue1` structure of Figure 11.16:

```
- open Queue2;
opening Queue2
  datatype 'a Queue = ...
  val createq : 'a Queue
  val enqueue : 'a Queue * 'a -> 'a Queue
  val frontq : 'a Queue -> 'a
  val dequeue : 'a Queue -> 'a Queue
  val emptyq : 'a Queue -> bool
- val q = enqueue(createq,3);
val q = Enqueue (Createq,3) : int Queue
- frontq q;
val it = 3 : int
- val q1 = dequeue q;
val q1 = Createq : int Queue
- emptyq q1;
val it = true : bool
```

ML signatures and structures satisfy most of the requirements of criteria 1 and 2 for abstract data types from this chapter's introduction. A signature collects a type and its associated operations in one place. The definitions do not depend on any implementation details. A structure collects the implementation details in one place and only exports the information contained in the associated signature. The main difficulty with the ML module mechanism from the point of view of abstract data types is that client code must explicitly state the implementation that is to be used in terms of the module name: Code cannot be written to depend only on the signature, with the actual implementation structure to be supplied externally to the code. Thus, we cannot write:

```
val x = QUEUE.createq;
```

Instead, we must write:

```
val x = Queue1.createq;
```

or

```
val x = Queue2.createq;
```

The main reason for this is that the ML language has no explicit or implicit separate compilation mechanism (or other code aggregation mechanism), so there is no way to externally specify an implementation choice.[8]

11.6 Modules in Earlier Languages

Historically, modules and abstract data type mechanisms began with Simula67 and made significant progress in the 1970s through language design experiments at universities and research centers. Among the languages that have contributed significantly to module mechanisms in Ada, ML, and other languages, are the languages CLU, Euclid, Modula-2, Mesa, and Cedar. To indicate how module mechanisms have developed, we sketch briefly the approaches taken by Euclid, CLU, and Modula-2.

11.6.1 Euclid

In the Euclid programming language, modules are types, so a complex number module is declared as a type:

```
type ComplexNumbers = module
   exports(Complex, add, subtract, multiply,
           divide, negate, makeComplex,
          realPart, imaginaryPart)
   type Complex = record
      var re, im: real
   end Complex

   procedure add (x,y: Complex, var z: Complex) =
   begin
       z.re := x.re + y.re
       z.im := x.im + y.im
   end add

   procedure makeComplex
          (x,y: real, var z:Complex) =
   begin
       z.re := x
       z.im := y
   end makeComplex
            ...
end ComplexNumbers
```

To be able to declare complex numbers, however, we need an actual object of type `ComplexNumbers`—a module type in itself does not exist as an object. Thus, we must declare:

```
var C: ComplexNumbers
```

[8]In fact, most ML systems have a compilation manager with separate compilation that can remove this problem, but such tools are not part of the language itself.

and then we can declare:

```
var z,w: C.Complex
```

and apply the operations of `C`:

```
C.makeComplex(1.0,1.0,z)
C.Add(z,z,w)
```

Note that this implies that there could be two different variables of type `ComplexNumbers` declared and, thus, two different sets of complex numbers:

```
var C1,C2: ComplexNumbers
var x: C1.Complex
var y: C2.Complex


C1.makeComplex(1.0,0.0,x)

C2.makeComplex(0.0,1.0,y)
(* x and y cannot be added together *)
```

When module types are used in a declaration, this creates a variable of the module type, or **instantiates** the module. In Euclid, two different instantiations of `ComplexNumbers` can, therefore, exist simultaneously. This is not true in languages such as Ada or ML, where modules are objects instead of types, with a single instantiation of each. (The Ada generic package can, of course, have several different instantiations, even for the same type; see the discussion in Section 11.4 and Chapter 9.)

11.6.2 CLU

In CLU, modules are defined using the **cluster** mechanism. In the case of complex numbers, the data type `Complex` can be defined directly as a cluster:

```
Complex = cluster is add, multiply,...,
          makeComplex, realPart, imaginaryPart
    rep = struct [re,im: real]
    add = proc (x,y: cvt ) returns (cvt)
      return
      (rep${re: x.re+y.re, im: x.im+y.im})
    end add
      ...
    makeComplex = proc (x,y: real) returns (cvt)
      return (rep${re:x, im:y})
    end makeComplex
```

(continues)

(continued)

```
    realPart = proc(x: cvt) returns (real)
      return(x.re)
    end realPart

end Complex
```

The principal difference of this abstraction mechanism from the previous ones is that the datatype `Complex` is defined directly as a cluster. However, when we define

```
x,y: Complex
```

we do not mean that they should be of the cluster type, but of the representation type within the cluster (given by the **rep** declaration). Thus, a cluster in CLU really refers to two different things: the cluster itself and its internal representation type. Of course, the representation type must be protected from access by the outside world, so the details of the representation type can be accessed only from within the cluster. This is the purpose of the `cvt` declaration (for `convert`), which converts from the external type `Complex` (with no explicit structure) to the internal `rep` type and back again. `cvt` can be used only within the body of a cluster.

Clients can use the functions from `Complex` in a similar manner to other languages:

```
x := Complex$makeComplex(1.0,1.0)
x := Complex$add (x,x)
```

Note the use of the "$" to qualify the operations instead of the dot notation used in Ada and ML.

11.6.3 Modula-2

In Modula-2, the specification and implementation of an abstract data type are separated into a `DEFINITION MODULE` and an `IMPLEMENTATION MODULE`. For the type `Complex`, a Modula-2 specification module would look like this:

```
DEFINITION MODULE ComplexNumbers;

TYPE Complex;

PROCEDURE Add (x,y: Complex): Complex;
PROCEDURE Subtract (x,y: Complex): Complex;
PROCEDURE Multiply (x,y: Complex): Complex;
PROCEDURE Divide (x,y: Complex): Complex;
PROCEDURE Negate (z: Complex): Complex;
PROCEDURE MakeComplex (x,y: REAL): Complex;
PROCEDURE RealPart (z: Complex) : REAL;

PROCEDURE ImaginaryPart (z: Complex) : REAL;

END ComplexNumbers.
```

A Modula-2 `DEFINITION MODULE` contains only definitions or declarations, and only the declarations that appear in the `DEFINITION MODULE` are **exported**, that is, are usable by other modules. The incomplete type specification of type `Complex` in the `DEFINITION MODULE` is called an opaque type; the details of its declaration are hidden in an implementation module and are not usable by other modules. These features are all similar to those of Ada and ML.

A corresponding `IMPLEMENTATION MODULE` in Modula-2 for the `DEFINITION MODULE` is as follows:

```
IMPLEMENTATION MODULE ComplexNumbers;

FROM Storage IMPORT ALLOCATE;

TYPE Complex = POINTER TO ComplexRecord
     ComplexRecord = RECORD
        re, im: REAL;
     END;

PROCEDURE add (x,y: Complex): Complex;
VAR t: Complex;
BEGIN
  NEW(t);
  t^.re := x^.re + y^.re
  t^.im := x^.im + y^.im;
  RETURN t;
END add;

    ...

PROCEDURE makeComplex (x,y: REAL): Complex;
VAR t: Complex;
BEGIN
  NEW(t);
  t^.re := x;
  t^.im := y;
  RETURN t;
END makeComplex;

PROCEDURE realPart (z: Complex) : REAL;
BEGIN
  RETURN z^.re;
END realPart;

PROCEDURE imaginaryPart (z: Complex) : REAL;
BEGIN
  RETURN z^.im;
END imaginaryPart;

END ComplexNumbers.
```

Because of the use of pointers and indirection, functions such as `makeComplex` have to include a call to `NEW` to allocate space for a new complex number, and this in turn requires that `ALLOCATE` be imported from a system `Storage` module. (The compiler converts a call to `NEW` to a call to `ALLOCATE`.)

A client module in Modula-2 uses type `Complex` by **importing** it and its functions from the `ComplexNumbers` module:

```
MODULE ComplexUser;

IMPORT ComplexNumbers;

VAR x,y,z: ComplexNumbers.Complex;

BEGIN
  x := ComplexNumbers.makeComplex(1.0,2.0);
  y := ComplexNumbers.makeComplex(-1.0,1.0);
  z := ComplexNumbers.add(x,y);
        ...
END ComplexUser.
```

Note the qualification in the use of the features from `ComplexNumbers`. An alternative to this is to use a **dereferencing** clause. In Modula-2, the dereferencing clause is the `FROM` clause:

```
MODULE ComplexUser;

FROM ComplexNumbers IMPORT Complex,add,makeComplex:
VAR x,y,z: Complex;
...
BEGIN
  x := makeComplex(1.0,2.0);
  y := makeComplex(-1.0,1.0);
  ...
  z := add(x,y);

END ComplexUser.
```

When a `FROM` clause is used, imported items must be listed by name in the `IMPORT` statement, and no other items (either imported or locally declared) may have the same names as those imported (Modula-2 does not support overloading).

The separation of a module into a `DEFINITION MODULE` and an `IMPLEMENTATION MODULE` in Modula-2 supports the first principle of an abstract data type mechanism: It allows the data type and operation definitions to be collected in one place (the `DEFINITION MODULE`) and associates the operations directly with the type via the `MODULE` name. The use of an opaque type in a `DEFINITION MODULE` supports the second principle of an abstract data type mechanism: the details of the `Complex` data type implementation are separated from its declaration and are hidden in an `IMPLEMENTATION MODULE`

together with the implementation of the operations. Further, a client is prevented from using any of the details in the `IMPLEMENTATION MODULE`.

Since the `DEFINITION MODULE` is independent of any of the details of the `IMPLEMENTATION MODULE`, the implementation can be rewritten without affecting any use of the module by clients, in a similar manner to Ada and C.

11.7 Problems with Abstract Data Type Mechanisms

Abstract data type mechanisms in programming languages use separate compilation facilities to meet protection and implementation independence requirements. The specification part of the ADT mechanism is used as an interface to guarantee consistency of use and implementation. However, ADT mechanisms are used to create types and associate operations to types, while separate compilation facilities are providers of services, which may include variables, constants, or any other programming language entities. Thus, compilation units are in one sense more general than ADT mechanisms. At the same time they are less general, however, in that the use of a compilation unit to define a type does not identify the type with the unit and, thus, is not a true type declaration. Furthermore, units are static entities—that is, they retain their identity only before linking, which can result in allocation and initialization problems. Thus, the use of separate compilation units, such as modules or packages, to implement abstract data types is a compromise in language design. However, it is a useful compromise, because it reduces the implementation question for ADTs to one of consistency checking and linkage.

In this section, we catalog some of the major drawbacks of the ADT mechanisms we have studied (and others we have not). Of course, not all languages suffer from all the drawbacks listed here. Many, but not all, of these drawbacks have been corrected in object-oriented languages. Thus, this section serves as an indication of what to look for in object-oriented languages.

11.7.1 Modules Are Not Types

In C, Ada, and ML, difficulties can arise because a module must export a type as well as operations. It would be helpful instead to define a module to *be a type*. This prevents the need to arrange to protect the implementation details of the type with an ad hoc mechanism such as incomplete or private declarations; the details are all contained in the implementation section of the type.

The fact that ML contains both an `abstype` and a module mechanism emphasizes this distinction. The module mechanism is more general, and it allows the clean separation of specification (signature) from implementation (structure), but a type must be exported. On the other hand, an `abstype` *is* a type, but the implementation of an `abstype` cannot be separated from its specification (although access to the details of the implementation is prevented), and clients of the `abstype` implicitly depend on the implementation.

11.7.2 Modules Are Static Entities

An attractive possibility for implementing an abstract data type is to simply not reveal a type at all, thus avoiding all possibility of clients depending in any way on implementation details, as well as preventing clients from any misuse of a type. For example, in Ada we could write a `Queue` package specification as follows. In the following example, note how the operations must change so that they never return a queue. This is pure imperative programming.

```
generic
  type T is private;
package Queues is
  procedure enqueue(elem:T);
  function frontq return T;
  procedure dequeue;
  function emptyq return Boolean;
end Queues;
```

The actual queue data structure is then buried in the implementation:

```
package body Queues is

  type Queuerep;
  type Queue is access Queuerep;

  type Queuerep is
  record
    data: T;
    next: Queue;
  end record;

  q: Queue;

  procedure enqueue(elem:T) is
    temp: Queue;
  begin
    temp := new Queuerep;
    temp.data := elem;
    if (q /= null) then
       temp.next := q.next;
       q.next := temp;
    else
       temp.next := temp;
    end if;
    q := temp;
  end enqueue;

  function frontq return T is
  begin
    return q.next.data;
  end frontq;
  procedure dequeue is
  begin
```

(continues)

(continued)

```
    if q = q.next then
       q := null;
    else
       q.next := q.next.next;
       q := q.next;
    end if;
  end dequeue;

  function emptyq return Boolean is
  begin
    return q = null;
  end emptyq;

begin
  q := null;
end Queues;
```

Note that this allows us to write initialization code internally within the package body (the last three lines of the above code), which is executed on "elaboration," or allocation time (at the beginning of execution). This eliminates the need for user-supplied code to initialize a queue, and so there is no `createq` procedure.

Normally this would imply that there can only be one queue in a client, since otherwise the entire code must be replicated (try this using a C++ namespace—see Exercise 11.28). This results from the static nature of most module mechanisms (including those of Ada and ML). In Ada, however, the generic package mechanism offers a convenient way to obtain several different queues (even for the same stored type) by using multiple instantiations of the same generic package:

```
with Ada.Text_IO; use Ada.Text_IO;
with Ada.Integer_Text_IO;
use Ada.Integer_Text_IO;
with Queues;

procedure Quser is

package Queue1 is new Queues(Integer);
package Queue2 is new Queues(Integer);

begin
  Queue1.enqueue(3);
  Queue1.enqueue(4);
  Queue2.enqueue(1);
  Queue2.enqueue(2);
  put(Queue1.frontq); -- prints 3
```

(continues)

(continued)

```
    new_line;
    Queue2.dequeue;
    put(Queue2.frontq); -- prints 2
    new_line;
  end Quser;
```

This is still a static alternative, and we could not create a new queue during execution. However, this can still be an extremely useful mechanism.

11.7.3 Modules That Export Types Do Not Adequately Control Operations on Variables of Such Types

In the C and Ada examples of data types `Queue` and `Complex` in Sections 11.3 and 11.4, variables of each type were pointers that had to be allocated and initialized by calling the procedure `createq` or `makeComplex`. However, the exporting module cannot guarantee that this procedure is called before the variables are used; thus, correct allocation and initialization cannot be ensured. Even worse, because queues and, most particularly, complex numbers are pointers in the sample implementations, copies can be made and deallocations performed outside the control of the module, without the user being aware of the consequences, and without the ability to return the deallocated memory to available storage. For instance, in the Ada code

```
z := makeComplex(1.0,0.0);
x := makeComplex(-1.0,0.0);
x := z;
```

the last assignment has the effect of making `z` and `x` point to the same allocated storage and leaves the original value of `x` as garbage in memory. Indeed, the implementation of type `Complex` as a pointer type gives variables of this type pointer semantics, subject to problems of sharing and copying (see Chapter 8).

Even if we were to rewrite the implementation of type `Complex` as a record or struct instead of a pointer (which would wipe out some encapsulation and information hiding in both C and Ada), we would not regain much control. In that case, the declaration of a variable of type `Complex` would automatically perform the allocation, and calling `makeComplex` would perform initialization only. Still, we would have no guarantee that initializations would be properly performed, but at least assignment would have the usual storage semantics, and there would be no unexpected creation of garbage or side effects.

Part of the problem comes from the use of assignment. Indeed, when `x` and `y` are pointer types, `x := y` (`x = y` in C) performs assignment by sharing the object pointed to by `y`, potentially resulting in some unwanted side effects. This is another example of an operation over which a module exporting types has no control. Similarly, the comparison `x = y` tests pointer equality, identity of location in memory, which is not the right test when `x` and `y` are complex numbers:

```
x := makeComplex(1.0,0.0);
y := makeComplex(1.0,0.0);
(* now a test of x = y returns false *)
```

We could write procedures `equal` and `assign` and add them to the operations in the package specification (or the header file in C), but, again, there would be no guarantee that they would be appropriately applied by a client.

In Ada, however, we can use a **limited private type** as a mechanism for controlling the use of equality and assignment. For example:

```
package ComplexNumbers is

type Complex is limited private;

-- operations, including assignment and equality
  ...
function equal(x,y: in Complex) return Boolean;
procedure assign(x: out Complex; y: in Complex);

private
   type ComplexRec;
   type Complex is access ComplexRec;

end ComplexNumbers;
```

Now clients are prevented from using the usual assignment and equality operations, and the package body can ensure that equality is performed appropriately and that assignment deallocates garbage and/or copies values instead of pointers. Indeed, the = (and, implicitly, the inequality / =) operator can be overloaded as described in Chapter 8, so the same infix form can be used after redefinition by the package. Assignment, however, cannot be overloaded in Ada, and there is still no automatic initialization and deallocation.

C++ has an advantage here, since it allows the overloading of *both* assignment (=) and equality (==). Additionally, object-oriented languages solve the initialization problem by the use of **constructors**.

ML also is able to control equality testing by limiting the data type in an `abstype` or `struct` specification to types that do not permit the equality operation. Indeed, ML makes a distinction between types that allow equality testing and types that do not. Type parameters that allow equality testing must be written using a double apostrophe `''a` instead of a single apostrophe `'a`, and a type definition that allows equality must be specified as an `eqtype` rather than just a type:

```
signature QUEUE =
   sig
   eqtype ''a Queue
   val createq: ''a Queue
   ...etc.
  end;
```

Without this, equality cannot be used to test queues. Additionally, assignment in ML is much less of a problem, since standard functional programming practice does away with most uses of that operation (see Chapter 4).

11.7.4 Modules Do Not Always Adequately Represent Their Dependency on Imported Types

Aside from assignment and equality testing, modules often depend on the existence of certain operations on type parameters and may also call functions whose existence is not made explicit in the module specification. A typical example is a data structure such as a binary search tree, priority queue, or ordered list. These data structures all require that the type of the stored values have an order operation such as the less-than arithmetic operation "<" available. Frequently this is not made explicit in a module specification.

For example, C++ templates mask such dependencies in specifications. A simple example is of a template `min` function, which depends on an order operation. In C++ this could be specified as:

```
template <typename T>
T min( T x, T y);
```

Naturally, the implementation of this function shows the dependency on the order operation (but the specification does not):

```
// C++ code
template <typename T>
T min( T x, T y)
// requires an available < operation on T
{ return x < y ? x : y;
}
```

(See Chapter 9 for a discussion on how to make this dependency explicit in C++.)

In Ada it is possible to specify this requirement using additional declarations in the generic part of a package declaration:

```
generic
type Element is private;
with function lessThan (x,y: Element) return Boolean;
package OrderedList is
...
end OrderedList;
```

Now an `OrderedList` package can be instantiated only by providing a suitable `lessThan` function, as in

```
package IntOrderedList is new
   OrderedList (Integer,"<");
```

Such a requirement is called **constrained parameterization**. Without explicit constraints in Ada, no operations are assumed for the type parameter except equality, inequality, and assignment (and, as noted in the previous section, even these are not assumed if the type is declared limited private).

ML also has a feature that allows structures to be explicitly parameterized by other structures; this feature is called a **functor**, because it is essentially a function on structures (albeit one that operates only statically). Given two signatures such as `ORDER` and `ORDERED_LIST` (lines 1–5 and 6–13 of Figure 11.18), a functor that takes an `ORDER` structure as an argument and produces an `ORDERED_LIST` structure as a result can be defined as follows (for a complete example, see lines 14–28 of Figure 11.18):

```
functor OListFUN (structure Order: ORDER):
ORDERED_LIST =
   struct
   ...
   end;
```

Then, given an actual `ORDER` structure value, such as `IntOrder` (lines 29–33), the functor can be applied to create a new `ORDERED_LIST` structure `IntOList`:

```
structure IntOList =
   OlistFUN(structure Order = IntOrder);
```

This makes explicit the appropriate dependencies, but at the cost of requiring an extra structure (`IntOrder`) to be defined that encapsulates the required features.

```
(1)  signature ORDER =
(2)     sig
(3)     type Elem
(4)     val lt: Elem * Elem -> bool
(5)     end;

(6)  signature ORDERED_LIST =
(7)     sig
(8)     type Elem
(9)     type OList
(10)     val create: OList
(11)    val insert: OList * Elem -> OList
(12)    val lookup: OList * Elem -> bool
(13)    end;
(14) functor OListFUN (structure Order: ORDER):
(15) ORDERED_LIST =
(16)    struct
(17)    type Elem = Order.Elem;
(18)    type OList = Order.Elem list;
(19)    val create = [];
(20)    fun insert ([], x) = [x]
(21)      | insert (h::t, x) = if Order.lt(x,h) then x::h::t
(22)                                 else h:: insert (t, x);
(23)    fun lookup ([], x) = false
```

Figure 11.18 The use of a functor in ML to define an ordered list *(continues)*

(continued)

```
(24)      | lookup (h::t, x) =
(25)          if Order.lt(x,h) then false
(26)          else if Order.lt(h,x) then lookup (t,x)
(27)          else true;
(28)    end;

(29) structure IntOrder: ORDER =
(30)    struct
(31)    type Elem = int;
(32)    val lt = (op <);
(33)    end;

(34) structure IntOList =
(35)    OListFUN(structure Order = IntOrder);
```

Figure 11.18 The use of a functor in ML to define an ordered list

With the code of Figure 11.18, the `IntOList` structure can be used as follows:

```
- open IntOList;
opening IntOList
  type Elem = IntOrder.Elem
  type OList = IntOrder.Elem list
  val create : OList
  val insert : OList * Elem -> OList
  val lookup : OList * Elem -> bool
- val ol = insert(create,2);
val ol = [2] : OList
- val ol2 = insert(ol,3);
val ol2 = [2,3] : OList
- lookup (ol2,3);
val it = true : bool
- lookup (ol,3);
val it = false : bool
```

11.7.5 Module Definitions Include No Specification of the Semantics of the Provided Operations

In the algebraic specification of an abstract data type, equations are given that specify the behavior of the operations. Yet in almost all languages no specification of the behavior of the available operations is required. Some experiments have been conducted with systems that require semantic specification and then attempt to determine whether a provided implementation agrees with its specification, but such languages and systems are still unusual and experimental.

One example of a language that does allow the specification of semantics is Eiffel, an object-oriented language. In Eiffel, semantic specifications are given by preconditions, postconditions, and invariants. Preconditions and postconditions establish what must be true before and after the execution of a procedure or function. Invariants establish what must be true about the internal state of the data in an abstract data type. For more on such conditions, see Chapter 13 on formal semantics. For more on Eiffel, see the Notes and References.

A brief example of the way Eiffel establishes semantics is as follows. In a queue ADT as described in Section 11.1, the enqueue operation is defined as follows in Eiffel:

```
enqueue (x:element) is
require
                    not full
ensure
                    if old empty then front = x
                    else front = old front;
                    not empty
end; -- enqueue
```

The `require` section establishes preconditions—in this case, to execute the enqueue operation correctly, the queue must not be full (this is not part of the algebraic specification as given in Section 11.1; see Exercise 11.24). The `ensure` section establishes postconditions, which in the case of the `enqueue` operation are that the newly enqueued element becomes the front of the queue only if the queue was previously empty and that the queue is now not empty. These requirements correspond to the algebraic axioms

frontq(enqueue(q,x)) = if emptyq(q) then x else frontq(q)
emptyq(enqueue(q,x)) = false

of Section 11.1.

In most languages, we are confined to giving indications of the semantic content of operations inside comments.

11.8 The Mathematics of Abstract Data Types

In the algebraic specification of an abstract data type, a data type, a set of operations, and a set of axioms in equational form are specified. Nothing is said, however, about the actual existence of such a type, which is hypothetical until an actual type is constructed that meets all the requirements. In the parlance of programming language theory, an abstract data type is said to have **existential type**—it asserts the existence of an actual type that meets its requirements. Such an actual type is a set with operations of the appropriate form that satisfy the given equations. A set and operations that meet the specification are a **model** for the specification. Given an algebraic specification, it is possible for no model to exist, or many models. Therefore, it is necessary to distinguish an actual type (a model) from a potential type (an algebraic specification). Potential types are called sorts, and potential sets of operations are called

signatures. (Note that this agrees with our previous use of the term signature to denote the syntax of prospective operations.) Thus, a sort is the name of a type, which is not yet associated with any actual set of values. Similarly, a signature is the name and type of an operation or set of operations, which exists only in theory, having no actual operations yet associated with it. A model is then an actualization of a sort and its signature and is called an algebra (since it has operations).

For this reason, algebraic specifications are often written using the sort-signature terminology:

sort queue(element) **imports** boolean

signature:

createq: queue
enqueue: queue × element → queue
dequeue: queue → queue
frontq: queue → element
emptyq: queue → boolean

axioms:

emptyq(createq) = true
emptyq (enqueue (q, x)) = false
frontq(createq) = error
frontq(enqueue(q,x)) = if emptyq(q) then x else frontq(q)
dequeue(createq) = error
dequeue(enqueue(q,x)) = if emptyq(q) then q else enqueue(dequeue(q), x)

Given a sort, a signature, and axioms, we would, of course, like to know that an actual type exists for that specification. In fact, we would like to be able to construct a *unique* algebra for the specification that we would then take to be *the* type represented by the specification.

How does one construct such an algebra? A standard mathematical method is to construct the **free algebra of terms** for a sort and signature and then to form the **quotient algebra** of the equivalence relation generated by the equational axioms. The free algebra of terms consists of all the legal combinations of operations. For example, the free algebra for sort queue(integer) and signature as above has terms such as the following:

createq
enqueue (createq, 2)
enqueue (enqueue(createq, 2), 1)
dequeue (enqueue (createq, 2))
dequeue (enqueue(enqueue (createq, 2), -1))
dequeue (dequeue (enqueue (createq, 3)))
etc.

Note that the axioms for a queue imply that some of these terms are actually equal, for example,

dequeue (enqueue (createq, 2)) = createq

In the free algebra, however, all terms are considered to be different; that is why it is called free. In the free algebra of terms, no axioms are true. To make the axioms true, and so to construct a type that models the specification, we need to use the axioms to reduce the number of distinct elements in the free algebra. The two distinct terms createq and dequeue(enqueue(createq,2)) need to be identified, for example, in order to make the axiom

dequeue(enqueue(q,x)) =
    if emptyq(q) then q else enqueue(dequeue(q),x)

hold. This can be done by constructing an **equivalence relation** == from the axioms: "==" is an equivalence relation if it is symmetric, transitive, and reflexive:

if x == y then y == x (symmetry)
if x == y and y == z then x == z (transitivity)
x == x (reflexivity)

It is a standard task in mathematics to show that, given an equivalence relation == and a free algebra F, there is a unique, well-defined algebra F/== such that $x = y$ in F/== if and only if x == y in F. The algebra F/== is called the quotient algebra of F by ==. Furthermore, given a set of equations, such as the axioms in an algebraic specification, there is a unique "smallest" equivalence relation making the two sides of every equation equivalent and hence equal in the quotient algebra. It is this quotient algebra that is usually taken to be "the" data type defined by an algebraic specification. This algebra has the property that the only terms that are equal are those that are provably equal from the axioms. Thus, in the queue example,

dequeue(enqueue(enqueue(createq,2),3)) =
enqueue (dequeue(enqueue(createq, 2)),3) =
enqueue(createq, 3)

but

enqueue(createq, 2) ≠ enqueue(createq, 3).

For mathematical reasons (coming from category theory and universal algebra), this algebra is called the **initial algebra** represented by the specification, and using this algebra as the data type of the specification results in what are called **initial semantics**.

How do we know whether the algebra we get from this construction really has the properties of the data type that we want? The answer is that we don't, unless we have written the "right" axioms. In general, axiom systems should be **consistent** and **complete**. Consistency says that the axioms should not identify terms that should be different. For example, empty(createq) = true and empty(createq) = false are inconsistent axioms, because they result in false = true; that is, false and true become identified in the initial semantics. Consistency of the axiom system says that the initial algebra is not "too small." Completeness of the axiom system, on the other hand, says that the initial algebra is not "too big"; that is, elements of the initial algebra that should be the same aren't. (Note that adding axioms identifies more elements, thus making the initial algebra "smaller" while taking away axioms makes it "larger.") Another useful but not as critical property for an axiom system is **independence**—that is, no axiom is implied by other axioms. For example, frontq(enqueue(enqueue(createq,x),y)) = x is not independent because it follows from other axioms:

frontq(enqueue(enqueue(createq,x),y)) = frontq(enqueue(createq,x))

since frontq(enqueue(q,y)) = frontq(q) for q = enqueue(createq,x), by the fourth axiom on page 496 applied to the case when $q \neq$ createq, and then

$$\text{frontq(enqueue (createq,}x\text{))} = x$$

by the same axiom applied to the case when q = createq.

Deciding on an appropriate set of axioms is in general a difficult process. In Section 11.1 we gave a simplistic method based on the classification of the operations into constructors and inspectors. Such methods, while useful, are not foolproof and do not cover all cases. Moreover, dealing with errors causes extra difficulties, which we do not study here.

Initial semantics for algebraic specifications are unforgiving because, if we leave out an axiom, we can in general get many values in our data type that should be equal but aren't. More forgiving semantics are given by an approach that assumes that any two data values that cannot be distinguished by inspector operations must be equal. An algebra that expresses such semantics is called (again for mathematical reasons) a **final algebra**, and the associated semantics are called **final semantics**.

A final algebra is also essentially unique, and can be constructed by means similar to the initial algebra construction. To see the difference between these two definitions, we take the example of an integer array abstract data type. An algebraic specification is as follows:

type IntArray **imports** integer

operations:

createIA: IntArray
insertIA: IntArray × integer × integer → IntArray
extractIA: IntArray × integer → integer

variables: A: IntArray; i,j,k: integer;

axioms:

extractIA(createIA,i) = 0
extractIA (insertIA (A, i, j), k) = if $i = k$ then j else extractIA(A, k)

This array data type is essentially an integer array type indexed by the entire set of integers: insertIA(A,i,j) is like A[i] = j, and extractIA(A, i) is like getting the value A[i]. There is a problem, however: In initial semantics, the arrays

$$\text{insertIA (insertIA (createIA,1), 2,2)}$$

and

$$\text{insertIA(insertIA(createIA,2,2),1,1)}$$

are not equal! (There is no rule that allows us to switch the order of inserts.) Final semantics, however, tells us that two arrays must be equal if all their values are equal:

$$A = B \text{ if and only if for all integers } i \text{ extractIA}(A,i) = \text{extractIA}(B,i)$$

This is an example of the **principle of extensionality** in mathematics: Two things are equal precisely when all their components are equal. It is a natural property and is likely to be one we want for abstract

data types, or at least for the IntArray specification. To make the final and initial semantics agree, however, we must add the axiom

insertIA (insertIA(A,i,j),k,m) = insertIA (insertIA(A,k,m),i,j)

In the case of the queue algebraic specification, the final and initial semantics already agree, so we don't have to worry: given two queues p and q,

$p = q$ if and only if p = createq and q = createq
or frontq(p) = frontq(q) and dequeue(p) = dequeue(q)

Exercises

11.1 Discuss how the following languages meet or fail to meet criteria 1 and 2 for an abstract data type mechanism from the beginning of this chapter:
(a) C
(b) Ada
(c) ML
(d) another language of your choice.

11.2 In the algebraic specification of the complex abstract data type in Section 11.1, axioms were given that base complex addition and subtraction on the corresponding real operations. Write similar axioms for **(a)** multiplication and **(b)** division.

11.3 An alternative to writing axioms relating the operations of complex numbers to the corresponding operations of their real and imaginary parts is to write axioms for all the usual properties of complex numbers, such as associativity, commutativity, and distributivity. What problems do you foresee in this approach?

11.4 Finish writing the `abstype Complex` definition of Figure 11.2 in ML (Section 11.2).

11.5 Finish writing the implementation of `package ComplexNumbers` in Section 11.4.

11.6 Write a complex number module in C using the header file approach described in Section 11.3.

11.7 Use your implementation of Exercise 11.4, 11.5, or 11.6 to write a program to compute the roots of a quadratic equation $ax2 + bx + c = 0$. (a, b, and c may be assumed to be either real or complex.)

11.8 Write a new implementation for complex numbers in C, Ada, or ML that uses polar coordinates. Make sure that your interface and client code (e.g., Exercise 11.7) do not change.

11.9 Write an implementation for the queue ADT in C or Ada that uses a doubly linked list with front and rear pointers instead of the singly linked list with rear pointer that was used in this chapter. Make sure that your interface and client code do not change.

11.10 Rewrite the abstype Complex in ML as a signature and structure.

11.11 Implement the Stack ADT as described in Section 11.1 in **(a)** C; **(b)** Ada; **(c)** ML.

11.12 A **double-ended queue**, or **deque**, is a data type that combines the actions of a stack and a queue. Write an algebraic specification for a deque abstract data type assuming the following operations: create, empty, front, rear, addfront, addrear, deletefront, and deleterear.

11.13 Which operations of the complex abstract data type of Section 11.1 are constructors? Which are inspectors? Which are selectors? Which are predicates? Based on the suggestions in Section 11.1, how many axioms should you have?

following operations:

create: SymbolTable
enter: SymbolTable × name × value → SymbolTable
lookup: SymbolTable × name → value
isin: SymbolTable × name → boolean

11.15 Which operations of the SymbolTable abstract data type in Exercise 11.14 are constructors? Which are inspectors? Which are selectors? Which are predicates? Based on the suggestions in Section 11.1, how many axioms should you have?

11.16 Write an algebraic specification for an abstract data type String; think up appropriate operations for such a data type and the axioms they must satisfy. Compare your operations to standard string operations in a language of your choice.

11.17 Write an algebraic specification for an abstract data type bstree (binary search tree) with the following operations:

create: bstree
make: bstree × element 3 bstree → bstree
empty: bstree → boolean
left: bstree → bstree
right: bstree → bstree
data: bstree → element
isin: bstree × element → boolean
insert: bstree × element → bstree

What does one need to know about the element data type in order for a bstree data type to be possible?

11.18 Write C, Ada, or ML implementations for the specifications of Exercises 11.12, 11.14, 11.16, or 11.17. (Use generic packages in Ada as appropriate; use type variables and/or functors in ML as appropriate.)

11.19 Describe the Boolean data type using an algebraic specification.

11.20 Show, using the axioms for a queue from Section 11.1, that the following are true:

(a) frontq(enqueue(enqueue(createq,*x*), *y*)) = *x*

(b) frontq(enqueue(dequeue(enqueue(createq,*x*)), *y*)) = *y*

(c) frontq(dequeue(enqueue(enqueue(createq,*x*), *y*))) = *y*

11.21 A delete operation in a SymbolTable (see Exercise 11.14),

delete: SymbolTable × name → SymbolTable

might have two different semantics, as expressed in the following axioms:

delete(enter(*s*,*x*,*v*),*y*) = if *x* = *y* then *s* else
enter(delete(*s*,*y*),*x*,*v*)

or

delete(enter(*s*,*x*,*v*),*y*) = if *x* = *y* then delete (*s*,*x*) else enter (delete(*s*,*y*),*x*,*v*)

(a) Which of these is more appropriate for a symbol table in a translator for a block-structured language? In what situations might the other be more appropriate?

(b) Rewrite your axioms of Exercise 11.14 to incorporate each of these two axioms.

11.22 Ada requires generic packages to be instantiated with an actual type before the package can be used. By contrast, some other languages (e.g., C++) allow for the use of a type parameter directly in a declaration as follows (in Ada syntax):

```
x: Stack(Integer);
```

Why do you think Ada did not adopt this approach?

11.23 One could complain that the error axioms in the stack and queue algebraic specifications given in this chapter are unnecessarily strict, since for at least some of them, it is possible to define reasonable nonerror values. Give two examples of such axioms. Is it possible to eliminate all error values by finding nonerror substitutes for error values? Discuss.

11.24 In the algebraic specification of a stack or a queue, no mention was made of the possibility that a stack or queue might become full:

$$\text{full: stack} \rightarrow \text{boolean}$$

Such a test requires the inclusion of another operation, namely,

$$\text{size: stack} \rightarrow \text{integer}$$

and a constant

$$\text{maxsize: integer}$$

Write out a set of axioms for such a stack.

