MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) Governments issue bonds as a way to borrow money to finance government expenses as opposed to printing more money which may result in inflation.

   (b) The long-term part of the yield curve may flatten during a period of high short-term interest rates and uncertain economic conditions in the future, causing investors to flock to longer-term bonds as a form of safe haven, driving their yields down.

   (c) Quantitative easing (QE) is a monetary policy that the Fed uses to influence the money supply so as to spur economic activity and is typically employed when interest rates are near zero. Since the outbreak of COVID-19, the Fed has used QE to print more money as well as purchase large amounts of debt securities.

2. A yield curve represents the term structure of interest rates for bonds with equal credit quality whilst a spot curve represents that of zero-coupon bonds. To construct 0-5 year yield and spot curves out of 10 bonds, we choose 2 bonds with equally spaced maturity dates each year up till 2027. The bonds chosen are as follows: 'CAN 0.25 May 23', 'CAN 0.50 Nov 23', 'CAN 1.50 May 24', 'CAN 0.75 Oct 24', 'CAN 1.50 Apr 25', 'CAN 0.50 Sep 25', 'CAN 0.25 Mar 26', 'CAN 1.00 Sep 26', 'CAN 1.25 Mar 27', 'CAN 2.75 Sep 27'.

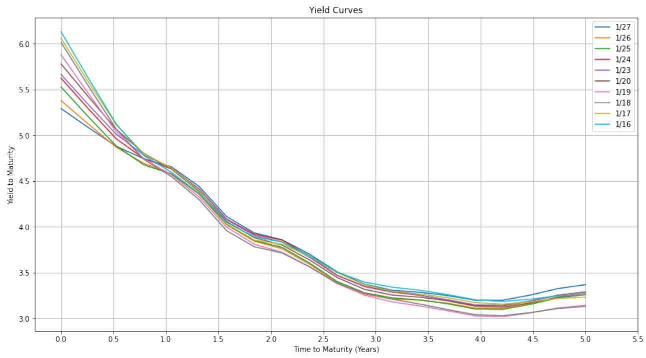
3. The goal of PCA is to reduce the dimension of a given feature space by projecting it onto a smaller subspace whose axes are the eigenvectors. In this case, the given feature space would be those stochastic processes along a stochastic curve. The eigenvectors associated with the covariance matrix of those stochastic processes represent the principal components and they determine the directions of the new feature space we intend to project those stochastic processes on. These eigenvectors (directions) capture and explain the most variability amongst the stochastic processes. Corresponding eigenvalues of eigenvectors then represent the magnitude and thus the amount of variability captured in each projection eigenvector.

Empirical Questions - 75 points

4.

   (a) Scipy's CubicSpline method was used to help us interpolate the yield to maturity data. This method was chosen as it interpolates using a cubic polynomial and avoids the Runge phenomenon which can result in catastrophic errors.

   (b) The spot rate of a bond is the current yield for a given term and is the yield to maturity of zero-coupon bonds with those terms. The spot rate is first calculated for the 6-month bond that

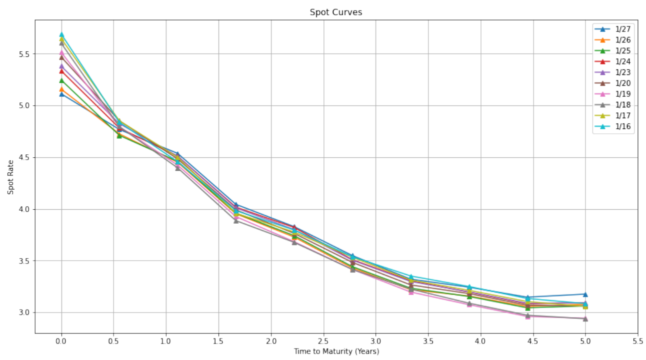


has a known market price that only has a single payment left. After the spot rate is calculated for the 6-month bond, we can use said spot rate to calculate the rate for the 2nd period of the 1-year bond. This process, known as bootstrapping is repeated for all subsequent bonds. We use the following equation to find all spot rates:

$$r(t_n) = \frac{lg(p_n) - lg(P - \sum_i^{n-1} p_i e^{r(t_i)t_i})}{t_n}$$

Algorithm 1 Finding spot rates

Initialize spotdf with first 6-month bond spot rates as 0th index
for $days = 1, 2, \ldots, 10$ **do**
  for i in range(1, len(days)) **do**
    prevSum $= \sum_i^{n-1} p_i e^{r(t_i)t_i}$
    spotdf.iloc[bond, date] $= \frac{lg(p_n) - lg(P - prevSum)}{t_n}$
  end for
end for

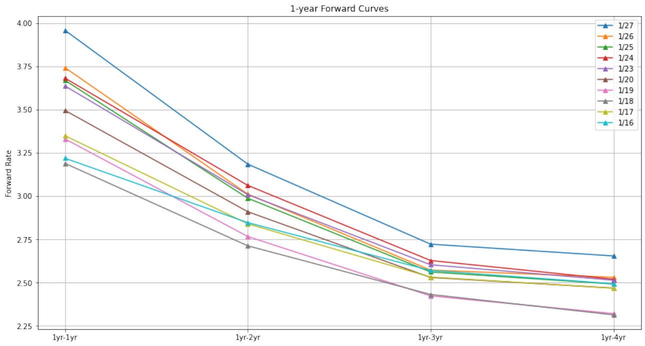


(c) Closely related to the spot rate is the forward rate, which is the interest rate for a certain term that begins in the future and ends later. The forward curve discounts a single payment, just not till today. In our case, we have to discount all future payments at varying maturities from 2-5 years till 1 year from now. We can find forward rates via the following formula:

$$f(t_1, t_2) = r_{t_1,t_2} = \frac{r_{t_2} \cdot t_2 - r_{t_1} \cdot t_1}{t_2 - t_1}$$

Algorithm 2 Finding forward rates

Initialize forwarddf with 5 of our bonds to calculate forward rates
for $days = 1, 2, \ldots, 10$ **do**
  for i in range(1, len(days)) **do**
    forwarddf.iloc[bond, date] $= \frac{r_{t_2} \cdot t_2 - r_{t_1} \cdot t_1}{t_2 - t_1}$
  end for
end for



5. The covariance matrix of the time series of daily log-returns of yield is a 5x5 matrix derived from a 5x9 time series matrix. The covariance matrix of the time series of daily log-returns of forward rates is a 4x4 matrix derived from a 4x9 time series matrix. This represents the amount of variability between each time series of yields or forward rates respectively and compares how much the bonds' time series move together.

6. The first eigenvector is the eigenvector with the largest associated eigenvalue and it is the direction of projection that explains the most variability in the data and preserves the most information. The eigenvalue thus represents the percent of total variance explained.

References and GitHub Link to Code

References

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GitHub Link

- https://github.com/rileyang1/APM466a1-Yield-Curves