11.25 The queue implementations of the text ignored the error axioms in the queue ADT specification (indeed, one of the ML implementations generated warnings as a result). Rewrite the queue implementations in **(a)** Ada or **(b)** ML to use exceptions to implement the error axioms.

11.26 If one attempts to rewrite the Complex abstype of ML as a signature and structure, a problem arises in that infix operators lose their infix status when used by clients, unlike the abstype definitions, which retain their infix status.

(a) Discuss possible reasons for this situation.

(b) Compare this to Ada's handling of infix operators defined in packages.

11.27 Compare the handling of infix operators defined in C++ namespaces to Ada's handling of infix operators defined in packages. Are there any significant differences?

11.28 Write a C++ namespace definition and implementation for a single queue that completely suppresses the queue data type itself. Compare this to the Ada `Queues` package in Section 11.7.2.

11.29 In the ML structure definition `Queue1` in Section 11.5 (Figure 11.16), we noted that, even though the `Queue` data type was protected from misuse by clients, the actual details of the internal representation were still visible in the printed values. Even worse, if we had used the built-in list representation directly (instead of with a `datatype` definition with constructor `Q`), a client could arbitrarily change a `Queue` value without using the interface.

(a) Rewrite the `Queue1` implementation to use a list directly, and show that the internal structure of the data is not protected.

(b) A 1997 addition to the ML module system is the opaque signature declaration, which uses the symbol `:>` instead of just the colon in a signature constraint:

```
structure Queue1:> QUEUE =
   struct
   ...
   end;
```

Show that the use of this opaque signature suppresses all details of the `Queue` type and that an ordinary list can now be used with complete safety.

11.30 The functor `OListFUN` defined in Section 11.7.4 is not safe, in that it exposes the internal details of the `Queue` type to clients, and allows clients full access to these details.

(a) Explain why. (*Hint*: See the previous exercise.)

(b) Could we use an opaque signature declaration to remove this problem, as we did in the previous exercise? Explain.

11.31 An implementation of abstract data type bstree of Exercise 11.17 is unsafe if it exports all the listed operations: A client can destroy the order property by improper use of the operations.

(a) Which operations should be exported and which should not?

(b) Discuss the problems this may create for an implementation and the best way to handle it in C, Ada, or ML.

(c) Write an implementation in C, Ada, or ML based on your answer to part (b).

11.32 Can the integers be used as a model for the Boolean data type? How do the integers differ from the initial algebra for this data type?

11.33 A **canonical form** for elements of an algebra is a way of writing all elements in a standard, or canonical, way. For example, the initial algebra for the queue algebraic specification has the following canonical form for its elements:

$$\text{enqueue(enqueue(. . . enqueue(create,}a1\text{). . .,}an-1\text{),}an\text{)}$$

Show that any element of the initial algebra can be put in this form, and use the canonical form to show that the initial and final semantics of a queue are the same.

11.34 Given the following algebraic specification:

type Item **imports** boolean

operations:

create: Item
push: Item → Item
empty: Item → boolean

variables: *s*: Item;

axioms:

empty(create) = true
empty(push(s)) = false

show that the initial and final semantics of this specification are different. What axiom needs to be added to make them the same?

11.35 Exercise 11.3 noted that the Complex abstract data type may need to have all its algebraic properties, such as commutativity, written out as axioms. This is not necessary if one uses the principle of extensionality (Section 11.8) and assumes the corresponding algebraic properties for the reals. Use the principle of extensionality and the axioms of Exercise 11.2 to show the commutativity of complex addition.

11.36 The principle of extensionality (Section 11.8) can apply to functions as well as data.
(a) Write an extensional definition for the equality of two functions.
(b) Can this definition be implemented in a programming language? Why or why not?
(c) Does the language C, Ada, or ML allow testing of functions for equality? If so, how does this test differ from your extensional definition?

Notes and References

The earliest language to influence the development of abstract data types was Simula67, which introduced abstract data types in the form of classes. The algebraic specification of abstract data types described in Section 11.1 was pioneered by Guttag [1977] and Goguen, Thatcher, and Wagner [1978], who also developed the initial algebra semantics described in Section 11.9. The alternative final algebra semantics described there were developed by Kamin [1983]. A good general reference for abstract data type issues, including mathematical issues discussed in Section 11.8, is Cleaveland [1986]. Namespaces in C++ are discussed in Stroustrup [1997]. Packages in Java are described in Arnold et al. [2000]. More details on ADT specification and implementation in Ada (Section 11.4) can be found in Booch et al. [1993], Barnes [1998], and Cohen [1996]. The ML module mechanisms (signatures, structures, and functors) are described further in Paulson [1996], Ullman [1998], and MacQueen [1988]. CLU examples (Section 11.6) can be found in Liskov [1984]; Euclid examples (Section 11.6) can be found in Lampson et al. [1981]; Modula-2 examples can be found in Wirth [1988a] and King [1988]. Other languages with interesting abstract data type mechanisms are Mesa (Mitchell, Maybury, and Sweet [1979]; Geschke, Morris, and Satterthwaite [1977]) and Cedar (Lampson [1983], and Teitelman [1984]).

For an interesting study of separate compilation and dependencies in ML with references to Ada, Modula-3, and C see Appel and MacQueen [1994].

CHAPTER

Formal Semantics

CHAPTER 12

In Chapters 7 through 10 we discussed the semantics, or meaning, of programs from an informal, or descriptive, point of view. Historically, this has been the usual approach, both for the programmer and the language designer, and it is typified by the language reference manual, which explains language features based on an underlying model of execution that is more implied than explicit.

However, many computer scientists have emphasized the need for a more mathematical description of the behavior of programs and programming languages. The advantages of such a description are to make the definition of a programming language so precise that programs can be **proven** correct in a mathematical way and that translators can be **validated** to produce exactly the behavior described in the language definition. In addition, the work of producing such a precise specification aids the language designer in discovering inconsistencies and ambiguities.

Attempts to develop a standard mathematical system for providing precise semantic descriptions of languages have not met with complete acceptance, so there is no single method for formally defining semantics. Instead, there are a number of methods that differ in the formalisms used and the kinds of intended applications. No one method can be considered universal and most languages are still specified in a somewhat informal way. Formal semantic descriptions are more often supplied after the fact, and only for a part of the language. Also, the use of formal definitions to prove correct program behavior has been confined primarily to academic settings, although formal methods have begun to be used as part of the specification of complex software projects, including programming language translators. The purpose of this chapter is to survey the different methods that have been developed and to give a little flavor of their potential application.

Researchers have developed three principal methods to describe semantics formally:

1. *Operational semantics*. This method defines a language by describing its actions in terms of the operations of an actual or hypothetical machine. Of course, this requires that the operations of the machine used in the description also be precisely defined, and for this reason a very simple hypothetical machine is often used that bears little resemblance to an actual computer. Indeed, the machine we use for operational semantics in Section 12.2 is more of a mathematical model, namely, a "reduction machine," which is a collection of permissible steps in reducing programs by applying their operations to values. It is similar in spirit to the notion of a Turing machine, in which actions are precisely described in a mathematical way.
2. *Denotational semantics*. This approach uses mathematical functions on programs and program components to specify semantics. Programs are translated into functions about which properties can be proved using the standard mathematical theory of functions.

3. *Axiomatic semantics*. This method applies mathematical logic to language definition. Assertions, or predicates, are used to describe desired outcomes and initial assumptions for programs. Language constructs are associated with **predicate transformers** that create new assertions out of old ones, reflecting the actions of the construct. These transformers can be used to prove that the desired outcome follows from the initial conditions. Thus, this method of formal semantics is aimed specifically at correctness proofs.

All these methods are syntax-directed—that is, the semantic definitions are based on a context-free grammar or Backus-Naur Form (BNF) rules as discussed in Chapter 6. Formal semantics must then define all properties of a language that are not specified by the BNF.

These include static properties such as static types and declaration before use, which a translator can determine prior to execution. Although some authors consider such static properties to be part of the syntax of a language rather than its semantics, formal methods can describe both static and dynamic properties, and we will continue to view the semantics of a language as everything not specified by the BNF.

As with any formal specification method, two properties of a specification are essential. First, it must be **complete**; that is, every correct, terminating program must have associated semantics given by the rules. Second, it must be **consistent**; that is, the same program cannot be given two different, conflicting semantics. Finally, though of much less concern, it is advantageous for the given semantics to be minimal, or **independent**, in the sense that no rule is derivable from the other rules.

Formal specifications written in the operational or denotational style have a nice additional property, in that they can be translated relatively easily into working programs in a language suitable for prototyping, such as Prolog, ML, or Haskell.

In the sections that follow, we give an overview of each of these approaches to formal semantics. To make the differences in approach clearer, we use a sample small language as a standard example. In the operational semantics section, we also show briefly how the semantics of this example might be translated into a Prolog program. (Prolog is closest in spirit to operational semantics.) In the denotational semantics section, we also show briefly how the semantics of the example might be translated into a Haskell program. (Haskell is closest in spirit to denotational semantics.)

First, however, we give a description of the syntax and informal semantics of this language.

12.1 A Sample Small Language

The basic sample language that we will use throughout the chapter is a version of the integer expression language used in Chapter 6 and elsewhere. BNF rules for this language are given in Figure 12.1.

```
expr → expr '+' term | expr '−' term | term
term → term '*' factor | factor
factor → '(' expr ')' | number
number → number digit | digit
digit → '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9'
```

Figure 12.1 Basic sample language

The semantics of such arithmetic expressions are particularly simple: The value of an expression is a complete representation of its meaning. Thus, 2 + 3 ∗ 4 means the value 14, and (2 + 3) * 4 means 20. Since this language is a little too simple to demonstrate adequately all the aspects of the formal methods, we add complexity to it in two stages, as follows.

In the first stage, we add variables, statements, and assignments, as given by the grammar in Figure 12.2.

A program in the extended language consists of a list of statements separated by semicolons, and a statement is an assignment of an expression to an identifier. The grammar of Figure 12.1 remains as before, except that identifiers are added to factors. The revised grammar is shown in Figure 12.2.

factor → '(' *expr* ')' | *number* | *identifier*
program → *stmt-list*
stmt-list → *stmt* ';' *stmt-list* | *stmt*
stmt → *identifier* ':=' *expr*
identifier → *identifier letter* | *letter*
letter → 'a' | 'b' | 'c' | . . . | 'z'

Figure 12.2 First extension of the sample language

The semantics of such programs are now represented not by a single value but by a set of values corresponding to identifiers whose values have been defined, or bound, by assignments. For example, the program

```
a := 2+3;
b := a*4;
a := b-5
```

results in the bindings $b = 20$ and $a = 15$ when it finishes, so the set of values representing the semantics of the program is $\{a = 15, b = 20\}$. Such a set is essentially a function from identifiers (strings of lowercase letters according to the foregoing grammar) to integer values, with all identifiers that have not been assigned a value undefined. For the purposes of this chapter we will call such a function an **environment**, and we will write:

$$Env: \text{Identifier} \rightarrow \text{Integer} \cup \{\text{undef}\}$$

to denote a particular environment *Env*. For example, the *Env* function given by the program example can be defined as follows:

$$Env(I) = \begin{cases} 15 \text{ if } I = a \\ 20 \text{ if } I = b \\ \text{undef otherwise} \end{cases}$$

The operation of looking up the value of an identifier *I* in an environment *Env* is then simply described by function evaluation *Env*(*I*). The operation of adding a new value binding to *Env* can also be defined in functional terms. We will use the notation *Env* & $\{I = n\}$ to denote the adding of the new value *n* for *I* to *Env*. In terms of functions,

$$(Env\& \{I = n\})(J) = \begin{cases} n \text{ if } J = I \\ Env(J) \text{ otherwise} \end{cases}$$

Finally, we also need the notion of the **empty environment**, which we will denote by Env_0:

$$Env_0(I) = \text{undef for all } I$$

This notion of environment is particularly simple and differs from what we called the environment in Chapters 7 and 10. Indeed, an environment as defined here incorporates both the symbol table and state functions from Chapter 7. We note that such environments do not allow pointer values, do not include scope information, and do not permit aliases. More complex environments require much greater complexity and will not be studied here.

This view of the semantics of a program as represented by a resulting, final environment has the effect that the consistency and completeness properties stated in the introduction have the following straightforward interpretation: Consistency means that we cannot derive two different final environments for the same program, and completeness means that we must be able to derive a final environment for every correct, terminating program.

The second extension to our sample language will be the addition of "if" and "while" control statements to the first extension. Statements can now be of three kinds, and we extend their definition accordingly; see Figure 12.3.

stmt → *assign-stmt* | *if-stmt* | *while-stmt*
assign-stmt → *identifier* ':=' *expr*
if-stmt → 'if' *expr* 'then' *stmt-list* 'else' *stmt-list* 'fi'
while-stmt → 'while' *expr* 'do' *stmt-list* 'od'

Figure 12.3 Second extension of the sample language

The syntax of the if statement and while statement borrow the Algol68 convention of writing reserved words backward—thus od and fi—to close statement blocks rather than using the begin and end of Pascal and Algol60.

The meaning of an *if-stmt* is that *expr* should be evaluated in the current environment. If it evaluates to an integer greater than 0, then the *stmt-list* after 'then' is executed. If not, the *stmt-list* after the 'else' is executed. The meaning of a *while-stmt* is similar: As long as *expr* evaluates to a quantity greater than 0, *stmt-list* is repeatedly executed, and *expr* is reevaluated. Note that these semantics are nonstandard!

Here is an example of a program in this language:

```
n := 0 - 5;
if n then i := n else i := 0 - n fi;
fact := 1;
while i do
  fact := fact * i;
  i := i - 1
od
```

The semantics of this program are given by the (final) environment $\{n = -5, i = 0, \text{fact} = 120\}$.

Loops are the most difficult of the foregoing constructs to give formal semantics for, and in the following we will not always give a complete solution. These can be found in the references at the end of the chapter.

Formal semantic methods frequently use a simplified version of syntax from that given. Since the parsing step can be assumed to have already taken place, and since semantics are to be defined only for syntactically correct constructs, an ambiguous grammar can be used to define semantics. Further, the nonterminal symbols can be replaced by single letters, which may be thought to represent either strings of tokens or nodes in a parse tree. Such a syntactic specification is sometimes called **abstract syntax**. An abstract syntax for our sample language (with extensions) is the following:

$$
\begin{aligned}
&P \to L\\
&L \to L_1\ \text{';'}\ L_2 \mid S\\
&S \to I\ \text{':='}\ E \mid \text{'if'}\ E\ \text{'then'}\ L_1, \text{'else'}\ L_2\ \text{'fi'}\\
&\qquad \mid \text{'while'}\ E\ \text{'do'}\ L\ \text{'od'}\\
&E \to E_1\ \text{'+'}\ E_2 \mid E_1\ \text{'-'}\ E_2 \mid E_1\ \text{'*'}\ E_2\\
&\qquad\qquad \mid \text{'('}\ E_1\ \text{')'} \mid N\\
&N \to N_1\ D \mid D\\
&D \to \text{'0'} \mid \text{'1'} \mid \ldots \mid \text{'9'}\\
&I \to I_1\ A \mid A\\
&A \to \text{'}a\text{'} \mid \text{'}b\text{'} \mid \ldots \mid \text{'}z\text{'}
\end{aligned}
$$

Here the letters stand for syntactic entities as follows:

P : Program
L : Statement-list
S : Statement
E : Expression
N : Number
D : Digit
I : Identifier
A : Letter

To define the semantics of each one of these symbols, we define the semantics of each right-hand side of the abstract syntax rules in terms of the semantics of their parts. Thus, syntax-directed semantic

definitions are recursive in nature. This also explains why we need to number the letters on the right-hand sides when they represent the same kind of construct: Each choice needs to be distinguished.

We note finally that the tokens in the grammar have been enclosed in quotation marks. This becomes an important point when we must distinguish between the symbol '1' and the operation of addition on the integers, or 1, which it represents. Similarly, the symbol '3' needs to be distinguished from its value, or the number 3.

We now survey the different formal semantic methods.

12.2 Operational Semantics

Operational semantics define the semantics of a programming language by specifying how an arbitrary program is to be executed on a machine whose operation is completely known.

We have noted in the introduction that there are many possibilities for the choice of a defining machine. The machine can be an actual computer, and the operational semantics can be specified by an actual translator for the language written in the machine code of the chosen machine. Such **definitional interpreters** or **compilers** have in the past been de facto language definitions (FORTRAN and C were originally examples). However, there are drawbacks to this method: The defining translator may not be available to a user, the operation of the underlying computer may not be completely specified, and the defining implementation may contain errors or other unexpected behavior.

By contrast, operational semantics can define the behavior of programs in terms of an **abstract machine** that does not need to exist anywhere in hardware, but that is simple enough to be completely understood and to be simulated by any user to answer questions about program behavior.

In principle, an abstract machine can be viewed as consisting of a program, a control, and a store or memory, as shown in Figure 12-4.

Figure 12-4 Three parts of an abstract machine

An operational semantic specification of a programming language specifies how the control of this abstract machine reacts to an arbitrary program in the language to be defined, and in particular, how storage is changed during the execution of a program.

The particular form of abstract machine that we will present is that of a **reduction machine**, whose control operates directly on a program to reduce it to its semantic "value." For example, given the expression $(3 + 4) * 5$, the control of the reduction machine will reduce it to its numeric value (which is its semantic content) using the following sequence of steps:

```
(3 + 4) * 5 => (7) * 5      — 3 and 4 are added to get 7
            => 7 * 5        — the parentheses around 7 are dropped
            => 35           — 7 and 5 are multiplied to get 35
```

In general, of course, the semantic content of a program will be more than just a numeric value, but as you will see, the semantic content of a program can be represented by a data value of some structured type, which operational semantic reductions will produce.

To specify the operational semantics of our sample language, we give **reduction rules** that specify how the control reduces the constructs of the language to a value. These reduction rules are given in a mathematical notation similar to logical inference rules, so we discuss such logical inference rules first.

12.2.1 Logical Inference Rules

Inference rules in logic are written in the following form:

$$\frac{\text{premise}}{\text{conclusion}}$$

That is, the premise, or condition, is written first; then a line is drawn, and the conclusion, or result, is written. This indicates that, whenever the premise is true, the conclusion is also true. As an example, we can express the commutative property of addition as the following inference rule:

$$\frac{a + b = c}{b + a = c}$$

In logic, such inference rules are used to express the basic rules of propositional and predicate calculus. As an example of an inference rule in logic, the transitive property of implication is given by the following rule:

$$\frac{a \rightarrow b, b \rightarrow c}{a \rightarrow c}$$

This says that if a implies b and b implies c, then a implies c.

Axioms are inference rules with no premise—they are always true. An example of an axiom is $a + 0 = a$ for integer addition. This can be written as an inference rule with an empty premise:

$$\frac{}{a + 0 = a}$$

More often, this is written without the horizontal line:

$$a + 0 = a$$

12.2.2 Reduction Rules for Integer Arithmetic Expressions

We use the notation of inference rules to describe the way the control operates to reduce an expression to its value. There are several styles in current use as to how these rules are written. The particular form we use for our reduction rules is called **structural operational semantics**; an alternative form, called **natural semantics**, is actually closer to the denotational semantics studied later; see the Notes and References.

We base the semantic rules on the abstract syntax for expressions in our sample language:

$$E \rightarrow E_1 \text{ '+' } E_2 \mid E_1 \text{ '−' } E_2 \mid E_1 \text{ '*' } E_2 \mid \text{'(' } E_1 \text{ ')'}$$
$$N \rightarrow N_1 D \mid D$$
$$D \rightarrow \text{'0'} \mid \text{'1'} \mid \ldots \mid \text{'9'}$$

For the time being we can ignore the storage, since this grammar does not include identifiers. We use the following notation: E, E_1, and so on to denote expressions that have not yet been reduced to values; V, V_1, and so on will stand for integer values; $E \Rightarrow E_1$ states that expression E reduces to expression E_1 by

some reduction rule. The reduction rules for expressions are the following, each of which we will discuss in turn.

(1) We collect all the rules for reducing digits to values in this one rule, all of which are axioms:

‘0’ => 0
‘1’ => 1
‘2’ => 2
‘3’ => 3
‘4’ => 4
‘5’ => 5
‘6’ => 6
‘7’ => 7
‘8’ => 8
‘9’ => 9

(2) We collect the rules for reducing numbers to values in this one rule, which are also axioms:

V ‘0’ => 10 * V
V ‘1’ => 10 * V + 1
V ‘2’ => 10 * V + 2
V ‘3’ => 10 * V + 3
V ‘4’ => 10 * V + 4
V ‘5’ => 10 * V + 5
V ‘6’ => 10 * V + 6
V ‘7’ => 10 * V + 7
V ‘8’ => 10 * V + 8
V ‘9’ => 10 * V + 9

(3) V_1 ‘+’ V_2 => $V_1 + V_2$

(4) V_1 ‘−’ V_2 => $V_1 - V_2$

(5) V_1 ‘*’ V_2 => $V_1 * V_2$

(6) ‘(’ V ‘)’ => V

(7) $$\frac{E \Rightarrow E_1}{E \text{ ‘+’ } E_2 \Rightarrow E_1 \text{ ‘+’ } E_2}$$

(8) $$\frac{E \Rightarrow E_1}{E \text{ ‘−’ } E_2 \Rightarrow E_1 \text{ ‘−’ } E_2}$$

(9) $$\frac{E \Rightarrow E_1}{E \text{ ‘*’ } E_2 \Rightarrow E_1 \text{ ‘*’ } E_2}$$

(10) $$\frac{E \Rightarrow E_1}{V \text{ ‘+’ } E \Rightarrow V \text{ ‘+’ } E_1}$$

(11) $$\frac{E \Rightarrow E_1}{E \text{ `}-\text{'} E \Rightarrow V \text{ `}-\text{'} E_1}$$

(12) $$\frac{E \Rightarrow E_1}{V \text{ `}*\text{'} E \Rightarrow V \text{ `}*\text{'} E_1}$$

(13) $$\frac{E \Rightarrow E_1}{\text{`(' } E \text{ `)'} \Rightarrow \text{`(' } E_1 \text{ `)'}}$$

(14) $$\frac{E \Rightarrow E_1,\ E_1 \Rightarrow E_2}{E \Rightarrow E_2}$$

Rules 1 through 6 are all axioms. Rules 1 and 2 express the reduction of digits and numbers to values: '0' => 0 states that the **character** '0' (a syntactic entity) reduces to the **value** 0 (a semantic entity). Rules 3, 4, and 5 say that whenever we have an expression that consists of two values and an operator symbol, we can reduce that expression to a value by applying the appropriate operation whose symbol appears in the expression. Rule 6 says that if an expression consists of a pair of parentheses surrounding a value, then the parentheses can be dropped.

The remainder of the reduction rules are inferences that allow the reduction machine to combine separate reductions together to achieve further reductions. Rules 7, 8, and 9 express the fact that, in an expression that consists of an operation applied to other expressions, the left subexpression may be reduced by itself and that reduction substituted into the larger expression. Rules 10 through 12 express the fact that, once a value is obtained for the left subexpression, the right subexpression may be reduced. Rule 13 says that we can first reduce the inside of an expression consisting of parentheses surrounding another expression. Finally, rule 14 expresses the general fact that reductions can be performed stepwise (sometimes called the **transitivity rule** for reductions).

Let us see how these reduction rules can be applied to a complicated expression to derive its value. Take, for example, the expression 2 * (3 + 4) − 5. To show each reduction step clearly, we surround each character with quotation marks within the reduction steps.

We first reduce the expression 3 + 4 as follows:

'3' '+' '4' => 3 '+' '4'	(Rules 1 and 7)
=> 3 '+' 4	(Rules 1 and 10)
=> 3 + 4 = 7	(Rule 3)

Hence by rule 14, we have '3' '+' '4' => 7. Continuing,

'(' '3' '+' '4' ')' => '(' 7 ')'	(Rule 13)
=> 7	(Rule 6)

Now we can reduce the expression 2 * (3 + 4) as follows:

'2' '*' '(' '3' '+' '4' ')' => 2 '*' '(' '3' '+' '4' ')'	(Rules 1 and 9)
=> 2 '*' 7	(Rule 12)
=> 2 * 7 = 14	(Rule 5)

And, finally,

‘2’ ‘*’ ‘(’ ‘3’ ‘+’ ‘4’ ‘)’ ‘−’ ‘5’ => 14 ‘−’ ‘5’ (Rules 1 and 8)
=> 14 ‘−’ 5 (Rule 11)
=> 14 − 5 = 9 (Rule 4)

We have shown that the reduction machine can reduce the expression 2 * (3 + 4) − 5 to 9, which is the value of the expression.

12.2.3 Environments and Assignment

We want to extend the operational semantics of expressions to include environments and assignments, according to the following abstract syntax:

$$
\begin{aligned}
&P \rightarrow L\\
&L \rightarrow L_1\ \text{';'}\ L_2 \mid S\\
&S \rightarrow I\ \text{':='}\ E\\
&E \rightarrow E_1\ \text{'+'}\ E_2 \mid E_1\ \text{'-'}\ E_2 \mid E_1\ \text{'*'}\ E_2\\
&\qquad\qquad \mid \text{'('}\ E_1\ \text{')'} \mid N\\
&N \rightarrow N_1\ D \mid D\\
&D \rightarrow \text{'0'} \mid \text{'1'} \mid \ldots \mid \text{'9'}\\
&I \rightarrow I_1\ A \mid A\\
&A \rightarrow \text{'}a\text{'} \mid \text{'}b\text{'} \mid \ldots \mid \text{'}z\text{'}
\end{aligned}
$$

To do this, we must include the effect of assignments on the storage of the abstract machine. Our view of the storage will be the same as in other sections; that is, we view it as an environment that is a function from identifiers to integer values (including the undefined value):

$$Env\text{: Identifier} \rightarrow \text{Integer} \cup \{\text{undef}\}$$

To add environments to the reduction rules, we need a notation to show the dependence of the value of an expression on an environment. We use the notation <*E* | *Env*> to indicate that expression *E* is evaluated in the presence of environment *Env*. Now our reduction rules change to include environments. For example, rule 7 with environments becomes:

(7)
$$\frac{<E \mid Env> \Rightarrow <E_1 \mid Env>}{<E\ \text{'+'}\ E_2 \mid Env> \Rightarrow <E_1\ \text{'+'}\ E_2 \mid Env>}$$

This states that if *E* reduces to E_1 in the presence of environment *Env*, then *E* ‘+’ E_2 reduces to E_1 ‘+’ E_2 in the same environment. Other rules are modified similarly. The one case of evaluation that explicitly involves the environment is when an expression is an identifier *I*:

(15)
$$\frac{Env(I) = V}{<I \mid Env> \Rightarrow <V \mid Env>}$$

This states that if the value of identifier *I* is *V* in environment *Env*, then *I* reduces to *V* in the presence of *Env*.

It remains to add assignment statements and statement sequences to the reduction rules. First, statements must reduce to environments instead of integer values, since they create and change environments. Thus, we have:

(16) $$\langle I \text{ ':=' } V \mid Env \rangle \Rightarrow Env \;\&\; \{I = V\}$$

which states that the assignment of the value V to I in environment Env reduces to a new environment where I is equal to V.

The reduction of expressions within assignments proceeds via the following rule:

(17) $$\frac{\langle E \mid Env \rangle \Rightarrow \langle E_1 \mid Env \rangle}{\langle I \text{ ':=' } E \mid Env \rangle \Rightarrow \langle I \text{ ':=' } E_1 \mid Env \rangle}$$

A statement sequence reduces to an environment formed by accumulating the effect of each assignment:

(18) $$\frac{\langle S \mid Env \rangle \Rightarrow Env_1}{\langle S \text{ ';' } L \mid \text{Env} \rangle \Rightarrow \langle L \mid Env_1 \rangle}$$

Finally, a program is a statement sequence that has no prior environment; it reduces to the effect it has on the empty starting environment:

(19) $$L \Rightarrow \langle L \mid Env_0 \rangle$$

(recall that $Env_0(I)$ = undef for all identifiers I).

We leave the rules for reducing identifier expressions to the reader; they are completely analogous to the rules for reducing numbers (see Exercise 12.4).

Let us use these rules to reduce the following sample program to an environment:

```
a := 2+3;
b := a*4;
a := b-5
```

To simplify the reduction, we will suppress the use of quotes to differentiate between syntactic and semantic entities. First, by rule 19, we have:

$a := 2 + 3; b := a * 4; a := b - 5 \Rightarrow$
$\langle a := 2 + 3; b := a * 4; a := b - 5 \mid Env_0 \rangle$

Also, by rules 3, 17, and 16,

$\langle a := 2 + 3 \mid Env_0 \rangle \Rightarrow$
$\langle a := 5 \mid Env_0 \rangle \Rightarrow$
$Env_0 \;\&\; \{a = 5\} = \{a = 5\}$

Then, by rule 18,

$\langle a := 2 + 3; b := a * 4; a := b - 5 \mid Env_0 \rangle \Rightarrow$
$\langle b := a * 4; a := b - 5 \mid \{a = 5\} \rangle$

Similarly, by rules 15, 9, 5, 17, and 16,

$<b := a * 4 \mid \{a = 5\}> => <b := 5 * 4 \mid \{a = 5\}> =>$
$<b := 20 \mid \{a = 5\}> => \{a = 5\} \;\&\; \{b = 20\} = \{a = 5, b = 20\}$

Thus, by rule 18,

$<b := a * 4; a := b - 5 \mid \{a = 5\}> =>$
$<a := b - 5 \mid \{a = 5, b = 20\}>$

Finally, by a similar application of the rules, we get:

$<a := b - 5 \mid \{a = 5, b = 20\}> =>$
$<a := 20 - 5 \mid \{a = 5, b = 20\}> =>$
$<a := 15 \mid \{a = 5, b = 20\}> =>$
$\{a = 5, b = 20\} \;\&\; \{a = 15, b = 20\}$

and the program reduces to the environment $\{a = 15, b = 20\}$.

12.2.4 Control

It remains to add the if- and while statements to our sample language, with the following abstract syntax:

$$S \rightarrow \text{`if'}\ E\ \text{`then'}\ L_1\ \text{`else'}\ L_2\ \text{`fi'}$$
$$\mid\ \text{`while'}\ E\ \text{`do'}\ L\ \text{`od'}$$

Reduction rules for if statements are the following:

(20)
$$\frac{<E \mid Env> => <E_1 \mid Env>}{<\text{`if'}\ E\ \text{`then'}\ L_1\ \text{`else'}\ L_2\ \text{`fi'} \mid Env> => <\text{`if'}\ E_1\ \text{`then'}\ L_1\ \text{`else'}\ L_2\ \text{`fi'} \mid Env>}$$

(21)
$$\frac{V > 0}{<\text{`if'}\ V\ \text{`then'}\ L_1\ \text{`else'}\ L_2\ \text{`fi'} \mid Env> => <L_1 \mid Env>}$$

(22)
$$\frac{V \leq 0}{<\text{`if'}\ V\ \text{`then'}\ L_1\ \text{`else'}\ L_2\ \text{`fi'} \mid Env> => <L_2 \mid Env>}$$

Reduction rules for while statements are as follows:

(23)
$$\frac{<E \mid Env> => <V \mid Env>,\ V \leq 0}{<\text{`while'}\ E\ \text{`do'}\ L\ \text{`od'} \mid Env> => Env}$$

(24)
$$\frac{<E \mid Env> => <V \mid Env>,\ V > 0}{<\text{`while'}\ E\ \text{`do'}\ L\ \text{`od'} \mid Env => < L;\ \text{`while'}\ E\ \text{`do'}\ L\ \text{`od'} \mid Env>}$$

Note that the last rule is recursive. It states that if, given environment *Env*, the expression *E* evaluates to a positive value, then execution of the while-loop under *Env* reduces to an execution of *L*, followed by the execution of the same while-loop all over again.

As an example, let us reduce the while statement of the program

```
n := 0 - 3;
if n then i := n else i := 0 - n fi;
fact := 1;
while i do
  fact := fact * i;
  i := i - 1
od
```

to its environment value. The environment at the start of the while-loop is $\{n = -3, i = 3, \text{fact} = 1\}$. Since $\langle i \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow \langle 3 \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle$ and $3 > 0$, rule 24 applies, so by rule 18 we must compute the environment resulting from the application of the body of the loop to the environment $\{n = -3, i = 3, \text{fact} = 1\}$:

$\langle \text{fact} := \text{fact} * i \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\langle \text{fact} := 1 * i \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\langle \text{fact} := 1 * 3 \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\langle \text{fact} := 3 \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\{n = -3, i = 3, \text{fact} = 3\}$

and

$\langle i := i - 1 \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\langle i := 3 - 1 \mid \{n = -3, i = 3, \text{fact} = 3\}\rangle \Rightarrow$
$\langle i := 2 \mid \{n = -3, i = 3, \text{fact} = 3\}\rangle \Rightarrow$
$\{n = -3, i = 2, \text{fact} = 3\}$

so

$\langle \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\langle \text{fact} := \text{fact} * i ; i := i - 1; \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 3, \text{fact} = 1\}\rangle \Rightarrow$
$\langle i := i - 1; \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 3, \text{fact} = 3\}\rangle \Rightarrow$
$\langle \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 2, \text{fact} = 3\}\rangle$

Continuing in this way, we get:

$\langle \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 2, \text{fact} = 3\}\rangle \Rightarrow$
$\langle \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 1, \text{fact} = 6\}\rangle \Rightarrow$
$\langle \text{while } i \text{ do } \ldots \text{ od} \mid \{n = -3, i = 0, \text{fact} = 6\}\rangle \Rightarrow$
$\{n = -3, i = 0, \text{fact} = 6\}$

so the final environment is $\{n = -3, i = 0, \text{fact} = 6\}$.

12.2.5 Implementing Operational Semantics in a Programming Language

It is possible to implement operational semantic rules directly as a program to get a so-called **executable specification**. This is useful for two reasons. First, it allows us to construct a language interpreter directly from a formal specification. Second, it allows us to check the correctness of the specification by testing the resulting interpreter. Since operational semantic rules as we have defined them are similar to logical inference rules, it is not surprising that Prolog is a natural choice as an implementation language. In this section, we briefly sketch a possible Prolog implementation for the reduction rules of our sample language.

First, consider how we might represent a sample language program in abstract syntax in Prolog. This is easy to do using terms. For example, 3 * (4 + 5) can be represented in Prolog as:

```
times(3,plus(4,5))
```

and the program

```
a := 2+3;
b := a*4;
a := b-5
```

as

```
seq(assign(a,plus(2,3)),
        seq(assign(b,times(a,4)),assign(a,sub(b,5))))
```

Note that this form of abstract syntax is actually a tree representation and that no parentheses are necessary to express grouping. Thus, all rules involving parentheses become unnecessary. Note that we could also take the shortcut of letting Prolog compute the semantics of integers, since we could also write, for example, `plus(23,345)`, and Prolog will automatically compute the values of 23 and 345. Thus, with these shortcuts, rules 1, 2, 6, and 13 can be eliminated.

Consider now how we might write reduction rules. Ignoring environments for the moment, we can write a general reduction rule for expressions as:

```
reduce(X,Y) :- ...
```

where `X` is any arithmetic expression (in abstract syntax) and `Y` is the result of a single reduction step applied to `X`. For example, rule 3 can be written as:

```
reduce(plus(V1,V2),R) :-
        integer(V1), integer(V2), !, R is V1 + V2.
```

Here the predicate `integer` tests its argument to make sure that it is an (integer) value, and then `R` is set to the result of the addition.

In a similar fashion, rule 7 can be written as:

```
reduce(plus(E,E2),plus(E1,E2)) :- reduce(E,E1).
```

and rule 10 can be written as:

```
reduce(plus(V,E),plus(V,E1)) :-
        integer(V), !, reduce(E,E1).
```

Rule 14 presents a problem. If we write it as given:

```
reduce(E,E2) :- reduce(E,E1), reduce(E1,E2).
```

then infinite recursive loops will result. Instead, we make a distinction between a single reduction step, which we call `reduce` as before, and multiple reductions, which we will call `reduce_all`. We then write rule 14 as two rules (the first is the stopping condition of the recursion):

```
reduce_all(V,V) :- integer(V), !.
reduce_all(E,E2) :- reduce(E,E1), reduce_all(E1,E2).
```

Now, if we want the final semantic value `V` of an expression `E`, we must write `reduce_all(E,V)`.

Finally, consider how this program might be extended to environments and control. First, a pair <*E* | *Env*> or < *L* | *Env*> can be thought of as a *configuration* and written in Prolog as `config(E,Env)` or `config(L,Env)`. Rule 15 can then be written as:

```
reduce(config(I,Env),config(V,Env)) :-
                    atom(I), !, lookup(Env, I, V).
```

`(atom(I)` tests for a variable, and the `lookup` operation finds values in an environment). Rule 16 can be similarly written as:

```
reduce(config(assign(I,V),Env),Env1) :-
     integer(V), !, update(Env, value(I,V), Env1).
```

where `update` inserts the new value `V` for `I` into `Env`, yielding new environment `Env1`.

Any dictionary structure for which `lookup` and `update` can be defined can be used to represent an environment in this code. For example, the environment $\{n = -3, i = 3, \text{fact} = 1\}$ can be represented as the list `[value(n,23)`, `value(i,3)`, `value(fact,1)]`. The remaining details are left to the reader.

12.3 Denotational Semantics

Denotational semantics use functions to describe the semantics of a programming language. A function describes semantics by associating semantic values to syntactically correct constructs. A simple example of such a function is a function that maps an integer arithmetic expression to its value, which we could call the *Val* function:

$$Val : \text{Expression} \rightarrow \text{Integer}$$

For example, *Val*(2 + 3 * 4) = 14 and *Val*((2 + 3) * 4) = 20. The domain of a semantic function such as *Val* is a **syntactic domain**. In the case of *Val*, it is the set of all syntactically correct integer arithmetic expressions. The range of a semantic function is a **semantic domain**, which is a mathematical structure. In the case of *Val*, the set of integers is the semantic domain. Since *Val* maps the syntactic construct 2 + 3 * 4 to the semantic value 14, 2 + 3 * 4 is said to **denote** the value 14. This is the origin of the name denotational semantics.

A second example may be useful before we give denotational semantics for our sample language from Section 12.1. In many programming languages a program can be viewed as something that receives input and produces output. Thus, the semantics of a program can be represented by a function from input to output, and a semantic function for programs would look like this:

$$P : \text{Program} \rightarrow (\text{Input} \rightarrow \text{Output})$$

The semantic domain to which P maps programs is a set of functions, namely, the functions from Input to Output, which we represent by Input → Output, and the semantic value of a program is a function. For example, if p represents the C program

```
main()
{ int x;
  scanf("%d",&x);
  printf("%d\n",x);
  return 0;
}
```

that inputs an integer and outputs the same integer, then p denotes the identity function f from integers to integers: $P(p) = f$, where f: Integer → Integer is given by $f(x) = x$.

Very often semantic domains in denotational descriptions will be function domains, and values of semantic functions will be functions themselves. To simplify the notation of these domains, we will often assume that the function symbol "→" is right associative and leave off the parentheses from domain descriptions. Thus,

$$P : \text{Program} \rightarrow (\text{Input} \rightarrow \text{Output})$$

becomes:

$$P : \text{Program} \rightarrow \text{Input} \rightarrow \text{Output}$$

In the following, we will give a brief overview of a denotational definition of the semantics of a programming language and then proceed with a denotational definition of our sample language from Section 12.1. A denotational definition of a programming language consists of three parts:

1. A definition of the **syntactic domains**, such as the sets Program and Expression, on which the semantic functions act
2. A definition of the **semantic domains** consisting of the values of the semantic functions, such as the sets Integer and Integer → Integer
3. A definition of the semantic functions themselves (sometimes called **valuation functions**)

We will consider each of these parts of the denotational definition in turn.

12.3.1 Syntactic Domains

Syntactic domains are defined in a denotational definition using notation that is almost identical to the abstract syntax described in Section 12.1. The sets being defined are listed first with capital letters denoting elements from the sets. Then the grammar rules are listed that recursively define the elements of the set. For example, the syntactic domains Number and Digit are specified as follows:

D: Digit

N: Number

$N \rightarrow N\ D \mid D$

$D \rightarrow$ '0' | '1' | . . . | '9'

A denotational definition views the syntactic domains as sets of syntax trees whose structure is given by the grammar rules. Semantic functions will be defined recursively on these sets, based on the structure of a syntax tree node.

12.3.2 Semantic Domains

Semantic domains are the sets in which semantic functions take their values. These are sets like syntactic domains, but they also may have additional mathematical structure, depending on their use. For example, the integers have the arithmetic operations "+," "−," and "*." Such domains are **algebras**, which need to be specified by listing their functions and properties. A denotational definition of the semantic domains lists the sets and the operations but usually omits the properties of the operations. These can be specified by the algebraic techniques studied in Chapter 11, or they can simply be assumed to be well known, as in the case of the arithmetic operations on the integers. A specification of the semantic domains also lists only the basic domains without specifically including domains that are constructed of functions on the basic domains.

Domains sometimes need special mathematical structures that are the subject of **domain theory** in programming language semantics. In particular, the term "domain" is sometimes reserved for an algebra with the structure of a complete partial order. Such a structure is needed to define the semantics of recursive functions and loops. See the references at the end of the chapter for further detail on domain theory.

An example of a specification of a semantic domain is the following specification of the integers:

Domain v: Integer $= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Operations

$+$: Integer $\times$ Integer $\rightarrow$ Integer

$-$: Integer $\times$ Integer $\rightarrow$ Integer

$*$: Integer $\times$ Integer $\rightarrow$ Integer

In this example, we restrict ourselves to the three operations "+," "−," and "*," which are the only operations represented in our sample language. In the foregoing notation, the symbols "v:" in the first line indicate that the name v will be used for a general element from the domain, that is, an arbitrary integer.

12.3.3 Semantic Functions

A semantic function is specified for each syntactic domain. Each semantic function is given a different name based on its associated syntactic domain. A common convention is to use the boldface letter corresponding to the elements in the syntactic domain. Thus, the value function from the syntactic domain Digit to the integers is written as follows:

$$\boldsymbol{D} : \text{Digit} \rightarrow \text{Integer}$$

The value of a semantic function is specified recursively on the trees of the syntactic domains using the structure of the grammar rules. This is done by giving a **semantic equation** corresponding to each grammar rule.

For example, the grammar rules for digits

$$D \rightarrow \text{`0'} \mid \text{`1'} \mid \ldots \mid \text{`9'}$$

give rise to the syntax tree nodes

$$\begin{array}{ccccc} D & & D & \ldots & D \\ | & & | & & | \\ \text{`0'} & & \text{`1'} & & \text{`9'} \end{array}$$

and the semantic function **D** is defined by the following semantic equations:

$$\boldsymbol{D}\left(\begin{array}{c} D \\ | \\ \text{`0'} \end{array}\right) = 0, \qquad \boldsymbol{D}\left(\begin{array}{c} D \\ | \\ \text{`1'} \end{array}\right) = 1, \ldots, \qquad \boldsymbol{D}\left(\begin{array}{c} D \\ | \\ \text{`9'} \end{array}\right) = 9$$

representing the value of each leaf.

This cumbersome notation is shortened to the following:

$$\boldsymbol{D}[[\text{`0'}]] = 0, \boldsymbol{D}[[\text{`1'}]] = 1, \ldots, \boldsymbol{D}[[\text{`9'}]] = 9$$

The double brackets [[. . .]] indicate that the argument is a syntactic entity consisting of a syntax tree node with the listed arguments as children.

As another example, the semantic function

$$N : \text{Number} \rightarrow \text{Integer}$$

from numbers to integers is based on the syntax

$$N \rightarrow N\,D \mid D$$

and is given by the following equations:

$$\begin{aligned} \boldsymbol{N}[[ND]] &= 10 * \boldsymbol{N}[[N]] + \boldsymbol{N}[[D]] \\ \boldsymbol{N}[[D]] &= \boldsymbol{D}[[D]] \end{aligned}$$

Here $[[ND]]$ refers to the tree node $\begin{array}{ccc} & N & \\ / & & \backslash \\ N & & D \end{array}$ and $[[D]]$ to the node $\begin{array}{c} N \\ | \\ D \end{array}$. We are now ready to give a complete denotational definition for the expression language of Section 12.1.

12.3.4 Denotational Semantics of Integer Arithmetic Expressions

Here is the denotational definition according to the conventions just described.

Syntactic Domains

E: Expression
N: Number
D: Digit

$E \rightarrow E_1$ '+' $E_2 \mid E_1$ '−' $E_2 \mid E_1$ '*' E_2
$\quad \mid$ '(' E ')' $\mid N$
$N \rightarrow N\,D \mid D$
$D \rightarrow$ '0' $\mid$ 1' $\mid \ldots \mid$ '9'

Semantic Domains

Domain v: Integer $= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
Operations
$+$: Integer $\times$ Integer $\rightarrow$ Integer
$-$: Integer $\times$ Integer $\rightarrow$ Integer
$*$: Integer $\times$ Integer $\rightarrow$ Integer

Semantic Functions

$\boldsymbol{E}$: Expression $\rightarrow$ Integer

$\boldsymbol{E}[[E_1$ '+' $E_2]] = \boldsymbol{E}[[E_1]] + \boldsymbol{E}[[E_2]]$
$\boldsymbol{E}[[E_1$ '−' $E_2]] = \boldsymbol{E}[[E_1]] - \boldsymbol{E}[[E_2]]$
$\boldsymbol{E}[[E_1$ '*' $E_2]] = \boldsymbol{E}[[E_1]] * \boldsymbol{E}[[E_2]]$
$\boldsymbol{E}[[$'(' E ')'$]] = \boldsymbol{E}[[E]]$
$\boldsymbol{E}[[N]] = \boldsymbol{N}[[N]]$

$\boldsymbol{N}$: Number $\rightarrow$ Integer

$\boldsymbol{N}[[ND]] = 10 * \boldsymbol{N}[[N]] + \boldsymbol{N}[[D]]$
$\boldsymbol{N}[[D]] = \boldsymbol{D}[[D]]$

$\boldsymbol{D}$: Digit $\rightarrow$ Integer

$\boldsymbol{D}[[$'0'$]] = 0, \boldsymbol{D}[[$'1'$]] = 1, \ldots, \boldsymbol{D}[[$'9'$]] = 9$

In this denotational description, we have retained the use of quotation marks to distinguish syntactic from semantic entities. In denotational semantics, this is not as necessary as in other semantic descriptions, since arguments to semantic functions are always syntactic entities. Thus, we could drop the quotation marks and write $\boldsymbol{D}[[0]] = 0$, and so on. For clarity, we will generally continue to use the quotation marks, however.

To see how these equations can be used to obtain the semantic value of an expression, we compute $\boldsymbol{E}$[[(2 + 3)∗4]] or, more precisely, $\boldsymbol{E}$[[‘(’ ‘2’ ‘+’ ‘3’ ‘)’ ‘∗’ ‘4’]]:

$\boldsymbol{E}$[[‘(’ ‘2’ ‘+’ ‘3’ ‘)’ ‘*’ ‘4’]]
= $\boldsymbol{E}$[[‘(’ ‘2’ ‘+’ ‘3’ ‘)’]] ∗ $\boldsymbol{E}$[[‘4’]]
= $\boldsymbol{E}$[[‘2’ ‘+’ ‘3’]] ∗ $\boldsymbol{N}$[[‘4’]]
= ($\boldsymbol{E}$[[‘2’]] + $\boldsymbol{E}$[[‘3’]]) ∗ $\boldsymbol{D}$[[‘4’]]
= ($\boldsymbol{N}$[[‘2’]] + $\boldsymbol{N}$[[‘3’]]) ∗ 4
= $\boldsymbol{D}$[[‘2’]] + $\boldsymbol{D}$[[‘3’]]) ∗ 4
= (2 + 3) ∗ 4 = 5 ∗ 4 = 20

12.3.5 Environments and Assignment

The first extension to our basic sample language adds identifiers, assignment statements, and environments to the semantics. Environments are functions from identifiers to integers (or undefined), and the set of environments becomes a new semantic domain:

Domain *Env*: Environment = Identifier → Integer ∪ {undef}

In denotational semantics the value undef is given a special name, *bottom*, taken from the theory of partial orders, and is denoted by a special symbol, ⊥. Semantic domains with this special value added are called **lifted domains** and are subscripted with the symbol ⊥. Thus, Integer ∪ {⊥} is written as Integer$_{\perp}$. The initial environment Env_0 defined in Section 12.1, in which all identifiers have undefined values, can now be defined as $Env_0(I) = \perp$ for all identifiers *I*.

The evaluation of expressions in the presence of an environment must include an environment as a parameter, so that identifiers may be associated with integer values. Thus, the semantic value of an expression becomes a function from environments to integers:

$$\boldsymbol{E} : \text{Expression} \rightarrow \text{Environment} \rightarrow \text{Integer } \perp$$

In particular, the value of an identifier is its value in the environment provided as a parameter:

$$\boldsymbol{E}[[I]](Env) = Env(I)$$

In the case of a number, the environment is immaterial:

$$\boldsymbol{E}[[N]](Env) = \boldsymbol{N}[[N]]$$

In other expression cases the environment is simply passed on to subexpressions.

To extend the semantics to statements and statement lists, we note that the semantic values of these constructs are functions from environments to environments. An assignment statement changes the environment to add the new value assigned to the identifier; in this case, we will use the same “&” notation for adding values to functions that we have used in previous sections. Now a statement-list is simply the composition of the functions of its individual statements (recall that the composition $f \circ g$ of two functions f and g is defined by $(f \circ g)(x) = f(g(x))$. A complete denotational definition of the extended language is given in Figure 12.5.

Syntactic Domains

P: Program
L: Statement-list
S: Statement
E: Expression
N: Number
D: Digit
I: Identifier
A: Letter

$P \rightarrow \mathrm{L}$
$L \rightarrow L_1$ ';' $L_2 \mid S$
$S \rightarrow I$ ':=' E
$E \rightarrow E_1$ '+' $E_2 \mid E_1$ '$-$' $E_2 \mid E_1$ '$*$' E_2
$\mid$ '(' E ')' $\mid I \mid N$
$N \rightarrow N\,D \mid D$
$D \rightarrow$ '0' | '1' |. . .| '9'
$I \rightarrow I\,A \mid A$
$A \rightarrow$ 'a' | 'b' |. . .| 'z'

Semantic Domains

Domain v: Integer $=$ {. . ., -2, -1, 0, 1, 2,. . .}
`Operations`

$+$: Integer $\times$ Integer $\rightarrow$ Integer
$-$: Integer $\times$ Integer $\rightarrow$ Integer
$*$: Integer $\times$ Integer $\rightarrow$ Integer

Domain *Env*: Environment $=$ Identifier $\rightarrow$ Integer$_\perp$

Semantic Functions

$\boldsymbol{P}$: Program $\rightarrow$ Environment

$\boldsymbol{P}[[L]] = \boldsymbol{L}[[L]](Env_0)$

$\boldsymbol{L}$: Statement-list $\rightarrow$ Environment $\rightarrow$ Environment

$\boldsymbol{L}[[L_1$ ';' $L_2]] = \boldsymbol{L}[[L_2]] \circ \boldsymbol{L}[[L_1]]$
$\boldsymbol{L}[[S]] = \boldsymbol{S}[[\mathrm{S}]]$

Figure 12.5 A denotational definition for the sample language extended with assignment statements and environments *(continues)*

(continued)

$\boldsymbol{S}$: Statement → Environment → Environment

$$\boldsymbol{S}[[\ I\ \text{':='}\ E\ ]](Env) = Env\ \&\ \{I = \boldsymbol{E}[[E]](Env)\}$$

$\boldsymbol{E}$: Expression → Environment → Integer$_\perp$

$$E[[E_1\ \text{'+'}\ E_2]](Env) = \boldsymbol{E}[[E_1]](Env) + \boldsymbol{E}[[E_2]](Env)$$
$$E[[E_1\ \text{'-'}\ E_2]](Env) = \boldsymbol{E}[[E_1]](Env) - \boldsymbol{E}[[E_2]](Env)$$
$$E[[\mathrm{E}_1\ \text{'*'}\ E_2]](Env) = \boldsymbol{E}[[E_1]](Env) * \boldsymbol{E}[[E_2]](Env)$$
$$E[[\text{'('}\ E\ \text{')'}]](Env) = \boldsymbol{E}[[E]](Env)$$
$$E[[I]](Env) = Env(\ I\ )$$
$$E[[N]](Env) = \boldsymbol{N}[[N]]$$

$\boldsymbol{N}$: Number → Integer

$$\boldsymbol{N}[[ND]] = 10{*}\boldsymbol{N}[[N]] + \boldsymbol{N}[[D]]$$
$$\boldsymbol{N}[[D]] = D[[D]]$$

$\boldsymbol{D}$: Digit → Integer

$$\boldsymbol{D}[[\text{'0'}]] = 0, \boldsymbol{D}[[\text{'1'}]] = 1, \ldots, \boldsymbol{D}[[\text{'9'}]] = 9$$

Figure 12.5 A denotational definition for the sample language extended with assignment statements and environments

12.3.6 Denotational Semantics of Control Statements

To complete our discussion of denotational semantics, we need to extend the denotational definition of Figure 12.5 to if- and while statements, with the following abstract syntax:

S: Statement
$S \rightarrow I$ ':=' E
| 'if' E 'then' L_1 'else' L_2 'fi'
| 'while' E 'do' L 'od'

As before, the denotational semantics of these statements must be given by a function from environments to environments:

$$\boldsymbol{S}\text{ : Statement} \rightarrow \text{Environment} \rightarrow \text{Environment}$$

We define the semantic function of the if statement as follows:

$$\boldsymbol{S}[[\text{'if'}\ E\ \text{'then'}\ L_1\ \text{'else'}\ L_2\ \text{'fi'}]](Env) =$$
$$\text{if } \boldsymbol{E}[[E]](Env) > 0 \text{ then } \boldsymbol{L}[[L_1]](Env) \text{ else } \boldsymbol{L}[[L_2]](Env)$$

Note that we are using the if-then-else construct on the right-hand side to express the construction of a function. Indeed, given F: Environment $\to$ Integer, G: Environment $\to$ Environment, and H: Environment $\to$ Environment, then the function "if F then G else H:" Environment $\to$ Environment is given as follows:

$$(\text{if } F \text{ then } G \text{ else } H)\ (Env) = \begin{cases} G(Env), \text{ if } F(Env) > 0 \\ H(Env), \text{ if } F(Env) \le 0 \end{cases}$$

The semantic function for the while statement is more difficult. In fact, if we let $F = \boldsymbol{S}$[['while' E 'do' L 'od']], so that F is a function from environments to environments, then F satisfies the following equation:

$$F(Env) = \text{if } \boldsymbol{E}[[E]](Env) \le 0 \text{ then } Env \text{ else } F(\boldsymbol{L}[[L]](Env))$$

This is a recursive equation for F. To use this equation as a specification for the semantics of F, we need to know that this equation has a unique solution in some sense among the functions Environment $\to$ Environment. We saw a very similar situation in Section 11.6, where the definition of the factorial function also led to a recursive equation for the function. In that case, we were able to construct the function as a set by successively extending it to a so-called **least-fixed-point solution**, that is, the "smallest" solution satisfying the equation. A similar approach will indeed work here too, and the solution is referred to as the least-fixed-point semantics of the while-loop. The situation here is more complicated, however, in that F is a function on the semantic domain of environments rather than the integers. The study of such equations and their solutions is a major topic of domain theory. For more information, see the references at the end of the chapter.

Note that there is an additional problem associated with loops: nontermination. For example, in our sample language, the loop

```
i := 1 ;
while i do i := i + 1 od
```

does not terminate. Such a loop does not define any function at all from environments to environments, but we still need to be able to associate a semantic interpretation with it. One does so by assigning it the "undefined" value $\perp$ similar to the value of an undefined identifier in an environment. In this case, the domain of environments becomes a lifted domain,

$$\text{Environment}_\perp = (\text{Identifier} \to \text{Integer}_\perp)_\perp$$

and the semantic function for statements must be defined as follows:

$$\boldsymbol{S} : \text{Statement} \to \text{Environment}_\perp \to \text{Environment}_\perp$$

We shall not discuss such complications further.

12.3.7 Implementing Denotational Semantics in a Programming Language

As we have noted previously, it is possible to implement denotational semantic rules directly as a program. Since denotational semantics are based on functions, particularly higher-order functions, it is not surprising that a functional language is a natural choice as an implementation language. In this section, we briefly sketch a possible Haskell implementation for the denotational functions of our sample

language. (We choose Haskell because of its minimal extra syntax; ML or Scheme would also be a good choice.)

First, consider how one might define the abstract syntax of expressions in the sample language, using a data declaration:

```
data Expr = Val Int | Ident String | Plus Expr Expr
                    | Minus Expr Expr | Times Expr Expr
```

Here, as with the operational semantics, we are ignoring the semantics of numbers and simply letting values be integers (`Val Int`).

Suppose now that we have defined an Environment type, with a lookup and update operation (a reasonable first choice for it, as in operational semantics, would be a list of string-integer pairs). Then the "***E***" evaluation function can be defined almost exactly as in the denotational definitions:

```
exprE :: Expr -> Environment -> Int
exprE (Plus e1 e2) env = (exprE e1 env) + (exprE e2 env)
exprE (Minus e1 e2) env = (exprE e1 env) - (exprE e2 env)
exprE (Times e1 e2) env = (exprE e1 env) * (exprE e2 env)
exprE (Val n) env = n
exprE (Ident a) env = lookup env a
```

Note that there is no rule for parentheses. Again in this case, the abstract syntax is in tree form, and parentheses simply do not appear.

Similar definitions can be written for statements, statement-lists, and programs. We leave the details as an exercise for the reader.

12.4 Axiomatic Semantics

Axiomatic semantics define the semantics of a program, statement, or language construct by describing the effect its execution has on assertions about the data manipulated by the program. The term "axiomatic" is used because elements of mathematical logic are used to specify the semantics of programming languages, including logical axioms. We discussed logic and assertions in the introduction to logic programming in Chapter 4. For our purposes, however, it suffices to consider logical assertions to be statements about the behavior of a program that are true or false at any moment during execution.

Assertions associated with language constructs are of two kinds: assertions about things that are true just before execution of the construct and assertions about things that are true just after the execution of the construct. Assertions about the situation just before execution are called **preconditions**, and assertions about the situation just after execution are called **postconditions**. For example, given the assignment statement

```
x := x + 1
```

we would expect that, whatever value `x` has just before execution of the statement, its value just after the execution of the assignment is one more than its previous value. This can be stated as the precondition that `x` $= A$ before execution and the postcondition that `x` $= A + 1$ after execution. Standard notation

for this is to write the precondition inside curly brackets just before the construct and to write the postcondition similarly just after the construct:

$$\{x = A\}\ x := x + 1\ \{x = A + 1\}$$

or:

$$\{x = A\}$$
$$x := x + 1$$
$$\{x = A + 1\}$$

As a second example of the use of precondition and postcondition to describe the action of a language construct, consider the following assignment:

```
x := 1 / y
```

Clearly, a precondition for the successful execution of the statement is that $y \neq 0$, and then x becomes equal to 1/y. Thus we have

$$\{y \neq 0\}$$
$$x := 1 / y$$
$$\{x = 1/y\}$$

Note that in this example the precondition establishes a restriction that is a requirement for successful execution, while in the first example, the precondition $x = A$ merely establishes a name for the value of x prior to execution, without making any restriction whatever on that value.

Precondition/postcondition pairs can be useful in specifying the expected behavior of programs—the programmer simply writes down the conditions he or she expects to be true at each step in a program. For example, a program that sorts the array a[1]..a[n] could be specified as follows:

{n ≥ 1 and for all i, 1 ≤ i ≤ n, a[i] = A[i]}
sort-program
{sorted(a) and permutation(a, A)}

Here, the assertions sorted(a) and permutation(a, A) mean that the elements of a are sorted and that the elements of a are the same, except for order, as the original elements of the array A.

Such preconditions and postconditions are often capable of being tested for validity during execution of the program, as a kind of error checking, since the conditions are usually Boolean expressions that can be evaluated as expressions in the language itself. Indeed, a few languages such as Eiffel and Euclid have language constructs that allow assertions to be written directly into programs. The C language also has a rudimentary but useful macro library for checking simple assertions: assert.h. Using this library and the macro assert allows programs to be terminated with an error message on assertion failure, which can be a useful debugging feature:

```
#include <assert.h>
...
assert(y != 0);
x = 1/y;
...
```

If y is 0 when this code is executed, the program halts and an error message, such as

```
Assertion failed at test.c line 27: y != 0
Exiting due to signal SIGABRT
...
```

is printed. One can get this same kind of behavior by using exceptions in a language with exception handling:

```
if (y != 0) throw Assertion_Failure();
```

An **axiomatic specification** of the semantics of the language construct *C* is of the form

$$\{P\}\ C\ \{Q\}$$

where *P* and *Q* are assertions; the meaning of such a specification is that, if *P* is true just before the execution of *C*, then *Q* is true just after the execution of *C*.

Unfortunately, such a representation of the action of *C* is not unique and may not completely specify all the actions of *C*. In the second example, for instance, we did not include in the postcondition the fact that y ≠ 0 continues to be true after the execution of the assignment. To specify completely the semantics of the assignment to x, we must somehow indicate that x is the only variable that changes under the assignment (unless it has aliases). Also, y ≠ 0 is not the only condition that will guarantee the correct evaluation of the expression; 1/y: y > 0 or y < 0 will do as well. Thus, writing an expected precondition and an expected postcondition will not always precisely determine the semantics of a language construct.

What is needed is a way of associating to the construct *C* a general relation between precondition *P* and postcondition *Q*. The way to do this is to use the property that programming is a **goal-oriented activity**: We usually know what we want to be true after the execution of a statement or program, and the question is whether the known conditions before the execution will guarantee that this becomes true. Thus, postcondition *Q* is assumed to be given, and a specification of the semantics of *C* becomes a statement of which preconditions *P* of *C* have the property that $\{P\}\ C\ \{Q\}$. To the uninitiated this may seem backward, but it is a consequence of working backward from the goal (the postcondition) to the initial requirements (the precondition).

In general, given an assertion *Q*, there are many assertions *P* with the property that $\{P\}\ C\ \{Q\}$. One example has been given: For 1/y to be evaluated, we may require that y ≠ 0 or y > 0 or y < 0. There is one precondition *P*, however, that is the **most general** or **weakest** assertion with the property that $\{P\}\ C\ \{Q\}$. This is called the **weakest precondition** of postcondition *Q* and construct *C* and is written *wp*(*C*,*Q*).

In the example, y ≠ 0 is clearly the weakest precondition such that 1/y can be evaluated. Both y > 0 and y < 0 are stronger than y ≠ 0, since they both imply y ≠ 0. Indeed, *P* is by definition weaker than *R* if *R* implies *P* (written in logical form as $R \rightarrow P$). Using these definitions we have the following restatement of the property $\{P\}\ C\ \{Q\}$:

$$\{P\}\ C\ \{Q\} \text{ if and only if } P \rightarrow wp(C,Q)$$

Finally, we define the axiomatic semantics of the language construct *C* as the function *wp*(*C*,_) from assertions to assertions. This function is a **predicate transformer** in that it takes a predicate as argument and returns a predicate result. It also appears to work backward, in that it computes the weakest precondition from any postcondition. This is a result of the goal-oriented behavior of programs as described earlier.

Our running example of the assignment can now be restated as follows:

$$wp(\texttt{x := 1/y, x = 1/y}) = \{\texttt{y} \neq 0\}$$

As another example, consider the assignment `x := x + 1` and the postcondition `x > 0`:

$$wp(\texttt{x := x + 1, x > 0}) = \{\texttt{x > -1}\}$$

In other words, for `x` to be greater than 0 after the execution of `x := x + 1`, `x` must be greater than -1 just prior to execution. On the other hand, if we have no condition on `x` but simply want to state its value, we have:

$$wp(\texttt{x := x + 1, x =} A) = \{\texttt{x =} A - 1\}$$

Again, this may seem backward, but a little reflection should convince you of its correctness. Of course, to determine completely the semantics of an assignment such as `x :=` E, where `x` is a variable and E is an expression, we need to compute wp(`x :=` E, Q) for any postcondition Q. This is done in Section 12.4.2, where the general rule for assignment is stated in terms of substitution. First, we will study *wp* a little further.

12.4.1 General Properties of *wp*

The predicate transformer $wp(C,Q)$ has certain properties that are true for almost all language constructs C, and we discuss these first, before giving axiomatic semantics for the sample language. The first of these is the following:

Law of the Excluded Miracle

$$wp(C,\text{false}) = \text{false}$$

This states that nothing a programming construct C can do will make false into true—if it did it would be a miracle!

The second property concerns the behavior of *wp* with regard to the "and" operator of logic (also called conjunction):

Distributivity of Conjunction

$$wp(C,P \text{ and } Q) = wp(C,P) \text{ and } wp(C,Q)$$

Two more properties regard the implication operator "$\rightarrow$" and the "or" operator (also called disjunction):

Law of Monotonicity

$$\text{if } Q \rightarrow R \text{ then } wp(C,Q) \rightarrow wp(C,R)$$

Distributivity of Disjunction

$$wp(C,P) \text{ or } wp(C,Q) \rightarrow wp(C,P \text{ or } Q)$$

with equality if C is deterministic.

The question of determinism adds a complicating technicality to the last law. Recall that some language constructs can be nondeterministic, such as the guarded commands discussed in Chapter 9. An example of the need for a weaker property in the presence of nondeterminism is discussed in Exercise 12.35. However, the existence of this exception serves to emphasize that, when one is talking about *any* language construct C, one must be extremely careful. Indeed, it is possible to invent complex

language mechanisms in which all of the foregoing properties become questionable without further conditions. (Fortunately, such situations are rare.)

12.4.2 Axiomatic Semantics of the Sample Language

We are now ready to give an axiomatic specification for our sample language. We note first that the specification of the semantics of expressions alone is not something that is commonly included in an axiomatic specification. In fact, the assertions involved in an axiomatic specificator are primarily statements about the side effects of language constructs; that is, they are statements involving identifiers and environments. For example, the assertion $Q = \{x > 0\}$ is an assertion about the value of x in an environment. Logically, we could think of Q as being represented by the set of all environments for which Q is true. Then logical operations can be represented by set theoretic operations. For example, $P \rightarrow Q$ is the same as saying that every environment for which P is true is in the set of environments for which Q is true—in other words, that P is contained in Q as sets.

We will not pursue this translation of logic into set theory. We will also skip over the specification of expression evaluation in terms of weakest preconditions and proceed directly to statements, environments, and control.

The abstract syntax for which we will define the *wp* operator is the following:

$$
\begin{aligned}
&P \rightarrow L \\
&L \rightarrow L_1 \text{ ';' } L_2 \mid S \\
&S \rightarrow I \text{ ':=' } E \\
&\qquad \mid \text{ 'if' } E \text{ 'then' } L_1 \text{ 'else' } L_2 \text{ 'fi'} \\
&\qquad \mid \text{ 'while' } E \text{ 'do' } L \text{ 'od'}
\end{aligned}
$$

Syntax rules such as $P \rightarrow L$ and $L \rightarrow S$ do not need separate specifications,[1] since these grammar rules simply state that the *wp* operator for a program P is the same as for its associated statement-list L, and similarly, if a statement-list L is a single statement S, then L has the same axiomatic semantics as S. The remaining four cases are treated in order. To simplify the description we will suppress the use of quotes; code will be distinguished from assertions by the use of a different typeface.

Statement-lists. For lists of statements separated by a semicolon, we have

$$wp(L_1; L_2, Q) = wp(L_1, wp(L_2, Q))$$

This states that the weakest precondition of a series of statements is essentially the composition of the weakest preconditions of its parts. Note that since *wp* works "backward" the positions of L_1 and L_2 are not interchanged, as they are in denotational semantics.

Assignment Statements. The definition of *wp* for the assignment statement is as follows:

$$wp(I := E, Q) = Q[E/I]$$

This rule involves a new notation: $Q[E/I]$. $Q[E/I]$ is defined to be the assertion Q, with E replacing all free occurrences of the identifier I in Q. The notion of "free occurrences" was discussed in Chapter 4; it also

[1] If we did have to write semantics for a program, we would have to change its designation from *P* to *Prog*, say, since we have been using *P* to refer to a precondition.

arose in Section 3.6 in connection with reducing lambda calculus expressions. An identifier *I* is **free** in a logical assertion *Q* if it is not **bound** by either the existential quantifier "there exists" or the universal quantifier "for all." Thus, in the following assertion, *j* is free, but *i* is bound (and thus not free):

$$Q = (\text{for all } i, a[i] \,.\, a[j])$$

In this case $Q[1/j] = (\text{for all } i, a[i] \,.\, a[1])$, but $Q[1/i] = \text{Q}$. In commonly occurring assertions, this should not become a problem, and in the absence of quantifiers, one can simply read $Q[E/I]$ as replacing all occurrences of *I* by *E*.

The axiomatic semantics $wp(I := E,Q) = Q[E/I]$ simply says that, for *Q* to be true after the assignment $I := E$, whatever *Q* says about *I* must be true about *E* before the assignment is executed.

A couple of examples will help to explain the semantics for assignment.

First, consider the previous example $wp(\texttt{x := x + 1 , x} > 0)$. Here $\text{Q} = (\texttt{x} > 0)$ and $\text{Q}[\texttt{x} + 1/\texttt{x}] = (\texttt{x} + 1 > 0)$. Thus,

$$wp(\texttt{x := x + 1, x} > 0) = (x + 1 > 0) = (x > -1)$$

which is what we obtained before. Similarly,

$$\begin{aligned} wp(\texttt{x ::= x + 1, x} = A) &= (\texttt{x} = A)[(\texttt{x} + 1)/\texttt{x}] \\ &= (\texttt{x} + 1 = A) \\ &= (\texttt{x} = A - 1) \end{aligned}$$

If statements. Recall that the semantics of the if statement in our sample language are somewhat unusual: `if` *E* `then` L_1 `else` L_2 `fi` means that L_1 is executed if the value of $E > 0$, and L_2 is executed if the value of $E \le 0$. The weakest precondition of this statement is defined as follows:

$$\begin{aligned} &wp(\texttt{if } E \texttt{ then } L_1 \texttt{ else } L_2 \texttt{ fi}, Q) = \\ &\qquad (E > 0 \rightarrow wp(L_1,Q)) \text{ and } (E \le 0 \rightarrow wp(L_2,Q)) \end{aligned}$$

As an example, we compute

$$\begin{aligned} &wp(\texttt{if x then x := 1 else x := -1 fi, x} = 1) = \\ &(\texttt{x} > 0 \rightarrow wp(\texttt{x =: 1, x} = 1)) \text{ and } (\texttt{x} \le 0 \rightarrow wp(\texttt{x := -1}, \texttt{x} = 1)) \\ &= (\texttt{x} > 0 \rightarrow 1 = 1) \text{ and } (\texttt{x} \le 0 \rightarrow -1 = 1) \end{aligned}$$

Recalling that $(P \rightarrow Q)$ is the same as *Q* or not *P* (see Exercise 12.1), we get

$$(\texttt{x} > 0 \rightarrow 1 = 1) = ((1 = 1) \text{ or not}(\texttt{x} > 0)) = \text{true}$$

and

$$\begin{aligned} &(\texttt{x} \le 0 \rightarrow 1 = 1) = (-1 = 1) \text{ or not}(\texttt{x} \le 0) = \\ &\text{not}(\texttt{x} \le 0) = (\texttt{x} > 0) \end{aligned}$$

so

$$wp(\texttt{if x then x :=1 else x := -1 fi, x} = 1) = (\texttt{x} > 0)$$

as we expect.

While statements. The while statement `while` E `do` L `od`, as defined in Section 12.1, executes as long as $E > 0$. As in other formal semantic methods, the semantics of the while-loop present particular problems. We must give an inductive definition based on the number of times the loop executes. Let H_i (`while` E `do` L `od`, Q) be the statement that the loop executes i times and terminates in a state satisfying Q. Then clearly

$$H_0(\texttt{while } E \texttt{ do } L \texttt{ od}, Q) = E \leq 0 \text{ and } Q$$

and

$$\begin{aligned} H_1(\texttt{while } E \texttt{ do } L \texttt{ od}, Q) &= E > 0 \text{ and } wp(L, Q \text{ and } E \leq 0) \\ &= E > 0 \text{ and } wp(L, H_0(\texttt{while } E \texttt{ do } L \texttt{ od}, Q)) \end{aligned}$$

Continuing in this fashion we have in general that

$$\begin{aligned} &H_{i+1}(\texttt{while } E \texttt{ do } L \texttt{ od}, Q) = \\ &E > 0 \text{ and } wp(L, H_i(\texttt{while } E \texttt{ do } L \texttt{ od}, Q)) \end{aligned}$$

Now we define

$$\begin{aligned} &wp(\texttt{while } E \texttt{ do } L \texttt{ od}, Q) \\ &= \text{there exists an } i \text{ such that } H_i(\texttt{while } E \texttt{ do } L \texttt{ od}, Q) \end{aligned}$$

Note that this definition of the semantics of the while requires the while-loop to terminate. Thus, a nonterminating loop always has false as its weakest precondition; that is, it can never make a postcondition true. For example,

$$wp(\texttt{while 1 do } L \texttt{ od}, Q) = \text{false, for all } L \text{ and } Q$$

The semantics we have just given for loops has the drawback that it is very difficult to use in the main application area for axiomatic semantics, namely, the proof of correctness of programs. In the next section we will describe an approximation of the semantics of a loop that is more usable in practice.

12.5 Proofs of Program Correctness

The theory of axiomatic semantics was developed as a tool for proving the correctness of programs and program fragments, and this continues to be its major application. In this section we will use the axiomatic semantics of the last section to prove properties of programs written in our sample language.

We have already mentioned in the last section that a specification for a program C can be written as $\{P\}\ C\ \{Q\}$, where P represents the set of conditions that are expected to hold at the beginning of a program and Q represents the set of conditions one wishes to have true after execution of the code C. As an example, we gave the following specification for a program that sorts the array `a[1]..a[n]`:

```
{n ≥ 1 and for all i, 1 ≤ i ≤ n, a[i] = A[i]}
 sort-program
{sorted(a) and permutation(a, A)}
```

Two easier examples of specifications for programs that can be written in our sample language are the following:

1. A program that swaps the value of x and y:

$$\{\mathtt{x} = X \text{ and } \mathtt{y} = Y\}$$
$$\mathtt{swapxy}$$
$$\{\mathtt{x} = Y \text{ and } \mathtt{y} = X\}$$

2. A program that computes the sum of integers less than or equal to a positive integer n:

$$\{\mathtt{n} > 0\}$$
$$\mathtt{sum\_to\_n}$$
$$\{\mathtt{sum} = 1 + 2 + \ldots + \mathtt{n}\}$$

We will give correctness proofs that the two programs we provide satisfy the specifications of (1) and (2).

Recall from the last section that C satisfies a specification $[P]\ C\ [Q]$ provided $P \rightarrow wp(C,Q)$. Thus, to prove that C satisfies a specification we need two steps: First, we must compute $wp(C,Q)$ from the axiomatic semantics and general properties of wp, and, second, we must show that $P \rightarrow wp(C,Q)$.

1. We claim that the following program is correct:

$$\{\mathtt{x} = X \text{ and } \mathtt{y} = Y\}$$
$$\mathtt{t := x;}$$
$$\mathtt{x := y;}$$
$$\mathtt{y := t}$$
$$\{\mathtt{x} = Y \text{ and } \mathtt{y} = X\}$$

We first compute $wp(C,Q)$ as follows:

$wp(\mathtt{t := x ; x := y ; y := t}, \mathtt{x} = Y \text{ and } \mathtt{y} = X)$
$= wp(\mathtt{t := x}, wp(\mathtt{x :=y; y := t}, \mathtt{x} = Y \text{ and } \mathtt{y} = X))$
$= wp(\mathtt{t := x}, wp(\mathtt{x :=y}, wp(\mathtt{y := t}, \mathtt{x} = Y \text{ and } \mathtt{y} = X)))$
$= wp(\mathtt{t := x}, wp(\mathtt{x :=y}, wp(\mathtt{y := t}, \mathtt{x} = Y)$
and $wp(\mathtt{y := t}, \mathtt{y} = X)))$
$= wp(\mathtt{t := x}, wp(\mathtt{x :=y}, wp(\mathtt{y := t}, \mathtt{x} = Y))$
and $wp(\mathtt{x := y}, wp(\mathtt{y := t}, \mathtt{y} = X)))$
$= wp(\mathtt{t := x}, wp(\mathtt{x :=y}, wp(\mathtt{y := t}, \mathtt{x} = Y)))$
and $wp(\mathtt{t := x}, wp(\mathtt{x :=y}, wp(\mathtt{y := t}, \mathtt{y} = X)))$

by distributivity of conjunction and the axiomatic semantics of statement-lists. Now

$$wp(\mathtt{t := x}, wp(\mathtt{x :=y}, wp(\mathtt{y := t}, \mathtt{x} = Y))) =$$
$$wp(\mathtt{t := x}, wp(\mathtt{x :=y}, \mathtt{x} = Y)) = wp(\mathtt{t :=x}, \mathtt{y} = Y) = (\mathtt{y} = Y)$$

and

$wp(\texttt{t := x}, wp(\texttt{x :=y}, wp(\texttt{y := t}, \texttt{y} = X))) =$
$wp(\texttt{t := x}, wp(\texttt{x :=y}, \texttt{t} = X)) = wp(\texttt{t :=}X, \texttt{t} = \texttt{x}) = (\texttt{x} = X)$

by the rule of substitution for assignments. Thus,

$wp(\texttt{t := x; x := y; y := t, x} = Y \text{ and } \texttt{y} = X) = (\texttt{y} = Y \text{ and } \texttt{x} = X)$

The second step is to show that $P \rightarrow wp(C,Q)$. But in this case P actually equals the weakest precondition, since $P = (\texttt{x} = X \text{ and } \texttt{y} = Y)$. Since $P = wp(C,Q)$, clearly also $P \rightarrow wp(C,Q)$. The proof of correctness is complete.

2. We claim that the following program is correct:

```
{n > 0}
i := n;
sum := 0;
while i do
sum := sum + i;
i := i - 1
od
{sum = 1 + 2 + . . . + n}
```

The problem now is that our semantics for while-statements are too difficult to use to prove correctness. To show that a while-statement is correct, we really do not need to derive completely its weakest precondition $wp(\texttt{while} \ldots, Q)$, but only an **approximation**, that is, some assertion W such that $W \rightarrow wp(\texttt{while} \ldots, Q)$. Then if we can show that $P \rightarrow W$, we have also shown the correctness of $\{P\}$ `while` . . . $\{Q\}$, since $P \rightarrow W$ and $W \rightarrow wp(\texttt{while} \ldots, Q)$ imply that $P \rightarrow wp(\texttt{while} \ldots, Q)$.

We do this in the following way. Given the loop `while` E `do` L `od`, suppose we find an assertion W such that the following three conditions are true:

(a) $W \text{ and } (E > 0) \rightarrow wp(L,W)$
(b) $W \text{ and } (E \leq 0) \rightarrow Q$
(c) $P \rightarrow W$

Then if we know that the loop `while` E `do` L `od` terminates, we must have $W \rightarrow wp(\texttt{while}\ E\ \texttt{do}\ L\ \texttt{od}, Q)$. This is because every time the loop executes, W continues to be true, by condition (a), and when the loop terminates, condition (b) says Q must be true. Finally, condition (c) implies that W is the required approximation for $wp(\texttt{while} \ldots, Q)$.

An assertion W satisfying condition (a) is said to be a **loop invariant** for the loop `while` E `do` L `od`, since a repetition of the loop leaves W true. In general, loops have many invariants W, and to prove the correctness of a loop, it sometimes takes a little skill to find an appropriate W, namely, one that also satisfies conditions (b) and (c).

In the case of our example program, however, a loop invariant is not too difficult to find:

$$W = (\texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} \geq 0)$$

is an appropriate one. We show conditions (a) and (b) in turn:

(a) We must show that W and $\texttt{i} > 0 \rightarrow wp(\texttt{sum := sum + i ; i := i - 1}, W)$. First, we have

$wp(\texttt{sum := sum + i; i := i - 1}, W)$
$= wp(\texttt{sum := sum + i; i := i - 1}, \texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n}$
and $\texttt{i} \geq 0)$
$= wp(\texttt{sum := sum + i}, wp(\texttt{i := i - 1}, \texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n}$
and $\texttt{i} \geq 0))$
$= wp(\texttt{sum := sum + i}, \texttt{sum} = ((\texttt{i} - 1) + 1) + \ldots + \texttt{n}$
and $\texttt{i} - 1 \geq 0)$
$= wp(\texttt{sum := sum + i, sum = i + . . . + n} \text{ and } \texttt{i} - 1 \geq 0)$
$= (\texttt{sum + i = i +} \ldots + \texttt{n} \text{ and } \texttt{i} - 1 \geq 0)$
$= (\texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} - 1 \geq 0)$

Now $(W \text{ and } \texttt{i} > 0) \rightarrow (W \text{ and } \texttt{i} - 1 \geq 0)$, since
$W \text{ and } \texttt{i} > 0 = (\texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} \geq 0 \text{ and } \texttt{i} > 0)$
$= (\texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} > 0) \rightarrow$
$(W \text{ and } \texttt{i} - 1 \geq 0)$

Thus, W is a loop invariant.

(b) We must show that $(W \text{ and } (\texttt{i} \leq 0)) \rightarrow (\texttt{sum} = 1 + \ldots + \texttt{n})$.
But this is clear:

$W \text{ and } (\texttt{i} \leq 0) = (\texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} \geq 0 \text{ and } \texttt{i} \leq 0)$
$= (\texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} = 0)$
$= (\texttt{sum} = 1 + \ldots + \texttt{n} \text{ and } \texttt{i} = 0)$

It remains to show that conditions just prior to the execution of the loop imply the truth of W.[2] We do this by showing that $\texttt{n} > 0 \rightarrow wp(\texttt{i := n; sum := 0}, W)$. We have

$wp(\texttt{i := n; sum := 0}, W)$
$= wp(\texttt{i := n}, wp(\texttt{sum := 0}, \texttt{sum} = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} \geq 0))$
$= wp(\texttt{i := n}, 0 = (\texttt{i} + 1) + \ldots + \texttt{n} \text{ and } \texttt{i} \geq 0)$
$= (0 = (\texttt{n} + 1) + \ldots + \texttt{n} \text{ and } \texttt{n} \geq 0)$
$= (0 = 0 \text{ and } \texttt{n} \geq 0)$
$= (\texttt{n} \geq 0)$

and of course $\texttt{n} > 0 \rightarrow \texttt{n} \geq 0$. In this computation we used the property that the sum $(\texttt{i} + 1) + \ldots + \texttt{n}$ with $\texttt{i} \geq \texttt{n}$ is 0. This is a general mathematical property: Empty sums are always assumed to be 0. We also note that this proof uncovered an additional property of our code: It works not only for $\texttt{n} > 0$, but for $\texttt{n} \geq 0$ as well.

This concludes our discussion of proofs of programs. A few more examples will be discussed in the exercises.

[2]In fact, we should also prove termination of the loop, but we ignore this issue in this brief discussion. Proving correctness while assuming termination is called proving **partial correctness**, which is what we are really doing here.

Exercises

12.1 Our sample language used the bracketing keywords "fi" and "od" for if statements and while-statements, similar to Algol68. Was this necessary? Why?

12.2 Add unary minuses to the arithmetic expressions of the sample language, and add its semantics to **(a)** the operational semantics and **(b)** the denotational semantics.

12.3 Add division to the arithmetic expressions of the sample language, and add its semantics to **(a)** the operational semantics and **(b)** the denotational semantics. Try to include a specification of what happens when division by 0 occurs.

12.4 The operational semantics of identifiers was skipped in the discussion in the text. Add the semantics of identifiers to the operational semantics of the sample language.

12.5 The denotational semantics of identifiers was also (silently) skipped. What we did was to use the set Identifier as both a syntactic domain (the set of syntax trees of identifiers) and as a semantic domain (the set of strings with the concatenation operator). Call the latter set Name, and develop a denotational definition of the semantic function $\boldsymbol{I}$: Identifier $\rightarrow$ Name. Revise the denotational semantics of the sample language to include this correction.

12.6 A problem that exists with any formal description of a language is that the description itself must be written in some "language," which we could call the **defining language**, to distinguish it from the defined language. For example, the defining language in each of the formal methods studied in this chapter are as follows:

operational semantics: reduction rules
denotational semantics: functions
axiomatic semantics: logic

For a formal semantic method to be successful, the defining language needs to have a precise description itself, and it must also be understandable. Discuss and compare the defining languages of the three semantic methods in terms of your perception of their precision and understandability.

12.7 One formal semantic method not discussed in this chapter but mentioned in Chapter 3 is the use of the defined language itself as the defining language. Such a method could be called **metacircular**, and metacircular interpreters are a common method of defining LISP-like languages. Discuss the advantages and disadvantages of this method in comparison with the methods discussed in this chapter.

12.8 The grammar of the sample language included a complete description of identifiers and numbers. However, a language translator usually recognizes such constructs in the scanner. Our view of semantics thus implies that the scanner is performing semantic functions. Wouldn't it be better simply to make numbers and identifiers into tokens in the grammar, thus making it unnecessary to describe something done by the scanner as "semantics?" Why or why not?

12.9 The axiomatic semantics of Section 12.4 did not include a description of the semantics of expressions. Describe a way of including expressions in the axiomatic description.

12.10 Show how the operational semantics of the sample language describes the reduction of the expression 23 * 5 - 34 to its value.

12.11 Compute $\boldsymbol{E}$[[23 * 5 - 34]] using the denotational definition of the sample language.

12.12 Show how the operational semantics of the sample language describes the reduction of the program `a := 2; b := a + 1; a := b * b` to its environment.

12.13 Compute the value *Env*(*a*) for the environment *Env* at the end of the program `a := 2; b := a + 1; a := b * b` using the denotational definition of the sample language.

12.14 Repeat Exercise 12.12 for the program

```
a := 0 - 11;
if a then a := a else a := 0 - a fi
```

12.15 Repeat Exercise 12.13 for the program of Exercise 12.14.

12.16 Use operational semantics to reduce the following program to its environment:

```
n := 2;
while n do n := n - 1 od
```

12.17 The sample language did not include any input or output statements. We could add these to the grammar as follows:

stmt → . . . | 'input' *identifier* | 'output' *expr*

Add input and output statements to the **(a)** operational semantics and **(b)** denotational semantics. (*Hint for denotational semantics:* Consider a new semantic domain IntSequence to be the set of sequences of integers. Then statement sequences act on states that are environments plus an input sequence and an output sequence.)

12.18 An often useful statement in a language is an empty statement, which is sometimes denoted by the keyword `skip` in texts on semantics. Add a `skip` statement to the sample language of this chapter, and describe its **(a)** operational, **(b)** denotational, or **(c)** axiomatic semantics. Why is such a statement useful?

12.19 The sample language has unusual semantics for if- and while-statements, due to the absence of Boolean expressions. Add Boolean expressions such as `x = 0`, `true`, `y > 2` to the sample language, and describe their semantics in **(a)** operational and **(b)** denotational terms.

12.20 Revise the axiomatic description of the sample language to include the Boolean expressions of Exercise 12.19.

12.21 Find *wp*(`a := 2; b := a + 1; a := b * b`, `a` = 9).

12.22 Find *wp*(`if x then x := x else x := 0 - x fi`, `x` ≤ 0).

12.23 Show that the following program is correct with respect to the given specification:

{true}
`if x then x := x else x := 0 - x fi`
{`x` ≥ 0}

12.24 Which of the following are loop invariants of the loop `while i do sum := sum + i; i := i - 1 od`?

(a) `-sum = i + ... + n`

(b) `-sum = (i + 1) + ... + n` and `i` > 0

(c) `sum` ≥ 0 and `i` ≥ 0

12.25 Prove the correctness of the following program:

```
{n > 0}
i := n;
fact := 1;
while i do
fact := fact * i;
i := i - 1
od
{fact = 1 * 2 * ... * n}
```

12.26 Write a program in the sample language that computes the product of two numbers `n` and `m` by repeatedly adding `n` `m`-times. Prove that your program is correct.

12.27 In Section 12.4 and 12.5 we used the following example of a program specification using preconditions and postconditions:

{`n` ≥ 1 and for all `i`, 1 ≤ `i` ≤ `n`, `a[i] = A[i]`}
sort-program
{sorted(`a`) and permutation(`a`, `A`)}

Write out the details of the assertions sorted(`a`) and permutation (`a`, `A`).

12.28 Show the correctness of the following program using axiomatic semantics:

{`n` > 0}
`while n do n := n - 1 od`
{`n` = 0}

12.29 Show using general properties of *wp* that *wp*(*C*, not *Q*) → not *wp*(*C*,*Q*) for any language construct *C* and any assertion *Q*.

12.30 Show that the law of monotonicity follows from the distributivity of conjunction. (*Hint:* Use the fact that $P \rightarrow Q$ is equivalent to (P and Q) = P.)

12.31 We did not describe how operations might be extended to lifted domains (domains with an undefined value) such as Integer$_\perp$. Give a definition for "+," "-," and "*" that includes the undefined value $\perp$.

12.32 Sometimes operations can ignore undefined values and still return a defined value. Give an example where the "*" operation can do this.

12.33 The formal semantic descriptions of the sample language in this chapter did not include a description of the semantics in the presence of undefined values (such as the use of an identifier without an assigned value or a loop that never terminates). Try to extend the semantic descriptions of each of the three methods discussed in this chapter to include a description of the effect of undefined values.

12.34 We might be tempted to define an environment in denotational terms as a semantic function from identifiers to integers (ignoring undefined values):

Env: Identifier → Integer

Would this be wrong? Why or why not?

12.35 The text mentions that in the presence of nondeterminism it is not true that *wp*(*C*,*P* or *Q*) = *wp*(*C*,*P*) or *wp*(*C*,*Q*). Let *C* be the following guarded if, in which either statement might be executed:

```
if
    true => x := x + 1
    true => x := x - 1
fi
```

Show that *wp*(*C*, (x > 0) or (x < 0)) is not equal to *wp*(*C*, x > 0) or *wp*(*C*, x < 0).

12.36 The operational semantics in Section 12.2 specified a left-to-right evaluation for expressions. Rewrite the reduction rules so that no particular evaluation order is specified.

12.37 The rule for reducing parentheses in operational semantics was written as the axiom '(' *V* ')' => *V*, where *V* stands for a numeric value. Why is it wrong to write the more general rule '(' *E* ')' => *E*, where *E* can be any expression?

12.38 In Section 12.2 we wrote three reduction rules for if statements, but only two for while-statements.

(a) Rewrite the rules for if-statements as only two rules.

(b) Can one write three rules for the while-statement? Why?

12.39 Using the `skip` statement (Exercise 12.18), write a *single* reduction rule for the operational semantics of the while statement.

12.40 Is it possible to express the evaluation order of expressions in denotational semantics? Explain.

12.41 In operational semantics, an environment must be given before an abstract machine can perform any reductions. Thus, one (extremely abstract) view of operational semantics is as a function Φ: Program × Environment→ Environment. In this sense, denotational semantics can be viewed as a Curried version (see Section 12.4.3) of operational semantics. Explain what is meant by this statement.

12.42 Complete the Prolog program sketched in Section 12.2.5 implementing the operational semantics for expressions in the sample language.

12.43 Extend the Prolog program of the previous exercise to include the full sample language.

12.44 Complete the Haskell program sketched in Section 12.3.7 implementing the operational semantics for expressions in the sample language.

12.45 Extend the Haskell program of the previous exercise to include the full sample language.

Notes and References

Formal semantic methods are studied in greater depth in Winskel [1993], Reynolds [1998], Gunter [1992], and Mitchell [1996]. A different operational method is the Vienna definition language, which is surveyed in Wegner [1972]. An outgrowth of VDL is the Vienna development method, or VDM, which is denotational in approach. A description of this method appears in Harry [1997], which includes a description of a related method, called Z.

Denotational semantics as they are presented here began with the early work of Scott and Strachey (see Stoy [1977]). An in-depth coverage of denotational semantics, including domain theory and fixed-point semantics, is given in Schmidt [1986]. Axiomatic semantics began with a seminal paper by Hoare [1969]. Weakest preconditions were introduced by Dijkstra [1975, 1976]. An in-depth coverage is given in Dijkstra and Scholten [1990] and Gries [1981]. For a perspective on nondeterminism and the law of the excluded miracle, see Nelson [1989].

Formal semantic methods as they apply to object-oriented languages are studied in Gunter and Mitchell [1994]. Perhaps the only production language for which a complete formal description has been given is ML (Milner et al. [1997]); a partial formal semantics for Ada83 can be found in Björner and Oest [1981].

One topic not studied in this chapter is the formal semantics of data types and type checking, which are usually given as a set of rules similar to the reduction rules of operational semantics. Details can be found in Hindley [1997] or Schmidt [1994].

CHAPTER

Parallel Programming

CHAPTER 13

The idea of **parallel processing**, or executing many computations in parallel, has been around at least since the 1960s, when the first **multiprogramming** became available. In these early systems, many processes shared a single processor, thus appearing to execute simultaneously. Pseudoparallelism of this sort represented a considerable advance in computer technology, and it is still the standard way most larger machines operate today. However, it has long been clear that **true parallelism** represents an even greater advance in computer technology; in fact, it is one way of solving the problem of the von Neumann bottleneck (see Chapter 1). In true parallelism, many processors are connected to run in concert, either as a single system incorporating all the processors (a **multiprocessor system**) or as a group of stand-alone processors connected together by high-speed links (a **distributed system**).

However, this seemingly simple idea has not been simple to put into practice, and truly parallel systems, in which a single process can share different processors, are even at the time of this writing relatively uncommon, despite extensive study of the issues involved.[1] Thus, a comprehensive study of parallel processing is beyond the scope of this text.

The situation has become even more complex with the advent of high-speed networks, including the Internet, in that physically distant computers are now capable of working together, using the same or similar programs to handle large problems. Thus, organized networks of independent computers can also be viewed as a kind of distributed system for parallel processing, and, indeed, may have already become more important than traditional styles of parallel computing.[2] Obviously, many of the issues involved in networked parallel processing are also beyond the scope of this text. (See the Notes and References for networking texts that provide more information.)

Nevertheless, programming languages have affected and been affected by the parallelism in a number of ways. First, programming languages have been used to express algorithms to solve the problems presented by parallel processing systems. Second, programming languages have been used to write operating systems that have implemented these solutions on various architectures. Also, programming languages have been used to harness the capabilities of multiple processors to solve application problems efficiently. Finally, programming languages have been used to implement and express communication across networks.

A survey of the principles and practice of programming language design would therefore not be complete without a study of the basic ways that programmers have used programming languages to express parallelism. In this study, we must also distinguish between the parallelism as expressed in a

[1]Larger computers often have multiple processors, but it is still rare for a single process to have access to more than one processor at a time.

[2]Indeed, the Internet can be viewed as a kind of gigantic parallel computer, and several initiatives have successfully used the Internet in that way; see the Notes and References.

programming language and the parallelism that actually exists in the underlying hardware. Programs that are written with parallel programming constructs do not necessarily result in actual parallel processing, but could simply be implemented by pseudoparallelism, even in a system with multiple processors. Thus, parallel programming is sometimes referred to as **concurrent programming** to emphasize the fact that parallel constructs express only the **potential** for parallelism, not that parallel processing actually occurs (which is decided by the architecture of the particular system, the operating system, and the pragmatics of the translator interface). However, we will suppress this distinction and will refer to concurrent programming as parallel programming, without implying that parallel processing must occur.

In this chapter, we briefly survey the basic concepts of parallelism, without which an understanding of language issues is impossible. We then survey basic language issues and introduce the standard approaches to parallelism taken by programming language designers. These include threads, semaphores and their structured alternative, the monitor; and message passing. Java and Ada are used for most of the examples. Finally, a brief look is taken at some ways of expressing parallelism in functional and logical programming languages.

13.1 Introduction to Parallel Processing

The fundamental notion of parallel processing is that of the **process**: It is the basic unit of code executed by a processor. Processes have been variously defined in the literature, but a simple definition is the following:

> A process is a program in execution.

This is not quite accurate, because processes can consist of parts of programs as well as whole programs, more than one process can correspond to the same program, and processes do not need to be currently executing to retain their status as processes. A better definition might be the following:

> A process is an instance of a program or program part that has been scheduled for independent execution.

Processes used to be called **jobs**, and in the early days of computing, jobs were executed in purely sequential, or **batch** fashion. Thus, there was only one process in existence at a time, and there was no need to distinguish between processes and programs.

With the advent of pseudoparallelism, several processes could exist simultaneously in one of three states. A process could be **executing** (that is, in possession of the processor), it could be **blocked** (that is, waiting for some activity such as input-output to finish, or some event such as a keypress to occur), or it could be **waiting** for the processor to execute it.[3] In such a system the operating system needs to apply some algorithm to schedule processes for execution, and to manage a data structure, usually a queue, to maintain waiting and blocked processes. It also needs a method to cause processes to relinquish the processor, or timeout. The principal method for accomplishing this is the **hardware interrupt**.

With the existence of multiple processes, a distinction can also be made between heavyweight processes and lightweight processes. A **heavyweight process** corresponds to the earlier notion of a

[3]This is a simplified description. Actual operating systems have a more complex state structure.

program in execution. It exists as a full-fledged independent entity, together with all of the memory and other resources that are ordinarily allocated by the operating system to an entire program. A **lightweight process**, on the other hand, shares its resources with the program it came from: It does not have an independent existence except insofar as it executes a particular sequence of instructions from its parent program independently of other execution paths, while sharing all memory and other resources. A lightweight process is also called a **thread**. Lightweight processes can be particularly efficient, since there is less overhead in their creation and management by the operating system.

In a true parallel processing system, where several processors are available, the notion of process and process state is retained much as in the pseudoparallel system. The complication is that each processor may individually be assigned to a process, and a clear distinction must be made between process and processor. Each processor may or may not be assigned its own queues for maintaining blocked and waiting processes.

The organization of the different processors is a critical issue to the operation of processes in a parallel processing system. Two primary requirements for the organization of the processors are

1. There must be a way for processors to synchronize their activities.
2. There must be a way for processors to communicate data among themselves.

For example, in a typical situation, one processor will be handling the input and sorting of data, while a second processor performs computations on the data. The second processor must not begin processing data before it is sorted by the first processor. This is a synchronization problem. Also, the first processor needs to communicate the actual sorted data to the second processor. This is a communication problem.

In some machines one processor is designated as a controller, which manages the operation of the other processors. In some cases this central control extends even to the selection of instructions, and all the processors must execute the same instructions on their respective registers or data sets. Such systems are called **single-instruction**, **multiple-data** (**SIMD) systems** and are by their nature multiprocessing rather than distributed systems. Such systems are also often synchronous, in that all the processors operate at the same speed and the controlling processor determines precisely when each instruction is executed by each processor. This implicitly solves the synchronization problem.

In other architectures all the processors act independently. Such systems are called **multiple-instruction**, **multiple-data**, or **MIMD systems** and may be either multiprocessor or distributed processor systems. In an MIMD system the processors may operate at different speeds, and therefore such systems are asynchronous. Thus, the synchronization of the processors in an MIMD system becomes a critical problem.

Hybrid systems are also possible, with each processor retaining some, but not complete, independence from the other processors. The difference between an SIMD and an MIMD system is illustrated in Figure 13.1.

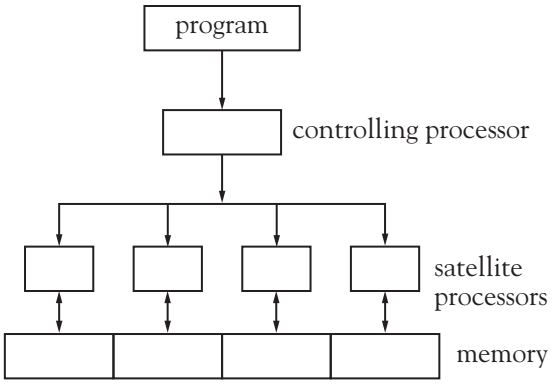


Figure 13.1a Schematic of an SIMD processor

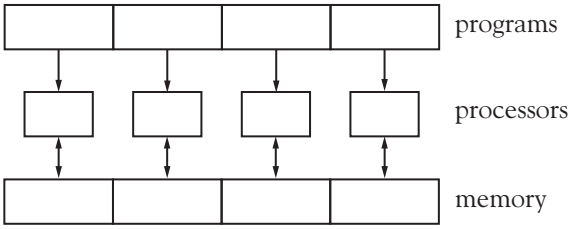


Figure 13.1b Schematic of an MIMD processor

Just as instructions are shared in an SIMD system, in a MIMD system, memory may be shared. A system in which one central memory is shared by all the processors is called a **shared-memory system** and is also by nature a multiprocessor rather than a distributed system, while a system in which each processor has its own independent memory is called a **distributed-memory system** (and may be either an actual distributed system or a multiprocessor system).

In a shared-memory system the processors communicate through the changes each makes to the shared memory. If the processors operate asynchronously, they may also use the shared memory to synchronize their activities. For this to work properly, each processor must have exclusive access to those parts of memory that it is changing. Without mutual exclusion, different processes may be modifying the same memory locations in an interleaved and unpredictable way, a situation that is sometimes called a **race condition**. Solving the mutual exclusion problem typically means blocking processes when another process is already accessing shared data using some kind of locking mechanism. This can cause a new problem to arise, namely, **deadlock**, in which processes end up waiting forever for each other to unblock. Detecting or preventing deadlock is typically one of the most difficult problems in parallel processing.

Distributed-memory systems do not have to worry about mutual exclusion, since each processor has its own memory inaccessible to the other processors. On the other hand, distributed processors have a **communication problem**, in that each processor must be able to send messages to and receive messages from all the other processors asynchronously. Communication between processors depends on the configuration of links between the processors. Sometimes processors are connected in sequence, and processors may need to forward information to other processors farther along the link. If the

number of processors is small, each processor may be fully linked to every other processor. Note that communicating processes may also block while waiting for a needed message, and this, too, can cause deadlock. Indeed, solving deadlock problems in a distributed system can be even more difficult than for a shared memory system, because each process may have little information about the status of other processes. For example, if another process is running on another computer connected to a network, and that computer crashes, it may be difficult to learn anything about what happened.

Again, it is possible for a system to be a hybrid between a shared memory and a distributed-memory system, with each processor maintaining some private memory in addition to the shared memory and the processors having some communication links separate from the shared memory. Figure 13.2 illustrates the difference between a shared-memory system and a fully linked distributed-memory system.

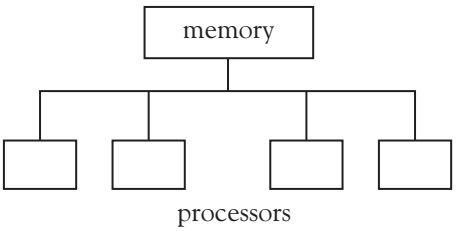


Figure 13.2a Schematic of a shared-memory system

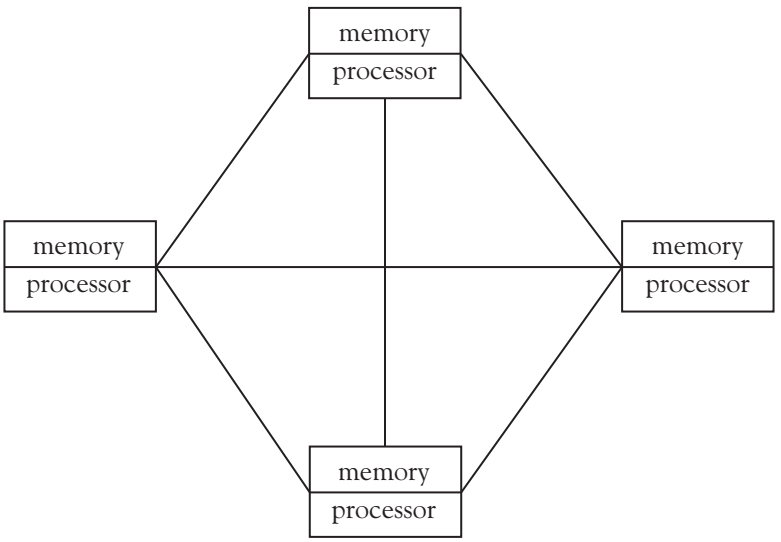


Figure 13.2b Schematic of a fully linked distributed-memory system

Regardless of the organization of the underlying machine, it is the task of the operating system to integrate the operation of the processors and to shield the user from needing to know too much about the particular configuration. The operating system can also change the view the user has of the hardware, depending on the utilities it makes available. For example, the operating system could assign distinct

parts of memory in a shared-memory system for exclusive use by each processor and use other parts of memory to simulate communications channels, thus making a shared-memory system appear to be a distributed-memory system. The operating system can even shield the user entirely from the fact that there is more than one processor and schedule users or processes on different processors automatically, according to an internal algorithm that attempts to allocate resources in an efficient way. However, automatic allocation of multiple processors is almost always less than optimal, and there is usually a need for operating systems to provide facilities for users to manage processes and processors manually. In general, an operating system will need to provide:

1. A means of creating and destroying processes.
2. A means of managing the number of processors used by processes (for example, a method of assigning processes to processors or a method for reserving a number of processors for use by a program).
3. On a shared-memory system, a mechanism for ensuring mutual exclusion of processes to shared memory. Mutual exclusion is used for both process synchronization and communication.
4. On a distributed-memory system, a mechanism for creating and maintaining communication channels between processors. These channels are used both for interprocess communication and for synchronization. A special case of this mechanism is necessary for a network of loosely connected independent computers, such as computers cooperating over the Internet.

In the next section, we will see how similar facilities must be provided by programming languages to make parallel processing available to programmers.

13.2 Parallel Processing and Programming Languages

Programming languages are like operating systems in that they need to provide programmers with mechanisms for process creation, synchronization, and communication. However, a programming language has stricter requirements than an operating system. Its facilities must be machine-independent and must adhere to language design principles such as readability, writability, and maintainability. Nevertheless, most programming languages have adopted a particular model of parallel organization in providing parallel facilities. Thus, some languages use the shared-memory model and provide facilities for mutual exclusion, typically by providing a built-in thread mechanism or thread library, while others assume the distributed model and provide communication facilities. A few languages have included both models, the designers arguing that sometimes one model and sometimes the other will be preferable in particular situations.

A language designer can also adopt the view that parallel mechanisms should not be included in a language definition at all. In that case, the language designer can still provide parallel facilities in other ways. Since this is the easiest approach to parallelism (from the point of view of the language designer), we will study this first. Also in this section, we will consider some approaches to process creation and destruction, since these are (more or less) common to both the shared-memory and the distributed models of parallelism. Model-specific facilities will be left to later sections in this chapter.

In particular, Section 13.3 will discuss **threads**, especially the Java thread implementation. Sections 13.4 and 13.5 will discuss **semaphores** and **monitors**, two approaches to the shared-memory model. Section 13.6 will study **message passing**, a mechanism that follows the distributed model, with examples from Ada. That section will also briefly mention a few of the issues surrounding the network model (which is a kind of distributed model).

We will also need a number of standard problems in parallel processing to demonstrate the use of particular mechanisms. We will use the following two problems throughout the remainder of this chapter (other problems are discussed in the exercises):

1. *The bounded buffer problem.* This problem assumes that two or more processes are cooperating in a computational or input-output situation. One (or more) process produces values that are consumed by another process (or processes). An intermediate buffer, or buffer process, stores produced values until they are consumed. A solution to this problem must ensure that no value is produced until there is room to store it and that a value is consumed only after it has been produced. This involves both communication and synchronization. When the emphasis is not on the buffer, this problem is called the *producer-consumer problem.*
2. *Parallel matrix multiplication.* This problem is different from the previous one in that it is an example of an algorithmic application in which the use of parallelism can cause significant speedups. Matrices are essentially two-dimensional arrays, as in the following declaration of integer matrices (we consider only the case where the size of both dimensions is the same, given by the positive integer constant `N`):

   ```
   typedef int Matrix [N][N];
   Matrix a,b,c;
   ```

   The standard way of multiplying two matrices `a` and `b` to form a third matrix `c` is given by the following nested loops:

```
for (i = 0; i < N; i++)
    for (j = 0; j < N; j++)
        c[i][j] = 0;
        for (k = 0; k < N; k++)
            c[i][j] = c[i][j] + a[i][k] * b[k][j];
```

This computation, if performed sequentially, takes N^3 steps. If, however, we assign a process to compute each `c[i][j]`, and if each process executes on a separate processor, then the computation can be performed in the equivalent of `N` steps. Algorithms such as this are studied extensively in courses on parallel algorithms. In fact, this is the simplest form of such an algorithm, since there are no write conflicts caused by the computation of the `c[i][j]` by separate processes. Thus, there is no need to enforce mutual exclusion in accessing the matrices as shared memory. There is, however,

a synchronization problem in that the product c cannot be used until all the processes that compute it have finished. A programming language, if it is to provide useful parallelism, must provide facilities for implementing such algorithms with a minimum of overhead.

13.2.1 Parallel Programming Without Explicit Language Facilities

As we have noted, one possible approach to parallelism is simply not to express it explicitly at all in the language. This is easiest in functional, logic, and object-oriented languages, which have a certain amount of inherent parallelism implicit in the language constructs. (We have mentioned this before, but we will review it again in Section 13.7.)

In theory it is possible for language translators, using optimization techniques, to automatically use operating system utilities to assign different processors to different parts of a program. However, as with operating systems, the automatic assignment of processors is likely to be suboptimal, and manual facilities are needed to make full use of parallel processors. In addition, a programming language may be used for purposes that require explicitly indicating the parallelism, such as the writing of operating systems themselves or the implementation of intricate parallel algorithms.

A second alternative to defining parallel constructs in a programming language is for the translator to offer the programmer **compiler options** to allow the explicit indicating of areas where parallelism is called for. This is usually better than automatic parallelization. One of the places where this is most effective is in the use of nested loops, where each repetition of the inner loop is relatively independent of the others. Such a situation is the matrix multiplication problem just discussed.

Figure 13.3 shows an example of a parallel loop compiler option in FORTRAN code for the matrix multiplication problem. The example is for a FORTRAN compiler on a Sequent parallel computer. The compiler option is:

```
C$doacross share(a, b, c), local(j, k)
```

that causes a preprocessor to insert code that parallelizes the outer loop. The `share` and `local` declarations indicate that `a`, `b`, and `c` are to be accessed by all processes, but that `j` and `k` are local variables to each process. The call to `m_set_procs` sets the number of processes (and processors) that are to be used, returning an error code if not enough processors are available. (Note that in the example the number of processes is 10, far below the number needed to compute optimally the matrix product; see Exercise 13.4.) The call to `m_kill_procs` synchronizes the processes, so that all processes wait for the entire loop to finish and that only one process continues to execute after the loop.

```
      integer a(100, 100), b(100, 100), c(100, 100)
      integer i, j, k, numprocs, err
      numprocs = 10
C code to read in a and b goes here
      err = m_set_procs(numprocs)
C$doacross share(a, b, c), local(j, k)
       do 10 i = 1,100
         do 10 j = 1,100
```

Figure 13.3 FORTRAN compiler options for parallelism *(continues)*

(continued)

```
          c(i, j) = 0
          do 10 k = 1,100
             c(i, j) = c(i, j) + a(i, k) * b(k, j)
  10 continue
      call m_kill_procs
C code to write out c goes here
      end
```

Figure 13.3 FORTRAN compiler options for parallelism

A third way of making parallel facilities available without explicit mechanisms in the language design is to provide a library of functions to perform parallel processing. This is a way of passing the facilities provided by an operating system directly to the programmer. This way, different libraries can be provided, depending on what facilities an operating system or parallel machine offers. Of course, if a standard parallel library is required by a language, then this is the same as including parallel facilities in the language definition.

Figure 13.4 is an example in C where library functions are used to provide parallel processing for the matrix multiplication problem. (C itself has no parallel mechanisms.) We note that the example of the use of a translator option to indicate parallelism (Figure 13.3) also used some of the same library procedures. (This example is also for a Sequent computer, for comparison purposes.)

```
#include <parallel/parallel.h>
#define SIZE 100
#define NUMPROCS 10

shared int a[SIZE][SIZE], b[SIZE][SIZE], c[SIZE][SIZE];

void multiply(){
    int i, j, k;
    for (i = m_get_myid(); i < SIZE; i += NUMPROCS)
        for (j = 0; j < SIZE; j++)
            for (k = 0 ; k < SIZE; k++)
                c[i][j] += a[i][k] * b[k][j];
}

main(){
    int err;
    /* code to read in the matrices a and b goes here */
    m_set_procs(NUMPROCS);
    m_fork(multiply);
    m_kill_procs();
    /* code to write out the matrix c goes here */
    return 0;
}
```

Figure 13.4 Use of a library to provide parallel processing

In Figure 13.4 the four procedures `m_set_procs`, `m_fork`, `m_kill_procs`, and `m_get_myid` are imported from a library (the parallel/parallel library). `m_set_procs` and `m_kill_procs` are as in the previous example. `m_fork` creates the 10 processes, which are all instances of the procedure multiply (the name `fork` comes from the UNIX operating system, discussed shortly). In procedure multiply, `m_get_myid` gets the number of the process instance (from 0 to 9). The remainder of the code then divides the work among the processes, so that process 0 calculates `c[0][i]`, `c[10][i]`, . . . , `c[90][i]` for all `i`, process 2 calculates `c[2][i]`, `c[12][i]`, . . . , `c[92][i]`, and so on.

A final alternative to introducing parallelism into a language is to simply rely on operating system features directly to run programs in parallel. Essentially, this requires that a parallel program be split up into separate, independently executable pieces and then set up to communicate via operating system mechanisms (thus, allowing only program-level parallelism as described shortly). A typical example of this is to string programs together in a UNIX operating system through the use of **pipes**, which is a method of streaming text input and output from one program to another without storing to intermediate files (which could be inefficient or impossible if these files are large). A simple example is the following, which will list all filenames in the current directory containing the string "java:"[4]

```
ls | grep "java"
```

This command runs the two UNIX utility programs `ls` and `grep` in parallel (`ls` is the directory listing program; `grep` is the "global regular expression print" program that finds string patterns in text). The output of `ls` is piped to `grep` using the pipe symbol (the vertical bar), and becomes the input to `grep`. As a more complex example, the pipe

```
cat *.java | tr -sc A-Za-z '\012' | sort | uniq -c | sort -rn
```

will list all of the words in Java programs in the current directory, with a count of their usage, and sorted in order of decreasing number of uses (we leave as an exercise for the reader the explanation of each of the five programs used in this pipeline).

13.2.2 Process Creation and Destruction

A programming language that contains explicit mechanisms for parallel processing must have a construct for creating new processes. You have seen this informally already in Figure 13.4, where calls to the library procedures `m_set_procs` and `m_fork` together created a fixed number of processes.

There are two basic ways that new processes can be created. One is to split the current process into two or more processes that continue to execute copies of the same program. In this case, one of the processes is usually distinguished as the **parent**, while the others become the **children**. The processes can execute different code by a test of process identifiers or some other condition, but the basic program is the same for all processes. This method of process creation resembles the SIMD organization of Section 13.1 and is, therefore, called **SPMD programming** (for single program multiple data). Note, however, that SPMD programs may execute different segments of their common code and so do not necessarily operate synchronously. Thus, there is a need for process synchronization.

[4]This can actually be condensed to the single command "`ls *java*`" in UNIX.

In the second method of process creation, a segment of code (commonly a procedure) is explicitly associated with each new process. Thus, different processes have different code, and we can call this method **MPMD programming**. A typical case of this is the so-called **fork-join** model, where a process creates several child processes, each with its own code (a fork), and then waits for the children to complete their execution (a join). Unfortunately, the name is confusing, because the UNIX system called `fork()` (studied shortly) is really an SPMD process creator, not a fork-join creator. We will, therefore refer to MPMD process creation rather than a fork-join creation. Note that Figure 13.4 is an example of MPMD programming (with `m_kill_procs` taking the place of the join).

An alternative view of process creation is to focus on the size of the code that can become a separate process. In some designs, individual statements can become processes and be executed in parallel. A second possibility is for procedures to be assigned to processes. This was the case in Figure 13.4, where the procedure multiply was assigned to processes via the call `m_fork(multiply)`. A third possibility is for processes to represent whole programs only. Sometimes the different size of the code assignable to separate processes is referred to as the **granularity** of processes. The three choices of constructs for parallel execution that we have just listed could be described as follows:

1. Statement-level parallelism: fine-grained
2. Procedure-level parallelism: medium-grained
3. Program-level parallelism: large-grained

Granularity can be an issue in program efficiency: Depending on the kind of machine, many small-grained processes can incur significant overhead in their creation and management, thus executing more slowly than fewer larger processes. On the other hand, large-grained processes may have difficulty in exploiting all opportunities for parallelism within a program. An intermediate case that can be very efficient is that of a thread, which typically represents fine-grained or medium-grained parallelism without the overhead of full-blown process creation.

Regardless of the method of process creation, it is possible to distinguish between process creator (the parent process) and process created (the child process), and for every process creation mechanism, the following two questions must be answered:

1. Does the parent process suspend execution while its child processes are executing, or does it continue to execute alongside them?
2. What memory, if any, does a parent share with its children or the children share among themselves?

In Figure 13.4, the assumption was that the parent process suspends execution while the child processes compute. It was also necessary to indicate explicitly that the global variables `a`, `b`, and `c` are to be shared by all processes, using the keyword `shared`.

In addition to process creation, a parallel programming language needs a method for process termination. In the simplest case, a process will simply execute its code to completion and then cease to exist. In more complex situations, however, a process may need to continue executing until a certain condition is met and then terminate. It may also be necessary to select a particular process to continue execution.

We will briefly study process creation and destruction mechanisms for each kind of granularity.

13.2.3 Statement-Level Parallelism

A typical construct for indicating that a number of statements can be executed in parallel is the **parbegin-parend** block:[5]

```
parbegin
  S1;
  S2;
  ...
  Sn;
parend;
```

In this statement the statements `S1`, . . . , `Sn` are executed in parallel. It is assumed that the main process is suspended during their execution, and that all the processes of the `Si` share all variables not locally declared within an `Si`.

An extension of this mechanism is the parallel loop construct of Fortran95, or **forall** construct, which indicates the parallel execution of each iteration of a loop, as in

```
    forall(i = 1:100, j = 1:100)
      c(i, j) = 0
      do 10 k = 1,100
                        c(i, j) = c(i, j) + a(i, k) * b(k, j)
10    continue
    end forall
```

This is similar to the `$doacross` compiler option of Figure 13.3.

13.2.4 Procedure-Level Parallelism

In this form of process creation/destruction, a procedure is associated with a process, and the process executes the code of the procedure. Schematically, such a mechanism has the form

```
x = newprocess(p);
...
...
killprocess(x);
```

where `p` is a declared procedure and `x` is process designator—either a numeric process number or a variable of type `process`. Figure 13.4 uses library procedures that create processes essentially this way. An alternative to this is to use declarations to associate procedures to processes:

```
process x(p);
```

[5]This example is taken from the programming language **Occam**, which is based on CSP—see the Notes and References.

Then the scope of x can be used as the region where x is active: x begins execution when the scope of its declaration is entered, and x is terminated on exit from its scope (if it has not already executed to completion). This is the method used by Ada in the declaration of tasks and task types, which is discussed more fully in Section 13.5. (The **task** is Ada's term for a process.)

13.2.5 Program-Level Parallelism

In this method of process creation only whole programs can become processes. Typically, this occurs in MPMD style, where a program creates a new process by creating a complete copy of itself. The typical example of this method is the `fork` call of the UNIX operating system. A call to `fork` causes a second child process to be created that is an exact copy of the calling process, including all variables and environment data at the moment of the fork. Processes can tell which is the child and which is the parent by the returned value of the call to `fork`: A zero value indicates that the process is the child, while a nonzero value indicates the process is the parent (the value itself is the process number of the child just created). By testing this value the parent and child processes can be made to execute different code:

```
if (fork() == 0)
  {/* ... child executes this part ...*/}
else
  {/* ... parent executes this part ...*/}
```

After a call to `fork`, a process can be terminated by a call to `exit`. Process synchronization can be achieved by calls to `wait`, which causes a parent to suspend its execution until a child terminates.

Figure 13.5 gives sample C code for parallel matrix multiplication using `fork`, `exit`, and `wait`. In this code, a `fork` is performed for each of the `NUMPROCS` child processes, and a `wait` is performed for each of the child processes as well. For simplicity, this code assumes that global variables are shared by all processes. (Warning! This is not true of processes in a standard UNIX implementation. See Exercises 13.15 and 13.16 for a discussion.)

```
#define SIZE 100
#define NUMPROCS 10

int a[SIZE][SIZE], b[SIZE][SIZE], c[SIZE][SIZE];

main(){
    int myid;
    /* code to input a,b goes here */
    for (myid = 0; myid < NUMPROCS; ++myid)
        if (fork() == 0){
            multiply(myid) ;
```

Figure 13.5 Sample C code for the fork construct *(continues)*

(continued)

```
            exit(0) ;
        }
    for (myid = 0; myid < NUMPROCS; ++myid)
        wait(0) ;
        /* code to output c goes here */
    return 0;
}

void multiply (int myid){
    int i, j, k;
    for (i = myid; i < SIZE; i+= NUMPROCS)
        for (j = 0; j < SIZE; ++j){
            c[i][j] = 0;
            for (k = 0; k < SIZE; ++k)
                c[i][j] += a[i][k] * b[k][j];
    }
}
```

Figure 13.5 Sample C code for the fork construct

13.3 Threads

As we have noted, threads can be an efficient mechanism for fine- or medium-grained parallelism in the shared memory model. Since Java has a widely used thread implementation, we shall study the thread implementation in Java in some detail here, as a good illustration of the principles we have discussed so far. In particular, we will describe process creation, destruction, synchronization, and mutual exclusion in Java, and illustrate these facilities with a variation of the bounded buffer problem.

13.3.1 Threads in Java

Threads are built into the Java language, in that the `Thread` class is part of the `java.lang` package, and the reserved word `synchronize` is used to establish mutual exclusion for threads.[6] A Java thread is created by instantiating a `Thread` object and by defining a `run` method that will be executed when the thread starts. This can be done in two ways, either by extending `Thread` through inheritance and overriding the (empty) `Thread.run` method,

[6]Actually, Java offers two thread packages, called **green threads** and **native threads**. Green threads (the default) do not use mechanisms provided by the underlying operating system, but all parallelism is managed by the Java Virtual Machine itself. Native threads, on the other hand, use special features of the underlying operating system, and may, therefore, operate more efficiently by taking advantage of system features, including hardware parallelism. Green threads are necessary if threads are to operate securely across an Internet connection.

```
class MyThread extends Thread{

    public void run()
    { ... }
}

...

Thread t = new MyThread();

...
```

or by defining a class that implements the `Runnable` interface (which only requires the definition of a `run` method), and then passing an object of this class to the `Thread` constructor:

```
class MyRunner implements Runnable{

    public void run()
    { ... }
}

...

MyRunner m = new MyRunner();
Thread t = new Thread(m);

...
```

This latter mechanism is more versatile, so we use it in our examples.

Defining a thread does not begin executing it. Instead, a thread begins running when the `start` method is called:

```
t.start(); // now t will execute the run method
```

The `start` method in turn will call `run`. Although one could call the `run` method directly, this will in general not work, since the system must perform some internal bookkeeping before calling `run`. Note also that every Java program already is executing inside a thread whose `run` method is `main`. Thus, when a new thread's execution is begun by calling `start`, the main program will still continue to execute to completion in its own thread (in other words, the `start` method of a thread immediately returns, while the thread itself continues to execute). However, the entire *program* will not finish execution until all of its threads complete the execution of their `run` methods.[7]

[7]There is also an alternative kind of thread in Java, called a **daemon**, which is killed off when the main program exits, regardless of whether it has completed execution. See the Notes and References.

How are threads destroyed? As we have just noted, the simplest mechanism is to let each thread execute its `run` method to completion, at which time it will cease to exist without any programmer intervention. Threads can also wait for other threads to finish before continuing by calling the `join` method on the thread object:

```
Thread t = new Thread(m);
t.start(); // t begins to execute
// do some other work
t.join(); // wait for t to finish
// continue with work that depends on t being finished
```

Thus, Java threads exhibit fork-join parallelism as previously defined (with the fork operation given by the `start` method).

An alternative to waiting for a thread to finish executing is to interrupt it using the `interrupt` method:

```
Thread t = new Thread(m);
t.start(); // t begins to execute
// do some other work
t.interrupt(); // tell t that we are waiting for it
t.join(); // continue to wait
// continue
```

The `interrupt` method, as this code indicates, does not actually stop the interrupted thread from executing, but simply sets an internal flag in the thread object that can be used by the object to test whether some other thread has called its `interrupt` method. This allows a thread to continue to execute some cleanup code before actually exiting. The semantics of the `interrupt` method are somewhat complex, so we defer a fuller discussion temporarily.

Note that, when `t.join()` is called, the current thread (i.e., the thread that makes this call) becomes blocked, and will only unblock once `t` exits its `run` method. If t is also in a blocked state waiting for an event, then deadlock can result. If there is a real potential for deadlock, then we could try to avoid it by waiting only a specified amount of time, and then timing out:

```
t.join(1000); // wait 1 second for t, then give up
```

In general, any Java method such as `join` that blocks the current thread has a timeout version such as the above, because the Java runtime system makes no attempt to discover or prevent deadlock. It is entirely up to the programmer to ensure that deadlock cannot occur.

So far we have not discussed how to enforce mutual exclusion for shared data in Java. Even with only the mechanisms described so far, however, Java threads can be useful, and indeed parallel matrix multiplication can already be written (see Exercise 13.17), since there is no need for mutual exclusion.

In general, Java threads will share some memory or other resources, typically by passing the objects to be shared to constructors of the `Runnable` objects that are used to create the threads.[8] For example, suppose several distinct threads are adding and removing objects using a shared queue:

[8]It is also possible to create inner `Runnable` classes and share data via nonlocal access.

```
class Queue{

    ...
    public Object dequeue(){
        if (empty()) throw EmptyQueueException ;
        ...
    }

    public void enqueue(Object obj) { ... }
    ...
}

class Remover implements Runnable{

    public Remover(Queue q){ ... }
    public void run(){ ... q.dequeue() ... }
    ...
}

class Inserter implements Runnable{

    public Inserter(Queue q){ ... }
    public void run(){ ... q.enqueue(...) ... }
    ...
}

Queue q = new Queue(...);
...
Remover r = new Remover(q);
Inserter i = new Inserter(q);
Thread t1 = new Thread(r);
Thread t2 = new Thread(i);
t1.start();
t2.start();
...
```

Now the queue `q` is shared between threads `t1` and `t2`, and some mechanism for ensuring mutual exclusion within `q` is necessary, so that it is not possible for one thread to be executing `q.dequeue()` while the other is executing `q.enqueue()`. In Java, this is done by using the `synchronized` keyword[9] in the definition of class `Queue`:

[9]In this text, we only discuss method synchronization. In fact, in Java any block of code can be synchronized on any object for which a local reference exists (synchronized methods implicitly synchronize on the current object `this`). Method synchronization is a special case of block synchronization; a method such as `enqueue` above can be synchronized by the following equivalent code: `public void enqueue(Object obj) { synchronized(this) {...} }`.

```
class Queue{
    ...
    synchronized public Object dequeue(){
        if (empty()) throw EmptyQueueException ;
        ...
    }

        synchronized public void enqueue(Object obj){ ... }
        ...
}
```

In Java, every object has a single **lock** (see Section 13.4) that is available to threads. When a thread attempts to execute a synchronized method of an object, it must first acquire the lock on the object. If the lock is in possession of another thread, it waits until the lock is available, acquires it, and then proceeds to execute the synchronized method. After exiting the method, it releases the lock before continuing its own execution.[10] Thus, no two threads can be simultaneously executing any synchronized methods of the same object. (Unsynchronized methods are not affected.) This solves the preceding mutual exclusion problem.

There is one more thread mechanism in Java that we discuss here before considering an extended example: how to cause threads to wait for certain explicit conditions and then resume execution. For example, suppose that, in the previous queue example, we know that if `dequeue` is ever called, then an object will eventually exist for the calling thread to remove from the queue. Then, it is undesirable to generate an exception on an empty queue—the thread should simply wait for the object to appear in the queue.

If a condition is testable in the code for a synchronized method, a thread executing that method can be manually stalled by calling the `wait()` method of the object (`wait` is a method of class `Object`, so every object has this method). Once a thread is waiting for a certain condition, it also needs to be manually reawakened by another thread (typically, the thread that establishes the desired condition). This is done using either the `notify()` or the `notifyAll()` methods of the object. Note that each object in Java has a wait-list containing all threads that have called `wait` on this object, in some unspecified order. The `notify()` method will wake up only one (arbitrary) thread in this list, while `notifyAll()` will wake up all threads in the list. Typically, `notifyAll()` is used in preference to `notify()`, since it is impossible to predict which thread will be awakened by `notify()`, and a thread in the wait-list may be waiting for a different condition than the one that has just been established.

Now let us reconsider the above queue example. Instead of generating an exception on an empty queue, the `dequeue` method should call `wait`, and the `enqueue` operation should call `notifyAll`, since it makes the queue nonempty:

[10]This is the simplest case; in fact, a thread can acquire multiple locks, or even the same lock multiple times; each synchronized exit releases only one "hold" on one lock.

```
(1)  class Queue
(2)  {   ...
(3)      synchronized public Object dequeue()
(4)                     {  try  // wait can generate InterruptedException
(5)         {  while (empty()) wait();
(6)            ...
(7)         }
(8)                         catch (InterruptedException e) // reset interrupt
(9)         { Thread.currentThread().interrupt(); }
(10)     }
(11)     synchronized public void enqueue(Object obj)
                            // tell waiting threads to try again
(12)     { // add obj into the queue
(13)       ...
(14)       notifyAll();
(15)     }
(16)     ...
(17) }
```

Two things deserve special mention here. First, a `while` loop is used on line 5 in `dequeue`, because `empty()` should be retested after a wake-up: Another thread may have already emptied the queue before this thread acquires the lock. Second, a call to `wait` can generate an `InterruptedException`, which must be handled or rethrown. This exception will be generated if another thread calls `interrupt` on a thread that is in the wait-queue, causing it to wake up and resume execution as though the exception occurred in the usual way.

We have noted previously that a call to `interrupt` does not actually stop a thread from executing but sets a flag that can be tested by the thread code, so that it may perform cleanup before exiting. However, if a thread is waiting for a lock prior to executing a synchronized method, or is in a wait-list, then it is not executing and so cannot test for an interrupt. This problem is solved by having a call to `interrupt` generate an exception if the thread is not currently executing and simultaneously wake the stalled thread. Also, the interrupt flag is cleared when the exception is generated. Thus, a thread typically will want to test for both the interrupt flag and `interruptedException`, if it is to handle `InterruptedException` itself. Alternatively (and this is often the simplest), the synchronized method itself can handle the exception, in which case it is reasonable to reset the interrupt flag before exiting, as in line 9, so that the client thread can still discover that it has been interrupted. Then, the client thread code could be as follows:

```
class Remover implements Runnable{
    public Remover(Queue q) { ... }
    public void run(){
       while (!Thread.currentThread().isInterrupted()){
         ... q.dequeue() ...
       }
       // perform some cleanup up
    } // and now exit
   ...
}
```

Note how the interrupt flag is set and tested by calls to the `interrupt` and `isInterrupted` methods of `Thread.currentThread()`. Indeed, `currentThread` is a static method of the `Thread` class (i.e., a class method) and must be called on the `Thread` class object, in which case it returns the current `Thread` object. This is necessary, since the `run` code inside a `Runnable` class has no way of identifying the actual thread that has been interrupted, but it will always be the *currently executing* thread.

13.3.2 A Bounded Buffer Example in Java

To finish our discussion of Java threads, we discuss in some detail a solution to the bounded buffer problem mentioned in Section 13.2. We will cast this problem as an input-output problem, with the producer reading characters from standard input and inserting them into the buffer, and the consumer removing characters from the buffer and writing them to standard output. To add interest to this example, we set the following additional requirements:

- The producer thread will continue to read until an end of file is encountered, whence it will exit.
- The consumer thread will continue to write until the producer has ended and no more characters remain in the buffer. Thus, the consumer should echo all the characters that the producer enters into the buffer.

The complete code for this Java program is given in Figure 13.6. We make the following explanatory remarks about this code.

```
(1) import java.io.*;

(2) class BoundedBuffer{

(3)     public static void main(String[] args){
(4)                     Buffer buffer = new Buffer(5); // buffer has size 5
(5)         Producer prod = new Producer(buffer);
(6)         Consumer cons = new Consumer(buffer);
(7)         Thread read = new Thread(prod);
(8)         Thread write = new Thread(cons);
(9)         read.start();
(10)        write.start();
(11)        try {
(12)          read.join();
(13)          write.interrupt();
(14)        }
(15)        catch (InterruptedException e) {}
(16)    }
(17) }

(18) class Buffer{
(19)     private final char[] buf;
```

Figure 13.6 Java code for a bounded buffer problem *(continues)*

(continued)

```
(20)     private int start = -1;
(21)     private int end = -1;
(22)     private int size = 0;

(23)     public Buffer(int length){
(24)         buf = new char[length];
(25)     }
(26)     public boolean more()
(27)     { return size > 0; }

(28)     public synchronized void put(char ch)
(29)     { try {
(30)         while (size == buf.length) wait();
(31)         end = (end+1) % buf.length;
(32)         buf[end] = ch;
(33)         size++;
(34)         notifyAll();
(35)       }
(36)       catch (InterruptedException e)
(37)       { Thread.currentThread().interrupt(); }
(38)     }

(39)     public synchronized char get()
(40)     { try {
(41)         while (size == 0) wait();
(42)         start = (start+1) % buf.length;
(43)         char ch = buf[start];
(44)         size--;
(45)         notifyAll();
(46)         return ch;
(47)       }
(48)       catch (InterruptedException e)
(49)       { Thread.currentThread().interrupt(); }
(50)       return 0;
(51)     }
(52) }

(53) class Consumer implements Runnable{
(54)     private final Buffer buffer;

(55)     public Consumer(Buffer b)
(56)     { buffer = b;
(57)     }
```

Figure 13.6 Java code for a bounded buffer problem *(continues)*

(continued)

```
(58)     public void run() {
(59)                  while (!Thread.currentThread().isInterrupted())
(60)       {   char c = buffer.get();
(61)           System.out.print(c);
(62)       }
(63)       while(buffer.more()) // clean-up
(64)       {   char c = buffer.get();
(65)           System.out.print(c);
(66)       }
(67)     }
(68) }

(69) class Producer implements Runnable{
(70)     private final Buffer buffer;
(71)   private final InputStreamReader in
(72)        = new InputStreamReader(System.in);

(73)     public Producer(Buffer b) { buffer = b; }

(74)     public void run() {
(75)       try {
(76)                 while (!Thread.currentThread().isInterrupted()) {
(77)         int c = in.read();
(78)         if (c == -1) break; // -1 is end of file
(79)         buffer.put((char)c);
(80)       }
(81)      }
(82)      catch (IOException e) {}
(83)     }
(84) }
```

Figure 13.6 Java code for a bounded buffer problem

The main program creates a buffer of five characters, a reader (the producer) for the buffer and a writer (the consumer) that prints characters. Two threads are created, one for the producer and one for the consumer (so that three threads in all exist for this program, including the main thread). The main thread then waits for the reader to finish (line 12), and then interrupts the writer (line 13). (Since `join` can, like `wait`, generate `InterruptedException`, we must also handle this exception.)

The bounded buffer class itself uses an array to hold the characters, along with two indices that traverse the array in a circular fashion, keeping track of the last positions where insertions and removals took place (this is a standard implementation technique discussed in many data structures books). Both the `put` (insert) and `get` (remove) operations are synchronized to ensure mutual exclusion. The `put` operation delays a thread if the buffer is full (line 30), while the `get` operation delays a thread if the

buffer is empty (line 41). As a result, both operations must handle the `InterruptedException`. Both also call `notifyAll` after performing their respective operations (lines 34 and 45).

The `Producer` class (lines 69–84) is relatively simple. Its `run` method checks for an interrupt—even though `interrupt` will never be called on its thread in this program, it is reasonable to write general code such as this. Its only other behavior is to check for an end of file (line 78, where EOF $= -1$), whence it exits.

The `Consumer` class is almost symmetrical to the `Producer` class. The major difference is that, when a `Consumer` is interrupted, we want to make sure that the buffer has been emptied before the `Consumer` exits. Thus, on reaching line 63, we know that the producer has finished, so we simply test for more characters in the buffer and write them before exiting.

13.4 Semaphores

A **semaphore** is a mechanism to provide mutual exclusion and synchronization in a shared-memory model. It was first developed by E. W. Dijkstra in the mid-1960s and included in the languages Algol68 and PL/I. A semaphore is a shared integer variable that may be accessed only via three operations: **InitSem**, **Signal**, and **Delay**. The **Delay** operation tests the semaphore for a positive value, decrementing it if it is positive and suspending the calling process if it is zero or negative. The **Signal** operation tests whether processes are waiting, causing one of them to continue if so and incrementing the semaphore if not. (These operations were originally called *P* and *V* by Dijkstra, but we will use the more descriptive names as given; note that **Signal** is analogous to `notify` in Java, and **Delay** is analogous to `wait`.)

Given a semaphore *S*, the **Signal** and **Delay** operations can be defined in terms of the following pseudocode:

Delay(*S*): if $S > 0$ then $S := S - 1$ else suspend the calling process
Signal(*S*): if processes are waiting then wake up a process else $S := S + 1$

Unlike ordinary code, however, the system must ensure that each of these operations executes **atomically**, that is, by only one process at a time. (If two processes were to try to increment *S* at the same time, the actual final value of *S* would be unpredictable; see Exercise 13.52.)

Given a semaphore *S*, we can ensure mutual exclusion by defining a **critical region**, that is, a region of code that can be executed by only one process at a time. If S is initialized to 1, then the following code defines such a critical region:

Delay(*S*);
{critical region}
Signal(*S*);

A typical critical region is code where shared data is read and/or updated. Sometimes semaphores are referred to as **locks**, since they lock out processes from critical regions.

Semaphores can also be used to synchronize processes. If, for example, process *p* must wait for process *q* to finish, then we can initialize a semaphore *S* to 0, call *Delay*(*S*) in *p* to suspend its execution, and call *Signal*(*S*) at the end of *q* to resume the execution of *p*.

An important question to be addressed when defining semaphores is the method used to choose a suspended process for continued execution when a call to *Signal* is made. Possibilities include making a random choice, using a first in, first out strategy, or using some sort of priority system. This choice has a major effect on the behavior of concurrent programs using semaphores.

As defined, a semaphore can be thought of as an abstract data type, with some added requirements. Indeed, semaphores can be defined using a Java class definition as given in Figure 13.7.[11]

```
class Semaphore{

    private int count;

    public Semaphore(int initialCount){
       count = initialCount;
    }

    public synchronized void delay() throws InterruptedException{
        while (count <= 0) wait();
        count--;
    }

    public synchronized void signal(){
        count++;
        notify();
    }
}
```

Figure 13.7 Simplified Java code for a semaphore

This code uses a call to `notify` rather than `notifyAll`, because each semaphore's wait-list is waiting on a single condition—that `count` should be greater than 0—so that only one waiting thread needs to be awakened at a time.

13.4.1 A Bounded Buffer Using Semaphores

We can write a version of the bounded buffer class that uses semaphores to enforce synchronization and mutual exclusion, as shown in Figure 13.8. (The rest of the program of Figure 13.6 can remain exactly the same.) We note a number of things about this solution.

The solution uses three semaphores. The first, `mutEx`, provides mutual exclusion to code that changes the private state of a `Buffer`. The semaphores `nonEmpty` and `nonFull` maintain the status of the number of stored items available. Note that all Java synchronization mechanisms—synchronized methods, calls to `wait` and `notifyAll`—are now gone from the `Buffer` code itself, since they are indirectly supplied by the semaphores.

[11]This simplified code ignores several Java issues, particularly those surrounding interrupts (as indicated by the `throws InterruptedException` declaration for `delay`); see Exercise 13.28.

```
(1)  class Buffer{
(2)      private final char[] buf;
(3)      private int start = -1;
(4)      private int end = -1;
(5)      private int size = 0;
(6)      private Semaphore nonFull, nonEmpty, mutEx;

(7)      public Buffer(int length) {
(8)          buf = new char[length];
(9)          nonFull = new Semaphore(length);
(10)         nonEmpty = new Semaphore(0);
(11)         mutEx = new Semaphore(1);
(12)     }

(13)     public boolean more()
(14)     { return size > 0; }

(15)     public void put(char ch){
(16)      try {
(17)         nonFull.delay();
(18)         mutEx.delay();
(19)         end = (end+1) % buf.length;
(20)         buf[end] = ch;
(21)         size++;
(22)         mutEx.signal();
(23)         nonEmpty.signal();
(24)       }
(25)       catch (InterruptedException e)
(26)       { Thread.currentThread().interrupt(); }
(27)     }
(28)     public char get(){
(29)      try{
(30)         nonEmpty.delay();
(31)         mutEx.delay();
(32)         start = (start+1) % buf.length;
(33)         char ch = buf[start];
(34)         size--;
(35)         mutEx.signal();
(36)         nonFull.signal();
(37)         return ch;
(38)       }
(39)       catch (InterruptedException e)
(40)       { Thread.currentThread().interrupt(); }
(41)       return 0;
(42)     }
(43) }
```

Figure 13.8 The bounded buffer problem using semaphores

13.4.2 Difficulties with Semaphores

The basic difficulty with semaphores is that, even though the semaphores themselves are protected, there is no protection from their incorrect use or misuse by programmers. For example, if a programmer incorrectly writes:

Signal(S);
. . .
Delay(*S*);

then the surrounded code is not a critical region and can be entered at will by any process. On the other hand, if a programmer writes:

Delay(*S*);
. . .
Delay(*S*);

then it is likely that the process will block at the second *Delay*, never to resume execution. It is also possible for the use of semaphores to cause deadlock. A typical example is represented by the following code in two processes, with two semaphores S_1 and S_2:

Process 1: *Delay*(S_1);
Delay(S_2);
. . .
Signal(S_2);
Signal(S_1);

Process 2: *Delay*(S_2);
Delay(S_1);
. . .
Signal(S_1);
Signal(S_2);

If Process 1 executes *Delay*(S_1) at the same time that Process 2 executes *Delay*(S_2), then each will block waiting for the other to issue a *Signal.* Deadlock has occurred.

To remove some of the insecurities in the use of semaphores, the monitor was invented; see Section 13.5.

13.4.3 Implementation of Semaphores

Generally, semaphores are implemented with some form of hardware support. Even on single-processor systems this is not an entirely trivial proposition, since an operating system may possibly interrupt a process between any two machine instructions. One common method for implementing semaphores on a single-processor system is the ***TestAndSet*** machine instruction, which is a single machine instruction that tests a memory location and simultaneously increments or decrements the location if the test succeeds.

Assuming that such a *TestAndSet* operation returns the value of its location parameter and decrements its location parameter if it is > 0, we can implement *Signal* and *Delay* with the following code schemas:

Delay(*S*): while *TestAndSet*(*S*) <= 0 do {nothing};
Signal(*S*) : *S* : = *S* + 1;

This implementation causes a blocked process to **busy-wait** or **spin** in a while-loop until *S* becomes positive again through a call to *Signal* by another process. (Semaphores implemented this way are sometimes called **spin-locks**.) It also leaves unresolved the order in which waiting processes are reactivated: It may be random or in some order imposed by the operating system. In the worst case, a waiting process may be preempted by many incoming calls to *Delay* from new processes and never get to execute despite a sufficient number of calls to *Signal.* Such a situation is called **starvation**. Starvation is prevented by the use of a scheduling system that is **fair**—that is, guarantees that every process will execute within a finite period of time. In general, avoiding starvation is a much more difficult problem to solve than avoidance of deadlock (which itself is not trivial). For example, neither Java nor Ada automatically provide for fair scheduling, although it is possible to achieve it with some effort. (See the Notes and References.)

Modern shared-memory systems often provide facilities for semaphores that do not require busy-waiting. Semaphores are special memory locations that can be accessed by only one processor at a time, and a queue is provided for each semaphore to store the processes that are waiting for it.

13.5 Monitors

A monitor is a language construct that attempts to encapsulate the mutual exclusion and synchronization mechanisms of semaphores. The idea is that a more structured construct will reduce programming errors and improve the readability and correctness of code. Originally designed by Per Brinch-Hansen and C. A. R. Hoare in the early 1970s, it has been used in the languages Concurrent Pascal and Mesa. It is also the basis for the concurrency mechanisms of both Java and Ada95, as you will see shortly.

A **monitor** is an abstract data type mechanism with the added property of mutual exclusion. It encapsulates shared data and operations on these data. At most one process at a time can be "inside" the monitor—using any of the monitor's operations. To keep track of processes waiting to use its operations, a monitor has an associated wait queue, which is organized in some fair fashion (so that processes do not wait in the queue forever as other processes arrive)—for example, as a regular first in, first out queue.

A monitor, therefore, can be viewed as a language entity with the schematic structure shown in Figure 13.9.

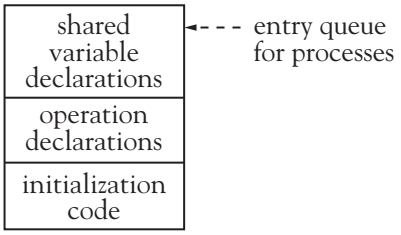


Figure 13.9 The structure of a monitor

This organization of a monitor provides for mutual exclusion in accessing shared data, but it is not adequate by itself to synchronize processes that must wait for certain conditions before continuing to execute. For example, in the bounded buffer problem, a consumer process must wait if no items are in the buffer, and a producer must wait if the buffer is full.

For this reason, a monitor must also provide **condition variables**, which are shared variables within the monitor resembling semaphores. Each has an associated queue made up of processes waiting for the condition, and each has associated **suspend** and **continue** operations, which have the effect of enqueuing and dequeuing processes from the associated queue (as well as suspending and continuing execution). Unfortunately, sometimes these operations are also called signal and delay, but their operation is different from the **Signal** and **Delay** operations of semaphores. If a condition queue is empty, a call to **continue** will have no effect, and a call to **suspend** will **always** suspend the current process.

What happens when a **continue** call is issued to a waiting process by a process in the monitor? In this situation, there are now potentially two processes active in the monitor, a situation that is forbidden. Two possibilities exist: (1) the suspended process that has just been awakened by the **continue** call must wait further until the calling process has left the monitor, or (2) the process that issued the **continue** call must suspend until the awakened process has left the monitor.

It is possible to imitate the behavior of a monitor using semaphores. (Indeed, in the last section you saw how to implement a semaphore in Java using what amounts to a monitor, and then used the semaphore to implement a bounded buffer monitor). Thus, monitors and semaphores are equivalent in terms of the kinds of parallelism they can express. However, monitors provide a more structured mechanism for concurrency than semaphores, and they ensure mutual exclusion. Monitors cannot guarantee the absence of deadlock, however. (See Exercise 13.26.)

Both Java and Ada have monitor-like mechanisms. In the next subsection, we describe how Java synchronized objects fit into the preceding description of monitors. After that, we discuss how the Java `Lock` and `Condition` interfaces, introduced in Java 1.5, provide a closer approximation to a true monitor than synchronized objects. In the final subsection of this section on Monitors, we briefly describe Ada's concurrency and monitor mechanisms, and provide an Ada bounded buffer example. Further Ada concurrency mechanisms will also be studied in Section 13.6 (message passing).

13.5.1 Java Synchronized Objects as Monitors

Java objects, all of whose methods are `synchronized`, are essentially monitors; for the purposes of this discussion, we will call such Java objects **synchronized objects**. Java provides an entry queue for each synchronized object. A thread that is inside the synchronized object (that is, that executes a synchronized method of the object) has the lock on the synchronized object. One immediate problem with these queues in Java, however, is that they do not operate in a fair fashion. No rules control which thread is chosen on a wait queue when the executing thread leaves a method (hence the tendency in Java to call `notifyAll()` instead of `notify()`.) Also, in Java an object may have both synchronized and unsynchronized methods. What's more, any of the unsynchronized methods may be executed without acquiring the lock or going through the entry queue.

Additionally, Java's synchronized objects do not have separate condition variables. There is only one wait queue per synchronized object for any and all conditions (which is of course separate from the entry queue). A thread is placed in a synchronized object's wait queue by a call to `wait` or `sleep`;

threads are removed from an object's wait queue by a call to `notify` or `notifyAll`. Thus, `wait` and `sleep` are **suspend** operations as described previously, while `notify` and `notifyAll` are **continue** operations. Note that a thread may only be placed in a synchronized object's wait queue if it has the lock on the synchronized object, and it gives up the lock at that time. When it comes off the wait queue, it must again acquire the object's lock, so that in general it must go back on the entry queue from the wait queue. Thus, awakened threads in Java must wait for the awakening thread to exit the synchronized code before continuing (and are given no priority over threads that may have just arrived at the entry queue).

13.5.2 The Java `Lock` and `Condition` Interfaces

Java 1.5 introduced the `java.concurrent.locks` package, which includes a set of interfaces and classes that support a more authentic monitor mechanism than synchronized objects. Using this new model, the programmer represents a monitor as an explicit lock object associated with a shared data resource. Methods that access the resource under mutual exclusion are now not specified as `synchronized`, but instead acquire the lock by running the `unlock` method and release it by running the `lock` method on the lock itself. After acquiring the lock, a method either finishes or waits on an explicit condition object (using the method `await`). Multiple condition objects can be associated with a single lock, and each condition object has its own queue of threads waiting on it. When a method finishes, it can signal other threads waiting on a condition object by using the method `signal` or `signalAll` with that object.

The next code segment shows modifications to the data for the bounded buffer implementation of Figure 13.6, which now includes an explicit lock and conditions:

```
import java.concurrent.locks.*;

class Buffer{

    private final char[] buf;
    private int start = -1;
    private int end = -1;
    private int size = 0;

    private Lock lock;                    // The lock for this resource
    private Condition okToGet;            // Condition with queue of
                                             waiting readers
    private Condition okToPut;            // Condition with queue of
                                             waiting writers


    public Buffer(int length){
        buf = new char[length];
        lock = new ReentrantLock();       // Instantiate the lock and its
                                             two conditions
        okToGet = lock.newCondition();
```

(continues)

(continued)

```
        okToPut = lock.newCondition();

    }

    public void put(char ch){            // Not synchronized on the Buffer
                                            object,
        ...                              // but unlocks and locks its lock
                                            instead
    }

    public char get(){
        ...
    }
}
```

Note that the methods `put` and `get` are no longer specified as `synchronized`. Instead, their first step is to acquire the buffer's lock, using the call `lock.unlock()`. The code to release the lock, `lock.unlock()`, should be placed in a `finally` clause associated with the `try-catch` statement. The method `put` then waits on the `okToPut` condition and signals the `okToGet` condition, whereas the method `get` waits on the `okToGetCondition` and signals the `okToPut` condition. The completion of the modified implementation is left as an exercise for the reader.

13.5.3 Ada95 Concurrency and Monitors

Concurrency in Ada is provided by independent processes called **tasks**, which are similar to Java threads. A task is declared using specification and body declarations similar to the package mechanism (see Chapter 11):

```
task T; -- task specification
-- see next section for more elaborate versions of this

task body T is
-- declarations
begin
-- code executed when task runs
end;
```

Unlike Java, an Ada task begins to execute as soon as the scope of its declaration is entered. It executes the code of its body in sequential fashion. When the end of the scope of the task declaration is reached, the program waits for the task to terminate before continuing execution. A task terminates by reaching the end of its code or by executing a `terminate` statement (discussed in Section 13.6). It is

also possible to declare task types and variables to get more than one task of a particular type or to get dynamic control over task execution. For example,

```
task type T is
...
end;

task body T is
...
begin
...
end T;

p,q: T;
```

declares a task type `T` and two tasks `p` and `q` of type `T`.

In addition to tasks, which are part of Ada83, Ada95 also has monitors, called **protected objects**, that correspond to the synchronized objects of Java. There is a similar separation of a protected type or object declaration into specification and body. For example, a protected queue type can be declared as follows:

```
protected type Queue is -- specification
   procedure enqueue (item: in ...);
   entry dequeue (item: out ...);
   function empty return boolean;
private:
   -- private data here
   size: natural;
   ...
end Queue;

protected body Queue is
  -- implementation of services here
end Queue;
```

As we see in this example, operations within a protected object are of three different kinds: functions, procedures, and **entries**. All three are synchronized in the Java sense, but with the following differences: Functions are not allowed to change the local state of a protected object but can be executed by any number of callers, provided that no procedures or entries are currently executing. Procedures and entries, however, can only be executed by a single caller at a time, and no functions can be executing simultaneously with a procedure or entry (this is a standard multi-reader, single-writer protocol).

The difference between a procedure and an entry in a protected object is that a procedure can always be executed (subject to the mutual exclusion rules), while an entry can only be executed under a certain condition, called the **entry barrier**. For example, the `dequeue` operation is an entry, because it can only be executed if the queue is nonempty, while an `enqueue` operation is a procedure, because it can always execute (assuming the queue size can be arbitrarily expanded). If a task calls an entry whose

barrier is closed (i.e., false), then the task is suspended and placed in a wait queue *for that entry* until the barrier becomes open (i.e., true), when a task in that entry's wait queue is reactivated. (Closed entries are reevaluated each time another task exits a procedure or entry that may have changed the value of the entry barrier.) Thus, Ada-protected object entries correspond to monitor condition variables (and associated code to execute when the condition is true).

Ada entries are similar to Java synchronized methods that call `wait` or `sleep`. The difference is that in Java there is only one wait queue per object, while in Ada there is a wait queue for each entry. Also, the Ada runtime system automatically recomputes the entry barrier at appropriate times and wakes a waiting task, so there are no `notify` or `notifyAll` calls in Ada.

As an example of an entry declaration, we can flesh out the previous `Queue` body a bit as follows:

```
protected body Queue is
   procedure enqueue (item: in ...) is
   begin
      ...
   end enqueue;

   entry dequeue (item: out ...) when size > 0 is
   begin
      ...
   end dequeue;

   function empty return boolean is
   begin
       return size > 0;
   end empty;

end Queue;
```

As a complete example of the use of protected types and tasks in Ada, Figure 13.10 gives a solution to the bounded buffer problem.

```
(1)  with Text_IO; use Text_IO;

(2)  procedure BoundedBuffer is

(3)  type StoreType is array (positive range <>) of character;

(4)  protected type Buffer (MaxBufferSize: positive) is
(5)    entry insert(ch: in character);
(6)    entry delete(ch: out character);
(7)    function more return boolean;
(8)  private
(9)    store: StoreType(1..MaxBufferSize);
```

Figure 13.10 A bounded buffer solution using Ada tasks and protected types *(continues)*

(continued)

```
(10)   bufferStart: integer := 1;
(11)   bufferEnd: integer := 0;
(12)   bufferSize: integer := 0;
(13) end Buffer;
(14) protected body Buffer is

(15)   entry insert(ch: in character)
(16)     when bufferSize < MaxBufferSize is
(17)   begin
(18)     bufferEnd := bufferEnd mod MaxBufferSize + 1;
(19)     store(bufferEnd) := ch;
(20)     bufferSize := bufferSize + 1;
(21)   end insert;

(22)   entry delete(ch: out CHARACTER)
(23)     when bufferSize > 0 is
(24)   begin
(25)     ch := store(bufferStart);
(26)     bufferStart := bufferStart mod MaxBufferSize + 1;
(27)     bufferSize := bufferSize - 1;
(28)   end delete;

(29)   function more return boolean is
(30)   begin
(31)     return bufferSize > 0;
(32)   end more;

(33) end Buffer;

(34) buf: Buffer(5); -- buffer of size 5

(35) task producer;
(36) task body producer is
(37) ch: character;
(38) begin
(39)   loop
(40)     if (end_of_file) then exit;
(41)     end if;
(42)     if (end_of_line) then
(43)       skip_line;
(44)-- use carriage return in buf to indicate new line:
(45)       buf.insert(character'(Standard.Ascii.CR));
(46)     else
(47)       get(ch);
(48)       buf.insert(ch);
```

Figure 13.10 A bounded buffer solution using Ada tasks and protected types *(continues)*

(continued)

```
(49)     end if;
(50)   end loop;
(51) end producer;

(52) task consumer;
(53) task body consumer is
(54) ch: character;
(55) begin
(56)     while (not producer'terminated or buf.more) loop
(57)       buf.delete(ch);
(58)       -- carriage return indicates new line:
(59)       if ch = character'(Standard.Ascii.CR) then
(60)               new_line;
(61)       else put(ch);
(62)       end if;
(63)     end loop;
(64) end Consumer;

(65) begin
(66)  null; -- no code needed, tasks execute automatically
(67) end BoundedBuffer;
```

Figure 13.10 A bounded buffer solution using Ada tasks and protected types

13.6 Message Passing

Message passing is a mechanism for process synchronization and communication using the distributed model of a parallel processor. It was introduced around 1970 by Brinch-Hansen and others.

In its most basic form, a message-passing mechanism in a language consists of two operations, ***send*** and ***receive***, which may be defined in C syntax as follows:

```
void send(Process to, Message m);
void receive(Process from, Message m);
```

In this form, both the sending process and the receiving process must be named. This implies that every sender must know its receiver, and vice versa. In particular, the sending and receiving processes must have names within the scope of each other. A less restrictive form of *send* and *receive* removes the requirement of naming sender and receiver:

```
void send(Message m);
void receive(Message m);
```

In this case, a sent message will go to any process willing to receive it, and a message will be received from any sender. More commonly, a message-passing mechanism will require *send* to name a receiver, but allow *receive* to receive from any process. This is asymmetrical, but it mimics the situation in a

procedure call, where only the caller must know the name of the called procedure, while the called procedure has in general no knowledge of its caller.

Other questions that must be answered about the *send* and *receive* operations revolve around the synchronization of processes that wish to communicate via *send* and *receive*:

1. Must a sender wait for a receiver to be ready before a message can be sent, or can a sender continue to execute even if there is no available receiver? If so, are messages stored in a buffer for later receipt?
2. Must a receiver wait until a message is available to be sent, or can a receiver receive a null message and continue to execute?

In the case where both sender and receiver must wait until the other is ready, the message-passing mechanism is sometimes called **rendezvous**. When messages are buffered, additional questions arise. For example, is there a size limit on the number of messages that can be buffered? And what process manages the buffer? If a separate process manages the buffer, then sometimes the buffer (or its managing process) is named in the *send* and *receive* calls, instead of the sending and receiving processes. In this case, we have a **mailbox** mechanism, where processes "drop off" and "retrieve" messages from named (or numbered) mailboxes. Sometimes mailboxes are assigned **owners** (for example, a process that creates a mailbox can become its owner). In this case, the mailbox may be managed by its owner instead of a separate process.

Essential to any message-passing mechanism are **control facilities** to permit processes to test for the existence of messages, to accept messages only on certain conditions, and to select from among several possible messages. Often these control structures are influenced by or are based on Dijkstra's **guarded if** and **guarded do** commands (see Chapter 9).

In the following, we will discuss Ada's version of message passing, which is a form of rendezvous. Other forms of message passing include Hoare's **Communicating Sequential Processes (CSP)** language framework, and its implementation in the language **occam**; mechanisms for distributing computation over a large network, such as **Remote Procedure Call (RPC)** or **Remote Method Invocation (RMI)**, which is supported by the Java networking library; and various forms of interfacing standards to facilitate the uniform sharing of code execution among different computers and operating systems, such as **MPI** (Message Passing Interface), **CORBA** (Common Object Request Broker Architecture) and **COM** (Common Object Model). See the Notes and References for information on these mechanisms, which are beyond the scope of this introductory chapter.

13.6.1 Task Rendezvous in Ada

So far you have seen Ada tasks as similar to Java threads, each executing a body of code, and communicating via protected (synchronized) code inside monitors. However, Ada tasks can also pass messages to each other via a rendezvous mechanism. Indeed, Ada83 did not have monitors, so rendezvous was the only way to share data among tasks in a synchronized way. (We will return to this issue shortly.)

Rendezvous points are defined by **task entries**, which are similar in appearance to protected type entries but are used (confusingly) in a completely different way. Entries are defined in a task's specification just as they are for protected types:

```
task userInput is
    entry buttonClick (button: out ButtonID);
    entry keyPress (ch: out character);
end userInput;
```

A task exports entry names to the outside world just as protected types do. Thus, another task can rendezvous with `userInput` by "calling" `userInput.buttonClick(b)` or `userInput.keyPress(c)`. However, entries do not have code bodies. Instead, entries must appear inside an `accept` statement, which provides the code to execute when the entry is called. `Accept` statements can only appear inside the body of a task, and their position inside the body determines when they will be executed. The caller of an entry will wait for the task to reach a corresponding `accept` statement, and, similarly, if no call has occurred, a task that reaches an `accept` statement will wait for a corresponding call. Thus, a rendezvous occurs at the point in the called task's body where the corresponding `accept` statement appears, and the message that is passed at the rendezvous is represented by the entry parameters. Each entry has an associated queue to maintain processes that are waiting for an `accept` statement to be executed. This queue is managed in a first in, first out fashion.

To continue with the previous example, the `userInput` body might, therefore, look as follows:

```
task body userInput is
begin
    ...
    -- respond to a button click request:
    accept buttonClick (button: out ButtonID) do
       -- get a button click while the caller waits
    end buttonClick; -- caller can now continue
    -- continue with further processing
    ...
    -- now respond to a keypress request:
    accept keyPress (ch: out character) do
      -- get a keypress while the caller waits
    end keyPress; – caller can now continue
    -- continue with further processing
end userInput;
```

An `accept` statement has the following form (in EBNF notation):

`accept` *entry-name* [*formal-parameter-list*]

[`do` *statement-sequence* `end` [*entry-name*]] ;

When an `accept` statement is executed, the caller remains suspended while the code in the accept body is executed (the statements between the `do` and the `end`). An `accept` statement does not need to name the caller of the entry. Thus, Ada's message-passing mechanism is asymmetric like procedure calls. The entry/accept mechanism can be used entirely for synchronization, in which case a body of code for the entry is unnecessary.

Typically, a task that operates as a server for other client tasks will want to maintain a set of possible entries that it will accept and wait for one of these entries to be called, whereupon it will process the entry and loop back to wait for the next entry. Which entries can be selected can depend on a number of conditions (such as whether a queue is empty or a buffer is full). Maintaining a set of entries to accept is done through the `select` statement. The EBNF for a `select` statement is:

```
select
[when condition =>] select-alternative
{or [when condition =>] select-alternative}
[else statement-sequence]
end select ;
```

where a *select-alternative* is an `accept` statement followed by a sequence of statements, or a `delay` statement (not described here) followed by a sequence of statements, or a `terminate` statement.

The semantics of the `select` statement are as follows. All the conditions in the `select` statement are evaluated, and those that evaluate to true have their corresponding select alternatives tagged as **open**. An open `accept` statement is selected for execution if another task has executed an entry call for its entry. If several accepts are available, one is chosen arbitrarily. If no open accepts are available and there is an `else` part, the statement sequence of the `else` part is executed. If there is no `else` part, the task waits for an entry call for one of the open accepts. If there are no open accepts, then the `else` part is executed if it exists. If there is no `else` part, an exception condition is raised (see Chapter 9).

Looping back to a set of `select` alternatives is not part of the basic select command but can easily be achieved by surrounding a `select` statement with a loop.

Continuing with the previous example, the `userInput` body might be structured as follows to allow it to behave as a server for `keyPress` and `buttonClick` requests:

```
(1)  task body userInput is
(2)  begin
(3)    loop
(4)      select
(5)        when buttonClicked =>
(6)          -- respond to a button click request:
(7)                     accept buttonClick (button: out ButtonID) do
(8)                       -- get and return a button click
(9)                       -- while the caller waits
(10)       end buttonClick; -- caller can now continue
(11)       -- do some buttonClick cleanup
(12)     or when keypressed =>
(13)       -- now respond to a keypress request:
(14)                    accept keyPress (ch: out character) do
(15)                      -- get a keypress while the caller waits
(16)                    end keyPress; -- caller can now continue
(17)       -- do some keyPress cleanup
```

(continues)

(continued)

```
(18)       or terminate;
(19)     end select;
(20)   end loop;
(21) end userInput;
```

Note the `terminate` alternative in the select statement (line 18). Termination of tasks in Ada can occur in one of two ways. Either the task executes to completion and has not created any dependent tasks (i.e., child processes) that are still executing, or the task is waiting with an open `terminate` alternative in a `select` statement, and its **master** (the block of its parent task in which it was created) has executed to completion. In that case, all the other tasks created by the same master must also have terminated or are waiting at a `terminate` alternative, in which case they all `terminate` simultaneously. This avoids the necessity of writing explicit synchronizing statements (such as `join` or `wait`) in a parent task that must wait for the completion of its children.

Of course, a task with the preceding behavior might, depending on circumstances, be better written as a protected type in Ada. For example, we can write a version of the bounded buffer (Figure 13.10) that uses a task and rendezvous to provide mutual exclusion and synchronization, even though a protected type is likely to be more appropriate. The code is in Figure 13.11. Note that, because tasks, unlike protected types, cannot have functions that return values, but must return values throughout parameters in entries, the buffer code in Figure 13.11 requires a small modification in the consumer code of Figure 13.10. We leave the details to the reader (Exercise 13.39).

We offer two additional examples of tasks in Ada. The first (Figure 13.12) is an implementation of a semaphore type as a task type. Note in that example that the code block in an `accept` of a parameterless entry such as `signal` or `wait` can be empty, in which case the `do . . . end` can be omitted. The second example (Figure 13.13) is an Ada package using tasks for parallel matrix multiplication.

```
task buf is
  entry insert(ch: in character);
  entry delete(ch: out character);
  entry more (notEmpty: out boolean);
end;
task body buf is
   MaxBufferSize: constant integer := 5;
   store: array (1..MaxBufferSize) of character;
   bufferStart: integer := 1;
   bufferEnd: integer := 0;
   bufferSize: integer := 0;
begin
   loop
     select
       when bufferSize < MaxBufferSize =>
          accept insert(ch: in character) do
             bufferEnd := bufferEnd mod MaxBufferSize + 1;
             store(bufferEnd) := ch;
```

Figure 13.11 A bounded buffer as an Ada task *(continues)*

(continued)

```
            end insert;
            bufferSize := bufferSize + 1;
         or when bufferSize > 0 =>
            accept delete(ch: out character) do
               ch := store(bufferStart);
            end delete;
            bufferStart := bufferStart mod MaxBufferSize + 1;
            bufferSize := bufferSize - 1;
         or
            accept more(notEmpty: out boolean) do
                 notEmpty := bufferSize > 0;
            end more;
         or terminate;
      end select;
   end loop;
end buf;
```

Figure 13.11 A bounded buffer as an Ada task

```
task type Semaphore is
  entry initSem (n: in integer);
  entry wait; -- use wait instead of delay, which is reserved in Ada
  entry wait;
end;
task body Semaphore is
  count : integer;
begin
  accept initSem (n: in integer) do
   count := n;
  end initSem;
  loop
   select
    when count > 0 =>
     accept wait;
     count := count - 1 ;
   or
    accept signal;
    count := count + 1;
   or
    terminate;
   end select;
  end loop;
end Semaphore;
```

Figure 13.12 A semaphore task type in Ada

```
generic Size: INTEGER;
package IntMatrices is
 type IntMatrix is array (1..Size,1..Size) OF INTEGER;
 function ParMult(a,b: in IntMatrix) return IntMatrix;
end;

package body IntMatrices is
 function ParMult(a,b: in IntMatrix) return IntMatrix is
   c:IntMatrix;

   task type Mult is
     entry DoRow (i: in INTEGER);
   end;
   task body Mult is
     iloc: INTEGER;
   begin

     accept DoRow (i: in INTEGER) do
       iloc := i;
     end;
     for j in 1..Size loop
       c(iloc,j) := 0;
       for k in 1.. Size loop
         c(iloc,j) := c(iloc,j) + a(iloc,k) * b(k,j);
       end loop;
     end loop;
  end Mult;

 begin -- ParMult
  declare m: array (1..Size) of Mult;
  begin
    for i in 1..Size loop
      m(i).DoRow(i);
    end loop;
  end;
  return c;
 end ParMult;
end IntMatrices;
```

Figure 13.13 Parallel matrix multiplication in an Ada package

13.7 Parallelism in Non-Imperative Languages

Parallel processing using functional or logic programming languages is still in an experimental and research phase, despite a significant amount of work since the mid-1980s. Nevertheless, a number of good implementations of research proposals exist. In this section we discuss several of these, including MultiLisp, QLisp, Parlog, and FGHC, after a brief general introduction to the area. Finally, we explore parallel programming with Erlang, a non-imperative language that supports a message-passing style. The reader is encouraged to consult the references at the end of the chapter for more information.

In earlier chapters, we have mentioned that non-imperative languages such as Lisp and Prolog offer more opportunities for automatic parallelization by a translator than do imperative languages. The opportunities for parallel execution in such languages fall into two basic classes: and-parallelism and or-parallelism. We'll consider and-parallelism first.

13.7.1 And-Parallelism

In and-parallelism, a number of values can be computed in parallel by child processes, while the parent process waits for the children to finish and return their values. This type of parallelism can be exploited in a functional language in the computation of arguments in a function call. For example, in Lisp, if a process executes a function call

```
(f a b c d e)
```

it can create six parallel processes to compute the values `f, a, ..., e`. It then suspends its own execution until all values are computed, and then calls the (function) value of `f` with the returned values of `a` through `e` as arguments. Similarly, in a let-binding such as:

```
(let ((a e1) (b e2) (c e3)) (...))
```

the values of `e1`, `e2`, and `e3` can be computed in parallel. In Prolog, a similar and-parallel opportunity exists in executing the clause

```
q :- p1, p2, ..., pn.
```

The `p1` through `pn` can be executed in parallel, and `q` succeeds if all the `pi` succeed. Implicit in this description is that the computations done in parallel do not interfere. In a purely functional language, the evaluation of arguments and let-bindings causes no side effects, so noninterference is guaranteed. However, most functional languages are not pure, and side effects or state changes require that and-parallelism be synchronized. In Prolog, the instantiation of variables is a typical and necessary side effect that can affect the behavior of and-parallelism. For example, in the clause

```
process(N,Data) :-
          M is N - 1,
          Data = [X|Data1],
          process(M,Data1).
```

the three goals on the right-hand side cannot in general be executed in parallel, since each of the first two contribute instantiations to the last. Thus, synchronization is also necessary here.

13.7.2 Or-Parallelism

In or-parallelism, execution of several alternatives can occur in parallel, with the first alternative to finish (or succeed) causing all other alternative processes to be ignored (and to terminate). In Lisp, an example of such parallelism can occur in the evaluation of a `cond` expression:

```
(cond (p1 e1) (p2 e2)...(pn en))
```

In this situation, it may be possible for the `ei` that correspond to true `pi` conditions to be evaluated in parallel, with the first value to be computed becoming the value of the `cond` expression. (It may also be possible to compute the `pi` themselves in parallel.) In this case, it may also be necessary to synchronize the computations. However, there is another problem as well: or-parallel computation makes the `cond` into a nondeterministic construct (similar to Dijkstra's guarded if), which may change the overall behavior of the program if the order of evaluation is significant. For example, an else-part in a `cond` should not be evaluated in parallel with the other cases.

In Prolog, or-parallelism is also possible in that a system may try to satisfy alternative clauses for a goal simultaneously. For example, if there are two or more clauses for the same predicate,

```
p(X) :- q(X).
p(X) :- r(X).
```

a system may try to satisfy `q` and `r` in parallel, with `p` succeeding with `X` instantiated according to the one that finishes first (or perhaps even saving other instantiations in a queue). In fact, this is consistent with the semantics of pure logic programming, since alternative goals are satisfied nondeterministically. However, in common Prolog implementations, the correct execution of a program may depend on the order in which alternatives are tried. In this case, a strict ordering may need to be applied. For example, in a common program for factorial in Prolog,

```
fact (X, Y) := X = 0 , ! , Y = 1.
fact (X, Y) := Z is X − 1, fact (Z, Y1), Y is X * Y1.
```

the first clause needs to be tried before the second (or reaching the cut in the first should terminate the second).

The synchronization and order problems we have mentioned for both and-parallelism and or-parallelism are difficult to solve automatically by a translator. For this reason, language designers have experimented with the inclusion of a number of manual parallel constructs in non-imperative languages. In some cases, these are traditional constructs like semaphores or mailboxes, with modified semantics to provide better integration into the language. In other cases, more language-specific means are used to indicate explicit parallelism.

There is, however, another argument for the inclusion of explicit parallelism in non-imperative languages. In many situations, one may wish to suppress the creation of small (i.e., fine-grained) processes when the computational overhead to create the process is greater than the advantage of computing the result in parallel. This may be the case, for example, in highly recursive processes, where as one approaches the base case, one may not want to create new processes but switch to ordinary computation. Or, one may wish to suppress parallel computation altogether when the values to be computed have a small computational overhead.

Next, we examine a few of the explicit parallel mechanisms employed in some of the parallel Lisps and Prologs mentioned earlier in this chapter.

13.7.3 Parallelism in Lisp

The more natural form of parallelism for Lisp seems to be and-parallelism, and this is the kind most often implemented. In the language Multilisp, which like Scheme (see Chapter 3) has static scoping and first-class function values, parallel evaluation of function calls is indicated by the syntax

```
(pcall f a b c...)
```

which is equivalent to the evaluation of `( f a b c...)` but with parallel evaluation of its subexpressions. Even more parallelism can be achieved in Multilisp by the use of a "future:" If `f` has already been computed in the function call `(f a b c)`, then the execution of `f` can proceed even before the values of `a, b,` and `c` have been computed, at least up to the point in `f` where those values are used. For example, in an `f` defined by:

```
(define (f a b) (cond ((= a 0)...)
                      ((> b 0)...)...))
```

the value of `b` is never used if `a` evaluates to 0, so the computation can proceed regardless of how long it takes to compute `b`.

A **future** is a construct that returns a pointer to the value of a not-yet-finished parallel computation. A future resembles (but is not identical to) delayed evaluation, as represented in the `delay` primitive of Scheme, studied in Section 10.5. In Multilisp a call

```
(pcall f (future a) (future b)...)
```

allows the execution of `f` to proceed before the values of `a` and `b` have been computed. When the execution of `f` reaches a point where the value of `a` is needed, it suspends execution until that value is available. Futures have been included in other parallel Lisps besides Multilisp—for example, Qlisp.

Qlisp includes a parallel let-construct called `qlet` to indicate and-parallelism in let-bindings:

```
(qlet p (bindings) exp)
```

This construct will evaluate the bindings in parallel under the control of the predicate `p`. If `p` evaluates to `nil`, ordinary evaluation occurs. If `p` evaluates to a non-nil value, then the bindings will be computed in parallel. If `p` evaluates to the special keyword `:eager`, then futures will be constructed for the values to be bound.

13.7.4 Parallelism in Prolog

The more natural form of parallelism for Prolog appears to be or-parallelism, although both forms have been implemented. In fact, or-paralellism fits so well with the semantics of logic programming (as noted above) that it is often performed automatically by a parallel Prolog system.

And-parallelism is more difficult in Prolog because of the interactions of instantiations mentioned earlier, and because there is no natural return point for backtracking. One version of and-parallelism uses guarded Horn clauses to eliminate backtracking. A guarded Horn clause is one of the form:

```
h : - g1, ..., gn | p1, ..., pm.
```

The g1, . . . , gn are **guards**, which are executed first and which are prohibited from establishing instantiations not already provided. If the guards succeed, the system **commits** to this clause for h (no backtracking to other clauses occurs), and the p1, . . . , pm are executed in parallel. Such a system is FGHC (for flat guarded Horn clauses). A sample FGHC program is the following, which generates lists of integers:

```
generate(N,X) :- N = 0 | X = [].
generate(N,X) :- N > 0 | X = [N|T], M is N-1,
                 generate(M,T).
```

The problem with FGHC is that it places severe constraints on variable instantiation and, hence, on unification. This results in a significant reduction in the expressiveness of the language. An alternative is to provide **variable annotations** that specify the kind of instantiations allowed and when they may take place. A language that does this is **Parlog**. In Chapters 3, 4, and 10 we discussed the difference between input variables and output variables of a procedure: Input variables are like value parameters; that is, they have incoming values but no outgoing values. Output variables, on the other hand, have only outgoing values. In Prolog this means that input variables may be instantiated when initiating a goal, but may not be instantiated during the process of satisfying the goal, and similarly for output variables. Parlog distinguishes input variables from output variables in a so-called **mode declaration** by writing input variables with a "?" and output variables with a "^." If during the process of satisfying a goal, an uninstantiated input variable must be unified, the process suspends until such time as the variable becomes instantiated. A further tool for controlling instantiations is "directional" unification: The goal X = Y has a variant X <= Y, which only allows X to be instantiated, not Y.

As an example of and-parallelism in Parlog, consider the following quicksort program:

```
mode qsort (P?,S^).
qsort([],[]).
qsort([H|T] S) :- partition(H,T,L,R),
                                qsort(L,L1),
                                qsort(R,R1),
                                append(L1,[ H|R],S).
mode partition(P?,Q?,R^,S^).
partition(P,[A|X],[A|Y],Z) :- A < P:
                                        partition(P,X,Y,Z).
partition(P,[A|X],Y,[A|Z]):- A >= P:
                                        partition(P,X,Y,Z).
partition(P,[], [], []).
```

Parlog selects an alternative from among clauses by the usual unification process and also by using guards. It then commits to one alternative. In the foregoing qsort, the partition predicate has guards to prevent incorrect choices. After selecting a clause, Parlog creates a process for each goal on the right-hand side. Thus, Parlog will execute the four goals on the right-hand side of qsort in parallel. Since L and R are input variables to the two right-hand calls to qsort, their associated processes will suspend as soon as these are needed for unification, until the first partition process produces them. The append process (whose definition is not shown) will also suspend until its first two arguments become available.

13.7.5 Parallelism with Message Passing in Erlang

Erlang is a functional language developed by Joe Armstrong at Ericsson, the telecommunications company, in 1986. Armstrong was assigned to develop software for the kinds of distributed, fault-tolerant, real-time applications needed in the telecommunications industry. Like Lisp, Erlang uses strict evaluation and dynamic typing. The language has a syntax similar to that of Haskell (Chapter 3), with powerful pattern-matching capability. However, Erlang extends this capability with variable binding via single assignment, and distinguishes between atoms and variables, as Prolog distinguishes between constant terms and logical variables. Finally, Erlang supports concurrency and parallelism based on message passing. The absence of side effects and the presence of message passing combine in Erlang to produce a very simple model of parallel programming. The examples of Erlang code that follow are adapted from Armstrong [2007], which provides an excellent introduction to the language.

Like the other functional languages we have studied, an Erlang implementation includes an interactive interpreter and a compiler. The following session with the Erlang interpreter shows the use of pattern matching with variables (capitalized names), numbers, constant terms (lowercase names), and tuples (items enclosed in `{}`):

```
1> Rectangle = {rectangle, 8, 4}
{rectangle, 8, 4}
2> Circle = {circle, 3.5}
{circle, 3.5}
3> {rectangle, Width, Height} = Rectangle.
{rectangle, 8, 4}
4> Width.
8
5> {circle, Radius} = Circle.
{circle, 3.5}
6> Radius.
3.5
```

In this code, rectangles and circles are represented as tuples containing numeric information and a constant term that serves as a type tag. Single assignment with the = operator allows a variable to be bound to a value just once but then be referenced multiple times thereafter. The assignments at prompts 3 and 5 illustrate pattern matching. The variables nested in the tuples on the left side of the = operator are matched and bound to the corresponding values in the structures referenced by the variables to the right of this operator.

Pattern matching and variable binding also form the basis of function applications in Erlang. The next code segment shows a `geometry` module containing the definition of an `area` function for the representations of rectangles and circles described earlier. This code is saved in the file `geometry.erl`.

```
-module(geometry).
-export([area/1].

area({rectangle, Width, Height}) -> Width * Height;
area({circle, Radius}) -> 3.14159 * Radius * Radius.
```

Note that the `area` function consists of two clauses, one for rectangles and the other for circles. When the programmer applies the `area` function to a value, the interpreter matches this value to the header of the appropriate clause of the `area` function as defined in the `geometry` module. Note that the form of the argument to the function must match the header of one of the two clauses specified in the module, or an error is raised. When an argument matches a function clause, the variables in the header of this clause are bound to the corresponding values contained in the argument. The clause's code following the `->` symbol is then executed in the context of these bindings. Here is an example session that compiles and tests the `geometry` module:

```
1> c(geometry).
{ok, geometry}
2> geometry:area({rectangle, 8, 4}).
32
3> geometry:area({circle, 3.5}).
38.4844775
```

Before we examine how Erlang can execute this code in parallel with other code, we need to see how functions can be passed as arguments to other functions. Mapping, which we have seen before in other functional languages, illustrates the use of function arguments. Erlang's standard `lists` module includes a `map` function for mapping other functions onto lists of arguments. A function argument to `lists:map` (or to any other function) must be a special type of Erlang function called a `fun`. The following session shows the use of `fun` in three applications of `lists:map`:

```
1> lists:map(fun(N) -> 2 * N end, [2, 4, 6]).
[4, 8, 12]
2> Double = fun(N) -> 2 * N end.
#Fun<erl_eval.6.57006448>
3> lists:map(Double, [2, 4, 6]).
[4, 8, 12]
4> lists:map(fun math:sqrt/0, [4, 9, 16]).
[2, 3, 4]
```

At prompt 1, `fun` is used, like `lambda` in Lisp, to build an anonymous function to pass as an argument to `lists:map`. At prompt 2, the variable `Double` is bound to a `fun`. This variable is then passed as an argument to `lists:map` at prompt 3. At prompt 4, the standard `math` function `sqrt` is wrapped in a `fun` before it is passed as an argument to `lists:map` at prompt 4.

Armed with these basic ideas, we are ready to see how Erlang supports parallel programming. An Erlang process is a very lightweight software object that runs independently of any other process and communicates with other processes by sending and receiving messages. The essential operations on processes are **spawn**, which creates a new process; **send**, for sending messages to other processes; and **receive**, for receiving messages. Table 13.1 describes the syntax and semantics of these operations in Erlang.

Table 13.1 The essential process operations in Erlang

Operation	What It Does
`Pid = spawn(Fun)`	Creates a new process to execute the function `Fun` in parallel with the caller and returns the process identifier for this process. `Pid` can then be used to send messages to the new process.
`Pid ! message`	Sends `message` (a pattern) to the process identified by `Pid`. The sender does not wait, but continues its execution.

`receive`
`  Pattern1 [ when Guard1 ] ->`
`  Expressions1`
`  Pattern2 [ when Guard2 ] ->`
`  Expressions2`
`  . . .`
`end`
	Receives a message (a pattern). If the message matches one of the given patterns, the corresponding code is executed. Otherwise, the message is saved for later processing.

Let's return to the example of the `area` function, defined earlier in a `geometry` module. In a parallel computing context, we can view this function as a **server** that executes in its own process. Another process, called a **client**, makes a request of this server for the area of a given geometric shape. The server handles this request by computing the area and returning it to the client.

The client process, which in our new example is just the interactive interpreter's process, begins by spawning a server process. When the server process is spawned, it runs a function that executes an infinite loop. This loop receives a message from a client, computes an area, and sends the result back to that client. In the new implementation, the `area` function is updated to handle the transaction between client and server. This function now expects the server's process identifier as well as the shape pattern as arguments. The next session demonstrates an interaction between the user/client and the area server.

```
1> c(area_server).
{ok, area_server}
2> Pid = spawn(fun area_server:loop/0).
<0.35.0>
3> area_server:area(Pid, {rectangle, 8, 4}).
32
4> area_server:area(Pid, {circle, 3.5}).
38.4844775
```

At prompt 1, the module of function definitions, called `area_server`, is compiled. At prompt 2, a server process is spawned. It is passed the `loop` function of the `area_server` module, suitably wrapped in a `fun`. The `loop` function, ready to receive and process messages, begins concurrent execution immediately in the new process, and the variable `Pid` is bound to the server's process identifier. At prompts 3 and 4, the new `area` function is called, with the server's process identifier and some shape patterns as arguments. Behind the scenes, the server process receives messages from this function, which returns the results of the server's computations.

Here is the code for the module of server functions, followed by an explanation.

```
-module(area_server).
-export([area/2, loop/0].

area(Pid, shape) ->
   Pid ! {self(), shape},
   receive
      {Pid, Result} ->
         Result
   end.

loop() ->
   receive
      {From, {rectangle, Width, Height}} ->
         From ! {self(), Width * Height},
         loop();
      {From, {circle, Radius}} ->
         From ! {self(), 3.14159 * Radius * Radius},
         loop();
      {From, Other} ->
         From ! {self(), error, Other},
         loop()
   end.
```

As mentioned earlier, the `area` function now expects the server's process identifier and the shape's pattern as arguments. This function creates a new pattern containing the shape and the client's own process identifier (obtained by a call to `self()`). The server will use the client's process identifier to send its result back to the client. This pattern is then sent as a message to the server (`Pid`). The `area` function then receives the result from the server. Note that this result is extracted from the pattern `{Pid, Result}`. This notation guarantees that the client receives, by a successful match, only a message sent by this particular server, rather than another message that might be sent by another process at this point.

The `loop` function handles requests from the client. This function is passed as an argument to the `spawn` function to start up the server's process (essentially telling the process to execute this function). Note that the `loop` function contains a single `receive` form, which in turn contains three guards for matching patterns sent as messages from the client. The variable `From` in each pattern is bound to the client's process identifier and is used to send the result back to that client. The first guard handles the case of rectangles, the second guard deals with circles, and the last guard recovers from the case of an unrecognized shape pattern. In each case, the message sent back to the client (`From`) includes the server's process identifier (`self()`), so that the client will receive only a message sent by this server. Finally, note that in each case, after the result is sent back to the client, the `loop` function executes a recursive call as its last logical step. This has the effect of continuing the infinite loop and also of freeing the server to receive other messages. Because the tail-recursive `loop` function compiles to a genuine loop, it can run forever without stack overflow.

Interactions between processes often are not as tightly coupled as in the area server example. On the one hand, a process may hand off work to other processes and go its merry way without waiting for a response. On the other hand, a process may wait longer than is desirable to receive a message, because

the sender has crashed or an error has occurred in the sender's algorithm. For this case, the receiver process can include an `after` clause to time out the `receive` operation. Its form is:

```
receive
   Pattern1 [ when Guard1 ] ->
      Expressions1
   Pattern2 [ when Guard2 ] ->
      Expressions2
   . . .
   after Time ->
      Expressions
end
```

The `Time` variable stands for the maximum number of milliseconds that the `receive` operation must wait for a message, after which control proceeds to the `Expressions`, which are executed. This feature can be used to define a `sleep` function, which essentially suspends the calling process for a given amount of time:

```
sleep(Time) ->
   receive
   after Time ->
      true
   end
```

The Erlang programmer also must be aware of the manner in which messages are scheduled for a process. Each process is associated with a **mailbox**, a queue-like data structure. When another process sends a message, it goes at the end of the receiver process's mailbox. The process's `receive` operation attempts to match the message at the front of its mailbox to one of its patterns. If a match occurs, the message is removed from the mailbox, discarded, and the expressions in the clause are executed. If no match occurs, the message is removed from the mailbox and put at the rear of the process's **save queue**. The next message in the mailbox is then tried, and so forth, until the mailbox becomes empty. If that happens, the process itself is suspended, until a new message arrives in the mailbox, at which point, the process itself is rescheduled for execution to examine this message. If a message is matched, all of the messages in the saved queue are transferred to the mailbox in their original order. The same transfer of messages from saved queue to mailbox is carried out if a timeout due to an `after` clause occurs.

Our final example defines a function `flush_mailbox`, which empties the mailbox of a process.

```
flush_mailbox() ->
   receive
      _Any ->
         flush_ mailbox()
   after 0 ->
      true
   end
```

In this code, the variable `_Any` matches any message/pattern in the mailbox, so the recursive calls of `flush_mailbox` continue until there are no more messages in the box. At this point, the `after` clause immediately times out the `receive` form, so that the process does not suspend forever.

Erlang also includes an array of resources for developing **distributed applications** that run on a network of independent computers. Independent processes are embedded in software objects called **nodes**, which allow messages to be sent and received across a network of machines. The reader is encouraged to consult the Armstrong text [2007] for further information.

Exercises

13.1 How many links does a fully linked distributed system with n processors need? How do you think this affects the design of fully linked systems with many processors?

13.2 Some parallel processors limit the number of processes that a user can create in a program to one less than the number of processors available. Why do you think this restriction is made? Is such a restriction more or less likely on an SIMD or MIMD system? Might there be a reason to create more processes than processors? Explain.

13.3 It is possible to compute the sum of n integers in fewer than n steps. Describe how you could use k processors to do this, where $k < n$. Would there be any advantage to having $k > n$?

13.4 The text mentioned that matrix multiplication can be done in n steps using $n2$ processors. Can it be done in fewer steps using more processors?

13.5 Describe the difference between SPMD and MPMD programming. Would you characterize the program of Figure 13.3 as SPMD or MPMD? Why?

13.6 In the chapter discussion, why did we use the term "MPMD" instead of "fork-join"?

13.7 Explain why both large-grained and small-grained parallelism can be less efficient than medium grained. Give programming examples to support your argument.

13.8 The C code of Figure 13.4 assumes that `NUMPROCS` is less than `SIZE`. What happens if `NUMPROCS > SIZE`? Rewrite the code of Figure 13.4 to take advantage of the extra processors if `NUMPROCS > SIZE`.

13.9 Explain what happens in each step of the following UNIX pipeline:

```
cat *.java | tr -sc A-Za-z '\012' | sort | uniq -c | sort -rn
```

13.10 Command-line windows in versions of Microsoft Windows also allow certain commands to be piped, such as `dir | more`. Determine if these pipes are true pipes in the UNIX sense (that is, each program in the pipe is a separate process, with no intermediate files saved to disk).

13.11 A typical process synchronization problem is that of resource allocation. For example, if a system has three printers, then at most three processes can be scheduled to print simultaneously. Write a program to allocate three printers to processes in

(a) Java

(b) Ada95

(c) Ada83

13.12 A standard problem (like the bounded buffer problem) that is used to test a concurrent language mechanism is the **readers-writers** problem. In this problem, stored data are continuously accessed (read) and updated (written) by a number of processes. A solution to this problem must allow access to the data by any number of readers, but by only one writer at a time. Write a solution to this problem using **(a)** Java; **(b)** Ada tasks.

13.13 The Java specification promises that getting the value of a single variable of any built-in type *except* `long` or `double`, or updating such a variable, is an atomic operation, but that such operations on `long` or `double` variables may not be atomic.

(a) Explain the reason for this difference.

(b) In what way did this fact affect the code for the bounded buffer problem (Figure 13.6)?

(c) Does this fact have any effect on the solution to the readers-writers problem? Explain.

13.14 Another standard concurrency problem is the **dining philosophers problem** of Dijkstra. In this problem, five philosophers spend their time eating and thinking. When a philosopher is ready to eat, she sits at one of five places at a table heaped with spaghetti. Unfortunately, there are only five forks, one between every two plates, and a philosopher needs two forks to eat. A philosopher will attempt to pick up the two forks next to her, and if she succeeds, she will eat for a while, leave the table, and go back to thinking. For this problem,

(a) Describe how deadlock and starvation can occur.

(b) Why are there five philosophers and not four or three?

(c) Write a deadlock-free solution to this problem in either Java or Ada.

(d) Is it possible to guarantee no starvation in your solution to part (c)? Explain.

13.15 The C code for matrix multiplication in Figure 13.5 doesn't work in a typical UNIX implementation, since a fork does not cause memory to be shared. This problem can be solved using files, since forked processes continue to share files. Rewrite the code to compute the matrix `c` correctly using files. Comment on the efficiency of your solution.

13.16 Can Figure 13.5 be corrected by using pointer variables to share the addresses of the arrays among several processes? Explain why or why not.

13.17 **(a)** Write a solution to the matrix multiplication problem in Java using one thread for each row and column of the answer.

(b) Write a solution to the matrix multiplication problem in Java that uses a fixed number of threads less than the number of elements in the answer matrix, where each thread looks for the next element in the answer matrix that has not yet been worked on.

13.18 Repeat the previous exercise in Ada.

13.19 Given two processes p and q, and a semaphore S initialized to 1, suppose p and q make the following calls:

p: *delay*(S);
. . .
delay(S);
q: *delay*(S);
. . .
signal(S);

Describe what will happen for all possible orders of these operations on S.

13.20 Describe how the busy-wait implementation of a semaphore can cause starvation.

13.21 In addition to deadlock and starvation, concurrent processes can experience **livelock**, where no process is blocked, all processes get to execute, but each continually repeats the same code without possibility of success. Write a simple program in **(a)** Java or **(b)** Ada that is an example of livelock.

13.22 Here is an alternative to the code for the semaphore operations:

Delay(*S*) : *S* : = *S* – 1;
if *S* < 0 then suspend the calling process;
Signal(*S*): *S* : = *S* + 1;
if *S* >= 0 then wake up a waiting process;

Compare this to the code in Section 13.4. Is there any difference in behavior?

13.23 Does it make any sense to initialize a semaphore to a negative value? Why or why not?

13.24 Sometimes a language will provide only a **binary semaphore** mechanism, in which the stored value is Boolean instead of integer. Then, the operations wait and signal become

Delay(*S*) : if *S* = true then *S* := false
else suspend the calling process
Signal(*S*): if processes are waiting then wake up a process
else *S* : = true;

(To distinguish them from binary semaphores, the semaphores described in the text are sometimes called **counting semaphores**.) Show how a counting semaphore can be implemented using binary semaphores.

13.25 A lock is sometimes distinguished from a semaphore by being nonblocking: Only one process can acquire a lock at a time, but if a process fails to acquire a lock, it can continue execution. Suppose that a lock L has two operations, a Boolean function **lock-lock(L)** that returns true if the lock has been acquired and false otherwise, and **unlock-lock(L)**, which unlocks the lock (having no effect if the lock is already unlocked). Write an implementation for a lock in:

(a) Java

(b) Ada

13.26 If one monitor entry calls another monitor entry, deadlock may result.

(a) Construct an example to show how this can happen.

(b) How is this problem dealt with in Java?

(c) In Ada?

13.27 Suppose that the code for the signal operation in the Java implementation of a semaphore (Figure 13.7) were changed to:

```
public synchronized void signal(){
    notify();
    count++;
}
```

Would this work? Explain.

13.28 The Java semaphore code in Figure 13.7 is overly simple, because of the possibility of interrupts. Here is an improved version for the `delay()` procedure:

```
// adapted from Lea [2000], p. 267

public void delay() throws InterruptedException{
    if (Thread.interrupted())
        throw new InterruptedException();
    synchronized(this){
        try{
            while (count <= 0) wait();
            count--;
        }catch(InterruptedException ie){
            notify(); throw ie;
        }
    }
}
```

Explain carefully the reason for each of the differences between this code and the code of Figure 13.7.

13.29 Explain the behavior of the Java code for the bounded buffer problem if the keyword `synchronized` is removed from the `put` operation (line 28 of Figure 13.6).

13.30 In Ada, you saw that the bounded buffer could be represented either by task or a monitor. Is one of these solutions preferred over the other? Why?

13.31 A shared-memory system can imitate the mailbox version of message passing by defining a mailbox utility using mutual exclusion. Write a package to implement mailboxes (i.e., queues of data with *send* and *receive* primitives) in:

(a) Java

(b) Ada

13.32 In Ada, the caller of an entry must suspend execution during the execution of the corresponding accept-statement. Why is this necessary?

13.33 In an Ada select alternative, an accept-statement can be followed by more statements. For example, in the bounded buffer solution in Ada, we wrote:

```
accept insert (ch: in character) do
    bufferEnd := bufferEnd mod MaxBufferSize + 1;
    store (bufferEnd) := ch;
end insert;
bufferSize := bufferSize + 1;
```

Note that `bufferSize` is incremented outside the `accept` statement. Is there a reason for this? Could the statement be put inside the `accept`? Could the statement `store(bufferEnd) := ch` be moved out of the `accept`? Why?

13.34 In Ada, a task type can be used in the declaration of other types. In particular, pointer (or access) types to task types can be declared, as in:

```
task type T is...end;
type A is access T;
...
x: A;
```

The variable `x` now represents a pointer to a task. When does the associated task begin executing? When might it terminate?

13.35 In Ada, a task is not automatically terminated when the end of the scope of its declaration is reached in its parent task. Instead, the parent suspends until the task completes. Why do you think this choice was made in the design of Ada?

13.36 In the Ada implementation of a semaphore, the `Signal` entry always incremented the semaphore value. Is this correct? Why?

13.37 An alternative for the Ada implementation of semaphores is to test for the existence of a waiting task using the attribute `COUNT`, which is predefined for entries. The code inside the loop would then read as follows:

```
select
    when count > 0 =>
        accept Wait;
        count := count - 1;
    or
        accept Signal;
        if Wait'COUNT > 0 then
            accept Wait;
        else
            count := count + 1;
        end if;
    or
       terminate;
end select;
```

Is this implementation preferable to the one given? Why or why not?

13.38 Write an implementation of a semaphore as an Ada protected type, and compare this with the task implementation of Figure 13.10.

13.39 Rewrite the Ada bounded buffer code of Figure 13.10 to use the buffer task of Figure 13.11.

13.40 In Figure 13.13 (parallel matrix multiplication in Ada), we used a local variable `iloc` inside task `Mult` to store the current row index of matrix multiplication for use outside the accept-statement. We could have avoided the need for `iloc` by writing the whole body of `Mult` inside the `accept` statement, as follows:

```
task body Mult is
    begin
        accept DoRow (i: in INTEGER ) do
            for j in 1..Size loop
                c(i, j) := 0;
                for k in 1..Size loop
                    c(i, j) := c(i, j) + a(i, k)*b(k, j);
                end loop;
            end loop;
    end;
end Mult;
```

What is wrong with this solution?

13.41 This chapter ignored the question of formal semantics for concurrent programs. (Some studies are mentioned in the references.) Can you think of problems that may be encountered in applying axiomatic semantics to concurrent programs? What about denotational semantics?

13.42 The matrix multiplication solution in Ada of Figure 13.13 uses the size of the matrix to determine the number of tasks. Rewrite the solution to use a number of tasks specified as an input parameter to the `ParMult` function.

13.43 Or-parallelism in Prolog can be viewed as a parallel search of the tree of alternatives (see Chapter 4), where at each node a new process is created to search each child. Describe how these processes can coordinate their activities to allow backtracking.

13.44 Write a procedure to perform a parallel search of an unordered binary tree in:
(a) Java
(b) Ada

13.45 Discuss the claim made in the text that or-parallelism is more natural for Prolog, whereas and-parallelism is more natural for Lisp.

13.46 Describe in detail the way or-parallel Prolog computes the three solutions to `delete([1,2,3],X,T)` in parallel. What time savings would you expect (in terms of numbers of steps)?

13.47 How much actual parallelism is performed in the and-parallel Prolog example of `generate` on page 625? Explain.

13.48 In and-parallel Prolog, if a guard encounters an uninstantiated variable, execution is suspended until the variable is instantiated (by another process). Why is this necessary? Are there any problems with this requirement?

13.49 Explain why backtracking is difficult in and-parallel Prolog.

13.50 In Figures 13.4 and 13.5, the main process creates new child processes to compute the matrix product and then suspends until all the child processes have finished. This is wasteful. Rewrite these programs so that the main process also performs some computations in parallel with its children.

13.51 Compare the notion of mailbox to that of a buffer.

13.52 Suppose that two processes both attempt to increment a shared variable *S* that is unprotected by a mutual exclusion mechanism. Suppose the assignment

$$S := S + 1$$

consists within each process of three machine instructions:

Load *S* into a reg
Increment the *reg*
Store the *reg* to *S*

What values might *S* have after both processes complete their increment operations? Show how each value can be obtained.

Notes and References

The field of parallel/concurrent programming is a huge one, involving operating system, architecture, and language issues. In this chapter, we have barely touched on specific language issues, using primarily Java and Ada as our examples. Typically, an entire course is devoted to parallel programming, often using one of the language-independent protocols supported by third-party libraries. Two examples of useful texts in this area are Andrews [2000] and Wilkinson and Allen [1997].

Surveys of parallel programming languages include the September 1989 issue of ACM Computing Surveys (Vol. 21, No. 3) and the July 1989 issue of IEEE Software. Some individual articles from these issues are also mentioned in the following. Another survey article is Andrews and Schneider [1983].

Programming using Java threads is studied in Lea [2000], Horstmann and Cornell [1999] (Vol. II), and Arnold et al. [2000]. For an alternative view of threads, see Butenhof [1997], which describes the Pthreads standard for UNIX. Programming using Ada tasks and Ada95 protected types is studied in Cohen [1996]; see also Wegner and Smolka [1983].

A survey of parallel architectures appears in Duncan [1990]. The diagrams in Section 13.1 are adapted from Karp [1987], where several methods of adding parallelism to FORTRAN are also studied. See also Karp and Babb [1988] for examples similar to those of Section 13.2.

Semaphores were introduced by Dijkstra [1968b]. See Stallings [2000] for a study of their use in operating systems. Monitors were introduced by Hoare [1974] and made part of the Concurrent Pascal language designed by Brinch-Hansen [1975].

The question of scheduling policies, fairness, and starvation were only mentioned briefly in this chapter, and the associated issue of process priorities was not discussed at all. For a discussion of the Java approach, see Lea [2000], especially Sections 3.4.1 and 3.7. For a discussion of the Ada95 approach, see Cohen [1996], Section 18.6.

Message passing, including a theoretical framework called Communicating Sequential Processes, or CSP, was introduced by Hoare [1978]. CSP is often used as a basis for formal semantics and verification of concurrent programs; see Schneider [2000]. CSP was also the inspiration for the programming language **Occam**, designed to run on a proprietary parallel system called the **transputer**; see Jones and Goldsmith [1988]. For a different approach to formal semantics of concurrent programs using a technique called **process algebra**, see Milner [1994].

A description of Multilisp is in Halstead [1985]. QLisp is described in Goldman and Gabriel [1989]. Surveys of parallel logic programming include Kergommeaux and Codognet [1994], Ciancarini [1992], Shapiro [1989], and Tick [1991]. Parlog is studied in Conlon [1989], Gregory [1987], and Ringwood [1988]. FGHC is described in Ueda [1987]. Concurrency and constraints in logic programming (Chapter 4) are closely related; see Ueda [2000] or Saraswat and Van Hentenryck [1995].

Information on various protocols for distributed parallel computing on networks can be found in Horstmann and Cornell [1999], Vol. II (Java's RMI), and Pacheco [1996] (MPI). Books on CORBA and COM are too numerous to mention, since these protocols are used for program interoperability as well as concurrency. A brief introduction can be found in Horstmann and Cornell [1999].

Several projects have used the Internet as a large parallel processing system; perhaps the best known is the Great Internet Mersenne Prime Search Project (see *www.mersenne.org*), which currently holds the record for the largest known prime number.

Erlang is described in Armstrong [2007] and [2010].

Abelson et al. 1998. "Revised Report on the Algorithmic Language Scheme," *Higher-Order and Symbolic Computation* 11(1): 5–105, August 1998. (Also known as R^{5}RS.)
Abelson, H. and G. J. Sussman with Julie Sussman. 1996. *Structure and Interpretation of Computer Programs* (2nd ed.). Cambridge, MA: MIT Press.
Aggoun, A. and N. Beldiceanu. 1991. "Overview of the CHIP Compiler System," in K. Furukawa (Ed.) *Logic Programming, Proceedings of the Eighth International Conference, Paris, France*, Cambridge: MIT Press, pp. 775–789.
Aho, A. V., J. E. Hopcroft, and J. D. Ullman. 1983. *Data Structures and Algorithms*. Reading, MA.: Addison-Wesley.
Aho, A. V., B. W. Kernighan, and P. J. Weinberger. 1988. *The AWK Programming Language*. Reading, MA: Addison-Wesley.
Aho, A. V., R. Sethi, and J. D. Ullman. 1986. *Compilers: Principles, Techniques, and Tools*. Reading, MA: Addison-Wesley.
Alexandrescu, A. 2001. *Modern C++ Design: Generic Programming and Design Patterns Applied*. Reading, MA: Addison-Wesley.
Andrews, G. R. 2000. *Foundations of Multithreaded, Parallel, and Distributed Programming*. Reading, MA: Addison-Wesley.
Andrews, G. R. and F. B. Schneider. 1983. "Concepts and Notations for Concurrent Programming." *ACM Computing Surveys* 15(1): 3–43.
ANSI-1815A. 1983. *Military Standard: Ada Programming Language*. Washington, DC: American National Standards Institute.
ANSI-X3.226. 1994. *Information Technology—Programming Language—Common Lisp*. Washington, DC: American National Standards Institute.
Antoniou, G. and M. Williams. 1997. *Nonmonotonic Reasoning*. Cambridge, MA: MIT Press.
Antoy, S. and Hanus, M. 2010. "Functional Logic Programming," *Communications of the ACM*, (53)4, pp. 74–85.
Appel, A. 1992. *Compiling with Continuations*. Cambridge, UK: Cambridge University Press.
Appel, A. and D. MacQueen. 1994. "Separate Compilation for Standard ML," *SIGPLAN Conference on Programming Language Design and Implementation*.
Apt, K. R., V. W. Marek, M. Truszczynski, and D. S. Warren (Eds.). 1999. *The Logic Programming Paradigm: A 25-Year Perspective*. New York: Springer-Verlag.
Armstrong, J. [2010] "Erlang," *Communications of the ACM*, (53)9: 68–75.
Armstrong, J. [2007] *Programming Erlang: Software for a Concurrent World*. Raleigh, NC: Pragmatic Bookshelf.
Arnold, K., J. Gosling, and D. Holmes. 2000. *The Java Programming Language* (3rd ed.). Reading, MA: Addison-Wesley.
Ashley, R. 1980. *Structured COBOL: A Self-Teaching Guide*. Hoboken, NJ: John Wiley & Sons.
Baase, S. 1988. *Computer Algorithms: Introduction to Design and Analysis* (2nd ed.). Reading, MA: Addison-Wesley.
Backus, J. W. 1981. "The History of FORTRAN I, II, and III." In Wexelblat [1981], pp. 25–45.
Backus, J. W. 1978. "Can Programming Be Liberated from the von Neumann Style? A Functional Style and Its Algebra of Programs." *Comm. ACM* 21(8): 613–641.
Backus, J. W. et al. 1957. "The FORTRAN Automatic Coding System." *Proceedings of the Western Joint Computing Conference*, pp. 188–198. Reprinted in Rosen [1967], pp. 29–47.
Barendregt, H. 1984. *The Lambda Calculus: Its Syntax and Semantics* (Revised Edition). Amsterdam: North-Holland.
Barnes, J. [2006] *Programming in Ada 2005*. London: Pearson Education Limited.
Barnes, J. G. P. 1980. "An overview of Ada." *Software Practice and Experience* 10, 851–887. Also reprinted in Horowitz [1987].
Barron, D. W. 1977. *An Introduction to the Study of Programming Languages*. Cambridge, UK: Cambridge University Press.
Bergin, T. J. and R. G. Gibson. 1996. *History of Programming Languages—II*. New York: ACM Press and Reading, MA: Addison-Wesley.
Bird, R., T. E. Scruggs, and M. A. Mastropieri. 1998. *Introduction to Functional Programming*. Englewood Cliffs, NJ: Prentice-Hall.
Birnes, W. J. (Ed.). 1989. *High-Level Languages and Software Applications*. New York: McGraw-Hill.
Birtwistle, G. M., O.-J. Dahl, B. Myhrhaug, and K. Nygaard. 1973. *Simula Begin*. Philadelphia: Auerbach.
Bishop, J. 1986. *Data Abstraction in Programming Languages*. Reading, MA: Addison-Wesley.
Björner, B. and O. N. Oest (Eds.). 1981. *Towards a Formal Description of Ada*, New York: Springer Verlag.
Black, D. 2009. *The Well-Grounded Rubyist*. Manning Publications.
Bobrow, D. G., L. G. DeMichiel, R. P. Gabriel, S. Keene, G. Kiczales, and D. A. Moon. 1988. "The Common Lisp Object System Specification." *ACM SIGPLAN Notices* 23(9) (special issue).
Bobrow, D. G. and M. Stefik. 1983. *The LOOPS Manual*. Palo Alto, CA: Xerox Palo Alto Research Center.
Böhm, C. and G. Jacopini. 1966. "Flow Diagrams, Turing Machines and Languages with Only Two Formation Rules." *Comm. ACM* 29(6), 471–483.
Booch, G. 1994. *Object-Oriented Analysis and Design with Applications* (Addison-Wesley Object Technology Series) Reading, MA: Addison-Wesley.

Booch, G., D. Bryan, and C. G. Petersen. 1993. *Software Engineering with Ada*. Reading, MA: Addison-Wesley.
Bratko, I. 2000. *PROLOG Programming for Artificial Intelligence* (3rd ed.) Reading, MA: Addison-Wesley.
Bridges, D. and F. Richman. 1987. *Varieties of Constructive Mathematics*, London Mathematical Society Lecture Note Series #97. Cambridge, UK: Cambridge University Press.
Brinch-Hansen, P. 1999. "Java's Insecure Parallelism." *ACM SIGPLAN Notices* 34(4), pp. 38–45.
Brinch-Hansen, P. 1996. "Monitors and Concurrent Pascal: A Personal History." In Bergin and Gibson [1996], pp. 121–172.
Brinch-Hansen, P. 1981. "The Design of Edison." *Software Practice and Experience* 11, pp. 363–396.
Brinch-Hansen, P. 1975. "The Programming Language Concurrent Pascal." *IEEE Transactions on Software Engineering* SE-1(2): 199–207.
Brodie, L. 1981. *Starting FORTH: An Introduction to the FORTH Language*. Englewood Cliffs, NJ: Prentice-Hall.
Brooks, F. 1996. "Language Design as Design." In Bergin and Gibson [1996], pp. 4–16.
Bruce, K. [2002] *Foundations of Object-Oriented Languages: Types and Semantics*. Cambridge, MA: MIT Press.
Budd, T. 1987. *A Little Smalltalk*. Reading, MA: Addison-Wesley.
Budd, T. 1997. *An Introduction to Object-Oriented Programming*. Reading, MA: Addison-Wesley.
Burks, A. W., H. H. Goldstine, and J. von Neumann. 1947. "Preliminary Discussion of the Logical Design of an Electronic Computing Instrument." In *John von Neumann: Collected Works*, Vol. V, pp. 34–79. New York: Macmillan, 1973.
Burstall, R., D. MacQueen, and D. Sanella. 1980. *HOPE: An Experimental Applicative Language*. Report CSR-62-80, Computer Science Department, Edinburgh University, Scotland.
Butenhof, D. R. 1997. *Programming with POSIX Threads*. Reading, MA: Addison-Wesley.
Cardelli, L., J. Donahue, L. Glassman, M. Jordan, B. Kaslow, and G. Nelson. 1992. "Modula-3 Language Definition," *SIGPLAN Notices* 27(8): 15–42.
Cardelli, L., J. Donahue, M. Jordan, B. Kaslow, and G. Nelson. 1989. "The Modula-3 Type System." *Sixteenth Annual ACM Symposium on Principles of Programming Languages*, pp. 202–212.
Cardelli, L. and P. Wegner. 1985. "On Understanding Types, Data Abstraction, and Polymorphism." *ACM Computing Surveys* 17(4): 471–522.
Carriero, N. and D. Gelernter. 1990. *How to Write Parallel Programs: A First Course*. Cambridge, MA: MIT Press.
Carriero, N. and D. Gelernter. 1989a. "How to Write Parallel Programs: A Guide to the Perplexed." *ACM Computing Surveys* 21(3): 323–357.
Carriero, N. and D. Gelernter. 1989b. "Linda in Context." *Comm. ACM* 32(4): 444–458.
Chapman, S. J. 1997. *Fortran 90/95 for Scientists and Engineers*. New York: McGraw-Hill.
Chomski, N. A. 1956. "Three Models for the Description of Language." *I. R. E. Transactions on Information Theory* IT-2(3); 113–124.
Church, A. 1941. *The Calculi of Lambda Conversion*. Princeton, NJ: Princeton University Press.
Ciancarini, P. 1992. "Parallel Programming with Logic Languages." *Computer Languages* 17(4): 213–239.
Clark, R. L. 1973. "A Linguistic Contribution to GOTO-less Programming." *Datamation* 19(12), 62–63. Reprinted in *Comm. ACM* 27(4), April 1984.
Clark, K. L. and S.A. Tarnlund (Eds.) 1982. *Logic Programming*. New York: Academic Press.
Cleaveland, J. C. 1986. *An Introduction to Data Types*. Reading, MA: Addison-Wesley.
Clocksin, W. F. and C. S. Mellish. 1994. *Programming in Prolog* (4th ed.). Berlin: Springer-Verlag.
Cohen, J. 1988. "A View of the Origins and Development of Prolog." *Comm. ACM* 31(1), 26–37.
Cohen, J. 1981. "Garbage Collection of Linked Data Structures." *ACM Computing Surveys* 13(3): 341–367.
Cohen, N. 1996. *Ada as a Second Language* (2nd ed.). New York: McGraw-Hill.
Colburn, D. R., C. H. Moore, and E. D. Rather. 1996. "The Evolution of FORTH." In Bergin and Gibson [1996], pp. 625–658.
Colmerauer, A. and P. Roussel. 1996. "The Birth of Prolog." In Bergin and Gibson [1996], pp. 331–352.
Colmerauer, A. 1982. "Prolog and Infinite Trees." In Clark and Tärnlund [1982].
Conlon, T. 1989. *Programming in PARLOG*. Reading, MA: Addison-Wesley.
Cooper, D. 1983. *Standard Pascal User Reference Manual*. New York: W. W. Norton.
Cox, B. 1986. *Object-Oriented Programming: An Evolutionary Approach*. Reading, MA: Addison-Wesley.
Cox, B. 1984. "Message/Object Programming: An Evolutionary Change in Programming Technology." *IEEE Software* 1(1): 50–69.
Curry, H. B. and R. Feys. 1958. *Combinatory Logic*, Vol. 1. Amsterdam: North-Holland.
Dahl, O.-J., E. W. Dijkstra, and C. A. R. Hoare. 1972. *Structured Programming*. New York: Academic Press.
Dahl, O.-J. and K. Nygaard. 1966. "SIMULA—An Algol-based Simulation Language." *Comm. ACM* 9(9): 671–678.
Dane, A. 1992. "Birth of an Old Machine." *Popular Mechanics*, March 1992, 99–100.
Davis, R. E. 1982. "Runnable Specifications as a Design Tool." In Clark and Tärnlund [1982].
Demers, A. J., J. E. Donahue, and G. Skinner. 1978. "Data Types as Values: Polymorphism, Type-checking, Encapsulation." *Conference Record of the Fifth Annual ACM Symposium on Principles of Programming Languages*, New York: ACM Press pp. 23–30.
Dijkstra, E. W. and C. S. Scholten. 1990. *Predicate Calculus and Program Semantics (Texts and Monographs in Computer Science)*. New York: Springer-Verlag.

Dijkstra, E. W. 1976. *A Discipline of Programming*. Englewood Cliffs, NJ: Prentice-Hall.
Dijkstra, E. W. 1975. "Guarded Commands, Nondeterminacy, and the Formal Derivation of Programs." *Comm. ACM* 18(8): 453–457.
Dijkstra, E. W. 1968a. "Goto Statement Considered Harmful" (letter to the editor). *Comm. ACM* 11(3): 147–148.
Dijkstra, E. W. 1968b. "Co-operating sequential processes." In F. Genuys (ed.), *Programming Languages: NATO Advanced Study Institute*. New York: Academic Press.
Donahue, J. E. and A. J. Demers. 1985. "Data Types Are Values." *ACM Transactions on Programming Languages and Systems* 7(3): 436–445.
Duncan, R. 1990. "A Survey of Parallel Computer Architectures." *IEEE Computer* 23(2): 5–16.
Dybvig, K. 1996. *The Scheme Programming Language: ANSI Scheme* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.
Ellis, M. A. and B. Stroustrup. 1990. *The Annotated C++ Reference Manual*. Reading, MA: Addison-Wesley.
Falkoff, A. D. and K. Iverson. 1981. "The Evolution of APL." In Wexelblat [1981], pp. 661–674.
Feinberg, N., Ed. 1996. *Dylan Programming : An Object-Oriented and Dynamic Language*. Reading, MA: Addison-Wesley.
Feldman, M. and E. Koffman. 1999. *ADA 95: Problem Solving and Program Design* (3rd ed.). Reading, MA: Addison-Wesley.
Flanagan, D. 1999. *Java in a Nutshell* (3rd ed.). O'Reilly & Associates, Inc.
Flanagan, D. and Matsumoto, Y. [2008] *The Ruby Programming Language*. Sebastopol: O'Reilly Media.
Friedman, D. P., C. T. Haynes, and E. Kohlbecker. 1985. "Programming with Continuations." In P. Pepper (Ed.), *Program Transformations and Programming Environments*. New York: Springer-Verlag.
Futatsugi, K., J. Goguen, J.-P. Jouannaud, and J. Meseguer. 1985. "Principles of OBJ2." *In Proceedings of the ACM Symposium on Principles of Programming Languages*, pp. 52–66.
Gabriel, R. P. 1985. *Performance and Evaluation of Lisp Systems*. Cambridge, MA: MIT Press.
Gabriel, R. 1996. *Patterns of Software: Tales from the Software Community*. Oxford, UK: Oxford University Press.
Gabriel, R. P., J. L. White, and D. G. Bobrow. 1991. "CLOS: Integrating Object-oriented and Functional Programming." *Comm. ACM* 34(9): 29–38.
Gehani, N. H. 1984. *Ada: Concurrent Programming*. Englewood Cliffs, NJ: Prentice-Hall.
Gelernter, D. and S. Jagannathan. 1990. *Programming Linguistics*. Cambridge, MA: MIT Press.
Geschke, C., J. Morris, and E. Satterthwaite. 1977. "Early Experience with Mesa." *Comm. ACM* 20(8): 540–553.
Ghezzi, C. and M. Jazayeri. 1997. *Programming Language Concepts* (3rd ed.). New York: John Wiley & Sons.
Goguen, J. and G. Malcolm. 1997. *Algebraic Semantics of Imperative Programs*. Cambridge, MA: MIT Press.
Goguen, J. and G. Malcolm (Eds.). 2000. *Software Engineering with OBJ: Algebraic Specification in Action*. Amsterdam: Kluwer Academic Publishers.
Goguen, J. A., J. W. Thatcher, and E. G. Wagner. 1978. "An Initial Algebra Approach to the Specification, Correctness, and Implementation of Abstract Data Types." In Yeh [1978], pp. 80–149.
Goldman, R. and R. P. Gabriel. 1989. "Qlisp: Parallel Processing in Lisp." *IEEE Software*, July 1989, 51–59.
Goodenough, J. B. 1975. "Exception Handling: Issues and a Proposed Notation." *Comm. ACM* 16(7): 683–696.
Gosling J., B. Joy, G. Steele, and G. Bracha. 2000. *The Java Language Specification* (2nd ed.). Reading, MA: Addison-Wesley.
Graham, P. 1996. *ANSI Common Lisp*. Englewood Cliffs, NJ: Prentice-Hall.
Graham, P. 2004. *Hackers & Painters: Big Ideas from the Computer Age*. Cambridge, MA: O'Reilly.
Gregory, S. 1987. *Parallel Logic Programming in PARLOG: The Language and Its Implementation*. Reading, MA: Addison-Wesley.
Gries, D. 1981. *The Science of Programming*. New York: Springer-Verlag.
Griswold, R. E. 1981. "A History of the SNOBOL Programming Languages." In Wexelblat [1981], pp. 601–645.
Griswold, R. E. and M. Griswold. 1996. "History of the ICON Programming Language." In Bergin and Gibson [1996], pp. 599–621.
Griswold, R. E. and M. Griswold. 1983. *The Icon Programming Language*. Englewood Cliffs, NJ: Prentice-Hall.
Griswold, R. E. and M. Griswold. 1973. *A SNOBOL4 Primer*. Englewood Cliffs, NJ: Prentice-Hall.
Griswold, R. E., J. F. Poage, and I. P. Polonsky. 1971. *The SNOBOL4 Programming Language* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.
Gunter, C. A. 1992. *Semantics of Programming Languages: Structures and Techniques (Foundations of Computing Series)*. Cambridge, MA: MIT Press.
Gunter, C. A. and J. C. Mitchell (Eds.). 1994. *Theoretical Aspects of Object-Oriented Programming: Types, Semantics, and Language Design (Foundations of Computing)*. Cambridge, MA: MIT Press.
Guttag, J. V. 1977. "Abstract Data Types and the Development of Data Structures." *Comm. ACM* 20(6): 396–404.
Halstead, R. H., Jr. 1985. "Multilisp: A Language for Concurrent Symbolic Computation." *ACM Transactions on Programming Languages and Systems* 7(4): 501–538.
Hankin, C. 1995. *Lambda Calculi : A Guide for Computer Scientists* (Graduate Texts in Computer Science, No 3). Oxford: Clarendon Press.
Hanson, D. R. 1981. "Is Block Structure Necessary?" *Software Practice and Experience* 11(8): 853–866.

Harper, R. and Mitchell, J. 1993. "On the Type Structure of Standard ML." *ACM Transactions on Programming Languages and Systems* 15(2), April 1993, 211–252.
Harry, A. 1997. *Formal Methods Fact File: Vdm and Z (Wiley Series in Software Engineering Practice)*, New York: John Wiley & Sons.
Henderson, P. 1980. *Functional Programming: Application and Implementation*. Englewood Cliffs, NJ: Prentice-Hall.
Henglein, F. [1993] "Type Inference with Polymorphic Recursion." *ACM Transactions on Programming Languages and Systems* 15(2), April 1993, 253–289.
Hindley, J. R. 1997. *Basic Simple Type Theory (Cambridge Tracts in Theoretical Computer Science, No 42)*. Cambridge, England: Cambridge University Press.
Hindley, J. R. 1969. "The Principal Type-scheme of an Object in Combinatory Logic." *Trans. Amer. Math. Soc.* 146(12): 29–60.
Hoare, C. A. R. 1981. "The Emperor's Old Clothes." *Comm. ACM* 24(2): 75–83.
Hoare, C. A. R. 1978. "Communicating Sequential Processes." *Comm. ACM* 21(8): 666–677.
Hoare, C. A. R. 1974. "Monitors: An Operating System Structuring Concept." *Comm. ACM* 17(10): 549–557.
Hoare, C. A. R. 1973. "Hints on Programming Language Design." *ACM SIGACT/SIGPLAN Symposium on Principles of Programming Languages*. Reprinted in Horowitz [1987], pp. 31–40.
Hoare, C. A. R. 1969. "An Axiomatic Basis for Computer Programming." *Comm. ACM* 12(10): 576–580, 583.
Hoare, C. A. R. and N. Wirth. 1966. "A Contribution to the Development of ALGOL." *Comm. ACM* 9(6): 413–431.
Hopcroft, J. E. and J. D. Ullman. 1979. *Introduction to Automata Theory, Languages, and Computation*. Reading, MA: Addison-Wesley.
Horn, A. 1951. "On Sentences Which Are True of Direct Unions of Algebras." *J. Symbolic Logic* 16: 14–21.
Horowitz, E. 1987. *Programming Languages: A Grand Tour* (3rd ed.). Rockville, MD.: Computer Science Press.
Horowitz, E. 1984. *Fundamentals of Programming Languages* (2nd ed.). Rockville, Md.: Computer Science Press.
Horowitz, E. and S. Sahni. 1984. *Fundamentals of Data Structures in Pascal*. Rockville, MD.: Computer Science Press.
Horstmann, C. 2006. *Object-Oriented Design & Patterns*. Hoboken, NJ: John Wiley & Sons.
Horstmann, C. and G. Cornell. 1999. *Core JAVA 1.2, Vols I and II*. Palo Alto, CA: Sun Microsystems Press/Prentice-Hall PTR.
Hudak, P. 1989. "Conception, Evolution, and Application of Functional Programming Languages." *ACM Computing Surveys* 21(3): 359–411.
Hudak, P. 2000. *The Haskell School of Expression: Learning Functional Programming through Multimedia*, New York: Cambridge University Press.
Hui, R. K. W. et al. 1990. "APL\?" in *APL90 Conf. Proc.*, 20(4): 192–200.
Ichbiah, J. D., J. G. P. Barnes, J. C. Heliard, B. Krieg-Brueckner, O. Roubine, and B. A. Wichmann. 1979. "Rationale for the Design of the Ada Programming Language." *ACM SIGPLAN Notices* 14(6), Part B.
IEEE. 1985. "ANSI/IEEE Std 754-1985: Standard for Binary Floating-Point Arithmetic." Reprinted in *ACM SIGPLAN Notices* 22(2): 9–25.
IEEE P1178. 1991. *IEEE Standard for the Scheme Programming Language*.
ISO 8652. 1995. ISO/IEC Standard—Information technology—Programming languages—Ada. International Organization for Standardization, Geneva, Switzerland.
ISO 9899. 1999. ISO/IEC Standard—Information technology—Programming languages—C. International Organization for Standardization, Geneva, Switzerland.
ISO 13211-1. 1995. ISO/IEC Standard—Information technology—Programming languages—Prolog-Part 1: General core. International Organization for Standardization, Geneva, Switzerland.
ISO 14882-1. 1998. ISO/IEC Standard—Information technology—Programming languages—C++. International Organization for Standardization, Geneva, Switzerland.
ISO 14977. 1996. ISO/IEC Standard—Information technology—Syntactic metalanguage—Extended BNF. International Organization for Standardization, Geneva, Switzerland.
Iverson, K. 1962. *A Programming Language*. Hobken, NJ: John Wiley & Sons.
Jaffar, J., S. Michaylov, P. Stuckey, and R. Yap. 1992. "The CLP(R) Language and System," *ACM TOPLAS*, 14(3): 339–395.
Jaffar, J. and Maher, M. 1994. "Constraint Logic Programming: A Survey," *Journal of Logic Programming*, 19/20, 503–581.
Johnson, S. C. 1975. "Yacc—Yet Another Compiler Compiler," Computing Science Technical Report No. 32. Murray Hill, N.J.: AT&T Bell Laboratories.
Jones, G. and M. Goldsmith. 1988. *Programming in Occam 2*. Englewood Cliffs, N.J.: Prentice-Hall.
Josuttis, N. 1999. *The C++ Standard Library—A Tutorial and Reference*. Reading, MA: Addison-Wesley.
Kamin, S. 1983. "Final Data Types and Their Specification." *ACM Trans. on Programming Languages and Systems* 5(1): 97–123.
Karp, A. H. 1987. "Programming for parallelism." *IEEE Computer* 21(5): 43–57.
Karp, A. H. and R. O. Babb II. 1988. "A Comparison of 12 Parallel Fortran Dialects." *IEEE Software*, September 1988, 52–67.
Kay, A. C. 1996. "The Early History of Smalltalk." In Bergin and Gibson [1996], pp. 511–579.

Kergommeaux, J. C. and P. Codognet. 1994. "Parallel LP Systems," *ACM Computing Surveys* 26(3), 295–336.
Kernighan, B. W. and D. M. Ritchie. 1988. *The C Programming Language* (ANSI Standard C) (2nd ed.). Englewood Cliffs, N.J.: Prentice-Hall.
King, K. N. 1988. *Modula-2: A Complete Guide*. Lexington, MA.: D. C. Heath.
Knuth, D. E. 1974. "Structured Programming with GOTO Statements." *ACM Computing Surveys* 6(4), 261–301.
Knuth, D. E. 1972. "Ancient Babylonian Algorithms." *Comm. ACM* 15(7), 671–677.
Knuth, D. E. 1967. "The Remaining Trouble Spots in Algol60." *Comm. ACM* 10(10), 611–617. Also reprinted in Horowitz [1987], pp. 61–68.
Knuth, D. E. and L. Trabb Pardo. 1977. "Early Development of Programming Languages." In *Encyclopedia of Computer Science and Technology*, Vol. 7, pp. 419–493. New York: Marcel Dekker.
Koenig, A. and B. Moo. 1996. *Ruminations on C++: A Decade of Programming Insight and Experience*. Reading, MA: Addison-Wesley.
Koenig, A. and B. Stroustrup. 1990. "Exception Handling for C++," *Proceedings of the USENIX C++ Conference* (pp. 149–176), San Francisco, April 1990; reprinted in *JOOP* Vol. 3, No. 2, July/Aug 1990, pp. 16–33.
Kowalski, R. A. 1988. "The Early Years of Logic Programming." *Comm. ACM* 31(1): 38–43.
Kowalski, R. A. 1979a. "Algorithm 5 Logic 1 Control." *Comm. ACM* 22(7): 424–436.
Kowalski, R. A. 1979b. *Logic for Problem Solving*. New York: Elsevier/North-Holland.
Kurtz, T. E. 1981. "BASIC." In Wexelblat [1981], pp. 515–537.
Lajoie, J. 1994a. "Exception Handling: Supporting the Runtime Mechanism." *C++ Report* (March/April 1994).
Lajoie, J. 1994b "Exception Handling: Behind the Scenes." *C++ Report* (June 1994).
Lalonde, W. 1994. *Discovering Smalltalk* (Addison-Wesley Object Technology Series). Reading, MA: Addison-Wesley.
Lambert, K. and M. Osborne. 1997. *Smalltalk in Brief: Introduction to Object-Oriented Software Development*. Boston: PWS.
Lambert, K, and D. Nance. 2001. *Fundamentals of C++*. Boston: Course Technology.
Lambert, K. 2010. *Fundamentals of Python: From First Programs through Data Structures*. Boston: Course Technology.
Lambert, K. and M. Osborne. 2010. *Fundamentals of Java*. Boston: Course Technology.
Lampson, B. W. 1983. "A Description of the Cedar Language," Technical Report CSL-83-15. Palo Alto, CA: Xerox Palo Alto Research Center.
Lampson, B. W., J. J. Horning, R. L. London, J. G. Mitchell, and G. J. Popek. 1981. "Report on the Programming Language Euclid," Technical Report CSL-81-12. Palo Alto, CA: Xerox Palo Alto Research Center.
Lampson, B. W. and D. Redell. 1980. "Experience with Processes and Monitors in Mesa." *Comm. ACM* 23(2): 105–117.
Landin, P. J. 1966. "The Next 700 Programming Languages." *Comm. ACM* 9(3), 157–165.
Lapalme, G. and F. Rabhi. 1999. *Algorithms: A Functional Programming Approach*. Reading, MA: Addison-Wesley.
Lea, D. 2000. *Concurrent Programming in Java: Design Principles and Patterns*. (2nd ed.). Reading, MA: Addison-Wesley.
Lenkov, D., D. Cameron, P. Faust, and M. Mehta. 1992. "A Portable Implementation of C++ Exception Handling," *Proceedings of the Usenix C++ Conference*, Portland, OR.
Lesk, M. E. 1975. "Lex—A Lexical Analyzer Generator," Computing Science Technical Report No. 39. Murray Hill, NJ: AT&T Bell Laboratories.
Lewis, J., M. Shields, E. Meijer, and J. Launchbury. 2000. "Implicit Parameters: Dynamic Scoping with Static Types." In *Proceedings of the 27th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, pp. 108–118. New York: ACM Press.
Lewis, S. 1995. *The Art and Science of Smalltalk*. Englewood Cliffs, NJ: Prentice-Hall.
Lindsey, C. H. 1996. "A History of Algol 68." In Bergin and Gibson [1996], pp. 27–84.
Lippman, S. and J. Lajoie 1998. *C++ Primer* (3rd ed.). Reading, MA: Addison-Wesley.
Liskov, B. 1996. "A History of CLU." In Bergin and Gibson [1996], pp. 471–497.
Liskov, B., R. Atkinson, T. Bloom, E. Moss, J. C. Schaffert, R. Scheifler, and A. Snyder. 1984. *CLU Reference Manual*. New York: Springer-Verlag.
Liskov, B. and A. Snyder. 1979. "Exception Handling in CLU." *IEEE Translations on Software Engineering* SE-5(6): 546–558. Also reprinted in Horowitz [1987], pp. 254–266.
Liskov, B., A. Snyder, R. Atkinson, and C. Schaffert. 1977. "Abstraction Mechanisms in CLU." *Comm. ACM* 20(8): 564–576. Also reprinted in Horowitz [1987], pp. 267–279.
Liu, C. 2000. *Smalltalk, Objects, and Design*. Lincoln, NE: iUniverse.com.
Lloyd, J. W. 1984. *Foundations of Logic Programming*. New York: Springer-Verlag.
Louden, K. 1987. "Recursion versus Non-recursion in Pascal: Recursion Can Be Faster." *ACM SIGPLAN Notices* 22(2): 62–67.
Louden, K. 1997. *Compiler Construction: Principles and Practice*. Boston: PWS.
Louden, K. 1997b. "Compilers and Interpreters." In Tucker [1997], pp. 2120–2147.
Louden, K. 2004 "Compilers and Interpreters," in *The Computer Science and Engineering Handbook*, 2nd Edition (2004), Allen B. Tucker (Editor), CRC Press.
Luckam, D. C. and W. Polak. 1980. "Ada Exception Handling: An Axiomatic Approach." *ACM Transactions on Programming Languages and Systems* 2(2), 225–233.
MacLennan, B. *Functional Programming: Practice and Theory*. 1990. Reading, MA: Addison-Wesley.

MacQueen, D. B. 1988. "The Implementation of Standard ML Modules." *ACM Conference on Lisp and Functional Programming*, 212–223. New York: ACM Press.
Malpas, J. *Prolog: A Relational Language and Its Applications*. 1987. Prentice Hall.
Mandrioli, D. and C. Ghezzi. 1987. *Theoretical Foundations of Computer Science*. Hoboken, NJ: John Wiley & Sons.
Marriott, K. and P. J. Stuckey (Eds.). 1997. *Programming with Constraints*. Cambridge, MA: MIT Press.
Martin-Löf, P. 1979. "Constructive Mathematics and Computer Programming." In L. J. Cohen et al. (Eds.) *Logic, Methodology and the Philosophy of Science*, Vol. VI. New York: North-Holland. 1982.
McCarthy, J. 1981. "History of LISP." In Wexelblat [1981], pp. 173–185.
Metcalf, M. and J. Reid. 1999. *Fortran 90/95 Explained* (2nd ed.). Oxford, UK: Oxford University Press.
Meyer, B. 1997. *Object-Oriented Software Construction* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.
Milner, R. 1978. "A Theory of Type Polymorphism in Programming." *J. Computer and System Sciences* 17(3): 348–375.
Milner, R. 1994. *Communication and Concurrency*. Englewood Cliffs, NJ: Prentice-Hall.
Milner, R. and M. Tofte. 1991. *Commentary on Standard ML*. Cambridge, MA.: MIT Press.
Milner, R., M. Tofte, R. Harper, and D. MacQueen. 1997. *The Definition of Standard ML (Revised)*. Cambridge, MA: MIT Press.
Minker J. (Ed.). 2000. *Logic-Based Artificial Intelligence* (The Kluwer International Series in Engineering and Computer Science Volume 597). Amsterdam: Kluwer Academic Publishers.
Mitchell, J. C. 1996. *Foundations for Programming Languages (Foundations of Computing Series)*. Cambridge, MA: MIT Press.
Mitchell, J. C. and R. Harper. 1988. "The Essence of ML." *Fifteenth ACM Symposium on Principles of Programming Languages*, New York: ACM Press, pp. 28–46.
Mitchell, J. G., W. Maybury, and R. Sweet. 1979. "Mesa Language Manual, Version 5.0," Technical Report CSL-79-3. Palo Alto, CA: Xerox Palo Alto Research Center.
Moon, D. A. 1986. "Object-oriented Programming with Flavors." *OOPSLA 1986, ACMSIGPLAN Notices* 21(11): 1–8.
Moon, D. A. 1984. "Garbage Collection in Large Lisp Systems." *Proceedings of the 1984 ACM Symposium on Lisp and Functional Programming*, New York: ACM Press, pp. 235–246.
Moore, D. L. 1977. *Ada, Countess of Lovelace: Byron's Legitimate Daughter*. London: Murray.
Morrison, P. and E. Morrison (Eds.). 1961. *Charles Babbage and His Calculating Engines*. New York: Dover.
Muhlbacher, J. (Ed.). 1997. *Oberon-2 Programming with Windows*. New York: Springer-Verlag, 1997.
Naur, P. 1981. "The European Side of the Last Phase of the Development of Algol 60." In Wexelblat [1981], pp. 92–139.
Naur, P. (Ed.). 1963a. "Revised Report on the Algorithmic Language Algol 60." *Comm. ACM* 6(1): 1–17. Also reprinted in Horowitz [1987], pp. 44–60.
Naur, P. 1963b. "GOTO Statements and Good Algol Style." *BIT* 3(3): 204–208.
Nelson, G. (Ed.). 1991. *Systems Programming with Modula-3*. Englewood Cliffs, NJ: Prentice-Hall.
Nelson, G. 1989. "A Generalization of Dijkstra's Calculus." *ACM Transactions on Programming Languages and Systems* 11(4): 517–561.
Nygaard, K. and O.-J. Dahl. 1981. "The Development of the SIMULA Languages." In Wexelblat [1981], pp. 439–480.
Okasaki, C. 1999. *Purely Functional Data Structures*. Cambridge, UK: Cambridge University Press.
O'Donnell, M. J. 1985. *Equational Logic as a Programming Language*. Cambridge, MA: MIT Press.
OOPSLA. 1986ff. ACM Conference on Object-Oriented Programming Systems and Languages. Also published as various issues of *ACM SIGPLAN Notices*.
Ousterhout, J. K. 1998. "Scripting: Higher-Level Programming for the 21st Century," *IEEE Computer* 31(3): pp. 23–30.
Pacheco, P. 1996. *Parallel Programming with MPI*. San Francisco: Morgan Kaufmann.
Pamas, D. L. 1985. "Software Aspects of Strategic Defense Systems." *Comm. ACM* 28(12): 1326–1335. Reprinted from the *American Scientist* 73(5): 432–440.
Paulson, L. 1996. *ML for the Working Programmer* (2nd ed.). Cambridge, UK: Cambridge University Press.
Perlis, A. J. 1981. "The American Side of the Development of Algol." In Wexelblat [1981], pp. 75–91.
Peyton Jones, S. L. 1987. *The Implementation of Functional Programming Languages*. Englewood Cliffs, NJ: Prentice-Hall.
Popek, G. J., J. J. Homing, B. W. Lampson, J. G. Mitchell, and R. L. London. 1977. "Notes on the Design of Euclid." *ACM SIGPLAN Notices* 12(3): 11–19.
Pountain, D. 1995. "Constraint Logic Programming: A Child of Prolog Finds New Life Solving Programming's Nastier Problems," *BYTE*, February 1995.
Radin, G. 1981. "The Early History and Characteristics of PL/I." In Wexelblat [1981], pp. 551–575.
Randell, B. and L. J. Russell. 1964. *Algol 60 Implementation*. New York: Academic Press.
Reade, C. 1989. *Elements of Functional Programming*. Reading, MA: Addison-Wesley.

Reynolds, J. C. 1998. *Theories of Programming Languages*. Cambridge, UK: Cambridge University Press.
Richards, M. and C. Whitby-Strevens. 1979. *BCPL—The Language and Its Compiler*. Cambridge, UK: Cambridge University Press.
Ringwood, G. A. 1988. "Parlog86 and the Dining Logicians." *Comm. ACM* 31(1): 10–25.
Ripley, G. D. and F. C. Druseikis. 1978. "A Statistical Analysis of Syntax Errors." *Computer Languages* 3(4): 227–240.
Ritchie, D. M. 1996. "The Development of the C Programming Language." In Bergin and Gibson [1996], pp. 671–687.
Roberts, E. [1995] "Loop Exits and Structured Programming: Reopening the Debate," *ACM SIGCSE Bulletin*, (27)1: 268–272.
Robinson, J. A. 1965. "A Machine-Oriented Logic Based on the Resolution Principle." *Journal of the ACM* 12(1): 23–41.
Rosen, S. 1972. "Programming Systems and Languages 1965–1975." *Comm. ACM* 15(7): 591–600.
Rosen, S. (Ed.). 1967. *Programming Systems and Languages*. New York: McGraw-Hill.
Rosser, J. B. 1982. "Highlights of the History of the Lambda Calculus." *Proceedings of the ACM Symposium on Lisp and Functional Programming*, New York: ACM Press, pp. 216–225.
Rubin, F. 1987. "'GOTO Statement Considered Harmful' Considered Harmful" (letter to the editor). *Comm. ACM* 30(3), pp. 195–196. Replies in the June, July, August, November, and December 1987 issues.
Sammet, J. E. 1981. "The Early History of COBOL." In Wexelblat [1981], pp. 199–243.
Sammet, J. E. 1976. "Roster of Programming Languages for 1974–75." *Comm. ACM* 19(12): 655–699.
Sammet, J. E. 1972. "Programming Languages: History and Future." *Comm. ACM* 15(7): 601–610.
Sammet, J. E. 1969. *Programming Languages: History and Fundamentals*. Englewood Cliffs, NJ: Prentice-Hall.
Saraswat, V. and P. Van Hentenryck (Eds.). 1995. *Principles and Practice of Constraint Programming: The Newport Papers*. Cambridge, MA: MIT Press.
Schmidt, D. A. 1994. *The Structure of Typed Programming Languages (Foundations of Computing Series)*. Cambridge, MA: MIT Press.
Schmidt, D. A. 1986. *Denotational Semantics: A Methodology for Language Development*. Dubuque, IA: Wm. C. Brown.
Schneider, S. 2000. *Concurrent and Real-time Systems: The CSP Approach*. Hoboken, NJ: John Wiley & Sons.
Schneiderman, B. 1985. "The Relationship between COBOL and Computer Science." *Annals of the History of Computing* 7(4): 348–352. Also reprinted in Horowitz [1987], pp. 417–421.
Schönfinkel, M. 1924. "Über die Bausteine der mathematischen Logik." *Mathematische Annalen* 92(3/4): 305–316.
Scott, M. L. 2000. *Programming Language Pragmatics*. San Francisco: Morgan Kaufmann.
Sebesta, R. W. 1996. *Concepts of Programming Languages* (3rd ed.). Reading, MA: Addison-Wesley.
Seibel, P. 2005. *Practical Common Lisp*. Berkeley: Apress.
Sethi, R. 1996. *Programming Languages Concepts and Constructs* (2nd ed.). Reading, MA: Addison-Wesley.
Shapiro, E. 1989. "The Family of Concurrent Logic Programming Languages." *ACM Computing Surveys* 21(3), 412–510.
Shapiro, E. and D. H. D. Warren (Eds.). 1993. "Special Section: The Fifth Generation Project: Personal Perspectives, Launching the New Era, and Epilogue," *Communications of the ACM*, 36(3): 46–103.
Sharp, A. 1997. *Smalltalk by Example: The Developer's Guide*. New York: McGraw-Hill.
Shaw, C. J. 1963. "A Specification of JOVIAL." *Comm. ACM* 6(12): 721–736.
Shaw, Mary (Ed.). 1981. *Alphard Form and Content*. New York: Springer-Verlag.
Sipser, M. 1997. *Introduction to the Theory of Computation*. Boston: PWS.
Snyder, A. 1986. "Encapsulation and Inheritance in Object-oriented Languages." *ACM SIGPLAN Notices* 21(11): 38–45.
Sperber, M., Dybvig, K., Flatt, M., and van Straaten, A. [2010] *Revised [6] Report of the Algorithmic Language Scheme*. Cambridge, UK: Cambridge University Press.
Springer, G. and D. P. Friedman. 1989. *Scheme and the Art of Programming*. Cambridge, MA: MIT Press.
Stallings, W. 2000. *Operating Systems: Internals and Design Principles*. Englewood Cliffs, NJ: Prentice-Hall.
Steele, G. L., Jr., and R. P. Gabriel. 1996. "The Evolution of Lisp." In Bergin and Gibson [1996], pp. 233–309.
Steele, G. 1984. *Common Lisp: The Language*. Burlington, MA: Digital Press.
Steele, G. 1982. "An Overview of Common Lisp." *Proceedings of the ACM Symposium on Lisp and Functional Programming*, pp. 98–107. New York: ACM Press.
Steele, G. 1977. "Debunking the 'Expensive Procedure Call' Myth." *Proceedings of the National Conference of the ACM*, pp. 153–162. New York: ACM Press.
Stein, D. 1985. *Ada: A Life and Legacy*. Cambridge, MA: MIT Press.
Sterling, L. and E. Shapiro. 1986. *The Art of Prolog*. Cambridge, MA: MIT Press.
Stoy, J. E. 1977. *Denotational Semantics: The Scott-Strachey Approach to Programming Language Semantics*. Cambridge, MA: MIT Press.
Stroustrup, B. 1997. *The C++ Programming Language* (3rd ed.). Reading, MA: Addison-Wesley.
Stroustrup, B. 1996. "A History of C++: 1979–1991." In Bergin and Gibson [1996], pp. 699–755.
Stroustrup, B. 1994. *The Design and Evolution of C++*. Reading, MA: Addison-Wesley.

Sussman, G. J. and G. L. Steele, Jr. 1975. "Scheme: An Interpreter for Extended Lambda Calculus." AI Memo 349. Cambridge, MA: MIT Artificial Intelligence Laboratory.
Swinehart, D. C., P. T. Zellweger, R. J. Beach, and R. B. Hagmann. 1986. "A Structural View of the Cedar Programming Environment." *ACM Transactions on Programming Languages and Systems* 8(4): 419–490.
Tanenbaum, A. S. 1976. "A Tutorial on Algol68." *ACM Computing Surveys* 8(2): 155–190. Also reprinted in Horowitz [1987], pp. 69–104.
Teitelman, W. 1984. "A Tour through Cedar." *IEEE Software*, April 1984, 44–73.
Tesler, L. 1985. "Object Pascal Report." *Structured Language World* 9(3): 10–14.
Thompson, S. 1999. *Haskell: The Craft of Functional Programming* (2nd ed.). Reading, MA: Addison-Wesley.
Tick, E. 1991. *Parallel Logic Programming*. Cambridge, MA: MIT Press.
Tucker, A. 1997. *The Computer Science and Engineering Handbook*. Boca Raton, FL: CRC Press.
Turner, D. A. 1986. "An Overview of Miranda." *ACM SIGPLAN Notices* 21(12): 158–166.
Turner, D. A. 1982. "Recursion Equations as a Programming Language." In Darlington J. et al. (Eds.), *Functional Programming and Its Applications*. Cambridge, UK: Cambridge University Press.
Ueda, K. 1987. "Guarded Horn Clauses." In E. Y. Shapiro (Ed.), *Concurrent Prolog: Collected Papers*, Cambridge, MA: MIT Press, pp. 140–156.
Ueda, K. 1999. "Concurrent Logic/Constraint Programming: The Next 10 Years." In Apt et al. [1999].
Ullman, J. 1998. *Elements of ML Programming, ML97 Edition*. Englewood Cliffs, NJ: Prentice-Hall.
Ungar, D. 1984. "Generation Scavenging: A Non-Disruptive High-Performance Storage Reclamation Algorithm." Proceedings of the ACM SIGSOFT/ SIGPLAN Symposium on Practical Software Development Environments, *ACM SIGPLAN Notices* 19(5): 157–167.
Ungar, D. and R. Smith. 1987. "SELF: The Power of Simplicity." *OOPSLA* 1987.
Wall, L., T. Christiansen, and J. Orwant. 2000. *Programming Perl* (3rd ed.). Sebastopol, CA, O'Reilly & Associates.
Warren, D. H. D. 1980. "Logic Programming and Compiler Writing." *Software Practice and Experience* 10(2): 97–125.
Wegner, P. 1976. "Programming Languages—The First 25 Years." *IEEE Transactions on Computers* C-25(12): 1207–1225. Also reprinted in Horowitz [1987], pp. 4–22.
Wegner, P. 1972. "The Vienna Definition Language." *ACM Computing Surveys* 4(1): 5–63.
Wegner, P. and S. A. Smolka. 1983. "Processes, Tasks, and Monitors: A Comparative Study of Concurrent Programming Primitives." *IEEE Transactions on Software Engineering* SE,9(4): 446–462. Also reprinted in Horowitz [1987], pp. 360–376.
Welch, B. B. 2000. *Practical Programming in Tcl and Tk*. Englewood Cliffs, NJ: Prentice-Hall.
Wexelblat, R. L. 1984. "Nth Generation Languages." *Datamation*, September 1, pp. 111–117.
Wexelblat, R. L. (Ed.). 1981. *History of Programming Languages*. New York: Academic Press.
Whitaker, W. A. 1996. "Ada—The Project: The DoD High Order Language Working Group." In Bergin and Gibson [1996], pp. 173–228.
Whitehead, A. N. 1911. *An Introduction to Mathematics*. Oxford, UK: Oxford University Press.
Wilkinson, B. and Allen, C. M. [1997]. *Parallel Programming: Techniques and Applications Using Networked Workstations and Parallel Computers*. Englewood Cliffs, NJ: Prentice-Hall.
Wilson, P. R. 1992. "Uniprocessor Garbage Collection Techniques." In Bekkers et al. (Eds.), *International Workshop on Memory Management*, Springer Lecture Notes in Computer Science #637, pp. 1–42. New York: Springer-Verlag.
Winograd, T. 1979. "Beyond Programming Languages." *Comm. ACM* 22(7): 391–401.
Winskel, G. 1993. *The Formal Semantics of Programming Languages: An Introduction (Foundations of Computing Series)*. Cambridge, MA.: MIT Press.
Wirth, N. 1996. "Recollections about the Development of Pascal." In Bergin and Gibson [1996], pp. 97–111.
Wirth, N. 1988a. *Programming in Modula-2* (4th ed.). Berlin: Springer-Verlag.
Wirth, N. 1988b. "From Modula to Oberon." *Software Practice and Experience* 18(7): 661–670.
Wirth, N. 1988c. "The Programming Language Oberon." *Software Practice and Experience* 18(7): 671–690.
Wirth, N. 1976. *Algorithms* 1 *Data Structures* 5 *Programs*. Englewood Cliffs, NJ: Prentice-Hall.
Wirth, N. 1974. "On the Design of Programming Languages." *Proc. IFIP Congress* 74, 386–393. Amsterdam: North-Holland. Reprinted in Horowitz [1987], pp. 23–30.
Wirth, N. and H. Weber. 1966a. "Euler: A Generalization of Algol, and Its Formal Definition," Part 1. *Comm. ACM* 9(1): 13–23.
Wirth, N. and H. Weber. 1966b. "Euler: A Generalization of Algol, and Its Formal Definition," Part II. *Comm. ACM* 9(2): 89–99.
Wulf, W. A., R. L. London, and M. Shaw. 1976. "An Introduction to the Construction and Verification of Alphard Programs." *IEEE Transactions on Software Engineering* 2(4): 253–265.
Wulf, W. A., D. B. Russell, and A. N. Habermann. 1971. "BLISS: A Language for Systems Programming." *Comm. ACM* 14(12): 780–790.
Yeh, R. T. (Ed.). 1978. *Current Trends in Programming Methodology, Vol. IV, Data Structuring*. Englewood Cliffs, NJ: Prentice-Hall.
Zuse, K. 1972. "Der Plankalkül." *Berichte der Gesellschaft für Mathematik und Datenverarbeitung*, No. 63, Part 3, Bonn.

A

B

C

D

E

F

G

N

O

T

U

V

W

Y

Z